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SIMULTANEOUS ANALYSIS OF NONLINEAR NETWORKS ON A DIGITAL COMPUTER

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ABSTRACT

The purpose of this investigation was to develop the necessary techniques and algorithms to allow the solution of a general nonlinear network on a large scale digital computer.

The network equations describing a general nonlinear network were developed from basic principles to obtain a final formulation that describes the network as a set of nonlinear differential equations. Since these equations must be solved numerically, a method that gives the minimum set of equations was chosen. This formulation is the state variable form with the capacitive voltages and inductive currents chosen as the variables.

The existence and uniqueness of the analytical solution was shown to exist if the partial derivatives of the parametric network functions are continuous functions. The stability of the system is guaranteed if the energy storage network parameters have bounded partial derivatives. Also, stability conditions, developed from a Liapunov function, was related to the network parameters, thus yielding a criteria for the stabiltiy of a general nonlinear network.

Computer algorithms were developed to accept the schematic network representation and the nonlinear parameter functions, and generated all the required topological and parametric relationships. These in turn are automatically put into the state variable formulation. All these relationships and algorithms result in a set of nonlinear differential equations which are solved numerically using Hammings predictor-corrector method with a Rung-Kutta routine used for initiation. The numerical integration algorithm is of the variable step type and adjusts the integration interval to the proper size.

The numerical experiments conducted during the course of this investigation verify the accuracy of simulation and the versatility of the methods developed in analyzing general nonlinear networks. The method also minimizes the number of differential equations to be solved and recomputes algebraic relationships only when needed for the nonlinear analysis, thereby minimizing the actual digital computer calculations required.

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CHAPTER I

INTRODUCTION

General Statement of the Problem

One of the most difficult types of electrical network analysis to perform is the transient analysis of nonlinear networks. Classically, the main analytical tool for obtaining the transient response of networks is the Laplace transform. In the general case, this method is restricted to linear or piecewise linear circuits. For nonlinear networks, the methods of solution involve linearization and approximation or specific techniques applied to certain classes of circuits. A generalized method for solving nonlinear networks is lacking. However, with the advent of digital computers the numerical solution of linear and nonlinear differential equations becomes practical, thus opening hitherto unavailable techniques for numerically solving nonlinear network problems. The selection and development of analytical and programming techniques and algorithms constitute the first half of the problem. The proofs of existence, uniqueness, and stability of the analytical and numerical solution define the other half.

Importance of the Study

In all phases of electronic analysis, the transient response of a circuit is of major importance. The logician must know whether or not levels are set in a given time; the circuit designer must know the shapes of voltage and current waveforms; the power supply designer must allow for transient conditions, etc. Many analyses are carried out by hand and result in approximate answers. If these answers are not accurate enough, an analog or digital solution can be obtained. In both of these cases, the programming and set-up time can be excessively long and, prior to using the program, all facets have to be carefully checked out. Furthermore, each new circuit and in many cases even minor circuit changes requires a new program. With a general computer program, the routine portions of network analysis can be delegated to the computer, thus allowing the engineer to perform his primary functions. In addition, the analyses can be obtained in a relatively short time. Thus, it can be seen that the case for developing a generalized digital computer analysis program is well-founded. From the myriad of analytical and numerical tools available, a consistent set needs to be chosen that will not only yield a solution for any real network but will provide some verification as to the uniqueness and stability of the solution. This type of program can be used not only for circuit analysis, but also for parametric studies, network model development, device simulation, and many other facets of electronic engineering.

Specific Statement of the Problem

The purpose of this research is to develop the methods of analysis and associated programming techniques required for general purpose nonlinear network analysis on a digital computer. It is assumed that the circuit topology is known in the form of a schematic diagram and all nonlinear relationships and parameter variations are known. The resultant program accepts the input parameters and circuit description and develops a set of nonlinear differential equations. These equations are subsequently solved for the network variables, current and voltage, as a function of time.

New topological algorithms are developed to set up the necessary equations automatically. Also, conditions for both numerical and analytical existence, uniqueness, and stability are developed and presented.

Previous Related Studies

Prior to the mid 1950's, network analysis using a digital computer was performed on a single circuit basis by established techniques. The circuit to be resolved was analyzed by classical loop or node equations. The resultant network equations were programmed and solved. This merely replaced the numerical portion of network analysis by computational techniques and did not take advantage of the capabilities of the computer except for its use as a large desk calculator. One of the first generalized computer analysis programs was TAP [4] developed by the International Business Machines Corporation. Since then, many programs have been developed, the most important ones being PECANS [2], NET-1 [13], ECAP [9], and ADNET [10]. The majority of these efforts were broad in nature, i.e., performed a-c, d-c and transient analysis, but are mainly applicable to linear circuits. In dealing with nonlinear transients, many questions arose as to the existence, uniqueness and stability of the solution need to be answered and, because of the scope of the programming efforts, these questions were left unanswered. Only in ADNET and PECANS are circuit nonlinearities allowed to be elements other than diodes and transistors. Furthermore, only in ADNET is any examination done with regard to existence and uniqueness of the solution.

Definition of Terms to be Used

To clarify the statement of the problem and to avoid any ambiguity, the following list of definitions will be adhered to in the course of this study:

Branch: A line segment in the network graph that replaces one network element.

Node: The intersections of two or more branches.

Circuit: Any closed contour selected in a graph.

Tree: Any connected set of branches which includes all nodes but no circuits of a given graph.

Tree Branch: A branch of a tree.

Link: A branch of a graph which does not belong to the tree.

Loop: A circuit formed by one link and where all the remaining branches are tree branches.

Cut-Set: That set of elements that dissociates two main portions of a network such that replacing any one element of the cut-set would destroy this property of dissociation.

Normal Tree: A network tree that contains the maximum number of capacitances and the minimum number of inductances possible. Thus a normal tree will contain the maximum allowable number of capacitances, the only capacitances being excluded are those whose addition would result in a loop that would contain only capacitive elements; an arbitrary number of resistors as topologically required for the formation of a tree; and a minimum number of inductors. The only inductors allowed are those whose addition to the set of links would result in a cut-set consisting entirely of inductive branches.

μ - Controlled Device: A device is said to be μ-controlled if, for all times and all values of μ in $(-\infty, \infty)$, the dependent variable y is a function F (here assumed to be single-valued) of μ (t) and t. This is written as $y(t) = F(\mu(t),t)$. This is consistent with the definitions given for current-controlled resistors, etc. These definitions are listed specifically by Doeser and Katzenelson [11].

Scope and Limitations of the Study

This research develops the required analytical techniques and computer algorithms for the analysis and simulation of nonlinear networks on a digital computer. The requirements for existence, uniqueness, and stability of the analytical and numerical solutions are developed and presented. Since the purpose of this study is to develop techniques and algorithms for nonlinear network analysis on a digital computer, the resultant computer program does not handle large networks (e.g., greater than ten branches). This is an arbitrary restriction that can be removed as computer memory capacity increases. The present program exists in three segments for the IBM 7074 computer, and is written in the DAFT language. Many programming esthetics have not been incorporated as they are not germane to the essential requirements of this study.

The type of circuits considered are nonlinear networks that are time invariant. The analytic techniques employed do not exclude time varying networks, per se, but have been developed on the assumption that they are not considered. For example, in equations of the type

$$i = \frac{d}{dt}(Cv) = C\frac{dv}{dt} + v\frac{dC}{dt} ,$$

it is assumed $\frac{dC}{dt} = 0$. This decision is based on the observation that most practical problems do not require time varying parameters and their inclusion would significantly increase the computer calculations required. The nonlinearities are all assumed to be single-valued in the independent variable. Furthermore, it is assumed that the network consists of branches that contain one of the following types of parameters: 1) a voltage dependent or constant capacitor, 2) a current dependent or constant inductor, 3) a voltage dependent or constant resistor, and 4) a current dependent or constant resistor.

In addition a branch may contain any of the following types of sources: 1) A constant voltage or current source, 2) a voltage or current source that is dependent upon another current or voltage, 3) a time-dependent source.

CHAPTER II

THEORETICAL DEVELOPMENT AND DISCUSSION OF THE ANALYTICAL MODEL

General Discussion

In this chapter, the network equations used in the analysis are developed from basic principles. The form of equations obtained is the state variable, or the mixed nodal-loop equation formulation.

Initially, no restrictions are placed on the type of network elements allowed except that they be two-pole resistors, capacitors, inductors, or current and voltage sources. The assumed definition of these parameters is as defined by Desoer and Katzenelson [6]. The nonlinear relationships and dependencies are restricted to single-valued functions of the independent variables. It is further assumed that resistors can be current or voltage controlled, inductors current controlled, capacitors voltage controlled, and the sources either current or voltage controlled or functions of time. Mutual relationships such as mutual inductors are not considered specifically since their effect can be represented by the allowable nonlinearities. The reason for excluding these devices in the mutual form is that their addition would add off-diagonal elements to the parametric matrices.

During the ensuing numerical calculations, these elements greatly increase computer calculation time, especially in the calculation of inverse matrices.

After the equations are developed, theorems are invoked and derived to assure existence, uniqueness, and stability of the analytical solution. The restrictions placed on the network and its elements in order to insure the existence of a unique stable solution are enumerated.

Derivation of the State Variable Equations

Let us consider a network that contains m branches and n nodes. If, in the schematic representation of the network, we replace each circuit element by a directed branch, its direction being the same as the assumed direction of current flow, the resultant graph will have m branches and n nodes. When a tree is chosen, it will contain b branches; hence, the set of links will contain $m-b = k$ branches. As each link is added back to the network, it will form a loop. Since the tree contains all the nodes, it will have one more branch than there are nodes or $b = n+1$. Hence, the number of equations obtained is $m-(n+1)$. This yields sufficient equations to solve the network equation and, furthermore, the set of equations thus obtained is independent since each loop contains a different (and only one) link.

Next, let us choose a particular type of tree, a normal tree. This tree will form the topological guide for the development of the state variable differential equations describing the network.

The topological formulation of the state variable equations follows the development by Kuh and Rohrer [11] quite closely, and uses essentially the same notation.

Let the tree-branch voltages and currents be defined by v_2 and i_2, respectively; likewise, the link voltages and currents can be designated v_1 and i_1.

Kirchhoff's voltage law can now be written in terms of the tree-branch and link voltages, and source voltages $[e]$, and the fundamental loop matrix $[I\ F]$, of the particular tree chosen. If it is assumed that the network consists of b branches, n nodes, and k links, there will be exactly $b-n+1$ independent equations. Since each link corresponds to an independent loop [15], [14], there are exactly $k = b-n+1$ links in the network. Thus, Kirchhoff's voltage law becomes:

$$[I\ F] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = [e] ,$$

where:

v_1 is a vector of length k,

v_2 is a vector of length $(b-k)$,

e is a vector of length b whose j^{th} element is the algebraic sum of all the sources in the j^{th} loop,

I is a $k \times k$ unit matrix,

F is a $k x(n-1)$ matrix expressing the topological relationships between the tree-branch and link elements.

The matrix F can be further partitioned as follows:

$$F = \begin{bmatrix} F_{SC} & F_{SG} & F_{SM} \\ F_{RC} & F_{RG} & F_{RM} \\ F_{LC} & F_{LG} & F_{LM} \end{bmatrix}$$

If we allow that there are:

a tree branches with capacitive elements

b tree branches with resistive elements

d tree branches with inductive elements

α tree links with capacitive elements

β tree links with resistive elements

λ tree links with inductive elements ,

then,

F_{SC} is an $(\alpha \times a)$ matrix relating the capacitive tree branches and the capacitive links

F_{RC} is a $(\beta \times a)$ matrix relating the capacitive tree branches and the resistive links

F_{LC} is a $(\lambda \times a)$ matrix relating the capacitive tree branches and the inductive links

F_{SG} is an $(\alpha \times b)$ matrix relating the resistive tree branches and the capacitive links

F_{RG} is a $(\beta \times b)$ matrix relating the resistive tree branches and the resistive links

F_{LG} is a $(\lambda \times b)$ matrix relating the resistive tree branches and the inductive links

F_{SM} is an $(\alpha \times d)$ matrix relating the inductive tree branches and the capacitive links

F_{RM} is a $(\beta \times d)$ matrix relating the inductive tree branches and the resistive links

F_{LM} is a $(\lambda \times d)$ matrix relating the inductive tree branches and the inductive links.

If the tree represented by F is a normal tree, the matrices F_{SG} , F_{SM} , and F_{RM} are nul-matrices. This follows immediately from the fact that a normal tree has a capacitive link only if all the tree branches that comprise the closed loop are capacitive branches; and likewise, a resistive link cannot be used in a loop that contains inductive tree branches.

Thus, the final form of the F-matrix is:

$$F = \begin{bmatrix} F_{SC} & 0 & 0 \\ F_{RC} & F_{RG} & 0 \\ F_{LC} & F_{LG} & F_{LM} \end{bmatrix} .$$

If $[j]$ is an n-vector representing the source current such that the i^{th} element of $[j]$ represents the algebraic sum of all the currents flowing in the i^{th} branch of the fundamental cut-set [-F' I], then Kirchhoff's current law can be written as:

$$[-F' \ I] \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = [J] ,$$

where I is an (n - 1) x (n - 1) unit matrix, and F' is the transpose of F, [15].

If the tree branch and link variables are further partitioned according to the elements in the particular branch, we can redefine the tree branch variables as:

$$v_1 = \begin{bmatrix} v_C \\ v_G \\ v_\lambda \end{bmatrix} \text{ and } i_1 = \begin{bmatrix} i_C \\ i_G \\ i_\lambda \end{bmatrix} ,$$

and the link variables as:

$$v_2 = \begin{bmatrix} v_S \\ v_R \\ v_L \end{bmatrix} \text{ and } i_L = \begin{bmatrix} i_S \\ i_R \\ i_L \end{bmatrix} .$$

Thus, all vectors are of the form:

capacitive elements

resistive elements

inductive elements .

The governing differential relationships are:

$$i_C = \frac{d}{dt}[C2\ v_C] \quad \text{and} \quad v_\lambda = \frac{d}{dt}[L2\ i_\lambda]$$

for the tree branch variables, and

$$i_S = \frac{d}{dt}[C1\ v_S] \quad \text{and} \quad v_L = \frac{d}{dt}[L1\ i_L]$$

for the link variables. The remaining algebraic equations complete the defining network equations:

$$i_G = G2\ v_G \quad \text{and} \quad v_R = R1\ v_R\ ,$$

where:

C2 is an $(\alpha \times \alpha)$ matrix representing the tree branch capacitors

G2 is a $(\beta \times \beta)$ matrix representing the tree branch conductors

L2 is a $(\lambda \times \lambda)$ matrix representing the tree branch inductors

C1 is an $(a \times a)$ matrix representing the link capacitors

R1 is a $(b \times b)$ matrix representing the link resistors

L1 is a $(d \times d)$ matrix representing the link inductors.

It is to be noted that C1, C2, R1, R2, G1, G2, L1, and L2 are diagonal matrices. Furthermore, $G1 = R1^{-1}$ and $G2 = R2^{-1}$. Also at this point, the tacit assumption that there are no mutual inductances becomes clear; otherwise, the inductive elements would include mutual terms and would be of the form:

$$\begin{bmatrix} v_L \\ v_\lambda \end{bmatrix} = \frac{d}{dt} \left\{ \begin{bmatrix} L11 & L12 \\ L21 & L22 \end{bmatrix} \begin{bmatrix} i_L \\ i_\lambda \end{bmatrix} \right\} .$$

From the point of view that the final equations are to be solved numerically on a digital computer, it can be seen that the nonlinear network elements can be evaluated in relatively small matrices. Also, any inverse matrix calculations can be readily carried out since the required matrices are in diagonal form.

To rearrange the network equations in a state variable form, we must first define which set of variables should be designated the state variables. Since the final output is to be the voltages and currents of the network as a function of time, a logical choice of state variables would be the set of capacitive tree voltages and inductive link currents. These variables represent independently specifiable initial conditions. It should be noted that this choice of state variables requires that the number of differential equations to be solved be equal to the number of tree-branch capacitors plus the number of link inductors less the number of capacitor loops and inductive cut-sets.

The final form of the equations will be:

$$\dot{X} = AX + BU ,$$

where, $X = \begin{bmatrix} v_C \\ i_L \end{bmatrix}$ is the state vector, and $\dot{X}$ is the time derivative of the vector X .

[A] is the $[(a + \lambda) \times (a + \lambda)]$ "A" matrix as described by Bashkow and Bryant [1], [5], which embeds the topological properties of the network along with the parameters.

[B] is an $[(a + \lambda) \times (b + k)]$ matrix expressing the parametric and topological relationships involved with the network sources, i.e. the drive voltages or currents.

[U] is a $(b + k) \times 1$ source vector, containing only the drive functions of the network (i.e., the current and voltage generators). It includes both dependent and independent sources.

The next task in the generation of the state variable formulation is the elimination of the algebraic equations from the branch and link equations to obtain the general form $\dot{X} = AX + BU$.

From Kirchhoff's laws and the topological relationships, the network equations form the following set:

$$v_S = -F_{SC} v_C + e_S \qquad (1\text{-a})$$

$$v_R = -F_{RC} v_C - F_{RG} v_G + e_R \qquad (1\text{-b})$$

$$v_L = -F_{RC} v_C - F_{LG} v_G - F_{LM} v_\lambda + e_L \qquad (1\text{-c})$$

$$i_C = F'_{SC} i_S + F'_{RC} i_R + F'_{LC} i_L + j_C \qquad (1\text{-d})$$

$$i_G = F'_{RG}\, i_R + F'_{LG}\, i_L + j_G \tag{1-e}$$

$$i_\lambda = F'_{LM}\, i_L + j_\lambda\,, \tag{1-f}$$

where F' denotes the transpose of F .

From these equations, along with the governing differential relationships, the state variable formulation can be obtained.

For the capacitive variables, it can be seen that:

$$- F'_{SC}\, i_S + i_C = - F'_{SC} \frac{d}{dt} [(C1)(F_{SC})(v_C)] + \frac{d}{dt} [C2\; v_C]$$

$$= \frac{d}{dt} [C2 + F'_{SC}\; C1\; F_{SC}]\; v_C\;.$$

Thus Equation (1-d) becomes:

$$\frac{d}{dt} [C2 + F'_{SC}\; C1\; F_{SC}][v_C] = F'_{RC}\, i_R + F'_{LC}\, i_L\;.$$

Let $C = C2 + F'_{SC}\; C1\; F_{SC}$.

Then

$$\frac{d}{dt} (C\; v_C) = F_{RC}\, i_R + F'_{LC}\, i_L\;.$$

Likewise for the inductive variables, let

$$L = L1 + F_{LM}\; L2\; F'_{LM}\;.$$

Then, $\frac{d}{dt} (L\; i_L) = - F_{LC}\, v_C - F_{LM}\, v_G$.

For the resistive variables, let:

$$R = R1 + F_{RG}\; R2\; F'_{RG}\;,$$

and

$$G = G2 + F'_{RG}\ G1\ F_{RG}\ .$$

At this point, it is assumed that the parametric matrices C1 , C2 , R1 , R2 , L1 , and L2 are not explicit functions of time (i.e., $\dot{C1} = \dot{C2} = \dot{L1} = \dot{L2} = \dot{R1} = \dot{R2} = 0$).

If Equation (1-a) is differentiated and added to Equation (1-d), recalling that $C2\ \dot{v}_C = i_C$ and $C1\ \dot{v}_S = i_S$, the result is:

$$C\ \dot{v}_C = j_C + F'_{SC}\ C1\ \dot{e}_S + F'_{RC}\ i_R + F'_{LC}\ i_L\ . \tag{2}$$

Equations (1-c) and (1-b) can be solved for i_R as

$$i_R = R^{-1}\ [e_R - F_{RC}\ v_C - F_{RG}\ R2\ F'_{LG}\ i_L - F_{RG}\ R2\ j_G]\ . \tag{3}$$

Differentiating (1-f) and adding to (1-c) yields:

$$L\ \frac{di_L}{dt} = -\ F_{RC} v_C - F_{LG}\ v_G - F_{LM}\ L2\ \frac{d}{dt}\ (j_\lambda) + e_L\ . \tag{4}$$

Equations (1-b) and (1-e) can be used to solve for v_G ,

$$v_G = G^{-1}\ [F_{RG}\ G1\ e_R - F'_{RC}\ G1\ F_{RC}\ v_C + j_G + F'_{LG}\ i_L]\ .$$

Let $Y = F'_{RC}\ R^{-1}\ F_{RC}$,

$Z = F_{LG}\ G^{-1}\ F'_{LG}$,

$H = F'_{LC} - F_{RC}\ R^{-1}\ F_{RG}\ R2\ F'_{LG}$.

The final form of the state equations can be arrived at from Equations (2) through (4).

Thus far, we have

$$\frac{d}{dt}\begin{bmatrix} C & 0 \\ 0 & L \end{bmatrix}\begin{bmatrix} v_C \\ i_L \end{bmatrix} = \begin{bmatrix} -Y & H \\ -H' & -Z \end{bmatrix}\begin{bmatrix} v_C \\ i_L \end{bmatrix} + [B^*]\begin{bmatrix} j_C \\ j_G \\ \frac{d}{dt}(j_\lambda) \\ \frac{d}{dt}(e_S) \\ e_R \\ e_L \end{bmatrix}, \quad (5)$$

where, $B^* =$

$$\begin{bmatrix} (I) & -(F'_{RC}\ R^{-1}\ F_{RC}\ R2) & (0) & (F'_{SC}\ C1) & (F'_{RC}\ R^{-1}) & (0) \\ (0) & -(F_{LG}\ G^{-1}) & -(F_{LM}\ L2) & (0) & -(F_{LG}\ G^{-1}\ F_{RG}\ G1) & (I) \end{bmatrix}.$$

Equation (5) can now be solved for the standard state-variable form.

$$\frac{d}{dt}\begin{bmatrix} v_C \\ i_L \end{bmatrix} = \begin{bmatrix} -C^{-1}\ Y & C^{-1}\ H \\ -L^{-1}\ H' & -L^{-1}\ Z \end{bmatrix} \cdot \begin{bmatrix} v_C \\ i_L \end{bmatrix} + [B] \cdot \begin{bmatrix} U_T \\ U_L \end{bmatrix}. \quad (6)$$

Where:

$$B = \begin{bmatrix} C^{-1} & 0 \\ 0 & L^{-1} \end{bmatrix} \cdot [B^*] ,$$

U_T is the vector containing all the tree branch drives, and U_L is the vector containing all the tree link drives.

These are further partitioned as follows:

$$U_T = \begin{bmatrix} j_C \\ j_G \\ \frac{dj_\lambda}{dt} \end{bmatrix} \quad \text{and} \quad U_L = \begin{bmatrix} \frac{de_S}{dt} \\ e_R \\ e_L \end{bmatrix},$$

where:

j_C is an $(a \times 1)$ vector containing all the current sources in the capacitive tree branches,

j_G is a $(b \times 1)$ vector containing all the current sources in the resistive tree branches,

j_λ is a $(d \times 1)$ vector containing all the current sources in the inductive tree branches,

e_S is an $(\alpha \times 1)$ vector containing all the voltage sources in the capacitive links,

e_R is a $(\beta \times 1)$ vector containing all the voltage sources in the resistive links, and

e_L is a $(\lambda \times 1)$ vector containing all the voltage sources in the inductive links.

The sign conventions observed with respect to the sources is that a voltage drop is positive in the direction of flow as determined by the link orientation in the loop.

At this point Equation (6) can be used to solve the network equations as long as all the nonlinearities whose independent variables are either capacitive tree branch or inductive link voltages or currents. Since these variables are explicitly solved for, they can be used to determine the nonlinear relationships required to update the parametric matrices or dependent sources. However, if the nonlinearities depend upon the resistive variables Equation (6) does not explicitly contain them since they have just been eliminated. Thus a solution is not obtainable. To require that all nonlinearities be functions of the state variables is much too restrictive to be of any practical use. The most common restriction [6], [11], [10] is to assume that the submatrix F_{RG} be a null matrix. This allows Equations (1) to be readily solved for the resistive variables along with the non-state energy variables (i.e., v_S, i_S, v_λ, i_λ). Examining this restriction shows that it excludes all networks that have loops consisting entirely of resistive branches. This seems an unduly harsh requirement for general networks; thus, an alternative method has been developed.

If i_G from Equation (1-e) is substituted into the defining equation for v_G namely, $v_G = R2\ i_G$, and this, in turn, substituted into Equation (1-a), the latter can be solved for v_R. The result is:

$$v_R = [I + F_{RG}\ R2\ F'_{RG}\ G1]^{-1}\ [e_R - F_{RC}\ v_C - F_{RG}\ R2(j_G + F'_{LG}\ i_L)]. \quad (7)$$

From this, the remaining variables can be determined.

Thus far the restrictions that have been placed on the network are in the nature of the nonlinearities involved. It is assumed that the nonlinearities are not explicit functions of time. This restriction could be removed and the analysis redone, keeping all the new derivative terms (e.g., $\frac{d}{dt}(C\,v_C) = C\,\frac{dv_C}{dt} + v_C\,\frac{dC}{dt}$), but it is felt that the practical need for such time varying parameters is not large enough to justify the resultant calculations. This is especially true in this study where a computer analysis is to be developed from the the techniques derived and all extraneous calculations should be eliminated.

It is also assumed that all nonlinear relationships are single-valued in the controlled variable. This is also true in regard to the time relationships involved in the source functions.

Uniqueness and Existence of the Solution to the Mathematical Model

Given a set of n-differential equations of the form $\dot{x} = X(x,t)$, where, x is an n-vector with components $x_1, x_2, \cdots, x_n$, and X is an n-vector with components $X_1, X_2, \cdots, X_n$, the existence and uniqueness of a solution is guaranteed by the Cauchy-Lipschitz Theorem [12].

Let $E^{n+1}_{x,t}$ denote the space with coordinates $x_1, x_2, \cdots, x_n, t$ and Q be a region contained in that space. Then, the existence theorem can be stated as follows:

Theorem 1

Assume there exists continuous partial derivatives $\frac{\partial X_j}{\partial x_i}$ at every point in Q. Let (x^o, t_o) be a point in Q . Then there exists a unique solution $x(t)$ of the system $\dot{x} = X(x,t)$ such that $x(t_o) = x^o$, and it may be extended throughout Q . Furthermore, this solution is a continuous function of (x^o, t_o) as this point varies throughout Q .

Applying Theorem 1 to the state variable system of equations, we equate:

$$x = \begin{bmatrix} v_C \\ i_L \end{bmatrix}$$

and

$$X(x,t) = A(x)\, x + B(x)\, U(x,t) .$$

The j^{th} element of $X(x,t)$ is given by

$$X_j = \sum_{i=1}^{n} A_{ji}(x)\, x_j + \sum_{i=1}^{n} b_{ji}(x)\, u_j(x,t) .$$

To apply Theorem 1, the partial derivative $\frac{\partial X_j}{\partial x_i}$ must be continuous at every point in Q .

The general form of the elements of X_j depends upon whether $j \leq a$ (i.e., the number of capacitance tree-voltages) or if $j > a$.

Let us first consider the case when $j \leq a$. Then:

$$X_j = \frac{dv_C}{dt} = -C^{-1}Y\ v_C + C^{-1}H\ i_L + C^{-1}\ [jC - F'_{RC}\ R^{-1}\ F_{RG}\ R2\ j_G$$

$$+ F'_{SC}\ C1\ \frac{de_S}{dt} + F'_{RC}\ R^{-1}\ e_R]\ . \qquad (7)$$

From the definition of C, Y, R, and H, it can be seen that they are sums, differences and products of the parametric circuit element matrices R1, R2, L1, L2, C1, and C2 . Likewise for the case when $j > a$,

$$X_j = \frac{di_L}{dt} = -L^{-1}\ H'\ v_C + -L^{-1}Z\ i_L + L^{-1}\ [-F_{LG}\ G^{-1}\ j_G$$

$$-F_{1\lambda}\ L2\ \frac{dj_\lambda}{dt} - F_{LG}\ G^{-1}\ F'_{RG}\ G1\ e_R + e_L]\ . \qquad (8)$$

Again from the definitions of L , and G , it can be seen that they are also sums, differences and products of the parametric matrices.

Thus, X_j can be represented as:

$$X_j = \sum_{i=1}^{k} [m_i(x,t)\ n_i(x,t) + q_i(x,t)],$$

where m(x,t), n(x,t) and q(x,t) are the circuit parameters: R1(v), R1(i), C1(v), L1(i), R2(v), R2(i), C2(v), L2(i), and the source drives J_C, j_G, e_R, e_L ; and the time derivatives of the sources j_λ and e_S .

Requiring $\partial X_j/\partial x_i$ to be continuous is equivalent to requiring the function:

$$\phi = \sum_{i=1}^{k} [m_i(x,t) \frac{\partial n_i}{\partial x}(x,t) + n_i(x,t) \frac{\partial m_i}{\partial x}(x,t) + \frac{\partial q_i}{\partial x}(x,t)]$$

be continuous.

The continuity of ϕ is assured if the partial derivatives are continuous. Thus, it is required that the partial derivatives of the parametric matrices with respect to the state variables exist and be continuous.

In the formulation being considered, no general a priori knowledge is known as to which variables will be selected as the state variables; thus, it is required that the restriction hold for all the parameters concerned. However, in most cases, the nonlinearity of an element will be a function of its terminal voltage or the current flowing through the device. It has further been assumed that a capacitance is voltage controlled, and an inductor current controlled. Thus, the restriction placed on the variables that will insure existence and uniqueness is that the partial derivatives of the parametric matrices with respect to the state variables exist and be continuous along with the partial derivatives of any sources which depend upon the state variables.

Most of the common circuit nonlinearities, (i.e., diodes, tunnel diodes, junction capacitances, etc.,) all satisfy these requirements; thus, the existence and uniqueness of the solution to the equation

$\dot{X} = A(x)\ X + B(x)\ U(x,t)$ is assured if $\frac{\partial C1(v)}{\partial x_i}$, $\frac{\partial L1(i)}{\partial x_i}$, $\frac{\partial C2(v)}{\partial x_i}$, $\frac{\partial L2(i)}{\partial x_i}$, $\frac{\partial R2}{\partial x_i}$, $\frac{\partial G1}{\partial x_i}$, and $\frac{\partial U}{\partial x_i}$ are continuous functions.

Stability of the Analytical Solution

In this section we shall consider the stability of a nonlinear time-invariant network. The stability in question is stability and asymptotic stability in the sense of Liapunov [12]. Liapunov's stability theorems can be stated as follows [12]:

Theorem 2 (Stability Theorem)

If there exists, in some neighborhood Q of the origin, a Liapunov function V(x) , then the origin is stable.

Theorem 3 (Asymptotic Stability Theorem)

If $-\dot{V}$ is likewise a positive definite function in Q , then the stability is asymptotic.

V(x) is a Liapunov function [12], if V(x) is:

(a) positive definite,

(b) V(x) is continuous and its first partial derivatives are continuous in an open region Q about the origin,

(c) $V(0) = 0$,

(d) $V(x) > 0$ for $x \neq 0$ and x in Q, and

(e) $\dot{V} \leq 0$ for x in Q.

In applying Liapunov's second method, the difficulties arise in the construction of a suitable function $V(x)$ and extending it to a system that contains nonlinear functions and possibly dependent sources.

To overcome the latter problem, we can apply a theorem given by Boyer [3] as adapted from LaSalle and Lefschetz [12]. This theorem is stated as follows:

<u>Theorem 4 (Stability Under Persistent Disturbances)</u>

Let $f(t,o) = 0$ and $r(t,o) = 0$ for $t \geq 0$. Suppose there exists a Liapunov function $V(x,t)$ for $\dot{x} = f(t,x)$ in the region $B(A) = \left\{ (t,x) : ||x|| < A \text{ and } t \geq 0 \right\}$. Suppose $V(x,t)$ is bounded by a $W_2(x)$; and $\dot{V}(t,x)$ is negative definite and that the first partial derivatives of V are bounded for $t \geq 0$ in $B(A)$. Then given ϵ, such that $0 < \epsilon < A$, there corresponds two numbers $n_1(\epsilon) > 0$, $n_2(\epsilon) > 0$, such that if $||x(0)|| < n_1(\epsilon)$, and $||r(t,x)|| < n_2(\epsilon)$ for all $||x|| < \epsilon$ and $t \geq 0$, then the origin is stable for:

$$\dot{x} = f(t,x) + r(t,x) ,$$

where $||x||$ is the Euclidean norm: $||x|| = \sqrt{x'x} = \left(\sum_{i=1}^{n} x_i^2\right)^{\frac{1}{2}}$.

Before anything can be determined, a Liapunov function has to be found. To this end, let us consider a function E which represents the total energy stored in the reactive elements.

The function E can be represented in the following form:

$$E = \frac{1}{2}[v_S' \; v_S'] \begin{bmatrix} C1 & 0 \\ 0 & C2 \end{bmatrix} \begin{bmatrix} v_S \\ v_C \end{bmatrix} + \frac{1}{2}[i_L' \; i_\lambda'] \begin{bmatrix} L1 & 0 \\ 0 & L2 \end{bmatrix} \begin{bmatrix} i_L \\ i_\lambda \end{bmatrix}$$

$$= \frac{1}{2} \; v_S' \; C1 \; v_S \; C2 \; v_C + i_L' \; L1 + i_\lambda' \; L2 \; i_\lambda \quad .$$

It can be recalled that:

$$v_S = -F_{SC} \; v_C \quad .$$

Thus,

$$v_S' = -(F_{SC} \; v_C)' = -v_C' \; F_{SC}' \quad .$$

Hence,

$$v_S' \; C1 \; v_S = (-v_S' \; F_{SC})(C1)(-F_{SC} \; v_C) = v_C' \; F_{SC}' \; C1 \; F_{SC} \; v_C \; ;$$

from which

$$v_S' \; C1 \; v_S + v_C' \; C2 \; v_C = v_C' \; F_{SC}' \; C1 \; F_{SC} \; v_C + v_C' \; C2 \; v_C \quad ,$$

$$= v_C' \; [F_{SC}' \; C1 \; F_{SC} + I] \; v_C = v_C' \; C v_C \quad .$$

Likewise,

$$i_\lambda = F_{LM} \; i_L \quad ,$$

$$i_\lambda' = i_L' \; F_{LM}' \quad ,$$

and

$$i_L' \, L1 \, i_L + i_\lambda' \, L2 \, i_\lambda = i_{LM}' \, L1 \, i_L + i_L' \, F_{LM}' \, L2 \, F_{LM} \, i_L$$

$$= i_L' \, (L1 + F_{LM}' \, L2 \, F_{LM}) \, i_L = i_L' \, L \, i_L \; .$$

Thus, the function E can be written:

$$\frac{1}{2} \left\{ v_C' \, C \, v_C + i_L' \, L \, i_L \right\} \; .$$

For $t = 0$, $v_C(0) = i_L(0) = 0$,

Hence, $E(0) = 0$.

Furthermore,

$$\frac{1}{2} v_C' \, C \, v_C + \frac{1}{2} i_L' \, L \, i_L = \frac{1}{2} [v_C' \; i_L'] \begin{bmatrix} C & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix}$$

$$= \frac{1}{2} x' \begin{bmatrix} C & 0 \\ 0 & L \end{bmatrix} x \; .$$

Let

$$\begin{bmatrix} C & 0 \\ 0 & L \end{bmatrix} = Q \text{ , then } E = \frac{1}{2} \, x'Qx = \sum_{i=1}^{n} \sum_{j=1}^{n} q_{ij} \, x_i \, x_j \; .$$

Since C and L are positive definite, it follows that Q is also positive definite and $E(x,t) > 0$ for all $t \geq 0$ and for $x \neq 0$.

Next, let us consider the time derivative of E:

$$\frac{dE}{dt} = \frac{1}{2}\frac{d}{dt}\left[\sum_{j=1}^{n}\sum_{i=1}^{n} q_{ij}\, x_i\, x_j\right]$$

$$= \frac{1}{2}\left[\sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij}\, \dot{x}_i\, x_j + \sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij}\, x_i\, \dot{x}_j\right]$$

$$= X'\, Q\, \dot{X}$$

$$= [v_C'\ i_L'] \begin{bmatrix} C & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \dot{v}_C \\ \dot{i}_L \end{bmatrix} = v_C'\, C\, \dot{v}_C + i_L'\, L\, \dot{i}_L \ .$$

From the defining equations:

$$\dot{v}_C = - C^{-1}\, Y\, v_C + C^{-1}\, H\, i_L$$

$$\dot{i}_L = - L^{-1}\, H'\, v_C - L^{-1}\, Z\, i_L \ .$$

Substituting these into the derivative expression yields:

$$\frac{dE}{dt} = - v_C'\, C\, C^{-1}\, Y\, v_C + v_C'\, C\, C^{-1}\, H\, i_L - i_L'\, L\, L^{-1}\, H'\, v_C - i_L'\, L\, L^{-1}\, Z i_L$$

$$= - v_C'\, Y\, v_C - i_L'\, Z\, i_L + v_C'\, H\, i_L - i_L'\, H'\, v_C$$

$$= -v_C'\, Y\, v_C - i_L'\, Z\, i_L$$

$$\frac{dE}{dt} = - \left\{ (v'_C \ i'_L) \begin{bmatrix} Y & 0 \\ 0 & Z \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} \right\} ,$$

$$\frac{dE(x,t)}{dt} = - \left\{ X' \begin{bmatrix} Y & 0 \\ 0 & Z \end{bmatrix} X \right\} .$$

Thus, $\frac{dE}{dt}$ is < 0 if $\begin{bmatrix} Y & 0 \\ 0 & Z \end{bmatrix} \geq 0$. This leads to the following theorem.

<u>Theorem 5</u>

A time-invariant nonlinear network with positive inductances and capacitances and without mutual inductances of the form $\dot{X} = AX$ is stable, in the sense of Liapunov, if:

$$Y = F'_{RC} \, (R1 + F_{RG} \, R2 \, F'_{RG})^{-1} \, F_{RC} \geq 0$$

and

$$Z = F_{LG} \, (G2 + F'_{RG} \, G1 \, F_{RG})^{-1} \, F'_{LG} \geq 0 \quad .$$

If the conditions of Theorem 5 are satisfied, a Liapunov function for the state variable formulation is given by:

$$V = \frac{1}{2} X' \begin{bmatrix} C & 0 \\ 0 & L \end{bmatrix} X . \qquad (9)$$

Having found a satisfactory Liapunov function, we can now proceed to apply Theorem 4. First, we equate:

$$f(t,x) = A(x) \quad \text{and}$$

$$r(t,x) = B(x)\, U(x,t) \quad .$$

The required function V is given by Equation (9); and since V of Equation (9) is positive definite and independent of t , the function $W_2(x)$ of Theorem 4 can be generated by adding a positive constant to V . Furthermore, if the conditions of Theorem 5 are met, the function $\dot{V}$ is negative definite.

Next, we must insure the boundedness of the first partial derivatives of V . For existence and uniqueness it was required that the partial derivatives of the parametric matrix functions with respect to x_i exist and be continuous. For stability, it will further be required that the partial derivatives be bounded. Since V is function of C and L , which in turn are functions of L1, L2, C1 and C2, the first partial derivatives of V are composed of sums and products of L1, L2, C1, C2, and their first partial derivatives. Thus, these partial derivatives of V are required to be bounded. Furthermore, V and, hence, W exist where A exists; thus, the region B(A) can be chosen as the entire state space. Therefore, we can conclude that the requirements for existence and uniqueness and the additional requirement of boundedness also suffice for Theorem 4 and hence the stability of the solution to

$$\dot{X} = A(x)X + B(x)\, U(x,t) \qquad (10)$$

is assured.

<u>Enumeration of the Conditions Required to Insure a Unique Stable Analytical Solution</u>

The following conditions will guarantee the existence of a unique stable solution to Equation (10).

(a) The first partial derivatives of the parametric matrices C1, C2, L1, L2, R2 and G1, and the source function U(x,t) exist and be continuous, and the partial derivatives C1, C2, L1 and L2 be bounded.

(b) R^{-1}, G^{-1}, C^{-1}, and L^{-1} exist,

(c) $Y = F'_{RC}\,(R1 + F_{RG}\, R2\, F'_{RG})^{-1}\, F_{RC} \geq 0$, and

(d) $Z = F_{LC}\,(G2 + F'_{RG}\, G1\, F_{RG})^{-1}\, F_{LC} \geq 0$.

CHAPTER III

DEVELOPMENT AND DISCUSSION OF THE COMPUTER MODELS AND MAJOR ALGORITHMS

General Discussion

The overall objectives of a computer oriented analysis are that it be fast, flexible, and solve the problem at hand with the highest degree of fidelity possible. These objectives have been met by using the state variable formulation of the network analysis problem. Using this approach, only the minimum number of differential equations need be solved numerically. Since this is the most time consuming operation in the numerical analysis, it is of prime importance. Furthermore, the state variable form by its nature partitions the network and the corresponding parametric matrices into many smaller matrices. This allows recalculating only those matrices in which nonlinearities occur. In addition, matrix inversion, where required, is greatly accelerated because the matrices are small and the time required to invert an $n \times n$ matrix is approximately proportional to n^3 .

Specifically, the program developed consists of three basic sections under control of executive routines. These sections logically perform initialization, topological analysis, and solve the resultant nonlinear differential equations. The output of the program consists of the topological and initial value parameters and matrices and a listing of all the currents and voltages as a function of time, from time $t = 0$ to a specified final time. A complete set of flow charts of all the computer programs is given in Appendix B.

Computer Algorithms

The initialization phase of the program reads in the schematic circuit description, nonlinear relationships, and parameter values. The circuit description and parameters are printed out for documentation and verification. All necessary programming housekeeping and initialization are carried out in this section.

The next portion of the program consists of the topological analysis section. Here, a vector containing all the nodes is formed along with a corresponding vector comprising all the branches connected to these nodes. These two vectors become the guide for the ensuing topological formulations. First, the list of branches is searched and compared to the parameter matrices to determine if the branch contains a resistance, a capacitance, or an inductance. If a branch contains an inductance, it is listed as a link and a forced-link. All branches are thus checked and, as the links are removed, the corresponding entries in the connection vector are labeled as used-branches. The actual formulation of the tree is initiated arbitrarily at the first node. The corresponding entries in the connection vector are searched for unused branches. If any of the unused branches contain a capacitor, it is chosen to be a tree branch and its terminal node chosen as the next node for examination. The initial node is labeled as used. If at any time the terminal node has been used, the branch is designated a link and another branch is examined. If all branches emanating from a node are used, the previously used node is examined. This process of retracing is continued back to the first node. When all tree branches have been chosen, the remaining branches are designated as links. After all branches have been used, a check is made to insure that the number of links equals the number of branches less the number of nodes plus one. When this condition is satisfied, the incidence matrix is formed.

