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PROPELLANT SLOSHING IN A CONICAL TANK UNDERGOING ARBITRARY FORCED TRANSLATIONAL MOTION

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FOREWORD

Studies of propellant sloshing in cylindrical tanks have been carried out in References 2 and 3. The present study is concerned with sloshing in conical tanks and is directed toward applications wherein the low level sloshing at the conical bottoms of missile tanks is encountered.

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SUMMARY

A solution to the perfect fluid equations for sloshing in a conical tank, presented in Reference 1, is extended to give the theoretical forces moments and surface wave forms. A mechanical analogy, in the form of a pendulum plus fixed mass, is found to yield identical responses.

INTRODUCTION

In large liquid fuelled missiles the propellants comprise the greatest portion of total mass throughout most of powered flight. The sloshing of these fluids, in response to missile accelerations, produces large destabilizing forces necessitating a study of fluid behavior and means for slosh control.

The present report represents a continuing effort in sloshing studies, this one being intended specifically to treat sloshing in a conical tank bottom. Mechanical analogies have proven useful for sloshing representations in missile stability studies (Reference 2, 3) and hence, the development of one is pursued herein also.

NOMENCLATURE

a	-	arbitrary constant
h_1	-	distance from cone apex to pendulum hinge in mechanical analogy
h_o	-	distance from cone apex to rigid mass in mechanical analogy
h	-	depth of fluid measured from cone apex
s	-	La Place operator
A_x	-	acceleration in X - direction
A_y	-	acceleration in vertical (y) direction
F, G	-	arbitrary functions
F_x	-	force in X - direction
K	-	arbitrary constant
L_p	-	length of pendulum in mechanical analogy
M_{AA}	-	moment about axis A-A
M	-	total fluid mass
M_o	-	rigid mass in mechanical analogy
M_1	-	pendulum mass in mechanical analogy
p	-	pressure
α	-	cone semi vertex angle
η	-	height of fluid surface wave
ρ	-	fluid density
γ_p	-	pendulum angle in mechanical analogy
ω	-	natural frequency (of fluid mode and/or pendulum)

NOMENCLATURE (CONTINUED)

y, r, θ -	cylindrical coordinates
x, y, z -	rectangular coordinates
w, v -	coordinate direction with respect to walls

ANALYSIS

The special case of a conical tank with a semi-vertex angle of 45° is considered here, with only translational tank motion involved. The fluid is considered incompressible and non-viscous. In addition, higher order terms, such as products of small perturbation velocities and accelerations are neglected. With these assumptions, the problem reduces to that of satisfying the following mathematical expressions (Reference 1).

1) $\nabla^2 \phi = 0$ throughout the fluid

2) $\phi_n = 0$ on the tank walls

3) $\phi_{tt} + A_y \phi_y + A'_x X = 0$ on the free surface, $y = h$

In cylindrical coordinates (y, ϕ, r), with origin at the apex (see Figure 1),

La Place's equation is

$$4)\quad \phi_{rr} + \frac{1}{r}\phi_r + \frac{1}{r^2}\phi_{\theta\theta} + \phi_{yy} = 0.$$

Now assume a trial solution of the form

$$5)\quad \phi = F(t) \cdot G(r, y) \cos\theta .$$

Substituting into Equation 4, one has

$$F(t) \cdot G_{rr} \cos\theta + \frac{1}{r} F(t) \cdot G_r \cos\theta - \frac{1}{r^2} F(t) G \cdot \cos\theta$$

$$+ F(t) G_{yy} \cos\theta = 0$$

HENCE,

$$6)\quad G_{rr} + \frac{1}{r} G_r - \frac{1}{r^2} G + G_{yy} = 0 \qquad \text{Equation 6}$$

corresponds to the result obtained in Reference 1.

By changing variables such that,

$$w = r + y$$

$$v = y - r \, ,$$

Levin (Reference 1), shows that Equation 6 becomes

$$7)\ (w^2 + 2wv + v^2)\left[G_{ww} + G_{vv}\right] + (w+v)\left[G_w + G_v\right] - 2G = 0$$

and condition (2) gives

8) $G_v = 0$ on $v = 0$ (for a 45° apex angle y = r on the tank walls).

