DUPLICATE

הטכניון - מכון טכנולוגי לישראל
הפקולטה להנדסה אירונוטית

TECHNION — ISRAEL INSTITUTE OF TECHNOLOGY
DEPARTMENT OF AERONAUTICAL ENGINEERING

HAIFA, ISRAEL

103

AF 61(052)-905

SR - 3

OCTOBER 1967.

SCIENTIFIC REPORT No. 3.

THE INFLUENCE OF STIFFENER GEOMETRY AND SPACING ON THE BUCKLING OF AXIALLY COMPRESSED CYLINDRICAL AND CONICAL SHELLS.

Josef Singer
Technion-Israel Institute of Technology,
Department of Aeronautical Engineering,
Haifa, Israel.

TAE REPORT 68.

Paper Presented at the Second IUTAM Symposium on the Theory of Thin Shells, Copenhagen, September 8, 1967.

The research reported in this document has been sponsored by the AIR FORCE OFFICE OF SCIENTIFIC RESEARCH OAR under Contract AF 61(052) - 905 through the European Office of Aerospace Research, United States Air Force.

SUMMARY

An experimental and theoretical study of the buckling of closely stiffened cylindrical and conical shells under axial compression has been undertaken to determine the influence of the stiffener geometry and spacing on the applicability of linear theory. Tests on integrally ring-stiffened cylinders, in which the spacing, cross-sectional area and eccentricity of the stiffeners is varied are described. The bounds of general instability are first determined by an elementary analysis of sub-shells and panels between stiffeners, in conjunction with "smeared" stiffener theory. The interaction between stiffeners and shell is then investigated with a linear discrete-stiffener theory. The experimental results are correlated with theory and approximate design criteria are developed. Experimental results and conclusions of other investigators are also discussed. The results of a test program of integrally ring-stiffened conical shells are briefly discussed and correlated with the results obtained for cylindrical shells.

The structural efficiency of closely stiffened cylindrical shells is then studied in view of the observed bounds of applicability of linear theory.

LIST OF CONTENTS.

LIST OF TABLES

LIST OF FIGURES

LIST OF FIGURES (CONT'D).

LIST OF SYMBOLS

a, b	distance between rings and stringers for a cylindrical shell, distance of top of truncated cone from vertex (see Fig. 1)
a_o	distance between rings for a conical shell (see Fig. 1).
a_n, b_n	defined by Eq. (16) of Ref. [12]
A_1, A_2	cross-sectional area of stringers and rings
c, d	the width and height of rings
D	$Eh^3/12(1-\nu^2)$
e_1 and e_2	eccentricity of stringers and rings (See Fig. 1)
E	modulus of elasticity
G	shear modulus
I_R	moment of inertia of the cross-section for the centroidal axis in the plane of the ring
J	torsional constant of ring cross-section
h	thickness of shell
$\bar{h}$	thickness of equivalent weight shell
I_{c1}, I_{o2}	moment of inertia of a stringer or ring cross-section about the middle line of the sheet respectively
I_{11}, I_{22}	moment of inertia of stiffener cross-section about its centroidal axis
L	length of shell between bulkheads
n	number of half longitudinal waves in cylindrical shell
P	axial force
$P_{c\ell}$	classical buckling load for isotropic cylinder for "classical" simple supports (SS3)

P_{cr}	linear theory buckling load for unstiffened cylinder (for short cylinders or boundary restraints)
P_{Gs}	linear theory general instability for stiffened cylinder with "smeared" stiffeners and "classical" simple supports (SS3)
$P_{Gs\ clamp}$	linear theory general instability for stiffened cylinder with "smeared" stiffeners and "classical" clamped boundaries (RF 2)
P_{GD}	linear theory general instability for stiffened cylinder with "discrete" stiffeners and "classical" simple supports (SS3)
P_B	empirical buckling load for isotropic cylindrical shell
R, R_1, R_2	radius of cylindrical shell, and radii of small or large end of truncated cone, respectively.
t	number of circumferential waves
u, v, w	non-dimensional displacements, in cylinder: $u = (u^*/R)$, $v = (v^*/R)$, $w = (w^*/R)$ in cone : $u = (u^*/a)$, $v = (v^*/a)$, $w = (w^*/a)$,(see Fig. 1)
x^*, z^*, ϕ	axial coordinate along a generator, radial and circumferential coordinates
x	non-dimensional axial coordinate, $x = (x^*/a)$ for a conical shell, $x = (x^*/R)$ for a cylindrical shell
Z	$=(1-\nu^2)^{1/2}(L/R)^2(R/h)$ Batdorf shell parameter

Z'	$(1-\nu^2)^{1/2}(a/R)^2(R/h)$ Batdorf shell parameter of sub-shell.
α	cone angle
β	$= (\pi R/L)$ for cylinder
$\bar{\delta}$	out of roundness (from reference line)
$\zeta_1, \eta_{o1}, \eta_{t1}, \mu_1, \chi_1$	changes in stiffnesses due to stringers and rings
$\zeta_2, \eta_{o2}, \eta_{t2}, \mu_2, \chi_2$	defined in Refs. [13] and [14]
η	structural efficiency
θ	$[12(1-\nu^2)]^{1/4}[b/2\pi(Rh)^{1/2}]$ Koiter's measure of total curvature
λ	$(PR/\pi D)$ axial compression parameter for cylindrical shell
ν	Poisson's ratio
ρ	$= (P_{exp}/P_{Gs})$, "linearity" or "knock down factor"
σ_{yp}	yield stress

I. INTRODUCTION

Closely stiffened shells are usually analysed by linear theory, with fairly good agreement between experiment and theory. A closer look at the experimental verification of linear theory shows that the agreement is good for closely and heavily stiffened shells whereas it is poor for moderate or sparse stiffening.

This is especially noticeable in the case of the worst "offender" of classical buckling theory - the cylindrical shell under axial compression. The very large discrepancies between experimental and theoretical buckling loads observed in unstiffened cylinders motivated extensive study of the problem (see for example [1] or [2]) as well as a major effort to stabilize the shell by internal pressure, stiffening or sandwich construction (see for example [3]). The designers also developed empirical "knock down factors", that were revised as the number of tests increased and summarized in various empirical formulae (see [4] - [6]). The axially compressed cylinder is therefore very suitable for study of the validity of linear theory in stiffened shells. A study of conical shells under axial compression may lend additional support to the results obtained in cylinders.

For many years the usual approach to the stability analysis of a stiffened shell was to replace it by an equivalent orthotropic shell (see for example [7] [3] or [9]. Such an approach, however, does not permit taking into account the eccentricity of stiffeners. As stiffeners became heavier, the importance of these eccentricity effects, observed already earlier [10] and [11], was realized and their influence studied.

Our group at the Technion has in recent years studied the behavior of stiffened cylindrical and conical shells with a linear theory that includes eccentricity effects (see [12],[13] and [14]). For cylindrical shells, the main merit of this theory, in which the stiffeners are "smeared", or "distributed" over the entire shell, lies in its simplicity . t has led to its adoption also by other investigators, in particular at NASA (for example [15] and [16] and at Lockheed (for example [17]). For conical shells this theory, [18] and [19], though less simple is manageable and even permitted some optimization studies [13] and [20].

These studies and their conclusions are, however, meaningful only if linear theory predicts the buckling load adequately and if the discreteness of the stiffeners has no noticeable effect. The second problem is the easier one, since it can be attacked by a linear discrete-stiffener-theory. Previous investigators, [21], [22] and [23], have shown that for ring stiffened cylindrical shells the discreteness effect is of importance only when the number of rings is very small, but eccentricity was not taken into account by them and stringers were not considered. Hence a more detailed discrete-stiffener analysis seems warranted.

The first problem - the adequacy of linear theory is more formidable and can be conclusively settled only by tests. The investigators differ in their opinions. In 1962 Van der Neut [22] considered, on the basis of a logical expected reduction in imperfection sensitivity, " that linear theory is adequate for the investigation of general instability of stiffened shells". Recently Hoff [3] has pointed out that " it is perhaps premature to state that the small displacement theory is rigorously applicable to reinforced shells" and Hutchinson and Amazigo [24] have presented imperfection sensitivity studies to " indicate to what extent the classical buckling results can be considered reliable".

Gerard and his group developed a linear theory for orthotropic cylindrical shells [9] and [25] and then embarked on an extensive test program [25] and [26] to show that linear orthotropic theory is adequate. Already in 1962 they pointed out the remarkable agreement of the buckling load under axial compression for Pugliese's integrally machined ring-stiffened cylinder, tested in 1959 [27], with linear orthotropic theory. Their later tests on carefully manufactured, and one can add beautiful, ring-and stringer-stiffened cylindrical shells [27] supported their contention of the adequacy of linear theory for closely stiffened shells. From one aspect, however, these otherwise excellent tests are inadequate - they were too closely stiffened. As the width of the stiffeners equalled the distance between them, they represented really thick cylinders with longitudinal or circumferential slots rather than stiffened thin cylinders. One cannot, therefore, rely on these tests to settle the problem of applicability of linear theory for reinforced cylindrical shells. A similar difficulty arises with another series of excellent tests in which Garkish [28] investigated the pronounced eccentricity effect that appears in longitudinally stiffened cylinders under axial compression. Here the stiffeners are back to back "L" sections, and the width of unstiffened skin is only 1/10 of the total width of the stringer.

Another series of tests on internal ring-stiffened cylindrical shells [29], though primarily concerned with buckling under hydrostatic pressure loading, includes some tests with predominant axial loading that support the adequacy of linear theory. As these tests, however, are for shells of rather low (R/h) values, about 250, and the wall thickness of the relevant specimens exhibit variations of up to + 36% and - 27% of the weighted average, they are not included in the discussion.

Three series of careful tests of stringer-stiffened cylinders present important evidence for evaluation of the applicability of linear theory. Card's tests with heavily integral-stiffened shells [30], the tests by Peterson, Dow, Card and Jones on more moderately stiffened shells, [31] and [16], and Katz's tests on large scale moderately stiffened cylinders [32]. Card's tests are by now the "classical" evidence of validity of linear theory for heavy stiffening and the importance of the eccentricity effect. Katz's tests are equally important when they are judiciously correlated with linear theory, as they give some indication on the stringer area required for linear theory to become valid. Some recent tests on ring-stiffened cylinders reported by Almroth [33] and a stringer-stiffened cylindrical panel tested by Len'ko [34] are additional evidence for the discussion.

It may be pointed out, that in stiffened shells the boundary conditions may be even more important than in isotropic shells, since two additional effects have to be considered. The eccentricity of the applied axial load (or end moment effects) may be important here [35], [32] and [36], and the in-plane boundary conditions have different effects on internal and external stiffeners [37].

If one now concurs with van der Neut's opinion and feels " that linear theory is adequate for closely spaced stiffeners ", and turns to the experimental evidence for support, one cannot discern the influence of the various geometrical parameters on the applicability of linear theory. A primary aim of the present investigation is to bring forth the predominant parameters. Answers are needed to the questions what is "closely stiffened" and how "heavy" have stiffeners to be.

The problem is not only one of validity of a theory. Since the main stability contribution of stiffeners in cylindrical and conical shells under axial compression, be they rings or stringers, is the raising of the buckling load to the classical one, the problem is one of structural efficiency. The question of how heavily does it pay to stiffen has also to be considered.

2. THEORETICAL CONSIDERATIONS

The discussion in the present paper is limited to ring or stringer-stiffened shells. Other forms of stabilization, such as internal pressure, sandwich construction, corrugated skin, 45° waffle-stiffening and orthotropic materials are not considered, though they may be of equal practical importance and hence justify serious study. The stiffeners are considered to be rigidly joined to the shell and the conclusions apply therefore best to integrally stiffened shells. Furthermore, stiffeners are here considered to be of the same material as the shell though the theory can easily accommodate different materials (see for example [14] or [16]) and different materials may sometimes be more efficient, for example [38].

2. 1. Ring-Stiffened Cylindrical Shells

A ring-stiffened cylindrical shell under axial compression may fail in two forms of instability, local buckling of the sub-shell between the rings or general instability of the stiffened shell as a whole. One type of general instability - buckling as an Euler column - occurs only in very long shells and may usually be excluded. In both forms of instability axisymmetric or asymmetric modes may occur, depending of the geometry of the shell. Furthermore, there may be a noticeable restraining effect of the rings on the local buckling and there may be interaction between the two forms of instability that may lower the general instability load.

An elementary linear analysis of the buckling of an axially compressed ring-stiffened cylindrical shell considers the sub-shell separately as a simply supported isotropic shell and then examines the general instability of a shell reinforced by the "distributed" or "smeared" stiffeness of the rings.

The classical formula for a simply supported isotropic cylinder

$$P_{c\ell} = [3(1-\nu^2)]^{-1/2} 2\pi h^2 E \qquad (1)$$

assumes no restrictions on the wave length parameter $[(n^2 + \beta^2)^2/n^2]$, see for example [39] or [40], and hence may not apply to the short sub-shells. Indeed for $Z' = (1-\nu^2)^{1/2}(a^2/Rh) < 2.85$ the sub-shell buckles into an axisymmetric pattern with one axial half-wave, $n = 1$, for which

$$P_{cr} = P_{c\ell} [1 + (12Z'^2/\pi^4)]/0.702 Z' \qquad (2)$$

For Z' slightly above 2.85, linear theory predicts an asymmetric patter with $n = 1$, till Z' is sufficiently large for an axisymmetric mode with $n = 2$, and then with further increase in Z' an asymmetric pattern will again appear with $n = 2$, and so forth. The accompanying fluctuations in buckling load are, however, negligible [41].

Contrary, however, to the large discrepancies between experimental and theoretical axial buckling loads universally observed for moderate length isotropic cylinders, tests on very short cylinders exhibit fairly good agreement with linear theory, see for example Fig. 4 of [6], Figs. 6 to 8 of [42] or Fig. 8.7 of [43]. The axisymmetric buckling pattern of the short shells is probably the main reason for their "linear" behavior, for the axisymmetric mode, also referred to as "ring-buckling" by some investigators, has a stable post-buckling behavior that makes it insensitive to imperfections, see [8]. The more pronounced influence of rotational restraint is the second reason. Consideration of the limiting case of Euler flat plate behavior, for $Z \to 0$, in which clamping raises the buckling load 4 times, or examination of the curves in [44] emphasize the importance of the boundary conditions for short shells.

Since for a clamped cylindrical shell no closed form solution is available even in linear theory, only an approximate estimate of the sub-shell geometry that ensures an axisymmetric buckling mode can be made. With aid of the second approximation of the Galerkin solution of Donnell's equations by Batdorf, Schildcrout and Stein [44] it is found that when $Z < 11.6$ the axisymmetric buckling pattern predominates. Though the rings, however heavy, will never represent fully clamped conditions, an indication of an upper bound for possible "linear" behavior of the sub-shells is thus obtained. In view of the recent work on the effect of the secondary boundary conditions (see for example [45] or [46]), consideration of the "classical" simple supports and clamped ends only out of the possible 8 end conditions may seem incomplete. For the short sub-shells, however, the rotational restraint is the prime boundary effect, [45] and [37], though the RF 4 B.C. (in the notation of [45]) extends its influence to much larger Z than the "classical" RF 2 B.C. [16].

The elementary analysis, therefore, immediately yields a conservative criterion to ensure local "linear" behavior. By taking "classical" simple supports as the weakest practical boundary conditions of the sub-shells one obtains from

$$Z' = (1-\nu^2)^{1/2}(a^2/Rh) < 2.85 \tag{3}$$

for safe ring spacing

$$\frac{a}{h} \leqslant [2.85(1-\nu^2)^{-1/2}(R/h)]^{1/2} \tag{4}$$

Since "linear" behavior extends also a little beyond the axisymmetric range, Eq. (4) is rather conservative.

One may note that a recent imperfection sensitivity study [33], which concludes that very short cylinders are insensitive to initial imperfections, lends further support to the expected "linear" behavior of the sub-shells.

The rings actually provide elastic rotational restraints that stiffen the sub-shells also within the framework of linear theory. The rings can provide two types of restraint: resistance to twist if the sub-shell buckles non-axisymmetrically and the cross-sections of ring resist rotation one relative to the other, or resistance to "rolling over" of ring that subjects the ring to out of plane bending [41]. The rings usually offer more restraint to twist than to "rolling over" which promotes the tendency towards axisymmetric buckling.

These rotational restraints are usually neglected in buckling analysis of cylindrical shells, though they were included in Reynold's careful study of ring stiffened shells under hydrostatic pressure [47], where appreciable restraint was observed (6 to 18% stiffening of sub-shells with Z' from 27 to 10 and R/h of about 100). Here, the effect of the rotational restraints will be analysed by a simple one-term Rayleigh-Ritz approach, that is an extension of earlier studies on the effect of axial restraint [48] and [49]. The details are given in Appendix A. The restraints are expressed as non-dimensional spring coefficients. For torsional resistance the spring coefficient is

$$(k_T/ER^4) = GJ/ER^4 = J/2(1+\nu)R^4 \qquad (5)$$

where $k_T = GJ$ is the torsional moment per twist per unit length (hence the dimensions of k_T are FL^2) and J is the torsional constant of the ring cross section. The effect of the torsional restraint appears in the final expression of the buckling load

$$(P_{cr}/Eh^2) = 2\pi\left\{\frac{1}{12(1-\nu^2)(R/h)}\frac{[n^2\beta^2+t^2]^2}{n^2\beta^2} + (R/h)\frac{n^2\beta^2}{[n^2\beta^2+t^2]^2} + (k_T/ER^4)2t^2(R/h)^2(R/L)\right\} \qquad (6)$$

as usual, the integer values of t and n that make P_{cr} a minimum have to be chosen. (A more precise analysis, an extension of the "discrete stiffener" theory [50] with emphasis on local buckling when the torsional stiffness is not neglected, is also in progress at the Technion).

For resistance to "rolling over" in the axisymmetric buckling mode the spring coefficient is, as in [41],

$$k_R = (EI_R/R^2) \tag{7}$$

where I_R is the moment of inertia of the cross-section for the centroidal axis in the plane of the ring. It may be noted that for rings of rectangular cross-section, as shown in Fig. 1, of a given area A_2, I_R and hence k_R is larger the smaller the eccentricity $|e_2/h|$. The buckling load is given by

$$(P_{cr}/Eh^2) = 2\pi\left\{\frac{n^2\beta^2}{12(1-\nu^2)(R/h)} + \frac{(R/h)}{n^2\beta^2} + (k_R/Eh^2)4(R/h)^2(R/L)\right\} \tag{8}$$

As the torsional restraint, when effective, is usually one order of magnitude larger than the restraint to "rolling over", the latter is neglected in the asymmetric buckling mode.

In the case of unrestrained sub-shells general instability can occur within the framework of linear theory, only when the sub-shells are in the short " axisymmetric mode" range, $Z' < 2.85$ where P_{cr} is given by Eq. (2). Otherwise the unstiffened sub-shell will always buckle locally at a lower load than the whole stiffened shell, as the critical load, according to Eq. (1) does not depend on the length of the shell. The rotational restraint determines therefore an upper bound to the ring spacing by the requirement that

$$P_{\text{general instability}} \leq P_{\text{local restrained}} \qquad (9)$$

It may be noted that in most of the specimens tested in the present program the influence of the rotational restraint was found to be very small. As an example of a noticeable influence of the rotational restraint three of the ring-stiffened shells of [33] are considered. In Table I the geometry of the shells and sub-shells are given. For simple supports (zero rotational restraint) an asymmetric local buckling pattern should appear, column (1). The heavy rings, however, offer considerable torsional resistance and hence in calculations of P_{cr} from Eq. (6) $t \rightarrow 0$ or, in other words, the sub-shell is forced to buckle axisymmetrically, column (2). The resistance to "rolling over", that was not taken into account in the asymmetrical mode, has to be considered now, yielding column (4). It is seen that $P_{rest.axisym.} > P_{Gs}$, the "smeared" general instability load, and that even $P_{axisym.}$ column (2), not considering the resistance to "rolling over", is not less than P_{Gs}, column (6). This example shows that by forcing the sub-shells to buckle axisymmetrically when $Z' > 2.85$, the torsional resistance of the rings increases the safe ring spacing given by Eq. (4). Hence heavier rings, especially with high torsional stiffness, may be advantageous. Note also in Table I that the test results are not far from the predicted buckling loads.

A linear theory analysis for general instability of stiffened cylindrical shells under axial compression is given in [14]. It is shown there that with inside rings non-axisymmetric buckling will occur and the positive eccentricity will lower the buckling load below that for centrally placed rings. With outside rings, however, the increase in buckling load that would result from the negative eccentricity if the shell were to buckle in a chess-board pattern is not realized, since the shell now buckles in the ring-shape pattern which is unaffected by eccentricity and yields a lower buckling load.

For classical simple supports, the linear theory general instability load of a stiffened cylindrical shell under axial compression can be obtained [14] from

$$\zeta_1(-n^3\beta^3 a_n) + \zeta_2(-2t^2 - b_n t^3) + (1 + \eta_{o1})n^4\beta^4 + (2 + \eta_{t1} + \eta_{t2})n^2\beta^2 t^2 +$$

$$+ (1 + \eta_{o2})t^4 + 12(R/h)^2 \left[(1 + \mu_2)(1 + b_n t) + \nu n\beta a_n\right] - \lambda(n^2\beta^2/2) = 0 \qquad (10)$$

where λ is a non-dimensional axial load parameter defined by

$$\lambda = (PR/\pi D) = \left[12(1-\nu^2)PR/\pi E h^3\right] \qquad (11)$$

μ_1, μ_2, η_{o1}, η_{o2}, η_{t1} and η_{t2} are the changes in stiffnesses due to stringer and frames and χ_1, χ_2, ζ_1 and ζ_2 are the changes in stiffnesses caused by the eccentricities of the stringers and rings as in [12] or [14].

For a ring-stiffened shell with outside rings, that buckles in the axisymmetric mode, Eq. (10) simplifies considerably, and when there are many waves in the axial direction, permitting n to be treated as a continous variable, the general instability load can be computed from the simple formula

$$P_G = [3(1-\nu^2)]^{-1/2}\, 2\pi h^3 E[1+(A_2/ah)]^{1/2} = P_{c\ell}[1+(A_2/ah)]^{1/2} \qquad (12)$$

In passing, it may be pointed out that the reduction in buckling for internal rings may be offset if the rings have high torsional stiffness, as then less energy will be absorbed in the axisymmetric buckling mode, that is unaffected by eccentricity, than in the non-axisymmetric mode that involves torsion of the rings. Some computations for a typical shell ($L/R = 0.5$, $R/h = 500$, $A_2/ah = 0.5$, I_{22}/ah^3, $e_2/h = 5$ and $\nu = 0.3$) show (see Fig.2) that when $n_{t2} > 15$ the shell buckles axisymmetrically with an accompanying increase of 14.7% in the general instability load, or - in other words - complete recovery of the reduction due to internal placing of the rings. For the typical shell considered, $n = 15$ can be obtained in practice (for a ring spacing of $a/h = 40$) by rings of tubular cross-section with a diameter of about 7h and a wall thickness of h.

In order to investigate the effect of discreteness of the rings, the buckling of ring-stiffened cylindrical shells is analysed by a linear "discrete" theory. Instead of being "smeared", the rings are now considered as linear discontinuities represented by the Dirac delta function but otherwise the analysis is similar to that of [14]. It may be noted that the delta function representation of rings has been employed by other investigators, [51], [21] and [23], but without consideration of the eccentricity of the rings. The details of the method used here, that is based on the formulation of [52], are given in [50], where also extensive parametric studies are discussed.

The analysis yields two sets of equations

$$b_{m1}B_m + c_{m1}C_m + \sum_{n=1}^{\infty} \sum_{i=1}^{q-1} (\beta_{i1}\beta_n + \gamma_{i1}C_n) \sin\frac{mi\pi}{q} \sin\frac{ni\pi}{q} = 0$$

$$b_{m2}B_m + c_{m2}C_m + \sum_{n=1}^{\infty} \sum_{i=1}^{q-1} (\beta_{i2}\beta_n + \gamma_{i2}C_n) \sin\frac{mi\pi}{q} \sin\frac{ni\pi}{q} +$$

$$+ \delta_{imn}C_n \cos\frac{mi\pi}{q} \cos\frac{ni\pi}{q} = 0 \qquad (13)$$

where b_{m1}, b_{m2}, c_{m1}, c_{m2}, β_{i1}, β_{i2}, γ_{i1}, γ_{i2} and δ_{imn} are expressions, involving shell and ring geometry as well as wave numbers, defined by Eqs. (9) of [50]. The buckling load is found from the vanishing of the determinant of Eqs. (13).

It should be pointed out that the Dirac delta function representation is satisfactory only as long as the width of the stiffeners is not comparable to the distance between them. Hence it could not be applied, for example, to the very closely stiffened shells of [26], and its reliability becomes doubtful in any shell with very wide stiffeners.

Though for buckling under hydrostatic pressure appreciable load reductions were found in [50] for discrete rings, the discreteness effect was always found to be very small for ring-stiffened cylinders under axial compression. A similar conclusion was reached in [23] for orthotropic ring-stiffened cylinders by an analysis that did not take ring eccentricity into account. For completeness, however, the "discrete" buckling load, in addition to the "smeared" one is computed for some of the test cylinders.

Hence, if the ring-spacing and the rotational restraint due to rings ensure axisymmetric local buckling and if the rings are placed on the outside or have high torsional stiffness to compensate for internal placing, an initially stable axisymmetric general instability should dominate and tests should agree well with predicted buckling loads.

2.2. Stringer-Stiffened Cylindrical Shells

A stringer-stiffened cylindrical shell under axial compression may again fail in two forms of instability, local buckling of the panel between the stringers or general instability of the stiffened shell as a whole. Axisymmetric buckling modes will occur, in both forms of instability, only for short shells, and hence has to be considered only in the case of stringer-stiffened shells reinforced also by strong rings. The present discussion is therefore limited to asymmetric modes, that include for general instability the n = 1 or "longitudinal" buckling modes mentioned in [26] and [53]. There may be an appreciable restraining effect of the stringers on the local buckling and there may be interaction between local and general instability.

The elementary analysis again separates the consideration of buckling of the panels between the stringers and the study of the general instability of the "smeared-stringer" shell.

The buckling and initial post buckling behavior of cylindrical panels has been studied by Koiter [54] for stringers that exert no rotational restraint on the panel. In the absence of restraining effects of the stringers except the radial one, the panel will buckle in the same mode and the same critical stress as the corresponding complete unstiffened cylindrical shell, provided the angle between the equally spaced stringers ϕ_o satisfies

$$\phi_o \geq \pi/m$$

where

$$m = (1/2)[12(1-\nu^2)]^{1/4}(\frac{R}{h})^{1/2}$$

as is well known (see for example [40] or [54]). When $\phi_o < \pi/m$ the panel is called narrow and will have a critical stress above that of the corresponding unstiffened cylinder. If a measure of the total curvature is introduced as in [54].

$$\theta = \frac{m\phi_o}{\pi} = \frac{[12(1-\nu^2)]^{1/4}}{2\pi} b\ (Rh)^{-1/2} \tag{14}$$

the critical stress can be written

$$\sigma_{cr} = \frac{\pi^2 E}{3(1-\nu^2)} (\frac{h}{b})^2 (1 + \theta^4) \tag{15}$$

With θ defined by Eq. (14) the width of the panel

$$b = R\phi_o = \theta R(\pi/m) \qquad 0 < \theta \leq 1 \tag{16}$$

and the stiffening of the panel due to "narrowness" is therefore

$$\frac{(\sigma_{cr})_{\text{narrow panel}}}{(\sigma_{cr})_{\text{complete cylinder}}} = (1/\theta^2)(1 + \theta^4) \tag{17}$$

Hence "narrowness" of the panel between stringers of a longitudinally stiffened cylindrical shell is somewhat analogous to the "shortness" of the sub-shell in a ring-stiffened shell and the stiffening of Eq. (17) is of a similar nature to that given in Eq. (2) for sub-shells.

