

AD 681701

GUIDELINES FOR MAKING REPAIR
EXPENDITURE DECISIONS

LMI TASK 68-6

November 1968

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LOGISTICS MANAGEMENT INSTITUTE
4701 Sangamore Road
Washington, D. C. 20016

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I. INTRODUCTION

A. BACKGROUND

The decision to repair or replace an item of equipment when it has failed or is malfunctioning has received much attention over the years by the Military Services, the Office of the Assistant Secretary of Defense (I&L), and other government agencies. Uniform criteria have been established by DoD Instructions for certain commercial type equipments such as materials handling equipment and motor vehicles in administrative use. Government-wide policy prevails in certain areas such as office equipment. Each of the Military Services or commodity managers within the Services has issued guidelines or established certain criteria for repair-replace decisions.

There are at least 2 DoD Directives, 4 DoD Instructions, 18 Army Regulations and assorted documents, 6 Navy Documents, 2 Marine Corps Orders, 4 Air Force Regulations and Orders, and 3 DSA Regulations that address the subject in one way or another.¹ The trend has been to achieve a greater degree of uniformity within and among the Military Services in the decision-making process affecting the repair vs. replacement of certain equipment.

¹These documents were identified by an "Ad Hoc Group on Repair Expenditure Limits" established on 25 July 1966 by the Equipment Maintenance and Readiness Council of the Department of Defense. These documents are listed by number, title, and date in Appendix E of the Ad Hoc Group's report submitted on 21 June 1967 and for the convenience of the reader are reproduced in the same format in Appendix I of this report.

The Logistics Management Institute has also addressed the subject of repair vs. replacement of equipment, but from a more fundamental point of view. The LMI study¹ emphasized the repair vs. discard decision at five stages during the equipment life cycle, including the development of design specifications, actual design, initial provisioning, design review, and time of equipment failure. The first four stages consider the decision to either normally repair a given type of equipment or to discard it at time of failure. The last stage presumes that the equipment has been designed and designated as a repairable item and considers the economics of repair vs. replacement with respect to an individual case. It is this latter decision process with which this task is concerned. While the earlier LMI study considered all five stages, emphasis was placed on the benefits of making a correct repair/discard decision during the earlier stages of the equipment life cycle.

During the earlier LMI study, a large number of previously completed studies related to the subject were examined and the more significant ones are identified in the report. Those studies were performed by various military and contractor organizations during the past five to ten years. All of the studies examined dealt with one or more facets of the repair/discard subject, although they did not all deal directly with the repair or replace decision. Most of the studies examined proposed mathematical decision models which appeared to be sound for the specific application for which they were developed. LMI could not recommend any of the decision models, however, as suitable

¹LMI Task 65-15, "Criteria for Repair vs. Discard Decisions," May 1966.

for general application because of one or more of the following reasons:

- (1) Scope of application too restricted.
- (2) Cost factors too detailed or not compatible with current cost accounting practices.
- (3) Decision models too complex making them difficult or costly to apply.

Despite the vast amount of research which has been directed toward the subject and notwithstanding the uniformity in certain areas which has been achieved through DoD and military policy guidance, there still appears to be a certain illusiveness in the repair vs. replace decision-making process. In May 1966 the Subcommittee on Foreign Aid Expenditures of the Senate Committee on Government Operations pointed out that variations still appeared to exist among the Military Services in criteria for repair versus replace decisions and questioned whether DoD policy in this area was adequate.

Prompted by the Senate Subcommittee hearings, the Assistant Secretary of Defense (I&L) in May 1966 requested the Deputy Assistant Secretary of Defense (Equipment Maintenance and Readiness) to prepare for his approval a Work Plan for reviewing repair expenditure limits among the Military Services to determine if further uniformity was feasible. The Work Plan was prepared and submitted to the Assistant Secretary of Defense (I&L) on 13 June 1966. The Work Plan defined the repair vs. replace decision process as one which depended initially on the comparison of two factors--the "repair expenditure limit" and the "cost to repair". Thus, the Work Plan proposed two tasks: 1) Establish uniform criteria for estimating the "cost to repair" consistent with cost accounting instructions issued under DoD

Instruction 7220.14; and 2) review repair expenditure limits and life expectancies for various types of equipment within the Services to determine the feasibility for greater uniformity.

The Work Plan subsequently proposed that Task 1 be accomplished by the Directorate of Maintenance Policy with coordination by the Military Services, and that Task 2 be accomplished by an inter-service ad hoc committee under the guidance of the Equipment Maintenance and Readiness Council.

The Work Plan was approved by the Assistant Secretary of Defense (I&L) on 16 June 1966 and an Ad Hoc Group was subsequently established for the following purpose: "To review current equipment repair expenditure limit policies and criteria among the Military Services and DSA to determine if further uniformity is feasible in the area of commercial design support-type equipment and military design cargo transport vehicles." In addition, the Ad Hoc Group's charter specified certain guidelines regarding the purpose, basis of determination, and use of repair expenditure limits. These guidelines provided the sine qua non for the LMI approach proposed in this report and will be described in detail later in the report.

In order to describe the objectives of this LMI task in proper perspective, it is useful to summarize briefly the findings, conclusions and recommendations of the Ad Hoc Group.

Principal Findings of the Ad Hoc Group

- "1. There is no published DoD general policy on repair expenditure limits which:
 - (a) Defines the purpose of repair expenditure limits,
 - (b) Provides guidance for their development, or

(c) Prescribes the uses of repair expenditure limits.

2. The military departments and DSA have published repair expenditure limits covering a wide range of equipments. The stated objective of these repair expenditure limits are similar in all cases.
3. Significant differences exist among the military departments and DSA in the methodology, including definitions of terms, used to establish and apply repair expenditure limits.
4. Repair versus replace decisions under current guidance vary significantly among military departments for the same or similar equipment in comparable condition."

Major Conclusions of the Ad Hoc Group

- "1. The differences found among the military departments in the application and use of repair expenditure limits result from a lack of general policy guidance.
2. Further uniformity is feasible under current organizations and maintenance concepts in the area of:
 - (a) General policy as to the purpose and use of repair expenditure limits.
 - (b) The methodology for development and application of repair expenditure limits.
3. Complete uniformity of results is not feasible now, and perhaps never will be, due to different uses, environment, and density of equipments among the military departments and DSA. By use of a common methodology, however, it can be expected that like equipment, used in a similar environment, in two or

more services will have comparable repair versus replace decisions applied.

4. Additional uniformity could probably be achieved, however, under different equipment support concepts, e.g.,

Use of standard commercial equipment at posts, camps and stations with common repair criteria and expenditure limits."

Recommendations by the Ad Hoc Group

- "1. Publish a DoD Instruction defining repair expenditure limits, their purpose and use, and prescribing broad guidance for their development and application.
2. Initiate a task for development of a common methodology for the establishment and application of specific repair expenditure limits."

The Ad Hoc Group report containing the above findings, conclusions and recommendations was submitted to the Equipment Maintenance and Readiness Council on 21 June 1967. The Council concurred with the recommendations of the Ad Hoc Group report and subsequently recommended that the Assistant Secretary of Defense (I&L) request the Logistics Management Institute to undertake the task of developing a common methodology for the establishment and application of specific repair expenditure limits. This task was assigned to LMI on 24 October 1967.

B. OBJECTIVES AND SCOPE OF THE TASK

The overall objective as stated in the Task Order¹ is
 "... to develop a common methodology for the establishment

¹A copy of the LMI Task, 68-6, "Guidelines for Making Repair Expenditure Decisions," is included in the report as Exhibit A.

and application of Repair Expenditure Limits which can be used throughout the Department of Defense in making repair versus replace decisions."

To achieve this objective, LMI was requested to assemble the data necessary to develop the relationship between the equipment acquisition price, life expectancy,¹ age, and other factors relevant to the repair versus replace decision; develop guidelines for establishing the economic repair limit for an item or class of material based on these factors; and develop a logic sequence for use by item managers in establishing repair expenditure limits using the economic repair limit as a base. In addition LMI was asked to consider the need for a follow-on task to prepare a handbook for item managers to describe the decision logic sequence.

The objectives of the LMI task and the subsequent approach proposed by LMI in this report are responsive to certain general guidelines provided in the Ad Hoc Group charter regarding the purpose, basis, and use of repair expenditure limits. These guidelines are:

Purpose of Repair Expenditure Limits

1. The objective in establishing equipment repair expenditure limits is to prevent unwarranted expenditures in the repair of equipment.
2. Repair expenditure limits should be set at or near the

¹The term "equipment life expectancy" has no standard definition which is universally accepted. The Ad Hoc Group on Repair Expenditure Limits proposed the following definition: "Life expectancy of an item is the planned average of the ages of the individual items of materiel at the time they probably will be retired from the service." LMI uses this definition of life expectancy with some qualification. See Section D, Chapter III of this report.

critical point where the remaining value of expected performance would be exceeded by further investment in repair cost.

Basis of Repair Expenditure Limits

3. The repair expenditure limit is a quantitative expression of the value of the estimated remaining useful life of the equipment at a point in time based on life expectancy, replacement cost, and other relevant factors, such as anticipated obsolescence and standardization.
4. The decision to repair should be based on the resulting remaining expected performance of the equipment in comparison with acceptable alternatives such as replacement.
5. Individual costs incurred in the past or accumulated repair costs are not a factor for consideration in the decision to repair.
6. Since replacement cost cannot be accurately determined, a standard inventory price will be used which is the average unit investment in the equipment including first destination transportation.¹

Use of Repair Expenditure Limits

7. The use of repair expenditure limits will cause an evaluation and decision in each case where a repair action is needed, before proceeding with repairs.