Assume the following expansion as a solution:

$$9)\ G = \sum_n^{\infty} \sum_m^{\infty} a_{mn} w^m v^n$$

It has been verified that by expanding the double recursion relations, "A_{ij}", through the first four terms and applying the conditions of Equations 7 and 8, only the a_{20} and a_{02} coefficients do not become zero. This results in

$$a_{20} = -a_{02}$$

then

10)

$$G = a_{20} w^2 + a_{02} v^2 = a_{20}(w^2 - v^2)$$

From the change of variables previously made and it can be found that,

$$w^2 - v^2 = 4ry \qquad \text{THEREFORE,}$$

11) $$G = 4a_{20}ry = Kry$$

Finally,

12) $$\phi = F(t)\,ry\cos\theta$$

Equation 12 now satisfies Equations 1 and 2. The remaining boundary condition, Equation 3, leads to

$$\ddot{F}ry\cos\theta + A_y F r\cos\theta + A_x r\cos\theta = 0$$

OR

13) $$h\ddot{F} + A_y F = -\dot{A}_x$$

In La Place operational notation,

$$F(s^2h - A_y) = -s^3 X(s)$$

or

14) $$F(s) = -\frac{1}{h}\frac{s^3 X}{s^2 + \omega^2}$$

where $\omega^2 = A_y/h$ is the natural frequency of slosh.

The final form for the scalar velocity potential is then:

15) $$\phi = \frac{s^3 X}{s^2 + \omega^2} \quad \frac{ry\cos\theta}{h}$$

Note that only a single sloshing mode, (only one natural frequency) is found hereby.

From the equation for the velocity potential one can successively derive expressions for the dynamic pressure, force, moment and surface wave form due to the oscillating propellants as follows:

Pressure

The fluid pressure can be expressed as follows:

$$p(s) = \rho\left[\phi_t + A_x r\cos\theta\right]$$

Now on the walls $r = y\,TAN\alpha = y$ Hence

$$16)\quad \phi_t = -\frac{s^4 \chi}{s^2+\omega^2}\,\frac{y^2}{h}\cos\theta$$

Then the pressure at the walls is

$$17)\quad p(s) = \rho\left[-\frac{s^4\chi}{s^2+\omega^2}\,\frac{y^2}{h}\cos\theta + y\cos\theta\, s^2\chi\right]$$

Forces

The forces exerted on the tank walls may now be obtained by an integration of the pressure in Equation 17 over the surface of the cone. By setting up a differential area for the cone, one obtains the integral:

$$F_x(s) = \int_0^h\int_0^{2\pi} p\cdot\cos\alpha\, dA\cdot\cos\theta = \int_0^h\int_0^{2\pi} p\cos\theta\cdot r y\, d\theta$$

Integration results in the following expression for the forces at the wall.

$$F_x(s) = +\rho \frac{h^3}{3} \pi s^2 x - \rho \frac{h^3 \pi}{4} \frac{s^4 x}{s^2 + \omega^2}$$

18) $$F_x(s) = M s^2 x - \frac{3}{4} M \frac{s^4 x}{s^2 + \omega^2}$$

where $M = \rho \pi h^3/3$ is the total mass of the fluid.

This result is identical in form with that obtained for the flat bottomed cylindrical tank undergoing translation only. (Reference 2).

