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RM-5470-SA
JANUARY 1968

AN INTRODUCTION TO EQUIPMENT COST ESTIMATING

J. P. Large

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PREFACE

In February 1967 RAND was commissioned by the Office of the Assistant Secretary of Defense (Systems Analysis) to prepare a text on the general subject of cost estimating procedures. This memorandum dealing with fundamentals of cost analysis constitutes the introductory portion of such a text. The complete report will present and illustrate methods and techniques for estimating aircraft and missile costs, a chapter on operating costs, and a discussion of cost models in addition to the material presented here. While the emphasis is to be on aircraft and missiles, the techniques illustrated are applicable to all types of major equipment; and it is hoped that the text will be useful throughout the Department of Defense.

SUMMARY

This memorandum discusses the fundamental problems of estimating major equipment costs and suggests that for many purposes, particularly for government cost analysts, a statistical approach is the most suitable. The kind of data required and the adjustments needed to make the data useful are discussed in some detail. The use of regression analysis in deriving cost-estimating relationships is described, but it is emphasized that unquestioning use of estimating relationships obtained in this manner can result in serious errors. The concepts underlying the cost-quantity relationship generally known as the learning curve are presented along with instructions for its use. Finally, the problem of uncertainty in cost estimating is discussed and a few suggestions for dealing with the problem are included.

ACKNOWLEDGEMENTS

The impetus for this project came from the Directorate for Cost Estimates in the Office of the Assistant Secretary of Defense (Systems Analysis), and much of the form and content of the present volume are due to the detailed suggestions of Donald B. Rice, Geri Ward and Keith Marvin of that directorate. Although a single person is listed as author, the volume is in a good part a compilation and adaptation of the ideas of other persons in the RAND Cost Analysis Department, principally, H. G. Campbell, G. H. Fisher, G. S. Levenson, M. A. Margolis, and C. Teng. The work could not have been accomplished, however, had not the entire department shared its ideas freely.

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I. COST ESTIMATING METHODS

A cost estimate is a judgment or opinion regarding the cost of an object, commodity, or service. This judgment or opinion may be arrived at formally or informally by a variety of methods, all of which are based on the assumption that experience is a reliable guide to the future. In some cases the guidance is clear and unequivocal, e.g.: bananas cost $.15/lb last week; one estimates they will cost about $.15/lb next week, barring unforeseen circumstances such as a freeze in Guatemala. At a slightly more sophisticated level average costs are calculated and used as factors to estimate the cost to excavate a cubic yard of earth, to fly an airplane for an hour, to drive an automobile a mile, etc. Much, perhaps most, estimating is of this general type, that is, where the relationship between past experience and future application is fairly direct and obvious.

The more interesting problems, however, are those where this relationship is unclear because the proposed item is different in some significant way from its predecessors. The challenge to cost analysts concerned with military hardware is to project from the known to the unknown, to use experience on existing equipment to predict the cost of next-generation missiles, aircraft and space vehicles. The challenge is not only in new equipment designs, since new materials, new production processes, and new contracting procedures also add to the uncertainty. Such innovations are frequently accompanied by an anticipation of cost-reduction, and these expectations have to be carefully evaluated.

The techniques used for estimating hardware costs range from intuition at one extreme to a detailed application of labor and material cost standards at the other. The Air Force Cost Estimating Manual (AFSC Manual 173-1) lists five basic estimating methods--industrial engineering standards; rates, factors and catalog prices; estimating relationships; specific analogies; and expert opinion. Other sources put the number at two (synthesis and analysis), three (round-table estimating, estimating by comparison, and detailed estimating) or four (analytical appraisal, comparative analysis, statistical, and standards). In this chapter we shall not attempt to be comprehensive but will limit our discussion to three techniques--the industrial engineering approach, analogy, and the statistical approach--and it is the latter that we will be primarily concerned with throughout the remainder of the book.

Estimating by industrial engineering procedures can be broadly defined as an examination of separate segments of work at a low level of detail and a synthesis of the many detailed estimates into a total. In the statistical approach, estimating relationships using explanatory variables such as weight, speed, power, frequency and thrust are relied upon to predict cost at a higher level of aggregation.* Figure I-1 illustrates this difference in level of detail. At the lowest level of detail the estimator begins with a set of drawings and specifies each engineering or production operation that will be required, the work stations where each operation will be performed, and the labor and material required. This is sometimes referred to as "grass-roots" or "bottom-up" estimating.

Figure I-2 illustrates the detail required at the lowest level of estimating, in this case for forming a center bracket of steel plate. The name and number of the operations and the machines that will be used are given along with estimates of the setup time and operating labor cost. Standard setup and operating costs are used in making the estimates wherever these exist, but if standards have not been established, as is frequently the case in the aerospace industry, a detailed study is made to determine the most efficient method of performing each operation. A standard may be a "pure" standard or an "attainable" standard, but essentially for some specified condition it is the minimum time required to complete a given operation and, theoretically, should be approached asymptotically when the planned production rate is attained.

*Statistical estimating is sometimes defined as a statistical extrapolation to produce an estimate-at-completion after some progress has been made on a job and costs or commitments experienced. This is not the sense in which the term is used here.

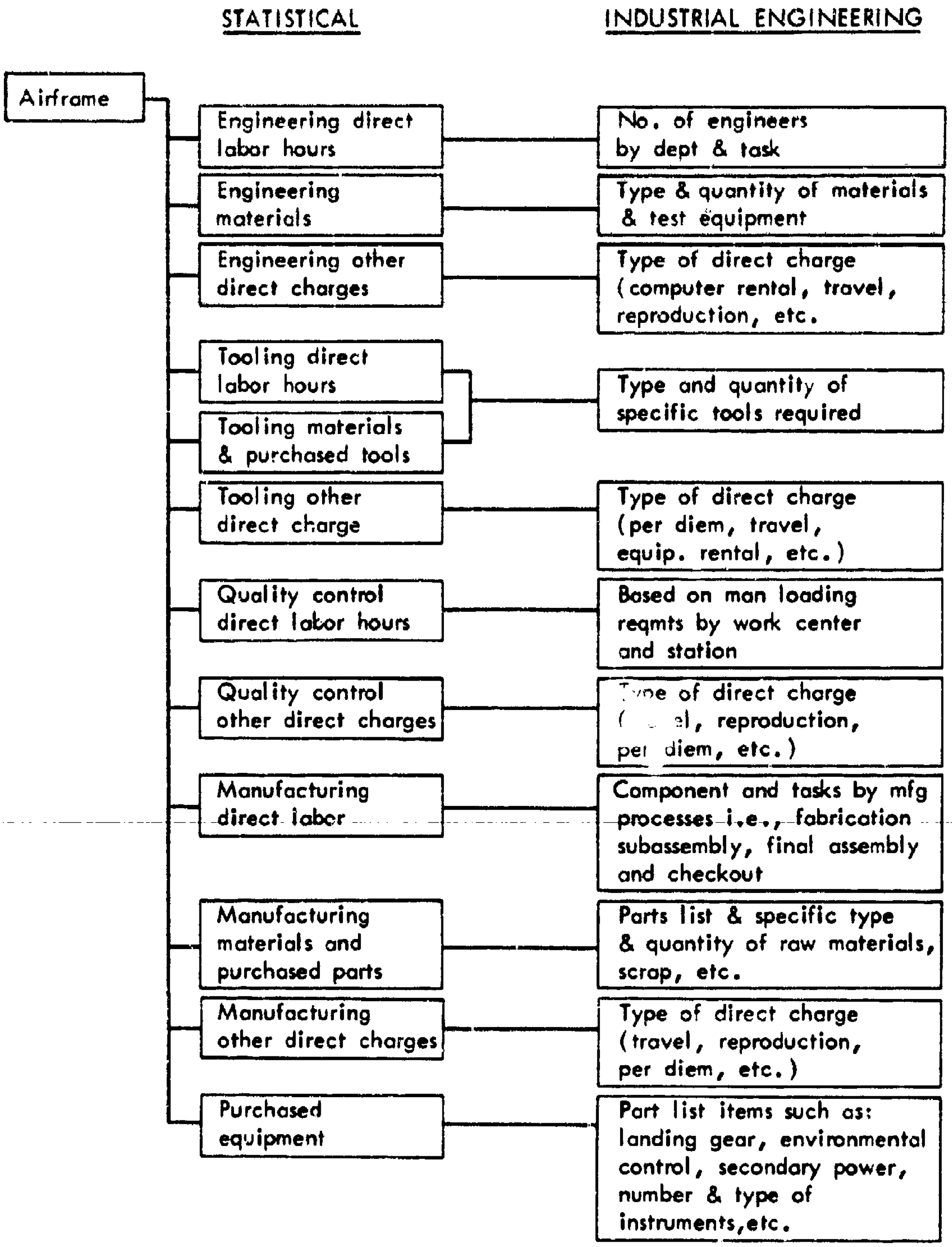

Fig. I-1—Levels of aggregation for estimating purposes

DATE Jan 1, 19-- NO. 12345

CUSTOMER Mechanical Products Co. PART NO. 6789 DRWG NO. 6613

DESCRIPTION Center Bracket MATERIAL 1/8" x 5" C.R. Steel Strip 20" long

Dept	Operation No.	Operation and Machine		Setup Cost			Operating Labor			Overhead		
				Setup hrs	Rate	Cost	Output per hr	Rate	Cost per 1000	Rate %	On setup labor	On operating labor
20	16241	Setup	Niagara 462	1/2	$3.40	$1.70						
20	16081	Shear to length	Niagara 462				3000	$3.40	$ 1.13			
20	16242	Setup	#4 Bliss press	2/3	3.40	2.27				175		
20	11571	Perforate and blank	#4 Bliss press				1100	3.30	3.00			
20	16243	Setup	#4 Bliss press	1	3.40	3.40						
20	12951	Form	#4 Bliss press				450	3.30	7.26		$12.90	$ 9.98
18	14351	Tap	Tap wheel				900	3.15	3.47			
18	16244	Setup	Plain mill	1/2	3.40	1.70				150		
18	14661	Mill slots	Plain mill				100	3.40	34.00			
18	15541	File burrs	Hand file				95	3.40	35.70		2.55	54.88
07	16245	Setup	Speed lathe	1/4	3.40	.85						
07	11542	Burr o slots (and mill)	Speed lathe				600	3.15	5.26			
07	16246	Setup	Multpl drill	1/2	3.40	1.70	900	3.15	3.47	125		
07	11941	Ct sink 2 holes	Multpl drill									
07	16247	Setup	Tapping machine	1/2	3.40	1.70						
07	16561	Tap 2 holes	Tapping machine				400	2.75	6.48		5.31	9.76
19	15151	Dull nickel plate					1000	3.50	3.50	100		1.75
	Quantity required 500			Totals		13.32			56.84		19.76	76.37

[a] Adapted from a figure in Production Handbook, edited by L. P. Alford and John R. Bangs, The Ronald Press Co., New York, 1953.

Fig. I-2—Detailed labor cost estimate

Standards are not widely used in the aerospace industry for estimating costs.* They are best applied where a long, stable production run of identical items is envisaged, whereas the emphasis in this industry is on development rather than production. The Gemini program provides an extreme example of this--12 spacecraft of varying configurations were developed and produced at a cost of about $700 million. Other examples would be less dramatic, but it is generally true that compared to other industries production runs of advanced military and space hardware tend to be short and that both design configurations and production processes may continue to evolve even after several hundred units have been completed. This means that standards are continually changing--one standard applies at unit 50, another at other production quantities. Because the changes are unpredictable, it is difficult to establish standards in advance of production experience that will be applicable at some specified production quantity.

Industrial engineering estimating procedures require considerably more personnel and data than are likely to be available to government agencies under any foreseeable conditions. One of the largest aerospace firms figures that to estimate the cost of an airframe using this approach about 4500 estimates are required, and for this reason it avoids making industrial engineering estimates whenever possible. They take too much time and are costly during a period of limited funds for both contractor and government. Moreover, for many purposes they have been found to be less accurate than estimates made statistically. One reason for this is simply that the whole generally turns out to be greater than the sum of 4500 parts. The detail estimator works under the same disadvantages as do all other estimators before an item has been produced. Working from sketches, blueprints, or word descriptions of some item that has not been completely designed, he can assign costs only to work that he knows about. (An attempt is sometimes made to estimate how complete the work statement is and this estimate becomes a factor to apply to the detail estimate, e.g., the work statement is

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*They are used extensively for other purposes, however, such as control of shop performance.

estimated to be 50 percent complete, so the detail estimate is multiplied by two.) The effect of a low estimate here is compounded because detail estimating is normally attempted only on a portion of production labor hours. A number of production labor elements, such as rework, planning time, coordination effort, etc., are usually factored in as percentages of the detail estimate. Then, other cost elements, such as sustaining effort, tool maintenance, quality control and manufacturing research, are factored in as percentages of production labor. Thus, small errors in the detail estimate can result in large errors in the total.

A second reason has already been suggested. This is the view that significant variability in the fabrication and assembly of successive production units is and will continue to be characteristic of the industry. Production runs of like models tend to be of limited length and to be characterized by numerous design changes. In the case of military aircraft, production rates have tended to vary frequently and at times unexpectedly. The proportion of new components in equipment is probably higher in the airframe industry than in any other. The effect of these factors can be represented statistically by the learning or progress curve so characteristic of this industry.* One set of fabrication and assembly modes is succeeded by more efficient production functions, thus lowering the total labor requirement. The introduction of engineering changes causes discontinuities in this process but does not interfere with the general trend. If new manufacturing processes and techniques are introduced, these may cause changes in past relationships. History, however, seems to show that changes in manufacturing and managements techniques, while they may have dramatic impacts in circumscribed areas, tend to result in only gradual changes over the entire process.

Because a private concern generally has data only on its own products, much of the estimating in industry is based on analogy, particularly when a firm is venturing into a new area. In the 1950s, for example, aircraft companies bidding on ballistic missile programs drew analogies between aircraft and missiles to develop estimates for

*Discussed in Chapter VI.

the latter. Douglas Aircraft Company (Now McDonnell-Douglas) made a good estimate on the Thor intermediate range ballistic missile by comparing Thor with the DC-4 transport airplane. The same company later, and less successfully, based its estimates of the Saturn S-IV stage on its Thor experience, adjusting for differences in size, the number of engines, higher performance, and insulation problems (the need to cope with liquid hydrogen as well as liquid oxygen).

At all levels of aggregation much estimating is of this type--System A required 100,000 hours; given the likenesses and differences in design and performance of proposed System B the requirement for B is estimated to be, say, 120,000 hours. Or, at a different level, engineers and shop foremen may rely on analogies when making a grass-roots estimate, and in this event analogy becomes part of the industrial engineering approach. The major drawback to estimating by analogy is that it is essentially an intuitive process, and as a consequence requires considerable experience and judgment to be done successfully.

Thus, while statistical procedures are preferable in most situations, there are circumstances where analogy or industrial engineering techniques are required because the data do not provide a systematic historical basis for estimating cost behavior. It may be that a new item is to be constructed of some unfamiliar material, or that some design consideration is so radically different that statistical procedures are inadequate. The employment of new structural material for aircraft often requires the development of special cutting and forming techniques with significantly different manufacturing labor requirements than those projected from a sample of essentially aluminum airframes. Faced with this problem on titanium, airframe companies developed standard-hour values for titanium fabrication on the basis of shop experience fabricating test parts and sections. Ratios of these values to those for comparable operations on aluminum aircraft were prepared and these ratios used in existing statistical estimating relationships. Thus, while industrial engineering procedures are used to provide input data, the approach remained statistical.

Another exception occurs in the case of industrial facilities. Requirements for these cannot be estimated without knowing the contractor's identity and the extent and availability of his existing plant. Consequently, facilities cost must be estimated from information available for each specific case.

There will always be exceptions of this kind, but in general the statistical approach is useful in a wide range of contexts, e.g., whether the purpose is long-range planning or contract negotiation. In the former a more highly aggregated procedure may be used, because it ensures comparability when little detailed knowledge about the equipment is available. Total hardware cost may be estimated as a function of one or more explanatory variables, e.g., engine cost as a function of thrust or transmitter cost as a function of power output and frequency, but this is often a matter of necessity, not choice. Even for long-range planning, it is sometimes desirable to estimate in some detail.

To say that statistical techniques can be used in a variety of situations does not imply that the techniques are the same for all situations. They will vary according to the purpose of the study and the information available. In a conceptual study it is necessary to have a procedure for estimating the total expected costs of a program, and this must include an allowance for the contingencies and unforeseen changes that seem to be an inherent part of most development and production programs.

Similarly, a long-range planning study would use industry-wide labor and burden rates and an estimated learning curve slope, while later in the acquisition cycle data that is specific for a particular contractor in a particular location can be used. In effect this merely states the obvious--that as more is known, fewer assumptions are required. When enough is known, and this means when a product is well into production, accounting type information and data can be taken directly from records of account and used with a minimum of statistical manipulation. This technique is useful only in those cases where the future product or activity under consideration is essentially the same (both in terms of configuration and scale of production or operation) as that for the past or current period.

In any situation the estimating procedure to be used should be determined by (1) the data available, (2) the purpose of the estimate, and (3) to a lesser extent by less relevant factors such as the time available to make an estimate. The essential idea we wish to convey in this chapter is that, when properly applied, statistical procedures are varied and flexible enough to be useful in most situations defense equipment cost analysts are likely to face. While no specified set of procedures can guarantee accuracy, decisions must be made and it is essential that they be made on the best possible information. What we are seeking here are the approaches which will give the best possible answers, given the basic information that is available.

II. DATA COLLECTION AND ADJUSTMENT

The government has been collecting cost and program data on weapon and support systems for many years, sometimes in detail, sometimes in highly aggregated form, but always in quantity. As a consequence, it is a little bit surprising that when an estimating job comes along, the right data seldom seem to be at hand. One can speculate about why this should be, but in our opinion the essential reason is that the needs of cost analysis have not always been considered in designing the many information systems that have been used over the years by the Army, Navy and Air Force. Data have been collected primarily for program control, for program management and for program audit, but this type of information was never systematically processed and stored. Instead, after a couple of years it has generally been discarded or stored in not readily accessible warehouses. Moreover, the data were inconsistent since they were gathered according to the requirements of each Service and each program manager. As a consequence, to obtain the kind of data necessary to develop estimating techniques, the analyst has had to go back to the contractor's records.

With the institution of CIR (Cost Information Report) in 1966, the situation should greatly change. This report was designed to collect costs and related data on aircraft, missile and space systems and their related components for the purpose of assisting both industry and government in estimating and analyzing the costs of these items. Information from other sources--contract records, GFE records, and the like--can be processed and spliced to CIR as it becomes available. Hence, over a period of years, as data are accumulated, the need for *ad hoc* collection efforts should diminish. These efforts will never disappear completely, however. Since it will never be possible to rely on CIR alone (or on any foreseeable information system) because it will not apply to all new hardware and will not provide all the cost information that might ever be required on the hardware it does cover, the subject of data collection is still one with which cost analysts must be concerned.

In the best of all possible worlds the analyst would have such a wealth of data that he could develop estimating techniques responsive to any demand. Such a world is unknown in the aerospace industry where even the largest contractors are reluctant to allocate the resources required to put estimators in such a favorable position. A government estimator is better placed in some regards, i.e., he has a much broader base of experience to draw upon, but he lacks the detail an industry estimator has on his own company's products. Data collection is expensive; hence, the estimator is generally in the position of having less than he wants and of having to design techniques to fit the data he has been able to accumulate.

Some minimum data requirement exists for any given job, however, and before data collection begins the analyst must consider the scope of his problem, define generally what he wants to do, and decide how he is going to do it. The data required to estimate equipment costs for a long-range planning study can be substantially less than those needed to prepare an independent cost estimate for contract negotiation. In the former, total equipment costs may suffice while in the latter costs must be collected at the level of detail in which the contract is to be negotiated. For major items this means a functional breakout, e.g., direct labor, materials, engineering, tooling, etc. One could postulate problems requiring even a greater amount of detail; suppose, for example, that two similar hardware items had substantially different costs. Only by examining the cost detail could this be explained.

In performing this initial appraisal of the job the analyst will be greatly aided by a thorough knowledge of the kind of equipment with which he will be dealing--its characteristics, the state of its technology, and the available sample. With this knowledge he can determine what types of data are required and available for what he wants to do, where the data are located, and what types of adjustments may be required to make the collected data base consistent and comparable. Only after the problem has been given this general consideration should one begin the task of data collection.

This is an important point. All too often a mountain of data is collected with little thought as to how it is going to be used. The result is that some portion may be unnecessary, unusable, or not completely understood. Data collection is generally the most troublesome and time-consuming part of any cost analysis. Consequently, careful planning in this phase of the overall effort is well worthwhile.

To develop a cost-estimating procedure, at least three different types of historical data are required. First, there are the resource data, usually in the form of expenditures or labor hours. It is customary to apply the word cost to both, and that practice is followed throughout this chapter. A second type of data describes the possible cost-explanatory elements; for hardware such as aircraft and missiles this means performance and physical characteristics. The third type is program data, i.e., information related to the development and production history of the hardware item.

Resource Data

Resource data are generally classified into end-item categories or functional categories. An example of the former in some of the various possible levels of detail would be:

System
Subsystem
Component
Part

The functional categories are engineering, tooling, manufacturing, quality control, purchased equipment, etc., and typically these are further broken down into labor, material, overhead, and other direct charges. The fountainhead of resource data is the contractor's plant. While the accounting systems will vary from one company to another, in general the amount of detail is immense. A typical airframe company, for example, sets up the production process on the basis of a number of different jobs or stations, each identified by a number or symbol. All manufacturing direct labor and/or material (depending on the type of cost accounting system) expended on a given job is recorded on a job order or, as is becoming increasingly more common, fed directly into a computer. Where such a system is used, the actual hours incurred for every operation are available to management; and these costs can be aggregated as needed. They cannot generally be attributed to a single unit, however, and some elements of cost, e.g., tooling and engineering, are not even identifiable by lot. And since different contractors do the work differently, they will have different job orders. This means in practice that data at more detailed levels may not be comparable from one contractor to another. Also, detailed information of this kind is unnecessary for most government estimating and, as a consequence, is rarely sought.

Parenthetically, it can be said that if there were a need to estimate in more detail the data required would increase by an order of magnitude or more, and data processing equipment would become a virtual necessity. The question of when to incorporate automatic data processing techniques into the data collection effort hinges primarily on the volume of data to be handled. The trend in the aerospace industry is to rely more and more on computers for internal data needs, and for some purposes data have been provided to the government on punch cards or magnetic tape. Thus, there are no technical reasons why cost data could not be obtained in this form should it be more convenient to the cost analyst, but as mentioned earlier, there are good reasons not to use excessive detail even if it is readily available--expense increases and accuracy is likely to decrease.

Theoretical considerations apart, the hard truth is that estimating techniques must be based on the resource data the analyst can lay his hands on, and in the past the availability of data has varied greatly from one type of equipment to another. As an illustration of this, aircraft estimating procedures tend to be different from those developed for missiles and spacecraft. An airframe model may contain the following cost elements:

Initial and sustaining engineering
Development support
Flight test operations
Initial and sustaining tooling
Manufacturing labor
Manufacturing material
Quality control

A list of cost elements something like this is desirable for all hardware estimating, but because of data limitations, present procedures for engines often include only two cost categories--development and production--and avionics procedures only one--procurement cost to the government. CIR should expand the possibilities in the future.

Performance and Physical Characteristics

Information about the physical and performance characteristics of aircraft, missile and space systems is just as important as resource data. This means that data collection in this area can be time-consuming, particularly since it is seldom clear in advance what the necessary data will be. The goal, of course, is to obtain a list of those characteristics which best explain differences in cost. Weight is the most commonly used explanatory variable, but weight alone is seldom enough. For airframes, speed is almost always included as a second explanatory variable, and one estimating procedure for aircraft uses all of the following:*

Maximum speed at optimal altitude
Maximum speed at sea level
Year of first delivery
Total airframe weight
Increase in airframe weight from unit 1 to unit n
Weight of installed equipment
Engine weight
Electronics complexity factor

In addition, the following characteristics were considered, but not used:

Maximum rate of climb
Maximum wing loading
Empty weight
Maximum altitude
Design load factor
Maximum range
Maximum payload

*Methods of Estimating Fixed-Wing Airframe Costs, Vol. I, Planning Research Corporation, PRC R-547, 1 February 1965.

At the outset of a study to develop an estimating relationship for aircraft costs, the cost analyst would not know which of all these characteristics would provide the best explanation of variations among the cost of different aircraft and would try to be as comprehensive as possible. An analyst who is familiar with the type of hardware under study should have some idea of what the most likely candidates are, but he will generally consider more characteristics than will eventually be used.

Program Data

A third type of essential data is drawn from the development and production history of hardware items. The acceptance date of the item, the significant milestones in the development program, the production rates, and the occurrence of major and minor modifications in its production--information such as this can contribute to the development of meaningful cost-estimating relationships. It will be noted that the list of explanatory variables in the previous section includes year of first delivery and increase in airframe weight from unit 1 to unit n, information that would be included in the category program data.

An airframe typically changes in weight during both development and production as a result of engineering changes. For example, the weight of the F-4D varied as follows:

Cumulative Plane Number	Airframe Unit Wt (lb)
1- 11	8456
12-186	8941
187-241	8541
242-419	9193

Since labor hours are commonly associated with weight to obtain hours-per-pound factors, it is important to have the weights correct and not to use a single weight.

The need for other kinds of program data will be made clear by the following pages on data adjustment. To cite one example here, one needs to know the year in which expenditures occur to adjust cost data for price level changes. (This is the reason for at least one CIR submission annually.) A certain amount of what we have chosen to call program data cannot be specified this definitely nor can its use be foretold, but it is important nonetheless. This is what might be termed background information--information about what else is going on in the contractor's plant at the time a particular hardware item is being built, unusual problems the contractor may be encountering, attempts to compress or stretch out the program, inefficiencies noted, etc. These facts may be useful in explaining what appear to be aberrations when the resource data are compared with those from other development and production programs. In addition a history of a contractor's overhead, G&A, and labor rates is useful both for analyzing and predicting costs.

DATA ADJUSTMENT

To be usable to the cost analyst data must be consistent and comparable, and in most cases the data as collected are neither. Hence, before estimating procedures can be derived the data have to be adjusted for such things as price level changes, definitional differences, production quantity differences, and so on. This section discusses some of the more common adjustments. It is by no means an exhaustive treatment of the subject, since the list of possible adjustments is long and many of them will apply only in a very small number of cases. Also, evidence on certain types of adjustments--for contractor efficiency, for contract type, for program stretch-out, etc.--consists largely of opinion rather than hard data and while we can allude to such adjustments the research necessary to treat them in some definitive way has not yet been done.

Definitional Differences

Different contractor accounting practices are one of the primary reasons that adjustment of the basic cost data is generally necessary. Companies record their costs in different ways, are often required to report costs to the government by categories somewhat different from those used internally, and the reporting categories change from time to time. Because of these definitional differences, one of the first steps in any cost analysis is to state the definition that is being used and then adjust all data to this one definition. With the inception of CIR, a standard set of definitions for airframes has been established for use throughout the Department of Defense. A primary purpose of CIR is to overcome the problem of definitional differences in hardware cost data. For the next few years, however, when most data will antedate CIR, some adjustment will be required.

As an example of what may be expected, a cost analyst may be examining data from a sample of 10 hardware items and discover that the cost element Quality Control is missing for some of the earlier items. He may conclude that no quality control was exercised back in the 1950's or that this function is included in some other cost element. The latter is correct of course. Traditionally, Quality Control was carried in the burden account, and it was only in the late 1950's that it began to appear (at the request of the Department of Defense) as a separate element. Hence to use cost data on equipment built prior to this change some portion of overhead cost has to be converted to Quality Control.

A more current example involves Planning, which in the CIR definition is included in Tooling. Planning consists of two components--tool planning and production planning--so some companies put the first in Tooling and the second in Manufacturing. Other practices are to include tool planning in Engineering, to put all planning in Manufacturing, or to include some portion in Overhead. In our view the CIR definition is the most logical.

Table II-1 illustrates this problem more concretely. On the left is a slightly abbreviated version of the CIR list of cost elements; on the right are the categories used by a large aerospace company and the non-recurring costs of a proposed airframe. The lists are different and, as shown by Table II-2, a simple rearrangement of the contractor cost elements does not solve the adjustment problem.

After this rearrangement four of the contractor cost elements--Developmental Material ($2.6 million), Outside Production ($70,000), Other Direct Charges ($2.7 million), and Manufacturing Overhead

Table II-1

COMPARISON OF CIR AND CONTRACTOR COST ELEMENTS

CIR

Cost Element	Contractor	Outside Production
1. Engineering		
Direct labor		
Overhead		
Material		
Other direct charges		
2. Tooling		
Direct labor		
Overhead		
Materials and purchased tools		
Other direct charges		
3. Quality control		
Direct labor		
Overhead		
Other direct charges		
4. Manufacturing		
Direct labor		
Overhead		
Materials and purchased parts		
Other direct charges		
5. Purchased equipment		
6. Material overhead		

Contractor

Cost Element	Cost (Thousands of dollars)
1. Engineering	8,600
2. Manufacturing	
Developmental direct labor	2,500
Tooling direct labor	11,600
Production direct labor	850
Developmental material	2,600
Tooling material	2,600
Production material	500
Purchased equipment	5
Outside production	70
3. Inspection	620
4. Other direct charges	2,700
5. Overhead	
Engineering	10,200
Manufacturing	28,940

Table II-2

CONTRACTOR COST ELEMENTS ARRANGED IN CIR FORMAT

CIR Cost Element	Contractor Cost Element	Cost (Thousands of $) In House	Cost (Thousands of $) Outside Production
1. Engineering			
Direct labor	Engineering	8,600	--------
Overhead	Engineering overhead	10,200	--------
Material		----	--------
Other direct charges		----	--------
2. Tooling			
Direct labor	Tooling direct labor	11,600	--------
Overhead		----	--------
Materials and purchased tools	Tooling material	2,600	--------
Other direct charges		----	--------
3. Quality control			
Direct labor	Inspection	620	--------
Overhead		----	--------
Other direct charges		----	--------
4. Manufacturing			
Direct labor	Developmental direct labor	2,500	--------
	Production direct labor	850	
Overhead		----	--------
Materials and purchased parts	Production material	500	--------
Other direct charges		----	--------
5. Purchased equipment	Purchased equipment	5	--------
6. Material overhead		----	--------

($28.94 million)--remain to be dealt with. Since these four categories can amount to well over half the total cost of a large production contract, we are not talking about trivial adjustments. Developmental Material presumably would be split between Engineering Material and Manufacturing Material; Other Direct Charges would have to be allocated among Engineering, Tooling, Quality Control and Manufacturing; and part of Manufacturing Overhead would be apportioned to Tooling Overhead and

Quality Control Overhead. In each of these instances the contractor furnishing Cost Information Reports would be able to make the necessary adjustments from his own accounting records. Outside Production costs, although small in this example, in some cases may comprise 30 to 40 percent of the total cost of an airframe. Where this is the case, the labor hours and material costs incurred by the prime contractor fall far short of the total required to build an airplane, and some method of arriving at a total must be devised. Ordinarily, the contractor would have a detailed breakout of costs only for subcontractors on cost-reimbursable contracts, and other Outside Production costs would have to be allocated to the specified categories. Production labor hours incurred out-of-plant, for example, are often estimated on the basis of the weight of that portion of the airframe being built out of plant. In using historical data, the analyst may be in a similar position occasionally, and where the amounts involved are large, he should be guided by whatever information the contractor can provide.

Physical and Performance Characteristics

A problem similar to the one discussed above concerns the need for consistency in definitions of physical and performance characteristics. "Speed," for example, can be defined in many ways--maximum speed at optimal altitude, true speed, equivalent speed, indicated speed, etc.--which differ in exact meaning and value. The weight of an aircraft or missile depends on what is included. Gross weight, empty weight and airframe unit weight are all used for aircraft. Some agencies include sweep volume in their definition of the physical volume of an aircraft fire control system; others exclude it. Examples of this kind are numerous, but the point hardly needs elaboration. It is raised here because differences such as these can lead an analyst unfamiliar with the equipment being investigated to use inconsistent or varying values inadvertently. When data are being collected from a variety of sources, an understanding of the terms used to describe physical and performance characteristics is at least as important as an understanding of the content of the various cost elements.

Nonrecurring and Recurring Costs

Another problem hinging on the question of definitions concerns nonrecurring and recurring costs. Recurring costs are a function of the number of items produced; nonrecurring costs are not. Thus, for estimating purposes it is useful to distinguish between the two and CIR provides for this distinction. Unfortunately, historical cost data frequently show such cost elements as initial and sustaining engineering as an accumulated item in the initial contract. Various analytical techniques have been developed for dividing the total into its two components synthetically, but it is not clear at this time whether the nonrecurring costs obtained by ex post facto methods will be comparable to those reported in CIR. The CIR instructions state:

> It is preferable to identify the point of segregation between nonrecurring and recurring engineering costs as a specific event or point in time. Ideally, the event used would be the point at which "design freeze" takes place as a result of a formal test or inspection, and after which formal Engineering Change Proposal (ECP) procedures must be followed to change design. If no reasonable event can be specified for this purpose, then all engineering costs incurred up to the date of 90 percent engineering drawing release may be used.