The "narrowness" of the panel has, however, an even more important influence on the buckling load carried, than the stiffening predicted by linear theory as given by Eq. (17). A wide panel, $\phi_o \gtrsim \pi/m$, buckles as a cylinder, and hence the large discrepancies between experimental and theoretical (linear theory) buckling loads observed for unstiffened cylinders will also appear in wide panels. The low buckling loads of axially compressed cylindrical shells are due primarily to their unstable post-buckling behavior. By investigating the initial post-buckling behavior of narrow panels and the effect of initial imperfections, Koiter [54] showed that θ is a suitable parameter for estimation of the expected "linear" buckling behavior of the panel. For perfect panels, the change from stable " plate type" behavior to unstable "cylindrical shell type" behavior occurs at $\theta = 0.64$ for prescribed load. Since the post-buckling tangent changes its direction rapidly after the transition (see Fig. 3 of [54]), stiffened cylinders with $\theta < 0.64$ are advisable for predominance of general instability. Note that for a shell with $(R/h) = 1000$, say, $\theta = 0.64$ corresponds to a stringer spacing $(b/h) = 70$ and that for $\theta = 0.64$ the "linear" stiffening, due to the "narrowness" of the panel, is 1.42.

The limiting value of $\theta = 0.64$ is, however, a conservative value since Koiter's analysis assumes zero torsional constraint. The finite torsional stiffness of the stringers will raise the limiting θ, as already pointed out by Koiter, and will also stiffen the panel within the bounds of linear theory.

An approximate estimate of the "linear" stiffening due to the rotational restraint of the stringers can be obtained from an analysis of the buckling of elastically restrained plates [55] in conjunction with an analysis of a clamped long curved panel [56]. A more precise analysis, that is an extension of the "discrete stiffener" theory of [50], is now being carried out at the Technion.

The general instability of the "smeared-stringer" stiffened cylindrical shell under axial compression is discussed in detail in [14]. For classical simple supports P_{Gs} may again be computed from Eq. (10), and $P_{Gs\ clamp}$ for "classical" clamped shells (RF. 2 boundary conditions) can be calculated from Eqs. (24) of [14] or for RF 4 boundary conditions from Eq. (A14) of [16]. For other boundary conditions the method of [37] can be applied. Slightly less accurate methods for calculation of the general instability are also given in [57] and [58]. It should be pointed out that in the case of stringer-stiffened shells very appreciable eccentricity effects appear and the boundary conditions affect the buckling loads considerably.

The effect of the discreteness of the stringers can again be investigated by a linear "discrete" theory in which the stringers are considered linear discontinuities represented by the Dirac delta function. The analysis of [50] has been extended to stringer-stiffened shells and calculations are in progress. Preliminary results indicate that for thin shells of practical dimensions the discreteness effect is negligible. This is not surprising on account of the large number of stringers required to prevent local buckling.

3. EXPERIMENTAL INVESTIGATION OF STIFFENED CYLINDRICAL SHELLS

Since the adequacy of linear theory can be reliably ascertained only by test, one has to turn to experiments if one aims at definition of some bounds of applicability of the theory. In this paper the experimental program is only briefly described, and details are given in [59].

3.1. Test Apparatus and Procedure

In order to be able to test specimens of large (R/h) ratios, the load frame employed earlier in tests on conical shells [20] and [60] was modified to accommodate the cylindrical shells. The load frame and the test set-up is shown in Fig. 3. The load is applied by two hydraulic jacks, controlled from an Amsler universal testing machine to a beam that moves a central load-transfer shaft with a thrust bearing on which the lower supporting disc fits. The upper supporting disc reacts against a load cell (or two load cells in series) that records the actual load applied to the test specimen. Motion along the vertical axis is preserved by a guide pin and mating sleeve fixed to the upper and lower supporting discs (except in two tests in which the sleeve was removed). To prevent friction due to misalignment of the load-transfer shaft a ball was introduced between the beam and the shaft.

Strain gages were distributed over each specimen. Usually, 38 gages were bonded to each shell, except some specimens with only 24 gages and one shell with 48 gages. The strain gages served to assist in the detection of incipient buckling and to check the symmetry of loading. 6 or 12 gages were distributed around the circumference at each vertical location. Strain measurements were recorded on a B & F multi-channel strain plotter.

The specimens are not clamped to the supporting discs. They are just put on the lower disc, which has a low central location platform with a clearance of about h, and the similar top disc is just put on top of the specimen. To prevent end moment effects, see [32], [35] and [36], the end rings of the specimens have ridges of width h that represent a continuation of the shell, see Fig. 5a. The boundary conditions are therefore not far from classical simple supports, probably somewhere between SS3 and SS4 (in the notation of [45]).

The dimensions of the shells are carefully measured before each test. The thickness is measured at about 300 points for each shell (for specimens MZ 5 - 18 the measurements were taken at least twice by different operators) and the stiffener dimensions were checked with a special gage for 4 shells at about 200 locations. Out-of-roundness is measured at 5 vertical stations prior to each test after the shell is in position. The maximum out-of-roundness A_o [61] was not computed from the readings since in earlier tests on conical shells (see [62], [60] and [20]) its significance could not be discerned. Correlation between out-of-roundness and strain readings was, however, studied for all specimens.

3.2. Test Specimens

18 integrally ring-stiffened cylindrical shells were tested in the present test program. The dimensions of the shells as defined in Fig. 1 are given in Table 2.

The specimens were machined from AISI 4130 steel alloy drawn tubes with a 1/4" wall-thickness. The mechanical properties of the material, and in particular E, were measured on 8 specimens cut out from the tubes before machining (both in the longitudinal and circumferential directions of the tubes), as well as on 16 specimens cut out from the shells after failure in buckling. The average mechanical properties found (with only slight scatter) were those that appear in the literature. The average value of Young's modulus found is $E = 2.0 \times 10^4$ kg/mm^2 (or 28.5×10^6 psi), the usual value of 4130 steel, and the yield stress $\sigma_{yp} \gtrsim 50$ kg/mm^2 (or 71000 psi), is considerably above the buckling stresses. Poisson's ratio is taken as $\nu = 0.3$.

In the interest of precision, the machining process is divided into stages. In the final stages the shell is mounted on a special "cooled" mandrel. This mandrel is made of cast aluminum with a high silicon content and has the shape of a reservoir with many fins around its inner surface (see Fig. 5b). The mandrel is fitted with special centering pivot. Liquid air poured into the reservoir of the mandrel cools it appreciably and as a result its diameter contracts 0.4 mm, enabling the shell to slide onto the mandrel. After returning to room temperature the shell sits well on the mandrel and permits accurate machining. After completion the shell is removed from the mandrel by another liquid air "cooling" and a second shell is immediately mounted.

This technique, combined with extreme care in the machining and continous measurements, has resulted in precise specimens in which the deviation of thickness (the most sensitive dimension) of the shell does not exceed 5% of the average in the worst case and is usually within 2.5% The accuracy of the height of the rings d is similar to that of the thickness, whereas larger deviations occur in the width of the rings c. Differences of up to 16% from the nominal width were measured, but these could, at worst, lead to an error of less than 3% in the computed buckling load.

3.3. Ring Stiffened Cylinders

Table 2 presents the important geometric parameters of the 18 ring-stiffened cylindrical shells tested, as well as the experimental and calculated buckling loads. The dimensions of the specimens have been chosen to yield the largest feasible (R/h) ratio and an average of 660 was achieved. The mean shell wall thickness is given in thousandths of a mm in the table, but though the thickness was measured in thousandth of a mm, the last figure is reliable only within ± 0.002 mm.

The ring spacing is chosen small enough for the sub-shell to be in the axisymmetrical range, or slightly above it to ensure local "ring-buckling" behavior. Except in shells Nos. MZ 3 and 4, the local shell geometry parameter $Z' < 2.85$. In shells MZ 3 and 4, however, the torsional stiffness of the rings is sufficiently large to force the sub-shell to buckle axisymmetrically in spite of $Z' > 2.85$. This is the same behavior as observed in the examples of Table 1, and indicates that Eq. (4) is a very conservative criterion and may be exceeded, provided the rings possess adequate torsional stiffness. The linear theory buckling load for very short shell exceeds the classical "moderate-length" buckling load appreciably, see Eq. (2). Hence for shells Nos. MZ. 5, 6, 9, 10, 13, 14, 15, 16, 17 and 18 local buckling is remote. In Shells Nos. MZ. 1, 2, 3, 4, 7, 8, 11 and 12, on the other hand, local buckling may be possible since, even after taking into account the resistance to "rolling over" (which is small in the test specimens - of the order of 1%) the local buckling load is slightly below the general instability load. In general, in the shells which were likely to buckle locally, the local buckling load was close enough to the general instability to make detection of local buckling behavior in the tests hardly feasible.

The general stability load is computed for some shells also by the "discrete theory" of [50], although the difference between "discrete" and "smeared"theory amounts here only to about 1% or less. The "discrete" general instability loads are not given for shells Nos. MZ. 1, 2, 3, 4, 7, 8, 11 and 12 since they are beyond the "cut off point" for which local buckling dominates (see also [50]).

At first sight, one may wonder why the ring spacing was not kept very low in all specimens to ensure predominance of general instability. As the aim of the present study was, however, to find the bounds of "linearity", the nominal design dimensions for some shells were intentionally chosen to be in the "doubtful" region. As a result the local buckling load nearly coincides with P_{GS} in some of the shells. In Table 2, the experimental buckling loads for shells Nos. MZ 1, 2, 3, 4, 7, 8, 11 and 12 are also correlated with the predicted local buckling loads. In the discussion, however, the correlation for all shells is for the general instability load that is of primary interest.

Before turning to the buckling loads carried by the specimens, one may discuss their observed buckling behavior. Figures 4, 6a, 6b, 7a, 7b and 8 show typical shells before and after failure. For all the shells, buckling occurred suddenly, and was rather violent in some. Visually, only the large displacement diamond patterns were detected. In shells MZ 1 - 14 they changed rapidly into typical plastic deformation pattern with sharp yield hinges. In some tests "travelling" of the diamond patterns could be seen momentarily, but in general the pattern appeared practically simultaneously around the whole circumference. The large inertia of the loading system is probably the main cause of the violence of the buckling process. In Shells MZ 15 - 18, a distance tube was incorporated to arrest the displacements much earlier, when the axial shortening was approximately 3 times the linear prebuckling one. Most of the plastic deformation was thereby eliminated, as can be seen by comparison of Fig. 8, which shows a buckled shell when the arresting tube was in position, with Figs. 4, 6 and 7 that depict the results in absence of an arresting device. In Shells MZ 15-18, the buckles practically disappeared upon release of load. Repeated loading, however, resulted in much lower buckling loads that indicate that some plastic deformation had originally taken place. Strain gages were attached to the arresting tube for control of possible load sharing (due to insufficient clearance) before buckling. However, no trouble of this type was encountered.

The buckling pattern is fairly uniform in most specimens, except some helical "climbing" of the diamonds in some shells, somewhat reminiscent of buckling patterns observed in pressurized unstiffened cylinders, see [8] or [42]. No axisymmetrical buckling patterns were observed even for shells that supported 95 percent of the linear theory buckling load P_{Gs} (in some of the ring-stiffened conical shells, discussed in Section 4, ring buckling was observed, but only for very heavy rings). It may, however, be possible that an axisymmetric pattern appears before the deformations become large, similar to the very shallow ripples indicating incipient buckling that were observed in pressurized cylinders [42]. Such an initial, briefly occuring axisymmetric pattern would fit the predictions of orthotropic theory [8] or [26].

Some preliminary attempts to obtain photographic records of the buckling process in order to obtain more information on the initial buckling are shown in Figs. 9a and 9b. These tests were carried out at night and the shells were not illuminated. The shutters of the two cameras were therefore kept open during each loading step (about 60 seconds). Flashlights were then triggered by contacts set at predetermined distances to produce the photographs. Though these attempts were not very successful, further similar investigations with more sophisticated instrumentation may provide more insight into the buckling process.

In passing, it may be of interest to mention one rather surprising insensitivity to imperfections, noticed accidentally during the tests. Shell MZ 7 was damaged upon removal from the mandrel. The damage was a small hole and a badly distorted area of a few square centimeters around it. Surprisingly, however, the buckling load of the specimen was not noticeably affected, and ρ was practically the same as that of a similar undamaged shell MZ 8 (see Table 2). This example lends additional support to the claim for low imperfection sensitivity of externally ring-stiffened cylindrical shells that results from the present test program.

The strain gages that "covered" the specimens proved to be excellent indicators of incipient buckling. In spite of the suddenness of the actual buckling, most of the gages showed signs of near-buckling. Hence buckling could sometimes be predicted from the gages during the test to within 5% of the load. From the strain gage readings Southwell plots could readily be made for most specimens. The computations follow the method proposed in [62] and the mean of the "perfect shell" buckling loads are given in Table 2. The intercept method [62] was also applied but was found to be less reliable. The results support the claims of Horton and his associates, [63] and [64], with regard to the applicability of Southwell's method.

An additional interesting preliminary result of the extensive use of strain gages is the spread over the shell of the indication of incipient buckling. One does not notice isolated local indications of near-buckling, but the gages become "lively" at many locations simultaneously. In Table 3 the count of the direction of deviation of the strain gages just before buckling is given according to horizontal rows of gages. The rows extend over complete circumferences and have 6 or 12 gages each. It can be seen that, with few exceptions, the deviations in each row are unidirectional, indicating axisymmetric deformation. With some stretching of the imagination one could "see" in these widespread indications of incipient buckling the initial axisymmetric pattern that is missing, or some confirmation that initial buckling has a complete periodic pattern as most theories assume and which the usual diamond pattern contradicts! Obviously, more substantial evidence is needed before one could make a definite claim, but it is an interesting thought. In one attempt to provide more evidence, the "Southwell loads" obtained from strain gages at various locations were compared. Figs. 16 and 17 show "good" examples - in which the "Southwell loads" at locations far away from the final buckling pattern location were similar to those near it. In Fig. 18 a "bad" example is shown in which the distant gages predict higher buckling loads. In most of the shells studied, however, the results seem to be encouraging.

One additional " behavior pattern" was studied - the correlation between initial out-of-roundness and strain gage readings. Figures 13 and 14 again show a "good" and a "bad" example. The studies produced no conclusive answer to the problem of prediction of the quality of a shell from its initial out-of-roundness. It appears from the present and similar previous studies, [62] and [68], that there is only a statistical answer.

The primary purpose of the present test series was to confirm the validity of linear theory for analysis of stiffened cylindrical shells and to obtain bounds on the stiffener parameters for upholding this validity. The ratio $\rho = (P_{exp}/P_{Gs})$, also referred to as "linearity", is therefore the primary criterion. For the present tests ρ is given in Table 2.

Two of the stiffener geometry parameters, the ring area ratio (A_2/ah) and the ring spacing (a/h) should predominate. The eccentricity (e_2/h) as well as $(\bar{I}_{22}/ah^3)$ do not affect the buckling of a shell with external rings, at least in theory. Hence ρ is plotted against (A_2/ah) in Fig. 10 and against (a/h) in Fig. 11. The ρ obtained from three ring-stiffened shells of [33] and from some of the shells of [26], given in Table 4, are also plotted for comparison in the figures. A tendency towards a slightly lower ρ for ring area ratios below 0.2 appears in Fig. 10, and there seems to be no additional gain for $(A_2/ah) > 0.4$. There is also a tendency towards a lower ρ with increasing (A/h) in Fig. 11, offset only by the three shells of [33] which, however, have much heavier rings that restrain the sub-shell appreciably, as has already been discussed. One of the attempts to arrive at a combined parameter is shown in Fig. 12. The values of ρ obtained in the present tests and in the experiments of other investigators on integral ring-stiffened cylindrical shells show that linear theory can be considered valid even for relatively weak rings, provided the ring spacing is not large enough to promote premature local buckling. The applicability of linear theory for ring-stiffened shells under axial compression appears to be similar to that under external pressure or torsion. However if one carries out structural efficiency studies (see Section 5) one finds that only light rings are advantageous.

3.4. Stringer-Stiffened Cylinders

A series of tests on stringer-stiffened cylinders of similar dimensions to the ring-stiffened cylinders tested has been initiated and will be reported separately [59]. The discussion will hence be limited to an evaluation of the tests of other investigators [16], [30], [31], [32] and [34], given in Table 5.

The measure of "linearity" ρ is again plotted versus the two primary stiffener geometry parameters, area ratio (A_1/bh) and spacing (b/h) in Figs. 19a and 19b. In the case of stringer-stiffened shells, however, the eccentricity (e_1/h) and $(\bar{I}_{11}/bh^3)$ are very important and their influence requires further study. Furthermore the boundary conditions, in particular clamping, are here more important (see for example [14], [16] or [57]). Hence the test results of [30] and [31], where the boundary conditions approached clamped ends, are compared instead of with P_{Gs}, with $P_{Gs\ clamp}$ and an arbitrarily chosen partial clamping $[(1/3)P_{Gs} + (2/3)P_{Gs\ clamp}]$. The comparison with $P_{Gs\ clamp}$ is conservative since clamping is not complete.

Koiter's Θ is also given in Table 5. It is below the limiting value 0.64 for all but two shells, and indeed general instability predominated in all the tests collected in the table. More tests near the limiting value are however needed

The results show that linear theory is also applicable to integral stringer-stiffened cylindrical shells, except for very weak stringers and wide stringer-spacing. In Fig. 19, the decrease in ρ with increase in (b/h) is pronounced. Heavy stringers show higher "linearity". Preliminary structural efficiency studies (see Section 5) point, however, towards weaker stringers. Further study of the relation between (A_l/bh) and (b/h), as well as the influence of (e_l/h), is therefore needed.

3.5. Further Remarks on Stiffened Cylindrical Shells

Recent imperfection studies [24] maintain that the reliability of linear theory for stiffened shells is doubtful for certain types of stiffening and shell geometries due to increased imperfection sensitivity. For ring-stiffened cylindrical shells, and in particular for shells with outside rings, Hutchinson and Amazigo [24] expect that the "shells may buckle at axial loads which are well below the classical buckling loads" The results of the present tests and those of other investigators discussed in sub-section 3.3. do not support this prediction. As for stringer-stiffened cylinders tested in [16], [30], [31] and [32] these can be classified as light to medium stiffening in the definition of Fig. 3 of [24]. The geometries of the shells tested (their Z) fall in the range, in Fig. 3 of [24], where imperfection sensitivity effects for outside stiffening should not be severe. This may be the reason for the high ρ values found in the evaluation of these tests. In Garkish's recent tests [28], cylinders with outside stringers buckled at loads that were appreciably below the prediction of linear theory. However, as the Z of the specimens puts them in a range in Fig. 3 of [24] where the imperfection sensitivity effects are hardly influenced by eccentricity, these low results do not lend support to the contention of [24], and are due to other causes. Therefore one can also conclude for stringer-stiffened cylindrical shells, although with less certainty, that tests to date do not verify the fears of [24].

The "knock-down factor" as ρ is sometimes called, is much larger for integrally stiffened shells than for unstiffened ones. If one compares some of the "knock-down factors" commonly used with the ρ found in tests of integrally stiffened shells, one finds those factors very conservative. In Table 6 the experimental ρ's of [32], [34], [30], [26] and some of the present shells are compared with ρ_{eff} obtained by different methods. First ρ_{eff} is computed by the methods of [2] and [3], based on Koiter's theory, with h replaced by h_{eff} as defined [3] for stringer stiffening

$$(h_{eff}/h) = 1 + (c/b)\left\{((I_{11}/bh^3) + (A_1/bh)(e_1/h)^2[1 + (A_1/bh)]^{-2}\}\ 12(A_1/bh)^{-1}\right\}^{\frac{1}{2}} \tag{18}$$

and by a similar expression for ring stiffening. ρ_{eff} is also calculated by Pflüger's formula, Eq. (20), in which h is replaced by h_{eff} of Eq. (18). Then h_{eff} and ρ_{eff} are computed by Eqs. (23) and (24) of [67], used at Lockheed Missiles and Space Company. Finally, $(R/h)_{eff}$ is calculated by the method proposed in [66] and ρ_{eff} is then found from Pflüger's formula for this $(R/h)_{eff}$. It is seen that ρ_{eff} is considerably smaller than ρ found in tests of closely and integrally stiffened shells, usually about half.

4. RING-STIFFENED CONICAL SHELLS

Before the present test program on cylindrical shells was initiated, a similar test program on ring-stiffened conical shells was carried out at the Technion, which continued parallel to the present program. The details of the conical shell program are reported in [20] and [68]. Only the main results are given here to support the conclusion arrived at for cylindrical shells.

The test set up for ring-stiffened conical shells is shown in Fig. 20. The axial load is applied directly in a 30 ton "Amsler" universal testing machine and the test arrangement is similar to that for cylindrical shells. The boundary conditions are different, however, since the cones are clamped in fixtures as explained in [20] and [68]. The test specimens were machined from 17-7 PH steel blanks that are first shear-spun to form thick cones. The accuracy of the machined shells was of the same order as that of the cylindrical shells, though slightly less - the thickness variations were up to ± 5%. Figs. 21, 22 and 23, reproduced from [68], show typical ring-stiffened conical shells tested and their buckling patterns. The rings were much heavier in some of the conical shells, up to $(A_2/a_o h) = 3$ in the shell shown in Fig. 23, and the heavier rings promoted axisymmetric buckling as can be seen in the figure.

The test results on conical shells are compared in [20] and [68] with an approximate formula for general instability of ring-stiffened conical shells which is a combination of Eq. (12) for ring-stiffened cylinders and an approximate formula for unstiffened conical shells proposed by Seide [69],

$$P_{cone} = P_{c\ell} \cos^2 \alpha \qquad (19a)$$

where α is the cone angle. The final form of the approximate expression is

$$P_{Gs\ cone} = 2\pi h^2 E\,[3(1-\nu^2)]^{-1/2}\,[1 + A_2/a_o h]^{1/2} \cos^2\alpha \qquad (19b)$$

and its validity has been checked with the theory of [18] for some typical shells.

In Fig. 24 the variation of ρ versus $(A_2 a_o h)$ is shown for the shells tested. It can be seen that above $(A_2/a_o h) = 0.2$ good "linearity" is obtained and that little is gained by increasing the ring area ratio beyond 1.0. Structural efficiency again advocates lighter rings. In Fig. 25 the results of the present tests on cylindrical shells are superimposed on Fig. 24 and, although of slightly better "linearity", the cylinders fit the trend of the conical shells well.

5. STRUCTURAL EFFICIENCY OF STIFFENED CYLINDRICAL SHELLS

The experimental study reported here and the work of other investigators that has been discussed show that cylindrical shells with closely spaced stiffeners buckle at axial loads not far from those predicted by linear theory. The next step is to study the structural efficiency of the stiffened shells to find out how the closely stiffened shell compares with an equivalent unstiffened shell from the weight point of view.

First, one has to establish a convenient standard of comparison. Since the buckling loads of unstiffened axially compressed cylindrical shells are much below the predictions of linear theory and no reliable theoretical estimate is available, one has to rely on empirical formulae that show the primary dependence of the buckling coefficient on (R/h). A very simple formula has been proposed by Pflüger [5] for R/h > 200,

$$(P_B/P_{c\ell}) = 1/[1 + (R/100h)]^{1/2} \qquad (20)$$

that, in addition to its simplicity, has the additional merit - for the purpose of comparison - of being unconservative for most test data. In Fig. 26 Pflüger's formula, Eq. (20), is superimposed on Fig. 3 of [6] that presents test results obtained by 14 investigators, and it can be seen that Eq. (20) is unconservative for practically all the shells tested. Hence P_B from Eq.(20) is a suitable buckling load for the "equivalent" unstiffened cylinders with which the stiffened cylinders are compared.

For ring-stiffened cylindrical shells with outside rings, axisymmetric buckling predominates and the simple formula for the general instability load, Eq. (12), makes the comparison with the "equivalent" unstiffened shell very easy. If linear theory is valid

$$(P_{Gs}/P_{c\ell}) = [1 + (A_2/ah)]^{1/2} \tag{21}$$

If only a fraction ρ of the "linear" load is achieved

$$(P_{Gs}/P_{c\ell}) = \rho[1 + A_2/ah)]^{1/2} \tag{22}$$

The thickness of the equivalent unstiffened shell (of identical weight) is

$$\bar{h} = [1 + (A_2/ah)]h \tag{23}$$

and the buckling load of the equivalent shell is given by

$$(\bar{P}_B/P_{c\ell}) = (\bar{h}/h)^2 [1 + (R/100\bar{h})]^{-1/2} \tag{24}$$

If Pflüger's empirical formula Eq. (20) is employed. Hence the efficiency of externally ring-stiffened cylindrical shells is given by

$$\eta = (P_{Gs}/\bar{P}_B) = \frac{\rho[\Delta_R + (R/100h)]^{1/2}}{\Delta_R^2} \tag{25}$$

where

$$\Delta_R = 1 + (A_2/ah) \tag{26}$$

With Eq. (25) design curves can readily be drawn that give η versus (R/h) for various values of ρ and (A_2/ah). In Fig. 27 a typical set of such curves is presented. It is immediately seen that even when only 60% "linearity" is achieved, weak ring-stiffening is very efficient for thin shells (large R/h); or, in other words, thin shells with many closely spaced rings (to prevent local buckling) and external rings of small cross-sectional area carry axial compression very efficiently.

For internal rings, asymmetrical buckling occurs and Eq. (12) is no longer valid, unless the rings have very high torsional stiffness. The critical load parameter λ has therefore to be computed from Eq.(10). Since, from Eq. (11),

$$P_{Gs} = \lambda[\pi E h^3/12(1-\nu^2)R] \tag{27}$$

one obtains, after substitution of Eq. (1) for $P_{c\ell}$, the efficiency for internally ring-stiffened cylindrical shell as

$$\eta = \frac{\rho\lambda}{8[3(1-\nu^2)]^{1/2}} \cdot \frac{[\Delta_R + (R/100h)]^{1/2}}{(R/h)\ \Delta_R^{2.5}} \tag{28}$$

where Δ_R is again given by Eq. (26). Design curves can then be drawn for internal rings. Internal rings will, naturally, be less efficient stiffeners than external ones, see [14].

For stringer-stiffened cylindrical shells a similar expression can be obtained for the structural efficiency,

$$\eta = \frac{\rho\lambda}{8[3(1-\nu^2)]^{1/2}} \cdot \frac{[\Delta_s + (R/100h)]^{1/2}}{(R/h)\ \Delta_s^{2.5}} \tag{29}$$

where

$$\Delta_s = 1 + (A_1/bh) \tag{30}$$

Some typical design curves for external stringers are presented in Fig. 28. Again the stiffening is seen to be more efficient for lighter stringers and a thinner shell, except for (R/h) below 500 where the efficiency may rise again slightly as (R/h) decreases.

For optimization of the stiffened shell, be it stringer-or ring-stiffened, one has to balance the likely "linearity" obtained for various stiffener areas with the η for the respective stiffener area and shell (R/h). From the present tests and the results of other investigators, the range of $0.2 < A_2/ah < 0.5$ appears most promising for rings and $0.3 < A_1/bh < 0.8$ for stringers, provided, obviously, that the stiffener spacing is small enough to eliminate local buckling.

Figures 29 and 30 show this for the ring-stiffened shells of the present tests and for tests of other investigators. The curves for 100% efficiency are obtained from Eq. (25) which for $\eta = 1$ immediately yields

$$\rho = \Delta_R^2/[\Delta_R + R/100h]^{1/2} \qquad (31)$$

Further study is needed for better definition of these ranges.

6. CONCLUSIONS

The present tests and the results of other investigators discussed lead to the conclusion that linear theory is applicable to integrally ring-and stringer-stiffened cylindrical shells under axial compression, provided the stiffeners are closely spaced. Applicability of linear theory means here that buckling loads can be estimated with the same reliability as say for unstiffened cylindrical shells under external pressure. Small design factors for imperfections of up to 30% are not excluded, but the customary large "knock down factors" are absent even for weak integral stiffening. Hence stiffener eccentricity effects and optimization studies based on linear theory may be relied upon.

Structural efficiency studies, which include the reduced "linearity" observed in tests, show that even ring-stiffening is advantageous under axial compression. The dominant geometrical parameters that determine the "linearity" are the stiffener spacing and stiffener cross-sectional area. Ring spacing that ensures axisymmetric local buckling, or stringer spacing with Koiter's $\theta < 0.64$, will give the required predominance of general instability; and stiffener cross-sectional area that yield $0.2 < (A_2/ah) < 0.5$ for rings, or $0.3 < A_1/bh < 0.8$ for stringers, appear most promising from the structural efficiency point of view.

The strain gage recordings in the tests suggest that the initial buckling pattern covers the whole shell and differs basically from the final visually observed pattern.

