

RAM-TR-57-65

AD663216

A MATHEMATICAL MODEL OF THE SAM ANIMAL
CALORIMETER RESPIRATORY GASES AND
RELATED COMPUTER TECHNIQUES



WILLIAM W. LUTNEY, JR.

HAROLD L. BITTER, Lieutenant Colonel, USAF, PhD

Volanti Subvenimus

USAF School of Aerospace Medicine
Aerospace Medical Division (AFPM)
Wright Air Force Base, Texas

Reproduced by the
CLEARINGHOUSE
for Federal Scientific & Technical
Information Springfield Va 22151

This document has been approved
for public release and is not
classified or restricted

**A MATHEMATICAL MODEL OF THE SAM ANIMAL CALORIMETER RESPIRATORY
GASES AND RELATED COMPUTER TECHNICS**

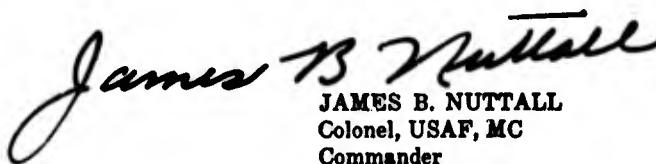
WILLIAM W. LACKEY, B.S.
HAROLD L. BITTER, Lieutenant Colonel, USAF, BSC

FOREWORD

This report was prepared in the Physiological Chemistry Section, covering work performed between 1965 and 1967 under task No. 630207. The paper was submitted for publication on 12 May 1967.

Leopold A. Broome and Franklin A. Chamness participated in the testing and recalibration; Edward J. Engelken and Al Zermeno of the Biometrics Branch gave assistance concerning computer technics and recording of data.

This report has been reviewed and is approved.


JAMES B. NUTTALL
Colonel, USAF, MC
Commander

ABSTRACT

Problems related to effects of air flow upon measurements of oxygen and carbon dioxide concentrations in the chamber caused us to modify the animal calorimeter system in use at the USAF School of Aerospace Medicine.

Differential equations were developed to describe the fractional concentrations of respiratory gases under "steady state" conditions. Their solutions gave the gas responses attributable to the subject. Sensor signals were amplified for increased sensitivity and scaled for magnetic tape recording by an analog computer. Radiant and evaporative heats were scaled to record values in units of Calories/minute. These parameters, along with signals of the oxygen and carbon dioxide sensors, corrected for equipment drift, were appropriately treated by a digital computer program to yield: oxygen consumption, carbon dioxide production, respiratory quotient, radiant heat, evaporative heat, total heat production, metabolic heat, and heat storage.

Recalibration data of the radiant and evaporative heat sensors are included.

BLANK PAGE

A MATHEMATICAL MODEL OF THE SAM ANIMAL CALORIMETER RESPIRATORY GASES AND RELATED COMPUTER TECHNIQS

I. INTRODUCTION

The "improved gradient layer animal calorimeter" in use at the USAF School of Aerospace Medicine incorporates both direct and indirect methods of whole-body calorimetry (1). The advantages of simultaneous use of direct and indirect calorimetry require no explanation; however, the ventilatory air flow of the calorimeter presents a problem affecting both methods. The evaporative heat, a component of the direct method, and the oxygen and carbon dioxide concentrations in the chamber require correction for the effects of air flow.

While this paper describes a simple mathematical model of the calorimeter subject's respiratory gases, the general equation describing the changes in partial pressure of the gases in the system was stated by Quattrone (1). Certain modifications made to the SAM calorimeter system, the treatment of response signals, and recalibration data are described in this report.

II. MATHEMATICAL MODEL

Symbols

To discuss in detail the calorimeter flow system, it is necessary to use various symbols and notations to show the mathematical relations. The symbols used are a variation of those adopted by a group of respiratory physiologists at Atlantic City in 1950 (2).

V = Volume of the calorimeter.

V_C = Volume flow rate of gas into and out of the calorimeter.

