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Aircraft Landing Speed and Accident Correlation

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This analysis* considers landing accidents for the three-year period 1962-64 in order to relate the frequency of these accidents to the magnitude of the landing speeds. In a recent unclassified report by Dressler [1], based upon all accidents occurring in U.S. Air Carriers for the four-year period 1961-64, it was shown that more meaningful statistics on aircraft safety require the introduction of at least three different accident rates: a rate for cruise, a separate rate for takeoffs, and a separate rate for landings. By using these separated rates (called "intrinsic rates" in [1]), it is possible to detect and analyze trends for each phase of flight in situations where the trends would otherwise be completely hidden if one used a conventional single rate (e.g., all accidents per flight hour) as is customary. As a result of a detailed examination of the intrinsic landing-accident rates, it was deduced in [1] that there is a strong correlation between the frequency of landing accidents and the average landing speeds for U.S. Air Carriers when one compares the three categories of winged aircraft: jets, turbo-props, and pistons. In that study, however, covering the four-year period 1961-64, there were only 174 landing accidents which could be used for the analysis. From this small number, the author tentatively concluded that

* This paper is an abridged, unclassified treatment of "USAF Landing Accidents and Landing Speeds," May 1966, Federal Aviation Agency [redacted].

[1] "New Perspectives on Air Safety," by R. F. Dressler, December 1965, Office of Policy Development, Federal Aviation Agency.

the landing accident rate was strongly correlated with at least the second power of the landing speed, or higher power. Because of the comparatively small number of accidents and the comparatively narrow range of landing speeds between the pistons and the jets used in commercial service, the conclusion reached there was unfortunately not as statistically conclusive as might have been desired. Therefore, a subsequent study was undertaken, which is the subject of this report, to analyze USAF landing accidents on the same basis in order to determine whether or not a similar conclusion could be validly inferred from Air Force data. There are considerable advantages in an analysis of USAF accidents not present in the analysis of U.S. Air Carrier accidents: (1) the number of accidents available for study is many times greater in the former case, (2) the range of variation in landing speeds is much wider, and (3) the number of different landing speeds available is much greater.

In [1] it was shown that, on the average for all winged aircraft, it requires about 3.7 hours of cruise to contain the same accident risk as one takeoff and landing. Also, it is well known that landing accidents completely overshadow in frequency both takeoff accidents (by more than two to one) and cruising accidents. It follows that landing accidents constitute the single most important category of aircraft accidents, and it is our purpose here to compare their frequency with landing speeds.

The basic data we have used in this analysis were obtained from [2]. We have not used USAF accidents earlier than 1962 because there had been a different method of reporting accidents in force prior to that date which would have introduced some inconsistency in our analysis. We have therefore restricted our attention to the most recent data available, the three-year period 1962-64. The data used include USAF accidents for all types of aircraft except helicopters, domestic and international, excluding only those accidents related to combat damage. Although we can use almost all of the total accident data, it is perfectly clear that a very small percentage must first be discarded, those representing the so-called "utility" flights. Such flights involve totally different and much more hazardous landing conditions than the normal types of operation. This naturally is reflected in an abnormally high landing accident rate which cannot be compared logically with the main body of data. We have therefore excluded all this utility flying which represents during this period only a very small percentage of total activity or accidents. The remaining activity comprises a statistically very large and reliable number of landing accidents, for many different aircraft types, including many pistons, turboprops, and jets. These types include 18 different landing speeds in knots in a range about 2 1/2 to 1 width. Just as in [1], "landing speed" means the speed at touchdown, which is defined as 1.3 times the stalling speed. The term "accident" as used by USAF is also consistent with the definition used in [1];

[2] "USAF Accident Bulletins 1962, 63, and 64," Directorate of Aerospace Safety, Norton Air Force Base, California [redacted].

in both cases the relatively minor accidents which are termed "incidents" are excluded, and the remaining data refer to aircraft accidents which are relatively serious.

