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PROPAGATION OF WAVES IN PLASMA ACROSS THE MAGNETIC FIELD

by

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ABSTRACT: The propagation of waves in a plasma across a magnetic field is being characterized by interesting features near gyro-frequencies of electrons and ions, as well as frequencies multiple to them. Frequencies of ordinary and extraordinary waves which propagate across a magnetic field in a non-relativistic plasma are determined, and the excitation of high frequency branches of these oscillations are investigated when an ion cyclotron wave passes through the plasma. Various dispersion equations for extraordinary waves are given. The behavioral frequency of various branches of extra-ordinary waves in hot plasma are described. ( )
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PROPAGATION OF WAVES IN PLASMA ACROSS THE MAGNETIC FIELD

A. B. Kitsenko and K. N. Stepanov

The effect of the thermal movement of plasma electrons and ions on wave propagation of a low pressure nonrelativistic plasma ($\varkappa_\alpha = 4\pi n_0 T\alpha | H_0^2 \leq 1$) across a magnetic field is investigated. The dispersion equation for an ordinary wave has solutions $\omega_s(k)$, similar to k $s\omega_\alpha$ (s = 1, 2, ...) at any given wavelength ratio ($\lambda = 1/k$) to the Larmorian radius ρ_α of α type particles having a thermal velocity (see Fig. 1). (ω_e and ω_i — gyro-frequency of electrons and ions.).

The dispersion equation for an extraordinary wave also has an analogous solution in the field of high frequencies ($\omega \approx \omega_e$), (see Fig. 5), as well as a solution, corresponding at $k^2 \rho_\alpha^2 \gg \varkappa_\alpha$ to the longitudinal plasma oscillations. Frequencies of longitudinal oscillations, in the case of a plasma with a greater density, when $\Omega_e \gg \omega_e$ (Ω_e — is the Langmuirs frequency of electrons) as a function of the wave vector, decrease at a rise in $k\rho_e$, decreasing from a value $\omega \approx s\,\omega_e$ (s = 2, 3, ...) at $k\rho_e \ll 1$ to $\omega \approx (s - 1)\omega_e$ at $k\rho_e \gg 1$ (see Fig. 5). For a plasma with a small density ($\Omega_e \lesssim \omega_e$), the behavior of plasma frequencies is shown in Fig. 2 and 3.

In the field of low frequencies ($\omega \lesssim \omega_i$) the dispersion equation for an extraordinary wave (in case of a cold plasma) determines the frequency $\omega = kV_A$, which correspond to the magneto-sonic wave (V_A — is the Alfven velocity). This expression appears to be unacceptable at $k\rho_i \approx s$ (s = 1,2,...), when $\omega \approx s\,\omega_i$. In this field it is necessary to compute the thermal movement of the ions. The behavior frequency of various branches of the extraordinary wave in a hot plasma at $\omega \approx \omega_i$ is shown in Fig. 6.

The auto-excitation of the electron (high frequency) branches of the examined oscillations, when an ion cyclotron wave passes through the plasma, due to bunched instability was investigated.

1. Introduction

The propagation of waves in a plasma across a magnetic field is characterized by interesting features near the gyro-frequencies of the electrons and ions, and frequencies multiple to them. In spite of the investigation of this question by a whole series of works [1—13], it cannot be considered fully explained. In the present work, the frequencies ω of ordinary and extraordinary waves which propagate across a magnetic field in a nonrelativistic plasma, are determined and also the excitation of high-frequency (electronic) branches of these oscillations when an ion cyclotron wave passes through the plasma is investigated. As is known, in the case of a transverse propagation of waves in a plasma situated in an outer magnetic field H_0, the dispersion equation, which determines the frequencies ω of these waves as a function of the wave vector k, breaks into two equations

$$(kc/\omega)^2 - \varepsilon_{33} = 0 \qquad (1)$$

$$\varepsilon_{11}(kc/\omega)^2 - \varepsilon_{11}\varepsilon_{22} - \varepsilon_{12}^2 = 0 \qquad (2)$$

where

$$\varepsilon_{11} = 1 - \sum_{\alpha, n} \frac{\Omega_\alpha^2}{\omega(\omega - n\omega_\alpha)} \frac{n^2 e^{-\mu} I_n}{\mu}$$

$$\varepsilon_{22} = 1 - \sum_{\alpha, n} \frac{\Omega_\alpha^2}{\omega(\omega - n\omega_\alpha)} [illegible] I_n + 2\mu(I_n - I_n')]$$

$$\varepsilon_{33} = 1 - \sum_{\alpha, n} \frac{\Omega_\alpha^2}{\omega(\omega - n\omega_\alpha)} [illegible] \qquad (3)$$

$$\varepsilon_{12} = \sum_{\alpha, n} \frac{[illegible]}{\omega(\omega - n\omega_\alpha)} [illegible] + I_n)$$

$k\,c/\omega$ -- is the index of refraction, $\Omega_\alpha = (4\pi e^2 n_0/m_\alpha)^{1/2}$, $\omega_\alpha = eH_0/m_\alpha c$ -- is the Langmuirian and cyclotronic frequencies of particles of type α, $\varepsilon_e = -1$, $\varepsilon_i = 1$, $I_n = I_n(\mu)$ -- is the Bessel function from the imaginary arguement, $v_\alpha = (T_\alpha/m_\alpha)^{1/2}$ -- is the thermal velocity, $\mu_\alpha = k^2 v_\alpha^2/\omega_\alpha^2 = k^2 \varrho_\alpha^2$, ϱ_α -- is the Larmorian radius of particles with thermal velocity.

Equation (1) determines the frequencies of an ordinary wave which appears to be purely transverse (the intensity of the electric field of this wave is perpendicular to the direction of propagation and parallel to the outer magnetic field H_0). Equation (2) determines the frequencies of an extraordinary longitudinal-transverse wave (the intensity of the electric field of that wave is perpendicular to H_0 and has a different (from zero) projection in the direction of propagation).