While it would be premature to consider the kinds of adjustments needed before a body of CIR data exists, splicing historical data to CIR data may involve an adjustment of some kind.

A more subtle problem arises when nonrecurring costs on one product are combined with recurring costs on another, i.e., when the contractor is allowed to fund development work on new products by charging it off as an operating expense against current production. This practice is especially prevalent in the aircraft engine industry. Separation of the nonrecurring and recurring costs in this instance means an adjustment of the production costs shown in contract or audit documents to exclude any amortization of development. The nonrecurring expense which had been amortized can then be attributed to the item for which it was incurred. This adjustment can only be accomplished in cooperation with the accounting department of the companies involved. It would be unnecessary, of course, for equipment on which CIR data are available.

Price-Level Changes

Figure II-1 shows the change in average hourly earnings of production on manufacturing payrolls from 1920 to 1965. Although these earnings

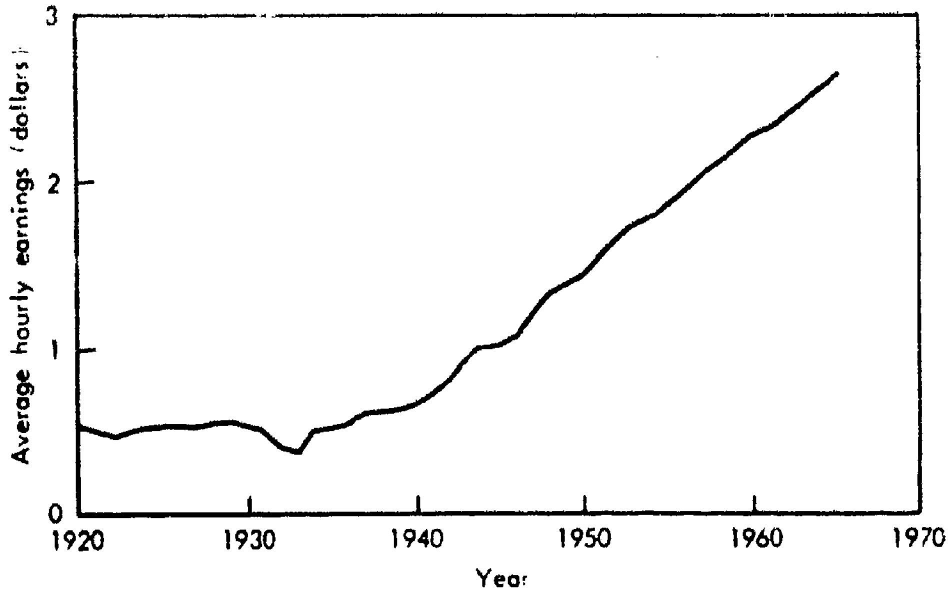

Fig. II-1—Change in hourly earnings

declined slightly during the early 1920's and again during the Depression, the trend has been steadily upward since 1934. The hourly wage rate has increased by a factor of 4.75 over a 45-year period, or put another way, in 1965 a manufacturer paid $4.75 for labor that would have cost him $1.00 back in 1920. The implication of this for equipment costs is clear. If the labor component of an automobile cost $500 in 1920, the cost for the same car today would be something over $2000 (the hours required in 1965 would be less because of increased productivity, but this effect will be discussed later).

The relevance of these observations to the subject of data adjustment is that the manufacturing date of the different hardware items in a sample are normally spread over a period perhaps as long as 10 to 15 years. To compare a missile built in 1955 when labor cost about $2.35 per hour with a missile built 10 years later when the labor rate had increased to over $3.35 per hour the labor cost of both must be adjusted to a common base. (This problem is obviated by dealing in hours rather than dollars but an adjustment would still be needed for raw material and purchased parts.) Adjustments of this kind are made by means of a price index constructed from a time-series of data by selecting one year as the base and expressing the value for that year as 100. The other years are then expressed as percentages of this base. The hourly earnings from 1950 to 1960 for production workers could be converted to an index using any of the years as the base; in the example below 1950 and 1960 have both been used as base years.

Year	Average Hourly Earnings	Index with 1950 as Base Year	Index with 1960 as Base Year
1950	$1.44	100	64
1951	1.56	108	69
1952	1.65	115	73
1953	1.74	121	77
1954	1.78	124	79
1955	1.86	129	82
1956	1.95	135	86
1957	2.05	142	91
1958	2.11	147	93
1959	2.19	152	97
1960	2.26	157	100

Information to construct a labor index such as this is available in the Bureau of Labor Statistics publication <u>Employment and Earnings</u>, and Table II-3 presents indexes based on this source. Changes in materials costs are available in another BLS publication, <u>Wholesale Prices and Price Indexes</u>, and these can be used to develop a materials price index for a given type of equipment by the following simple procedure. A list of materials representative of those used in constructing the equipment is chosen from the commodity groups in the Wholesale Price Index, and these materials weighted according to estimates of the amount of each in fabricating the equipment. A composite aircraft raw materials index might be based on the following materials and weights:

Table II-3

LABOR PRICE INDEX

Year	Aircraft	Aircraft Engines and Engine Parts	Other Aircraft Parts and Equipment	Motor Vehicles and Equipment	Electrical Equipment and Supplies	Ship and Boat Building
1952	.59	.62	NA[a]	.61	.64	.63
1953	.63	.63	NA[a]	.64	.67	.68
1954	.66	.66	NA[a]	.66	.69	.68
1955	.69	.68	NA[a]	.74	.71	.71
1956	.72	.71	NA[a]	.75	.75	.75
1957	.75	.75	NA[a]	.73	.79	.80
1958	.80	.80	.81	.82	.82	.83
1959	.84	.84	.85	.81	.85	.86
1960	.86	.87	.88	.84	.89	.89
1961	.89	.90	.90	.85	.91	.93
1962	.91	.93	.93	.89	.93	.97
1963	.94	.95	.94	.93	.95	.98
1964	.98	.98	.98	.96	.98	1.00
1965	1.00	1.00	1.00	1.00	1.00	1.00

[a]Not available (for years 1952-1957 it is suggested that the labor price index for aircraft be used).

Finished steel	.02
Stainless steel sheet	.04
Titanium sponge	.07
Aluminum sheet	.29
Aluminum rod	.11
Aluminum extrusions	.20
Wire and Cable	,12
Rivets, etc.	.15

For any given year a price index for each of these is obtained and a composite index constructed by summing the individual index numbers multiplied by the weightings, e.g.:

Commodity	1967 Index Number[a]	Weight	Index Number Times Weight
Finished steel	105.8	.02	2.12
Stainless steel sheet	108.0	.04	43.2
Titanium sponge	60.3	.07	4.22
Aluminum sheet	99.8	.29	28.94
Aluminum rod	110.4	.11	12.14
Aluminum extrusions	75.6	.20	15.12
Wire and cable	126.0	.12	15.12
Rivets, etc.	133.2	.15	19.98
Composite index number			101.96

[a]1957-1959 = 100.

Weights in an index such as this need to be updated from time to time to reflect changing technology, and it may be that those shown here, are only applicable to current aircraft. This simple example is included only to illustrate the principle of deriving a composite index; the reader who wishes to pursue the matter further will find index numbers discussed in most textbooks on economic statistics.* Another type of composite index is used in those instances where labor and

*See, for example, W. A. Spurr, L. S. Kellogg, and J. H. Smith, Business and Economic Statistics, rev. ed., Richard D. Irwin, Inc., Homewood, Illinois, 1961.

material costs cannot be separated and the price-level adjustment has to be made to the total cost of an engine, airframe, missile, etc. Such an index can be derived in the manner illustrated above with the labor and material elements weighted according to whatever pattern has been found to exist in the past, e.g., labor, 80 percent; materials, 20 percent.

Overhead, which is a mixture of labor, materials, and items such as rent, utilities, taxes, etc., in most cases is adjusted by the same percentage as direct labor. To decide in any particular case whether a different adjustment factor should be used, an examination of each component of overhead--indirect labor, fringe benefits, etc.--would be required. This cannot be done by reference to the various indexes published by BLS and other governmental agencies.

Adjustment of costs for price level changes is not always as straightforward as the foregoing discussion may imply. One problem is that price indexes are inherently inexact and their use, while necessary, can introduce errors into the data. The average hourly earnings for all aircraft production workers may increase by $.05 in a given year but at any particular company they will increase more or less than that amount. Use of the average number to adjust the data for a given company will bias the data up or down. Also, for many specialized items of equipment, a good published price index does not exist. In fact, the usual indexes are oriented toward the civilian economy and may be misleading, i.e., they may understate the change experienced in defense and space industries. The United States, along with many other countries, furnishes the Office of Economic Cooperation and Development (OECD) in Paris with an index applicable to government defense expenditures in general. This index, shown below for 1952-1964, is useful to refer to when detailed index numbers seem questionable or are nonexistent.

Year	Index Number	Year	Index Number
1952	84	1959	102
1953	83	1960	104
1954	84	1961	105
1955	88	1962	106
1956	93	1963	108
1957	97	1964	113
1958	100		

Another problem is that of identifying the years in which expenditures occur when the only data available show total contract cost. Production and cash flow may have been spread out over a period of several years, and in principle the costs should be adjusted for each year separately. Although CIR will provide the information needed to do this in the future, it may be unavailable today, and some reasonable approximation of the expenditure pattern must suffice.

One method of doing this is to use a percent-of-cost versus percent-of-time curve of the type illustrated in Fig. II-2. These curves are developed from historical data on a number of programs involving the same kind of hardware--in this case, large ballistic missiles--and can be used to break total research and development or total production cost into annual expenditures. For example, to determine the annual expenditures in a five-year R&D program amounting to a total of $50 million the following percentages would be obtained from the R&D curve of Fig. II-2:

Time	Expenditures
20	6.5
40	23.0
60	65.0
80	92.0
100	100.0

These percentages are cumulative, of course, so the annual percentages and the amount they represent would be:

Year	Expenditures	
	Percent	Dollars (millions)
1	6.5	3.25
2	16.5	8.25
3	42.0	21.00
4	27.0	13.50
5	8.0	4.00

In the production phase a technique which can be used is to develop "lag" factors by examining delivery schedules and production lead times. Costs are then lagged behind delivery dates by some reasonable factor.

A more fundamental question than any of those raised above is

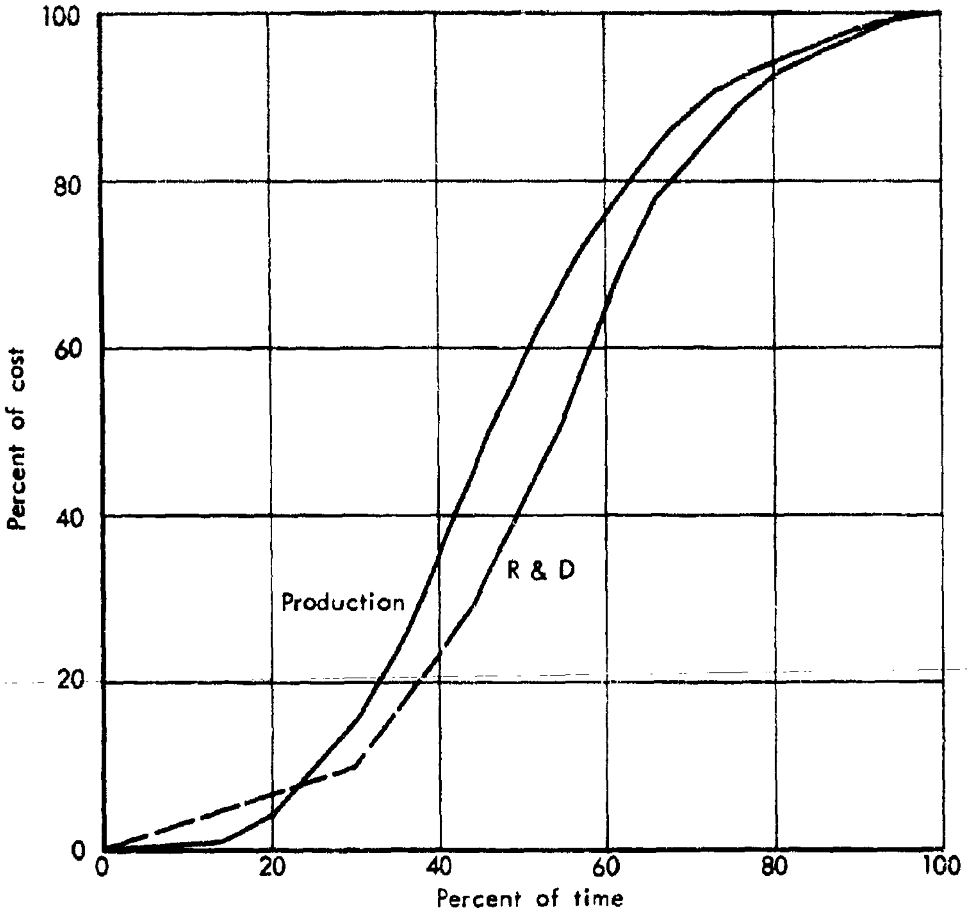

Fig. II-2—Percent-of-cost versus percent-of-time curves whether price-level changes should be made at all. The argument is sometimes made that the upward trend in wage rates has been accompanied by a parallel trend in the output per employee, or productivity rate. This implies that there has been little change in the real costs of aerospace equipment since increases in wages and materials costs have been offset by a decrease in the number of employees required per dollar of output. The real dollar output per man is difficult to measure, however, in an industry where continual change rather than standardization is the rule. Certainly the growth in productivity is not uniform for aircraft, missiles, ships, and tanks, and to develop a productivity index for each would be a difficult and contentious task. Present practice, therefore, is to apply the price-level adjustment factors to obtain constant dollars while remaining alert to any obvious inequities that may be introduced by doing so.

Cost-Quantity Adjustments

Chapter VI of this volume discusses the cost-quantity relationship, generally known in the aerospace industry as the learning curve, at some length. For those persons unfamiliar with this concept it states in brief that each time the total quantity of items produced doubles, the cost per item is reduced to some constant percentage of its previous value. Whether one accepts this particular formulation or not, the fact is that for most production processes costs are in some way a function of quantity: as the number of items produced increases, cost normally decreases. Thus, in speaking of cost it is essential that some quantity be associated with that cost. An equipment item can be said to cost \$100,000, \$80,000, \$64,000, or \$51,200 and all of these numbers will be correct.

Which cost should be used by the cost analyst? The answer to that question will depend on a number of factors; if his purpose is to compare one missile with another the cumulative quantity must be the same for both missiles. The adjustment to a specific quantity can be made very simply if the slope of the learning curve is known or can be inferred from the data. To illustrate, costs for three missiles are shown below. The cost is the same for each item, but the quantity is different. To compare the costs for the items, they must be adjusted

Missile	Unit Number	Cost/Unit
1	50	$1000
2	100	1000
3	200	1000

to a common quantity. If the quantity 100 is chosen and an 80 percent learning curve assumed for all three missiles, the adjusted costs will be:

Missile	Unit Number	Cost/Unit
1	100	$ 800
2	100	1000
3	100	1250

Projecting labor requirements for the 100th unit when only 50 units have been produced is somewhat uncertain, of course, but ignoring the cost-quantity relationship will in most instances result in greater error than such a projection introduces.

The learning curve is most frequently depicted as a straight line on log log paper as in Fig. II-3. The points above the curve illustrate a point made earlier. They show the effect of adjusting production costs incurred over the period 1954-1958 to 1965 dollars.

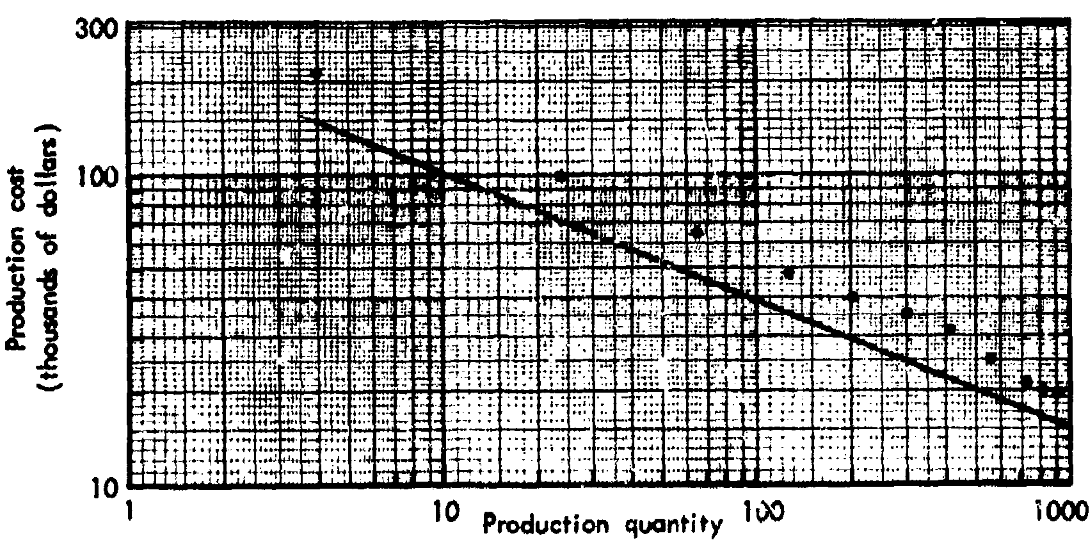

Fig. II-3—Learning curve and adjustment for price-level changes

Other Possible Cost Adjustments

As exemplified earlier by the mention of productivity changes over time and the lack of a way to adjust cost data for such changes, many more kinds of adjustments can be theorized than have been quantified. It has been suggested, for example, that some adjustment may be required because of differences in contract type--fixed price, fixed price incentive, cost plus fixed fee, etc.--or differences in the type of procurement--competitive bidding or sole source. The hypothesis here is that the type of contract or procurement procedure will bias costs up or down, but this has been an exceedingly difficult hypothesis to substantiate.

Another suggestion concerns manufacturing techniques. What are the effects of varying amounts of capital investment or capital improvement and of changes in manufacturing state of the art? A related question concerns the efficiency of the contractor. We may suspect that Contractor A has been a lower cost producer than Contractor B on similar items, but this is extremely difficult to substantiate. A low-cost producer may be one who because of his geographical location pays lower labor rates. Contractors in Fort Worth, Texas and Atlanta, Georgia may have a considerable advantage in this regard over their competitors in Los Angeles, San Francisco and Seattle. The table below does not give a fair picture of comparative rates because differences between industries in the various cities tend to be more important than differences in location. But it can be seen for two cities as close together as Los Angeles and San Francisco that labor rates differ by about 10 percent. Thus while it might not be possible to adjust cost data on the basis of contractor efficiency, it is possible to make adjustments for differences in location by using the specific area labor rates.

Table II-4

AVERAGE HOURLY EARNINGS OF PRODUCTION WORKERS
ON MANUFACTURING PAYROLLS--NOVEMBER 1965[a]

Atlanta	$2.69
Boston	2.69
Chicago	2.91
Detroit	3.45
Los Angeles	3.04
New Orleans	2.72
New York	2.63
Philadelphia	2.79
St. Louis	2.96
San Francisco	3.35
Seattle	3.25

[a]From Employment and Earnings, Bureau of Labor Statistics, January 1966.

III. USING STATISTICS IN THE DEVELOPMENT OF ESTIMATING RELATIONSHIPS

As stated in a previous chapter, many, perhaps most, estimating relationships are simple statements indicating that the cost of some commodity is directly proportional to the weight, area, volume or some other physical characteristic of that commodity. These estimating relationships are simple averages--very useful in a variety of situations but because of their simplicity requiring little explanation here. Our concern is with the derivation of more complex relationships, i.e., equations that describe the basic data better than a simple factor can and that can reflect the influence on cost of more than one variable. The intent is to illustrate a general approach to the development of such relationships and to introduce certain basic concepts of statistical analysis. The emphasis is not on statistics *per se*, and the basic mathematical statistical theory involved as well as the computational aspects of regression analysis are generally ignored. This chapter merely presents some of the statistical considerations involved in developing estimating relationships for advanced equipment estimating. While statistical procedures are stressed, the intent is not to suggest that regression analysis offers a quick and easy solution to all the problems of estimating cost. Statistical analysis can help provide an understanding of factors which influence cost, but estimating relationships are no substitute for understanding.

The outstanding characteristic of a cost factor is that the relationship between cost and the explanatory variable is direct and obvious; thus, cost per pound is widely used because of the generally satisfying thesis that as a ship, tank, or aircraft increases in weight it becomes more costly. Weight changes do not always explain cost changes, however, and many other explanatory variables are used. The problem is to find these, and this is done first by deciding what variables are logically or theoretically related to cost and then by looking for patterns in the data that suggest a relationship between cost and these variables. A simple array, as in Table III-1, may reveal such patterns.

Table III-1

TEN AIRBORNE RADIO COMMUNICATION SETS

Cost ($)	Weight (lb)	Power Output (w)	Frequency (mh)
22,200	90	20	400
17,300	161	400	30
11,800	40	30	400
9,600	108	10	400
8,800	82	10	400
7,600	135	100	25
6,800	59	6	400
3,200	68	8	156
1,700	25	8	42
1,600	24	.5	258

In this table, the costs of 10 airborne radio communications sets are given along with the weight, power output and frequency of each. A priori, one might expect cost to increase with weight or with power output. Frequency is included because, historically, higher and higher frequencies have been sought to increase communications capacity, and in general for a given power output higher frequency sets have been more costly.

From Table III-1 it is clear that cost is not a simple linear function of any of the three possible explanatory variables shown. Cost tends to increase with weight, but there are notable exceptions to the trend as shown in the scatter diagram of Fig. III-1a. Cost plotted against power output (Fig. III-1b) is even less promising, partially because of the scale which does not enable an observer to distinguish among the points between .5 and 30 watts. Changing from an arithmetic to a logarithmic scale (Fig. III-2) distinguishes better among points in the low power range and indicates that a trend does exist but, again, with a very wide scatter.

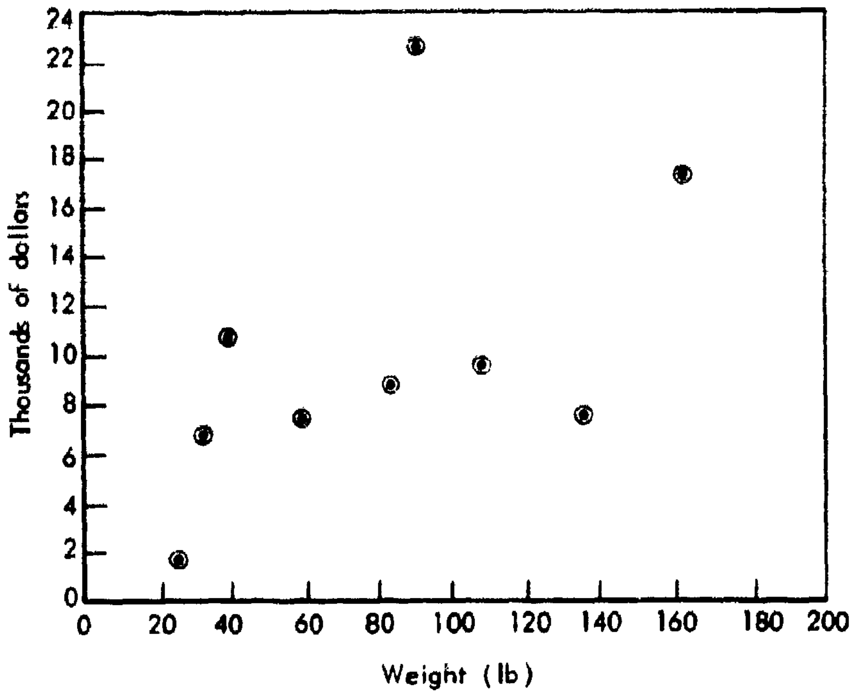

Fig. III - 1a—Cost versus weight

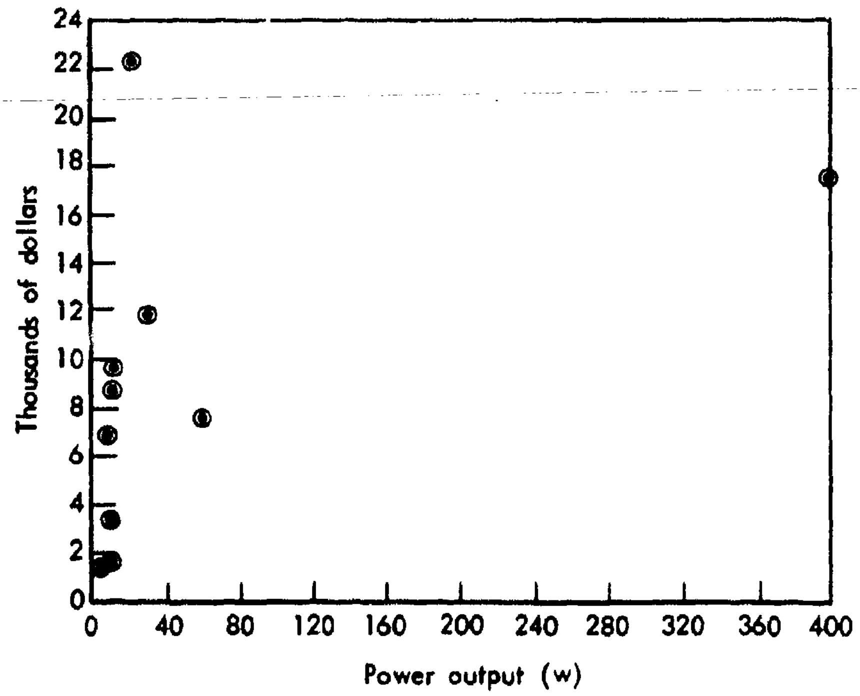

Fig. III - 1b—Cost versus power output

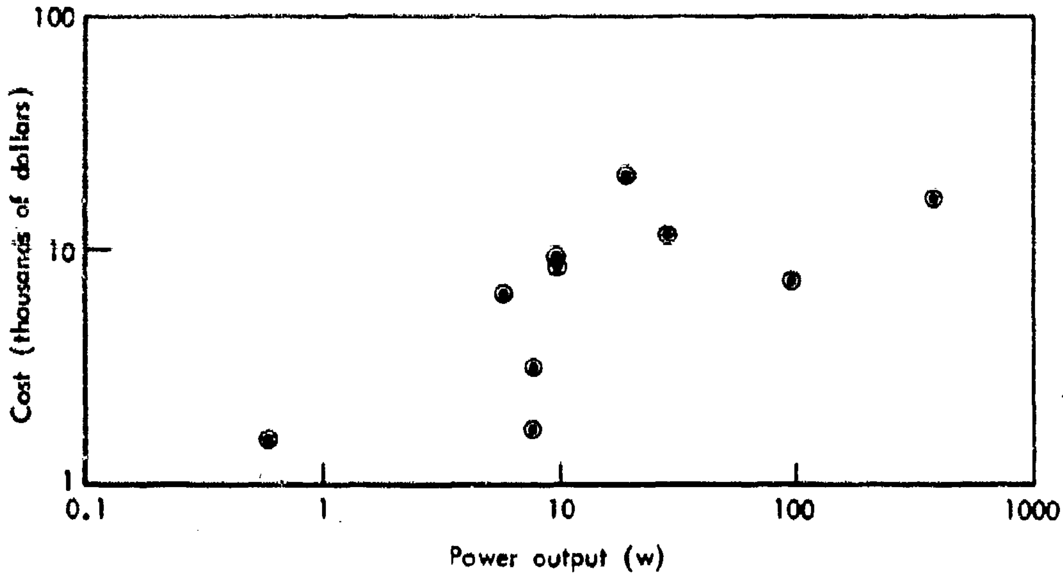

Fig. III-2—Cost versus power output (logarithmic grid)

It appears that the scatter may be explained to some extent by the effect of frequency and in Fig. III-3 each point is identified to a frequency class:

HF - up to 30 mh
VHF - 30 to 300 mh
UHF - above 300 mh

A clearer relationship exists between cost and power output within each frequency class than would seem to exist for the whole sample scattered without regard to frequency. This suggests that the sample is not homogeneous. Each frequency band may constitute a separate sample, or possibly HF and VHF costs are on one level and UHF costs on another.

With a larger data base each sample could be examined separately and a regression line drawn for each. Given a maximum of five points in each of two samples, however, regression analysis techniques are not warranted. The justification for regression analysis (as distinct from simply drawing a line of best fit through the points either by a

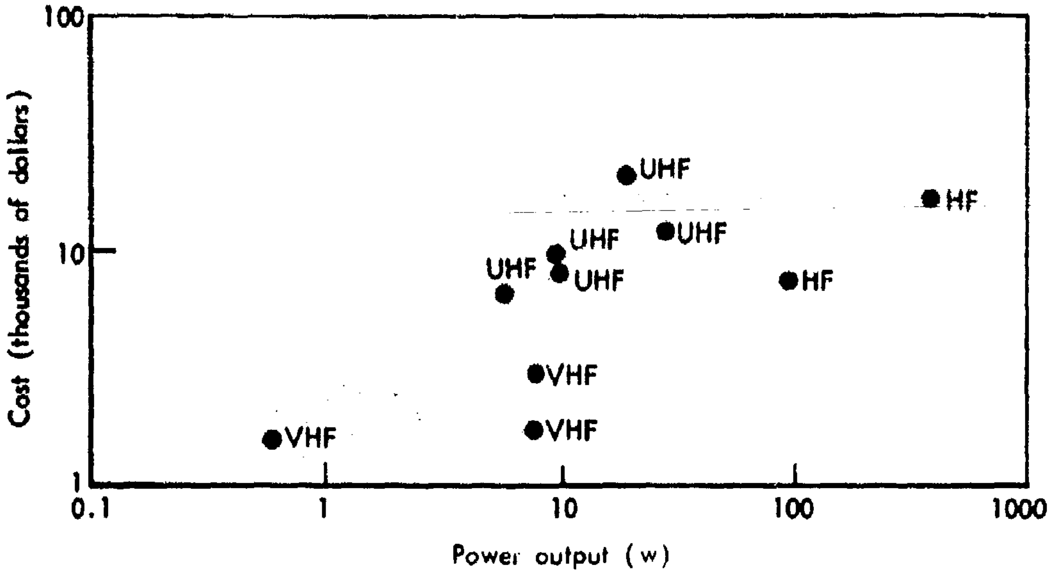

Fig. III-3—Frequency class identified

least-squares or freehand technique) is to be able to say something about the reliability of the regression line; in this case statistical measures of reliability would have little meaning.

At this point it is not clear that any of the possible explanatory variables, either singly or in combination, will yield a useful estimating relationship. But as a means of illustrating some of the techniques commonly used in deriving such relationships, let us begin with the assumption that cost can be related to a single predictive variable--weight--and examine the results of a linear normal regression model. In a later example we shall consider several variables in a multiple regression analysis.

Regression theory has become a widely accepted tool for cost analysts and is often used to develop estimating relationships. In simple regression analysis we are interested in estimating the value of one variable based on its relationship to a second variable. Regression theory provides a means for examining whether a relationship exists; and when it does, for measuring the nature and extent of the relationship.

According to classical statistics a population (or universe) defines the totality of all pertinent values that any variable or variables can achieve. It follows that the true relationship between two variables must be embodied within a population. (It is seldom known, however, whether the set of values available in any given problem constitutes a population or is only a subset (sample) of a larger population. Generally, these values are considered to be a sample which can be used to estimate relationships for an actual population.)

The form of the regression function depends, of course, upon the problem. It may reflect an underlying physical law or perhaps some other structural relationship. When no particular functional form is suspected, the simple linear-regression model is frequently used to describe the relationship between two variables. The equation of this model is:

$$y = a + bx$$

Where y is the dependent variable and x the independent variable. The symbols a and b are parameters or constants whose values are to be calculated from the data. Here y could be the cost of a radio communication set and x the weight. The model then indicates that heavier equipment will cost more than lighter equipment. The values of a, b and x allow a computation of a value for the cost for any equipment if we know its weight.

To make statistical predictions, certain assumptions must be made about this model. The classical requirement is that x values are fixed and y values are random variables for given x values. This is graphically illustrated in Fig. III-4. Specifically, for the population it is assumed that (1) the variance of y-values about the regression line is the same for all x-values (x_1, x_2, x_3, x_4, etc.) and (2) y-values for a given x value are normally distributed about the regression line. For the sample it is assumed that y-values are simple random samples taken from the total population.*

*For a more complete statement of the assumptions about the sample see W. A. Spurr and C. P. Bonini, Statistical Analysis for Business Decisions, Richard D. Irwin, Inc., 1967, pp. 564-565.

