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The Cyclotron Resonance Amplification of Whistlers in the Magnetosphere

Harold B. Liemohn

GEO-ASTROPHYSICS LABORATORY APRIL 1968

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D1-82-0713

THE CYCLOTRON RESONANCE AMPLIFICATION OF WHISTLERS IN THE MAGNETOSPHERE

by

Harold B. Liemohn
Boeing Scientific Research Laboratories
Seattle, Washington 98124

April 1968

Presented at NATO Advanced Study Institute Plasma Waves in Space and in the Laboratory, Røros, Norway 17-26 April 1968.

ABSTRACT

Electromagnetic waves and charged particles in the magnetosphere may exchange energy through the cyclotron resonance interaction. If the nonthermal particles have an anisotropic pitch angle distribution, VLF and ULF signals may be amplified over well-defined broad bands of frequencies. An analytic solution of the dispersion equation is obtained for a particular anisotropic distribution which simulates magnetospheric conditions, and the instability is found to be unique and convective. Frequency spectra of the net power transfer along whistler paths are presented for a class of distributions which are thought to be typical of quiescent conditions in the magnetosphere. Finally, the theory is applied to experimental amplitude measurements of a VLF whistler event to illustrate its potential for estimating the phase space distribution of nonthermal particles in the magnetosphere.

ACKNOWLEDGMENT

I wish to thank Dr. R. A. Gerwin for helpful discussions of the instability criterion in Section 3.

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1. Introduction

The cyclotron resonance interaction between electromagnetic fields and charged particles plays an important role in the physics of the magnetosphere. In general, the dominant interactions are between electrons and VLF (1 - 50 kHz) signals and protons and ULF (0.1 - 5.0 Hz) signals. The resonance occurs when the wave frequency as seen by the particle is equal to its cyclotron frequency and the wave polarization as seen by the particle has the same sense as the particle gyration. Significant amounts of energy may be exchanged at this Doppler-shifted frequency depending on the nature of the phase space distribution for the particles.

In general a wide variety of electromagnetic and hydromagnetic modes can propagate in the magnetosphere. Only the VLF and ULF whistler modes (c.f. Helliwell, 1965; Jacobs and Watanabe, 1964) will be considered here. These signals propagate along geomagnetic field-line paths and their characteristic dispersion is a measure of certain path properties. The wave polarization is right handed for the VLF whistler mode and its propagation frequencies are restricted to the band below the cyclotron frequency for electrons. The ULF (Alfven) mode is left handed and its frequencies are restricted to the band below the cyclotron frequency for ions.

Theoretical applications of the cyclotron resonance interaction to phenomena in the magnetosphere generally treat either the generation of emissions, or whistler amplitudes, or particle scattering. Emission thecries for VLF (Kimura, 1961; Dowden, 1963; Trakhtengerts, 1963; Bell and Buneman, 1964; Fung, 1966a, 1966b; Helliwell, 1967; Kimura, 1967; and Das, 1968) and ULF (Cornwall, 1965; Hultqvist, 1965; and Watanabe, 1966) have attributed the signals to the cyclotron instability of a particle beam streaming through a thermal background plasma. A resonance with particles in the nonthermal tail of the background plasma has been suggested for the amplification and absorption bands of whistler signals at VLF (Scarf, 1962; Liemohn and Scarf, 1964; Guthart, 1965; Kennel, 1966; Liemohn, 1967; and Kellen and Thorne, 1967) and ULF (Hultqvist, 1966; Gurnett and Brice, 1966; Jacobs and Watanabe, 1966; Cornwall, 1966; Kennel and Wong, 1967a, 1967b; and Cocke and Cornwall, 1967). The interaction may also be responsible for significant scattering of trapped radiation belt particles (Dungey, 1963, 1964, 1968; Roberts and Buchsbaum, 1964; Cornwall, 1964; and Roberts, 1966) and may produce a state of quasi-equilibrium between wave and particle energy densities (Andronov and Trakhtengerts, 1964; and Kennel and Petschek, 1966; Trakhtengerts, 1966, 1967).

In this paper attention is restricted to the interaction between individual electromagnetic wave packets and distributions of nonthermal charged particles which have anisotropic pitch angle distributions. It has been shown (Liemohn, 1967) that such distributions may give substantial wave amplification over well-defined broad bands of frequencies. Results of this earlier analysis will be reviewed briefly, some interesting aspects will be illustrated by a new analytic solution, and an experimental application of the theory to VLF whistlers will be demonstrated.

2. Theory

The physics of the cyclotron-resonance interaction is described by the well-known Vlasov and Maxwell equations. The analysis is restricted to the special case of propagation parallel to a uniform magnetic field in a homogeneous plasma so that the characteristic modes are circularly polarized. This assumption is a good approximation for propagation of whistlers in field aligned ducts of enhanced ionization (Smith, 1961; and Dowden, 1966). The duration of the interaction between an individual particle and a wave packet is assumed to be sufficiently short that the energy transfer is small and nonlinear effects on f_0 may be ignored.

