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Technical Memorandum

BI-CHI: A COMPUTER PROGRAM FOR THE
CHI-SQUARE GOODNESS OF FIT TEST FOR A BIVARIATE
NORMAL DISTRIBUTION

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U. S. NAVAL WEAPONS LABORATORY

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While the contents of this memorandum are considered to be correct,
they are subject to modification upon further study.

Distribution of this document is unlimited.

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ABSTRACT

Frequently the applied statistician is confronted with the problem of investigating the distribution of experimental data sampled from a continuous two-dimensional distribution. A prime example in the area of weapons system analysis is the investigation of the distribution of hit points of missiles or projectiles. Because of the desirable characteristics of the bivariate normal distribution and because of the frequency of occurrence of these characteristics in physical phenomena, the applied statistician is often interested in determining if the sample distribution of the experimental data can be approximated by a bivariate normal distribution. The BI-CHI (Bivariate Chi-Square Test) computer program described in this report is for assessing the agreement of a continuous two-dimensional sample distribution with a parent bivariate normal distribution. The program computes the Chi-square statistic for testing the null hypothesis "The random sample is from a bivariate normal parent population." The minimum and maximum sample values of each random variable are determined, and the sample estimates of the five parameters (two means, two variances, and the correlation coefficient) of the bivariate normal distribution are computed. Also, the sample estimates of the two regression lines (x_2 on x_1 and x_1 on x_2) are computed. An optional feature allows additional output of a plot of the sample data and the contour ellipses.

The program is coded in FORTRAN IV for the IBM 7030 (STRETCH) computer.

FOREWORD

The formulation of the BI-CHI computer program was performed in the Mathematical Statistics Branch of the Operations Research Division. The coding of the program was performed in the Operations Sciences Branch of the Computer Programming Division. The project was supported under Foundational Research Project No. 29Y, "Computer Programs for Statistical Analyses."

The original program, used for exploratory computations, was coded by Mr. Rick Ball; the final program as documented in this report was coded by Mrs. Jo Ann Brown. Miss Nancy Dean coded the computer routine which generated the c_p^2 -values tabulated within the BI-CHI program.

The date of completion was 31 October 1967.

I. INTRODUCTION

BI-CHI (Bivariate Chi-Square Test) is a computer program for the Chi-Square Goodness of Fit Test for a bivariate normal distribution. The program performs a statistical analysis of the discrepancy between sampled data from a continuous two-dimensional distribution and the hypothesized bivariate normal distribution. This discrepancy is measured by classifying the n randomly sampled (x_{1i}, x_{2i}) -observations into k mutually exclusive and exhaustive categories (areas between coaxial contour ellipses) and comparing the observed frequency with the theoretical frequency of each of the k areas. The formulation of the Goodness of Fit Test is described in Bates (1966) and therefore is not repeated in its entirety in this report.

The joint probability density function (pdf) of two jointly normally distributed random variables x_1 and x_2 is

$$f(x_1, x_2) = K^{-1} e^{-\frac{1}{2}Q}; -\infty < x_1 < \infty, -\infty < x_2 < \infty, \quad (1)$$

where

$$Q = \frac{1}{(1-\rho^2)} \left[\left(\frac{x_1 - \mu_1}{\sigma_1} \right)^2 - 2\rho \left(\frac{x_1 - \mu_1}{\sigma_1} \right) \left(\frac{x_2 - \mu_2}{\sigma_2} \right) + \left(\frac{x_2 - \mu_2}{\sigma_2} \right)^2 \right]. \quad (2)$$

A horizontal plane parallel to the $x_1 x_2$ -plane intersects the surface of equation (1) in the shape of an ellipse (contour ellipse). The equation of the general contour ellipse is

$$Q = c^2, \quad (3)$$

where c is a constant. The contour ellipses are centered at (μ_1, μ_2) , and their general shape is determined by σ_1, σ_2 , and ρ . By performing an orthogonal transformation, the quadratic form (Q) can be shown to be distributed as χ^2 with two degrees of freedom. That is,

$$Q \sim \chi^2(2). \quad (4)$$

Therefore, the probability that (x_1, x_2) falls in the contour ellipse of equation (3) is

$$P[\chi^2(2) < c_p^2] = p, \quad (5)$$

where c_p^2 is defined by $p = \int_0^{c_p^2} f(\chi^2) d\chi^2$, where $f(\chi^2)$ is the pdf of $\chi^2(2)$. Additionally, the probability that (x_1, x_2) falls inside the p'' contour ellipse and outside the p' contour ellipse is

$$P[c_{p'}^2 < \chi^2(2) < c_{p''}^2] = p'' - p'; \quad 0 < p' < p'' \leq 1. \quad (6)$$

The classification of the (x_{1i}, x_{2i}) -observations is then performed by evaluating $Q_i (i=1, 2, \dots, n)$. If $c_{p, j-1}^2 \leq Q_i < c_{p, j}^2$, $(j=1, 2, \dots, k)$, the i^{th} observation falls within the j^{th} contour ellipse and outside the $(j-1)^{\text{th}}$ contour ellipse. Denoting the observed number of (x_{1i}, x_{2i}) falling within the j^{th} contour ellipse and outside the $(j-1)^{\text{th}}$ contour ellipse by O_j and the expected number by $E_j = n(p_j - p_{j-1})$, the statistic

$$\chi^2 = \sum_{j=1}^k (O_j - E_j)^2 / E_j \quad (7)$$

is approximately distributed as $\chi^2(k-1)$.

In practice, however, the population parameters of the hypothesized bivariate normal distribution are unknown. That is, the population is not completely specified under the null hypothesis; only the type of the parent population is known or assumed. In such case, the population parameters are estimated from the sample data; Q_i is replaced with $\hat{Q}_i$; and equation (7) becomes

$$X^2 = \sum_{j=1}^k (O_j - \hat{E}_j)^2 / \hat{E}_j, \quad (8)$$

where $\hat{E}_j$ denotes the use of sample statistics to estimate population parameters. The degrees of freedom of X^2 is reduced by one for each parameter estimated. Therefore, when all five parameters of the bivariate normal distribution are estimated, X^2 is approximately distributed as $\chi^2(k-6)$.

BI-CHI computes the five estimators of the population parameters, classifies the $\hat{Q}_i$ ($i=1,2,\dots,n$), and computes X^2 . Estimators of the two "true" regression lines are computed, and the two minimum and maximum sample values are determined. In addition to the printout of the above computed statistics, optional features provide for printout of the categorization, a plot of the data, and a plot of the data with contour ellipses.

II. COMPUTATIONAL PROCEDURE

The estimators of the five population parameters are

$$\hat{\mu}_v = \bar{x}_v = \sum x_{v1} / n, \quad \hat{\sigma}_v^2 = s_v^2 = \sum (x_{v1} - \bar{x}_v)^2 / (n-1), \quad (9)$$

$$\hat{r} = r = \frac{\sum (x_{11} - \bar{x}_1)(x_{21} - \bar{x}_2)}{\sqrt{\sum (x_{11} - \bar{x}_1)^2 \sum (x_{21} - \bar{x}_2)^2}},$$

where $v=1,2$ and all summation signs run from 1 to n . The minimum and maximum sample values of each random variable are the respective order statistics $x_{1(1)}, x_{1(n)}$ and $x_{2(1)}, x_{2(n)}$. Estimates of the two "true" regression lines are

$$\begin{aligned} x_2 &= \bar{x}_2 + r \frac{s_2}{s_1} (x_1 - \bar{x}_1) = a_1 + b_1 x_1, \\ x_1 &= \bar{x}_1 + r \frac{s_1}{s_2} (x_2 - \bar{x}_2) = a_2 + b_2 x_2. \end{aligned} \quad (10)$$

The five statistics of equation (9) are substituted for the five parameters of equation (2), giving

$$\hat{Q}_i = \frac{1}{(1-r^2)} \left[\left(\frac{x_{11} - \bar{x}_1}{s_1} \right)^2 - 2r \left(\frac{x_{11} - \bar{x}_1}{s_1} \right) \left(\frac{x_{21} - \bar{x}_2}{s_2} \right) + \left(\frac{x_{21} - \bar{x}_2}{s_2} \right)^2 \right]. \quad (11)$$

After evaluating $\hat{Q}_i$ ($i=1,2,\dots,n$), the classification of the n observations is performed by comparing each $\hat{Q}_i$ with $(k-1)$ specified c_{pj}^2 -values (see card type 5) which define k mutually exclusive and exhaustive areas between coaxial contour ellipses. If $c_{p,j-1}^2 \leq \hat{Q}_i < c_{pj}^2$ ($j=1,2,\dots,k$), the i^{th} observation (x_{11}, x_{21}) falls within the

j^{th} contour ellipse and outside the $(j-1)^{\text{th}}$ contour ellipse. The number of (x_{1j}, x_{2j}) -observations, $0_j (j=1, 2, \dots, k)$, falling within each of the k areas is then determined, and X^2 as given by equation (6) is computed.

If the quantity $(k-6) > 0$, the statement "CHI-SQUARE = X^2 WITH $(k-6)$ DEGREES OF FREEDOM" is printed (see page 19); if the quantity $(k-6) \leq 0$, the statement "CHI-SQUARE COULD NOT BE COMPUTED BECAUSE OF INSUFFICIENT NUMBER OF INTERVALS" is printed.

III. INPUT PREPARATION

A. Deck Setup

The input deck is listed below by card type. Multiple jobs may be processed by stacking sets of card types 1 through 10. Up to twenty runs may be performed for each set of sample data by stacking sets of card types 4 through 9 behind card type 10.

CARD TYPE 1 - JOB IDENTIFICATION CARD

CARD TYPE 2 - VARIABLE FORMAT CARD

CARD TYPE 3 - NUMBER OF RUNS CARD

CARD TYPE 4 - MAIN CONTROL CARD

CARD TYPE 5 - INTERVAL IDENTIFICATION CARD

CARD TYPE 6 - CONTOUR IDENTIFICATION CARD

CARD TYPE 7 - PLOT HEADING CARD (optional)

CARD TYPE 8 - x_1 TRANSFORMATION CONSTANT CARD

CARD TYPE 9 - x_2 TRANSFORMATION CONSTANT CARD

CARD TYPE 10- SAMPLE DATA CARD

B. Input Deck Description

CARD TYPE 1 - JOB IDENTIFICATION CARD

<u>Column</u>	<u>Format</u>	<u>Program Variable</u>	<u>Explanation</u>
1-80	10A8	JOB(1)-JOB(10)	For job identification.

CARD TYPE 2 - VARIABLE FORMAT CARD

<u>Column</u>	<u>Format</u>	<u>Program Variable</u>	<u>Explanation</u>
1-80	10A8	FMT(1)-FMT(10)	The format by which the (x_1, x_2) -values (card type 10 or input record if data is on tape) are to be read. The format specifications must be enclosed by parentheses.

CARD TYPE 3 - NUMBER OF RUNS CARD

<u>Column</u>	<u>Format</u>	<u>Program Variable</u>	<u>Explanation</u>
4-5	I2	NRUN	The number of runs to be performed per set of sample data. $1 \leq \text{NRUN} \leq 20$.

CARD TYPE 4 - MAIN CONTROL CARD

<u>Column</u>	<u>Format</u>	<u>Program Variable</u>	<u>Explanation</u>
5	I1	NPRINT	0 or blank - Categorization is <u>not</u> to be printed. 1 - Categorization <u>is</u> to be printed.
10	I1	JPL0T	0 or blank - <u>No</u> plotting is to be performed. 1 - Both data and contours are to be plotted. 2 - Data only is to be plotted.
14-15	I2	N0TIC	The number of desired tick marks on the axis corresponding to the larger range of x_1 and x_2 . If N0TIC = 0 or blank, N0TIC is set equal to 15.

<u>Column</u>	<u>Format</u>	<u>Program Variable</u>	<u>Explanation</u>
20	I1	INPUT	0 or blank - Sample data is on punched cards. 1 - Sample data is on tape.
25	I1	NTYPES	0 or blank - Plot headings (card type 7) are <u>not</u> to be input. 1 - Plot headings <u>are</u> to be input.
29-30	I2	ITRAN1	Number corresponding to the transformation to be performed on the x_1 -values. $1 \leq \text{ITRAN1} \leq 13$.
34-35	I2	ITRAN2	Number corresponding to the transformation to be performed on the x_2 -values. $1 \leq \text{ITRAN2} \leq 13$.
39-40	I2	IRA	The number of decimal digits to be used in labeling abscissa tick marks. $0 \leq \text{IRA} \leq 14$.
44-45	I2	IR $\emptyset$	The number of decimal digits to be used in labeling ordinate tick marks. $0 \leq \text{IR}\emptyset \leq 14$.
49-50	I2	NPAIR	The number of pairs of (x_1, x_2) -values per sample data card or sample data input record if data is on tape.
55	I1	NN	The number of sets of probability values to be used to define interval bounds for the Chi-Square Test. $1 \leq \text{NN} \leq 5$.
60	I1	NL	The number of sets of probability values to be used to define contours to be plotted. $1 \leq \text{NL} \leq 5$.

TRANSFORMATIONS

The following numbers are used to identify the thirteen available transformations (ITRAN1 and ITRAN2).

<u>Transformation Number</u>	<u>Transformation</u>
1	$x \leftarrow x$
2	$x \leftarrow \ln x$
3	$x \leftarrow \ln(\ln x)$
4	$x \leftarrow \ln(A+x)$
5	$x \leftarrow \ln(B+\ln(C+x))$
6	$x \leftarrow \sqrt{x}$
7	$x \leftarrow 1/x$
8	$x \leftarrow 1/(D+x)$
9	$x \leftarrow \sin^{-1} x$
10	$x \leftarrow 2 \sin^{-1} \sqrt{x}$
11	$x \leftarrow x/E$
12	$x \leftarrow \sin x$
13	$x \leftarrow \cos x$

Constants A,B,C,D, and E are input on card type 8 for x_1 and card type 9 for x_2 .

CARD TYPE 5 - INTERVAL IDENTIFICATION CARD

<u>Column</u>	<u>Format</u>	<u>Program Variable</u>	<u>Explanation</u>
1-5	F5.3	PS(1)	The smallest probability value for the 1st set of probability values defining interval bounds for the Chi-Square Test.
6-10	F5.3	PDEL(1)	The increment value for the 1st set of probability values defining interval bounds for the Chi-Square Test.
11-15	F5.3	PE(1)	The largest probability value for the 1st set of probability values defining interval bounds for the Chi-Square Test.
16-20	F5.3	PS(2)	"
21-25	F5.3	PDEL(2)	"
26-30	F5.3	PE(2)	"
31-35	F5.3	PS(3)	"
36-40	F5.3	PDEL(3)	"
41-45	F5.3	PE(3)	"
46-50	F5.3	PS(4)	"
51-55	F5.3	PDEL(4)	"
56-60	F5.3	PE(4)	"
61-65	F5.3	PS(5)	The smallest, increment, and largest probability value for the 5th set of probability values defining interval bounds for the Chi-Square Test.
66-70	F5.3	PDEL(5)	
71-75	F5.3	PE(5)	

CARD TYPE 6 - CONTOUR IDENTIFICATION CARD

This card type is omitted if JPL~~OT~~ = 0 or JPL~~OT~~ = 2 on card type 4.

<u>Column</u>	<u>Format</u>	<u>Program Variable</u>	<u>Explanation</u>
1-5	F5.3	PPS(1)	The smallest probability value for the 1st set of probability values defining contours to be plotted.