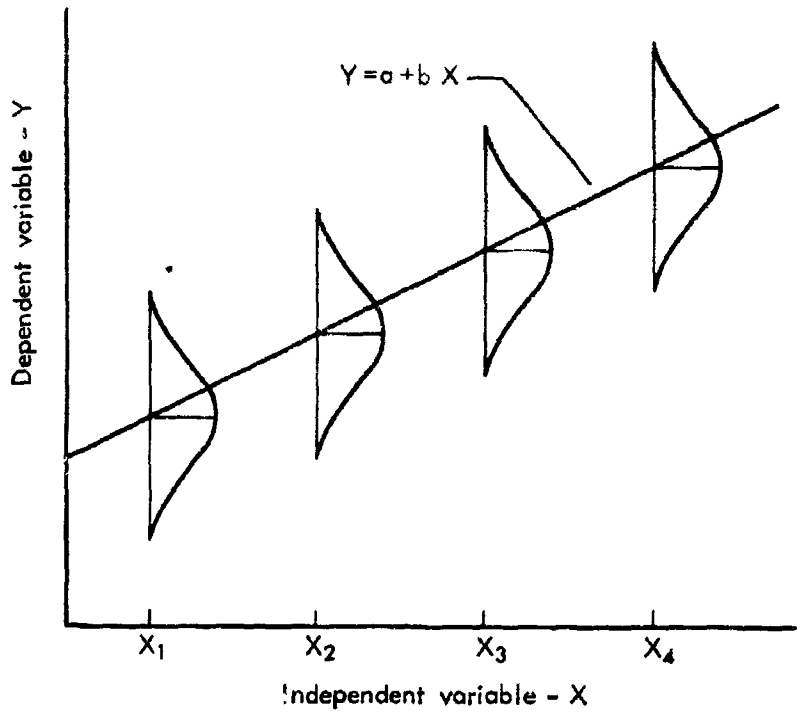

Fig. III-4—Simple linear population regression model

Given the regression model shown above, the basic problem is to derive estimates of the parameters a and b such that the regression equation will approximate the sample data as closely as possible. One procedure for doing this uses the method of maximum likelihood.* In normal linear regression it turns out that the maximum likelihood method is exactly equivalent to a least-squares procedure. The values of a and b are determined by the requirement that the sum of the square of the deviations of the sample observations from the regression line will be at a minimum. The two normal equations for linear regression are:

$$\Sigma y = na + b\Sigma x$$

$$\Sigma yx = a\Sigma x + b\Sigma x^2$$

---

*The principle of maximum likelihood is discussed in <u>Introduction to the Theory of Statistics</u> by A. F. Mood, McGraw-Hill, 1950, pp. 152-154.

In this example:

> y = cost of airborne radio equipment (in thousands of dollars)
>
> x = weight of airborne radio equipment (in pounds)
>
> n = number of items in sample
>
> Σ = sum of (e.g., Σy = the sum of all y's)

Table III-2 shows the relevant numerical values to be substituted in the above equations. They are:

$$n = 10$$
$$\Sigma y = 90.6$$
$$\Sigma x = 792$$
$$\Sigma yx = 8739.4$$
$$\Sigma x^2 = 81,540$$

Substituting these numbers in the normal equations, we obtain:

$$90.6 = 10a + 792b$$
$$8739.4 = 792a + 81,540b$$

Table III-2

DATA FOR REGRESSION ANALYSIS OF COST AND WEIGHT

X	Y	X^2	Y^2	XY
90	22.2	8,100	492.84	1998.0
161	17.3	25,921	299.29	2785.3
40	11.8	1,600	139.24	472.0
108	9.6	11,664	92.16	1036.8
82	8.8	6,724	77.44	721.6
135	7.6	18,225	57.76	1026.0
59	6.8	3,481	46.24	401.2
68	3.2	4,624	10.24	217.6
25	1.7	625	2.89	42.5
24	1.6	576	2.56	38.4
792	90.6	81,540	1220.66	8739.4

Solving these simultaneously gives:

$$a = 2.477$$
$$b = .083$$

Or:

$$y = 2.477 + .083x$$

The regression line represented by the equation is shown in Fig. III-5 as the solid line. Its usefulness for predictive purposes depends on the extent of the dispersion of the observations about it--the greater the dispersion of observed values of y about the line, the less accurate estimates based on the line are likely to be. The measure of the dispersion of the actual observations is the <u>standard error of estimate</u> (S) of the regression equation.

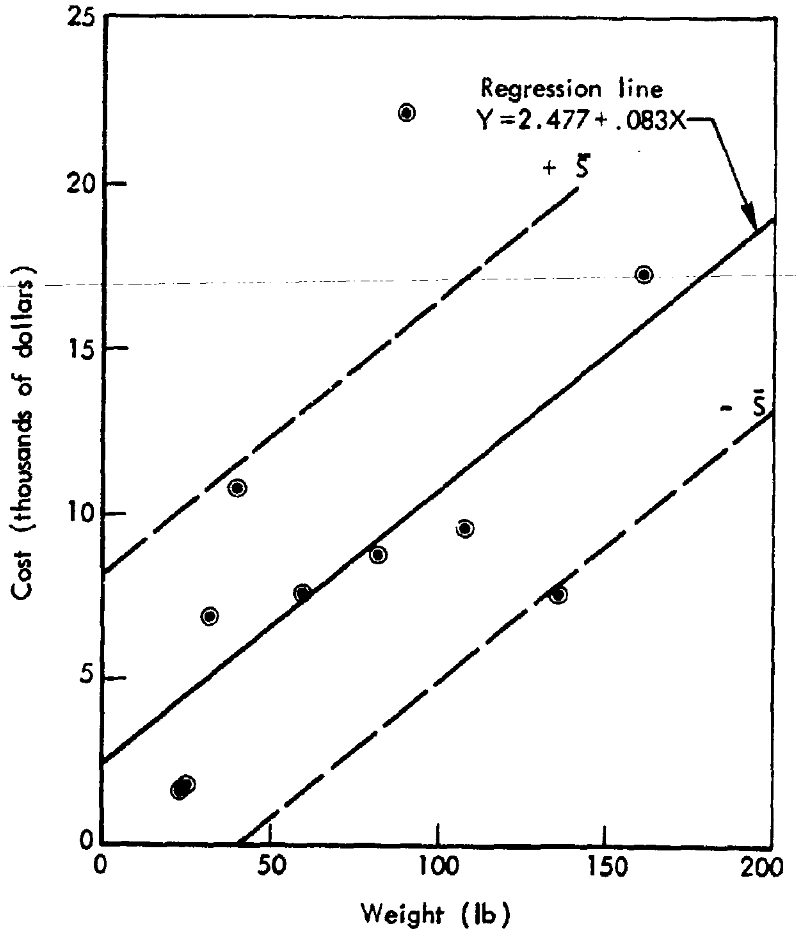

Fig. III-5—Regression line and standard error of estimate

The standard error of estimate is defined as the square root of the unexplained variance of the y's in the sample. This unexplained variance is derived from the difference between the observed y values (from Table III-1) and the computed y values (computed from the regression equation). This is illustrated in Fig. III-6.

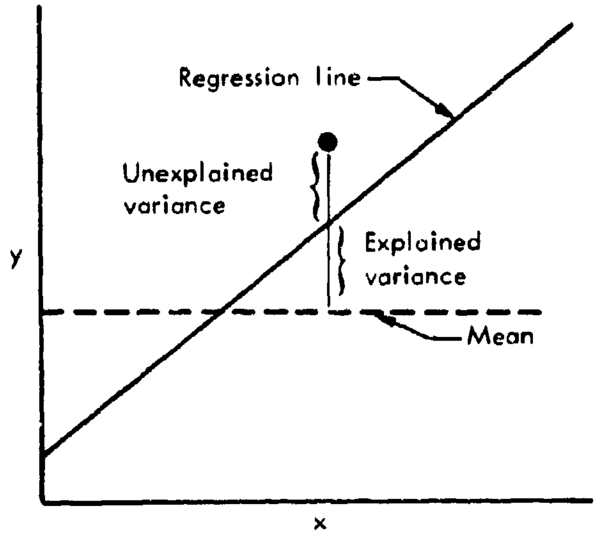

Fig. III-6—Unexplained and explained variance

Explained variance, which we will deal with later, is derived from the difference between the computed y values and the mean of the observed values. Total variance is the sum of the two.

Expressed mathematically, unexplained variance is:

$$\sigma_u^2 = \frac{\Sigma(y - y_c)^2}{n}$$

Thus, the unadjusted standard error of estimate is the square root of this expression, or:

$$S = \sqrt{\frac{\Sigma(y - y_c)^2}{n}}$$

To compensate for the fact that standard errors calculated for small samples typically understate the dispersion in the population, an adjustment is required. The adjusted standard error of estimate ($\bar{S}$) is obtained by subtracting the number of parameters in the regression equation from the sample size (n) in the formula for S. In this case the number of parameters is two (a and b). Therefore the formula for $\bar{S}$ is:

$$\bar{S} = \sqrt{\frac{\Sigma(y - y_c)^2}{n - 2}}$$

From this it is clear that for large sample sizes the adjustment is of no importance. In small samples--particularly very small samples such as we are dealing with here--the adjustment can make quite a difference.

The standard error of estimate for the estimating equation $y = 2.477 + .083x$ is \$5,800 and in Fig. III-5 a band of $\pm \bar{S}$ from the regression line has been plotted. In interpreting the standard error of estimate the main point is that in normal linear regression analyses one might expect about two-thirds of the sample observations to fall within a region bounded by $\pm \bar{S}$ from the regression line. Virtually all observations should fall within $\pm 3\bar{S}$. In practice these generalizations do not tend to hold up very well in very small sample cases.

For some purposes--particularly in comparing one $\bar{S}$ with another--it is useful to compute a relative standard error of estimate. One such measure is the coefficient of variation (C), which relates the standard error of estimate to the mean of the sample y's:

$$C = \frac{\bar{S}}{\bar{Y}}$$

In the example the mean of the y's is \$9,060. The value of C, therefore, is:

$$\frac{\$5,800}{\$9,060} = .64$$

which is quite high. While the question of reliability of an estimating equation is a relative matter, that is, it is relative to the context in which the equation is to be used, something like 10 to 20 percent would be more desirable.

The standard error of estimate and the coefficient of variation indicate how well the regression equation describes the sample observation, but this is rarely the area of greatest interest. The analyst is usually more interested in using the estimating equation to predict costs in the population or universe of items that the sample supposedly represents, and the standard error of estimate does not furnish a good measure of the reliability of the regression equation for predictive purposes. The subject of reliability raises several additional considerations. First, is the question of whether x and y are actually related in the manner indicated by the regression equation. A particular sample could show such a relationship out of pure chance when in fact none exists. Second, the regression equation obtained from the sample is only one of a family that could be obtained from different samples within the same population. This means that the predicted y may not be the true y. Both questions are dealt with by statistical inference, the first by a test of statistical significance and the second by establishing a prediction interval for the regression line.

While the subject of statistical testing is too complex to treat in any detail here, basically what is involved is to set up the hypothesis that x and y are not related (the null hypothesis), and then let the testing procedure indicate whether the hypothesis is accepted or rejected at some specified level of probability. The particular test to be used here is commonly known as the t-test because it uses the t-ratio, or ratio of a coefficient to its standard error. This ratio is expressed:

$$t_b = \frac{b}{s_b}$$

where b = the regression coefficient (from the linear regression model $y = a + bx$)

s_b = the standard error of b

The value obtained for t_b is 1.96, and this is interpreted by reference to a table of t-values.* The relevant row from such a table is shown below.

Degrees of Freedom	Level of Significance (or Probability)				
	.20	.10	.05	.02	.01
8	1.397	1.860	2.306	2.896	3.355

Note that the first column is headed "Degrees of Freedom" instead of n, the number of items in the sample. In a regression analysis the term "degrees of freedom" means the sample size minus the number of parameters (values to be estimated, i.e., a and b) in the regression equation, or in this case, 10 - 2 = 8. The value of 1.96 is seen to lie between the .1 and .05 levels of significance. This means that the chances are between 5 and 10 percent that a sample taken from a population in which x and y have zero correlation could have a t as high as 1.96. Hence, if we establish the required level of probability at 10 percent, the hypothesis that there is no correlation in the population is rejected. On the other hand if a .05 level of significance seems appropriate, the hypothesis must be accepted.

A reasonable question at this point is: What should be the level of probability for accepting or rejecting the hypothesis? Unfortunately, no simple answer is possible. The 10, 5, and 1 percent values are probably most commonly used, but the analyst must make his own judgment based on the risk assumed by rejecting a true hypothesis (a Type I error) or accepting a false hypothesis (a Type II error).** For our

*All the references at the end of the chapter contain t-tables.

**For a good discussion of this see Business and Economic Statistics by W. A. Spurr, L. S. Kellogg and J. H. Smith, Richard D. Irwin, Inc., 1961, pp. 251-255.

purpose here we will accept a 10 percent value both here and in establishing a confidence or prediction interval for the regression line. The procedure for that is as follows:

For a given value of the explanatory variable, say $\hat{x}$, the estimating equation is used to obtain a predicted value of the dependent variable:

$$\hat{y} = a + b\hat{x}$$

Then we can put a boundary around $\hat{y}$, say $\hat{y} \pm A$--such that there is a certain level of confidence that the established interval does indeed bracket the true value of y in the population.

In the case of normal linear regression, a $100(1 - \varepsilon)$ percent prediction interval for an estimated value of the dependent variable can be constructed as follows:

$$\hat{y} \pm A,$$

where

$$A = \bar{S}t_{\varepsilon}\sqrt{\frac{n+1}{n} + \frac{(\hat{x} - \bar{x})^2}{\Sigma(x - \bar{x})^2}}$$

and:

$\bar{S}$ = standard error of the estimating equation from which $\hat{y}$ was obtained,

t_{ε} = the value obtained from a table of t-values for the ε significance level,

n = size of the sample,

$\hat{x}$ = the specified value of the explanatory variable used as a basis for obtaining $\hat{y}$,

$\bar{x}$ = the mean of the x's in the sample,

$\Sigma(x - \bar{x})^2$ = the sum of squared deviations of the sample x's from their mean.

Using the estimating equation derived previously, the cost of a communications set weighing 100 lb is estimated to be \$10,777. To establish a 90 percent prediction interval around this value the necessary data are:

$\bar{S}$ = \$5,800

ϵ = 0.1 (Since a .90 prediction interval is to be computed, $1 - \epsilon = .9$ or $\epsilon = .1$)

$n = 10$

x = 100 lb

$\bar{x}$ = 79.2 lb

$\Sigma(x - \bar{x})^2$ = 18,893 lb

Substituting in the above equation and solving for A gives:

$$A = \$12,380$$

Therefore, for $\hat{x}$ = 100 lb, the 90 percent prediction interval is:

$$\hat{y} \pm S = \$10,777 \pm \$12,380$$

This means that when all the underlying assumptions about the sample are met, we have a subjective confidence of 90 percent that this interval brackets the true or population value of y when x = 100. It should be emphasized that a 90 percent prediction interval does not mean that the probability is 0.90 that the true value of y lies within the interval. Rather, it means that if we were to repeat the prediction procedure a number of times, we would expect that 90 percent of the time our prediction intervals would include the true value of y. The point is that the true value of y, while unknown to us, is a constant and not a random variable that could take on many values. Therefore, the relevant probability concept is that 90 percent of the intervals computed as this one has been will include the true value of y. This statement, of course, depends on the assumptions depicted in Fig. III-4, p. 39.

Using the prediction interval procedure outlined above, we can compute 90 percent prediction intervals for other values of x and plot these numbers to obtain a 90 percent confidence band around the regression line as in Fig. III-7. In this case it is clear from the figure that the 90 percent confidence region is fairly wide, reflecting graphically a measure of the uncertainty associated with the estimating equation. This is typical of analyses based on small samples. The equation for the prediction interval is constructed so that the width of the interval is quite sensitive to variation in sample size when n is small. Sensitivity to small values of n is logical, since generalizations based on very small samples should be subject to greater uncertainty than those founded on a larger data base.

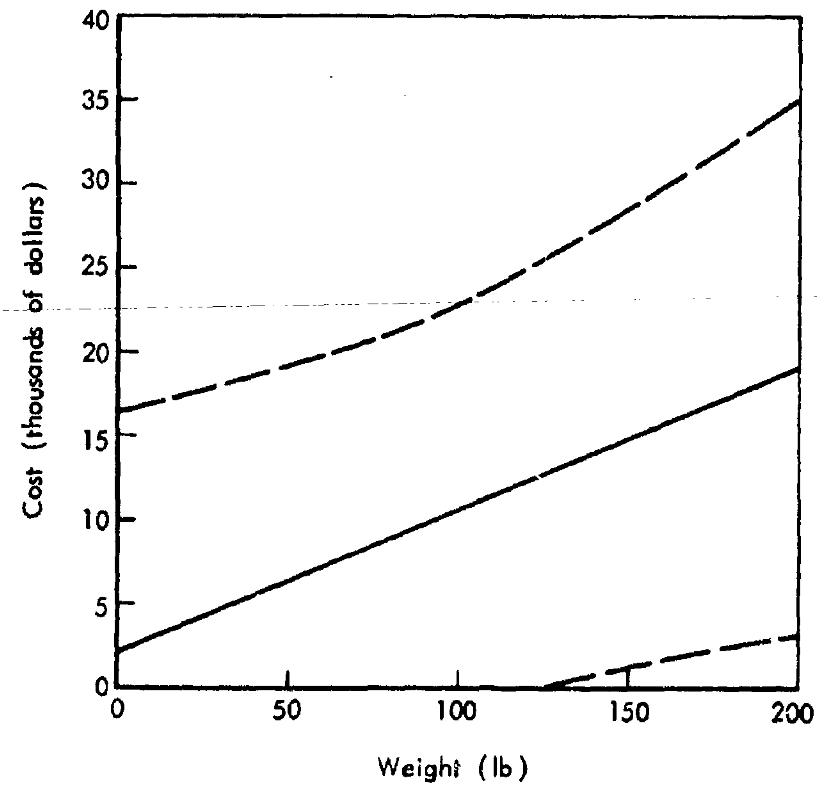

Fig. III-7—Ninety percent prediction interval

It should also be noted that the prediction interval becomes wider as values of x farther from the mean value and the sample are selected. Thus, for example the prediction interval for the mean (79 lb) is \$9,300 ± \$12,500, while for x = 200 lb it is \$19,000 ± \$15,990. The width of the interval in the latter case is about 1.3 times the width for the mean weight. This illustrates in a rough way how our confidence in the estimate decreases as we extrapolate beyond the range of the sample data--something that we often do in estimating the cost of advanced equipment.

The width of the prediction interval is also sensitive to the level of confidence specified. Up to now that level has been set at 90 percent (i.e., $\epsilon = .1$). Suppose that only a 70 percent level of confidence is desired ($\epsilon = 0.3$). The only thing that changes in the inputs used in the previous calculations is the value of t. Before, we used $t_{.1} = 1.86$; now we use $t_{.3} = 1.108$. This will make quite a difference in the width of the prediction interval. Since our confidence is lower, the prediction interval can be narrower, and for lower levels of confidence, the band would be even narrower. However, except for very low levels of confidence the interval obtained by the prediction interval procedure will always be wider than an interval established on the basis of the standard error of estimate alone.*

Up to this point the discussion has been confined largely to statistical regression analyses--developing an estimating equation and various measures of uncertainty pertaining to that equation. From an estimating point of view, this indeed is the most important part of the analysis. There is, however, another form of statistical analysis called correlation analysis. Correlation analysis is concerned with developing an abstract measure of the degree of association between the dependent variable and the explanatory variable or variables. In simple linear regression the most commonly used measure of degree of association is the correlation coefficient (r). The coefficient r is constructed in such a way that it is bounded by the interval ± 1. The sign indicates

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*But recall the point made previously: $\bar{S}$ can only be used to measure variations of y <u>in the sample</u>, not for describing the uncertainty of a <u>predicted</u> y.

whether the slope of the regression line is positive or negative--i.e., whether the regression coefficient b is positive or negative. At the boundaries of the interval for r we have the cases of perfect correlation: $r = +1$ (perfect positive correlation); $r = -1$ (perfect negative correlation). In these instances all of the sample points would lie exactly on the regression line. When there is no correlation between the variables whatsoever, $r = 0$.

While correlation is a somewhat different type of analysis from that discussed previously, it is nevertheless related in a definite way to regression analysis. To see this let us return to the concepts of total variance, explained variance, and unexplained variance referred to earlier in the discussion of the standard error of estimate and illustrated in Fig. III-6. Total variance (σ_t^2) pertains to the deviations of the y values in the sample from their mean, and is measured by:

$$\sigma_t^2 = \frac{\Sigma(y - \bar{y})^2}{n}$$

Explained variance (σ_e^2) refers to the deviations from $\bar{y}$ of the computed y values (calculated from the regression equation) corresponding to the values of the independent variable x in the sample, and is measured by:

$$\sigma_e^2 = \frac{\Sigma(y_c - \bar{y})^2}{n}$$

As explained previously, the standard error of estimate (unadjusted) is the square root of the unexplained variance. The coefficient of correlation (r), on the other hand, is related to the explained variance. It is defined as the square root of the proportion of total variance that is represented by the expla[illegible]d variance.* That is:

$$r = \sqrt{\frac{\Sigma(y_c - y)^2}{\Sigma(y - \bar{y})^2}}$$

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* r^2 is sometimes referred to as the coefficient of determination.

We now see the interrelationship among r, S, and the regression equation. The regression equation is used to determine the computed y's, which are inputs to the calculation of both r and S. Also, since r^2 is defined as a proportion of total variance, r and S in a sense have an inverse relationship to one another.

Just as S had to be adjusted for sample size--particularly so in the case of small samples--r should also be corrected. The value of r corrected for sample size is as follows:

$$\bar{r} = \sqrt{\frac{r^2 (n - 1) - 1}{n - 2}}$$

As is obvious from this equation, the effect of the correction dampens out as n becomes large. For very small samples the correction should most certainly be made.

The correlation coefficient adjusted for sample size in our illustrative example is .48. This is quite low and tends to substantiate the evidence already seen that weight alone is not a good predictor of the cost of airborne radio communication equipment. However, it should be kept in mind that a high correlation coefficient, say .95, can be misleading. Mere correlation does not allow an analyst to infer a cause-and-effect relationship between x and y. Spurious correlations are common. For example, the number of bathtubs in the United States has been increasing steadily and so has the crime rate as reported by the FBI. One might very well find a statistical correlation between the two much better than that found between cost and weight in the above sample. Another point is that the coefficient of correlation may be high but the reliability of an estimating equation as measured by the standard error of estimate may be low. The explanation hinges on the fact that r is based on a ratio while S is based on an absolute quantity:

$$r = \sqrt{\frac{\text{explained variance}}{\text{total variance}}}$$

$$S = \sqrt{\text{unexplained variance}}$$

Thus, even if the explained variance represents a high fraction of the total variance, it is still possible for the unexplained variance to be large.

CURVILINEAR ANALYSIS: LOGARITHMIC REGRESSION

Up to this point the analysis has been confined to simple linear regression. While a first examination of the scatter diagram of cost vs weight indicates that a linear relationship might be as good as anything else, it still cannot be concluded definitely that some type of non-linear relationship might not be preferable. Several such relationships can be tried. One that is very frequently used, and that we will be dealing with in discussing cost-quantity relationships in Chapter V, is of the form:

$$y = ax^b$$

Since this equation is difficult to deal with statistically, usually we make a logarithmic transformation of the variables, obtaining an equation which is linear in the logarithms of the variables:

$$\text{Log } y = \log a + b(\log x)$$

The procedure here is to conduct the statistical analysis in terms of the logarithms of the variables, that is, obtaining estimates of log a and b from a least squares fit of this equation. This approach has several advantages over dealing directly with $y = ax^b$, the most important ones being:

1. We can proceed almost identically to the simple linear regression case.
2. No additional degrees of freedom are lost--an important consideration when the sample size is small.

The first step is to take the original data for y and x contained in Table III-1 and convert these data to logarithms. The next step is a simple linear regression analysis of the data in logarithmic form. This means that a linear regression equation is derived such that the sum of the squares of the logarithms of the variables around the regression line is at a minimum. Solving as before, the estimates of log a and b are found and the regression equation for the logarithms of the variable is:

$$\log y = -1.0425 + 1.0241 \log x$$

This equation is shown as a solid line on the scatter diagram in Fig. III-8. Note that here the original values (arithmetic form) of x and y

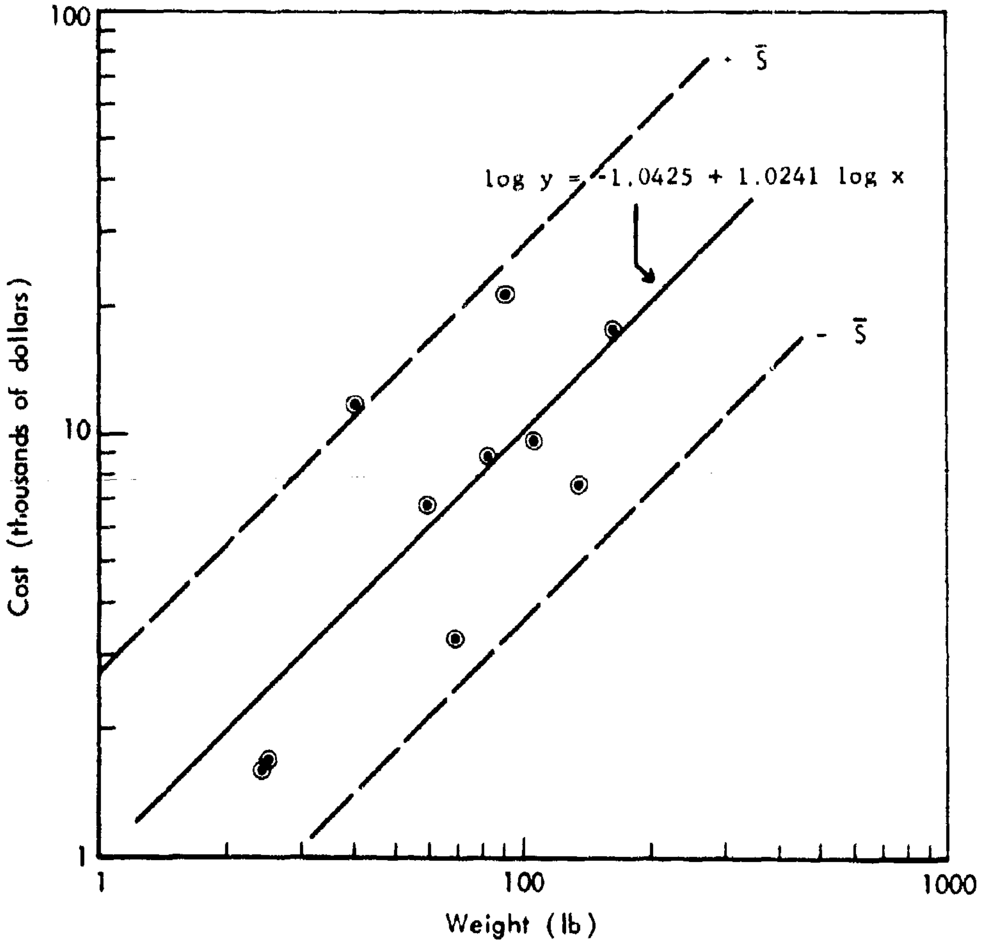

Fig. III-8—Logarithmic regression

are plotted on a chart having logarithmic scales on both axes. This is exactly equivalent to plotting the logarithms of the variables on an arithmetic chart. Note also that the regression line slopes upward. This is because the b-value is greater than one. With a b-value of less than one, the curve would slope down.

The standard error of estimate is computed as before but in log terms:

$$\bar{S}_{\log} = .2763$$

In Fig. III-8 the dashed lines indicate a band representing $\pm\ \bar{S}_{\log}$ around the regression line.

For perspective, the value $\bar{S}_{\log}$ may be related to the mean of the log y's in the sample to obtain the coefficient of variation for the log equation. The procedure is the same as that shown on P. 40.

$$\frac{\bar{S}_{\log}}{\frac{\Sigma \log y}{n}} = .335$$

At this point it would appear that things have improved somewhat over the simple linear regression case. The picture portrayed in Fig. III-8 suggests a better fit to the data. Also, the standard error of estimate in relation to the mean of the log y's is substantially lower than in the simple linear regression example: 34 percent as compared with 64 percent.

But this is not the whole story, since up to now the analysis has dealt with the logarithms of the data, and the analyst is interested in making estimates in terms of the original data. We therefore have to transform the logarithmic analysis back to an arithmetic form. When this transformation is made, the estimating equation becomes:

$$y = .09056(x^{1.0241})$$

where .09056 is the anti-log of log a = -1.0425. This equation is plotted on the scatter diagram contained in Fig. III-9. It should be noted that the equation plots as a straight line over the range of weights shown. Since the exponent of x is close to unity, the curvilinearity implied by the form of the equation does not show up. Note also that the regression line does not appear to be a particularly good fit to the original data--no better than the simple linear estimating equation obtained previously.

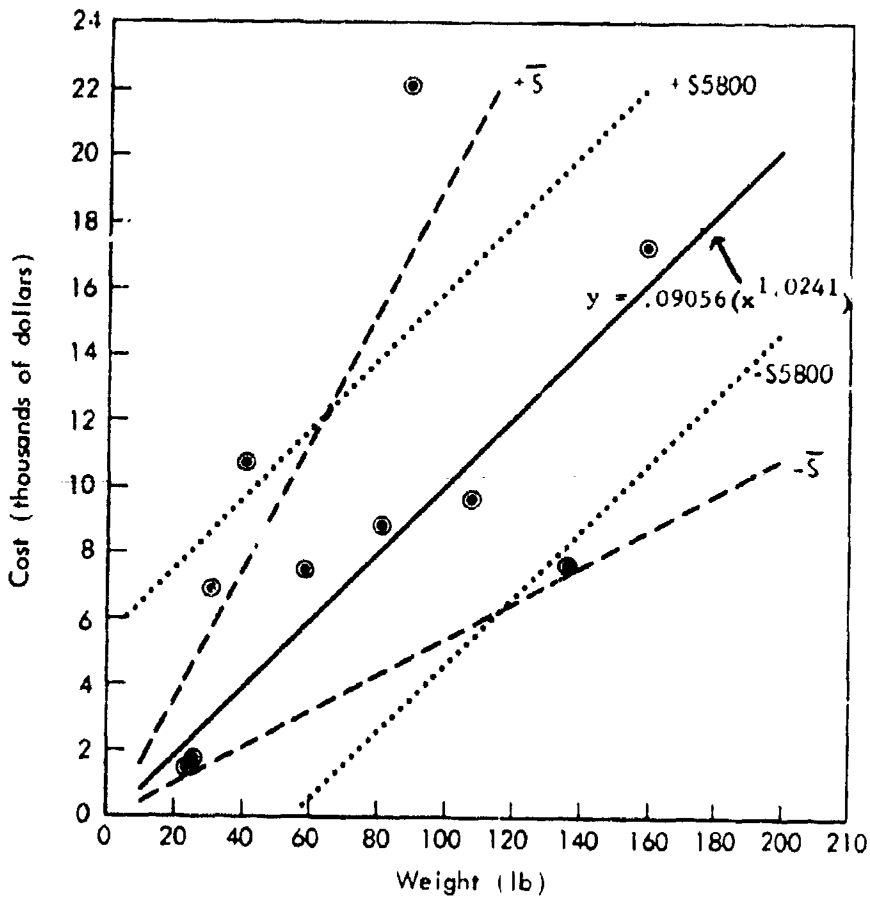

Fig. III-9—Cost versus weight on arithmetic grid

To gain further insight, let us turn to the standard error of estimate and compute a $\pm$ 1 $\bar{S}$ band about the regression line. This band is illustrated by the dashed lines in Fig. III-9. We now have a much different picture than that indicated in Fig. III-8 for the logarithmic analysis. In Fig. III-9 the $\bar{S}$ interval is an ever-widening one defined in terms of linear homogeneous functions of x. Recall that in our simple linear regression analysis $\bar{S}$ = \$5,800. If we lay off $\pm$ \$5,800 around the regression line, the results are the dotted lines in Fig. III-9. We conclude, therefore, that in this case the logarithmic regression offers no improvement over the linear regression.

The situation portrayed in Fig. III-9 has sometimes led to the suggestion that the curvilinear equation be used for small values of x (because the standard error of estimate is small) and the linear equation for large values. It is important to keep in mind that the difference between the two standard errors of estimate in Fig. III-9 stems from different basic assumptions about the variance of y-values about the regression line, not from any change in the real distribution of the variance. In the linear case, as pointed out previously, it is assumed that the variance of the y-values about the regression line is constant. In the curvilinear case the variance is still constant, but it is constant in logarithmic terms, which means that it actually increases with the magnitude of the dependent variable.