Let us note, that the expressions of (3), for the tensor of dielectric permeability ε_{ij}, can be used only for the fulfillment of the inequality

$$|\omega - s\,\omega_\alpha| \gg \beta_\alpha^2 \omega_\alpha \qquad (4)$$

where $\beta_\alpha = v_\alpha/c$. This condition denotes, that the dispersion of frequencies of the cyclotron radiation particles with thermal velocity, due to the relativistic (transverse) Doppler effect, is small in comparison with ω_α. When fulfilling the conditions of (4), the cyclotron oscillation damping is exponentially small.

Below, we will investigate plasma oscillations close to gyroresonances. If the length of the wave is on the order of the Larmorian radius of electrons or ions ($\mu_e \approx 1$ or $\mu_\alpha \approx 1$), then the square of the refractive index, when $\omega \approx \omega_\alpha$ is on the order c^2/v_α^2.

Values $\varepsilon_{ij} \approx \Omega_\alpha^2/\omega_\alpha^2$, if ω is not close to $s\omega_\alpha$. It is obvious the resonances $\omega \approx s\omega_\alpha$, are separated only when $c^2/v_\alpha^2 \gg \Omega_\alpha^2/\omega_\alpha^2$, in the case when the gas kinetic pressure of the plasma is considerably lower than the magnetic pressure,

$$\varkappa_\alpha = 4\pi n_\alpha T_\alpha / H_0^2 \ll 1 \tag{5}$$

We will confine ourselves to the investigation of this case.

When $\varkappa_\alpha \approx 1$, the qualitative pattern of the frequency dependence ω the wave vector is also the same when $\varkappa_\alpha \lessapprox 1$; however, for the determination of frequencies $\omega(k)$, a numerical solution of eq. 1 and 2 is needed. Upon the fulfillment of condition (5) it is possible to obtain simple expressions for frequencies $\omega(k)$, which are close to $s\omega_\alpha$.

2. Ordinary Wave Frequencies

Let us first of all examine Equation 1 in the case of high frequency (electronic) oscillations when the movement of ions can be disregarded. In the case of low density plasma ($\Omega_\alpha \lesssim \omega_\alpha$) the value ε_{33} is comparable to $\omega \approx \omega_e$, $\mu_e \approx 1$ with $(kc/\omega)^2 \approx c^2/v_\alpha^2$ only when $|\omega - s\omega_\alpha|/\omega_\alpha \approx \beta_e^2$. But in this case the nonrelativistic expressions of equation 3 for ε_{ij} cannot be used. Therefore, we will examine the case of dense plasma ($\Omega_e \gg \omega_e$). In this case maintaining only a resonance component in the expression for ε_{33}, we will obtain

$$(\omega - s\omega_e)/\omega_e = -\varkappa_e f_s(\mu_e) \quad (s = 1, 2, \ldots) \tag{6}$$

where

$$f_s(\mu_e) = s\, e^{-\mu_e} I_s(\mu_e)/\mu_e \tag{7}$$

Expression 6 can be used only for $\mu_e \gg \varkappa_e$, if $\mu_e \lesssim \varkappa_e$, then in ε_{33} it is also necessary to maintain that a component with $n = 0$ in addition to the resonance member. Then we will obtain

$$(\omega - s\omega_e)/\omega_e = -\varkappa_e \mu_e f_s(\mu_e)/(\varkappa_e + \mu_e) \tag{8}$$

where

$$f_s(\mu_e) \approx \mu_e^{s-1}/2^s (s-1)!$$

The behavior of frequencies depending on the wave vector is shown schematically in Fig. 1. The frequency close to ω_e at low $\mu_e < \varkappa_e^{1/2}$ decreases $(\Delta\omega|\omega_e \approx -\varkappa_e\mu_e|[\varkappa_e + \mu_e])$, it reaches minimum at $\mu_e = \varkappa_e^{1/2}$ $(\Delta\omega/\omega_e \approx -\varkappa_e/1)$, and then it increases, approaching asymptotically to ω_e at $\mu_e \gg 1$. Frequencies close to $s\omega_e (s = 2, 3, \ldots)$, decrease upon an increase in μ_e, reaching a minimum at $k = k_{min} \approx 1/\varrho_e$, and then increase, approaching $s\omega_e$. With an increase in the number s of branch oscillations, value k_{min} increases, and value $|\omega - s\omega_e|/\omega_e$ decreases in a minimum, whereby functions $(\omega - s\omega_e)/\omega_e$ change slowly at $k\varrho_e \approx 1$.

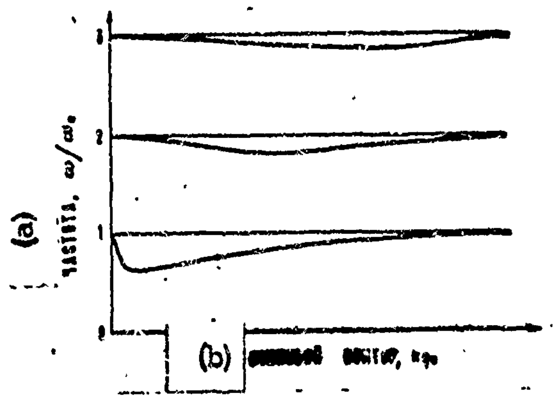

Fig. 1.

KEY: (a) frequency; (b) wave vector.

Clear expressions for the refraction index as a frequency functions a $\omega \approx s\omega_e$ can be obtained in case of long wave oscillations ($\mu_e \ll 1$) at $s = 1, 2$ and in case of short wave oscillations ($\mu_e \gg 1$) for any $s \ll \mu_e$.