¹The standard inventory price is defined on Page 10 as proposed by the Ad Hoc Group on repair expenditure limits.

8. Where the estimated cost to repair is less than the repair expenditure limit, repair is normally authorized unless other specific guidance is provided.
9. Where the estimated cost to repair is more than the repair expenditure limit, the facts must be referred to the appropriate inventory manager for a decision based on total knowledge of requirements, asset position, replacement availability and other factors.
10. An estimated cost to repair in excess of the repair expenditure limits will not, of itself, constitute authorization to discard or to otherwise dispose of equipment.

C. STUDY APPROACH

It was recognized at the outset of the task that the normal life expectancy of an item of equipment should itself be determined primarily on the basis of an economic analysis with due consideration given to equipment obsolescence. The Ad Hoc Group report indicated that no common methodology was currently followed by the Military Services in determining equipment life expectancy. The objectives of the task include the development of a methodology for establishing the economic repair limit based on, among other things, the equipment life expectancy. Therefore, LMI considered it necessary to develop a methodology for establishing equipment life expectancy as well as a methodology for establishing the economic repair limit.

The task was pursued in five principal steps as follows:

- (1) Identification of economic factors and allied considerations affecting repair versus replace decisions.

- (2) Examination of equipment cost versus use relationships including the analysis of sample data.
- (3) Development of a methodology for establishing equipment life expectancy based on various cost/use relationships.
- (4) Development of a methodology for establishing economic repair limits based on equipment life expectancy and various cost/use relationships.
- (5) Development of a proposed decision logic network for use by item managers in establishing repair expenditure limits.

The results of our analysis are presented in Section III of this report. The decision logic network is presented in the report before the proposed methodologies for establishing equipment life expectancy and economic repair limits in order to provide the reader with an initial overview of the intended application of such factors.

II. CONCLUSIONS AND RECOMMENDATION

A. CONCLUSIONS

This task called for the development of a common methodology for the establishment and application of repair expenditure limits based on a conclusion by the Ad Hoc Group on Repair Expenditure Limits that further uniformity in this area among the Military Services is feasible and desirable. The task order explicitly required that repair expenditure limits be based on economic repair limits and that the economic repair limit be based on certain factors including equipment acquisition price, life expectancy and equipment age. The methodology proposed in this report is applicable to commercial design support type equipment and military design cargo transport vehicles in accordance with the general guidance given to the Ad Hoc Group on Repair Expenditure Limits. In addition the methodology can also be applied to any other types of equipment or components thereof where the cost/use relationships required to apply the methodology can be approximated with reasonable accuracy.

In developing a methodology compatible with the requirements set forth in the task order, several conclusions were drawn with respect to applying a more disciplined approach in determining equipment life expectancy and economic repair limits. These are:

1. If equipment life expectancy and economic repair limits are to be determined on a sound economic basis, cost vs. equipment use or age behavior patterns must be approximated with reasonable accuracy.

2. Since cost/use behavior patterns associated with different types of equipment vary, no single formula can be applied to all equipments for determining equipment life expectancy and economic repair limits.
3. Repair expenditure limits should be periodically re-calculated to reflect changes in cost/use behavior patterns.
4. Since the methodology developed in this report represents a departure from current practices, the preparation of a handbook for Item Managers should be deferred until the methodology can be applied to specific categories of equipment, tested, and the results compared with the results achieved under current practices.

B. RECOMMENDATION

It is recommended that the Department of Defense establish test applications of the methodology proposed in this report with respect to several selected types of equipment.

III. FINDINGS AND ANALYSIS

A. PRINCIPAL ECONOMIC FACTORS

Four of the principal factors which should be considered in determining equipment life expectancy and economic repair limits are:

- (1) Acquisition cost.
- (2) Support cost.
- (3) Equipment downtime.
- (4) Continued use requirements (or, conversely, equipment obsolescence).

1. Acquisition Cost

The acquisition cost is the standard inventory price of a replacement item less the disposal value, if any, of the item retired from service.

The Ad Hoc Group on Repair Expenditure Limits proposed that the standard inventory price of a unit of equipment be defined in a Department of Defense Instruction as follows:

"The Standard Inventory Price is the published inventory unit price which represents the latest purchase or production cost of the item including first destination transportation costs to the user when the purchase was representative as to quantity, terms and other conditions and which is considered to reflect the probable unit cost of future procurement.

For items, not purchased for over three years, but for which new procurements are anticipated in the future, a new inventory price should be established which represents the estimated price of the quantities anticipated."

2. Support Cost

Support cost may be classified into two basic categories--operating cost and corrective maintenance cost. Operating cost includes all those elements of cost incident to the operation of the equipment and which occur on a relatively constant and repetitive basis such as: fuel or power consumption; normal replacement of consumable components such as tires, batteries, spark plugs, etc.; and normal preventive maintenance. Corrective maintenance cost includes all those costs incident to repair of the equipment due to failure or malfunction including materials, labor, handling, and overhead burden incurred at the repair facility.¹

Support cost is identified in these two categories because operating cost with respect to many equipments may be considered to vary at a constant rate with the use of the equipment, while the rate at which corrective maintenance cost is incurred generally increases with equipment use or age. In any event the aggregate support cost will be considered as a function of equipment use.

¹The terms preventive and corrective maintenance as used in the report are in agreement with the basic definition provided in MIL-STD-778. For a more detailed description of preventive and corrective maintenance, see LMI Report 65-15, "Criteria for Repair vs. Discard Decisions, May 1966."

3. Equipment Downtime

In some cases the cost associated with equipment downtime is significant in determining the economic life of an item of equipment. Equipment downtime can result from either preventive or corrective maintenance. For example, suppose that a special vehicle has a standard inventory price of \$30,000 and has a normal life expectancy of 20 years. Suppose further that during the normal service life the vehicle is inoperative for an aggregate period of two years because of corrective and preventive maintenance, and that while the vehicle is inoperative, another identical vehicle is used to perform the intended function. Thus, one might reason that the acquisition cost of the vehicle should include not only the initial purchase price but also an additional cost of providing a replacement vehicle during periods of non-operation. It might further be reasoned that this additional cost is to the standard inventory price as the downtime is to the operational time during the normal service life of the vehicle. Thus in the example cited the additional cost of providing a replacement vehicle during periods of downtime for corrective and preventive maintenance is \$3,333; and hence, the acquisition cost to provide 20 years of needed service is \$33,333.

It should be recognized that the relationship between cost and equipment use may vary depending on the preventive maintenance concept applied in any given case including the extent and frequency of scheduled overhauls. While the optimum economic life of an item of equipment can be determined for any given scheduled overhaul concept, it is best to consider the total cost associated with different preventive maintenance concepts before establishing the life expectancy.

In some cases the cost attributable to equipment downtime for maintenance may be insignificant, and thus unnecessary to consider in establishing equipment life expectancy. In other cases downtime for maintenance may be considered to vary directly with use of the equipment. In still other cases, downtime for maintenance may occur at an increasing rate with equipment use. In the latter two cases the cost associated with equipment downtime should be considered in establishing equipment life expectancy.

Three categories are therefore considered with respect to equipment downtime for maintenance in developing a methodology for establishing equipment life expectancy. These are:

- (a) Downtime for maintenance insignificant;
- (b) Downtime for maintenance varies at a constant rate with equipment use; and
- (c) Downtime for maintenance occurs at an increasing rate with equipment use.

4. Continued Use Requirements

The continued use of a given type of equipment to perform some intended function must be considered in determining life expectancy. For example, if a given type of equipment is required indefinitely, the optimum life expectancy of a single unit might be determined to be five years. However, if the same type of equipment is known to be required for only seven years, it might be more economical to stretch the life expectancy to seven years or reduce it to three and one-half years. Consideration of the overall time a given type of equipment is expected to be required is actually consideration of equipment obsolescence. Except in those cases where a given type of equipment

is expected to become obsolete after a definite period, continued use of a particular type of equipment will be considered to be unending. Thus, use requirements will be specified in one of two categories--indefinite use or limited use. If the latter is specified, specific limits must be identified; e.g., ten years, one million miles, sixty thousand operating hours, etc.

5. Allied Considerations

In addition to the principal economic factors described above, there are a number of other considerations which should be taken into account by the Item Manager before establishing repair expenditure limits. Some of these factors are:

- (a) Current and projected requirements for the equipment, including mobilization requirements.
- (b) Availability and lead time of replacements for the item.
- (c) Availability or over-stockage of repair parts.
- (d) Requirements for, or feasibility of, modernization or standardization of the equipment.
- (e) Feasibility of revising current preventive maintenance or scheduled overhaul plan in order to achieve a lower rate of support cost.
- (f) Feasibility of obtaining new equipment warranties.
- (g) Current and projected depot workload.
- (h) Feasibility of centralized depot repair or overhaul.

These allied considerations might also impose constraints on the decision to dispose of a given unit of equipment when the

repair cost exceeds the established repair expenditure limit for that category of equipment.

It is not intended that the above areas of consideration represent an exhaustive list of all related areas of concern; nor is it intended to prescribe a precise method of evaluating each allied area of consideration. It is rather intended that the above list illustrates typical allied areas of consideration which should be taken into account by the Item Manager, where applicable and to whatever extent is required, before establishing repair expenditure limits for a category of equipment. Similar allied areas should be considered, where applicable, with respect to individual units of equipment of high value when the repair cost exceeds the established repair expenditure limit.