APPENDIX A

EFFECT OF ROTATIONAL RESTRAINTS

The analysis is an extension of that given in [49] for axial restraints. The effect of rotational restraint is found approximately by a Rayleigh-Ritz approach, using the displacements of the unrestrained shell. The rotational restraints are assumed to come into action only at onset of buckling - type (b) restraints in the classification of [49]. Two forms of rotational restraint are provided by the rings: resistance to twist, if the sub-shell buckles non-axisymmetrically, and resistance to "rolling over" that appears also in axisymmetric buckling. Since the torsional restraint, when effective, is usually much larger than the restraint to "rolling over", the latter is neglected for asymmetrical buckling.

Note that in the appendix the displacements and coordinates are written u, v, w, x instead of u^*, v^*, w^*, x^* and the origin is at the midlength of the shell or sub-shell of length L.

For uniform axial compression, the additional total potential energy (after buckling occurs) of a thin cylindrical shell with elastic rotational restraints due to torsional resistance of the rings k_T is:

$$U + V = [hER/2(1-\nu^2)] \int_0^{2\pi} \int_{-L/2}^{L/2} \Big\{ [u_{,x} + (v_{,\phi}/R) - (w/R)]^2 -$$

$$-2(1-\nu)\{u_{,x}[(v_{,\phi}/R) - (w/R)] - (1/4)[(u_{,\phi}/R) + v_{,x}]^2\} \Big\} \, dx\,d\phi \, +$$

$$+ [h^3ER/24(1-\nu^2)] \int_0^{2\pi} \int_{-L/2}^{L/2} \{[w_{,xx} + (\nu/R^2)w_{,\phi\phi}]w_{,xx} +$$

$$+ [\nu w_{,xx} + (w_{,\phi\phi}/R^2)] (w_{,\phi\phi}/R^2) + [2(1-\nu)/R^2 w_{,x\phi}{}^2\} dx d\phi -$$

$$-(P/4\pi) \int_0^{2\pi} \int_{-L/2}^{L/2} (w_{,x})^2 dx d\phi + (k_T/R) \int_0^{2\pi} [(w_{,x\phi})_{L/2}]^2 d\phi \qquad \text{(A1)}$$

The displacement boundary conditions of the problem are

$$\left.\begin{array}{l} w = 0 \\ \\ v = 0 \end{array}\right\} \text{at } x = \pm L/2 \qquad \text{(A2)}$$

A cylinder under uniform axial compression may buckle in an axisymmetrical pattern or in a more general form. For the more general buckled shape, the displacements functions

$$\begin{aligned} u &= A_m \sin t\phi \sin (m\pi x/L) && \text{where} \\ v &= B_m \cos t\phi \cos (m\pi x/L) && m = 1, 3, 5\ldots \qquad \text{(A3)} \\ w &= C_m \sin t\phi \cos (m\pi x/L) && t = 1, 2, 3\ldots \end{aligned}$$

satisfy the boundary conditions, Eqs. (A2), and are admissible.

Substitution of the assumed displacement functions, Eqs. (A3), into the total potential energy expression Eq. (A1) and minimization with respect to the free parameters yields the usual stability determinant from which Eq. (6) of Section 2 is obtained.

For axisymmetrical buckling the last term of Eq. (A1), representing the elastic restraint, has to be replaced by

$$Rk_R \int_0^{2} [(w_{,x})_{L/2}]^2 d\phi \qquad \text{(A4)}$$

the strain energy absorbed by the "rolling over" mechanism of the rings. The corresponding displacement functions are

$$
\begin{aligned}
u &= A_m \sin (m\pi x/L) \\
v &= 0 \\
w &= C_m \cos (m\pi x/L)
\end{aligned}
\tag{A5}
$$

and the stability determinant that results from the minimization yields Eq.(8) of Section 2.

Note that when Eqs. (6) and (8) are applied to the local buckling of a ring-stiffened shell, L represents the length of the sub-shells, denoted "a" elsewhere.

REFERENCES

1. Fung, Y.C. and Sechler, E.E., " Instability of Thin Elastic Shells" Structural Mechanics, Proceedings of the First Symposium on Naval Structural Mechanics, Stanford University, August 1958, edited by Goodier,T.N. and Hoff, N.J., Pergamon Press, 1960, pp. 115-168.

2. Hoff, N.J., " The Perplexing Behavior of Thin Circular Cylindrical Shells in Axial Compression", Second Theodore von Karman Memorial Lecture, Proceedings 8th Israel Annual Conference on Aviation and Autronautics, Israel Journal of Technology, Vol. 4, No. 1, pp. 1-28, February 1966.

3. Hoff, N.J., " Thin Shells in Aerospace Structures", 4th AIAA Von Karman Lecture, Astronautics and Aeronautics, Vol. No. 2, pp. 26-45, February 1967.

4. Kanemitsu, S. and Nojima, N.M., " Axial Compression Tests of Thin Circular Cylinders, M.S. Thesis, California Institute of Technology, 1939.

5. Pflüger, A., " Zur praktischen Berechnung der axial gedrückten Kreiszylinderschale", Der Stahlbau, Vol. 32, No. 6, pp. 161-165, June 1963.

6. Weingarten, V.I., Morgan, E.J. and Seide, P. " Elastic Stability of Thin Walled Cylindrical and Conical Shells under Axial Compression" AIAA Journal, Vol. 3, No. 3, pp. 500-505, March 1965.

REFERENCES (CONT'D)

7. Bodner, S.R., " General Instability of a Ring-Stiffened Cylindrical Shell under Hydrostatic Pressure" Journal of Applied Mechanics, Vol.24 No. 2, pp. 269-277, June 1957.

8. Thielemann, W.F., " New Developments in the Nonlinear Theories of the Buckling of Cylindrical Shells ", Aeronautics and Astronautics, Proceedings of the Durand Centennial Conference 1959 (Pergamon Press, Oxford, 1960), pp. 76-119.

9. Becker, H. and Gerard, G., "Elastic Stability of Orthotropic Shells", Journal of the Aerospace Sciences, Vol. 29, No. 5, pp. 505-512, May 1962.

10. Flügge, W., " Die Stabilitat der Kreiszylinderschale", Ingenieur, Archiv, Vol. 3, pp. 463-506, 1932.

11. Van der Neut, A., " The General Instability of Stiffened Cylindrical Shells Under Axial Compression, Report S-314, National Luchtvaart-laboratorium, Amsterdam, Report and Transactions, Vol. 13, S.57--84, 1947.

12. Baruch, M. and Singer, J., " Effect of Eccentricity of Stiffeners on the General Instability of Stiffened Cylindrical Shells under Hydrostatic Pressure", Journal of Mechanical Engineering Science, Vol. 5, No. 1, pp. 23-27, March 1963.

13. Singer, J. and Baruch, M., " Recent Studies on Optimization for Elastic Stability of Cylindrical and Conical Shells", presented at the Royal Aeronautical Society Centenary Congress, London, September 12-16, 1966.

REFERENCES (CONT'D)

14. Singer, J., Baruch, M. and Harari, O., " On the Stability of Eccentrically Stiffened Cylindrical Shells under Axial Compression. International Journal of Solids and Structures, Vol. 3, No. 4, pp. 445-470, July 1967. Also TAE Report No. 44, Technion Research and Development Foundation, Haifa, Israel, December 1965.

15. Block, D.L., Card, M.F. and Mikulas, M.M., "Buckling of Eccentrically Stiffened Orthotropic Cylinders", NASA TN D-2960, August, 1965.

16. Card, M.F. and Jones, R.M., "Experimental and Theoretical Results for Buckling of Eccentrically Stiffened Cylinders", NASA TND-3639, October, 1966.

17. Burns, A.B., " Structural Optimization of Axially Compressed Cylinders, Considering Ring-Stringer Eccentricity Effects" Journal of Spacecraft and Rockets, Vol. 3 , No. 8 pp. 1263-1268 August 1966.

18. Baruch, M. and Singer, J., "General Instability of Stiffened Circular Conical Shells under Hydrostatic Pressure, The Aeronautical Quarterly, Vol. 26, Part 2, May 1965, p. 187. Also TAE Report 28, Technion Research and Development Foundation, Haifa, Israel, June 1963.

19. Baruch, M., Singer, J. and Harari, O., " General Instability of Conical Shells with non-Uniformly Spaced Stiffeners under Hydrostatic Pressure", Proceedings of the 7th Israel Annual Conference on Aviation and Astronautics, Israel Journal of Technology, Vol. 3, No. 1, pp. 62 - 71, February 1965.

REFERENCES (CONT'D)

20. Singer, J., Berkovits, A., Weller, T., Ishai, O., Baruch, M. and Harari,O., "Experimental and Theoretical Studies on Buckling of Conical and Cylindrical Shells under Combined Loading", TAE Report 48, Technion Research and Development Foundation, Haifa, Israel, June 1966.

21. Moe, J., " Stability of Ring-Reinforced Cyli 'rical Shells Under Lateral Pressure", Publications, International Association for Bridge and Structural Engineering, Vol. 18, pp. 113-136, 1958.

22. Van der Neut, A., " General Instability of Orthogonally Stiffened Cylindrical Shells" Collected Papers on Instability of Shell Structures - 1962. NASA TND-1510, pp. 309-319, December 1962.

23. Block, D.L., " Influence of Ring Stiffeners on Instability of Orthotropic Cylinders in Axial Compression", NASA TND-2482, October 1964.

24. Hutch'nson, J.W. and Amazigo, J.C., " Imperfection-Sensitivity of Eccentrically Stiffened Cylindrical Shells", AIAA Journal, Vol. 5, No.3, pp.392-401, March 1967.

25. Gerard, George, " Elastic and Plastic Stability of Orthotropic Cylinders", Collected Papers on Instability of Shell Structures - 1962 NASA TN D- 1510, pp. 277 - 295 December 1962.

26, Milligan R., Gerard, G., Lakshmikantham, C., and Becker, H., " General Instability of Orthotropic Stiffened Cylinders under Axial Compression", AIAA Journal, Vol. 4, No. 11, pp. 1906-1913, November 1966, Also Report AFFDL-TR-65-161, Air Force Flight Dynamics Laboratory, USAF, Wright Patterson Air Force Base, Chio, July 1965.

REFERENCES (CONT'D).

27. Pugliese, P.J., " Tank Compression Test-Model DM-18" Douglas Aircraft Co. Report SM-27650, February 1959.

28. Garkisch H.D., " Experimentelle Untersuchung des Beulverhaltens Kreiszylinderschalen mit exzentrischer Längsversteifungen (to be published as a DFL Forschungsbericht 1967).

29. Midgley, W.R. and Johnson, A.E., " Experimental Buckling of Internal Integral Ring-Stiffened Cylinders", Experimental Mechanics, Vol. 7, No. 4, pp. 145-153, April 1967.

30. Card, M.F., "Preliminary Results of Compression Tests on Cylinders with Eccentric Longitudinal Stiffeners", NASA TMX-104, September, 1964.

31. Peterson, J.P. and Dow, M.B., " Compression Tests on Circular Cylinders Stiffened Longitudinally by Closely Spaced Z-Section Stringers", NASA MEMO 2-12-59L, 1959.

32. Katz, L., " Compression Tests on Integrally Stiffened Cylinders" NASA TMX-55315, August 1965.

33. Almroth, B.O.,"Influence of Imperfections and Edge Restraint on the Buckling of Axially Compressed Cylinders", presented at the AIAA/ASME 7th Structures and Materials Conference, Cocoa Beach, Florida, April 18-20, 1966.

34. Len'ko O.N., " The Stability of Orthotropic Cylindrical Shells" Raschet Prostranstvennykh Konstruktsii, Issue IV, pp.499-524, Moscow 1958, Translation NASA TT F-9826, July 1963.

REFERENCES (CONT'D)

35. De Luzio, A., Stuhlman, C.E., and Almroth, B.O., "Influence of Stiffener Eccentricity and End Moment on Stability of Cylinders in Compression", AIAA Journal, Vol. 4, No. 5, pp. 872-877, May 1966.

36. Block, D.L., " Influence of Prebuckling Deformations, Ring Stiffeners and Load Eccentricity on the Buckling of Stiffened Cylinders", presented at the AIAA/ASME 8th Structures , Structural Dynamics and Materials Conference, Palm Springs, California, March 29-31, 1967.

37. Soong, T.C., " Influence of Boundary Constraints on the Buckling of Eccentrically Stiffened Orthotropic Cylinders", presented at the 7th International Symposium on Space Technology and Science, Tokyo, May, 1967.

38. Burns, A.B., "Optimization Cylinders with Contrasting Materials and Various Ring/Stringer Configurations", Journal of Spacecraft and Rockets, Vol. 4, No. 3, pp. 375-385, March 1967.

39. Batdorf, S.R.," A simplified Method of Elastic Stability Analysis for Thin Cylindrical Shells",NACA Report 874, 1947.

40. Timoshenko, S.P. and Gere, G.M., "Theory of Elastic Stability" Second Ed., McGraw-Hill Book Company, Inc., 1961, pp. 465-467.

41. Flügge, W. "Stresses in Shells", Springer-Verlag, Berlin, 1960, p. 428 and p. 480.

42. Harris, L.A., Suer, H.S., Skene, W.T. and Benjamin, R.J., " The stability of Thin-Walled Unstiffened Circular Cylinders under Axial Compression, Including the Effects of Internal Pressure",Journal of the Aeronautical Sciences, Vol. 24, No.8, pp.587-596, August 1957.

REFERENCES (CONT'D)

43. Gerard, G., "Introduction to Structural Stability Theory" McGraw-Hill Book Co., New-York, 1962, p. 145.

44. Batdorf, S.B., Schildcrout, M. and Stein M., " Critical Stress in Thin-Walled Cylinders in Axial Compression, NACA Report 887, 1947.

45. Hoff, N.J. and Soong, T.C., " Buckling of Circular Cylindrical Shells in Axial Compression", International Journal of Mechanical Sciences Vol. 7., No. 7, pp.489-520, July 1965.

46. Almroth, B.O., " Influence of Edge Conditions on the Stability of Axially Compressed Cylindrical Shells", AIAA Journal, Vol. 4, No. 1, pp. 134-140, January 1966.

47. Reynolds, T.E., " Elastic Lobar Buckling of Ring-Supported Cylindrical Shells under Hydrostatic Pressure", David Taylor Model Basin Report 1614, September 1962.

48. Singer, J., " The Effect of Axial Constraint on the Instability of Thin Circular Cylindrical Shells under External Pressure", Journal of Applied Mechanics, Vol. 27, No. 4, pp. 737-739, December 1960.

49. Singer, J., " The Effect of Axial Constraint on the Instability of Thin Cylindrical Shells under Uniform Axial Compression", International Journal of Mechanical Sciences, Vol. 4, No. 2, pp. 253-258, May 1962.

REFERENCES (CONT'D)

50) Singer, J. and Haftka, R. " Buckling of Discretely Ring-Stiffened Cylindrical Shells", TAE Report 67, Technion Research and Development Foundation, Haifa, Israel, August 1967.

51) Stein, M., Sanders , S.L. and Crate, H., " Critical Stress of Ring-Stiffened Cylinders", NACA Report 989, 1951.

52) Baruch, M., " Equilibrium and Stability Equations for Discretely Stiffened Shells", Israel Journal of Technology, Vol. 3, No. 2, pp. 138-146, June 1965.

53) Appel, H. " Buckling Modes for Orthotropic Circular Cylinders under Axial Compression for various Combinations of Stiffness Parameters", Deutsche Forschungsanstalt für Luft-und Raumfahrt Report DLR FB 65-47.

54) Koiter, W.T. " Buckling and Post-Buckling Behavior of a Cylindrical Panel under Axial Compression", Report S. 476, National Luchtvaartlaboratorium, Amsterdam, Reports and Transactions, Vol. 20, 1956.

55) Lundquist, E.E. and Stowell, E.Z., " Compressive Stress for Flat Rectangular Plates Supported Along All Edges and Elastically Restrained Against Rotation Along the Unloaded Edges", NACA Report 733, 1942.

56) Leggett, D.M., " The Buckling of a Long Curved Panel under Axial Compression", Reports and Memorandum No. 1899, Ministry of Aviation, ARC, 1942.

REFERENCES (CONT'D)

57. Hedgepeth, J.M., and Hall, D.B., "Stability of Stiffened Cylinders", AIAA Journal, Vol. 3, No. 12, pp. 2275-2286, December 1965.

58. Geier, B., "Beullasten versteifter Kreiszylinderschalen", Jahrbuch WGLR 1965, Vieweg, 1966, pp. 440-447.

59. Singer, J., Weller, T., Frum, J. and Nachmani, S., "Experimental Studies on Buckling of Integrally Stiffened Cylindrical Shells under Axial Compression", to be published as a TAE Report, Technion Research and Development Foundation, Haifa, Israel.

60. Berkovits, A. and Singer, J., "Buckling of Unstiffened Conical Shells under Combined Torsion and Axial Compression or Tension", Proceedings of 7th Israel Annual Conference on Aviation and Astronautics, Jerusalem Academic Press, pp. 15 - 24, February 1965.

61. Holt, M., " A Procedure for Determining the Allowable Out-of-Roundness for Vessels under External Pressure", Transactions ASME Vol. 74, pp. 1225 - 1230, 1952.

62. Singer, J. and Eckstein, A., " Experimental Investigations of the Instability of Conical Shells under External Pressure", Proceedings of the Fourth Annual Conference on Aviation and Astronautics, February 1962, Bulletin of the Research Council of Israel, Vol. IIC, No. 1, pp. 97 - 122, April 1962.

REFERENCES (CONT'D)

63. Galletly G.D. and Reynolds, T.E., "A Simple Extension of Southwell's Method for Determining the Elastic General Instability Pressure for Ring-Stiffened Cylinders Subject to External Hydrostatic Pressure", Proceedings of the Society for Experimental Stress Analysis, Vol. 13, No. 2, 141-152, 1956.

64. Horton, W.H., Cundari, F.L. and Johnson, R.W., " The Analysis of Experimental Data Obtained from Stability Studies on Elastic Column and Plate Structures", Proceedings of the 9th Israel Annual Conference on Aviation and Astronautics, Israel Journal of Technology, Vol. 5, Nos. 1 - 2, pp. 104 - 113, February 1967.

65. Horton, W.H. and Cundari F.L., " On the Applicability of the Southwell Plot to the Interpretation of Test Data Obtained from Instability Studies of Shell Bodies", SUDAAR No. 290, Department of Aeronautics and Astronautics, Stanford University, August 1966.

66. Peterson, J.P., "Weight-Strength Studies of Structures Representative of Fuselage Construction" NACA TN 4114, October, 1957.

67. Burns, A.B. and Almroth, B.O., " Structural Optimization of Axially Compressed Ring-Stringer Stiffened Cylinders", Journal of Spacecraft and Rockets, Vol. 3, No. 1, pp. 19 - 25, January 1966.

68. Weller, T. and Singer, J., "Further Experimental Studies on Buckling of Ring-Stiffened Conical Shells under Axial Compression", TAE Report 70, Technion Research and Development Foundation, Haifa, Israel, December 1967.

REFERENCES (CONT'D).

69. Seide, P., " Axisymmetrical Buckling of Circular Cones under Axial Compression", Journal of Applied Mechanics, Vol. 23, No. 4, p. 625, December 1956.

ACKNOWLEDGEMENT

The author would like to thank Messrs. T. Weller, J. Frum, S. Nachmani, S. Regenstreif, A. Klausner, A. Greenwald and G. Shiner for their assistance during the course of the tests, to Miss A. Adler and Mrs. M. Herbst-Rosiano for assistance with computations and the staff of the Technion Computing Center for their valuable help.

TABLE 1.

EFFECT OF RESTRAINTS ON LOCAL BUCKLING IN RING-STIFFENED SHELLS [33]

$R = 8''$ $\quad E = 10.5 \times 10^6$ psi $\quad k_T/ER^4 = 4.20 \times 10^{-9}$

$L/R = 2.03$ $\quad \nu = 0.33$ $\quad k_R/ER^2 = 1.63 \times 10^{-8}$

Shell Almroth [33]	Buckling Load					Lbs					
	R/h	A_2/ah	e_2/h	a/h	Z'	(1) $P_{classical}$ (Asym.) Lbs.	(2) $P_{axisym.}$ Lbs.	(3) $\frac{P_{restrained}}{P_{axisym.}}$	(4) $P_{rest.axisym.}$ Lbs.	(5) P_{exp} Lbs.	(6) P_{Gs} Lbs.
A1	395	.878	3.0	57	7.764	16150	17370	1.107	19230	15300	17060
A2	384	.900	2.93	56	7.710	16960	18350	1.100	20180	14830	17950
A3	415	.834	3.16	60	8.189	14570	15340	1.124	17240	15030	15400

TABLE 2.

RING-STIFFENED CYLINDRICAL SHELLS - DIMENSIONS OF SPECIMENS AND RESULTS

R = 175.6 mm L = 200 mm

Shell No.	Mean Thickness h** [mm]	Thickness Deviation [±mm]	a [mm]	a/h	R/h	Number of Rings	e_2/h	A_2/ah	I_{22}/ah^3	Z'	P_{cl} unstiffened [kg]	$P_{G.s.}$ Approx. [kg]	$P_{G.s.}$ [14] [kg]	n	P_{discr}
MZ - 1*	.283	-(.01)+(.02)	10	35.3	620	17	-2.28	.535	.568	1.92	6090	7540	7580	18	-
2	.280	-(.01)+(.01)	10	35.7	627	17	-2.32	.546	.604	1.94	5960	7410	7430	18	-
3	.285	-(.01)+(.02)	15	52.6	616	11	-2.31	.361	.393	4.29	6180	7210	7230	18	-
4	.284	-(.01)+(.02)	15	52.8	618	11	-2.30	.359	.366	4.30	6130	7150	7170	18	-
5	.274	-(.01)+(.02)	7	25.6	641	25	-1.56	.373	.148	.97	5710	6690	6690	19	66
6	.253	-(.02)+(.02)	7	27.7	694	25	-1.68	.405	.187	1.05	4870	5770	5770	19	57
7	.268	-(.02)+(.02)	10	37.3	655	17	-2.06	.373	.301	2.03	5460	6400	6410	18	-
8	.258	-(.02)+(.02)	10	38.8	681	17	-2.04	.370	.293	2.11	5060	59[illegible]	5920	19	-
9	.281	-(.01)+(.02)	7	24.9	625	25	-1.29	.271	.057	.95	6000	[illegible]	6780	18	-
10	.281	-(.01)+(.01)	7	24.9	625	25	-1.29	.271	.057	.95	6000	[illegible]	6780	18	-
11	.271	-(.01)+(.01)	10	36.9	648	17	-1.34	.202	.048	2.00	5580	6120	6130	18	-
12	.270	-(.01)+(.01)	10	37.0	650	17	-1.33	.198	.045	2.01	5540	6070	6080	18	-
13	.246	-(.01)+(.01)	7	28.5	714	25	-1.77	.435	.235	1.08	4600	5510	5510	20	54
14	.236	-(.01)+(.01)	7	29.7	744	25	-1.12	.211	.027	1.13	4230	4660	4660	19	46
15	.249	-(.01)+(.01)	8	32.1	705	22	-1.01	.165	.014	1.40	4710	5090	5090	18	50
16	.295	-(.01)+(.01)	8	27.1	595	22	-0.88	.123	.006	1.18	6620	7010	7030	17	70
17	.255	-(.01)+(.02)	8	31.4	689	21	-1.38	.264	.068	1.36	4940	5560	5560	18	-
18***	.245	-(.02)+(.025)	8	32.7	717	21	-1.52	.328	.130	1.42	4560	5260	5260	19	-

* In the test of shell MZ - 1 a dynamic load was introduced inadvertently and hence it is not considered a valid test point.

** The last figure in h is only approximate

*** Shell MZ 18 had more pronounced non-uniformities in thickness than the other test specimens

		L = 200 mm				L/R = 1.14			ν = 0.3		E = 2 x 10^4 kg/mm^2		
Z'	P_{cl} unstiffened [kg]	$P_{G.s.}$ Approx. [kg]	$P^{*}_{G.s.}$ [14] [kg]	n	$P_{discrete}$	$P_{Loc.}$ s.s. [kg]	$P_{Loc.}$ corrected for "Springs" [kg]	P_{exp} [kg]	t_{exp}	tiers	P_{exp} Southwell [kg]	$\frac{P_{exp}}{P_{Gs}}$	$\frac{P_{exp}}{P_{Loca}}$
1.92	6090	7540	7580	18	-	6560	6640	5060	9	2	-	.670	.762
1.94	5960	7410	7430	18	-	6400	6490	6260	9	2	6570	.844	.965
4.29	6180	7210	7230	18	-	6700	6760	5900	9		6470	.818	.872
4.30	6130	7150	7170	18	-	6660	6720	5940	9	3	6560	.830	.884
.97	5710	6690	6690	19	6680	9340	9390	6070	13	2	6390	.877	
1.05	4870	5770	5770	19	5720	7480	7530	4510	12	2	4790	.781	
2.03	5460	6400	6410	18	-	5780	5830	4670	12	2	-	.729	.801
2.11	5060	5920	5920	19	-	5290	5320	4420	12	2	5240	.746	.831
.95	6000	6770	6780	18	-	10030	10060	5780	12-13	2	-	.854	
.95	6000	6770	6780	18	-	10030	10060	6260	13	3	6980	.925	
2.00	5580	6120	6130	18	-	5930	5950	5150	13	3	5920	.841	.866
2.01	5540	6070	6080	18	-	5870	5890	4400	13	3	4710	.724	.747
1.08	4600	5510	5510	20	5450	6930	6960	5370	13	2	5730	.952	
1.13	4230	4660	4660	19	4650	6180	6200	3140	12	2	3430	.673	
1.40	4710	5090	5090	18	5070	5960	5980	3850	14	2-3	-	.757	
1.18	6620	7010	7030	17	7020	9360	9380	5950	14	3	6380	.848	
1.36	4940	5560	5560	18	-	6350	6380	4650	16	3	4810	.837	
1.42	4560	5260	5260	19	-	6710	6740	3630	15	2	3960	.691	

ce it is not considered a

st specimens

TABLE 3

STRAIN GAGE DEVIATIONS AS INDICATION OF AXISYMMETRIC INITIAL BUCKLING

	CIRCUMFERENTIAL GAGES			AXIAL GAGES		
Shell No.	Gage Nos. (Rows)	Count of Deviations Tension	Count of Deviations Compression	Gage Nos. (Rows)	Count of Deviation Tension	Count of Deviation Compression
MZ-2	25-30		6	1-12	12	
	31-36		6			
	37-42		6			
	43-48	6				
MZ-3	25-30	6		1-12	11	
	31-36	2	4			
	37-42	4	1			
	43-48	4	2			
MZ-4	25-30	1	4	1-12	12	
	31-36	6				
	37-42	5				
	43-48	6				
MZ-5	25-30		4	1-12	5	5
	31-36	5				
	37-42		6			
	43-48	6				
MZ-6	25-30	1	5	1-12	11	
	31-36	6				
	37-42	4	2			
	43-48	3	2			
MZ-8	25-30		6	1-12	4	7
	31-36		6			
	37-42		6			
	43-48		6			
MZ-9	25-30	2	4	1-12	1	10
	31-36		6			
	37-42		6			
	43-48		6			
MZ-10	25-30		6	1-12	3	8
	31-36	1	2			
	37-42	3	1			
	43-48		6			
MZ-11	7-12	6		19-24	6	
	13-18		6			
MZ-12	25-30	5		1-12		12
	31-36	6				
	37-42	1	1			
	43-48	6				
MZ-13	13-18	5	1	1-12	11	
	19-24		6			
	25-30	2	4			
	31-36	1	5			
	37-42	3	1			
	43-48	6				
MZ-14				1-6	3	1
				7-12	5	
				13-18	6	
				19-24	5	1
MZ-15	1-12	2	8	19-24	4	2
	13-18	6		25-30	3	3
MZ-16	1-12	8	2	19-24	5	1
	13-18	5		25-30	4	1
MZ-17	1-6	5		7-12	4	1
	13-18	5				
	19-24	2	4			
MZ-18	1-6	6		7-12	4	
	13-18		6			
	19-24	6				

NOTE: Gage Numbers refer to Nos. on B & F recorder and hence numbering is usually not continuous.

In some cases not all the gages showed noticeable deviations.

TABLE 4

"LINEARITY" OBTAINED IN TESTS ON RING-STIFFENED CYLINDRICAL SHELLS BY OTHER INVESTIGATORS.