The logarithmic example contained in this section illustrates a point that is often forgotten. A logarithmic transformation of the variables has a tendency to compress and shape the original data in such a way that a statistical fit to the logarithms looks good. Very often, however, when the logarithmic analysis is transformed back into terms of the original data, the results do not appear so impressive. In sum, logarithmic transformations can be tricky and misleading. The analyst must be cautious when using them.

CURVILINEAR ANALYSIS: SECOND-DEGREE EQUATION

We have just seen that for our illustrative example a logarithmic regression does not seem to offer any improvement over the simple linear regression case. Here another type of curvilinear regression analysis will be attempted using a second-degree equation of the form:

$$y = a + bx + bx^2$$

Solving for a and b we obtain:*

$$y = .0743 + 1.6138x - .0457\ x^2$$

This equation is shown as a solid line on Fig. III-10.

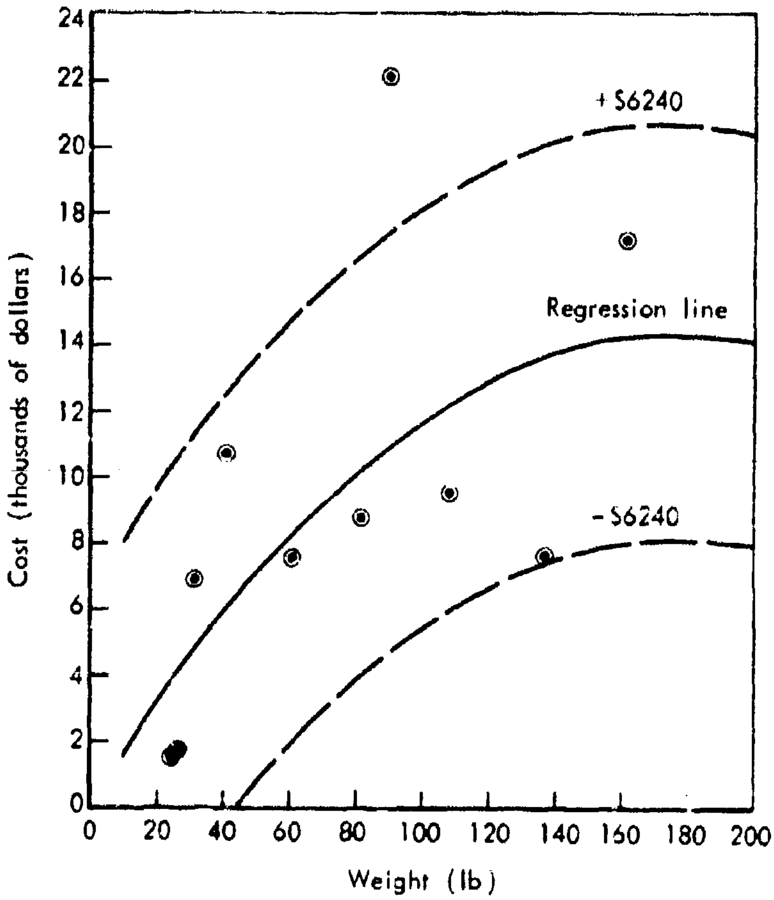

Fig. III-10—Second-degree equation

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*The procedure is given in Applied General Statistics, Third Edition, by F. E. Croxton, D. J. Cowden, and S. Klein, Prentice-Hall, Inc., pp. 419-422.

The standard error of estimate is calculated as before, except here we must add a term for x^2 and take into account the loss of the additional degree of freedom. The result is that $\bar{S}$ is greater than that obtained for the linear regression equation--\$6,240 vs \$5,800. An area bounded by $\pm 1\ \bar{S}$ around the regression line is presented in Fig. III-10.

Relating $\bar{S}$ to the mean of the sample y's gives a coefficient of variation of:

$$C = \frac{\bar{S}}{\bar{y}} = \frac{\$6,240}{\$9,069} = .69$$

Should it be desired, a prediction interval may be calculated for a value of y obtained from the estimating equation for specified values of x and x^2, but for a second-degree regression the calculation is somewhat complicated and in the present example is unlikely to add anything to the analysis.

Insofar as measures of correlation are concerned, in curvilinear analysis the coefficient of curvilinear correlation is usually referred to as the index of correlation and is denoted by the symbol ρ. ρ^2 is called the index of determination and in this example is equal to .37. To adjust this for degrees of freedom the following formula may be used:*

$$\rho^2 = \frac{\rho^2(n - 1) - (m - 1)}{n - m}$$

where m is the number of coefficients in the regression equation (m = 3 in the case of second-degree regression).

Comparing the results of the statistical analysis for the second-degree regression case with those obtained for the simple linear regression example suggests that the second-degree regression offers no improvement over the simple linear case. The standard error of estimate is increased by \$430 and the coefficient of variation is higher by 7 percentage points. The explained variation is higher by 5 percentage points, but it is questionable whether such an improvement is significant in a statistical sense.

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*This equation is shown in slightly different form in Methods of Correlation and Regression Analysis, Third Edition, by M. Ezekial and K. A. Fox, John Wiley and Sons, 1959, p. 301.

	Simple linear regression	Second-degree regression
Standard error of estimate	$5,800	$6,240
Coefficient of variation	.64	.71
Coefficient (index) of determination (unadjusted)	.32	.37
Coefficient (index) of correlation (unadjusted)	.57	.61

It is conceivable when dealing with a small sample of data that the differences in statistical measures presented above could be due purely to sampling error. In this case, for example, the difference between two (unadjusted) coefficients of determination is .05. A statistical test might indicate that the chances are very small that two random samples drawn from the assumed population would have a difference as large as this. In other words it would seem highly unlikely that the observed difference could be due to sampling variation. If this were the case, the difference between the linear regression and the second-degree regression would be considered significant.

A simple test to determine whether the incremental increase in explained variance associated with the addition of the variable x^2 (or any additional variable) is significant involves the use of the statistic F.* An F-test indicates whether the increase in explained variance is significant in relation to the remaining unexplained variance. In this case:

$$F = \frac{\text{increment of explained variance} \div \text{degrees of freedom}}{\text{remaining unexplained variance} \div \text{degrees of freedom}}$$

This can be rewritten

$$F = \frac{(r_2^{\,2} - r_1^{\,2})/1}{(1 - r_2^{\,2})/7}$$

*See Croxton, et al, p. 627.

where

$r_1^2 = r^2$ of linear regression

$r_2^2 = r^2$ of 2d degree regression

As explained earlier, the degrees of freedom are generally the sample size minus the number of parameters in the regression equation, and this holds true for the denominator of the above expression (10-3 = 7). In the numerator only one degree of freedom is involved, the incremental degree of freedom lost by adding another constant to the estimating equation.

Substituting r^2 values in the above formula·

$$F = \frac{.37 - .32}{(1 - .37)/7} = \frac{.05}{.63/7}$$

$$= .56$$

This falls far short of the critical F value of 5.59 (at a .05 level of significance),* indicating that the additional explained variance is not considered significant. In other words the net increment of explained variance associated with the introduction of x^2 (after allowance for the loss of an additional degree of freedom) is not sufficient to allow us to be reasonably confident that the improvement is not due to chance.

MULTIPLE REGRESSION ANALYSIS

Previously the simple linear regression example was extended by introducing the variable x^2 into the estimating equation. At this point we shall go back to the simple linear case and consider some of the possibilities in a multivariate analysis, e.g.:

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*Most statistics texts contain an F table showing values for levels of significance from .05 to .001. The F value of 5.59 is given for a numerator of 1 degree of freedom and a denominator of 7.

1. Introduce power or frequency into the equation.

2. Abandon weight in favor of power and frequency.

3. Use three explanatory variables, *i.e.*, power, weight, and frequency.

At this point, two technical considerations must be raised. The first is a stipulation that in the multiple regression model to be used, the explanatory variables must be non-correlated. If, for example, weight and power output were correlated, the addition of weight would not make a statistically significant contribution to the explanation of cost. The inclusion of a non-significant variable is undesirable for a very practical reason: it is almost as likely to move the result away from an accurate estimate as toward it.

Hence, before deciding whether weight can be used in conjunction with power output and frequency the relationship between them must be examined. While there are statistical techniques for testing whether or not a significant correlation exists between two variables, a simpler procedure is to examine scatter diagrams for one plotted against the other. From Fig. III-11 it is clear that no association exists between weight and frequency and very little between weight and power output.

The second consideration is that a sample of 10 will barely support simultaneous inferences about the effects of two explanatory variables. To obtain a regression equation of satisfactory reliability with three independent variables the sample should contain at least 20 observations. Consequently, we shall limit our exploration here to the following combinations of variables: weight and power output, weight and frequency, and power output and frequency.

The estimating equation for linear multiple regression analysis is of the form:

$$z = a + bx + cy$$

And for the above combinations of variables the regression equations are as follows:

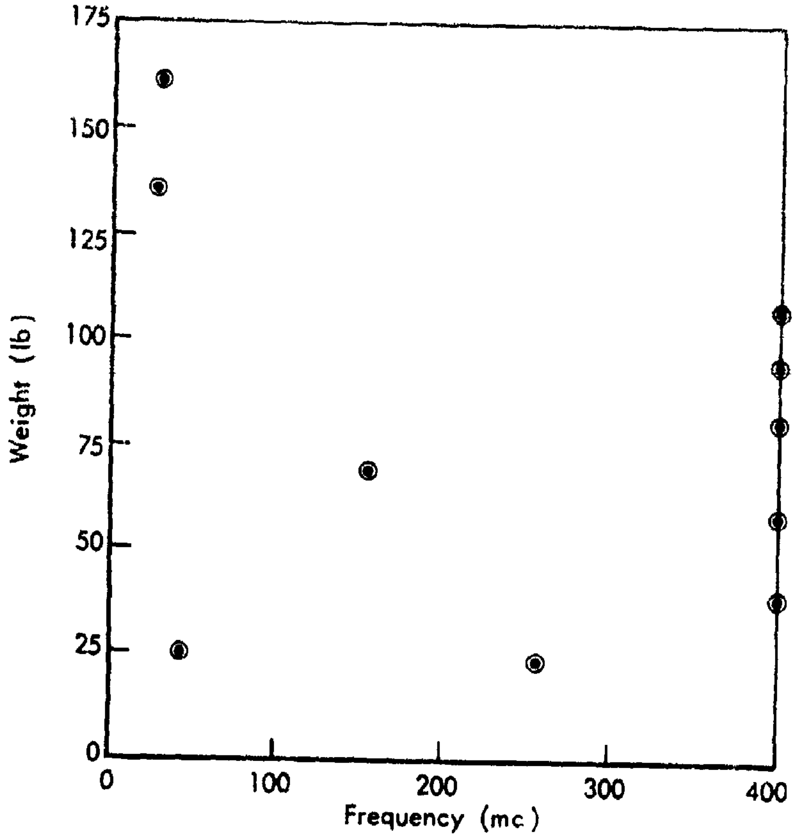

Fig. III-11a—Weight versus frequency

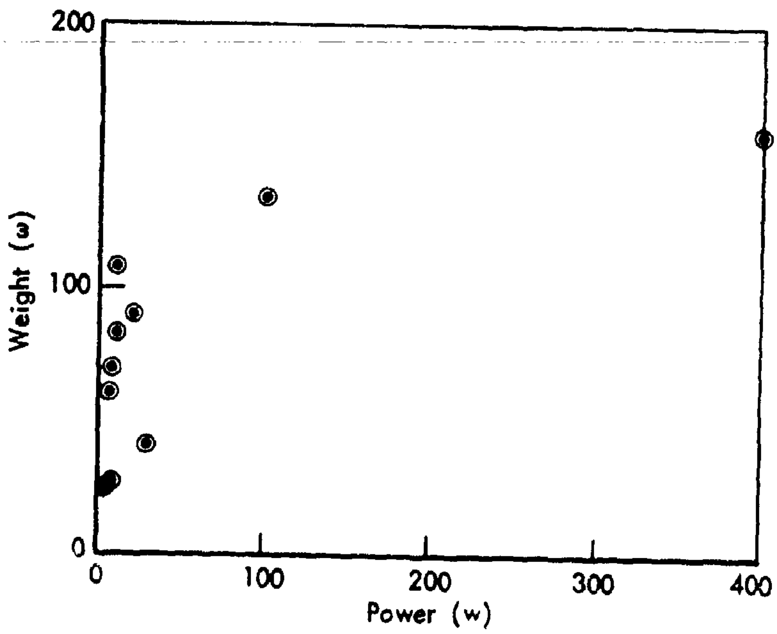

Fig. III-11b—Weight versus power

$$C = 113.85 - .4523\ W - .1308\ F$$

$$C = 2.9303 + .07338\ W + .004705\ P$$

$$C = -.5257 + .04258\ P + .02749\ F$$

where:

C = cost F = frequency

W = weight P = power output

The various statistical measures of each are compared below with those obtained for weight alone.

	Weight	Weight + Frequency	Weight + Power	Power + Frequency
Standard error of estimate ($\bar{S}$)	\$5,800	\$137,145	\$6,190	\$5,000
Coefficient of variation (C)	.64	2.83	.68	.55
Coefficient of determination $(\bar{R})^2$	.32	.04	.33	.56
Coefficient of correlation ($\bar{R}$)	.57	.2	.57	.75

The addition of frequency degrades the estimating relationship tremendously, giving a coefficient of correlation close to zero. Weight and power together are not as good as weight alone, and the only improvement seen is for the combination of power output and frequency. While it would be preferable to have a lower value for C and a higher value for $\bar{R}$, this combination should do a somewhat better job of predicting cost than would weight alone.

Earlier, we examined a curvilinear function with two variables. A non-linear relationship of that type using three variables can be examined here in an attempt to improve the reliability of the equation. With three variables the equation would be of the form

$$z = ax^b y^c$$

Again making a logarithmic transformation of the variables to facilitate computation and solving for the constant a, b, and c, we obtain

$$\log C: \ -1.1933 + .5756 \log P + .6085 \log F$$

where:

C = cost

P = power output

F = frequency

or

$$C = .00641\ P^{.5756}\ F^{.6085}$$

This equation improves the fit considerably as shown by the comparison below and is generally satisfactory on logical grounds as well since

	Linear	Curvilinear
Standard error of estimate ($\overline{S}$.	±\$5,000	+\$3,200, - \$2,370[a]
Coefficient of variation (C)	.55	+ .35, - .26[a]
Coefficient (index) of determination $(\overline{R})^2$	.51	.88
Coefficient (index) of correlation ($\overline{R}$)	.71	.94

[a]Values at the sample mean (\$9,060).

power and frequency should be causally related to cost. Given the limitations inherent in a sample of 10, the above estimating relationship is probably as good as can be derived.

DOCUMENTATION

Once an estimating relationship has been developed, a written report documenting the major data, assumptions, and analytical results is indispensable. The following guidelines for such a report are suggested.

1. The scope and coverage of the study and the resulting equations should be fully and clearly described.

2. Assuming that the study has made provision for a survey of work already performed in the area of interest (a very desirable part of a cost research study), a summary of the survey results should be presented.

3. The major input data used in the study should be provided. Both the raw and adjusted data should be documented to the extent feasible. This includes data for both the dependent and independent variables. Data should be included not only for those cost categories and characteristics included in the final estimating equations, but also for those major characteristics which were considered but were dropped in the analysis. Any adjustments to the raw data which are made should be fully described and explained. The limitations and some indication of the accuracy of the data should be provided. Since one of the outputs of a cost research study is the data base itself it should be sufficiently described so as to be usable in future studies.

4. The sources and dates of the data should be specifically identified.

5. Each dependent and independent variable considered in the study should be fully and clearly defined. Unambiguous definitions of weapon system characteristics and cost elements are usually considerably more involved than appears at first glance.

6. The major dependent versus single-independent variable scatter diagrams utilized in the study should be provided. The points on the diagrams should be labeled to identify the particular items.

7. The final equations plus other major equation forms examined in the study should be presented along with such statistics as the standard error, correlation coefficient, coefficient of variation and prediction intervals (to the extent derived) for each equation. Any other criteria felt appropriate for indicating the goodness of fit and prediction capabilities of the equations should be provided.

8. For the major final equations, tables such as Table III-3 should be presented which show the observed values of the dependent variables, the estimated values, the deviations, and the percent deviation from the observed. The average percent deviation for the sample should also be presented. This not frequently used statistic is felt to provide a good and easily understood measure of the goodness of fit.*

Table III-3

ACTUAL AND ESTIMATED COSTS OF AIRBORNE COMMUNICATIONS EQUIPMENT

Actual Cost	Estimated Cost	Deviation	Percent Deviation
$22,200	$13,700	-$8,500	-38
17,300	16,000	- 1,300	- 8
11,800	17,400	+ 5,600	+47
9,600	9,200	- 400	- 4
8,800	9,200	+ 400	+ 5
7,600	6,400	- 1,200	-16
6,800	6,900	+ 100	+ 1
3,200	4,600	- 1,400	-44
1,700	2,000	+ 300	+18
1,600	1,300	- 300	-19
		Average percent deviation	20

*Note, however, that this is not the function minimized when using the least squares technique for obtaining the equation coefficients.

In addition, a scatter diagram plotting the observed versus estimated values for the sample should be presented (see Fig. III-12). The points on the diagram should be labeled to identify the particular items.

9. The major alternative equations which were considered in the study, but rejected, should be described sufficiently for the reader to understand which were considered and why rejected. The reader should be given some feeling for the improvement gained by the selection of the final recommended forms over these other major alternatives. Alternative equations could involve such aspects as:

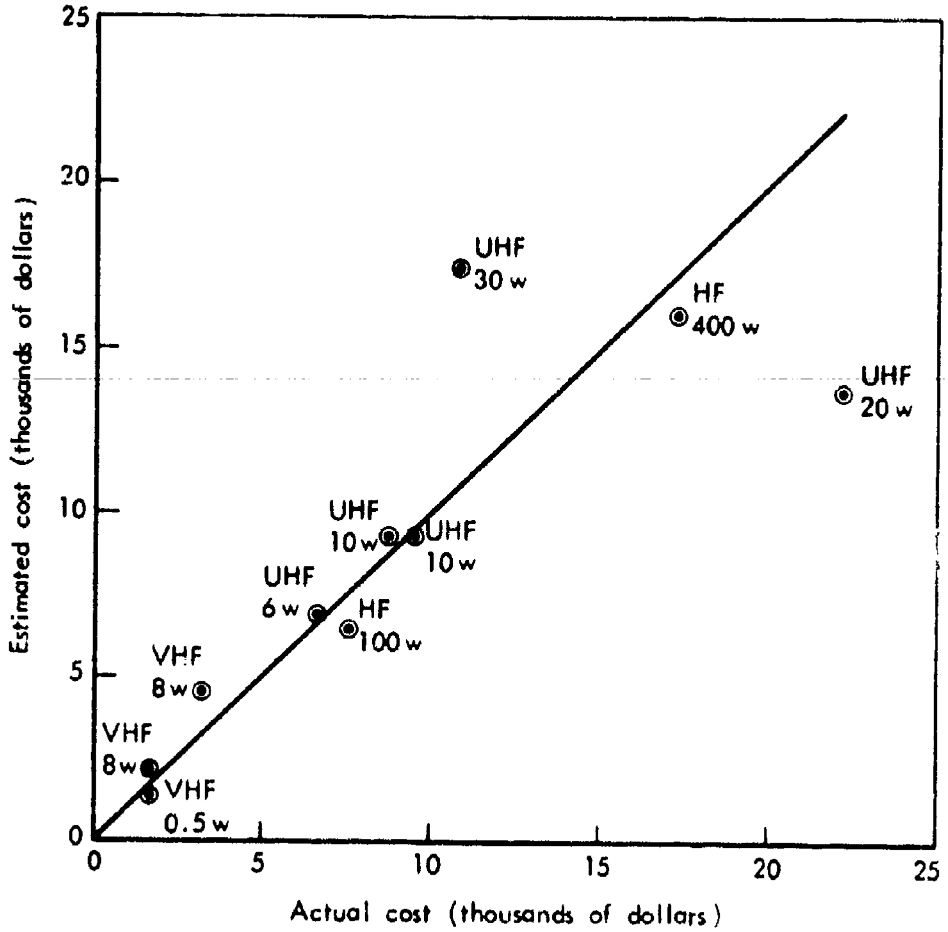

Fig. III-12—Estimated versus actual costs

a. The use of different independent variables;

b. Different forms of the equations, e.g., linear, multiplicative (i.e., linear in the logs) or non-linear forms;

c. The use of different forms of the dependent variables, e.g., cost per pound or cost per item;

d. The use of stratified dependent variables grouped into sub-categories determined by such factors as ship or missile type, weight, frequency, speed regime, etc.

10. Any special methodology should be described, perhaps in an appendix if only of specialized interest (such as a sophisticated mathematical approach).

11. The methods used should be described fully and clearly. It should be possible from the information presented in the report for a reader to reconstruct from the same data base (though not necessarily agree with) the results of the study. The major assumptions, both statistical and otherwise, used in the derivation of the equations should be explicitly stated.

12. An example to illustrate the procedure for using the final cost-estimating relations is always helpful.

13. The limitations of the final equations should, to the extent possible, be clearly delineated and be as specific as possible. The range of characteristics over which the estimating procedure applies should be clearly stated as well as any other restrictions on the population covered by the equations.

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IV. USING ESTIMATING RELATIONSHIPS

The widespread use of estimating relationships in the form of simple cost factors, equations, curves, nomograms, and rules of thumb attests to their value and to the variety of situations in which they can be helpful. Yet an estimating relationship can only be derived from information on what has occurred in the past, and the past is not always a reliable guide to the future. As all horseplayers know, the favorite runs out of the money often enough to prove that an estimate based on past performance is quite likely to be wrong. Admittedly, there may be other factors at work in the case of the racehorse, but the problem remains the same as that encountered in any attempt to predict the course of future events, i.e., how much confidence can be put in the prediction? This question dominates all others in any discussion of the use of estimating relationships.

These remarks are not intended to depreciate the value of estimating relationships. They comprise the most important tool in an estimator's kit and are in many cases the only tool. This being the case, it is essential that their limitations be understood so that they will not be used improperly. These limitations stem from two sources: (1) the uncertainty inherent in any application of statistics and (2) the uncertainty that an estimating relationship is applicable to a particular article. The first pertains primarily to articles well within the bounds of the sample on which the relationship is based and says that uncertainty exists even here. The second refers to those cases where the article in question has characteristics somewhat different from those of the sample. Extrapolating beyond the sample, although universally deplored by statisticians, is universally practiced by cost analysts dealing with advanced hardware since in most cases it is precisely those systems outside the range of the sample that are of interest. The question is whether the equation is relevant to the case at hand even though good statistical practice would question its use.

UNDERSTANDING THE ESTIMATING RELATIONSHIP

Sometimes so much emphasis is placed on statistical treatment of the data that a fundamental point is overlooked--an estimating relationship must be reasonable and must have predictive value.

Reasonableness can be tested in various ways--by inspection, by simple plots, and by some fairly complicated techniques which involve an examination of each variable over a range of possible values. Inspection will often suffice to indicate that an estimating relationship is not structurally sound. For example, the following equation resulted from an exercise at the Air Force Institute of Technology in which students were asked to develop cost-estimating relationships for small missiles:

$$C = 8347.5 + 150.6W - 1149.1R$$

where

C = cost of airframe + guidance and control
W = weight (lb)
R = range (mi)

This equation fits the data very well, but it says that as range increases, cost decreases, and this intuitively seems wrong. If cost is a function of range, we would expect the relationship to be direct rather than inverse. To investigate further, we can choose two hypothetical but reasonable values for W and R within the range of the sample data (38.5 - 157 lb for W; 5.0 - 14.8 mi for R). As the Table below shows, Missile A, although heavier and with greater range than Missile B, is estimated to be the cheaper of the two. This is contrary to most experience and suggests that a re-examination of the sample data and the equation is in order.

Hypothetical Missile	Weight of Airframe + Guidance and Control (lb)	Range (mi)	Estimated Cost of Airframe + Guidance and Control ($)
A	50	5	11,133
B	75	10	8,153

Sometimes an estimating relationship is developed to make a particular estimate, but has no predictive value outside a very narrow range. As an example of this, consider the following equation for estimating the cost of solid propellant motors for small missiles:

$$\text{Cost} = 1195.6 + .000003\ I^2$$

where I

I = total impulse

The equation fits the sample data very well:

Missile Motor	Observed Cost	Estimated Cost
A	$2600	$2660
B	1700	1693
C	1250	1265
D	1750	1781

If it were appropriate to use statistical measures for a sample of four, one could say that this relationship explains over 99 percent of the total variance. But, note that the constant 1195.6 accounts for 94 percent of the cost of Motor C and that the cost of all motors smaller than D will be about $1200. On the other hand, because of the I^2 term, the influence of total impulse is likely to be too pronounced for motors larger than those in the sample.

A common method of examining the implications of an estimating relationship for values outside the range of the sample is to plot a scaling curve as illustrated in Fig. IV-1.* The theory underlying a scaling curve is that as an item increases in weight (or some other dimension) the incremental cost of each additional pound (or square foot, watt, horsepower, etc.) will decrease or increase in some predictable way. Thus, in Fig. IV-1 the cost per pound of an electrical power subsystem in a manned spacecraft decreases from about $4200 to $1400 as the total weight

*Scaling curves may be plotted on either arithmetic or logarithmic graph paper as shown in Fig. IV-1. Because the log-linear representation is more convenient to work with, this is the one generally used by cost analysts.

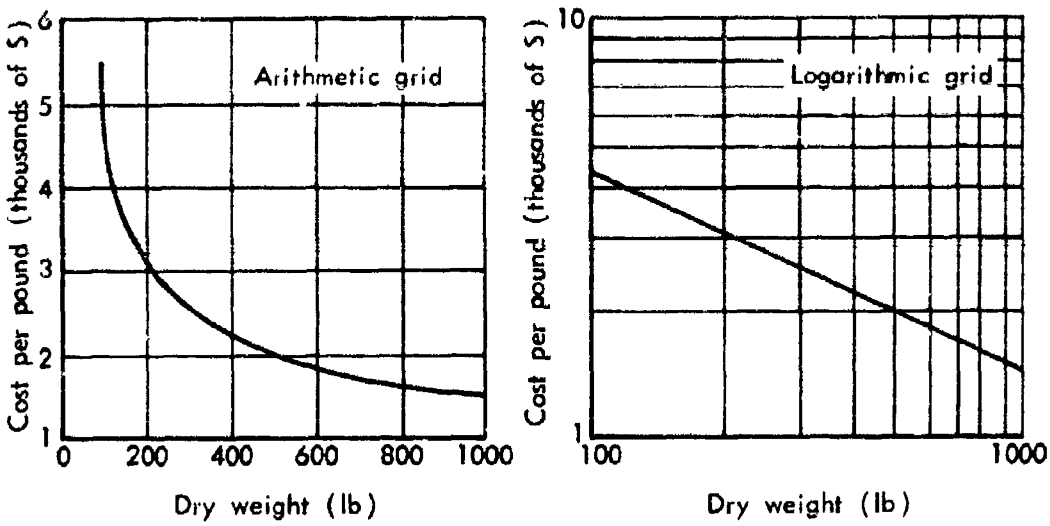


Fig. IV-1 — Scaling curve: cost per pound versus dry weight

increases from 100 to 1000 lb. The slope of the curve is fairly steep, and if the curve were extended to the right, one might expect to see some flattening. Eventually, the cu[illegible] might become completely flat when no more economies of scale can b[illegible] rea[illegible]ed, but it is unlikely that the slope would ever become positive.

Now examine Fig. IV-2 where total impulse is plotted against cost per pound-second based on values obtained from the estimating relationship above. Two differences are immediately seen. First, the left-hand portion of the curve is unusually steep. Second, the slope becomes positive when total impulse exceeds about 24,000 lb-sec. In some instances, fabrication problems increase with the size of the object being fabricated and a positive slope may result. No such problems are encountered in the manufacture of small solid propellant rocket motors, however, and continued economies of scale are to be expected.

A final point to be made about Fig. IV-2 is that a more useful estimating relationship could have been obtained by drawing a trend line than by fitting a curve to the four data points. With a small sample, it is often possible to write an equation that fits the data perfectly, but is useless outside the range of the sample. Statistical manipulation of a sample this size rarely produces satisfactory results.

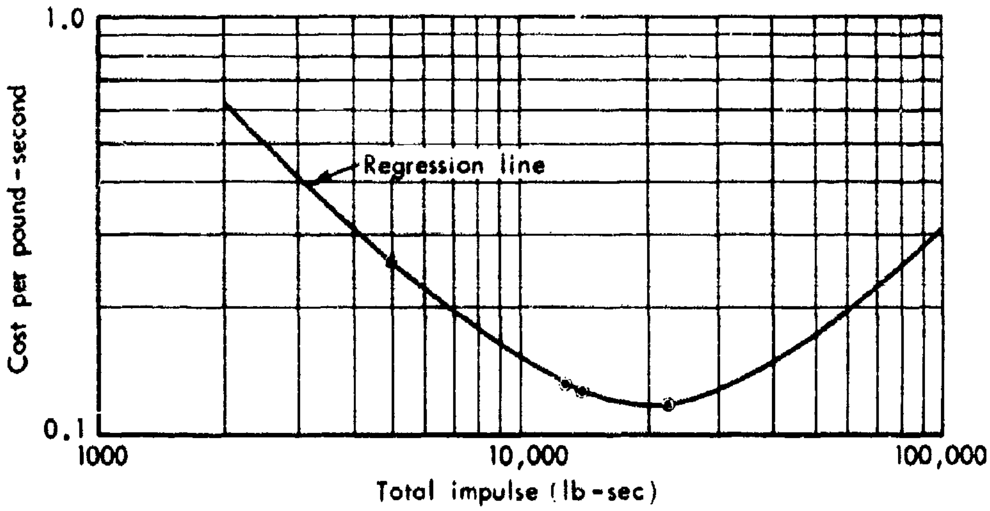


Fig. IV-2—Cost per pound-second versus total impulse

A final example of the kind of error that undue reliance on statistical measures of fit may give rise to is based on an estimating equation for aircraft airframes. Initially, an equation for estimating airframe production labor hours was based on a sample of 44 aircraft. It then seemed that grouping aircraft by type should give better correlation, and in fact by considering bombers, fighters, trainers, etc., separately the average deviation between estimates and actual values was markedly reduced. In the case of trainer aircraft, for example, average deviation was reduced from 20 to 6 percent, and a more useful estimating relationship obtained. In the case of fighters, however, while average deviation was reduced from 15 to 11 percent, the estimating equation, shown below, had a visible flaw:

$$\text{Manhours/lb} = 4.28\ (\text{weight})^{1.08}\ (\text{speed})^{.4}$$

The flaw is that the exponent of weight is greater than 1.0, and this means that when speed is held constant and weight increased, the manhours per pound of airframe weight will increase. This can be seen in Fig. IV-3. The dashed lines show scaling curves derived from the total sample of 44 aircraft These portray the normal relationship--

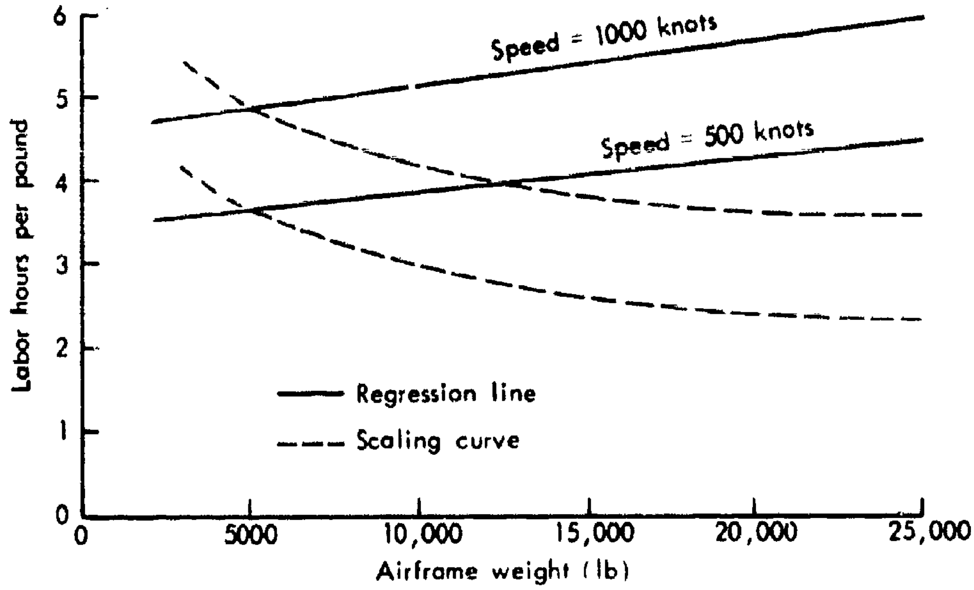


Fig. IV-3—Comparison of regression lines with scaling curves

as weight increases, hours per pound decrease. The regression equation gives the opposite results because the general trend in fighter aircraft has been for increased speed to be accompanied by increased weight, and this causes an emphasis on the weight variable. One cannot assume, however, that all new fighters will conform to this trend; and the equation, if used at all, would have to be used with great care.

