A NEW FAMILY OF LIFE DISTRIBUTIONS

by

Z. W. Birnbaum*
University of Washington
Seattle, Washington

and

Sam C. Saunders
Boeing Scientific Research Laboratories
Seattle, Washington

Technical Report No. 52

March 22, 1968

Contract N-onr-477(38)

Laboratory of Statistical Research
Department of Mathematics
University of Washington
Seattle, Washington

This research was supported by the Office of Naval Research.

A NEW FAMILY OF LIFE DISTRIBUTIONS

by

Z. W. Birnbaum*
University of Washington
Seattle, Washington

and

Sam C. Saunders
Boeing Scientific Research Laboratories
Seattle, Washington

*The research of this author was supported in part by the Office of Naval Research and in part by The Boeing Company.

SUMMARY

A new two parameter family of life length distributions is presented which is derived from a model for fatigue. This derivation follows from considerations of renewal theory for the number of cycles needed to force a fatigue crack extension to exceed a critical value. Some closure properties of this family are given and some comparisons made with other families such as the lognormal which have been previously used in fatigue studies.

1. INTRODUCTION

It is well known that for the amount of fatigue data which can usually be obtained almost any two dimensional parametric family of distributions can be made to fit reasonably well. In fact, in the region of central tendency the lognormal, the Weibull, the Gamma, etc., can all be fitted by parametric estimation and because of the relatively small sample sizes hardly any can be rejected by, say a Chi-square Goodness of Fit test. However, when it becomes a question of predicting the "safe life" say the one thousandth percentile, there is a wide discrepancy between these models.

For this reason a family of distributions which is obtained from considerations of the basic characteristics of the fatigue process should be more persuasive in its implications than any ad hoc family chosen for extraneous reasons. In this paper we derive, using some elementary renewal theory, a two parameter family of nonnegative random variables as an idealization of the number of cycles necessary to force a fatigue crack to grow to a critical value. We then examine some of its relevant properties.

2. A MODEL FOR THE DISTRIBUTION OF LIFE

We propose a basic framework and notation which is similar to that used previously in [3]. We consider only standardized material specimens which are subjected to fluctuating stresses by a periodic loading. By a load (or load oscillation) we mean a continuous unimodal function on the unit interval, the value of which at any time gives the stress imposed by the deflection of the specimen. Let $\ell_1, \ell_2, \ldots$ be the sequence of loads which are to be applied at each oscillation so that at the i^{th} oscillation load ℓ_i is imposed. We suppose that the loading is *cyclic* in the sense that for some $m > 1$ and all $i=1,\ldots,m$

$$\ell_{jm+i} = \ell_{km+i} \quad \text{for all} \quad j \neq k \tag{2.1.1}$$

and the loading is continuous so that for all $i=1,2,\ldots$

$$\ell_{i+1}(0) = \ell_i(1). \tag{2.1.2}$$

Hence the $(j+1)^{st}$ *cycle* is the loading $(\ell_{jm+1},\ldots,\ell_{jm+m})$.

We assume that fatigue failure is due to the initiation, growth and ultimate extension of a dominant crack. At each oscillation this crack is extended by some amount which is a random function due to the variation in the material, the magnitude of the imposed stress and a certain number of the prior loads and perhaps the actual crack extensions caused by the prior loads in that cycle.

Thus we now make our first assumption

1° The incremental crack extension X_i following the application of the i^{th} oscillation is a random variable with a distribution which depends upon all the loads and actual crack extensions which have preceded it in that cycle.

The crack extension during the $(j+1)^{st}$ cycle is

$$Y_{j+1} = X_{jm+1} + \cdots + X_{jm+m} \quad \text{for} \quad j=0,1,\ldots,$$

where X_{jm+i} is the (possibly microscopic) crack extension following the load l_i applied in the i^{th} oscillation of the $(j+1)^{st}$ cycle.