Investigator	Shell No.	R/h	A_2/ah	a/h	e_2/h	I_{22}/ah^3	P_{Gs} [lb]	P_{exp} [lb]	$\rho = P_{exp}/P_{Gs}$
MILLIGAN et al [26]	5	373	.347	10.9	- .877	.0169	4880	3720	.762
	9	314	.175	9.2	- .690	.0021	6410	5180	.808
	20	404	.447	11.8	- .968	.0226	4320	4460	1.032
	32	384	.410	11.2	+ .914	.2830	3990	4740	1.188
	33	667	.878	19.5	+1.386	.0031	1340	1480	1.104
ALMROTH [33]	A1	395	.878	57.0	+3.00	1.83	17060	15300	.897
	A2	384	.900	55.6	+2.93	1.70	17950	14880	.829
	A3	415	.834	60.0	+3.16	2.13	15400	15030	.976

TABLE 5.

"LINEARITY" OBTAINED IN TESTS ON STRINGER-STIFFENED CYLINDRICAL SHELLS BY OTHER INVESTIGATORS AND CORRELATION WITH KOITER'S MEASURE OF TOTAL CURVATURE θ.

Investigator	Shell	R/h	L/R	A_1/bh	b/h	e_1/h	I_{11}/bh^3	$\rho = \frac{P_{exp}}{P_{Gs}}$	$\rho_1 = \frac{P_{exp}}{P_{Gs\ clamp}}$	$\rho_2 = \frac{P_{exp}}{\frac{1}{3} P_{Gs} + \frac{2}{3} P_{Gs\ clamp}}$	Koiter's θ
	1	338	3.98	1.03	35.3	-5.83	9.79	1.639	.922	1.080	.555
	2	345	3.98	1.06	36.0	+5.96	10.5	1.165	.803	.895	.561
	3	347	2.49	1.07	36.3	-6.0	10.8	1.600	.817	.976	.564
	4	341	2.49	1.05	35.7	+5.89	10.1	1.339	.789	.915	.559
	5	388	3.71	.824	30.5	-7.07	20.4	1.421	.741	.882	.448
CARD [16]	6	397	3.71	.84	31.2	+7.22	21.8	1.299	.848	.959	.453
	7	407	2.63	.862	32.0	-7.41	23.5	1.145	.712	.858	.459
	8	397	2.63	.84	31.2	+7.22	21.8	1.630	.911	1.068	.453
	9	406	1.86	.862	31.9	-7.40	23.4	1.734	.797	.972	.458
	10	395	1.86	.837	31.0	+7.19	21.5	2.041	.943	1.149	.451
	11	403	1.30	.854	31.6	-7.34	22.8	1.828	.764	.948	.455
	12	397	1.30	.84	31.2	+7.22	21.8	2.658	.960	1.219	.453
	5A-1	688	2.12	.244	63.8	3.29	.632	.608			.705
	5A-2	707	2.12	.251	65.6	3.38	.694	.606			.714
KATZ [32]	5B-1	685	2.13	.349	45.1	3.29	.904	.562			.498
	5B-2	685	2.13	.339	45.1	3.29	.879	.541			.498
	5C-1	668	2.12	.481	43.8	3.68	1.08	.663			.491
	5C-2	686	2.12	.492	45.1	3.79	1.17	.825			.498
LENKO [34]		243	2.14	.72	32.5	0	.781	.965			.384
	15	317	.526	.174	9.9	- .675	.000911	.850			.161
	16	286	.534	.141	8.9	- .639	.167	.798			.152
MILLIGAN	23	691	.789	.937	21.6	-1.427	.020	1.065			.238
ET AL [26]	24	380	1.053	.394	11.9	- .890	.00143	.976			.173
	25	380	.789	.163	11.9	- .665	.0142	.860			.173
	38	314	.787	.199	9.8	+ .698	.00259	.924			.160
	41	314	.889	.330	9.8	+ .825	.0117	1.188			.160

TABLE 6

COMPARISON OF "KNOCK DOWN FACTOR" ρ_{eff} (BASED ON EFFECTIVE THICKNESS - [3], [65] and [66]) WITH THAT OBTAINED IN TESTS OF INTEGRALLY STIFFENED SHELLS.

	Shell	R/h	A_1/bh	I_{11}/bh^3	e_1/h	$(R/h)_{eff}$ [3]	ρ_{eff} [3]	ρ_{eff} [3]&[5]	$(R/h)_{eff}$ [65]	ρ_{eff} [65]&[5]	$(R/h)_{eff}$ [66]	ρ_{eff} [66]	ρ obtained in tests
STRINGER STIFFENED	KATZ 5A-1	688	.244	.632	- 3.29	469	.513	.419	292	.505	191	.380	.608
	5A-2	707	.251	.694	- 3.38	478	.510	.416	295	.503	190	.381	.606
	5B-1	685	.349	.904	- 3.29	419	.532	.439	276	.516	173	.401	.562
	5B-2	685	.339	.879	- 3.29	423	.530	.437	277	.516	174	.400	.541
	5C-1	668	.481	1.08	- 3.68	421	.532	.438	251	.534	157	.423	.663
	5C-2	686	.492	1.17	- 3.79	430	.528	.434	253	.532	156	.425	.825
	LENKO	243	.72	.781	0	106	.725	.697	151	.631	210	.361	.965
	CARD 1	338	1.03	9.79	- 5.83	140	.692	.646	92.3	.721	39.5	.889	1.639
	2	345	1.06	10.5	+ 5.96	142	.690	.642	93.1	.719	39.3	.892	1.165
	3	347	1.07	10.8	- 6.0	142	.690	.642	93.2	.719	39.2	.894	1.600
	4	341	1.05	10.1	+ 5.89	141	.691	.644	92.6	.720	39.3	.892	1.339
	MILLIGAN 15	317	.174	.0009	- .675	158	.676	.623	272	.518	290	.303	.850
	16	286	.141	.167	- .639	91.2	.743	.723	207	.571	268	.317	.798
	23	691	.937	.020	- 1.427	299	.585	.501	416	.440	343	.277	1.065
	24	380	.394	.0014	- .890	179	.661	.599	287	.508	293	.302	.976
	38	314	.199	.0026	+ .698	154	.680	.627	265	.524	283	.308	.924
	41	314	.330	.0117	+ .826	147	.686	.636	243	.540	255	.325	1.188
			A_2/ah	I_{22}/ah^3	e_2/h								
RING STIFFENED	MZ 5	641	.373	.148	- 1.56	362	.555	.465	374	.459	336	.281	.877
	9	625	.271	.057	- 1.29	377	.549	.458	413	.442	393	.258	.854
	13	714	.435	.235	- 1.77	386	.545	.454	386	.454	314	.291	.952
	15	705	.165	.014	- 1.01	478	.510	.416	544	.394	557	.214	.757
	16	595	.123	.006	- .88	418	.532	.439	492	.411	516	.222	.848
	17	689	.264	.068	- 1.38	424	.529	.437	444	.429	412	.251	.837

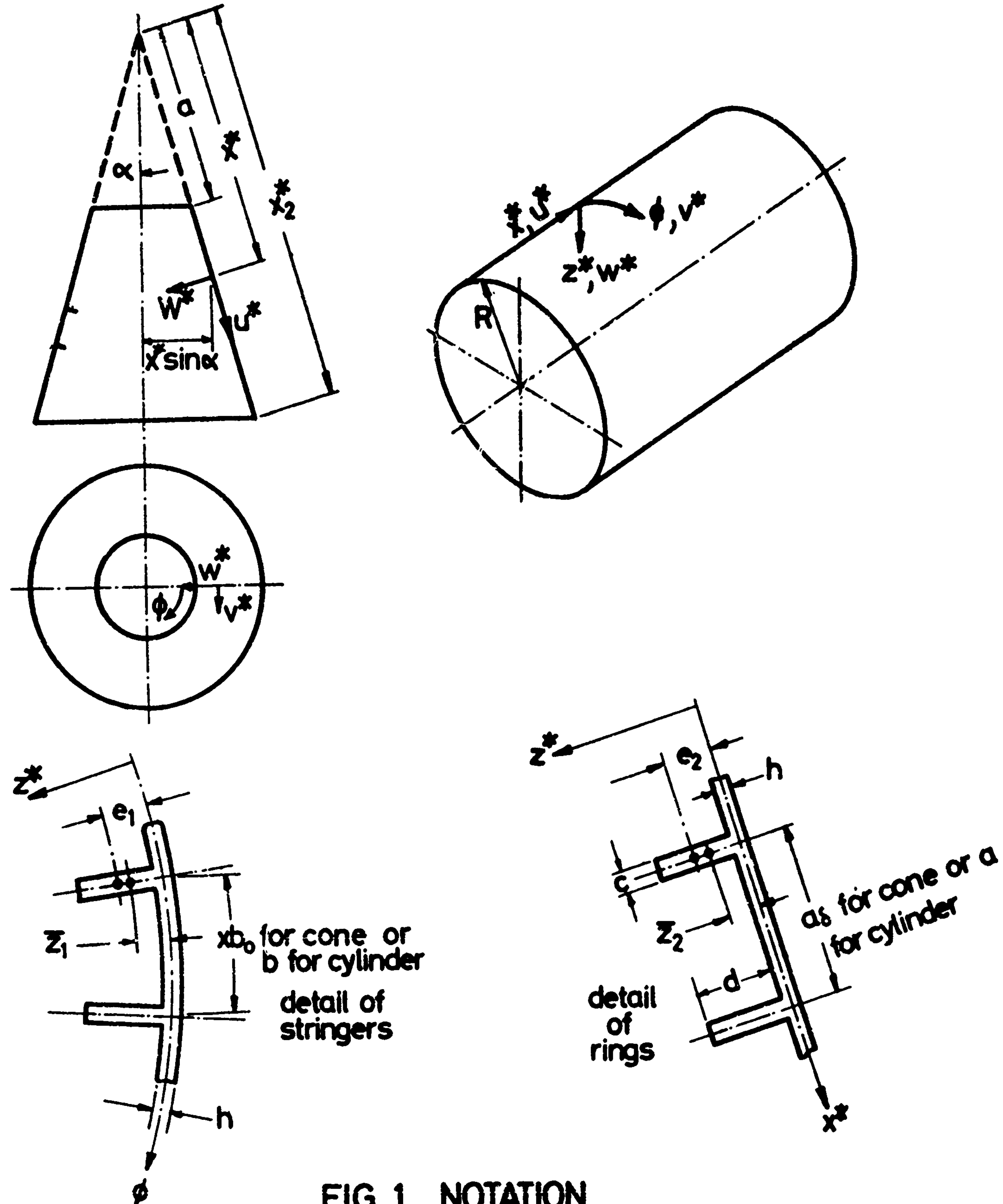
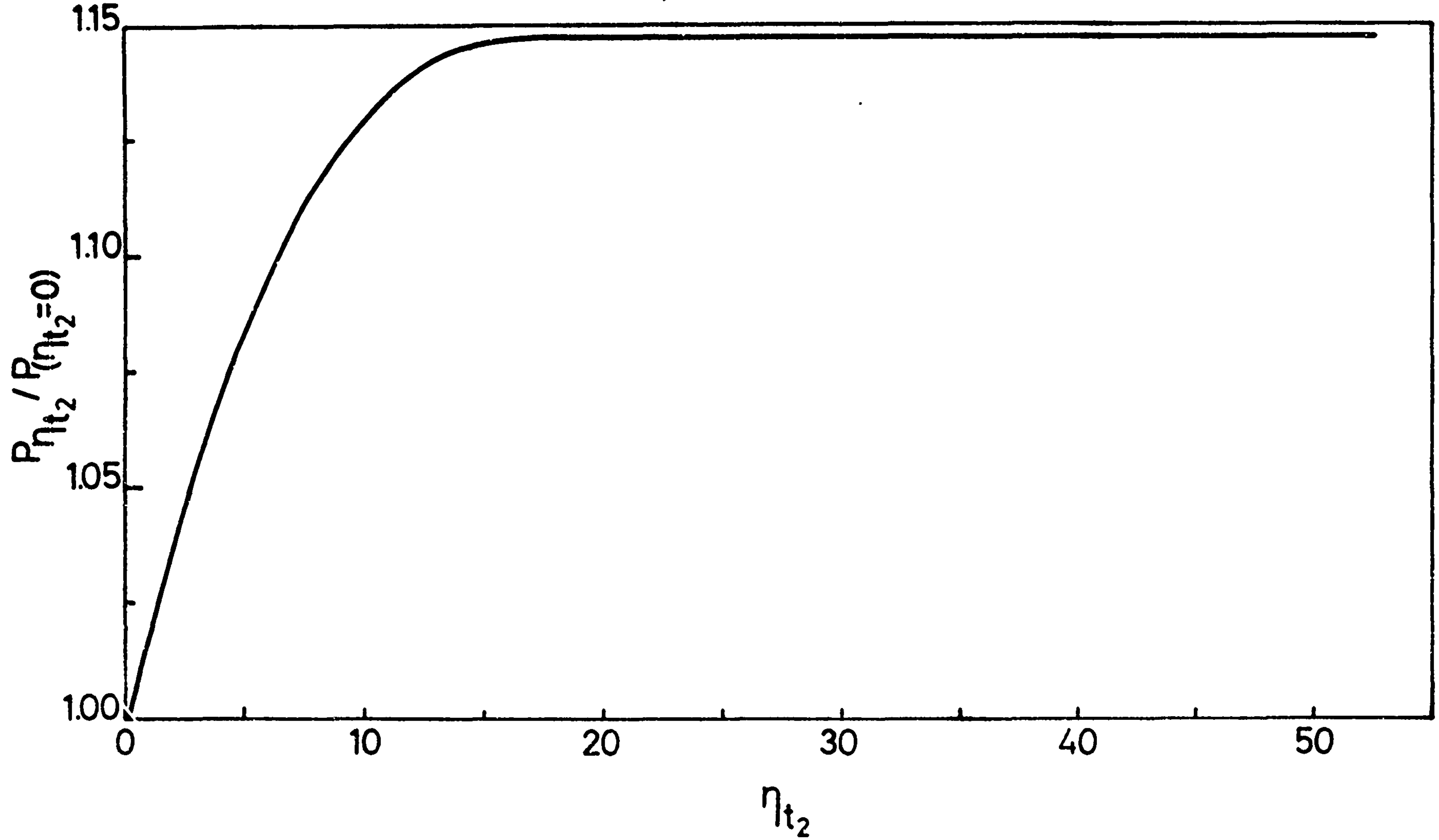


FIG. 1 NOTATION



THE INFLUENCE OF TORSIONAL RIGIDITY ON CRITICAL AXIAL LOAD OF INTERNAL-RING-STIFFENED CYLINDERS

FIG. 2

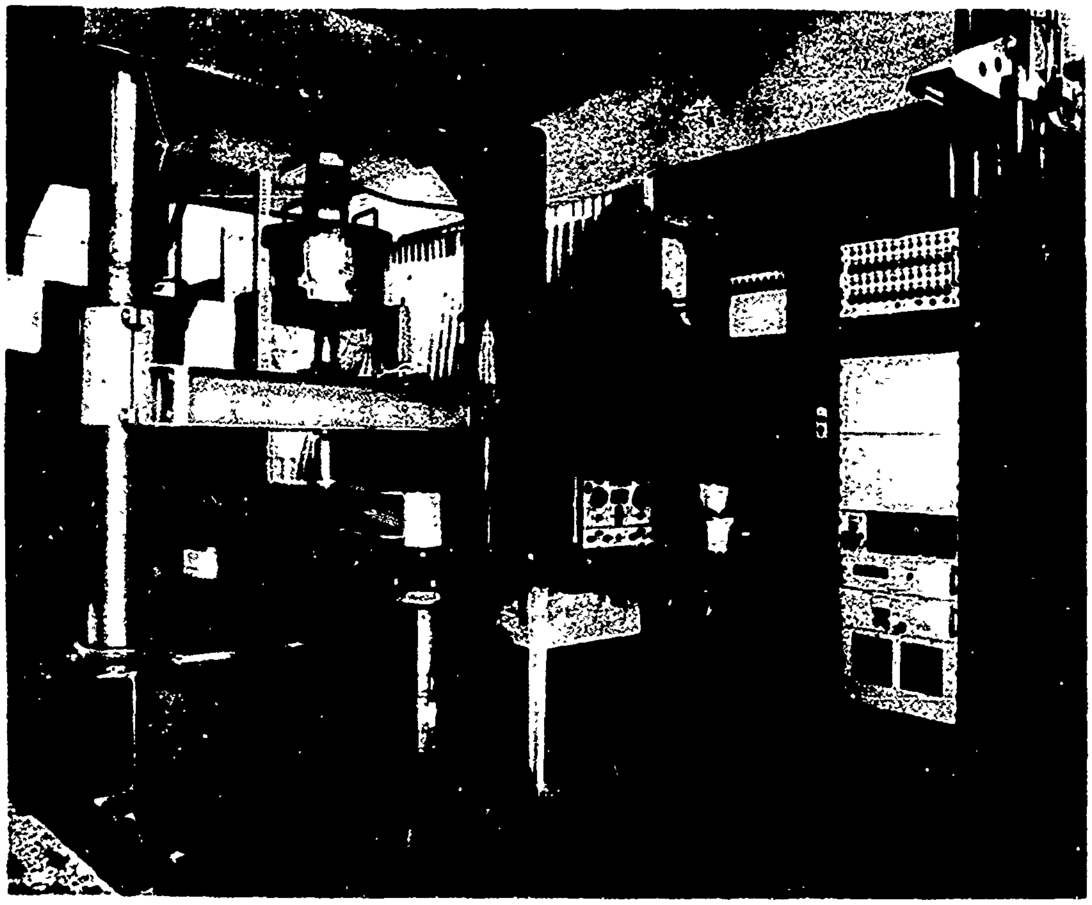


FIG. 3. TEST SET UP FOR STIFFENED CYLINDRICAL SHELLS UNDER AXIAL COMPRESSION.

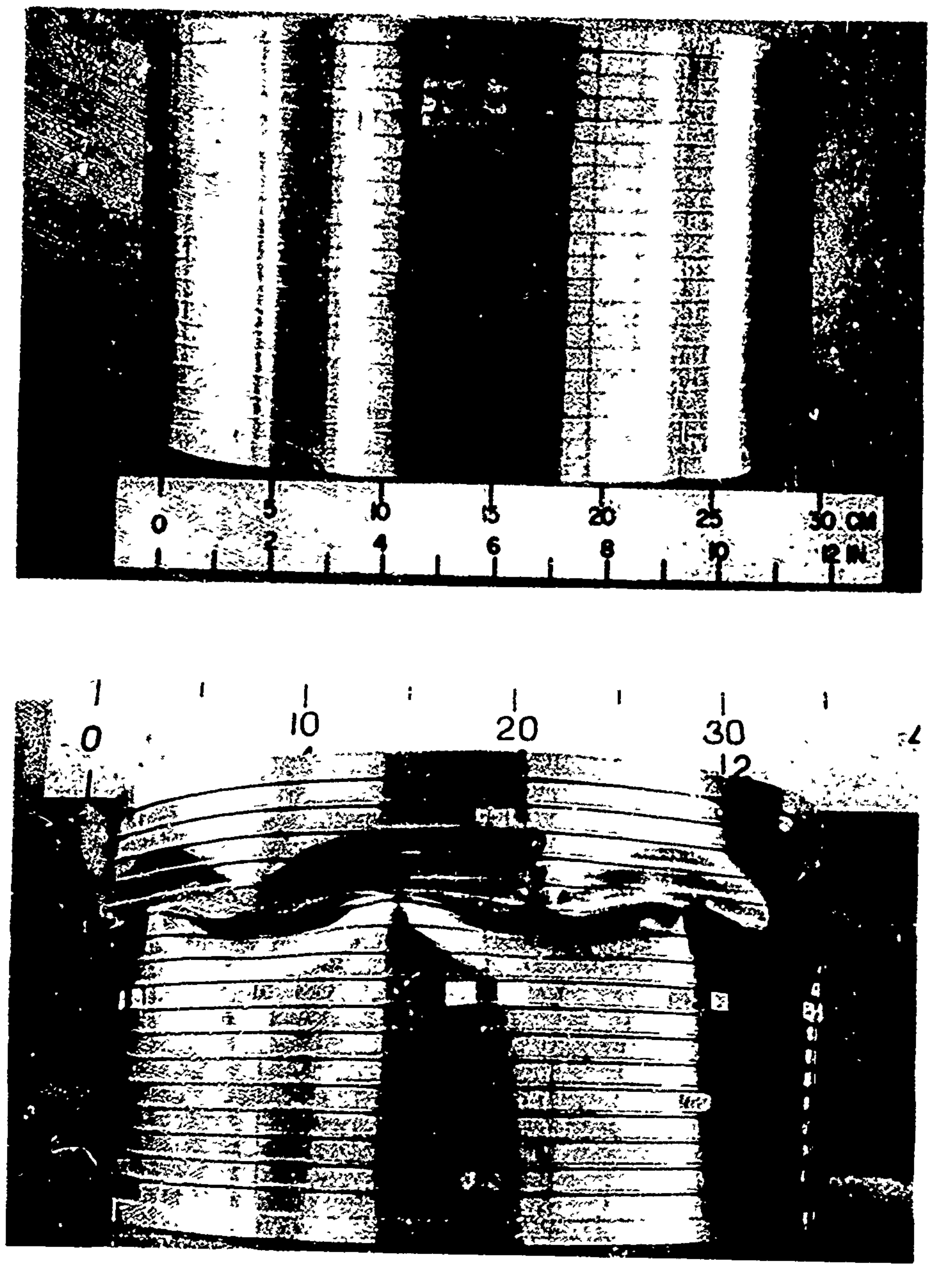
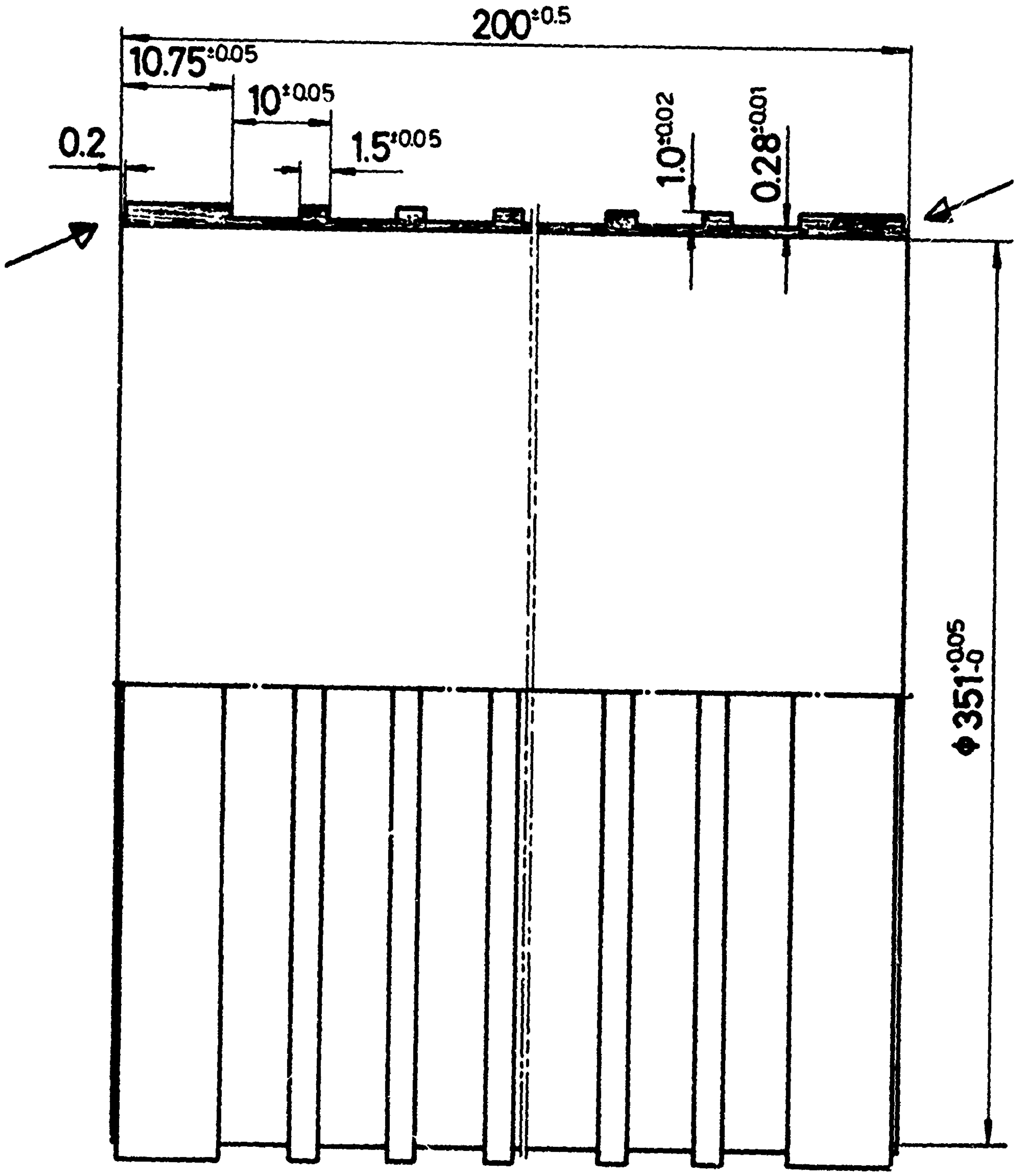


FIG. 4. SPECIMEN MZ 2



TYPICAL SPECIMEN (MZ-2)

