NELC/REPORT 1550

OPTIMUM BINARY INTEGRATION

The fixed-sample binomial test is analyzed for the Normal, Rayleigh, Swerling Case 4, and Rice distributions

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THE PROBLEM

Apply statistical decision theory to the design of automatic radar detection systems. The specific phase of the problem reported here is the analysis of a fixed-sample-size statistical test which uses binary data.

RESULTS

1. Binary integration of statistically independent observations is discussed. Distribution functions and other appropriate equations are given for the Normal (test of the mean and test of the variance), Rayleigh, Swerling Case 4, and Rice distributions.

2. For the distributions listed above, curves are presented which give the optimum threshold versus the number of observations for four combinations of error probabilities.

3. It is shown that the optimum value of the threshold depends considerably on the functional form of the distribution but only slightly on the values (under the two hypotheses) of the parameter of the distribution, i.e., on the signal-to-noise ratio.

4. Curves of the required signal-to-noise ratio versus the number of pulses and of the miss probability versus the signal-to-noise ratio are given for several signal detection cases.

RECOMMENDATIONS

Compute optimum thresholds for applications in which the observations are statistically dependent; in particular, for standard signal-fluctuation models involving complete or partial correlation between successive pulses.

ADMINISTRATIVE INFORMATION

Work was performed under SF 001 02 05, Task 6072 (NELC D11771) by members of the Electronic Circuits and Signal Processing Division. The report covers work from February 1967 to February 1968 and was approved for publication 28 March 1968. The computations were made by R.F. Arenz.

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INTRODUCTION

The testing procedure known under the names *binary integration, coincidence detection, binomial fixed-sample-size testing, double-threshold detection, K out of N detection, the method of Bernoulli trials*, etc., has received considerable attention in the literature as a method of detecting signals in noise.[1-12] (See list of references at end of report.) The primary purpose of this report is to present graphical results as aids in selecting optimum thresholds for this test. Additional graphs give, for certain signal-detection cases, the required signal-to-noise ratio as a function of the number of pulses and the miss probability as a function of the signal-to-noise ratio.

THE TEST

Although we are concerned primarily with the problem of determining the presence or absence of a signal at a receiver, it is convenient to formulate the testing procedure in terms of testing a hypothesis H_0 against a single alternative H_1. In the particular hypothesis tests to be treated here, the functional form of the distribution of a certain random variable is known, but the distribution has an unknown parameter θ; other parameters of the distribution, if there are any, are known. It is assumed that the parameter θ can take one of only two values, and these values θ_0 and θ_1 (arbitrarily, let $\theta_1 > \theta_0$) are exactly specified. Under the simple hypothesis H_0 that $\theta = \theta_0$, the probability density function of x is $f(x;\theta_0)$, and under the simple alternative hypothesis H_1 that $\theta = \theta_1$, the probability density function is $f(x;\theta_1)$.

The test is performed by making N statistically independent observations on x, counting how many of these observations exceed a "quantization" threshold q, and comparing this count with a "counter" threshold K.* If the count is less than K, the decision is made that H_0 is true; otherwise the decision is that H_1 is true. A *type I error* is committed if H_1 is accepted as true when H_0 is true, and a *type II* error is committed if H_0 is accepted when H_1 is true.

*Binary integration with dependent sampling is treated in references 11 and 12. References 9, 10, and 11 deal with "median detectors," where the quantization threshold is the median of the noise distribution.

APPLICATION TO SIGNAL DETECTION

In signal detection, the hypotheses H_0 and H_1 correspond, respectively, to the presence of noise alone and the presence of signal-plus-noise. The type I error is then a *false alarm* and the type II error a *miss*. Typically, the N observations on x are voltage amplitudes at a receiver output and correspond to N pulses. Refer to the block diagram in figure 1. Here the "second" detector is a type that extracts the amplitude modulation of the input narrowband process and rejects the i-f carrier; its output is some instantaneous monotone function of this envelope. Examples are a linear envelope detector and a square-law envelope detector. The "binary integration system" in figure 1 is any system which employs tests of the type described. In a radar application, for example, it might perform a number of tests simultaneously, one for each range or doppler bin. The results on optimum thresholds shown later are based on the optimization of a single test, therefore are valid only for systems in which the tests are independent (i.e., the data used in each test are independent of the data used in all other tests). However, these optimum

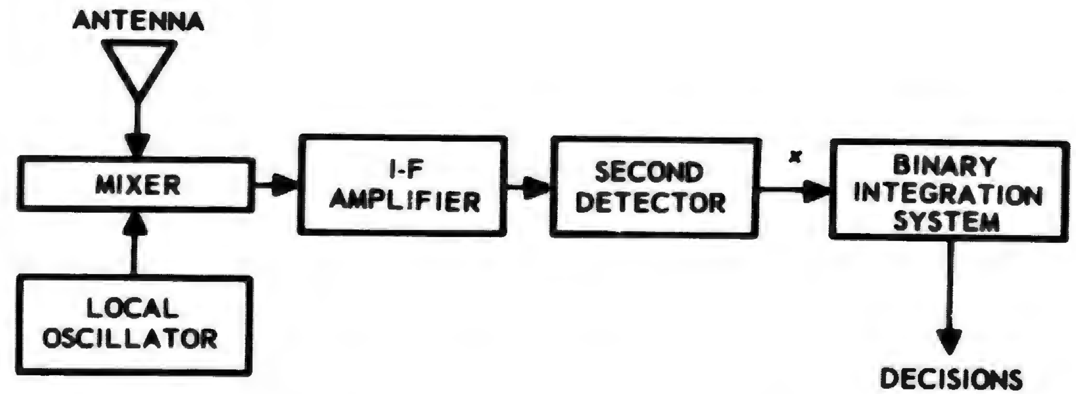

Figure 1. Typical receiver and binary integration system.

thresholds are indicative of those for systems in which the tests are dependent, such as the moving-window detector discussed in reference 8.

In practice, the assumptions (for an individual test) that the parameter θ (a measure of the signal strength) has the same value for all N pulses and that the N observations are statistically independent are not always met under signal-plus-noise conditions. However, the assumptions commonly are made whenever approximately valid, since an analysis of the test is difficult otherwise.

CALCULATION PROCEDURE

The probabilities under the two hypotheses that a given observation of x exceeds the threshold q are, respectively,

$$p_0 = \int_q^{\infty} f(x;\theta_0)\,dx \qquad (H_0) \tag{1}$$

$$p_1 = \int_q^{\infty} f(x;\theta_1)\,dx \qquad (H_1) \tag{2}$$

The probability α of a false alarm (of accepting H_1 when H_0 is true) is the cumulative binomial probability*

$$\alpha = \sum_{i=K}^{N} \binom{N}{i} p_0^i (1-p_0)^{N-i} \tag{3}$$

and the probability of a miss (of accepting H_0 when H_1 is true) is

*The false-alarm probability α is related to Marcum's *false-alarm number* n (ref. 13; p.77) by the equation $(1-\alpha)^{n/N} = ½$. For small α this relation becomes $\alpha n/N = \ln 2$.

$$\beta = \sum_{i=0}^{K-1} \binom{N}{i} p_1^i (1-p_1)^{N-i} \tag{4}$$

When values of α and N are specified, each possible value of the threshold K ($K = 1, \ldots, N$) determines by (3) a value of p_0, and thus by (1) a value of q. Then each K and its associated q determine by (2) a value of p_1, which in turn determines by (4) a value of β. We denote by K_{opt} the value of K for which β is minimized. For our purposes, a special computation of q is unnecessary; combining (1) and (2), p_1 can be written in terms of p_0 and the parameter values.

DISTRIBUTIONS FOR WHICH RESULTS ARE GIVEN

Normal Distribution

The probability density function of a random variable x that is normally or Gaussian distributed with mean μ and variance σ^2 is

$$f(x;\mu,\sigma) = (2\pi\sigma^2)^{-1/2} \exp[-(x-\mu)^2/2\sigma^2] \tag{5}$$

Note that $p \equiv \text{prob}\{x \geq q\} = \text{prob}\{(x-\mu)/\sigma \geq (q-\mu)/\sigma\} = \text{erfc}((q-\mu)/\sigma)$, where

$$\text{erfc } y = (2\pi)^{-1/2} \int_y^\infty \exp(-t^2/2)\, dt \tag{6}$$

In a test of the mean of a normal distribution, the density functions $f(x;\mu_0,\sigma)$ and $f(x,\mu_1,\sigma)$ describe the distribution of x under the hypotheses H_0 and H_1, respectively. From the expressions $p_0 = \text{erfc}[(q-\mu_0)/\sigma]$ and $p_1 = \text{erfc}[(q-\mu_1)/\sigma]$, we obtain the following equation relating p_1 to p_0 and the parameter values.

$$p_1 = \text{erfc}[\text{erfc}^{-1}(p_0) - (\mu_1-\mu_0)/\sigma] \tag{7}$$

For a test of the variance of a normal distribution, where $f(x;\mu,\sigma_0)$ and $f(x;\mu,\sigma_1)$ are the density functions under the two hypotheses,* it follows from the expressions $p_0 = 2\,\text{erfc}[(q-\mu)/\sigma_0]$ and $p_1 = 2\,\text{erfc}[(q-\mu)/\sigma_1]$ that

$$p_1 = 2\,\text{erfc}[(\sigma_0/\sigma_1)\,\text{erfc}^{-1}(p_0/2)] \tag{8}$$

Rayleigh and Exponential Distributions

In a test of the parameter ψ of a Rayleigh distribution, the density functions under H_0 and H_1 are $f_R(x_R;\psi_0)$ and $f_R(x_R;\psi_1)$, where

$$f_R(x_R;\psi) = (x_R/\psi)\exp(-x_R^2/2\psi), \quad x_R \geq 0 \tag{9}$$
$$= 0, \quad x_R < 0$$

From (1) and (2) we have $p_0 = \exp(-q_R^2/2\psi_0)$ and $p_1 = \exp(-q_R^2/2\psi_1)$, and thus

$$p_1 = p_0^{\psi_0/\psi_1} \tag{10}$$

In a test of the mean ψ of an exponential distribution, the density functions under H_0 and H_1 are $f_T(x_T;\psi_0)$ and $f_T(x_T;\psi_1)$, where

*For a test of the variance, observations on $|x|$ rather than on x are compared with q, and the distribution of $|x|$ is used in equations 1 and 2. This is equivalent to a two-sided quantization of x.

$$f_T(x_T;\psi) = (1/\psi)\exp(-x_T/\psi), \quad x_T \ge 0$$
$$= 0, \quad x_T < 0 \tag{11}$$

From (1) and (2), we have $p_0 = \exp(-q_T/\psi_0)$ and $p_1 = \exp(-q_T/\psi_1)$, and thus equation 10 is valid also for the exponential distribution.

The exponential density (11) is obtained from the Rayleigh density (9) by the transformation $x_T = \frac{1}{2}x_R^2$. More will be said on the subject of transformations later.

Consider now a receiver of the type illustrated in figure 1, where the i-f amplifier has a passband which is narrow compared to the midfrequency, and assume that the noise entering the i-f amplifier is Gaussian (i.e., normally) distributed with mean zero. Under no-signal conditions and under certain signal-plus-noise conditions, the Rayleigh and exponential distributions describe the output of the second detector when it is, respectively, a linear envelope detector and a square-law envelope detector.*

For a conventional pulsed radar, these distributions describe the output envelope of the two detectors when the radar cross section fluctuates according to the density function model

$$w(\sigma;\bar{\sigma}) = (1/\bar{\sigma})\exp(-\sigma/\bar{\sigma}), \quad \sigma \ge 0$$
$$= 0, \quad \sigma < 0 \tag{12}$$

where $\bar{\sigma}$ is the average cross section (see refs. 5, p.50-56; 6, p.22, 81; 7, p.131; 14; 15; 16, p.67, 75-77; 17-19). Some authors refer to this as the "Rayleigh" fluctuational model, since the amplitude of the sine-wave signal is proportional to $\sqrt{\sigma}$ and therefore is Rayleigh-distributed. In this signal-fluctuation case, the parameter ψ (note that $\psi = \frac{1}{2}E(x_R^2) = E(x_T)$ for densities (9) and (11), respectively) is the mean square value of the voltage in the i-f. In the signal-plus-noise case, ψ must be considered an average also over all fluctuations. Denote by $\bar{S}$ the average, over all target fluctuations, of the (single-pulse) mean-power ratio

$$S = \frac{\text{Mean signal power in i-f}}{\text{Mean noise power in i-f}}$$

Relating the new parameter $\bar{S}$ to the parameter ψ, we have* $\bar{S} = (\psi/\psi_0) - 1$, with $\bar{S} = \bar{S}_0 = 0$ under H_0 and $\bar{S} = \bar{S}_1 = (\psi_1/\psi_0) - 1$ under H_1. Equation 10 can then be written

$$p_1 = p_0^{1/(1+\bar{S}_1)} \tag{13}$$

The mean-power signal-to-noise ratio per pulse (after averaging over all fluctuations) under H_1 is given in decibels by

$$\text{SNR} = 10\log_{10}\bar{S}_1 = 10\log_{10}\left(\frac{\psi_1}{\psi_0} - 1\right) \tag{14}$$

If the output of the linear detector is normalized in units of the rms noise voltage, the Rayleigh density (9) becomes

$$f_r(x_r;\bar{S}) = [x_r/(1+\bar{S})]\exp[-x_r^2/2(1+\bar{S})], \quad x_r \ge 0 \tag{15}$$

where $x_r = x_R/\sqrt{\psi_0}$ is the normalized variable.