The advice is frequently given that one should not use an estimating relationship mechanically. This implies two things: (1) that the function must be thoroughly understood and (2) that the hardware involved must be understood as well. To illustrate the former, let us examine an estimating relationship for direct manufacturing hours derived from a sample of Navy and Air Force airframes:

$$H_{100} = 1.45W^{.74}S^{.43},$$

where H_{100} = manufacturing labor hours required to produce the 100th airframe,

W = gross takeoff weight (lb),

S = maximum speed (kn).

The multiple correlation coefficient is 0.98 and the coefficient of variation is .016 (in logarithmic terms). Despite these very satisfactory measures of fit, it is always interesting to compare the actual hours for each airframe in the sample with those estimated by the equation to get a better understanding of how the relationship relates to the real world. In such a comparison, as shown by the summary table below, 33 percent of the estimates differ from the actuals by more than 20 percent, and 7 percent differ by more than 30 percent. These figures imply that a person who has nothing to rely on but the estimating relationship may or may not come up with a good estimate. However, if

Difference Between Actual Hours and Estimated Hours	Number of Airframes	Percentage of Sample
10% or less	15	56
11-20%	3	11
21-30%	7	26
31-40%	2	7

the poorer results can be explained in some way, the analyst is then in a much better position to understand the strengths and weaknesses of the equation.

Since this estimating relationship is based on gross takeoff weight and maximum speed, an initial hypothesis to explain the variations might be that at one end of the weight or speed range or for some combinations of weight and speed, the estimates decrease in quality. In this case, however, as shown by Fig. IV-4, the poorer estimates are scattered throughout the sample, thus indicating no consistent bias because of the explanatory variables.

A second hypothesis might be that the manufacturing history of the airframes in the sample should explain the discrepancies, and, in

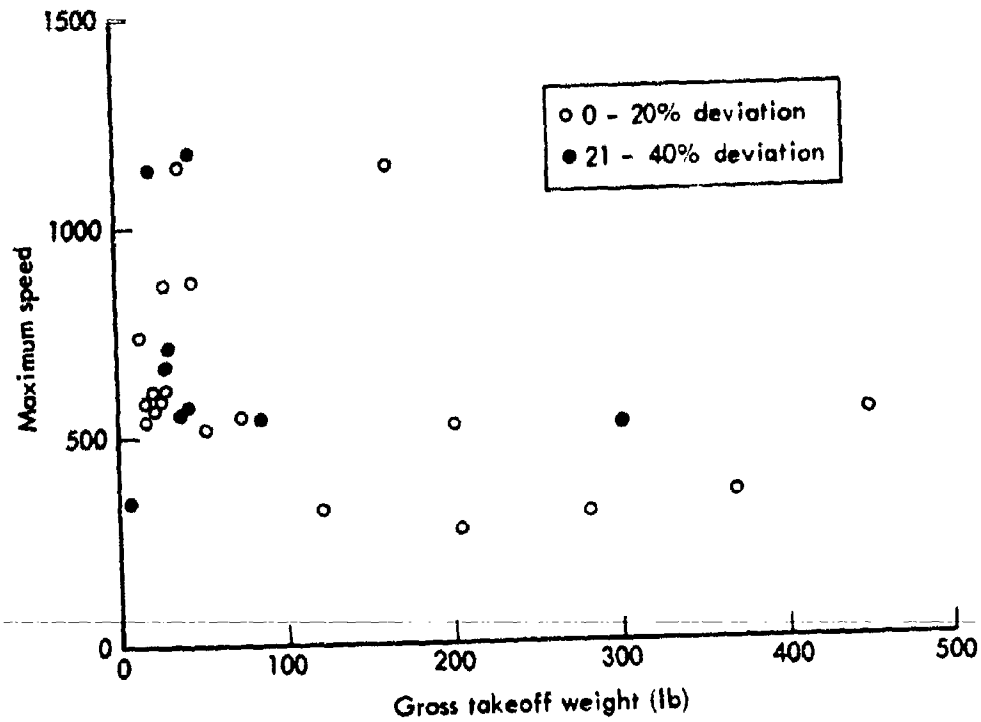


Fig. IV-4—Plot of data sample

general, this hypothesis seems valid. Of the nine airframes in the sample for which estimates differed from actuals by 20 percent or more, several were considered "problem" airframes, that is, airframes where the manufacturer had an abnormal number of problems in meeting weight and/or performance specifications. Interestingly enough, these were not aircraft in which a major state-of-the-art advance was being attempted. Another cause for discrepancy was discovered to be interspersion of different models of the same aircraft in a single lot--e.g., reconnaissance versions of a bomber were interspersed among bomber airframes--and changes of this kind increase direct labor requirements. The two airframes for which the estimates were the poorest, requiring almost 40 percent less labor than the equation predicts, were vastly different ones--a large transport and a supersonic fighter. One of these benefited from the manufacturer's concurrent experience with a commercial aircraft of similar configuration. The other cannot be explained; it simply appears that the labor content of this aircraft was unusually low.

However, while it never is possible to resolve all the uncertainties, with information such as this, an estimator can feel reasonably confident that the estimating relationship does not contain a systematic bias, that it should be applicable to normal production programs, and that it provides reasonable estimates throughout the breadth of the sample.

UNDERSTANDING THE HARDWARE

This sample included aircraft with gross takeoff weights of 6,100 lb to 450,000 lb and maximum speeds of from 300 kn to 1,200 kn. Suppose a proposed new aircraft has a gross weight of 500,000 lb or a maximum speed of 1,700 kn. Should the estimating equation be used here? The same question could arise for an aircraft whose weight and speed are in the sample range, but is to be fabricated by a new process or out of a new material. Again, the estimator must decide whether the equation is relevant or how it can be modified to be useful. All of this points to the fact that an estimating relationship can be used properly only by a person familiar with the type of equipment whose cost is to be estimated. To say that a person estimating the cost of a destroyer should know something about destroyers may be a truism, but an estimator is sometimes far removed from the actual hardware. Further, he may be expected to provide costs for everything from air-to-air missiles one week to a new anti-ballistic missile system the next. The tendency in such a situation may be to use whatever equation looks best without taking a detailed look to determine whether it really is applicable or not.

To illustrate the problem, let us assume that a new bomber is proposed with a gross weight of 450,000 lb and a maximum speed of 1,700 kn. The estimating equation discussed above may be inappropriate because the speed is so far beyond the range of the sample. On the other hand, no equation exists for aircraft in that speed range, and an estimate is required. This may be regarded as the normal situation, and one has no choice but to make do with what is available. In this example, use of the equation gives 542,000 direct labor manufacturing hours.

The next step is to compare the result with other somewhat similar systems to see if the estimate appears reasonable. Thus, in this instance one might plot hours versus gross weight for several other large aircraft as in Fig. IV-5. The supersonic bomber (SSB_1) is substantially

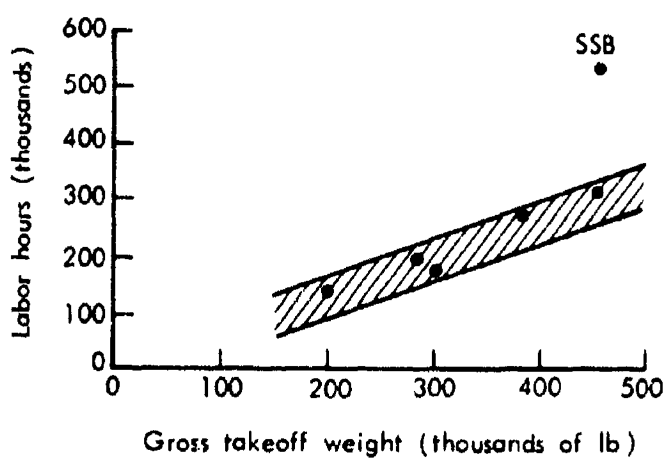


Fig. IV-5 —Trend line for large aircraft above the trend, and this is as it should be. A 1,700-kn airframe is going to be more difficult to build than a subsonic airframe of the same size, and lacking any other information an estimator might be inclined to accept the figure of 542,000 hr. In this case, however, all the airframes in the sample were fabricated almost entirely of aluminum, while an airframe built to withstand the heat generated by sustained flight in the atmosphere of around Mach 3 will require a metal such as stainless steel or titanium. The question that arises is whether the speed variable in the equation fully accounts for this change in technology.

One way to approach this question is to plot a second scatter diagram, this time with speed as the independent variable. Figure IV-6 shows labor hours per pound of airframe weight plotted against speed

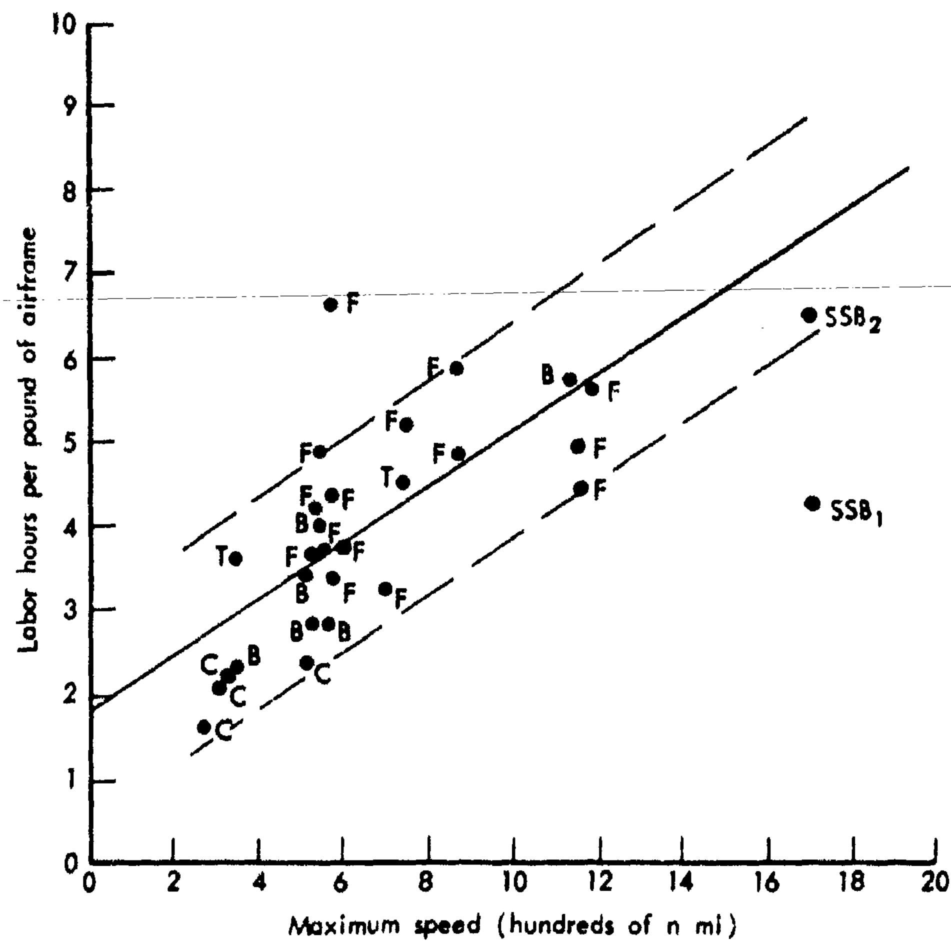


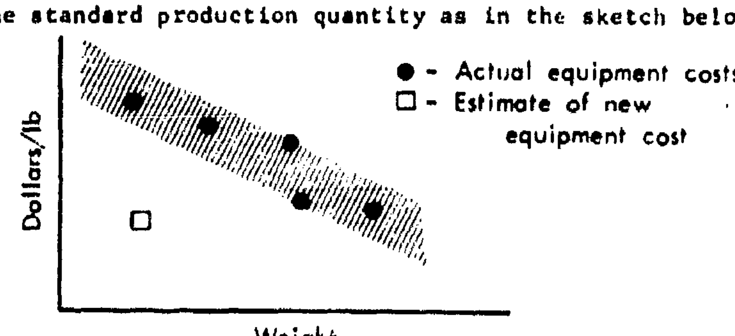
Fig. IV-6—Labor hours per pound versus maximum speed with a calculated line of best fit drawn through the scatter. Assuming an airframe weight of 125,000 lb out of a gross weight of 450,000 lb, the estimate of 542,000 hr is equal to 4.3 hr/lb of airframe which (shown on Fig. IV-6 as SSB_1) is not only below the calculated trend line, it is below any reasonable trend line that can be drawn through the sample. At this point, we might say that we have three estimates: 542,000 hr based on speed and weight, about 300,000 hr based on weight alone (from Fig. IV-5) and about 925,000 hr based on speed alone (from Fig. IV-6--7.4 hr/lb × 125,000 lb = 925,000 hr). More information is needed to narrow this range, and although information on this subject is something less than abundant, several experimental and prototype aircraft have been fabricated using stainless steel and titanium.

One manufacturer, on the basis of his experience with several prototypes, maintains that a titanium airframe requires twice the hours of an aluminum airframe. This is interesting but not very helpful information because manufacturing hours for an aluminum airframe can vary considerably. A second indication is more precise. An examination of the fragmentary data available on several different airframes with speeds of Mach 3 and above tends to show that they require about 1.5 times as many hours as are estimated by the estimating relationship above. This implies 813,000 hr or 6.5 hr/lb for the supersonic bomber. This point is shown as SSB_2 on Fig. IV-6. On the basis of what is currently known, this appears to be a reasonable estimate. One could go further, of course, and make another independent estimate using a different estimating relationship. For most kinds of hardware, an estimator does not have this option because estimating relationships are not all that plentiful. In the case of airframes, however, a number of equations have been developed over the years, and it is a good idea to use one to confirm an estimate made with another.

JUDGMENT

The need for judgment is often mentioned in connection with the use of estimating relationships, and while this need may be self-evident, one of the problems in the past is that there has been too much judgment and too little reliance on estimating relationships. One problem, that of introducing personal bias along with judgment, has been studied in other contexts and the conclusions are probably applicable here. In brief, it appears that a person's occupation or position strongly influences his forecasts. Thus, one can expect to find a consistent tandancy toward low estimates among those persons whose best interests are served by low estimates, e.g., proponents of a new weapon or support system whether in industry or government. Similarly, there are people, again both in industry and government, whose bread is buttered on the side of caution. As a consequence, their estimates are likely to run higher than would be the case were they free from all external pressures. (In all fairness to this latter group, however, it must be said that overestimates are sufficiently rare to suggest that caution is not a quality to be despised.)

The primary use of judgment should be to decide (1) whether an estimating relationship can be used for an advanced system, and (2) if so, what adjustments may be necessary to take into account the impact of technology not present in the sample. Judgment is also required to decide whether the results obtained from an estimating relationship are reasonable. This does not mean reasonable according to some preconception of what the cost ought to be, but reasonable when compared to what similar hardware has cost in the past. A typical test for reasonableness is to look at a scattergram of costs of analogous equipment at some standard production quantity as in the sketch below.



The estimate of the article may be outside the trend lines of the scattergram and still be correct, but an initial presumption exists that a discrepancy has been discovered and this discrepancy must be investigated. An analyst who emerges from his deliberations with an estimate implying that new, higher performance equipment can be procured for less than existing hardware knows his task is not finished. If after some research he is convinced that the estimate is correct, he should then be prepared to explain what new development is responsible for the decrease in cost.

What he should not do is raise the cost arbitrarily by some percent to make it appear more acceptable or because he has a visceral feeling that the estimate is too low. (Visceral judgments are the province of management and are generally occasioned by reasons somewhat removed from those discussed here.) Judgments based on evidence of some kind that an estimate is too high or too low are another matter, and the only injunction to be observed is that the change by fully documented so that: (1) the estimate can be thoroughly understood by others, and (2) the equations can be re-examined in the light of the new data.

V. THE LEARNING CURVE

For many years now it has been standard practice throughout the aerospace industry to make use of what have been variously called "learning," "progress," "improvement," or "experience" curves to predict reductions in cost as the number of items produced increases. This learning process is a phenomenon which exists in many industries; its existence has been verified by empirical data and controlled tests. While there are several different hypotheses about the exact manner in which this learning or cost reduction occurs, the main content of learning curve theory is that each time the total quantity of items produced doubles, the cost per item is reduced to some constant percentage of its previous value. Alternative forms of the theory refer to the incremental (unit) cost of producing an item at a given quantity or to the average cost of producing all items up to a given quantity. If, for example, the cost of producing the 200th unit of an item is 80 percent of the cost of producing the 100th item, the cost of the 400th unit is 80 percent of the cost of the 200th, and so forth, then the production process is said to follow an 80 percent <u>unit</u> learning curve. If the average cost of producing all 200 units is 80 percent of the average cost of producing the first 100 units, etc., then the process follows an 80 percent <u>cumulative</u> average learning curve.

Either formulation of the theory results in an exponential function that is linear on logarithmic grids. Figure V-1 shows a unit curve for which the reduction in cost is 20 percent with each doubling of cumulative output, the upper figure showing the curve on arithmetic grids and the lower on logarithmic grids. The arithmetic plot emphasizes an important point--that the percentage reduction in cost in each unit is most pronounced for the early units. On an 80 percent curve, for example, cost decreases to 28 percent of the original value over the first 50 units. Over the next 50 units, it declines only five more percentage points, i.e., down to about 23 percent of unit one cost.

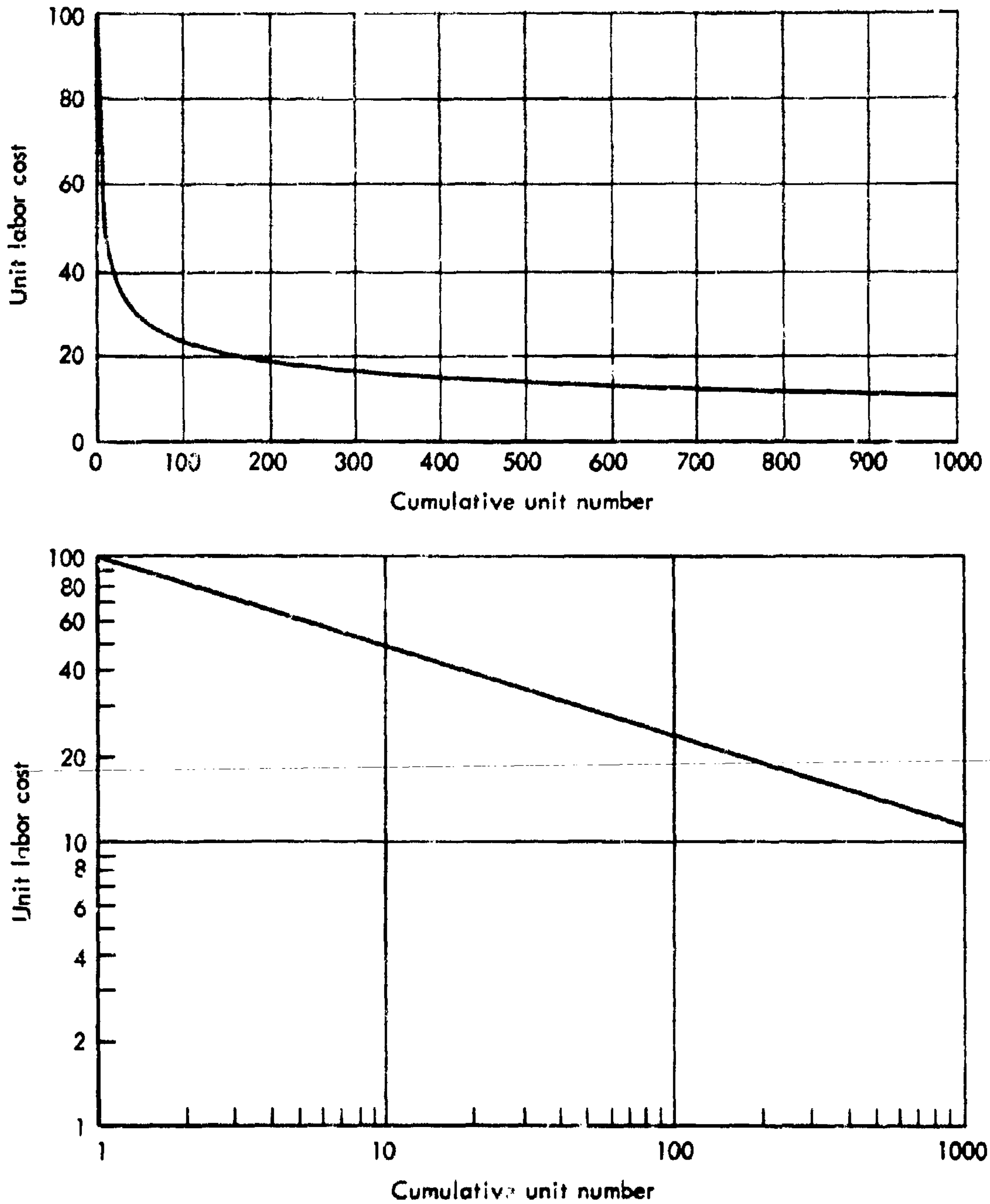


Fig. V-1—The 80 percent learning curve on arithmetic and logarithmic grids

The factors that account for the decline in unit cost as cumulative output increases are numerous and not completely understood. Those most commonly mentioned are:

1. Job familiarization by workmen, which results from the repetition of manufacturing operations.
2. General improvement in tool coordination, shop organization, and engineering liaison.
3. Development of more efficiently produced sub-assemblies.
4. Development of more efficient parts-supply systems.
5. Development of more efficient tools.
6. Substitution of cast or forged components for machined components.

This is not a complete list of the relevant factors, of course, and it tends to understate the importance of the item sometimes considered the most important--labor learning. Labor cost, however, cannot decline through experience gained by workmen unless management also becomes more efficient. In other words, it is also necessary for management to organize and coordinate the work of all manufacturing departments more efficiently so that parts and assemblies will flow through the plant smoothly.

Labor cost is not the only element of manufacturing that declines as cumulative output increases. A learning curve also exists for unit materials cost. The materials category frequently includes a great deal of purchased equipment, which in turn includes a substantial number of engineering, tooling, and labor hours. These hours decline as production quantities increase, and the contractor who buys in successive lots is generally able to negotiate a lower price for each lot. Decreases in raw material costs are generally attributed to two factors: as cumulative output increases, (1) the workmen learn to work the raw materials more efficiently and so cut down spoilage and reduce the rejection rate, and (2) management learns to order materials from suppliers in shapes and sizes that reduce the amount of scrap that must be shaved and cut from the pieces of sheet, bar, etc., to fabricate the item of equipment. Substitution of forgings for machined parts also reduces the amount of scrap material. An additional factor probably responsible to a lesser extent for the decline in materials cost is the pricing policy of the raw material suppliers. These suppliers generally reduce the price per pound for the various kinds of raw materials if an order is sufficiently large. While the learning curve pertains to cost reductions as materials are applied to successive lots and not to reductions due to volume purchases, segregation of the two effects is imperfect. This accounts for some of the difference in learning curve slopes.

A third major component of cost--overhead--also declines with cumulative output, but as a result of the method of allocating overhead, not because of a perceptible relationship between overhead rate and cumulative output. Direct labor hours per unit decline as cumulative output increases and overhead is often distributed to each unit on the basis of direct labor cost or hours. As a consequence, it is inappropriate to talk about a learning curve for this element of cost.

THE LINEAR HYPOTHESIS

This relationship between cost and quantity may be represented by an exponential (log-linear) equation of the form

$$C = aX^b$$

where X equals cumulative production quantity. The relationship corresponds to a unit or a cumulative average learning curve according to whether C is the cost of the Xth unit or the average cost of the first X units. The constant <u>a</u> is the cost of the first unit produced. The exponent <u>b</u> which measures the slope of the learning curve, bears a simple relationship to the constant percentage to which cost is reduced as the quantity is doubled. If the fraction to which cost decreases when quantity doubles is represented by <u>p</u>, we have

$$p = \frac{C(2X)}{C(X)} = \frac{a(2X)^b}{aX^b} = 2^b\text{, or,}$$

$$b = \frac{\log p}{\log 2}$$

For example, if the percentage reduction in cost for each doubling of quantity is 80 percent, the corresponding value of b is: log .80/log 2, or -.322.*

If a production process follows a unit learning curve of the form $C = aX^b$, the cumulative cost of producing N units is given by

$$C = a\sum_{x=1}^{N} X^b$$

The cumulative average cost, A, of producing N units is then

$$A = \frac{a}{N}\sum_{x=1}^{N} X^b$$

The relationship between the unit curve and the cumulative average curve is shown by Fig. V-2. The relationship between A and N is not log-linear; however, as N becomes larger, A approaches asymptotically the value

$$\frac{a}{1+b}\ X^b$$

*In learning curve literature the term "slope" has not only its usual meaning but also refers to this percentage reduction, e.g., an 80 percent slope means a curve with a b value of -.322.

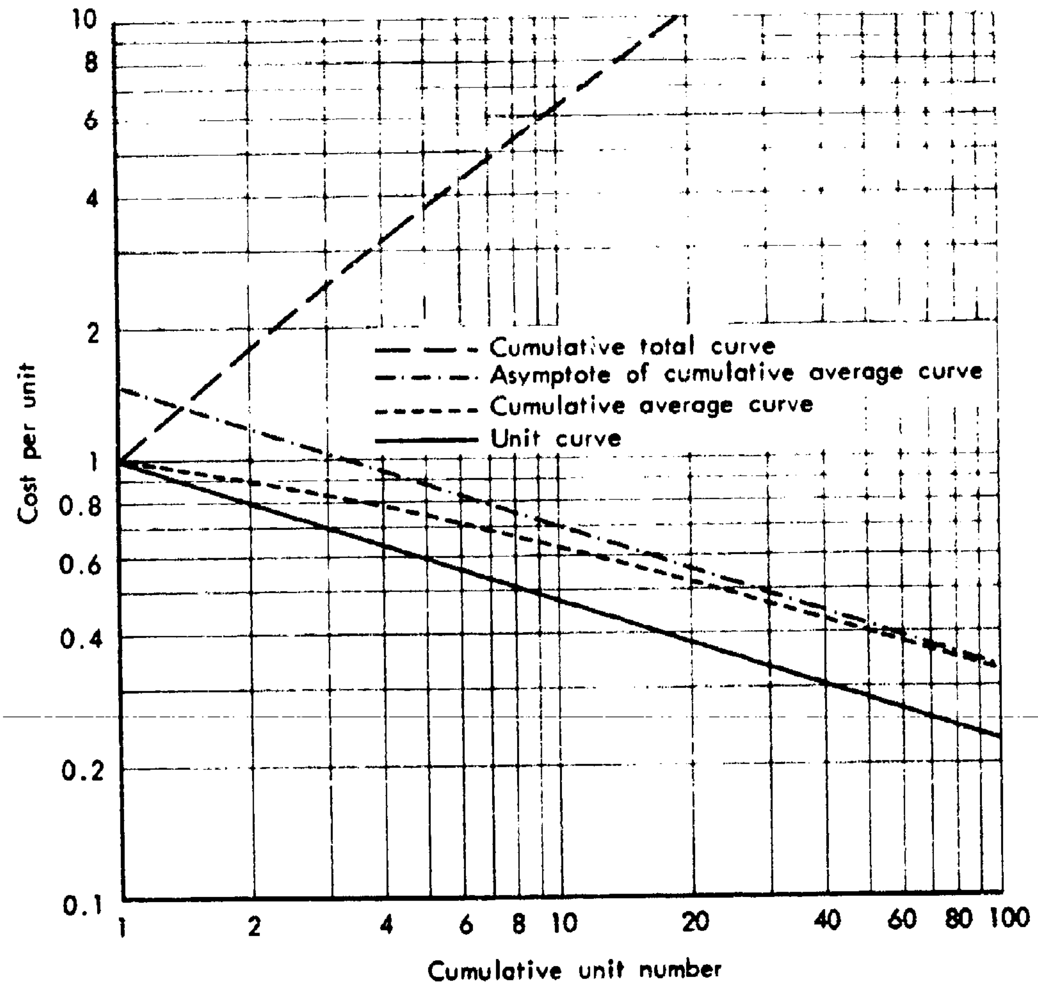


Fig. V-2—Linear unit curve

which differs from the expression for unit cost only by the constant factor, $1/(1+b)$. Consequently, if unit cost has been estimated at a sufficiently large quantity, the cumulative average cost for the same quantity may be approximated by multiplying the unit measure by $1/(1+b)$.*

When a production process follows a cumulative average curve rather than a unit curve, the basic functional form is still $C = aX^b$ but can be written $A = aX^b$ where A is the average cost of the first X units. The cumulative cost for producing N units is simply AN, and the unit cost is obtained from the equation

$$a\left[X^{1+b} - (X-1)^{1+b}\right]$$

The relationship between a linear cumulative average curve and the resulting unit curve is illustrated by Fig. V-3.

These equations may appear cumbersome to work with but in practice much of the work involved in using learning curves has been made simpler by the preparation of tables giving the relationship between cumulative total, cumulative average, and unit cost for a range of slopes and quantities.** Tables V-1 and V-2 give values of these equations for selected slopes and quantities when a is equal to one. Use of more detailed tables is recommended, but to determine approximate solutions for values not listed, one may interpolate between given values of quantity and slope.

To illustrate how the tables are used, assume a linear unit curve with a slope of 95 percent. From the first row in Table V-1, it can be seen that the cost of unit 2 is 95 percent of the cost of unit 1. Similarly, the cost of unit 4 is 95 percent of the cost of unit 2

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*Whether or not a quantity is sufficiently large so that the asymptotic method will provide a good approximation depends on the slope of the learning curve. For the 80 percent curve, the asymptotic method produces an error of about 1 percent at quantity 100; for a 75 percent curve, the error at quantity 100 is almost 5 percent and does not decrease to 1 percent until a quantity in excess of 1,000 has been reached.

**See for example, The Experience Curves, Vol. I (67-84%) and Vol. II (85-99%), Army Missile Command, Redstone Arsenal, Alabama (available from the Defense Documentation Center).

