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SUMMATION OF A FINITE SERIES OCCURRING IN KILL
PROBABILITY FOR A CIRCULAR TARGET ENGAGED BY
A SALVO OF AREA KILL WEAPONS

by

Harold J. Breaux

December 1967

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SUMMATION OF A FINITE SERIES OCCURRING IN
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Computing Laboratory

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December 1967

SUMMATION OF A FINITE SERIES OCCURRING IN
KILL PROBABILITY FOR A CIRCULAR TARGET ENGAGED BY A SALVO OF
AREA KILL WEAPONS

ABSTRACT

The probability function expressing the fractional kill of a circular target engaged by a salvo of area kill weapons can be expressed in terms of a finite series. When the salvo contains many rounds, the nature of the series makes it useless in computing the kill probability. In cases where the distribution of hits is bivariate normal, with equal or near equal standard deviations, this difficulty may be overcome by expressing the series in terms of a definite integral, readily solved by numerical quadrature. Additionally, the definite integral is expressed in terms of a different finite series having properties making it easily summable on a digital computer.

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I. INTRODUCTION

An important problem in weapon systems analysis is the computation of the expected fractional kill of a circular target when engaged by a salvo of area kill weapons. The solution of this problem, f_N , usually is in the form of a finite series

$$f_N = C \sum_{i=1}^N (-1)^{i+1} \binom{N}{i} [x^i/i] g(i), \quad (1)$$

where C and x are constants, $0 < x < 1$, N is the number of rounds in the salvo, and $g(i)$ is a function dependent on the summation index i . This series is developed by Groves^{1*} and by Dr. F. E. Grubbs in an unpublished work. This sum can usually be obtained quite easily on a digital computer; however, for large N the nature of the series makes it useless^{**} in computing f_N . In cases where the distribution of hits is bivariate normal with equal or near equal standard deviations, $g(i)$ can be expressed in the form

$$g(i) = (1 - e^{-\lambda i}), \quad (2)$$

where λ is a constant. For this form of $g(i)$, an alternate form for f_N can be obtained which is more amenable to computation. The procedure developed by Whelson^{2,3} leads to a definite integral which can be expressed in terms of a finite series. This series, in contrast to the original, can easily be summed for large N on a digital computer.

The procedure consists of equating part of the general term of the series, $H(i)$, with the Laplace transform of a function $h(y)$. Let

$$H(i) = (1 - e^{-\lambda i})/i = \int_0^\infty e^{-iy} i^{-1} \{(1 - e^{-\lambda i})/i\} dy, \quad (3)$$

* Superscript numbers denote references which may be found on page 11.

** The nature of this computational difficulty and the presentation of a method to overcome it for a more general problem than the one solved here are contained in a forthcoming report by the author and Lynn S. Mohler.

where $\mathcal{L}^{-1}\{(1-e^{-\lambda i})/i\}$ is the inverse Laplace transform of $(1-e^{-\lambda i})/i$, and i is treated as the transform variable. Now

$$\mathcal{L}^{-1}\{(1-e^{-\lambda i})/i\} = [1-S_{\lambda}(y)], \quad (4)$$

where $S_{\lambda}(y)$ is the Heaviside step function, i.e.,

$$S_{\lambda}(y) = \begin{cases} 0, & 0 < y < \lambda, \\ 1, & y > \lambda. \end{cases} \quad (5)$$

If we let $F_N = f_N/C$, F_N may be expressed in the form

$$\begin{aligned} F_N &= \sum_{i=1}^N (-1)^{i+1} \binom{N}{i} x^i \int_0^{\infty} e^{-iy} [1-S_{\lambda}(y)] dy \\ &= \int_0^{\infty} [1-S_{\lambda}(y)] \sum_{i=1}^N (-1)^{i+1} \binom{N}{i} x^i e^{-iy} dy. \end{aligned} \quad (6)$$

Letting $w = xe^{-y}$, we note that

$$\begin{aligned} \sum_{i=1}^N (-1)^{i+1} \binom{N}{i} x^i e^{-iy} &= - \sum_{i=1}^N (-1)^i \binom{N}{i} w^i \\ &= [1-(1-w)^N]. \end{aligned} \quad (7)$$

Hence

$$\begin{aligned} F_N &= \int_0^{\infty} [1-S_{\lambda}(y)] [1-(1-xe^{-y})^N] dy \\ &= \int_0^{\lambda} [1-(1-xe^{-y})^N] dy, \end{aligned} \quad (8)$$

or

$$F_N = \lambda - \int_0^\lambda (1 - xe^{-y})^N dy. \quad (9)$$

Due to the form of the result in Equation (7), it becomes apparent that proper substitutions in the original multiple integral¹ and the use of appropriate transformations of variables would lead to the result of Equation (9). Nevertheless the approach used seems instructive in itself.

II. AN ALTERNATE SERIES

If in Equation (9) we let

$$u = (1 - xe^{-y}), \quad (10)$$

we obtain

$$F_N = \lambda - \int_{u_1}^{u_2} u^N (1-u)^{-1} du, \quad (11)$$

where $u_1 = (1-x)$ and $u_2 = (1 - xe^{-\lambda})$. Since

$$(1-u)^{-1} = \sum_{i=0}^{\infty} u^i, \quad 0 < u < 1,$$

F_N can be written as

$$\begin{aligned} F_N &= \lambda - \sum_{i=0}^{\infty} \int_{u_1}^{u_2} u^{i+N} du \\ &= \lambda - \sum_{i=N+1}^{\infty} (u_2^i - u_1^i) / i. \end{aligned} \quad (12)$$

Since

$$\sum_{i=1}^{\infty} u^i/i = -\ln(1-u), \quad |u| < 1, \quad (13)$$

Equation (12) becomes

$$\begin{aligned} F_N &= \lambda - \ln [1-(1-x)] + \ln [1-(1-xe^{-\lambda})] + \sum_{i=1}^N (u_2^i - u_1^i)/i \\ &= \sum_{i=1}^N [(1-xe^{-\lambda})^i - (1-x)^i]/i. \end{aligned} \quad (14)$$

The terms of the series in Equation (14) are well behaved, i.e., they decrease monotonically with the summation index i and are therefore readily summable.

This series should be useful in computing kill probability for large N where $g(i)$ can be expressed by Equation (2). Additionally, it has proved useful in checking the accuracy of more general methods used to solve the problem in cases where $g(i)$ cannot be expressed by Equation (2).

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