AD 664 166

THE DISPERSION OF GEOMORPHIC DATA AROUND MEASURES OF CENTRAL TENDENCY AND ITS APPLICATION

Walter F. Wood, et al

Quartermaster Research and Development Center
Natick, Massachusetts

March 1957

UNCLASSIFIED

HEADQUARTERS QUARTERMASTER RESEARCH & DEVELOPMENT COMMAND
Quartermaster Research & Development Center, US Army
Natick, Massachusetts

Environmental Protection Research Division

THE DISPERSION OF GEOMORPHIC DATA AROUND MEASURES OF CENTRAL TENDENCY AND ITS APPLICATION

Walter F. Wood
Geographer

Joan B. Snell
Geographer

Environmental Analysis Branch

Research Study Report EA-8

7-83-01-001A-2b

March 1957

12

ABSTRACT

A voluminous amount of data were gathered from a random sample of over 200 USGS and AMS topographic maps of the United States. As a first step in the analysis of this information, means of several terrain factors were computed.

The mean relief of a 5/16 square mile area is 240 feet, and for a 160 square miles area is 1,420 feet. A successive doubling of areas from the smallest of the limits to the largest produces means of relief which fall into a regular progression. This permits the construction of a nomograph for determining the mean relief of areas of any size lying between 5/16 and 160 square miles.

Other information of interest computed from these data include an estimate of about 2,300 feet for the average elevation of the United States. A 20 foot contour is crossed on the average of every 290 feet of random traverse. On a similar traverse, 3 ridges averaging 150 feet above valleys would be encountered about every mile. Hilltops rising sufficiently above other land to be represented by a closed contour occur on the average of three every square mile.

The above mean values probably seem extreme, but modes and medians have values of a smaller magnitude. The strong skewness of geomorphic data indicated by the spacing between the means and medians calls for a kind of analysis different from that used with normal distributions.

THE DISPERSION OF GEOMORPHIC DATA AROUND MEASURES OF CENTRAL TENDENCY AND ITS APPLICATION

Statistical analysis is becoming increasingly important as a technique for solving geomorphic problems. Much work still remains to be done, however, to learn more about the distributions of geomorphic elements and to broaden application of such information. The present study was undertaken as another step towards increasing the knowledge about quantitative analysis of landforms. Because the United States provides a great diversity of landforms and adequate map coverage is available, a random sample of topographic maps of the United States was selected as a basis for study. Information from these sheets was analyzed to determine the statistical distribution of physiographic data; to learn the nature of relations among physiographic measure; and to discover ways to use this knowledge for predicting various geomorphic factors when actual data is lacking.

Topographic sheets at a scale of 1:62,500 or 1:50,000 were selected for analysis at intersections of each parallel with each odd numbered meridian. In cases where map coverage was lacking at a particular point, an adjacent sheet was chosen at random. The proximity of large water areas and recent alluvium in many cases limited the amount of land area from which data could be drawn so that it was necessary to discard some of the original sample. A total of 204 maps remained for analysis. These sheets represent every physiographic province of the United States and include a wide variety of terrain features. (Fig. 1 - Distribution of Samples)

Six elements of terrain were considered. Four of these are taken by area: the highest elevation within a sample unit, the lowest elevation within a sample unit, relief of the sample unit (difference between highest and lowest elevations), and number of hilltops (individual heights of land as represented by a closed contour). Two sets of data were taken by traverse: the number of 20 foot contour crossings and the number of valleys and divides encountered on the same traverse. The first of these is a measure of slope. The latter is a measure of dissection and, for lack of a better term, is collectively referred to as slope direction changes.

For the elements studied on an areal basis, the smallest sample unit was a circle with an area of 5/16 square mile and mid-point in the center of the topographic sheet. Successive doublings of the smallest unit produced sample circles with areas of 5/8, 1¼, 2½, 5, 10, 20, 40, 80, and 160 square miles. For the two factors related to traverse, four lines were drawn from the mid-point of the concentric circles in the cardinal directions, giving two diameters to the ten concentric circles; the combined lengths of the two diameters for each circle are: 1.2, 1.8, 2.5, 3.6, 5.1, 7.1, 10.1, 14.3, 20.2, and 28.5 miles.