It follows from Assumption 1*, regardless of how much dependence exists between the successive random extensions per oscillation in each cycle, that the random total crack extensions per cycle are independent. Our notation will be

$$W_n = \sum_{j=1}^{n} Y_j,$$

which has distribution function

$$H_n(w) = P[W_n \leq w], \quad \text{for} \quad n=1,2,\ldots .$$

It follows from elementary probability considerations, see p. 169, [6], that the distribution of C, the number of such cycles until failure, in the case failure is defined as the crack length exceeding some fixed critical length ω for the first time, is

$$P[C\leq n] = 1 - H_n(\omega). \tag{2.2}$$

If we suppose that we are dealing with a long cycle of oscillations which is as complex as the ground-air-ground cycle in aeronautical failure studies, it might be reasonable to assume that each cycle itself consists of a large number of distinct phases of loading. Even though the total crack extension per cycle is the sum of random variables which are not necessarily independent or identically distributed, the number of summands might be sufficiently large to make it reasonable to assume that the total crack extension per cycle is normally distributed.

Thus we could formally make a second assumption

2° The total crack extension Y_j due to the j^{th} cycle is a normal random variable with mean μ and variance σ^2 for all $j=1,2,\ldots$.

As a possible alternative one could postulate that the distribution of the crack extension might be different for the earlier cycles than it would be for the later ones. One of the first suppositions might be to make the distribution of the extension per cycle depend upon the size of the crack at the start of the cycle. One such assumption and the resulting distribution of the total crack length at the end of n cycles is presented in Section 4. We do not make that assumption here but instead, for reasons of simplicity, proceed with the study of the implications of the one above.

Note that the crack extension per cycle must be a nonnegative random variable. Thus in order for the Assumption 2° to apply we must regard as negligible the probability with which this normal variable becomes negative e.g. by assuming

$$\mu > 3\sigma > 0. \qquad (2.2.1)$$

It now follows from Assumptions 1° and 2° that the Y_j are independent and identically distributed normal random variables. Thus we have as the distribution of C from (2.2)

$$P[C \leq n] = 1 - \mathfrak{N}\left(\frac{\omega - n\mu}{\sqrt{n}\,\sigma}\right) = \mathfrak{N}\left(\frac{\sqrt{n}\,\mu}{\sigma} - \frac{\omega}{\sqrt{n}\,\sigma}\right) \qquad (2.3)$$

where $\mathfrak{N}$ is the distribution function of the standard normal variate with zero mean and unit variance defined for $-\infty < y < \infty$ by

$$\mathfrak{N}(y) = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt. \qquad (2.4)$$

We write

$$\alpha = \frac{\sigma}{\sqrt{\mu\omega}}, \quad \beta = \frac{\omega}{\mu} \qquad (2.5)$$

and replace n by the nonnegative real variable $t > 0$. If we now denote the continuous extension of the discrete random variable C by T, a continuous nonnegative random variable, then it follows by (2.3) and (2.5) that T has the life distribution

$$F(t:\alpha,\beta) = \mathfrak{N}[\frac{1}{\alpha}\,\xi(t/\beta)] \quad \text{for} \quad t > 0 \qquad (2.6)$$

where

$$\alpha > 0, \quad \beta > 0$$

and

$$\xi(t) = t^{\frac{1}{2}} - t^{-\frac{1}{2}}. \tag{2.7}$$

This two-parameter family of distributions is a plausible model for the distribution of fatigue life. The set of all life distributions of the form (2.6) for $\alpha,\beta > 0$ will be denoted by $\mathcal{J}$ and most of this paper will be devoted to the study of its properties. We shall also refer to the law which has the distribution (2.6) with the somewhat shorter notation $F(\alpha,\beta)$.

Of course, there are motivational derivations for other distributions as well. The Weibull distribution, which is well known for its applications as a life length distribution and for fatigue life in particular, is obtained as a special case of the extreme value distributions, see p. 302 [7]. The Gamma family has also been obtained as a distribution of life by utilizing a model of a bundle of strands which are supporting a tensile load, see [4].

It is instructive to make a comparison between the derivation of the family of distributions $\mathcal{J}$ and an appropriate adaptation of the classical heuristic argument, found for example p. 219, Cramér [5], as it might be used to obtain the lognormal distribution of the time until failure in fatigue. We present this derivation in Section 5.