In the square-law case, the normalized variable $x_t = x_T/\psi_0$ is conventionally used.** The exponential density (11) then has the form

$$f_t(x_t;\bar{S}) = [1/(1+\bar{S})]\exp[-x_t/(1+\bar{S})], \quad x_t \ge 0 \tag{16}$$

The case where the cross section follows the density (12) and fluctuates rapidly (resulting in pulse-to-pulse independence) is sometimes called Swerling Case 2.

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*The detector law $x_T = \frac{1}{2}x_R^2$, where x_T and x_R represent the outputs of a square-law detector and a linear detector, respectively, is used in the literature more often than $x_T = x_R^2$, and therefore is used here.

*Mean-square voltage is proportional to mean power, and the mean-square value ψ of the sum of the signal and noise voltages (in i-f) is the sum of their mean-square values.

**Since $[\overline{x_T^2} - (\overline{x_T})^2]^{1/2} = \psi$ when x_T has the exponential density (11), with $\psi = \psi_0$ in the noise case, we can consider the normalized variable x_T to be in units of the rms value of the ac component of noise <u>after</u> the detector (ref. 17, p.173-174).

Swerling Case 4

For certain kinds of fluctuating targets (see refs. 5, p.50-56; 6, p.24, 82; 14; 15; 16, p.67, 77-78 – "one dominant plus Rayleigh"; 17-19), the density-function model used to describe the radar cross section σ is the chi-square distribution with 4 degrees of freedom

$$w(\sigma;\bar{\sigma}) = (4\sigma/\bar{\sigma}^2)\exp(-2\sigma/\bar{\sigma}), \quad \sigma \geq 0$$
$$= 0, \quad \sigma < 0 \tag{17}$$

where $\bar{\sigma}$ is the average cross section. In the conventional pulsed radar situation, the pulse-to-pulse independence case of (17) is often known as Swerling Case 4.

When the cross section fluctuates according to the above model, the normalized output of a square-law envelope detector* has the (single-pulse) density function (ref. 15, p. 286).

$$f_t(x_t;\bar{S}) = \frac{1}{(1+\bar{S}/2)^2}\left(1+\frac{x_t}{1+2/\bar{S}}\right)\exp\left(\frac{-x_t}{1+\bar{S}/2}\right), \quad x_t \geq 0$$
$$= 0, \quad x_t < 0 \tag{18}$$

where $\bar{S}$ is the mean power signal-to-noise ratio averaged over all fluctuations. (As before, the subscripts r and t will refer to the normalized forms for the linear envelope detector and the square-law envelope detector, respectively.) Under the hypothesis H_0 that $\bar{S} = \bar{S}_0 = 0$, i.e., that noise alone is present, $p = p_0 = \exp(-q_t)$. Under the hypothesis H_1 that $\bar{S} = \bar{S}_1$ (for graphical purposes, SNR = 10 $\log_{10} \bar{S}_1$), i.e., that a signal of specified average strength is present,

$$p_1 = \left(1+\frac{\bar{S}_1 \ln(1/p_0)}{2(1+\bar{S}_1/2)^2}\right)\exp\left(-\frac{\ln(1/p_0)}{1+\bar{S}_1/2}\right) \tag{19}$$

*We again assume narrowband Gaussian noise, and we again can consider the normalized variable to be in units of the rms value of the ac component of noise after the detector.

If we again consider the square-law output to be one-half the square of the linear detector output, from (18) we can write for the case of a linear envelope detector

$$f_r(x_r;\bar{S}) = \frac{x_r}{(1+\bar{S}/2)^2}\left(1+\frac{x_r^2}{2+4/\bar{S}}\right)\exp\left(\frac{-x_r^2}{2+\bar{S}}\right), \quad x_r \geq 0$$
$$= 0, \quad x_r < 0 \tag{20}$$

The equation for p_1 given by (19) is valid also for the linear detector.

Rice Distribution

The density function

$$f_r(x_r;a) = x_r \exp[-(x_r^2+a^2)/2]\, I_0(ax_r), \quad x_r \geq 0$$
$$= 0, \quad x_r < 0 \tag{21}$$

where $I_0(\cdot)$ is the modified Bessel function of the first kind of order zero, appears in several fields of scientific study under different names and in various forms. In signal detection problems, where it is sometimes known as the Rice, modified Rayleigh, noncentral Rayleigh, or just Rayleigh distribution, it is the single-pulse distribution of the normalized output (in units of rms noise voltage in i-f) of a linear envelope detector whose input is narrowband Gaussian noise and an additive steady sine-wave signal at the center frequency of the noise band.[20] The parameter a is the ratio of peak i-f signal voltage to rms i-f noise voltage and is zero under no-signal conditions. Under the hypothesis H_0 that $a = a_0 = 0$, equation 21 reduces to a form of the Rayleigh density function.

$f_r(x_r;0) = x_r \exp(-x_r^2/2)$. The value of p under H_0 is $p_0 = \exp(-q_r^2/2)$ and under H_1 is the so-called Q function (see refs. 20; 21, p.159; 22) evaluated at $a = a_1$ and $q_r = \sqrt{-2 \ln p_0}$

$$p_1 = \int_{q_r}^{\infty} x_r \exp[-(x_r^2 + a_1^2)/2]\, I_0(a_1 x_r)\, dx_r \qquad (22)$$

The mean power signal-to-noise ratio per pulse (in dB) is SNR = $10 \log_{10}(a_1^2/2)$.

The graphical results for the Rice case are also valid for the density function

$$f_t(x_t; d) = \exp(-x_t - d)\, I_0(2\sqrt{x_t d}\,), \quad x_t \geq 0$$
$$= 0, \quad x_t < 0 \qquad (23)$$

which is a form of the noncentral chi-square with two degrees of freedom. With d the ratio of mean signal power in i-f to mean noise power in i-f (SNR = $10 \log_{10} d_1$), equation 23 is the distribution of the normalized output of a square-law detector in the nonfluctuating signal case.

TRANSFORMATIONS

Let x_a be a one-dimensional continuous random variable with density function $f_a(x_a)$, and let $x_b = g(x_a)$ be a monotone increasing function such that the derivative $dx_b/dx_a = g'(x_a)$ exists and is different from 0 except, possibly, at a finite number of points. Let, furthermore,

$$\lim_{x_a \to -\infty} g(x_a) = c \quad \text{and} \quad \lim_{x_a \to +\infty} g(x_a) = d$$

Then x_b is a continuous random variable with the density function (from p.63-64 of ref. 23).

$$f_b(x_b) = f_a(x_a) \left| \frac{dx_a}{dx_b} \right| = \frac{f_a[g^{-1}(x_b)]}{g'[g^{-1}(x_b)]}, \quad c < x_b < d$$
$$= 0, \quad x_b < c \text{ and } x_b > d \qquad (24)$$

The graphical results given for each distribution are valid also for any distribution obtained from it by a transformation $x_b = g(x_a)$ satisfying the conditions above. This can be seen by noting that when p_0 is assigned a value, the quantization thresholds q_a and $q_b = g(q_a)$ obtained from

$$p_0 = \text{prob}\{x_a \geq q_a \mid \theta = \theta_0\} = \text{prob}\{g(x_a) \geq g(q_a) \mid \theta = \theta_0\}$$
$$= \text{prob}\{x_b \geq q_b \mid \theta = \theta_0\}$$

will result in the same value of p_1 for both the random variable x_a and the transformed variable x_b, since

$$p_1 = \text{prob}\{x_a \geq q_a \mid \theta = \theta_1\} = \text{prob}\{g(x_a) \geq g(q_a) \mid \theta = \theta_1\}$$
$$= \text{prob}\{x_b \geq q_b \mid \theta = \theta_1\}$$

For each of the three signal situations considered (nonfluctuating signals and two types of fluctuating signals), receiver-output distributions were given only for linear envelope and square-law envelope detectors, but it follows from the above that the results based on the distributions for these two detectors are valid also, for example, for a receiver whose output is proportional to the logarithm of the input envelope (i.e., the results for all three detectors are identical).

NUMERICAL RESULTS AND THEIR INTERPRETATION

For the distributions under consideration, figures 2A and 2B give for the $N = 100$ case the optimum thresholds as function of β for $\alpha = 10^{-8}$ and $\alpha = 10^{-4}$. For other large values of N, the curves (not shown) of optimum thresholds (normalized to N) versus β have roughly the same small slope; the curves for small N (see figs. 2C and 2D) tend to have a slightly greater slope. (The curves for the smaller values of N were smoothed by judging for each point how good relative to K_{opt} the next best K is.) These curves were obtained in the manner described immediately after equation 4, using the equations for p_1 given in terms of p_0 and the parameter values under H_0 and H_1. As can be seen after an inspection of these equations for p_1, all possible combinations of the two parameter values of a given distribution are taken into consideration when we vary β (the minimum β over K, $K = 1, \ldots, N$) by varying $(\mu_1 - \mu_0)/\sigma$ for a test of the mean of a normal distribution, σ_1/σ_0 for a test of the variance of a normal distribution, ψ_1/ψ_0 or $\bar{S}_1$ for the Rayleigh and exponential distributions, $\bar{S}_1$ for the Swerling Case 4, or a_1 for the Rice distribution.

In a signal-detection application involving one of these distributions, the SNR, however defined, should be a one-to-one, monotone increasing, continuous function of the aforementioned appropriate expression. Some authors have defined the optimum threshold, for given values of α, β, and N, to be that K which minimizes the required SNR. The use of that definition of the optimum threshold yields exactly the same results as those found here, provided the definition of the SNR satisfies the condition just above.

The important fact we learn from figures 2A-D (understanding the abscissa to be the value of β for $K = K_{opt}$ as the SNR varies) is that the effect of the SNR on K_{opt} is small, especially within the range of SNR values corresponding to $0.05 < \beta < 0.95$. Similar observations were made for the Rice case by Swerling[1] and by Schwartz.[3,7]

Figures 3A-D present optimum thresholds for four combinations of α and β; these curves were obtained graphically from the K_{opt} versus β curves described earlier. (Recall that the curves for smaller N were smoothed.) Note that the optimum thresholds decrease as α is increased from 10^{-8} to 10^{-4}, though this decrease is negligible for some of the distributions.

To indicate how much poorer than K_{opt} each of the other values of K is (and to do this for all distributions on the same graph), the parameter values which result in 0.1 as the minimum value of β (minimum over K for $N = 100$ and $\alpha = 10^{-8}$) were found for each distribution, and the corresponding curves of β versus K were plotted as figure 4. Similar curves for $N = 10$ (not shown) have roughly the same shape (assuming a normalized abscissa, K/N), but are shifted somewhat to the right and tend to give near minimum values over a wider range of K/N.

We recall that in figure 2A the optimum threshold changes only a small amount as β (for that optimum threshold) varies from zero to one, or, equivalently, as the SNR varies; then, in figure 4, we note that (for the fixed, appropriate SNR) β remains close to its minimum value over a relatively wide range of threshold values. Thus it appears (and this is supported by other results) that a fixed-threshold device, which is certainly the easiest to implement, will perform efficiently for all signal strengths. For example, the same threshold can be used for all of a number of range-bin tests, and the miss probabilities will not differ appreciably from those in the ideal case where the threshold for each bin is chosen to optimize detection for the SNR of the design target at that range.