To further clarify this procedure, consider the circuit shown in Figure 3-1.

The nodal vector would contain the nodes a, b, ..., m . The corresponding connection vector would be:

branches: $\underbrace{1,\ 2,\ 4,}_{a}\ \ \underbrace{9,\ 12,\ 3,\ 1,}_{b}\ \ \ldots\ \underbrace{18,\ 19\ ,}_{m}$

for node a b ... m .

Initially, the branches 3, 5, 10, 11, and 16 would be listed as forced-links. It should be noted that branches 10, 5, and 11 form an inductive cut-set.

Let us arbitrarily assume a starting point at node (a). Since no capacitive branches have node (a) as an initial or terminal node, a resistive branch would be chosen. Because of its numerical order, branch 1 would be taken. At node (b), branch 9 would be selected. The tree generation would continue through the network choosing branches via a C-R-L hierarchy. This procedure will continue through the network to node (l), at which time the tree would appear as branches 1, 9, 18, 19, and 17. At node (i) branch 2 was rejected since its terminal node is (a) which has already been included in the tree. From node (l), a back-up process would start retracing through the used nodes searching for unused branches. At node (h), branch 13 would be taken into the tree. From node (d), a retrace would again terminate at node (h). This time, branches 14 and 15 would be added. Once more, a back-up retrace would start; however, this retrace would continue all the way back to node (a) and the existing tree would consist of branches 1, 9, 18, 19, 17, 13, 14, 12, and 15. When the number of nodes and branches are checked against the links formed, it becomes evident that part of the tree is missing. The forced-links are examined to determine whether any of them have one node in the present tree and the other node not used. If this case exists, as it does here, the link in question is removed from the forced-link and link lists. This procedure would take branch 5 into the tree, thus removing the inductive cut-set. From node (e), the tree generation would continue picking up branches 7 and 6. A check of the consistence requirements would yield agreement and the final tree would be as shown in Figure 3-2.

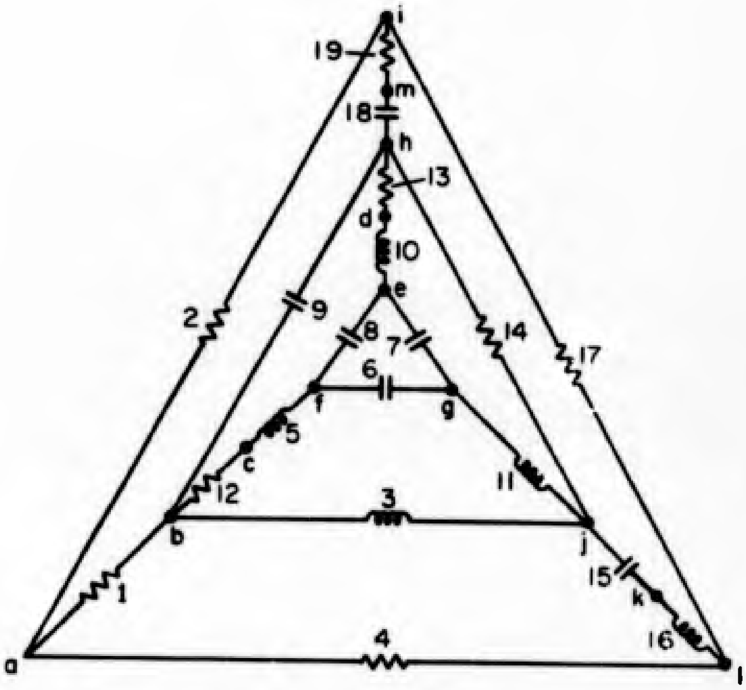


Figure 3-1. Topological Test Circuit Containing a Capacitive Loop and an Inductive Cut-Set

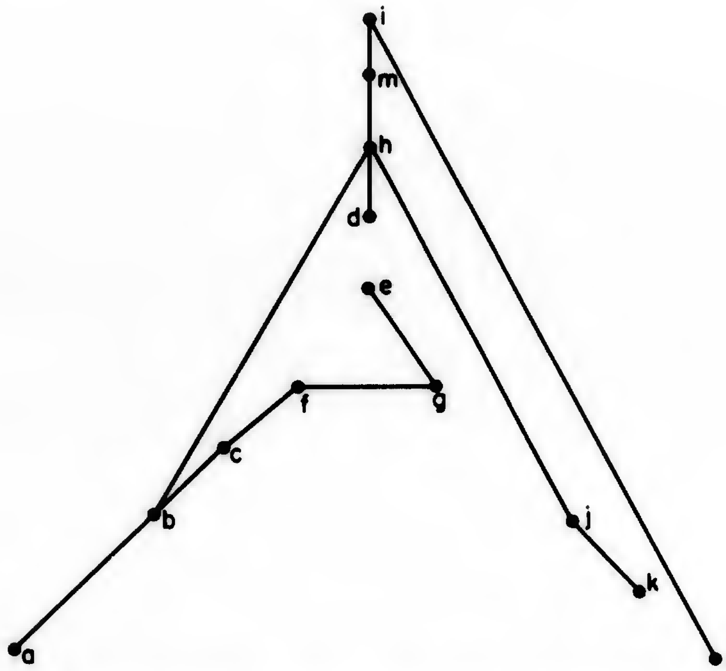


Figure 3-2. Network Tree Selected for Circuit of Figure 3-1

The circuit of Figure 3-1 was chosen to illustrate the procedure of selecting a normal tree from a network containing a capacitive loop and an inductive cut-set. To proceed to illustrate the formation of the incidence and parameter matrices with this circuit would unnecessarily complicate the issue, so let us consider a less complex circuit such as shown in Figure 3-3.

In the previous example, the network was represented as an undirected graph whereas, in reality, it is directed. The assumed direction is from the initial node to the terminal node. This is also the assumed direction of positive current flow and potential drop.

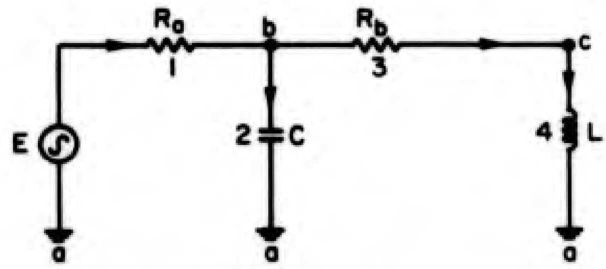
The fundamental loop matrix is formed from the normal tree. It is an incidence matrix whose columns represent tree branches and whose rows represent tree links. The entries are f_{ij} .

f_{ij} = + 1, if the j^{th} branch is included in the loop formed by closing the i^{th} link, and if the direction of the j^{th} branch in the tree is the same direction as that of the loop;

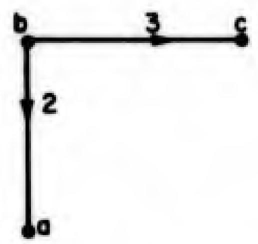
f_{ij} = - 1, if the direction of the j^{th} tree branch opposes that of the loop; and

f_{ij} = 0, if the j^{th} branch is not included in the loop formed by the i^{th} link.

The direction of the loop is chosen to be such that the link is positively oriented. Both the columns and rows of the loop matrix are arranged according to the C-R-L hierarchical structure.



CIRCUIT SCHEMATIC DIAGRAM



NETWORK NORMAL TREE

Figure 3-3. An R-L-C Circuit Used to Illustrate Topological Techniques

For the circuit of Figure 3-3, the loop matrix is given below:

$$F = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} .$$

The parametric matrices associated with the network are formulated at this point.

For the circuit of Figure 3-3, these matrices are:

$R1 = R_a$,	$C1 = 0$,	$L1 = L$,
$R2 = R_b$,	$C2 = C$,	$L2 = 0$.

Since this case is so elementary, the matrices are scalars.

At this point, the topological analysis of the network is completed.

However, prior to considering the next facet of the analysis a special case was encountered in the testing of these algorithms. This case is pictured in Figure 3-4 where the network can be considered as two separate graphs, S_1 and S_2, connected by a resistive branch and having one node in common which is already included in the tree. By topological consideration, both nodes (a) and (b) are required to be in the tree. As the tree-generating algorithm proceeds to node (a), it rejects the branch with C_2 because node (o) has already been incorporated into the tree. The algorithm will choose the branch with R and incorporate it into the tree. From this point, the algorithm proceeds to select a good network tree. However, included with the links is C_4 and this violates the condition for a normal tree (i.e., a capacitive link must form a loop with only capacitive tree branches). The algorithm does not detect this condition, but in the formation of the F-matrix, the submatrix F_{SG} is not empty. This condition is checked for in the matrix formulation section of the analysis. If this condition is detected, a reentry to the tree generating routine occurs. Everything is re-initialized and a forced-tree branch section is entered. Here all the tree branches are searched and any that contain capacitors are listed in a forced-tree list and the tree list. When the search is completed, a topological review of the forced-branch list is conducted to determine if any capacitance loops exist. All existing loops are opened by removing a branch and listing it as a link. The remainder of the algorithm proceeds as before. It can be seen for the circuit of Figure 3-4 the requirement that the capacitive branches be in the tree forces in the branch R into being a link as it must for the normal-tree formulation.

The next major algorithm encountered is the generation of the parametric matrices. Using the tree as a guide, the parameters are separated into tree and link parameters and the resultant inductance, resistance, and capacitance matrices are formed.

The next group of algorithms use the incidence matrix and the parametric matrices to generate the state variable formulation as developed in Chapter II.

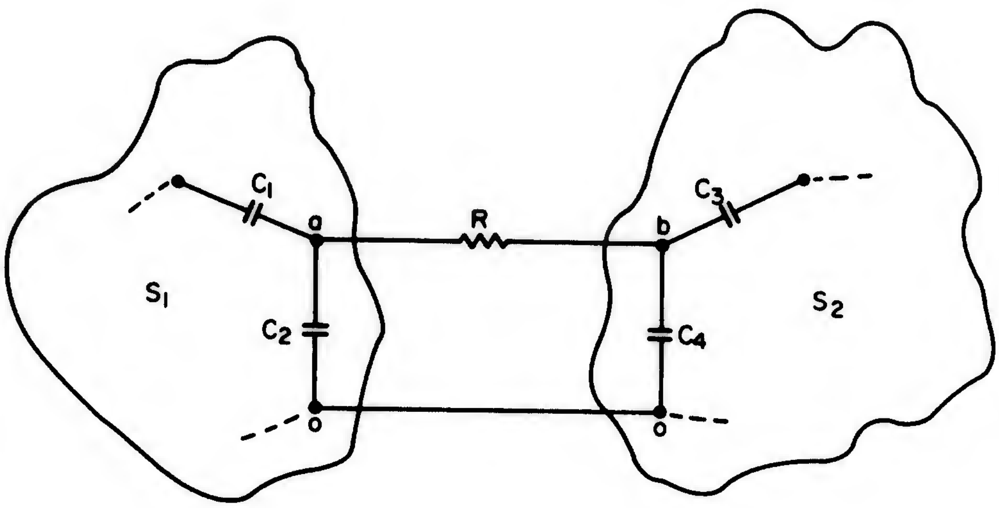


Figure 3-4. Network Subgraphs Connected by a Resistive Branch

One of the major sections of the computational analysis is the solution of the nonlinear differential equations. This is accomplished by numerical integration. In selecting a numerical integration scheme for obtaining the solution of the differential equations that describe the nonlinear network, several factors had to be considered. First, the scheme or routine had to be able to operate at a variable time increment. Second, the routine had to have as small an error term as possible or by the same token had to be of the highest order in accordance with a given criterion. Third and last, the criterion required in the second requirement had to be determined.

To choose a routine that has a variable step requires that a comparison be made to determine where and how the step is to be altered. The ideal situation would be to compare the solution with the known actual solution, but this can be realized only in the trivial case. Thus, another comparison had to be made. The only practical choice was to compare a predicted (P_n) solution for the n^{th} step with the corrected value (C_n); hence, the method should be of the predictor-corrector type. This eliminates the one-step methods such as the Runge-Kutta methods. Having decided on varying the interval by examining the relationship between the predictor and corrector, the exact relationship had to be determined as well as the amount and direction of change required. The latter choice can be made by saying that the interval can be halved or doubled. This is a standard choice. However, when considering electrical networks where such functions as square waves, pulsed drives and negative resistance regions exist, it is wise to consider the possibility of continually halving the interval to obtain a suitable one. With this in mind, a logical solution is to halve the interval at first, but if two successive halvings are required, accept the last good point as a new initial starting point and start out at some small increment. Also, doubling the interval must be kept in check so that some significant data is not overlooked (e.g., a narrow pulsed driving function). Both of these criteria can be met by the following scheme which was used. A given maximum interval h is supplied. This can be chosen as naively as the spacing desired for the actual computer printout. Initially, this interval is reduced by some factor 2^N ($N = 10$ is not unreasonable). At this point, the philosophy is to double the interval whenever possible up to the maximum value. If at any point the interval cannot be doubled, it is either halved or remains at its present value. If halving is required twice in succession, the last accepted point is taken as a new starting point, using $h/2^N$ as the interval. This method allows the running interval to adjust to a proper value regardless of the initial h chosen. The next question to be resolved was: what specific relationship between predictor and corrector should be used? Since the error is measured by Pn - Cn from step to step, it was decided to use the following scheme:

If $\quad |p_{n+1} - c_{n+1}|/|p_{n+1}| \leq \epsilon_1$, then double h ;

if $\quad |p_{n+1} - c_{n+1}|/|p_{n+1}| \geq \epsilon_2$, then halve h ;

if $\epsilon_1 \quad |p_{n+1} - c_{n+1}|/|p_{n+1}| < \epsilon_2$, then allow h to remain as it is. Here it was assumed that these criteria would have to be met by all n equations in an n^{th}-order system.

With the variable interval now well in hand, the second factor, namely, how high an order system can be practically chosen, was considered next. Since a myriad of predictor-corrector methods exist, many of which are about the same order, a choice had to be made by another means than coin-flipping. Once again, considering the fact that the equations arise from nonlinear electrical networks which can result in a sizable system with lengthy calculations required for evaluation, a reasonable criterion would be to keep the required substitutions into the system at a minimum. Indeed, this was the criterion chosen as a figure of merit, along with the requirements of stability of the solution. Since the Milne method is unstable for many occurrences, it was disregarded in its unmodified form. This left such methods as Adams-Bashforth, Adams-Moulton, and Hamming's Modified Predictor Corrector Method [7]. The latter system was chosen since it was of high order and required minimal substitution.

This procedure is as follows:

Predict: $$p_{n+1} = \frac{2y_{n-1} + y_{n-2}}{3}$$

$$+ \frac{h}{72} (191\, y'_n - 107\, y'_{n-1} + 109\, y'_{n-2} - 25\, y'_{n-3}) + \frac{707}{2,160} h^5 y^{(5)} .$$

Modify:

$$m_{n+1} = p_{n+1} - 707/750\, (p_n - c_n)$$

Correct:

$$c_{n+1} = \frac{2y_{n-1} + y_{n-2}}{3} + \frac{h}{72} (25m'_{n+1} + 91y'_n + 43y'_{n-1} + 9y'_{n-2})$$

$$- \frac{43}{2,160} h^5 y^{(5)} .$$

Final Value:

$$y_{n+1} = c_{n+1} + 43/750\, (p_{n+1} - c_{n+1}) .$$

In this process we get an exact answer if $y^{(5)}(x) = \text{constant}$, so that it may be called a sixth-order method.

Since the method is not self starting, it requires four starting values. The first is supplied by the initial conditions, the next three by a Runge-Kutta method. These formulas for $y' = f(t,y)$ are:

$$y_{n+1} = y_n + h\,\phi\,(t, y, h)\ ,$$

where

$$\phi(t, y, h) = \frac{1}{6}[k_1 + 2\,k_2 + 2\,k_3 + k_4]\ ,$$

$$k_1 = f(t, y)\ ,$$

$$k_2 = f(t + h/2,\ y + \frac{1}{2}\,h\,k_2)\ ,$$

$$k_3 = f(t + h/2,\ y + \frac{1}{2}\,h\,k_2)\ ,$$

$$k_4 = f(t + h,\ y + h\,k_3)\ .$$

Finally, formulas were required for halving and doubling. The latter is most easily accomplished by carrying sufficient back values. For a suitable halving formula, the value of $y_{n-1/2}$ can be obtained from y_n, y_{n-1}, y_{n-2}, and their derivatives, while making it exact for $1, t, \ldots t^5$. Thus, the error depends on $y^{(6)}$. The required formulas are:

$$y_{n-1/2} = 1/128[45y_n + 72y_{n-1} + 11y_{n-2} + h(-9y'_n + 36y'_{n-1} + 3y'_{n-2})]\ .$$

$y_{n-3/2}$ was obtained by reversing the formula. This results in

$$y_{n-3/2} = 1/128[11y_n + 72y_{n-1} + 45y_{n-2} - h(3y'_n + 36y'_{n-1} - 9y'_{n-2})]\ .$$

Table 1 shows the data distribution required for running. Where $j = n - 2$, no interval change is allowed and the next three points are calculated. If, after calculation y_{n+1}, an interval change is required, it is performed and the values just calculated are discarded. This resets j to the value $n-2$. If no change is made for y_{n+1}, it then replaces y_n, and y_{n-1} is replaced by y_n, and so on through the whole list. A similar table applies for the derivatives.

Experimental Verification

To more closely examine the advantages of using the Hamming method, several comparisons were made with the Runge-Kutta method. The problem chosen for this purpose was the second-order equation:

$\ddot{x} + 2\,ab\,\dot{x} + bx = 0$, $\dot{x}(0) = 3/2$, $x(0) = 0$, with values $a = 1/2$ and $b = 1$ the exact solution is:

$$x = e^{-t/2} \sin \frac{\sqrt{3}}{2} t \ .$$

Since both the Runge-Kutta and Hamming's methods are for the system of equations of the form:

$$y' = f(x, y) \ , \ x > 0$$

$$y = y_0 \ , \ x = 0 \ .$$

TABLE 1

STORAGE OF PAST HISTORY REQUIRED BY THE NUMERICAL INTEGRATION ALGORITHM

y_j	Starting	Doubling	Halving
j = n-6	Initial value	y_{n-6}	$y_{n-3/2}$
j = n-5	Runge-Kutta	y_{n-4}	y_{n-1}
j = n-4	Runge-Kutta	y_{n-2}	$y_{n-1/2}$
j = n-3	Runge-Kutta	y_n	y_n
j = n-2	Hamming	Hamming	Hamming
j = n-1	Hamming	Hamming	Hamming
j = n	Hamming	Hamming	Hamming

The second-order equation was made into two first-order equations. Thus, the system solved was:

$$\dot{x} = y \ , \ x(0) = 0$$

$$\dot{y} = x - y \ , \ y(0) = \sqrt{3/2} \ .$$

Table 2 shows the tabulation of the cases considered.

In each case, the exact solution was also calculated for comparison. Examination of these results showed that for the cases where the increment was chosen judiciously $(h = 0.1)$, both the Runge-Kutta and Hamming methods yielded the exact solution to five places. However, when the increment was chosen an order of magnitude larger, the Runge-Kutta essentially failed and yielded only one significant digit, whereas the Hamming method still yielded five correct digits. Furthermore, in the latter case (case 4), the interval was continually doubled until it reached a value of 0.25 where it remained. From these four cases, it can be seen that the Hamming method has a decided advantage over the Runge-Kutta or any other fixed increment method. This advantage is especially important in nonlinear network analysis where time constants, as such, are nonexistent and hence no easily obtainable criterion for choosing an interval is available.

The remainder of the algorithms deal with recovering the eliminated resistive and non-state energy variables as required

TABLE 2

PARAMETERS USED IN THE HAMMING AND RUNGE-KUTTA COMPARISON

Case	Method	h	ϵ1	ϵ2
1	Runge-Kutta	0.1	--	--
2	Hamming with interval halving	0.1	10^{-2}	10^{-4}
3	Runge-Kutta	1.0	--	--
4	Hamming with interval halving	1.0	10^{-4}	10^{-7}

by the nonlinear calculations. At initialization, the independent variables with the exception of time are specified as functions of a branch variable, i.e., voltage or current. Likewise, all dependent variables are related to the network branches. However, at execution time, the variables are related to the tree branches and links. Thus, a mapping of network branches to links, and network branches to tree branches had to be constructed. In the case of nonlinear resistance, the instantaneous resistance is replaced by a resistance equal to $\frac{dv}{di}$ at the point in question along with a series voltage source equal to the intercept on the voltage axis.

Uniqueness and Existence of the Numerical Solution

With the exception of the numerical integration algorithm, the restrictions of Chapter II are sufficient to insure a unique stable solution. For numerical integration, the unique existence of the initial value problem is assured by the following theorem [8].

Theorem 6

Let the real function $f(t,y)$ be defined in the strip $0 \leq t \leq t_f$, $-\infty < y < \infty$ and satisfy a Lipshitz condition; i.e., there exists a constant L such that for any t in $[0,t_f]$ and any two numbers y and y^*, $|f(t,y^*) - f(t,y)| \leq L|y^* - y|$, and let y_0 be a given number, then there exists exactly one function $y(t)$ with the following properties:

(a) $y(t)$ is continuous and differentiable for t in $[0,t_f]$;

(b) $\dot{y}(t) = f(t, y(t))$, t in $[0,t_f]$;

(c) $y(0) = y_o$.

The numerical integration formulas are in the form of a general linear k-step method [15], i.e.,

$$\sum_{j=o}^{k} \alpha_j y_{n+j} = h \sum_{j=o}^{k} \beta_j Y'_{n+j} (Y_{n+j}, t_{n+j}) \quad n = 0,1,2,\ldots \quad (11)$$

In the case of the predictor equation, $k = 4$, and for the corrector equation, $k = 3$.

It has already been required by Theorem 6 and the restrictions of Chapter II that $f(x,t) = A(x) X + B(x) U(x,t)$ satisfy a Lipshitz condition. If the relation (11) considered as an equation for y_{n+k} has a unique solution for arbitrary values of $y_n, y_{n+1},\ldots,y_{n+k-1}$, then the difference Equation (11) has a unique solution y_n (t_n in $[0,t_f]$) for arbitrarily chosen initial values $y_o, y_1,\ldots,y_k$. For the predictor equation $\beta_4 = 0$, thus Equation (11) can be solved explicitly for y_{n+4} as a function of $y_n, y_{n+1},\ldots,y_{n+3}$ which is defined for all values of its arguments. However, for the correction equation $\beta_k = \beta_3 \neq 0$ and Equation (11) is a nonlinear equation in y_{n+k} . To analyze this situation, recalling $\alpha_k = \alpha_3 = 1$ and $\alpha_o = 0$, let

$$y = F(y) = h\,\beta_3\, f(y,\, t_{n+3}) + h\Big\{[\beta_2\, f_{n+2} + \beta_1\, f_{n+1} + \beta_o f_n] - [\alpha_2\, y_{n+2} + \alpha_1\, y_{n+1}]\Big\}\ , \qquad (12)$$

where

$$y = y_{n+k}$$

and $\frac{dy}{dt} = f(y,t)$.

If $y(0)$ is the first approximation, a generalized iteration procedure is described by:

$$y^{(m+1)} = F\,(y^{(m)}),\ m = 0,\ 1,\ 2, \ldots \ . \qquad (13)$$

Then, for sufficiently small values of h , the following theorem proves the convergence of the sequence $\left\{ y^{(m)} \right\}$ to a solution of Equation (12) and also assures the uniqueness of the solution [15].

Theorem 7

Let $F\,(y)$ be defined for $-\infty < y < \infty$ and let a constant K exist such that $0 \leq K < 1$, and $|F(y^*) - F(y)| \leq K|y^* - y|$ for arbitrary y^* and y . Then the following statements hold:

(a) Equation (12) has a unique solution y.

(b) For arbitrary $y^{(o)}$ the sequence defined by $y^{(m+1)} = F\left(y^{(m)}\right)$, $m = 0, 1, \ldots$ converges to y.

(c) For $m = 1, 2, \ldots$ the following estimates hold:

$$|y - y^{(m)}| \le \frac{K}{1-K} |y^{(m)} - y^{(m-1)}| \le \frac{K^m}{1-K} |y^{(1)} - y^{(o)}| \quad .$$

Since $f(t,y)$ satisfies a Lipshitz condition with respect to y with a Lipshitz constant L, the constant K required for the theorem is given by $K = |h\beta_k \alpha^k| L = |h\beta_3| L$, and this can be made less than 1 .

Convergence of the Numerical Solution

A linear multistep method is convergent if for all functions $f(t,y)$ satisfying the conditions of Theorem 6 and if for all values y_o

$$\dot{y} = f(t,y), \; y(0) = y_o \quad ,$$

then the $\lim(y_n) = y(t)$ as $h \to o$ and $t_o = t$ holds for all t in $[o,t_f]$ and for all solutions $\{y_n\}$ of the difference Equation (11) having starting values $y_x = N_x(h)$ satisfying

$$\lim_{h \to o} N_x(h) = y_o, \; x = 0, 1, \ldots, k-1 \quad .$$

Let us denote by $P(x)$ the characteristic polynomial of a linear multistep method. $P(x)$ is defined as:

$$P(x) = \sum_{j=o}^{k} \alpha_j x^j \quad .$$

The condition of stability imposed on the characteristic polynomial $P(x)$ is a necessary condition for convergence as stated in the following Theorem [15].

Theorem 8

A necessary condition for convergence of the linear multistep method, Equation (11), is that the modulus of no root of the characteristic polynomial $P(x)$ exceeds 1, and that the roots of modulus 1 be simple. For the predictor equation, the polynomial is:

$$P(x) = x^4 - \frac{2}{3}x^3 - \frac{1}{3}x^2 = 0 ,$$

which has roots of modulus 0, 0, $\frac{1}{3}$, and 1.

For the corrector, the polynomial $P(x)$ is given by:

$$P(x) = x^3 - \frac{2}{3}x^2 - \frac{1}{3}x = 0 ,$$

which has roots of modulus 0, $\frac{1}{3}$, and 1 .

Hence, the predictor and corrector both satisfy the conditions imposed by Theorem 8 and the method converges.

Thus, the numerical solution exits uniquely and converges to the true solution.

CHAPTER IV

NUMERICAL EXPERIMENTS AND VERIFICATIONS

General Discussion

Many numerical experiments were conducted during the development and testing of the computational algorithms. Topological tests were run to insure the selection of a normal tree under varied circumstances. The numerical integration program was tested separately to insure its performance under the most adverse as well as normal conditions. The system as a whole was tested and run with various circuits. Three of these circuits are described here and the results evaluated in detail. The first of these circuits is a simple linear R-L-C circuit. This circuit was chosen to show the comparison to a closed form solution. It also tests out the trivial case which many elegant systems fail to test and become disturbingly difficult to solve using elaborate techniques. The second circuit tested as a nonlinear circuit that has a closed-form approximate solution for comparison. The final circuit tested is of the type the system was designed for; i.e., a small circuit with multiple nonlinearities that does not possess a closed-form analytical solution. The circuit chosen for this test was a tunnel diode switch with a nonlinear junction capacitance.

These numerical experiments verify the analytical techniques and computational algorithms. Computer output listings given in Appendix C show the detailed values of these experiments.

Topological Experiments

Other than the computer debugging tests, the topological algorithms were tested mainly to insure the selection of a normal tree. The main violations tested for were the existence of an inductive cut-set consisting of links, capacitive circuits consisting exclusively of tree branches, and capacitive loops with a resistive or inductive link.

A typical test of the simultaneous existence of a capacitive loop and an inductive cut-set is given in Figures 3-1 and 3-2. The results are analyzed in detail in Chapter III.

The circuits of Figure 4-1 and 4-2 were used to test for capacitive loops. The former circuit possesses a tight loop consisting of branches 5 and 7, while the latter has a larger loop consisting of branches 5, 6, and 8, and a second loop consisting of branches 2, 4, 6, and 8.

Tests of the Numerical Integration Algorithm

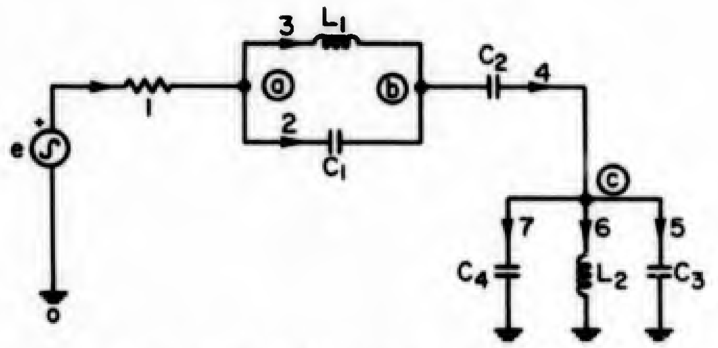
Aside from the initial comparisons described in Chapter III, the integration algorithm was tested against the linear fourth-order equation:

$$y^{iv} - y^{iii} - 7y'' + y' + 6y = 0 \ , \ y(o) = 4, \ y'(o) = 1, \ y''(o) = 15$$

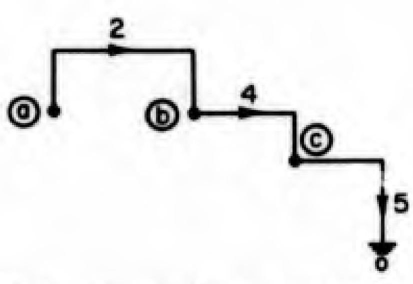
and $y^{iii}(o) = 19$,

with the known solution:

$$y = e^{-2x} + e^{3x} + e^{x} + e^{-x} \ .$$

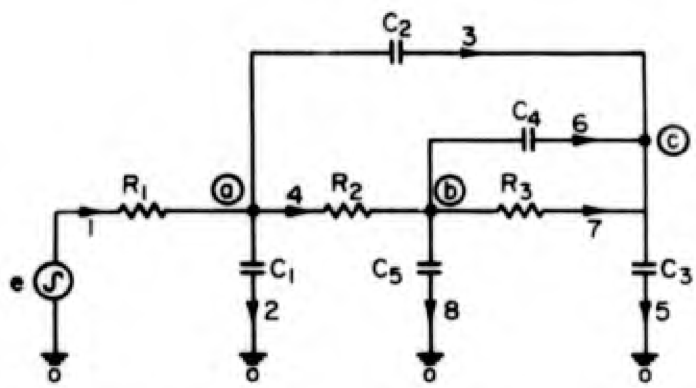


SCHEMATIC DIAGRAM OF THE NETWORK

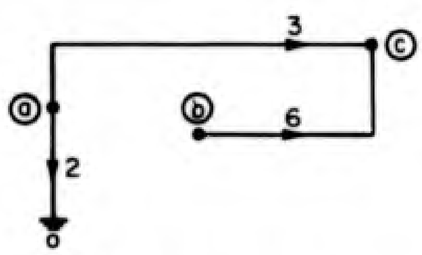


NETWORK TREE SELECTED

Figure 4-1. Topoligical Test for a Tight Capacitive Loop



SCHEMATIC DIAGRAM OF THE NETWORK



NETWORK TREE SELECTED

Figure 4-2. Topological Test for a Large Capacitive Loop

The actual system tested is the fourth-order system.

$$y' = r \quad , \quad y(o) = 4$$

$$r' = s \quad , \quad r(o) = 1$$

$$s' = f \quad , \quad s(o) = 15$$

$$f' = f + 7s - r - 6y \quad , \quad f(o) = 19$$

As the results shown in Appendix C indicate, the solution is good to five significant figures with $h = 0.1$, $\epsilon_1 = 10^{-7}$ and $\epsilon_2 = 10^{-5}$.

A more rigorous test was performed in the nature of a circuit consisting of a series resistor, capacitor, and voltage source. The voltage source had a wave-form as shown in Figure 4-3. The resultant output defined as the voltage across the capacitor is plotted in Figure 4-4. The time constant of the circuit was chosen as unity and $h = 0.1$, $\epsilon_1 = 10^{-5}$, and $\epsilon_2 = 10^{-3}$.

As can be seen from the input waveform, many conditions were tested. Initially, the voltage was constant and then rose as a slow S-curve; i.e., $v = K(1 - 2e^{-dt} + e^{-2dt})$. This was followed by a succession of positive and negative spikes. The spikes had a duration of 0.101, and 0.2 time units and were included at such intervals that the numerical integration algorithm would have reached a maximum value. The response as seen in Figure 4-4, along with the results listed in Appendix C, show that the algorithm followed the desired waveform and restarted when necessary. An

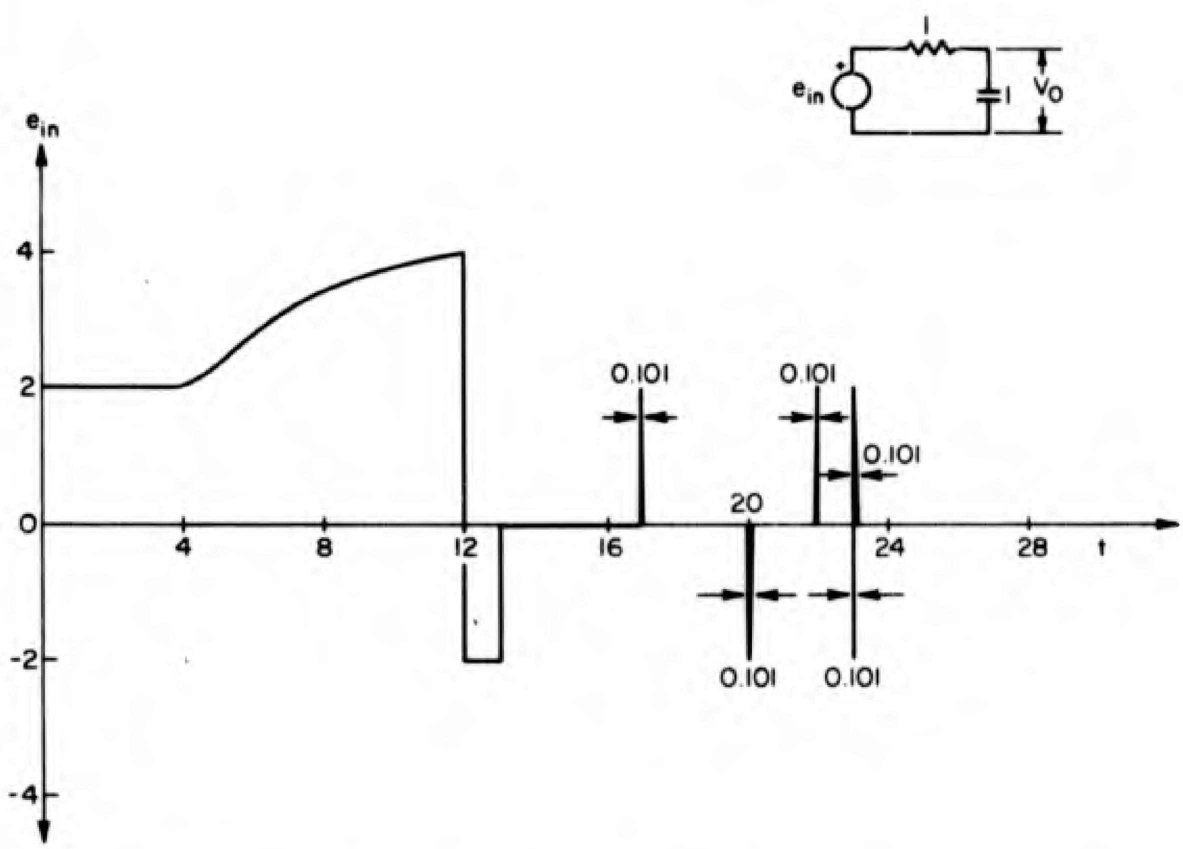


Figure 4-3. Waveform of Voltage Input to Test Circuit to Test the Numerical Integration Algorithm

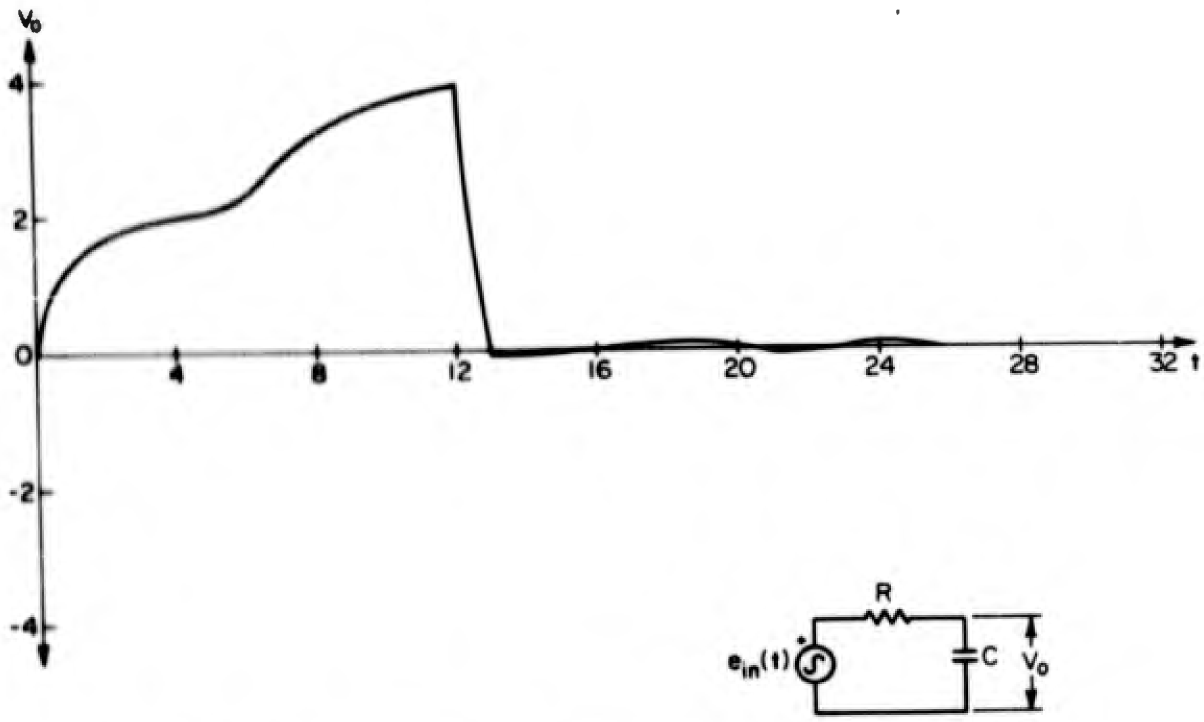


Figure 4-4. Output Waveform of Circuit Used to Test the Numerical Integration Algorithm

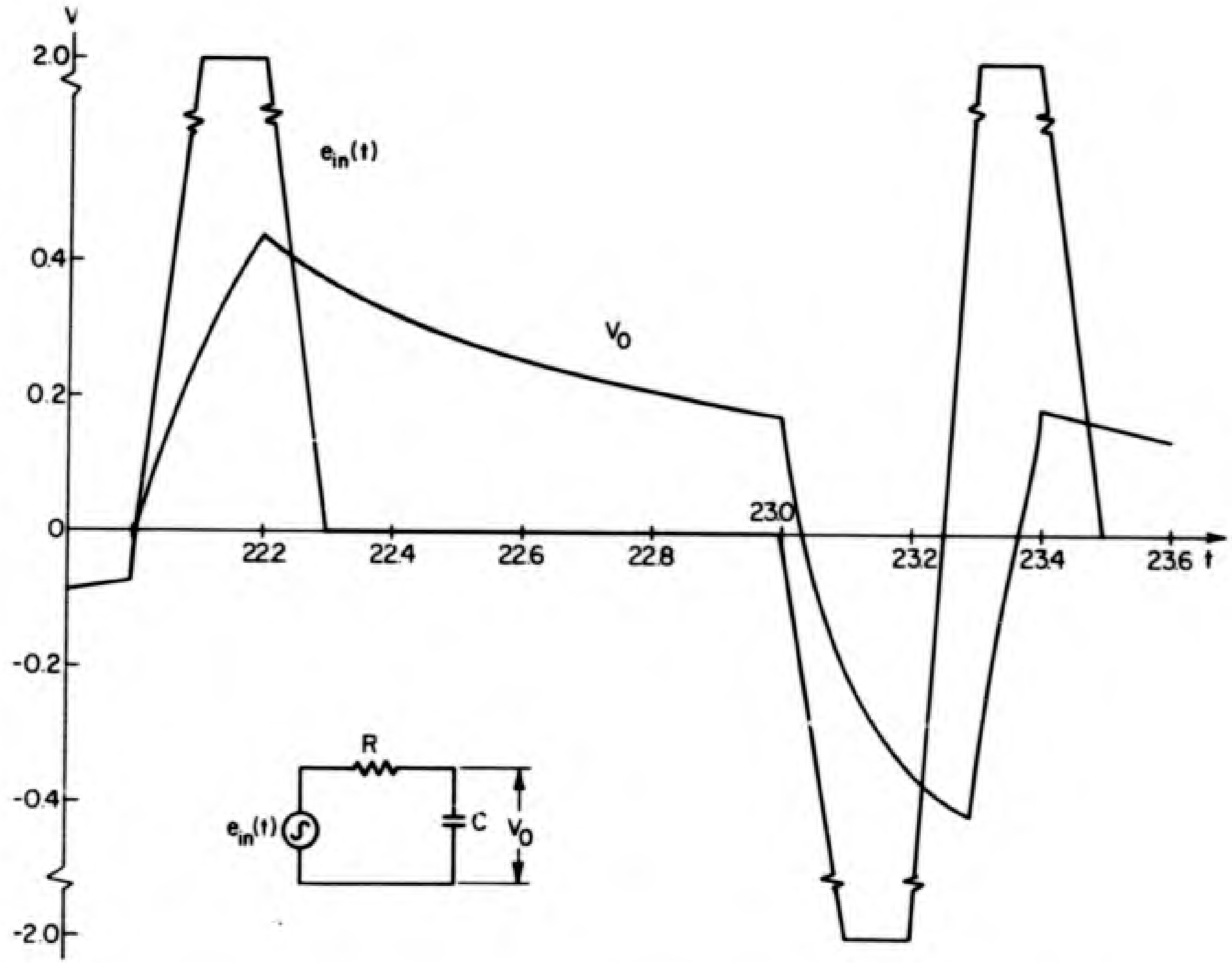


Figure 4-5. Enlarged View of the Excitation and Response in the Region t = 21.8 to t = 23.6 Time Units

enlargement of the region where the input voltage is a rapidly changing spike voltage is shown in Figure 4-5.