B. EQUIPMENT COST VERSUS USE RELATIONSHIPS

In order to determine the equipment life expectancy on a sound economic basis and the economic repair limit during that life, certain normal cost/use relationships must be known or at least be capable of approximation. These are:

- Support cost vs. equipment use or age.
- Disposal value vs. equipment use or age.
- Downtime for maintenance cost vs. equipment use or age.

We found in this and previous studies that accumulated support costs related to specific increments of equipment age or use were not readily available with respect to most types of equipment other than motor vehicles. However, for those equipments where the age of individual units can be identified,

sufficient cost data are generated in the repair operation to enable the Item Manager to develop, on a sampling basis, repair cost/equipment age relationships. Similarly other cost/use relationships could be developed on a sampling basis, such as operating costs vs. use, disposal value vs. use, and downtime for maintenance vs. use.

To illustrate the type of cost/use relationship which might be developed, we compiled a data sample from the Monthly Motor Vehicle Operating and Maintenance Costs Report prepared by Bolling Air Force Base. This report provides actual accumulated repair cost and actual mileage at the end of the reporting period for individual vehicles. Figure 1 depicts the cost vs. use relationship for 122 vehicles in the "Light Sedan Automobile" category. The solid line represents the normal cost/use relationship and may be expressed, in the illustration, by the equation:¹

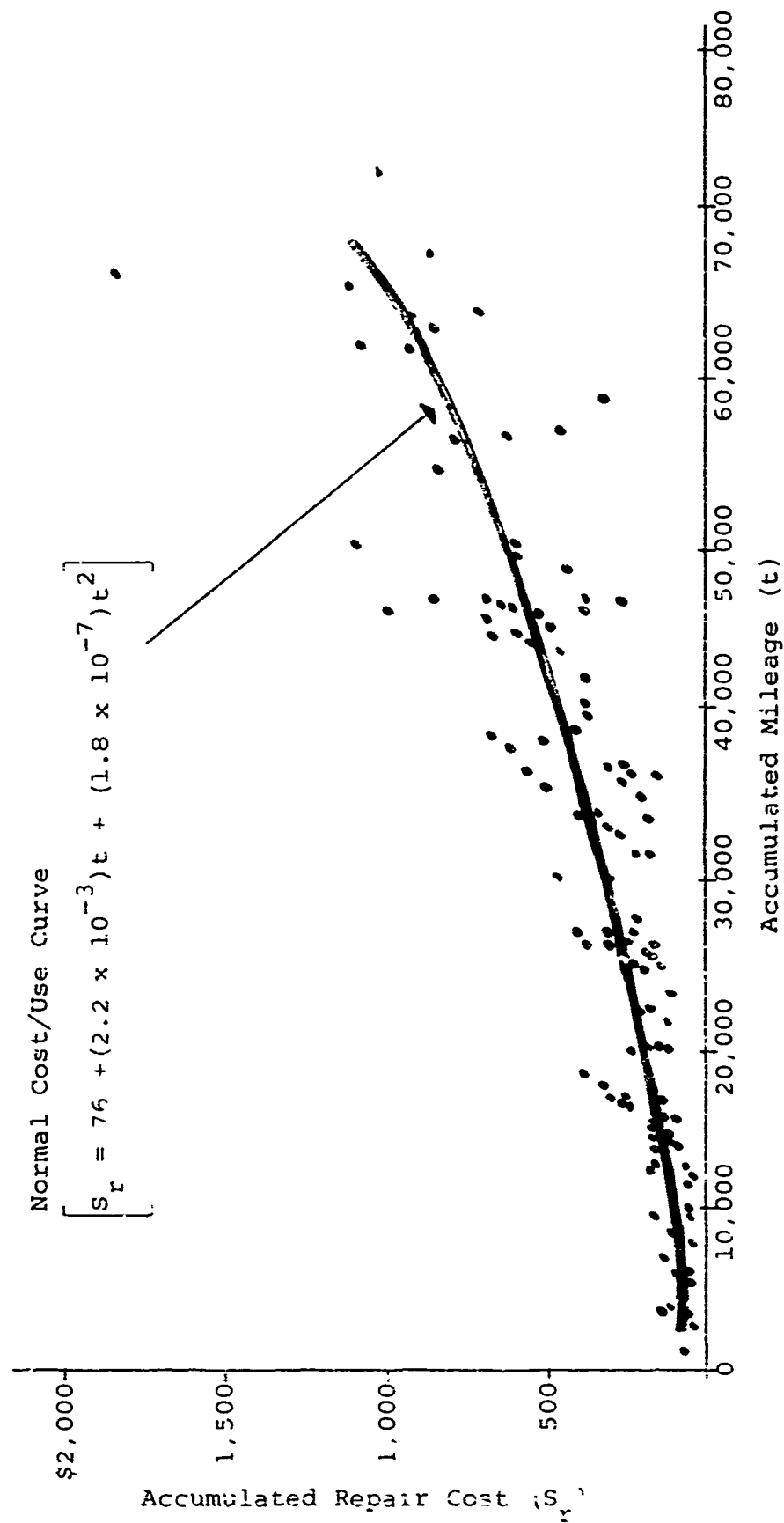
$$S_r = 76 + (2.2 \times 10^{-3})t + (1.8 \times 10^{-7})t^2$$

where S_r = the accumulated repair cost in dollars, and
 t = equipment use in miles.

Data were also obtained on a number of other types of vehicles. Our analysis indicates that no general equation with the same coefficients is feasible to satisfy the cost/use relationship of even similar type equipments. Although the cost/use relationships for "Light Sedan Automobiles" and

¹This equation is presented only for illustrative purposes. The data used were taken from a single source and may not represent a typical sample. The equation was determined by using a Burroughs B 5000 computer program that allows curve-fitting on an arbitrary number of points to a power series of a variable number of terms which resulted in an 89.36% Goodness-of-Fit.

Figure 1 - ACCUMULATED REPAIR COST vs. ACCUMULATED MILEAGE
FOR A SAMPLE OF 122 LIGHT SEDAN AUTOMOBILES



"heavy Sedan Automobiles" may be satisfied by the same general type of equation, the coefficients are unique to each category. Thus, it is necessary to compile separate data samples for each type of equipment for which economic repair limits and equipment life expectancies are desired. However in developing a methodology it is sufficient to examine only a limited sample.

It should be noted that cost/use relationships with respect to most equipments are subject to some degree of control. For example, the extent and frequency of preventive maintenance actions can be manipulated to control the overall support costs vs. equipment use relationship. It is therefore important to recognize that the equipment life expectancy will tend to represent an optimum service life under the maintenance concepts from which the various cost/use relationships are derived. Applying different maintenance concepts will most likely result in different values of equipment life expectancy. The general approach proposed herein can, therefore, be used as a basis for comparing various maintenance concepts in addition to providing a methodology for establishing equipment life expectancy.

In selecting the curve which satisfactorily approximates the cost/use relationship for a given type of equipment, one should use the simplest form available which will be reasonably accurate.

Although each type of equipment may have its own unique set of cost/use relationships, there are several relatively simple types of equations which would appear to satisfy the cost/use relationships for most types of equipment. These equations are described in Appendix II of this report.

C. PROPOSED DECISION LOGIC NETWORK

The decision logic network for establishing repair expenditure limits proposed herein is compatible with the general guidelines regarding the purpose, basis, and use of repair expenditure limits as set forth by the Equipment Maintenance and Readiness Council.¹ The Item Manager will develop repair expenditure limits for categories of equipment under his cognizance and make the results available to field and depot repair personnel for use in determining whether or not a particular item of equipment is authorized for repair when it has failed or malfunctioned. The repair expenditure limits for a specific type of equipment will identify the equipment and indicate a specific dollar limitation on the repair action at various values of equipment age or some other measurement of equipment use.

In developing repair expenditure limits, six principal steps are required by the Item Manager. These are:

- Step 1. Classification of Equipment.
- Step 2. Identification of Constraints.
- Step 3. Establishment of Equipment Life Expectancy.
- Step 4. Establishment of Economic Repair Limits.
- Step 5. Evaluation of Economic and Non-Economic Factors.
- Step 6. Establishment of Repair Expenditure Limits.

Once the repair expenditure limits have been established,

¹These guidelines are set forth in the charter for the Ad Hoc Group on Repair Expenditure Limits and are re-stated on Pages 7, 8, and 9 of this report.

two subsequent steps are required by field or depot repair personnel in applying the repair expenditure limits. These are:

Step 7. Estimate Repair Cost and Disposal Value.

Step 8. Make Repair/Replace Analysis.