Figures 2 and 4 suggest also that it is important to make a reasonably correct assumption as to the functional form of the distribution, even though an accurate prediction of the signal strength is not required.

Inasmuch as the quantization threshold q is completely determined by the decision threshold K when the distribution under H_0 is specified and α and N are fixed, the statements concerning the sensitivity of K_{opt} to the SNR can be read optionally with optimum q in mind. The graphical results are presented in terms of K rather than q because those for q would be limited, in direct usage, to one particular form of each distribution.

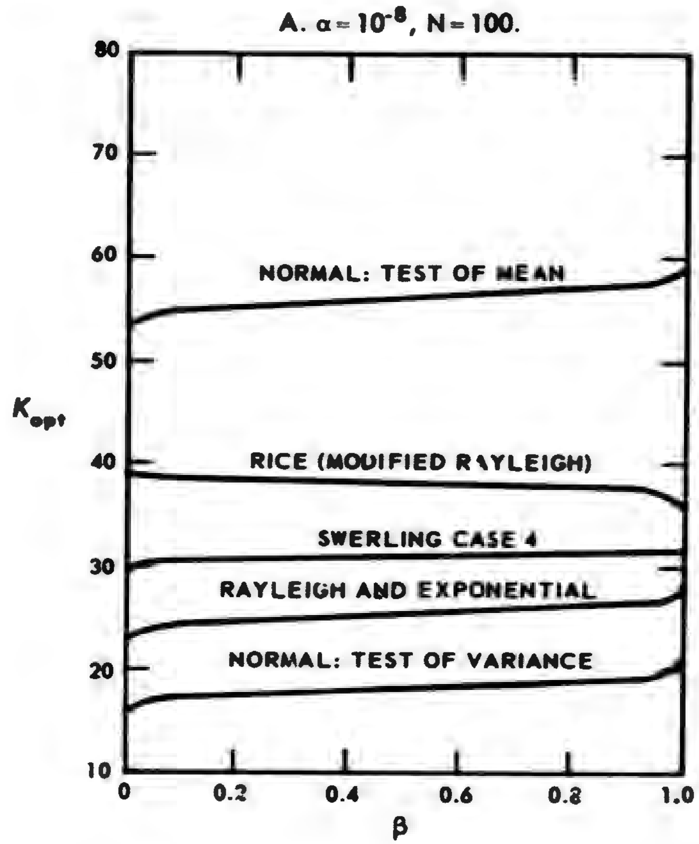


Figure 2. Optimum threshold versus the miss probability for that threshold. All possible pairs of parameter values (thus all values of the SNR) are represented as β varies from 0 to 1. (α = false-alarm probability; N = number of observations.)

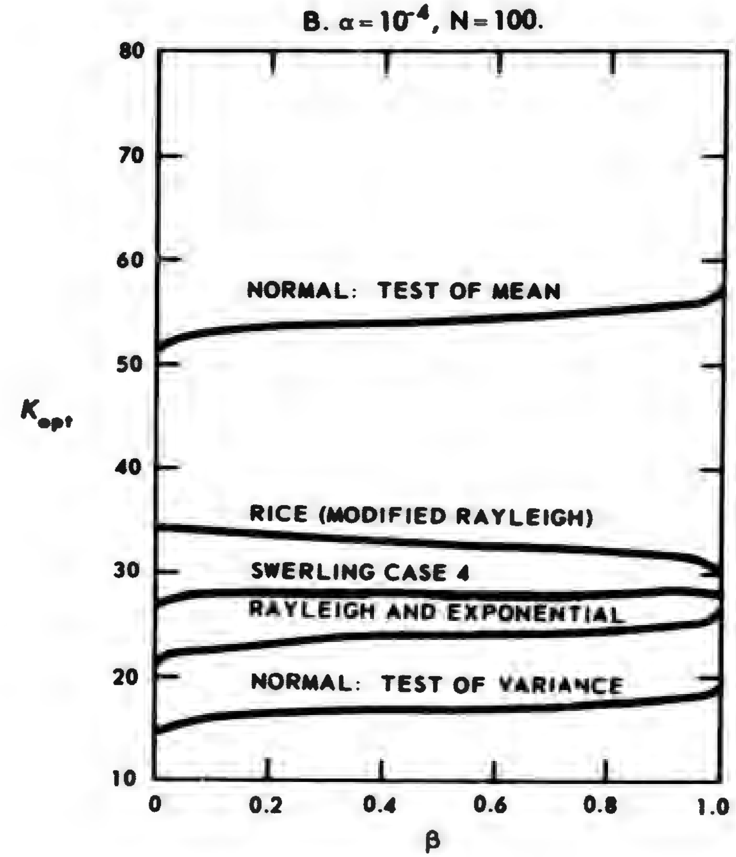
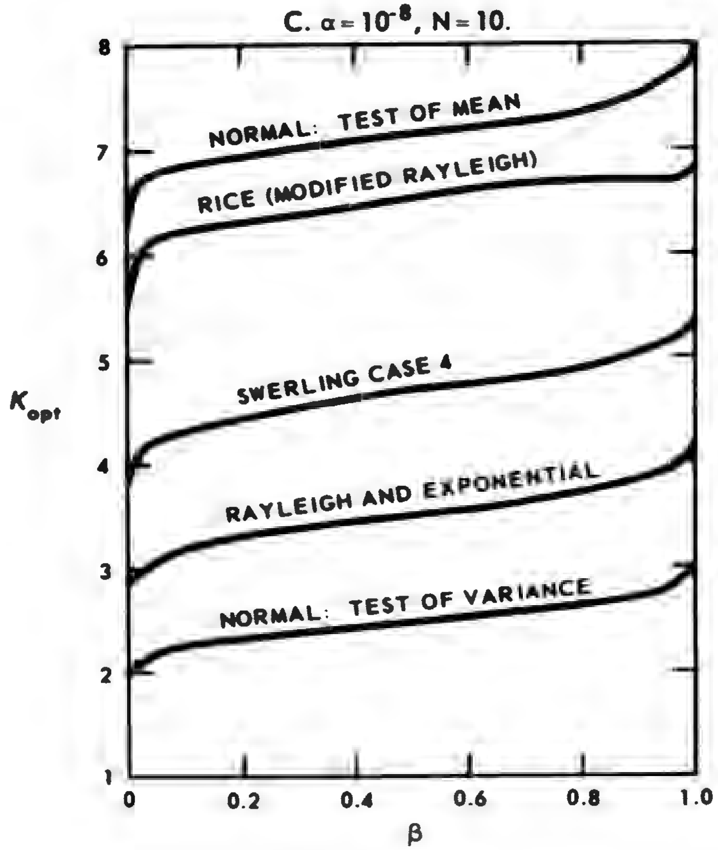


Figure 2 (Continued).



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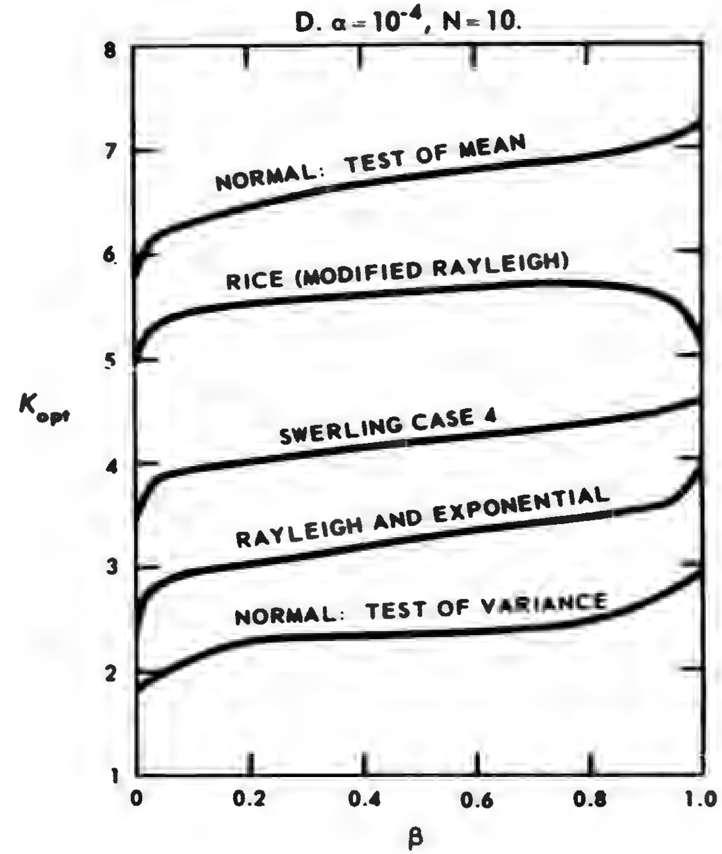

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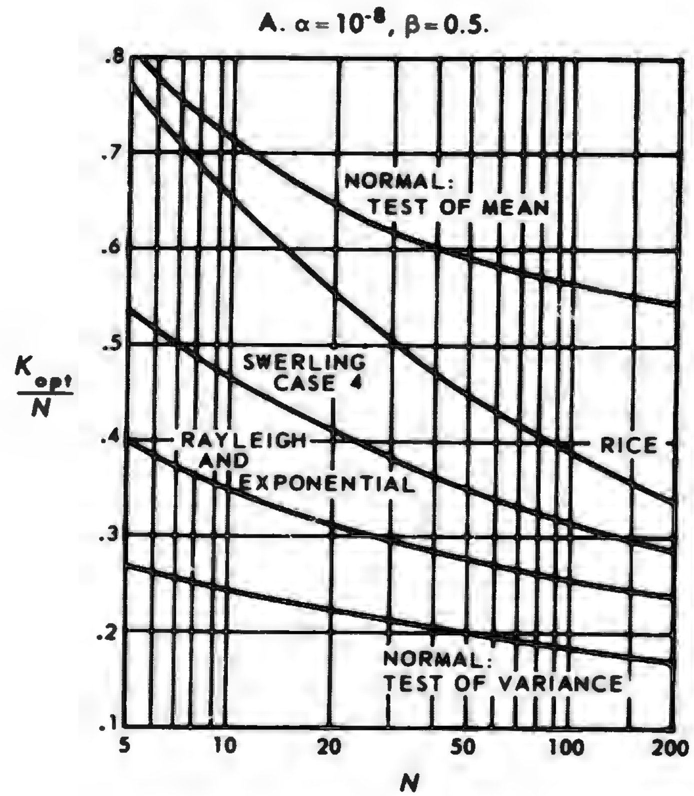


Figure 3. Ratio of optimum threshold to number N of observations versus N, where β has the stated value under optimum thresholding.

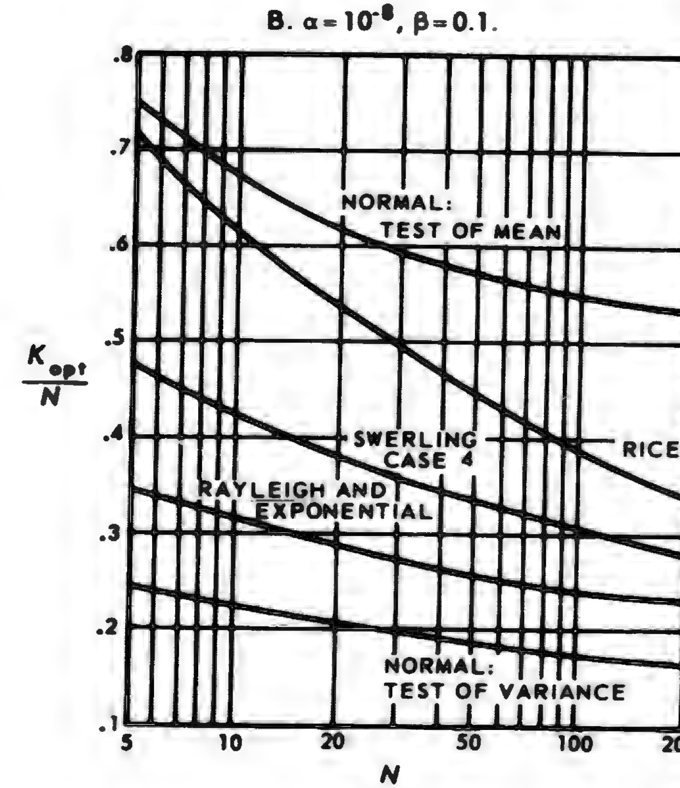


Figure 3 (Continued).