These tests verified the effectiveness of the numerical integration algorithm in integrating arbitrary and rapidly changing waveforms.

A Numerical Test Evaluating the Linear Circuit Case

In order to evaluate the accuracy of the network analysis system developed, a comparison had to be made to a network with a known solution. This was done by solving the linear network of Figure 4-6 by the Laplace transform method. In the s-plane, $I_1(s)$ and $I_2(s)$ can be solved for as:

$$I_1(s) = \frac{1}{s} + \frac{0.507}{(s + 0.331)} - \frac{0.007}{(s + 2.269)} ,$$

and

$$I_2(s) = \frac{1}{s} + \frac{1.170}{(s + 0.331)} - \frac{0.170}{(s + 2.269)} ,$$

which transforms into

$$i_1(t) = 1.000 + 0.507e^{-0.331t} - 0.007e^{-2.269t} \text{ ma} ,$$

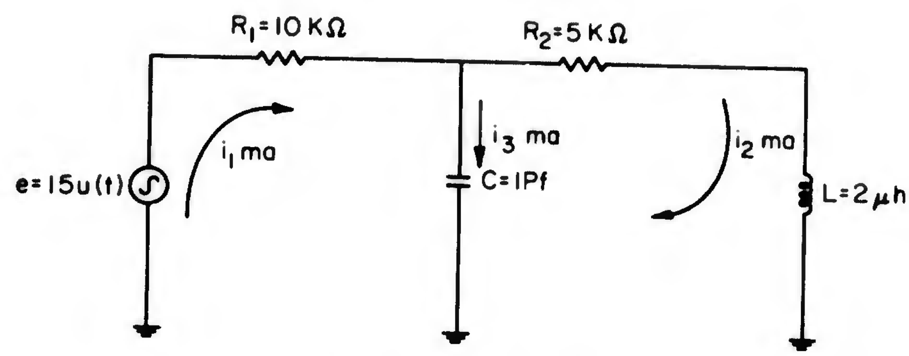
$$i_2(t) = 1.000 - 1.17e^{-0.331t} + 0.170e^{-2.269t} \text{ ma} ,$$

and

$$i_3(t) = i_1(t) - i_2(t)$$

$$= 1.677e^{-0.331t} - 0.177e^{-2.269t} \text{ ma} .$$

These solutions are plotted in Figure 4-7.



STATE VARIABLES = v_C AND i_2
COMPUTATIONAL VARIABLES
$i_1 = i_R$, $i_2 = i_L = i_G$, $i_3 = i_C$

Figure 4-6. Linear R-L-C Circuit Analyzed by Laplace Transform and Numerical State Variable Methods

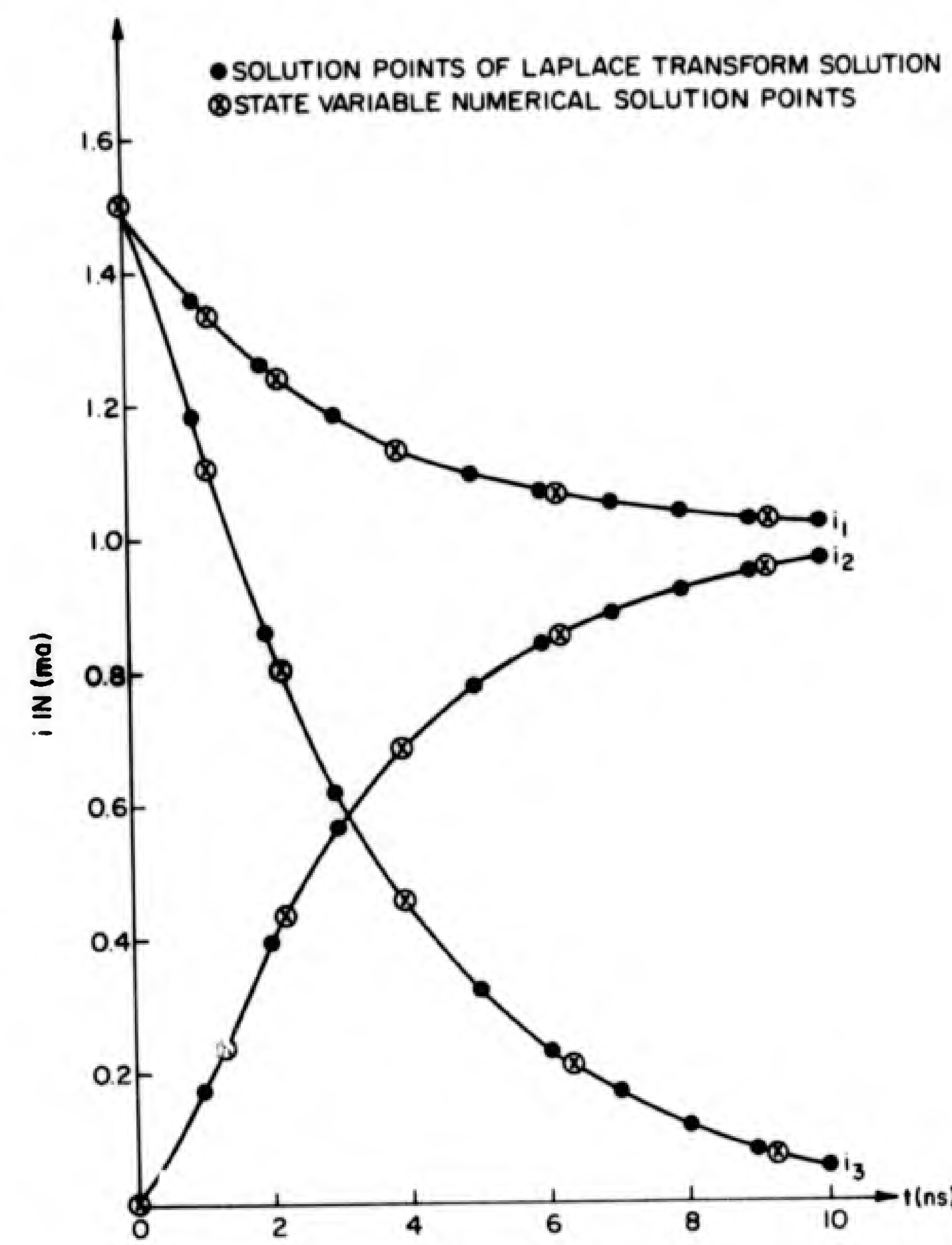


Figure 4-7. Comparison of Classical and Numerical Solutions for the Linear R-L-C Circuit of Figure 4-6

This same circuit was analyzed by the network analysis program. The results are listed in Appendix C, and plotted in Figure 4-7.

A comparison of the two sets of solutions shows that the two methods yield identical results to within five place accuracy.

Comparison to a Closed-Form Solution of a Nonlinear Circuit

To verify the nonlinear capabilities of the system, a comparison was made to a nonlinear circuit for which a closed-form solution could be obtained. The circuit considered was the negative resistance oscillator shown in Figure 4-8. The current-voltage relationship of the nonlinear element is given by a cubic equation of the form:

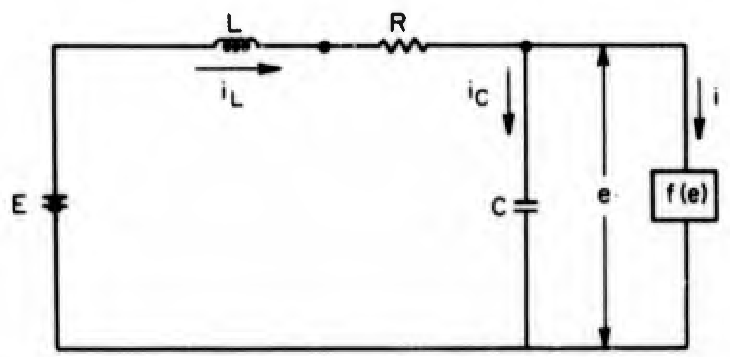
$$i = -ae + be^3 . \tag{14}$$

For oscillations to occur, several conditons must be satisfied.

1) The value of the resistance R must be small enough that it will form a load line that will intersect the nonlinear characteristic only once.

2) The biasing voltage E must be such that the intersection of the load line and the characteristic curve must occur in the region where $di/de < 0$.

3) The single intersection of the load line and the negative resistance element should be an unstable focus.



$i = f(e) = -ae + be^3$

Figure 4-8. Negative Resistance Oscillator Used for Comparison of Nonlinear Solutions

The following conditions will assure these three conditions:
If,

$$\frac{1}{a} = |r| < \frac{L}{R\,C} \quad , \qquad (15)$$

where

$$|r| = \frac{\Delta e}{\Delta i} \text{ in the region where } \frac{di}{de} < 0 \ ,$$

then the magnitude of the negative resistance is less than the impedances of the parallel L-R-C at its resonance frequency.

Further, if $R < |r|$, the singularity cannot be a saddle, since the conditon for the characteristic roots to be real and of opposite sign is that $R > |r|$.

To insure that the singularity be a focus, the characteristic roots must be complex conjugates. This condition is met if:

$$\frac{4}{LC} \left(1 + \frac{R}{|r|}\right) > \left(\frac{1}{|r|C} + \frac{R}{L}\right)^2 \ . \qquad (16)$$

If the details of the steady state biasing voltage are omitted, the equation for the circuit of Figure 4-8 can be obtained from the Kirchhoff current equation written at the node. Thus,

$$i + i_C - i_L = 0 \ , \qquad (17)$$

where

$$i_C = C\dot{e} \quad ,$$

$$i = -\,ae + be^3 \quad ,$$

and

$$\frac{di_L}{dt} = -(e + i_L R) \ .$$

Differentiating Equation (17) yields:

$$\frac{di}{dt} + \frac{di_C}{dt} - \frac{di_L}{dt} = 0 \qquad (18)$$

where

$$\frac{di_C}{dt} = C\ddot{e}\ , \ \frac{di}{dt} = (-a + 3be^2)\,\dot{e} \qquad (19)$$

and

$$\frac{di_L}{dt} = -\frac{1}{L}(e + i_L R)$$

$$= -\frac{1}{L}[e + R(i + i_C)]$$

$$= -\frac{1}{L}[e + R(Ce - ae + be^3)] \ . \qquad (20)$$

Substituting Equations (19) and (20) into Equation (18) gives:

$$C\ddot{e} - a\dot{e} + 3be^2\dot{e} + \frac{1}{L}[e + R(Ce - ae + be^3)] = 0,$$

$$\text{or} \quad \ddot{e} - \frac{a}{C}\left(1 - \frac{3be^2}{a}\right)\dot{e} + \frac{e}{LC} + \frac{R}{L}\left(e - \frac{ae}{C} + \frac{be^3}{C}\right) = 0. \qquad (21)$$

If the assumption is made that R is very small, the last term in the equation can be omitted and the resulting equation greatly simplifies to:

$$\ddot{e} - \frac{a}{C}\left(1 - \frac{3be^2}{a}\right)\dot{e} + \left(\frac{1}{LC}\right)e = 0\ , \qquad (22)$$

or

$$\ddot{e} - \alpha(1 - \beta e^2)\,\omega_o \dot{e} + \omega_o^2 e = 0 \,, \qquad (23)$$

where

$$\alpha = \frac{a}{C\omega_o} \,,$$

$$\omega_o^2 = \frac{1}{LC} \,,$$

$$\beta = \frac{3b}{a} \,.$$

Equation (23) then is essentially Van der Pol's equation.

If the desired oscillation is to be nearly sinusoidal, α should be small. This can be quickly verified by the fact that if α is negligible, Equation (23) will approach:

$$\ddot{e} + \omega_o^2 e = 0 \,,$$

which has the solution $e = A \sin \omega_o t + B \cos \omega_o t$. If it is assumed that $\alpha \leq 0.1$, the following restriction is placed on the circuit parameters:

$$\alpha = \frac{a}{C\omega_o} \leq 0.1 \,,$$

but

$$\omega_o = \frac{1}{\sqrt{LC}} \,.$$

Thus, $\alpha = a\sqrt{\frac{L}{C}} \leq 0.1$

or $$L \leq \frac{0.01}{a^2} C \quad . \tag{24}$$

At this point the parameters a and b in Equation (14) can be assumed. If a is chosen as 2 and b = 1/4 , Equation (14) becomes:

$$i = -2e + 0.25e^3 \text{ ma} ,$$

from which the value of $|r|$ is 0.5 k-ohms in the negative resistance region.

Furthermore, it is assumed that R is small, let R = 0.001 k-ohms. If the value of C is chosen, the value of L will be fixed by the restrictions impressed on the circuit in order to obtain nearly sinusoidal self-oscillations.

Let C = 10 pf.

Then, Equation (24) becomes:

$$L \leq 0.025 \ \mu h ,$$

and if Equation (15) is rearranged as:

$$|r| \cdot (RC) < L ,$$

which, when evaluated with the assumed values, requires that:

$$0.005 < L \ \mu h \quad .$$

Hence,

$$0.005 < L \leq 0.025 \ \mu h ;$$

assume $\alpha = 0.1$,

then, $L = 0.025\ \mu h$.

To sustain oscillation, Equation (16) must be satisfied. Evaluating Equation (16) with the chosen circuit values gives the following inequality:

$$\frac{4}{(0.025)(10)}\left[1 + \left(\frac{0.001}{0.5}\right)\right] > \left[\frac{1}{(0.5)(10)} + \frac{0.001}{0.025}\right]^2 .$$

or

$$16 > 0.0576 .$$

Thus, all the conditions are satisfied.

It was assumed that the analysis would neglect the biasing voltage; however, for the actual computer analysis, this voltage must be considered since the circuit must receive energy from some source. To accomplish this, a source can be included and the e-i characteristic of the negative resistance device can be shifted by a constant amount. Thus, if the source voltage is chosen to be 3.6 volts, the function describing the negative resistance device becomes:

$$i = -2\,(e - 3.6) + \frac{1}{4}\,(e - 3.6)^3 \ \text{ma} .$$

All these assumptions lead to the following parametric values for the components of Figure 4-7:

$E = 3.6$ volts

$L = 0.025\ \mu h$

$$C = 10.0 \text{ pf}$$

$$R = 0.001 \text{ k-ohms}$$

$$f(e) = i = -2\,(e - 3.6) + \frac{1}{4}\,(e - 3.6)^3 \text{ ma} .$$

The current voltage relationship $f(e)$ is plotted in Figure 4-8.

At $e = 0$, $\frac{de}{di} = -0.5$ k-ohms. This is the value of resistance represented in the computer analysis input and corresponds to the initial value of resistance.

To obtain an approximation to the solution of Equation (23) let us apply the perturbation method and keep only those terms required to give a first-order approximation to the solution.

A solution is sought that will be of the form:

$$e = e_o(t) + \alpha\, e_1(t) . \qquad (25)$$

With the possibility of a frequency change of the form $\omega = \omega_o + \alpha\,\omega_1$ or solving for ω_o, gives:

$$\omega_o = \omega - \alpha\,\omega_1 . \qquad (26)$$

Substituting Equations (25) and (26) into (23) gives:

$$\ddot{e}_o + \alpha\,\ddot{e}_1 - \alpha\,\omega\,\dot{e}_o - \alpha^2\omega\,e_1 + \alpha^2\omega_1\,\dot{e}_o + \alpha\,\beta\,\omega\,e_o^2\,\dot{e}_o$$
$$+ 2\alpha^2\beta\,\omega\,e_o\,e_1\,\dot{e}_o - \alpha^2\beta\,\omega_1\,e_o^2\,\dot{e}_o + \alpha^2\beta\,\omega\,e_o^2\,\dot{e}_1 + \omega^2\,e_o$$
$$+ \alpha\,\omega^2\,e_1 - 2\alpha\,\omega_1\,\omega\,e_o - 2\alpha^2\omega_1\,\omega\,e_1 = 0 . \qquad (27)$$

From the α^o terms: $\ddot{e}_o + \omega^2\,e_o = 0$.

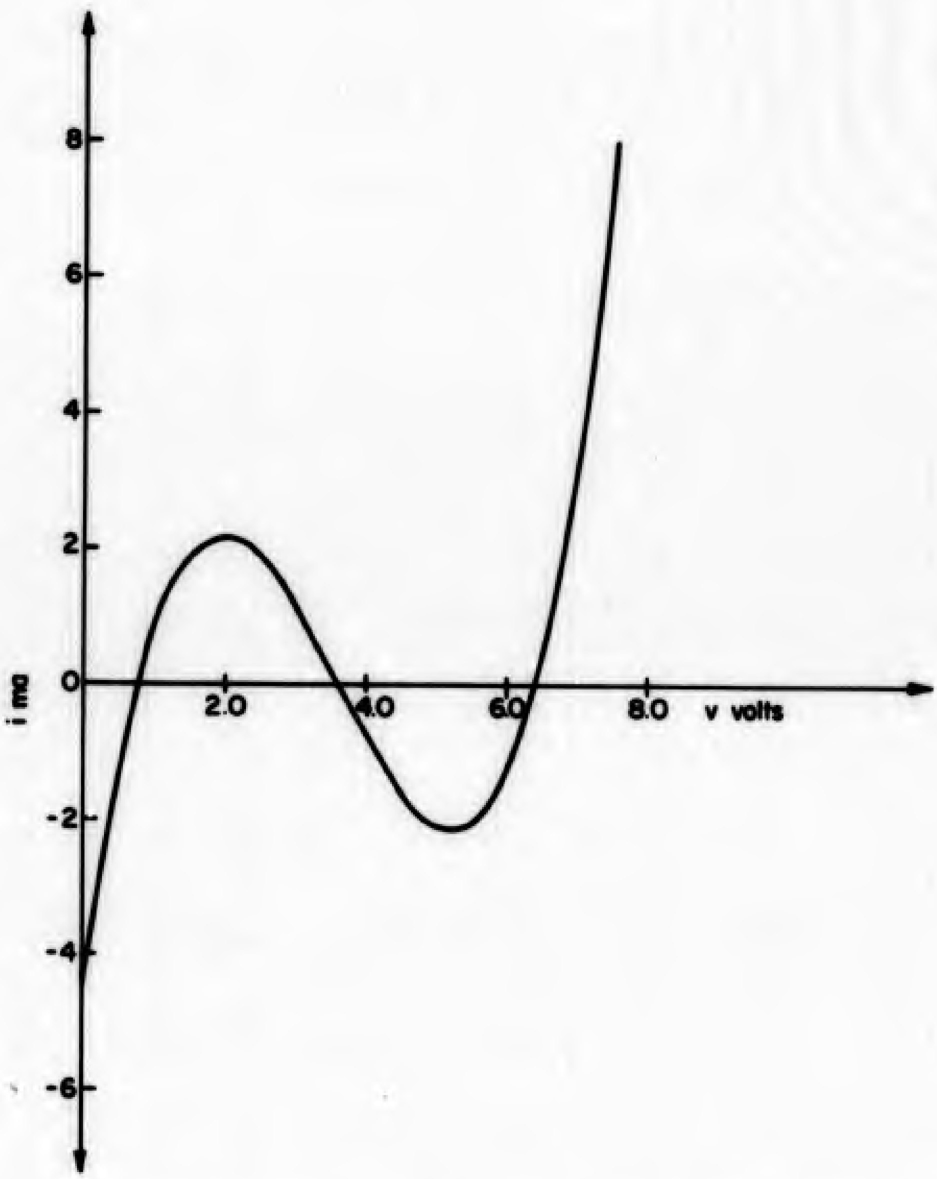


Figure 4-9. Voltage-Current Relationship Defining the Negative Resistance Oscillator Characteristic

The solution to this equation is:

$$e_o = A \sin \omega t + B \cos \omega t \; .$$

When the initial conditions $e_o = E$, and $\dot{e}_o = 0$ at $t = 0$, are applied,

$$e_o = E \cos \omega t \; .$$

From the α^1 terms, the first-order correction is found to be

$$\ddot{e}_1 + \omega^2 e_1 = \omega \dot{e}_o - \beta \omega e_o^2 \dot{e}_o + 2\omega_1 \omega e_o$$

$$= -\left(\omega^2 E + \frac{\beta \omega^2 E^3}{4}\right) \sin \omega t$$

$$+ \; 2\omega_1 \omega E \cos \omega t + \frac{\beta \omega^2 E^3}{4} \sin 3\omega t \; . \qquad (28)$$

Since the solution of the homogeneous equation,

$$\ddot{e}_1 + \omega^2 e_1 = 0 \; ,$$

is given by:

$$e_1 = A_1 \sin \omega t + B_2 \cos \omega t \; ,$$

it is required that

$$\omega^2 E + \frac{\beta \omega^2 E^3}{4} = 2\omega_1 \omega E = 0$$ in order to avoid secular terms.

Thus,

$$E = \pm 2\beta^{-1/2} \; , \qquad (29)$$

and

$$\omega_1 = 0 \; . \qquad (30)$$

With these conditions, the solution to Equation (28) is

$$e_1 = A_1 \sin \omega t + B_1 \cos \omega t - \frac{\beta E^3}{32} \sin 3\omega t$$

subject to the initial conditions $e_1 = 0$, and $\dot{e}_1 = 0$ at $t = 0$.

Thus, $B_1 = 0$

and $A_1 = \frac{3\beta E^2}{32}$, with the substitution of Equation (29)

$A_1 = \frac{3}{8}$, and the first-order correction term becomes:

$$e_1 = \frac{E}{8}(3 \sin \omega t - \sin 3\omega t).$$

Thus, to a first-order of approximation, a solution is:

$$e = E \cos \omega t + \frac{\alpha E}{8}(3 \sin \omega t - \sin 3 \omega t) ,$$

$$\omega = \omega_o + \alpha(0) = \omega_o ,$$

where $E = \pm 2/\beta^{-1/2} = \pm 2(a/3b)^{1/2}$. Substituting numerical values in the above relationships gives:

$$e = 3.27 \cos (2.00t) + 0.123 \sin (2.00t)$$
$$- 0.041 \sin (6.00t) \text{ volts} . \quad (31)$$

Equation (31) is plotted in Figure 4-10.

The circuit Figure 4-8 was also analyzed by means of the computer program. The complete numerical solution, along with a listing of the input data cards, is given in Appendix C.

A plot of the numerical results is shown in Figure 4-11. It should be noted that the numerical solution starts off at zero

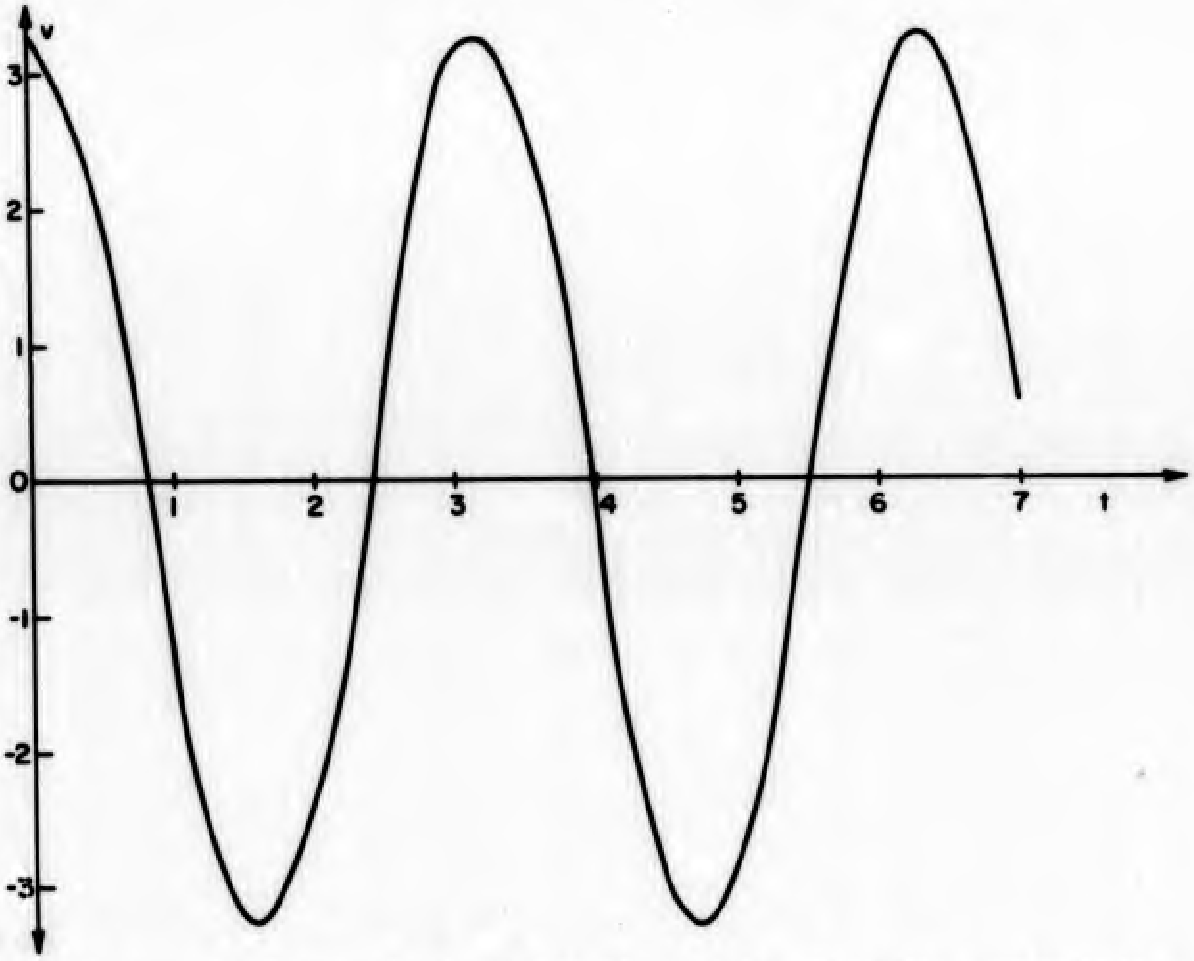


Figure 4-10. Analytical Solution of the Negative Resistance Oscillator with $\alpha = 0.1$

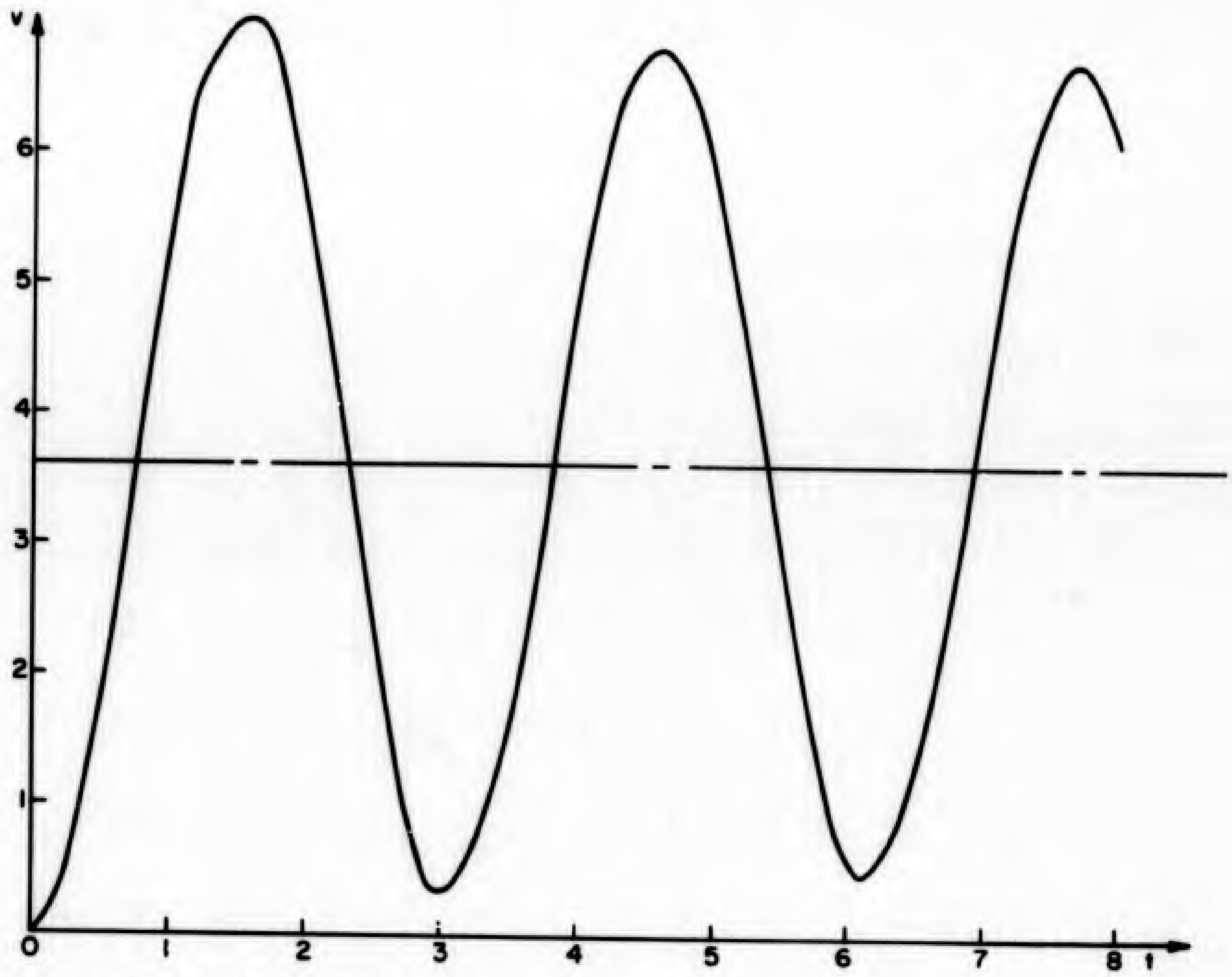


Figure 4-11. Numerical Solution of the Negative Resistance Oscillator with $\alpha = 0.1$

and has a transient build-up to the final oscillatory state. This transient results from the assumption that the initial conditions are all zero. This assumption causes the constant voltage source to act as a 3.6 volt step function applied to an initially relaxed circuit.

A comparison of Figures 4-10 and 4-11 shows that, ignoring the initial transient and the biasing effects, the two solutions yield the same results within an accuracy of four significant figures. However, it should be noted that the classical analysis is limited by the assumptions made during the analysis. This is not the case in the analysis developed during the course of this investigation. To show the limitation of the classical nonlinear method and at the same time point out the inherent advantages of the numerical method, the value of α was increased to unity. The classical solution to the new problem is:

$$e = 3.27 \cos (0.633t) + 1.23 \sin (0.633t) - 0.41 \sin (1.899) \text{ volts.} \tag{32}$$

Equation (32) is plotted in Figure 4-12.

To implement the equivalent change for the numerical solution, the capacitor was changed to 1 pf. and the inductor to 0.25 μh . The numerical solution to this problem is plotted. in Figure 4-13.

A comparison of Figures 4-12 and 4-13 show that the two methods no longer yield the same solution. The reason for this is, of course, that $\alpha = 1$ violates the original assumption that α is small. In this case, the cause and effect is obvious, but it is conceivable that an analytic solution is obtained and a parametric study carried out in which the capacitor C and the inductor L are varied to the point where the solution is no longer valid. With the digital solution, this problem does not arise since no assumptions about smallness have been made. As long as the restrictions of Chapters II and III are met, a unique stable solution is assured regardless of the parametric values chosen. Examining these conditions, it can be seen that the parameters satisfy a Lipshitz condition and, thus, a unique solution exists. For this circuit, the conditions of Theorem 5 become:

$$Y = F'_{RC} \, R1^{-1} \, F_{RC} \geq 0 \ ,$$

and

$$Z = F_{LG} \, G2^{-1} \, F'_{LG} \geq 0 \ .$$

Evaluating these conditions yields:

$$Y = R1^{-1} = \frac{1}{r} \geq 0 \ ,$$

and

$$Z = G2^{-1} = R \geq 0 \ .$$

Since R is a non-zero constant, as expected, the only possibility for instability is for r to be negative.

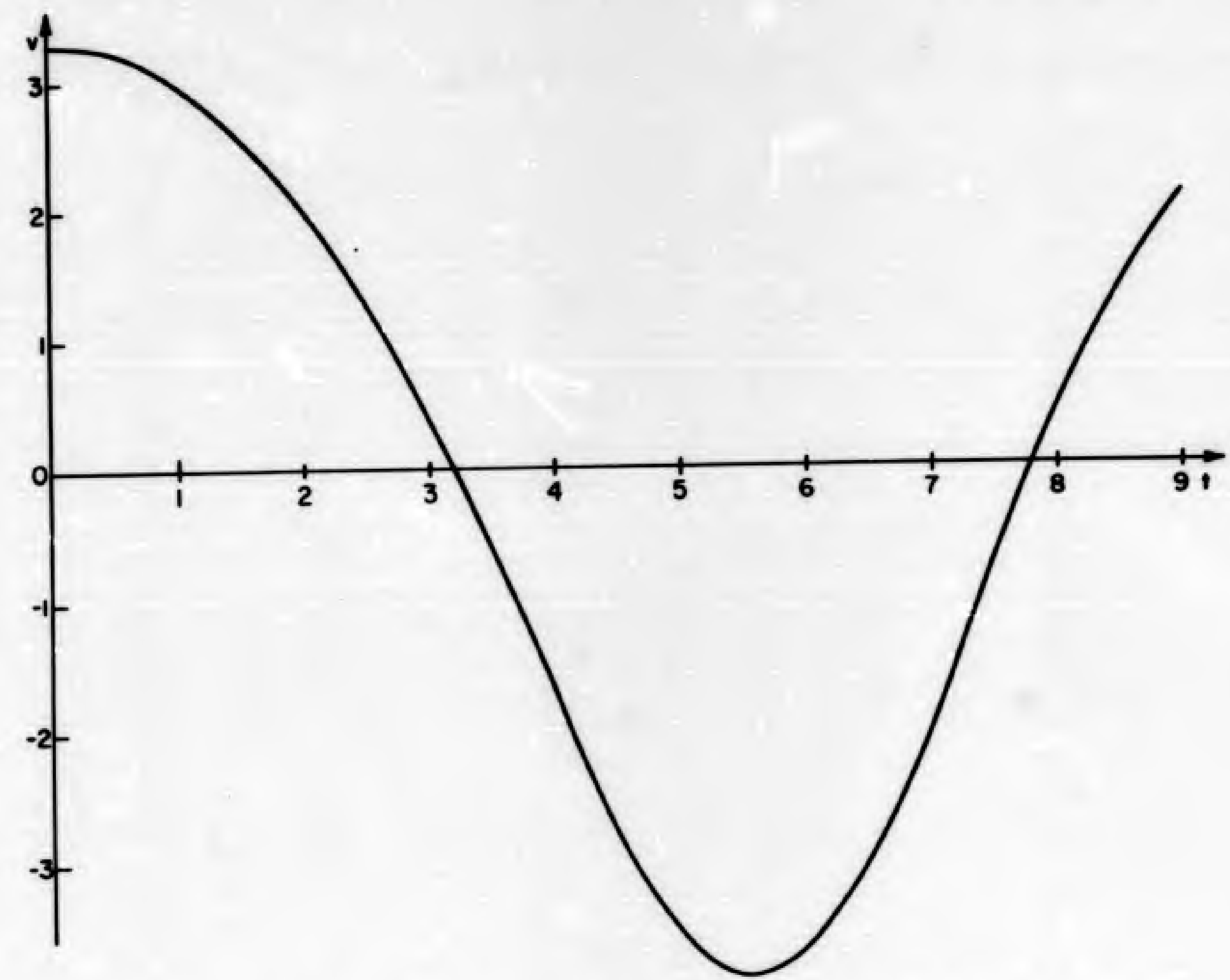


Figure 4-12. Analytical Solution of the Negative Resistance Oscillator with $\alpha = 1$.

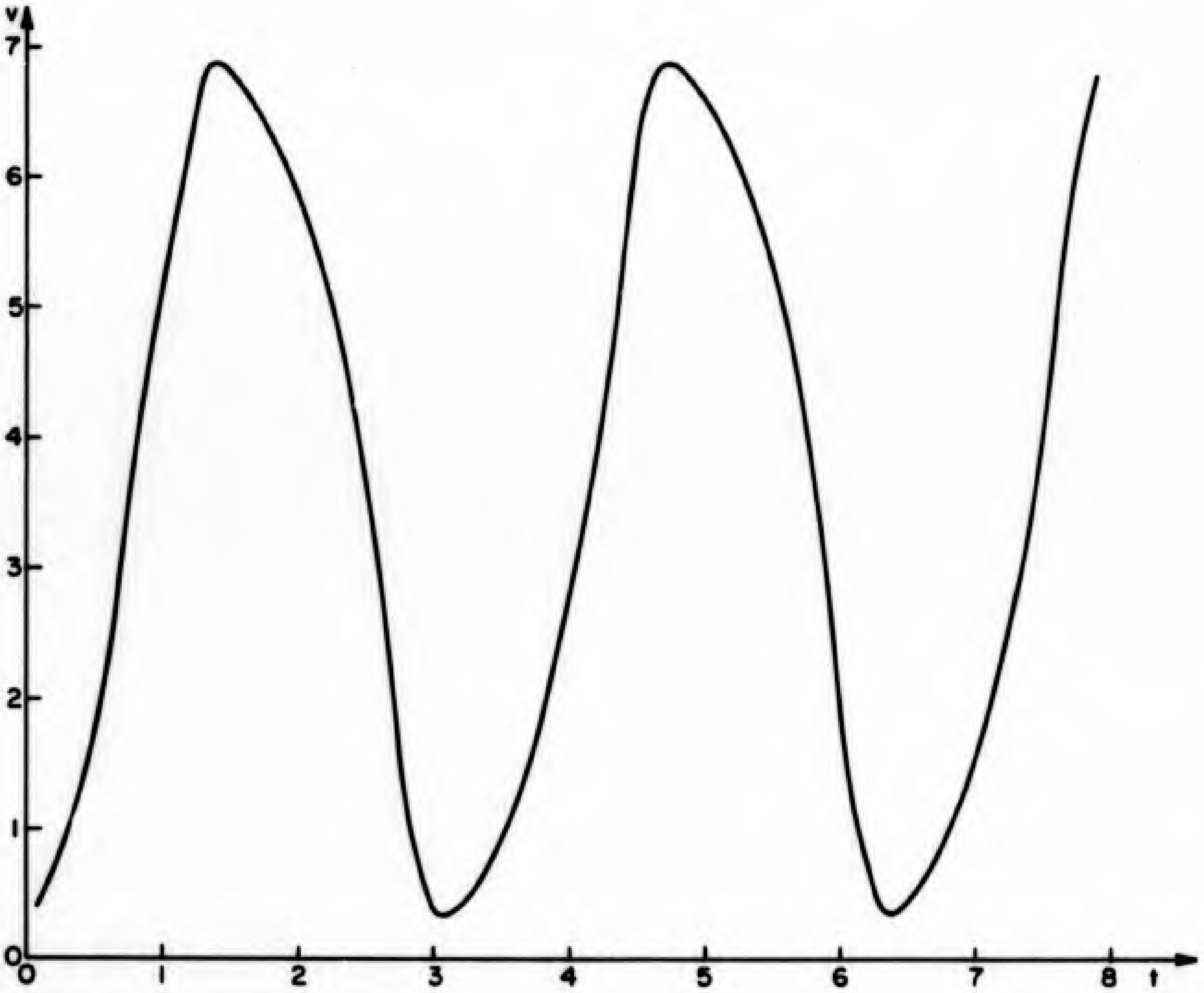


Figure 4-13. Numerical Solution of the Negative Resistance Oscillator with $\alpha = 1$.

Tunnel Diode Switch Analysis

To illustrate the use of the techniques developed during the course of this investigation, a circuit which does not possess a closed-form classical solution was analyzed. This circuit is the tunnel diode switch of Figure 4-14. Two cases were analyzed to exhibit the type of investigations that can be done by these methods. First, the switch was analyzed using a constant value of 10 pf. to represent the junction capacitance. These results are listed in Appendix C and a plot of the output and input voltages is shown in Figure 4-17. The voltage-current characteristic, f(v) of Figure 4-14, is shown in Figure 4-15. The value of the resistance R used in the analysis is 70 ohms. From Figure 4-15, it can be seen that the output voltage (Vc in the listing shown in Appendix C) rises to 0.020 volts when the input is at 0.3 volts; then as the input switches to 0.9 volts, the output switches to 0.57 volts. Examination of Figure 4-17 and the numerical results shows that the solution behaves exactly as predicted by theory. However, the plot of Figure 4-17 could not be obtained by classical theory.

To further exemplify the advantages and facility of use of the developed method of analysis, the capacitance C of Figure 4-14 was changed from a constant of 10 pf. to a voltage dependant function as shown in Figure 4-16. A plot of this analysis is shown in Figure 4-18. This case has the same on and off states as the previous analysis, but the transient behavior is quite different. In the later case, both the turn-on and turn-off times are increased due to the increased capacitance effects.