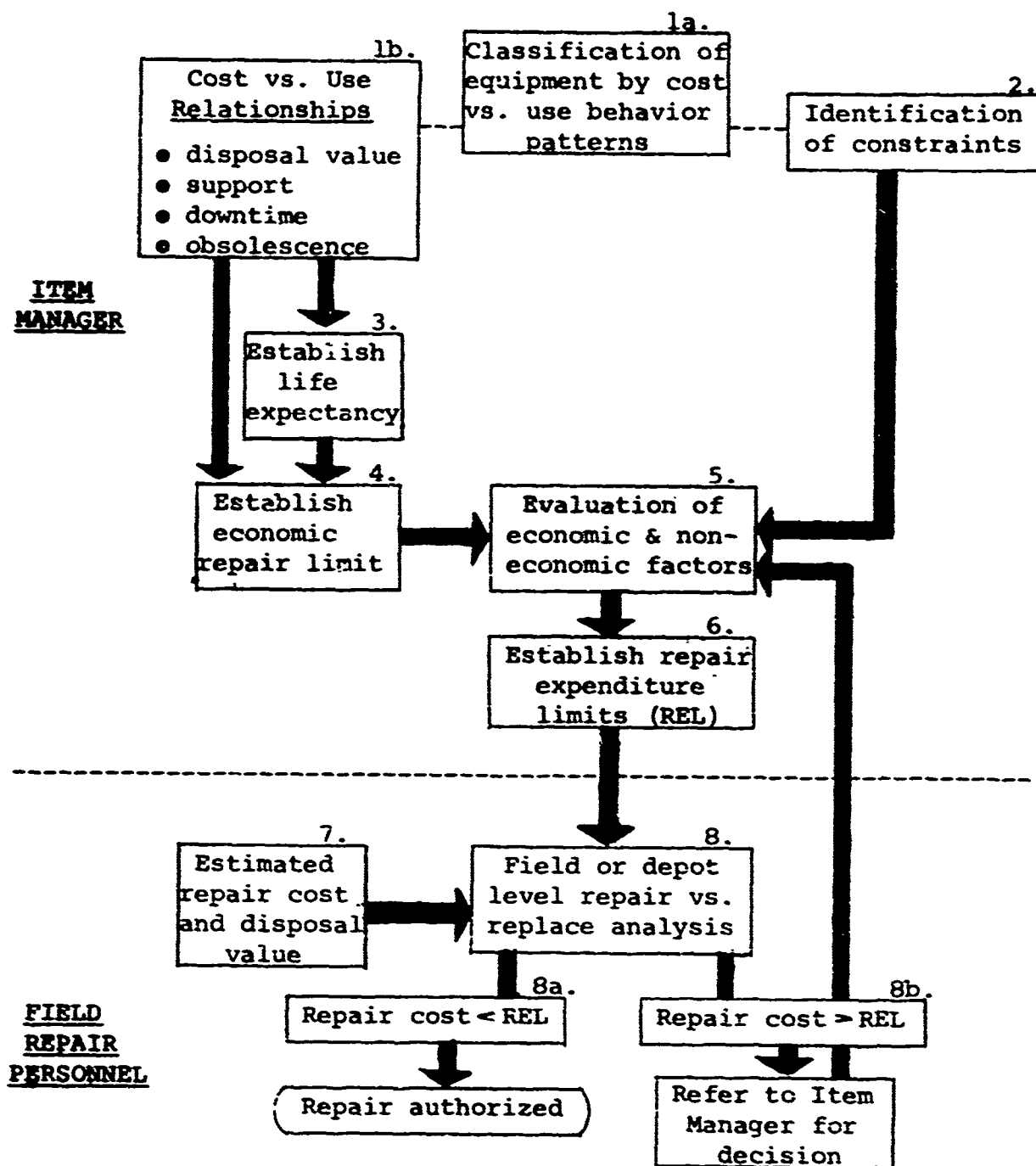
Figure 2 shows a flow diagram of the decision logic network, including the use of repair expenditure limits by repair personnel, which will aid in describing the above steps.

Step 1. Classification of Equipment

Each type of equipment for which repair expenditure limits are to be developed should be classified into categories of similar cost vs. use behavior patterns. In order to do this, certain cost vs. use relationships must be developed (Box 1b in Figure 2). It would seem reasonable to expect that certain cost/use relationships associated with many different equipments could be expressed by equations of the same general form where only the coefficients, constants, or standard inventory price are unique to the particular type of equipment. In such cases, formulae can be developed for a particular category of equipment which will aid the Item Manager in establishing equipment life expectancy and economic repair limits.

Some types of cost/use relationships of relatively simple forms, which are believed to be typical of certain types of equipment, are presented in Appendix II of this report. These cost/use relationships are then grouped into nine different categories to illustrate the methodologies proposed in this report for calculating equipment life expectancy and economic repair limits. (See Table 2, Appendix III and Table 4, Appendix IV.)

Figure 2 - FLOW DIAGRAM OF DECISION LOGIC NETWORK
FOR ESTABLISHING REPAIR EXPENDITURE LIMITS
BY ITEM MANAGERS



In determining the appropriate cost/use relationships for a specific type of equipment, the Item Manager should perform the following tasks:

- (1) Select an appropriate unit of measurement for equipment use, such as years of ownership, miles driven, or hours of operation.
- (2) Compile a sample of data reflecting accumulated support costs at various quantities of equipment use. The sample size should be sufficient to cover a reasonable range of equipment use and extreme variations of support costs within that range. This type of data is generally readily available with respect to most types of vehicles. For other types of equipment, it will probably be necessary to develop the data sample from depot level maintenance records on a case-by-case basis. For new equipments that have no historical data available, cost/use relationships would have to be approximated by analyzing the design characteristics of the equipment. In such cases the cost/use relationships approximated should be reviewed and revised as actual cost data become available.
- (3) Using the data sample compiled in Task 2 above, develop a support cost versus equipment use relationship which expresses accumulated support cost as a function of equipment use. These cost/use relationships can be determined by applying any of a number of curve-fitting programs

readily available in most computer libraries.¹

- (4) Repeating Tasks 2 and 3 above, develop cost/use relationships for a) disposal value vs. equipment use, and b) equipment downtime costs vs. equipment use.
- (5) Determine whether the equipment will be used indefinitely or for a limited period. If the requirement for the equipment is known to be limited or if the equipment is suspect of soon becoming obsolete, then a definite period of use should be identified. Otherwise, the equipment should be considered to be required indefinitely. Where obsolescence is to be considered, the Item Manager might examine the probability that the equipment will become obsolete at different periods (e.g., 2, 5, or 10 years hence), and then identify the period of limited use at that point when the equipment is most likely to become obsolete.

Step 2. Identification of Economic Constraints

Concurrent with the classification of equipment by cost/use behavior patterns, the Item Manager should evaluate the current and projected equipment requirements and support posture for each type of equipment being considered in order to determine any economic constraints which should be imposed on the repair vs. replace decision (Box 2 of Figure 2). Several

¹The form of equation used to describe the normal cost/use relationship may be any one of those discussed in Appendix II of this report, or, if more appropriate, it may be of some other form.

areas are indicated on Page 17 of this report which should be considered as a minimum by the Item Manager.

Step 3. Establishment of Equipment Life Expectancy

Applying the cost/use relationships developed in Step 1 above, the Item Manager will determine for each type of equipment the normal equipment life expectancy. The methodology proposed for accomplishing this is described in Section D and Appendix III of the report. Depending on the nature of the cost/use relationships, formulae for calculating the equipment life expectancy with respect to certain categories of equipment can be obtained from Table 2, Appendix III.

Step 4. Establishment of Economic Repair Limits

Applying the cost/use relationships developed in Step 1 above, and utilizing the equipment life expectancy developed in Step 2, the Item Manager will determine for each type of equipment the economic repair limit at various ratios of equipment age to equipment life expectancy. The methodology proposed for accomplishing this is described in Section E and Appendix IV of the report. Depending on the nature of the cost/use relationships, formulae for calculating the economic repair limit with respect to certain categories of equipment can be obtained from Table 4, Appendix IV. The economic repair limit is determined without consideration of any economic or non-economic constraints which might have been identified in Step 2. Thus the economic repair limit developed here may not necessarily become the repair expenditure limit.

Step 5. Evaluation of Economic and Non-Economic Factors

At this point the Item Manager will consider the economic repair limit in conjunction with any constraints

identified in Step 2 above, which might be applicable to a specific type of equipment. For example, if a sufficient quantity of replacement equipment is not available, the Item Manager might establish repair expenditure limits above the economic repair limits for a period of time necessary to procure additional replacement equipment.

Step 6. Establishment of Repair Expenditure Limits

After consideration of any economic constraints which might be applicable, the Item Manager will establish repair expenditure limits for each type of equipment.

The repair expenditure limits should be published in a concise format for field and depot use, and should contain the following:

- (a) Identification of equipment.
- (b) Normal equipment life expectancy.
- (c) Dates repair expenditure limits are effective.
- (d) Date of next review of the repair expenditure limits by Item Manager.
- (e) A specific dollar limitation on the repair action which may be undertaken without prior approval at various values of equipment age or other measurement of equipment use.

Step 7. Estimate Repair Cost and Disposal Value

When an item of equipment for which repair expenditure limits have been established enters the repair facility, the cost to repair the item is estimated by repair personnel in accordance with Department of Defense Instruction 7220.21.

In addition the disposal value of the item in its unrepaired condition is estimated.

Step 8. Field or Depot Level Repair/Replace Analysis

Repair personnel will compare the estimated repair cost with the repair expenditure limit for the appropriate equipment age. If the estimated repair cost is less than the repair expenditure limit, then repair is authorized without further approval. If not, the decision to repair is referred to the Item Manager.

D. ESTABLISHING EQUIPMENT LIFE EXPECTANCY

1. Definition

The Ad Hoc Group on Repair Expenditure Limits proposed the following definition for equipment life expectancy:

"Life expectancy of an item is the planned average of the ages of the individual items of materiel at the time they probably will be retired from the service."