3. SOME PROPERTIES OF $\mathcal{T}$

Let T have the distribution defined in (2.6). Note that T is a two-parameter random variable with β as a location parameter since it is the median of the distribution. (We show later that β is neither the mode nor the mean.) Notice also that α is a shape parameter and β a scale parameter. As we have seen $\frac{1}{\alpha}\xi(T/\beta)$ is a standard normal random variable with mean zero and unit variance. If we let X be $\mathfrak{N}(0, \frac{\alpha^2}{4})$ we see that, in distribution,

$$2X = \xi(T/\beta). \tag{3.1}$$

If we define the function ψ by

$$\psi(x) = \xi^{-1}(2x) \quad \text{for all real } x, \tag{3.1.5}$$

then

$$T = \beta\psi(X) \tag{3.2}$$

From elementary algebra we find that

$$\psi(x) = [\rho(x)]^2 \tag{3.3}$$

where

$$\rho(x) = x + \sqrt{x^2+1}\ . \tag{3.4}$$

So by (3.2)

$$T = \beta[1 + 2X^2 + 2X\sqrt{1+X^2}\ ] \tag{3.5}$$

where X is $\mathfrak{N}(0, \frac{\alpha^2}{4})$. Hence we have immediately

$$E(T) = \beta(1 + \frac{\alpha^2}{2}) \tag{3.5.1}$$

$$E(T^2) = \beta^2(1 + 2\alpha^2 + \frac{3\alpha^4}{2})$$

$$\text{var}(T) = (\alpha\beta)^2(1 + \frac{5\alpha^2}{4}) \qquad (3.5.2)$$

and we note that for fixed α the variance of T increases as the scale parameter (median) β increases. This is not true for the lognormal distribution but empirical evidence shows that it is true for the fatigue lives themselves.

By noting that

$$\frac{1}{\rho(x)} = -x + \sqrt{1+x^2} = \rho(-x) \qquad (3.5.3)$$

we see that, whenever $-X$ has the same distribution as X, we have by (3.3), in distribution,

$$\frac{1}{\rho(X)} = \rho(X) \quad \text{and} \quad \frac{1}{\psi(X)} = \psi(X). \qquad (3.6)$$

Thus there follows immediately from (3.2) the

Theorem 3.1. If T has a fatigue life distribution $F(\alpha,\beta)$, in $\mathcal{F}$ then $\frac{1}{T}$ has a distribution in $\mathcal{F}$ given by $F(\alpha, \frac{1}{\beta})$. Moreover, for any real $a > 0$, the random variable aT has a distribution in $\mathcal{F}$ given by $F(\alpha, a\beta)$.

It is known that every random variable with distribution defined by (2.2) for which the Y_j are nonnegative and have densities which are Pólya frequency functions of order 2, has an increasing failure rate, see [1]. We now show that the random variable T does not have this property and moreover the failure rate does not even increase on the average which is a weaker condition, see [2]. However, it just barely fails to have this last condition satisfied.

Remark 3.2. T does not have a failure rate which increases on the average.

Proof. Without loss of generality take $\alpha = \beta = 1$ and define

$$Q(t) = -\ln\{1 - \mathfrak{N}[\xi(t)]\}.$$

Writing Mill's ratio

$$M(\xi) = \xi[1 - \mathfrak{N}(\xi)]/\mathfrak{N}'(\xi)$$

then applying L'Hospital's rule to $Q(t)/t$ for $t \to \infty$ and using (4.1.1) of the next section we obtain

$$\frac{Q(t)}{t} \simeq \frac{\mathfrak{N}'[\xi(t)]}{1-\mathfrak{N}[\xi(t)]}\,\xi'(t) = \frac{(1-t^{-2})}{2M[\xi(t)]}\,.$$

We know that as $\xi \to \infty$ we have

$$M(\xi) \simeq 1 - \frac{1}{\xi^2} + \frac{3}{\xi^4} + O(\xi^{-5}).$$

Hence $\lim_{t\to\infty} \frac{Q(t)}{t} = \frac{1}{2}$ but $Q(1) =$ [illegible] $> \frac{1}{2}$. This proves our contention that $\frac{Q(t)}{t}$ does not always increase. ||

Actual numerical calculation shows that $\frac{Q(t)}{t}$ decreases slowly for $t > 1.64$ as Figure 1 shows. Since normal random variables do have densities which are Pólya frequency functions of order 2 the reason the failure rate can decrease is that our summands fail to be nonnegative with probability one.