<u>Column</u>	<u>Format</u>	<u>Program Variable</u>	<u>Explanation</u>
6-10	F5.3	PPDEL(1)	The increment value for the 1st set of probability values defining contours to be plotted.
11-15	F5.3	PPE(1)	The largest probability value for the 1st set of probability values defining contours to be plotted.
16-20	F5.3	PPS(2)	"
21-25	F5.3	PPDEL(2)	"
26-30	F5.3	PPE(2)	"
31-35	F5.3	PPS(3)	"
36-40	F5.3	PPDEL(3)	"
41-45	F5.3	PPE(3)	"
46-50	F5.3	PPS(4)	"
51-55	F5.3	PPDEL(4)	"
56-60	F5.3	PPE(4)	"
61-65	F5.3	PPS(5)	The smallest, increment, and largest probability value for the 5th set of probability values defining contours to be plotted.
66-70	F5.3	PPDEL(5)	
71-75	F5.3	PPE(5)	

CARD TYPE 7 - PLOT HEADING CARD

This card type is omitted if NTPES = 0 on card type 4. Two card types 7 are used for identification.

<u>Column</u>	<u>Format</u>	<u>Program Variable</u>	<u>Explanation</u>
1-80	10A8	XID1(1) - XID1(10)	The first line of the plot identification.
1-80	10A8	XID2(1) - XID2(10)	The second line of the plot identification.

CARD TYPE 8 - x, TRANSFORMATION CONSTANT CARD

This card type is omitted if ITRAN1 = 1 on card type 4. If $2 < \text{ITRAN1} < 13$, but the transformation corresponding to the transformation

number does not contain one of the constants A,B,C,D, or E, a blank card must replace card type 8.

<u>Column</u>	<u>Format</u>	<u>Program Variable</u>	<u>Explanation</u>
1-14	E14.8	ATRA1	The constant A in transformation number 4 for transforming x_1 -values.
15-28	E14.8	BTRA1	The constant B in transformation number 5 for transforming x_1 -values.
29-42	E14.8	CTRA1	The constant C in transformation number 5 for transforming x_1 -values.
43-56	E14.8	DTRA1	The constant D in transformation number 8 for transforming x_1 -values.
57-70	E14.8	ETRA1	The constant E in transformation number 11 for transforming x_1 -values.

CARD TYPE 9 - x_2 TRANSFORMATION CONSTANT CARD

This card type is omitted if ITRAN2 = 1 on card type 4. If $2 \leq \text{ITRAN2} \leq 13$, but the transformation corresponding to the transformation number does not contain one of the constants A,B,C,D, or E, a blank card must replace card type 9.

<u>Column</u>	<u>Format</u>	<u>Program Variable</u>	<u>Explanation</u>
1-14	E14.8	ATRA2	The constant A in transformation number 4 for transforming x_2 -values.
15-28	E14.8	BTRA2	The constant B in transformation number 5 for transforming x_2 -values.
29-42	E14.8	CTRA2	The constant C in transformation number 5 for transforming x_2 -values.
43-56	E14.8	DTRA2	The constant D in transformation number 8 for transforming x_2 -values.
57-70	E14.8	ETRA2	The constant E in transformation number 11 for transforming x_2 -values.

CARD TYPE 10 - SAMPLE DATA CARD

<u>Column</u>	<u>Format</u>	<u>Program Variable</u>	<u>Explanation</u>
Variable	Specified on card type 2	IEND	Indicates termination of the sample data cards.*
Variable	Specified on card type 2	X1(I) & X2(I)	The i^{th} pair of (x_1, x_2) - values; $i \leq 4,000$.

*If IEND = 0 or blank, NPAIR (x_1, x_2) -values are on the card and at least one more sample data card follows.

If IEND > 0, IEND (x_1, x_2) -values are on the card and this card is the last sample data card.

C. Request Sheets

If the sample data (card type 10) is input on punched cards,
the job request sheet is prepared as shown below.

SECURITY CLASSIFICATION ☐ TS ☐ S. ☐ C. ☒ U.		NAME J. BROWN		IDENTY. NO. A3		ROOM A141	BLDG. 1200	PHONE 7309	SETUP # _____ OF _____
☐ COMPILE ☒ GO ☐ CK OUT		JOB CARD CHARGE CODE IDENT. PROGRAMMER				JOB TITLE BI - CHI			
☐ COMPILE GO ☒ PROC		2	1	5	8	B	C	A	3
						EST. COMPILER TIME	EST. EXECUTION TIME 30 SEC.	DATE 10/20/67	
TAPES CALLED FOR BY PROGRAM PROGRAM									
TAPE NUMBER	Scratch								
FILE PROTECT ON									
PROGRAMMER NUMBER									
SPECIAL HANDLING (See attached inst.)									
OPERATOR'S COMMENTS									
☐ ABEQJ	☐ HOLD	☐ LO-P	☐ IF CHECKED, SEE REVERSE SIDE FOR ADDITIONAL COMMENTS.						
SPECIAL INSTRUCTIONS (continued on reverse)									

7030 JOB REQUEST NDW.NWL-5230-29 (REV. 11-66)

If the sample data is input on tape, the tape number is written on
the job request sheet instead of "Scratch" and the tape number is
punched in place of XXXX on the "REEL, PULXXXX" card (second card)
of the IOD deck.

If plots are requested (JPL~~OT~~ = 1 or 2 on card type 4), the CRT request sheet is prepared as shown below.

TRAID CAMERA OUTPUT FILM OR PAPER COPIES REQUEST
NDW-NWL-5600 2 (REV. 5-66)

PROGRAMMER	J. BROWN	ROOM	A141	BLDG.	1200	PHONE	7309	DATE	10/20/67
FILM IDENTIFICATION									
A3-BC									
APPROXIMATE NUMBER OF FRAMES									
The total number of plots requested for jobs being processed.									
NUMBER OF PAPER COPIES PER FRAME									
The number of copies of plots desired.									
FOR OPERATORS ONLY									
COUNTER READING START					FINISH				
DATE AND TIME PROBLEM RAN									

IV. OUTPUT FORMAT

A. System Output

CONTROL CARDS

(Job identification as given on card type 1)

VARIABLE FORMAT CARD _____

NRUN = _____

IRUN = _____

NPRINT = _____ JPLOT = _____ NOTIC = _____ INPUT = _____ NTYPES = _____ ITRAN1 = _____

ITRAN2 = _____ IBA = _____ IBO = _____ NPAIR = _____ NN = _____ NL = _____

P-VALUES IDENTIFICATION FOR CHI-SQUARE TEST
(Identification as given on card type 5)

P-VALUES IDENTIFICATION FOR PLOTS
(Identification as given on card type 6)

TRANSFORMATION CODE FOR X1 VALUES IS _____
TRANSFORMATION CONSTANTS FOR X1 VALUES ARE -
ATRAN1 = _____ BTRAN1 = _____ CTRAN1 = _____ DTRAN1 = _____ ETRAN1 = _____

TRANSFORMATION CODE FOR X2 VALUES IS _____
TRANSFORMATION CONSTANTS FOR X2 VALUES ARE -
ATRAN2 = _____ BTRAN2 = _____ CTRAN2 = _____ DTRAN2 = _____ ETRAN2 = _____

ORIGINAL DATA			TRANSFORMED DATA		
I	X1	X2	X1	X2	
1	x_{11}	x_{21}	x'_{11}	x'_{21}	
2	x_{12}	x_{22}	x'_{12}	x'_{22}	
:	:	:	:	:	
:	:	:	:	:	
:	:	:	:	:	
:	:	:	:	:	
1	x_{11}	x_{21}	x'_{11}	x'_{21}	
:	:	:	:	:	
:	:	:	:	:	
:	:	:	:	:	
:	:	:	:	:	
n	x_{1n}	x_{2n}	x'_{1n}	x'_{2n}	

NOTE: If ITRAN1 and ITRAN2 equal 1, the transformed and original are identical; i.e., $x'_{vi} = x_{vi}$; $v=1,2$; $i=1,2,\dots,n$.

$$\text{MIN XI} = \frac{x_1(1)}{}$$

$$\text{MAX } X1 = \frac{X1(n)}{\quad}$$

$$\text{MIN } X_2 = \frac{x_2(1)}{1}$$

$$\text{MAX } X_2 = \frac{X_2(n)}{n}$$

$$R = \frac{r}{\text{MEAN XI} = \frac{\Sigma x_1}{n}}$$

$$\text{MEAN } X_2 = \frac{X_2}{n}$$

$$\text{VARIANCE } X1 = \frac{s_1^2}{}$$

$$\text{VARIANCE X2} = \frac{s_2^2}{n_2}$$

STAN DEV X1 = S1

$$\text{STAN DEV X2} = \frac{S_2}{\dots}$$

$$X2 = \frac{a_1}{b_1} + \frac{b_1}{*X1}$$

$$X1 = \frac{a_2}{b_2} + \frac{b_2}{*X2}$$

$$C2(1) = \frac{c_{p1}^2}{(all\ i; x_{11}, x_{21}; \hat{Q}_1 \text{ for which } 0 \leq \hat{Q}_1 < c_{p1}^2)} \quad P(1) = \frac{P_1}{}$$

$$C2(2) = \frac{c_{p2}^2}{(all\ i; x_{11}, x_{21}; \hat{Q}_1 \text{ for which } c_{p1}^2 \leq \hat{Q}_1 < c_{p2}^2)} \quad P(2) = \frac{P_2}{}$$

:
:
:

$$C2(J) = \frac{c_{pJ}^2}{(all\ i; x_{11}, x_{21}; \hat{Q}_1 \text{ for which } c_{p_{j-1}}^2 \leq \hat{Q}_1 < c_{pJ}^2)} \quad P(J) = \frac{P_J}{}$$

:
:
:

$$C2(K-1) = \frac{c_{p_{K-1}}^2}{(all\ i; x_{11}, x_{21}; \hat{Q}_1 \text{ for which } c_{p_{K-2}}^2 \leq \hat{Q}_1 < c_{p_{K-1}}^2)} \quad P(K-1) = \frac{P_{K-1}}{}$$

$$C2(K) = \frac{\underline{INFINITY}}{(all\ i; x_{11}, x_{21}; \hat{Q}_1 \text{ for which } c_{p_{K-1}}^2 \leq \hat{Q}_1 < \infty)} \quad P(K) = \frac{1.000}{}$$

NOTE: The categorization illustrated on this page is optional output and is obtained only when
NPRINT = 1 (see card type 4).

CHI-SQUARE TEST

INTERVAL INDEX(J)	UPPER BOUND C2(J)	RUN NUMBER <u>i</u> WITH <u>n</u> OBSERVATIONS AND <u>k</u> INTERVALS	PROBABILITY P(J)	OBS FREQ	THEO FREQ	CHI-SQU CONTR
1	$c_{p_1}^2$		p_1	0_1	$\hat{E}_1$	$(0_1 - \hat{E}_1)^2 / \hat{E}_1$
2	$c_{p_2}^2$		p_2	0_2	$\hat{E}_2$	$(0_2 - \hat{E}_2)^2 / \hat{E}_2$
:	:		:	:	:	:
:	:		:	:	:	:
j	$c_{p_j}^2$		p_j	0_j	$\hat{E}_j$	$(0_j - \hat{E}_j)^2 / \hat{E}_j$
:	:		:	:	:	:
:	:		:	:	:	:
k-1	$c_{p_{k-1}}^2$		p_{k-1}	0_{k-1}	$\hat{E}_{k-1}$	$(0_{k-1} - \hat{E}_{k-1})^2 / \hat{E}_{k-1}$
k	∞		1.000	0_k	$\hat{E}_k$	$(0_k - \hat{E}_k)^2 / \hat{E}_k$

CHI-SQUARE = χ^2 WITH $(k-6)$ DEGREES OF FREEDOM

B. CRT Output

The plots are performed by the system "GRF Plot Subroutine" and are plotted by the CRT printer units. The plots are in an ordinary Cartesian coordinate system, and the origin is at $(x_{1(1)}, x_{2(1)})$.

The abscissa of the system is the x_1 -axis; the ordinate of the system is the x_2 -axis. An asterisk (*) is used to denote an (x_{1i}, x_{2i}) -observation; a zero (0) is used to denote the center $(\bar{x}_1, \bar{x}_2)$ of the distribution of the observations. In addition, a zero (0) is used to denote points on each of the estimated regression lines, $x_2 = f(x_1)$ and $x_1 = f(x_2)$. Up to four points (depending upon the range of the particular plot) are plotted for each regression line. The plotted points on the regression lines, however, are not connected.

C. Optional Output

The output illustrated on pages 15, 16, 17, and 19 of Section IV.A is automatically printed for each run processed. The categorization (page 18) and the CRT output is controlled by the analyst. This flexibility is provided by the program variables, NPRINT and JPLOT, on card type 4. The allowable input values (two for NPRINT and three for JPLOT) for these program variables permit six different output options. A quick reference of the available output options is given in the following table.

<u>NPRINT</u>	<u>JPLOT</u>	<u>OUTPUT</u>
0	0	1. Chi-Square Test
0	1	1. Chi-Square Test 2. Plot of data with contours
0	2	1. Chi-Square Test 2. Plot of data
1	0	1. Chi-Square Test 2. Categorization
1	1	1. Chi-Square Test 2. Categorization 3. Plot of data with contours
1	2	1. Chi-Square Test 2. Categorization 3. Plot of data

D. Program Running Time

The running time is printed at the end of each run processed.

The following table of observed times may be used as a guide for estimating program running times.

<u>Sample Size</u>	<u>No. of Intervals</u>	<u>Categorization</u>	<u>Plots</u>	<u>Time in Seconds</u>
50	10	Yes	Data & Contours	8.2
100	14	No	Data & Contours	9.8
150	12	Yes	Data & Contours	14.3
500	100	No	No	16.2
1000	150	Yes	Data & Contours	56.5
2500	200	No	Data & Contours	81.2

V. NUMERICAL EXAMPLE

A. Problem Description

The example problem consists of 100 observations (pairs of (x_1, x_2) -values) randomly sampled from a continuous two-dimensional distribution. Two runs were performed, one on the original observations and one on the logarithmically transformed values. Because some of the original observations (both x_1 -values and x_2 -values) were negative, transformation number 4 was used. For transforming x_1 , the constant A was set equal to 25; for transforming x_2 , A was set equal to 50. In both runs, 14 intervals were specified for the Chi-Square Test. Categorization was requested for the first run, but it was not requested for the second run. The heading of the plot (card type 7) was input for the first run only.

The input deck setup is illustrated on the following Data Card Layout Sheet.