In this medium the characteristic modes which propagate as $\exp i(kz - \omega t)$ have a dispersion equation of the form (c.f., Montgomery and Tidman, 1964)

$$\frac{c^2k^2}{\omega^2} = 1 - \sum_{i,e} \frac{\pi\omega_p^2}{k\omega} \int_0^\infty dv_\perp \int_{-\infty}^{+\infty} dv_\parallel I_0/(v_\parallel - v_c) \qquad (1)$$

where

$$I_0(v_\perp, v_\parallel) = v_\perp^2 \frac{\partial f_0}{\partial v_\perp} - \frac{k}{\omega} v_\perp^2 \left(v_\parallel \frac{\partial f_0}{\partial v_\perp} - v_\perp \frac{\partial f_0}{\partial v_\parallel} \right) \qquad (2)$$

and

$$v_c = (\omega \pm \omega_c)/k \, . \qquad (3)$$

Here $\omega_p^{i,e} = (4\pi Ne^2/m^{i,e})^{\frac{1}{2}}$ and $\omega_c^{i,e} = (\pm e)B/m^{i,e}c$ are the plasma and cyclotron frequencies for ions (i) and electrons (e) in a medium of particle density N and magnetic field $B\hat{z}$, and the velocity components $v_\perp$ and $v_\parallel$ are perpendicular and parallel to $\hat{z}$, respectively.

Particle labels (i.e) are generally omitted to simplify notation in the equations.

In general (1) has many roots so that a variety of modes could be excited, but the details are unknown in the magnetosphere. Therefore, it is necessary at this point to assume that a "dominant" mode carries the signal energy. Furthermore it is necessary to assume that the propagation characteristics obtained from (1) by a well-established expansion do indeed describe the dominant mode. Although a general proof is not available, these assumptions are verified by a specific algebraic solution of (1) in the next section.

Since energy exchange is anticipated from (1) it is pertinent to inquire about the nature of the instability (c.f. Briggs, 1964; in these proceedings see also Rowlands, 1968 and Crawford, Lee, and Tataronis, 1968). If the wave amplitude grows in time at a given point in space, the instability is nonconvective and the system is termed "absolutely unstable" to any perturbations. For this case the solution of (1) is given by a complex $\omega = \omega_r + i\omega_i$ and a real k. However, if the amplitude grows in space at a given point in time, the instability is convective and the system behaves like an amplifier. For this case the solution is described by a complex $k = k_r + i\, k_i$ and a real ω. In the next section a particular example is found to be convective, and it is suggested without proof that the cyclotron resonance is a convective instability for all quasi-static conditions in the magnetosphere.

The cyclotron-resonance velocity v_c in (3) is the value of $v_\parallel$ which Doppler shifts ω to ω_c in the rest frame of the particle to allow the interaction. The plus and minus signs in v_c refer to right-hand (R) and left-hand (L) circularly polarized modes, respectively. Evidently there are four possible combinations for interactions between the modes and particles. The right-hand electron resonance (Re) and the left-hand ion resonance (Li) are *normal* interactions in the sense that $v_c < 0$ for $\omega < |\omega_c|$ and the wave and particle rotate in the same direction. However, the right-hand ion resonance (Ri) and the left-hand electron resonance (Le) are *anomalous* interactions since $v_c > \omega/k$ and the particle must overtake the wave which consequently reverses its apparent sense of rotation as seen by the particle.

Frequencies near $|\omega_c|$ are not generally detected in the magnetosphere so that only nonthermal particles contribute to the resonance. This circumstance allows the distribution f_0 to be divided conveniently into a nonthermal tail f_q which is normalized to $\varepsilon << 1$ and a thermal part f_t which is normalized to $1 - \varepsilon$. Such a subdivision permits (1) to be solved approximately for a wide variety of f_q .

It is assumed and subsequently justified numerically that $|k_i| << k_r$ (arbitrarily positive for propagation along $+ \hat{z}$) for conditions in the magnetosphere. Then the imaginary part of v_c is very small and (1) may be expanded in a Taylor series in the complex $v_{\parallel}$ plane (c.f. Jackson, 1960, Appendix 1). By equating real and imaginary parts, the propagation characteristics to lowest order in $|k_i|/k_r$ are given by

$$\frac{c^2 k_r^{\ 2}}{\omega^2} = 1 - \sum_{i,e} \frac{\omega_p^{\ 2}}{\omega(\omega \pm \omega_c)} \tag{4}$$

$$\frac{ck_i}{\omega} = \frac{\pi^2}{2c^2 k_r^{\ 2}} \sum_{i,e} \omega_p^{\ 2} \int_0^\infty dv_\perp\ I_q(v_\perp, v_c) \tag{5}$$

In the latter expression only f_q is substituted in (2) to obtain I_q, and v_c is the real part of (3). (I am indebted to Dr. R. W. Fredricks for correcting an error of $\sim 3/2$ in the expression for k_i which appeared in equation (9) of Liemohn, 1967.)

Regimes of wave growth $(k_i < 0)$ and decay $(k_i > 0)$ for anisotropic distributions can be derived formally. For this purpose it is convenient to introduce the pitch angle between the velocity and magnetic field explicitly, $\alpha = \tan^{-1}(-v_\perp/v_\parallel)$. Then (5) can be expressed in the form

$$\frac{ck_i}{\omega} = \sum_{i,e} \frac{\pi^2 \omega_p^{\ 2}}{c^2 k_r^{\ 2}} \left[1 + (1 \pm \omega_c/\omega)a\right] b\ , \tag{6}$$

where

$$a = \frac{1}{2b} \int_0^\infty dv_\perp\ v_\perp [\tan\alpha\ \partial f_q/\partial\alpha]_{v_\parallel = v_c} \tag{7}$$

$$b = \int_0^\infty dv_\perp\ v_\perp f_q(v_\perp, v_c)\ . \tag{8}$$

For an isotropic distribution, $a = 0$ so that its summation term in (6) is always positive. However, for $a \neq 0$, its term may be either positive or negative. The special class of anisotropies $f_q \propto \sin^m\alpha$ gives a particularly simple result. In this case $a = m/2$ and the normal interaction terms Re and Li yield amplification when $\omega/|\omega_c| < m/(m + 2)$, whereas the anomalous terms Ri and Le give absorption for all $\omega < |\omega_c|$.