If $\omega \approx \omega_e$, $\mu_e \ll 1$, then

$$(kc/\omega)^2 = (\Omega_e^2/\omega_e^2 \varkappa_e^{1/2})[x \pm (x^2 - 1)^{1/2}] \qquad (9)$$
$$x = (\omega/\omega_0 - 1)/\varkappa_e^{3/2}; \ \omega_0 = \omega_e(1 - \varkappa_e/2)$$

At $|x| \gg 1$ we will thus obtain

$$\left(\frac{kc}{\omega}\right)^2 = \frac{\Omega_e^2}{\omega_e^2} \frac{\omega^2/\omega_e^2 - 1}{\varkappa_e + 1 - \omega_e^2/\omega^2} \qquad (10)$$

If $\omega \approx 2\omega_e$ and $\mu_e \ll 1$, then

$$(kc/\omega)^2 = (\Omega_e^2/2\omega_e^2)[-x \pm (x^2 - x)^{1/2}] \qquad (11)$$
$$x = 2(1 - 2\omega_e/\omega)/\varkappa_e^2$$

At $\omega \approx s\omega_e$ and $\mu_e \gg 1$ the refraction index equals

$$\frac{kc}{\omega} = \frac{\Omega_e/\omega_e}{[(2\pi\varkappa_e)^{1/2} s^3 (s - \omega/\omega_e)]^{1/2}} \qquad (12)$$

The conclusion about the possibility of wave propagation in plasma with a greater density at $\omega \approx \omega_e$ was made by Drummond [1], who obtained expression 10. (Formula 10 was already obtained in Gershman's work [2]. Let us note that the numerical results for kc/ω, obtained in report 1 from formula 10, are erroneous, i.e. they lie outside the applicability area of expression 10 either condition $\mu_e \leq 1$ away from frequency $\omega = \omega_e$ is disrupted, or condition $|x| = |1 - \omega_e/\omega| \varkappa_e^{3/2} \gg 1$ close to $\omega = \omega_e$).

Expressions 9 and 11 were obtained in Stepanov's work [3], and expression 12 were obtained in Ramashvili and Rukhadze's report [4], and with the consideration of collisions, they were obtained in the report by Demidov and Frank-Kamenetskiy [5] also.

An analysis of equation 1 without decomposition by degrees μ_e or $1/\mu_e$ for plasma with an arbitrary ratio of gas kinetic pressure to the magnetic pressure was mentioned in the report by Dnestrovskiy and Kostomarov [6], in which the appearance of new branches for the index of refraction at $\omega \approx s\omega_e$ was indicated, and the entire picture of the behavior of refraction indices is explained depending on the frequency. Let us note that the results of numerical calculations of the refraction index in the area $\omega \approx s\omega_e$, given in report 6 are incorrect, i.e. they belong to the case $\Omega_e \lesssim \omega_e$ and lie outside of the area of applicability of the initial dispersion equation.

In the low frequency area at $\omega \approx s\omega_i$ and $\mu_e \ll 1$ we will obtain the following expression for the frequency from equation 1.

$$\frac{\omega - s\omega_i}{\omega_i} = -\frac{\mu_i s (\exp - \mu_i) I_s(\mu_i)}{\mu_i + \varkappa_i m_i/m_e} \qquad (13)$$

The schematic behavior of the frequencies depending on $k\rho_i$ is shown in Fig. 1 for which it is necessary to replace ω_e by ω_i and ρ_e by ρ_i.

3. High-Frequency Longitudinal Oscillations

We will now go to the investigation of equation 2 in the case of high-frequency (electronic) oscillations. If the refraction index is great $(kc/\omega)^2 \approx \mu_e/\varkappa_e \gg 1$, then in equation 2 a branch of longitudinal oscillations, whose dispersion equation $\varepsilon_{11} = 0$, will be presented in form of the following , can be separated

$$\omega_e^2/\Omega_e^2 = f(\omega) = \frac{\omega_e^2}{\omega^2}\sum_{n=-\infty}^{\infty}\frac{n^2(\exp-\mu_e)I_n(\mu_e)}{\mu_e}\frac{\omega}{\omega-n\omega_e}. \quad (14)$$

As is evident from Fig. 2, on which the form of the function $y = f(\omega)$ is shown schematically, this equation has solutions

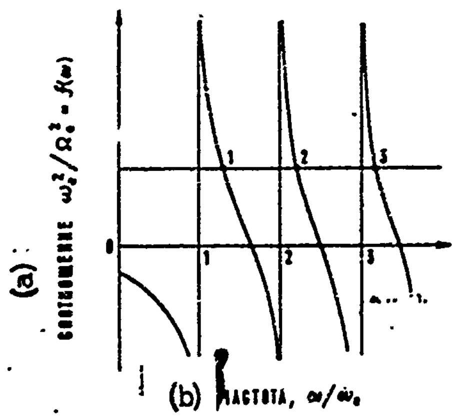

Fig. 2.
KEY: (a) ratio; (b) frequency.

$\omega = \omega_s(k)$ ($s = 1, 2, \ldots$) corresponding to intersection points 1, 2, ... of curves $y = f(\omega)$ with a straight line $y = \omega_e^2/\Omega_e^2$. In the area $\mu_e \ll 1$ from (14) we will obtain

$$\frac{\omega - s\omega_e}{\omega_e} = \frac{sI_s(\mu_e)/\mu_e}{\omega_e^2/\Omega_e^2 - 1/(s^2-1)} \; (s = 2, 3, \ldots) \quad (15)$$

or $\omega = \omega_0(1 + \xi)$, where

$$\omega_0 = (\Omega_e^2 + \omega_e^2)^{1/2}, \quad \xi = 3k^2 v_e^2 \Omega_e^2/2\omega^2(\omega^2 - 4\omega_e^2) \quad (16)$$

If $\mu_e \gg 1$, then for frequencies close to $s\,\omega_e$, we will obtain the expression

$$(\omega - s\,\omega_e)/\omega_e = s\,\Omega_0^2/\omega_e^2\,(2\pi\mu_e^2)^{1/2} \qquad (16a)$$