Consistent with the analysis in [1], we here consider all flights as divided into three phases: the takeoff including climb, the cruise, and the landing operation including descent and holding. The basic data in terms of the aircraft types, numbers of landings, and numbers of landing accidents for each type together with touchdown speed in knots, were taken from [2].

We are of course using the touchdown velocities as single numbers to describe the entire landing phase for comparisons of aircraft types. Although this procedure cannot be precise for any portion of the entire landing phase except at touchdown, nevertheless, aircraft types which are faster in touchdown will normally also be faster in descent, holding pattern, etc. These single touchdown velocities at least should therefore give us relative numerical ratings for the entire landing phase, and are used here in that context.

We first divide the totality of the many aircraft types into <u>three</u> major sub-categories. The first category embraces all aircraft with landing speeds in the range up to and including 120 knots; the second category ranges from 121 through 150 knots; the third category includes all landing speeds above 150 knots. The numbers of accidents represented in these three categories have extremely high statistical reliability. Because of the irregular way in which the landing speeds are distributed over the range, it is not always possible to divide so that sub-categories will always contain precisely equal numbers of accidents, but our choice here and in the next section has been made so that the numbers of accidents in each category are roughly of the same magnitude insofar as this is possible. The left-hand section of Figure 1 plots the landing accident rates for these three major categories against the weighted average landing speeds within each category. This graph is made on double-log paper in order to ascertain the proper exponent for the variation exhibited. The least-squares straight line through these points has a linear slope of about 3.15 which means of course that the variation between accident rate y and landing speed x is approximately $y = ax^3$. These same three points are next plotted in Figure 2 on a rectangular grid. The best fitting polynomial of the form ax^3 in the sense of least squares has been computed for these three points and is shown in Figure 2. Here our computations give $a = 2.16 \times 10^{-6}$ for the units used. The standard deviation of the three points from this cubic curve is only $\sigma = 0.47$.

One sees from both Figure 1 and Figure 2 that, in terms of this 3-category grouping, the variation of landing accident rate with landing speed follows almost precisely a cubic law.

Next, we have taken the same total data and divided the 41 aircraft types into <u>seven</u> sub-categories ranging from lowest to highest landing speeds as follows:

First Group	All landing speeds less than 100 knots
Second Group	From 100 to 105 knots
Third Group	" 106 " 115 "
Fourth Group	" 116 " 140 "
Fifth Group	" 141 " 155 "
Sixth Group	" 156 " 175 "
Seventh Group	Above 175 knots

The numbers of accidents represented by these groups are still sufficiently large to have excellent statistical reliability. The average accident rate for each group has again been plotted against the weighted average landing speeds within each group on double-log paper on the right side of Figure 1. Again, a least-squares straight line has been shown which has linear slope about 3.11. Thus our conclusion now based upon a <u>sevenfold subdivision of the total data still shows almost precisely a power variation of the form $y = ax^3$</u>. In Figure 3, the seven rates versus the landing speeds have been plotted on a rectangular grid. A separate calculation for the best fitting least-squares cubic has been made with respect to these seven points; the computations now indicate $a = 2.27 \times 10^{-6}$. Even for this finer subdivision of the total data compared with Figure 2, we again see that the conformity of the points to the cubic curve is still very close. Here the standard deviation of the seven points from the curve is only $\sigma = 1.12$. The lower curve in Figure 3 consisting of the three points marked by triangles is taken from Figure 13 of [1] and shows the decomposition of all U. S. Air Carrier landing accidents in the period 1961-64 divided into three separate groupings of pistons, turboprops, and jets, respectively, with weighted averages of landing speeds within each category. The author had previously concluded in [1] on the basis of these three points that the relationship was at least quadratic; the dashed curve shown is the best fitting parabola through the three points. With the much more copious data furnished by the USAF accidents, we can now conclude that the variation is more rapid than quadratic and is in fact approximately cubic as shown. One sees also that the landing accident rates for U. S. Air Carriers are consistently lower than the counterparts in USAF. This is to be expected in view of the different requirements and different environments of the two services.