The physics of the interaction may be explained in terms of particle trapping in the apparent potential well of the wave fields. A particle that has $v_\parallel$ sufficiently close to v_c finds itself subjected to electric and magnetic forces which modify its velocity so that $v_\parallel \to v_c$. The mechanics of the energy exchange have been described by Brice (1964). The net transfer of energy between nonthermal plasma and the wave depends on the shape of f_q in the vicinity of $v_\parallel = v_c$ as measured by $I_q(v_\perp, v_c)$. If more particles lose energy than gain at a particular ω, the wave is amplified, otherwise it is absorbed. This nonlinear distortion of f_q may be ignored in the theory for individual wave packets but it is an important consideration for the gross properties of the steady state (Engel, 1965; Kennel and Petschek, 1966).

These theoretical expressions are strictly valid only for propagation vectors parallel to the local magnetic field. When the wave normal is not parallel to the local field the wave is also subject to conventional Landau damping (Kennel, 1966) due to the longitudinal component of the electric vector. Such an effect actually enhances the probability that the detected signal is caused by wave normals approximately parallel to the local field.

3. An Analytic Solution

The dispersion relation in (1) is amenable to a complete analytic solution for anisotropic Cauchy (resonance) distributions of the form $\left(v_\perp^2 + v_q^2\right)^\mu / \left(v_\perp^2 + v_\parallel^2 + v_q^2\right)^\nu$ where μ and ν are integers and v_q is constant. These distributions have the useful asymptotic form $E^{-\nu+\mu}\sin^{2\mu}\alpha$ which is frequently used to describe energy and pitch angle observations in the magnetosphere. For simplicity in this analysis the lowest exponents which give finite energy $(\mu \leq \nu - 2)$ are used for f_q and a Dirac delta function suffices for f_t. The complete distribution has the explicit form

$$f_0 = \frac{(1-\varepsilon)\delta(v_\perp)\delta(v_\parallel)}{2\pi v_\perp} + \frac{24\varepsilon v_q{}^3(v_\perp{}^2 + v_q{}^2)}{5\pi^2(v_\perp{}^2 + v_\parallel{}^2 + v_q{}^2)^3} , \tag{9}$$

where v_q is a constant "nonthermal" speed defined by the mean kinetic energy, $13\, mv_q{}^2/10 = 3\kappa T/2$.

The integrals in (1) may be performed analytically for (9) by residue calculus. Since the solution is intended to illustrate some qualitative effects, only the electron interaction with the right-hand mode will be considered. The general dispersion equation for complex k and ω has an algebraic form

$$\frac{c^2k^2}{\omega^2} = 1 + \frac{(1-\varepsilon)\omega_p{}^2}{\omega(|\omega_c| - \omega)}$$

$$+ \frac{\varepsilon\omega_p{}^2}{5\omega^2} \frac{5\omega(|\omega_c| - \omega)^2 \mp 15ikv_q\omega(|\omega_c| - \omega) - k^2v_q{}^2(10\omega + 2|\omega_c|) \pm 2ik^3v_q{}^3}{(|\omega_c| - \omega \mp ikv_q)^3} \tag{10}$$

where the upper and lower signs apply for the conditions

$$Im(v_c) = \frac{k_r\omega_i + k_i(|\omega_c| - \omega_r)}{k_r{}^2 + k_i{}^2} \gtrless 0 , \tag{11}$$

respectively. The solution on each branch of (11) may be analytically continued into the other one by suitably deforming the branch cut in the complex k plane but this does not alter (10).

The simple form of (10) and (11) permits a complete solution for all the propagation modes of the system. Thus it is straightforward to isolate the "dominant" mode explicitly and to study the nature of the instabilities. In general (10) has five roots $k(\omega)$ or six roots $\omega(k)$ in each branch, but only those which satisfy (11) and its analytic continuation are valid. The arbitrary Van Kampen modes from the branch cut are assumed to be suppressed by formally requiring the distribution to vanish at an arbitrarily large finite velocity (Derfler, 1962). Due to the form of (10) and (11) it is readily apparent that every valid root k in the upper branch is paired with a valid root $-k$ in the lower branch. A numerical evaluation of the complex roots for parameters appropriate to the magnetosphere has revealed that only *one* root is valid in each branch. This uniqueness precludes the need for a "dominant" mode and suggests that a wide class of distributions also have unique solutions for cyclotron resonance.

To determine whether the interaction is an absolute or convective instability, characteristics of the roots in the complex k and ω planes must be studied (Briggs, 1964, Appendix B). For an absolute instability there must be two roots $k(\omega)$ in different halves (branches) of the complex plane which merge into a double root when $\omega_i > 0$ (this inequality is opposite that of Briggs due to a different convention for the phase of the wave). If such a double root does not exist the instability is convective. In this application the valid root from each branch can only merge into a double root at the origin $k = 0$ which proves to correspond to the trivial case $\omega = 0$. Thus the cyclotron resonance is a convective instability here, and probably for many similar distributions. The roots also satisfy alternative hot plasma criteria established for a convective instability (Sudan, 1965; Dysthe, 1966; Hall and Heckrotte, 1968; and Baldwin and Rowlands, 1968).

Therefore, the wave is appropriately described by complex k and real ω. The roots in opposite branches describe two waves propagating in opposite directions with the same growth or decay as anticipated from symmetry conditions. It is notable that the branch cut in the k plane is indented by these roots only when amplification is present. In this case the valid roots do not satisfy (11) explicitly and the correct ones must be selected at large ω_i where (11) is satisfied and traced by analytic continuity to the limit $\omega_i = 0$.