C. $\alpha = 10^{-4}$, $\beta = 0.5$.

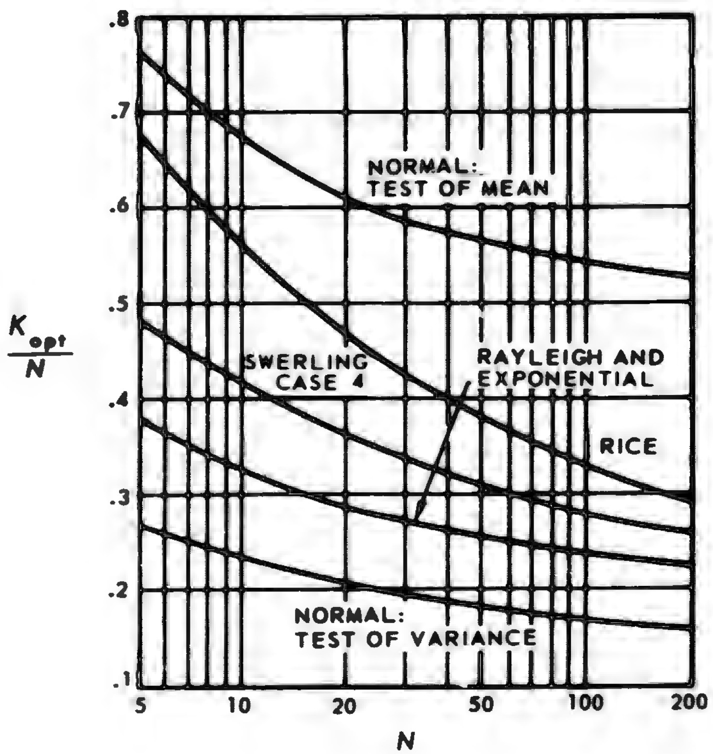


Figure 3 (Continued).

D. $\alpha = 10^{-4}$, $\beta = 0.1$.

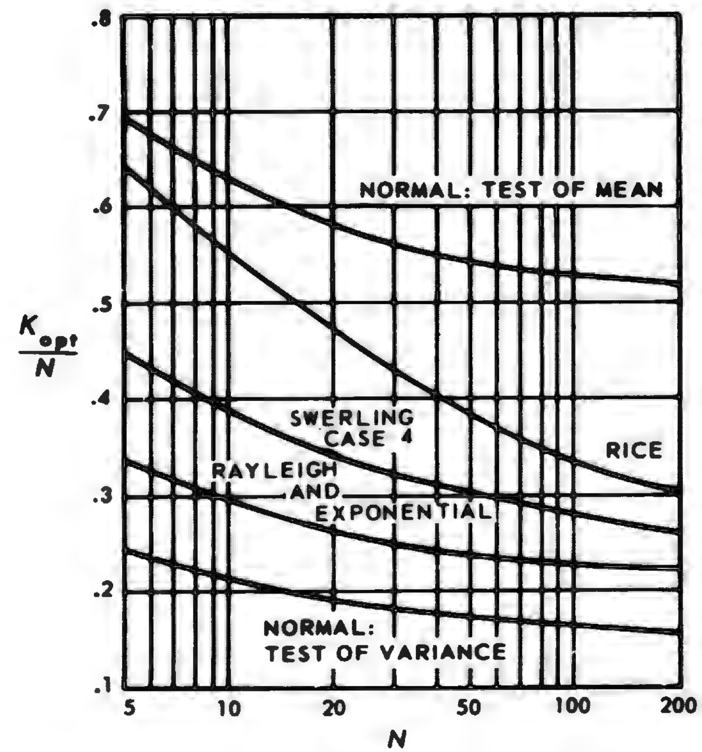


Figure 3 (Continued).

CURVE	DISTRIBUTION	PARAMETER
1	NORMAL: TEST OF VARIANCE	$\sigma_1/\sigma_0 = 1.75$
2	RAYLEIGH AND EXPONENTIAL	$\psi_1/\psi_0 = 2.25$ (SNR = 0.98 dB)
3	SWERLING CASE 4	$\bar{S}_1 = 1.19$ (SNR = 0.74 dB)
4	RICE (MODIFIED RAYLEIGH)	$a_1 = 1.49$ (SNR = 0.44 dB)
5	NORMAL: TEST OF MEAN	$(\mu_1 - \mu_0)/\sigma = 0.86$

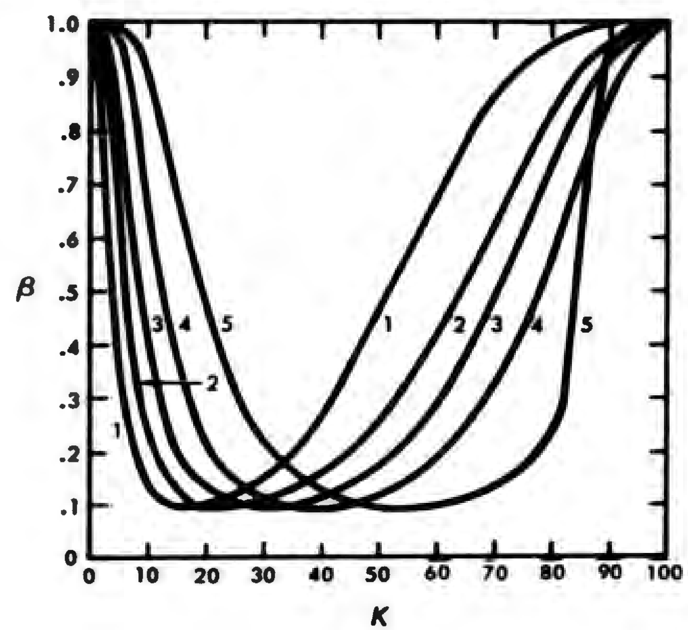


Figure 4. Miss probability versus threshold. For each of the distributions, the parameter values used are those for which 0.1 is the minimum (over K) value of β when $N = 100$ and $\alpha = 10^{-8}$.

For any value of N between 5 and 200, the value of p_0 necessary to give a false-alarm probability $\alpha = 10^{-8}$ can be determined from figure 5A for each value of the threshold K ($0.1N \leq K \leq 0.8N$); interpolation is required for values of N other than 5, 10, 20, 50, 100, and 200. Figure 5B shows a similar graph for $\alpha = 10^{-4}$. The primary purpose of figures 5A and 5B is to give the corresponding optimum value of p_0 for each value of K_{opt} found from one of the figures 3A through 3D. From the p_0 value, the associated value of q is easily calculated with the use of a normal or (depending upon the distribution) exponential table.

For the Rice distribution and for the Rayleigh and exponential distributions, the per-pulse SNR required to give $\beta = 0.1$ and $\beta = 0.5$ (assuming optimum thresholds are used) is plotted as functions of N for $\alpha = 10^{-8}$ and 10^{-4} in figures 6A and 6B. The curves for Swerling Case 4 (not shown) fall between those for the corresponding Rayleigh and Rice cases.

Figures 7A-7L give β as a function of the per-pulse SNR for the Rice distribution, the Rayleigh and exponential distributions, and the Swerling Case 4.

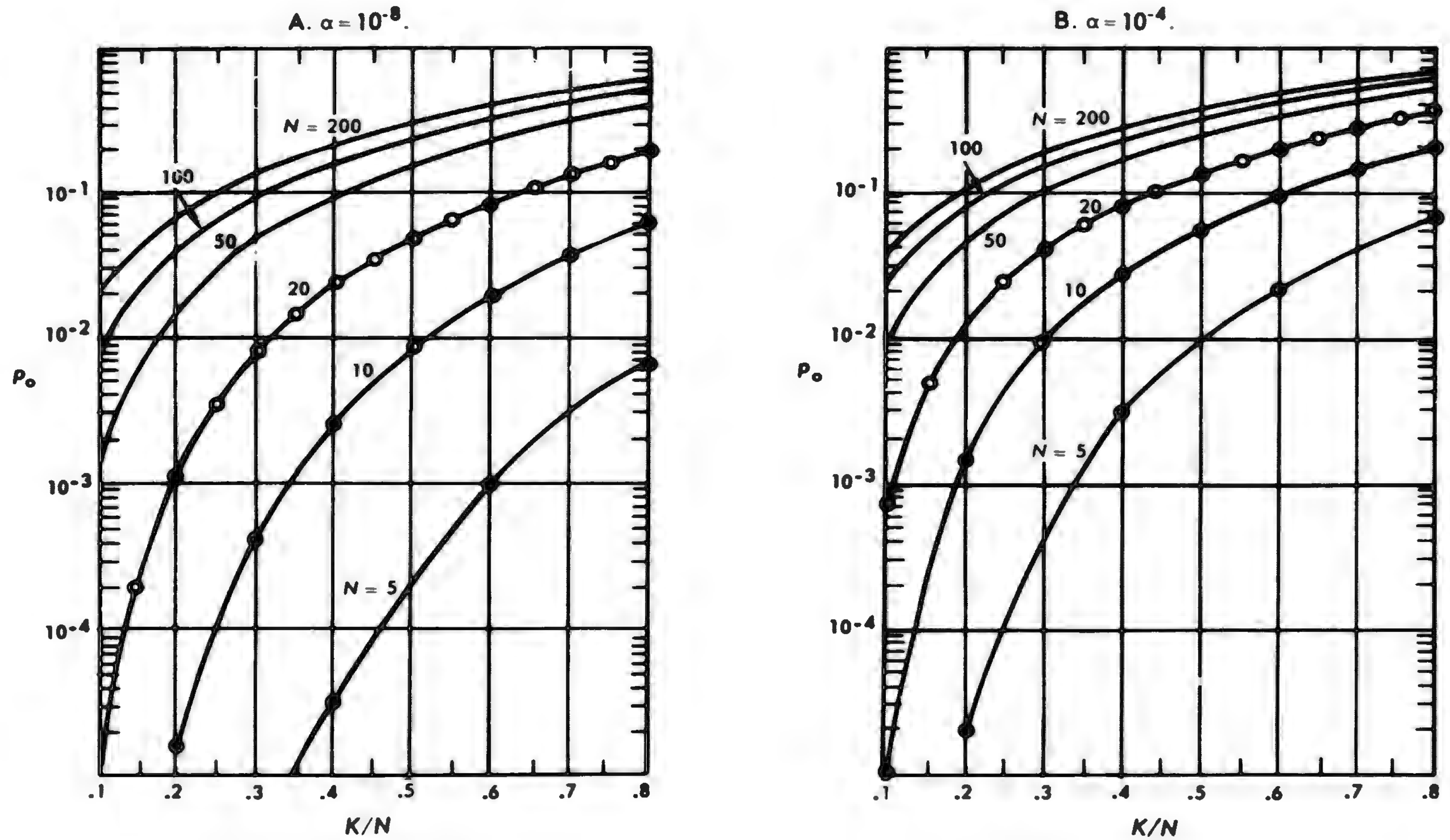


Figure 5. Probability under no-signal conditions that observation exceeds quantization threshold versus ratio of decision threshold to number of observations.

Figure 5 (Continued).

A. $\alpha = 10^{-8}$.

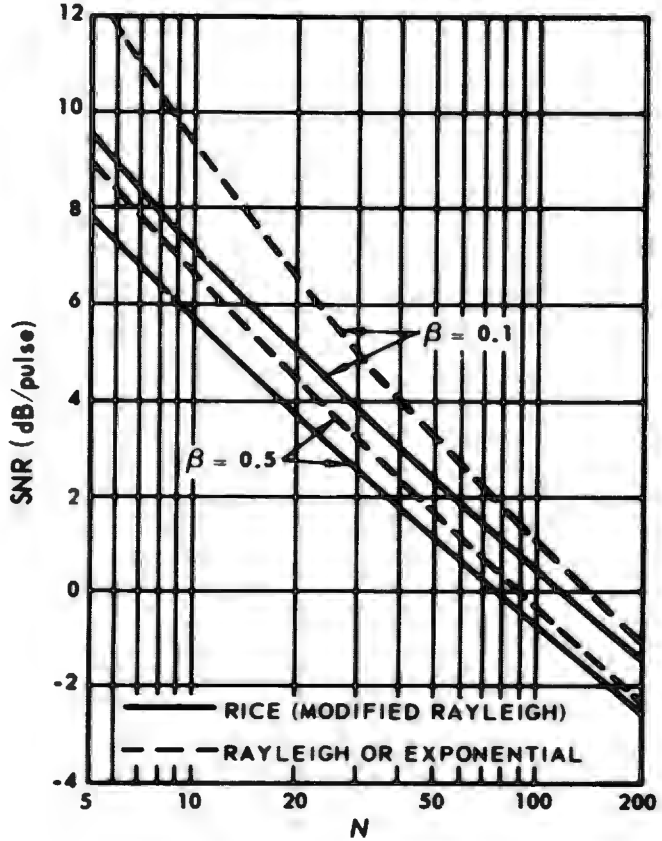


Figure 6. Required mean power signal-to-noise ratio per pulse versus number of pulses, assuming threshold K optimum for each point.