The behavior of frequencies $\omega = \omega_s(k)$ is shown schematically in Fig. 3 at $\Omega_e^2/\omega_e^2 < 3$ and in Fig. 4 $\ell^2 < \Omega_e^2/\omega_e^2/ + 1 < (\ell + 1)^2$. As is evident from Fig. 4, frequencies $\omega_s(k)$ at $s < \ell$ decrease monotonrically upon an increase of k, tending $\mu_e \geq 1$ and $s < \ell$ toward $s\,\omega_e$. Frequency $\omega_1(k)$ ($\ell\,\omega_e < (\Omega_e^2 + \omega_e^2)^{1/2} < (\ell + 1)\,\omega_e$) at low μ_e rises with an increase of k, if $\ell > 1$, it reaches maximum at $k\varrho_e \approx 1$, then it monotonrically decreases, tending toward $\ell\,\omega_e$. At $\ell = 1$, this frequency monotonrically decreases, approaching ω_e with a rise in $k\varrho_e$. Frequencies greater than $\omega_s(k)$ at $s > \ell$ increase at low $k\varrho_e$, reaching maximum at $k\varrho_e \approx 1$, and then again approach $s\,\omega_e$. Expressions 15 $s = \ell$ or $s = \ell + 1$ and equation 16 are inapplicable, when the hybrid frequency ω_0 is close to $(\ell + 1)\,\omega_e$. In this case, maintaining the components in equation 14 with $n = \pm 1$ and the resonance member $n = \ell$ or $n = \ell + 1$, we will obtain

$$\omega = \omega_0(1 + \xi) \qquad (17)$$

where

$$\xi = \xi_{\pm} = \frac{1}{2}\left(\frac{s\omega_e}{\omega_0} - 1\right) \pm \frac{1}{2}\left[\left(\frac{s\omega_e}{\omega_0} - 1\right)^2 + \frac{2(s^2 - 1)^2}{s^2\mu_e} I_s(\mu_e)\right]^{1/2} \qquad (18)$$

From equations 17 and 18 it follows, that frequency ω_ℓ, as well

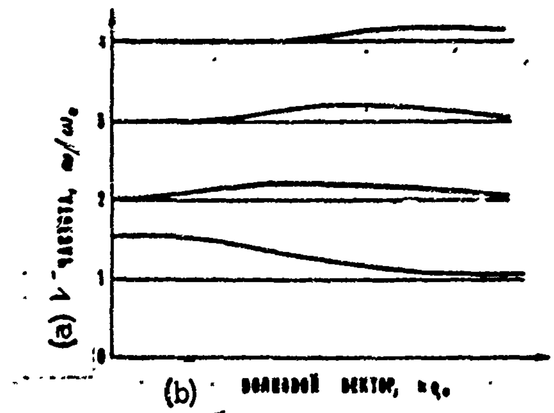

Fig. 3.
KEY: (a) frequency; (b) wave vector.

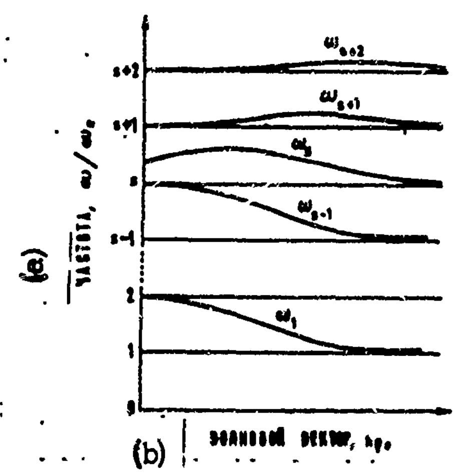

Fig. 4.
KEY: (a) frequency; (b) wave vector.

as the adjoining frequency ω_l-1, or ω_l+1, cannot be so conveniently close to $l\ \omega_e$ at a given $\mu_e \ll 1$ even in the case, when $\omega_0 = l\ \omega_e$ ("slots" by Gross). At $\omega_0 = l\ \omega_e$ and $l = 2$, expressions 17 and 18 were obtained in Gross report [7] and at $l = 2, 3, \ldots$ — in the Sitenko and Stepanov report [8].

The expressions for the refraction index of longitudinal oscillations can be obtained in a clear form at $\mu_e \ll 1$ and $\mu_e \gg 1$. If $\omega \approx s\omega_e$, $\mu_e \ll 1$, then.

$$\frac{kc}{\omega} = \frac{1}{\beta_e s}\left[\frac{2^s s!}{s^s}\left(\frac{\omega_e^2}{\Omega_e^2} - \frac{1}{s^2-1}\right)\left(\frac{\omega}{\omega_e} - s\right)\right]^{1/(2s-2)} \tag{19}$$

At $\mu_e \geq 1$ and $\omega \approx s\,\omega_e$ we have [4]

$$\frac{kc}{\omega} = \left[\frac{\Omega_e^2}{(2\pi)^{1/2}\omega_e(\omega - s\omega_e)s^3\beta_e^3}\right]^{1/3} \tag{20}$$

4. An Extraordinary Wave (High Frequencies)

Let us now examine equation 2 in the general case of plasma with greater density in the field of high frequencies. At $\mu_e \gg \varkappa_e$ in expressions 3 for ε_{11}, ε_{22} and ε_{12}, it is possible to leave only the resonance components $\sim \omega/(\omega - s\,\omega_e)$. Then we will obtain

$$\frac{\omega - s\omega_e}{\omega_e} = -i\varkappa_e\varphi_s(\mu_e) \quad (s = 1, 2, \ldots) \tag{21}$$

where

$$\varphi_s(\mu_e) = s(\exp - \mu_e)\left[(s^2/\mu_e^2 + 1)I_s - I_s'^2/I_s\right] \tag{22}$$

At small μ_e the function $\varphi_s(\mu_e) \approx s\,\mu_e^s/2^s\,(s+1)!$ increase in μ_e, reaching maximum at $\mu_e \approx 1$, and then decreases, approaching $\varphi_s(\mu_e) \approx s/(2\pi)^{1/2}\mu_e^{3/2}$. (At $\mu_e \gg 1$ function $\varphi_s(\mu_e)$ and $f_s(\mu_e)$ coincide asymptotically, therefore the refraction index of the extraordinary and ordinary wave is determined by one and the very same formula 12 [4]).