Approximate expressions for the propagation characteristics may be derived for $\omega < |\omega_c|$ in the low temperature limit $v_q k_r \ll (|\omega_c| - \omega)$, which ensures that $|k_i| \ll |k_r|$ as well. By expanding (10) and equating real and imaginary parts, the root for propagation in the $+\hat{z}$ direction is found to be

$$\frac{c^2 k_r^{\,2}}{\omega^2} = \left[1 + \frac{\omega_p^{\,2}}{\omega(|\omega_c| - \omega)}\right]\left[1 - \frac{\varepsilon}{5}\frac{v_q^{\,2}}{c^2}\frac{\omega_p^{\,2}(2|\omega_c| - 5\omega)}{(|\omega_c| - \omega)^3} + \mathcal{O}\left(\frac{v_q^{\,4}}{c^4}\right)\right], \tag{12}$$

$$\frac{ck_i}{\omega} = -\frac{2\varepsilon}{5}\frac{v_q^{\,3}}{c^3}\frac{c^2 k_r^{\,2}}{\omega^2}\frac{\omega_p^{\,2}\omega(|\omega_c| - 2\omega)}{(|\omega_c| - \omega)^4} + \mathcal{O}\left(\frac{v_q^{\,5}}{c^5}\right). \tag{13}$$

The spatial phase factor k_r is just the zero-temperature expression with a small "nonthermal" correction. The amplitude factor k_i reveals amplification below $|\omega_c|/2$ and absorption above, which is in agreement with (6) since $m = 2\mu = 2$ in the high-energy asymptotic limit of f_q. The magnitude of k_i is proportional to $\epsilon v_q{}^3$ which is a relative measure of the number of electrons in the tail of (9).

4. Whistler Amplification

On occasion VLF and ULF whistlers (c.f. Helliwell, 1965; Jacobs and Watanabe, 1964) may complete many hops from one hemisphere to the other without evidence of appreciable energy loss. (ULF whistlers are frequently called pearls or Pc 1 micropulsations.) Since the reflection process at each base of the field-line path must involve some ionospheric losses, a mechanism must be amplifying the signals. The cyclotron resonance interaction between these waves and anisotropic particle distributions is the only known source of amplification energy. To illustrate its properties the results of an earlier application of the general theory (Liemohn, 1967) will be reviewed briefly.

Anisotropic distributions of the form $f_q \propto E^{-n-0.5}\sin^m\alpha$ were used because they typify quiescent conditions in the magnetosphere (O'Brien, 1963, 1966; Davis and Williamson, 1963; and Frank, 1967a, 1967b), when most VLF and ULF whistlers are detected. Certainly other distributions are possible and may indeed be essential to simulate special conditions, but they are omitted here. The pitch angle exponent $m \geq 0$ and energy exponent $n \geq 1$ are convenient parameters for labeling the shape of a wide variety of distributions. In actual practice the distributions were modified to take into account a loss cone and a high-energy cutoff. Both the proton and electron distributions were normalized to experimental observations which have nearly identical differential densities.

When these distributions were substituted in (5), it was found that the normal interaction dominated the anomalous one. This is due primarily to the difference in the minimum resonance energy $\frac{1}{2} m v_c^2$ and secondarily to the shape and normalization of the distributions. For typical whistler paths near the equator the normal resonances Re and Li have $\frac{1}{2} m v_c^2 \sim 1$ keV for $\omega \sim |\omega_c|/2$ whereas the anomalous resonances Ri and Le have $\frac{1}{2} m v_c^2 \sim 1$ MeV. Thus these distributions which decrease monotonically with energy have many more particles which can contribute at normal-resonance energies. However, during disturbed conditions when the proton and electron normalizations are different or high-energy spikes in the distribution are temporarily present (Frank, 1967a, 1967c) the anomalous interactions may be important.

To make numerical estimates of the amplification, (5) was evaluated along the path $L = 4$ (surface latitude = 60°) where the equatorial cyclotron frequency is $|\omega_{c0}^{e}| = 8.60 \times 10^4$ rad/sec. The background plasma was assumed to be in hydrostatic equilibrium, with the normalization $\omega_{p0}^{e} = 10\ |\omega_{c0}^{e}|$. The latitude variation of k_i for the VLF R-mode along this path is shown in Figure 1 for the distribution $m = 2$, $n = 2$. Substantial amplification away from the equator, which is due to the enhanced loss cone, requires path integrals to get the net effect. Somewhat above $\omega/|\omega_{c0}^{e}| = 0.5$ the equatorial attenuation