B. $\alpha = 10^{-4}$.

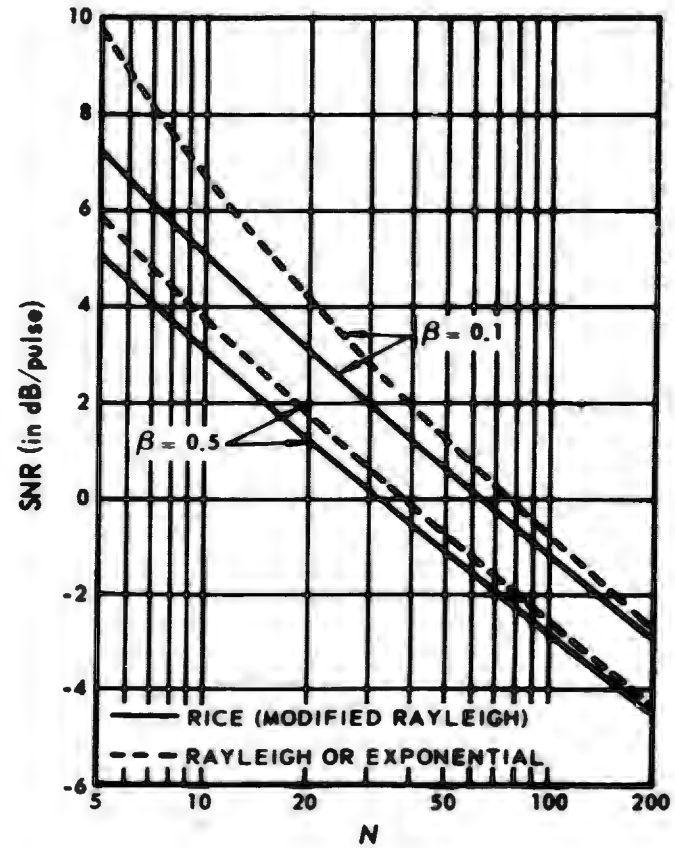


Figure 6 (Continued).

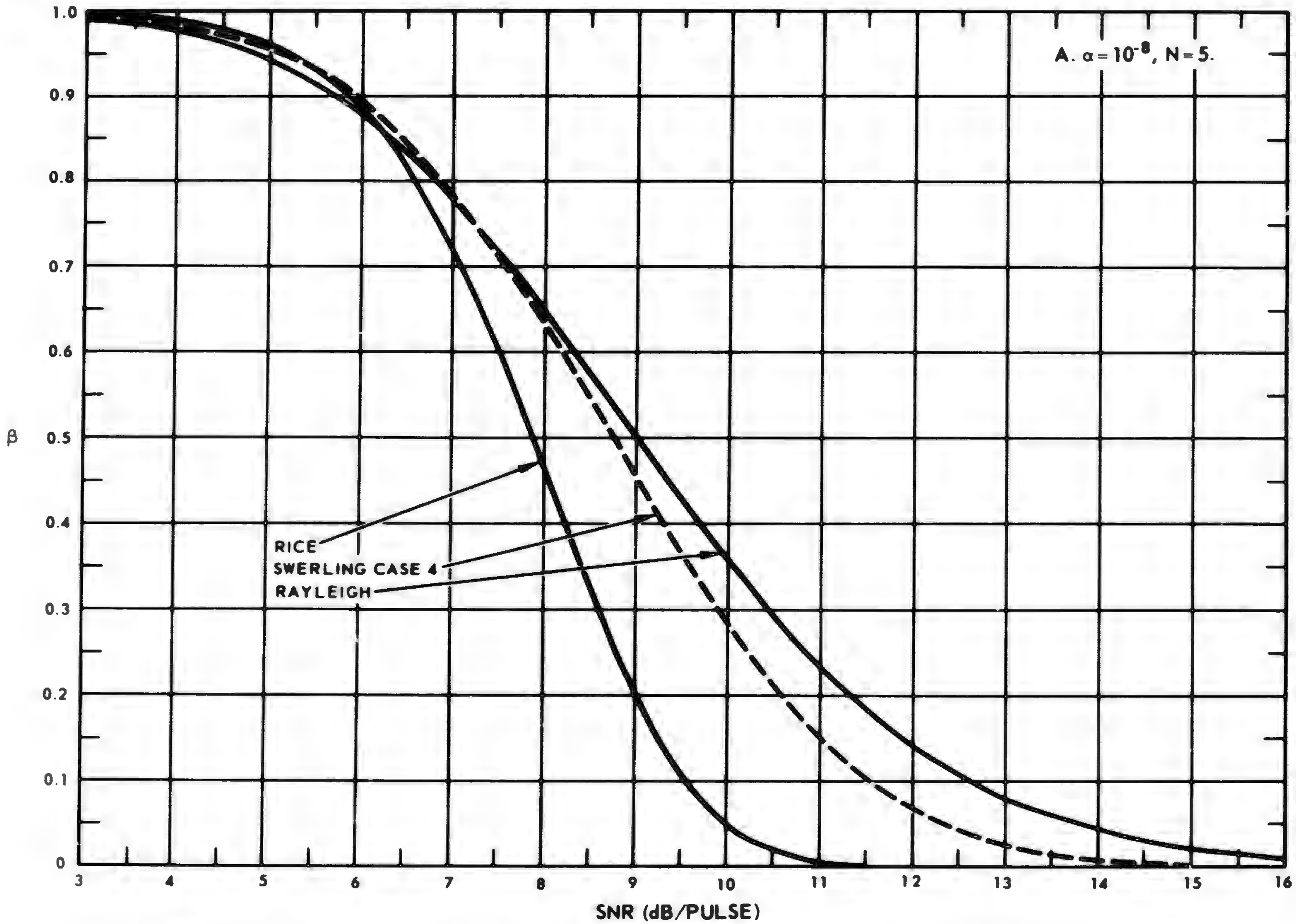


Figure 7. Miss probability versus mean power signal-to-noise ratio per pulse, assuming threshold K optimum for each point.

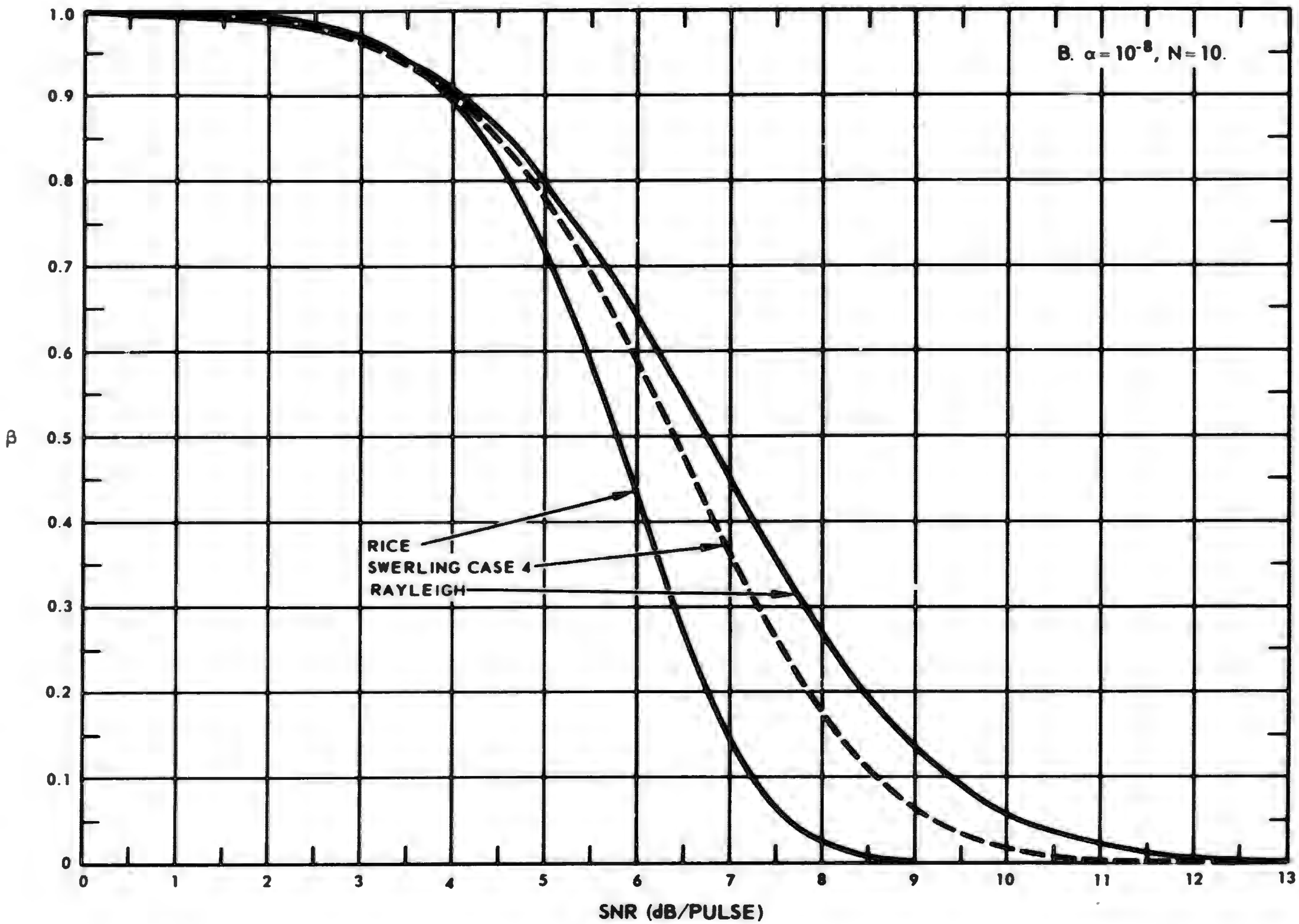


Figure 7 (Continued).