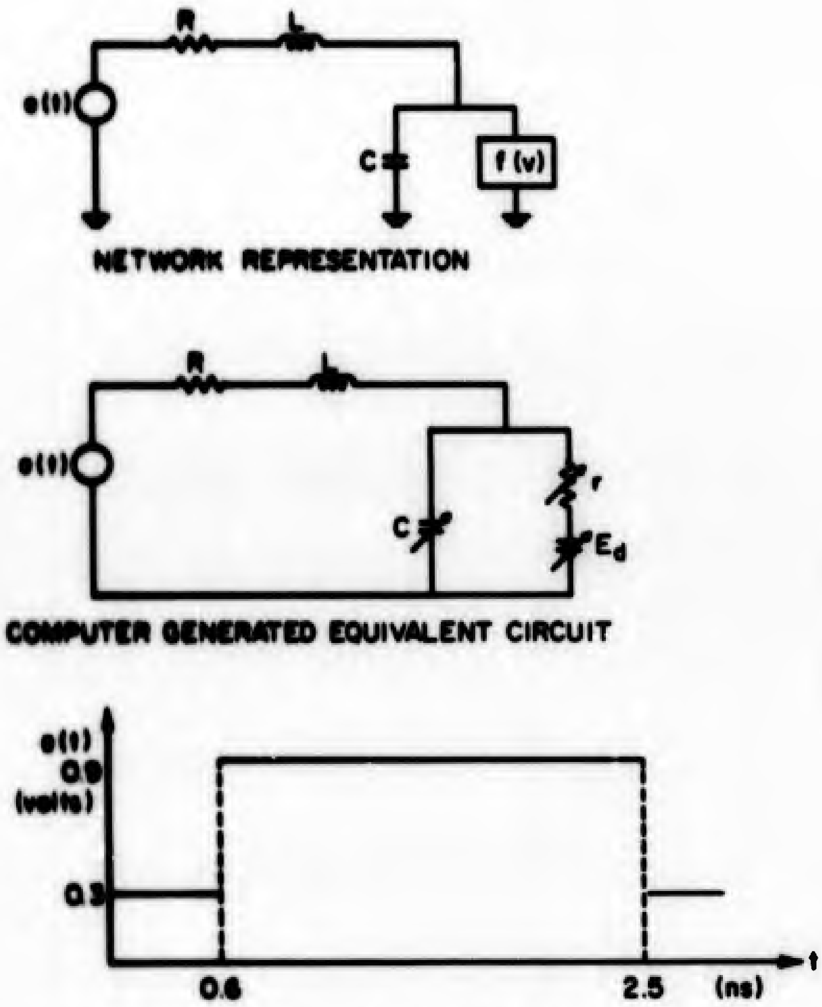


Figure 4-14. Tunnel Diode Switching Circuit

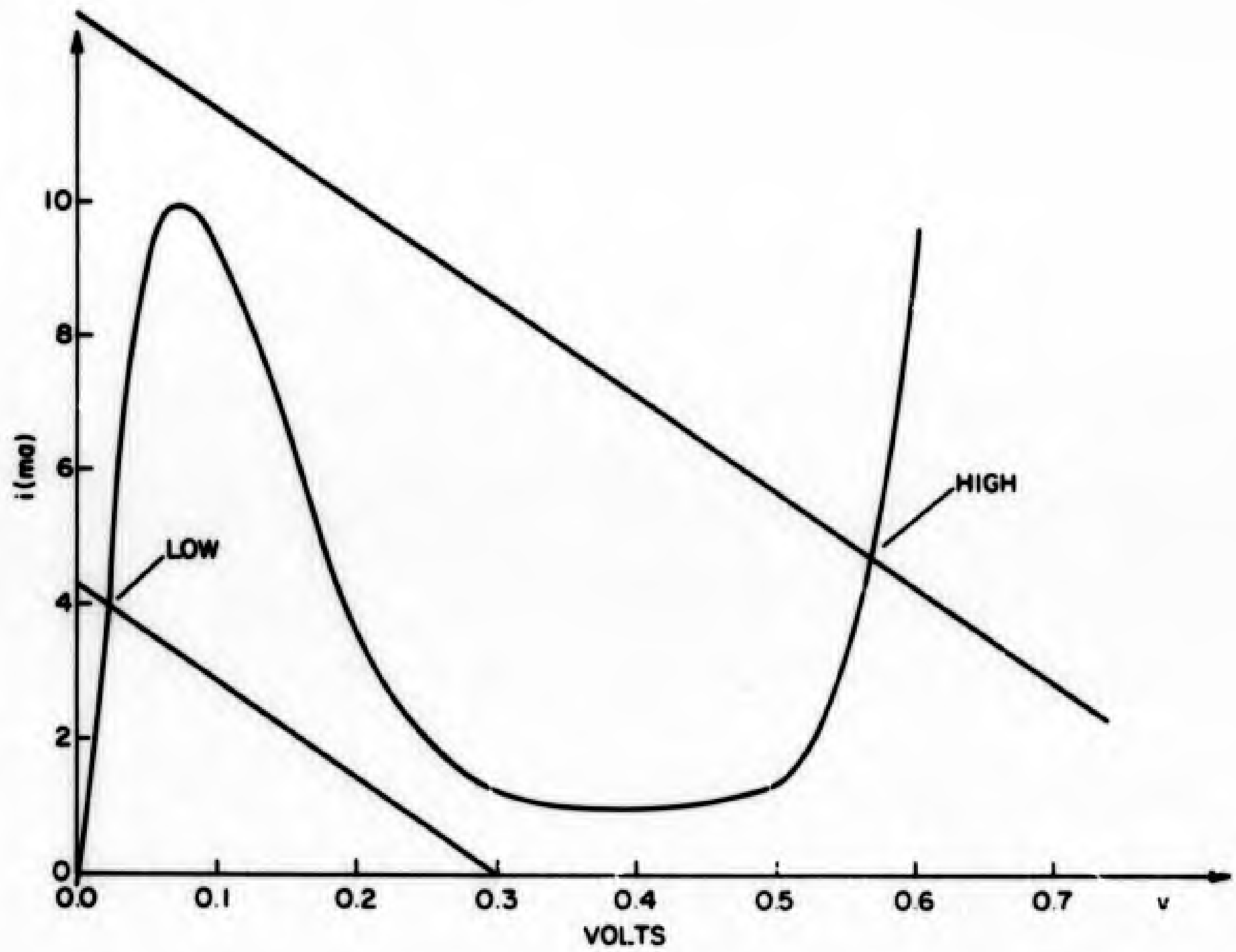


Figure 4-15. Voltage-Current Relationship Used in the Tunnel Diode Analysis

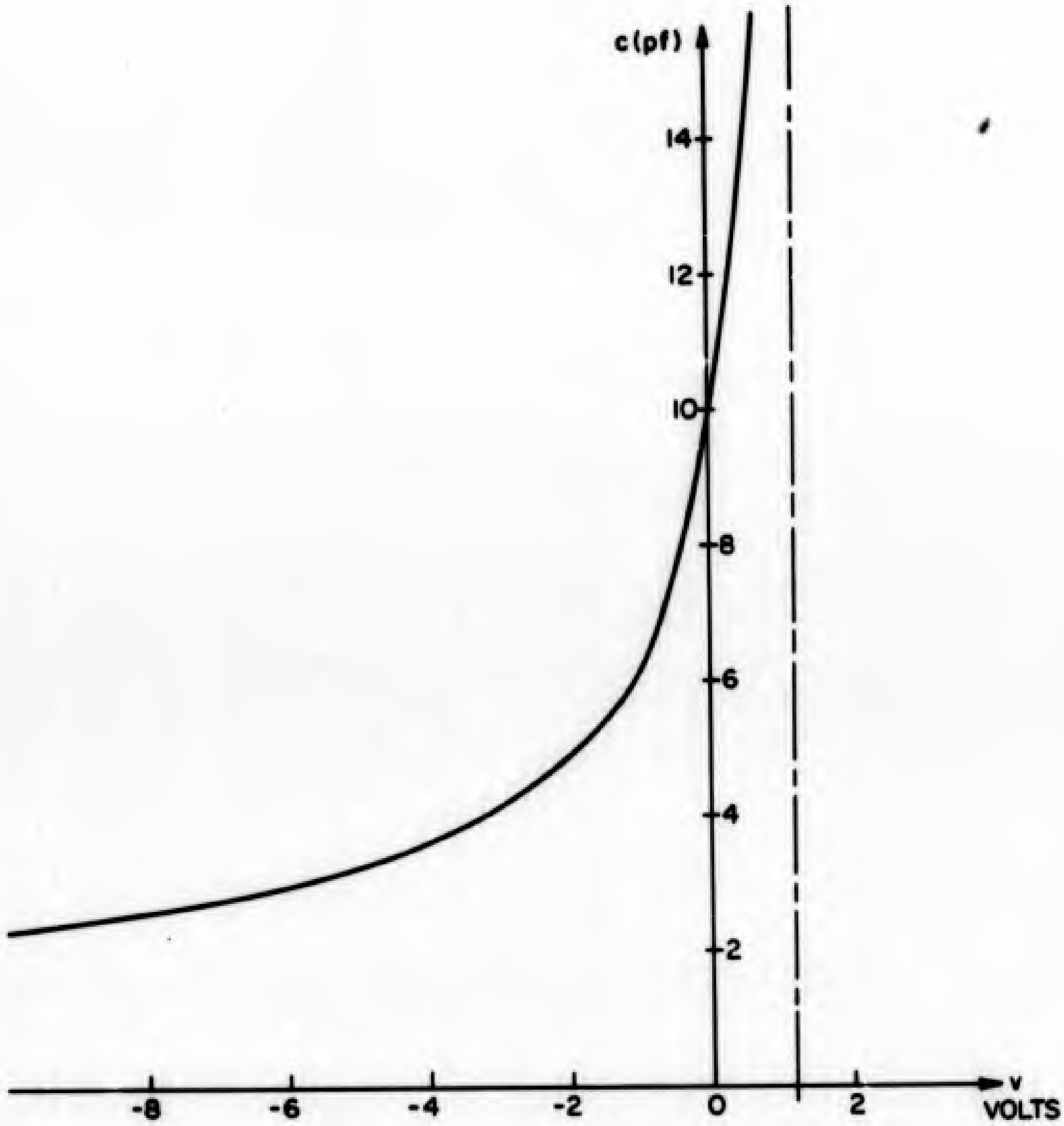


Figure 4-16. Capacitance-Voltage Relationship Used in the Tunnel Diode Analysis

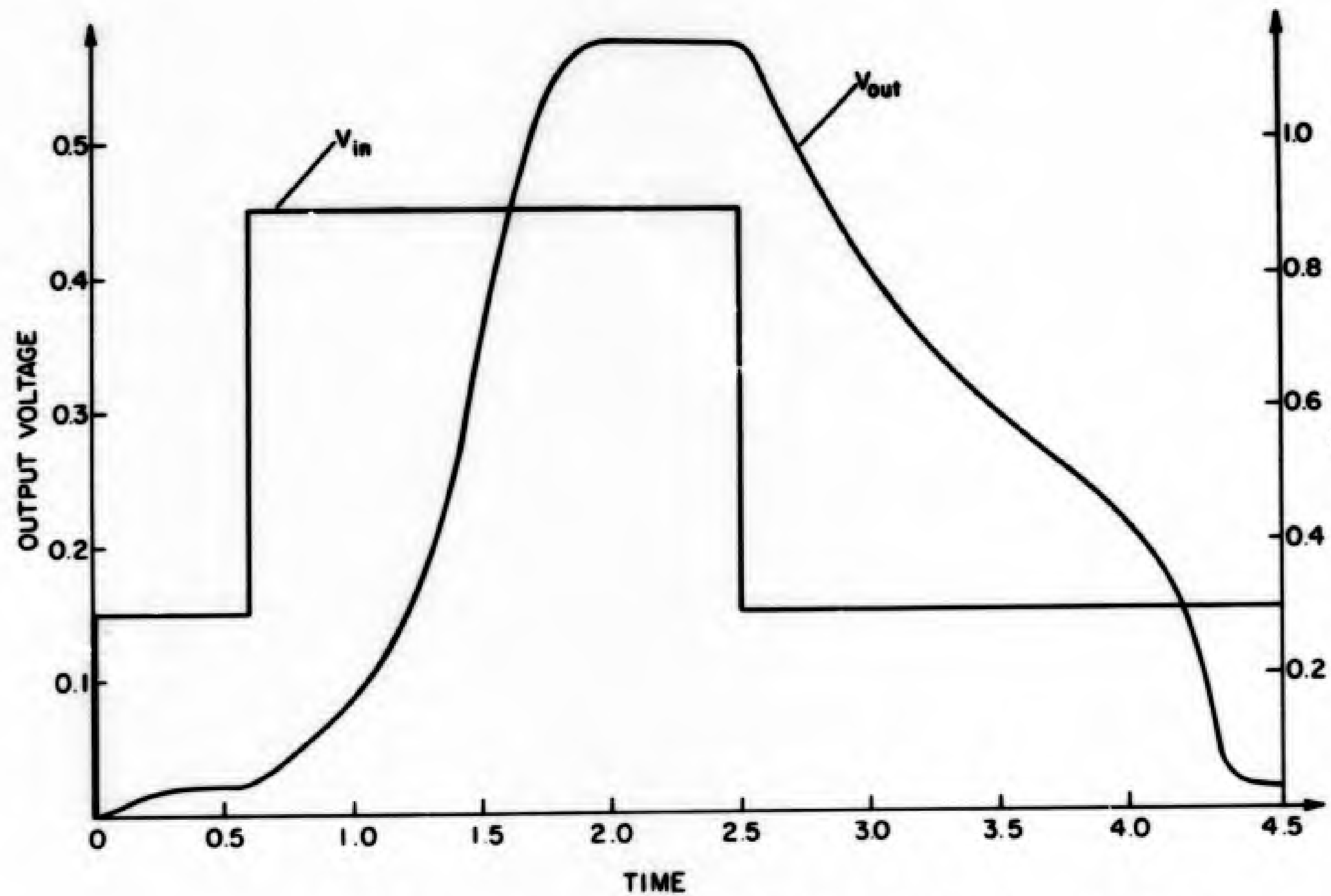


Figure 4-17. Numerical Solution of the Tunnel Diode Analysis, Constant Capacitance Case

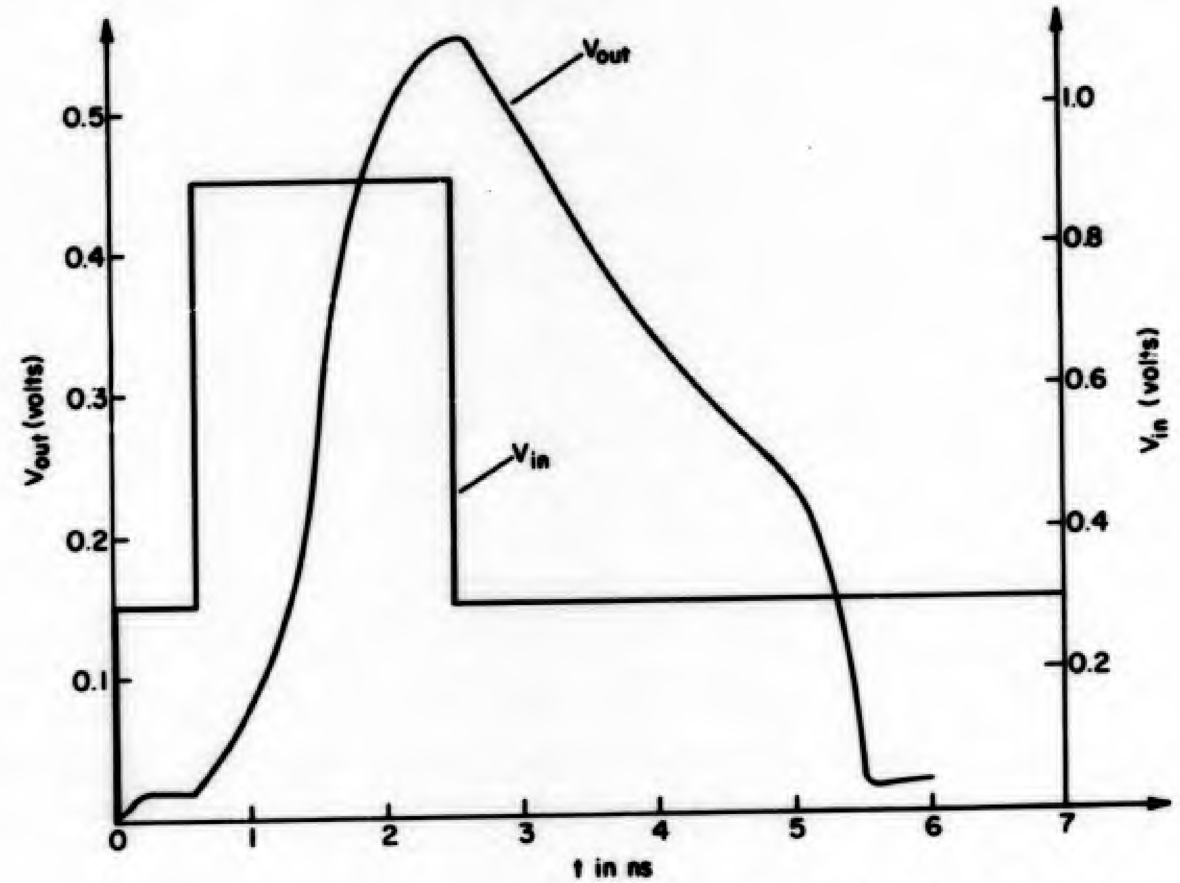


Figure 4-18. Numerical Solution of the Tunnel Diode Analysis, Variable Capacitance Case

CHAPTER V

SUMMARY

Statement of the Problem

The purpose of this investigation was to develop the necessary techniques and algorithms to allow the solution of a general nonlinear network on a large scale digital computer. From this analysis, the required numerical and computational algorithms were developed. These in turn were programmed, coded, and debugged into a working system. In parallel with the aforementioned analysis, proofs for the existence, uniqueness, and stability of the analytical and numerical solutions were developed.

Importance of the Study

Prior to the advent of the digital computer, investigations of nonlinear networks have been, by and large, limited to classical examples or special circuits which could be reduced to a special case that could be solved. However, these answers are generally only approximations. When the approximations are no longer accurate enough, an analog or digital computer solution must be obtained. In both these cases, the programming and initial set-up time can be long and each new circuit requires a new program. With a general computer program, the routine portion of network analysis can be delegated to the computer, thus freeing the engineer to perform his primary functions. The generalized program has the additional feature that a new analysis can be performed in a short time. This type of program can be used not only for circuit analysis, but also for parameter studies, network model development, device simulation, and many other facets of electronic engineering.

Procedure of Investigation

The network equations describing a general nonlinear network were developed from basic principles, using as a guide the fact that the final method of solution was to be a digital computer program. All the various approaches yield a final formulation which describes the network as a set of nonlinear defferential or integro-differential equations. Since these equations are to be solved numerically, a method which yielded the minimum set of equations was chosen. This formulation is the state variable form with the capacitive voltages and inductive currents chosen as the variables. By its basic formulation, the state variable approach eliminates all the algebraic equations. However, for a general nonlinear analysis, some of the relationships involve these algebraic variables, so algorithms were developed to recover these variables as they are needed. With the method of analysis selected, the next phase of the investigation involved the establishment of conditions to guarantee the existence of a unique and stable solution. The Cauchy-Lipschitz theorem insures the existence of a solution to a set of n differential equations of the form $\dot{x} = X(x,t)$, where

x is the state vector $\begin{matrix} V_c \\ i_L \end{matrix}$,

and

$X(x,t) = A(x)\,x + B(x)\,U(x,t).$

The matrices A (x) and B (x) are functions of the parametric matrices:

C1 consisting of the link capacitances,

C2 consisting of the branch capacitances,

L1 consisting of the link inductances,

L2 consisting of the branch inductances,

R1 consisting of the link resistances, $G1 = R1^{-1}$,

R2 consisting of the branch resistances, $G2 = R2^{-1}$

These matrices can be functions of circuit voltages and currents. U (x,t) is the source vector containing all the voltage and current sources. Application of the Cauchy-Lipschitz theorem leads to the requirement that the partial derivatives of C1, C2, L1, L2, R2, G1 and U with respect to x be continuous functions.

The stability of the system was investigated by application of Liapunov's stability theorems. These theorems insure stability of a system defined by $\dot{X} = A(x)\,X$.

In order to apply Liapunov's theorems, a Liapunov function had to be found. It was shown that the energy function E, satisfies all the requirements of a Liapunov function.

E is defined as follows:

$$E = \frac{1}{2} V' \begin{bmatrix} C1 & 0 \\ 0 & C2 \end{bmatrix} V + \frac{1}{2} I' \begin{bmatrix} L1 & 0 \\ 0 & L2 \end{bmatrix} I ,$$

where

$$V = \begin{bmatrix} v_S \\ v_C \end{bmatrix} , \text{ and } I = \begin{bmatrix} i_L \\ i_\gamma \end{bmatrix} ,$$

and

v_S = capacitive link voltages,

v_C = capacitive branch voltages,

i_L = inductive link currents,

i_γ = inductive branch currents,

The stability of the complete formulation was assured by considering it as a system under a persistent disturbance.

Application of these theorems assured existence of a unique stable solution to the nonlinear state variable formulation of the network analysis problem, provided the partial derivatives of C1, C2, L1, L2, R2, G1 and U with respect to x are continuous functions, and the partial derivatives of L1, C1, L2, and C2 are bounded.

The next area of investigation was to develop the computer algorithms and programs. Also, the existence, uniqueness, and convergence of the numerical problem had to be insured. As the computer algorithms and programs were developed, they were thoroughly tested. In this manner, when the final programming system was formed, only the interfaces needed to be tested. Algorithms were developed to perform all the required topological analyses and generate the normal tree required in the formulation of the A-Matrix.

With the exception of the numerical integration algorithm, the requirements for the existence, uniqueness and stability of the analytical solution also suffice for the numerical solution.

The numerical integration scheme used is of the predictor-corrector type, with a Runge-Kutta starting routine to give the required starting values. Since the system to be solved involves many abrupt discontinuities, a variable step size numerical integrator was developed. Initially, a maximum step size h is specified and this step is divided by 2^{10} to begin the solution. After a sufficient number of starting values are obtained, a comparison between the predictor and corrector is made at each step and, if the difference is small enough, the step size is doubled. The interval is always doubled to the maximum size unless the difference between the predictor and corrector becomes too large. A comparison is continually made between two error criteria $\epsilon 1$ and $\epsilon 2$

if $\left|\frac{\text{Predictor-Corrector}}{\text{Predictor}}\right| \leq \epsilon 1$ double h,

if $\left|\frac{\text{Predictor-Corrector}}{\text{Predictor}}\right| \geq \epsilon 2$ halve h

if $\epsilon 1 < \left|\frac{\text{Predictor-Corrector}}{\text{Predictor}}\right| < \epsilon 2$ allow h to remain the same.

In halving the interval, the possibility of continually halving exists. Thus if halving is required twice in succession, the last accepted point is taken as a new starting point using

$h/2^{10}$ as the interval. This method allows the running interval to adjust to a proper value regardless of the initial h chosen. It was shown that the integration algorithm yields a unique solution and converges to that solution. The numerical integration algorithm was also tested exhaustively for accuracy and its ability to solve highly nonlinear problems.

The last phase of the investigation was to run numerical experiments on the whole system. The results of three types of circuits were described in detail. The first circuit was a linear R-L-C circuit that allowed comparison to a known closed-form solution. The second circuit was a negative resistance oscillator. A classical nonlinear analysis was performed under suitable approximations. The approximations were then violated by varying the parameter values. The third circuit was a tunnel diode switch. This circuit does not possess a closed-form classical solution. This circuit was chosen to show the effectiveness of the developed method of analysis.

Results and Conclusions

The numerical experiments conducted during the course of this investigation verify the accuracy of simulation and the versatility of the methods developed in analyzing general nonlinear networks. The method also minimizes the number of differential equations to be solved and recomputes algebraic relationships only when needed for the nonlinear analysis, thus reducing the actual digital computer calculations required.

The only restrictions placed on the types of elements allowed arose from the requirements for the existence of a unique stable solution, and for the existence of a unique convergent numerical analog. These restrictions are:

(a) the partial derivatives of the capacitance, inductance, resistance, and source matrix functions with respect to state variables exists and be continuous, and the partial derivatives of the capacitance and inductance function be bounded.

(b) the inverse of the network resistive (R), conductive (G), capacitive (C), and inductive (L), matrix function as defined in Chapter II exist;

(c) the network admittance (Y) and impedance (Z) functions as defined in Chapter II be greater than or equal to zero;

(d) a parameter relationships are required not to be explicit functions of time;

(e) nonlinear capacitors are voltage controlled, nonlinear inductors are current controlled, and nonlinear resistors are voltage or current controlled;

(f) all sources are either constant, time dependent, or dependent upon a voltage or current defined in the network and consistent with restriction (a);

(g) Each network branch contains one and only one inductive, or capacitive, or resistive type element;

(h) all parameters are specified in a consistent set of units; for most high speed electronic work, the following set is very satisfactory and was used in most of the analyses in this study:

capacitance in picofarads,
inductance in microhenries,
resistance in kilohms,
current in milliamperes,
and time in nanoseconds.

If condition (c) is violated, then the existence of a stable solution is no longer guaranteed; thus, it can be used to examine the network formulation for possible regions of instability.

For solving the nonlinear differential equations numerically, the variable step algorithm developed succeeds very well, and the Hamming method used in this investigation yields a unique converging solution to the network problems as developed during the course of this study.

Thus, it can be concluded that the state variable approach as formulated in this investigation yields a general method for solving arbitrarily specified nonlinear network problems on a digital computer.

BIBLIOGRAPHY

1. Bashkow, T. R., "The A Matrix, New Network Description," IRE Transactions on Circuit Theory, Vol. CT-4, September, 1957, pp. 117-120.

2. Beaudette, J. H., and P. A. Honkanen, "Circuit Analysis of Nonlinear Systems, PECANS," Scientific Computation Department, International Business Machines Corporation, Poughkeepsie, New York, Program write-up, August 14, 1961.

3. Boyer, A. I., "Liapunov Stability for Nonlinear Differential Equations," M.S. Thesis, Mathematics, University of Utah, 1966.

4. Brooks, N., and H. S. Long, "A Program for Computing the Transient Response of Transistor Switching Circuits - PETAP," International Business Machines Corporation, Poughkeepsie, New York, 1959.

5. Bryant, P. R. "The explicit Form of Bashkow's A Matrix," IRE Transactions on Circuit Theory, Vol. CT-9, September, 1962, pp. 303-306.

6. Desoer, C. A., and J. Katzenelson, "Nonlinear RLC Networks," Bell Systems Technical Journal, Vol. 44, January, 1965, pp. 161-198.

7. Hamming, R. W., Numerical Methods for Engineers and Scientists, (McGraw-Hill, New York, 1962).

8. Henrici, P., Discrete Variable Methods in Ordinary Differential Equations, (John Wiley and Sons, New York, 1962).

9. International Business Machines Corporation, "1620 Electronic Circuit Analysis Program - ECAP," Internternational Business Machines Corporation, Data Processing Division, White Plains, New York, 1965.

10. Katzenelson, J., "ADNET. A Simulator for Nonlinear Networks," Proceedings of the IEEE, November, 1966, pp. 1536-1552.

11. Kuh, E. S., and R. A. Rohrer, "The State-Variable Approach to Network Analysis," Proceedings of the IEEE, July, 1965, pp. 672-686.

12. La Salle, J., and S. Lefschetz, Stability by Liapunov's Direct Method with Applications, (Academic Press, New York, 1961).

13. Malenburg, A. F., and F. L. Conwell, "NET-1: Network Analysis Program," University of California, Los Alamos Scientific Laboratory, Los Alamos, New Mexico, April, 1963.

14. Reza, F., and S. Seely, Modern Network Analysis, (McGraw-Hill, New York, 1959).

15. Seshu, S., and M. B. Reed, Linear Graphs and Electrical Networks, (Addison-Wesley, Reading, Massachusetts, 1961).

APPENDIX A

USAGE OF THE COMPUTER PROGRAM

To specify a network to be analyzed, the program requires four sets of data; these are:

(a) topological and parametric descriptions,

(b) nonlinear tabular data,

(c) computer time-control data,

(d) source function data.

The topological and parametric data specifies the initial node, final node, type of parameter in the branch and its value (or initial value for a nonlinear element). For nonlinear elements, an additional coded number specifying the nonlinear relationship must be included. This number takes the form NMXXYY,

where N = 1, if the element is a resistor,
2, if the element is a inductor,
3, if the element is a capacitor,
4, if the element is a current source,
5, if the element is a voltage source,
M = 1, if the independent variable is time,
2, if the independent variable is a voltage,
3, if the independent variable is a current.
XX is the number of the branch of the independent variable,

XX = 0 if the independent variable is time.

YY is the number of points in the nonlinear tabular relationship.

For example, a nonlinear resistor that depends upon the voltage in the third branch and is specified by twenty pairs of points in a table would have NMXXYY = 120340.

The tabular data has pairs of independent and dependent variables starting with the lowest value of the independent variable and proceeding in ascending order of the independent variable.

The time-control data specifies the final time to which the analysis is to run, the time of the first printout, the time increment between printouts, the maximum step size for the numerical integration scheme, and a constant K. If $K \neq 0$, only the state variables v_C, and i_L along with their associated values i_C, and v_L will be printed out, if $K = 0$ all the currents and voltages will be printed out.

The source function data specifies the branch where the source is, the type of source involved, and three parameters A, ω, and ϕ. The types of sources allowed are:

$$\epsilon\ (t) = A \sin (\omega t + \phi)$$

$$i\ (t) = A \sin (\omega t + \phi)$$

$$e\ (t) = A\ (1 - 2e^{-\omega t} + e^{-2\omega t})$$

$$i\ (t) + A\ (1 - 2e^{-\omega t} + e^{-2\omega t}).$$

Specifically the data is to appear as follows:

Card number 1 through n is to contain the topological and parametric data for branches 1 through n. On these cards, columns 1 and 2 specify the initial node, and columns 4 and 5 the final node. The numbers 00 and 99 are non-allowable node numbers, otherwise any numbers may be used. All parameter values are specified in a FORTRAN F8.3 format:

a) the value of R is given starting in column 7,

b) the value of L is given starting in column 15,

c) the value of C is given starting in column 23,

d) the value of E is given starting in column 31,

e) the value of I is given starting in column 39,

f) the value of RS is given starting in column 47,

where RS is the value of resistance associated with the voltage or current source.

The number NMXXYY is to start in column 56. All nonexisting values are left blank. Following the last topological data card, a control card with 99 in columns 1 and 2 must be inserted to signify the end of the data.

The next set of cards contains the tabular data with ten numbers per card, again in the F8.3 format. Each separate set of data must begin a new card.

The next card is the computer time-control card containing the four numbers; final time, time of first printout, integration step size, and print interval. All these numbers are in the F8.3 format. The number K is to go in columns 33 and 34 in I2 format.

The final group of cards contains the source information given as follows:

In columns 1 and 2 is the branch number of the source.

In columns 4 and 5 is the type of the source.

Starting in column 7 is the number A.

Starting in column 15 is the number ω.

Starting in column 23 is the number ϕ.

There are as many of these cards as there are sources. Once more A, ω, and ϕ are in F8.3 format.

The final card is another control with 99 in columns 1 and 2.

Any cards not required can be omitted with the exception of the three control cards. In addition, it is assumed that there will be at least two topological description cards.

In Appendix C the data cards for the runs listed are shown. These should serve as a sufficient set of examples.

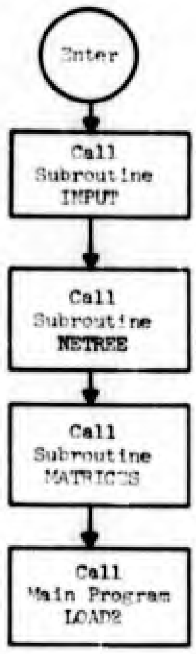
APPENDIX B

FLOW CHARTS FOR THE COMPUTER PROGRAMS

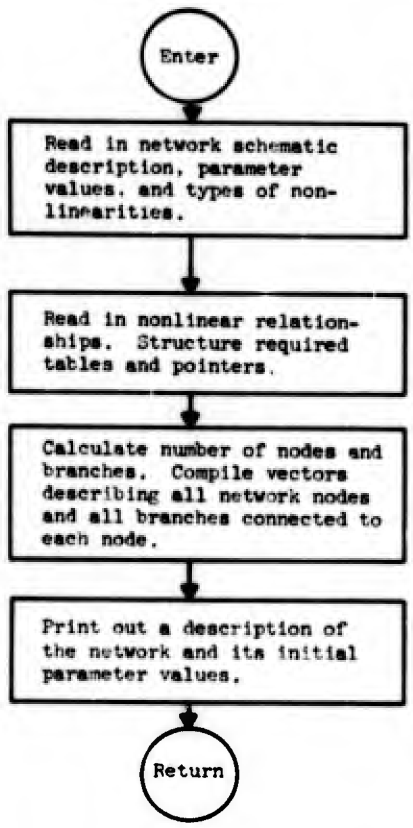
The computer programs associated with this study are written in the DAFT language for the IBM 7074 Computer. The DAFT language is a special version of the FORTRAN II language developed at The Pennsylvania State University Computer Center. Because of the small memory of the IBM 7074, the program consists of three segments. These segments LOAD1, LOAD2, and LOAD3 are main programs brought into core under the control of a library program CHAIN. Each of these segments are in turn controlled by main programs bearing the same name as the segment itself.

Other subroutines not shown in the flow charts are obtained from the library routines by the DAFT compiler. These routines perform the required mathematical functions of matrix inversion and multiplication, evaluation of trigmonetric and exponential functions, etc.

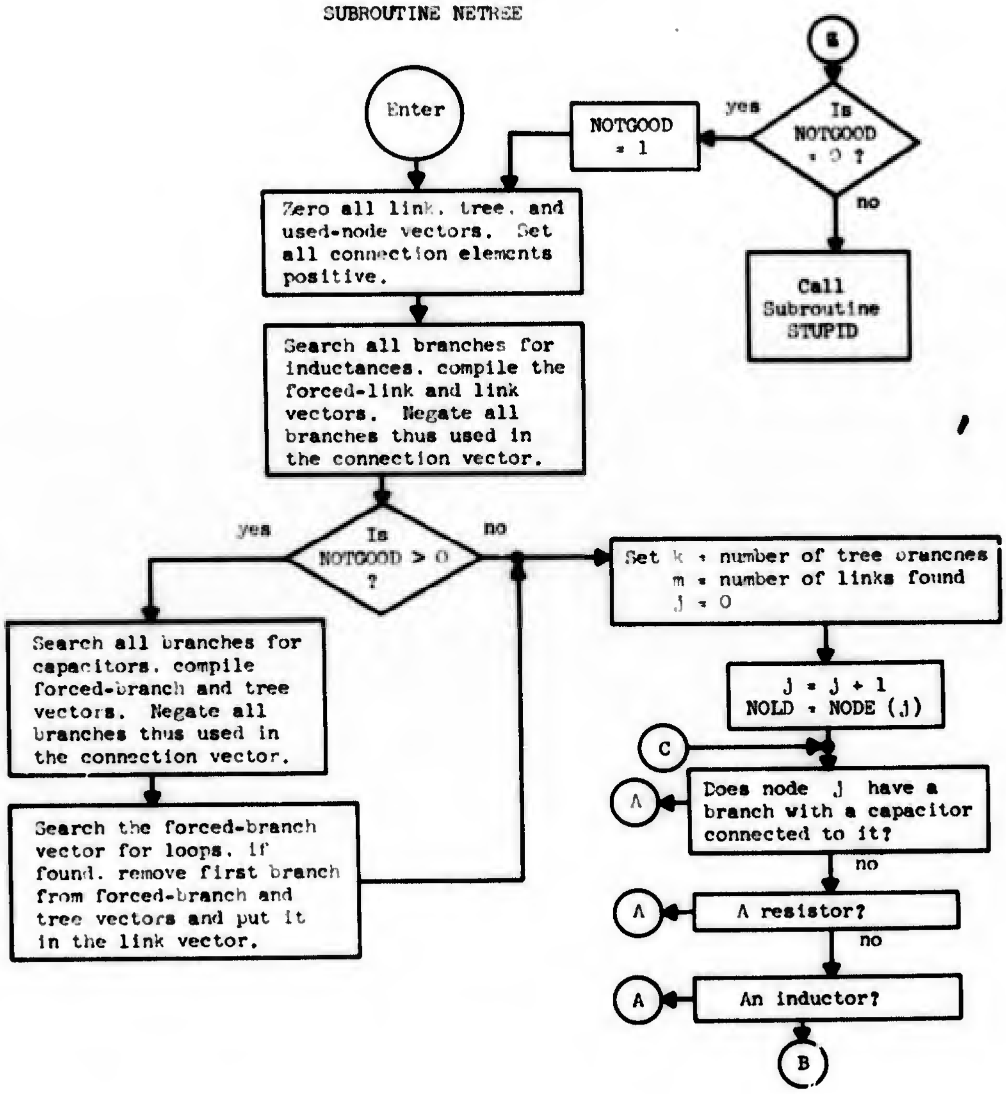
MAIN PROGRAM LOAD1



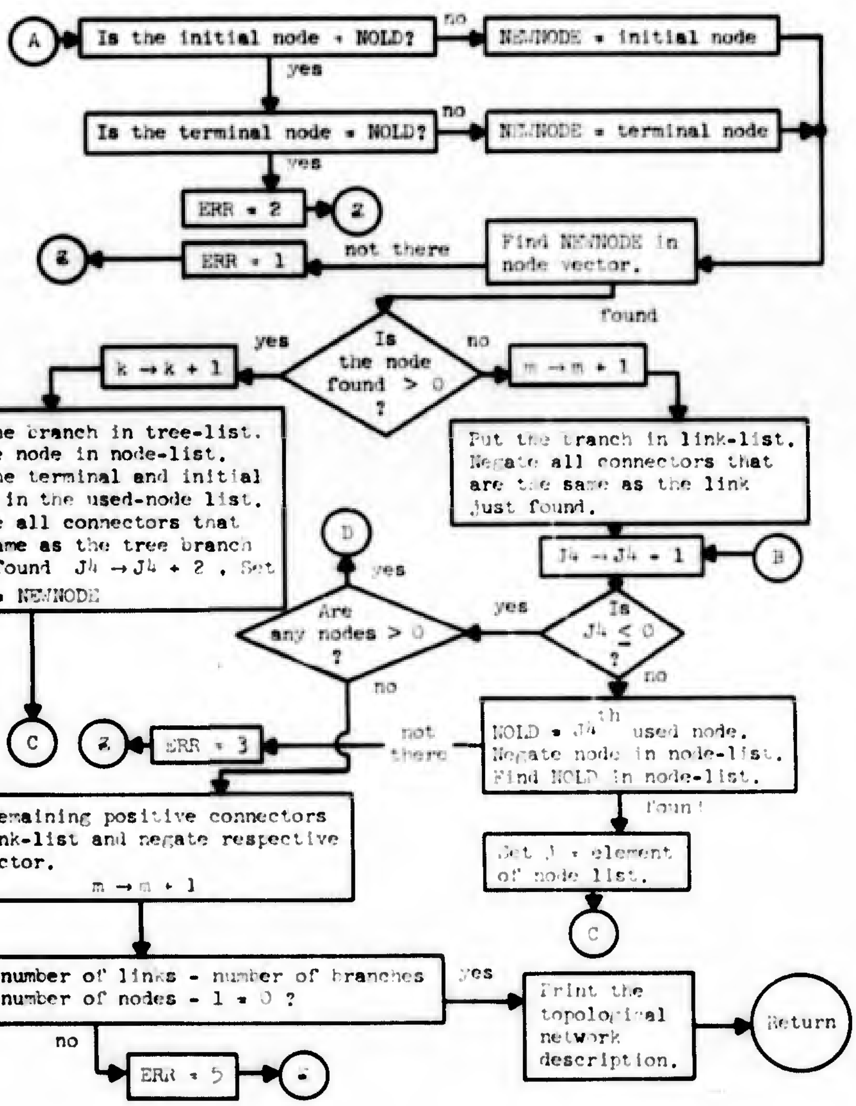
SUBROUTINE INPUT



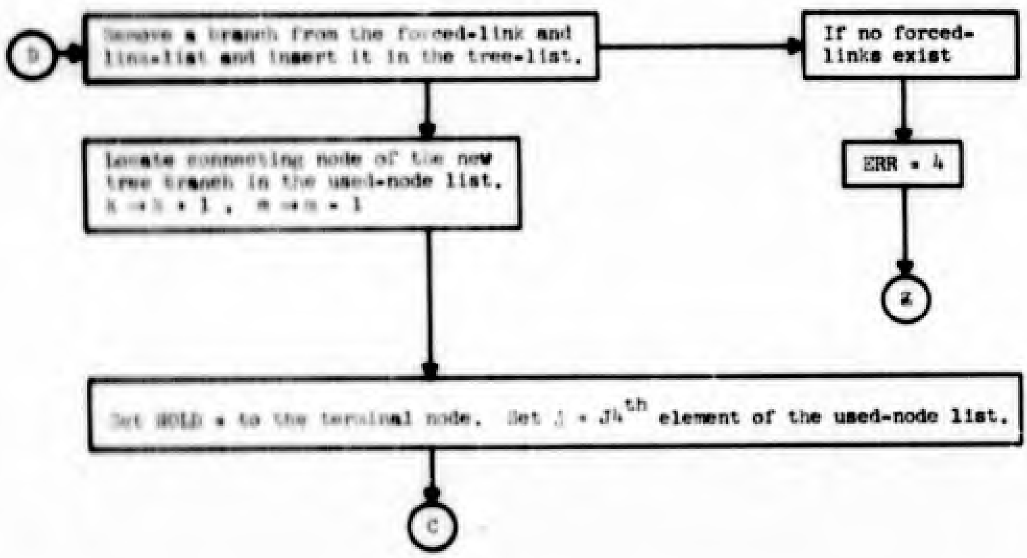
SUBROUTINE NETREE



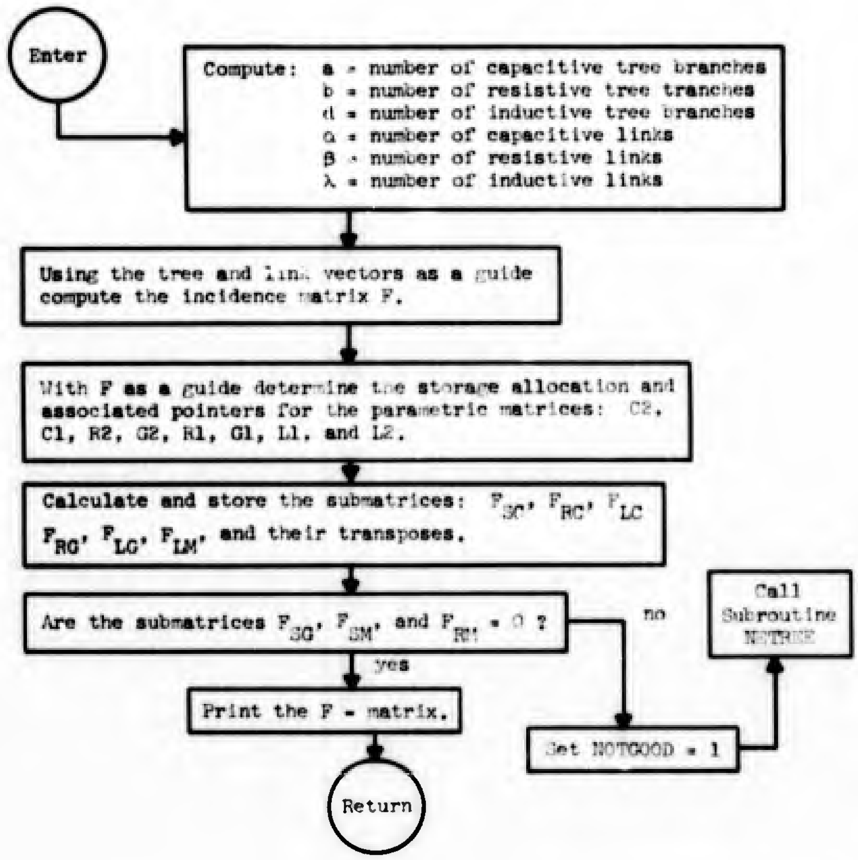
SUBROUTINE NETREE (CONTINUED)