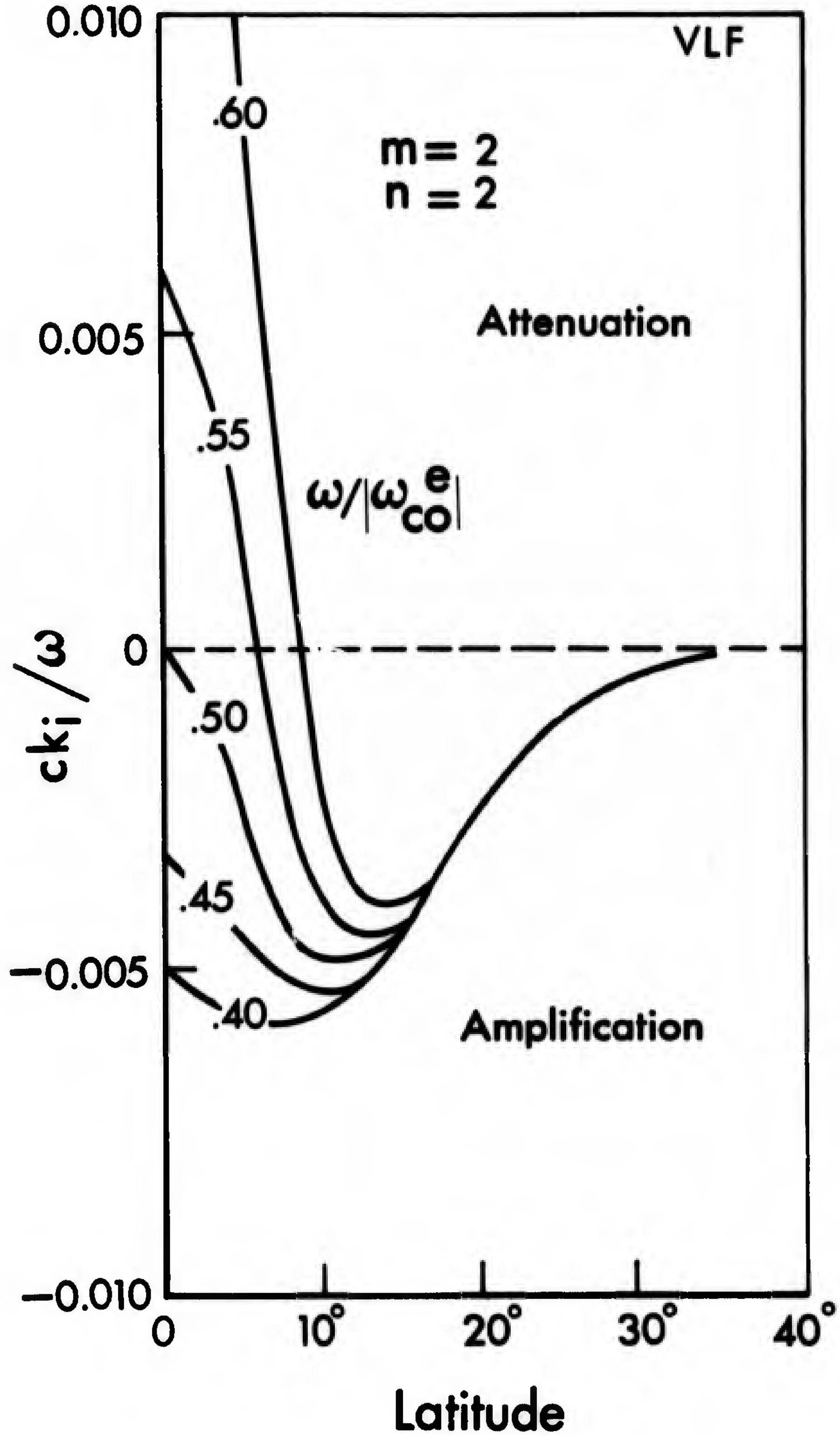

Fig. 1. Typical frequency and latitude variations of the amplitude factor k_i for the R-mode.

becomes dominant and interhemisphere propagation terminates. Far from the equator the interaction is negligible due to the large Doppler shift.

The net amplification or absorption in decibels along this path is given by

$$A = -10 \log_{10}\left[\exp \int_{path} 2k_i(\omega,s)ds\right] . \qquad (14)$$

Several frequency spectra of the power transfer function A for $n = 2$ and various m are shown in Figure 2 for the VLF R-mode. The anisotropic cases $(m \neq 0)$ always have a broad band of amplification with a relatively sharp upper cutoff. The magnitude of A is relative due to the imprecise normalization of f_q. In practice an upper limit of 40 db is anticipated since background noise from incoherent emissions would seriously hamper VLF satellite receivers if more amplification were present.

The general form of the spectra in Figure 2 is typical of both VLF and ULF power transfer for the model distributions $E^{-n-0.5}\sin^m\alpha$. In an earlier analysis (Liemohn, 1967) A was computed for the parameter values $m = 0, 0.5, 1, 2, 4, 8$ and $n = 1, 1.5, 2, 2.5$, and 3. The spectral characteristics were summarized by four properties: the maximum amplitude $\bar{A}$, the half-log-power bandwidth $\Delta\omega_{1/2}$ where $A = \bar{A}/2$, the frequency of maximum amplification $\bar{\omega}$ where $A = \bar{A}$ and the cutoff frequency ω_0 where $A = 0$. Contours of $\Delta\omega_{1/2}$ and $\bar{\omega}$ for a wide range of m and n are shown in Figure 3 for the VLF R-mode.