FIG. 5a

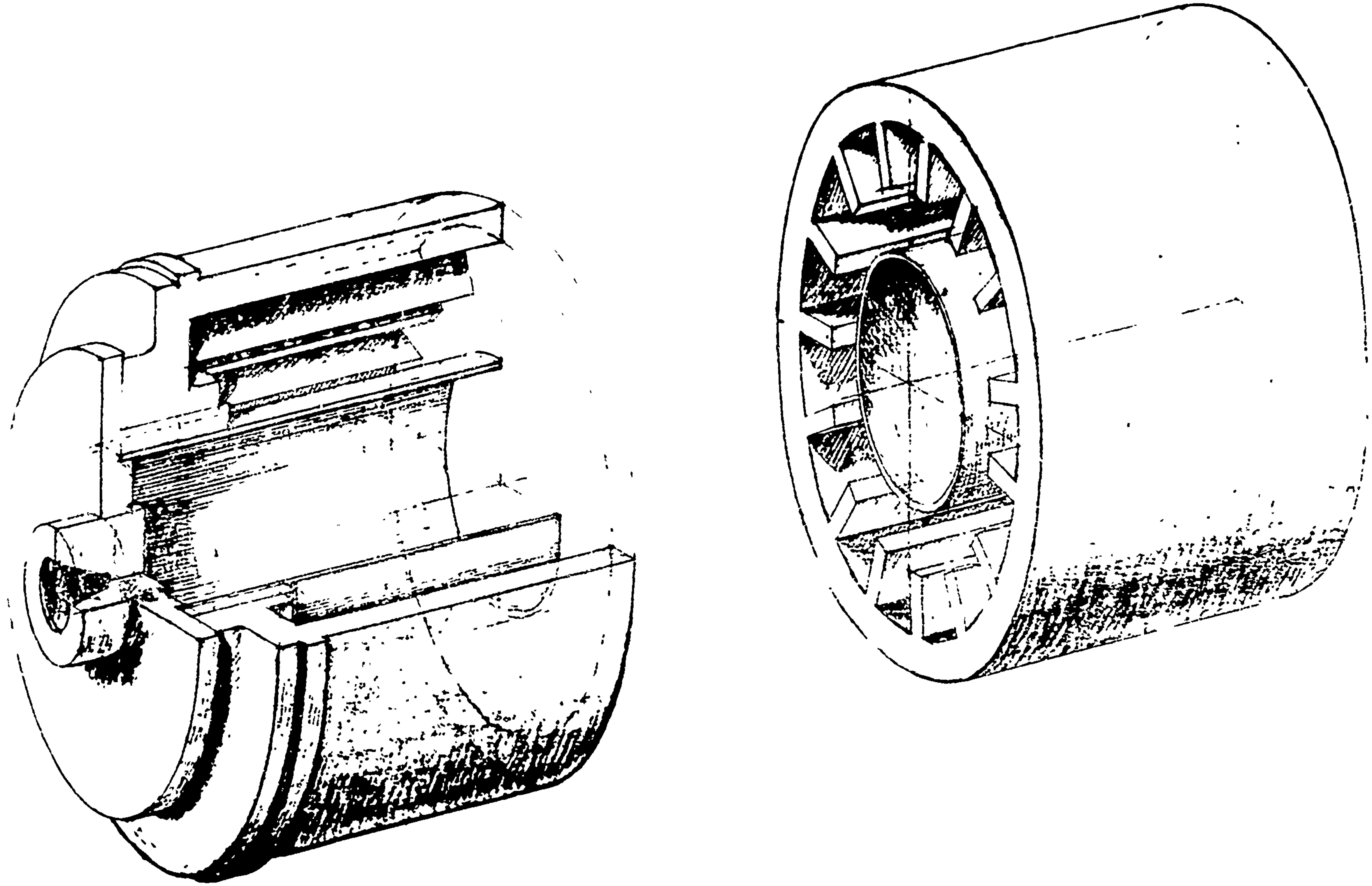


FIG. 5b THE 'COOLED' MANDREL

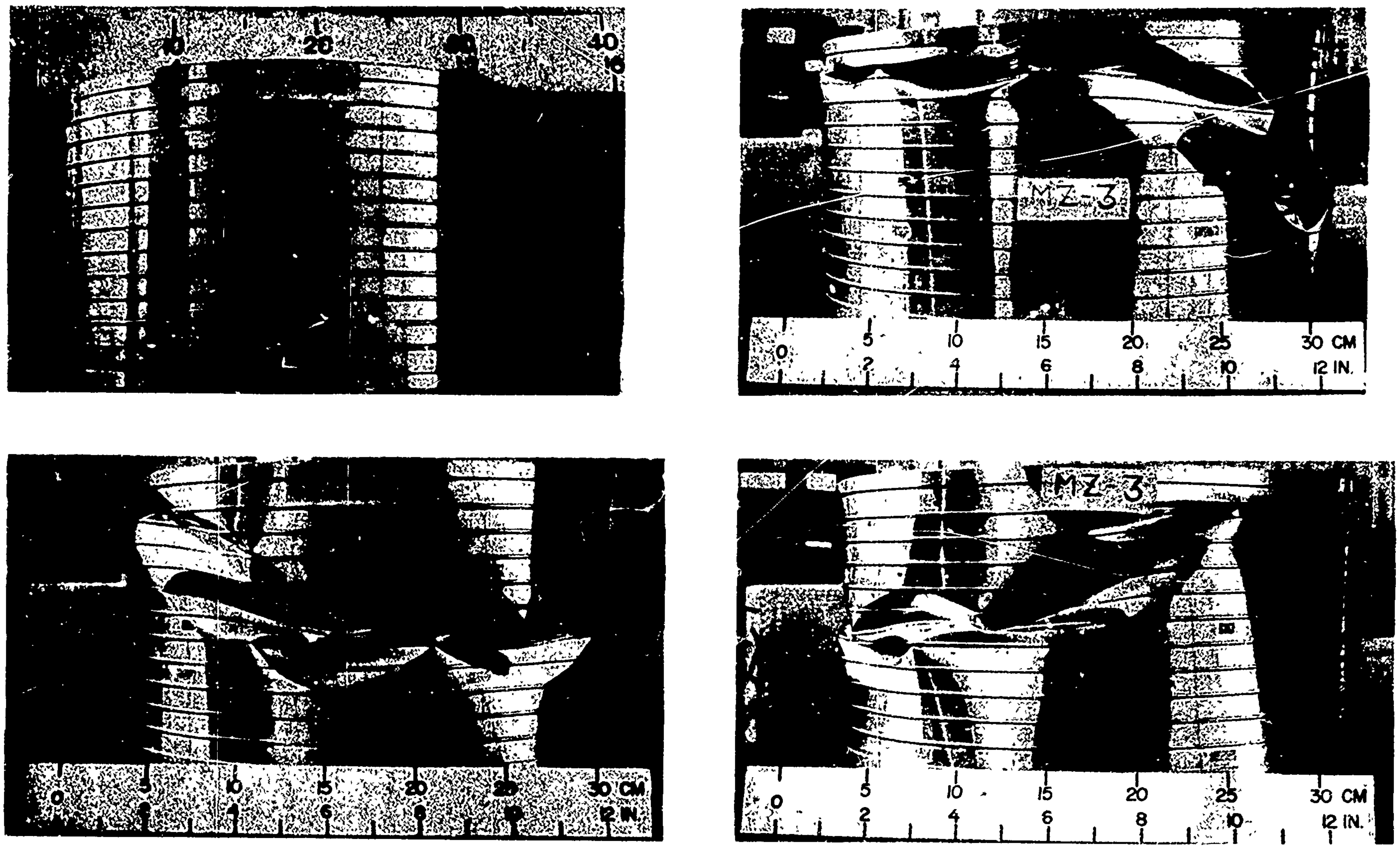


FIG. 6a. SPECIMEN MZ 3

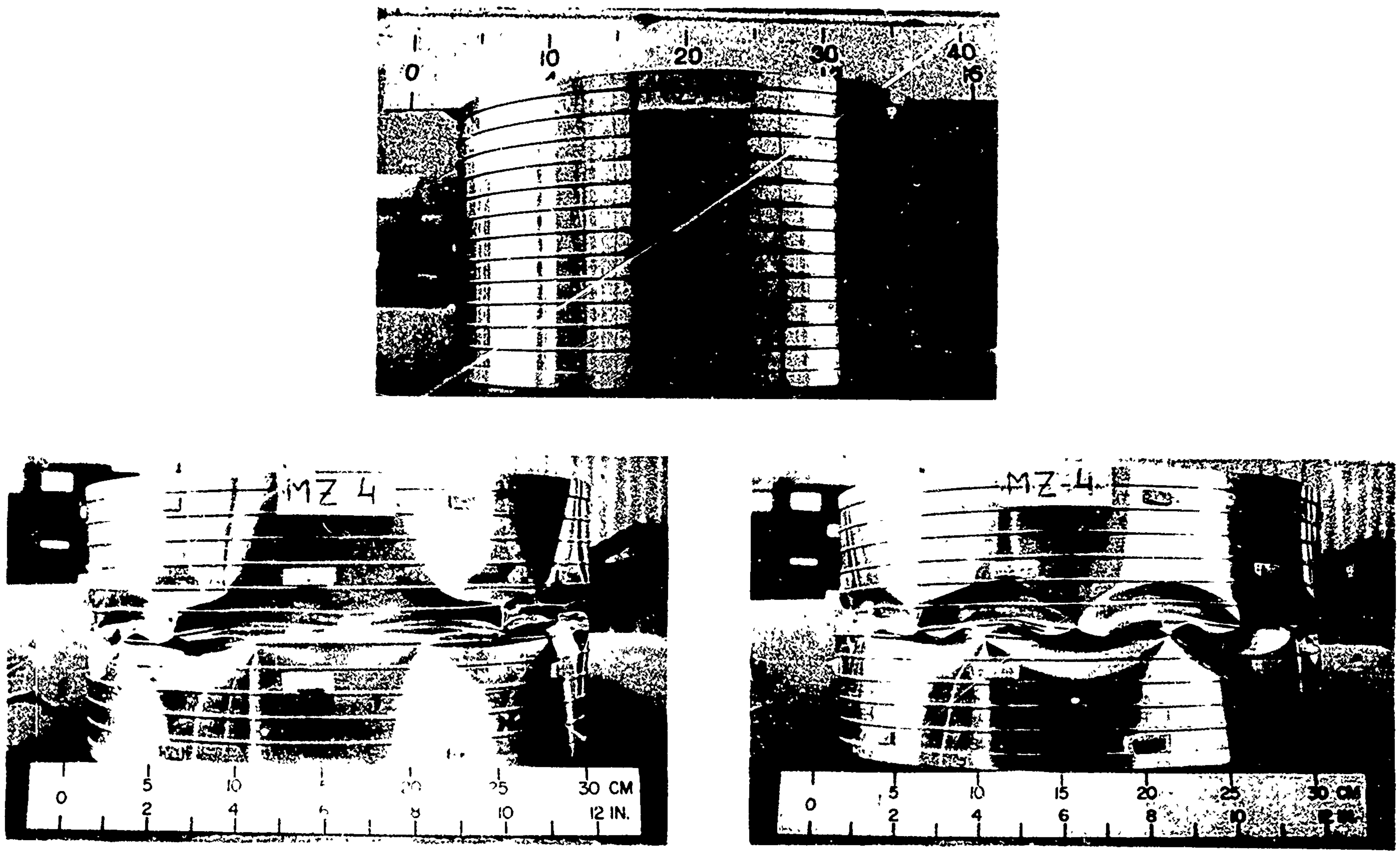


FIG. 6b. SPECIMEN MZ 4.

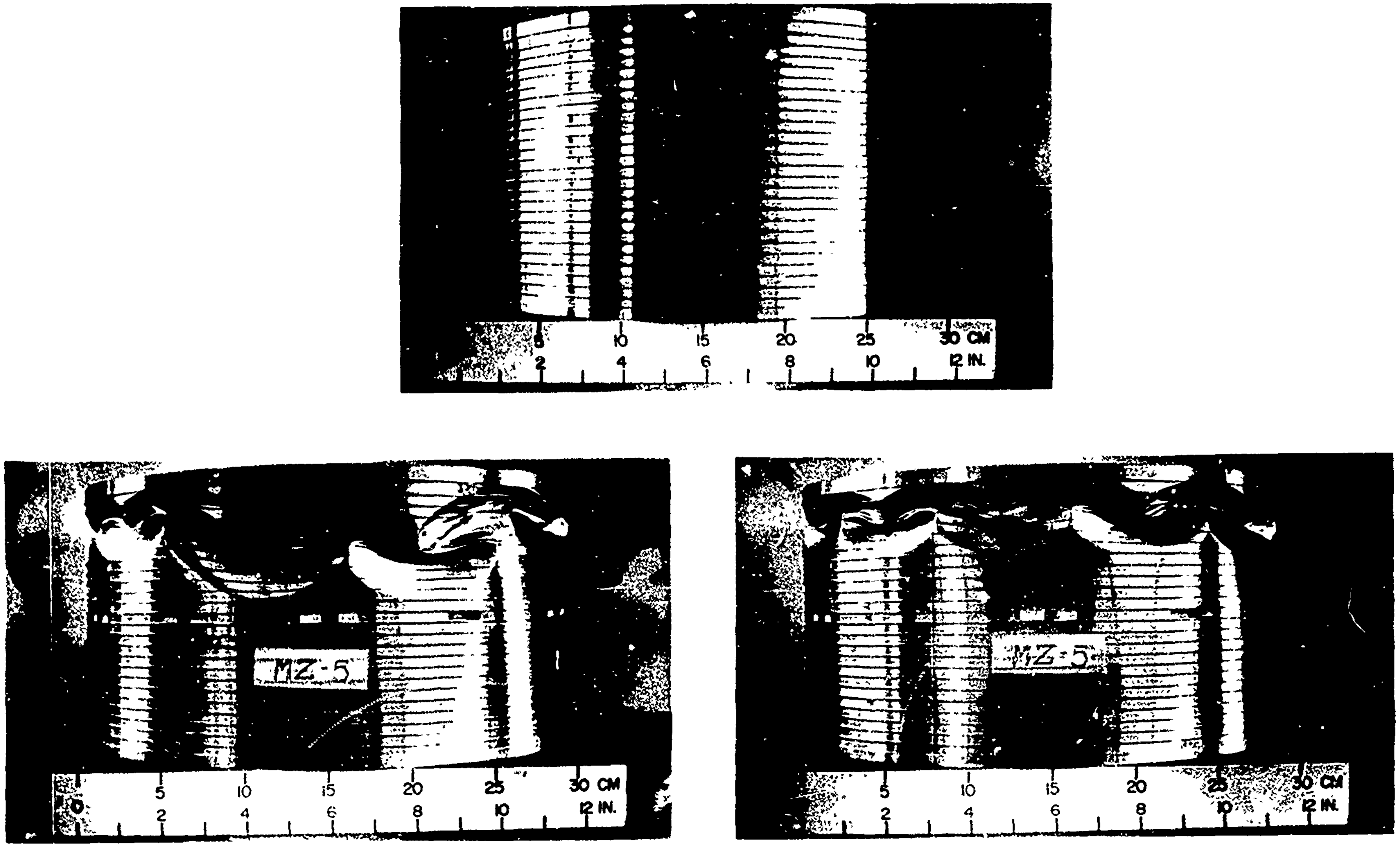


FIG. 7a. SPECIMEN MZ 5

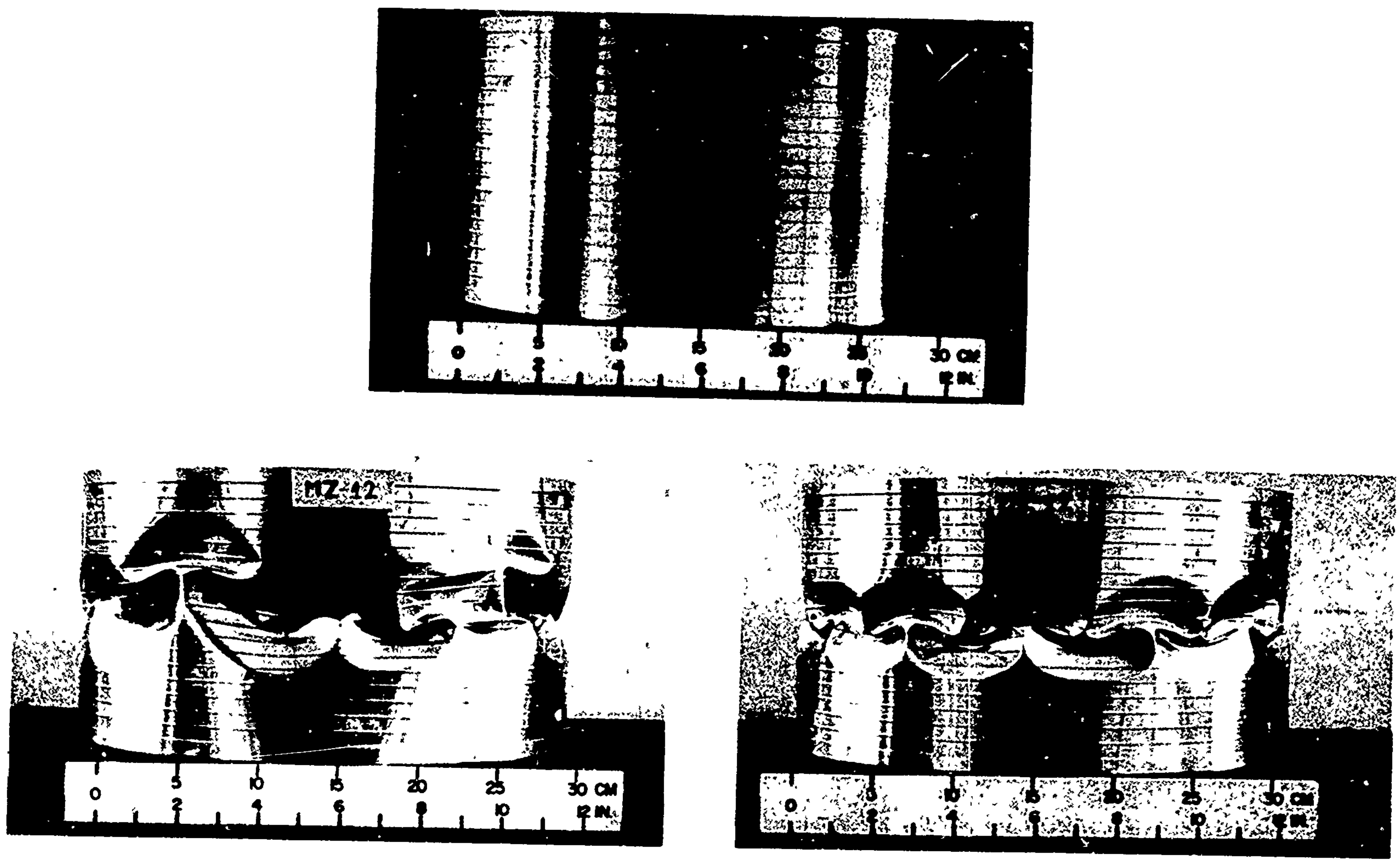


FIG. 7b. SPECIMEN MZ 12

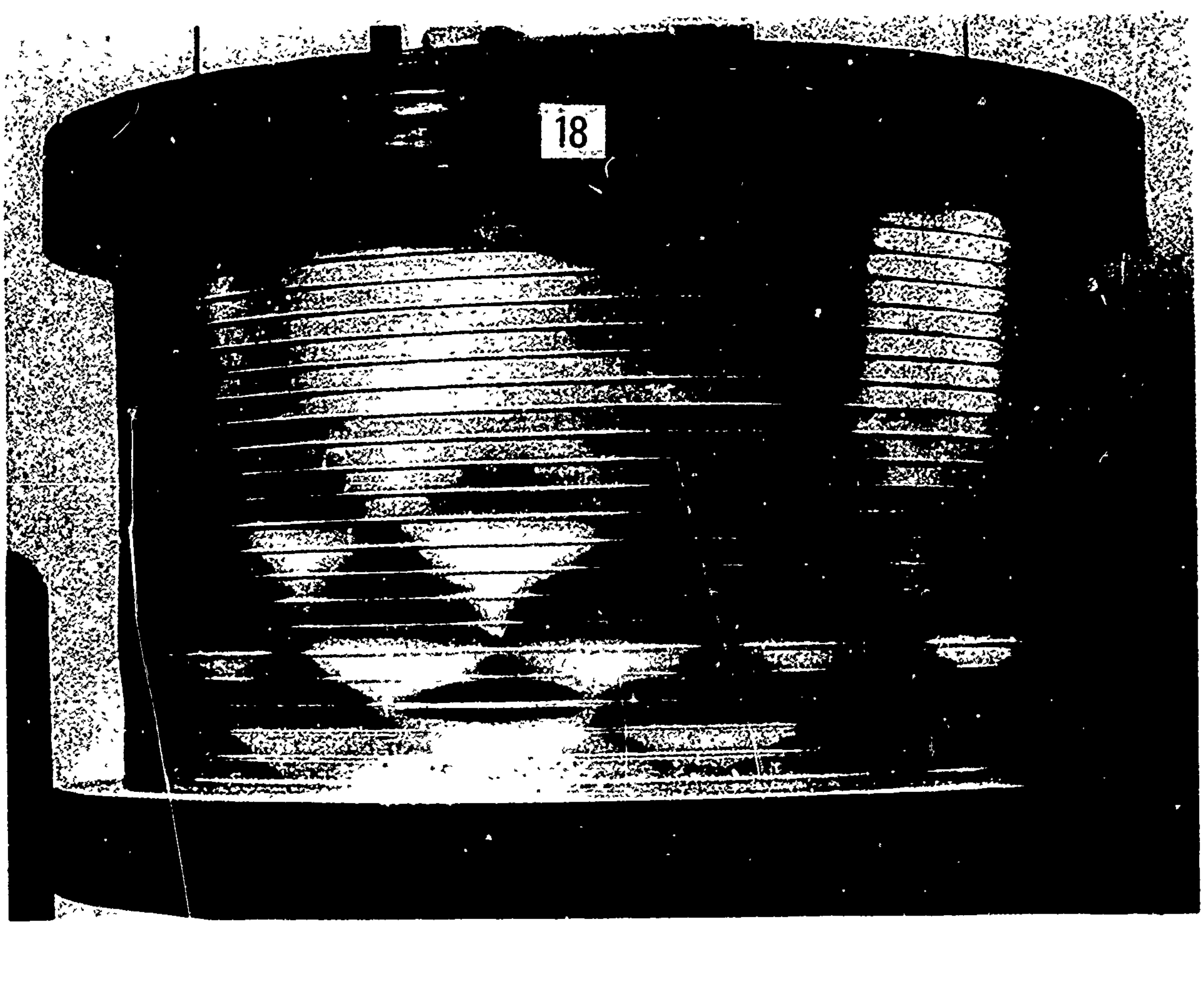


FIG. 8. SPECIMEN BUCKLED WITH AXIAL DISPLACEMENT ARRESTING TUBE IN POSITION MZ - 18.

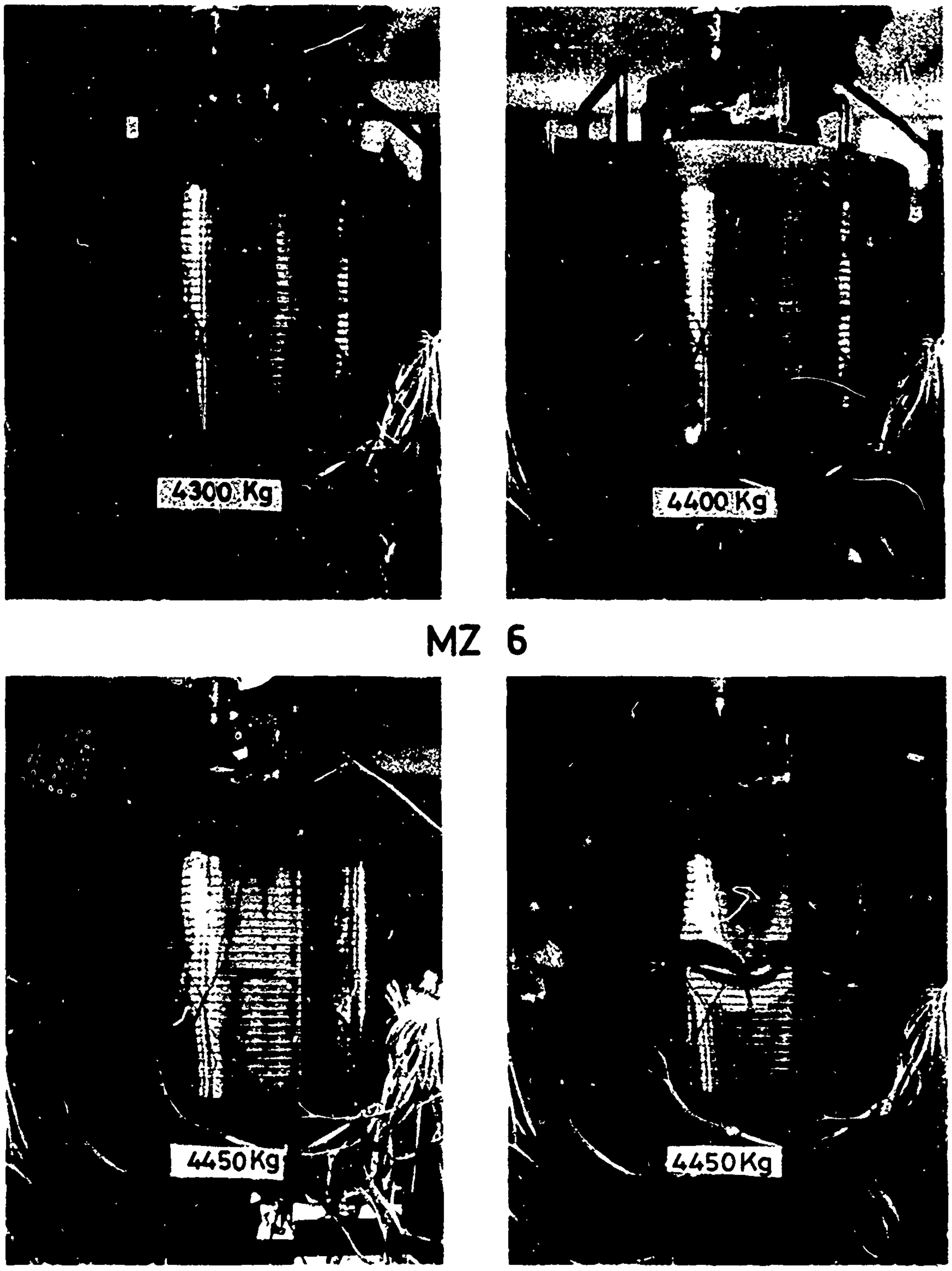


FIG. 9a ONSET OF BUCKLING - SHELL MZ 6.

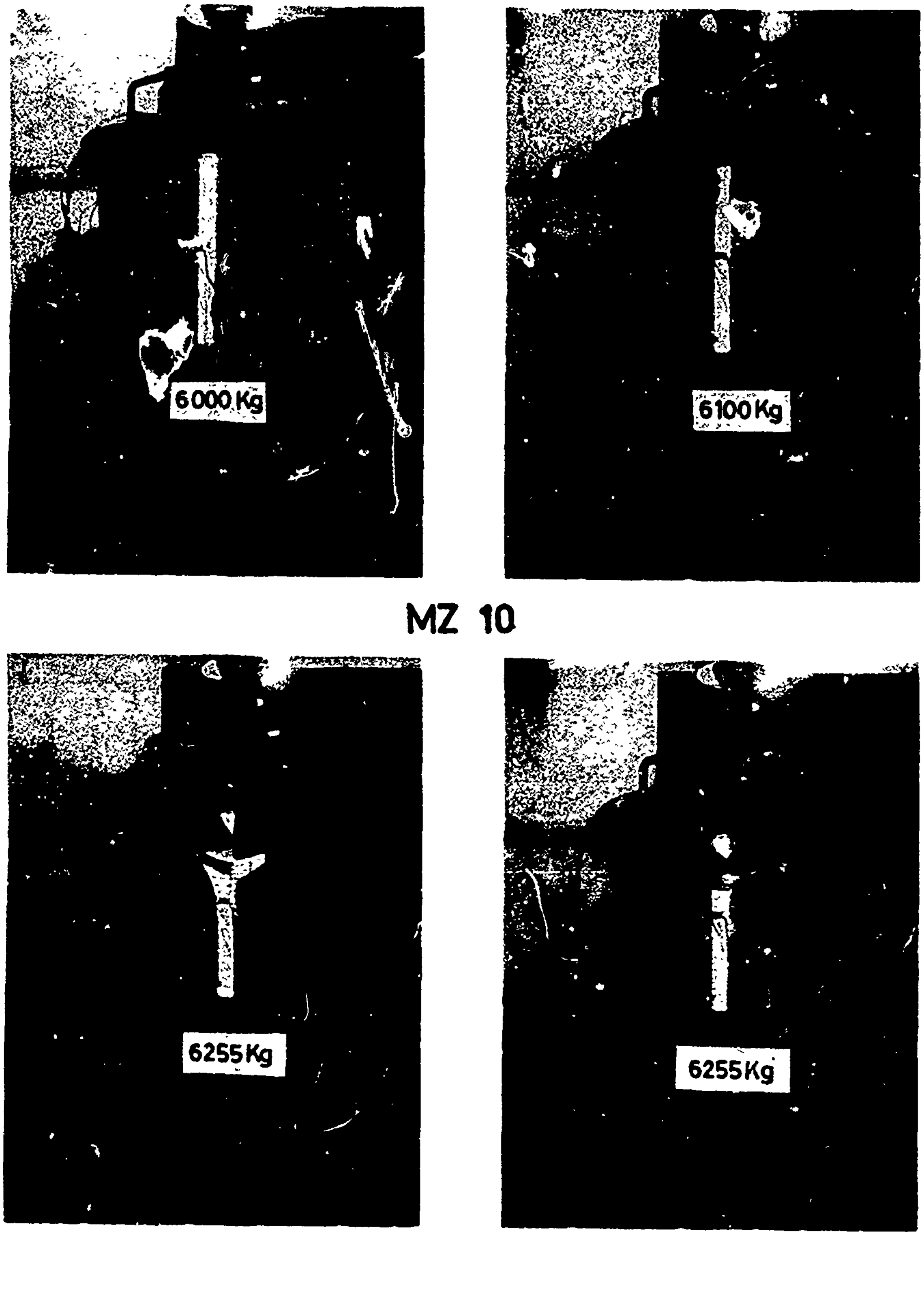


FIG 9b. ONSET OF BUCKLING - SHELL MZ 10.