In the area $\mu_e \lesssim \varkappa_e$ in the tensor ε_{ij} it is necessary to consider also the components with $n = \pm 1$ in addition to the resonance components $\sim \omega/(\omega - s\,\omega_e)$. In this case we will obtain

$$\frac{\omega - s\omega_e}{\omega_e} = -\frac{\varkappa_e \varphi_s(\mu_e)}{1 + 2s\varkappa_e/\mu_e(s+1)} \qquad (23)$$

At $\mu_e \gg \varkappa_e$ this formula coincides with formula 21

In addition to the oscillations branch in the field of low μ_e there is still another branch, whose frequency is determined by expression

$$\frac{\omega - s\omega_e}{\omega_e} = -\frac{s(s-1)[(s+1)\mu_e + 2s\varkappa_e]}{\mu_e(\varkappa_e + \mu_e)} I_s(\mu_e) \quad (s = 2, 3, \ldots) \qquad (24)$$

The frequency of that branch settles away much farther from $s\,\omega_e$ than the frequency determined by formula 21. In the area $\mu_e \gg \varkappa_e$, expression 24 transforms into expression 15 for a frequency of longitudinal waves (in the denominator of equation 15 in the case in question of dense plasma ω_e^2/Ω_e^2 in comparison with $1/(s^2 - 1)$ can be disregarded. In this way waves with the frequencies of equation 24 are the long wave part of the branch of plasma oscillations.

The behavior of frequencies $\omega_s(k)$ depending on $k\varrho_e$ for both branches is shown schematically in Fig. 5.

Equation 2 has been numerically solved at low μ_e and $\omega \approx \omega_e$ in Drummond's report [1], which mentioned the possibility of wave passage with a frequency of $\omega \approx \omega_e$ through dense plasma. (The numerical results of the report [1] are erroneous, because at $\mu_e \approx 1$ for calculations a formula suitable only for $\mu_e \ll 1$ is used). The expressions for the refraction index of an extraordinary wave at

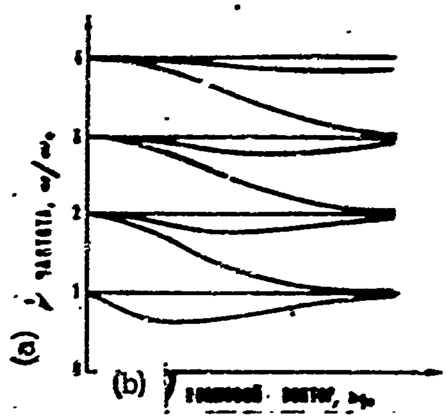

Fig. 5.
KEY: (a) frequency; (b) wave vector.

$\omega \approx \omega_e$ and $\omega \approx 2\,\omega_e$ and $\mu_e \ll 1$ were obtained in report [3]. In the Dnestrovski-Kostomarov report [9] an analysis of equation 2 in the general case ($\mu_e < 1$, $\mu_e \approx 1$, $\mu_e > 1$) was made and shown for the existence of new oscillation branches at $\omega \approx s\,\omega_e$. Let us note, that the numerical calculations of the refraction index, developed in report 9 at $\omega \approx s\,\omega_e$ are invalid, i.e. for them condition 4 of the applicability of the expressions 3 for ε_{ij} used in this case is disrupted. Schematic graphs for $k\,c/\omega$, which correspond to waves with the frequencies equation 21, are given in the Demidov report [10].

5. Low-Frequency Plasma Oscillations

In case of low frequency plasma oscillations ($\omega \ll \omega_e$) equation $\varepsilon_{11} = 0$ has the following form at $\mu_e \ll 1$

$$\frac{\omega_i^2}{\Omega_i^2}\left(1+\frac{\Omega_e^2}{\omega_e^2}\right) = f(\omega) = \frac{\omega_i^2}{\omega^2}\sum_{n=-\infty}^{\infty}\frac{n^2 \exp(-\mu_i)\, I_n(\mu_i)}{\mu_i}\,\frac{\omega}{\omega - n\omega_i} \qquad (25)$$

It is obvious that equation 25, as well as equation 14 has a number of solutions $\omega = \omega_s(k)$ ($s = 1, 2, 3, \ldots$), lying in the interval

$s\,\omega_i < \omega_s < (s+1)\,\omega_i.$

In case of long wave oscillations ($k\rho_i \ll 1$) we find from equation 25

$$\frac{\omega - s\omega_i}{\omega_i} = \frac{s I_s(\mu_i)/\mu_i}{(\omega_i^2/\Omega_i^2 - 1)(s^2-1)} \quad (s = 1, 2, \ldots) \tag{26}$$

or

$$\omega = \omega_k (1 + \xi) \tag{27}$$

where

$$\omega_k = [\omega_i^2 + \Omega_i^2/(1 + \Omega_e^2/\omega_e^2)]^{1/2}$$

$$\xi = \frac{3\Omega_i^2 k^2 \rho_i^2}{2\omega_k^2(1 + \Omega_e^2/\omega_e^2)}\left[\frac{1}{\omega_k^2 - 4\omega_i^2} + \frac{T_e(\omega_k^2 - \omega_i^2)}{4T_i\,\omega_i^2\,\omega_e^2}\right]$$

Expression 27 for a plasma oscillation frequency at $T_\alpha = 0$ was derived by Koerper [11]. A consideration of the thermal movement of plasma particles, which leads to the limitation of refraction indices of an unusual wave at $\omega \to \omega_k$, is given in report 3, 12, and 13. (Let us note, that in report 3 the thermal movement of electrons was not considered; in report 12 the thermal movement of ions was not considered; in addition the coefficient at N^4 in formula 12 of report [12] should be multiplied by 2.)