As a first step in analyzing the data, means and standard deviations of each of the six terrain factors were computed. (Fig. 2 - Means and Standard Deviations of Terrain Elements)

MEANS AND STANDARD DEVIATIONS OF TERRAIN ELEMENTS

AREA	HIGHEST ELEVATION feet		LOWEST ELEVATION feet		RELIEF feet		HILL-TOPS number		TRAV-ERSE	CONTOUR COUNTS number		DIRECTION CHANGES number	
	M	SD	M	SD	M	SD	M	SD		M	SD	M	SD
5/16 sq mi	2387	2444	2147	2264	240	355	1	2	1.2 mi	25	32	3	3
5/8	2430	2454	2124	2236	305	441	2	3	1.8	36	45	5	4
1¼	2471	2476	2098	2211	372	520	4	6	2.5	50	61	7	5
2½	2527	2520	2068	2188	459	618	9	11	3.6	68	81	10	6
5	2634	2599	2056	2150	578	753	17	21	5.1	97	116	14	8
10	2724	2667	2011	2115	713	877	35	43	7.1	137	156	21	11
20	2834	2736	1972	2082	861	1039	67	78	10.1	191	214	29	1[illegible]
40	2943	2861	1921	2032	1022	1186	132	148	14.3	266	288	42	21
80	3084	2984	1869	1987	1214	1366	264	302	20.2	378	398	59	27
160	3221	2099	1800	1930	1421	1573	534	607	28.5	532	540	82	36

Fig. 2

When means are plotted against sample size, the straight-line arithmetic progression of number of hilltops, contour counts and direction changes is readily noted. Relief exhibits more complex relationships than this, in that the logs of mean and median relief are directly proportional to the log of the area of sample units.

Translating some of the figures from the above table into more easily understood terms, mean local relief on a 5/16 square mile area, averaged over the entire United States, is 240 feet. Average elevation for the country is estimated as 2300 feet. In every 290 feet of random traverse, one 20 foot contour is crossed; three ridges rising 150 feet above valleys are encountered every mile; about three hilltops occur in every square mile . Mean values of the various factors are fairly high and perhaps appear unrealistic. It is to be remembered, however, that samples from all sections of the country have been included in the averages and a few extreme cases will pull the means toward the higher values. The Grand Canyon sheet has this effect on the averages for relief and contour counts, while a sheet from the glacial drift area such as Barrington, Illinois, will similarly unbalance means for number of direction changes and hilltops.

To obtain more information about the distribution, data for the six terrain factors were arrayed in frequency distribution graphs for each of the ten areas. (Fig. 3 - Frequency Distributions) As the sample graphs indicate, the geomorphic data are strongly skewed, with great discrepancy among the positions of the measures of central tendency. (Fig. 4 - Central Tendencies of Terrain Elements)

CENTRAL TENDENCIES OF TERRAIN ELEMENTS

AREA OF SAMPLE IN SQ. MI.		5/16	5/8	1¼	2½	5	10	20	40	80	160
RELIEF (feet)	MEAN	239	305	372	459	578	713	861	1022	1214	1421
	MEDIAN	100	125	160	181	234	288	340	444	598	685
	MODE	17	18	53	77	84	166	232	249	255	289
HIGHEST ELEVATION (feet)	MEAN	2387	2430	2471	2527	2634	2724	2834	2943	3084	3221
	MEDIAN	1388	1410	1440	1455	1505	1535	1560	1640	1745	1940
	MODE	926	927	919	926	976	975	991	1042	1058	1075
LOWEST ELEVATION (feet)	MEAN	2147	2124	2098	2068	2056	2011	1973	1921	1869	1800
	MEDIAN	1260	1200	1133	1118	1085	1080	1050	1020	985	960
	MODE	808	712	690	684	660	658	325	321	92	69
HILLTOPS (number)	MEAN	1	2	4	9	17	35	67	132	264	534
	MEDIAN	0	1	2	4	10	21	46	91	177	376
	MODE	0	0	0	1	1	3	3	7	17	163
CONTOUR COUNTS (number)	MEAN	25	36	50	68	97	137	191	266	378	532
	MEDIAN	15	21	28	40	56	80	115	165	236	329
	MODE	3	4	5	8	14	18	45	70	92	124
DIRECTION CHANGES (number)	MEAN	3	5	7	10	14	21	29	42	58	82
	MEDIAN	3	4	6	9	14	20	28	40	57	80
	MODE	0	5	5	9	14	18	27	37	63	93

Fig. 4

For example, mean relief on 160 square mile areas, is 1421 feet. The median - that value which occurs at the central position in an array - is 685 feet, and the mode - the most common magnitude - is only 289 feet. In other words, in a large number of cases, relief on 160 square miles would be found to have values around 288 feet, half of the cases would have values of less than 685 feet and yet the average would be as high as 1421 feet. The other terrain factors follow this same pattern, with the exception of direction change, which does show a coincidence of mean, median and mode. Relief appears to exhibit this skewness to the greatest degree.