Moments

The moments produced by the oscillating fluid are next obtained from an integration of the forces on the tank. The clockwise moments about the z axis (Figure 1) are as follows:

$$M_{AA} = \int_0^h \int_0^{2\pi} \rho \, dA \cdot \sqrt{2}\, r \cos\theta$$

$$= \int_0^h \int_0^{2\pi} \rho \cdot \frac{r\, d\theta\, dy}{\cos\alpha} \sqrt{2}\, r \cos\theta$$

$$= 2\rho \int_0^h \int_0^{\pi} \left[s^2 x - \frac{s^4 x}{(s^2+\omega^2)} \cdot \frac{y}{h} \right] r^3 \cos^2\theta \, dy \, d\theta$$

$$= h^4 \pi \rho \left[\frac{1}{2} s^2 x - \frac{2}{5} \frac{s^4 x}{s^2+\omega^2} \right]$$

or

$$19)\quad M_{AA} = 1.5\, h\, M\, s^2 \chi - 1.2\, h\, M \frac{s^4 \chi}{s^2 + \omega^2}$$

Surface Wave Form

As in Reference 2, the surface wave form can be derived by letting the pressure term in Euler's equation be zero. Then by denoting the surface elevation by η , one has:

$$\eta(s) = -\frac{1}{A_y}\left[\phi_t - A_x r \cos\theta\right]_{y=h}$$

$$= \frac{1}{A_y}\left[s^2\chi - \frac{s^4 \chi}{(s^2+\omega^2)} \cdot \frac{y}{h}\right] r \cos\theta$$

or at the walls where $r = h = y$

$$20)\quad \eta(s) = \frac{h \cos\theta}{A_y}\left[s^2 \chi - \frac{s^4 \chi}{s^2+\omega^2}\right]$$

Mechanical Analogy

The mechanical analogy used here will be similar to that used in Reference 3, so only the final equations will be given along with the determination of the arbitrary constants. Figure 2 shows the mechanical system.

Forces

For the mechanical system the forces may be written directly as:

$$F = s^2 X M_0 - M_1 A_y \gamma_p$$

where γ_p , the pendulum angle, has a response

$$21)\quad \gamma_p = \frac{-s^2 X}{L_p(s^2+\omega^2)}$$

Equation 21 can be manipulated such that the force equation is put into the form

$$22)\quad F(s) = (M_0 + M_1)\, s^2 X - M_1 \frac{s^4 X}{s^2 + A_y/L_p}$$

Equations 18 and 22 are equal if and only if:

$$M = M_0 + M_1$$

$$\frac{3}{4} M = M_1$$

$$\omega^2 = A_y/L_p$$

follows that $M_0 = \frac{1}{4} M$

Moments

By choosing various pivot points for the pendulums, the force analogy can be extended to include the moment equation. The moment equation for the mechanical system is as follows:

$$M = s^2 X M_0 h_0 - M_1 A_y h_1 \delta_p$$

again by using Equation 21 one can rewrite this as the equation,

$$23) \quad M = (M_0 h_0 + M_1 h_1) s^2 X - M_1 h_1 \frac{s^4 X}{s^2 + \omega^2}$$

By comparing Equations 19 and 23 one finds:

$$M_0 h_0 + M_1 h_1 = 1.5\, h M$$
$$M_1 h_1 = 1.2\, h M$$

Using the values for M_0 and M_1 obtained previously one finds,

$$h_1 = 1.6 h$$
$$h_0 = 1.2 h$$

The mechanical analogy is thus completely defined. Table I summarizes the parameters.

TABLE I

Mechanical	Hydrodynamic
h_1	$1.6h$
h_o	$1.2h$
M_o	$0.25M$
M_1	$0.75M$
L_p	h
ω^2	α_T/h

Surface Wave Form

The surface wave form of Equation 20 can also be expressed in terms of the displacement of the equivalent pendulum as follows. From Reference 1, Equation 102,

$$s^2 x = -(s^2+\omega^2) L_p \gamma_p$$

and

$$s^4 x = -s^2 \gamma_p L_p (s^2+\omega^2)$$

Substituting into Equation 20 and rearranging terms gives,

$$\eta(s) = \frac{L_p}{A_y} r \cos\theta \left[s^2 \gamma_p - \gamma_p (s^2+\omega^2) \right]$$

or at the tank walls

$$26)\quad \eta = -h \gamma_p \cos\theta$$

Note that the sloshing surface is a planar surface, unlike that found for the cylindrical tank, Reference 2.

CONCLUSIONS

The equations for the forces, moments and surface wave forms for a conical tank (of 45° semi-vertex angle) being subjected to arbitrary horizontal, translational motions have been obtained.