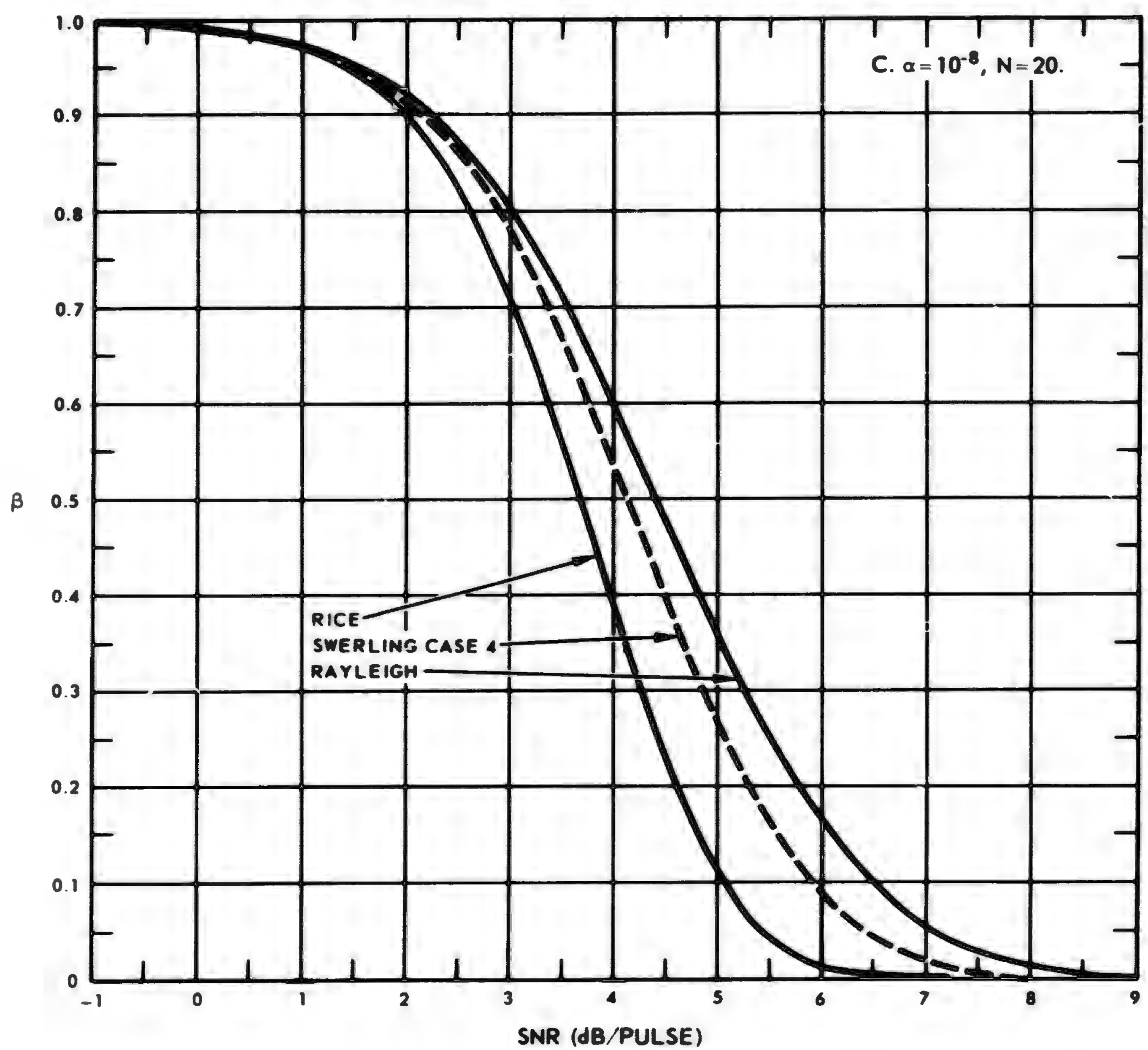


Figure 7 (Continued).

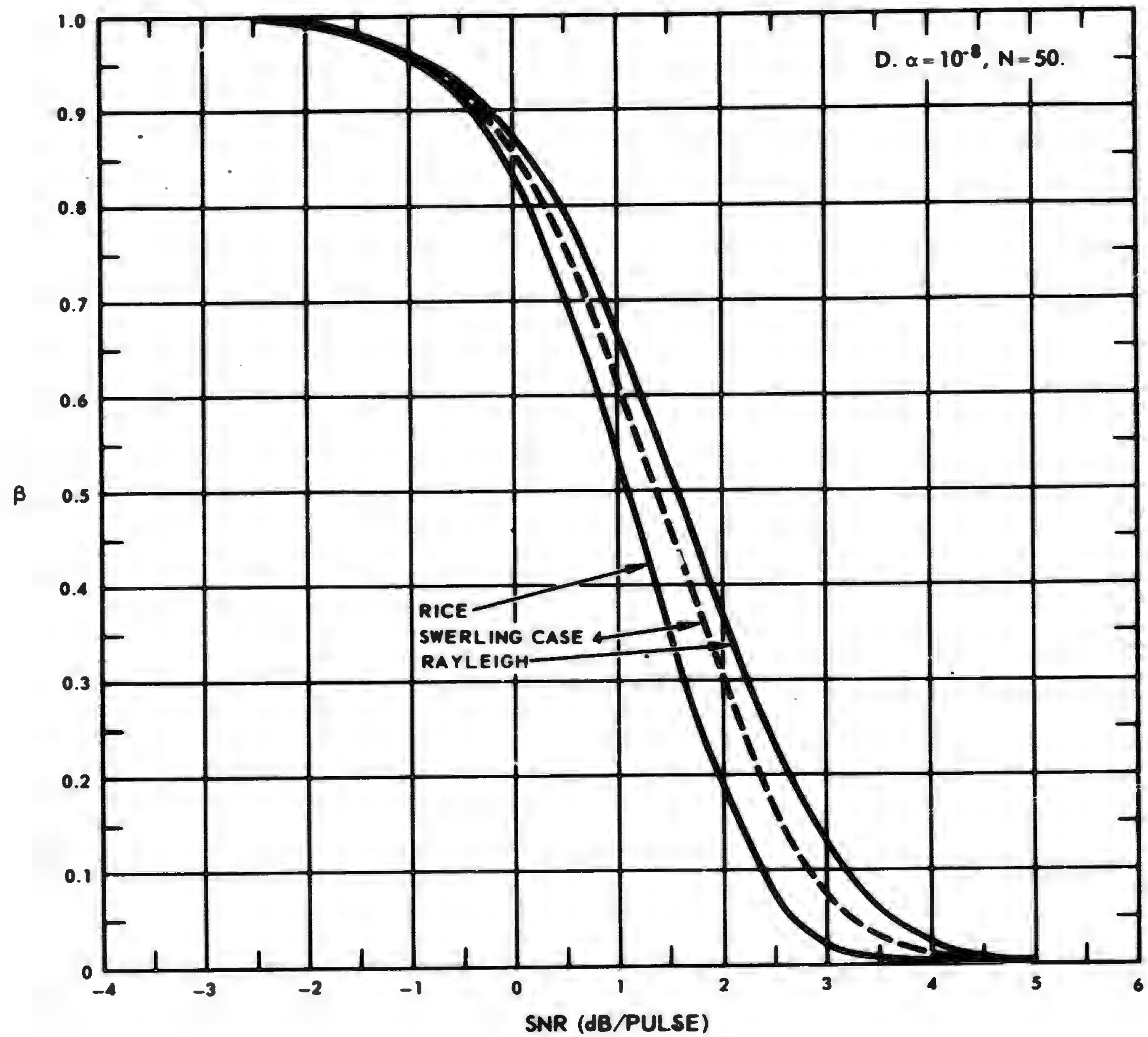
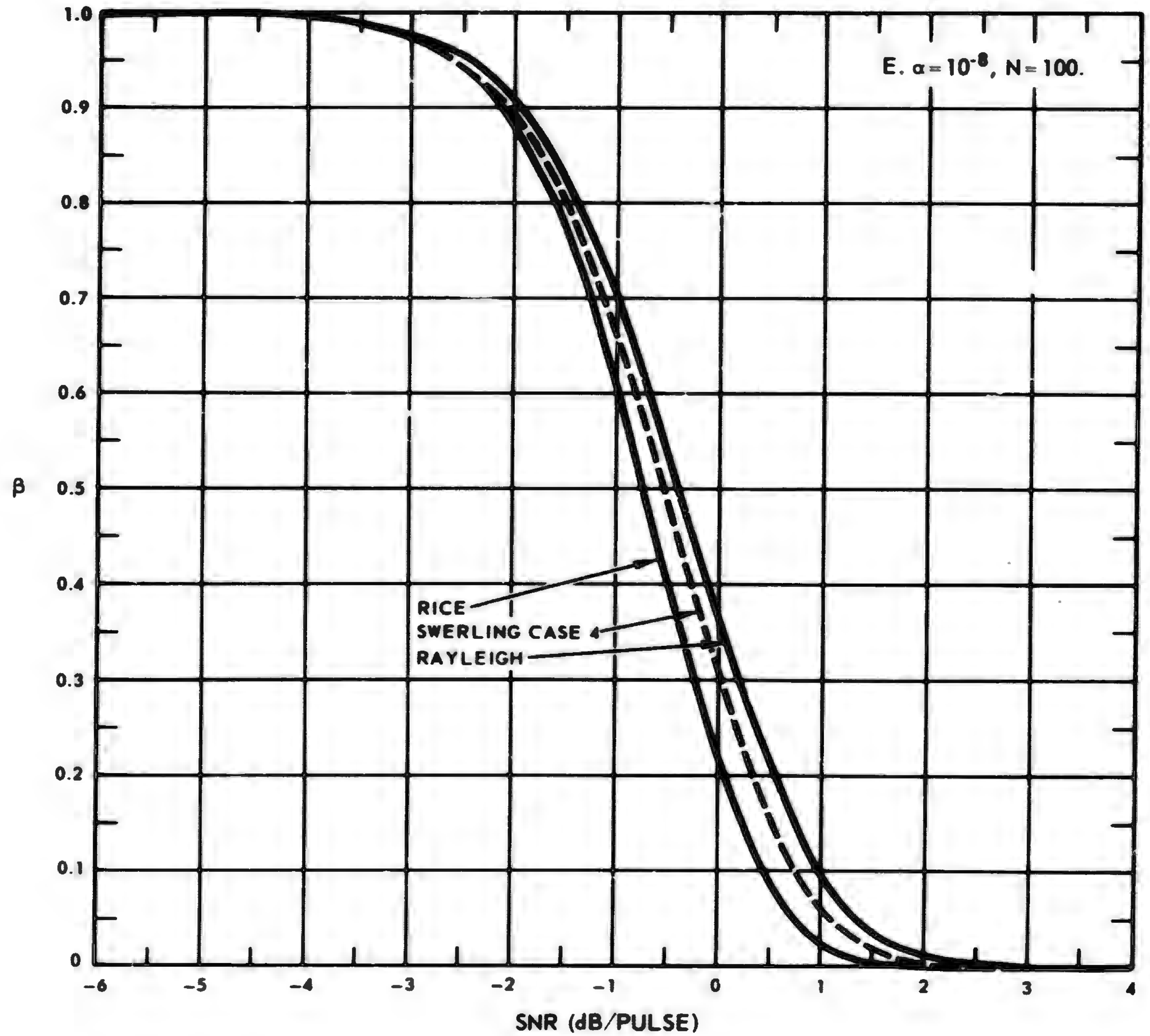


Figure 7 (Continued).



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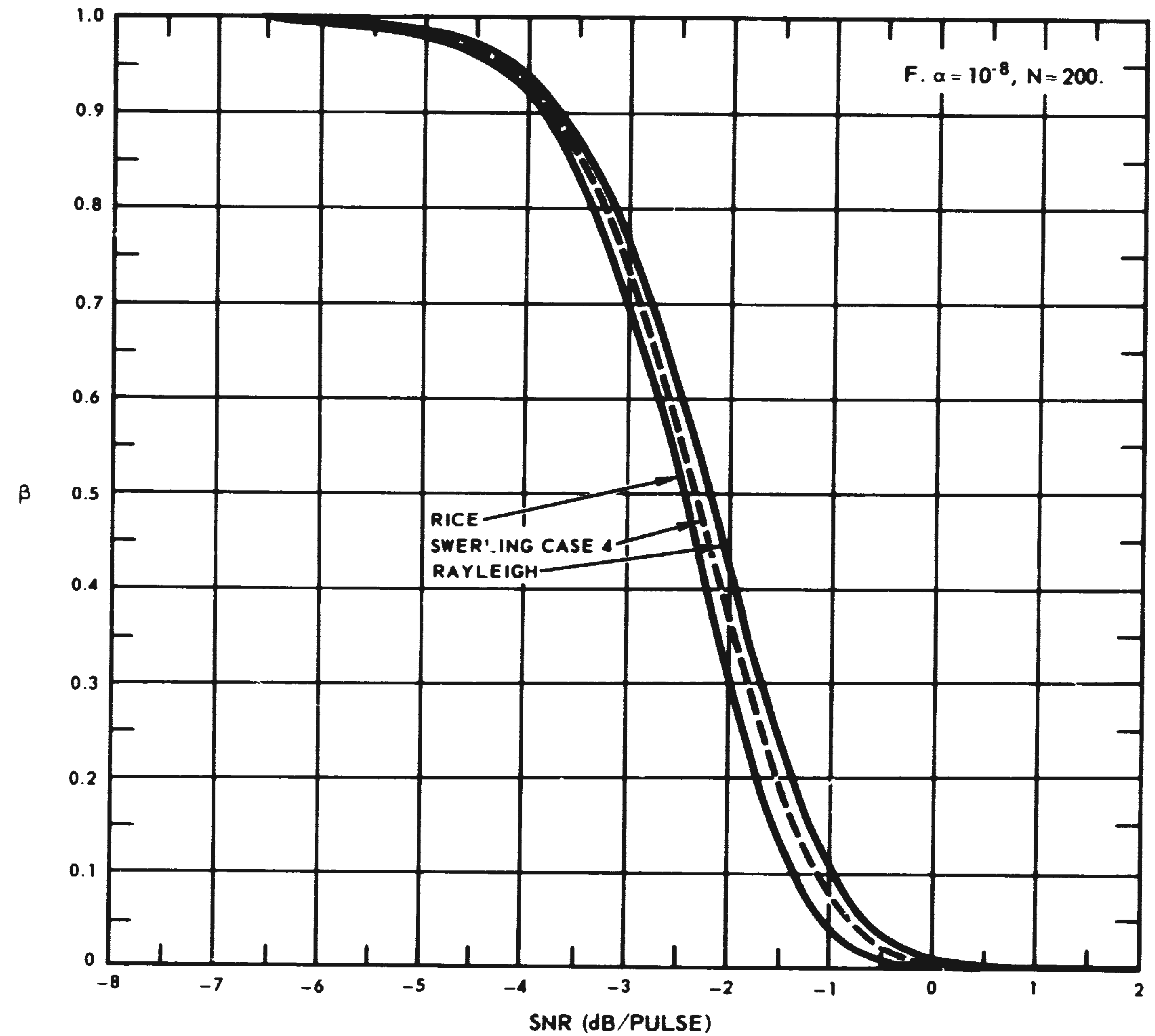

Figure 7 (Continued).