V_I = Volume flow rate of gas inspired by the subject.

V_E = Volume flow rate of gas expired by the subject.

F_{O_2} = Fractional concentration of oxygen in the ventilatory air.

F_{CO_2} = Fractional concentration of carbon dioxide in the ventilatory air.

F_{H_2O} = Fractional concentration of water vapor in the ventilatory air.

$F_{I_{O_2}}$ = Fractional concentration of oxygen inspired by the subject.

$F_{I_{CO_2}}$ = Fractional concentration of carbon dioxide inspired by the subject.

$F_{I_{H_2O}}$ = Fractional concentration of water vapor inspired by the subject.

$F_{E_{O_2}}$ = Fractional concentration of oxygen expired by the subject.

$F_{E_{CO_2}}$ = Fractional concentration of carbon dioxide expired by the subject.

$F_{E_{H_2O}}$ = Fractional concentration of water vapor expired by the subject.

$F_{I(H_2O)_A}$ = Fractional concentration of water vapor of ambient air.

t = Time in minutes.

Basic assumptions

The basic assumptions required in developing the working equations are:

a. There is complete mixing of gases in the calorimeter.

b. The volume flow rate of gas into and out of the calorimeter is constant during any experiment.

c. The fractional concentrations of the gases in the ventilatory air are constant during any experiment; i.e., F_{O_2} , F_{CO_2} , and F_{H_2O} are fixed prior to an experiment.

d. The subject is in a steady state. (Although it is realized that a true "steady state" cannot be maintained for any length of time in an experimental animal, this assumption allows a solution to the differential equations describing the system. The solutions of the equations reflect a net change in all monitored parameters at a given time.)

e. In maintaining stable conditions for the subject, the fractional concentration of CO_2 in the chamber is not great enough to induce an involuntary increase in the respiratory rate; i.e., $F_{I_{O_2}} < 1\%$ (3).

Derivation of equations

The product of flow rate and concentration is taken as the instantaneous rate of change; and if the fractional concentrations of the inspired gases are considered the time-dependent variables, then the differential equations

$$\frac{d F_{I_{O_2}}}{dt} + V_c F_{I_{O_2}} = V_c F_{O_2} - V_{O_2} \quad (1)$$

$$\frac{d F_{I_{CO_2}}}{dt} + V_c F_{I_{CO_2}} = V_c F_{CO_2} + V_{CO_2} \quad (2)$$

$$\frac{d F_{I_{H_2O}}}{dt} + V_c F_{I_{H_2O}} = V_c F_{H_2O} + V_{H_2O} \quad (3)$$

define the system. Here the oxygen consumption, carbon dioxide production, and water vapor production are defined as $V_{O_2} = V_I F_{I_{O_2}} - V_E F_{E_{O_2}}$, $V_{CO_2} = V_E F_{E_{CO_2}} - V_I F_{I_{CO_2}}$, and $V_{H_2O} = V_E F_{E_{H_2O}} - V_I F_{I_{H_2O}}$, respectively.

By the basic assumptions, the right sides of equations 1, 2, and 3 are to be treated as constants; therefore, the equations are readily

solved by the first order method of integrating factors (4). The solutions are

$$V_{O_2} = V_c (F_{O_2} - F_{I_{O_2}}) / (1 - e^{-kt}) \quad (4)$$

$$V_{CO_2} = V_c (F_{I_{CO_2}} - F_{CO_2}) / (1 - e^{-kt}) \quad (5)$$

$$V_{H_2O} = [V_c (F_{I_{H_2O}} - F_{I_{(H_2O)_A}} e^{-kt}) / (1 - e^{-kt})] - V_c F_{H_2O} \quad (6)$$

where $k = V_c/V$ and the constants of integration have been evaluated at initial conditions which are: $t = 0$, $F_{I_{O_2}} = F_{O_2}$, $F_{I_{CO_2}} = F_{CO_2}$, and $F_{I_{H_2O}} = F_{I_{(H_2O)_A}}$. These equations give the gas responses attributable to the subject by accounting for the effects of the ventilatory air flow and chamber volume.