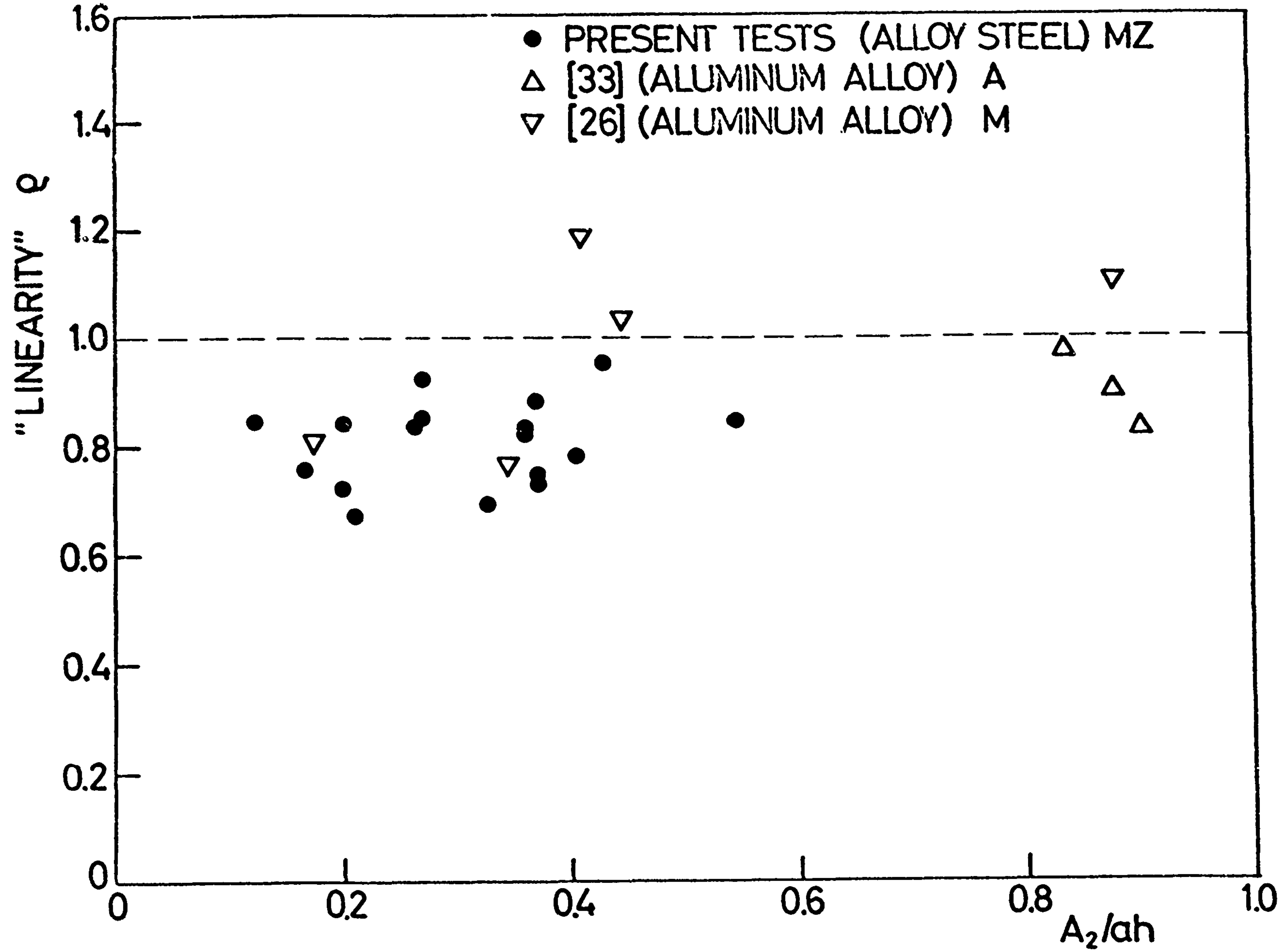


FIG. 10. "LINEARITY" OF RING-STIFFENED CYLINDRICAL SHELLS AS A FUNCTION OF RING AREA.

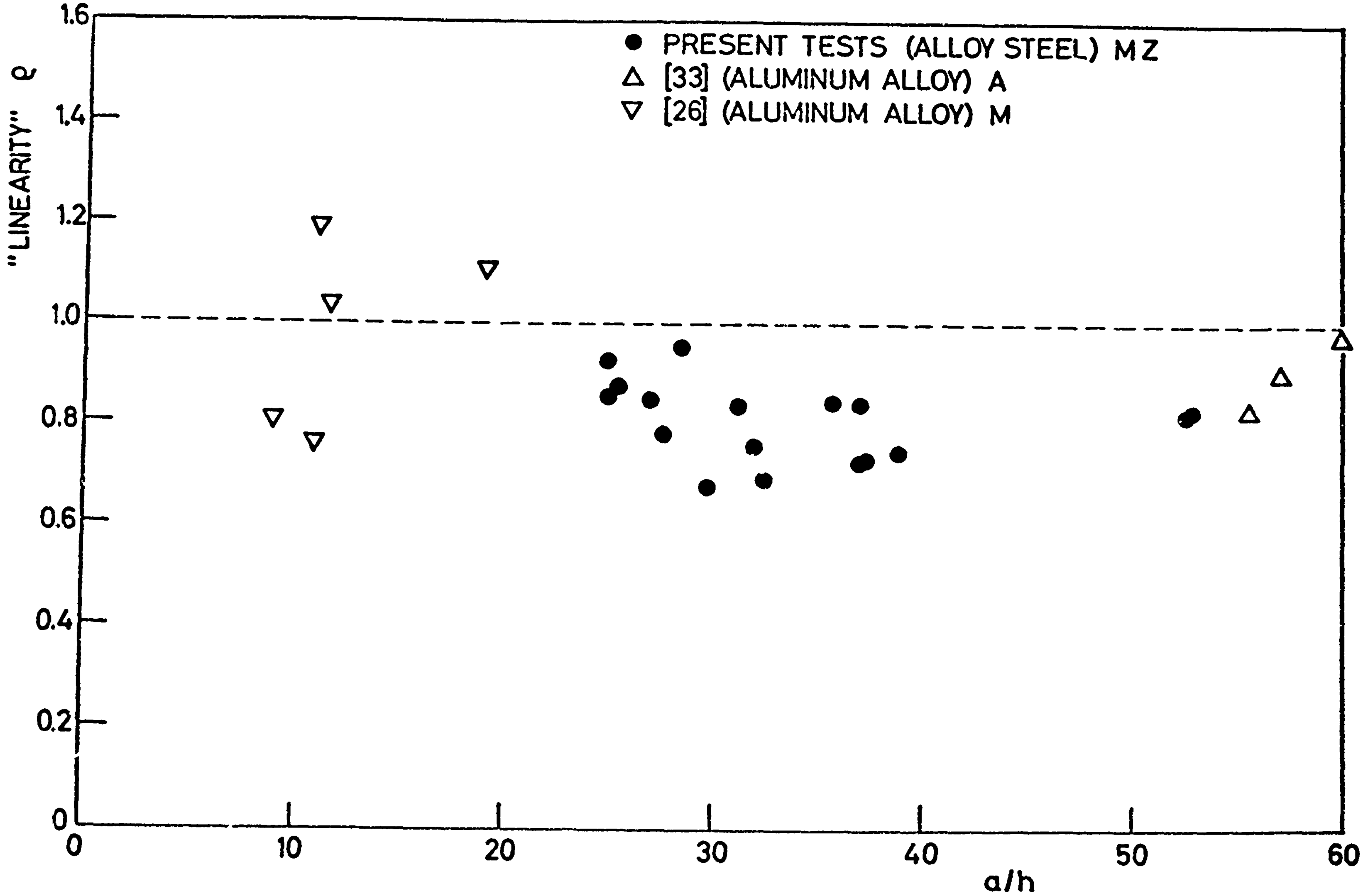


FIG. 11. "LINEARITY" OF RING STIFFENED CYLINDRICAL SHELLS AS FUNCTION OF RING SPACING

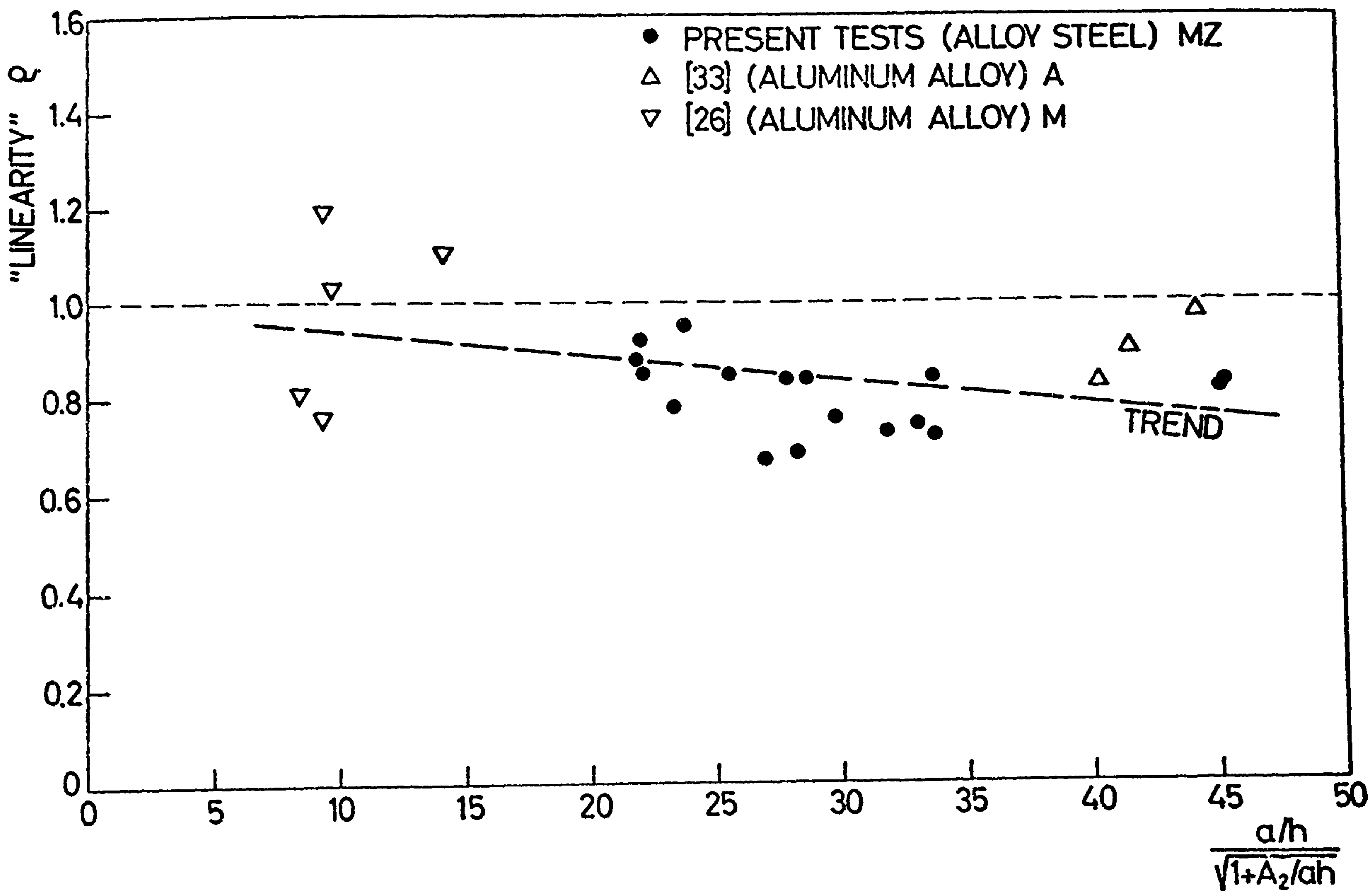
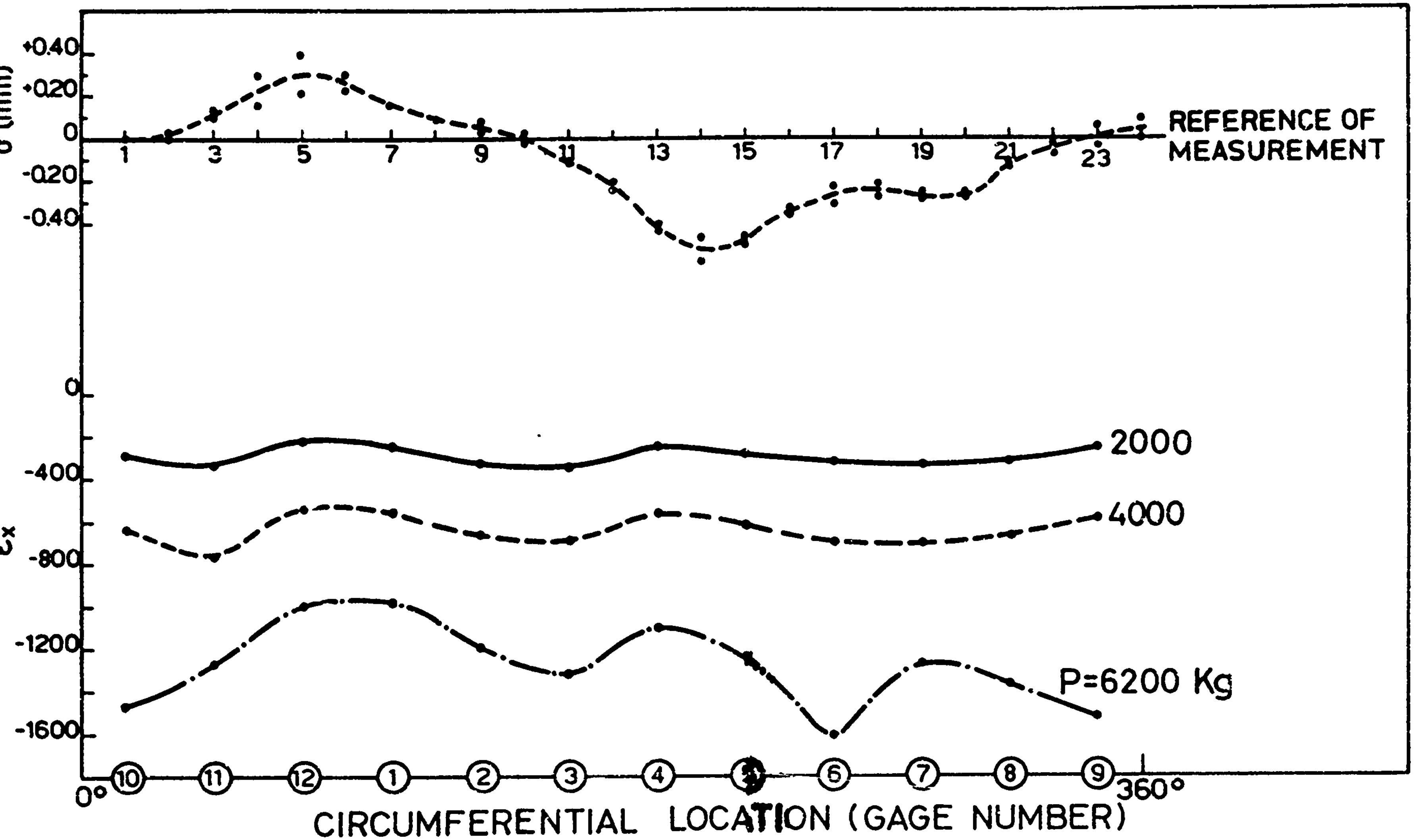
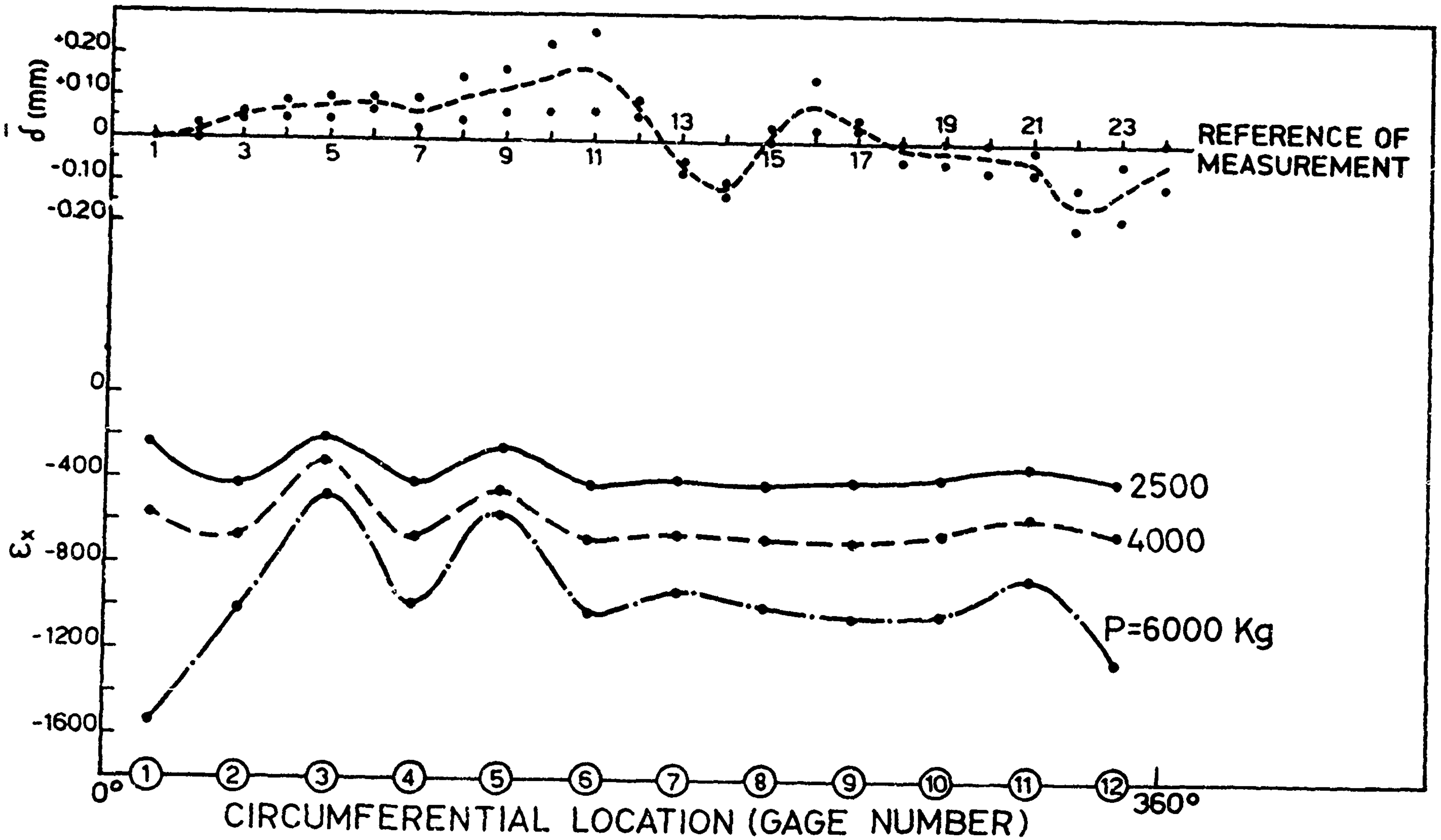


FIG. 12. "LINEARITY" OF RING-STIFFENED SHELLS



CORRELATION OF OUT-OF-ROUNDNESS AND STRAIN READINGS SHELL MZ-2

FIG. 13



CORRELATION OF OUT-OF-ROUNDNESS
AND STRAIN READINGS SHELL MZ-5

FIG. 14

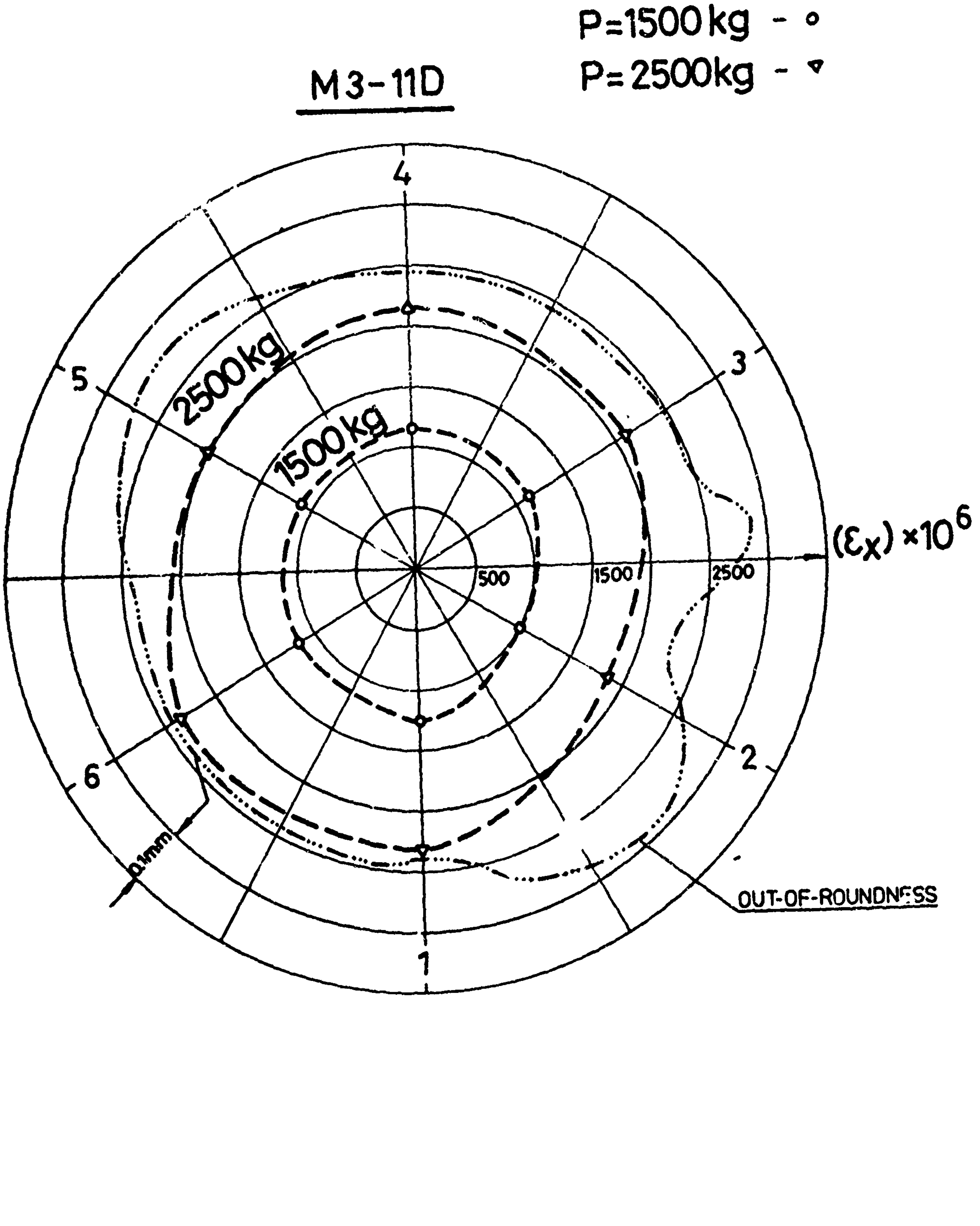


FIG. 15 - CORRELATION OF OUT-OF-ROUNDNESS AND STRAIN READINGS FOR TYPICAL CONICAL SHELL.

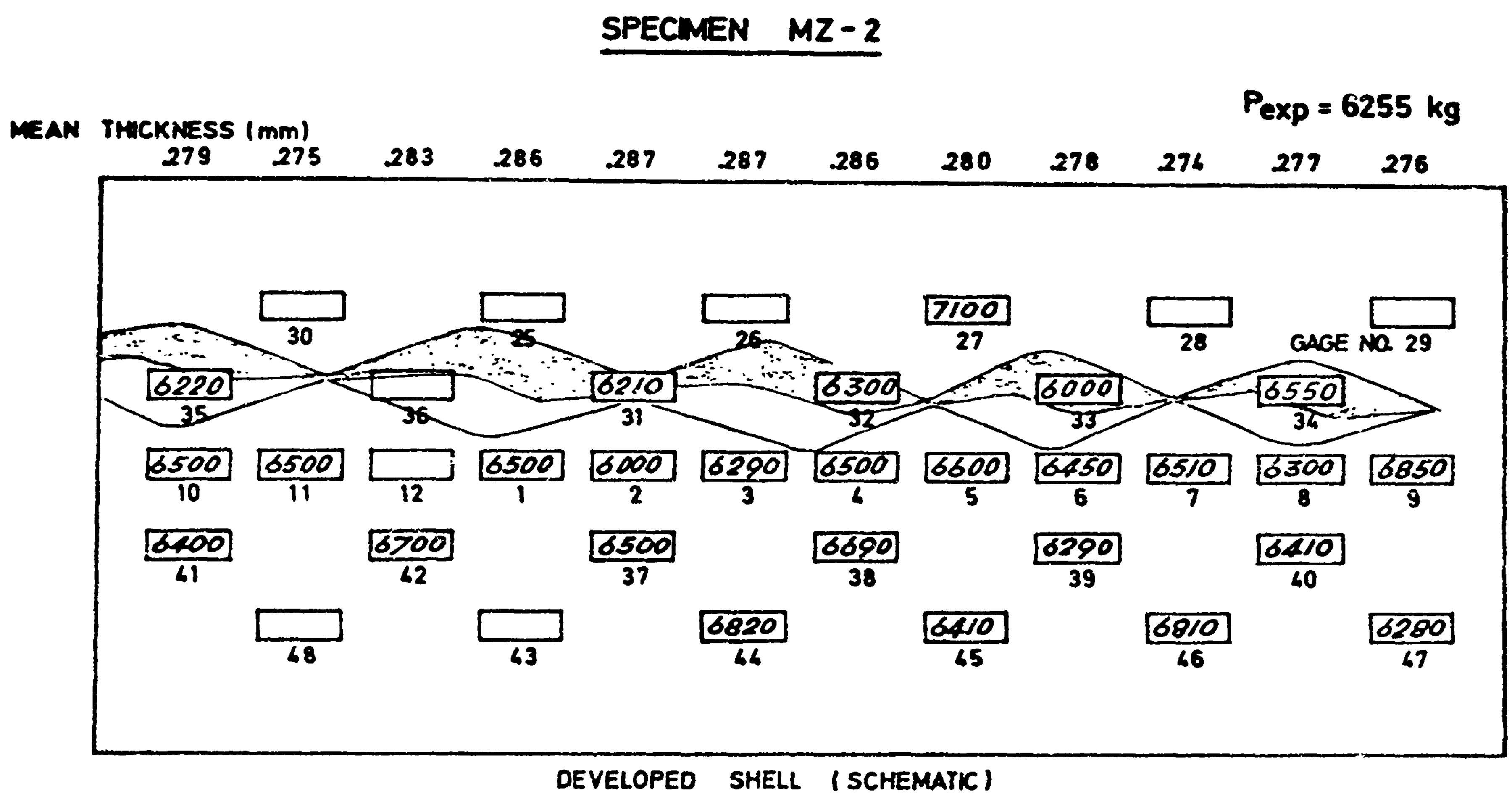
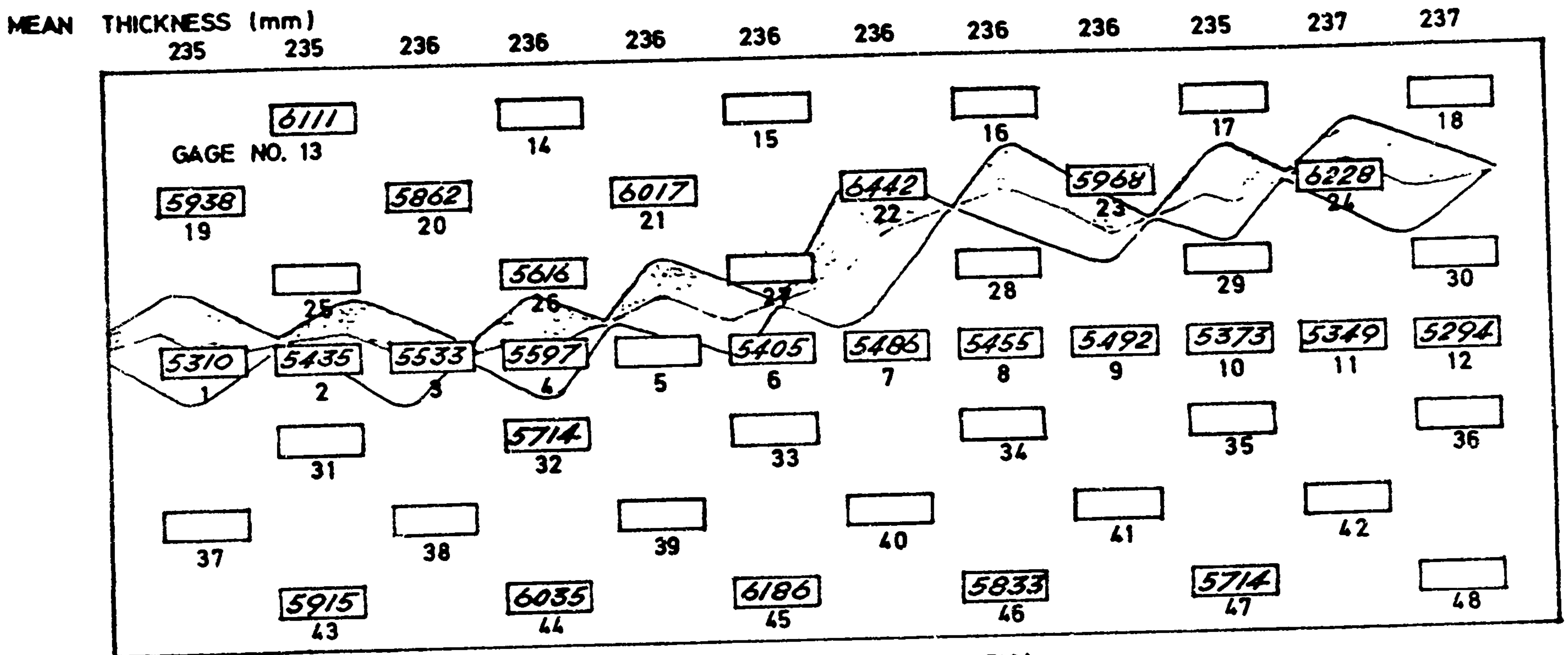