Not only do the geomorphic data not posses a normal symmetrical distribution, but they do not have the characteristics of a normal logarithmic distribution. Previous work, especially that of Strahler, indicates that samples drawn from localized homogeneous areas can be normalized by simple methods. However, several attempts to normalize this distribution by the use of logs, cubes, squares or square roots were unsuccessful. The samples used in this study were drawn from places having great variety in climate and lithology as well as in other factors which govern earth configuration. Hence, it was found very difficult to normalize the distribution pattern of these samples through known means of transformation.

Additional knowledge concerning the nature of geomorphic data can be gained from percentile graphs. Data for each of the six terrain factors were divided into ten equal parts, or deciles, and the class limit of each decile, or each tenth percentile, graphed. (Fig. 5 - Central Tendency and Percentiles of Relief and Fig. 6 - Percentile Distribution) The percentiles plotted by area of samples, as in the example of percentiles of relief (Fig. 5) show a fairly uniform straight line progression from the smallest sample to the largest. The greatest irregularities are shown in the tenth and hundredth percentile[3]. Percentiles graphed for any single unit area, as in the example of the percentile distribution graphs (Fig.6), exhibit a fairly uniform increase in value between the twentieth and ninetieth percentiles. There is, however, great irregularity from the tenth to the twentieth and from the ninetieth to the hundredth percentile. The median is necessarily the fiftieth percentile; the modes weave irregularly between the tenth and the fortieth percentile but in general lie between the thirtieth and fortieth. With the exception of data on direction changes, the means regularly fall between the sixtieth and seventieth percentiles.

With a clearer understanding of the distribution of geomorphic data, it is possible to make practical use of the findings in an attempt to predict various factors in areas where data is lacking. Nomographs can be constructed for determining mean values of the six terrain factors for areas of any size lying between 5/16 and 160 square miles. This is a relatively simple and direct matter. For predicting values in areas smaller than 5/16 square miles and larger than 160 square miles, the method is more complex. The greatest possibility for estimating here seems to be the method of rank correlation.

To experiment with the possibility of using rank correlation as a means of prediction, a test was made on relief values in a group of twenty stratified samples drawn from the 204 samples. The relief values of the twenty samples for each size of area were ranked and these rank values for each size of area correlated with those of every other size. To extend the test further, values of relief were added for larger units of area - 320, 640, 1280, 2560, 5120, and 10,240 square miles - and correlated. The following table of correlation coefficients resulted. (Fig. 7 - Rank Correlation of Micro-Macro Relief)

The accompanying graph results from taking the average values of rank correlation r_r from successive doublings of the unit areas. With successive doubling of areas, the mean values of r_r diminish at a fairly regular rate. This orderly progression suggests that reliable results could be expected if an attempt is made to estimate relief on areas too small to be measured except on large-scale aerial photos or by field checking, by using estimates of the regression coefficient between relief on a 5/16 square mile circle with that in smaller circles. This then permits the formulation of a theory of micro-macro relief relationships having possible wide application. This theory, after field check, would save a great deal of time in estimating the relief of small areas.

The foregoing study presents pertinent information concerning statistical distributions of geomorphic data. Data from diversified terrain, treated collectively, shows strong skewness and large differences among means, medians, and modes. Within the limits of the sample areas it is possible to make predictions of mean and percentile values of six geomorphic factors. Criteria now exist for describing a given plot as follows: a 20 square mile area from the Springfield, Ohio sheet is represented by the twentieth percentile of relief and number of hilltops, the twenty-fifth percentile of contour counts or average slope, and the thirtieth percentile of direction change or dissection. Beyond these limits, estimates can be made by rank correlation methods. The information to be found in the data collected for this study has not been exhausted; further analysis will disclose more knowledge of the relationships among physiographic measures.