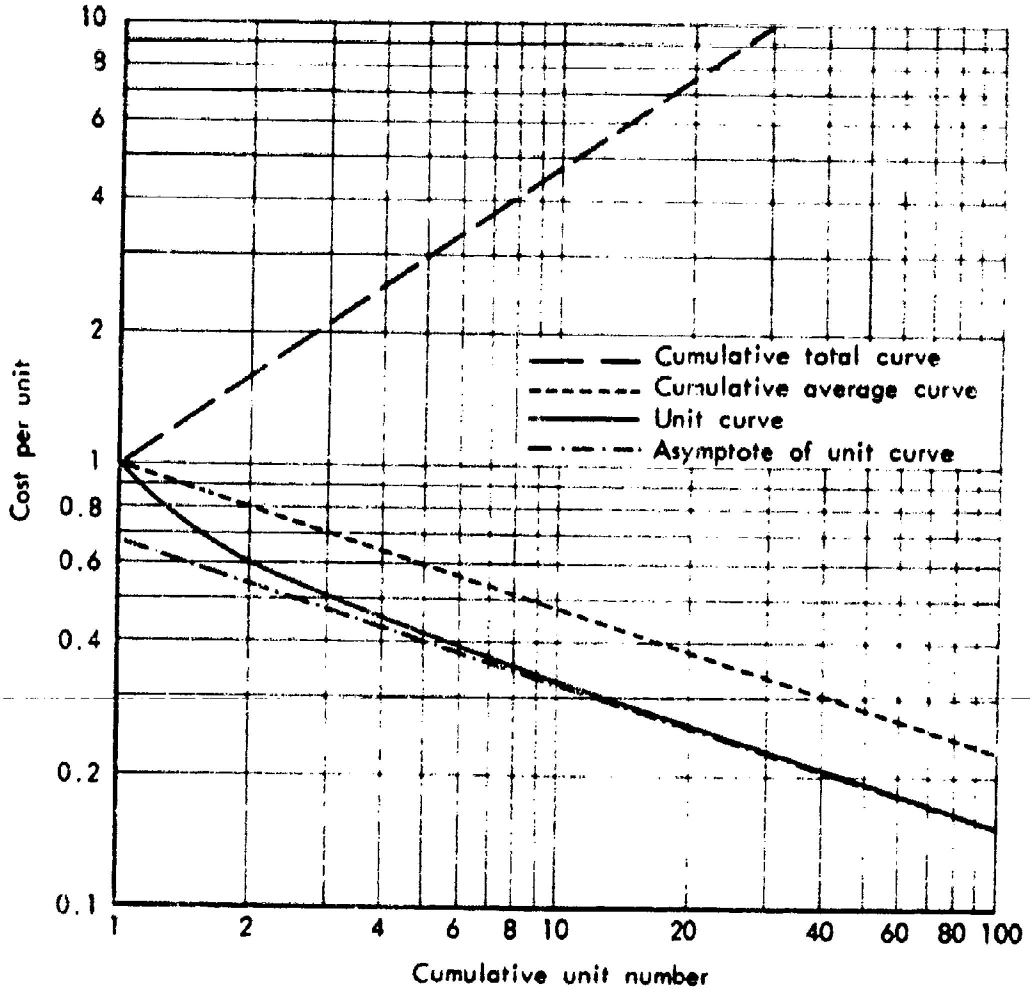

Fig. V-3—Linear cumulative average curve

Table V-1

SLOPE — QUANTITY FACTORS FOR THE LOG-LINEAR UNIT CURVE

Item	Slope	Quantity												
		2	3	4	5	7	10	25	50	100	250	500	1000	2000
Log-Linear Unit Curve (C)	95	.950	.922	.903	.888	.866	.843	.788	.749	.711	.665	.631	.600	.670
	90	.900	.846	.810	.783	.744	.705	.613	.552	.497	.432	.389	.350	.315
	85	.850	.773	.723	.686	.634	.583	.470	.400	.340	.274	.233	.198	.188
	80	.800	.702	.640	.596	.535	.477	.355	.284	.227	.169	.135	.108	.0866
	75	.750	.634	.563	.513	.446	.385	.263	.197	.148	.101	.0758	.0569	.0427
	70	.700	.568	.490	.437	.367	.306	.191	.134	.0935	.0584	.0408	.0286	.0200
	65	.650	.505	.423	.368	.298	.239	.135	.0879	.0571	.0323	.0210	.0137	.0088
Cumulative Average Curve (A)	95	.975	.957	.944	.932	.915	.895	.844	.804	.766	.716	.681	.647	.615
	90	.950	.915	.889	.868	.835	.799	.708	.643	.581	.508	.457	.412	.371
	85	.925	.874	.836	.806	.762	.712	.692	.510	.438	.355	.303	.258	.213
	80	.900	.834	.786	.747	.690	.632	.492	.402	.327	.246	.198	.159	.127
	75	.875	.794	.737	.692	.626	.559	.408	.315	.242	.168	.127	.0961	.0723
	70	.850	.756	.690	.639	.566	.493	.336	.246	.178	.114	.0818	.0574	.0405
	65	.825	.718	.644	.589	.510	.434	.276	.191	.130	.0771	.0514	.0340	.0224
Ratio (A/C)	95	1.03	1.04	1.05	1.05	1.06	1.06	1.07	1.07	1.08	1.08	1.08	1.08	1.08
	90	1.06	1.08	1.10	1.11	1.12	1.13	1.15	1.16	1.17	1.18	1.18	1.18	1.18
	85	1.09	1.13	1.16	1.17	1.20	1.22	1.26	1.28	1.29	1.30	1.30	1.30	1.30
	80	1.13	1.19	1.23	1.25	1.29	1.32	1.39	1.42	1.44	1.46	1.47	1.47	1.47
	75	1.17	1.25	1.31	1.35	1.40	1.45	1.55	1.60	1.64	1.66	1.68	1.69	1.69
	70	1.21	1.33	1.41	1.46	1.54	1.61	1.76	1.84	1.90	1.95	1.99	2.00	2.02
	65	1.27	1.42	1.52	1.60	1.71	1.82	2.04	2.17	2.28	2.39	2.45	2.48	2.52

Table V-2

SLOPE — QUANTITY FACTORS FOR THE LOG-LINEAR CUMULATIVE AVERAGE CURVE

Item	Slope	Quantity												
		2	3	4	5	7	10	25	50	100	250	500	1000	2000
Unit Curve (C)	95	.900	.866	.844	.829	.806	.784	.731	.694	.659	.615	.585	.555	.528
	90	.800	.739	.701	.675	.638	.602	.521	.469	.421	.366	.330	.297	.267
	85	.700	.619	.571	.538	.494	.452	.362	.307	.260	.210	.178	.152	.129
	80	.600	.506	.454	.418	.371	.329	.242	.193	.154	.115	.0917	.0734	.0587
	75	.500	.402	.348	.314	.269	.230	.155	.116	.0867	.0592	.0444	.0333	.0250
	70	.400	.305	.255	.224	.185	.152	.0936	.0652	.0455	.0284	.0198	.0139	.00972
	65	.300	.216	.174	.149	.118	.0935	.0519	.0335	.0217	.0123	.00796	.00517	.00336
Log-Linear Cumulative Average Curve (A)	95	.950	.922	.903	.888	.866	.843	.788	.749	.711	.665	.631	.600	.570
	90	.900	.846	.810	.783	.744	.705	.613	.552	.497	.432	.389	.350	.315
	85	.850	.773	.723	.686	.634	.583	.470	.400	.340	.274	.233	.198	.168
	80	.800	.702	.640	.596	.535	.477	.355	.284	.227	.169	.135	.108	.0866
	75	.750	.634	.563	.513	.446	.385	.263	.197	.148	.101	.0758	.0569	.0427
	70	.700	.568	.490	.437	.367	.306	.191	.184	.0935	.0584	.0408	.0286	.0200
	65	.650	.505	.423	.368	.298	.239	.135	.0879	.0571	.0323	.0210	.0137	.00888
Ratio (A/C)	95	1.05	1.06	1.07	1.07	1.07	1.08	1.08	1.08	1.08	1.08	1.08	1.08	1.08
	90	1.13	1.14	1.15	1.16	1.17	1.17	1.18	1.18	1.18	1.18	1.18	1.18	1.18
	85	1.21	1.25	1.27	1.28	1.28	1.29	1.30	1.30	1.30	1.30	1.30	1.30	1.30
	80	1.33	1.39	1.41	1.42	1.44	1.45	1.47	1.47	1.47	1.47	1.47	1.47	1.46
	75	1.50	1.58	1.62	1.63	1.66	1.67	1.70	1.70	1.71	1.71	1.71	1.71	1.71
	70	1.75	1.86	1.92	1.95	1.98	2.01	2.04	2.06	2.06	2.06	2.06	2.06	2.06
	65	2.17	2.34	2.43	2.47	2.53	2.56	2.60	2.62	2.63	2.63	2.64	2.64	2.64

(.95 x .95 = .903). Thus, if the cost of any unit is known, the cost of any other can be calculated from this table. For example, given the value of unit 25, unit 100 cost would be obtained from the ratio .711/.788 or .902, i.e., the 100th unit would be 90.2 percent of the cost of unit 25.

Since the cumulative average curve is always above the unit curve, the cumulative average cost at any given quantity will be greater than the unit cost. As shown in Table V-1, the cumulative average cost of unit 2 is .975 (the average of unit costs of 1.0 at unit 1 and .95 at unit 2). To move quickly from the unit curve to the cumulative average curve, a simple ratio is provided in the bottom portion of Table V-1.

It is probably fair to say that in actual practice the unit cost is most frequently considered to be linear, but there are sufficient exceptions to this statement to suggest that the choice is a matter of preference rather than necessity. Once the choice is made, however, it is of the utmost importance to apply the technique consistently. As is evident from the example above, confusing one type of curve for the other could result in large errors.

NONLINEAR HYPOTHESES

Throughout succeeding sections of this chapter it is assumed that the linear hypothesis applies, i.e., that the learning curve is linear when plotted on logarithmic grids. It must be mentioned, however, that this is not the only possible formulation of the learning curve. A number of studies have indicated that the curve is not linear. One of the best known of these is the Stanford Research Institute investigation of 20 World War II aircraft. This study proposed

$$C = \frac{A}{\sqrt{X+b}}$$

as a more reliable expression of the relationship between manhour cost and cumulative output. Here the decision to find a substitute function was apparently prompted by a visual inspection of several series that seemed to indicate a concavity in the unit learning curve.* This concavity early in the series has been recognized independently in other studies.

On the other hand, it has been noted in some cases that beyond certain values of cumulative output, both the labor and the production cost curves develop convexities. The theory of a linear unit curve implicitly assumes that its component curves (e.g., fabrication, subassembly, and major and final assembly) are parallel to the linear unit curve, and this implies that the rate of learning on all production jobs in all departments is the same. One would expect, however, that the departmental learning curves would have different slopes from each other (e.g., fabrication might be 90 percent; subassembly, 85 percent; and major and final assembly, 70 percent). If this is the case, the sum of these curves (the unit curve) would approach as a limit the flattest of the departmental curves.

A considerable amount of literature is available describing the bases for and hypotheses about learning curves, and it is beyond the scope of this chapter to attempt to cover this background material in any detail.** A list of some of the most useful reports on the subject is appended for those interested in pursuing the matter further. For our purpose here, we stipulate that the learning curve has become a useful and accepted estimating tool, particularly in the aerospace industry, that the log-linear curve is the one most commonly used, and that a knowledge of its mechanics is indispensable to persons making or using cost estimates.

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*"Concavity" in this context means that when viewed on log-log paper the curve declines at an increasingly steep slope as it moves away from the y-axis. In the formulation $C = \frac{a}{\sqrt{X+b}}$ the curve becomes essentially linear as X becomes large relative to b.

**One subject not discussed at all concerns the effect of production rate on unit cost. Economic theory generally holds that this relationship can be described by a U-shaped function: cost declines as production rate increases, then is insensitive to rate over some range and eventually begins to rise again. In learning curve applications, on the other hand, an implicit assumption is that cost is not affected by rate of output (or that the rate is constant). Empirical evidence of the interaction between the volume and rate effects is scanty, but for a good illustration of the problem see: Preston, L., and E. Keachie, "Cost Functions and Progress Functions: An Integration," American Economic Review, March 1964, pp. 100-107.

PLOTTING A CURVE

The graphical representation of learning curves involves the problem of representing the average cost for a lot or a complete contract, since, typically, manhours or costs are not recorded by unit. The following sample illustrates this.

Lot	Units	Manufacturing Hours per Lot
1	1-10	5,830
2	11-20	4,370
3	21-50	10,550
4	51-100	14,750

To plot a cumulative average curve from these data the cumulative average hours at the final unit in each lot are computed, as shown below. The cumulative average at the 10th unit is 583 hours; and this is the first plot point. Successive plot points are at the end of each lot, since these are the points at which the cumulative average hour figures apply.

Plot Point (Unit)	Manufacturing Hours per Lot	Computation	Cumulative Average Hours
10	5,830	(5,830÷10)	583
20	4,370	(10,200÷20)	510
50	10,550	(20,750÷50)	415
100	14,750	(35,500÷100)	355

To plot the unit curve, however, it is necessary first to compute the unit hours and then to establish plot points. The unit hours can be taken as an average for each lot, that is:

Lot	Computation	Unit Hours
1	(5,830 ÷ 10)	583
2	(4,370 ÷ 10)	437
3	(10,550 ÷ 30)	352
4	(14,750 ÷ 50)	295

The lots can be represented by these unit hour values. The question is, where should the values be plotted? To plot at the lot midpoint is to assume that the learning curve can be approximated by a linear curve on arithmetic grids, but as we have seen from Fig. V-1, this assumption only becomes reasonable after a number of units have been produced. The effect of choosing the arithmetic midpoint as the plot point for the first lot is illustrated in Fig. V-4. This figure shows that for a learning curve plotted on arithmetic grids, the area under the curve from A to the midpoint is greater than that from the midpoint to B. Only when the algebraic midpoint is chosen, which is somewhat to the left of the arithmetic midpoint, will the area under the curve be equal for the two segments.

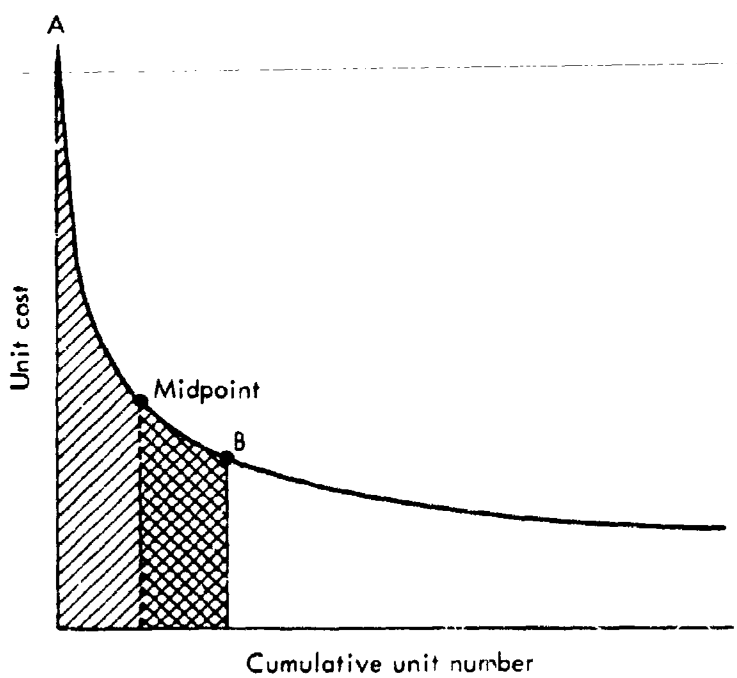

Fig. V-4—Learning curve on arithmetic grids

It is the algebraic midpoint, then, instead of the arithmetic midpoint through which the unit curve should be drawn for the first few lots. This can be obtained from the following equation:

$$K = \left[\frac{L\ (1+b)}{N_1^{\ 1+b} - N_2^{\ 1+b}} \right]^{-1/b}$$

where K = algebraic lot midpoint,

N_2 = first unit in lot minus .5,

N_1 = last unit in lot plus .5,

L = number of units in lot,

b = slope of learning curve.

Tables allowing rapid computation of lot midpoints for specific slopes and lot quantities are also available.* Note that this procedure assumes a knowledge of the learning curve slope. Actually, an approximation of slope is all that is required since the results are not very sensitive to this parameter.

Less precise, but somewhat handier than the above equation, is the graph shown in Fig. V-5 which provides plot points for early lot quantities of less than 100. These points represent an average of the range obtained from 65 and 95 percent slopes. The graph is used as follows:

1. First unit of contract lot is found on the 45-degree line.
2. The curve extending out from this line is followed to the point on the horizontal axis which represents the last unit of the lot.

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*See, for example, PAMPER (Practical Application of Mid-Points for Exponential Regression) Tables, Army Missile Command, Redstone Arsenal, Alabama.

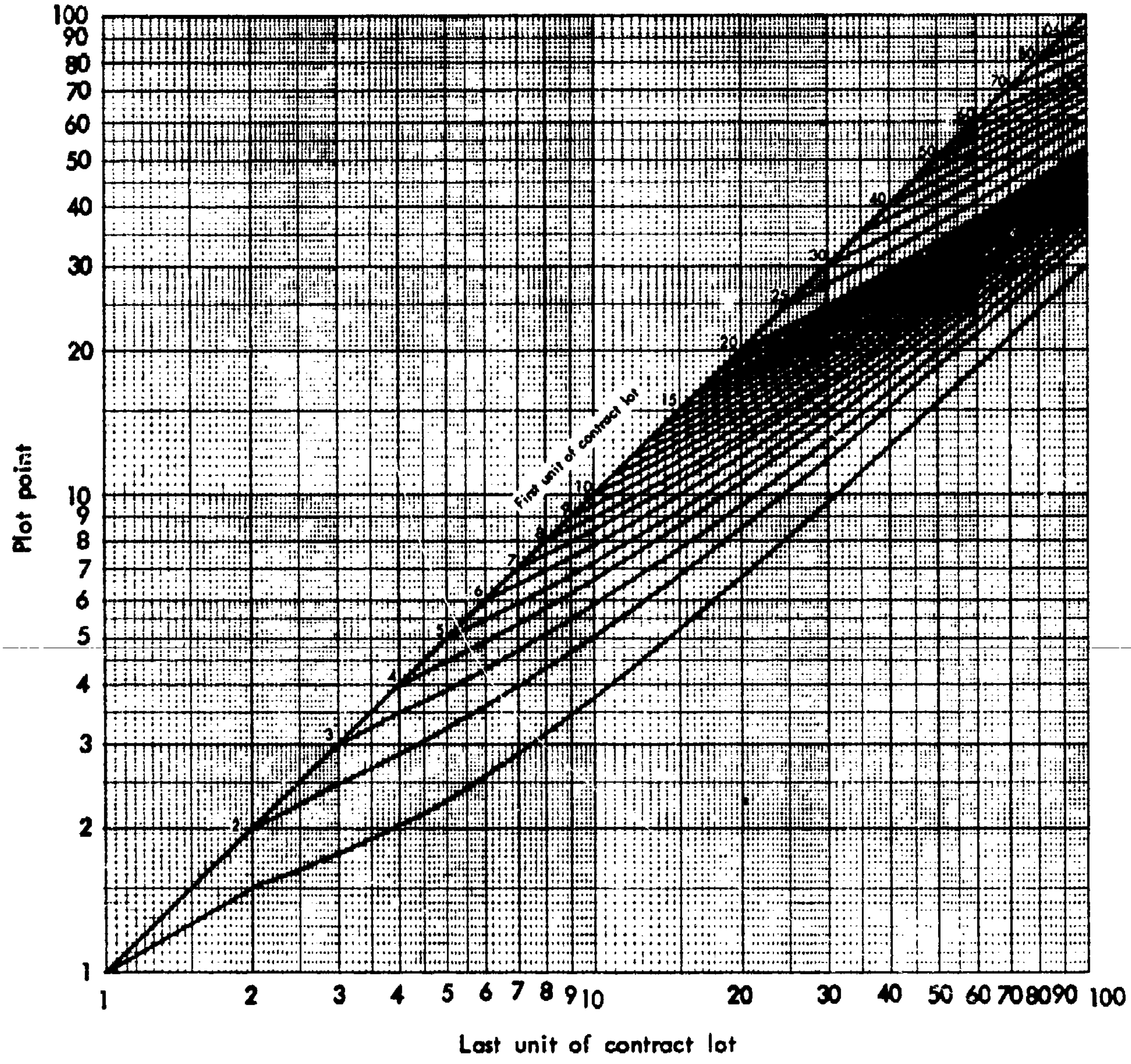

Fig. V-5—Plot points for average costs

3. The plot point is read off the vertical axis at that point. Thus, for a first lot of 10 units, the plot point would be 3.75.

In practice, plot points for only the first two or three lots, or only the first if that lot comprises more than about 25 units, need be taken from the graph. For succeeding lots, the arithmetic lot midpoint is quite adequate.

The point here is not to recommend any particular technique, but rather to underline that the problem of how best to plot the representative unit costs for lots is important. Gross misplacement of early points could lead to improper conclusions about the cost-quantity relationships the curves are intended to represent.

VARIATIONS

The examples used earlier for illustrative purposes tend to suggest that data points generally fall along a straight line as one would expect from the linear hypothesis. The sad truth is that plots of the type illustrated in Fig. V-6 are not unusual and that fitting a curve to these points is more than a matter of understanding the least-squares method of curve fitting. The types of plots seen in Fig. V-6 are common enough to have been given names in the airframe industry. The "scallop" is generally caused by a model change or some other major interruption in the production process. The characteristic of a scallop is that an abrupt rise in manufacturing hours is followed by a rapid decline and the basic slope of the curve is relatively unchanged.

When a model change is sufficiently great, as in the case of the change to the F-106 from the F-102, the result is not a scallop but a change to a new curve. In this case, a "leveling-off" or "follow-on" is characteristic of the initial portion of the new curve. This is attributed to learning from a previous model that carries over and flattens the curve during initial production. This can also occur when production is halted for a long period or where production is transferred to a new facility.

"Bottoming-out" is the tendency for a learning curve to flatten at high production quantities. Intuitively, it seems reasonable that

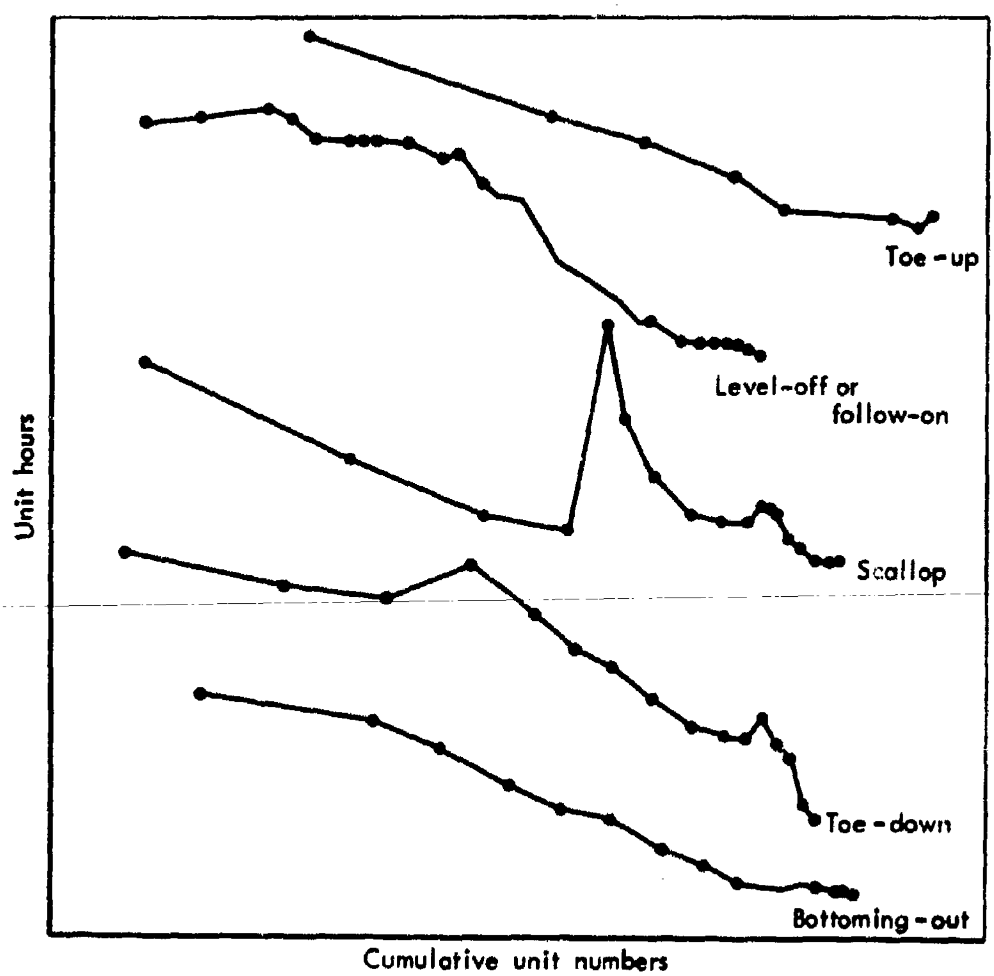

Fig. V-6—Illustrative examples of learning curve slopes at some point no further learning should occur or that whatever slight learning does occur would be offset by the effect of small changes. And empirically it can be established that bottoming-out has occurred in a number of cases. There are those who argue, however, that learning <u>can</u> continue indefinitely, or at least as long as the attempt is made to obtain manhour reductions, and empirical evidence can be cited to support this point of view. The classic case is probably that of the operation involving the assembly of candy boxes where the learning curve was found to have continued for the preceding 16 years during which 16 million boxes were assembled by one person.* The problem for the estimator, of course, is that while bottoming-out may occur in any given case, it is difficult to predict where it will occur. One study found that for the sample of airframes examined it was fairly typical for some flattening to begin at the 300th unit,** but this has not been true for many airframes in the past. The B-17 curve maintained about a 70 percent slope out to the 6000th unit and then exhibited a toe-up.

"Toe-ups" and "toe-downs" are the names given to the rather sharp rises or falls in hours that sometimes occur at the end of a production series. The upward trend has been explained as resulting from the transfer of experienced workers to other production lines, an increase in the amount of handwork as machines are disassembled, failure to replace or repair worn tooling at the normal rate, tool disassembly, or from labor becoming less productive at the end of a program so as not to work itself out of a job.*** Toe-downs are felt to be caused by fewer engineering changes at the end of a production run and also by the ability of the manufacturer to salvage certain types of items fabricated in previous lots.

While the names given to these particular variations are unimportant, it is important to know that such variations occur--not occasionally but frequently. In the analysis of manhour or cost data use of the

*Glen E. Ghormley, "The Learning Curve," <u>Western Industry</u>, September 1952.

**<u>Methods of Estimating Fixed-Wing Airframe Costs</u>, Vol. I (Revised), Planning Research Corporation, R-547A, April 1967.

***G. M. Brewer, <u>The Learning Curve in the Airframe Industry</u>, Air Force Institute of Technology, Report SLSR-18-65, 1965.

unit curve reveals these variations and is generally preferred for this reason. The cumulative average curve tends to smooth out aberrations to such an extent that even major changes can be obscured. Figure V-7 illustrates this. The data points are taken from a fighter aircraft production program which had more than its share of problems. The solid line shows how a cumulative average curve damps out the effect of these problems. The choice between working with the unit or the cumulative average curve depends upon the purpose at hand. The unit curve better describes the data and is sometimes preferred for this

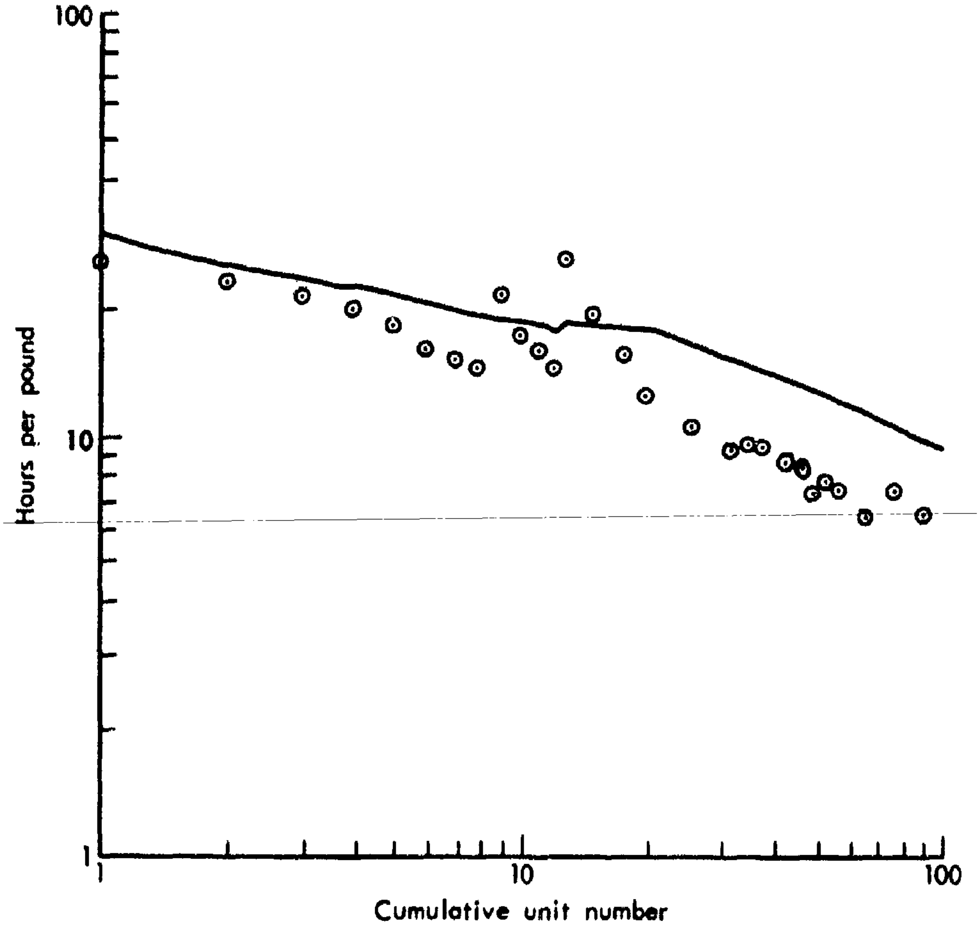

Fig. V-7—Smoothing effect of cumulative average curve reason. On the other hand the cumulative average curve is widely preferred in predictive models because of its computational simplicity, i.e., the cost of N items is simply the cumulative average cost of the Nth items times N. The important point is to understand both well enough to be able to choose intelligently between them.

APPLICATIONS

The learning curve is used for a variety of purposes and in a variety of contexts; and how the curve is drawn will depend on the purpose and the context. In long-range planning studies, for example, the curve must be constructed on the basis of generalized historical data and the possible error is considerable. Empirical evidence does not support the concept of a single slope for all fighter aircraft, all solid propellant missiles, all spacecraft, etc. The practice, therefore, of assuming that manufacturing hours on the airframe will follow an 80 percent curve (as was common for many years) or that electronic equipment will follow, say, a 90 percent curve, can lead to very large estimating errors.

In regard to airframes, Table V-3 shows the slope of the manufacturing-hour curves for 25 post-World War II Air Force and Navy aircraft and indicates that a slope steeper than 80 percent is the rule. Since the learning-curve slopes of Table V-3 show important differences it would be desirable to relate slope to aircraft characteristics. In a sense a technique suggested by Planning Research Corporation does this.* Separate estimating equations based on aircraft characteristics are derived for four different production quantities--10, 30, 100 and 300--and a learning curve is developed from the estimates at these four points. On a theoretical level, however, the concern is with those aircraft characteristics which influence the rate of learning. In this regard it seems reasonable to expect relatively little learning for a model which represents a small modification over some preceding type since the previous model would have already absorbed a considerable learning effect. On the other hand, if an aircraft contains radically new design features, one would expect a high initial cost followed by a rapid

*Op. cit.

Table V-3

LEARNING CURVES FOR MANUFACTURING
(Labor--Airframe Only)

	LEARNING CURVE PERCENTAGE
Aircraft	
Fighter	77
Fighter	73
Fighter	74
Fighter	73
Fighter	78
Fighter	71
Fighter	74
Fighter	76
Fighter	77
Fighter	79
Fighter	82
Fighter	76
Fighter	75
Fighter	74
Bomber	76
Bomber	73
Bomber	70
Bomber	71
Bomber	79
Cargo	74
Cargo	78
Cargo	77
Cargo	75
Trainer	74
Trainer	75
Mean	75
Standard Deviation	2.7

*G. S. Levenson and S. M. Barro, Cost-Estimating Relationships for Aircraft Airframes, The RAND Corporation, RM-4845-PR (Abridged), May 1966.

decline with increased production quantities. In other words it has been suggested that the "newness" of an aircraft should be a major determinant of learning-curve slope, but explicit techniques for taking newness into account have yet to be developed.

For good estimating, then, learning curves must be established on the basis of historical data relevant to the problem at hand. They are equally applicable to missiles, electronic equipment, aircraft, ships, and other types of equipment, but the slopes may be quite different for each of these. (A recent study of avionics, for example, showed slopes ranging from 84 percent to 91 percent with a median value of 88 percent.) If a comparison is being made between two weapon systems, one involving aircraft and the other missiles, the learning curve slope chosen for each could play a significant part in the total system cost comparison. In an appendix to this chapter the effect of slight variations in slope is shown to be much greater than is generally recognized. To cite two examples: The effect of using a 92 percent rather than a 90 percent cumulative average curve is an increase of 25 percent in the total cost of 1,500 items. As one would guess, the situation is much worse when steeper slopes are involved. Assuming a slope of 62 percent instead of 60 percent results in a 42 percent overstatement of the cost of 1,500 items and a 25 percent overstatement of the cost of 100 items.

As a practical matter, errors of this type can be minimized by originating the curve at the estimated cost of the 100th unit rather than the first. The table below shows how this reduces the effect of a two percent change in slope on total cost.