We do not regard this as being a serious shortcoming to our family of distributions when applied to fatigue, any more than the negativity of the normal distribution vitiates its usefulness when applied to such things as measurements, which theoretically cannot be negative.

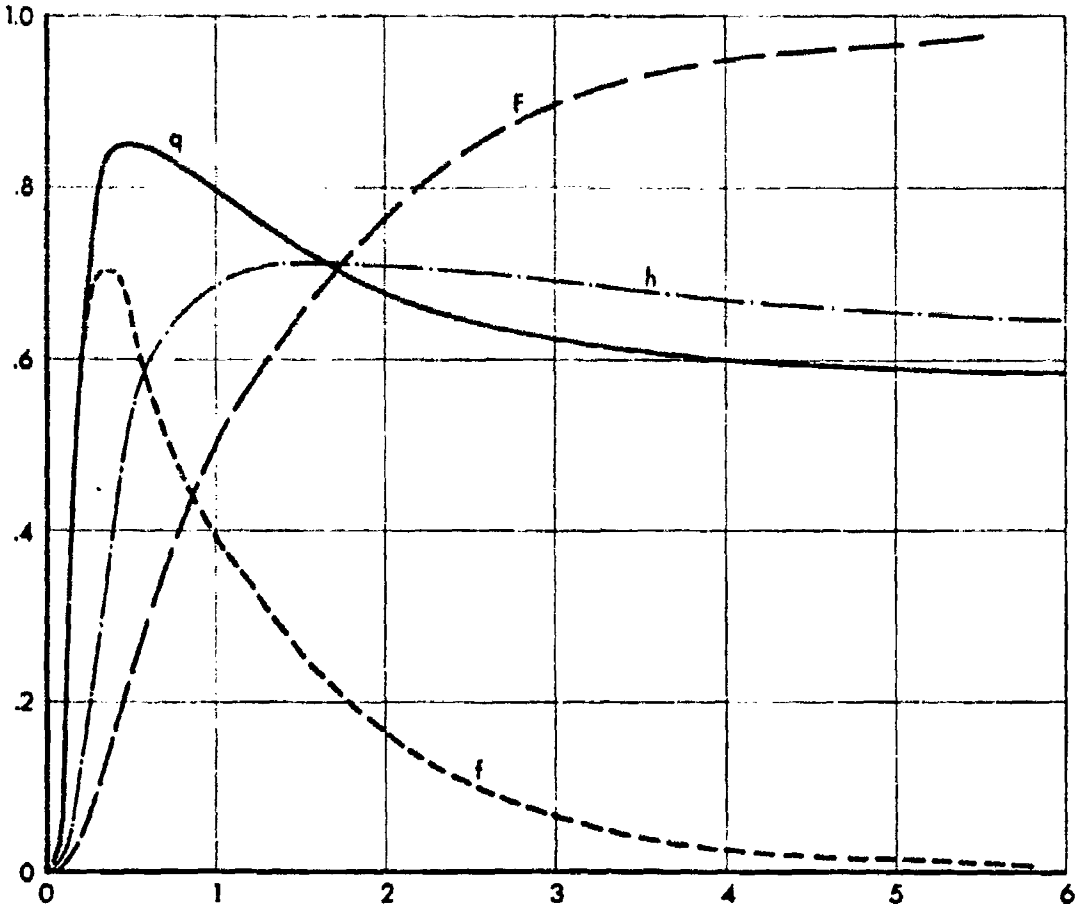

Figure 1

Graphs of the density f, the distribution F, the hazard rate q, and the average hazard rate h where for $t > 0$

$$f(t) = \frac{t^{\frac{1}{2}}+t^{-\frac{1}{2}}}{2t\sqrt{2\pi}} \exp\left\{-\frac{t}{2} - \frac{1}{2t} + 1\right\}$$

$$F(t) = \int_0^t f(x)dx, \quad q(t) = \frac{f(t)}{1-F(t)}, \quad h(t) = \frac{1}{t}\int_0^t q(x)dx.$$