Since the respiratory quotient, RQ, furnishes qualitative information regarding the substances utilized in the metabolic pool, it has important physiologic meaning and should be evaluated in terms of the SAM system. RQ is defined as the ratio of the volume of CO_2 produced per unit time to the volume of O_2 consumed per unit time, or in terms of this model, equation 5 divided by equation 4, which yields

$$RQ = V_{CO_2}/V_{O_2} = (F_{I_{CO_2}} - F_{CO_2}) / (F_{O_2} - F_{I_{O_2}}) \quad (7)$$

Therefore, RQ is a function of the O_2 and CO_2 concentration changes which occur in the calorimeter and is not complicated by the type of ventilatory air flow system or the design of the calorimeter chamber.

III. MODIFICATION OF SYSTEM

Ventilatory system

In the present system of the SAM calorimeter, the ventilatory air is supplied by a large capacity air compressor rather than by compressed air cylinders. The compressor air is passed through a refrigeration-type drier, set at $4^\circ C$, before it passes through the flowmeter of the system. The platometer, originally constructed to measure the evaporative heat loss of the subject, has been replaced by two dew-probe-type sensors (Honeywell, SSP129D). The two identical dew probe sensors, one in the

ventilatory air stream and the other in the exhaust gas stream, are connected in a bridge circuit so that the output is a measure of the water vapor loss of the subject. This system results in a reproducible linear response (see fig. 3) which allows measurement of small energy inputs at low ventilatory flow rates. Quattrone (1) reported that the platometer has a nonlinear response under these conditions.

The oxygen and carbon dioxide concentrations in the exhaust gas stream are measured continuously, dry and at room temperature. The exhaust gases from the dew probe sensor are passed through a large copper tube which allows equilibration to room temperature, and then through a system of valves where a small amount is diverted to the gas sensors and the unused portion discarded. The fraction diverted to the gas sensors is passed through a drier, a flowmeter, an oxygen analyzer (Beckman model F-3), a length of tubing, and finally, the carbon dioxide analyzer (Beckman model LB-1). The length of tubing between the two analyzers was adjusted so that the analyzers responded simultaneously to changes in gas concentration. The system of valves mentioned allows standard gases to be passed through the analyzers without disrupting the gas flow through the calorimeter.

Computer circuitry

The output signals from the oxygen and carbon dioxide analyzers along with the output signals from the calorimeter thermopile and dew probe bridge are passed through an analog computer (TR-20 Analog Computer) to obtain the desired amplification and control of the signals, and are then recorded by a magnetic tape recorder (Ampex model FR-1200). The complete circuit of the computer is shown in figure 1.

The arrangement of the computer circuit is such that, with ventilatory air flowing through the sensors, the analogs of $\dot{V}O_2$ and $\dot{V}CO_2$ (see equations 4 and 5) may be set equal to zero and the differences from these set points, occurring during an experiment, amplified for good sensitivity to small changes. The amplified analogs of the changes in concentration of

O_2 and CO_2 are simultaneously passed into a divider circuit and into separate summing amplifiers (see fig. 1). The summing amplifiers are needed to maintain output signals within the limits of the tape recorder. The quotient of CO_2/O_2 , where $O_2 = \dot{V}O_2 - \dot{V}I_{O_2}$ and $CO_2 = \dot{V}I_{CO_2} - \dot{V}CO_2$, appearing at the output of amplifier 13, is passed into an attenuator (potentiometer 10) which is set according to the ratio of the sensitivities of the two analyzers (O_2 sensitivity/ CO_2 sensitivity) resulting in a continuous recording of RQ simultaneously with the corresponding analogs of O_2 and CO_2 .