FIG. 16 P_{cr} FROM SOUTHWELL PLOT AT DIFFERENT GAGE LOCATIONS

SPECIMEN MZ-13

$P_{exp} = 5365$ kg



DEVELOPED SHELL (SCHEMATIC)

FIG. 17 P_{cr} FROM SOUTHWELL PLOT AT DIFFERENT GAGE LOCATIONS

SPECIMEN MZ-8

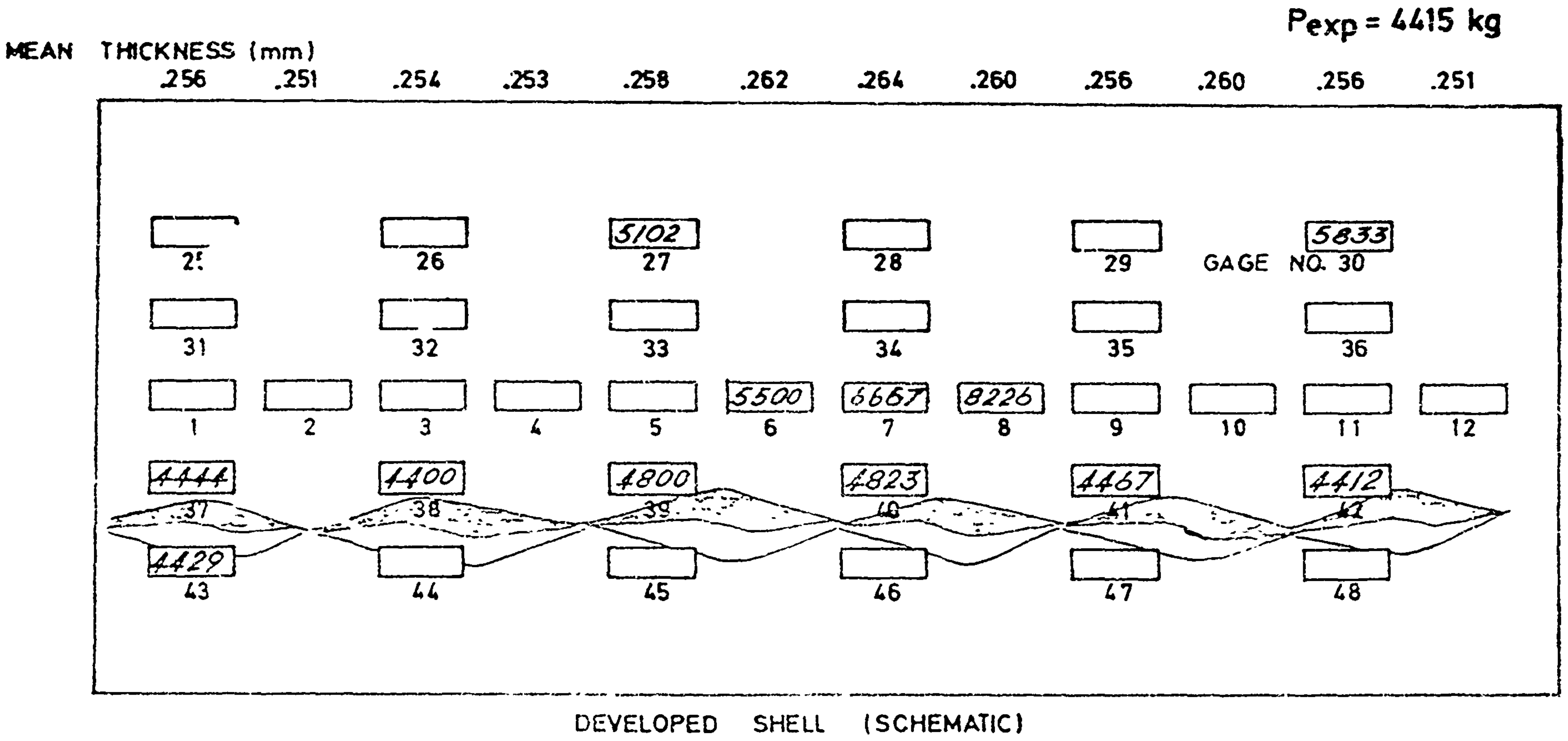


FIG. 18 P_{cr} FROM SOUTHWELL PLOT AT DIFFERENT GAGE LOCATIONS

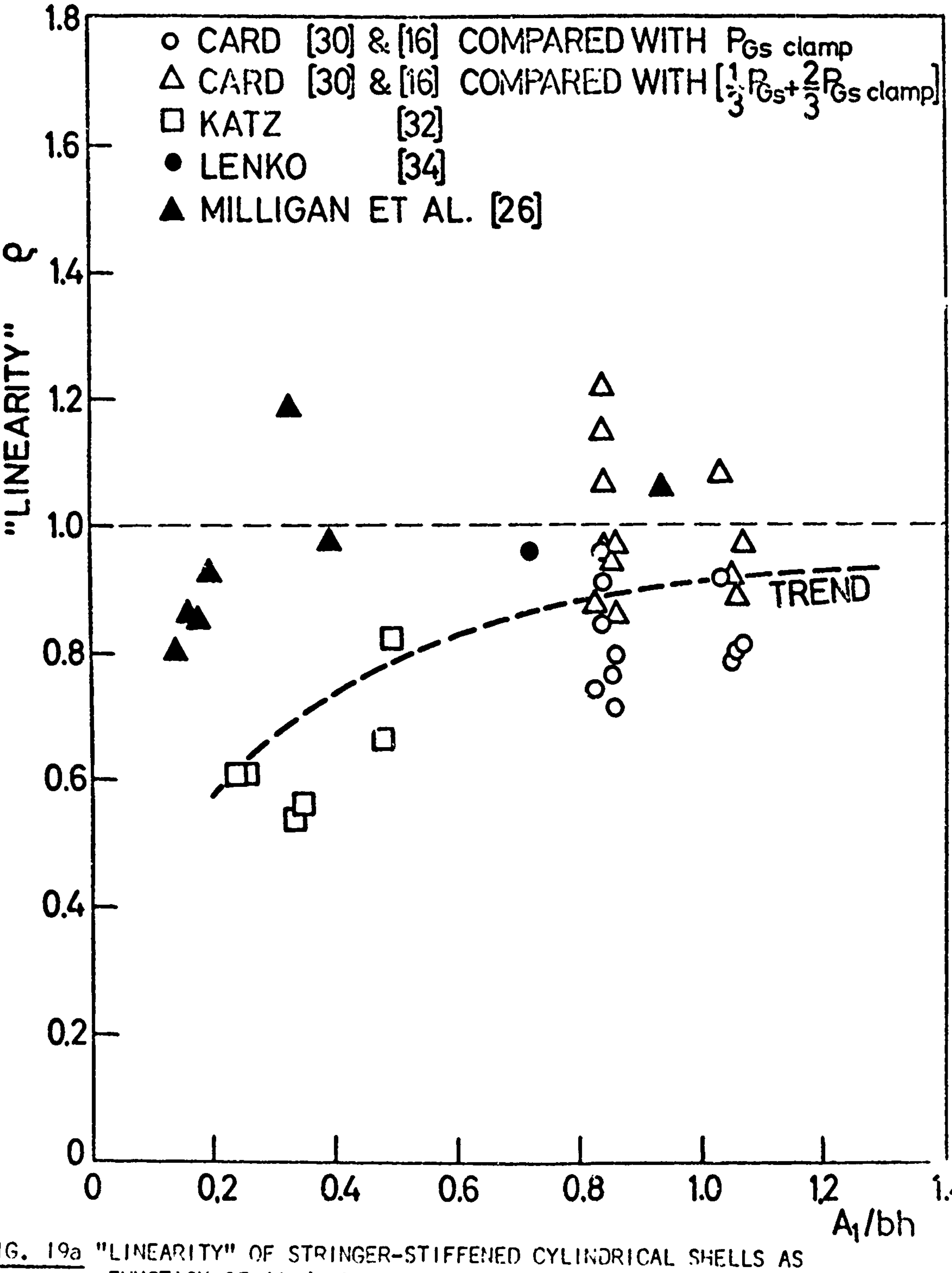


FIG. 19a "LINEARITY" OF STRINGER-STIFFENED CYLINDRICAL SHELLS AS FUNCTION OF (A_1/bh).

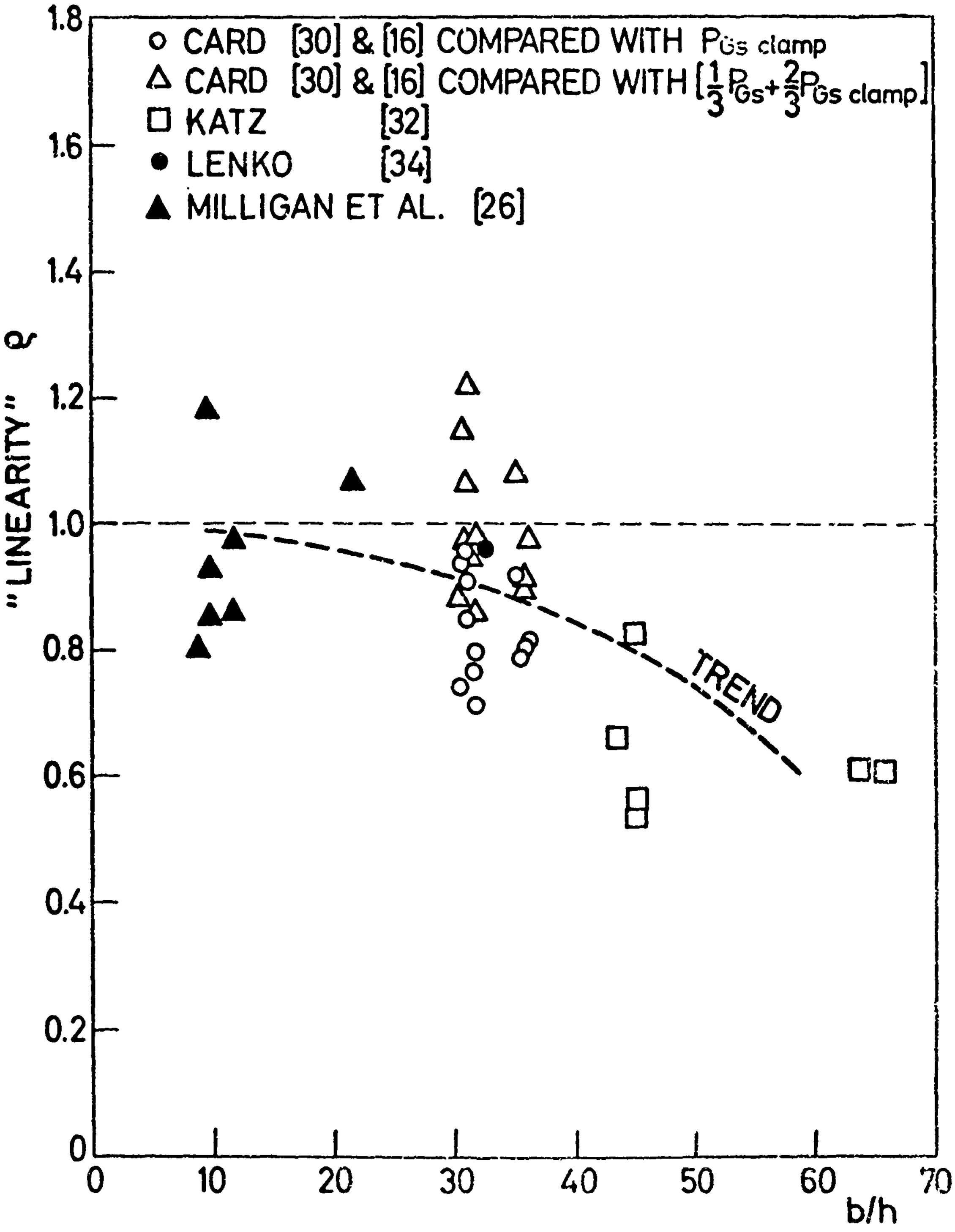


FIG. 19b "LINEARITY" OF STRINGER-STIFFENED CYLINDRICAL SHELLS AS FUNCTION OF (b/h)

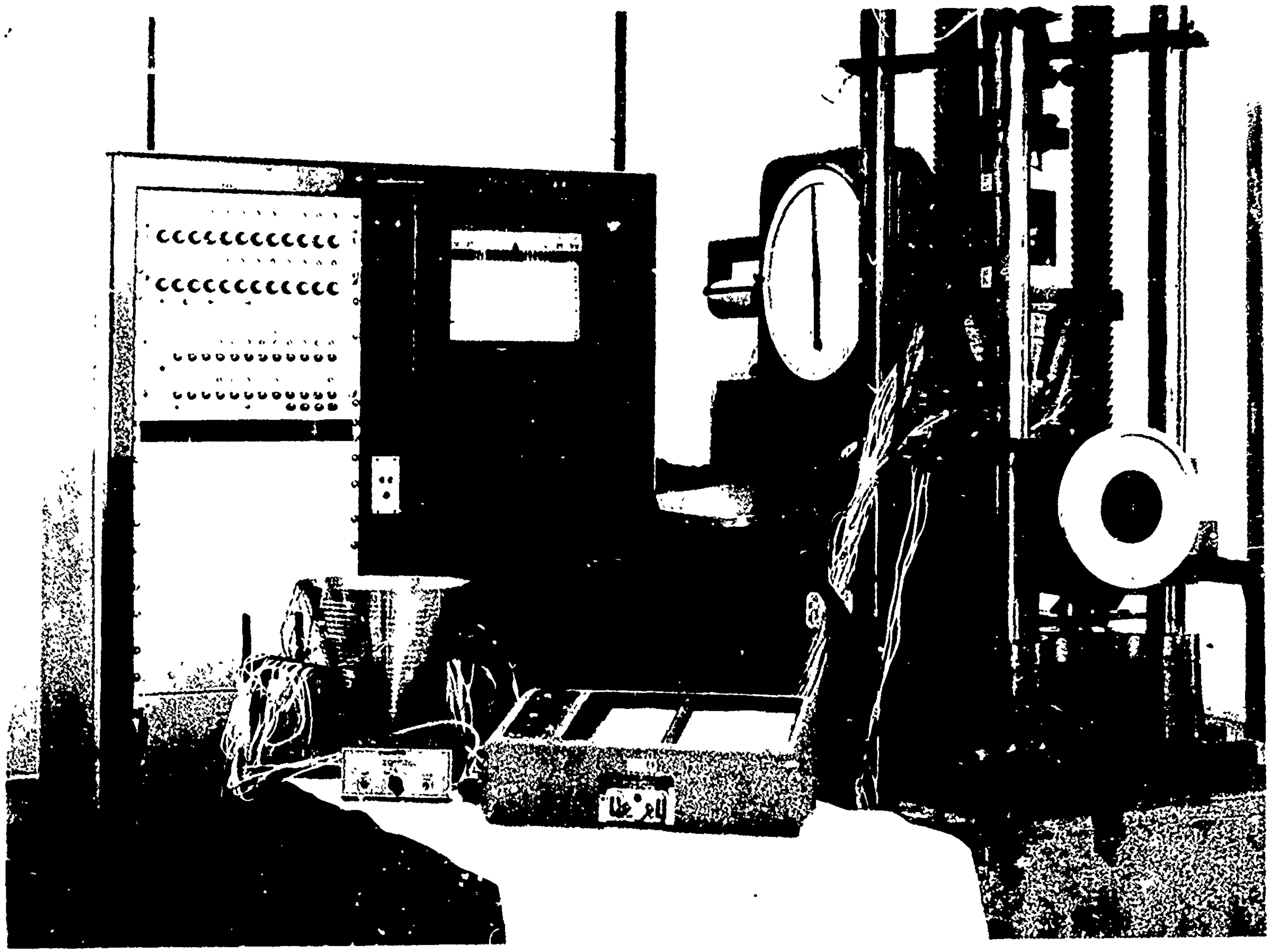


FIG. 20. TEST SET UP FOR CONICAL SHELLS UNDER AXIAL COMPRESSION

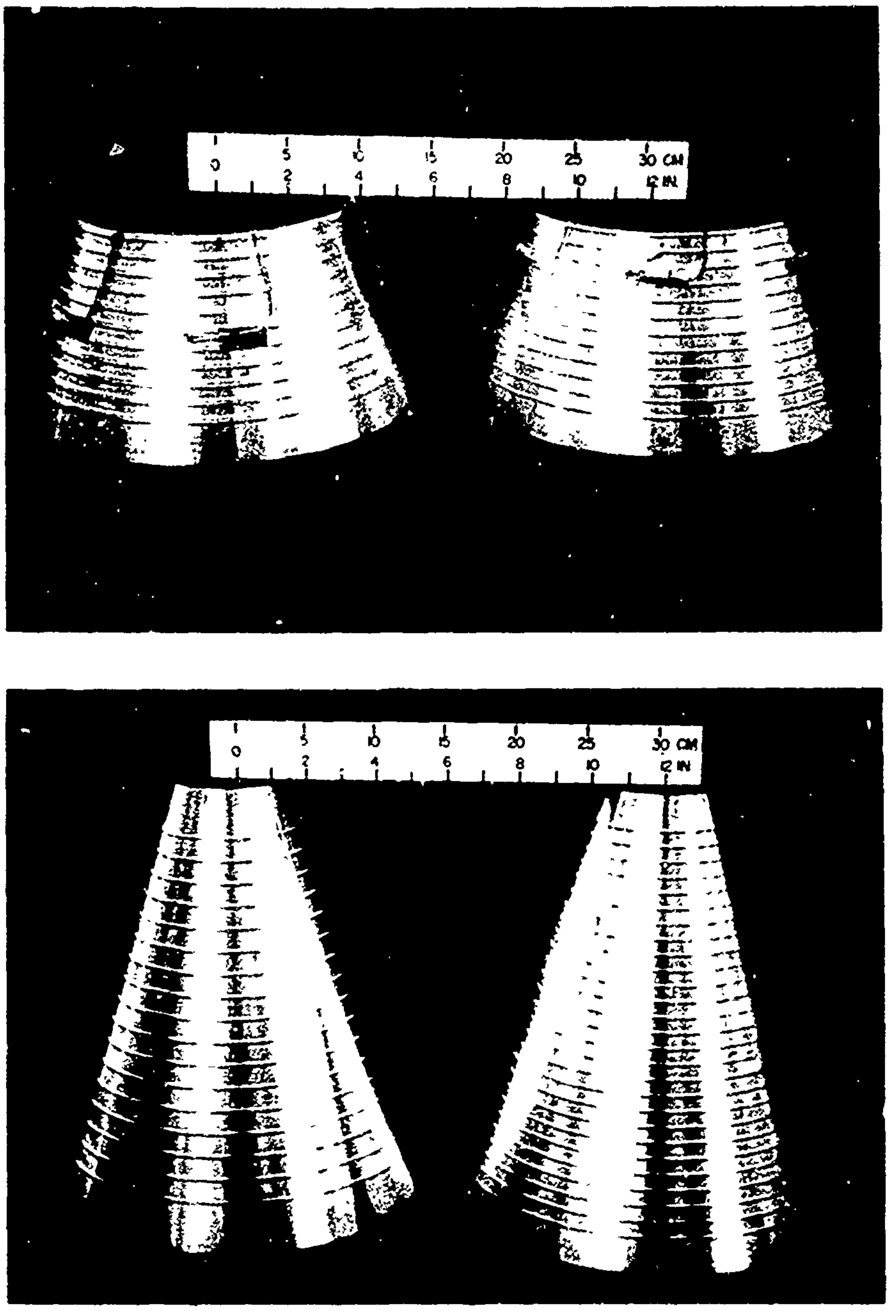


FIG. 21. TYPICAL RING-STIFFENED CONICAL SHELLS TESTED

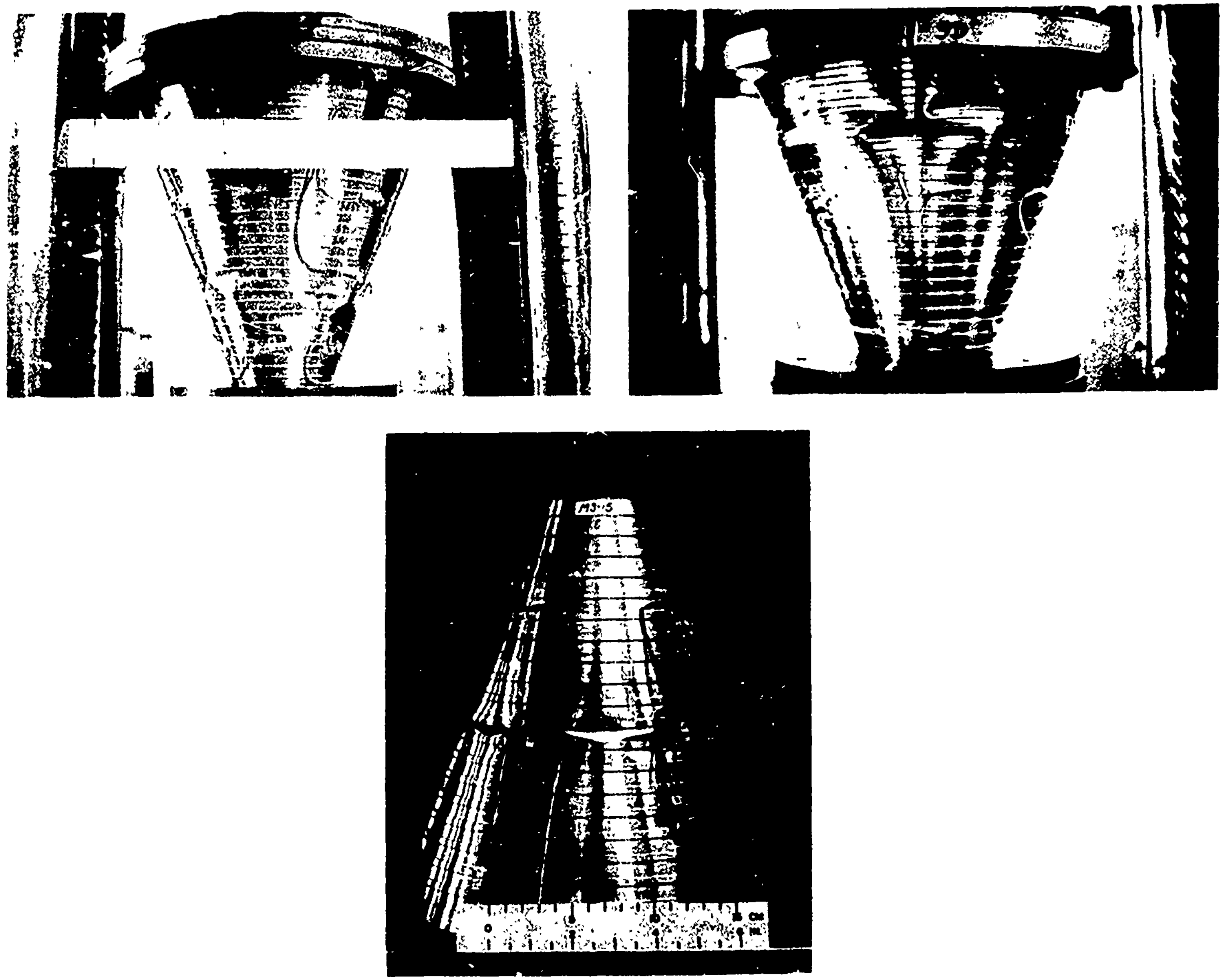


FIG. 22. ASYMMETRICAL BUCKLING PATTERNS IN RING-STIFFENED CONICAL SHELLS

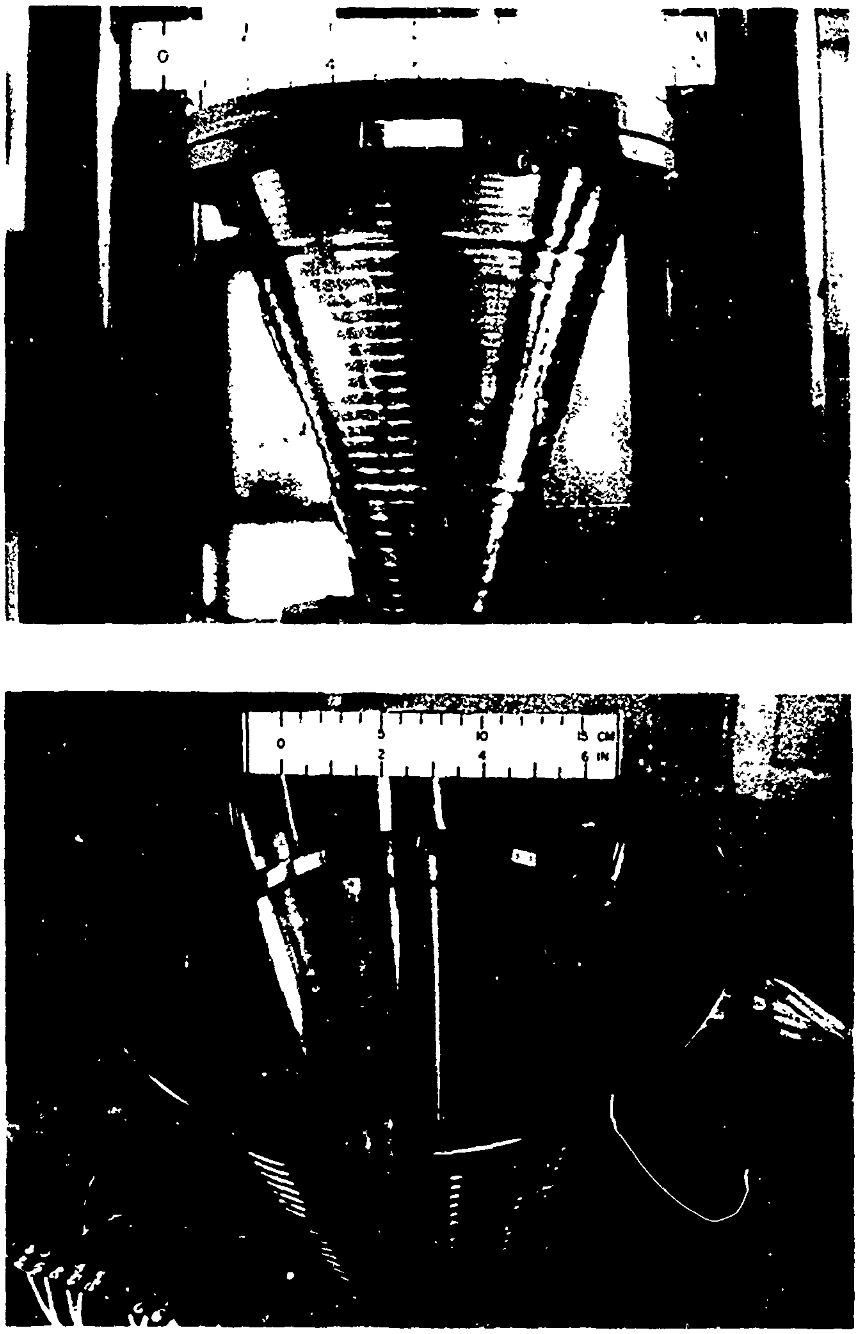
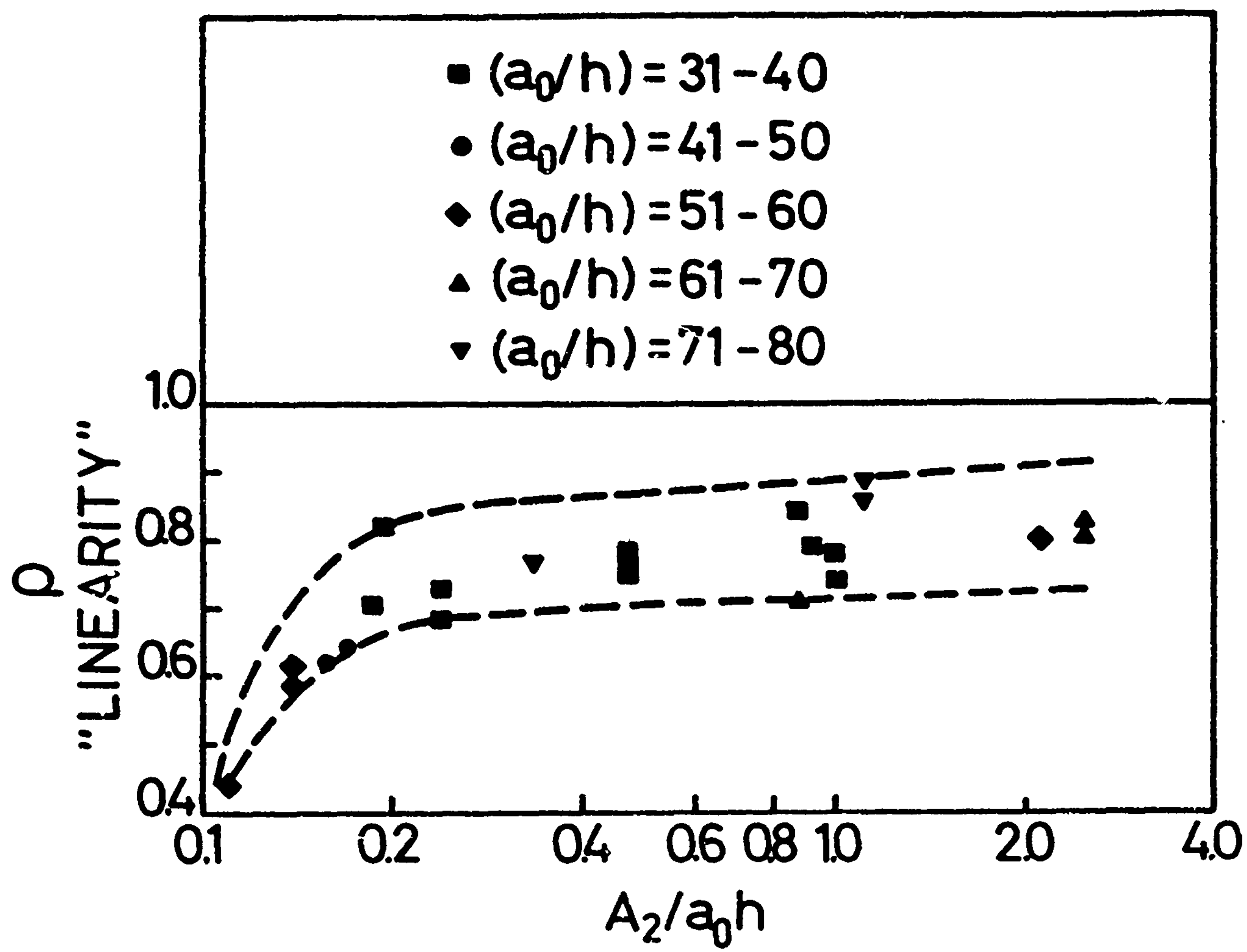


FIG. 23. SYMMETRICAL BUCKLING PATTERNS IN HEAVILY STIFFENED CONICAL SHELLS.