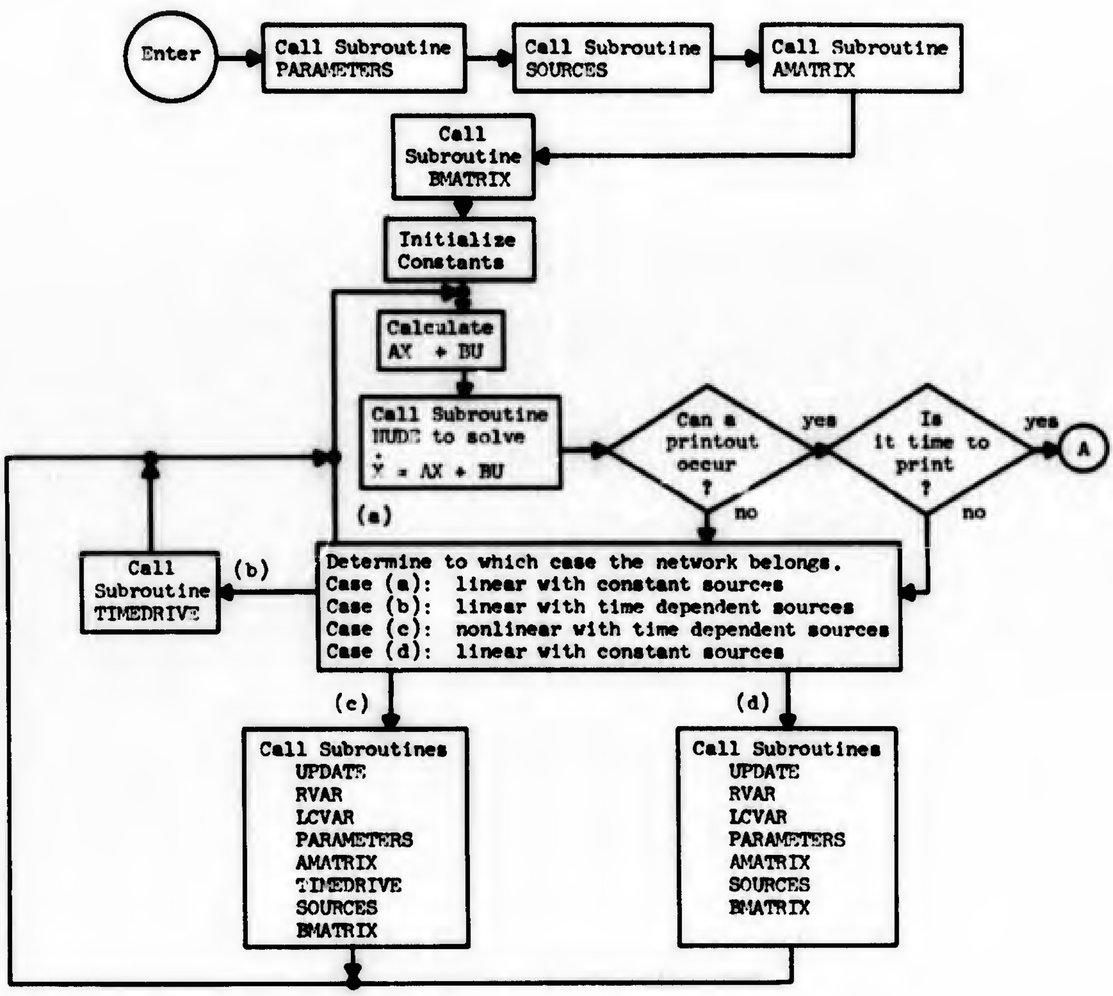
SUBROUTINE NETREE (CONTINUED)



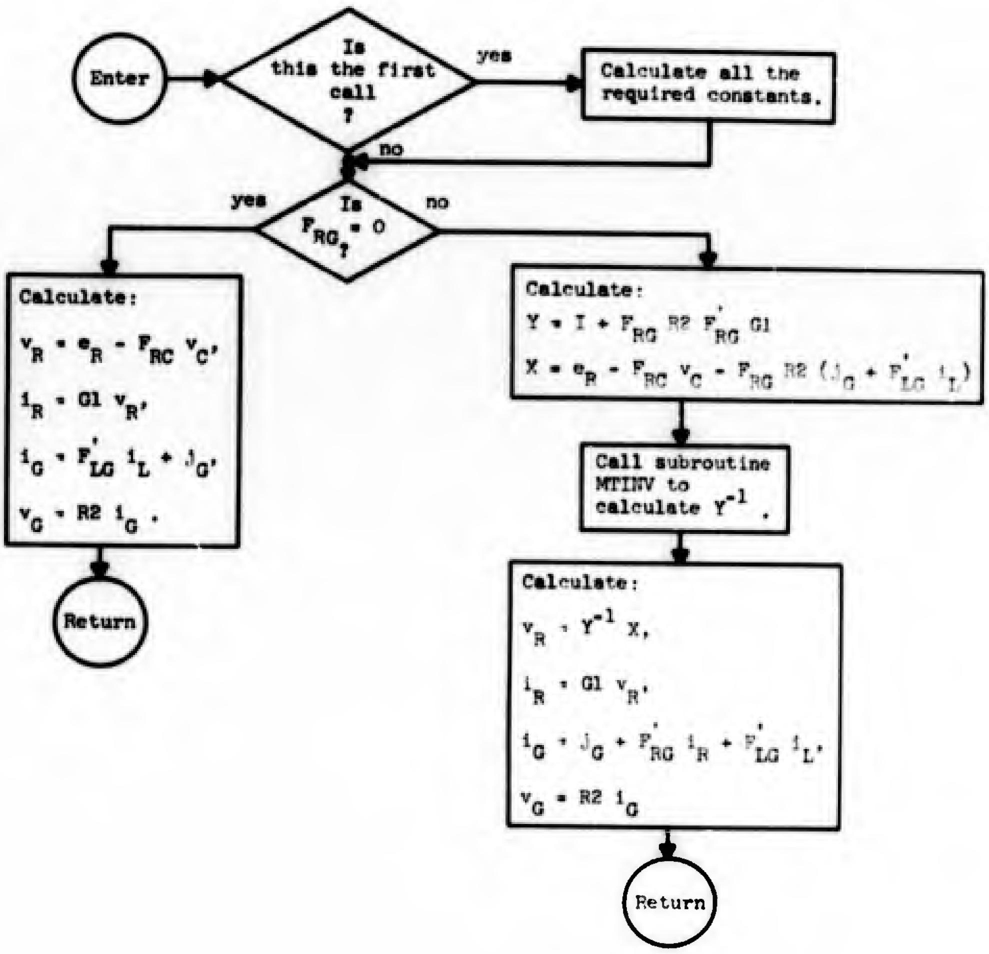
SUBROUTINE MATRICES



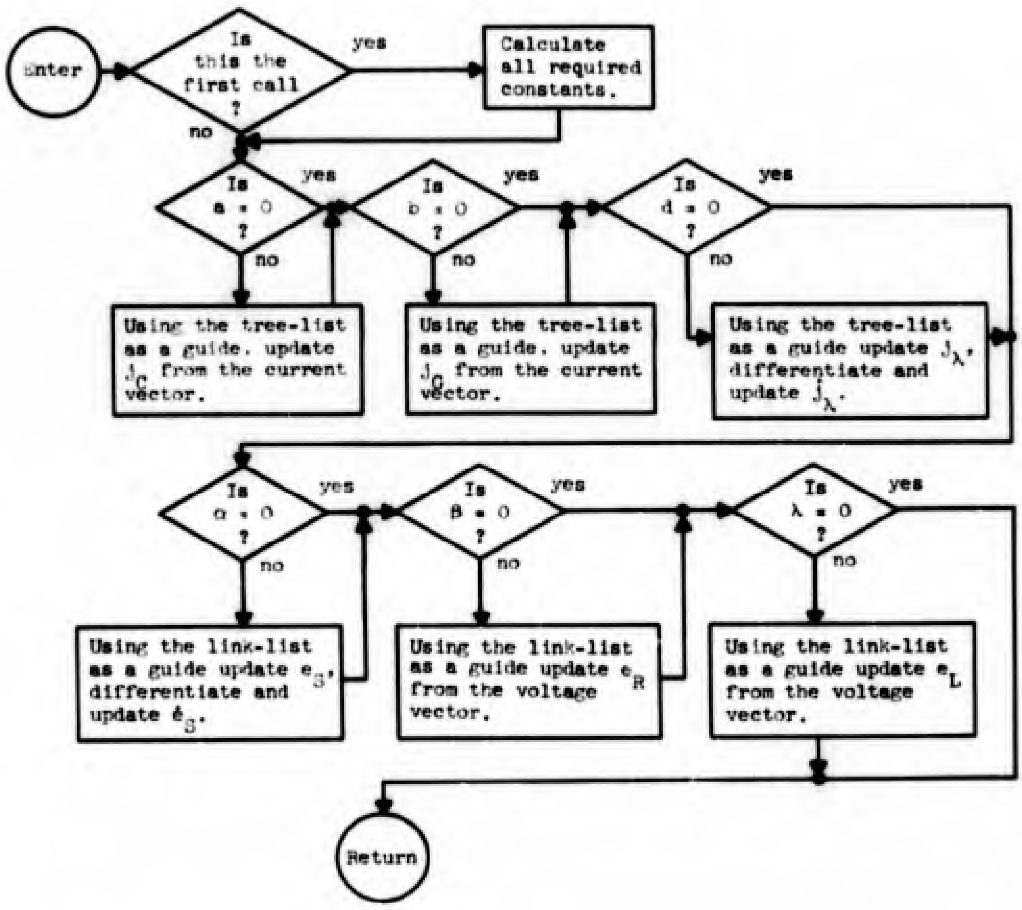
MAIN PROGRAM LOAD2



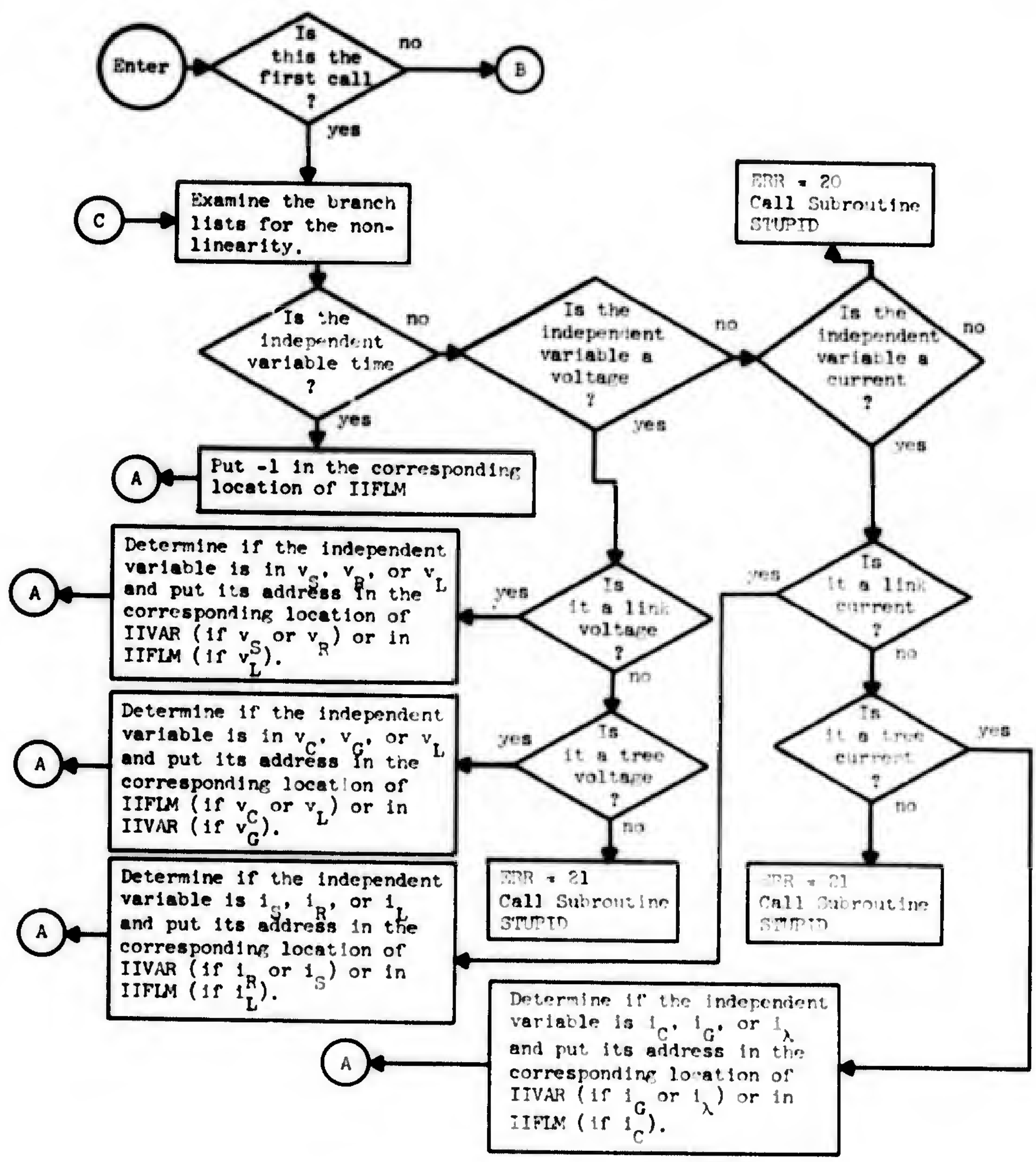
SUBROUTINE RVAR



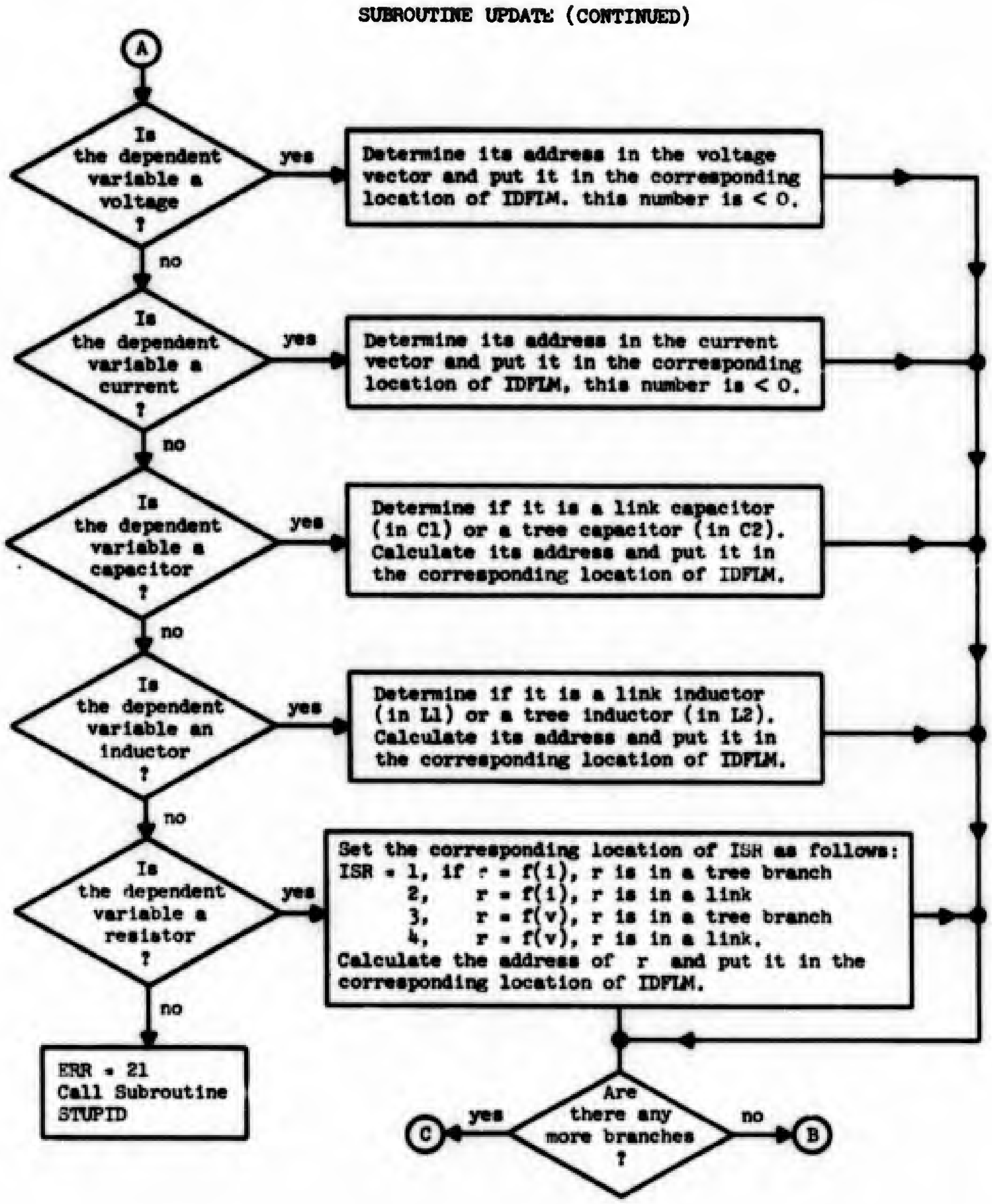
SUBROUTINE SOURCES



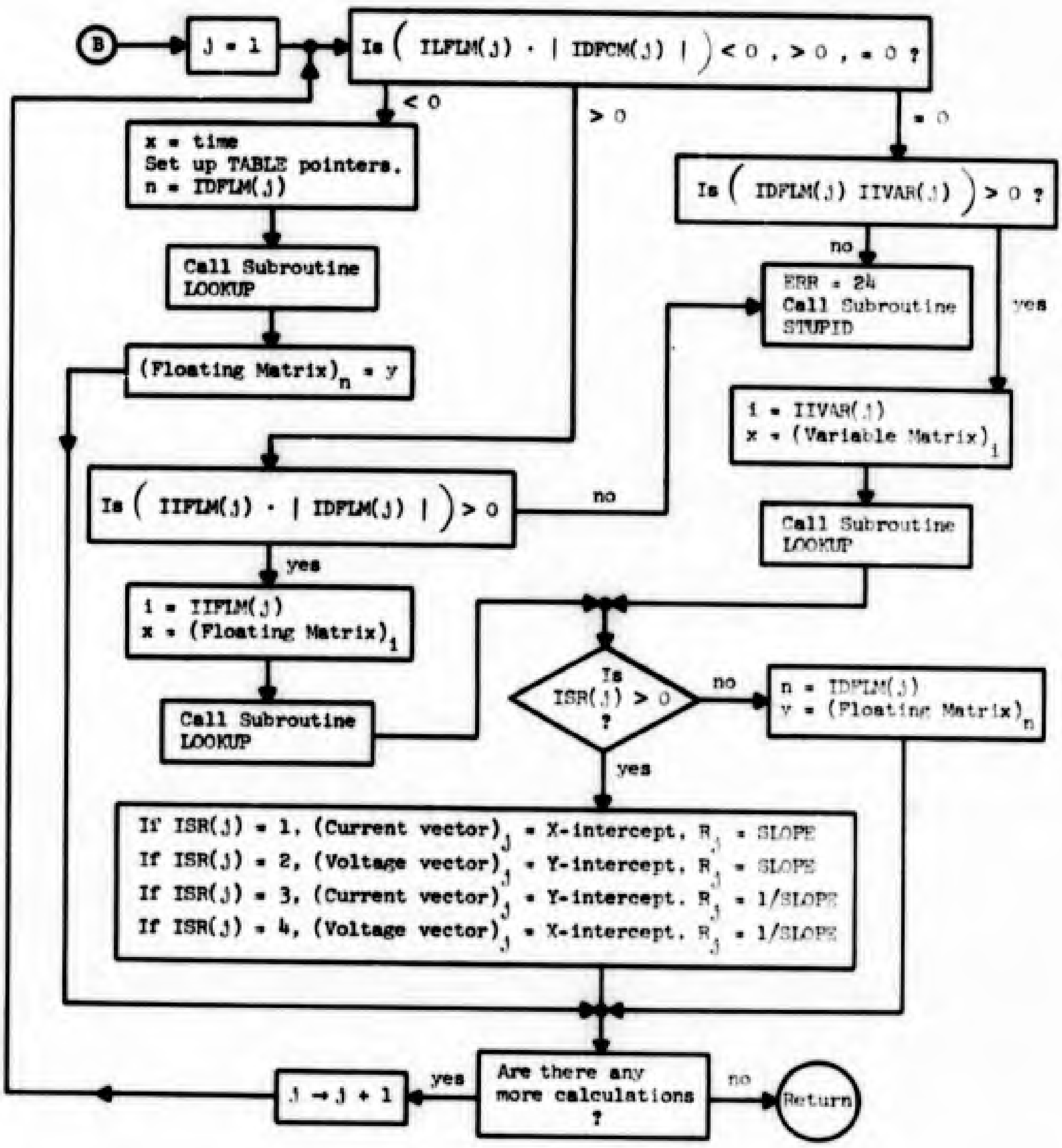
SUBROUTINE UPDATE



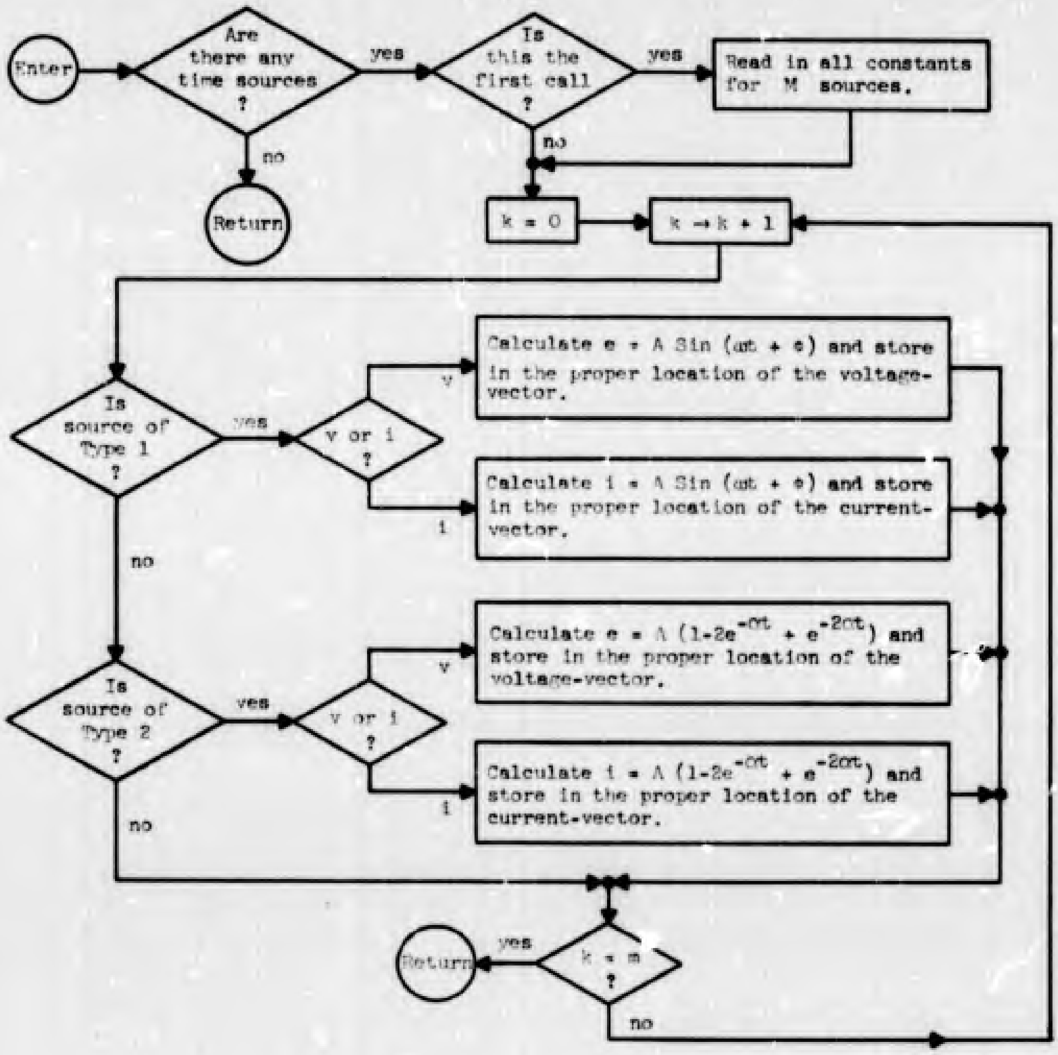
SUBROUTINE UPDATE (CONTINUED)



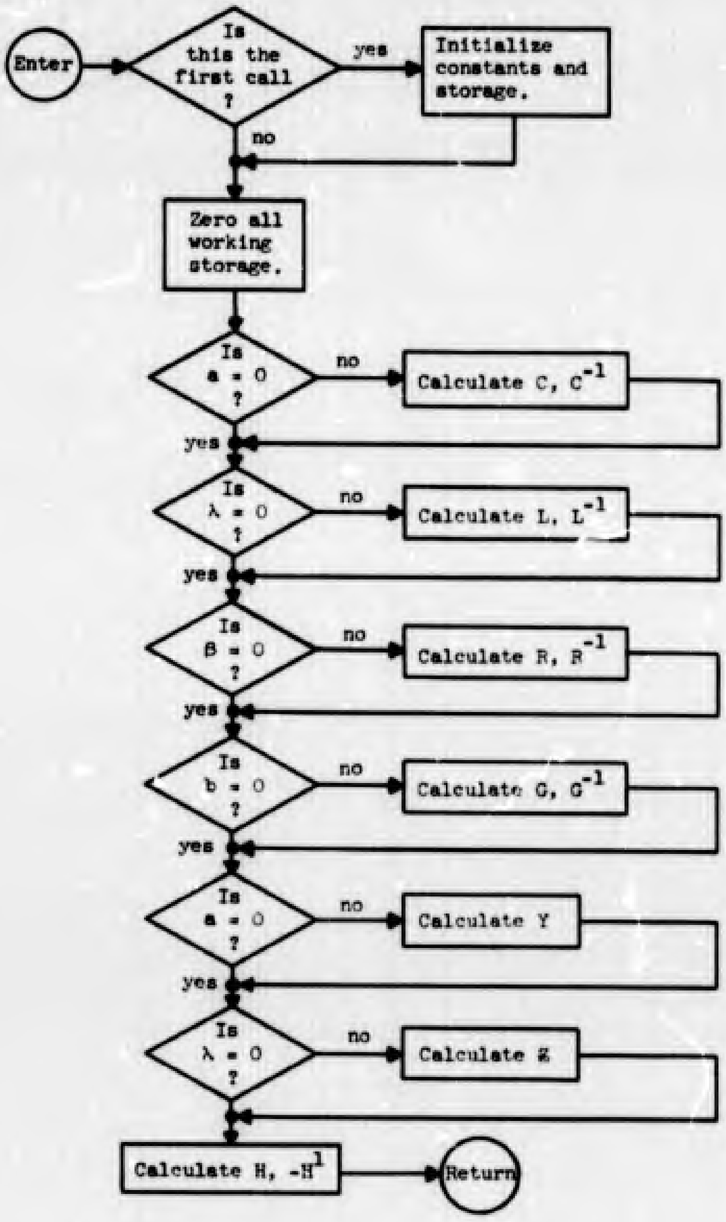
SUBROUTINE UPDATE (CONTINUED)



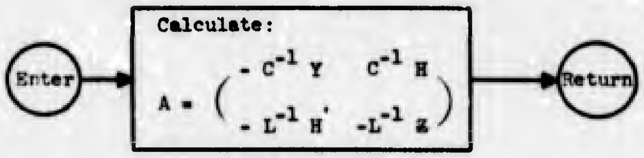
SUBROUTINE TIMEDRIVE



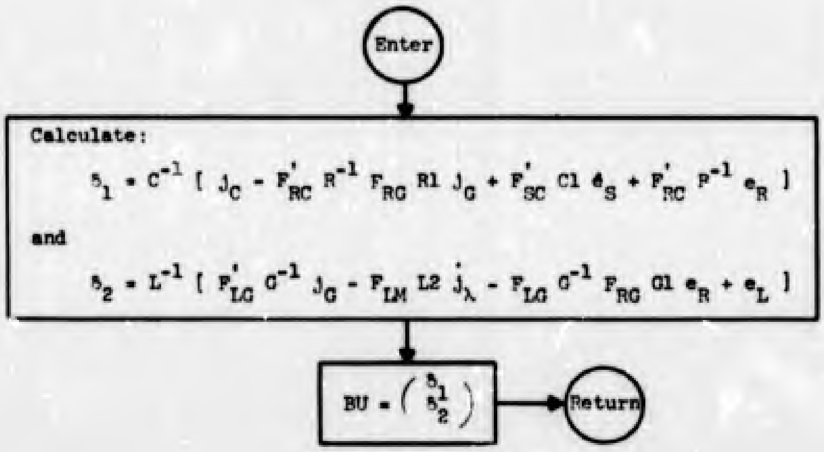
SURBOUTINE PARAMETERS



SUBROUTINE AMATRIX

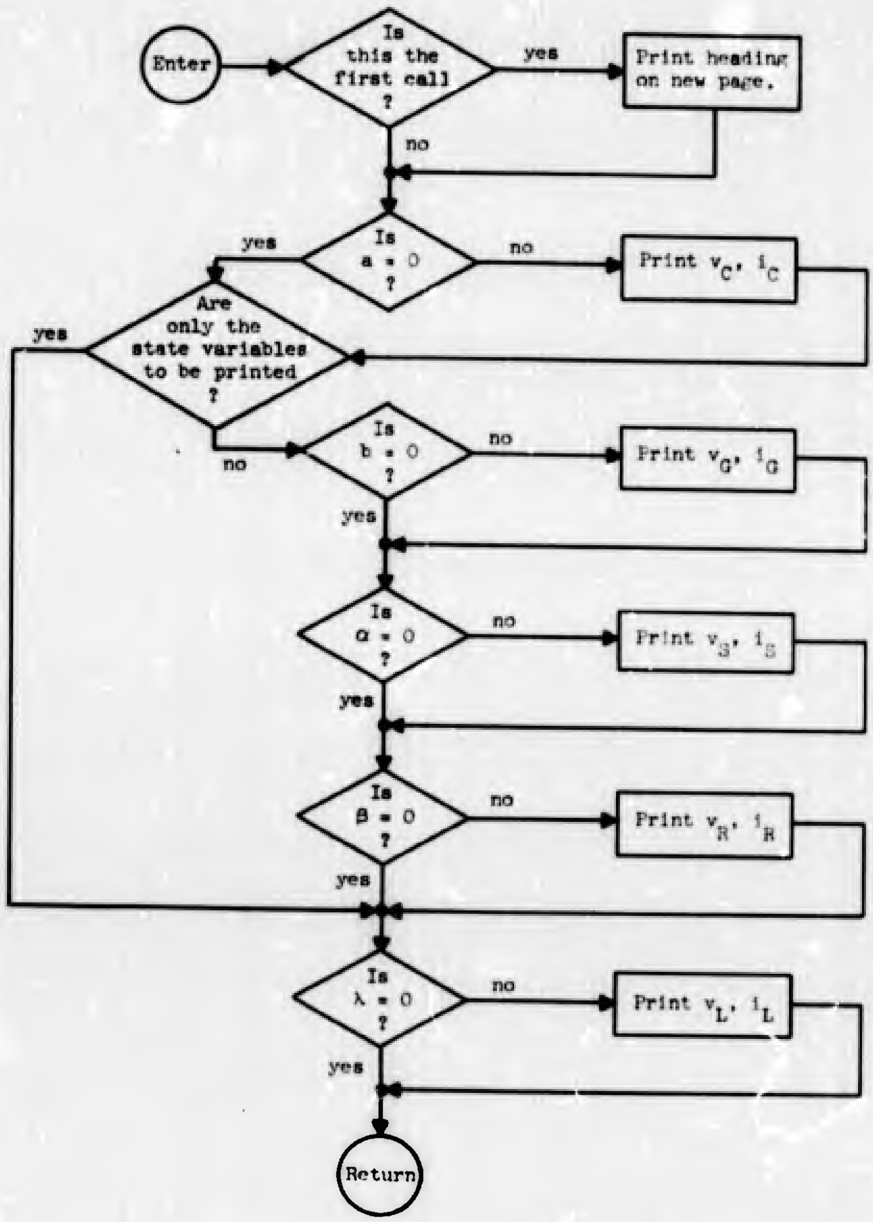


SUBROUTINE BMATRIX

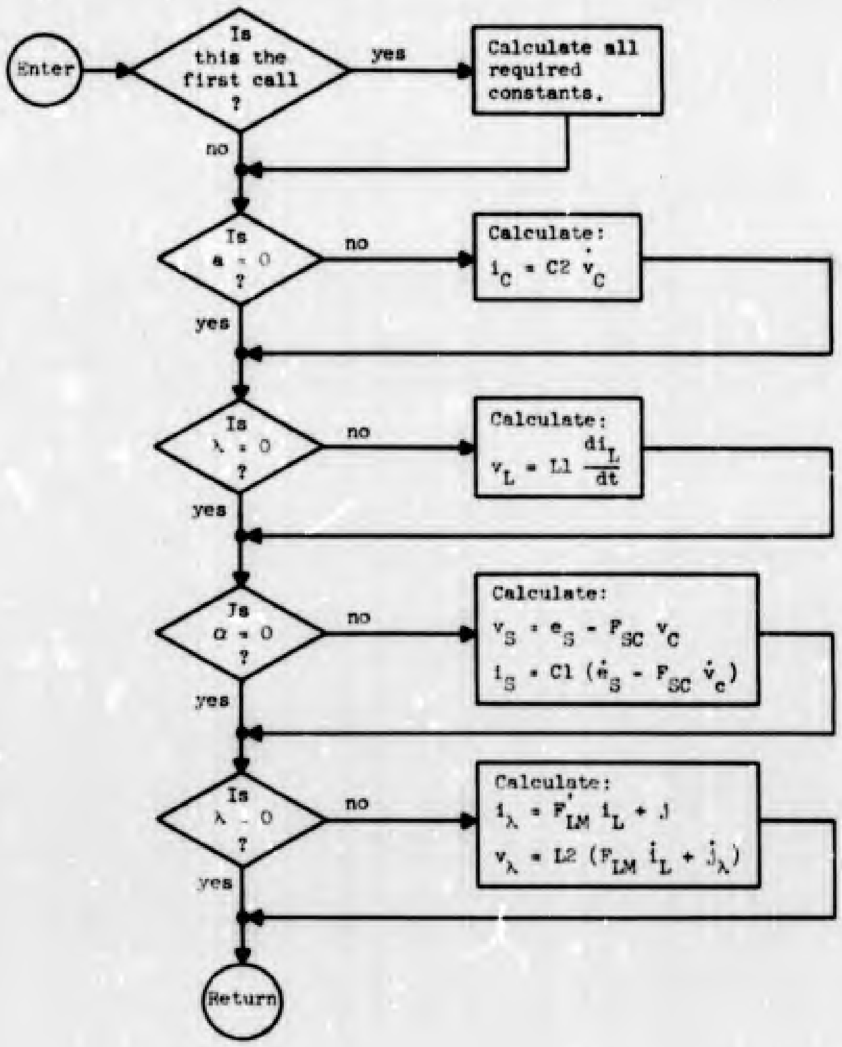


SUBROUTINE STUPID

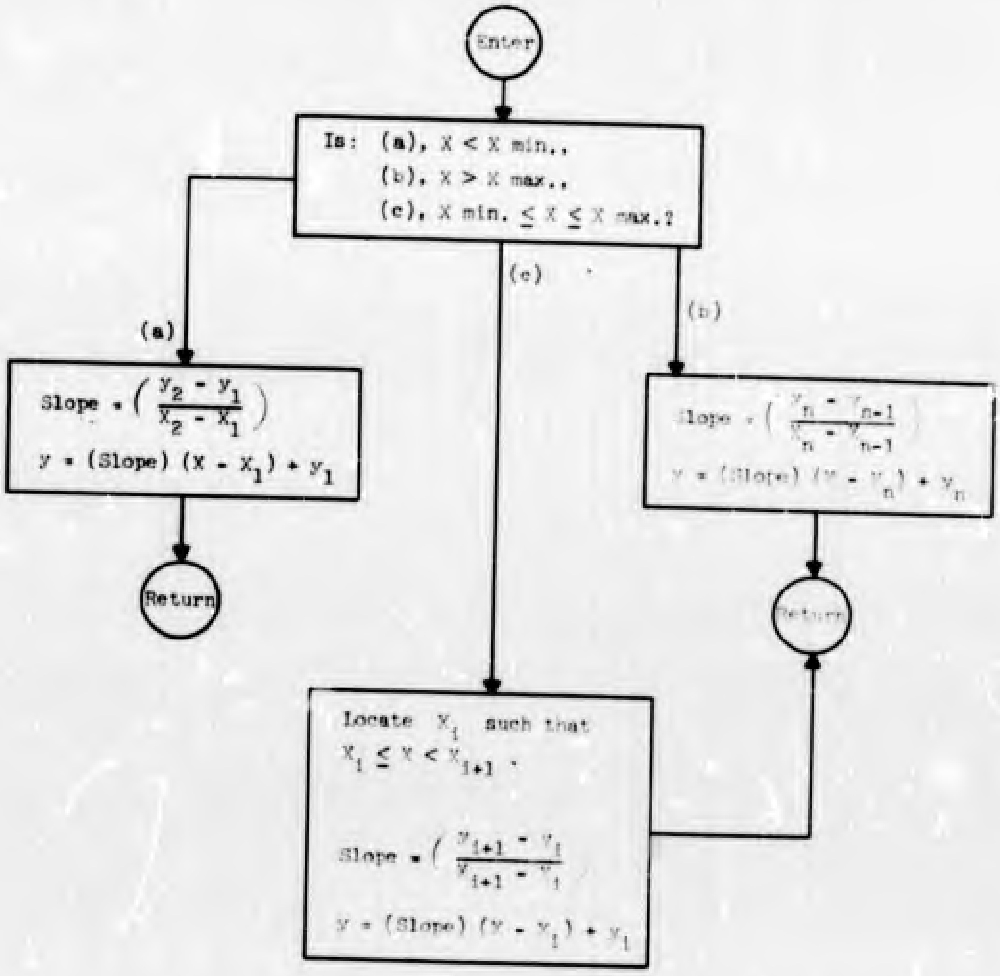
SUBROUTINE WORDS



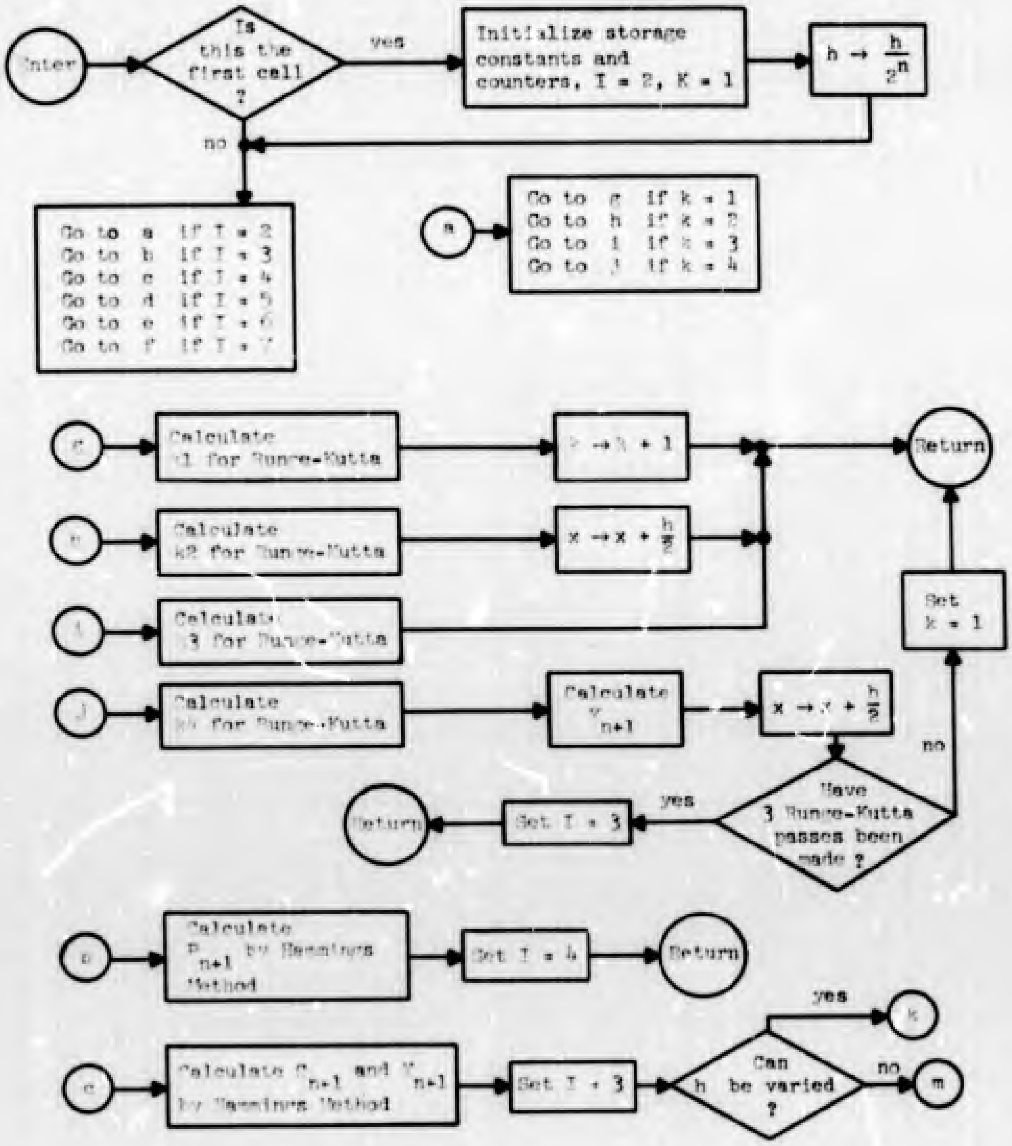
SUBROUTINE LCVAR



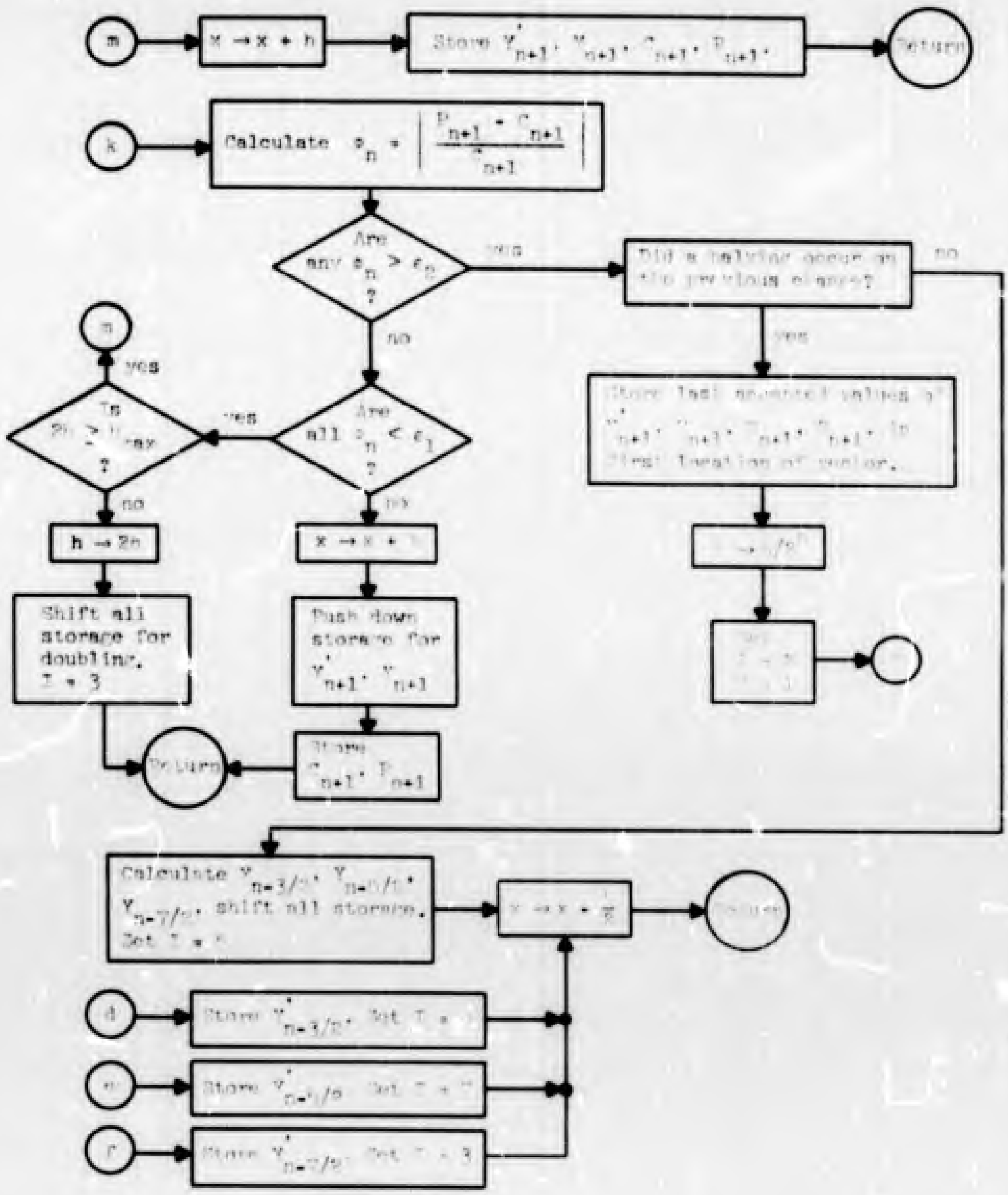
SUBROUTINE LOOKUP



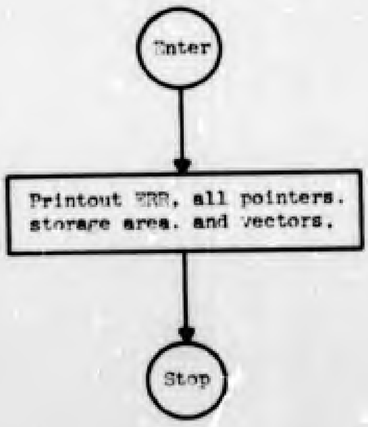
SUBROUTINE MUDE



SUBROUTINE (CONTINUED)



MAIN PROGRAM LOAD3



APPENDIX C

COMPUTER OUTPUT LISTINGS

The following pages of computer output lists the comparison of the numerical integration of a fourth-order differential equation to the known solution.