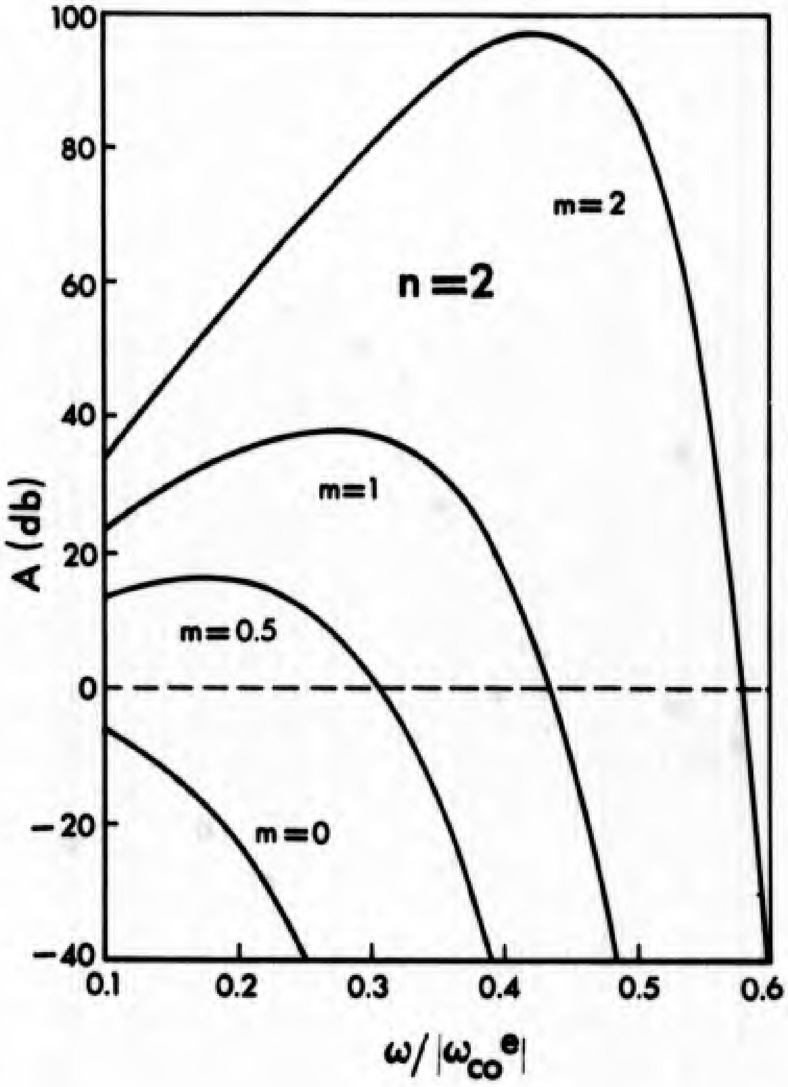

Fig. 2. VLF spectra of the power transfer function A for several electron distributions.

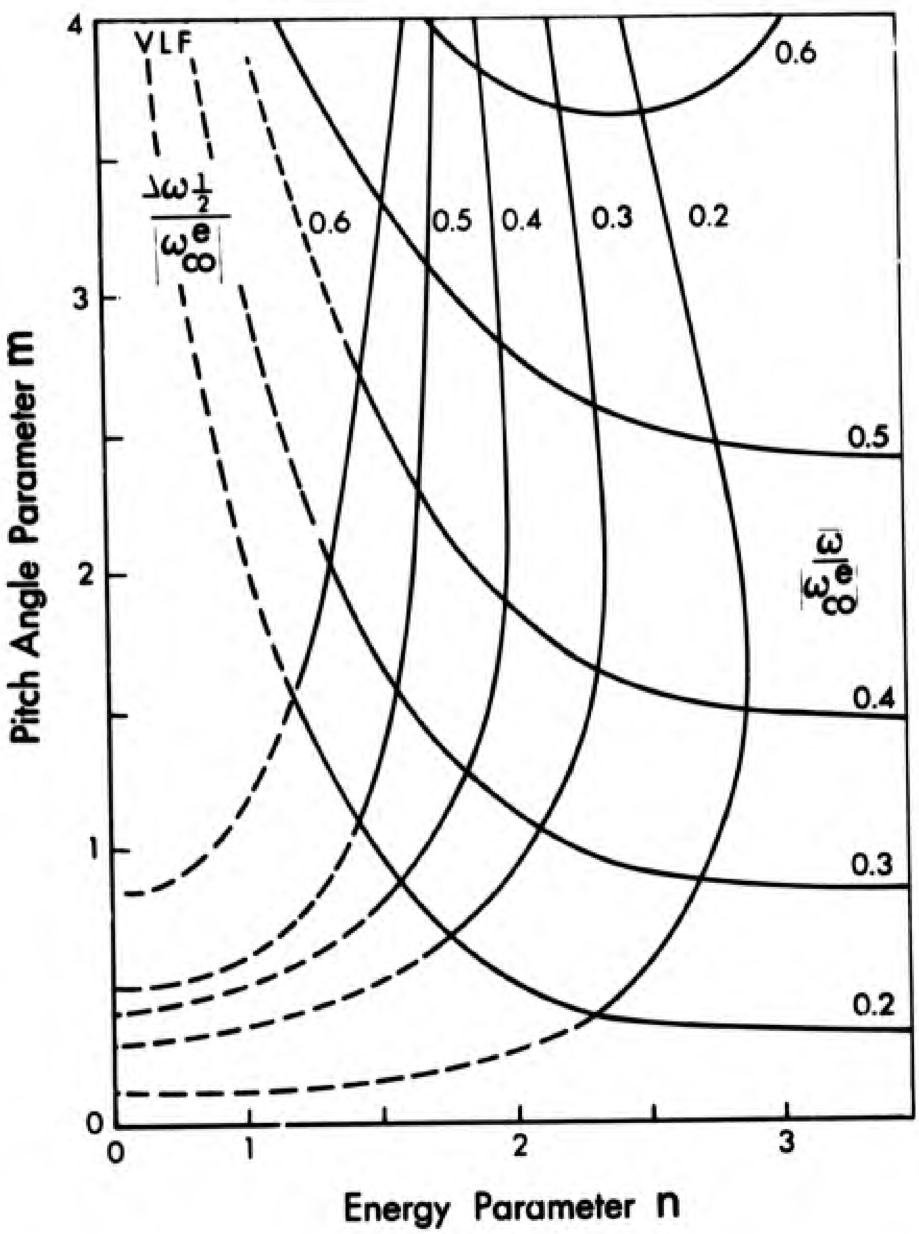

Fig. 3. Properties of VLF spectra of A for the R-mode. Half-log power bandwidth $\Delta\omega_{\frac{1}{2}}$ and frequency of maximum amplitude $\bar{\omega}$ as functions of the distribution parameters m and n .