Change in Slope	Change in Total Cost of 1,500 Units
From 90% to 92%	
Curve originated at	
Unit 1	25%
Unit 100	9%
From 60% to 62%	
Curve originated at	
Unit 1	42%
Unit 100	14%

Once a few data points are available either for developmental or production items, the situation should be better, but, as illustrated by Fig. V-8, the first few points may be misleading. Suppose an estimator

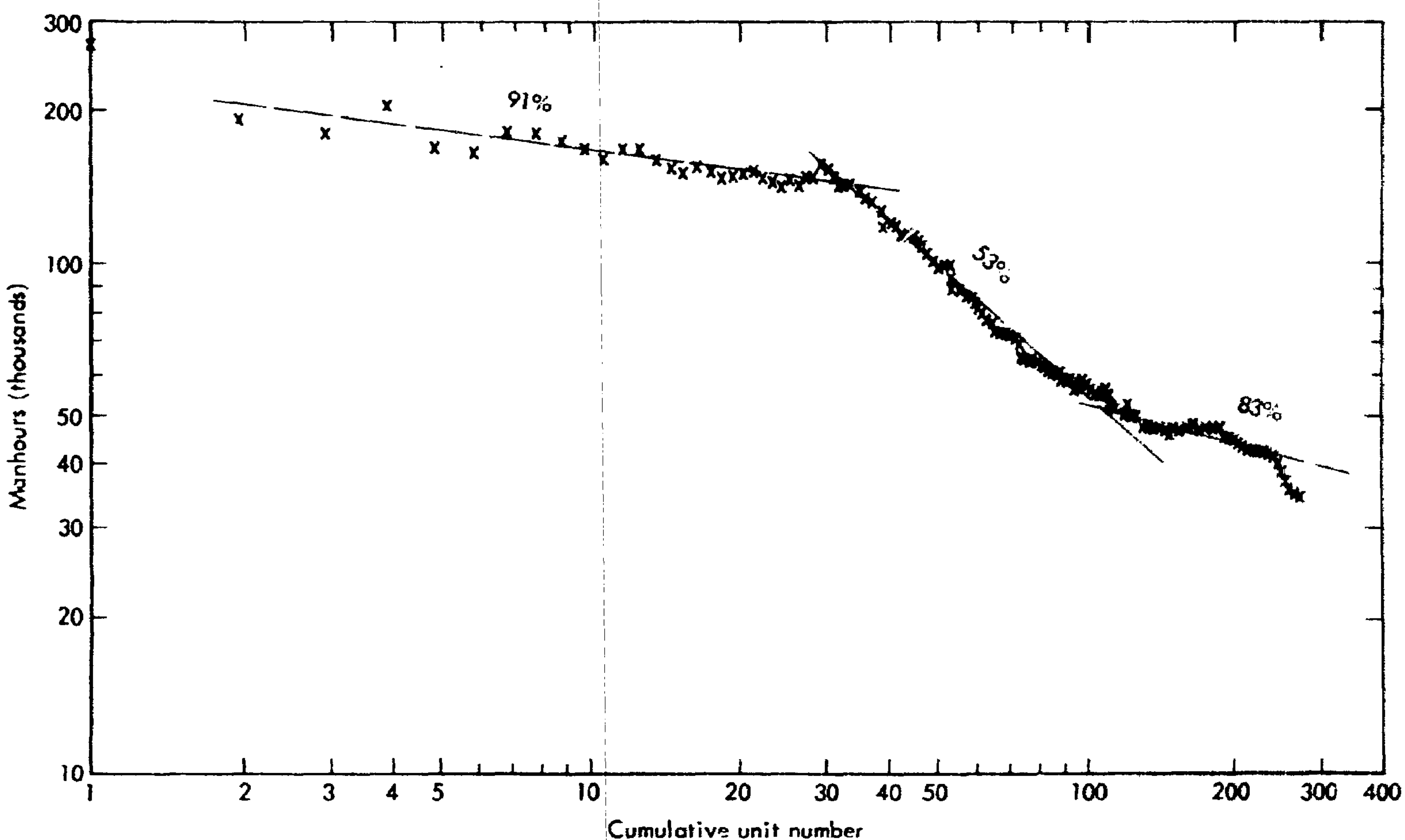

Fig. V-8—Direct labor nours for a transport aircraft had been asked to estimate the cost of a large production contract after the fabrication of the first 30 units. By fitting a curve to the existing data he would have projected a learning curve with about an 88 percent slope and at a level considerably higher than that later experienced. In such a situation it is important to realize that an 88 percent learning curve for airframe production is unlikely. In effect, one should have some idea of what the answer should be and differences should be investigated.

This can also be taken as an example of the small sample problem. Where a learning curve is fitted to a few points, the correlation may be perfect, that is, all the points may lie on the fitted line, but the results can still be unreliable. The points used in fitting must be sufficiently numerous and reasonably homogeneous with the points implied by extending the curve to offer some statistical probability of success in predicting costs.

The most important information an estimator faced with the above problem could have would be a manufacturing history of the item involved. Variations from the norm may be caused by particular problems, configurations changes, or changes in manufacturing methods. In the curve of Fig. V-8, the initially flat portion (out to the 30th airframe) is explained by the manufacturer as being typical of the initial production period. In this manufacturer's experience, the curve begins to steepen when:

1. Manpower has stabilized or reached its peak,
2. The engineering configuration has stabilized, and
3. The parts flow has stabilized.

Thus, it may be preferable to explain some points and exclude them rather than to include them and bias the curve in height or slope.*

Whether or not to include all the points depends, in addition, on the anticipated use of the resulting curve. If a unit cost curve that includes all costs including changes is desired, a line of best fit through the unit plot points may be appropriate. If the curve is to be used in negotiating a follow-on contract, the effect of changes should be eliminated by constructing a curve through the lower portion

*It is also possible to have a segmented unit curve as implied by Fig. V-8 and some manufacturers subscribe to this concept.

of the plotted individual u it points as in Fig. V-9. In effect, this assumes that the introducti n of changes raises the hours initially but that these decrease ag in to the level of the original curve.

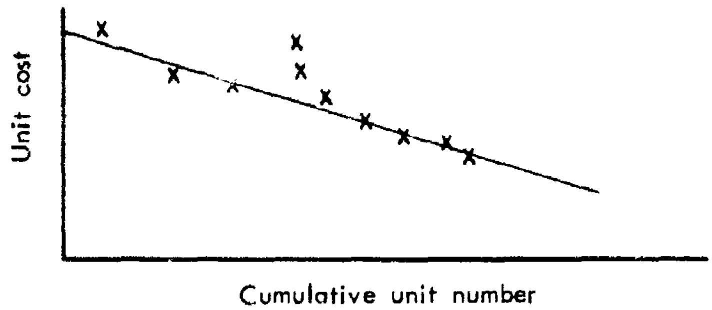

Fig. V-9—Eliminating the effect of changes

Whatever the basic technique, it is important to remember that on logarithmic grids the points at the right are much more important than those at the left. In visually fitting a line, one should avoid the tendency to be unduly influenced by plot points for small early lots. Early units are often incomplete because they are used for test purposes. Also, the early units are apt to include certain nonrecurring problems incident to startup, and for this reason may be above the level suggested by later plot points (CIR should help reduce this problem).

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APPENDIX*

Assume a cumulative average cost-quantity curve of the form

$$A = aX^b \tag{1}$$

where a is the cost of the first item produced,

X is the number of items produced,

b is an exponent that measures slope,

A is the average cost of all items produced up to and including X.

In cost-quantity curve parlance, the rate of change of cost with respect to X is referred to as the slope (S) of the curve instead of b. S has special meaning in that it describes the average cost of 2X items as a fraction of the average cost of X items. As aX^b represents the average cost of X items, $a(2X)^b$ must equal the average cost of 2X items. Thus, given the above definition, the following relationship between b and S must hold

$$S = \frac{a(2X)^b}{a(X)^b}$$

$$S = 2^b .$$

Using logarithms to solve for b results in

$$b = \frac{\log S}{\log 2} .$$

Substitution of this expression for b in equation (1) results in

$$A = aX^{\left(\frac{\log S}{\log 2}\right)} . \tag{2}$$

The cumulative average cost is but an input to the calculation of the total cost of X items which is of particular interest. It is therefore logical, for analytical purposes, to work with the total cost

*This appendix is the work of R. L. Petruschell.

equation itself which can be developed from the equation for the cumulative average cost as follows. A, the average cost of X items, when multiplied by X gives the total cost (T) of the same X items. This follows from the fundamental idea of an average. Carrying out the required manipulations in symbolic form results in the following expression for T.

$$T = AX$$

and substituting equation (2) for C

$$T = aX^{\left(\frac{\log S}{\log 2}\right)}X$$

and simplifying

$$T = aX^{\left(1 + \frac{\log S}{\log 2}\right)} \tag{3}$$

At this point, observe that changes in the value of a are reflected in T in relative fashion. If the value of a were to increase 10 percent, the value of T would likewise increase 10 percent and furthermore do so independently of the value of either X or S.

The effect of X and S on T is more complex. Rather than try to display these effects by partial differentiation, etc., which is possible, graphics are employed exclusively. Figure V-10 portrays the solutions of equation (3) for values of S between .70 and 1.00, an a equal to 1, and X between 10 and 400, chosen to display the varying shapes of the different curves.

It appears that as X becomes larger, T becomes more sensitive to changes in S. For example, a shift in S from 0.85 to 0.90 causes a 16 point change in the cost of 100 items and a 65 point change in the cost of 400 items. Also, each of the curves levels off as S decreases leading to the conclusion that the sensitivity of total cost to changes in S decreases with S.

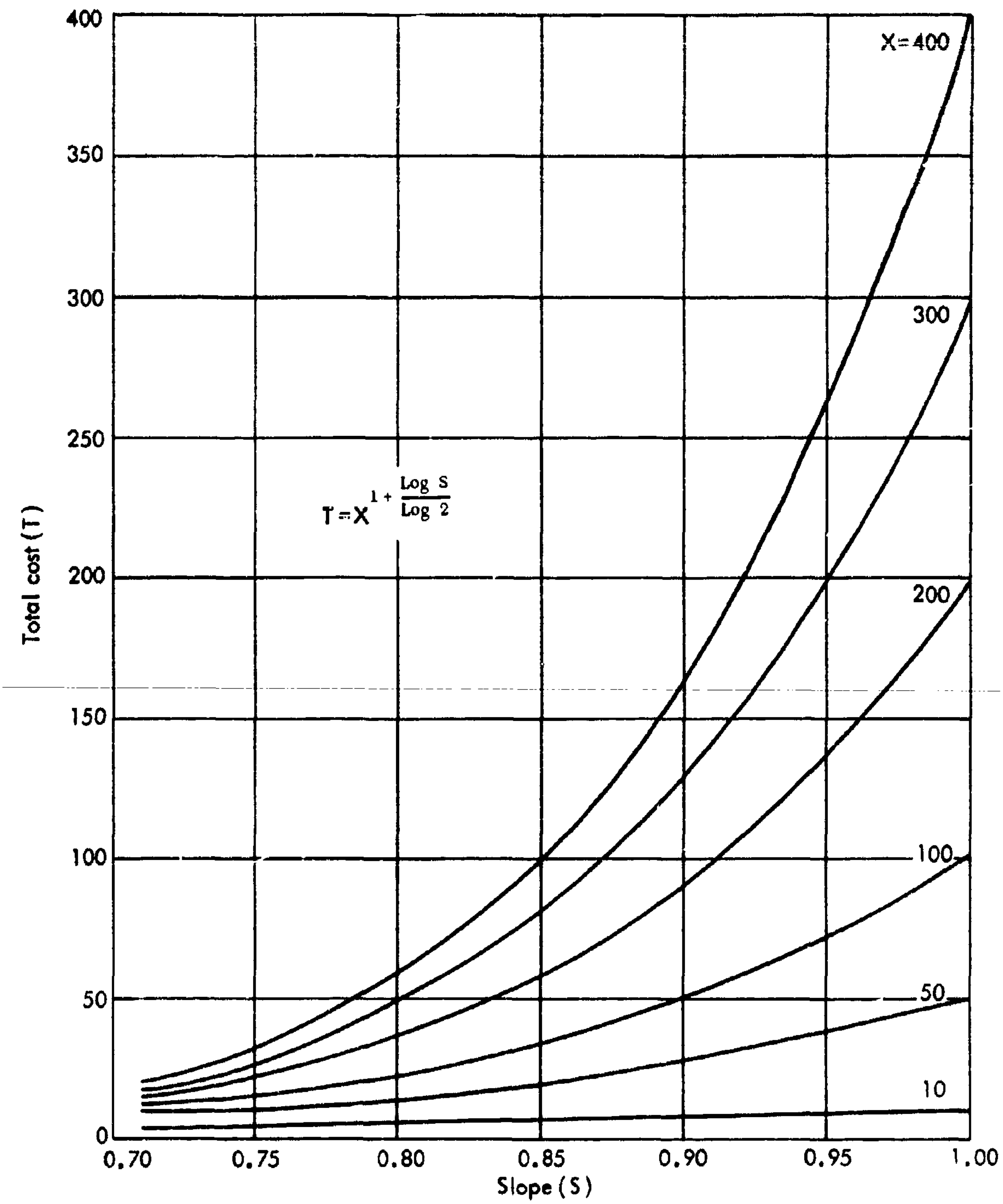

Fig. V - 10—Total cost versus slope

An examination of these sensitivities in relative terms provides some additional insights as is often the case when dimensions are removed. Figure V-11, which is largely a simplified copy of Fig. V-10, illustrates, in part, the calculation of an index (T_R) to measure the variation in T with respect to X and S. A value of S designated S_N and a corresponding value of T likewise designated T_N are selected. These values as the subscript implies are regarded as norms, or base points around which variation is allowed (indicated by shift to S and T). T_R the index of relative change in T is defined as the fractional change in T resulting from and absolute change in S, or in equation form

$$T_R = \frac{T - T_N}{T_N} \qquad (4)$$

$$\text{or} \quad T_R = \frac{T}{T_N} - 1.$$

The following substitutions and simplifications result in the expression that was actually used to evaluate T_R.

$$T = aX^{\left(1 + \frac{\log S}{\log 2}\right)} \qquad (3)$$

$$T_N = aX^{\left(1 + \frac{\log S_N}{\log 2}\right)} \qquad (3')$$

$$T_R = \frac{aX^{\left(1 + \frac{\log S}{\log 2}\right)}}{aX^{\left(1 + \frac{\log S_N}{\log 2}\right)}} - 1$$

$$T_R = X^{\frac{\log S - \log S_N}{\log 2}} - 1 \qquad (5)$$

The fact that the a's cancel out indicates that the sensitivity of T_R to S and X is independent of the value of a. Figure V-12 shows the results of solving equation (5) assuming $S_N = .90$, $.86 \le S \le .94$, and

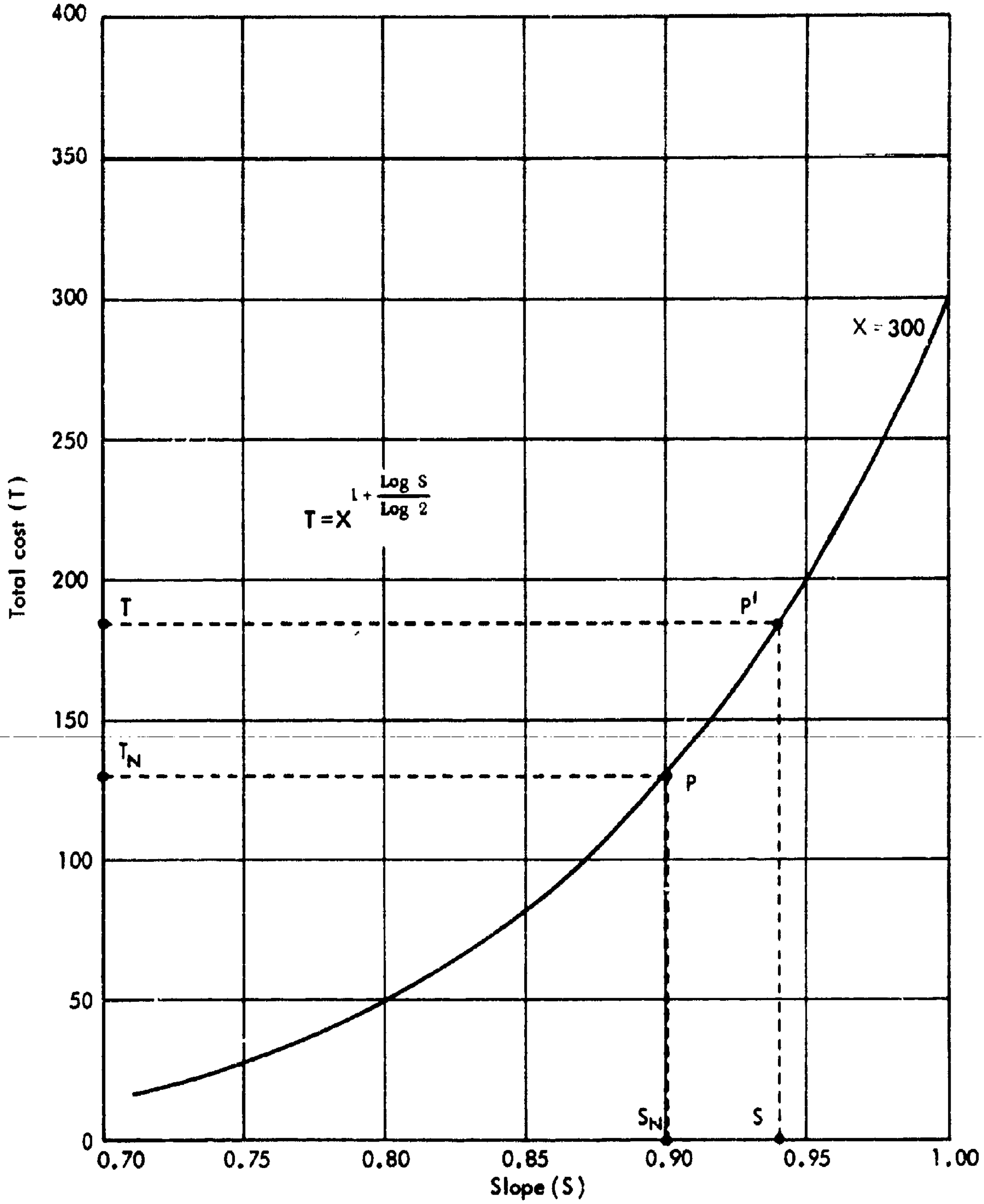

Fig. V-11—The calculation of T_R

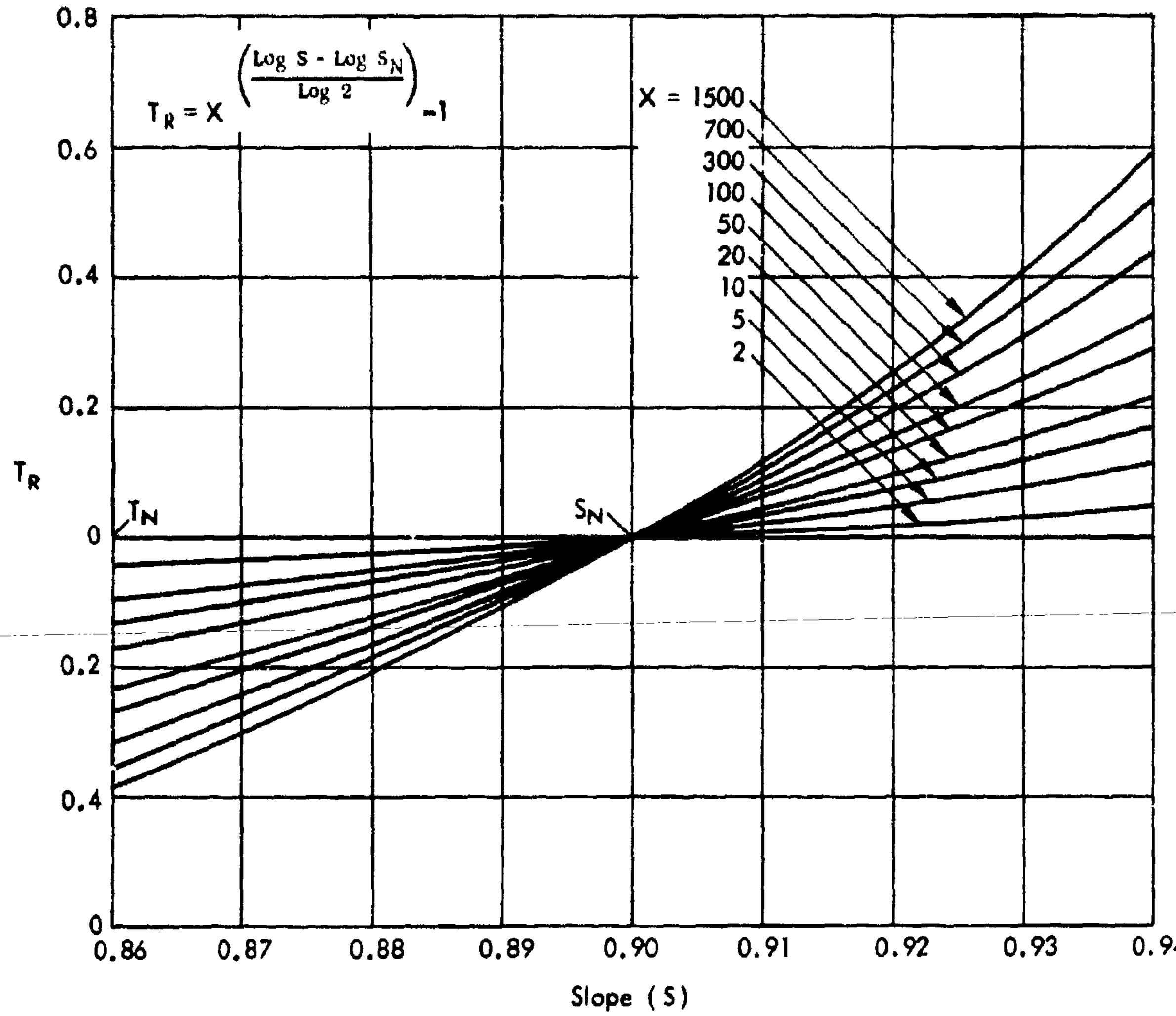

Fig. V-12—Values of T_R when S = 0.86 - 0.94

$1 < X < 1500$. The vertical axis (T_R) indicates decimal fractions of T_N by which T differs. The origin at the center, allows changes both above and below T_N to be indicated. The horizontal scale (S) is similarly marked. Figure V-13, V-14, and V-15 present similar displays for different values of S_N. The range of S, in each case, was restricted to $S_N \pm 4$ units thus permitting coverage of the relevant spectrum without overlapping from figure to figure.

An examination of Fig. V-12 shows that the relative difference between using an S of .90 and an S of .92 would be + 25 percent in the total cost of 1500 items. Alternately, if an S of .89 rather than an S of .91 had been used, the difference relative to an average S of .90 would be approximately 23 percent.

Carrying out the same kind of exercise using Fig. V-15 results in significantly greater differences. For example, assuming an S of .62 instead of .60 results in a 42 percent overstatement of the cost of 1500 items and a 25 percent overstatement in the cost of 100 items.

We must conclude that when using equations of this type to estimate cost as a function of quantity, significant percentage variations in the total cost can result from what are apparently much less significant changes in S. In addition, the impact of a unit change in S on T_R is inversely proportional to the size of S.

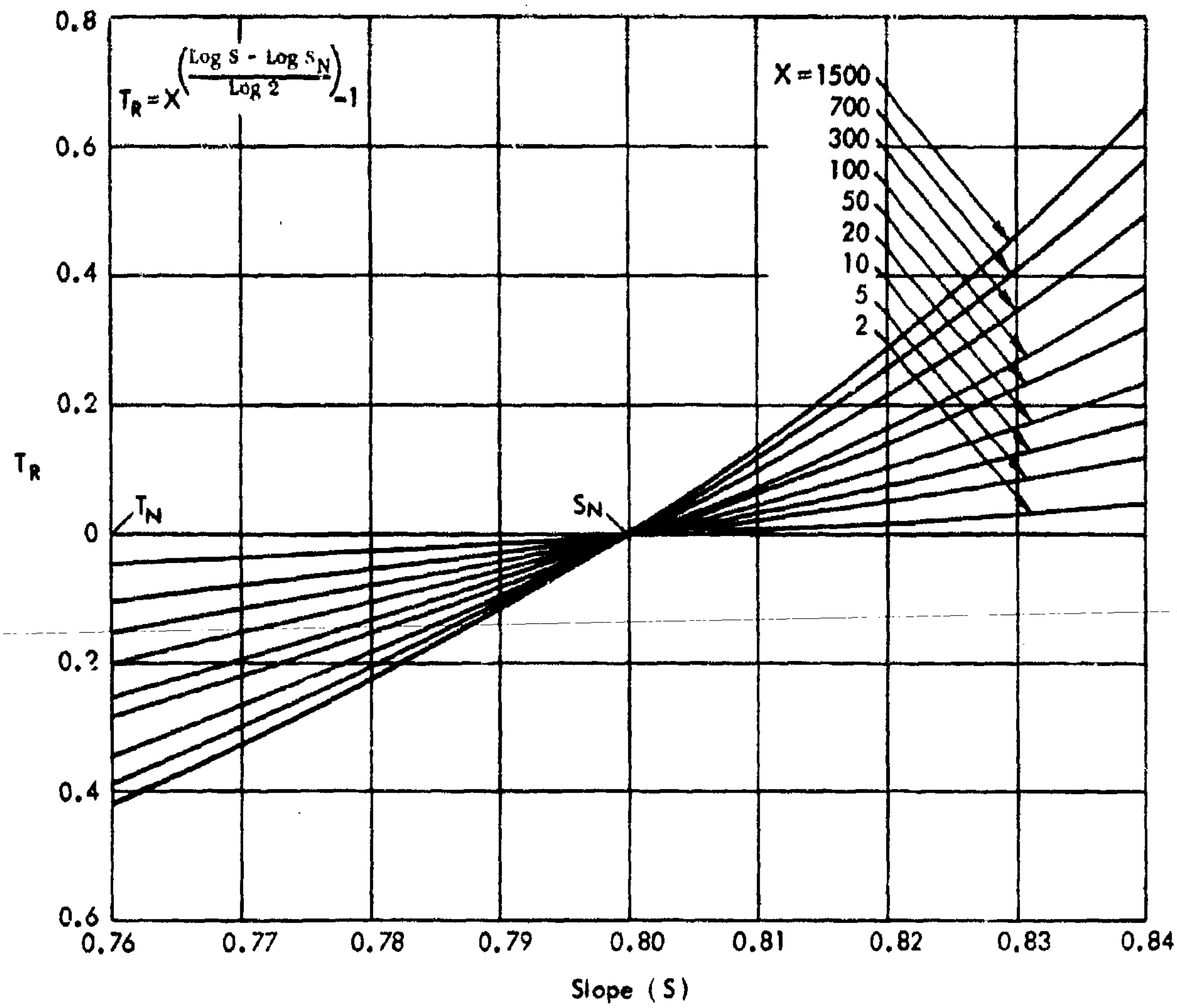

Fig. V-13—Values of T_R when S = 0.76 - 0.84

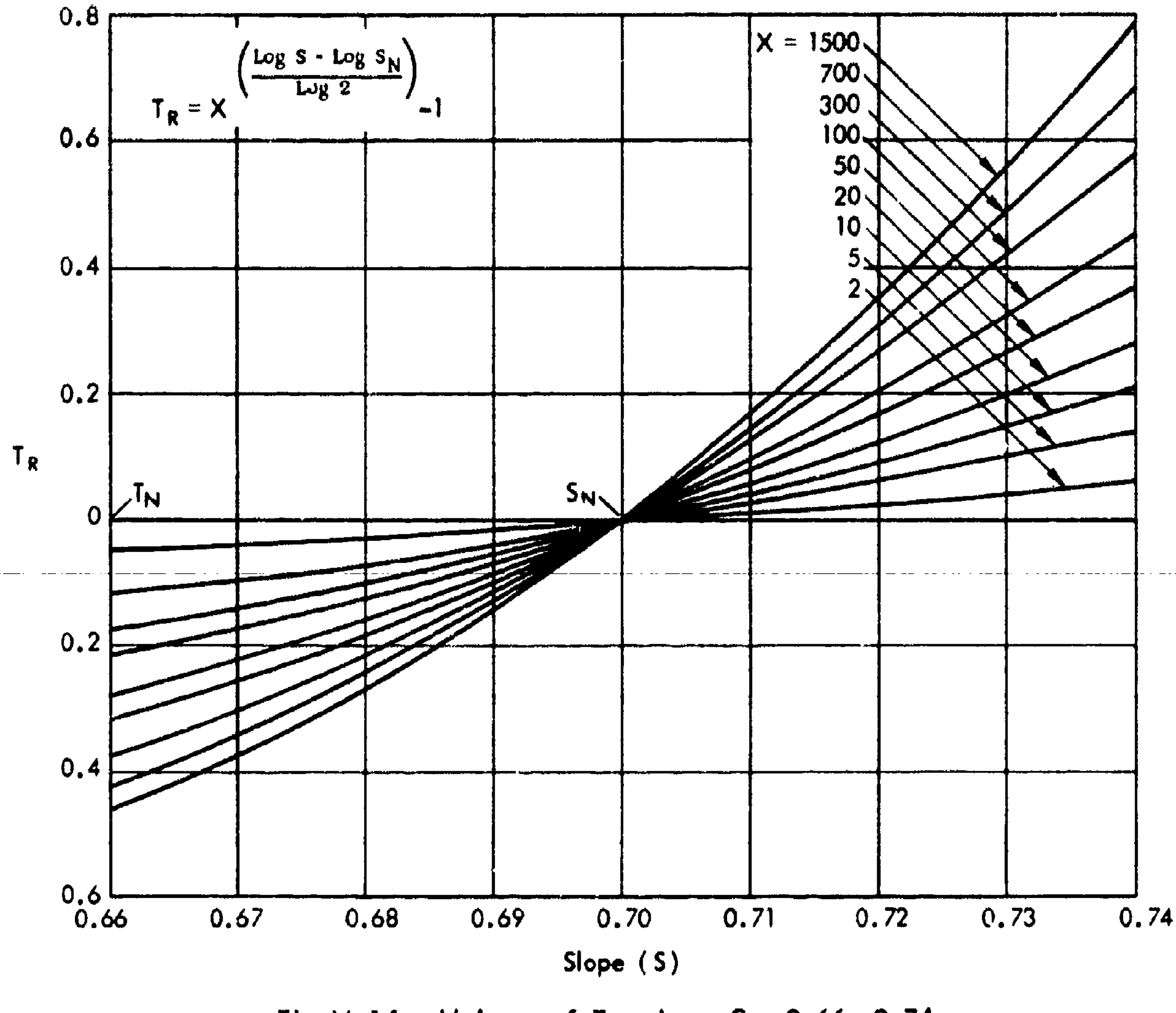

Fig. V-14—Values of T_R when S = 0.66 - 0.74

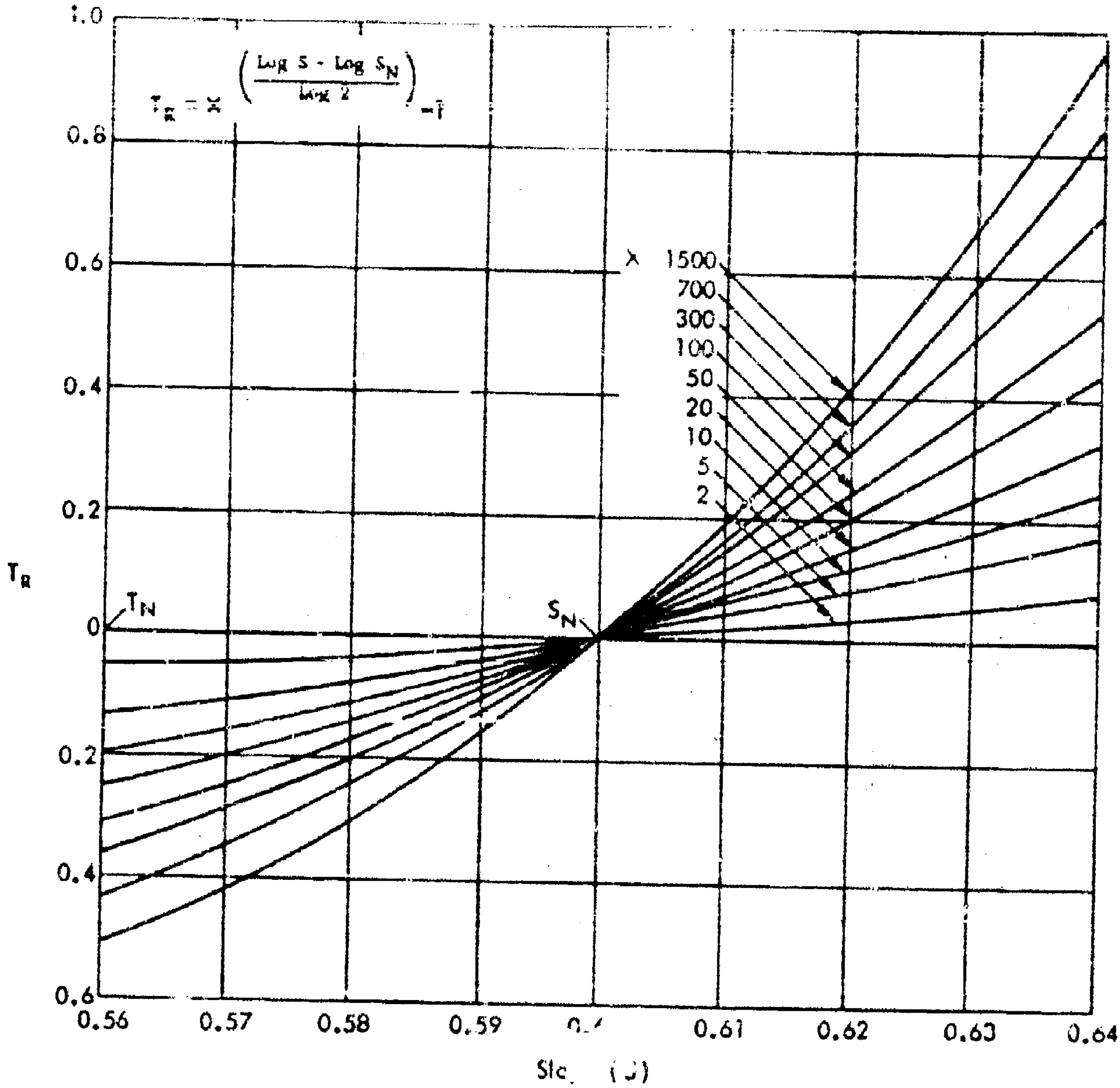

Fig. V-15—Values of T_R when S = 0.56 - 0.64

VI. UNCERTAINTY

During the 1950's the difference between the original estimate and the final cost of a number of weapon systems was so great that in the latter part of that decade various agencies began looking at case histories of the major equipment items involved in an attempt to identify the reasons for the discrepancies. The problem is illustrated by the table below (Table VI-1). Here for 16 aircraft and 6 missiles developed prior to 1958 the ratio of late estimate or actual cost to early estimate has been computed and is shown as the factor increase.