B. Input for Example Problem

THIS DATA CARD LAYOUT
APPLIES TO THE
PROGRAMS: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

PAGE 1 OF 1

CARD TYPE		DATA CARD LAYOUT																				PROG. LOG. NUMBER																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																								
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C. Program Output

CONTROL CARDS

EXAMPLE PROBLEM WITH 100 PAIRS OF $X(1), X(2)$ -VALUES

VARIABLE FORMAT CARD (15,1459,0)

NRUM = 2

IRUM = 1

NRQ1M7 = 1 JPL07 = 1 WOTIC = 10 INPUT = 0 NTYPES = 1 ITRAM1 = 1

ITRAM2 = 1 IRA = 3 IRD = 0 NPAIR = 7 NN = 3 NL = 2

P-VALUES IDENTIFICATION FOR CHI-SQUARE TEST

0.05010.05010.200 0.30010.10010.800 0.85010.05010.950

P-VALUES IDENTIFICATION FOR PLOTS

0.10010.10010.800 0.90010.05010.950


```

MIN X1 =-.2000000E+02      MAX X1 = .4000000E+02
MIN X2 =-.4000000E+02      MAX X2 = .4000000E+02
R =-.1563169E-02      MEAN X1 =-.1250000E+01      MEAN X2 =-.2000000E+02
STAY DEV X1 = .1464349E+02      STAY DEV X2 = .1827892E+02
X2 =-.2124415E+02 + -.1951245E-02 *X1
X1 =-.5810919E+01 + -.1252275E-02 *X2
VARIANCE X1 = .2144318E+03      VARIANCE X2 = .3341190E+03

```

C2(1) = .12258659E+00 P(1) = 0.050
 30-(-.4700000E+01,-.2100000E+02)-.3505739E-02
 40-(-.7700000E+01,-.1800000E+02)-.7440370E-01
 51-(-.4700000E+01,-.2400000E+02)-.4096943E-01
 87-(-.5700000E+01,-.2200000E+02)-.2347946E-01
 C2(2) = .21672103E+00 P(2) = 0.100
 7-(.7000000E+01,-.1500000E+02)-.1385693E-00
 24-(.5700000E+01,-.2600000E+02)-.1150419E-00
 58-(.5700000E+01,-.2700000E+02)-.1522970E-00
 C2(3) = .3250378E+00 P(3) = 0.150
 1-(.5000000E+01,-.2100000E+02)-.3215731E-00
 5-(.1100000E+02,-.2100000E+02)-.2912016E-00
 19-(.2000000E+01,-.2800000E+02)-.2528519E-00
 53-(.1070000E+01,-.1300000E+02)-.2222691E-00
 77-(.2000000E+01,-.1500000E+02)-.2637234E-00
 95-(.1100000E+02,-.1600000E+02)-.3095411E-00
 C2(4) = .44628717E+00 P(4) = 0.200
 15-(.4700000E+01,-.9000000E+01)-.4057937E-00
 65-(.3070000E+01,-.3200000E+02)-.3448958E-00
 66-(.1000000E+02,-.2600000E+02)-.35049021E-00
 74-(.3700000E+01,-.3100000E+02)-.4401711E-00
 80-(.5000000E+01,-.2800000E+02)-.4199104E-00
 99-(.1200000E+02,-.2400000E+02)-.4347413E-00
 100-(.2000000E+01,-.3000000E+02)-.3432394E-00
 C2(5) = .7133498E+00 P(5) = 0.300
 35-(.0000000E+00,-.3400000E+02)-.5484424E+00
 48-(.1070000E+01,-.3700000E+02)-.4617851E-00
 49-(.1200000E+02,-.2700000E+02)-.55820154E+00
 55-(.4000000E+01,-.3200000E+02)-.4716580E-00
 92-(.1000000E+02,-.7000000E+01)-.6760891E+00
 98-(.1500000E+02,-.1500000E+02)-.6774131E+00
 C2(6) = .10216512E+01 P(6) = 0.400
 2-(.1100000E+02,-.3200000E+02)-.7908067E+00
 4-(.1300000E+02,-.3500000E+02)-.9493722E+00
 12-(.9000000E+01,-.3300000E+02)-.7143044E+00
 13-(.8000000E+01,-.3200000E+02)-.6495971E+00
 17-(.1300000E+02,-.3200000E+02)-.9853903E+00
 29-(.0000000E+00,-.3600000E+02)-.7172491E+00
 32-(.1700000E+02,-.2400000E+02)-.9948376E+00
 34-(.4700000E+01,-.3700000E+02)-.8419447E+00
 44-(.1600000E+02,-.2500000E+02)-.9032344E+00
 53-(.1200000E+02,-.6000000E+01)-.8677895E+00
 54-(.1500000E+02,-.2700000E+02)-.8751408E+00
 58-(.4700000E+01,-.3600000E+02)-.7431488E+00
 60-(.6000000E+01,-.4000000E+01)-.8278755E+00
 63-(.1200000E+02,-.8000000E+01)-.7217244E+00
 67-(.2000000E+01,-.3700000E+02)-.8528754E+00
 77-(.3000000E+01,-.3700000E+02)-.8195319E+00
 75-(.1500000E+02,-.2700000E+02)-.8751408E+00
 79-(.1300000E+02,-.2900000E+02)-.7683211E+00

94-(-.10000000E+02,-.30000000E+02)-.96078554E+00
 C2(7) = .13862944E+01 PI 7) = 0.500
 3-(-.12000000E+02,-.37000000E+02)-.13618305E+01
 6-(-.12000000E+02,-.27000000E+02)-.11203172E+01
 9-(-.12000000E+02,-.37000000E+02)-.13618305E+01
 17-(-.67000000E+01,-.39000000E+02)-.12028955E+01
 14-(-.17000000E+02,-.29000000E+02)-.12579484E+01
 18-(-.11000000E+02,-.37000000E+02)-.12623327E+01
 20-(-.17000000E+02,-.29000000E+02)-.12579484E+01
 23-(-.70000000E+01,-.36000000E+02)-.10400073E+01
 27-(-.17000000E+02,-.29000000E+02)-.12579484E+01
 33-(-.11000000E+02,-.31000000E+02)-.11246239E+01
 51-(-.16000000E+02,-.28000000E+02)-.10574492E+01
 71-(-.19000000E+02,-.26000000E+02)-.12243260E+01
 90-(-.19000000E+02,-.15000000E+02)-.11765792E+01
 93-(-.11000000E+02,-.35000000E+02)-.13604719E+01
 C2(8) = .18325615E+01 PI 8) = 0.600
 21-(-.11000000E+02,-.37000000E+02)-.19152108E+01
 38-(-.19000000E+02,-.32000000E+02)-.16351566E+01
 42-(-.15000000E+02,-.31000000E+02)-.15546111E+01
 43-(-.12000000E+02,-.39000000E+02)-.15961861E+01
 45-(-.16000000E+02,-.28000000E+02)-.17686244E+01
 56-(-.19000000E+02,-.20000000E+02)-.14606553E+01
 57-(-.16000000E+02,-.28000000E+02)-.17686244E+01
 59-(-.12000000E+02,-.39000000E+02)-.15961861E+01
 43-(-.17000000E+02,-.34000000E+02)-.16742545E+01
 85-(-.11000000E+02,-.20000000E+02)-.16456777E+01
 C2(9) = .24079456E+01 PI 9) = 0.700
 11-(-.14000000E+02,-.30000000E+02)-.19577029E+01
 14-(-.18000000E+02,-.26000000E+02)-.21106095E+01
 22-(-.00000000E+00,-.30000000E+02)-.18925107E+01
 28-(-.15000000E+02,-.40000000E+02)-.23219173E+01
 36-(-.19000000E+02,-.36000000E+02)-.20384171E+01
 84-(-.19000000E+02,-.35000000E+02)-.20924377E+01
 86-(-.20000000E+02,-.30000000E+02)-.19356173E+01
 49-(-.19000000E+02,-.33000000E+02)-.18904044E+01
 94-(-.17000000E+02,-.37000000E+02)-.19977280E+01
 97-(-.16000000E+02,-.37000000E+02)-.18514273E+01
 C2(10) = .32188759E+01 PI 10) = 0.800
 26-(-.90000000E+01,-.20000000E+02)-.24778933E+01
 46-(-.17000000E+02,-.70000000E+02)-.27879579E+01
 75-(-.17000000E+02,-.50000000E+02)-.24943340E+01
 78-(-.99000000E+01,-.80000000E+02)-.24722741E+01
 C2(11) = .37942400E+01 PI 11) = 0.850
 31-(-.21000000E+02,-.80000000E+02)-.32784126E+01
 91-(-.14000000E+02,-.20000000E+02)-.36617422E+01
 C2(12) = .46051703E+01 PI 12) = 0.900
 37-(-.13000000E+02,-.16000000E+02)-.41389660E+01
 41-(-.27000000E+02,-.28000000E+02)-.42741702E+01
 73-(-.67000000E+01,-.13000000E+02)-.42468380E+01

```

C2( 13) = .59914645E+01      P( 13) =      0.997
46-( .29000000E+02, -.23000000E+02) - .48970471E+01
47-( .30000000E+02, -.37000000E+02) - .54545903E+01
98-( .80000000E+01, .21000000E+02) - .51829757E+01
91-( .31000000E+02, -.37000000E+02) - .57556372E+01

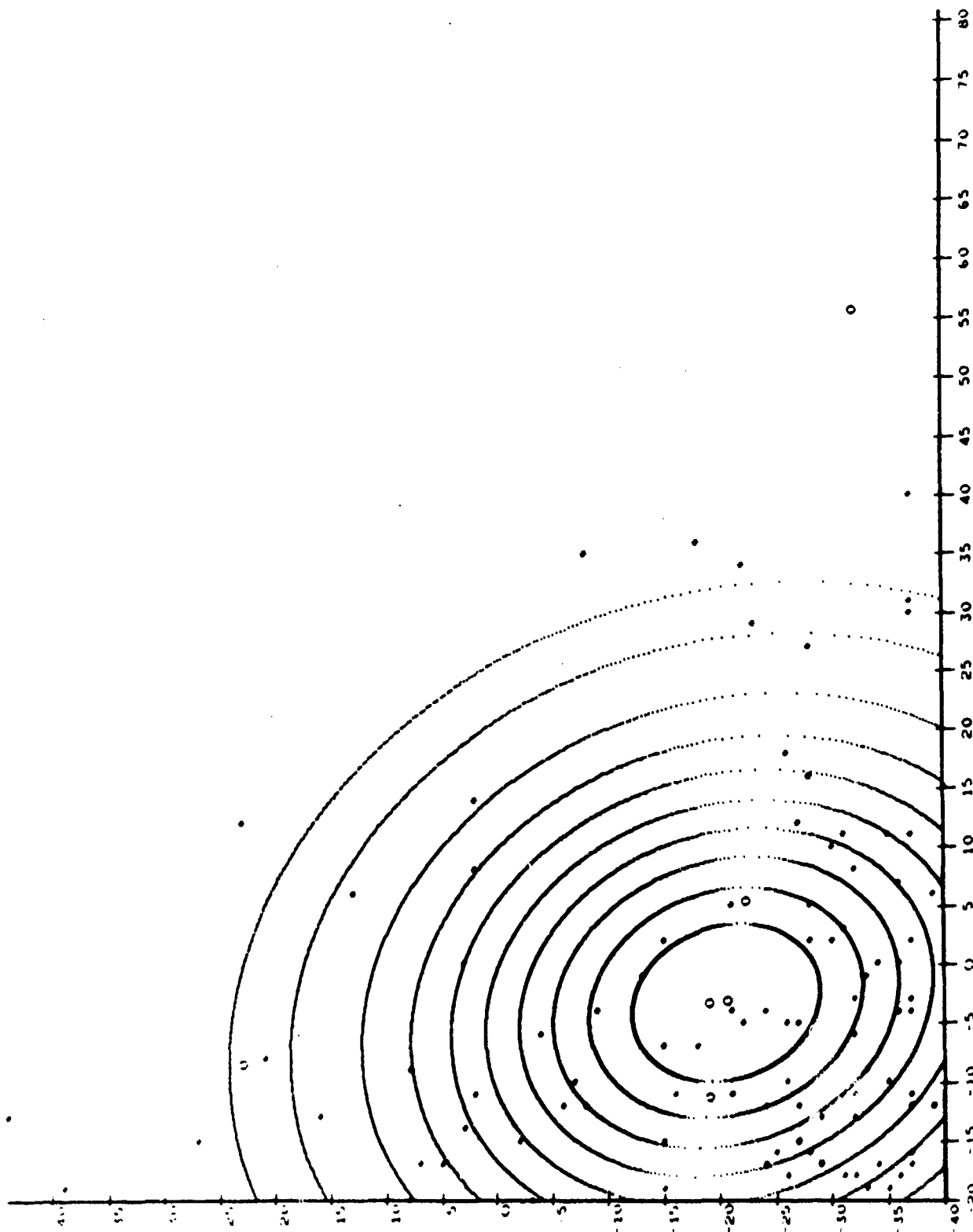
C2( 14) = INFINITY          P( 14) =      1.000
8-( .36000000E+02, -.18000000E+02) - .75074897E+01
25-( .34000000E+02, -.22000000E+02) - .65768927E+01
38-( .13000000E+02, .44000000E+02) - .12507062E+02
52-( .19000000E+02, .39000000E+02) - .10963127E+02
62-( .12000000E+02, .23000000E+02) - .77425981E+01
69-( .35000000E+02, -.80000000E+01) - .80551745E+01
70-( .15000000E+02, .27000000E+02) - .69442840E+01
82-( .49000000E+02, -.37000000E+02) - .49172856E+01

```

CHI-SQUARE TEST						
INTERVAL INDEX (J)	UPPER 90%ND C2(J)	RUN NUMBER 1 WITH 100 OBSERVATIONS AND 14 INTERVALS		THEO FREQ	CHI-SQU CONTR	
		PROBABILITY P(J)	Q25 FREQ			
1	.10258659E-00	0.050	4	5.0000	0.2000	
2	.21072103E-00	0.100	3	5.0000	0.8000	
3	.32503786E-00	0.150	6	5.0000	0.2000	
4	.44628710E-00	0.200	7	5.0000	0.8000	
5	.71334989E+00	0.300	6	10.0000	1.6000	
6	.70216512E+01	0.400	19	10.0000	8.1000	
7	.13862944E+01	0.500	14	10.0000	1.4000	
8	.48325815E+01	0.670	10	10.0000	0.0000	
9	.24079456E+01	0.770	10	10.0000	0.0000	
10	.21887585E+01	0.870	4	10.0000	3.6000	
11	.17942400E+01	0.900	2	5.0000	1.8000	
12	.76051702E+01	0.950	3	5.0000	0.8000	
13	.59914645E+01	1.000	4	5.0000	0.2000	
14	9.99999999E+14		8	5.0000	1.8000	

CHI-SQUARE = 21.503000 WITH 8 DEGREES OF FREEDOM

11M CONTOUR ELLIPSES
 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90, 0.95



EXAMPLE 0008, FM WITH 100 PAGES OF X(1), X(2)-VALUES

VA31291F FERRYAT CARY (15,14FS.C)

NRUY = ?