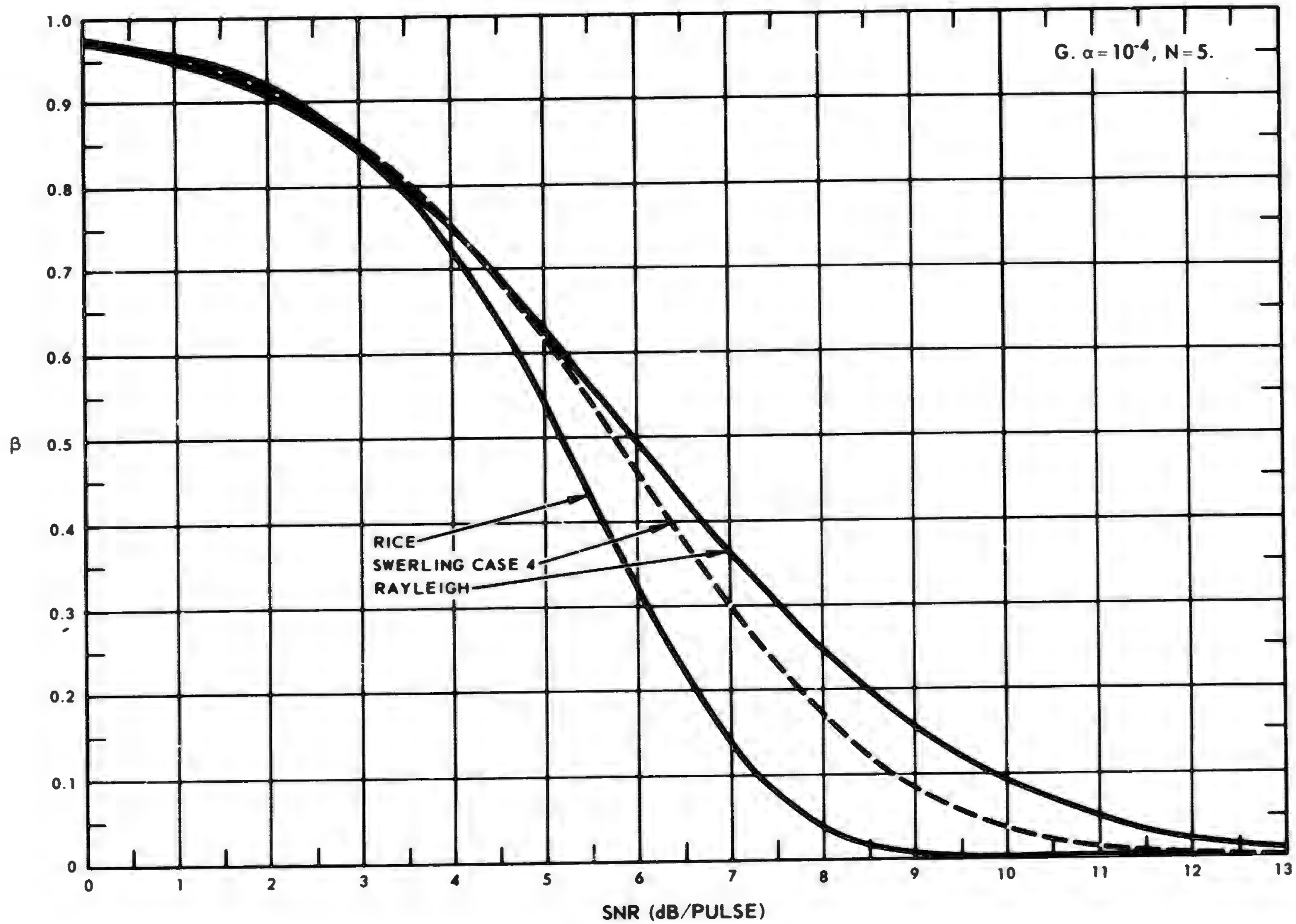

Figure 7 (Continued).

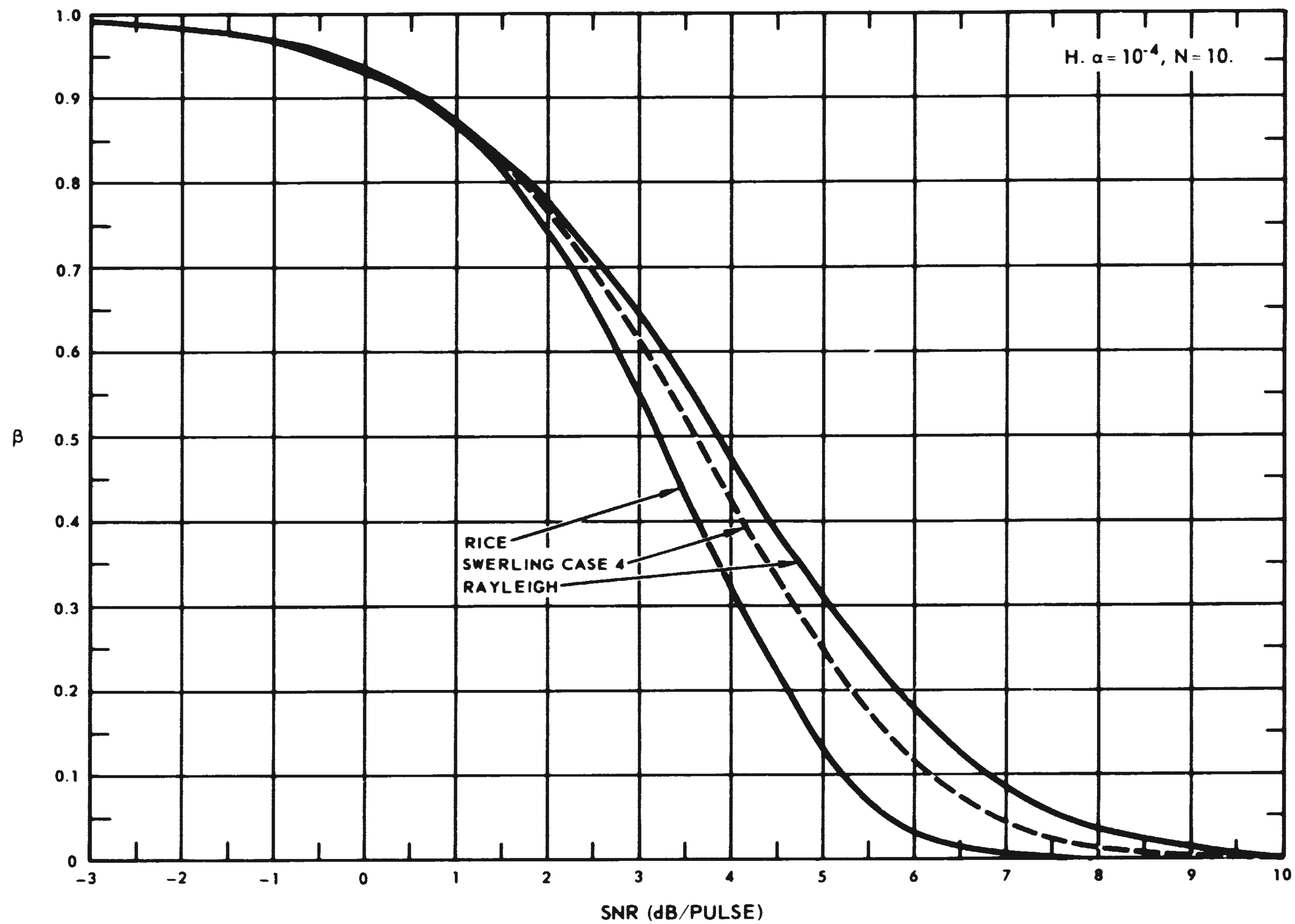

Figure 7 (Continued).

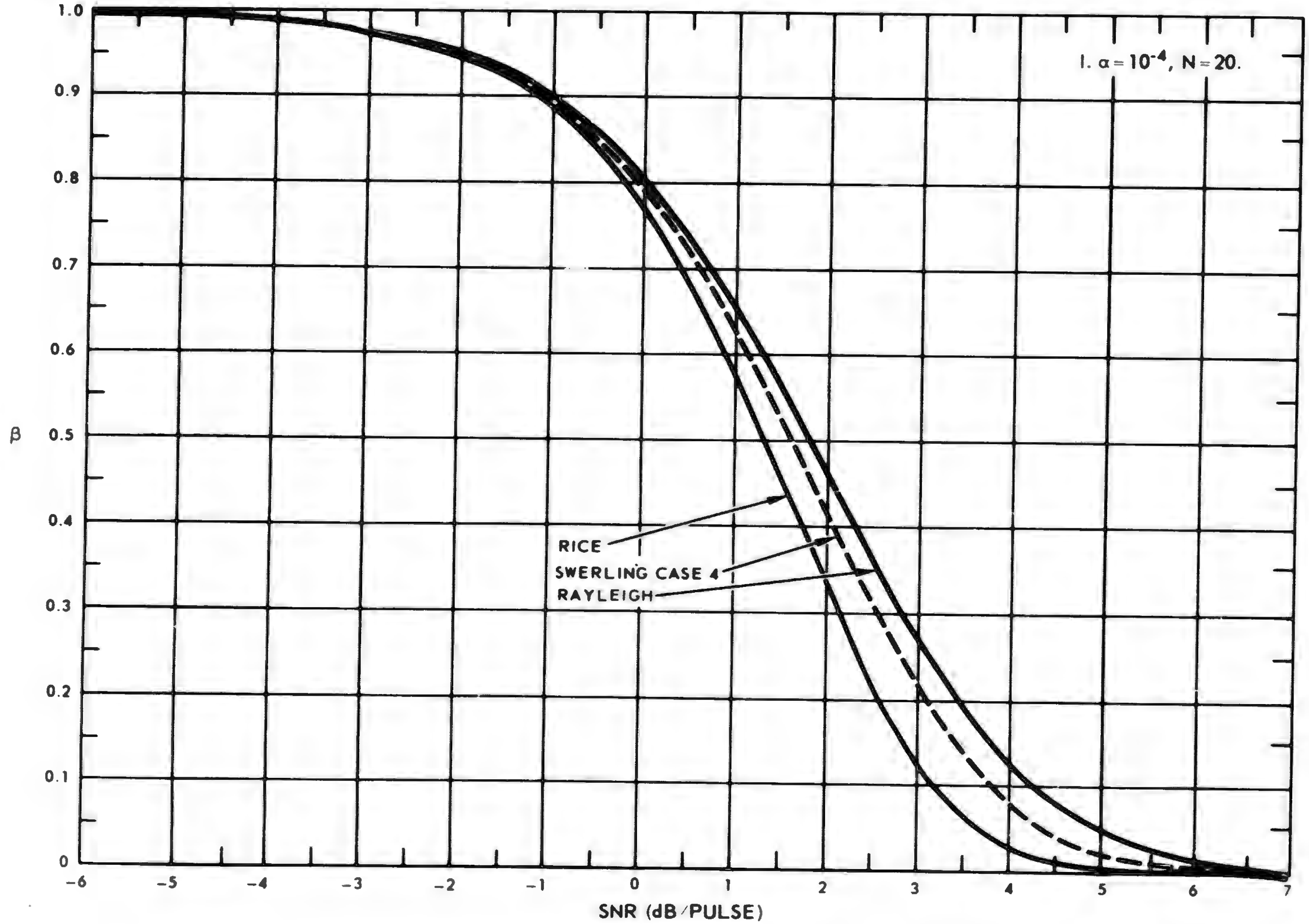

Figure 7 (Continued).