The differential equation used was:

$$y^{iv} - y^{iii} - 7y'' + y' + 6y = 0.$$

Subject to the following initial conditions:

$y'(0) = 1,$

$y''(0) = 15,$

and $y^{iii}(0) = 19.$

TEST OF NUDE FOR D4-D3-7D2+C+6Y

X	Y	Y-EXACT	H
0.000000E 00	0.400000E 01	0.400000E 01	0.100000E 00
0.976563E-04	0.400010E 01	0.400010E 01	0.976563E-04
0.100000E 00	0.417860E 01	0.417860E 01	0.250000E-01
0.200000E 00	0.453257E 01	0.453257E 01	0.250000E-01
0.300000E 00	0.509909E 01	0.509909E 01	0.250000E-01
0.400000E 00	0.593159E 01	0.593159E 01	0.250000E-01
0.500000E 00	0.710482E 01	0.710482E 01	0.250000E-01
0.600000E 00	0.872177E 01	0.872177E 01	0.250000E-01
0.700000E 00	0.109231E 02	0.109231E 02	0.250000E-01
0.800000E 00	0.138999E 02	0.138999E 02	0.250000E-01
0.900000E 00	0.179112E 02	0.179112E 02	0.250000E-01
0.100000E 01	0.233070E 02	0.233070E 02	0.250000E-01
0.110000E 01	0.305605E 02	0.305605E 02	0.250000E-01
0.120000E 01	0.403102E 02	0.403103E 02	0.250000E-01
0.130000E 01	0.534185E 02	0.534186E 02	0.250000E-01
0.140000E 01	0.710489E 02	0.710489E 02	0.250000E-01
0.150000E 01	0.947717E 02	0.947717E 02	0.250000E-01
0.160000E 01	0.126706E 03	0.126706E 03	0.250000E-01
0.170000E 01	0.169712E 03	0.169712E 03	0.250000E-01
0.180000E 01	0.227649E 03	0.227649E 03	0.250000E-01
0.190000E 01	0.305725E 03	0.305725E 03	0.250000E-01
0.200000E 01	0.410971E 03	0.410972E 03	0.250000E-01
0.210000E 01	0.552875E 03	0.552876E 03	0.250000E-01
0.220000E 01	0.744243E 03	0.744243E 03	0.250000E-01
0.230000E 01	0.100236E 04	0.100236E 04	0.250000E-01
0.240000E 01	0.135055E 04	0.135055E 04	0.250000E-01
0.250000E 01	0.182031E 04	0.182031E 04	0.250000E-01
0.260000E 01	0.245414E 04	0.245415E 04	0.250000E-01
0.270000E 01	0.330942E 04	0.330942E 04	0.250000E-01
0.280000E 01	0.446357E 04	0.446358E 04	0.250000E-01
0.290000E 01	0.602114E 04	0.602114E 04	0.250000E-01
0.300000E 01	0.812322E 04	0.812322E 04	0.250000E-01
0.310000E 01	0.109603E 05	0.109603E 05	0.250000E-01
0.320000E 01	0.147893E 05	0.147894E 05	0.250000E-01
0.330000E 01	0.199575E 05	0.199575E 05	0.250000E-01
0.340000E 01	0.269332E 05	0.269332E 05	0.250000E-01
0.350000E 01	0.363486E 05	0.363486E 05	0.250000E-01
0.360000E 01	0.490574E 05	0.490574E 05	0.250000E-01
0.370000E 01	0.662116E 05	0.662116E 05	0.250000E-01
0.380000E 01	0.893664E 05	0.893664E 05	0.250000E-01
0.390000E 01	0.120621E 06	0.120621E 06	0.250000E-01
0.400000E 01	0.162809E 06	0.162809E 06	0.250000E-01
0.410000E 01	0.219756E 06	0.219756E 06	0.250000E-01
0.420000E 01	0.296625E 06	0.296625E 06	0.250000E-01
0.430000E 01	0.400386E 06	0.400386E 06	0.250000E-01
0.440000E 01	0.540446E 06	0.540446E 06	0.250000E-01
0.450000E 01	0.729506E 06	0.729506E 06	0.250000E-01
0.460000E 01	0.984708E 06	0.984709E 06	0.250000E-01
0.470000E 01	0.132919E 07	0.132919E 07	0.250000E-01
0.480000E 01	0.179419E 07	0.179420E 07	0.250000E-01
0.490000E 01	0.242188E 07	0.242188E 07	0.250000E-01
0.500000E 01	0.326916E 07	0.326917E 07	0.250000E-01
0.510000E 01	0.441287E 07	0.441288E 07	0.250000E-01

0.520000E 01	0.595671E 07	0.595672E 07	0.250000E-01
0.530000E 01	0.804068E 07	0.804069E 07	0.250000E-01
0.540000E 01	0.108537E 08	0.108537E 08	0.250000E-01
0.550000E 01	0.146509E 08	0.146510E 08	0.250000E-01
0.560000E 01	0.197767E 08	0.197767E 08	0.250000E-01
0.570000E 01	0.266956E 08	0.266957E 08	0.250000E-01
0.580000E 01	0.360352E 08	0.360353E 08	0.250000E-01
0.590000E 01	0.486424E 08	0.486425E 08	0.250000E-01
0.600000E 01	0.656603E 08	0.656604E 08	0.250000E-01
0.610000E 01	0.886320E 08	0.886321E 08	0.250000E-01
0.620000E 01	0.119641E 09	0.119641E 09	0.250000E-01
0.630000E 01	0.161498E 09	0.161498E 09	0.250000E-01
0.640000E 01	0.217999E 09	0.217999E 09	0.250000E-01
0.650000E 01	0.294268E 09	0.294268E 09	0.250000E-01
0.660000E 01	0.397220E 09	0.397220E 09	0.250000E-01
0.670000E 01	0.536191E 09	0.536191E 09	0.250000E-01
0.680000E 01	0.723781E 09	0.723782E 09	0.250000E-01
0.690000E 01	0.977003E 09	0.977004E 09	0.250000E-01
0.700000E 01	0.131882E 10	0.131882E 10	0.250000E-01
0.710000E 01	0.178021E 10	0.178022E 10	0.250000E-01
0.720000E 01	0.240304E 10	0.240304E 10	0.250000E-01
0.730000E 01	0.324376E 10	0.324376E 10	0.250000E-01
0.740000E 01	0.437862E 10	0.437862E 10	0.250000E-01
0.750000E 01	0.591052E 10	0.591052E 10	0.250000E-01
0.760000E 01	0.797836E 10	0.797837E 10	0.250000E-01
0.770000E 01	0.107697E 11	0.107697E 11	0.250000E-01
0.780000E 01	0.145375E 11	0.145375E 11	0.250000E-01
0.790000E 01	0.196236E 11	0.196236E 11	0.250000E-01
0.800000E 01	0.264891E 11	0.264891E 11	0.250000E-01
0.810000E 01	0.357565E 11	0.357566E 11	0.250000E-01
0.820000E 01	0.482663E 11	0.482663E 11	0.250000E-01
0.830000E 01	0.651526E 11	0.651527E 11	0.250000E-01
0.840000E 01	0.879469E 11	0.879470E 11	0.250000E-01
0.850000E 01	0.118716E 12	0.118716E 12	0.250000E-01
0.860000E 01	0.160250E 12	0.160250E 12	0.250000E-01
0.870000E 01	0.216314E 12	0.216315E 12	0.250000E-01
0.880000E 01	0.291994E 12	0.291994E 12	0.250000E-01
0.890000E 01	0.394150E 12	0.394151E 12	0.250000E-01
0.900000E 01	0.532047E 12	0.532048E 12	0.250000E-01
0.910000E 01	0.718189E 12	0.718190E 12	0.250000E-01
0.920000E 01	0.969453E 12	0.969455E 12	0.250000E-01
0.930000E 01	0.130863E 13	0.130863E 13	0.250000E-01
0.940000E 01	0.176646E 13	0.176646E 13	0.250000E-01
0.950000E 01	0.238447E 13	0.238447E 13	0.250000E-01
0.960000E 01	0.321870E 13	0.321870E 13	0.250000E-01
0.970000E 01	0.434479E 13	0.434480E 13	0.250000E-01
0.980000E 01	0.586485E 13	0.586486E 13	0.250000E-01
0.990000E 01	0.791672E 13	0.791674E 13	0.250000E-01
0.100000E 02	0.106865E 14	0.106865E 14	0.250000E-01
0.101000E 02	0.144252E 14	0.144252E 14	0.250000E-01

The following pages of computer output lists the results of the tests of the numerical integration algorithm.

This tests the response of the system to the highly nonlinear source drive of Figure 4-3.

TIME	VDOT	V	V(T)	H
0.00010	0.19998E 01	0.19530E-03	C.2000CE 01	0.97656E-04
0.10000	0.18097E 01	0.19033E 00	C.2000CE 01	0.25000E-01
0.20000	0.16375E 01	0.36254E 00	C.2000CE 01	C.50000E-01
0.30000	0.14816E 01	0.51836E 00	C.20000E 01	0.10000E 00
0.40000	0.13406E 01	0.65936E 00	0.20000E 01	0.10000E 00
0.50000	0.12131E 01	0.78694E 00	0.20000E 01	0.10000E 00
0.60000	0.10976E 01	0.90238E 00	0.2000CE 01	0.10000E 00
0.70000	0.99317E 00	0.10068E 01	0.2000CE 01	0.10000E 00
0.80000	0.89866E 00	0.11013E 01	C.20000E 01	0.10000E C0
0.90000	0.81314E 00	0.11869E 01	0.20000E 01	0.10000E 00
1.00000	0.73576E 00	0.12642E 01	C.20C00E 01	0.10000E 00
1.10000	0.66574E C0	0.13343E 01	0.20C00E 01	0.10000E 00
1.20000	0.60239E 00	0.13976E C1	0.20000E 01	0.10000E 00
1.30000	0.54506E 00	0.14549E C1	C.20000E 01	0.10000E 00
1.40000	0.49319E 00	0.15068E C1	C.20000E 01	0.10000E CC
1.50000	0.44626E 00	0.15537E 01	0.20000E 01	0.1000CE 00
1.60000	0.40379E 00	0.15962E 01	C.20000E 01	0.10000E 00
1.70000	0.36537E 00	0.16346E 01	C.20C00E 01	0.10000E 00
1.80000	0.33060E 00	0.16694E 01	C.20000E 01	0.10000E 00
1.90000	0.29914E 00	0.17009E C1	C.20C00E 01	0.10000E 00
2.00000	0.27067E 00	0.17293E 01	C.20000E 01	0.10000E 00
2.10000	0.24491E 00	0.17551E C1	C.20C00E 01	0.1C000E 00
2.20000	0.22161E 00	0.17784E 01	C.20000E 01	0.10000E 0C
2.30000	0.20052E 00	0.17995E C1	C.20C00E C1	0.10000E 00
2.40000	0.18144E 00	0.18186E 01	C.20000E 01	0.10000E C0
2.50000	0.16417E 00	0.18358E C1	C.20C00E 01	0.10000E 00
2.60000	0.14855E 00	0.18515E 01	C.20000E 01	0.10000E C0
2.70000	0.13441E 00	0.18656E 01	C.20000E 01	0.10000E 00
2.80000	0.12162E 00	0.18784E C1	C.2000CE 01	0.1000CE 00
2.90000	0.11005E 00	0.18900E C1	C.20C0CE 01	C.10000E 00
3.00000	0.99574E-01	0.19004E 01	0.20000E 01	0.1C000E C0
3.10000	0.90099E-01	0.19099E 01	0.20000E 01	0.1000CE 00
3.20000	0.81525E-01	0.19185E 01	0.20000E 01	C.1C00CE 0C
3.30000	0.73766E-01	0.19262E 01	C.20C00E 01	C.1C0C0E C0
3.40000	0.66747E-01	0.19333E 01	0.20000E 01	0.10000E 00
3.50000	0.60395E-01	0.19396E 01	C.20000E 01	0.1C000E 00
3.60000	0.54648E-01	0.19454E C1	C.20C00E 01	C.10000E 00
3.70000	0.49447E-01	0.19506E 01	0.20000E 01	0.1000OE 00
3.80000	0.44742E-01	0.19553E 01	C.20000E 01	0.1000OE 00
3.90000	0.40484E-01	0.19595E 01	0.20000E 01	0.10000E 00
4.00000	0.36631E-01	0.19634E C1	C.20000E 01	0.10000E 00
4.10000	0.33146E-01	0.19669E 01	C.20C00E 01	0.1000OE 0C
4.20000	0.41173E-01	0.19729E 01	C.20140E 01	0.25000E-01
4.30000	0.58493E-01	0.19781E 01	C.20359E 01	C.25000E-01
4.41250	0.76018E-01	0.19860E 01	C.20620E 01	0.25000E-01
4.51250	0.98115E-01	0.19954E C1	C.20936E 01	0.25000E-01
4.60000	0.11924E 00	0.20056E C1	C.21249E 01	0.25000E-01
4.70000	0.14437E 00	0.20197E 01	C.21641E 01	0.250C0E-01
4.80000	0.17769E 00	0.20364E C1	0.22119E 01	0.25000E-01
4.91250	0.19866E 00	0.20582E 01	C.22569E 01	0.25000E-01
5.01250	0.22326E 00	0.20803E 01	0.23037E 01	0.25000E-01
5.10000	0.24403E 00	0.21017E C1	C.23458E 01	0.25000E-01
5.20000	0.26634E 00	0.21283E 01	C.23948E 01	C.25000E-01
5.30000	0.29714E 00	0.21572E C1	C.24506E 01	0.25000E-01
5.41250	0.30865E 00	0.21919E 01	C.25006E 01	0.25000E-01
5.51250	0.32569E 00	0.22248E 01	C.25506E 01	0.25000E-01
5.60000	0.33923E 00	0.22549E 01	C.25941E 01	0.25000E-01
5.70000	0.35279E 00	0.22906E 01	C.26435E 01	0.25000E-01
5.80000	0.37538E 00	0.23276E C1	C.26983E 01	0.25000E-01

5.91250	0.37581E 00	0.23704E 01	0.27462E 01	0.25000E-01
6.01250	0.38378E 00	0.24095E C1	0.27933E 01	0.25000E-01
6.10000	0.38954E 00	0.24442E 01	0.28338E 01	0.25000E-01
6.20000	0.39447E 00	0.24844E 01	0.28790E C1	0.25000E-01
6.30000	0.40858E 00	0.25251E 01	C.29286E 01	0.25000E-01
6.41250	0.40033E 00	0.25711E 01	0.29714E 01	0.25000E-01
6.51250	0.40090E 00	0.26120E 01	0.30130E 01	0.25000E-01
6.60000	0.40054E 00	0.26479E 01	0.30485E 01	0.25000E-01
6.70000	0.39896E 00	0.26888E 01	C.30878E 01	0.25000E-01
6.80000	0.40623E 00	C.27294E 01	0.31306E 01	0.25000E-01
6.91250	0.39254E 00	0.27747E 01	0.31673E 01	0.25000E-01
7.01250	0.38809E 00	0.28145E 01	0.32C27E 01	0.25000E-01
7.10000	0.38371E 00	0.28490E 01	C.32327E 01	0.25000E-01
7.20000	0.37796E 00	0.28878E 01	0.32658E 01	0.25000E-01
7.30000	0.38040E 00	0.29261E C1	0.33017E 01	0.25000E-01
7.41250	0.36407E 0C	0.29682E 01	C.33323E 01	0.25000E-01
7.51250	0.35675E 00	0.30049E 01	0.33617E 01	0.25000E-01
7.60000	0.35013E 00	0.30364E 01	0.33865E 01	0.25000E-01
7.70000	0.34218E 00	0.30716E 01	0.34138E 01	0.25000E-01
7.80000	0.34154E 00	0.31060E 01	0.34433E 01	0.25000E-01
7.91250	0.32463E 00	0.31437E 01	C.34683E 01	0.25000E-01
8.01250	0.31604E 00	0.31763E 01	C.34924E 01	0.25000E-01
8.10000	0.30852E 00	0.32040E 01	C.35126E 01	0.25000E-01
8.20000	0.29976E 00	0.32350E 01	C.35348E 01	0.25000E-01
8.30000	0.29737E 00	0.32650E 01	0.35586E 01	0.25000E-01
8.41250	0.28119E 00	0.32977E 01	0.35789E 01	0.25000E-01
8.51250	0.27243E 00	0.33258E 01	C.35983E 01	0.25000E-01
8.60000	0.26487E 00	0.33497E 01	C.36145E 01	0.25000E-01
8.70000	0.25623E 00	0.33761E 01	C.36324E 01	0.25000E-01
8.80000	0.25300E 00	0.34017E 01	C.36515E 01	0.25000E-01
8.92500	0.23328E 00	0.34324E 01	0.36658E 01	0.50000E-01
9.00000	0.23712E 00	0.34503E 01	C.36813E 01	C.50000E-01
9.10000	0.21937E 00	0.34732E 01	0.36926E 01	0.50000E-01
9.22500	0.20966E 00	0.35008E 01	0.37105E 01	0.50000E-01
9.30000	0.20386E 00	0.35168E 01	C.37207E 01	0.50000E-01
9.42500	0.19455E 00	0.35424E 01	C.37371E 01	0.50000E-01
9.50000	0.19720E 00	0.35573E 01	0.37495E 01	0.50000E-01
9.60000	0.18205E 00	0.35763E 01	C.37584E 01	0.50000E-01
9.72500	0.17340E 00	0.35992E 01	C.37726E 01	0.50000E-01
9.80000	0.16829E 00	0.36123E 01	0.37807E 01	0.50000E-01
9.92500	0.16011E 00	0.36334E 01	0.37936E 01	0.50000E-01
10.00000	0.16191E 00	0.36456E 01	0.38C34E 01	0.50000E-01
10.10000	0.14922E 00	0.36613E 01	C.38105E 01	0.50000E-01
10.22500	0.14175E 00	0.36799E 01	0.38217E 01	0.50000E-01
10.30000	0.13736E 00	0.36907E 01	0.38281E 01	0.50000E-01
10.42500	0.13036E 00	0.37079E 01	0.38383E 01	0.50000E-01
10.50000	0.13158E 00	0.37178E 01	C.38460E 01	0.50000E-01
10.60000	0.12111E 00	0.37305E 01	0.38516E 01	0.50000E-01
10.72500	0.11480E 00	0.37456E 01	0.38604E 01	0.50000E-01
10.80000	0.11111E 00	0.37543E 01	0.38655E 01	0.50000E-01
10.92500	0.10525E 00	0.37682E 01	0.38735E 01	0.50000E-01
11.00000	0.10607E 00	0.37762E 01	0.38796E 01	0.50000E-01
11.10000	0.97523E-01	0.37864E 01	0.38839E 01	0.50000E-01
11.22500	0.92287E-01	0.37986E 01	0.38909E 01	0.50000E-01
11.30000	0.89232E-01	0.38055E 01	0.38948E 01	0.50000E-01
11.42500	0.84391E-01	0.38167E 01	0.39011E 01	0.50000E-01
11.52500	0.80741E-01	0.38251E 01	0.39059E 01	0.10000E 00
11.62500	0.74912E-01	0.38332E 01	0.39082E 01	G.10000E 00
11.72500	0.71695E-01	0.38409E 01	C.39126E 01	0.10000E 00
11.87500	0.66889E-01	0.38518E 01	0.39189E 01	0.10000E 00

11.97500	0.63986E-01	0.38588E 01	0.39228E 01	0.10000E 00
12.02500	0.67881E-01	0.38621E 01	0.39265E 01	0.10000E 00
12.12500	-0.42960E 01	0.29294E 01	-0.20000E 01	0.10000E 00
12.22500	-0.60649E 01	0.27203E 01	-0.20000E 01	0.10000E 00
12.30000	-0.39907E 01	0.19907E 01	-0.20000E 01	0.25000E-01
12.42500	-0.35276E 01	0.15221E 01	-0.20000E 01	0.50000E-01
12.50001	-0.31864E 01	0.11862E 01	-0.20000E 01	0.62500E-02
12.60001	-0.28823E 01	0.88300E 00	-0.20000E 01	0.25000E-01
12.72501	-0.25428E 01	0.54428E 00	-0.20000E 01	0.50000E-01
12.80001	-0.23017E 01	0.30170E 00	-0.20000E 01	0.25000E-01
12.90001	-0.20568E 01	0.56714E-01	-0.20000E 01	0.12500E-01
13.01252	-0.18265E 01	-0.17359E 00	-0.20000E 01	0.25000E-01
13.10002	0.14847E 00	-0.14844E 00	0.00000E 00	0.12500E-01
13.23752	0.12926E 00	-0.12937E 00	0.00000E 00	0.50000E-01
13.33752	0.11705E 00	-0.11706E 00	0.00000E 00	0.10000E 00
13.43752	0.10570E 00	-0.10591E 00	0.00000E 00	0.10000E 00
13.53752	0.96054E-01	-0.95830E-01	0.00000E 00	0.10000E 00
13.63752	0.86706E-01	-0.86715E-01	0.00000E 00	0.10000E 00
13.73752	0.78460E-01	-0.78459E-01	0.00000E 00	0.10000E 00
13.80002	0.70032E-01	-0.70050E-01	0.00000E 00	0.12500E-01
13.93752	0.61126E-01	-0.61056E-01	0.00000E 00	0.50000E-01
14.03752	0.55246E-01	-0.55244E-01	0.00000E 00	0.10000E 00
14.13752	0.50135E-01	-0.49991E-01	0.00000E 00	0.10000E 00
14.23752	0.45085E-01	-0.45235E-01	0.00000E 00	0.10000E 00
14.30002	0.40374E-01	-0.40384E-01	0.00000E 00	0.12500E-01
14.43752	0.35240E-01	-0.35199E-01	0.00000E 00	0.50000E-01
14.53752	0.31850E-01	-0.31849E-01	0.00000E 00	0.10000E 00
14.63752	0.28903E-01	-0.28820E-01	0.00000E 00	0.10000E 00
14.73752	0.25992E-01	-0.26078E-01	0.00000E 00	0.10000E 00
14.80003	0.23276E-01	-0.23281E-01	0.00000E 00	0.12500E-01
14.93753	0.20316E-01	-0.20292E-01	0.00000E 00	0.50000E-01
15.03753	0.18361E-01	-0.18361E-01	0.00000E 00	0.10000E 00
15.13753	0.16663E-01	-0.16615E-01	0.00000E 00	0.10000E 00
15.23753	0.14984E-01	-0.15034E-01	0.00000E 00	0.10000E 00
15.30003	0.13418E-01	-0.13422E-01	0.00000E 00	0.12500E-01
15.43753	0.11712E-01	-0.11699E-01	0.00000E 00	0.50000E-01
15.53753	0.10585E-01	-0.10585E-01	0.00000E 00	0.10000E 00
15.63753	0.96061E-02	-0.95785E-02	0.00000E 00	0.10000E 00
15.73753	0.86385E-02	-0.86672E-02	0.00000E 00	0.10000E 00
15.80003	0.77358E-02	-0.77377E-02	0.00000E 00	0.12500E-01
15.93753	0.67520E-02	-0.67442E-02	-0.00000E 00	0.50000E-01
16.03753	0.61025E-02	-0.61023E-02	0.00000E 00	0.10000E 00
16.13753	0.55379E-02	-0.55220E-02	0.00000E 00	0.10000E 00
16.23753	0.49801E-02	-0.49967E-02	0.00000E 00	0.10000E 00
16.30004	0.44597E-02	-0.44608E-02	0.00000E 00	0.12500E-01
16.43754	0.38926E-02	-0.38881E-02	0.00000E 00	0.50000E-01
16.53754	0.35181E-02	-0.35180E-02	0.00000E 00	0.10000E 00
16.63754	0.31926E-02	-0.31835E-02	0.00000E 00	0.10000E 00
16.73754	0.28711E-02	-0.28806E-02	0.00000E 00	0.10000E 00
16.80004	0.25710E-02	-0.25717E-02	0.00000E 00	0.12500E-01
16.93754	0.22441E-02	-0.22415E-02	0.00000E 00	0.50000E-01
17.03754	0.20282E-02	-0.20281E-02	0.00000E 00	0.10000E 00
17.13754	0.14713E 01	0.31497E 00	0.20000E 01	0.10000E 00
17.23754	0.59667E 00	0.70808E-01	0.00000E 00	0.10000E 00
17.30004	-0.21043E 00	0.20941E 00	0.00000E 00	0.12500E-01
17.40005	-0.18702E 00	0.18702E 00	0.00000E 00	0.12500E-01
17.51255	-0.16712E 00	0.16712E 00	0.00000E 00	0.50000E-01
17.61255	-0.15117E 00	0.15122E 00	0.00000E 00	0.50000E-01
17.76255	-0.13024E 00	0.13016E 00	0.00000E 00	0.10000E 00
17.86255	-0.11768E 00	0.11777E 00	0.00000E 00	0.10000E 00

17.96255	-0.10657E 00	0.10656E 00	0.00000E 00	0.10000E 00
18.06255	-0.96422E-01	0.96423E-01	0.00000E 00	0.10000E 00
18.16255	-0.87245E-01	0.87246E-01	0.00000E 00	0.10000E 00
18.26255	-0.78944E-01	0.78944E-01	0.00000E 00	0.10000E 00
18.36255	-0.71431E-01	0.71431E-01	0.00000E 00	0.10000E 00
18.46255	-0.64634E-01	0.64634E-01	0.00000E 00	0.10000E 00
18.56255	-0.58483E-01	0.58483E-01	0.00000E 00	0.10000E 00
18.66255	-0.52917E-01	0.52917E-01	0.00000E 00	0.10000E 00
18.76255	-0.47882E-01	0.47882E-01	0.00000E 00	0.10000E 00
18.86255	-0.43325E-01	0.43325E-01	0.00000E 00	0.10000E 00
18.96255	-0.39202E-01	0.39202E-01	0.00000E 00	0.10000E 00
19.06255	-0.35472E-01	0.35472E-01	0.00000E 00	0.10000E 00
19.16255	-0.32096E-01	0.32096E-01	0.00000E 00	0.10000E 00
19.26255	-0.29042E-01	0.29042E-01	0.00000E 00	0.10000E 00
19.36255	-0.26278E-01	0.26278E-01	0.00000E 00	0.10000E 00
19.46255	-0.23777E-01	0.23777E-01	0.00000E 00	0.10000E 00
19.56255	-0.21515E-01	0.21515E-01	0.00000E 00	0.10000E 00
19.66255	-0.19467E-01	0.19467E-01	0.00000E 00	0.10000E 00
19.76255	-0.17615E-01	0.17615E-01	0.00000E 00	0.10000E 00
19.86255	-0.15938E-01	0.15938E-01	0.00000E 00	0.10000E 00
19.96255	-0.14422E-01	0.14422E-01	0.00000E 00	0.10000E 00
20.06255	-0.13049E-01	0.13049E-01	0.00000E 00	0.10000E 00
20.10005	-0.17374E 01	-0.26262E 00	-0.20000E 01	0.12500E-01
20.20005	-0.15526E 01	-0.44739E 00	-0.20000E 01	0.31250E-02
20.30318	0.40170E 00	-0.40175E 00	0.00000E 00	0.25000E-01
20.40318	0.36341E 00	-0.36351E 00	0.00000E 00	0.50000E-01
20.50318	0.32892E 00	-0.32892E 00	0.00000E 00	0.10000E 00
20.60318	0.29741E 00	-0.29761E 00	0.00000E 00	0.10000E 00
20.70318	0.26950E 00	-0.26929E 00	0.00000E 00	0.10000E 00
20.80318	0.24366E 00	-0.24367E 00	0.00000E 00	0.10000E 00
20.90318	0.22048E 00	-0.22048E 00	0.00000E 00	0.10000E 00
21.00318	0.19950E 00	-0.19950E 00	0.00000E 00	0.10000E 00
21.10318	0.18051E 00	-0.18051E 00	0.00000E 00	0.10000E 00
21.20318	0.16334E 00	-0.16333E 00	0.00000E 00	0.10000E 00
21.30318	0.14779E 00	-0.14779E 00	0.00000E 00	0.10000E 00
21.40318	0.13373E 00	-0.13373E 00	0.00000E 00	0.10000E 00
21.50318	0.12100E 00	-0.12100E 00	0.00000E 00	0.10000E 00
21.60318	0.10949E 00	-0.10949E 00	0.00000E 00	0.10000E 00
21.70318	0.99067E-01	-0.99067E-01	0.00000E 00	0.10000E 00
21.80318	0.89640E-01	-0.89640E-01	0.00000E 00	0.10000E 00
21.90318	0.81109E-01	-0.81109E-01	0.00000E 00	0.10000E 00
22.00318	0.73391E-01	-0.73391E-01	0.00000E 00	0.10000E 00
22.10318	0.17326E 01	0.26674E 00	0.20000E 01	0.25000E-01
22.20318	0.15669E 01	0.43173E 00	0.20000E 01	0.50000E-01
22.30319	-0.36420E 00	0.36462E 00	0.00000E 00	0.12500E-01
22.40319	-0.32989E 00	0.32990E 00	0.00000E 00	0.50000E-01
22.50319	-0.30019E 00	0.29848E 00	0.00000E 00	0.50000E-01
22.60319	-0.26339E 00	0.26333E 00	0.00000E 00	0.25000E-01
22.70319	-0.23840E 00	0.23827E 00	0.00000E 00	0.50000E-01
22.80319	-0.21560E 00	0.21560E 00	0.00000E 00	0.10000E 00
22.90319	-0.19535E 00	0.19509E 00	0.00000E 00	0.10000E 00
23.00319	-0.17625E 00	0.17652E 00	0.00000E 00	0.10000E 00
23.10319	-0.18213E 01	-0.17805E 00	-0.20000E 01	0.25000E-01
23.20319	-0.16473E 01	-0.35148E 00	-0.20000E 01	0.50000E-01
23.30319	0.20415E 01	-0.42142E-01	0.20000E 01	0.12500E-01
23.41570	0.18136E 01	0.18652E 00	0.20000E 01	0.25000E-01
23.50320	-0.16017E 00	0.16014E 00	0.00000E 00	0.12500E-01
23.64070	-0.13946E 00	0.13956E 00	0.00000E 00	0.50000E-01
23.74070	-0.12628E 00	0.12628E 00	0.00000E 00	0.10000E 00
23.84070	-0.11405E 00	0.11426E 00	0.00000E 00	0.10000E 00

23.94070	-0.10360E 00	0.10338E 00	0.00000E 00	0.10000E 00
24.04070	-0.93541E-01	0.93550E-01	0.00000E 00	0.10000E 00
24.14070	-0.84644E-01	0.84644E-01	0.00000E 00	0.10000E 00
24.20320	-0.75552E-01	0.75571E-01	0.00000E 00	0.12500E-01
24.34070	-0.65944E-01	0.65868E-01	0.00000E 00	0.50000E-01
24.44070	-0.59601E-01	0.59599E-01	0.00000E 00	0.10000E 00
24.54070	-0.54087E-01	0.53931E-01	0.00000E 00	0.10000E 00
24.64070	-0.48639E-01	0.48800E-01	0.00000E 00	0.10000E 00
24.70321	-0.43556E-01	0.43567E-01	0.00000E 00	0.12500E-01
24.84071	-0.38017E-01	0.37973E-01	0.00000E 00	0.50000E-01
24.94071	-0.34360E-01	0.34359E-01	0.00000E 00	0.10000E 00
25.04071	-0.31181E-01	0.31091E-01	0.00000E 00	0.10000E 00
25.14071	-0.28040E-01	0.28133E-01	0.00000E 00	0.10000E 00
25.20321	-0.25110E-01	0.25116E-01	0.00000E 00	0.12500E-01
25.34071	-0.21917E-01	0.21892E-01	0.00000E 00	0.50000E-01
25.44071	-0.19809E-01	0.19808E-01	0.00000E 00	0.10000E 00
25.54071	-0.17976E-01	0.17924E-01	0.00000E 00	0.10000E 00
25.64071	-0.16165E-01	0.16219E-01	0.00000E 00	0.10000E 00
25.70321	-0.14476E-01	0.14480E-01	0.00000E 00	0.12500E-01
25.84071	-0.12635E-01	0.12621E-01	0.00000E 00	0.50000E-01
25.94071	-0.11420E-01	0.11419E-01	0.00000E 00	0.10000E 00
26.04071	-0.10363E-01	0.10333E-01	0.00000E 00	0.10000E 00
26.14071	-0.93194E-02	0.93503E-02	0.00000E 00	0.10000E 00
26.20322	-0.83455E-02	0.83476E-02	0.00000E 00	0.12500E-01
26.34072	-0.72842E-02	0.72758E-02	0.00000E 00	0.50000E-01
26.44072	-0.65835E-02	0.65833E-02	0.00000E 00	0.10000E 00
26.54072	-0.59744E-02	0.59573E-02	0.00000E 00	0.10000E 00
26.64072	-0.53727E-02	0.53905E-02	0.00000E 00	0.10000E 00
26.70322	-0.48112E-02	0.48124E-02	0.00000E 00	0.12500E-01
26.84072	-0.41994E-02	0.41945E-02	0.00000E 00	0.50000E-01
26.94072	-0.37954E-02	0.37953E-02	0.00000E 00	0.10000E 00
27.04072	-0.34443E-02	0.34344E-02	0.00000E 00	0.10000E 00
27.14072	-0.30974E-02	0.31076E-02	0.00000E 00	0.10000E 00
27.20322	-0.27737E-02	0.27744E-02	0.00000E 00	0.12500E-01
27.34072	-0.24210E-02	0.24182E-02	0.00000E 00	0.50000E-01
27.44072	-0.21881E-02	0.21880E-02	0.00000E 00	0.10000E 00
27.54072	-0.19856E-02	0.19799E-02	0.00000E 00	0.10000E 00
27.64072	-0.17856E-02	0.17916E-02	0.00000E 00	0.10000E 00
27.70322	-0.15990E-02	0.15994E-02	0.00000E 00	0.12500E-01
27.84072	-0.13957E-02	0.13941E-02	0.00000E 00	0.50000E-01
27.94072	-0.12614E-02	0.12614E-02	0.00000E 00	0.10000E 00
28.04072	-0.11447E-02	0.11414E-02	0.00000E 00	0.10000E 00
28.14072	-0.10294E-02	0.10328E-02	0.00000E 00	0.10000E 00
28.20323	-0.92185E-03	0.92208E-03	0.00000E 00	0.12500E-01
28.34073	-0.80462E-03	0.80369E-03	0.00000E 00	0.50000E-01
28.44073	-0.72722E-03	0.72719E-03	0.00000E 00	0.10000E 00
28.54073	-0.65994E-03	0.65804E-03	0.00000E 00	0.10000E 00
28.64073	-0.59347E-03	0.59544E-03	0.00000E 00	0.10000E 00
28.70323	-0.53145E-03	0.53158E-03	0.00000E 00	0.12500E-01
28.84073	-0.46386E-03	0.46333E-03	0.00000E 00	0.50000E-01
28.94073	-0.41924E-03	0.41923E-03	0.00000E 00	0.10000E 00
29.04073	-0.38046E-03	0.37936E-03	0.00000E 00	0.10000E 00
29.14073	-0.34213E-03	0.34327E-03	0.00000E 00	0.10000E 00
29.20323	-0.30638E-03	0.30646E-03	0.00000E 00	0.12500E-01
29.34073	-0.26742E-03	0.26711E-03	0.00000E 00	0.50000E-01
29.44073	-0.24169E-03	0.24169E-03	0.00000E 00	0.10000E 00
29.54073	-0.21933E-03	0.21870E-03	0.00000E 00	0.10000E 00
29.64073	-0.19724E-03	0.19790E-03	0.00000E 00	0.10000E 00
29.70324	-0.17663E-03	0.17667E-03	0.00000E 00	0.12500E-01
29.84074	-0.15417E-03	0.15399E-03	0.00000E 00	0.50000E-01

29.94074	-0.13934E-03	0.13933E-03	0.00000E 00	0.10000E 00
30.04074	-0.12645E-03	0.12608E-03	0.00000E 00	0.10000E 00
30.14074	-0.11371E-03	0.11409E-03	0.00000E 00	0.10000E 00
30.20324	-0.10183E-03	0.10185E-03	0.00000E 00	0.12500E-01
30.34074	-0.88878E-04	0.88776E-04	0.00000E 00	0.50000E-01
30.44074	-0.80329E-04	0.80326E-04	0.00000E 00	0.10000E 00
30.54074	-0.72897E-04	0.72687E-04	0.00000E 00	0.10000E 00
30.64074	-0.65554E-04	0.65772E-04	0.00000E 00	0.10000E 00
30.70324	-0.58704E-04	0.58719E-04	0.00000E 00	0.12500E-01
30.84074	-0.51239E-04	0.51179E-04	0.00000E 00	0.50000E-01
30.94074	-0.46310E-04	0.46308E-04	0.00000E 00	0.10000E 00
31.04074	-0.42025E-04	0.41905E-04	0.00000E 00	0.10000E 00
31.14074	-0.37792E-04	0.37918E-04	0.00000E 00	0.10000E 00
31.20325	-0.33843E-04	0.33852E-04	0.00000E 00	0.12500E-01
31.34075	-0.29539E-04	0.29505E-04	0.00000E 00	0.50000E-01
31.44075	-0.26698E-04	0.26697E-04	0.00000E 00	0.10000E 00
31.54075	-0.24228E-04	0.24158E-04	0.00000E 00	0.10000E 00
31.64075	-0.21787E-04	0.21860E-04	0.00000E 00	0.10000E 00
31.70325	-0.19511E-04	0.19516E-04	0.00000E 00	0.12500E-01
31.84075	-0.17029E-04	0.17010E-04	0.00000E 00	0.50000E-01
31.94075	-0.15391E-04	0.15391E-04	0.00000E 00	0.10000E 00
32.04075	-0.13967E-04	0.13927E-04	0.00000E 00	0.10000E 00
32.14075	-0.12561E-04	0.12602E-04	0.00000E 00	0.10000E 00
32.20325	-0.11248E-04	0.11251E-04	0.00000E 00	0.12500E-01
32.34075	-0.98175E-05	0.98062E-05	0.00000E 00	0.50000E-01
32.44075	-0.88731E-05	0.88728E-05	0.00000E 00	0.10000E 00
32.54075	-0.80522E-05	0.80291E-05	0.00000E 00	0.10000E 00
32.64075	-0.72412E-05	0.72652E-05	0.00000E 00	0.10000E 00
32.70325	-0.64845E-05	0.64861E-05	0.00000E 00	0.12500E-01
32.84075	-0.56598E-05	0.56533E-05	0.00000E 00	0.50000E-01
32.94075	-0.51154E-05	0.51152E-05	0.00000E 00	0.10000E 00
33.04075	-0.46421E-05	0.46288E-05	0.00000E 00	0.10000E 00

The following pages of computer output lists the input data cards and the program output for the topological tests circuit of Figure 3-1.