4. A SECOND MODEL FOR THE DISTRIBUTION OF LIFE

In this section we replace the Assumption 2° by the more general one

2' The crack extension Y_{n+1} during the $(n+1)^{st}$ cycle, given that the total crack length was s at the start of the cycle, is a normal random variable with mean $\mu + \delta s$, for some constant $\delta \geq 0$, and variance σ^2 for each $n=1,2,\ldots$.

By formula (2.2), to find the distribution of C it is necessary and sufficient that we find the distribution of W_n, called H_n.

By assumption

$$P[Y_{n+1} \leq y \mid W_n = s] = \mathfrak{N}\left(\frac{y-\mu-\delta s}{\sigma}\right)$$

and for $n=1$

$$H_1(y) = \mathfrak{N}\left(\frac{y-\mu}{\sigma}\right) \qquad -\infty < y < \infty. \tag{4.1}$$

Now by definition, setting $y = x - s$

$$H_{n+1}(x) = \int_{-\infty}^{\infty} \mathfrak{N}\left(\frac{y-\mu-\delta s}{\sigma}\right) dH_n(s). \tag{4.2}$$

We can now prove the

Theorem 4.1 The total crack length W_n, at the end of the n^{th} cycle, is normal with mean μ_n and variance σ_n^2 where

$$\mu_n = \frac{\mu}{\delta}\,[(1+\delta)^n - 1], \qquad \sigma_n^2 = \sigma^2\,\frac{(1+\delta)^{2n}-1}{(1+\delta)^2-1}. \tag{4.3}$$

Proof by induction. The statement is true for $n=1$. Assume it true for n, then from (4.2)

$$H_{n+1}(x) = \int_{-\infty}^{\infty} \mathfrak{N}\left(\frac{u-s}{v}\right) d_s\, \mathfrak{N}\left(\frac{s-\mu_n}{\sigma_n}\right)$$

where

$$u = \frac{x-\mu}{1+\delta} \qquad v = \frac{\sigma}{1+\delta} .$$

But this we recognize as the usual convolution of two normal random variables and hence the result is

$$H_{n+1}(x) = \mathfrak{N}\left(\frac{u-\mu_n}{\sqrt{v^2+\sigma_n^2}}\right).$$

Hence by simplification we find that

$$\mu_{n+1} = \mu + (1+\delta)\mu_n, \qquad \sigma_{n+1}^2 = \sigma^2 + (1+\delta)^2\sigma_n^2$$

and one checks that formulas given do satisfy the recursion relations (4.3).||

Strictly speaking a proper distribution, analogous to that obtained in (2.6), which would be the continuous extension of the present case, cannot be generated since $P[C < \infty] < 1$.

For, one can see that

$$P[C \leq n] = \mathfrak{N}\left(\frac{\mu_n-\omega}{\sigma_n}\right)$$

and as $n \to \infty$ we have $\mu_n/\sigma_n \to \frac{\mu}{\delta\sigma}\sqrt{(1+\delta)^2-1} < \infty$.

Of course this would not be of practical significance, since we are restricting ourselves to situations where $\frac{\mu}{\sigma} > 3$, see (2.2.1), hence $\lim_{n\to\infty} \frac{\mu_n}{\sigma_n} > 3\sqrt{2/\delta}$ for $0 < \delta < 1$, and $\lim \frac{\mu_n}{\sigma_n} > 3$ for any δ.

5. A DERIVATION OF THE LOGNORMAL DISTRIBUTION

In our notation the i^{th} oscillation is caused by the loading function ℓ_i, and the crack extension depends upon both the loading function and the size of the crack already attained at the time when the oscillation was begun.