We have thus far shown the correlation by considering the accidents first as being divided into three large groups and then as being divided into seven groups of moderate size. Now we go to the other extreme and consider the total data when divided into its fine-grained structure. There are 18 different landing speeds listed when this quantity is expressed to the nearest integer in knots. We therefore take a subdivision of the total data on this basis into 18 points. The resulting scatter-plot is shown in Figure 4. Here, however, the various points have widely differing degrees of statistical reliability. This is indicated by the legend of solid circles, open circles, and crosses. In order now to pass a least-squares curve through these points, a weighting procedure must first be established. Such a weighting procedure will result in a very large number of points, all of equal weight. Since we did not have available a computer program for least-squares computations for cubic curves involving many hundreds of points, we have been content in Figure 4 to use the least-squares cubic as computed in Figure 3. Although this procedure is no longer exact, it will give a sufficiently good approximation for our purposes. It should be emphasised that the solid circles, open circles, and crosses shown in Figure 4 do not themselves represent quantitatively the exact weights. In Figure 4 we have used the legend merely to indicate the numbers of accidents involved in the rate determination at each point. In order to change this into an exact quantitative weighting procedure, one must consider the variance V for a binomial distribution. An accident rate y represents a proportion p in samples taken from a set conforming to the binomial distribution. If n represents the number of landings at each graph point, then we have

$$V(p) = \frac{p\,(1-p)}{n} \doteq \frac{p}{n}$$

The proper least-squares computation on samples having different degrees of reliability (different standard deviations) is obtained by minimizing the summation of d_i^2/σ_i^2 where the d's are vertical distances between the plotted points and the approximating curve. Since $p \doteq y = a/n$ this amounts to considering each point to be a multiple point with weight

$$w = 1/\sigma = n/\sqrt{a}$$

where a is the number of landing accidents. We have therefore weighted each of the 18 points by the appropriate quantity $n/\sqrt{a}$ and have disregarded fractional parts in the multiplicities. Next we must consider how to obtain the correlation coefficient r when the correlation is to be taken, not with respect to the usual linear regression line, but rather with respect to a third degree curve. This can be accomplished by the transformation $X = ax^3$. In terms of the new independent variable X, the points from Figure 4 have been replotted in Figure 5. In Figure 5, the legend of solid circles, open circles, and crosses now indicates the weights actually given to each of the 18 points. In terms of the multiple points thus produced, we are now considering a scatter-plot of 280 points, all taken with equal weight. Having made the transformation to X, the correct approximating curve becomes therefore a straight line which, ideally, should pass through the origin and have slope + 1. For these 280 points, we have therefore computed the linear regression line and its corresponding linear correlation coefficient. The result is