This Ad Hoc Group definition is not specific with regard to whether or not the "planned average age" is determined on the basis of an economic analysis with due consideration given to equipment obsolescence. In order to insure this for the purpose of establishing equipment life expectancy we have re-defined the term to emphasize the goal of achieving an optimum economic life. Our definition is:

The life expectancy of an item of equipment is the average amount of use per unit which will cause a minimum total system cost in fulfilling a given requirement.

Total system cost as used in the above definition includes the cost of acquiring, operating, and maintaining a number of like items over a specified period.

The amount of use and the requirement for such use is usually expressed in units of elapsed time such as years or months. However, in some cases it may be more appropriate to express the use of equipment in terms of performance delivered, such as miles driven or hours of operation.

2. Objective

The objective is to develop a methodology for establishing the life expectancy of a specific type of equipment within a specified category of equipment in accordance with the above definition of equipment life expectancy. The methodology will consist of (1) mathematical models for calculating the life expectancy of a given item within a specified equipment category, and (2) procedural guidelines for determining the specific equipment category within which a given item of equipment should be classified. The methodology for establishing equipment life expectancy is intended to be applied by Item Managers within the Department of Defense. The equipment life expectancy will subsequently be used as a basis for determining the economic repair limit for an item of equipment when such items require repair during the normal life expectancy period.

3. General Approach

The general approach to establishing equipment life expectancy is to determine the length or amount of service for which a single unit of equipment should normally be retained before disposal in order that the total cost associated with the use of such equipment over a specified period will be

at a minimum. A mathematical model is developed for this purpose in Appendix III.

E. ESTABLISHING THE ECONOMIC REPAIR LIMIT

1. Definition

The economic repair limit is defined in this study as the maximum expense allowed in returning a failed or malfunctioned item of equipment back to serviceable condition at a given point during the normal equipment life expectancy so that the overall cost of retaining the item does not exceed the overall cost of replacing it with a new like item.

2. Assumptions

- (a) The failed equipment has, up to the time of failure, followed a normal pattern of cost/use relationship for the general category of equipment in which it is classified.
- (b) The failed equipment, if repaired, will continue to follow a normal pattern of cost/use relationship for the general category of equipment in which it is classified.
- (c) The failed equipment, if repaired, will provide satisfactory performance.
- (d) The anticipated repair is basically corrective in nature and is not part of the preventive maintenance or scheduled overhaul plan.
- (e) The standard inventory price of a new item is the same as the standard inventory price of the failed item.

(f) The normal cost/use relationships of a new item are the same as the cost/use relationships of the failed item.

(g) A new item will provide satisfactory performance.

3. General Approach

The general approach is to: (1) determine the anticipated cost of retaining the failed item for the duration of its normal life expectancy period, including the cost to repair the item at any given point during that period; (2) determine the anticipated cost of discarding the failed item and replacing it with a new one for the remaining period of normal life expectancy; and (3) by equating these two costs, determine the maximum amount that could be expended for a single repair at a given point during the normal life expectancy period.

F. ILLUSTRATION

A simple hypothetical case is presented in order to illustrate the application of the methodology. Suppose that a special military design cargo vehicle has a standard inventory price of \$6,000 per unit, and because of its unique application has no commercial value, so that the disposal value at any age of the equipment is equal to the salvage value of its components which we will suppose is \$1100. Suppose further that the vehicle is assumed to be required indefinitely and that the cost of equipment downtime is considered insignificant. Finally, suppose that a data sample of accumulated support cost at various values of equipment age has been analyzed and the average support cost/age relationship can be approximated by the equation:

$$S = 100t + 81t^2$$

where S is the accumulated support cost and t is equipment age in years.

Based on the above, the following conditions may be stated:

- Support cost/use relationship: $S = 100t + 81t^2$
- Disposal value/use relationship: $V = 1100$
- Downtime/use relationship: $D = 0$
- Equipment obsolescence: Indefinite use of equipment.

Referring to Table 1 and Table 2 in Appendix III, the vehicle in our example is characteristic of Category II for which a formula for calculating equipment life expectancy has already been developed, namely:

$$e = \sqrt{\frac{U - b}{c_2}}$$

where e = equipment life expectancy

U = standard inventory price, \$6,000 in this case

b = constant disposal value, \$1100 in this case, and

c_2 = the coefficient (81) in the support cost/use equation.

The equipment life expectancy in this case solved by the above equation:

$$e = \sqrt{\frac{6000 - 1100}{81}} = 7.7 \text{ years}$$

Now referring to Table 4 in Appendix IV, the economic repair limit for the vehicle in our example can be determined by the following equation:

$$(ERL) = (1 - r)^2 (U - b) + b - V_a$$

where (ERL) = the economic repair limit

r = ratio of equipment age to equipment life expectancy,
and

V_a = disposal value of the equipment in its unrepaired condition at equipment age (a).

In this example $V_a = b$, so that the equation may be stated:

$$(ERL) = (1 - r)^2(U - b)$$

Applying the above equation the following table can be developed:

$r = \frac{\text{equipment age}}{\text{equipment life expectancy}}$	Economic Repair Limit (ERL)
.0	\$4900
.1	3969
.2	3136
.3	2401
.4	1764
.5	1225
.6	784
.7	441
.8	196
.9	49
1.0	0

Assuming there are no overriding constraints, the repair expenditure limits in this example may be established in accordance with the economic repair limits indicated in the above table.

ASSISTANT SECRETARY OF DEFENSE
Washington, D. C.

Installations and Logistics

DATE: 24 October 1967

TASK ORDER SD-271-80
(TASK 68-6)

1. Pursuant to Articles I and III of the Department of Defense Contract No. SD-271 with the Logistics Management Institute, the Institute is requested to undertake the following task:

A. TITLE: Guidelines for Making Repair Expenditure Decisions

B. SCOPE OF WORK: The objective of the task is to develop a common methodology for the establishment and application of Repair Expenditure Limits which can be used throughout the Department of Defense in making repair versus replace decisions. Three major segments of the task are:

1) To assemble the data necessary to develop the relationship between the equipment acquisition price, life expectancy, age, and other relevant factors to the repair versus disposal decision.

2) To develop guidelines for establishing the economic repair limit for an item or class of material based on equipment acquisition price, life expectancy, age, and other relevant factors.

3) Development of a logic sequence for use by item managers in establishing Repair Expenditure Limits. This decision process will consider the economic repair limit as a base, but provide for consideration of additional factors on a systematic basis.

As a part of this task LMI will consider the need for a follow-on task to prepare a suitable handbook for Item Managers to describe the decision logic sequence. The OSD(I&L) will be advised as to the results of this consideration.

2. SCHEDULE: This task will be completed approximately four months after work is begun.*

s/Thomas D Morris

ACCEPTED s/Barry J. Shillito

DATE October 24, 1967

* Subsequently revised to 31 October 1968

APPENDIX I

DIRECTIVES - INSTRUCTIONS - REGULATIONS

REPAIR EXPENDITURE LIMITS

DEPARTMENT OF DEFENSE

- DoD Directive 3232.1, "Department of Defense Maintenance Engineering Program" dated November 3, 1965.
- DoD Directive 4151.2, "Management of Depot Maintenance Activities" dated October 3, 1960.
- DoD Instruction 4150.1, "Maintenance of Office Machines and Office Furniture", Change 1 dated July 7, 1960.
- DoD Instruction 4150.2, "Repair Limits and Life Expectancies for Materials Handling Equipment" dated April 28, 1955.
- DoD Instruction 4150.4, "Replacement and Repair Guidance and Life Expectancies for Commercial Design Vehicles" dated April 5, 1963.
- DoD Instruction 7220.21, "Uniform Criteria for Repair Cost Estimates Used in Determination of Economical Repair", dated December 23, 1966.

ARMY

- AR 750-1, "Maintenance of Supplies and Equipment-Maintenance Concepts" dated October 30, 1963.
- AR 750-5, "Maintenance of Supplies and Equipment - Organization, Policies, and Responsibilities for Maintenance Operation" dated August 3, 1964. Change 1 dated November 27, 1964.
- AR 750-520, "Maintenance of Supplies and Equipment Modernization Program, Repair or Overhaul Criteria, Corps of Engineers Equipment" dated May 19, 1959; Change 3 dated February 13, 1964.
- AR 750-612, "Maintenance of Supplies and Equipment Maintenance Expenditure Limitations, Signal Corps Equipment" dated August 14, 1961.

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- AR 750-713, "Maintenance of Supplies and Equipment Repair and Overhaul of Army Aircraft" dated February 16, 1962; Change 2 dated November 6, 1962.
- AR 750-808, "Maintenance of Supplies and Equipment Repair Limitations for Medical Equipment" dated April 29, 1960; Change 2 dated September 20, 1962.
- AR 750-3900-1, "Maintenance of Supplies and Equipment Materials Handling Equipment" dated August 24, 1955; Change 2 dated July 13, 1960.
- AR 750-2300-7, "Maintenance of Supplies and Equipment Limits for Repair of Military Type Transport Vehicles" dated June 23, 1961.
- AR 750-3900-9, "Maintenance of Supplies and Equipment-Repair of Ordnance Towed Wheeled Vehicles" dated February 2, 1960.
- AR 750-2300-11, "Maintenance of Supplies and Equipment Maintenance Policy for 1/4-Ton Military-Type Trucks" dated February 23, 1962.
- SB 710-1, "Supply Control Replacement Factors for Army Materiel" dated October 1, 1962.
- AG Ltr, "Replacement and Repair Guidance, and Life Expectancies for Commercial Design Vehicles, FSC-2300 Class" dated October 23, 1963.
- SB 5-75, "Repair Criteria for Corps of Engineers Mechanical and Special Type Equipment" Change No. 1 dated April 16, 1963. Change No. 2 dated December 28, 1965.
- AG Ltr, "Military Type Transport Vehicle Maintenance Policy" dated July 10, 1962.
- AG Ltr, "Depot Maintenance Policy for Military Design Transport Type Tactical Support Wheeled Motor Vehicles" dated September 30, 1966.
- AG Ltr, "Repair Expenditure Limits, Truck 3/4 Ton, 4x4, M-37-M-43" dated April 14, 1965.
- AR 742-2300-1, "Inspection and Classification of Military Type Transport Vehicles" dated June 25, 1958.