100% = 2

```

IPICCT = 1      NOTIC = 15      INPUT = 0      NTYPES = 0      ITFANI = 4

```

100 - 2 NP418 = 7 MM = 3 NL = 3

p-VALUES IDENTIFICATION FOR CHI-SQUARE TEST

0.055(0.050)0.270 0.300(0.100)0.800 0.050(0.050)0.950

P-VALUES IDENTIFICATION FOR PLOTS

0.25% INFORMATION FOR FEES

TRANSMISSION CODE FOR Y1 VALUES IS 4

TRANSFORMATION CONSTANTS FOR XI VALVES ARE-

```

CONVERSION FACTORS FOR
INSPIRATION CONSTANT FOR
TRAM1 = 250.000000E+02
CYTRAM1 = -.00000000E 00
CYTRAM1 = -.00000000E 00
OTRAN1 = -.00000000E 00
ETRAN1 = -.00000000E 00

```

TRANSMFORMATION CODE FOR V2 VALUES IS 4

TRANSCFORMATION CONSTANTS FOR X2 VALVES ARE-

```

ATRAN2 = .50000000E+02
ATRAN2 = -.00000000E 00
CTRAN7 = -.00000000E 00
OTRAN2 = -.00000000E 00
EIRANZ = -.00000000E 00

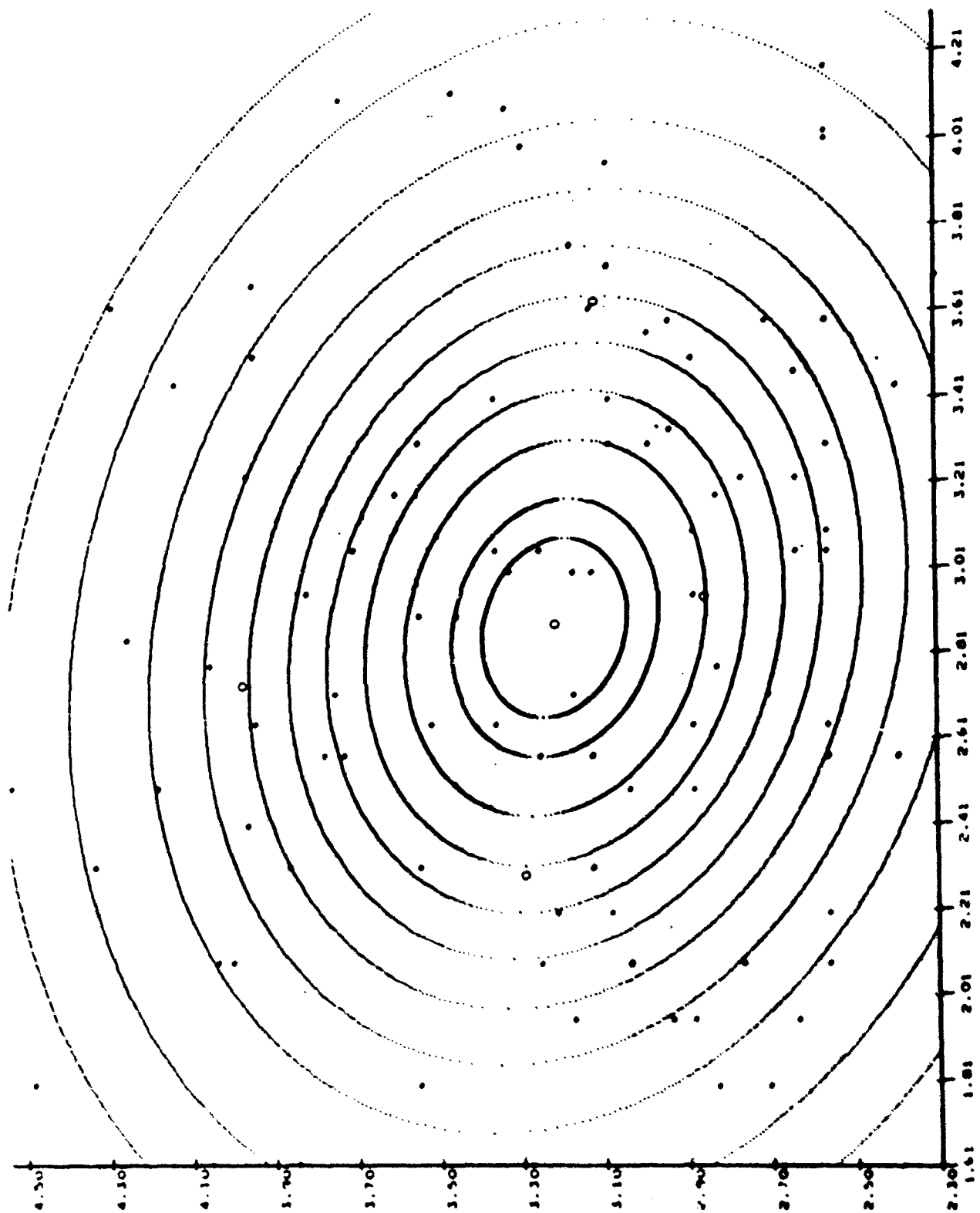
```

I	ORIGINAL DATA		TRANSFORMED DATA	
	X1	X2	X1	X2
1	9000000E+01	-2100000E+02	34611974E+01	33672558E+01
2	-11000000E+02	-32000000E+02	2690573E+01	28903718E+01
3	-12000000E+02	-37000000E+02	25649494E+01	25649494E+01
4	-10000000E+02	-15000000E+02	27080502E+01	27080502E+01
5	-11000000E+02	-21000000E+02	2690573E+01	33672558E+01
6	12000000E+02	-27000000E+02	34109179E+01	31354942E+01
7	-70000000E+01	-15000000E+02	2803718E+01	35533481E+01
8	90000000E+02	-18000000E+02	41108799E+01	34617359E+01
9	-12000000E+02	-37000000E+02	25649494E+01	25649494E+01
10	60000000E+01	-39000000E+02	34339872E+01	23978933E+01
11	-14000000E+02	30000000E+01	23578933E+01	39702919E+01
12	90000000E+01	-13000000E+02	27725987E+01	28337213E+01
13	80000000E+01	-32000000E+02	34565076E+01	28903718E+01
14	-17700000E+02	-29000000E+02	20794415E+01	30445224E+01
15	-40000000E+01	-90000000E+01	30445224E+01	37135721E+01
16	18000000E+02	-26000000E+02	37612901E+01	31750538E+01
17	-13700000E+02	-32000000E+02	24649066E+01	28903718E+01
18	-11000000E+02	-37000000E+02	2839573E+01	25649494E+01
19	20000000E+01	-28000000E+02	3258369E+01	30910425E+01
20	-17000000E+02	-29000000E+02	20794415E+01	30445224E+01
21	11000000E+02	-17000000E+02	3585189E+01	25649494E+01
22	00000000E+00	30000000E+01	32188758E+01	39702919E+01
23	70700000E+01	-36000000E+02	34657359E+01	26390573E+01
24	50000000E+01	-26000000E+02	29557323E+01	31780538E+01
25	34700000E+02	-22000000E+02	40174374E+01	33322045E+01
26	83000000E+01	-20000000E+02	34565076E+01	39312437E+01
27	-17000000E+02	-29000000E+02	20794415E+01	30445224E+01
28	15000000E+02	-40000000E+02	34618795E+01	23978933E+01
29	00000000E+00	-36000000E+02	32188758E+01	26390573E+01
30	-40000000E+01	-21000000E+02	30445224E+01	33672558E+01
31	-20700000E+02	80000000E+01	18094379E+01	40604630E+01
32	-17000000E+02	-24000000E+02	20794415E+01	32590965E+01
33	11700000E+02	-11000000E+02	3585189E+01	29444390E+01
34	-40000000E+01	-17000000E+02	30445224E+01	25649494E+01
35	90000000E+00	-34000000E+02	72188758E+01	27725987E+01
36	-14700000E+02	-16000000E+02	19459101E+01	76390573E+01
37	-13700000E+02	-16000000E+02	24649066E+01	41896547E+01
38	-18700000E+02	-32000000E+02	19459101E+01	28903718E+01
39	-13700000E+02	44000000E+02	24649066E+01	45433294E+01
40	-77000000E+01	-19000000E+02	28503718E+01	34617359E+01
41	27000000E+02	-28000000E+02	3912437E+01	30910425E+01
42	-14700000E+02	-11000000E+02	19459101E+01	29444390E+01
43	-12700000E+02	-19000000E+02	25649494E+01	23978933E+01
44	-16700000E+02	-75000000E+02	21572746E+01	32188758E+01
45	16700000E+02	-28000000E+02	17135721E+01	30910425E+01
46	29700000E+02	-23000000E+02	19689840E+01	32958368E+01
47	30700000E+02	-17000000E+02	40071132E+01	25649494E+01
48	-19700000E+01	-37000000E+02	31780538E+01	28337213E+01
49	-12000000E+02	-27000000E+02	25649494E+01	31354942E+01
50	-10000000E+01	-13700000E+02	31780538E+01	36109179E+01

37

MIN X1 = .16J94179E+01	MAX X1 = .41743873E+01	VARIANCE X1 = .43577068E-00	VARIANCE X2 = .30139972E-00
MIN X2 = .2J25931E+01	MAX X2 = .45432946E+01		
R = -.15454747E-00	MEAN X1 = .28648214E+01	MEAN X2 = .32213889E+01	
STAN DEV X1 = .66012929E+00	STAN DEV X2 = .54899884E+00		
X2 = .35993861E+01	+ -.13185247E-00		
X1 = .34809310E+01	+ -.19063324E-00		

EXAMPLE PROBLEM WITH 100 PAIRS OF $x(1), x(2)$ VALUES
 RUN NUMBER 2 WITH 100 OBSERVATIONS AND 11 CONTOURS
 0.050(0.050)0.100(0.100)0.200(0.200)0.300(0.300)0.400(0.400)0.500(0.500)0.600(0.600)0.700(0.700)0.800(0.800)0.900(0.900)1.000(1.000)1.100(1.100)1.200(1.200)1.300(1.300)1.400(1.400)1.500(1.500)1.600(1.600)1.700(1.700)1.800(1.800)1.900(1.900)2.000(2.000)2.100(2.100)2.200(2.200)2.300(2.300)2.400(2.400)2.500(2.500)2.600(2.600)2.700(2.700)2.800(2.800)2.900(2.900)3.000(3.000)3.100(3.100)3.200(3.200)3.300(3.300)3.400(3.400)3.500(3.500)3.600(3.600)3.700(3.700)3.800(3.800)3.900(3.900)4.000(4.000)4.100(4.100)4.200(4.200)4.300(4.300)4.400(4.400)4.500(4.500)



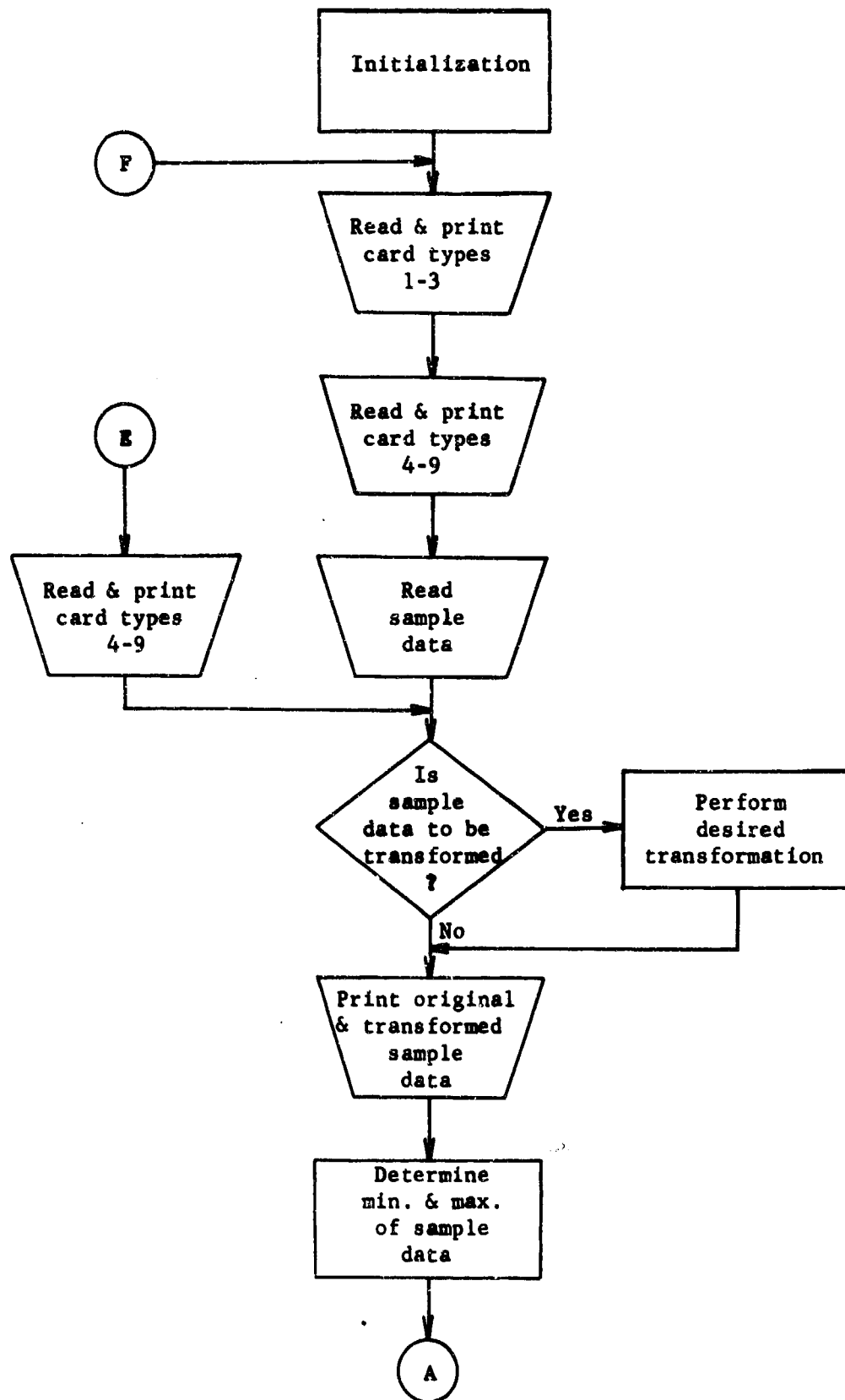
RANGE OVERFLOW
RANGE OVERFLOW
RANGE OVERFLOW
RANGE OVERFLOW
RANGE OVERFLOW
RUNNING TIME IN SECONDS = .95068214E+01