The output signals from the dew probe bridge and the calorimeter thermopile are passed through a series of amplifiers to obtain good sensitivities to small changes in evaporative and radiant heat. Then the amplified signals are passed through circuits representing the calibration curves (see recalibration) for each of the sensors so that the output of amplifiers 3 and 4 are in Calories per minute. Therefore, the evaporative heat, radiant heat, and the sum of the two heats are recorded simultaneously (see fig. 1).

The treatment of these data on magnetic tape will be mentioned only briefly. A digital program performs various operations upon the taped data as it is sampled each minute. The program corrects the data for any equipment drift occurring during the experiment and then, utilizing equations 4, 5, and 6 appropriately, calculates the O_2 consumption, CO_2 production (ml./min./kg. wt.) STPD, and RQ from these data. Then from the O_2 consumption, the metabolic heat (gm. cal./min./kg. wt.) is calculated and this figure is compared to the sum of the radiant and evaporative heats (cal./min./kg. wt.). The difference between the metabolic heat and the measured heat is used as an estimate of heat loss or heat storage.

Other modifications

The calorimeter water bath system has been modified. The previous two baths have been replaced by one of approximately equal volume. A larger, more efficient refrigeration unit, installed with the new water bath, allows greater

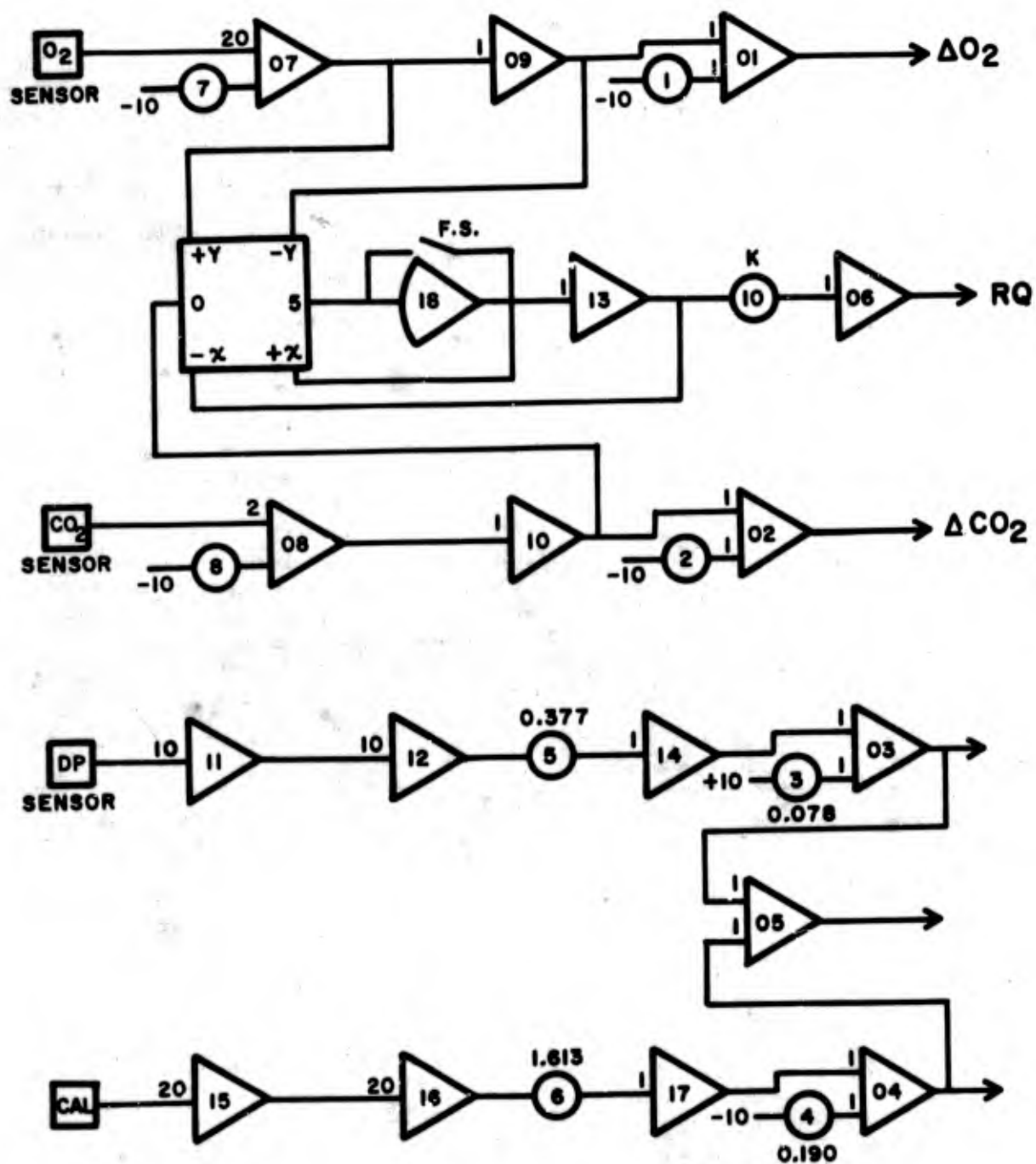


FIGURE 1

Schematic of analog computer patching used to amplify and control sensor signals.

heat inputs while maintaining a temperature of $20 \pm 0.01^\circ \text{C}$. With this ability to handle more heat, the flow of water through the calorimeter walls has been increased to approximately 28 liters per minute per wall by installation of pumps with greater volume flow. The direction of water flow has been reversed so that the water is pulled from the bath, passed through the calorimeter walls and then through the pumps which return the water to the bath. Therefore, any heat added to the system by the pumps is transferred to the bath where it is carried off by the cooling system.