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TE ORD 245, 'Overhaul Standards Including Test Requirements for Tank Automotive Vehicles" dated December 23, 1965.

NAVY

NavDocks P-300, "Management of Transportation Equipment dated June 1964.

BuDocks 11200.19B, "Automotive Vehicles and Construction Equipment (Civil Engineering Support Equipment); Mechanized Processing of Annual Reviews for" dated October 15, 1963.

BuDocks Inst. 11200.12D, "Automotive Vehicles, Construction, Weight Handling and Railway Equipment (Civil Engineering Support Equipment); Administration and Control of" dated July 1, 1965.

BUSANDA Instruction 10490.22, "Materials Handling Equipment for the Naval Shore Establishment and Land Based Operating Forces; Administration and Control of" dated October 22, 1965.

NAVSANDA Publication 289, "Materials Handling Equipment Maintenance Manual" dated October 6, 1961.

BuWEPS Instruction 4710.3, "Rework of Damaged Aircraft; Cost Reporting Limitations" dated May 15, 1958.

Marine Corps Order 4710.2B, "Engineer Equipment Repair Criteria" dated May 11, 1965.

Marine Corps Order 11240.50, "Replacement and Repair Guidance and Life-Expectancies for Commercial-Design Motor Vehicles" dated September 12, 1963.

AIR FORCE

AFR 66-27, "Depot Field and Organizational Maintenance Base Self-Sufficiency Program" dated April 18, 1963.

T.O. 36A-1-70, "Maximum Repair Allowances, Condition and Management Codes for USAF Vehicles" dated January 15, 1965.

T.O. 1-1-638, "Expeditious Repair and Disposal of Aerospace Vehicles" dated February 23, 1962.

AFLC Regulation 65-2, "Economic Repair Policy and Maintenance Repair Level Coding" dated June 1964.

DEFENSE SUPPLY AGENCY

DSA Regulation 4145.18, "Utilization, Acquisition, Reporting, Repair and Disposal of Powered and Nonpowered Materials Handling Equipment (MHE) and Storage Aids" dated November 4, 1964.

DSA Regulation 4151.4, "Technical Maintenance Standards for Defense Supply Agency Items Requiring Depot Maintenance" dated October 30, 1964.

DSA Regulation 4500.6, "Administration, Control and Reporting of DSA Equipment" dated August 25, 1966.

TYPICAL COST/USE RELATIONSHIPS OF
RELATIVELY SIMPLE FORMS

1. Support Cost vs. Use

Certain types of support cost may increase at a constant rate with the use of equipment while other types may increase at an increasing rate with equipment use. With regard to the latter, the support cost for a specific category of equipment can normally be approximated by assuming that the cost increases as a power function of the equipment use. Thus, let

$t \geq 0$ represent an appropriate measure of equipment use, such as years, and

S = accumulated support costs after (t) units of equipment use for a specific type of equipment.

Let support costs (S) be approximated by a function of equipment use (t) by:

$$S = c_1 t + c_2 t^n \quad \text{Eq. (1)}$$

where c_1 and c_2 represent constant coefficients applicable to a particular type of equipment, and the exponent $n \geq 0$ has a specific value for a specific type of equipment.

After determining the appropriate value of the coefficients c_1 and c_2 and the exponent n for a category of equipment, an equation of the type above could probably be used to approximate the behavior pattern of most types of equipment.

Our analysis of certain types of commercial vehicles, although not conclusive, suggests that support costs for commercial

vehicles could be approximated by an equation of the above type when $n = 2$, in which case

$$S = c_1 t + c_2 t^2 \quad \text{Eq. (2)}$$

The entire support cost for certain types of equipment may increase at a constant rate with use of the equipment. For example, operating cost rates may be constant and worn out parts or components may be continually replaced, resulting in a relatively constant corrective maintenance cost rate. In such cases the coefficient c_2 may be considered equal to zero, and hence

$$S = c_1 t \quad \text{Eq. (3)}$$

2. Disposal Value vs. Use

As stated earlier, the acquisition cost of equipment is the difference between the standard inventory price and the disposal value. Thus, the manner in which the disposal value varies with equipment use can have a significant effect on the optimum economic life of the equipment. The simplest relationship between disposal value and equipment use occurs in those cases where the disposal value is constant. In such cases:

$$V = b \quad \text{Eq. (4)}$$

where (V) is the disposal value and (b) is a constant.

In many cases, however, the disposal value of a unit of equipment will decrease from its standard inventory price (U) , when it is first put into operation, to a constant value or zero after some quantity or period of use (t) . This is shown in Figure 1 where the value approached is zero. Here the disposal value (V) may be approximated by the general equation:

$$V = \frac{U - b}{1 + qt^k} + b \quad \text{Eq. (5)}$$

where V = the disposal value after some period of use (t) ;

$t \geq 0$ = an appropriate measure of equipment use, such as years;

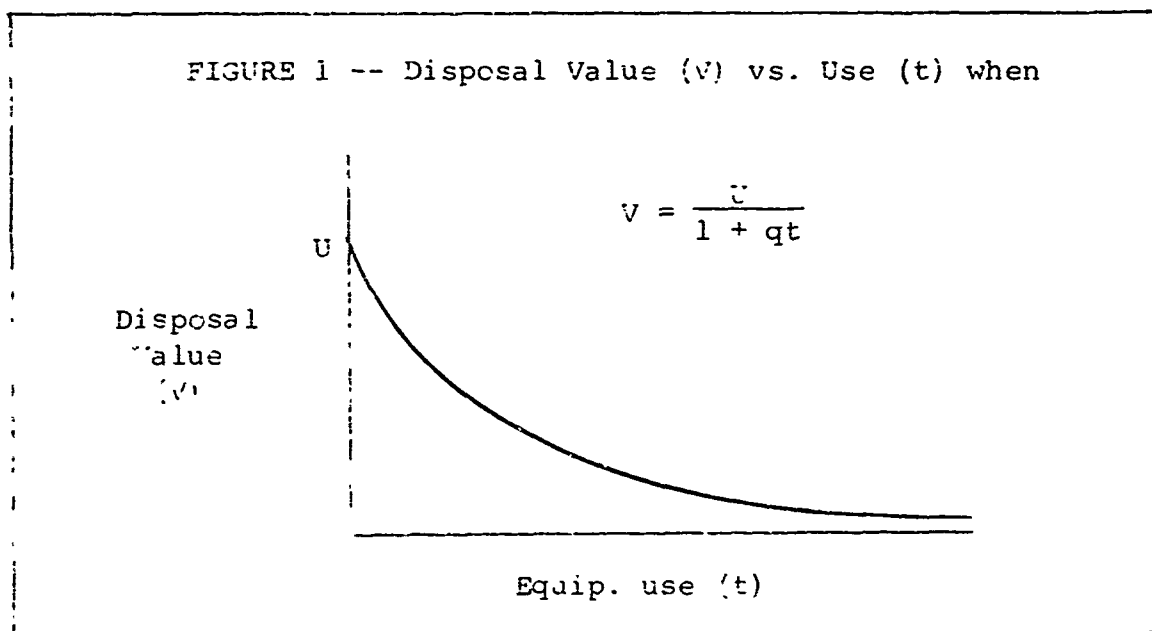
U = the standard inventory price of a single unit of equipment;

q = a constant coefficient applicable to a specific type of equipment;

k = a constant exponent of the variable (t) applicable to a specific type of equipment, and

b = the minimum disposal value of the equipment.