In the area of short wave oscillations ($\mu_i \gg 1$) from eq. 25, we have

$$\frac{\omega - s\omega_i}{\omega_i} = \frac{s\,\Omega_i^2}{\omega_i^2(1 + \Omega_e^2/\omega_e^2)(2\pi\mu_i^3)^{1/2}} \tag{28}$$

The dependence of the frequencies $\omega_s(k)$ on $k\rho_i$ is schemtically represented in Fig. 3 and Fig. 4 (it is only necessary to make a replacement of ω_e by ω_i ρ_e by ρ_i and ω_0 by ω_k).

If the hybrid frequency ω_k is close to s ω_i then expressions 26 and 27 are inapplicable. In this case

$$\omega = \omega_k (1 + \xi_\pm) \tag{29}$$

where

$$\xi_\pm = \frac{1}{2}\left(\frac{s\,\omega_i}{\omega_k} - 1\right) \pm \frac{1}{2}\left[\left(\frac{s\,\omega_i}{\omega_k} - 1\right)^2 + \frac{2(s^2-1)^2}{s^2 \mu_i} I_s(\mu_i)\right]^{1/2} \tag{30}$$

6. The Extraordinary Wave (Low Frequencies)

Let us first of all mention, that eq. 2 in a hydrodynamic approximation ($\mu_e \ll 1$, $\mu_i \ll 1$) has a solution $\omega \ll \omega_e$ in the area corresponding to a magnetic audio wave

$$\omega = kV_A = \omega_i \frac{k\rho_i}{\mu_i^{1/2}} \tag{31}$$

where

$$V_A = H_0/(4\pi n_0 m_i)^{1/2} \tag{31a}$$

On the other hand, maintaining the resonance components in ε_{ij}, we will obtain

$$\frac{\omega - s\,\omega_i}{\omega_i} = -\frac{\mu_i \varphi_s(\mu_i)}{1 - \mu_0/\mu_i}; \quad \mu_0 = \mu_i \frac{2s^2}{s+1} \; (s = 1, 2, \ldots) \tag{32}$$

where function $\varphi_s(\mu_i)$ is determined by formula 22, or (at low μ_i)

$$\frac{\omega - s\,\omega_i}{\omega_i} = -\frac{s(s^2-1) I_s(\mu_i)(\mu_i - \mu_s)}{\mu_i(\mu_i - s^2 \mu_e)} \; (s = 2, 3, \ldots) \tag{33}$$

Function 33 has an extremum at points $\mu_i = \mu_\pm$, where

$$\mu_{\pm} = [\varkappa s^2/2(s+1)][s+4 \pm (s^2+8)^{1/2}] \quad (s=2,3,\ldots) \tag{33a}$$

and function 32 -- has in points $\mu_i = \mu^{\dagger} = 2\,s\,\varkappa_i (s = 1,2, \ldots)$.
The branch of oscillations with a frequency of eq. 33 passes in the area $\mu_i \gg \varkappa_i$ into longitudinal oscillations, investigated above (see formula 26).

Expressions 31 and 33 are applicable in the narrow area at $\mu_i \approx s^2 \varkappa_i$. In this case instead of eq. 31 and 33 we will obtain $\omega = \omega_{\pm}$, where

$$\frac{\omega_{\pm} - s\omega_i}{\omega_i} = \frac{1}{2}\left[\left(\frac{\mu_i}{\varkappa_i}\right)^{1/2} - s\right] \pm \frac{1}{2}\left\{\left[\left(\frac{\mu_i}{\varkappa_i}\right)^{1/2} - s\right]^2 + 2s^2(s-1)^2\frac{I_s(\mu_i)}{\mu_i}\right\}^{1/2} \quad (s=2,3,\ldots) \tag{34}$$

$$\frac{\omega_{\pm} - \omega_i}{\omega_i} = \frac{1}{2}\left[\left(\frac{\mu_i}{\varkappa_i}\right)^{1/2} - 1\right] \pm \frac{1}{2}\left\{\left[\left(\frac{\mu_i}{\varkappa_i}\right)^{1/2} - 1\right]^2 + \frac{\mu_i^2}{2}\right\}^{1/2} \quad (s=1) \tag{34'}$$

At an increase in differential $|(\mu_i/\varkappa_i)^{1/2} - 1|$ formulas 34 and 34' change respectively into formulas 31, 33 and 31, 32.

$$\frac{\omega_{\pm}^{\dagger} - s\omega_i}{\omega_i} = (s+1)^{1/2} I_s(\mu_i)\, \varphi_{\pm}^{\dagger}(x) \tag{35}$$

where

$$\varphi_{\pm}^{\dagger}(x) = x \pm (x^2+1)^{1/2}$$
$$x = s(s+1)^{1/2}(\mu_i - \mu_0)/\mu_0^2$$

Formulas 35 at $|x| \gg 1$ changes into formulas 32 and 33.