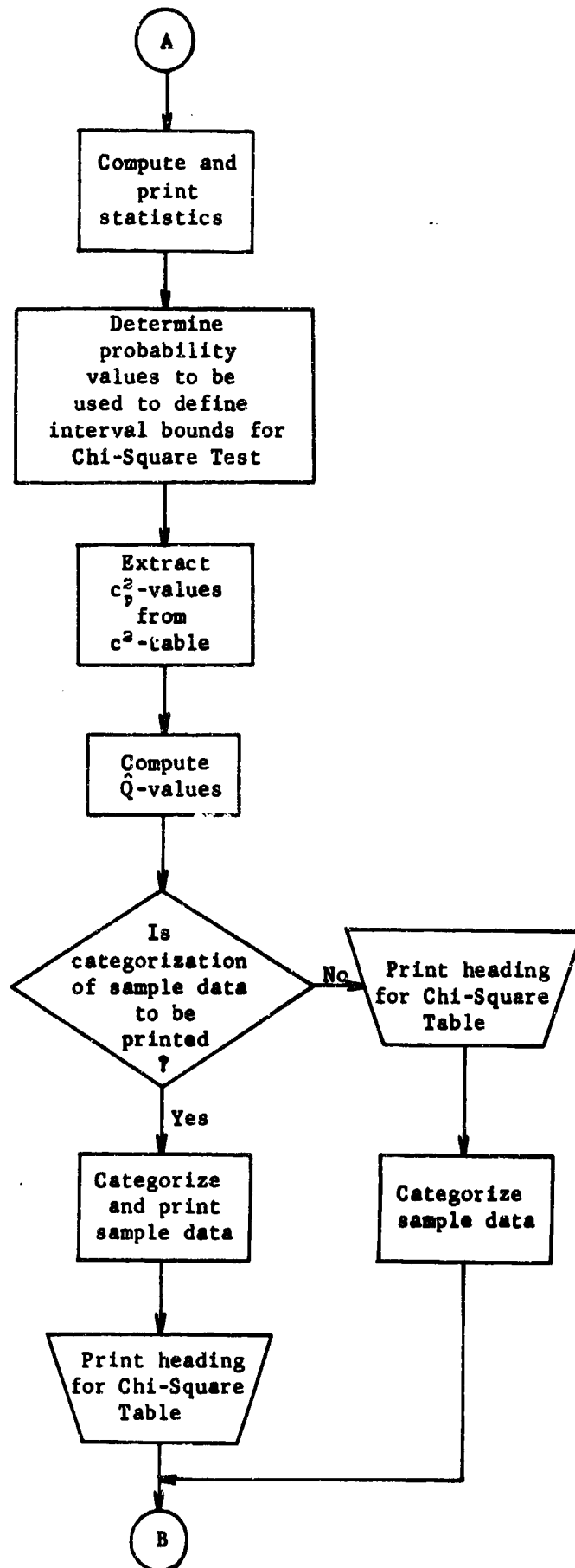
EOF ON \$READER

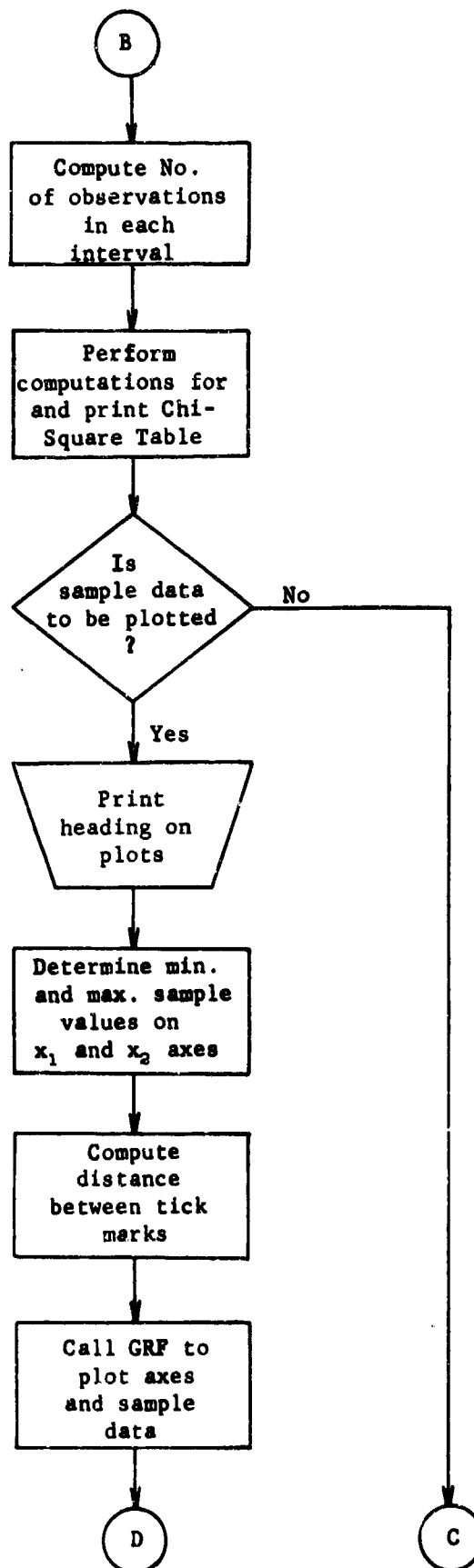
D. Discussion of Test Results

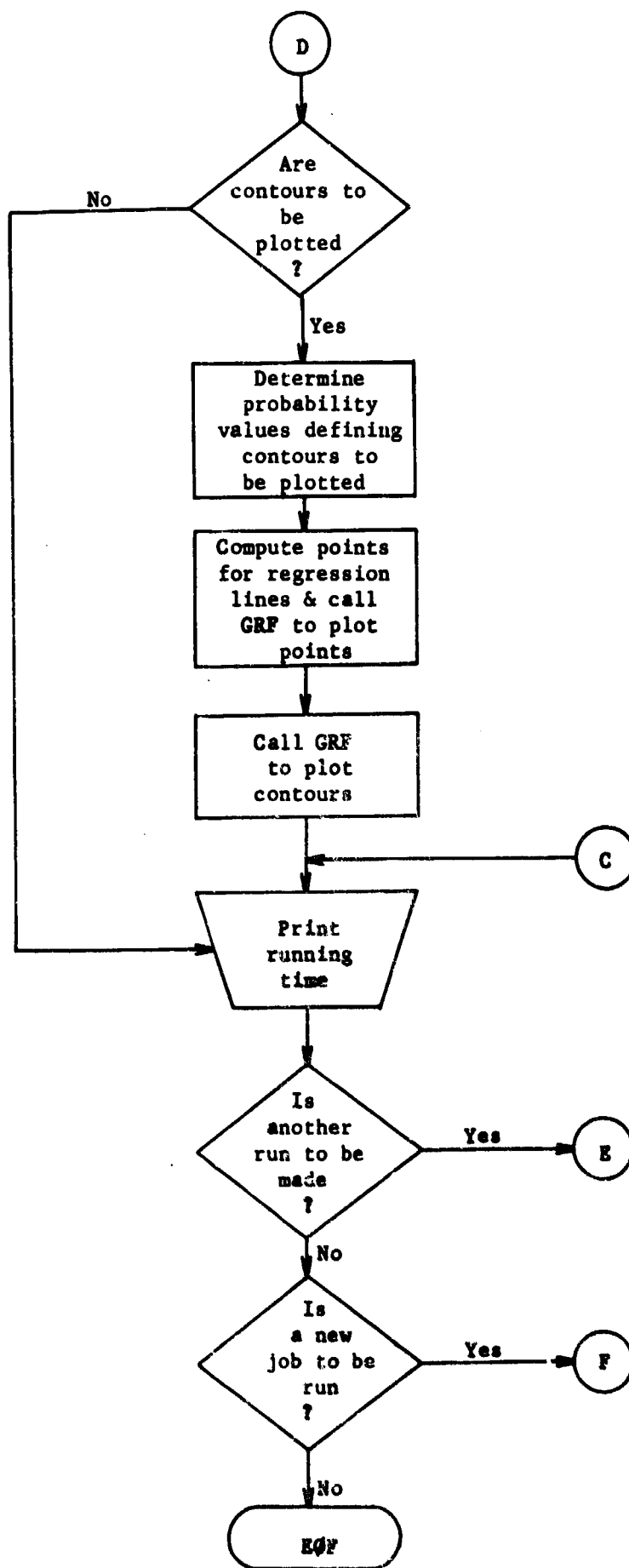
Suppose the 0.05-level of significance is the preselected level for testing the null hypothesis of normality. To test the null hypothesis "The 100 randomly sampled observations are from a parent bivariate normal population," $X^2 = 21.5$ (from the first run) is compared with $\chi^2(0.95, 8) = 15.5$. The null hypothesis is, therefore, rejected at the 0.05-level of significance. To test the null hypothesis of normality of the transformed random variables, $X^2 = 13.5$ (from the second run) is compared with $\chi^2(0.95, 8) = 15.5$. Because the latter null hypothesis cannot be rejected at the 0.05-level of significance, we conclude that the two original random variables are approximately jointly log normally distributed.

VI. FLOW CHART









VII. PROGRAM LISTING

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DIMENSION TABLEC(999),JOB(10),      OBS(999),X1ORIG(4000),      BC000010
1      X2ORIG(4000),PVAL(10),PPVAL(10),TABP(10),TABN(10)      BC000020
COMMON PS(10),PDEL(10),PE(10),PPS(10),PPDEL(10),PPE(10),      BC000030
1      X1(4000),X2(4000),P(999),C2(999),Q(4000),HANDY(18),      BC000040
2 FMT(10),XID1(10),XID2(10),IOBS(999),PP(999),E(4000),C2P(999)      BC000050
DATA (TABP(I),I=1,10)(1.,10.,100.,1000.,10000.,100000.,1000000.,      BC000060
1      10000000.,100000000.,1000000000.)      BC000070
DATA(TABN(I),I=1,10)(.000000001,.00000001,.0000001,.000001,      BC000080
1      .0001,.0001,.001,.01,.1,1.)      BC000090
DATA (TABLEC(I),I=1,36)      BC000100
1(.20010007E-02,.40040053E-02,.60090180E-02,.80160428E-02,      BC000110
2 .10025084E-01,.12036145E-01,.14049230E-01,.16064343E-01,      BC000120
3 .18081489E-01,.20100672E-01,.22121895E-01,.24145162E-01,      BC000130
4 .26170479E-01,.28197849E-01,.30227276E-01,.32258764E-01,      BC000140
5 .34292318E-01,.36327941E-01,.38365639E-01,.40405415E-01,      BC000150
6 .42447273E-01,.44491218E-01,.46537254E-01,.48585385E-01,      BC000160
7 .50635616E-01,.52687951E-01,.54742394E-01,.56798949E-01,      BC000170
8 .58857621E-01,.60918415E-01,.62981334E-01,.65046383E-01,      BC000180
9 .67113567E-01,.69182890E-01,.71254355E-01,.73327969E-01)      BC000190
DATA (TABLEC(I),I=37,72)      BC000200
1(.75403734E-01,.77481657E-01,.79561740E-01,.81643989E-01,      BC000210
2 .83728408E-01,.85815002E-01,.87903775E-01,.89994732E-01,      BC000220
3 .92087877E-01,.94183215E-01,.96280751E-01,.98380488E-01,      BC000230
4 .10048243E-00,.10258659E-00,.10469296E-00,.10680155E-00,      BC000240
5 .10891237E-00,.11102542E-00,.11314070E-00,.11525823E-00,      BC000250
6 .11737799E-00,.11950001E-00,.12162428E-00,.12375081E-00,      BC000260
7 .12587960E-00,.12801066E-00,.13014399E-00,.13227961E-00,      BC000270
8 .13441750E-00,.13655768E-00,.13870016E-00,.14084493E-00,      BC000280
9 .14299200E-00,.14514139E-00,.14729308E-00,.14944709E-00)      BC000290
DATA (TABLEC(I),I=73,108)      BC000300
1(.15160343E-00,.15376209E-00,.15592308E-00,.15808641E-00,      BC000310
2 .16025209E-00,.16242011E-00,.16459049E-00,.16676322E-00,      BC000320
3 .16893831E-00,.17111578E-00,.17329561E-00,.17547783E-00,      BC000330
4 .17766243E-00,.17984942E-00,.18203880E-00,.18423058E-00,      BC000340
5 .18642476E-00,.18862136E-00,.19082037E-00,.19302180E-00,      BC000350
6 .19522566E-00,.19743195E-00,.19964067E-00,.20185184E-00,      BC000360
7 .20406545E-00,.20628152E-00,.20850004E-00,.21072103E-00,      BC000370
8 .21294449E-00,.21517042E-00,.21739883E-00,.21962973E-00,      BC000380
9 .22186312E-00,.22409901E-00,.22633740E-00,.22857829E-00)      BC000390
DATA (TABLEC(I),I=109,144)      BC000400
1(.23082170E-00,.23306763E-00,.23531609E-00,.23756707E-00,      BC000410
2 .23982059E-00,.24207666E-00,.24433527E-00,.24659643E-00,      BC000420
3 .24886016E-00,.25112645E-00,.25339531E-00,.25566674E-00,      BC000430
4 .25794076E-00,.26021737E-00,.26249657E-00,.26477838E-00,      BC000440
5 .26706279E-00,.26934981E-00,.27163945E-00,.27393171E-00,      BC000450
6 .27622660E-00,.27852413E-00,.28082431E-00,.28312713E-00,      BC000460
7 .28543260E-00,.28774074E-00,.29005154E-00,.29236502E-00,      BC000470
8 .29468118E-00,.29700002E-00,.29932155E-00,.30164578E-00,      BC000480
9 .30397271E-00,.30630236E-00,.30863472E-00,.31096981E-00)      BC000490
DATA (TABLEC(I),I=145,180)      BC000500
1(.31330762E-00,.31564817E-00,.31799146E-00,.32033750E-00,      BC000510
2 .32268630E-00,.32503786E-00,.32739219E-00,.32974929E-00,      BC000520
3 .33210917E-00,.33447184E-00,.33683730E-00,.33920557E-00,      BC000530
4 .34157664E-00,.34395053E-00,.34632724E-00,.34870677E-00,      BC000540
5 .35108915E-00,.35347436E-00,.35586242E-00,.35825333E-00,      BC000550
6 .36064711E-00,.36304375E-00,.36544327E-00,.36784568E-00,      BC000560
7 .37025097E-00,.37265916E-00,.37507025E-00,.37748425E-00,      BC000570
8 .37990117E-00,.38232101E-00,.38474379E-00,.38716950E-00,      BC000580
9 .38959816E-00,.39202977E-00,.39446434E-00,.39690188E-00)      BC000590

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DATA (TABLEC(I),I=181,216)				BC000600
1 (.39934239E-00,	.40178588E-00,	.40423237E-00,	.40668185E-00,	BC000610
2 .40913433E-00,	.41158983E-00,	.41404834E-00,	.41650988E-00,	BC000620
3 .41897445E-00,	.42144206E-00,	.42391272E-00,	.42638644E-00,	BC000630
4 .42886322E-00,	.43134307E-00,	.43382600E-00,	.43631202E-00,	BC000640
5 .43880113E-00,	.44129334E-00,	.44378866E-00,	.44628710E-00,	BC000650
6 .44878867E-00,	.45129336E-00,	.45380120E-00,	.45631219E-00,	BC000660
7 .45882633E-00,	.46134364E-00,	.46386411E-00,	.46638777E-00,	BC000670
8 .46891462E-00,	.47144467E-00,	.47397792E-00,	.47651438E-00,	BC000680
9 .47905406E-00,	.48159697E-00,	.48414312E-00,	.48669252E-00)	BC000690
DATA (TABLEC(I),I=217,252)				BC000700
1 (.48924517E-00,	.49180108E-00,	.49436026E-00,	.49692272E-00,	BC000710
2 .49948847E-00,	.50205751E+00,	.50462986E+00,	.50720552E+00,	BC000720
3 .50978450E+00,	.51236681E+00,	.51495246E+00,	.51754146E+00,	BC000730
4 .52013381E+00,	.52272953E+00,	.52532862E+00,	.52793109E+00,	BC000740
5 .53053696E+00,	.53314622E+00,	.53575889E+00,	.53837498E+00,	BC000750
6 .54099450E+00,	.54361745E+00,	.54624384E+00,	.54887369E+00,	BC000760
7 .55150700E+00,	.55414379E+00,	.55678405E+00,	.55942781E+00,	BC000770
8 .56207506E+00,	.56472582E+00,	.56738010E+00,	.57003791E+00,	BC000780
9 .57269925E+00,	.57536414E+00,	.57803259E+00,	.58070460E+00)	BC000790
DATA (TABLEC(I),I=253,288)				BC000800
1 .58338019E+00,	.58605936E+00,	.58874212E+00,	.59142849E+00,	BC000810
2 .59411847E+00,	.59681207E+00,	.59950931E+00,	.60221019E+00,	BC000820
3 .60491472E+00,	.60762291E+00,	.61033477E+00,	.61305032E+00,	BC000830
4 .61576956E+00,	.61849250E+00,	.62121915E+00,	.62394953E+00,	BC000840
5 .62668364E+00,	.62942149E+00,	.63216309E+00,	.63490846E+00,	BC000850
6 .63765760E+00,	.64041053E+00,	.64316725E+00,	.64592777E+00,	BC000860
7 .64869211E+00,	.65146028E+00,	.65423228E+00,	.65700813E+00,	BC000870
8 .65978784E+00,	.66257142E+00,	.66535888E+00,	.66815022E+00,	BC000880
9 .67094547E+00,	.67374463E+00,	.67654772E+00,	.67935474E+00)	BC000890
DATA (TABLEC(I),I=289,324)				BC000900
1 (.68216570E+00,	.68498062E+00,	.68779950E+00,	.69062237E+00,	BC000910
2 .69344923E+00,	.69628008E+00,	.69911495E+00,	.70195385E+00,	BC000920
3 .70479677E+00,	.70764375E+00,	.71049478E+00,	.71334989E+00,	BC000930
4 .71620907E+00,	.71907235E+00,	.72193974E+00,	.72481124E+00,	BC000940
5 .72768687E+00,	.73056664E+00,	.73345056E+00,	.73633865E+00,	BC000950
6 .73923091E+00,	.74212736E+00,	.74502802E+00,	.74793288E+00,	BC000960
7 .75084197E+00,	.75375530E+00,	.75667288E+00,	.75959472E+00,	BC000970
8 .76252084E+00,	.76545124E+00,	.76838595E+00,	.77132496E+00,	BC000980
9 .77426830E+00,	.77721598E+00,	.78016801E+00,	.78312441E+00)	BC000990
DATA (TABLEC(I),I=325,360)				BC001000
1 (.78608518E+00,	.78905034E+00,	.79201990E+00,	.79499388E+00,	BC001010
2 .79797228E+00,	.80095513E+00,	.80394244E+00,	.80693421E+00,	BC001020
3 .80993047E+00,	.81293122E+00,	.81593648E+00,	.81894626E+00,	BC001030
4 .82196058E+00,	.82497945E+00,	.82800288E+00,	.83103089E+00,	BC001040
5 .83406349E+00,	.83710070E+00,	.84014252E+00,	.84318998E+00,	BC001050
6 .84624009E+00,	.84929586E+00,	.85235630E+00,	.85542143E+00,	BC001060
7 .85849127E+00,	.86156583E+00,	.86464451E+00,	.86772791E+00,	BC001070
8 .87081797E+00,	.87391155E+00,	.87700992E+00,	.88011311E+00,	BC001080
9 .88322111E+00,	.88633395E+00,	.88945164E+00,	.89257421E+00)	BC001090
DATA (TABLEC(I),I=361,396)				BC001100
1 (.89570165E+00,	.89883399E+00,	.90197125E+00,	.90511343E+00,	BC001110
2 .90826056E+00,	.91141265E+00,	.91456971E+00,	.91773177E+00,	BC001120
3 .92089883E+00,	.92407092E+00,	.92724804E+00,	.93043023E+00,	BC001130
4 .93361748E+00,	.93680982E+00,	.94000726E+00,	.94320982E+00,	BC001140
5 .94641752E+00,	.94963037E+00,	.95284839E+00,	.95607160E+00,	BC001150
6 .95930001E+00,	.96253364E+00,	.96577251E+00,	.96901663E+00,	BC001160
7 .97226602E+00,	.97552070E+00,	.97878069E+00,	.98204599E+00,	BC001170
8 .98531664E+00,	.98859264E+00,	.99187402E+00,	.99516079E+00,	BC001180