$$y = .90\,X - .27$$

$$r = .83$$

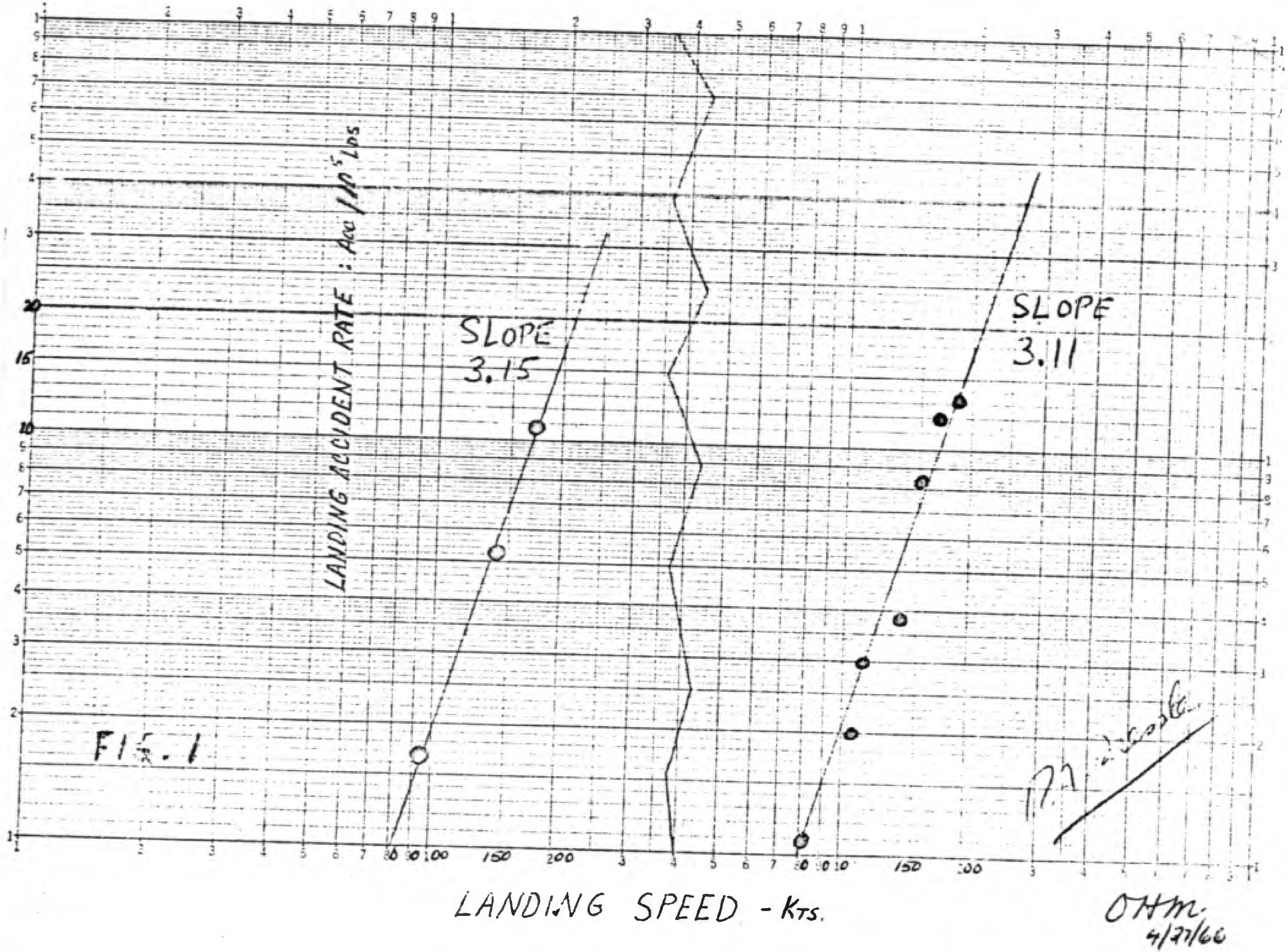
Since we have used only an approximation to the exact least-squares cubic taken from Figure 3, our result here gives a slope of only + .90 rather than + 1, and an intercept which is not precisely zero. Our new value <u>r = .83</u> can therefore be regarded as a <u>lower bound</u> for the true cubical correlation which will be somewhat higher than this value. In any case, the value .83

obtained is certainly sufficiently high to indicate a very strong correlation between landing speeds and landing accident rates.

In Figure 4, if we instead merely calculate in elementary fashion the linear regression line with respect to the 18 points with the weights $n/\sqrt{a}$, the resulting coefficient of linear correlation becomes $r = .78$. The higher value .83 which we have obtained for a cubical correlation thus confirms that the relationship between abscissa and ordinate is actually of third degree rather than linear.

Summary

On the basis of all non-utility landing accidents for fixed-wing aircraft in USAF in the three years 1962-64, it has been demonstrated that there is a very high correlation between landing accident rates and landing speeds. Furthermore, this relationship varies more rapidly than linearly, in fact approximately as a third-degree curve. It is not intended to suggest that landing speeds are the "cause" of landing accidents, but the high correlation demonstrated does imply that landing speed is one of the significant parameters involved in the process. Showing that a statistical correlation exists sheds no light on how speed and operational, mechanical, aerodynamic, and other factors may be causally related to the occurrence of accidents. The finding of a statistical correlation does, however, point to the desirability of studying these interactions to see why the correlation exists and what, if anything, can be done about it.