IV. RECALIBRATION

Recalibration conditions

During all stages of recalibration, the calorimeter water bath and the air chamber were regulated at 20°C ., and the ventilatory air flow rate was maintained at 20.2 liters per minute (STPD). For all calibrations, the calorimeter thermopile and the dew probe bridge signals were monitored with a digital voltmeter (Hewlett-Packard model 405) after the signals had been passed through the various amplifiers of the analog computer (already described).

Calorimeter

Several different types of radiant heat sources were used to check the calorimeter thermopile output, and at increased gains for good sensitivity there was no detectable difference in the output regardless of which source was used. The method of calibration reported by Hammel and Hardy (5)—i.e., an alcohol lamp as a source of radiant heat and water vapor in the calorimeter—was used to establish the standard curves in figures 2 and 3.

As previously reported (1), the calorimeter thermopile has a linear response as shown by the recalibration data plotted in figure 2. These data were obtained by an amplification of 400 times the thermopile output. These data were fitted by the method of least squares to an equation of the form

where

$$H_c = a + bE_c$$

and

$$H_c = \text{calories input}$$

to yield

$$E_c = \text{volts output}$$

and

$$a = 19.015 \pm 11.655$$

$$b = 161.268 \pm 2.011$$

The estimates of precision are given as standard deviations for each constant, these being calculated from the least squares equations for a linear function, with one variable, H_c , subject to error.

The differences between this calibration curve and that previously reported (1) can be accounted for by a number of modifications made to the system; i.e., the increased flow rate of water through the calorimeter walls, the method of measurement, amplification of thermopile signal, and the associated circuitry.

Dew probe

Calibration of the dew probe bridge to monitor the evaporative heat loss was done simultaneously with the calibration of the calorimeter thermopile. As pointed out by Hammel and Hardy (5), the advantage of using