FIGURE 1 -- Disposal Value (V) vs. Use (t) when

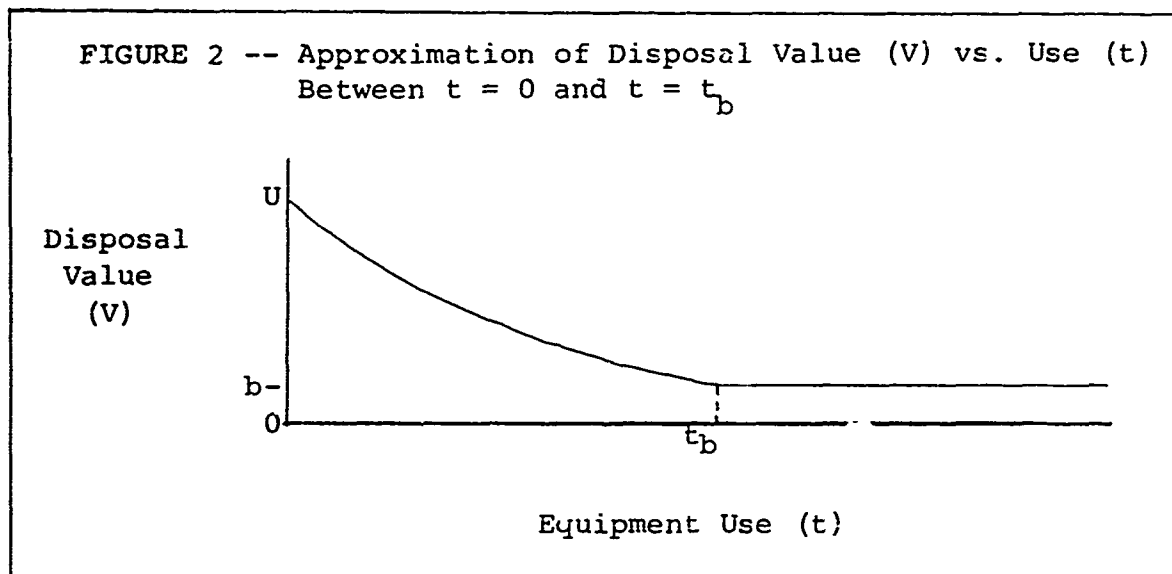


A special case of the above equation may satisfy many types of equipment when $k = 1$ and $b = 0$; hence under these conditions

$$V = \frac{U}{1 + qt} \quad \text{Eq. (6)}$$

The disposal value indicated by Eq. (5) and Eq. (6) is always decreasing and approaches some minimum value (b or zero), but never actually reaches a minimum value. In many cases such behavior is sufficiently accurate to provide a practical approximation of the

disposal value at any unit of use (t). However, in many other cases the minimum disposal value is reached more abruptly and remains constant thereafter. When such is the case the relationship between disposal value and equipment use may generally be approximated by taking the following approach.



Referring to Figure 2, suppose that the disposal value of a given type of equipment is equal to its inventory price (U) when it is first put to use and decreases to some constant value (b) after (t_b) years of use. In addition, when $t \geq t_b$, the disposal value remains equal to (b). The equipment life expectancy (e) can then be developed without knowing the precise relationship between disposal value and equipment use, providing $e \geq t_b$. On the other hand, if $e < t_b$, some approximation of the curve between $t = 0$ and $t = t_b$ is required. This curve might be satisfactorily approximated by Equation (7) after determining the coefficient (q) in the equation:

$$\left. \begin{aligned} V &= \frac{(U-b)(t_b-t)}{t_b+qt} + b, \text{ for } t < t_b \\ V &= b, \text{ for } t \geq t_b \end{aligned} \right\} \text{Eq. (7)}$$

where V = the disposal value of a given type of equipment after (t) units of use;

U = the standard inventory price of a given type of equipment;

t = an appropriate measure of equipment use, such as years;

t_b = a constant unit of equipment use after which the disposal value of the equipment remains constant,

b = the minimum disposal value of the equipment after t_b units of use; and

q = a constant coefficient applicable to a particular type of equipment.

Since the disposal value (V) = U when $t = 0$, and $V = b$ when $t = t_b$, two points on the curve are established (see Figure 2). A third point can be fixed by determining the coefficient (q) which will satisfy the equation at a given value of $0 < t < t_b$. For example, suppose a certain type of vehicle which initially cost \$10,000 depreciates 25% of its initial cost after the first year of use and its disposal value reaches a minimum of \$200 after 10 years of use and remains constant thereafter. Thus, the coefficient for this type of vehicle can be determined from Eq. (7) as follows:

$$V = \frac{(U-b)(t_b-t)}{t_b+qt} + b$$

$$(1 - .25)(10,000) = \frac{(10,000 - 200)(10 - 1)}{10 + q(1)} + 200$$

$$q = 2.1$$

Using the calculated value of the coefficient (q) for a given type of equipment, the disposal value of the equipment may be approximated for all values of (t) by applying Eq. (7). In applying Eq. (7), however, it should be noted that the curve is only an approximation of the relationship between disposal value and equipment use, and, as such, the coefficient (q) should be determined at that point which will result in the best overall approximation.

3. Downtime for Maintenance vs. Use

The costs attributable to equipment downtime for maintenance may or may not be significant, and hence may or may not be considered in determining equipment life expectancy. If such costs are believed to be significant, two cost/use relationships are considered. First, consider the cost attributable to downtime for maintenance when the rate at which equipment downtime occurs is constant throughout the service life of the equipment.

Let D = cost attributable to equipment downtime due to maintenance;

U = the standard inventory price of a single unit of equipment;

$t \geq 0$ = an appropriate measure of equipment use, such as years; and

j = the average ratio of downtime per unit of equipment use.

Thus, jt = the aggregate equipment downtime over a period of use (t).

The additional cost of providing replacement equipment during

periods of equipment downtime for maintenance is to the standard inventory price of the equipment as the equipment downtime is to the equipment operating time.

$$\text{Thus, } D = \left(\frac{jt}{t - jt} \right) U$$

$$D = \frac{jU}{1 - j} \quad \text{Eq. (8)}$$

In many types of equipment, the rate of equipment downtime increases significantly with equipment use. Thus, as the equipment becomes older the frequency and length of the periods of downtime increase, leading to an increasing value of (D). In such cases, this increase in downtime cost should influence the decision regarding equipment life expectancy and repair expenditure limits.

To approximate the future downtime, the nature of the equipment as well as its past history of downtime should be considered. In many cases the accumulated downtime (g) over a period of use (t) may be approximated by the equation:

$$g = pt^m \quad \text{Eq. (9)}$$

where p = a constant coefficient applicable to a specific type of equipment, and (m) = a constant exponent applicable to a specific type of equipment. Thus, in cases where the accumulated downtime can be approximated by Eq. (9) above:

$$D = \left(\frac{g}{t - g} \right) U$$

$$D = \left(\frac{pt^m}{t - pt^m} \right) U \quad \text{Eq. (10)}$$

DEVELOPMENT OF MATHEMATICAL MODEL FOR ESTABLISHING
EQUIPMENT LIFE EXPECTANCY

1. General Approach

The general approach to establishing equipment life expectancy is to determine the length or amount of service for which a single unit of equipment should normally be retained before disposal in order that the total cost associated with the use of such equipment over a specified period will be minimum. Figures 3, 4, and 5 will aid in illustrating the general approach.

Let C = total costs associated with the use of a single unit of a specific type of equipment consisting of acquisition costs (A), downtime for maintenance costs (D), and support costs (S); so that

$$C = A + D + S \quad \text{Eq. (1)}$$

Let t = an appropriate measure of equipment use, such as years, so that each of the elements of total costs can be expressed as a function of (t) .

Thus,

$A = U - f_1(t)$, where U is the standard inventory price and $f_1(t)$ is the disposal value expressed as a function of (t) :

$D = f_2(t)$; and

$S = f_3(t)$.