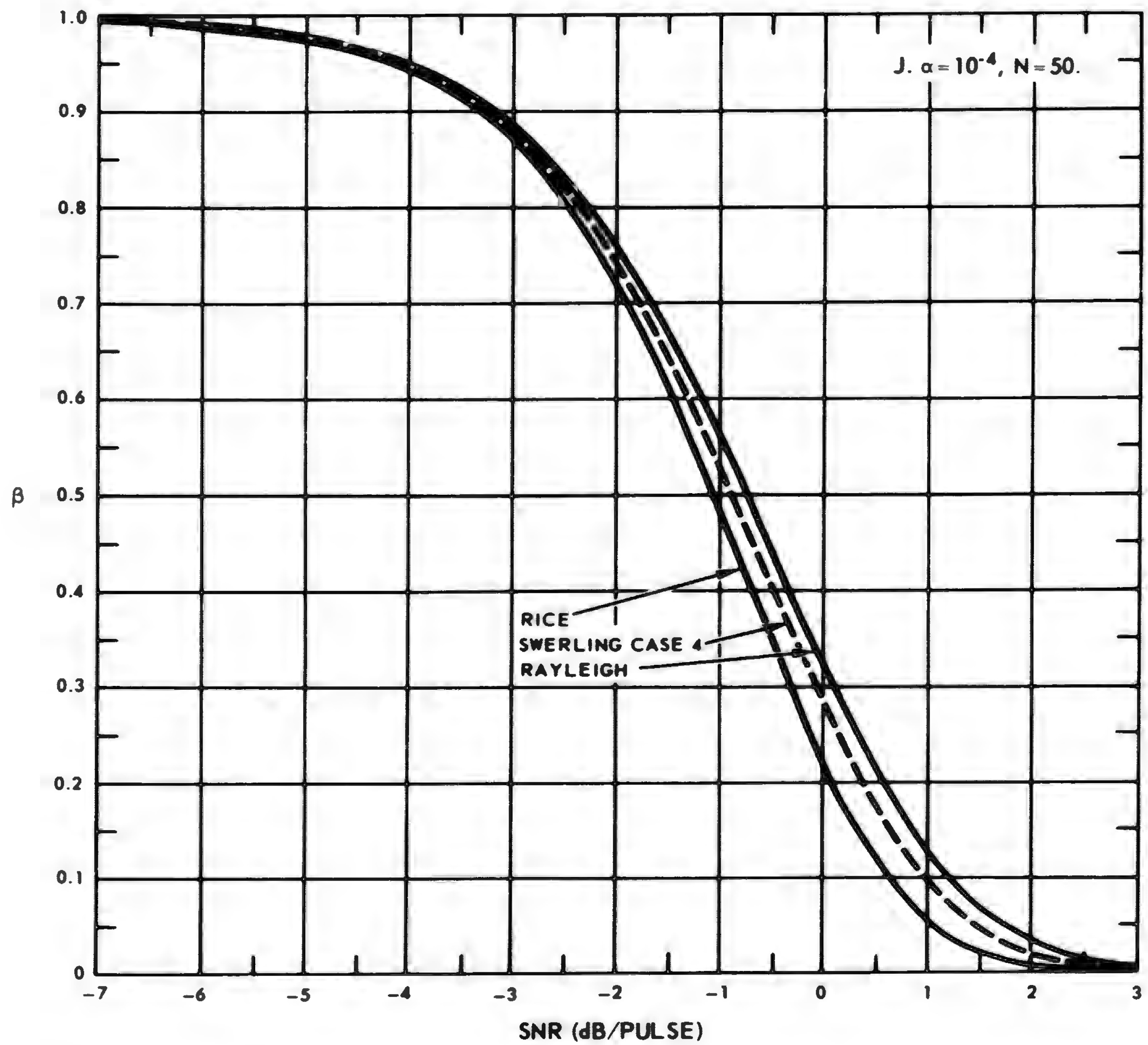

Figure 7 (Continued).

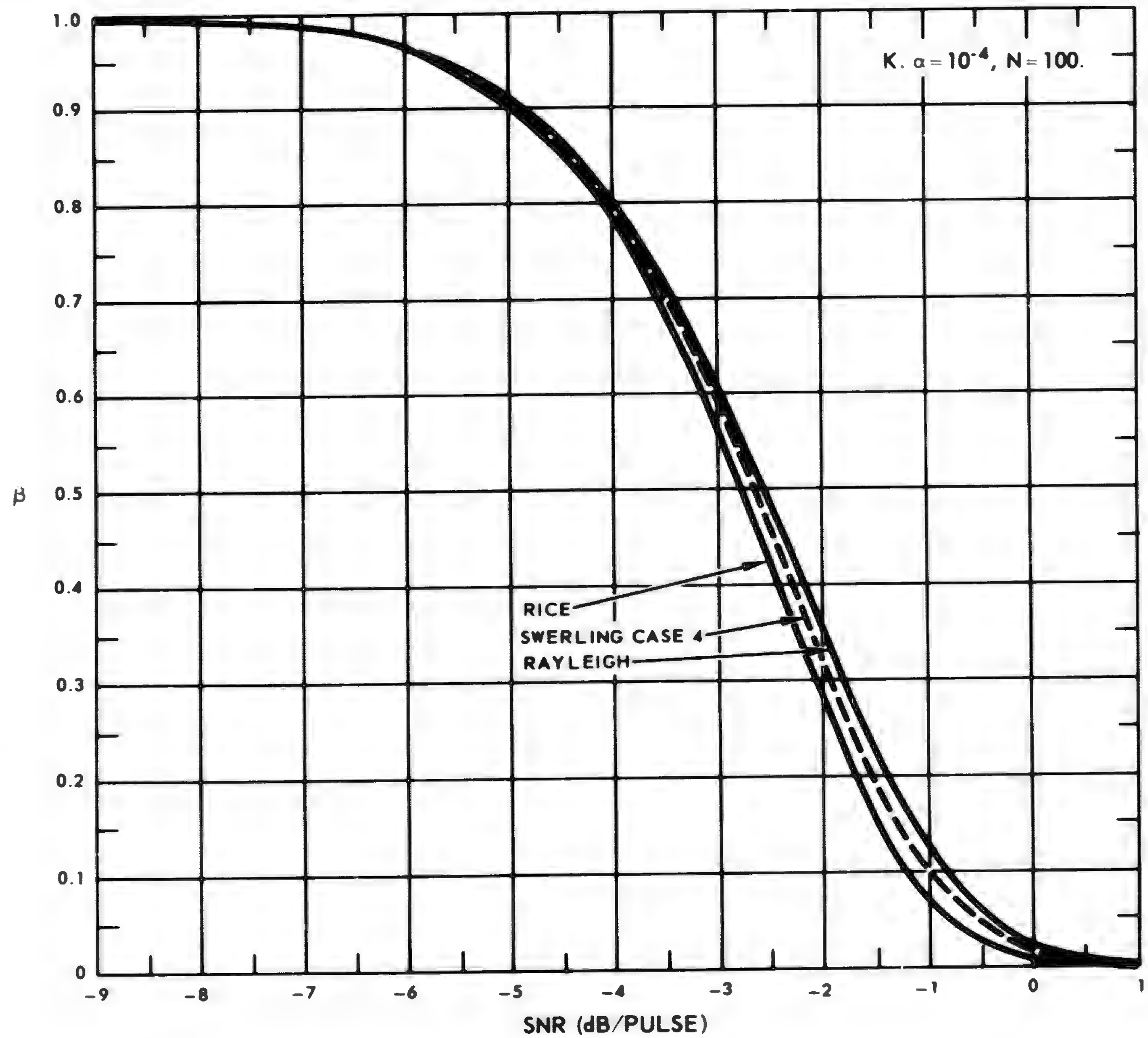

Figure 7 (Continued).

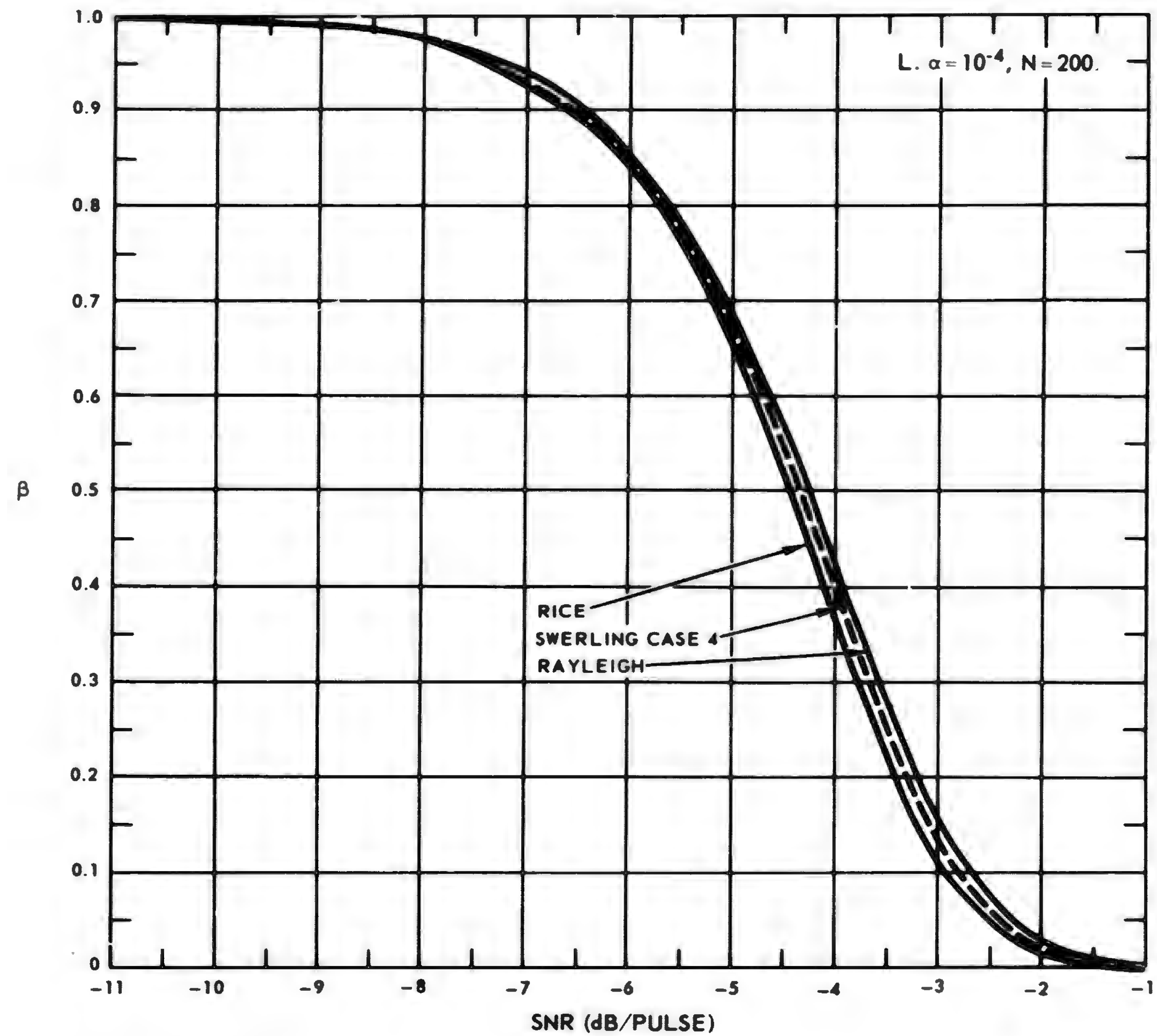

Figure 7 (Continued).

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Some graphical results are presented as aids in selecting optimum thresholds for the testing procedure known under the names binary integration, coincidence detection, binomial fixed-sample-size testing, double-threshold detection, K out of N detection, etc. Optimum thresholds are given for the Normal (test of the mean and test of the variance), Rayleigh, Swerling Case 4, and Rice distributions. For the latter three distributions, curves of the required signal-to-noise ratio versus the number of pulses and of the miss probability versus the signal-to-noise ratio are given. All of the results assume statistically independent observations.

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