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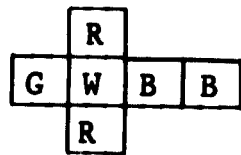
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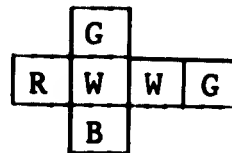
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A NOTE ON "INSTANT INSANITY"

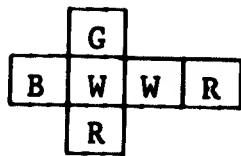
An interesting puzzle has recently appeared in toy stores under the name "Instant Insanity". It consists of four multi-colored unit cubes. Each cube has its faces painted red, blue, white, and green, the exact coloring is indicated in Fig. 1. The puzzle is



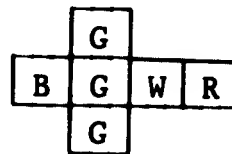
Cube I



Cube II



Cube III



Cube IV

G = green face
W = white face

B = blue face
R = red face

Figure 1

to assemble these four cubes into a $1 \times 1 \times 4$ rectangular prism such that all four colors appear on each of the four long faces of the prism. The charm of this puzzle lies in the fact that it sounds very easy but is, in fact, quite difficult. Each cube may be given 24 different orientations, and thus there are 82,944 possible rectangular prisms which can be arranged

from them (not counting permutation of the order of the cubes or rotation of the prism about its long axis). As we shall see in a moment, only two of these arrangements satisfy the conditions of the puzzle.

Some people will (after manipulating the cubes for an hour or so) give up or come to the conclusion that the puzzle is actually impossible to solve. The purpose of this note is to present a method of analyzing puzzles of this type which will permit the solution of this particular puzzle in a minute or two, and which can be used to design similar puzzles which are easier, harder, or actually impossible.

The first fact to note is that, for purposes of this type of puzzle, any colored cube may be replaced by its mirror image (in our particular example Cubes I and IV are their own mirror images, but II and III are not). To see this, imagine that we have assembled the cubes in a rectangular prism on the table in front of us; notice that four faces of each cube appear on long faces of the prism, while two faces of each cube are "buried" or constitute one of the two square faces of the prism (and thus are ignored in determining whether the prism is or is not a solution to the puzzle). If one of the cubes (say II) is replaced by its mirror image (call it II') then it is obvious that II' can be rotated in such a way that the four of its faces which appear on long faces of the prism are exactly the same

colors in exactly the same positions as before, while its "buried" faces interchange colors. Thus we may characterize a cube (from the standpoint of the puzzle) by naming the three color-pairs appearing on opposite faces. We may set up a correspondence between a subset of the natural numbers and sets of sets of n colored objects in the following way:

Let 1 stand for red.

Let 2 stand for white.

Let 3 stand for blue.

Let 5 stand for green.

Then we let the "characteristic number" of a set of n colored objects be simply the product of the n integers corresponding to their individual colors. For example, the characteristic integer of two blue objects, a red one, and a green one would be $3^2 \cdot 1 \cdot 5 = 45$. Provided we know the number of objects in the set, knowledge of its characteristic number obviously tells us its distribution of colors. Using this code, we easily prepare a list of the characteristic numbers of pairs of opposite faces of each cube:

	(1)		(2)		(3)
Cube I	1	—	6	—	15
Cube II	2	—	10	—	15
Cube III	2	—	5	—	6
Cube IV	5	—	6	—	25

We wish to select a set of four pairs of opposite faces (one pair from each cube) which will constitute the "top" and "bottom" of our prism. Since each color must be represented once on the top and once on the bottom, we want the characteristic number of the total set of eight faces to be $1^2 \cdot 2^2 \cdot 3^2 \cdot 5^2 = 900$. Thus we must select a set of four numbers, one from each row in the 4×3 matrix above, whose product is 900. We must select two such sets: one to determine the "top and bottom" and one to determine the "front and back". The two sets must be disjoint, of course. It is not hard to carry out this task by inspection; however, if you wish to be systematic, it is useful to prepare a list of the nine possible products from cubes I and II, and compare it to a list of the nine possible products from cubes III and IV, as follows:

I	II		III	IV
(1) x (1)	2		(1) x (1)	
(1) x (2)	10		(1) x (2)	
(1) x (3)	<u>15</u>		(1) x (3)	
(2) x (1)	12		(2) x (1)	
(2) x (2)	60		(2) x (2)	
(2) x (3)	<u>90</u>		(2) x (3)	
(3) x (1)	<u>30</u>		(3) x (1)	
(3) x (2)	150		(3) x (2)	
(3) x (3)	225		(3) x (3)	

			<u>10</u>	
			12	
			<u>50</u>	
			25	
			<u>30</u>	
			<u>125</u>	
			30	
			36	
			150	

The only two numbers in the first column which have a multiplicative complement with respect to 900 in the second column are 90 (whose multiplicative complement with respect to 900 is 10) and 30 (whose multiplicative complement with respect to 900 is 30). It is now easy to deduce the following solution to the puzzle:

Cube	"Top and Bottom"	"Front and Back"
I	white - blue	green - blue
II	blue - green	white - red
III	red - white	red - green
IV	green - red	blue - white

The calculations above show that this solution is unique (up to rotation of the prism about its long axis) except for the fact that the colors on the top and bottom may be interchanged without altering the colors on the front and back. Thus there are only two essentially different solutions to the puzzle.

The reader may enjoy using the methods of this paper to solve the following three puzzles, which are presented in increasing order of difficulty.

1. Determine whether or not it is possible to arrange the three cubes of Fig. 2 in a $1 \times 1 \times 3$ prism such that the colors red, white, and blue appear on each of the four long faces.

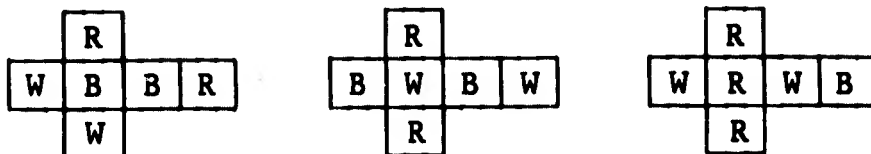
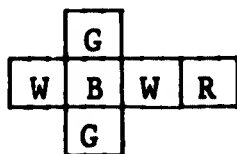


Figure 2

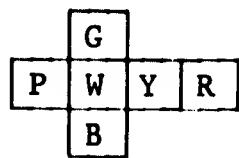
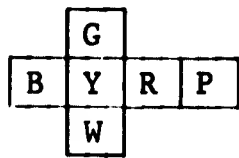
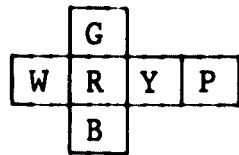
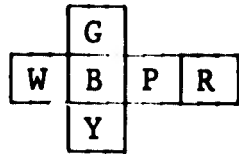
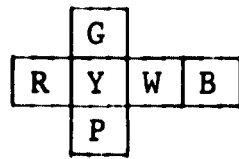
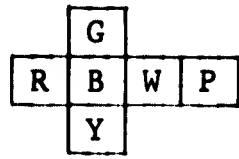
2. Show that the "Instant Insanity" puzzle has no solution if cube IV is replaced by cube IV* (shown in Fig. 3).



Cube IV*

Figure 3

3. Find a way to arrange the six cubes shown in Fig. 4 in a $1 \times 1 \times 6$ prism such that all six colors will appear on each long face of the prism. Note that every color appears on every cube; this implies that if there is one solution there must be at least six essentially different solutions. Why?



Y = yellow face P = purple face

Figure 4