The dependence of frequencies $\omega_s(k)$, determined by formulas 31-36, on the wave vector k is shown in Fig. 6. Let us discuss the individual sections of curves 1, 2, 3, ... in Fig. 5 in greater detail. Curve 1 in area $\mu_i < \varkappa_i$ corresponds to magnetic audio oscillations with the frequency of eq. 31. In the area $\mu_i \approx \varkappa_i$ the frequency of this wave is determined by formula 34' for ω_+ (in this

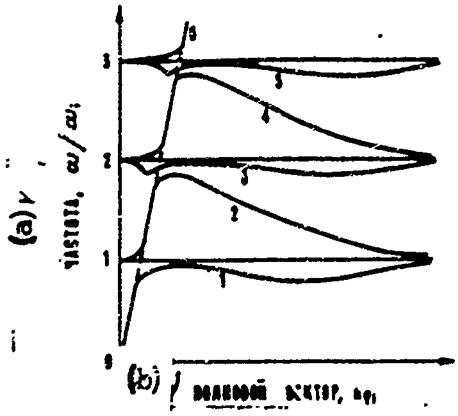

Fig. 6.
KEY; (a) frequency; (b) wave vector.

case $\omega - \omega_1 \approx -\varkappa_1^{1/2}\omega_1$ if $|\mu_1 - \varkappa_1| \lesssim \varkappa_1^2$), and at $\mu_1 > \varkappa_1$ ($|\mu_1 - \varkappa_1| \gg \varkappa_1^2$) by formula 32. Curve 1 reaches maximum at $\mu_1 \approx 2\varkappa_1$ (in this case $\omega - \omega_1 \approx -\varkappa_1^2\omega_1$), and then drops. At $\mu_1 \approx 1$ curve 1 has a minimum $\omega - \omega_1 \approx -\varkappa_1\omega_1$). With a further increase of μ_1 curve 1 approaches asymptotically to ω_1.

The frequency of the branch of oscillations corresponding to curve 2 is determined by formulas: 32 at $\mu_1 < \varkappa_1$; at $\mu_1 \approx \varkappa_1$ 34' for ω_+ (in this case $\omega - \omega_1 \approx \varkappa_1^{1/2}\omega_1$, if $|\mu_1 - \varkappa_1| \lesssim \varkappa_1^2$); 31 — in area $\varkappa_1 < \mu_1 < 4\varkappa_1$; 34 for ω_- ($s = 2$) and 33 in the area $4\varkappa_1 < \mu_1 \ll 1$ ($s = 2$). Curve 2 reaches maximum at $\mu_1 = \mu_+ = 6.3\varkappa_1$; (in this case $\omega - 2\omega_1 \approx -\varkappa_1\omega_1$), and then decreases, converting into a branch of longitudinal oscillations, whose frequency tends toward ω_1, when $\mu_1 \gg 1$.

Curves $f = 2s - 1 = 3, 5, 7$, whose form at $\mu_1 < \mu_0$ are determined by formula 33, decrease at low μ_1, reaching maximum at $\mu_1 = \mu_-$ (in this case $\omega - s\omega_1 \approx -\varkappa_1^s\omega_1$), and then they increase. At $\mu_1 \approx \mu_0$ the form of these curves is determined by formula 35 for ω_-^+ ($\omega - s\omega_1 \approx -\varkappa_1^s\omega_1$, if $|\mu_1 - \mu_0| \lesssim \varkappa_1^2$). When

$|\mu_1 - \mu_0| \gg \varkappa_1^2$ the form of the curve is determined by formula 32. That curve reaches maximum at $\mu = \mu^+$ (in this $\omega - s\,\omega_1 \approx - \varkappa_1^{s+1}\omega_1$); it then decreases, reaching minimum at $\mu_1 = \mu_s \approx 1$ ($\omega - s\,\omega_1 \approx - \alpha_s \varkappa_1 \omega_1$), and then approaches $s\,\omega_1$.

Point μ_s is displaced to the right with an increase of s, and value α_s at an increase of s decreases).

Curves $\int = 2\,s = 4, 6, \ldots$ are determined by formulas: 32 at $\mu_1 < \mu_0$, 35 for ω_+ at $\mu_1 \approx \mu_0$ (in this case $\omega - s\,\omega_1 \approx \varkappa_1^s \omega_1$), 33 at $\mu_0 < \mu_1 < \varkappa_1 s^2$ ($|\mu_1 - \mu_0| \gg \varkappa_1^2$); 34 for $\omega_+^\dagger$ at $\mu_1 \approx s^2 \varkappa_1$; 31 at $s^2\varkappa_1 < \mu_1 < (s + 1)^2\varkappa_1$, 34 at $\mu \approx (s + 1)^2\varkappa_1$ (in this case it is necessary to replace s by s + 1 in formula 34, 33 at $(s + 1)^2 \varkappa_1 < \mu_1 \ll 1$ (here it is also necessary to replace s by s + 1). In this area $\mu_1 \gg \varkappa_1$ this curve determines the frequency of plasma oscillations which decreases with an increase in μ_1 approaching $s\,\omega_1$. Curve $\int = 2\,s$ has a maximum at $\mu_1 = \mu_+$ (in this case $\omega - (s + 1)\,\omega_1 \approx - \varkappa_1^{s+1}\omega_1$), which lies below the minimum of curve $\int = 2\,s + 1$ at point $\mu_1 = \mu_-$, where μ_- is determined by formula 33, in which s should be replaced by s + 1.

The obtained expressions for frequencies can be used only at $s^2 \ll \Omega_1^2/\omega_1^2$.

Let us note that for curve sections $\omega(k)$, which decrease with an increase in k, group and phase velocities of the waves are directed in opposite directions.

7. Excitation of Electronic Oscillations When Ions Cyclotron Waves Pass Through Plasma

When an ion cyclotron wave passes through plasma, electrons and ions acquire a relative velocity u in the field after that wave has passed. If $u \gg v_1$, then in the plasma beam instability may originate [14]. We will examine the excitation of extraordinary and ordinary waves in dense plasma investigated above by an ion stream caused by the field of a cyclotron wave. We will assume that the wave length of the excited waves is considerably smaller than the length of an ion-cyclotron wave and the time of developing the instability is considerably less than the time of turning the ions in the magnetic field H_0. Then the value u(t) can be considered as a constant value and the effect of the magnetic field on the perturbed movement of ions may be disregarded.