<u>Prepared by</u>:

Walter F. Wood
Geographer
Joan B. Snell
Geographer
Environmental Analysis
Branch

APPROVED:

D. H. MILLER
Chief, Environmental Analysis Branch
Environmental Protection Research
Division

AUSTIN HENSCHEL
Chief
Environmental Protection Research
Division

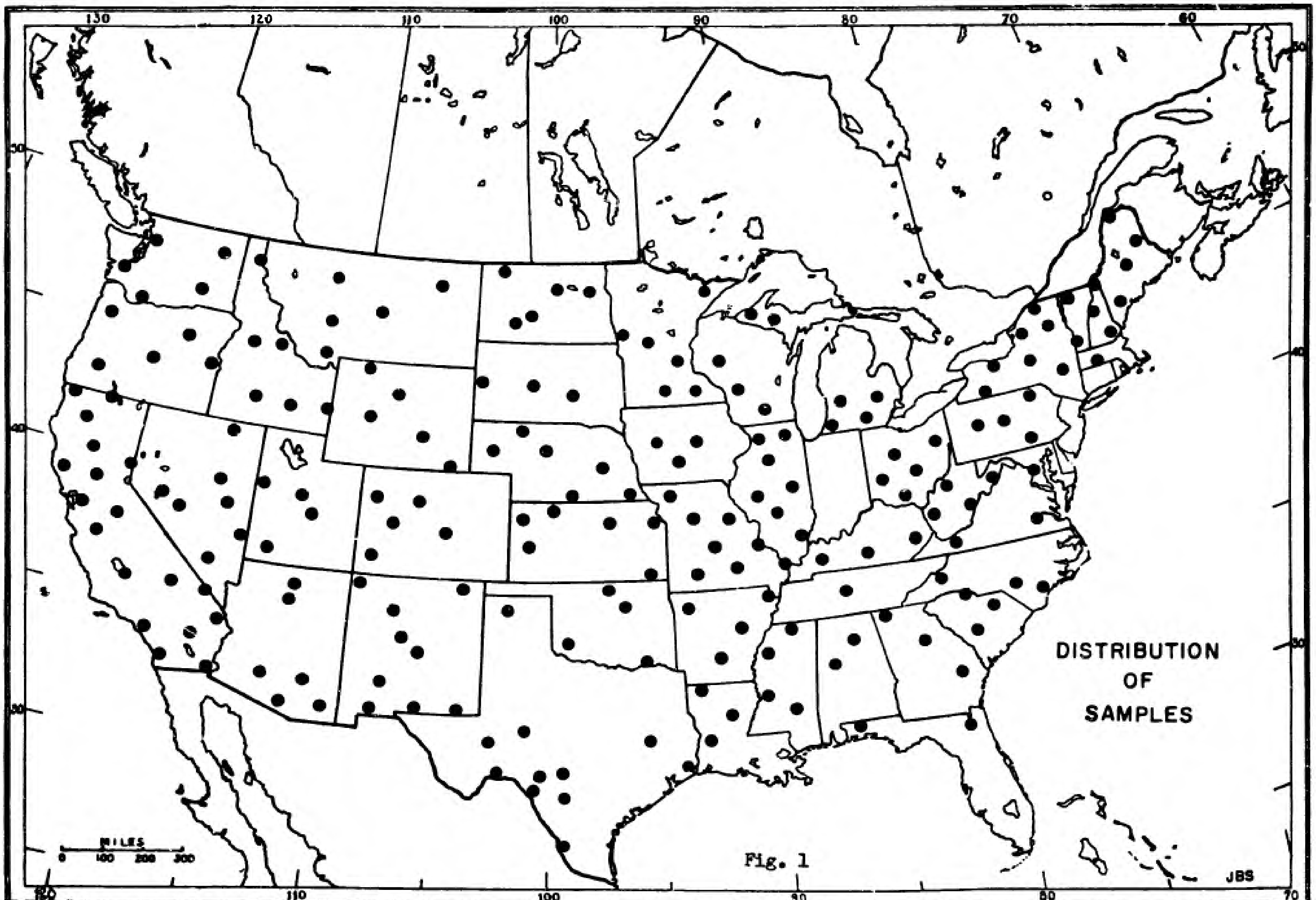

DISTRIBUTION