EFFECT OF RING AREA ON THE "LINEARITY" IN BUCKLING OF RING-STIFFENED CONICAL SHELLS

FIG. 24

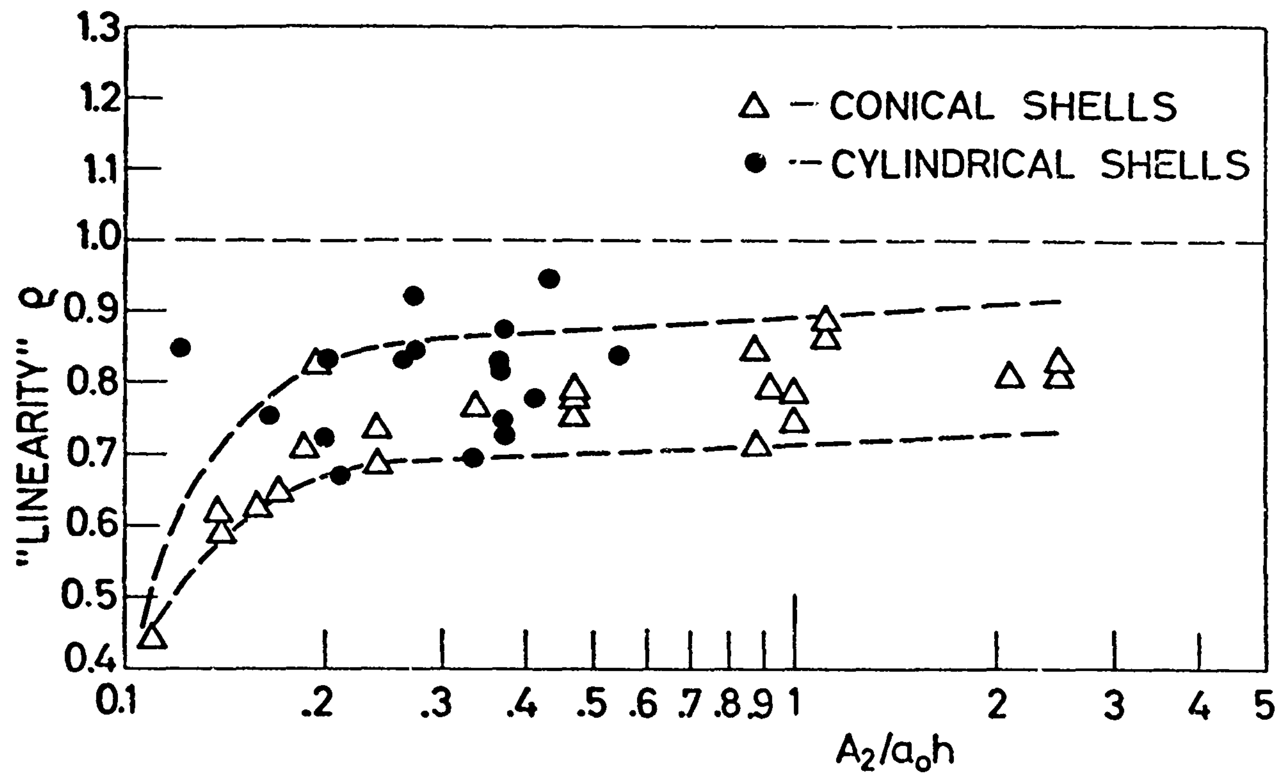


FIG. 25. EFFECT OF RING AREA ON "LINEARITY" - COMPARISON OF CONICAL AND CYLINDRICAL SHELLS.

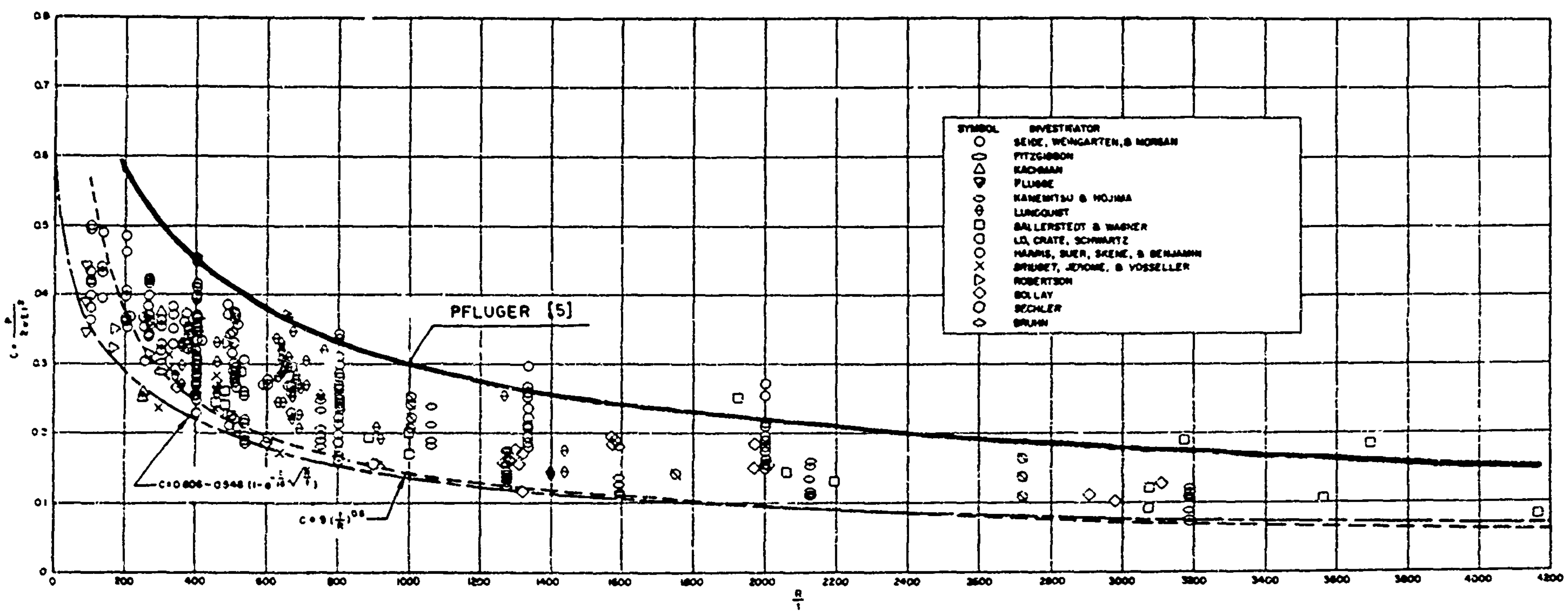


FIG. 26 COMPARISON OF EXPERIMENTAL DATA (REPRODUCED FROM [6]) AND PFLÜGERS FORMULA

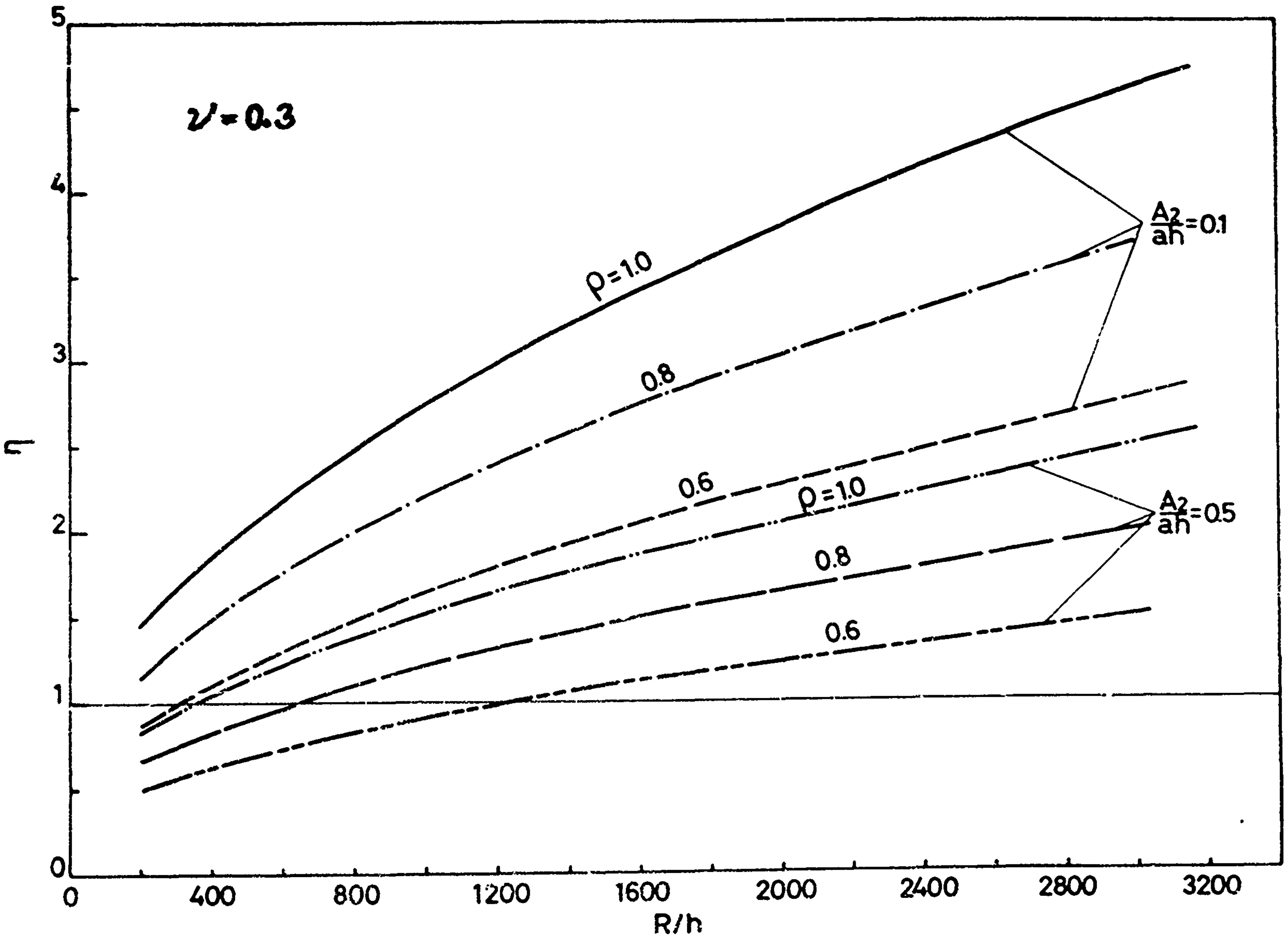
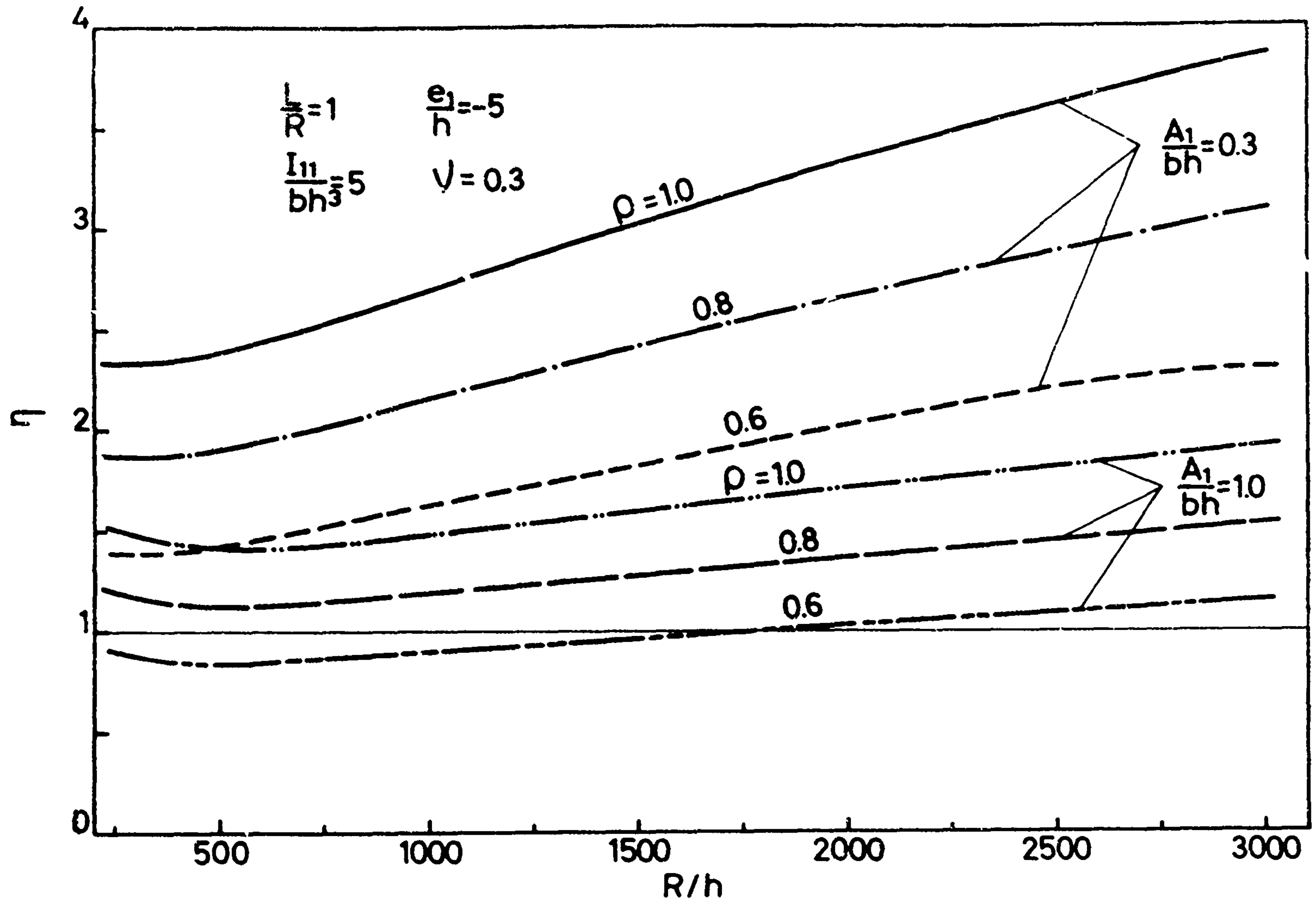


FIG. 27 STRUCTURAL EFFECIENCY OF RING-STIFFENED CYLINDRICAL SHELLS (OUTSIDE RINGS)



STRUCTURAL EFFICIENCY OF STRINGER-STIFFENED CYLINDRICAL SHELLS (OUTSIDE STRINGERS)

FIG. 28

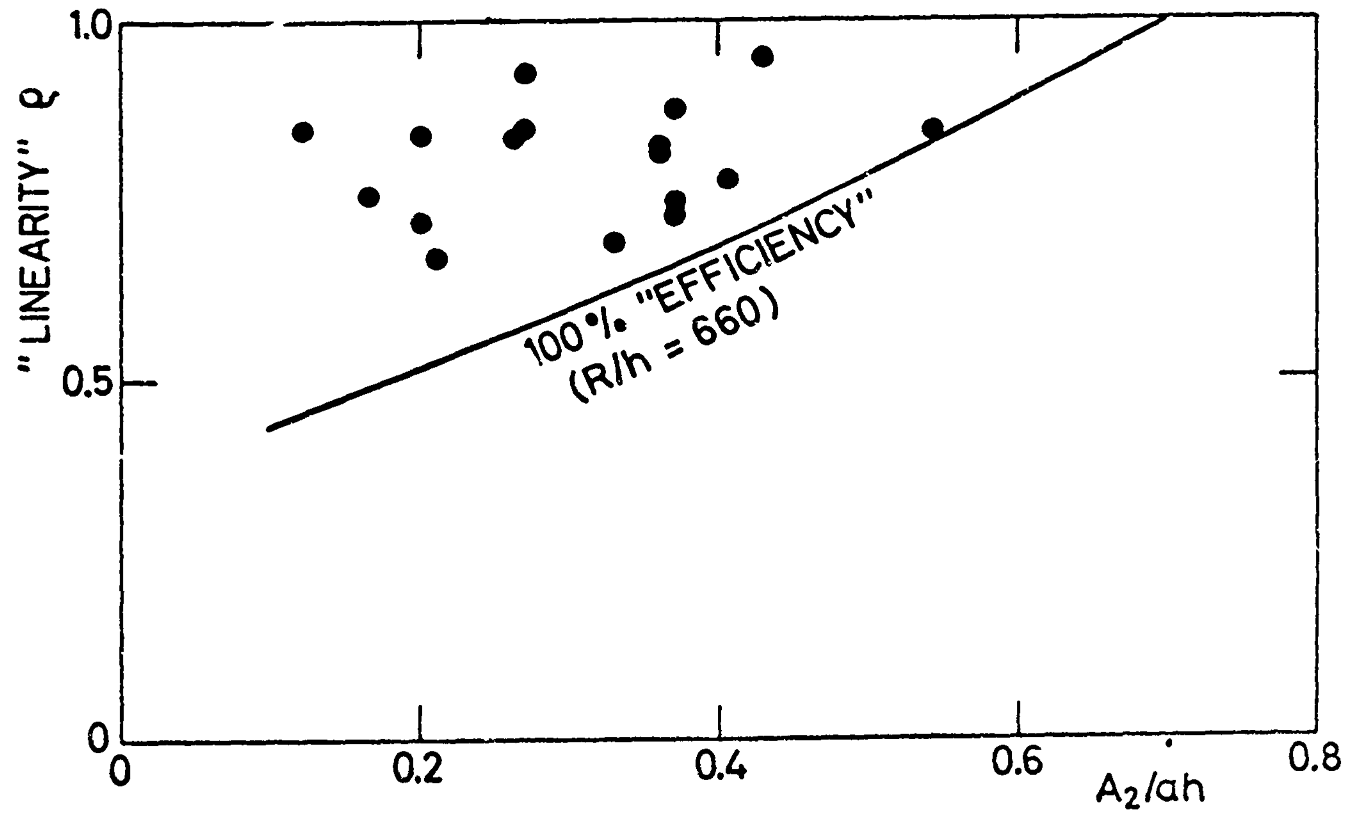
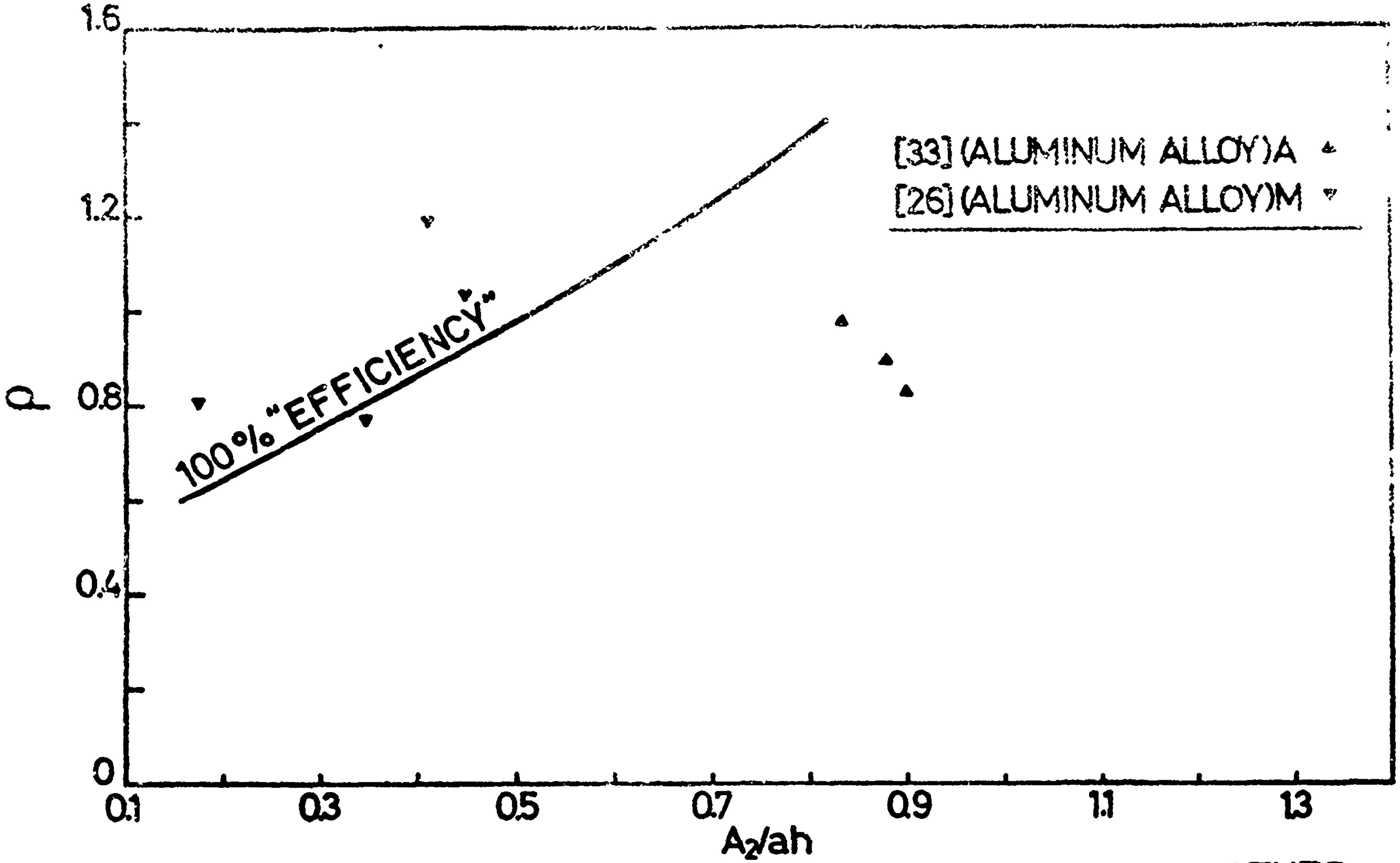


FIG. 29. "LINEARITY" OF RING-STIFFENED CYLINDRICAL SHELLS TESTED SHOWING THEIR STRUCTURAL EFFICIENCY.



"LINEARITY" OF RING-STIFFENED CYLINDERS (TESTS OF OTHER INVESTIGATORS) SHOWING STRUCTURAL EFFICIENCY

FIG. 30

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1 ORIGINATING ACTIVITY (Corporate author) TECHNION RESEARCH AND DEVELOPMENT FOUNDATION, HAIFA, ISRAEL	2a. REPORT SECURITY CLASSIFICATION 2b GROUP

3 REPORT TITLE

THE INFLUENCE OF STIFFENER GEOMETRY AND SPACING ON THE BUCKLING OF AXIALLY COMPRESSED CYLINDRICAL AND CONICAL SHELLS.

4 DESCRIPTIVE NOTES (Type of report and inclusive dates)

Scientific Interim

5 AUTHOR(S) (Last name, first name, initial)

SINGER Josef

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
OCTOBER 1967	56 + 30 Figures	69

8a CONTRACT OR GRANT NO. AF 61(052)-905 b. PROJECT NO. 9782-01 c. 61445014 d 681307	9a. ORIGINATOR'S REPORT NUMBER(S) TAE REPORT 68 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. AVAILABILITY/LIMITATION NOTICES

This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY AIR FORCE OFFICE OF SCIENTIFIC RESEARCH (SREM), 1400 Wilson Blvd. Arlington, Va. 22209.

13 ABSTRACT

An experimental and theoretical study of the buckling of closely stiffened cylindrical and conical shells under axial compression has been undertaken to determine the influence of the stiffener geometry and spacing on the applicability of linear theory. Tests on integrally ring-stiffened cylinders, in which the spacing, cross-sectional area and eccentricity of the stiffeners is varied are described. The bounds of general instability are first determined by an elementary analysis of sub-shells and panels between stiffeners, in conjunction with "smeared" stiffener theory. The interaction between stiffeners and shell is then investigated with a linear discrete-stiffener theory. The experimental results are correlated with theory and approximate design criteria are developed. Experimental results and conclusions of other investigators are also discussed. The results of a test program of integrally ring-stiffened conical shells are briefly discussed and correlated with the results obtained for cylindrical shells.

The structural efficiency of closely stiffened cylindrical shells is then studied in view of the observed bounds of applicability of linear theory.

DD FORM 1 JAN 64 1473

UNCLASSIFIED

Security Classification

UNCLASSIFIED

Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
1. Stiffened Cylindrical and Conical Shells.						
2. Local Buckling and General Instability.						
3. Experimental Study of Buckling Under Axial Compression.						
4. Validity of Linear Theory for Buckling.						

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization *(corporate author)* issuing the report.

2a. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7a. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8a. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b, 8c, & 8d. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers *(either by the originator or by the sponsor)*, also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through ________________."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through ________________."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through ________________."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS)*, *(S)*, *(C)*, or *(U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.

UNCLASSIFIED

Security Classification