LANDING ACCIDENT RATE : Acc / 10^5 Lds
SLOPE 3.15
SLOPE 3.11
FIG. 1
LANDING SPEED - Kts.
OHM. 4/27/66

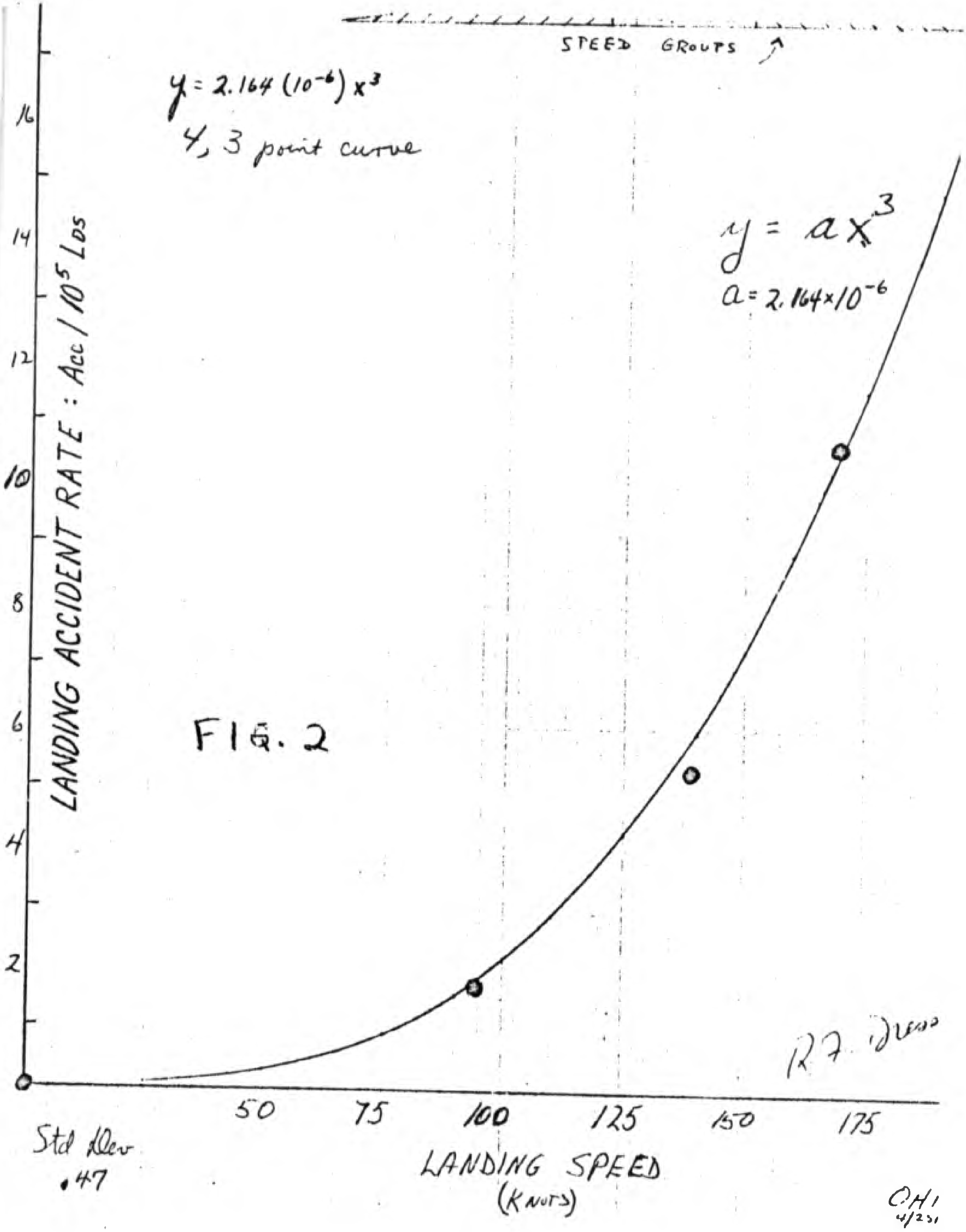