9 .99845298E+00. .10017506E+01. .10050536E+01. .10083622E+01)	BC001190
DATA (TABLEC(I),I=397,432)	BC001200
1 (.10116762E+01. .10149957E+01. .10183207E+01. .10216512E+01.	BC001210
2 .10249874E+01. .10283291E+01. .10316763E+01. .10350292E+01.	BC001220
3 .10383877E+01. .10417519E+01. .10451218E+01. .10484973E+01.	BC001230
4 .10518785E+01. .10552655E+01. .10586582E+01. .10620567E+01.	BC001240
5 .10654609E+01. .10688710E+01. .10722869E+01. .10757086E+01.	BC001250
6 .10791362E+01. .10825697E+01. .10860090E+01. .10894544E+01.	BC001260
7 .10929056E+01. .10963628E+01. .10998260E+01. .11032952E+01.	BC001270
8 .11067705E+01. .11102518E+01. .11137391E+01. .11172326E+01.	BC001280
9 .11207321E+01. .11242378E+01. .11277497E+01. .11312677E+01)	BC001290
DATA (TABLEC(I),I=433,468)	BC001300
1 (.11347920E+01. .11383224E+01. .11418591E+01. .11454021E+01.	BC001310
2 .11489513E+01. .11525069E+01. .11560687E+01. .11596370E+01.	BC001320
3 .11632116E+01. .11667926E+01. .11703801E+01. .11739740E+01.	BC001330
4 .11775743E+01. .11811812E+01. .11847946E+01. .11884145E+01.	BC001340
5 .11920409E+01. .11956740E+01. .11993137E+01. .12029600E+01.	BC001350
6 .12066130E+01. .12102726E+01. .12139390E+01. .12176121E+01.	BC001360
7 .12212919E+01. .12249786E+01. .12286720E+01. .12323723E+01.	BC001370
8 .12360794E+01. .12397934E+01. .12435144E+01. .12472422E+01.	BC001380
9 .12509771E+01. .12547199E+01. .12584677E+01. .12622236E+01)	BC001390
DATA (TABLEC(I),I=469,504)	BC001400
1 (.12659865E+01. .12697565E+01. .12735337E+01. .12773180E+01.	BC001410
2 .12811095E+01. .12849081E+01. .12887140E+01. .12925272E+01.	BC001420
3 .12963476E+01. .13001754E+01. .13040105E+01. .13078529E+01.	BC001430
4 .13117028E+01. .13155601E+01. .13194248E+01. .13232970E+01.	BC001440
5 .13271768E+01. .13310640E+01. .13349589E+01. .13388613E+01.	BC001450
6 .13427714E+01. .13466891E+01. .13506145E+01. .13545477E+01.	BC001460
7 .13584886E+01. .13624372E+01. .13663937E+01. .13703580E+01.	BC001470
8 .13743302E+01. .13783103E+01. .13822984E+01. .13862944E+01.	BC001480
9 .13902984E+01. .13943104E+01. .13983305E+01. .14023587E+01)	BC001490
DATA (TABLEC(I),I=505,540)	BC001500
1 (.14063950E+01. .14104395E+01. .14144922E+01. .14185500E+01.	BC001510
2 .14226223E+01. .14266998E+01. .14307856E+01. .14348797E+01.	BC001520
3 .14389823E+01. .14430933E+01. .14472128E+01. .14513407E+01.	BC001530
4 .14554773E+01. .14596223E+01. .14637760E+01. .14679384E+01.	BC001540
5 .14721094E+01. .14762891E+01. .14804776E+01. .14846748E+01.	BC001550
6 .14888809E+01. .14930959E+01. .14973198E+01. .15015526E+01.	BC001560
7 .15057944E+01. .15100452E+01. .15143050E+01. .15185740E+01.	BC001570
8 .15228520E+01. .15271393E+01. .15314357E+01. .15357415E+01.	BC001580
9 .15400564E+01. .15443808E+01. .15487145E+01. .15530576E+01)	BC001590
DATA (TABLEC(I),I=541,576)	BC001600
1 (.15574101E+01. .15617722E+01. .15661438E+01. .15705249E+01.	BC001610
2 .15749157E+01. .15793162E+01. .15837263E+01. .15881462E+01.	BC001620
3 .15925759E+01. .15970154E+01. .16014648E+01. .16059241E+01.	BC001630
4 .16103934E+01. .16148727E+01. .16193620E+01. .16238614E+01.	BC001640
5 .16283710E+01. .16328908E+01. .16374200E+01. .16419611E+01.	BC001650
6 .16465117E+01. .16510727E+01. .16556442E+01. .16602261E+01.	BC001660
7 .16648185E+01. .16694215E+01. .16740351E+01. .16786594E+01.	BC001670
8 .16832944E+01. .16879401E+01. .16925967E+01. .16972642E+01.	BC001680
9 .17019425E+01. .17066319E+01. .17113322E+01. .17160436E+01)	BC001690
DATA (TABLEC(I),I=577,612)	BC001700
1 (.17207662E+01. .17254999E+01. .17302449E+01. .17350011E+01.	BC001710
2 .17397687E+01. .17445477E+01. .17493381E+01. .17541400E+01.	BC001720
3 .17589535E+01. .17637786E+01. .17686154E+01. .17734639E+01.	BC001730
4 .17783241E+01. .17831962E+01. .17880802E+01. .17929762E+01.	BC001740
5 .17978842E+01. .18028042E+01. .18077364E+01. .18126808E+01.	BC001750
6 .18176374E+01. .18226064E+01. .18275877E+01. .18325815E+01.	BC001760
7 .18375877E+01. .18426065E+01. .18476380E+01. .18526821E+01.	BC001770

8 .1817390E+01. .18628087E+01. .18678913E+01. .18729869E+01.	BC001780
9 .18780954E+01. .18832171E+01. .18883519E+01. .18934999E+01)	BC001790
DATA (TABLEC(I),I=613,648)	BC001800
1 (.18986612E+01. .19038358E+01. .19090239E+01. .19142255E+01.	BC001810
2 .19194406E+01. .19246693E+01. .19299118E+01. .19351681E+01.	BC001820
3 .19404381E+01. .19457222E+01. .19510202E+01. .19563323E+01.	BC001830
4 .19616585E+01. .19669990E+01. .19723537E+01. .19777228E+01.	BC001840
5 .19831064E+01. .19885045E+01. .19939173E+01. .19993447E+01.	BC001850
6 .20047869E+01. .20102439E+01. .20157159E+01. .20212028E+01.	BC001860
7 .20267049E+01. .20322221E+01. .20377546E+01. .20433025E+01.	BC001870
8 .20488658E+01. .20544446E+01. .20600390E+01. .20656491E+01.	BC001880
9 .20712750E+01. .20769167E+01. .20825744E+01. .20882482E+01)	BC001890
DATA (TABLEC(I),I=649,684)	BC001900
1 (.20939381E+01. .20996442E+01. .21053667E+01. .21111056E+01.	BC001910
2 .21168610E+01. .21226330E+01. .21284217E+01. .21342272E+01.	BC001920
3 .21400497E+01. .21458891E+01. .21517456E+01. .21576193E+01.	BC001930
4 .21635103E+01. .21694188E+01. .21753447E+01. .21812882E+01.	BC001940
5 .21872495E+01. .21932286E+01. .21992256E+01. .22052406E+01.	BC001950
6 .22112738E+01. .22173252E+01. .22233951E+01. .22294833E+01.	BC001960
7 .22355902E+01. .22417158E+01. .22478602E+01. .22540235E+01.	BC001970
8 .22602059E+01. .22664075E+01. .22726283E+01. .22788686E+01.	BC001980
9 .22851284E+01. .22914078E+01. .22977070E+01. .23040261E+01)	BC001990
DATA (TABLEC(I),I=685,720)	BC002000
1 (.23103653E+01. .23167446E+01. .23231042E+01. .23295042E+01.	BC002010
2 .23359247E+01. .23423660E+01. .23488280E+01. .23553110E+01.	BC002020
3 .23618151E+01. .23683404E+01. .23748870E+01. .23814552E+01.	BC002030
4 .23880449E+01. .23946565E+01. .24012900E+01. .24079456E+01.	BC002040
5 .24146234E+01. .24213236E+01. .24280463E+01. .24347916E+01.	BC002050
6 .24415598E+01. .24483510E+01. .24551653E+01. .24620030E+01.	BC002060
7 .24688640E+01. .24757487E+01. .24826572E+01. .24895896E+01.	BC002070
8 .24965461E+01. .25035269E+01. .25105322E+01. .25175621E+01.	BC002080
9 .25246168E+01. .25316964E+01. .25388012E+01. .25459314E+01)	BC002090
DATA (TABLEC(I),I=721,756)	BC002100
1 (.25530870E+01. .25602683E+01. .25674755E+01. .25747088E+01.	BC002110
2 .25819684E+01. .25892543E+01. .25965670E+01. .26039064E+01.	BC002120
3 .26112729E+01. .26186666E+01. .26260878E+01. .26335366E+01.	BC002130
4 .26410132E+01. .26485179E+01. .26560509E+01. .26636124E+01.	BC002140
5 .26712025E+01. .26788216E+01. .26864697E+01. .26941473E+01.	BC002150
6 .27018544E+01. .27095914E+01. .27173584E+01. .27251557E+01.	BC002160
7 .27329835E+01. .27408420E+01. .27487316E+01. .27566524E+01.	BC002170
8 .27646047E+01. .27725887E+01. .27806048E+01. .27886531E+01.	BC002180
9 .27967339E+01. .28048475E+01. .28129941E+01. .28211741E+01)	BC002190
DATA (TABLEC(I),I=757,792)	BC002200
1 (.28293877E+01. .28376351E+01. .28459167E+01. .28542327E+01.	BC002210
2 .28625835E+01. .28709692E+01. .28793903E+01. .28878469E+01.	BC002220
3 .28963395E+01. .29048683E+01. .29134337E+01. .29220358E+01.	BC002230
4 .29306751E+01. .29393519E+01. .29480666E+01. .29568193E+01.	BC002240
5 .29656105E+01. .29744406E+01. .29833098E+01. .29922185E+01.	BC002250
6 .30011670E+01. .30101558E+01. .30191852E+01. .30282555E+01.	BC002260
7 .30373671E+01. .30465204E+01. .30557159E+01. .30649537E+01.	BC002270
8 .30742345E+01. .30835585E+01. .30929262E+01. .31023380E+01.	BC002280
9 .31117943E+01. .31212955E+01. .31308421E+01. .31404344E+01)	BC002290
DATA (TABLEC(I),I=793,828)	BC002300
1 (.31500730E+01. .31597582E+01. .31694906E+01. .31792706E+01.	BC002310
2 .31890986E+01. .31989752E+01. .32089007E+01. .32188758E+01.	BC002320
3 .32289009E+01. .32389765E+01. .32491031E+01. .32592812E+01.	BC002330
4 .32695114E+01. .32797942E+01. .32901302E+01. .33005198E+01.	BC002340
5 .33109637E+01. .33214624E+01. .33320165E+01. .33426264E+01.	BC002350
6 .33532933E+01. .33640172E+01. .33747989E+01. .33856390E+01.	BC002360