OF
SAMPLES
MILES
0 100 200 300
JBS
130
120
110
100
90
80
70
60
50
40
30

Fig. 1

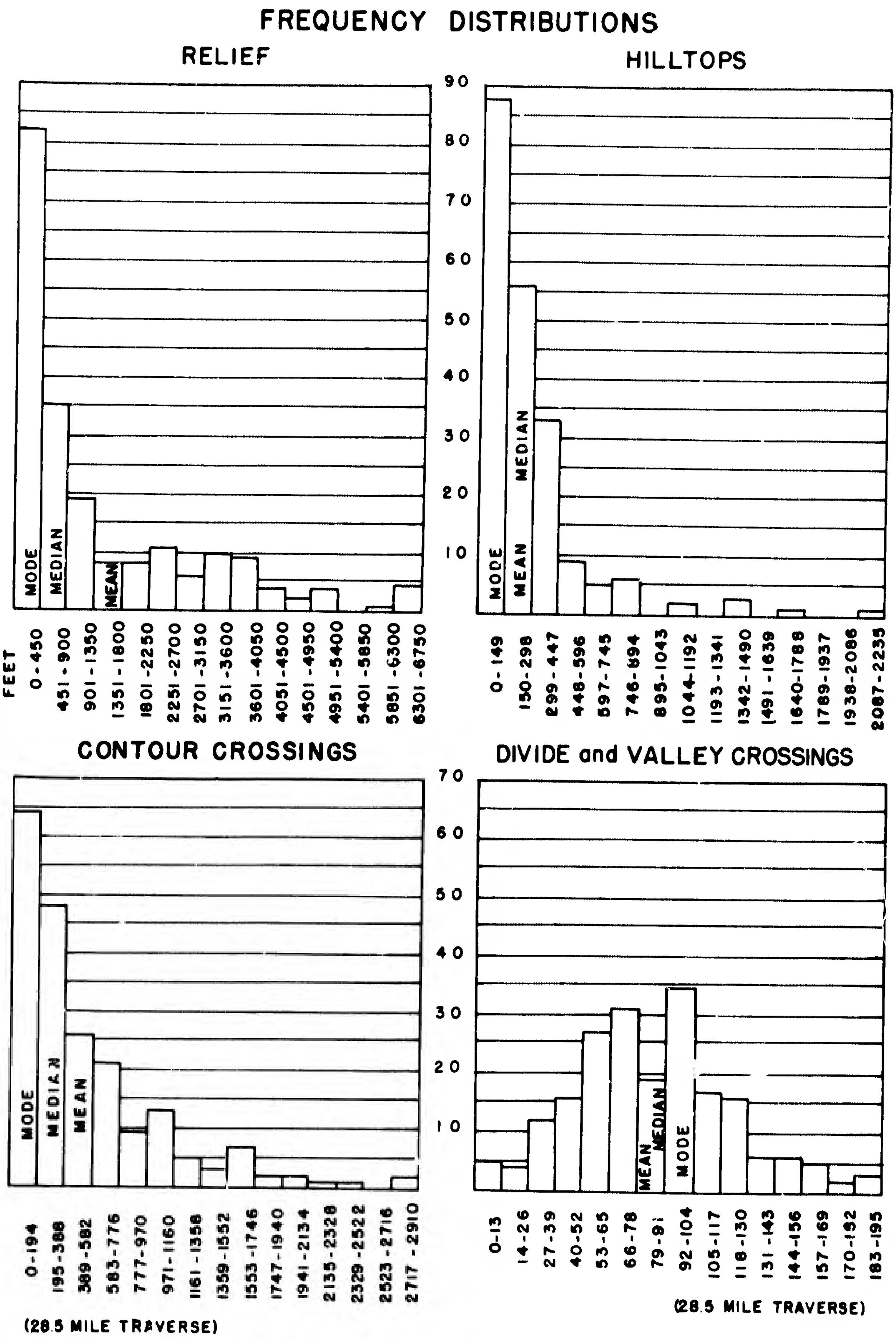
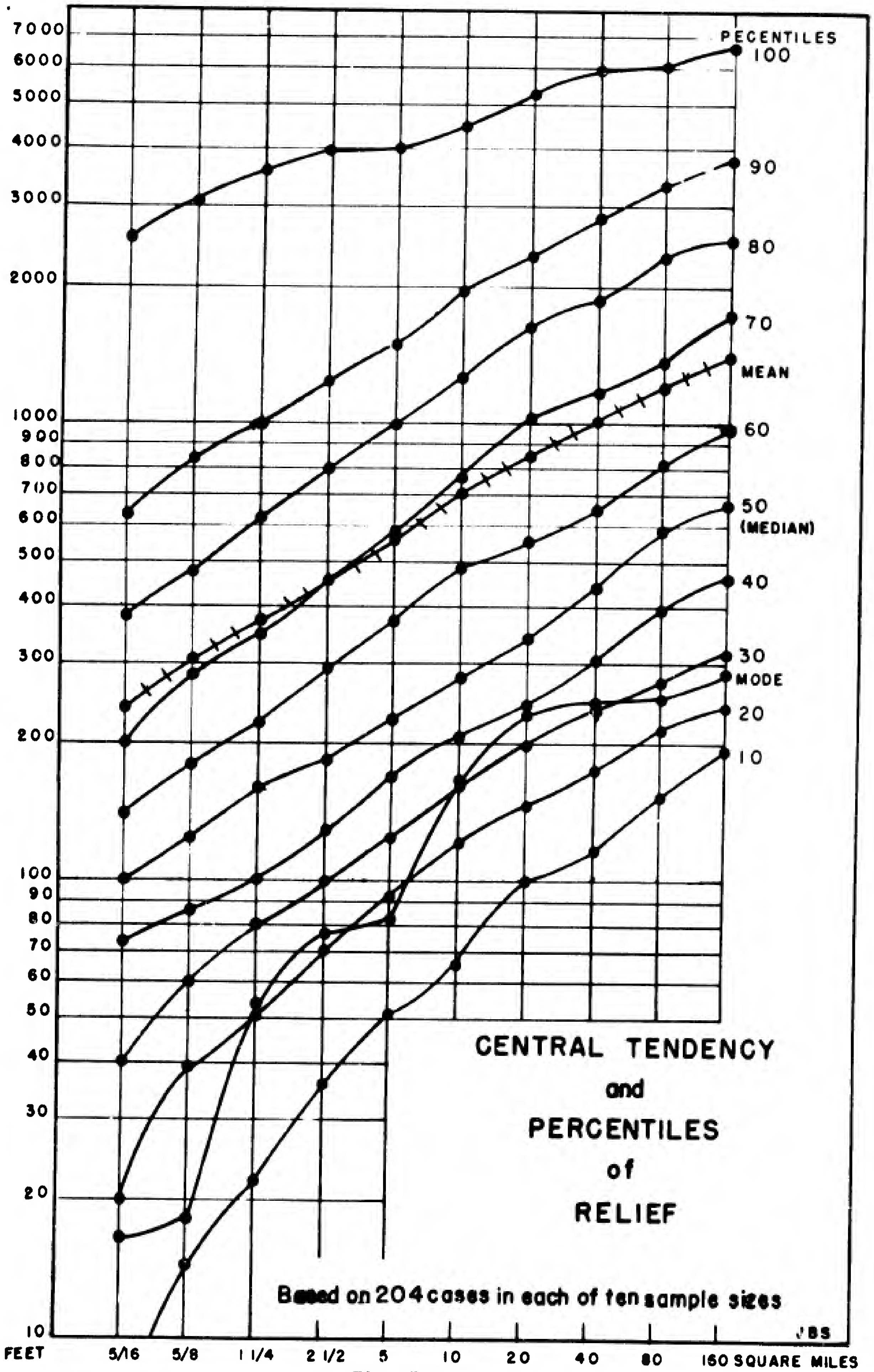