Although the curves vary with both m and n the physical explanation is readily identified. There are relatively more high-energy particles to broaden $\Delta\omega_{\frac{1}{2}}$ as n decreases, and a stronger anisotropy permits lower energies to contribute effectively to the amplification so that $\bar{\omega}$ increases with m.

These spectral characteristics of the cyclotron resonance interaction are consistent with the observed frequency bands of VLF and ULF whistlers. There are wide variations in the bandwidth and its upper and lower cutoffs which suggest variations in the amplification band and particle distribution in both space and time. The power spectra of VLF and ULF whistlers may also be affected by other cutoff mechanisms. In particular, the duct theory (Smith, 1961) predicts a lack of guidance for $\omega \geq 0.5|\omega_c^e|$, which is supported by some VLF data (Carpenter, 1968).

5. Experimental Measurements

By comparing experimental amplification spectra with the foregoing theoretical results, the actual energy and pitch angle distributions of the nonthermal components in the magnetosphere during magnetically quiet periods may be estimated. To determine such amplification spectra the power spectra of two successive echoes of a multihop VLF or ULF whistler event must be measured and corrected for ionospheric losses. Both satellite and ground records have been measured recently (Kenney, Knaflich, and Liemohn, 1967), and the method and results will be summarized.

The transfer equations for the power spectra of a multihop whistler event are linear. If the initial signal above the ionosphere is P_w, then the "source" spectrum at the receiver is $P_s = TP_w$ where T is the ionospheric transmission coefficient. Part of the signal is reflected as R_1P_w where R_1 is the local ionospheric reflection coefficient. The energy traverses the magnetospheric path which has a power transfer coefficient M so that MR_1P_w reaches the opposite hemisphere. After reflection, magnetospheric return, and transmission it appears at the receiver as a "response" spectrum $P_r = TM^2R_1R_2P_w$ where R_2 is the reflection coefficient in the opposite hemisphere. Thus the successive spectra yield the relation $M^2R_1R_2 = P_r/P_s$. The power transfer function for the magnetosphere therefore has the decibel form

$$A_{exp} = 10 \log_{10} M = 5 \log_{10}(P_r/P_s) - 5 \log_{10}(R_1R_2) \ . \qquad (15)$$

Similar formulas are readily derived for the comparison of VLF whistlers and their atmospheric sources and for fractional hop conditions encountered with satellites.

The power ratio P_r/P_s was measured by a new method (Kenney, Deaton, and Miller, 1967) which is a modification in the output of the conventional sonagraph instrument. A tape loop of the signal is played repeatedly through a swept frequency filter (sonagraph), and the narrow band signal amplitude is displayed on an oscilloscope or a high speed strip chart. The spectral detail obtainable by this method is illustrated in Figure 4 which contains a portion of a multihop VLF event