7 .33965383E+01. .34074972E+01. .34185165E+01. .34295969E+01.	BC002370
8 .34407389E+01. .34519435E+01. .34632111E+01. .34745426E+01.	BC002380
9 .34859386E+01. .34974000E+01. .35089274E+01. .35205216E+01)	BC002390
DATA (TABLEC(1),I=829,864)	
1 (.35321834E+01. .35439137E+01. .35557131E+01. .35675826E+01.	BC002400
2 .35795229E+01. .35915350E+01. .36036196E+01. .36157777E+01.	BC002410
3 .36280102E+01. .36403179E+01. .36527018E+01. .36651629E+01.	BC002420
4 .36777022E+01. .36903205E+01. .37030189E+01. .37157985E+01.	BC002430
5 .37286603E+01. .37416054E+01. .37546347E+01. .37677495E+01.	BC002440
6 .37809509E+01. .37942400E+01. .38076179E+01. .38210860E+01.	BC002450
7 .38346454E+01. .38482973E+01. .38620431E+01. .38758840E+01.	BC002460
8 .38898213E+01. .39038564E+01. .39179908E+01. .39322257E+01.	BC002470
9 .39465627E+01. .39610032E+01. .39755487E+01. .39902008E+01)	BC002480
DATA (TABLEC(1),I=865,900)	
1 (.40049610E+01. .40198310E+01. .40348123E+01. .40499067E+01.	BC002490
2 .40651159E+01. .40804417E+01. .40958857E+01. .41114500E+01.	BC002500
3 .41271364E+01. .41429467E+01. .41588831E+01. .41749474E+01.	BC002510
4 .41911418E+01. .42074685E+01. .42239295E+01. .42405271E+01.	BC002520
5 .42572636E+01. .42741413E+01. .42911627E+01. .43083302E+01.	BC002530
6 .43256463E+01. .43431137E+01. .43607349E+01. .43785128E+01.	BC002540
7 .43964502E+01. .44145498E+01. .44328148E+01. .44512481E+01.	BC002550
8 .44698529E+01. .44886324E+01. .45075899E+01. .45267288E+01.	BC002560
9 .45460526E+01. .45655649E+01. .45852695E+01. .46051702E+01)	BC002570
DATA (TABLEC(1),I=901,936)	
1 (.46252709E+01. .46455756E+01. .46660886E+01. .46868142E+01.	BC002580
2 .47077568E+01. .47289210E+01. .47503116E+01. .47719334E+01.	BC002590
3 .47937915E+01. .48158912E+01. .48382378E+01. .48608369E+01.	BC002600
4 .48836943E+01. .49068160E+01. .49302080E+01. .49538770E+01.	BC002610
5 .49778293E+01. .50020721E+01. .50266122E+01. .50514573E+01.	BC002620
6 .50766149E+01. .51020929E+01. .51278997E+01. .51540439E+01.	BC002630
7 .51805343E+01. .52073804E+01. .52345917E+01. .52621793E+01.	BC002640
8 .52901508E+01. .53185201E+01. .53472975E+01. .53764951E+01.	BC002650
9 .54061253E+01. .54362011E+01. .54667360E+01. .54977444E+01)	BC002660
DATA (TABLEC(1),I=937,972)	
1 (.55292411E+01. .55612418E+01. .55937628E+01. .56268214E+01.	BC002670
2 .56604357E+01. .56946243E+01. .57294080E+01. .57648072E+01.	BC002680
3 .58008442E+01. .58375425E+01. .58749267E+01. .59130231E+01.	BC002690
4 .59518593E+01. .59914645E+01. .60318700E+01. .60731085E+01.	BC002700
5 .61152154E+01. .61582278E+01. .62021856E+01. .62471313E+01.	9C002710
6 .62931103E+01. .63401713E+01. .63883664E+01. .64377516E+01.	BC002720
7 .64883873E+01. .65403382E+01. .65936747E+01. .66484727E+01.	BC002730
8 .67048144E+01. .67627895E+01. .68224954E+01. .68840388E+01.	BC002740
9 .69475361E+01. .70131158E+01. .70809189E+01. .71511015E+01)	BC002750
DATA (TABLEC(1),I=973,999)	
1 (.72238768E+01. .72993175E+01. .73777589E+01. .74594029E+01.	BC002760
2 .75445221E+01. .76304257E+01. .77264557E+01. .78240460E+01.	BC002770
3 .79266326E+01. .80247670E+01. .81490839E+01. .82703331E+01.	BC002780
4 .83994102E+01. .85373090E+01. .86856118E+01. .88456973E+01.	BC002790
5 .90197200E+01. .92103404E+01. .94210614E+01. .96566275E+01.	BC002800
6 .99236273E+01. .10271992E+02. .10596635E+02. .11042922E+02.	BC002810
7 .11618286E+02. .12429216E+02. .13815511E+02)	BC002820
EQUIVALENCE (OPS,IC05)	
CALL OPT11	BC002830
CALL CRT10 (4HA39C)	BC002840
C506=5.76.	BC002850
C605=6.75.	BC002860
LA=1	BC002870
LO=1	BC002880
50 READ LOG,JOB,FMT	BC002890
	BC002900
	BC002910
	BC002920
	BC002930
	BC002940
	BC002950

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100 FORMAT (10A8/10A8)
READ 115,NRUN
110 FORMAT (15)
DO 120 OF IRUN=1,NRUN
PRINT 110,JOB,FMT
110 FORMAT (1H1,56X,13HCONTROL CARDS//10A8//2X,20HVARIBLE FORMAT CARDBC002960
1,5X,10A8)
PRINT 117,NRUN
117 FORMAT (1H0, 6HN RUN =,13)
120 READ 150,NPRINT,JPLOT,NOTIC,INPUT,NTYPES,ITRAN1,ITRAN2,IRA,IRO,
1 NPAIR,NN,NL,(PS(JJ),PDEL(TJJ),PE(JJ),JJ=1,NN)
150 FORMAT (1215/15F5.3)
PRINT 155,IRUN,NPRINT,JPLOT,NOTIC,INPUT,NTYPES,ITRAN1,ITRAN2,IRA,
1 IRO,NPAIR,NN,NL
155 FORMAT (1H0,6HIRUN =,13/1H0,9HNPRINT = ,11,5X,8HJPLOT = ,11,5X,
1 8HNOTIC = ,12,5X,8HINPUT = ,11,5X,8HNTYPES = ,11,5X,
2 9HITRAN1 = ,12/1H0,9HITRAN2 = ,12,4X,6HIRA = ,11,7X,
3 6HIRO = ,11, 8X,8HNPAIR = ,12,4X,5HNN = ,11,9X,5HNL = ,11)
PRINT 157,(PS(JJ),PDEL(TJJ),PE(JJ),JJ=1,NN)
157 FORMAT (1H0,43HP-VALUES IDENTIFICATION FOR CHI-SQUARE TEST/1H ,5X,
1 5(F5.3,1H(,F5.3,1H),F5.3,2X))
IF (JPLOT.LE.0) GO TO 163
IF (JPLOT.EQ.2) GO TO 163
READ 160, (PPS(JJ),PPDEL(TJJ),PPE(JJ),JJ=1,NL)
160 FORMAT ((15F5.3))
PRINT 162,(PPS(JJ),PPDEL(TJJ),PPE(JJ),JJ=1,NL)
162 FORMAT (1H0,33HP-VALUES IDENTIFICATION FOR PLOTS/1H ,5X,5(F5.3,
1 1H(,F5.3,1H),F5.3,2X ))
163 IF (NTYPES.LE.0) GO TO 165
READ 164, XID1,XID2
164 FORMAT (10A8/10A8)
165 IF (ITRAN1.EQ.1) GO TO 168
READ 166,ATRAN1,BTRAN1,CTRAN1,DTRAN1,ETRAN1
166 FORMAT (5E14.8)
PRINT 167,ITRAN1,ATRAN1,BTRAN1,CTRAN1,DTRAN1,ETRAN1
167 FORMAT (1H0,37HTRANSFORMATION CODE FOR X1 VALUES IS ,12/1H ,
1 43HTRANSFORMATION CONSTANTS FOR X1 VALUES ARE-/1H ,3X,
2 8HATRAN1 =,E15.8,3X,8HBTRAN1 =,E15.8,3X,8HCTRAN1 =,E15.8
3 ,3X,8HDTRAN1 =,E15.8,3X,8HETRAN1 =,E15.8)
168 IF (ITRAN2.EQ.1) GO TO 180
READ 169,ATRAN2,BTRAN2,CTRAN2,DTRAN2,ETRAN2
169 FORMAT (5E14.8)
PRINT 170,ITRAN2,ATRAN2,BTRAN2,CTRAN2,DTRAN2,ETRAN2
170 FORMAT (1H0,37HTRANSFORMATION CODE FOR X2 VALUES IS ,12/1H ,
1 43HTRANSFORMATION CONSTANTS FOR X2 VALUES ARE-/1H ,3X,
2 8HATRAN2 =,E15.8,3X,8HBTRAN2 =,E15.8,3X,8HCTRAN2 =,E15.8,
3 3X,8HDTRAN2 =,E15.8,3X,8HETRAN2 =,E15.8)
180 PRINT 190
190 FORMAT (1H1,15X,13HORIGINAL DATA,19X,16HTRANSFORMED DATA/1H ,3X,
1 1H1,8X,2HX1,14X,2HX2,15X,2HX1,14X,2HX2)
IF (IRUN.GT.1) GO TO 701
200 I1=1
I2=NPAIR
IF (INPUT.LE.0) GO TO 500
300 READ (5,FMT) IEND,(X1(I),X2(I),I=1,12)
IF (IEND.NE.0) GO TO 400
I1=I1+NPAIR
I2=I2+NPAIR
GO TO 300

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400	NPTS=I1+IEND-1	BC003550
	GO TO 700	BC003560
500	READ FMT,IEND,(X1(I),X2(I),I=I1,I2)	BC003570
	IF (IEND.NE.0) GO TO 600	BC003580
	I1=I1+NPAIR	BC003590
	I2=I2+NPAIR	BC003600
	GO TO 500	BC003610
600	NPTS =I1+IEND-1	BC003620
700	CALL SEND (NPTS,X1,X1ORIG)	BC003630
	CALL SEND (NPTS,X2,X2ORIG)	BC003640
701	IF (ITRAN1.LE.1) GO TO 825	BC003650
	GO TO (825,705,715,725,735,745,755,765,775,785,795,805,815),ITRAN1	BC003660
705	DO 710 K=1,NPTS	BC003670
710	X1(K)=ALOG(X1(K))	BC003680
	GO TO 825	BC003690
715	DO 720 K=1,NPTS	BC003700
720	X1(K)=ALOG(ALOG(X1(K)))	BC003710
	GO TO 825	BC003720
725	DO 730 K=1,NPTS	BC003730
730	X1(K)=ALOG(ATRAN1+X1(K))	BC003740
	GO TO 825	BC003750
735	DO 740 K=1,NPTS	BC003760
740	X1(K)=ALOG(BTRAN1+ALOG(CTRAN1+X1(K)))	BC003770
	GO TO 825	BC003780
745	DO 750 K=1,NPTS	BC003790
750	X1(K)=SQRT(X1(K))	BC003800
	GO TO 825	BC003810
755	DO 760 K=1,NPTS	BC003820
760	X1(K)=1.0/X1(K)	BC003830
	GO TO 825	BC003840
765	DO 770 K=1,NPTS	BC003850
770	X1(K)=1.0/(DTRAN1+X1(K))	BC003860
	GO TO 825	BC003870
775	DO 780 K=1,NPTS	BC003880
780	X1(K)= ATAN(X1(K)/SQRT(1.0-X1(K)**2))	BC003890
	GO TO 825	BC003900
785	DO 790 K=1,NPTS	BC003910
790	X1(K)=2.0*ATAN(SQRT(X1(K))/SQRT(1.0-X1(K)))	BC003920
	GO TO 825	BC003930
795	DO 800 K=1,NPTS	BC003940
800	X1(K)=X1(K)/ETRAN1	BC003950
	GO TO 825	BC003960
805	DO 810 K=1,NPTS	BC003970
810	X1(K)=SIN(X1(K))	BC003980
	GO TO 825	BC003990
815	DO 820 K=1,NPTS	BC004000
820	X1(K)=COS(X1(K))	BC004010
825	IF (ITRAN2.LE.1) GO TO 950	BC004020
	GO TO (950,830,840,850,860,870,880,890,900,910,920,930,940),ITRAN2	BC004030
830	DO 835 K=1,NPTS	BC004040
835	X2(K)=ALOG(X2(K))	BC004050
	GO TO 950	BC004060
840	DO 845 K=1,NPTS	BC004070
845	X2(K)=ALOG(ALOG(X2(K)))	BC004080
	GO TO 950	BC004090
850	DO 855 K=1,NPTS	BC004100
855	X2(K)=ALOG(ATRAN2+X2(K))	BC004110
	GO TO 950	BC004120
860	DO 865 K=1,NPTS	BC004130

865 X2(K)=ALOG(BTRAN2+ALOG(CTAN2+X2(K)))	BC004140
GO TO 950	BC004150
870 DO 875 K=1,NPTS	BC004160
875 X2(K)=SQRT(X2(K))	BC004170
GO TO 950	BC004180
880 DO 885 K=1,NPTS	BC004190
885 X2(K)=1.0/X2(K)	BC004200
GO TO 950	BC004210
890 DO 895 K=1,NPTS	BC004220
895 X2(K)=1.0/(DTRAN2+X2(K))	BC004230
GO TO 950	BC004240
900 DO 905 K=1,NPTS	BC004250
905 X2(K)=ATAN(X2(K))/SQRT(1.0-X2(K)**2)	BC004260
GO TO 950	BC004270
910 DO 915 K=1,NPTS	BC004280
915 X2(K)=2.0*ATAN(SQRT(X2(K))/SQRT(1.0-X2(K)))	BC004290
GO TO 950	BC004300
920 DO 925 K=1,NPTS	BC004310
925 X2(K)=X2(K)/ETAN2	PC004320
GO TO 950	BC004330
930 DO 935 K=1,NPTS	BC004340
935 X2(K)=SIN(X2(K))	BC004350
GO TO 950	BC004360
940 DO 945 K=1,NPTS	BC004370
945 X2(K)=COS(X2(K))	BC004380
950 IPAGE=50	BC004390
DO 960 I=1,NPTS	BC004400
IF (IPAGE.NE.0) GO TO 954	BC004410
IPAGE=50	BC004420
PRINT 951	BC004430
951 FORMAT (1H1)	BC004440
954 PRINT 955,1,X1ORIG(I),X2ORIG(I),X1(I),X2(I)	BC004450
955 FORMAT (1H ,14,2X,E14.8,2X,E14.8,3X,E14.8,2X,E14.8)	BC004460
960 IPAGE=IPAGE-1	BC004470
X1MIN=X1(I)	BC004480
X2MIN=X2(I)	BC004490
SIGX12=X1(I)**2	BC004500
SIGX22=X2(I)**2	BC004510
R=X1(I)*X2(I)	BC004520
X1MEAN=X1(I)	BC004530
X2MEAN=X2(I)	BC004540
X1MAX=X1MIN	BC004550
X2MAX=X2MIN	BC004560
DO 1350 I=2,NPTS	BC004570
IF (X1(I).GT.X1MAX) GO TO 970	BC004580
IF (X1(I).LE.X1MIN) GO TO 990	BC004590
GO TO 1000	BC004600
970 X1MAX=X1(I)	BC004610
GO TO 1000	BC004620
990 X1MIN=X1(I)	BC004630
1000 IF (X2(I).GT.X2MAX) GO TO 1100	BC004640
IF (X2(I).LE.X2MIN) GO TO 1200	BC004650
GO TO 1300	BC004660
1100 X2MAX=X2(I)	BC004670
GO TO 1300	BC004680
1200 X2MIN=X2(I)	BC004690
1300 SIGX12=SIGX12+X1(I)**2	BC004700
SIGX22=SIGX22+X2(I)**2	BC004710
R=R+X1(I)*X2(I)	BC004720