Fig. 3

Based on 204, 160 square mile samples

JBS



CENTRAL TENDENCY
and
PERCENTILES
of
RELIEF
Based on 204 cases in each of ten sample sizes
PECENTILES
100
90
80
70
MEAN
60
50
(MEDIAN)
40
30
MODE
20
10
7000
6000
5000
4000
3000
2000
1000
900
800
700
600
500
400
300
200
100
90
80
70
60
50
40
30
20
10
FEET
5/16
5/8
1 1/4
2 1/2
5
10
20
40
80
160 SQUARE MILES

Fig. 5

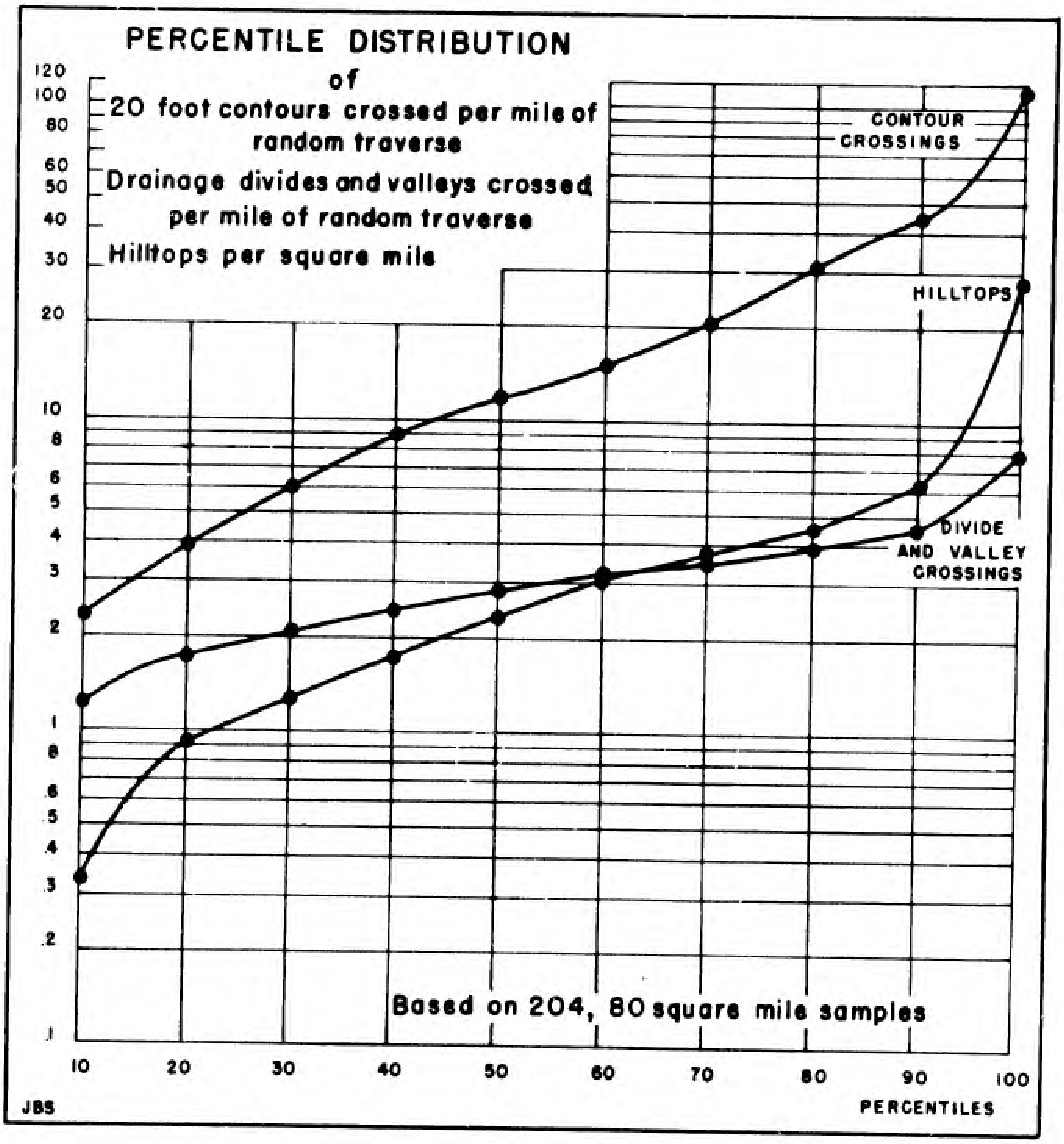

PERCENTILE DISTRIBUTION
of
20 foot contours crossed per mile of random traverse
Drainage divides and valleys crossed per mile of random traverse
Hilltops per square mile
CONTOUR CROSSINGS
HILLTOPS
DIVIDE AND VALLEY CROSSINGS
Based on 204, 80 square mile samples
120
100
80
60
50
40
30
20
10
8
6
5
4
3
2
1
.8
.6
.5
.4
.3
.2
.1
10
20
30
40
50
60
70
80
90
100
PERCENTILES
JBS

Fig. 6

RANK CORRELATION OF MICRO-MACRO RELIEF

	5/16	5/8	1¼	2½	5	10	20	40	80	160	320	640	1280	2560	5120	10240
5/16		.993	.985	.986	.940	.864	.822	.818	.762	.761	.661	.576	.536	.521	.507	.591
5/8			.986	.981	.924	.853	.812	.796	.750	.756	.657	.650	.550	.517	.507	.539
1¼				.978	.904	.813	.777	.759	.704	.706	.609	.535	.504	.479	.470	.533
2½					.949	.882	.842	.821	.782	.779	.681	.595	.565	.553	.546	.594
5						.967	.912	.914	.874	.868	.747	.642	.617	.638	.642	.682
10							.945	.929	.919	.902	.776	.699	.690	.716	.726	.760
20								.977	.955	.936	.855	.788	.778	.800	.798	.830
40									.973	.962	.898	.823	.791	.827	.820	.848
80										.983	.937	.883	.869	.899	.886	.895
160											.959	.899	.890	.908	.896	.910
320												.952	.943	.940	.923	.904
640													.976	.956	.932	.914
1280														.970	.944	.917
2560															.983	.959
5120																.971
10240																