The forces, moments and surface wave forms can be duplicated with the use of an equivalent pendulum system whose parameters, in terms of the fluid depth, are given herein.

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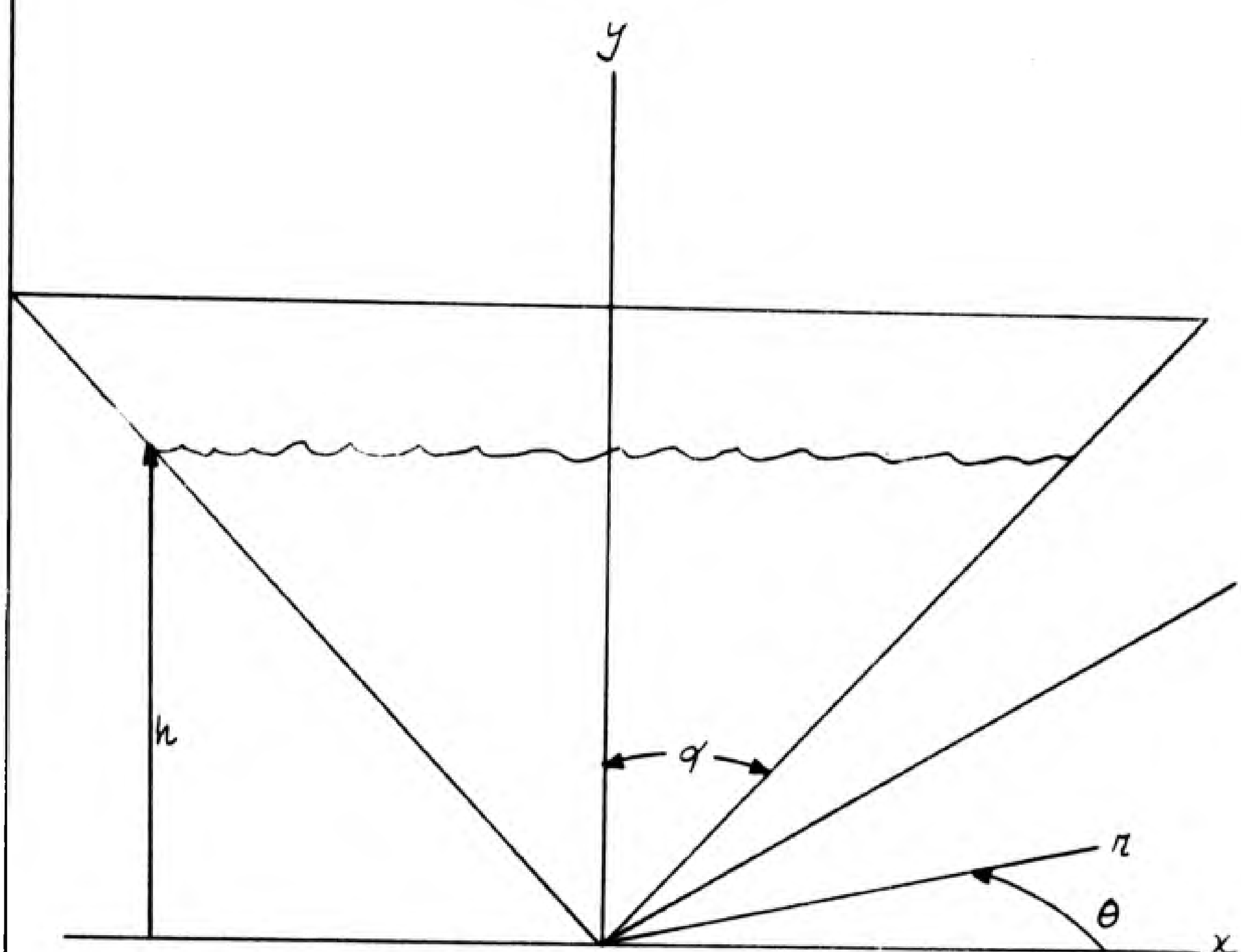