Consider the n loads $\ell_1,\ldots,\ell_n$ which have resulted in a crack of size W_n, then

$$W_{n+1} - W_n = \frac{L_{n+1}}{\delta(W_n)} ,$$

assuming the incremental growth of the crack is proportional to some random functional L_{n+1} of the loading function ℓ_{n+1} imposed at that time, and some function of the size of the crack $\delta(W_n)$. Now we have

$$L_1 + \cdots + L_n = \sum_{j=1}^{n} (W_j - W_{j-1})\delta(W_{j-1}).$$

Letting n become large we then have the left-hand side becoming asymptotically a normal random variable, due to the periodicity of loading, while the right-hand side becomes in the limit

$$\int_{Z_0}^{Z} \delta(t)dt.$$

If we take $\delta(t) = 1/t$ then Z becomes asymptotically lognormal.

The important distinction between this derivation and that which we previously considered is that while a given number of identically distributed summands does approach normality as the number gets large, a random number with large mean of identically distributed summands does not necessarily do so. Moreover, we maintain that what is at question here is the number of impulses required to exceed a certain critical crack size and it is indeed a random number of cycles which will be necessary to accomplish this.

6. CONCLUSION

A derivation, based on plausible physical considerations, for a family of distributions is, by itself, not a conclusive argument that such a particular family should be used in life studies. No family, however reasonable its derivation, can be accepted for use in fatigue life studies until it is confronted with actual fatigue data obtained under various conditions and the distribution is shown to represent adequately the life length Δ which are obtained.

In order to do this one must have the theory of estimation for this family completed. The derivation of parametric estimators and the ancillary computing formulas for this family will be presented in a later study. Also further studies of the application of this distribution to the calculation of "safe life" will be made. Thus the confrontations of this family with actual data will be carried out, to provide the justification for this presentation.

BIBLIOGRAPHY

[1] Barlow, Richard E., and Proschan, Frank. Mathematical Theory of Reliability, John Wiley and Sons, New York, 1965.

[2] Birnbaum, Z. W., Esary, J. D., and Marshall, A. W. Stochastic Characterization of Wear-Out for Components and Systems, *Ann. Math. Statist.*, *37*, 816-825, 1966.

[3] Birnbaum, Z. W., and Saunders, S. C. A Probabilistic Interpretation of Miner's Rule. Boeing document D1-82-0603, April 1967; to appear in *J. Soc. Indust. Appl. Math.*

[4] Birnbaum, Z. W., and Saunders, S. C. A Statistical Model for Life-Length of Material, *J. Amer. Statist. Assoc.*, *53*, 151-160, 1958.

[5] Cramér, Harold. Mathematical Methods of Statistics, Princeton University Press, 1951.

[6] Feller, William. An Introduction to Probability Theory and Its Applications, Volume II, John Wiley and Sons, New York, 1966.

[7] Gumbel, E. J. Statistics of Extremes, Columbia University Press, New York, 1958.

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Laboratory of Statistical Research Department of Mathematics University of Washington, Seattle, Wash.	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

A New Family of Life Distributions

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Technical Report No. 52, March 22, 1968

5. AUTHOR(S) (Last name, first name, initial)

Z. W. Birnbaum and S. C. Saunders

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
March 22, 1968	15	7

8a. CONTRACT OR GRANT NO. N-onr-477(38) b. PROJECT NO. NR 042038 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) 52 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. AVAILABILITY LIMITATION NOTICES

Qualified requesters may obtain copies from DDC

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	U. S. Navy Office of Naval Research Washington, D.C.

13. ABSTRACT

A new two-parameter family of life length distributions is presented which is derived from a model for fatigue. This derivation follows from considerations of renewal theory for the number of cycles needed to force a fatigue crack extension to exceed a critical value. Some closure properties of this family are given and some comparisons made with other families such as the lognormal which have been previously used in fatigue studies.

DD FORM 1473

Security Classification

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Life distributions						
Fatigue						
Renewal theory						
Crack extension						
Cycles to failure						

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (*corporate author*) issuing the report.

2a. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7a. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8a. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b, 8c, & 8d. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers (*either by the originator or by the sponsor*), also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through ______."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through ______."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through ______."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring (*paying for*) the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as (*TS*), (*S*), (*C*), or (*U*).

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.

DD FORM 1473 (BACK)

Security Classification