CORRELATIONS OF r_r

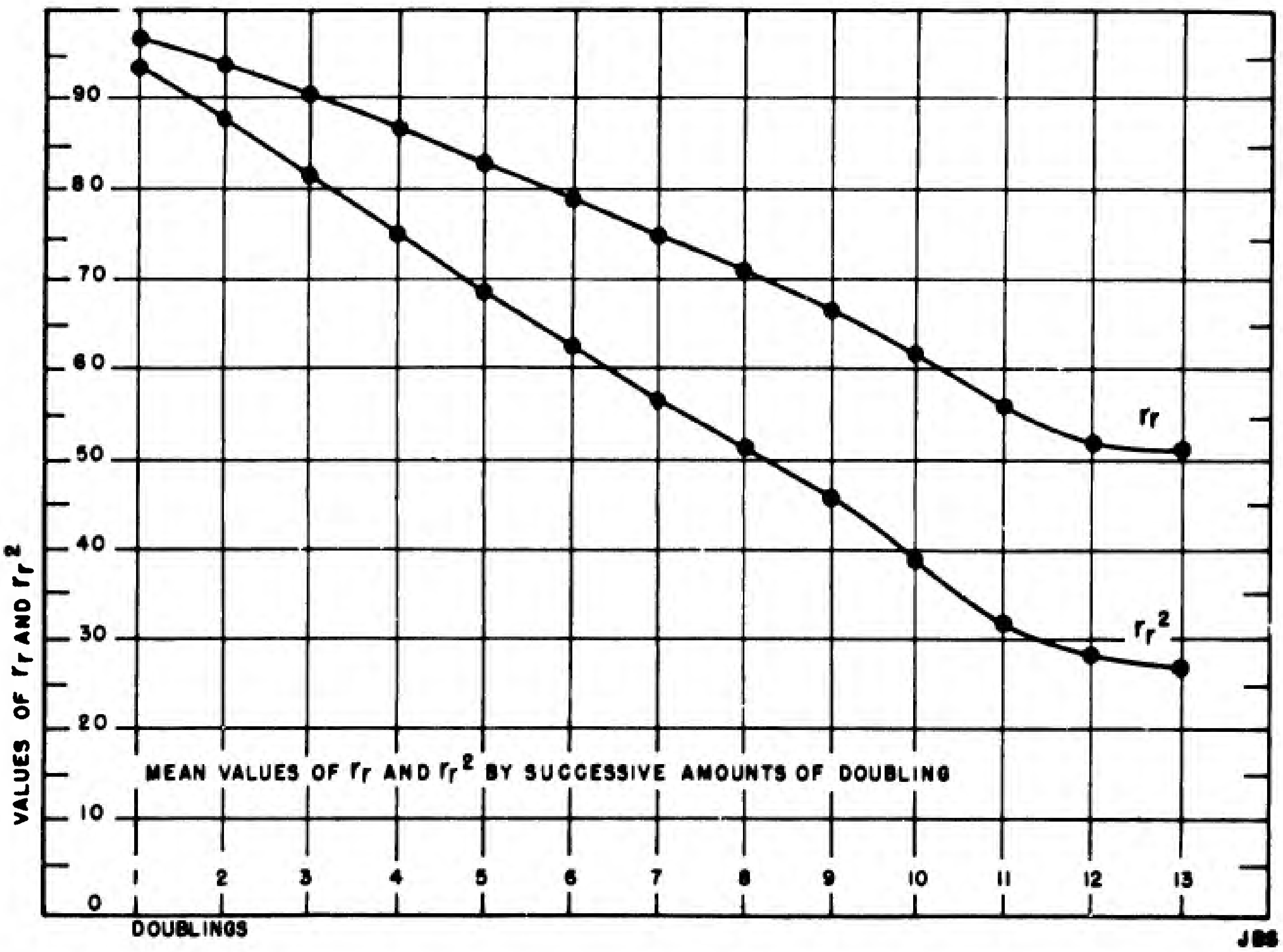


Fig. 7