The circuit contains a capacitive loop and an inductive cut-set.

THESE ARE THE DATA CARDS FOR THE ANALYSIS OF THE CIRCUIT OF FIGURE 3- 1
THIS IS THE TOPOLOGICAL TEST WITH A CAPACITIVE LOOP SET AND AN INDUCTIVE CUT SET

```
01-03    1.0
01-02    2.0
03-11              3.0
01-13    4.0
08-09              5.0
09-10                        6.0
10-07                        7.0
07-09                        9.0
03-05                        8.0
06-07             10.0
10-11             11.0
03-08   12.0
05-06   13.0
05-11   14.0
11-12                       15.0
12-13             16.0
13-02   17.0
04-05                       18.0
02-04   19.0
99
                      1.0       1.0
99
```

INPUT PARAMETERS

BRANCH	FROM	TO	R	L	C	I	E	R-SOURCE
1	1	3	1.000	0.000	0.000	0.000	0.000	0.000
2	1	2	2.000	0.000	0.000	0.000	0.000	0.000
3	3	11	0.000	3.000	0.000	0.000	0.000	0.000
4	1	13	4.000	0.000	0.000	0.000	0.000	0.000
5	8	9	0.000	5.000	0.000	0.000	0.000	0.000
6	9	10	0.000	0.000	6.000	0.000	0.000	0.000
7	10	7	0.000	0.000	7.000	0.000	0.000	0.000
8	7	9	0.000	0.000	9.000	0.000	0.000	0.000
9	3	5	0.000	0.000	8.000	0.000	0.000	0.000
10	6	7	0.000	10.000	0.000	0.000	0.000	0.000
11	10	11	0.000	11.000	0.000	0.000	0.000	0.000
12	3	8	12.000	0.000	0.000	0.000	0.000	0.000
13	5	6	13.000	0.000	0.000	0.000	0.000	0.000
14	5	11	14.000	0.000	0.000	0.000	0.000	0.000
15	11	12	0.000	0.000	15.000	0.000	0.000	0.000
16	12	13	0.000	16.000	0.000	0.000	0.000	0.000
17	13	2	17.000	0.000	0.000	0.000	0.000	0.000
18	4	5	0.000	0.000	18.000	0.000	0.000	0.000
19	2	4	19.000	0.000	0.000	0.000	0.000	0.000

TOPOLOGICAL NETWORK DESCRIPTION

TREE BRANCHES	LINKS
1	3
9	10
18	11
19	16
17	2
13	4
14	8
15	
12	
5	
6	
7	

THE TOTAL NUMBER OF BRANCHES IS 19,LINKS IS 7,AND TREE BRANCHES IS 12

THE F-MATRIX IS PARTITIONED INTO THE FOLLOWING SUB-MATRICES

SUB MATRIX			WITH THE DIMENSIONS
FSC	0	0	(1X 5)
FRC	FRG	0	(2X 5) (2X 6)
FLC	FLG	FLM	(4X 5) (4X 6) (4X 1)

FOR THIS NETWORK THE F-MATRIX IS

0	0	0	1	1	0	0	0	0	0	0	0
-1	1	0	0	0	-1	1	0	0	0	0	0

-1	1	0	0	0	-1	1	1	0	0	0	0
-1	0	0	0	0	0	0	0	0	-1	0	0
1	0	0	-1	-1	0	0	0	1	0	-1	-1
-1	0	0	1	0	0	0	0	0	-1	1	1
0	1	1	0	0	0	1	1	0	1	0	0

The following pages of computer output lists the input data cards and the program output for the linear R-L-C circuits of Figure 4-6.

THESE ARE THE DATA CARDS FOR THE ANALYSIS OF THE CIRCUIT OF FIGURE 4- 6
THIS CIRCUIT WAS USED TO TEST THE LINEAR CIRCUIT ANALYSIS CAPABLITIES .

```
01-02   10.                  15.
02-01               1.
02-03    5.
03-01          2.
99
  10.000    0.000    1.000    1.000
99
```

INPUT PARAMETERS

BRANCH	FROM	TO	R	L	C	I	E	R-SOURCE
1	1	2	10.000	0.000	0.000	0.000	15.000	0.000
2	2	1	0.000	0.000	1.000	0.000	0.000	0.000
3	2	3	5.000	0.000	0.000	0.000	0.000	0.000
4	3	1	0.000	2.000	0.000	0.000	0.000	0.000

TOPOLOGICAL NETWORK DESCRIPTION

TREE BRANCHES	LINKS
2	4
3	1

THE TOTAL NUMBER OF BRANCHES IS 4,LINKS IS 2,AND TREE BRANCHES IS 2

THE F-MATRIX IS PARTITIONED INTO THE FOLLOWING SUB-MATRICES

SUB MATRIX WITH THE DIMENSIONS

```
FSC  0   C     ( 0X 1)
FRC FRG  C     ( 1X 1) ( 1X 1)
FLC FLG FLM    ( 1X 1) ( 1X 1) ( 1X 0)
```

FOR THIS NETWORK THE F-MATRIX IS

```
  1   0
 -1   1
```

```
 THE STATE VARIABLES ARE VC AND IL
THE BRANCH VARIABLES ARE (VC,IC),(VG,IG),AND (VX,IX)
THE LINK VARIABLES ARE (VS,IS),(VR,IR),AND (VL ,IL),
ALL VARIABLES ARE IN C-R-L ORDER.

 TIME   0.001
  VC          0.001
  IC          1.500
  VG          0.000
  IG          0.000
  VR         14.999
  IR          1.500
  VL          0.001
  IL          0.000
 TIME   1.063
  VC          1.432
  IC          1.165
  VG          0.957
  IG          0.191
  VR         13.568
  IR          1.357
  VL          0.475
  IL          0.191
 TIME   2.188
  VC          2.536
  IC          0.813
  VG          2.166
  IG          0.433
  VR         12.464
  IR          1.246
  VL          0.370
  IL          0.433
 TIME   3.188
  VC          3.229
  IC          0.585
  VG          2.960
  IG          0.592
  VR         11.771
  IR          1.177
  VL          0.269
  IL          0.592
 TIME   4.188
  VC          3.727
  IC          0.421
  VG          3.533
  IG          0.707
  VR         11.273
  IR          1.127
  VL          0.194
  IL          0.707
 TIME   5.188
  VC          4.085
  IC          0.302
  VG          3.946
  IG          0.789
  VR         10.915
  IR          1.091
  VL          0.140
  IL          0.789
```

```
TIME    6.188
 VC          4.343
 IC          0.217
 VG          4.243
 IG          0.849
 VR         10.657
 IR          1.066
 VL          0.100
 IL          0.849
TIME    7.188
 VC          4.528
 IC          0.156
 VG          4.456
 IG          0.891
 VR         10.472
 IR          1.047
 VL          0.072
 IL          0.891
TIME    8.188
 VC          4.661
 IC          0.112
 VG          4.609
 IG          0.922
 VR         10.339
 IR          1.034
 VL          0.052
 IL          0.922
TIME    9.188
 VC          4.756
 IC          0.081
 VG          4.719
 IG          0.944
 VR         10.244
 IR          1.024
 VL          0.038
 IL          0.944
TIME   10.188
 VC          4.825
 IC          0.058
 VG          4.799
 IG          0.960
 VR         10.175
 IR          1.018
 VL          0.026
 IL          0.960
```

The following pages of computer output lists the input data cards and the program output for the topological tests circuit of Figure 4-2. This circuit contains a large capative loop. To further exhibit the form of the printout of a larger circuit, the output also contains the results of the analysis.

THESE ARE THE DATA CARDS FOR THE ANALYSIS OF THE CIRCUIT OF FIGURE 4- 2
THIS CIRCUIT CONTAINS A LARGE CAPACITIVE LOOP.

```
01-02    1.                      10.
02-01                 1.
02-04                 2.
02-03    2.
03-01                 3.
03-04                 4.
03-04    3.
03-04    4.
99
  20.000   0.000   1.000   0.100
99
```

INPUT PARAMETERS

BRANCH	FROM	TO	R	L	C	I	E	R-SOURCE
1	1	2	1.000	0.000	0.000	0.000	10.000	0.000
2	2	1	0.000	0.000	1.000	0.000	0.000	0.000
3	2	4	0.000	0.000	2.000	0.000	0.000	0.000
4	2	3	2.000	0.000	0.000	0.000	0.000	0.000
5	3	1	0.000	0.000	3.000	0.000	0.000	0.000
6	3	4	0.000	0.000	4.000	0.000	0.000	0.000
7	3	4	3.000	0.000	0.000	0.000	0.000	0.000
8	3	4	4.000	0.000	0.000	0.000	0.000	0.000

TOPOLOGICAL NETWORK DESCRIPTION

TREE BRANCHES	LINKS
2	5
3	4
6	7
	1
	8

THE TOTAL NUMBER OF BRANCHES IS 8,LINKS IS 5,AND TREE BRANCHES IS 3

THE F-MATRIX IS PARTITIONED INTO THE FOLLOWING SUB-MATRICES

SUB MATRIX WITH THE DIMENSIONS

```
FSC  0   0     ( 1X 3)
FRC FRG  0     ( 4X 3) ( 4X 0)
FLC FLG FLM    ( 0X 3) ( 0X 0) ( 0X 0)
```

FOR THIS NETWORK THE F-MATRIX IS

```
-1   1  -1
 0  -1   1
 0   0  -1
 1   0   0
 0   0  -1
```

THE STATE VARIABLES ARE VC AND IL
THE BRANCH VARIABLES ARE (VC,IC),(VG,IG),AND (VX,IX)
THE LINK VARIABLES ARE (VS,IS),(VR,IR),AND (VL,IL),
ALL VARIABLES ARE IN C-R-L ORDER.

TIME	0.001			
VC	0.005	0.003	-0.001	
IC	4.717	5.277	-5.276	
VS	0.001			
IS	2.279			
VR	0.004	-0.001	9.995	-0.001
IR	0.002	-0.000	9.995	-0.000
TIME	0.125			
VC	0.565	0.317	-0.157	
IC	4.330	4.868	-4.776	
VS	0.091			
IS	2.105			
VR	0.474	-0.157	9.435	-0.157
IR	0.237	-0.052	9.435	-0.039
TIME	0.250			
VC	1.084	0.609	-0.299	
IC	3.972	4.490	-4.316	
VS	0.176			
IS	1.944			
VR	0.908	-0.299	8.916	-0.299
IR	0.454	-0.100	8.916	-0.075
TIME	0.313			
VC	1.327	0.747	-0.365	
IC	3.804	4.313	-4.100	
VS	0.215			
IS	1.869			
VR	1.111	-0.365	8.673	-0.365
IR	0.555	-0.122	8.673	-0.091
TIME	0.500			
VC	1.996	1.128	-0.543	
IC	3.343	3.825	-3.509	
VS	0.325			
IS	1.661			
VR	1.670	-0.543	8.004	-0.543
IR	0.835	-0.181	8.004	-0.136
TIME	0.625			
VC	2.396	1.358	-0.647	
IC	3.068	3.534	-3.156	
VS	0.392			
IS	1.536			
VR	2.004	-0.647	7.604	-0.647
IR	1.002	-0.216	7.604	-0.162
TIME	0.750			
VC	2.764	1.570	-0.740	
IC	2.815	3.266	-2.834	
VS	0.454			
IS	1.421			
VR	2.310	-0.740	7.236	-0.740
IR	1.155	-0.247	7.236	-0.185
TIME	1.000			
VC	3.411	1.948	-0.899	
IC	2.371	2.795	-2.270	
VS	0.563			
IS	1.218			

VR	2.847	-0.899	6.589	-0.899
IR	1.424	-0.300	6.589	-0.225
TIME	1.250			
VC	3.955	2.272	-1.026	
IC	1.998	2.398	-1.799	
VS	0.658			
IS	1.047			
VR	3.298	-1.026	6.045	-1.026
IR	1.649	-0.342	6.045	-0.257
TIME	1.500			
VC	4.414	2.550	-1.126	
IC	1.684	2.064	-1.407	
VS	0.739			
IS	0.902			
VR	3.676	-1.126	5.586	-1.126
IR	1.838	-0.375	5.586	-0.281
TIME	1.750			
VC	4.801	2.790	-1.203	
IC	1.420	1.782	-1.080	
VS	0.808			
IS	0.778			
VR	3.993	-1.203	5.199	-1.203
IR	1.997	-0.401	5.199	-0.301
TIME	2.000			
VC	5.128	2.997	-1.262	
IC	1.198	1.544	-0.808	
VS	0.869			
IS	0.674			
VR	4.259	-1.262	4.872	-1.262
IR	2.130	-0.421	4.872	-0.315
TIME	2.250			
VC	5.404	3.177	-1.305	
IC	1.012	1.343	-0.582	
VS	0.921			
IS	0.584			
VR	4.483	-1.305	4.596	-1.305
IR	2.241	-0.435	4.596	-0.326
TIME	2.500			
VC	5.636	3.334	-1.336	
IC	0.855	1.174	-0.395	
VS	0.967			
IS	0.509			
VR	4.670	-1.336	4.364	-1.336
IR	2.335	-0.445	4.364	-0.334
TIME	2.750			
VC	5.833	3.472	-1.355	
IC	0.723	1.030	-0.240	
VS	1.006			
IS	0.444			
VR	4.827	-1.355	4.167	-1.355
IR	2.414	-0.452	4.167	-0.339
TIME	3.000			
VC	6.000	3.593	-1.366	
IC	0.612	0.909	-0.112	
VS	1.041			
IS	0.388			
VR	4.959	-1.366	4.000	-1.366
IR	2.479	-0.455	4.000	-0.342
TIME	3.250			
VC	6.141	3.700	-1.370	

IC	0.518	0.806	-0.007	
VS	1.071			
IS	0.341			
VR	5.070	-1.370	3.859	-1.370
IR	2.535	-0.457	3.859	-0.342
TIME	3.500			
VC	6.260	3.795	-1.367	
IC	0.440	0.719	0.079	
VS	1.098			
IS	0.300			
VR	5.162	-1.367	3.740	-1.367
IR	2.581	-0.456	3.740	-0.342
TIME	3.750			
VC	6.362	3.880	-1.360	
IC	0.374	0.645	0.149	
VS	1.121			
IS	0.265			
VR	5.240	-1.360	3.638	-1.360
IR	2.620	-0.453	3.638	-0.340
TIME	4.250			
VC	6.521	4.026	-1.335	
IC	0.271	0.529	0.250	
VS	1.161			
IS	0.209			
VR	5.361	-1.335	3.479	-1.335
IR	2.680	-0.445	3.479	-0.334
TIME	4.750			
VC	6.637	4.147	-1.299	
IC	0.197	0.442	0.316	
VS	1.191			
IS	0.165			
VR	5.446	-1.299	3.363	-1.299
IR	2.723	-0.433	3.363	-0.325
TIME	5.250			
VC	6.722	4.249	-1.257	
IC	0.145	0.379	0.354	
VS	1.216			
IS	0.132			
VR	5.506	-1.257	3.278	-1.257
IR	2.753	-0.419	3.278	-0.314
TIME	5.750			
VC	6.785	4.338	-1.211	
IC	0.108	0.332	0.374	
VS	1.236			
IS	0.107			
VR	5.549	-1.211	3.215	-1.211
IR	2.775	-0.404	3.215	-0.303
TIME	6.250			
VC	6.832	4.416	-1.164	
IC	0.081	0.296	0.383	
VS	1.252			
IS	0.086			
VR	5.580	-1.164	3.168	-1.164
IR	2.790	-0.388	3.168	-0.291
TIME	6.750			
VC	6.868	4.487	-1.116	
IC	0.062	0.269	0.382	
VS	1.265			
IS	0.070			
VR	5.603	-1.116	3.132	-1.116

IR	2.801	-0.372	3.132	-0.279
TIME	7.250			
VC	6.895	4.551	-1.069	
IC	0.048	0.247	0.376	
VS	1.276			
IS	0.057			
VR	5.620	-1.069	3.105	-1.069
IR	2.810	-0.356	3.105	-0.267
TIME	7.750			
VC	6.917	4.610	-1.022	
IC	0.038	0.229	0.367	
VS	1.284			
IS	0.045			
VR	5.633	-1.022	3.083	-1.022
IR	2.816	-0.341	3.083	-0.256
TIME	8.250			
VC	6.934	4.666	-0.977	
IC	0.031	0.215	0.355	
VS	1.291			
IS	0.036			
VR	5.643	-0.977	3.066	-0.977
IR	2.821	-0.326	3.066	-0.244
TIME	8.750			
VC	6.947	4.718	-0.933	
IC	0.025	0.202	0.342	
VS	1.296			
IS	0.028			
VR	5.651	-0.933	3.053	-0.933
IR	2.826	-0.311	3.053	-0.233
TIME	9.750			
VC	6.968	4.814	-0.851	
IC	0.019	0.183	0.313	
VS	1.303			
IS	0.015			
VR	5.665	-0.851	3.032	-0.851
IR	2.833	-0.284	3.032	-0.213
TIME	10.750			
VC	6.982	4.900	-0.776	
IC	0.012	0.166	0.287	
VS	1.306			
IS	0.004			
VR	5.676	-0.776	3.018	-0.776
IR	2.838	-0.259	3.018	-0.194
TIME	11.750			
VC	6.994	4.980	-0.708	
IC	0.008	0.151	0.263	
VS	1.306			
IS	-0.004			
VR	5.688	-0.708	3.006	-0.708
IR	2.844	-0.236	3.006	-0.177
TIME	12.250			
VC	6.999	5.018	-0.675	
IC	0.010	0.153	0.261	
VS	1.305			
IS	-0.003			
VR	5.693	-0.675	3.001	-0.675
IR	2.847	-0.225	3.001	-0.169
TIME	12.750			
VC	7.003	5.053	-0.646	
IC	0.007	0.140	0.236	

VS	1.304			
IS	-0.010			
VR	5.699	-0.646	2.997	-0.646
IR	2.849	-0.215	2.997	-0.161
TIME	13.250			
VC	7.006	5.088	-0.616	
IC	0.007	0.135	0.225	
VS	1.302			
IS	-0.013			
VR	5.704	-0.616	2.994	-0.616
IR	2.852	-0.205	2.994	-0.154
TIME	13.750			
VC	7.009	5.121	-0.589	
IC	0.006	0.129	0.215	
VS	1.300			
IS	-0.016			
VR	5.710	-0.589	2.991	-0.589
IR	2.855	-0.196	2.991	-0.147
TIME	14.250			
VC	7.012	5.153	-0.563	
IC	0.005	0.125	0.204	
VS	1.297			
IS	-0.018			
VR	5.716	-0.563	2.988	-0.563
IR	2.858	-0.188	2.988	-0.141
TIME	14.750			
VC	7.015	5.183	-0.538	
IC	0.005	0.120	0.194	
VS	1.294			
IS	-0.020			
VR	5.721	-0.538	2.985	-0.538
IR	2.861	-0.179	2.985	-0.135
TIME	15.250			
VC	7.017	5.212	-0.514	
IC	0.004	0.115	0.185	
VS	1.290			
IS	-0.021			
VR	5.727	-0.514	2.983	-0.514
IR	2.863	-0.171	2.983	-0.129
TIME	15.750			
VC	7.019	5.241	-0.492	
IC	0.004	0.111	0.176	
VS	1.287			
IS	-0.023			
VR	5.733	-0.492	2.981	-0.492
IR	2.866	-0.164	2.981	-0.123
TIME	16.250			
VC	7.021	5.268	-0.470	
IC	0.003	0.107	0.168	
VS	1.283			
IS	-0.024			
VR	5.738	-0.470	2.979	-0.470
IR	2.869	-0.157	2.979	-0.118
TIME	16.750			
VC	7.022	5.294	-0.450	
IC	0.003	0.103	0.160	
VS	1.278			
IS	-0.025			
VR	5.744	-0.450	2.978	-0.450
IR	2.872	-0.150	2.978	-0.112

TIME	17.250			
VC	7.C24	5.319	-0.430	
IC	0.003	0.099	0.152	
VS	1.274			
IS	-0.C26			
VR	5.750	-C.430	2.976	-0.430
IR	2.875	-C.143	2.976	-0.108
TIME	18.250			
VC	7.C26	5.367	-0.394	
IC	0.C02	C.092	0.138	
VS	1.265			
IS	-0.C28			
VR	5.761	-C.394	2.974	-0.394
IR	2.880	-0.131	2.974	-0.099
TIME	19.250			
VC	7.028	5.411	-0.361	
IC	0.0C1	0.085	0.126	
VS	1.256			
IS	-0.029			
VR	5.772	-0.361	2.972	-0.361
IR	2.886	-0.120	2.972	-0.C90
TIME	2C.250			
VC	7.C29	5.452	-0.331	
IC	0.C01	0.079	0.114	
VS	1.246			
IS	-0.C30			
VR	5.783	-0.331	2.971	-0.331
IR	2.891	-0.110	2.971	-0.083

The following pages of computer output lists the input data cards and the program for the topological test circuit of Figure 4-1, which contains a tight capacitive loop.

THESE ARE THE CATA CARDS FOR THE ANALYSIS OF THE CIRCUIT OF FIGURE 4- 1
THIS CIRCUIT CCNTAINS A TIGHT CAPACITIVE LOOP.

```
01-02    1.                    10.
02-03                1.0
02-03          2.
C3-04                1.
03-01                1.
C3-01          1.
03-01                2.0
99
  20.000   0.0C0   1.000   0.100
99
```

INPUT PARAMETERS

BRANCH	FROM	TO	R	L	C	I	E	R-SOURCE
1	1	2	1.000	0.000	0.000	0.000	10.000	0.000
2	2	3	0.000	0.000	1.000	0.000	0.000	0.000
3	2	3	0.000	2.000	0.000	0.000	0.000	0.000
4	3	4	0.000	0.000	1.000	0.000	0.000	0.000
5	3	1	0.000	0.000	1.000	0.000	0.000	0.000
6	3	1	0.000	1.000	0.000	0.000	0.000	0.000
7	3	1	0.000	0.000	2.000	0.000	0.000	0.000

TOPOLOGICAL NETWORK DESCRIPTION

TREE BRANCHES	LINKS
5	3
2	6
4	1
	7

THE TOTAL NUMBER OF BRANCHES IS 7,LINKS IS 4,AND TREE BRANCHES IS 3

THE F-MATRIX IS PARTITIONED INTO THE FOLLOWING SUB-MATRICES

SUB MATRIX WITH THE DIMENSIONS

```
FSC  0   C    ( 1X 3)
FRC FRG  C    ( 1X 3) ( 1X 0)
FLC FLG FLM   ( 2X 3) ( 2X 0) ( 2X 0)
```

FOR THIS NETWORK THE F-MATRIX IS

```
 -1   0   0
  1   1   0
  0  -1   0
 -1   0   0
```

The following analyses are for the nonlinear circuits which contain a small number of parameters. Since the general program lists the output row-wise, small circuits, such as these, tend to have a single column of output. In order to avoid this, a special output subroutine was written especially for these listings. This output form is used for the remainder of this Appendix.

The following pages of computer output lists the input data cards and the program output of the negative resistance oscillator of Figure 4-8, with $\alpha = 0.1$.

THESE ARE THE DATA CARDS FOR THE ANALYSIS OF THE CIRCUIT OF FIGURE 4- 8
THIS CIRCUIT IS THE NEGATIVE RESISTANCE OSCILLATOR WITH ALPHA=0.1.

```
01-02              .025            3.6
02-03      .001
03-01                    10.0
03-01     -.50                                               120370
99
  -5.000-141.814  -4.500-116.660  -4.000 -94.544  -3.500 -75.278  -3.000 -58.674
  -2.500 -44.545  -2.000 -32.704  -1.500 -22.963  -1.000 -15.134  -0.500  -9.030
   0.000  -4.464   0.500  -1.248   1.000   0.806   1.500   1.885   2.000   2.176
   2.500   1.867   3.000   1.146   3.500   0.200   4.000  -0.784   4.500  -1.618
   5.000  -2.114   5.500  -2.085   6.000  -1.344   6.500   0.297   7.000   3.026
   7.500   7.030   8.000  12.496   8.500  19.612   9.000  28.566   9.500  39.545
  10.000  52.736  10.500  68.327  11.000  86.506  11.500 107.460  12.000 131.376
  25.000             .1      .1
99
```

INPUT PARAMETERS

BRANCH	FROM	TO	R	L	C	I	E	R-SOURCE
1	1	2	0.000	0.025	0.000	0.000	3.600	0.000
2	2	3	0.001	0.000	0.000	0.000	0.000	0.000
3	3	1	0.000	0.000	10.000	0.000	0.000	0.000
4	3	1	-0.500	0.000	0.000	0.000	0.000	0.000

TOPOLOGICAL NETWORK DESCRIPTION

TREE BRANCHES	LINKS
3	1
2	4

THE TOTAL NUMBER OF BRANCHES IS 4,LINKS IS 2,AND TREE BRANCHES IS 2

THE F-MATRIX IS PARTITIONED INTO THE FOLLOWING SUB-MATRICES

```
SUB MATRIX    WITH THE DIMENSIONS

FSC  0   0    ( 0X 1)
FRC FRG  0    ( 1X 1) ( 1X 1)
FLC FLG FLM   ( 1X 1) ( 1X 1) ( 1X 0)
```

FOR THIS NETWORK THE F-MATRIX IS

```
 -1   0
  1   1
```

```
TIME =    0.000  VC =    0.000  IC =    4.478  VL =    3.600   IL =    0.014
VR =   -0.694  IR =   -4.464  VG =    0.000  IG =   0.014
TIME =    0.106  VC =    0.125  IC =   18.719  VL =    3.460   IL =   15.057
VR =   -0.569  IR =   -3.663  VG =    0.015  IG =  15.057
TIME =    0.206  VC =    0.372  IC =   30.486  VL =    3.199   IL =   28.415
VR =   -0.322  IR =   -2.071  VG =    0.028  IG =  28.415
TIME =    0.306  VC =    0.729  IC =   40.816  VL =    2.830   IL =   40.509
VR =   -0.075  IR =   -0.306  VG =    0.041  IG =  40.509
TIME =    0.419  VC =    1.373  IC =   52.763  VL =    2.172   IL =   54.374
VR =    0.747  IR =    1.612  VG =    0.054  IG =  54.374
TIME =    0.519  VC =    1.937  IC =   59.803  VL =    1.601   IL =   61.943
VR =    3.676  IR =    2.140  VG =    0.062  IG =  61.943
TIME =    0.619  VC =    2.565  IC =   65.324  VL =    0.968   IL =   67.099
VR =   -1.230  IR =    1.774  VG =    0.067  IG =  67.099
TIME =    0.744  VC =    3.411  IC =   69.468  VL =    0.120   IL =   69.838
VR =   -0.195  IR =    0.369  VG =    0.070  IG =  69.838
TIME =    0.844  VC =    4.110  IC =   69.885  VL =   -0.579   IL =   68.919
VR =    0.580  IR =   -0.967  VG =    0.069  IG =  68.919
TIME =    0.994  VC =    5.128  IC =   64.469  VL =   -1.590   IL =   62.369
VR =  -36.321  IR =   -2.107  VG =    0.062  IG =  62.369
TIME =    1.094  VC =    5.735  IC =   56.521  VL =   -2.189   IL =   54.783
VR =   -1.172  IR =   -1.736  VG =    0.055  IG =  54.783
TIME =    1.194  VC =    6.248  IC =   45.520  VL =   -2.693   IL =   44.978
VR =   -0.162  IR =   -0.531  VG =    0.045  IG =  44.978
TIME =    1.294  VC =    6.638  IC =   32.357  VL =   -3.072   IL =   33.405
VR =    0.193  IR =    1.052  VG =    0.033  IG =  33.405
TIME =    1.394  VC =    6.891  IC =   18.158  VL =   -3.313   IL =   20.589
VR =    0.446  IR =    2.433  VG =    0.021  IG =  20.589
TIME =    1.400  VC =    6.982  IC =   10.197  VL =   -3.395   IL =   13.127
VR =    0.537  IR =    2.929  VG =    0.013  IG =  13.127
TIME =    1.506  VC =    7.011  IC =   -4.508  VL =   -3.410   IL =   -1.391
VR =    0.389  IR =    3.118  VG =   -0.001  IG =  -1.391
TIME =    1.644  VC =    6.828  IC =  -21.786  VL =   -3.208   IL =  -19.700
VR =    0.382  IR =    2.088  VG =   -0.020  IG = -19.700
TIME =    1.744  VC =    6.555  IC =  -32.593  VL =   -2.923   IL =  -31.998
VR =    0.109  IR =    0.596  VG =   -0.032  IG = -31.998
TIME =    1.844  VC =    6.180  IC =  -42.203  VL =   -2.540   IL =  -42.952
VR =   -0.229  IR =   -0.752  VG =   -0.043  IG = -42.952
TIME =    1.944  VC =    5.715  IC =  -50.417  VL =   -2.059   IL =  -52.172
VR =   -1.191  IR =   -1.766  VG =   -0.052  IG = -52.172
TIME =    2.044  VC =    5.177  IC =  -57.239  VL =   -1.520   IL =  -59.364
VR =  -36.272  IR =   -2.104  VG =   -0.059  IG = -59.364
TIME =    2.144  VC =    4.576  IC =  -62.557  VL =   -0.910   IL =  -64.231
VR =    1.707  IR =   -1.693  VG =   -0.064  IG = -64.231
TIME =    2.244  VC =    3.932  IC =  -65.931  VL =   -0.266   IL =  -66.599
VR =    0.330  IR =   -0.650  VG =   -0.067  IG = -66.599
TIME =    2.344  VC =    3.265  IC =  -66.983  VL =    0.402   IL =  -66.327
VR =   -0.341  IR =    0.645  VG =   -0.066  IG = -66.327
TIME =    2.444  VC =    2.602  IC =  -65.102  VL =    1.061   IL =  -63.394
VR =   -1.193  IR =    1.720  VG =   -0.063  IG = -63.394
TIME =    2.544  VC =    1.973  IC =  -60.054  VL =    1.684   IL =  -57.885
VR =    3.712  IR =    2.160  VG =   -0.058  IG = -57.885
TIME =    2.644  VC =    1.412  IC =  -51.704  VL =    2.238   IL =  -50.010
VR =    0.785  IR =    1.695  VG =   -0.050  IG = -50.010
TIME =    2.706  VC =    0.890  IC =  -39.308  VL =    2.750   IL =  -38.913
VR =    0.086  IR =    0.355  VG =   -0.039  IG = -38.913
TIME =    2.806  VC =    0.562  IC =  -26.248  VL =    3.065   IL =  -27.241
VR =   -0.242  IR =   -0.993  VG =   -0.027  IG = -27.241
TIME =    2.906  VC =    0.368  IC =  -12.478  VL =    3.246   IL =  -14.571
```

```
VR =  -0.326  IR =  -2.097  VG =  -0.015  IG = -14.571
TIME =   3.006  VC =   0.311  IC =   1.013  VL =   3.291  IL =  -1.452
VR =  -0.383  IR =  -2.462  VG =  -0.001  IG =  -1.452
TIME =   3.106  VC =   0.385  IC =  13.563  VL =   3.203  IL =  11.577
VR =  -0.309  IR =  -1.987  VG =   0.012  IG =  11.577
TIME =   3.206  VC =   0.578  IC =  24.944  VL =   2.998  IL =  24.016
VR =  -0.225  IR =  -0.926  VG =   0.024  IG =  24.016
TIME =   3.306  VC =   0.880  IC =  35.101  VL =   2.685  IL =  35.414
VR =   0.076  IR =   0.313  VG =   0.035  IG =  35.414
TIME =   3.406  VC =   1.276  IC =  43.968  VL =   2.278  IL =  45.369
VR =   0.650  IR =   1.402  VG =   0.045  IG =  45.369
TIME =   3.506  VC =   1.754  IC =  51.503  VL =   1.794  IL =  53.537
VR =   3.493  IR =   2.033  VG =   0.054  IG =  53.537
TIME =   3.606  VC =   2.300  IC =  57.623  VL =   1.242  IL =  59.621
VR =  -3.221  IR =   1.990  VG =   0.060  IG =  59.621
TIME =   3.706  VC =   2.901  IC =  62.088  VL =   0.637  IL =  63.386
VR =  -0.894  IR =   1.289  VG =   0.063  IG =  63.386
TIME =   3.806  VC =   3.536  IC =  64.529  VL =  -0.000  IL =  64.664
VR =  -0.066  IR =   0.130  VG =   0.065  IG =  64.664
TIME =   3.906  VC =   4.183  IC =  64.456  VL =  -0.647  IL =  63.368
VR =   0.653  IR =  -1.089  VG =   0.063  IG =  63.368
TIME =   4.006  VC =   4.815  IC =  61.446  VL =  -1.275  IL =  59.514
VR =   1.947  IR =  -1.931  VG =   0.060  IG =  59.514
TIME =   4.106  VC =   5.402  IC =  55.326  VL =  -1.856  IL =  53.233
VR = -36.046  IR =  -2.091  VG =   0.053  IG =  53.233
TIME =   4.206  VC =   5.912  IC =  46.256  VL =  -2.356  IL =  44.780
VR =  -0.995  IR =  -1.475  VG =   0.045  IG =  44.780
TIME =   4.306  VC =   6.319  IC =  34.821  VL =  -2.752  IL =  34.522
VR =  -0.091  IR =  -0.298  VG =   0.035  IG =  34.522
TIME =   4.406  VC =   6.604  IC =  22.046  VL =  -3.028  IL =  22.917
VR =   0.159  IR =   0.867  VG =   0.023  IG =  22.917
TIME =   4.506  VC =   6.758  IC =   8.771  VL =  -3.170  IL =  10.480
VR =   0.313  IR =   1.707  VG =   0.010  IG =  10.480
TIME =   4.606  VC =   6.781  IC =  -4.087  VL =  -3.179  IL =  -2.257
VR =   0.336  IR =   1.832  VG =  -0.002  IG =  -2.257
TIME =   4.706  VC =   6.679  IC = -16.060  VL =  -3.065  IL = -14.784
VR =   0.234  IR =   1.277  VG =  -0.015  IG = -14.784
TIME =   4.856  VC =   6.317  IC = -31.849  VL =  -2.687  IL = -32.152
VR =  -0.092  IR =  -0.302  VG =  -0.032  IG = -32.152
TIME =   4.956  VC =   5.953  IC = -40.761  VL =  -2.308  IL = -42.169
VR =  -0.954  IR =  -1.414  VG =  -0.042  IG = -42.169
TIME =   5.056  VC =   5.506  IC = -48.448  VL =  -1.858  IL = -50.532
VR =  -1.401  IR =  -2.076  VG =  -0.051  IG = -50.532
TIME =   5.156  VC =   4.988  IC = -54.836  VL =  -1.329  IL = -56.920
VR =   2.119  IR =  -2.102  VG =  -0.057  IG = -56.920
TIME =   5.256  VC =   4.415  IC = -59.617  VL =  -0.754  IL = -61.111
VR =   0.885  IR =  -1.476  VG =  -0.061  IG = -61.111
TIME =   5.356  VC =   3.802  IC = -62.518  VL =  -0.139  IL = -62.903
VR =   0.200  IR =  -0.395  VG =  -0.063  IG = -62.903
TIME =   5.456  VC =   3.172  IC = -63.012  VL =   0.489  IL = -62.204
VR =  -0.433  IR =   0.820  VG =  -0.062  IG = -62.204
TIME =   5.556  VC =   2.551  IC = -60.800  VL =   1.108  IL = -59.000
VR =  -1.244  IR =   1.794  VG =  -0.059  IG = -59.000
TIME =   5.656  VC =   1.967  IC = -55.545  VL =   1.686  IL = -53.392
VR =   3.705  IR =   2.157  VG =  -0.053  IG = -53.392
TIME =   5.756  VC =   1.450  IC = -47.383  VL =   2.195  IL = -45.600
VR =   0.823  IR =   1.776  VG =  -0.046  IG = -45.600
TIME =   5.806  VC =   1.020  IC = -36.911  VL =   2.614  IL = -36.100
VR =   0.393  IR =   0.849  VG =  -0.036  IG = -36.100
TIME =   5.919  VC =   0.682  IC = -23.034  VL =   2.942  IL = -23.536
```

```
VR =   -0.122  IR =   -0.502  VG =   -0.024  IG = -23.536
TIME =    6.044  VC =    0.494  IC =   -7.022  VL =    3.115  IL =   -8.311
VR =   -0.200  IR =   -1.290  VG =   -0.008  IG =  -8.311
TIME =    6.144  VC =    0.487  IC =    5.511  VL =    3.109  IL =    4.179
VR =   -0.207  IR =   -1.334  VG =    0.004  IG =   4.179
TIME =    6.244  VC =    0.601  IC =   17.233  VL =    2.982  IL =   16.401
VR =   -0.203  IR =   -0.833  VG =    0.016  IG =  16.401
TIME =    6.344  VC =    0.827  IC =   27.794  VL =    2.745  IL =   27.890
VR =    0.024  IR =    0.097  VG =    0.028  IG =  27.890
TIME =    6.444  VC =    1.152  IC =   37.096  VL =    2.410  IL =   38.229
VR =    0.526  IR =    1.135  VG =    0.038  IG =  38.229
TIME =    6.544  VC =    1.565  IC =   45.126  VL =    1.990  IL =   47.051
VR =    3.303  IR =    1.923  VG =    0.047  IG =  47.051
TIME =    6.644  VC =    2.051  IC =   51.893  VL =    1.494  IL =   54.038
VR =   -3.470  IR =    2.144  VG =    0.054  IG =  54.038
TIME =    6.744  VC =    2.598  IC =   [illegible]  VL =    0.942  IL =   58.930
VR =   -1.196  IR =    1.725  VG =   [illegible]  IG =  58.930
TIME =    6.844  VC =    3.190  IC =   [illegible]  VL =    0.348  IL =   61.525
VR =   -0.416  IR =    0.787  VG =    0.062  IG =  61.525
TIME =    6.944  VC =    3.806  IC =   62.096  VL =   -0.267  IL =   61.692
VR =    0.204  IR =   -0.402  VG =    0.062  IG =  61.692
TIME =    7.044  VC =    4.423  IC =   60.871  VL =   -0.882  IL =   59.387
VR =    0.893  IR =   -1.489  VG =    0.059  IG =  59.387
TIME =    7.144  VC =    5.014  IC =   56.765  VL =   -1.468  IL =   54.672
VR =  -36.435  IR =   -2.113  VG =    0.055  IG =  54.672
TIME =    7.244  VC =    5.549  IC =   49.735  VL =   -1.996  IL =   47.718
VR =   -1.358  IR =   -2.013  VG =    0.048  IG =  47.718
TIME =    7.344  VC =    6.000  IC =   40.165  VL =   -2.438  IL =   38.816
VR =   -0.907  IR =   -1.344  VG =    0.039  IG =  38.816
TIME =    7.444  VC =    6.345  IC =   28.567  VL =   -2.775  IL =   28.350
VR =   -0.064  IR =   -0.211  VG =    0.028  IG =  28.350
TIME =    7.544  VC =    6.568  IC =   16.132  VL =   -2.983  IL =   16.796
VR =    0.123  IR =    0.670  VG =    0.017  IG =  16.796
TIME =    7.606  VC =    6.680  IC =    1.875  VL =   -3.086  IL =    3.148
VR =    0.234  IR =    1.280  VG =    0.003  IG =   3.148
TIME =    7.722  VC =    6.621  IC =  -11.959  VL =   -3.010  IL =  -11.004
VR =    0.175  IR =    0.955  VG =   -0.011  IG = -11.004
TIME =    7.822  VC =    6.446  IC =  -22.825  VL =   -2.823  IL =  -22.705
VR =    0.036  IR =    0.120  VG =   -0.023  IG = -22.705
TIME =    7.922  VC =    6.167  IC =  -32.656  VL =   -2.534  IL =  -33.452
VR =   -0.242  IR =   -0.795  VG =   -0.033  IG = -33.452
TIME =    8.022  VC =    5.798  IC =  -41.214  VL =   -2.155  IL =  -42.857
VR =   -1.109  IR =   -1.644  VG =   -0.043  IG = -42.857
TIME =    8.122  VC =    5.348  IC =  -48.488  VL =   -1.699  IL =  -50.585
VR =  -36.100  IR =   -2.094  VG =   -0.051  IG = -50.585
TIME =    8.222  VC =    4.832  IC =  -54.398  VL =   -1.177  IL =  -56.351
VR =    1.963  IR =   -1.948  VG =   -0.056  IG = -56.351
TIME =    8.322  VC =    4.266  IC =  -58.697  VL =   -0.606  IL =  -59.927
VR =    0.736  IR =   -1.227  VG =   -0.060  IG = -59.927
TIME =    8.422  VC =    3.665  IC =  -61.025  VL =   -0.005  IL =  -61.153
VR =    0.064  IR =   -0.125  VG =   -0.061  IG = -61.153
TIME =    8.522  VC =    3.053  IC =  -60.989  VL =    0.607  IL =  -59.946
VR =   -0.553  IR =    1.046  VG =   -0.060  IG = -59.946
TIME =    8.622  VC =    2.455  IC =  -58.212  VL =    1.201  IL =  -56.318
VR =   -3.066  IR =    1.895  VG =   -0.056  IG = -56.318
TIME =    8.722  VC =    1.899  IC =  -52.508  VL =    1.751  IL =  -50.392
VR =    3.638  IR =    2.117  VG =   -0.050  IG = -50.392
TIME =    8.822  VC =    1.414  IC =  -44.100  VL =    2.228  IL =  -42.402
VR =    0.787  IR =    1.699  VG =   -0.042  IG = -42.402
TIME =    8.922  VC =    1.024  IC =  -33.550  VL =    2.608  IL =  -32.692
```

```
VR =    0.398  IR =    0.858  VG =  -0.033  IG = -32.692
TIME =    9.003  VC =    0.733  IC = -20.702  VL =    2.889  IL = -20.989
VR =   -0.071  IR =   -0.291  VG =  -0.021  IG = -20.989
TIME =    9.122  VC =    0.575  IC =  -5.917  VL =    3.032  IL =  -6.858
VR =   -0.229  IR =   -0.940  VG =  -0.007  IG =  -6.858
TIME =    9.222  VC =    0.577  IC =   6.213  VL =    3.018  IL =   5.281
VR =   -0.227  IR =   -0.932  VG =   0.005  IG =   5.281
TIME =    9.322  VC =    0.697  IC =  17.567  VL =    2.886  IL =  17.127
VR =   -0.107  IR =   -0.440  VG =   0.017  IG =  17.127
TIME =    9.472  VC =    1.075  IC =  32.399  VL =    2.492  IL =  33.371
VR =    0.448  IR =    0.968  VG =   0.033  IG =  33.371
TIME =    9.572  VC =    1.442  IC =  40.857  VL =    2.116  IL =  42.612
VR =    0.816  IR =    1.760  VG =   0.043  IG =  42.612
TIME =    9.672  VC =    1.888  IC =  48.081  VL =    1.660  IL =  50.188
VR =    3.627  IR =    2.111  VG =   0.050  IG =  50.188
TIME =    9.772  VC =    2.399  IC =  53.887  VL =    1.145  IL =  55.823
VR =   -3.122  IR =    1.929  VG =   0.056  IG =  55.823
TIME =    9.872  VC =    2.960  IC =  58.084  VL =    0.581  IL =  59.286
VR =   -0.834  IR =    1.203  VG =   0.059  IG =  59.286
TIME =    9.972  VC =    3.554  IC =  60.330  VL =   -0.015  IL =  60.425
VR =   -0.047  IR =    0.093  VG =   0.060  IG =  60.425
TIME =   10.072  VC =    4.159  IC =  60.207  VL =   -0.618  IL =  59.158
VR =    0.629  IR =   -1.049  VG =   0.059  IG =  59.158
TIME =   10.172  VC =    4.749  IC =  57.365  VL =   -1.204  IL =  55.502
VR =    1.880  IR =   -1.865  VG =   0.056  IG =  55.502
TIME =   10.272  VC =    5.297  IC =  51.680  VL =   -1.746  IL =  49.580
VR = -36.151  IR =   -2.097  VG =   0.050  IG =  49.580
TIME =   10.372  VC =    5.774  IC =  43.310  VL =   -2.215  IL =  41.628
VR =   -1.133  IR =   -1.679  VG =   0.042  IG =  41.628
TIME =   10.472  VC =    6.156  IC =  32.823  VL =   -2.588  IL =  31.985
VR =   -0.254  IR =   -0.832  VG =   0.032  IG =  31.985
TIME =   10.572  VC =    6.426  IC =  21.025  VL =   -2.847  IL =  21.075
VR =    0.016  IR =    0.054  VG =   0.021  IG =  21.075
TIME =   10.603  VC =    6.566  IC =  11.030  VL =   -2.979  IL =  11.680
VR =    0.126  IR =    0.656  VG =   0.012  IG =  11.680
TIME =   10.709  VC =    6.613  IC =  -2.011  VL =   -3.012  IL =  -1.097
VR =    0.167  IR =    0.913  VG =  -0.001  IG =  -1.097
TIME =   10.834  VC =    6.497  IC = -16.193  VL =   -2.882  IL = -15.904
VR =    0.088  IR =    0.289  VG =  -0.016  IG = -15.904
TIME =   10.934  VC =    6.282  IC = -26.594  VL =   -2.655  IL = -27.013
VR =   -0.127  IR =   -0.417  VG =  -0.027  IG = -27.013
TIME =   11.034  VC =    5.970  IC = -35.634  VL =   -2.333  IL = -37.020
VR =   -0.937  IR =   -1.388  VG =  -0.037  IG = -37.020
TIME =   11.134  VC =    5.573  IC = -43.590  VL =   -1.926  IL = -45.567
VR =   -1.334  IR =   -1.977  VG =  -0.046  IG = -45.567
TIME =   11.234  VC =    5.102  IC = -50.240  VL =   -1.449  IL = -52.343
VR = -36.346  IR =   -2.108  VG =  -0.052  IG = -52.343
TIME =   11.334  VC =    4.573  IC = -55.406  VL =   -0.915  IL = -57.092
VR =    1.704  IR =   -1.690  VG =  -0.057  IG = -57.092
TIME =   11.434  VC =    4.000  IC = -58.834  VL =   -0.340  IL = -59.616
VR =    0.398  IR =   -0.784  VG =  -0.060  IG = -59.616
TIME =   11.534  VC =    3.403  IC = -60.173  VL =    0.257  IL = -59.787
VR =   -0.203  IR =    0.383  VG =  -0.060  IG = -59.787
TIME =   11.634  VC =    2.805  IC = -58.993  VL =    0.853  IL = -57.564
VR =   -0.990  IR =    1.427  VG =  -0.058  IG = -57.564
TIME =   11.734  VC =    2.232  IC = -55.037  VL =    1.421  IL = -53.003
VR =   -3.289  IR =    2.033  VG =  -0.053  IG = -53.003
TIME =   11.834  VC =    1.713  IC = -48.283  VL =    1.934  IL = -46.272
VR =    3.452  IR =    2.009  VG =  -0.046  IG = -46.272
TIME =   11.934  VC =    1.274  IC = -39.046  VL =    2.365  IL = -37.647
```

```
VR =    0.648  IR =    1.397  VG =  -0.038  IG = -37.647
TIME =  12.009  VC =    0.935  IC = -28.071  VL =    2.692  IL = -27.531
VR =    0.132  IR =    0.541  VG =  -0.028  IG = -27.531
TIME =  12.147  VC =    0.664  IC = -11.338  VL =    2.949  IL = -11.914
VR =   -0.140  IR =   -0.575  VG =  -0.012  IG = -11.914
TIME =  12.200  VC =    0.610  IC =  -1.827  VL =    2.993  IL =  -2.623
VR =   -0.194  IR =   -0.796  VG =  -0.003  IG =  -2.623
```