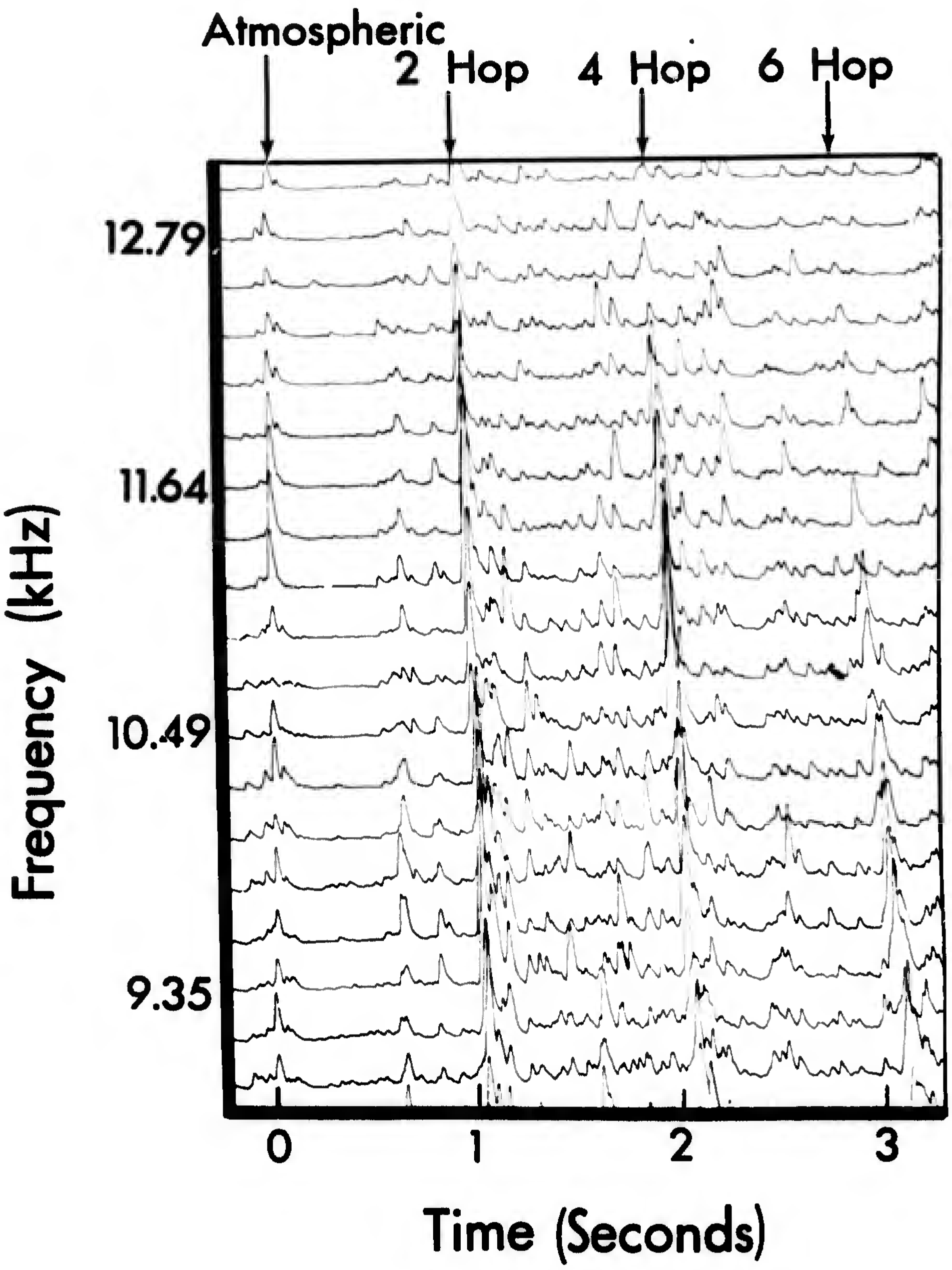

Fig. 4. Amplitude-frequency-time display of a multihop VLF whistler event (1235 UT, 7 October 1960, Seattle).

recorded at a ground station near Seattle (courtesy D. L. Carpenter; a sonagram of the complete event is shown by Helliwell, 1965, Figure 4-8). Although the record contains excessive atmospheric noise, the causative atmospheric and the second, fourth and sixth hops are clearly discernible, and the power spectra may be measured conveniently.

The ionospheric reflection coefficients provide the major source of uncertainty at this time. The theoretical estimates for VLF (Helliwell, 1965; Pitteway and Jesperson, 1966) are not sufficiently complete, and no experimental measurements from satellites are available yet. Thus, to complete the analysis of the VLF record in Figure 4, the reflection was assumed to occur at the base of the E layer, and the ionospheric absorption (Helliwell, 1965) was doubled to estimate $R = R_1 = R_2$.

The amplification for the atmospheric and five echoes in this VLF event are shown in Figure 5 after some smoothing. Because the power ratios in this event were near unity, the shape of the amplification curve is due almost entirely to the ionospheric absorption. Conventional dispersion analysis on this event revealed, however, that $\bar{\omega}/|\omega_{c0}^{e}| \sim 0.27$ and $\Delta\omega_{1/2}/|\omega_{c0}^{e}| \sim 0.15$ which is essentially independent of the reflection process. From Figure 3 these values give $m \sim 0.5$ and $n \sim 3.0$ for pitch angle and energy exponents. Similar analysis of other VLF and ULF whistlers is nearing completion (Liemohn, Kenney, Knaflich, 1968).

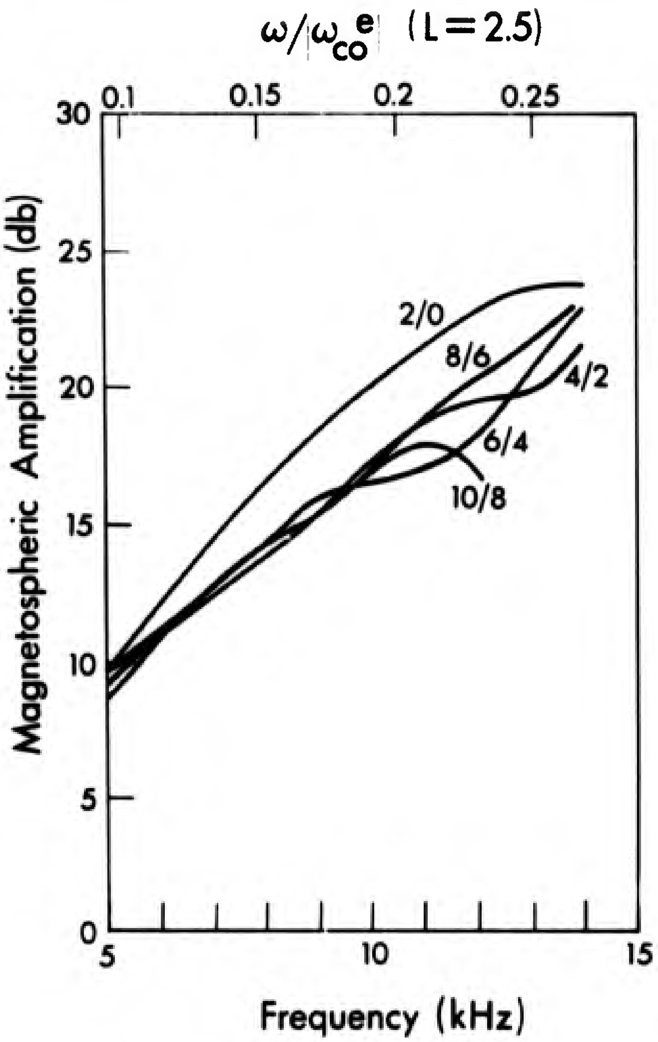

Fig. 5. Magnetospheric amplification deduced from power ratios of a multihop VLF whistler event (1235 UT, 7 October 1960, Seattle) and ionospheric absorption data. The ratio numbers refer to whistler hops.

Although the pitch angle and energy distributions determined from whistler power spectra agree in an average sense with the satellite observations, there have been no simultaneous measurements to test the theory. Hopefully such an experiment can be completed in the near future. A geostationary satellite located on a field line connecting conjugate VLF and ULF ground stations offers a particularly good system for simultaneous field and particle observations. The pitch angle and energy distribution could be measured very near the equator; the half-hop amplification could be determined from the wave power spectra at the satellite; and the ground station spectra could provide an independent check on the theory and an estimate of the ionospheric properties.

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