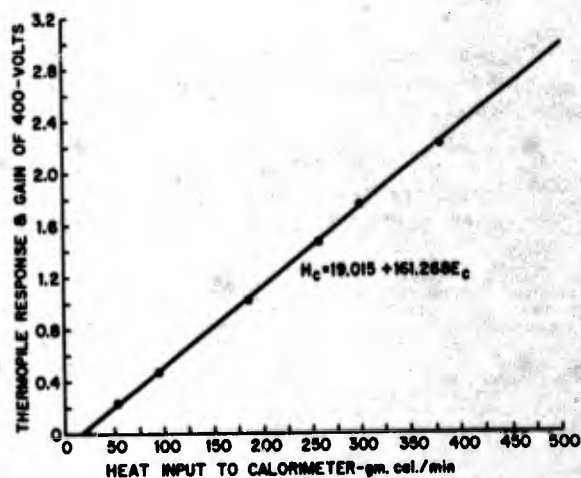


FIGURE 2

Calorimeter calibration curve.

an alcohol lamp for calibration is that oxygen consumption, carbon dioxide production, water vapor production, and radiant heat may be checked simultaneously.

The dew probe bridge has a linear response as shown in figure 3. These data were obtained at an amplification of 100 times the bridge output. A least squares solution of these data yields

where $H_d = a + bE_d$

H_d = calories input

E_d = volts output

and $a = 7.826 \pm 9.011$

$b = 37.707 \pm 1.550$

As in the case of the calorimeter recalibration, the estimates of precision are given as standard

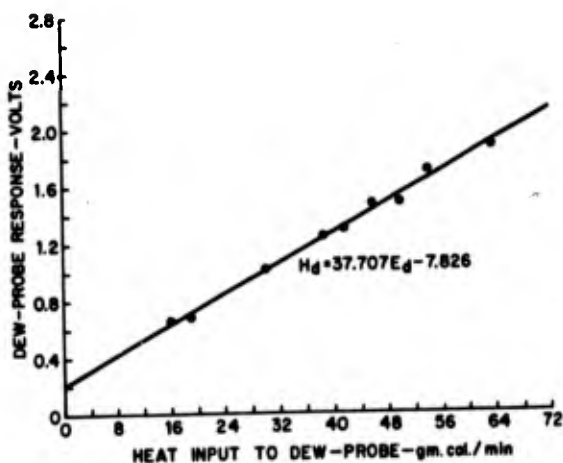


FIGURE 3

Dew probe calibration curve.

deviations for each constant, these being calculated as before with variable, H_d , subject to error.

REFERENCES

1. Quattrone, P. D. An improved gradient layer animal calorimeter. SAM-TDR-64-53, Sept. 1964.
2. Pappenheimer, J. R. Standardization of definitions and symbols in respiratory physiology. Fed. Proc. 9:602 (1950).
3. Lambersten, C. J. Chemical factors in respiratory control. In Bard, P. (ed.). Medical physiology, ch. 39, p. 634. St. Louis: C. V. Mosby, 1961.
4. Spiegel, M. R. Applied differential equations, pp. 41-42. Englewood Cliffs, N. J.: Prentice-Hall, 1958.
5. Hammel, H. T., and J. D. Hardy. A gradient type of calorimeter for measurement of thermoregulatory responses in the dog. In Herzfeld, C. M. (ed.). Temperature. Its measurement and control in science and industry, vol. 3, part 3, ch. 4, pp. 31-42. New York: Reinhold, 1963.