The following pages of computer output lists the input data cards and the program output of the negative resistance oscillator of Figure 4-8 with $\alpha = 1$

```
THESE ARE THE DATA CARDS FOR THE ANALYSIS OF THE CIRCUIT OF FIGURE 4- 8
THIS CIRCUIT IS THE NEGATIVE RESISTANCE OSCILLATOR WITH ALPHA=1.0.


01-02               .250            3.6
02-03      .001
03-01                       1.0
03-01     -.50                                                 120370
99
  -5.000-141.814  -4.500-116.660  -4.000 -94.544  -3.500 -75.278  -3.000 -58.674
  -2.500 -44.545  -2.000 -32.704  -1.500 -22.963  -1.000 -15.134  -0.500  -9.030
   0.000  -4.464   0.500  -1.248   1.000   0.806   1.500   1.885   2.000   2.176
   2.500   1.867   3.000   1.146   3.500   0.200   4.000  -0.784   4.500  -1.618
   5.000  -2.114   5.500  -2.085   6.000  -1.344   6.500   0.297   7.000   3.026
   7.500   7.030   8.000  12.496   8.500  19.612   9.000  28.566   9.500  39.545
  10.000  52.736  10.500  68.327  11.000  86.506  11.500 107.460  12.000 131.376
  25.000             .1      .1
99
```

INPUT PARAMETERS

BRANCH	FROM	TO	R	L	C	I	E	R-SOURCE
1	1	2	0.000	0.250	0.000	0.000	3.600	0.000
2	2	3	0.001	0.000	0.000	0.000	0.000	0.000
3	3	1	0.000	0.000	1.000	0.000	0.000	0.000
4	3	1	-0.500	0.000	0.000	0.000	0.000	0.000

TOPOLOGICAL NETWORK DESCRIPTION

TREE BRANCHES	LINKS
3	1
2	4

THE TOTAL NUMBER OF BRANCHES IS 4,LINKS IS 2,AND TREE BRANCHES IS 2

THE F-MATRIX IS PARTITIONED INTO THE FOLLOWING SUB-MATRICES

SUB MATRIX	WITH THE DIMENSIONS
FSC 0 0	(0X 1)
FRC FRG 0	(1X 1) (1X 1)
FLC FLG FLM	(1X 1) (1X 1) (1X 0)

FOR THIS NETWORK THE F-MATRIX IS

-1	0
1	1

```
TIME =   0.000  VC =   0.000  IC =   4.463  VL =   3.600  IL =   0.001
VR =  -0.694  IR =  -4.462  VG =   0.000  IG =  0.001
TIME =   0.101  VC =   0.388  IC =   3.336  VL =   3.210  IL =   1.369
VR =  -0.306  IR =  -1.967  VG =   0.001  IG =  1.369
TIME =   0.201  VC =   0.698  IC =   3.025  VL =   2.899  IL =   2.590
VR =  -0.106  IR =  -0.434  VG =   0.003  IG =  2.590
TIME =   0.301  VC =   0.995  IC =   2.904  VL =   2.601  IL =   3.690
VR =   0.191  IR =   0.786  VG =   0.004  IG =  3.690
TIME =   0.401  VC =   1.302  IC =   3.183  VL =   2.279  IL =   4.670
VR =   0.675  IR =   1.458  VG =   0.005  IG =  4.670
TIME =   0.501  VC =   1.635  IC =   3.559  VL =   1.957  IL =   5.523
VR =   3.374  IR =   1.964  VG =   0.006  IG =  5.523
TIME =   0.626  VC =   2.121  IC =   4.280  VL =   1.478  IL =   6.385
VR =  -3.400  IR =   2.101  VG =   0.006  IG =  6.385
TIME =   0.726  VC =   2.592  IC =   5.141  VL =   1.004  IL =   6.882
VR =  -1.203  IR =   1.735  VG =   0.007  IG =  6.882
TIME =   0.826  VC =   3.164  IC =   6.342  VL =   0.425  IL =   7.172
VR =  -0.442  IR =   0.836  VG =   0.007  IG =  7.172
TIME =   0.926  VC =   3.866  IC =   7.732  VL =  -0.275  IL =   7.207
VR =   0.265  IR =  -0.521  VG =   0.007  IG =  7.207
TIME =   1.026  VC =   4.697  IC =   8.748  VL =  -1.103  IL =   6.936
VR =   1.828  IR =  -1.814  VG =   0.007  IG =  6.936
TIME =   1.126  VC =   5.567  IC =   8.307  VL =  -1.972  IL =   6.319
VR =  -1.340  IR =  -1.986  VG =   0.006  IG =  6.319
TIME =   1.226  VC =   6.287  IC =   5.779  VL =  -2.692  IL =   5.377
VR =  -0.122  IR =  -0.401  VG =   0.005  IG =  5.377
TIME =   1.301  VC =   6.718  IC =   2.721  VL =  -3.122  IL =   4.208
VR =   0.273  IR =   1.488  VG =   0.004  IG =  4.208
TIME =   1.426  VC =   6.884  IC =   0.250  VL =  -3.275  IL =   2.593
VR =   0.438  IR =   2.392  VG =   0.003  IG =  2.593
TIME =   1.526  VC =   6.844  IC =  -0.892  VL =  -3.245  IL =   1.283
VR =   0.399  IR =   2.177  VG =   0.001  IG =  1.283
TIME =   1.601  VC =   6.757  IC =  -1.382  VL =  -3.158  IL =   0.321
VR =   0.312  IR =   1.702  VG =   0.000  IG =  0.321
TIME =   1.720  VC =   6.566  IC =  -1.792  VL =  -2.965  IL =  -1.135
VR =   0.120  IR =   0.656  VG =  -0.001  IG = -1.135
TIME =   1.820  VC =   6.371  IC =  -2.158  VL =  -2.769  IL =  -2.283
VR =  -0.038  IR =  -0.126  VG =  -0.002  IG = -2.283
TIME =   1.920  VC =   6.139  IC =  -2.465  VL =  -2.538  IL =  -3.345
VR =  -0.271  IR =  -0.888  VG =  -0.003  IG = -3.345
TIME =   2.020  VC =   5.882  IC =  -2.795  VL =  -2.281  IL =  -4.309
VR =  -1.025  IR =  -1.519  VG =  -0.004  IG = -4.309
TIME =   2.120  VC =   5.581  IC =  -3.197  VL =  -1.975  IL =  -5.162
VR =  -1.026  IR =  -1.965  VG =  -0.005  IG = -5.162
TIME =   2.220  VC =   5.235  IC =  -3.785  VL =  -1.632  IL =  -5.885
VR = -36.213  IR =  -2.100  VG =  -0.006  IG = -5.885
TIME =   2.320  VC =   4.824  IC =  -4.519  VL =  -1.216  IL =  -6.457
VR =   1.955  IR =  -1.940  VG =  -0.006  IG = -6.457
TIME =   2.420  VC =   4.326  IC =  -5.521  VL =  -0.719  IL =  -6.847
VR =   0.796  IR =  -1.327  VG =  -0.007  IG = -6.847
TIME =   2.520  VC =   3.712  IC =  -6.802  VL =  -0.104  IL =  -7.016
VR =   0.110  IR =  -0.216  VG =  -0.007  IG = -7.016
TIME =   2.620  VC =   2.965  IC =  -8.109  VL =   0.642  IL =  -6.913
VR =  -0.830  IR =   1.197  VG =  -0.007  IG = -6.913
TIME =   2.720  VC =   2.117  IC =  -8.591  VL =   1.489  IL =  -6.488
VR =  -3.405  IR =   2.104  VG =  -0.006  IG = -6.488
TIME =   2.801  VC =   1.262  IC =  -7.046  VL =   2.345  IL =  -5.675
VR =   0.635  IR =   1.370  VG =  -0.006  IG = -5.675
TIME =   2.907  VC =   0.669  IC =  -3.973  VL =   2.939  IL =  -4.541
```

```
VR =  -0.135  IR =  -0.555  VG =  -0.005  IG =  -4.541
TIME =   3.007  VC =   0.384  IC =  -1.150  VL =   3.219  IL =  -3.144
VR =  -0.310  IR =  -1.994  VG =  -0.003  IG =  -3.144
TIME =   3.107  VC =   0.352  IC =   0.354  VL =   3.250  IL =  -1.846
VR =  -0.342  IR =  -2.199  VG =  -0.002  IG =  -1.846
TIME =   3.207  VC =   0.431  IC =   1.133  VL =   3.170  IL =  -0.559
VR =  -0.263  IR =  -1.693  VG =  -0.001  IG =  -0.559
TIME =   3.307  VC =   0.589  IC =   1.713  VL =   3.009  IL =   0.834
VR =  -0.214  IR =  -0.880  VG =   0.001  IG =   0.834
TIME =   3.407  VC =   0.782  IC =   2.091  VL =   2.816  IL =   2.000
VR =  -0.022  IR =  -0.091  VG =   0.002  IG =   2.000
TIME =   3.507  VC =   1.001  IC =   2.279  VL =   2.597  IL =   3.083
VR =   0.198  IR =   0.811  VG =   0.003  IG =   3.083
TIME =   3.607  VC =   1.252  IC =   2.724  VL =   2.344  IL =   4.074
VR =   0.625  IR =   1.350  VG =   0.004  IG =   4.074
TIME =   3.707  VC =   1.539  IC =   3.047  VL =   2.056  IL =   4.955
VR =   3.278  IR =   1.908  VG =   0.005  IG =   4.955
TIME =   3.807  VC =   1.874  IC =   3.610  VL =   1.720  IL =   5.712
VR =   3.612  IR =   2.102  VG =   0.006  IG =   5.712
TIME =   3.907  VC =   2.266  IC =   4.315  VL =   1.325  IL =   6.324
VR =  -3.255  IR =   2.012  VG =   0.006  IG =   6.324
TIME =   4.007  VC =   2.739  IC =   5.247  VL =   0.852  IL =   6.765
VR =  -1.056  IR =   1.522  VG =   0.007  IG =   6.765
TIME =   4.107  VC =   3.321  IC =   6.458  VL =   0.271  IL =   6.995
VR =  -0.285  IR =   0.540  VG =   0.007  IG =   6.995
TIME =   4.207  VC =   4.033  IC =   7.805  VL =  -0.440  IL =   6.966
VR =   0.504  IR =  -0.840  VG =   0.007  IG =   6.966
TIME =   4.307  VC =   4.864  IC =   8.604  VL =  -1.269  IL =   6.626
VR =   1.995  IR =  -1.979  VG =   0.007  IG =   6.626
TIME =   4.407  VC =   5.698  IC =   7.741  VL =  -2.104  IL =   5.949
VR =  -1.209  IR =  -1.791  VG =   0.006  IG =   5.949
TIME =   4.507  VC =   6.355  IC =   5.131  VL =  -2.765  IL =   4.967
VR =  -0.054  IR =  -0.178  VG =   0.005  IG =   4.967
TIME =   4.607  VC =   6.776  IC =   1.692  VL =  -3.173  IL =   3.467
VR =   0.330  IR =   1.803  VG =   0.003  IG =   3.467
TIME =   4.707  VC =   6.850  IC =   0.024  VL =  -3.240  IL =   2.175
VR =   0.405  IR =   2.209  VG =   0.002  IG =   2.175
TIME =   4.807  VC =   6.794  IC =  -1.014  VL =  -3.194  IL =   0.883
VR =   0.348  IR =   1.899  VG =   0.001  IG =   0.883
TIME =   4.920  VC =   6.602  IC =  -1.680  VL =  -3.001  IL =  -0.826
VR =   0.156  IR =   0.853  VG =  -0.001  IG =  -0.826
TIME =   5.020  VC =   6.420  IC =  -2.036  VL =  -2.821  IL =  -1.991
VR =   0.011  IR =   0.036  VG =  -0.002  IG =  -1.991
TIME =   5.120  VC =   6.198  IC =  -2.405  VL =  -2.603  IL =  -3.076
VR =  -0.212  IR =  -0.695  VG =  -0.003  IG =  -3.076
TIME =   5.220  VC =   5.948  IC =  -2.645  VL =  -2.345  IL =  -4.064
VR =  -0.959  IR =  -1.421  VG =  -0.004  IG =  -4.064
TIME =   5.320  VC =   5.660  IC =  -3.093  VL =  -2.052  IL =  -4.945
VR =  -1.247  IR =  -1.848  VG =  -0.005  IG =  -4.945
TIME =   5.420  VC =   5.331  IC =  -3.609  VL =  -1.727  IL =  -5.702
VR = -36.117  IR =  -2.095  VG =  -0.006  IG =  -5.702
TIME =   5.520  VC =   4.939  IC =  -4.264  VL =  -1.332  IL =  -6.317
VR =   2.070  IR =  -2.053  VG =  -0.006  IG =  -6.317
TIME =   5.620  VC =   4.466  IC =  -5.198  VL =  -0.859  IL =  -6.759
VR =   0.936  IR =  -1.561  VG =  -0.007  IG =  -6.759
TIME =   5.720  VC =   3.885  IC =  -6.432  VL =  -0.278  IL =  -6.990
VR =   0.284  IR =  -0.559  VG =  -0.007  IG =  -6.990
TIME =   5.845  VC =   2.976  IC =  -8.091  VL =   0.631  IL =  -6.911
VR =  -0.819  IR =   1.181  VG =  -0.007  IG =  -6.911
TIME =   5.945  VC =   2.129  IC =  -8.585  VL =   1.475  IL =  -6.491
```

```
VR =   -3.392  IR =    2.096  VG =  -0.006  IG =  -6.491
TIME =    6.001  VC =    1.457  IC =  -7.699  VL =    2.149  IL =  -5.907
VR =    0.831  IR =    1.793  VG =  -0.006  IG =  -5.907
TIME =    6.107  VC =    0.785  IC =  -4.756  VL =    2.822  IL =  -4.840
VR =   -0.019  IR =   -0.078  VG =  -0.005  IG =  -4.840
TIME =    6.201  VC =    0.434  IC =  -1.883  VL =    3.170  IL =  -3.557
VR =   -0.260  IR =   -1.674  VG =  -0.004  IG =  -3.557
TIME =    6.320  VC =    0.349  IC =    0.198  VL =    3.253  IL =  -2.022
VR =   -0.345  IR =   -2.220  VG =  -0.002  IG =  -2.022
TIME =    6.445  VC =    0.444  IC =    1.191  VL =    3.156  IL =  -0.415
VR =   -0.250  IR =   -1.605  VG =  -0.000  IG =  -0.415
TIME =    6.501  VC =    0.555  IC =    1.614  VL =    3.044  IL =    0.594
VR =   -0.248  IR =   -1.020  VG =   0.001  IG =    0.594
TIME =    6.620  VC =    0.779  IC =    2.099  VL =    2.821  IL =    1.989
VR =   -0.025  IR =   -0.102  VG =   0.002  IG =    1.989
TIME =    6.720  VC =    0.998  IC =    2.273  VL =    2.598  IL =    3.073
VR =    0.195  IR =    0.800  VG =   0.003  IG =    3.073
TIME =    6.820  VC =    1.249  IC =    2.724  VL =    2.348  IL =    4.064
VR =    0.622  IR =    1.342  VG =   0.004  IG =    4.064
TIME =    6.920  VC =    1.536  IC =    3.042  VL =    2.060  IL =    4.947
VR =    3.275  IR =    1.906  VG =   0.005  IG =    4.947
TIME =    7.020  VC =    1.870  IC =    3.606  VL =    1.724  IL =    5.706
VR =    3.608  IR =    2.100  VG =   0.006  IG =    5.706
```

The following pages of computer output lists the input data cards and the program output of the tunnel diode circuit of Figure 4-14.

This is the constant capacitance case and is presented in the abrieviated form, i.e., the state variables and their associated functions are printed out as a function of time.

THESE ARE THE CATA CARDS FOR THE ANALYSIS OF THE CIRCUIT OF FIGURE 4-14
THIS IS THE TUNNEL DIODE ANALYSIS WITH THE CONSTANT JUNCTION CAPACITANCE.

```
01-02              0.01              0.30                      510012
C2-03     0.07
03-01                        1C.0
03-01     0.01
99                                                             120342
   0.000    0.300    0.600    0.300    C.601    0.90C    2.5C0    0.900    2.501    0.300
  10.000    0.300
 -0.0341  -7.00C0   0.0000   0.0C00   0.0341   7.0C00   0.0511   9.2000   0.0750  1C.0000
  C.1023   9.2000   0.1364   7.5C0C   0.1707   5.400C   0.2045   3.6000   0.2386   2.1000
  0.2727   1.4000   0.3068   1.2C00   0.3409   1.1000   0.3750   1.0000   0.4091   0.9000
  0.4432   1.0000   0.4773   1.2C00   0.5114   1.4000   0.5455   2.7000   0.5795   5.4000
  0.6136   9.60C0
   7.000    0.1C0    0.100    0.10011
99
```

INPUT PARAMETERS

BRANCH	FROM	TO	R	L	C	I	E	R-SOURCE
1	1	2	0.000	0.010	0.000	0.000	0.300	0.000
2	2	3	0.070	0.000	0.000	0.000	0.000	0.000
3	3	1	0.000	0.000	10.000	0.000	0.000	0.000
4	3	1	0.010	0.000	0.000	0.000	0.000	0.000

TOPOLOGICAL NETWORK DESCRIPTION

TREE BRANCHES	LINKS
3	1
2	4

THE TOTAL NUMBER OF BRANCHES IS 4,LINKS IS 2,AND TREE BRANCHES IS 2

THE F-MATRIX IS PARTITIONED INTO THE FOLLOWING SUB-MATRICES

SUB MATRIX	WITH THE DIMENSIONS
FSC 0 0	(0X 1)
FRC FRG 0	(1X 1) (1X 1)
FLC FLG FLM	(1X 1) (1X 1) (1X 0)

FOR THIS NETWORK THE F-MATRIX IS

-1	0
1	1

TIME =	0.100	VC =	0.006	IC =	0.824	VL =	0.158	IL =	2.135
TIME =	0.200	VC =	0.013	IC =	0.521	VL =	0.074	IL =	3.143
TIME =	0.300	VC =	0.016	IC =	0.284	VL =	0.037	IL =	3.610
TIME =	0.400	VC =	0.018	IC =	0.134	VL =	0.017	IL =	3.824
TIME =	0.500	VC =	0.019	IC =	0.075	VL =	0.010	IL =	3.923
TIME =	0.600	VC =	0.019	IC =	0.034	VL =	0.004	IL =	3.968
TIME =	0.702	VC =	0.032	IC =	2.561	VL =	0.274	IL =	8.853
TIME =	0.802	VC =	0.050	IC =	1.554	VL =	0.122	IL =	10.561
TIME =	0.921	VC =	0.069	IC =	1.603	VL =	0.047	IL =	11.367
TIME =	1.009	VC =	0.083	IC =	1.701	VL =	0.012	IL =	11.551
TIME =	1.121	VC =	0.107	IC =	2.296	VL =	-0.009	IL =	11.519
TIME =	1.221	VC =	0.138	IC =	3.511	VL =	-0.025	IL =	11.307
TIME =	1.321	VC =	0.190	IC =	5.829	VL =	-0.046	IL =	10.901
TIME =	1.421	VC =	0.269	IC =	8.489	VL =	-0.076	IL =	10.218
TIME =	1.521	VC =	0.354	IC =	8.400	VL =	-0.100	IL =	9.269
TIME =	1.621	VC =	0.432	IC =	7.567	VL =	-0.107	IL =	8.212
TIME =	1.721	VC =	0.497	IC =	6.198	VL =	-0.102	IL =	7.174
TIME =	1.846	VC =	0.553	IC =	4.118	VL =	-0.089	IL =	6.058
TIME =	1.909	VC =	0.569	IC =	1.082	VL =	-0.056	IL =	5.478
TIME =	2.021	VC =	0.574	IC =	0.202	VL =	-0.032	IL =	5.043
TIME =	2.121	VC =	0.574	IC =	-0.059	VL =	-0.016	IL =	4.850
TIME =	2.221	VC =	0.573	IC =	-0.099	VL =	-0.007	IL =	4.760
TIME =	2.346	VC =	0.572	IC =	-0.076	VL =	-0.003	IL =	4.716
TIME =	2.446	VC =	0.571	IC =	-0.046	VL =	-0.001	IL =	4.705
TIME =	2.546	VC =	0.571	IC =	-0.025	VL =	-0.000	IL =	4.702
TIME =	2.604	VC =	0.553	IC =	-3.587	VL =	-0.246	IL =	-0.282
TIME =	2.704	VC =	0.518	IC =	-3.502	VL =	-0.103	IL =	-1.817
TIME =	2.804	VC =	0.483	IC =	-3.548	VL =	-0.033	IL =	-2.330
TIME =	2.904	VC =	0.448	IC =	-3.438	VL =	0.009	IL =	-2.330
TIME =	3.004	VC =	0.416	IC =	-3.107	VL =	0.028	IL =	-2.091
TIME =	3.104	VC =	0.387	IC =	-2.788	VL =	0.035	IL =	-1.757
TIME =	3.204	VC =	0.362	IC =	-2.579	VL =	0.036	IL =	-1.397
TIME =	3.304	VC =	0.339	IC =	-2.291	VL =	0.035	IL =	-1.044
TIME =	3.404	VC =	0.319	IC =	-2.012	VL =	0.033	IL =	-0.715
TIME =	3.504	VC =	0.301	IC =	-1.754	VL =	0.030	IL =	-0.417
TIME =	3.604	VC =	0.282	IC =	-1.456	VL =	0.025	IL =	-0.089
TIME =	3.704	VC =	0.268	IC =	-1.339	VL =	0.022	IL =	0.143
TIME =	3.804	VC =	0.254	IC =	-1.404	VL =	0.021	IL =	0.355
TIME =	3.929	VC =	0.236	IC =	-1.492	VL =	0.021	IL =	0.617
TIME =	4.004	VC =	0.222	IC =	-1.891	VL =	0.022	IL =	0.782
TIME =	4.104	VC =	0.198	IC =	-2.626	VL =	0.027	IL =	1.042
TIME =	4.204	VC =	0.161	IC =	-4.053	VL =	0.038	IL =	1.391
TIME =	4.301	VC =	0.095	IC =	-7.401	VL =	0.066	IL =	1.974
TIME =	4.401	VC =	0.030	IC =	-4.898	VL =	0.081	IL =	2.742
TIME =	4.510	VC =	0.017	IC =	-0.222	VL =	0.048	IL =	3.414
TIME =	4.610	VC =	0.018	IC =	0.101	VL =	0.023	IL =	3.728
TIME =	4.735	VC =	0.019	IC =	0.080	VL =	0.012	IL =	3.901
TIME =	4.810	VC =	0.019	IC =	0.091	VL =	0.015	IL =	3.949
TIME =	4.910	VC =	0.019	IC =	0.018	VL =	0.002	IL =	3.980
TIME =	5.023	VC =	0.019	IC =	0.008	VL =	0.001	IL =	3.996
TIME =	5.123	VC =	0.019	IC =	0.005	VL =	0.001	IL =	4.002
TIME =	5.223	VC =	0.020	IC =	0.002	VL =	0.000	IL =	4.005
TIME =	5.323	VC =	0.020	IC =	0.001	VL =	0.000	IL =	4.006
TIME =	5.423	VC =	0.020	IC =	0.000	VL =	0.000	IL =	4.006
TIME =	5.523	VC =	0.020	IC =	0.000	VL =	0.000	IL =	4.007
TIME =	5.623	VC =	0.020	IC =	0.000	VL =	0.000	IL =	4.007
TIME =	5.723	VC =	0.020	IC =	-0.001	VL =	-0.000	IL =	4.007
TIME =	5.823	VC =	0.020	IC =	0.000	VL =	0.000	IL =	4.007
TIME =	5.923	VC =	0.020	IC =	0.000	VL =	0.000	IL =	4.007

The following pages of computer output lists the input data cards and the program output of the tunnel diode circuit of Figure 4-14. This case is for the variables junction capacitance. The output is also in the abbreviated form, i.e., only the state variables are printed out.

THESE ARE THE DATA CARDS FOR THE ANALYSIS OF THE CIRCUIT OF FIGURE 4-14
THIS IS THE TUNNEL DIODE ANALYSIS WITH THE VARIABLE JUNCTION CAPACITANCE.

```
01-02             0.01              0.30                        510012
02-03     0.07
03-01                      10.0                                 320332
03-02     0.01                                                  120342
99
   0.000     0.300     0.600     0.300     0.601     0.900     2.500     0.900     2.501     0.300
  10.000     0.300
 -10.000     2.370    -5.000     3.250    -4.000     3.610    -3.000     4.100    -2.000     4.830
  -1.000     6.120    -0.500     7.370     0.000    10.000     0.100    10.900     0.200    12.200
   0.300    14.150     0.400    17.300     0.500    24.400     0.550    36.100     0.590    77.460
  0.5999   774.600
 -0.0341   -7.0000    0.0000    0.0000    0.0341    7.0000    0.0511    9.2000    0.0750   10.0000
  0.1023    9.2000    0.1364    7.5000    0.1707    5.4000    0.2045    3.6000    0.2386    2.1000
  0.2727    1.4000    0.3068    1.2000    0.3409    1.1000    0.3750    1.0000    0.4091    0.9000
  0.4432    1.0000    0.4773    1.2000    0.5114    1.4000    0.5455    2.7000    0.5795    5.4000
  0.6136    9.6000
   7.000     0.100     0.100     0.10011
99
```

INPUT PARAMETERS

BRANCH	FROM	TO	R	L	C	I	E	R-SOURCE
1	1	2	0.000	0.010	0.000	0.000	0.300	0.000
2	2	3	0.070	0.000	0.000	0.000	0.000	0.000
3	3	1	0.000	0.000	10.000	0.000	0.000	0.000
4	3	1	0.010	0.000	0.000	0.000	0.000	0.000

TOPOLOGICAL NETWORK DESCRIPTION

TREE BRANCHES	LINKS
3	1
2	4

THE TOTAL NUMBER OF BRANCHES IS 4,LINKS IS 2,AND TREE BRANCHES IS 2

THE F-MATRIX IS PARTITIONED INTO THE FOLLOWING SUB-MATRICES

SUB MATRIX WITH THE DIMENSIONS

```
FSC  0   0     ( 0X 1)
FRC FRG  0     ( 1X 1) ( 1X 1)
FLC FLG FLM    ( 1X 1) ( 1X 1) ( 1X 0)
```

FOR THIS NETWORK THE F-MATRIX IS

```
-1  0
 1  1
```

TIME =	0.100	VC =	0.006	IC =	0.826	VL =	0.158	IL =	2.135
TIME =	0.200	VC =	0.013	IC =	0.526	VL =	0.074	IL =	3.143
TIME =	0.300	VC =	0.016	IC =	0.289	VL =	0.038	IL =	3.610
TIME =	0.400	VC =	0.018	IC =	0.166	VL =	0.021	IL =	3.825
TIME =	0.500	VC =	0.019	IC =	0.063	VL =	0.008	IL =	3.923
TIME =	0.600	VC =	0.019	IC =	0.028	VL =	0.004	IL =	3.969
TIME =	0.701	VC =	0.032	IC =	1.829	VL =	0.306	IL =	8.411
TIME =	0.801	VC =	0.049	IC =	1.558	VL =	0.140	IL =	10.351
TIME =	0.913	VC =	0.065	IC =	1.610	VL =	0.059	IL =	11.269
TIME =	1.013	VC =	0.080	IC =	1.547	VL =	0.026	IL =	11.544
TIME =	1.101	VC =	0.100	IC =	2.266	VL =	-0.008	IL =	11.556
TIME =	1.201	VC =	0.125	IC =	3.065	VL =	-0.020	IL =	11.395
TIME =	1.301	VC =	0.161	IC =	4.628	VL =	-0.034	IL =	11.094
TIME =	1.401	VC =	0.214	IC =	7.021	VL =	-0.052	IL =	10.621
TIME =	1.501	VC =	0.276	IC =	8.682	VL =	-0.070	IL =	9.961
TIME =	1.601	VC =	0.334	IC =	8.463	VL =	-0.078	IL =	9.194
TIME =	1.701	VC =	0.383	IC =	7.774	VL =	-0.075	IL =	8.434
TIME =	1.801	VC =	0.423	IC =	7.240	VL =	-0.067	IL =	7.739
TIME =	1.901	VC =	0.455	IC =	6.408	VL =	-0.058	IL =	7.138
TIME =	2.026	VC =	0.487	IC =	5.771	VL =	-0.048	IL =	6.524
TIME =	2.126	VC =	0.507	IC =	5.397	VL =	-0.040	IL =	6.125
TIME =	2.226	VC =	0.523	IC =	4.657	VL =	-0.033	IL =	5.798
TIME =	2.326	VC =	0.535	IC =	3.729	VL =	-0.026	IL =	5.542
TIME =	2.426	VC =	0.543	IC =	3.059	VL =	-0.020	IL =	5.343
TIME =	2.526	VC =	0.550	IC =	2.492	VL =	-0.015	IL =	5.189
TIME =	2.604	VC =	0.547	IC =	-2.879	VL =	-0.251	IL =	-0.269
TIME =	2.716	VC =	0.535	IC =	-4.050	VL =	-0.117	IL =	-1.996
TIME =	2.816	VC =	0.521	IC =	-4.280	VL =	-0.048	IL =	-2.623
TIME =	2.916	VC =	0.505	IC =	-4.038	VL =	-0.013	IL =	-2.827
TIME =	3.016	VC =	0.488	IC =	-4.063	VL =	0.006	IL =	-2.810
TIME =	3.141	VC =	0.466	IC =	-3.799	VL =	0.015	IL =	-2.634
TIME =	3.241	VC =	0.449	IC =	-3.524	VL =	0.020	IL =	-2.436
TIME =	3.341	VC =	0.433	IC =	-3.228	VL =	0.022	IL =	-2.218
TIME =	3.441	VC =	0.417	IC =	-2.963	VL =	0.023	IL =	-1.994
TIME =	3.541	VC =	0.401	IC =	-2.697	VL =	0.022	IL =	-1.769
TIME =	3.641	VC =	0.386	IC =	-2.567	VL =	0.022	IL =	-1.547
TIME =	3.741	VC =	0.372	IC =	-2.392	VL =	0.022	IL =	-1.330
TIME =	3.841	VC =	0.358	IC =	-2.224	VL =	0.021	IL =	-1.119
TIME =	3.941	VC =	0.344	IC =	-2.060	VL =	0.020	IL =	-0.917
TIME =	4.041	VC =	0.332	IC =	-1.904	VL =	0.019	IL =	-0.725
TIME =	4.141	VC =	0.320	IC =	-1.754	VL =	0.018	IL =	-0.542
TIME =	4.241	VC =	0.309	IC =	-1.612	VL =	0.017	IL =	-0.368
TIME =	4.341	VC =	0.298	IC =	-1.489	VL =	0.016	IL =	-0.204
TIME =	4.401	VC =	0.290	IC =	-1.373	VL =	0.015	IL =	-0.072
TIME =	4.516	VC =	0.279	IC =	-1.284	VL =	0.014	IL =	0.098
TIME =	4.641	VC =	0.268	IC =	-1.202	VL =	0.014	IL =	0.271
TIME =	4.716	VC =	0.261	IC =	-1.258	VL =	0.013	IL =	0.372
TIME =	4.816	VC =	0.251	IC =	-1.322	VL =	0.014	IL =	0.507
TIME =	4.941	VC =	0.237	IC =	-1.389	VL =	0.014	IL =	0.685
TIME =	5.041	VC =	0.224	IC =	-1.641	VL =	0.015	IL =	0.841
TIME =	5.116	VC =	0.212	IC =	-2.134	VL =	0.019	IL =	0.976
TIME =	5.216	VC =	0.190	IC =	-2.878	VL =	0.024	IL =	1.201
TIME =	5.316	VC =	0.158	IC =	-4.154	VL =	0.033	IL =	1.509
TIME =	5.401	VC =	0.108	IC =	-6.918	VL =	0.055	IL =	1.948
TIME =	5.504	VC =	0.040	IC =	-6.039	VL =	0.075	IL =	2.644
TIME =	5.607	VC =	0.018	IC =	-0.533	VL =	0.052	IL =	3.343
TIME =	5.707	VC =	0.018	IC =	0.075	VL =	0.026	IL =	3.692
TIME =	5.807	VC =	0.018	IC =	0.085	VL =	0.014	IL =	3.861
TIME =	5.907	VC =	0.019	IC =	0.060	VL =	0.008	IL =	3.940

TIME =	6.001	VC =	0.019	IC =	0.021	VL =	0.003	IL =	3.975
TIME =	6.107	VC =	0.019	IC =	0.011	VL =	0.001	IL =	3.993
TIME =	6.232	VC =	0.019	IC =	0.005	VL =	0.001	IL =	4.002
TIME =	6.332	VC =	0.019	IC =	0.002	VL =	0.000	IL =	4.004
TIME =	6.432	VC =	0.020	IC =	0.001	VL =	0.000	IL =	4.006
TIME =	6.582	VC =	0.020	IC =	0.001	VL =	0.000	IL =	4.007
TIME =	6.682	VC =	0.020	IC =	0.000	VL =	0.000	IL =	4.007
TIME =	6.782	VC =	0.020	IC =	-0.000	VL =	0.000	IL =	4.007
TIME =	6.832	VC =	0.020	IC =	0.000	VL =	0.000	IL =	4.007
TIME =	6.907	VC =	0.020	IC =	0.000	VL =	0.000	IL =	4.007
TIME =	7.007	VC =	0.020	IC =	-0.000	VL =	-0.000	IL =	4.007

VITA

Pentti A. Honkanen was born November 25, 1932, in Brooklyn, New York. He was graduated from Manual Training High School, Brooklyn, New York in 1950. He served in the United States Navy from 1951 to 1955. He graduated from the University of Colorado in 1959 with a B. S. degree in Electrical Engineering and a B. S. degree in Applied Mathematics. In 1959 he was employed by the International Business Machine Corporation, Poughkeepsie, New York, where he served as staff engineer In 1962 he received the Master of Electrical Engineering degree from Syracuse University. From 1964 to the present, he has been a research assistant at the Ordnance Research Laboratory of The Pennsylvania State University. He is a member of the Institute of Electrical and Electronic Engineers, the Research Society of America, Tau Beta Pi, Eta Kappa Nu, and Sigma Tau

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