FIG. 2

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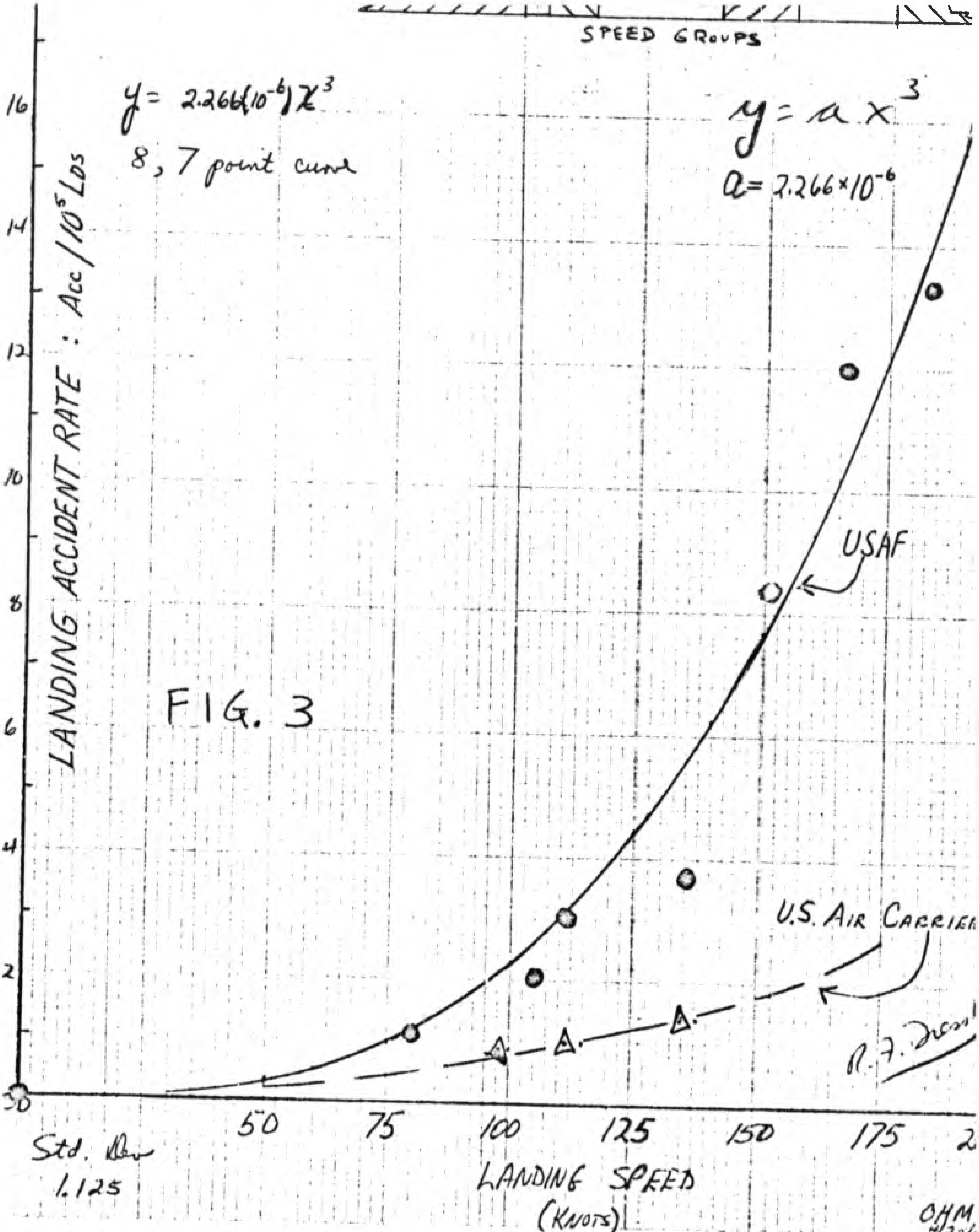