Unclassified
Security Classification

DOCUMENT CONTROL DATA - R&D		
(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)		
1. ORIGINATING ACTIVITY (Corporate author) USAF School of Aerospace Medicine Aerospace Medical Division (AFSC) Brooks Air Force Base, Texas		2a. REPORT SECURITY CLASSIFICATION Unclassified
		2b. GROUP
3. REPORT TITLE A MATHEMATICAL MODEL OF THE SAM ANIMAL CALORIMETER RESPIRATORY GASES AND RELATED COMPUTER TECHNIQS		
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) 1965 - 1966		
5. AUTHOR(S) (Last name, first name, initial) Lackey, William W. Bitter, Harold L., Lieutenant Colonel, USAF, BSC		
6. REPORT DATE July 1967	7a. TOTAL NO. OF PAGES 6	7b. NO. OF REFS 5
8a. CONTRACT OR GRANT NO. b. PROJECT NO. 6302 c. Task No. 630207 d.		9a. ORIGINATOR'S REPORT NUMBER(S) SAM-TR-67-65 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
10. AVAILABILITY/LIMITATION NOTICES This document has been approved for public release and sale; its distribution is unlimited.		
11. SUPPLEMENTARY NOTES		12. SPONSORING MILITARY ACTIVITY USAF School of Aerospace Medicine Aerospace Medical Division (AFSC) Brooks Air Force Base, Texas
13. ABSTRACT Problems related to effects of air flow upon measurements of oxygen and carbon dioxide concentrations in the chamber caused us to modify the animal calorimeter system in use at the USAF School of Aerospace Medicine. Differential equations were developed to describe the fractional concentrations of respiratory gases under "steady state" conditions. Their solutions gave the gas responses attributable to the subject. Sensor signals were amplified for increased sensitivity and scaled for magnetic tape recording by an analog computer. Radiant and evaporative heats were scaled to record values in units of Calories/minute. These parameters, along with signals of the oxygen and carbon dioxide sensors, corrected for equipment drift, were appropriately treated by a digital computer program to yield: oxygen consumption, carbon dioxide production, respiratory quotient, radiant heat, evaporative heat, total heat production, metabolic heat, and heat storage. Recalibration data of the radiant and evaporative heat sensors are included.		

DD FORM 1473
1 JAN 64

Unclassified
Security Classification

Unclassified

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Physiology						
Calorimetry, gradient						
Calorimetry, computer technics						
Respiratory gases, a mathematical model						

INSTRUCTIONS

1. **ORIGINATING ACTIVITY:** Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (*corporate author*) issuing the report.

2a. **REPORT SECURITY CLASSIFICATION:** Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. **GROUP:** Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. **REPORT TITLE:** Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. **DESCRIPTIVE NOTES:** If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. **AUTHOR(S):** Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. **REPORT DATE:** Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7a. **TOTAL NUMBER OF PAGES:** The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. **NUMBER OF REFERENCES:** Enter the total number of references cited in the report.

8a. **CONTRACT OR GRANT NUMBER:** If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b, 8c, & 8d. **PROJECT NUMBER:** Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. **ORIGINATOR'S REPORT NUMBER(S):** Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. **OTHER REPORT NUMBER(S):** If the report has been assigned any other report numbers (*either by the originator or by the sponsor*), also enter this number(s).

10. **AVAILABILITY/LIMITATION NOTICES:** Enter any limitations on further dissemination of the report, other than those

imposed by security classification, using standard statements such as:

- (1) "Qualified requesters may obtain copies of this report from DDC."
- (2) "Foreign announcement and dissemination of this report by DDC is not authorized."
- (3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through _____."
- (4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through _____."
- (5) "All distribution of this report is controlled. Qualified DDC users shall request through _____."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. **SUPPLEMENTARY NOTES:** Use for additional explanatory notes.

12. **SPONSORING MILITARY ACTIVITY:** Enter the name of the departmental project office or laboratory sponsoring (*paying for*) the research and development. Include address.

13. **ABSTRACT:** Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as (TS), (S), (C), or (U).

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. **KEY WORDS:** Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.

Unclassified

Security Classification

AD 663216

AD 663216

CORRECTIONS

("A Mathematical Model of the SAM Animal Calorimeter Respiratory Gases and Related Computer Technics," by William W. Lackey and Harold L. Bitter, SAM-TR-67-65, July 1967)

Page 2, par. 4, line 5: Reference is made to carbon dioxide--not oxygen.

$$\dots \text{i.e., } F_{\text{ICO}_2} < 1\% \quad (3).$$

On page 2, equations 1, 2, and 3 should have the symbol "V" preceding each of the $\frac{d(\quad)}{dt}$ terms.

In all such symbols of V (volume) having subscript letters (V_c , V_I , V_E , V_{O_2} , V_{CO_2} , V_{H_2O}) there should be a dot over the V to indicate "flow per minute."