The contribution of ions into the tensor ε_{ij} in the reading system in which electrons rest, equals

$$\begin{aligned}
\varepsilon_{11}' &= -\frac{\Omega_i^2}{(\omega-ku_x)^2}; & \varepsilon_{12}' = \varepsilon_{21}' &= -\frac{\Omega_i^2 k u_y}{\omega(\omega-ku_x)^2} \\
\varepsilon_{22}' &= -\frac{\Omega_i^2[(\omega-ku_x)^2+k^2u_y^2]}{\omega^2(\omega-ku_x)^2}; & \varepsilon_{13}' = \varepsilon_{31}' &= -\frac{\Omega_i^2 k u_z}{\omega(\omega-ku_x)^2} \\
\varepsilon_{33}' &= -\frac{\Omega_i^2[(\omega-ku_x)^2+k^2u_z^2]}{\omega^2(\omega-ku_x)^2}; & \varepsilon_{23}' = \varepsilon_{32}' &= -\frac{\Omega_i^2 k^2 u_y u_z}{\omega^2(\omega-ku_x)^2}
\end{aligned} \tag{36}$$

Disregarding the components $\propto \varepsilon_{ij}'^2$, we will obtain from the dispersion equation

$$\det[(k_i k_j - k^2\delta_{ij}) + (\omega^2/c^2)(\varepsilon_{ij}+\varepsilon_{ij}')] = 0 \tag{36a}$$

two equations

$$(kc/\omega)^2 - \varepsilon_{33} - \varepsilon_{33}' = 0 \tag{37}$$

$$(\varepsilon_{11}+\varepsilon_{11}')(kc/\omega)^2 - (\varepsilon_{11}+\varepsilon_{11}')(\varepsilon_{22}+\varepsilon_{22}') - \varepsilon_{12}^2 = 0 \tag{38}$$

where ε_{11}, ε_{22}, ε_{33} and ε_{12} are determined by formulas 3, in which it is only necessary to consider the contribution of the electrons.

Let us first of all examine the excitation of a usual wave. Assuming

$$\omega = \omega_s(k) + \varepsilon, \quad |\varepsilon| \ll \omega(k) \tag{39}$$

where $\omega_s(k)$ is a solution of eq. 37 at $\Omega_1 = 0$, determined by formula 6, we will obtain $\omega(k) = k_x u$, at resonance which

$$\varepsilon = \frac{-1 + i3^{1/2}}{2}\left[\frac{m_e k^2 u_z^2 (\omega_s - s\omega_e)^2}{m_i (\exp - \mu_e) I_s(\mu_e) s\omega_e}\right]^{1/3} \tag{40}$$

At $u_z \sim u_x \sim v_e$ $(k\rho_e \approx 1)$ by the order of magnitude $\varepsilon \approx (\varkappa_e^2 m_e/m_i)^{1/3} \omega_s$.

In case of the excitation of a longitudinal-transverse extraordinary wave, whose frequency is determined by formula 21, we will find $\omega_s(k) = k u_x$, in resonance conditions that

$$\varepsilon = [(-1 + i3^{1/2})/2](\varkappa_e^2 m_e/m_i)^{1/3} \varphi_s^{1/3}(\mu_e)[\mu_e(1 - I_s'/I_s)^2 + u_y^2/v_e^2]^{1/3} \omega_e \tag{41}$$

At $u_x \approx v_e$, $\varkappa_e \approx 1$ we have $\varepsilon \approx (\varkappa_e^2 me/m_1)^{1/3} \omega_e$.

Let us now investigate the excitation of longitudinal oscillations. The dispersion equation of these oscillations, $\varepsilon_{11} + \varepsilon_{11}' = 0$, has the form

$$1 - \frac{\Omega_e^2}{\omega^2} \sum_{n=-\infty}^{\infty} \frac{n^2 (\exp - \mu_e) I_n(\mu_e)}{\mu_e} \frac{\omega}{\omega - n\omega_e} - \frac{\Omega_i^2}{(\omega - k u_x)^2} = 0 \tag{42}$$

The component $\alpha\,\Omega_i$ plays a substantial role in equation 42 at $\omega \approx k\,u_x$. Assuming $\omega = k\,u_x + \varepsilon$, we will find that

$$\varepsilon = \frac{i\Omega_i}{\left[(\Omega_e^2/k^2 u_x^2)\sum_{n=-\infty}^{\infty}[n^2(\exp-\mu_e)I_n/\mu_e]\frac{k u_x}{k u_x - n\omega_e} - 1\right]^{1/3}} \qquad (43)$$

Hence it is evident that the excitation of oscillations ($\mathrm{Im}\,\varepsilon > 0$) takes place at $s\,\omega_e < k\,u_x < \omega_s$, where ω_s is the frequency of HF longitudinal oscillations ($\varepsilon_{11} = 0$ at $\omega = \omega_s(k)$). The increment of increase in expression 43 increases at $k\,u_x \to \omega_s$. However, expression 43 can be utilized only at $|\omega_s - k\,u_x| \gg |\varepsilon|$. At $|\omega_\varepsilon - k\,u_x| \ll \varepsilon$ the increment reaches a maximum value

$$\varepsilon = \frac{-1+i3^{1/2}}{2^{4/3}}\,\omega_e\left(\frac{m_e}{m_i}\right)^{1/3}\left[\sum_{n=1}^{\infty}\frac{n^2(\exp-\mu_e)I_n}{\mu_e}\,\frac{\omega_s^4}{(\omega_s^2-n^2\omega_e^2)^2}\right]^{-1/3} \qquad (44)$$

If $\mu_e \approx 1$, $u_x \approx v_e$, then $\varepsilon \approx (m_e/m_i)^{1/3}\omega_e$. At $v_i \ll u_x \ll v_e$ and $\mu_e \gg 1$ formula 44 coincides with the result of report 14.

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