FIG. 3

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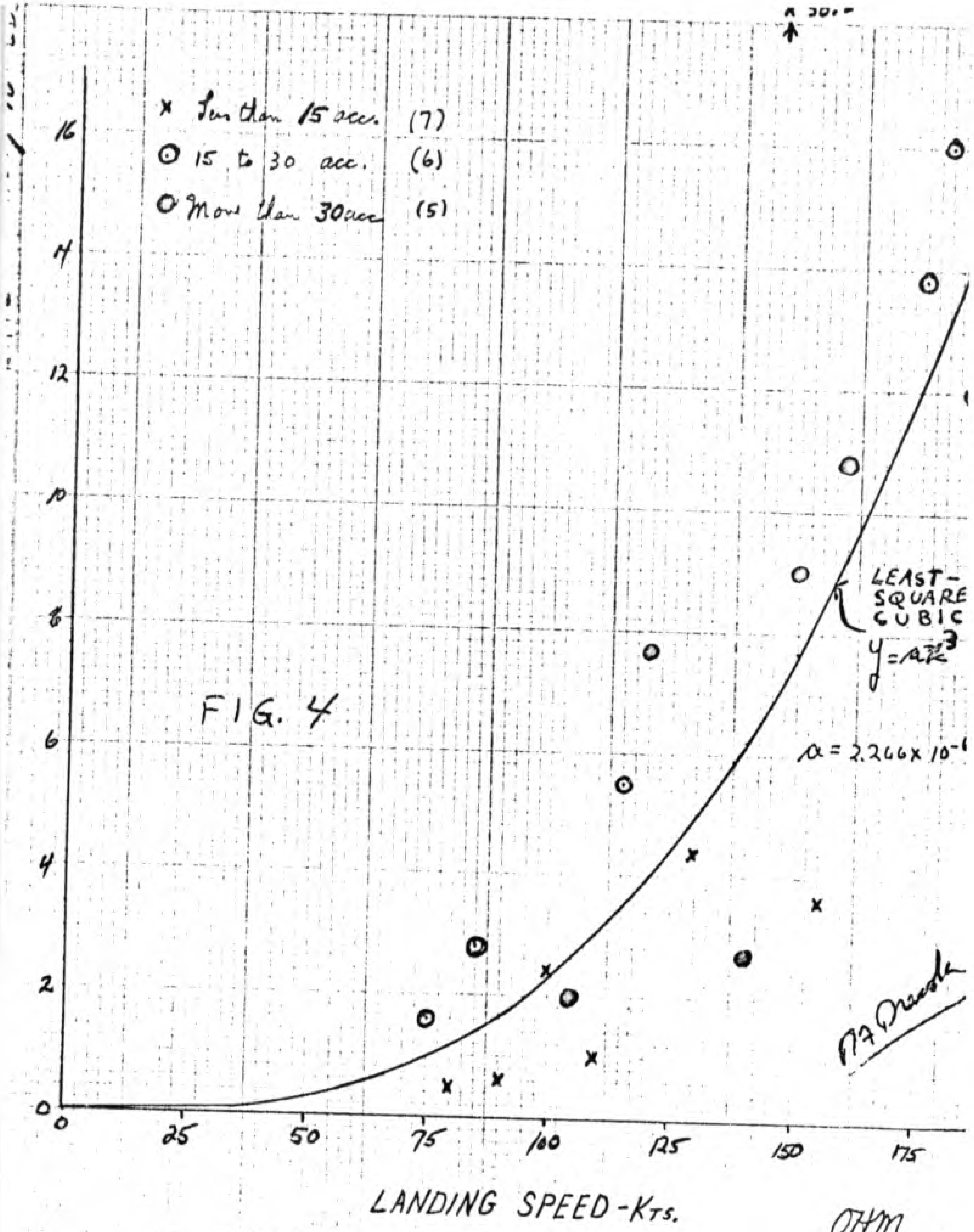
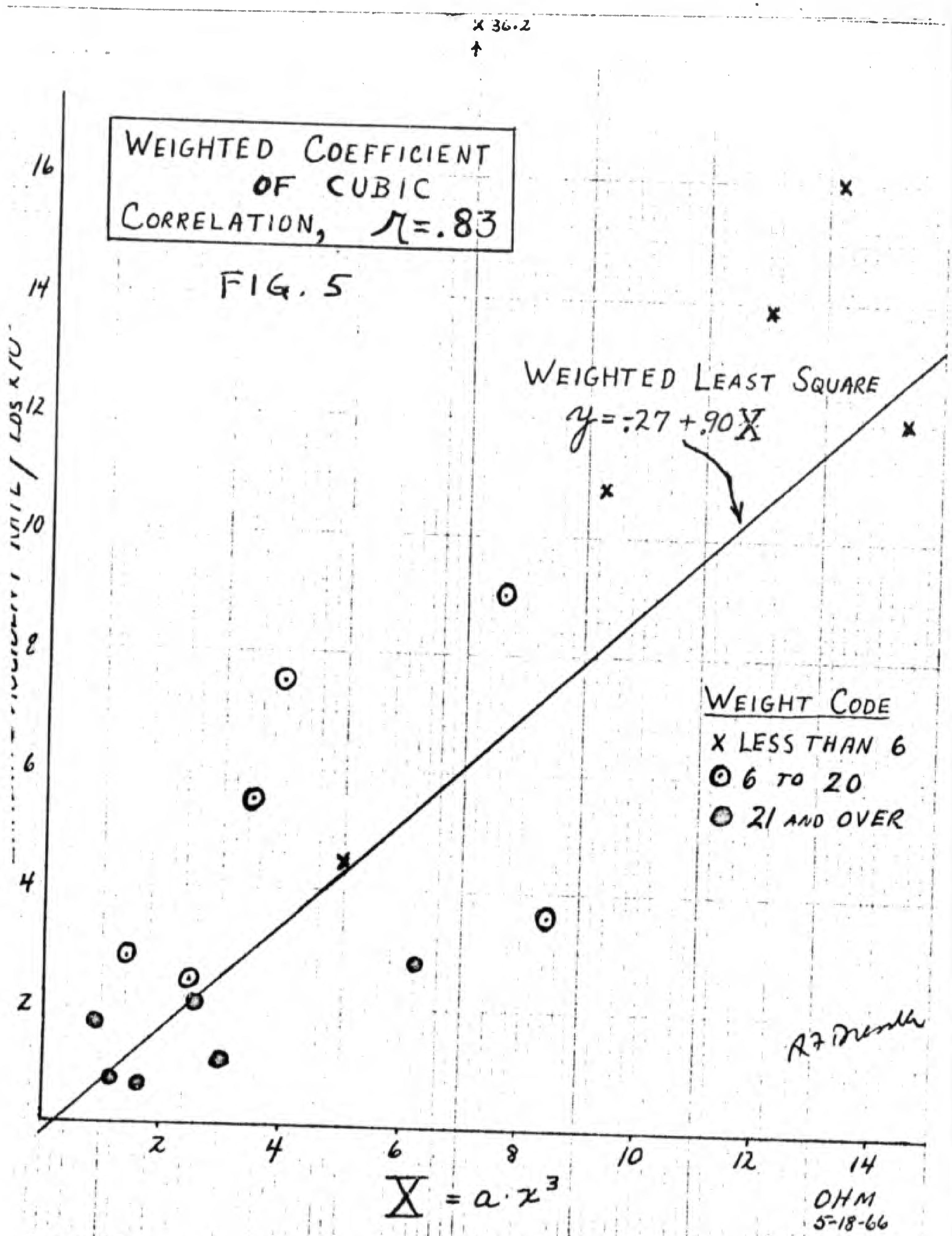


FIG. 4

OHM 4/27/66


x 36.2
WEIGHTED COEFFICIENT OF CUBIC CORRELATION, r=.83
WEIGHTED LEAST SQUARE
y=-.27+.90X
WEIGHT CODE
x LESS THAN 6
⊙ 6 TO 20
● 21 AND OVER
16
14
12
10
8
6
4
2
2
4
6
8
10
12
14
X = a·x³
OHM
5-18-66
FIG. 5