FIGURE 1

PROBLEM COORDINATE SYSTEM

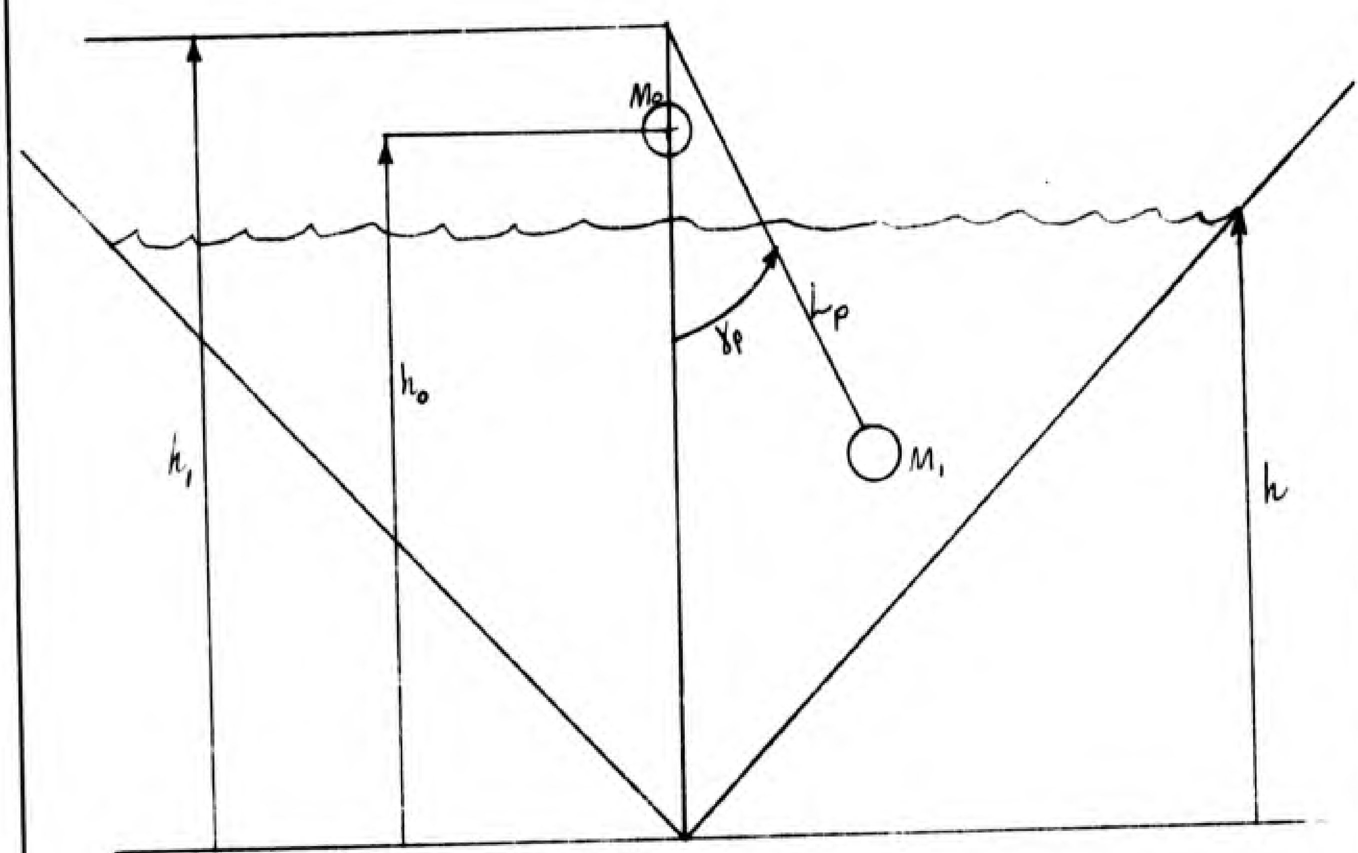

FIGURE 2

MECHANICAL ANALOGY USED TO

DUPLICATE FLUID RESPONSES