	X1MEAN=X1MEAN+X1(I)	BC004730
	X2MEAN=X2MEAN+X2(I)	BC004740
1350	CONTINUE	BC004750
	PRINT 1250, X1MIN,X1MAX,X2MIN,X2MAX	BC004760
1250	FORMAT (1H1,8HMIN X1 =,E14.8,10X,8HMAX X1 =,E14.8/1H ,8HMIN X2 =,	BC004770
1	E14.8,10X,8HMAX X2 =,E14.8)	BC004780
	XNPTS=NPTS	BC004790
	SIGX12=SIGX12-X1MEAN**2/XNPTS	BC004800
	SIGX22=SIGX22-X2MEAN**2/XNPTS	BC004810
	R=(R*XNPTS-X1MEAN*X2MEAN)/(XNPTS*SQRT(SIGX12*SIGX22))	BC004820
	SIGX12= SIGX12/(XNPTS-1.)	BC004830
	SIGX22= SIGX22/(XNPTS-1.)	BC004840
	X1MEAN = X1MEAN/XNPTS	BC004850
	X2MEAN = X2MEAN/XNPTS	BC004860
	SIGX1 = SQRT(SIGX12)	BC004870
	SIGX2 = SQRT(SIGX22)	BC004880
	PRINT 1400,R,X1MEAN,X2MEAN,SIGX12,SIGX22,SIGX1,SIGX2	BC004890
1400	FORMAT (1H0,2HR=,E14.8,3X,8HMEAN X1=,E14.8,4X,8HMEAN X2=,E14.8,4X,	BC004900
1	12HVARIANCE X1=,E14.8,4X,12HVARIANCE X2=,E14.8,/ 1H ,	BC004910
2	12HSTAN DEV X1=,E14.8,5X,12HSTAN DEV X2=,E14.8)	BC004920
	A1=R*SIGX2/SIGX1	BC004930
	A2=X2MEAN-A1*X1MEAN	BC004940
	A3=R*SIGX1/SIGX2	BC004950
	A4=X1MEAN-A3*X2MEAN	BC004960
	PRINT 1450,A2,A1,A4,A3	BC004970
1450	FORMAT (1H0,3HX2=,E14.8,3H + ,E14.8,4H *X1/4H X1=,E14.8,3H + ,E14.	BC004980
	18,4H *X2)	BC004990
	PRINT 1475	BC005000
1475	FORMAT (1H1)	BC005010
	NT=0	BC005020
	DO 1600 KN=1,NN	BC005030
	NT=NT+1	BC005040
	IF (PDELT(KN).EQ.0.) GO TO 1550	BC005050
	NT1= (PE(KN)-PS(KN))/PDELT(KN)+0.0001	BC005060
	P(NT)= PS(KN)	BC005070
	NTP1 =NT+1	BC005080
	NT = NT+NT1	BC005090
	PROB = PS(KN)	BC005100
	DO 1500 KM=NTP1,NT	BC005110
	PROB = PROB+PDELT(KN)	BC005120
1500	P(KM)= PROB	BC005130
	GO TO 1600	BC005140
1550	P(NT)=PS(KN)	BC005150
1600	CONTINUE	BC005160
	NPROB =NT	BC005170
	DO 1700 LM=1,NT	BC005180
	IS = P(LM)*1000.+0.0005	BC005190
1700	C2(LM+1) = TABLEC(IS)	BC005200
	NT=NT+1	BC005210
	RCOMP=1./(1.-R**2)	BC005220
	DO 1800 I=1,NPTS	BC005230
	AA=((X1(I)-X1MEAN)/SIGX1)	BC005240
	AB=((X2(I)-X2MEAN)/SIGX2)	BC005250
1800	Q(I)=RCOMP*(AA**2-2.*R*AA*AB+AB**2)	BC005260
	C2(I)=C	BC005270
	IF (NPRINT.LE.0) GO TO 2650	BC005280
	ILINE=50	BC005290
	DO 2200 J=2,NT	BC005300
	JMI=J-1	BC005310

IF (ILINE.NE.0) GO TO 1875	BC005320
ILINE=50	BC005330
PRINT 1850	BC005340
1850 FORMAT (1H1)	BC005350
1875 PRINT 1900,JM1,C2(J),JM1,P(JM1)	BC005360
1900 FORMAT (1H ,4H C2(,13,4H) = ,E14.8,5X,2HP(,13,4H) = ,F8.3)	BC005370
ILINE=ILINE-1	BC005380
2000 DO 2200 I=1,NPTS	BC005390
IF (ILINE.NE.0) GO TO 2050	BC005400
ILINE=50	BC005410
PRINT 1850	BC005420
2050 IF ((C2(J-1).GT.Q(I)).OR. (C2(J).LT.Q(I))) GO TO 2200	BC005430
PRINT 2100,I,X1(I),X2(I),Q(I)	BC005440
2100 FORMAT (1H ,15,2H-(,E14.8,1H,,E14.8,2H)-,E14.8)	BC005450
ILINE=ILINE-1	BC005460
2200 CONTINUE	BC005470
IF (ILINE.NE.0) GO TO 2250	BC005480
ILINE=50	BC005490
PRINT 1850	BC005500
2250 PRINT 2300,NT,NT	BC005510
2300 FORMAT (1H0,4H C2(,13,12H) = INFINITY,11X,2HP(,13,12H) = 1.000)	BC005520
ILINE=ILINE-1	BC005530
DO 2500 I=1,NPTS	BC005540
IF (ILINE.NE.0) GO TO 2400	BC005550
ILINE=50	BC005560
PRINT 1850	BC005570
2400 IF (Q(I).LE.C2(NT)) GO TO 2500	BC005580
PRINT 2100,I,X1(I),X2(I),Q(I)	BC005590
ILINE=ILINE-1	BC005600
2500 CONTINUE	BC005610
2650 PRINT 2700,IRUN,NPTS,NT	BC005620
2700 FORMAT (1H1,39X,15HCHI-SQUARE TEST/1H0,18X,11H RUN NUMBER ,13,	BC005630
1 6H WITH ,15,18H OBSERVATIONS AND ,14,10H INTERVALS/1H0,	BC005640
2 8HINTERVAL,3X,11HUPPER BOUND,6X,11HPROBABILITY,3X,	BC005650
3 8 HOBBS FREQ,7X,9HTHEO FREQ,6X,13HCHI-SQU CONTR /1H	BC005660
4 8 HINDEX(J),6X,5HC2(J),13X,4HP(J)/)	BC005670
P(NT)=1.000	BC005680
CALL MOVE (0,IOBS(1),999)	BC005690
DO 2850 IP=1,NPTS	BC005700
DO 2800 IC=2,NT	BC005710
IF (C2(IC).GT.Q(IP).AND.C2(IC-1).LE.Q(IP))GO TO 2750	BC005720
GO TO 2800	BC005730
2750 IOBS(IC-1)=IOBS(IC-1)+1	BC005740
GO TO 2850	BC005750
2800 CONTINUE	BC005760
IOBS(NT) = IOBS(NT)+1	BC005770
2850 CONTINUE	BC005780
E(1)=XNPTS*P(1)	BC005790
DO 2900 IE=2,NT	BC005800
2900 E(IE) = XNPTS* (P(IE)-P(IE-1))	BC005810
2975 CSUM=0	BC005820
DO 3000 IJ=1,NT	BC005830
IC =IJ+1	BC005840
CSC = (IOBS(IJ)-E(IJ))*2/E(IJ)	BC005850
CSUM = CSUM +CSC	BC005860
3000 PRINT 3100, IJ,C2(IC),P(IJ),IOBS(IJ),E(IJ),CSC	BC005870
3100 FORMAT (1H ,2X,13,5X,E14.8,5X,F8.3,7X,14,10X,F8.4,9X,F8.4)	BC005880
K1=NT-6	BC005890
IF (K1.LE.0) GO TO 3710	BC005900

PRINT 3700,CSUM,K1	BC005910
3700 FORMAT (1H0,12HCHI-SQUARE =,F12.6,6H WITH ,13,19H DEGREES OF FREEDOM/1H2)	BC005920
IF(K1.GT.0) GO TO 3730	BC005930
3710 PRINT 3725	BC005940
3725 FORMAT (1H0,76HCHI-SQUARE COULD NOT BE COMPUTED BECAUSE OF INSUFFICIENT NUMBER OF INTERVALS)	BC005950
3730 IF (JPL0T.LE.0) GO TO 9000	BC005960
IF (JPL0T.EQ.2) GO TO 3826	BC005970
NP=0	BC005980
DO 3800 KN=1,NL	BC005990
NP=NP+1	BC006000
IF (PPDELT(KN).EQ.0.) GO TO 3775	BC006010
NP1 = (PPE(KN)-PPS(KN))/PPDELT(KN)+0.0001	BC006020
PP(NP)=PPS(KN)	BC006030
NPP1 = NP+1	BC006040
NP = NP+NP1	BC006050
PROB1 = PPS(KN)	BC006060
DO 3750 KM=NPP1,NP	BC006070
PROB1 = PROB1+PPDELT(KN)	BC006080
3750 PP(KM) = PROB1	BC006090
GO TO 3800	BC006100
3775 PP(NP)=PPS(KN)	BC006110
3800 CONTINUE	BC006120
NPROB1 = NP	BC006130
DO 3825 LP=1,NP	BC006140
ISP=PP(LP)*1000.+0.0005	BC006150
3825 C2P(LP)=TABLEC(ISP)	BC006160
GO TO 3827	BC006170
3826 IF (NTYPES.EQ.1) GO TO 3829	BC006180
PRINT 3828,J0B,1RUN,NPTS	BC006190
3828 FORMAT (2H\$2,10A8/2H\$,11HRUN NUMBER ,13,6H WITH ,14,13H OBSERVATIONS)	BC006200
GO TO 4100	BC006210
3827 IF (NTYPES.LE.0) GO TO 3840	BC006220
3829 PRINT 3830,X1D1,X1D2	BC006230
3830 FORMAT (2H\$2,10A8/2H\$,10A8)	BC006240
GO TO 4100	BC006250
3840 PRINT 3850,J0B,1RUN,NPTS,NP,(PPS(JJ),PPDELT(JJ),PPE(JJ),JJ=1,NL	BC006260
1)	BC006270
3850 FORMAT (2H\$2,10A8/2H\$,11HRUN NUMBER ,13,6H WITH ,14,18H OBSERVATIONS AND ,13,9H CONTOURS/2H\$,5(F5.3,1H(,F5.3,1H),F5.3))	BC006280
4100 IF (NOTIC.LE.0) GO TO 4200	BC006290
PNOTIC = NOTIC	BC006300
GO TO 4300	BC006310
4200 PNOTIC = 15.	BC006320
4300 X1RANG = X1MAX-X1MIN	BC006330
X2RANG = X2MAX-X2MIN	BC006340
R1=AMAX1(X1RANG,X2RANG)	BC006350
D= R1/PNOTIC	BC006360
IF (D.LT.1.) GO TO 4900	BC006370
DO 4500 IP=1,9	BC006380
IF (D.EQ.TABP(IP))GO TO 4600	BC006390
IF (TABP(IP).LT.D.AND.TABP(IP+1).GT.D) GO TO 4700	BC006400
4500 CONTINUE	BC006410
IF (D.GT.TABP(10)) D=TABP(10)	BC006420
GO TO 5300	BC006430
4600 ID=TABP(IP)+.005	BC006440
GO TO 4750	BC006450
	BC006460
	BC006470
	BC006480
	BC006490

4700 ID1=D/TABP(IP)+.9999	BC006500
D=TABP(IP)*FLOAT(ID1)	BC006510
GO TO 5300	BC006520
4750 D=ID	BC006530
GO TO 5300	BC006540
4900 IF (D.LT.TABN(1)) GO TO 5100	BC006550
DO 5000 INEG=1,9	BC006560
IF (D.EQ.TABN(INEG))GO TO 5100	BC006570
IF (TABN(INEG).LT.D.AND.TABN(INEG+1).GT.D)GO TO 5200	BC006580
5000 CONTINUE	BC006590
5100 D=TABN(1)	BC006600
GO TO 5300	BC006610
5200 ID2=D/TABN(INEG) +.9999	BC006620
D =TABN(INEG)*FLOAT(ID2;	BC006630
5300 DX=D	BC006640
DY=D	BC006650
XOR=X1MIN	BC006660
YOR=X2MIN	BC006670
3875 YP=C5D6*(X1MAX-X1MIN)+X2MIN	BC006680
IF (YP-X2MAX) 4000,5375,3900	BC006690
3900 X2MAX=YP	BC006700
GO TO 5375	BC006710
4000 X1MAX=C6D5*(X2MAX-X2MIN)+X1MIN	BC006720
5375 CALL GRF (X1MIN,X2MIN,X1MAX,X2MAX,XOR,YOR,DX,DY,IRA,IRO,0,NPTS,	BC006730
1 X1(1),X2(1),LA,LO,2H54,-1.,0)	BC006740
IF (JPLOT.EQ.2) GO TO 9000	BC006750
IF (JPLOT.LE.0) GO TO 9000	BC006760
QRT1=(X1MAX-X1MIN)/4.	BC006770
X11=X1MIN+QRT1	BC006780
X12=X1MAX-QRT1	BC006790
HANDY(5)=X1MEAN	BC006800
HANDY(6)=X2MEAN	BC006810
TEMP1=(X11-X1MEAN)*SIGX2/SIGX1	BC006820
TEMP2=(X12-X1MEAN)*SIGX2/SIGX1	BC006830
HANDY(2)=X2MEAN+R *TEMP1	BC006840
HANDY(4)=X2MEAN+TEMP1/R	BC006850
HANDY(8)=X2MEAN+R *TEMP2	BC006860
HANDY(10)=X2MEAN+TEMP2/R	BC006870
HANDY(1)=X11	BC006880
HANDY(3)=X11	BC006890
HANDY(7)=X12	BC006900
HANDY(9)=X12	BC006910
QRT2=(X2MAX-X2MIN)/4.	BC006920
X21=X2MIN+QRT2	BC006930
X22=X2MAX-QRT2	BC006940
HANDY(12)=X21	BC006950
HANDY(14)=X21	BC006960
HANDY(16)=X22	BC006970
HANDY(18)=X22	BC006980
TEMP3=(X21-X2MEAN)*SIGX1/SIGX2	BC006990
TEMP4=(X22-X2MEAN)*SIGX1/SIGX2	BC007000
HANDY(11)=X1MEAN+R *TEMP3	BC007010
HANDY(13)=X1MEAN+TEMP3/R	BC007020
HANDY(15)=X1MEAN+R *TEMP4	BC007030
HANDY(17)=X1MEAN+TEMP4/R	BC007040
CALL GRF (0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,1,9,HANDY(1),HANDY(2),2,2,	BC007050
1 2H00,-1,0)	BC007060
XOMR2=1.-R **2	BC007070
DO 5500 J=1,NP	BC007080

V=C2P(J)	BC007090
TEMP1= SIGX1*SQRT(V)	BC007100
XINTVL=1.999*TEMP1/499.	BC007110
XORD=X1MEAN-TEMP1-XINTVL	BC007120
DO 5400 L=1,500	BC007130
XORD=XORD+XINTVL	BC007140
XTMP=(XORD-X1MEAN), SIGX1	BC007150
DISC=SQRT(ABS((V-XTMP**2)*XOMR2))	BC007160
X1(2*L)=XORD	BC007170
X1(2*L-1)=XORD	BC007180
X2(2*L-1)=X2MEAN+SIGX2*(R *XTMP+DISC)	BC007190
5400 X2(2*L)=X2MEAN+SIGX2*(R *XTMP-DISC)	BC007200
CALL GRF (0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,1,1000,X1(1),X2(1),1,1,2H75,	BC007210
1 -1,0)	BC007220
5500 CONTINUE	BC007230
9000 CALL INTVL(Y)	BC007240
PRINT 11000,Y	BC007250
11000 FORMAT (27H RUNNING TIME IN SECONDS = ,E14.8)	BC007260
CALL SEND(NPTS,X1ORIG,X1)	BC007270
CALL SEND (NPTS,X2ORIG,X2)	BC007280
10000 CONTINUE	BC007290
GO TO 50	BC007300
STOP	BC007310
END	BC007320

VIII. REFERENCES

Bates, C. D. (1966), The Chi-Square Test of Goodness of Fit for a Bivariate Normal Distribution, NWL Technical Memorandum No. K-77/66, U. S. Naval Weapons Laboratory, Dahlgren, Virginia.

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A description is given of the BI-CHI (Bivariate Chi-Square Test) computer program. The program is for assessing the agreement of a continuous two-dimensional sample distribution with a parent bivariate normal distribution. The program computes the Chi-square statistic for testing the null hypothesis "The random sample is from a bivariate normal parent population." The minimum and maximum sample values of each random variable are determined, and the sample estimates of the five parameters (two means, two variances, and the correlation coefficient) of the bivariate normal distribution are computed. Also, the sample estimates of the two regression lines (x_2 on x_1 and x_1 on x_2) are computed. An optional feature allows additional output of a plot of the sample data and the contour ellipses. In addition to an example problem illustrating the input and the output format, a flow chart and program listing are included in the report. The program is coded in FORTRAN IV for the IBM 7030 (STRETCH) computer.			