Table VI-1

FACTOR INCREASES OF THE PRODUCTION COST OF EQUIPMENT*

Equipment	Factor Increase	Equipment	Factor Increase
Fighter	3.9	Cargo	1.4
Fighter	2.6	Cargo	1.5
Fighter	2.0	Cargo	1.0
Fighter	1.5	Cargo	1.0
Fighter	1.7	Missile	14.7
Fighter	1.7	Missile	9.4
Fighter	1.0	Missile	4.4
Fighter	1.0	Missile	8.2
Fighter	1.1	Missile	1.5
Bomber	6.2	Missile	1.1
Bomber	2.8		
[illegible]	[illegible]		

This table is of more than historical interest because factor increases are still being experienced on some types of hardware, particularly spacecraft, being procured by the government. For our purpose, the main point of interest is the reason for these increases,

*Taken from A. W. Marshall and W. H. Meckling, Predictability of the Costs, Time, and Success of Development, The RAND Corporation, Paper P-1821, December 1959.

and, specifically, we would like to know if they are due to bad cost-estimating. If the problem is simply this, presumably the estimator can learn to do better and the situation can be improved. If, on the other hand, the problem turns out to be poor management, bad design, inadequate guidance or something of that sort, the cost estimator can do little except hope that the future will be better. A study of the development histories of the equipment in the above table in an attempt to answer such questions led to the following conclusions:

> When early estimates are made of what it will cost to produce or develop something new, the estimator typically bases his estimate on the current design and the currently planned program for development. If he is estimating cost of production, he gets a total cost by costing the various components as <u>presently conceived</u> and aggregating those. If he is estimating the cost of development, he estimates the cost of test articles, engineering man-hours, etc., as <u>presently planned</u> and aggregates those. He does not specify what performance he is associating with the particular design nor does he indicate the date at which this performance is to be operationally available. He is simply costing a physical configuration and/or the physical resources contemplated in the current development plan.
>
> As development proceeds, however, these initial designs and plans are almost invariably changed, either because of unforeseen technical difficulties that forestall meeting performance requirements, or because the customer decides it is essential that the equipment be modified so as to keep pace with changing predictions of enemy capabilities, new operational concepts, and new technological possibilities.
>
> . . . In principle it would be possible to factor into two parts the total error in cost estimates as they are prepared: (1) the part due to errors in the costing of the configuration supplied to the cost estimator (i.e., the intrinsic error in cost estimating) and (2) the part due to changes in the configuration as development progresses. In practice it has not been possible to carry out this separation. However, it is our belief that the intrinsic errors in costing a fixed configuration tend to be small relative to the other source of error in the costing of most major items of military equipment.*

In other words, of the two kinds of errors mentioned above, <u>requirements uncertainty</u>, i.e., variations in cost estimates stemming

*Marshall and Meckling, <u>op. cit</u>.

from changes in the configuration being costed, is generally held to be responsible for the major portion of factor increases. It should be understood that this kind of uncertainty is not found in the United States alone--the estimated cost of the joint British-French strike/trainer aircraft, the Jaguar, had at the end of 1966 increased from $1 million to $2.9 million because of changes in requirements and the final cost was still uncertain. Nor is requirements uncertainty confined to weapon systems--the House of Representatives' Rayburn Office Building, originally expected to cost about $50 million, exceeded $120 million when finished, largely because of design changes after the original estimate was made. While it may be impossible to eliminate discrepancies of this kind entirely, the Department of Defense has attempted to minimize them by initiating the Contract Definition Phase (CDP) for major defense contracts. A rigorous definition of requirements prior to source selection should reduce the importance of this kind of uncertainty in the future.

<u>Cost-estimating uncertainty</u> refers to variations in cost estimates of a system for which the configuration is essentially fixed and can arise for a variety of reasons:

1. Variations in cost estimates of a given set of requirements can occur purely because of differences between cost analysts in interpreting the given requirements, in methodological approach to the problem, in specific techniques used, and so on, even if the analysts are of comparable competency.

2. Cost-estimating relationships used in cost analysis cannot be assumed to hold exactly. This simply means that in estimating a certain cost component as a function of some variable (or variables), we usually cannot assume that these variables will predict the particular cost with certainty.

3. Cost-estimating error can arise from the fact that data used as a basis for cost analysis are themselves subject to error. Putting it another way, the observations used in deriving cost-estimating relationships invariably contain errors--even if these data come from carefully kept historical records.

4. In costing advanced military systems, the cost analyst very often uses cost-estimating relationships derived from past or current experience. Here, one cannot be very confident that a structural relation that holds reasonably well now will continue to hold satisfactorily for the advanced system being costed. In fact, we frequently of necessity have to extrapolate beyond the range of the sample or data base from which the estimating relationship was derived.

5. Usually in making cost estimates for use in analyses where comparative costs are of prime concern, the estimates are made in terms of constant dollars, i.e., in terms of price levels prevailing in some base year. Hence price level uncertainty is not a significant factor. However, there are occasions when estimates for future systems may have to be made in terms of price levels expected to prevail in future years. Here there is obviously a potential source of error arising from the possibility that future price levels may in fact turn out differently than originally expected.

6. The price-level factor may cause difficulties of a different nature. Sometimes, for example, the cost analyst may obtain data to be used in cost analyses, and from the source it may not be clear whether the data are in terms of constant or current price levels. A case in point is contractor data--either historical or projected. Very often contractor projections make provisions for possible wage rate changes and/or material price changes. To be useful for purposes of analysis, the analyst should be able to determine the bases used for making these projected price changes. Also, with respect to correcting historical data for price level changes, some error is bound to arise because of the deficiencies inherent in most price indexes.

The above listing is no doubt incomplete, but it does give an indication of the main sources of cost-estimating uncertainty. In the absence of a definitive empirical study, it is difficult to say which of the sources are generally of greatest relative importance. In an overall context, the following might be singled out:

Errors in cost-estimating relationships

Errors in data

Extrapolation errors

PROPOSALS FOR TREATMENT OF UNCERTAINTY

Proposals for treatment of uncertainty in cost analysis range from conventional statistical tools to the application of what are commonly known as "fudge" factors. The latter we rule out--not because of a high sense of morality that says use of fudge factors is wrong but on pragmatic grounds. To multiply a carefully worked out cost estimate by some factor because, on the average, estimates of a certain type of hardware have been low by that amount may or may not improve the quality of the estimate. For example, use of an average factor for the cases of Table VI-1 would have the following results:

	Number of Estimates Improved	Number of Estimates Degraded
Fighters	5	4
Bombers	2	1
Cargo	2	2
Missiles	4	2
Total	13	9

To improve the quality of some estimates it is necessary to degrade that of others. Hence in any particular case the cost analyst cannot know in advance whether use of a factor will be beneficial or harmful.

Conventional statistical tools are of only limited value in coping with the problem of uncertainty in cost analysis because the occasions on which they can be used rigorously are quite rare. First of all, to derive the conventional statistical measures of uncertainty, e.g., confidence intervals, prediction intervals, and the like, one must draw a representative sample from a designated population to be used as a basis for the statistical analysis. In cost analysis of advanced hardware by the nature of the case, we usually do not have such a population from which to draw representative samples. (In fact we sometimes deal with the entire universe.)

Even where samples of a sort can be drawn, the size of the sample is invariably very small--two or three observations, five or six if we are lucky. Sample sizes this small strain the applicability of most statistical theory to the limit--even small sample theory.

In the rare instances where the objections above can be reasonably overcome, we may still have problems because of difficulty in justifying the assumptions of the statistical model in our particular application. For example, the model may require specifications of the form of the distribution function in the population from which the sample is drawn. We are usually not in a position to make such a designation--for example, to make the assumption of normality. The normality assumption would not be so serious if the sample size were large. But as indicated above, in our work exceedingly small sample size is the rule rather than the exception. One possibility for dealing with this problem in the future is to use non-parametric or distribution-free methods of estimation.* While these methods are still relatively new and the theory not fully developed, the possible usefulness of distribution-free methods in the future should not be overlooked.

In addition to this problem, other technical difficulties are apt to arise. Consider the case of a regression model using the "errors in the equation" approach--i.e., that the estimating equation holds subject to a random disturbance (μ), but that the variables contain no error or at least errors of relatively minor significance. A usual specification on μ is that successive values of this variable are mutually independent (non-autocorrelated) and that μ is independent of the explanatory variables. This assumption may be somewhat difficult to justify in certain applications. However, in doubtful cases, the non-autocorrelation assumption can be subjected to statistical test.**

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*Distribution-free methods do not require an assumption about a specific form of probability distribution function. E.g., see A. M. Mood, Introduction to the Theory of Statistics, New York, McGraw-Hill Book Co., Inc., 1950, Chap. 16.

**E.g., see Lawrence R. Klein, Econometrics, Evanston, Illinois, Row, Peterson & Co., 1953, pp. 89-90. Also, see B. I. Hart and J. von Neumann, "Tabulation of the Probabilities for the Ratio of the Mean Square Successive Difference to the Variance," Annals of Mathematical Statistics, XIII (1942), pp. 207-214.

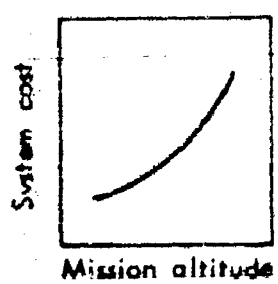
Finally, where conventional statistical methods are applicable, it is most likely to be in the treatment of cost-estimating uncertainty rather than requirements uncertainty. Yet, as was stated earlier, requirements uncertainty has been the more important of the two. Thus, many of the technical statistical points raised above may not always be of major significance in practical applications. Nevertheless we must be aware of these matters in dealing with cost-estimating uncertainty, so that our interpretations of standard errors, prediction intervals, and the like may be kept in proper context. Finally, it should be pointed out that even in cases where such statistical measures are not subject to rigorous interpretation, they may still be of considerable help in forming subjective judgments about the reliability of statistically derived estimating relationships.

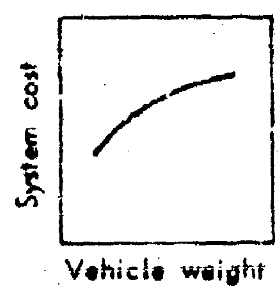
Cost-Sensitivity Analysis

One basis for approaching the problem is an acceptance of the idea that a certain amount of uncertainty is inevitable in any action occurring in the future. Having admitted this, it is possible to look at a proposed weapon or support system, single out the areas of greatest uncertainty and assign some limits to them. This process is sometimes known as cost-sensitivity analysis. It appears promising because it highlights the uncertainty inherent in future system costs and gives the planner a full view of the cost implications of decisions affecting system configuration and operations.

This type of analysis is primarily useful in the long-range planning phase of a program where system parameters are still tentative. It is also of more value, perhaps, in looking at total system cost than at hardware cost only. As an example, consider making an estimate of the total system cost of an aerospace plane--a manned aircraft that can take off from a runway, fly into orbit, and after completing its mission there, fly back to earth and land. Among the characteristics that are unknown are: the size of the vehicle, the number of flights it could make per year, the missions it would be used for, and the attrition and wear-out rates. The unknowns far outnumber the knowns, if indeed there are any knowns in a system as far-out as this one. But let us assume it would be desirable for planning purposes to compare the cost of using an aerospace plane for performing a number of missions with the cost of using several different expendable boosters. It is possible, using a range-of-values approach, to come up with a range of cost estimates.

Using this approach means looking at a range of vehicle weights, a range of utilization rates, and so on. Thus we would have a series of displays like those sketched below. Further, an analysis of these will indicate the particular system characteristics to which total system cost is sensitive and those to which it is insensitive. In our aerospace plane example, we might find that for a range of vehicle weights, utilization rates, missions, and attrition and wear-out rates, the range of total system cost is so great as to be meaningless. Closer scrutiny might reveal, however, that a major part of this variation comes from a single system characteristic, say, mission altitude. By limiting the system to low-orbit missions, the variation in cost might be reduced to a range small enough to make meaningful comparisons with other systems possible.





This example is concerned with requirements uncertainty in a total system context. If we are interested only in the cost of the aerospace plane itself, a similar analysis could be performed to establish the cost implications of changes in weight, speed, payload in orbit, type of propulsion, etc. Or, if interest centers on cost-estimating uncertainty, one could examine a range of material or fabrication costs as in the following example where new technology makes estimating more uncertain than usual.

The aircraft industry is continually searching for new materials that will be stronger, lighter, have a higher heat resistance, or offer some other advantage over materials now used. At present, boron-fiber reinforced composite appears to offer potential weight savings as a replacement for some portion of the aluminum commonly used, but it also, being in an experimental stage, is very expensive--about $700 per pound. Fabrication techniques are just being worked on now, and they are also very expensive. At some time in the future, however, boron material may become available in quantity for use in aircraft production, and it is always of interest to examine the possible effect of a new material on cost (consider, for example, the speculation about the cost of using titanium in the F-111 and supersonic transport).

To examine the effect substitution of boron materials would have on the production cost of aircraft, a range-of-values approach provides more information than a single-value estimate as well as emphasizing the uncertainty of the numbers. In this example, then, in which manufacturing costs only are considered, a range of costs is stipulated wherever appropriate. Manufacturing costs are largely a function of weight and for a large modern fighter aircraft are estimated to run about $60 per pound (at the 400th unit). Considerable uncertainty exists about the cost of fabricating sheets and panels of boron, even assuming that computer-controlled machines will be available. To allow for this uncertainty we postulate a range of fabrication costs, from $72/lb to $121/lb based on optimistic and pessimistic prediction of persons having some experience with fabrication of boron composite.

The material cost is comprised of aluminum, purchased parts and equipment, and boron composite. These can also be estimated on a cost-per-pound basis, and for aluminum the cost should be about what it is today--$10/lb--with no variation considered. For purchased parts and equipment there is some uncertainty about what would go on the boron airplane, so a range of $60/lb to $100/lb is chosen (compared with $60/lb for an aluminum aircraft). While boron costs are still in the realm of conjecture, Fig. VI-1 shows a prediction of how they might decrease over time. For this example, we have taken the cost at three different times--$325/lb in 1968, $50/lb in 1974, and $25/lb in 1980--with the expectation that the real range of interest is comprised of the final two. The 1968 figure is included as a reminder of current

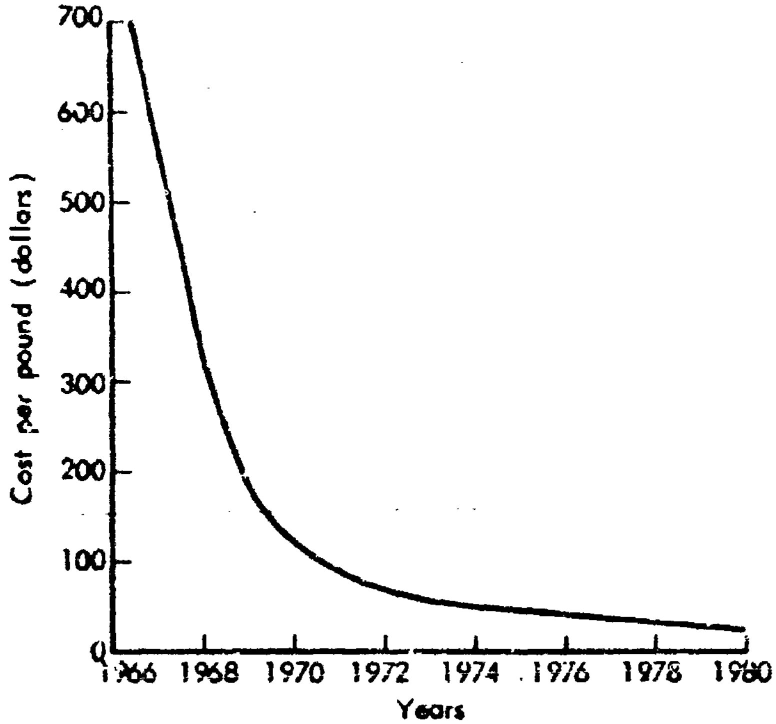


Fig. VI - 1—Projected boron material cost reality. The manufacturing and material costs (in millions of dollars) resulting from these cost factors are shown below:

Boron Cost	$325/lb		$50/lb		$25/lb	
	High	Low	High	Low	High	Low
Manufacturing	2.00	1.45	2.00	1.45	2.00	1.45
Material	3.32	3.09	1.01	.78	.80	.57
Total	5.32	4.54	3.01	2.23	2.80	2.02

These figures show a possible range of $5.32 million to $2.02 million, and a likely range of $3.01 million to $2.02 million. They also show that total manufacturing cost is relatively insensitive to changes in the cost of boron once this cost has declined to the $50/lb level.

The procedure illustrated above is applicable to any situation in which costs and/or requirements are uncertain and limits can be assigned to the uncertainty with some assurance. The major drawback to cost-sensitivity analysis is implied by this latter condition, since there is no guarantee that in any given analysis all the relevant alternatives will be included. Regardless of its limitations, cost-sensitivity analysis is probably one of the best currently available techniques for helping deal with the uncertainty problem in estimating the cost of equipment and weapon systems.

Monte Carlo Techniques

One method proposed for dealing with uncertainty begins with the assumption that a cost analyst can describe each input parameter with a probability distribution.* This distribution is then treated as a theoretical population from which random samples are obtained. The methods of taking such samples, as well as problems which rely on these sampling techniques, are often referred to as Monte Carlo methods.

To illustrate the Monte Carlo procedure for simulating cost input uncertainty, consider the example depicted in Fig. VI-2.

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*This method is described in more detail in a report by P. F. Dienemann, Estimating Cost Uncertainty Using Monte Carlo Techniques, The RAND Corporation, RM-4854-PR, January 1966.

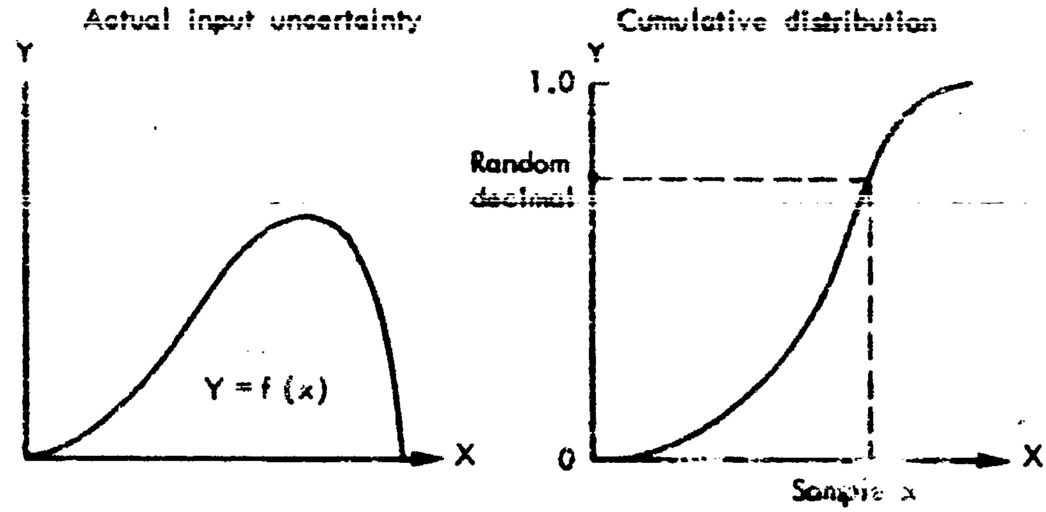

Fig. VI-2—Monte Carlo sampling

From the probability density, Y = f(x), describing the actual (or estimated) input uncertainty, a cumulative distribution is plotted. Next, a random decimal between zero and one is selected from a table of random digits. By projecting horizontally from the point on the Y-axis corresponding to the random decimal to the cumulative curve, we find the value of x corresponding to the point of intersection. This value is taken as a sample of value of x.

The result, if this procedure is repeated numerous times, is a sample of input values that approximates the required input uncertainty. As seen in Fig. VI-3, the more repetitions, the better the simulated input distribution.

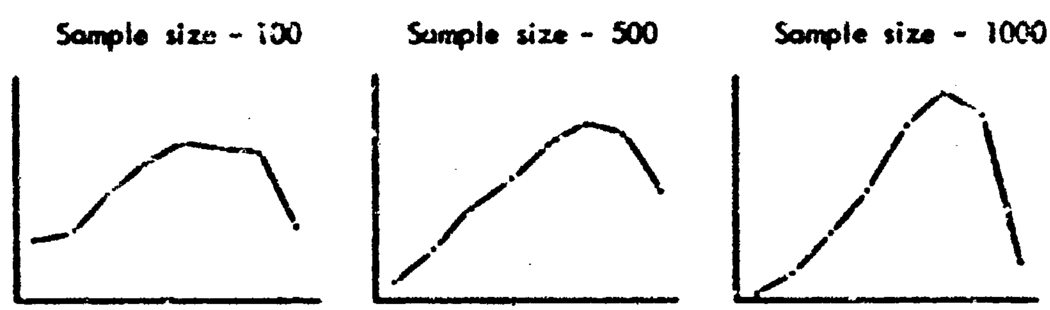

Fig. VI-3—Simulated input distribution

The procedure for estimating cost uncertainty follows readily once simulated input values have been made. To illustrate, consider the following simple estimating relationship:

$$C = W \times P$$

where C = cost,

W = weight,

P = cost per pound.

Assume the actual uncertainty of the input parameters can be represented with probability distributions as shown in Fig. VI-4, with L, M, and H denoting the lowest possible, most-likely, and highest possible values, respectively. Furthermore, assume that these values are as follows:

Item	L	M	H
Weight	75	100	125
Cost/lb	300	400	700

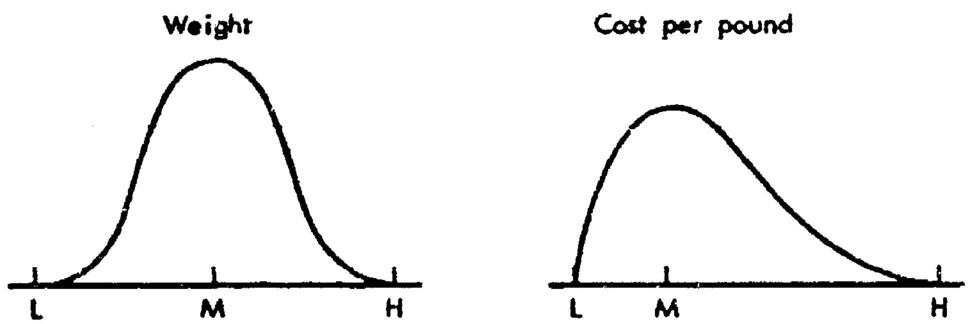

Fig. VI-4—Input uncertainty distributions

From the input distributions, a sample value for both the weight and the cost per pound is generated by means of the Monte Carlo technique. Using these two sample values, a cost is calculated. The procedure is repeated again and again until the nature of the output uncertainty has been established. Table VI-2 summarizes the procedure for 1000 iterations.

Table VI-2

MONTE CARLO SIMULATION OF COST UNCERTAINTY

Iteration	W	P	C
1	83	405	33,615
2	108	633	68,364
3	103	374	38,522
4	101	452	45,652
5	92	387	35,604
.	.	.	.
.	.	.	.
.	.	.	.
1,000			
Mean Values	100	450	45,000

From the set of cost estimates, a frequency distribution as shown in Fig. VI-5 can be prepared to portray the cost uncertainty. It is interesting to note that the mean value of the cost is higher than the single-value cost estimate ($40,000)--the product of the most-likely values for each input factor. The difference between the two estimates occurs because the uncertainty about the cost per pound is skewed to the right. If the uncertainty distributions of both input factors were symmetric, the two cost estimates would be identical.

Although this example depicts a very simple costing problem, the techniques are applicable to more realistic situations. However, when the scope of the problem is expanded it is expedient that the costing model be programmed for a computer.

It must be noted that using the Monte Carlo technique to estimate cost uncertainty in this manner requires that all input parameters be mutually independent. With cost factor inputs, we can probably conclude that the assumption of independence is true. However, with system

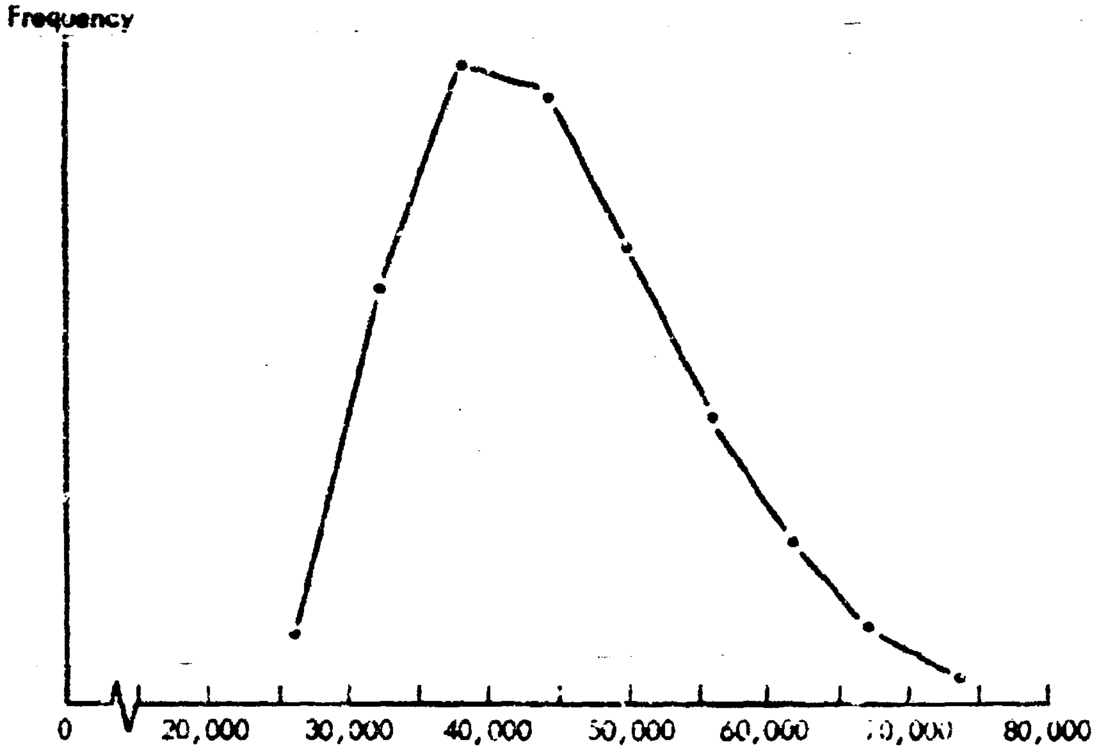

Fig. VI-5—Frequency versus cost

requirements we must be more careful. In cases where a function relationship does exist between two or more inputs, we can often circumvent the interdependence problem by incorporating the relationship within the cost model; or if the problem demands, one could explore more sophisticated techniques for sampling from joint frequency distributions.*

Cost Estimate Confidence Rating

In an entirely different approach to the problem the Air Force Systems Command has instituted a Cost Estimate Confidence Rating (CECR), AFSC Form 27, which attempts to establish subjective limits on the confidence to be placed in each separate segment of an estimate, e.g., airframe, propulsion, etc.** In this procedure the estimator is asked to assign a value of from 1 to 5 to each of the following factors:

Estimating Conditions
- Estimating and information access
- Ground rules and assumptions
- Other (specify)

Nature of the Item
- State of the art
- Production experience
- Other (specify)

Item Description
- Specification status
- Operating program characteristics

Cost Methods and Data
- Methods
- Data

A rating of 1 on Estimating Time and Information Access, for example, means "there was complete access to available data needed to cost

*D. J. Finney, "Frequency Distribution of Deviation from Means and Regression Lines in Samples from a Multi-variate Normal Population," The Annals of Mathematical Statistics, Vol. 17, 1946.

**Described in AFSCL 173-1A, Attachments 3-8 through 3-14.

this item and there was ample time to thoroughly research these sources." A rating of 5, on the other hand, implies that "the dominating source of uncertainty has been the completely inadequate amount of time provided to make the estimate and/or the complete lack of access to useful data sources." From the ratings assigned to each factor a consolidated confidence rating is determined (normally the arithmetic mean of the rating assigned to the individual factors) which expresses the estimator's overall confidence. In addition to the ratings AFSC Form 27 calls for an estimate of the most likely cost, lower-bound cost, and upper-bound cost. These upper and lower bounds presumably stem from the uncertainties previously specified. A sample form is shown in Fig. VI-6.

While from an operational point of view it is not clear what the recipient of an estimate does when he is told to give the estimate little credence, documentation of the sources and extent of uncertainty in an estimate should be helpful. Also, the need to specify which estimates he is most uncertain about and why may spur the estimator to do a better job on these items. Thus, while the AFSC CECR is still experimental and cannot be evaluated empirically as yet, it does represent a constructive step in the right direction.

Better Information

One better solution is sometimes feasible, given the same condition necessary to use cost-sensitivity and Monte Carlo techniques, i.e., that the areas of uncertainty can be defined. This solution is to reduce or eliminate uncertainty by obtaining better knowledge, which in effect is the purpose of the Contract Definition Phase of hardware procurement. A careful spelling out of requirements and design specifications can eliminate much of the uncertainty that pervades a conceptual study. Or actual tests may be performed to obtain more knowledge, as in the case of the supersonic transport where both Boeing and Lockheed fabricated a number of parts out of titanium to gain information on the cost of working with this metal. In that situation, the need to reduce cost-estimating uncertainty impelled both companies to spend several millions of dollars. The government cost estimator may never have the

SYSTEM	COST ESTIMATE CONFIDENCE RATING	PAGE OF PAGES
WBS ITEM	PREPARED BY	DATE PREPARED

1. COST MAGNITUDE (Enter % in appropriate cost category)			2. RANGE OF COST (Estimated)		3. CONSOLIDATED CONFIDENCE RATING
COST CATEGORIES	a. DEVELOPMENT	%	a. MOST LIKELY COST	$	
	b. INVESTMENT	%	b. LOWER BOUND COST	$	
	c. OPERATIONAL	%	c. UPPER BOUND COST	$	

4 EVALUATION OF FACTORS

FACTORS	RATING	EXPLANATION
a. ESTIMATING CONDITIONS	1 2 3 4 5	
(1) ESTIMATING TIME AND INFORMATION ACCESS		
(2) GROUND RULES AND ASSUMPTIONS		
(3) OTHER (Specify)		
b. NATURE OF THE ITEM	1 2 3 4 5	
(1) STATE OF THE ART		
(2) PRODUCTION EXPERIENCE		
(3) OTHER (Specify)		
c. ITEM DESCRIPTION	1 2 3 4 5	
(1) SPECIFICATION STATUS		
(2) OPERATING PROGRAM CHARACTERISTICS		
d. COST METHODS AND DATA	1 2 3 4 5	
(1) METHODS		
(2) DATA		
e. CONSOLIDATED CONFIDENCE RATING	1 2 3 4 5	

AFSC FORM MAY 66 27 PREVIOUS EDITIONS OF THIS FORM ARE OBSOLETE AFSC (AWB)

Fig. VI-6—Cost estimate confidence rating resources for a similarly massive attack on his own problem, but the example is instructive nonetheless. Uncertainty can be reduced in some instances by experimentation, in others by better definition, and in all by increased knowledge. Nevertheless, the cautionary note sounded by The World Meteorological Organization in 1966 on the subject of weather forecasting is probably applicable here:

> The basic characteristics of uncertainty will almost surely continue to be operationally significant for the foreseeable future.

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10. ABSTRACT

This Memorandum is the introductory portion of a text on the general subject of cost estimating procedures being prepared at the request of the Office of the Assistant Secretary of Defense (Systems Analysis). The study discusses the fundamental problems of estimating major equipment costs and suggests that for many purposes, particularly for government cost analysts, a statistical approach is the most suitable. The kind of data required and the adjustments needed to make the data useful are discussed in some detail. The use of regression analysis in deriving cost estimating relationships is described, but it is emphasized that unquestioning use of estimating relationships obtained in this manner can result in serious errors. The concepts underlying the cost-quantity relationship generally known as the learning curve are presented along with instructions for its use. Finally, the problem of uncertainty in cost estimating is discussed, and a few suggestions for dealing with the problem are included.

11. KEY WORDS

Cost analysis
Cost effectiveness studies
Cost estimating relationships
Statistical methods and processes
Uncertainty
Probability