Therefore,

$C = f_4(t) = U - f_1(t) + f_2(t) + f_3(t)$ and

$$C_e = f_4(e) \text{ when } t = e. \quad \text{Eq. (2)}$$

FIGURE 3

COST VS. USE OF A SINGLE UNIT OF EQUIPMENT

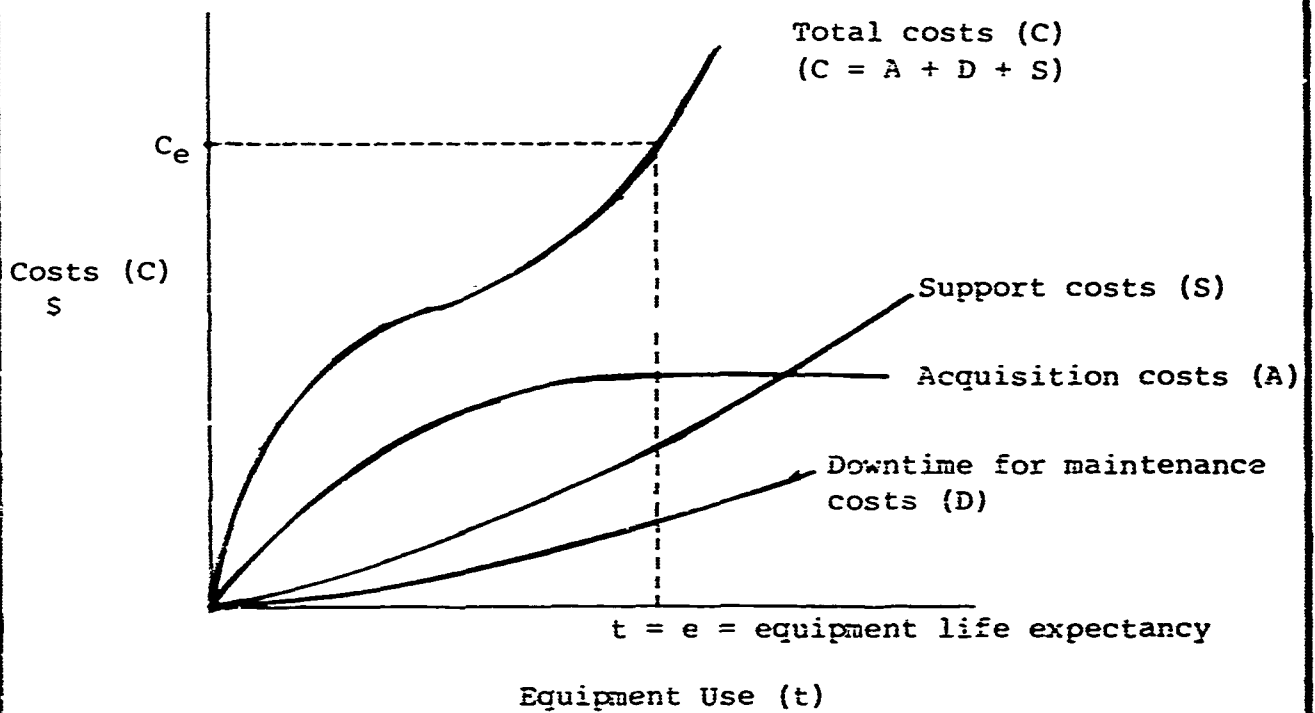


FIGURE 4

RATE OF COST VS. USE OF N UNITS OF EQUIPMENT
CONSECUTIVELY

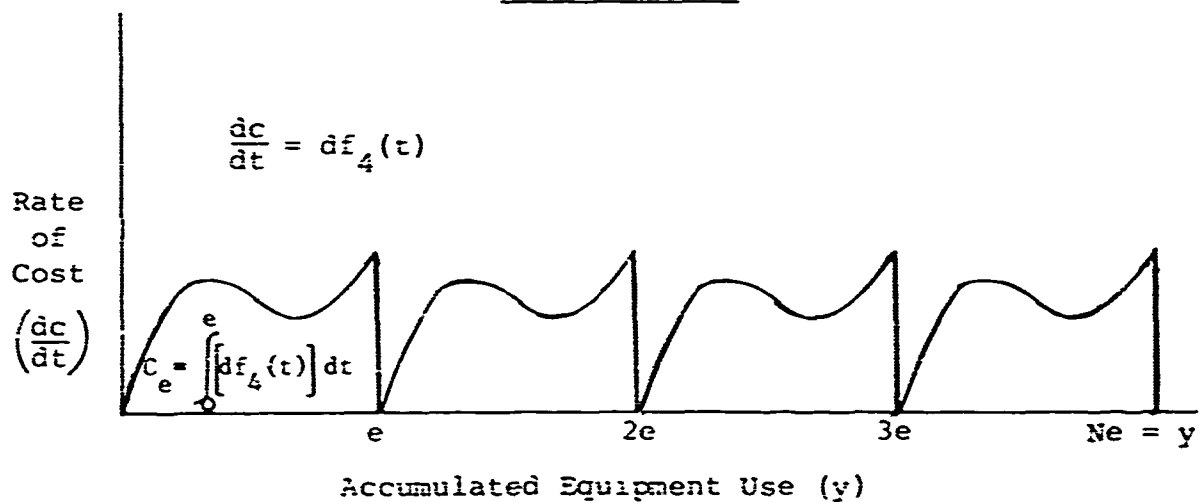


Figure 5

TOTAL COST OF (N) UNITS VS. USE OF EQUIPMENT PER UNIT

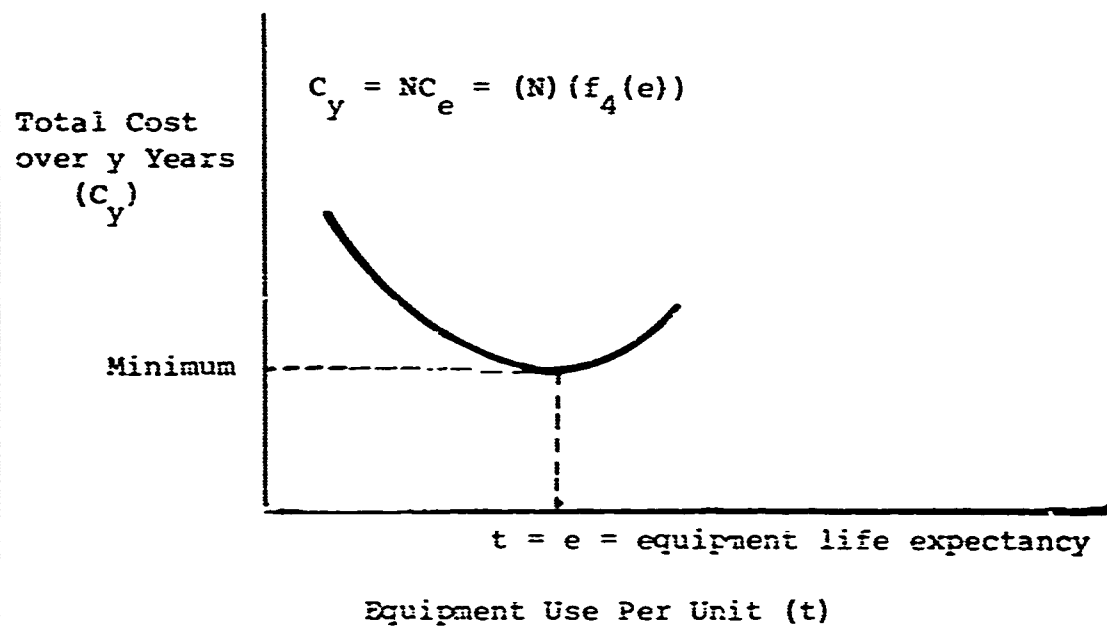


Figure 3 illustrates the total cost (C_e) for a single unit of equipment when $t = e$.

Now, let e = equipment life expectancy of a single unit of equipment which is to be determined, and

N = the number of units which will be required over a period of (y) years,

so that $N = y$. Eq. (3)

Let C_y = the total cost over (y) years,

so that $C_y = NC_e$. Eq. (4)

Figure 4 depicts the rate at which total costs associated with each unit are incurred if each unit is retained for (e) number of years, where the total costs associated with each unit are represented by the area under each respective curve.

Now, if the total costs over (y) years (C_y) are plotted at various values of (t) , the curve will show a minimum cost at some value of (t) as indicated in Figure 5. The corresponding value of (t) which results in a minimum value of C_y , therefore represents the optimum life expectancy of a single unit of equipment. To determine this point, express C_y as a function of (e) ; find the derivative of C_y with respect to (e) , set the derivative equal to zero and solve for the value of (e) .

Thus, substituting Eq. (2) and Eq. (3) into Eq. (4):

$$C_y = \frac{y}{e} [f_4(e)]$$

Now, expressing $f_4(e)$ in its component parts:

$$C_y = \frac{y}{e} [U - f_1(e) + f_2(e) + f_3(e)] \quad \text{Eq. (5)}$$

where

U = the standard inventory price

$f_1(e)$ = the disposal value (V) = $f_1(t)$ when $t = e$

$f_2(e)$ = downtime for maintenance costs (D) = $f_2(t)$
when $t = e$

$f_3(e)$ = support costs (S) = $f_3(t)$ when $t = e$, and

$\frac{V}{e}$ = number of units required over a period of (y)
years if each unit is retained for (e) years.

2. Stratification of Equipment Categories

The general approach for establishing equipment life expectancy, described above, can be applied for any given functions of disposal value, downtime for maintenance costs, and support costs provided such functions are expressed in common units of measurement of equipment use, and provided the use of like equipment is considered to be required indefinitely. The equations resulting from the derivation, however, will be complex for complex functions of disposal value, downtime for maintenance costs, and support costs, and, hence may require the use of a computer for expeditious solution. However, the normal cost/use relationships with respect to many items of equipments should be accurately approximated by relatively simple functions, in which case relatively simple equations can be developed for calculating the equipment life expectancy. It is, therefore, desirable to consider several different cost/use functions which appear to be applicable to many types of equipment and for which relatively simple equations can be developed for calculating equipment life expectancy. Table 1 describes the various cost/use relationships considered under four major categories (A through D).¹ Any combination of these relationships may be

¹The cost/use relationships presented in Table 1 are discussed in some detail in Appendix II.

applicable to a given type of equipment. For example, a given type of equipment may have a cost/use relationship as indicated in Table 1 by category A2, B2, C1, D1.

Formulae are developed in Appendix V for each category that lends itself to a relatively simple derivation. Table 2 summarizes the results and indicates those categories where no simple formula is possible.

3. Equipment Obsolescence

Equipment obsolescence is considered only when the equipment being analyzed has a special application and is known to have a limited use requirement. Therefore, all equipments subjected to equipment life expectancy analyses should be classified into either one of the two following categories:

- a) Indefinite use of like equipment
- b) Limited use of like equipment

If the equipment being analyzed falls into the category of indefinite use, then the procedures for calculating equipment life expectancy (e) remain as previously prescribed and the formulae for calculating (e) are indicated in categories I through VII of Table 2.

On the other hand, if the equipment falls into the category of limited use, then the procedures previously described are modified slightly as follows:

First, calculate the equipment life expectancy under the conditions prevailing if the equipment were to be used indefinitely and identify such life expectancy as (e_1) . In other words, using the same cost/use relationships that

TABLE 1

CATEGORIES OF VARIOUS COST/USE RELATIONSHIPS
CONSIDERED IN DEVELOPING FORMULAE FOR
ESTABLISHING EQUIPMENT LIFE EXPECTANCY

CATEGORY/DESCRIPTION

A. Support Costs vs. Use Relationships

1. $S = c_1 t$

2. $S = c_1 t + c_2 t^2$

3. $S = c_1 t + c_2 t^n$

B. Disposal Value vs. Use Relationships

1. $V = b$

2. $V = \frac{U}{1+qt}$

3. $V = \frac{(U-b)(t_b-t)}{t_b + qt} + b$

4. $V = \frac{(U-b)}{1 + qt^k} + b$

C. Equipment Obsolescence

1. Use of like equipment indefinitely

2. Use of like equipment limited

D. Equipment Downtime for Maintenance Costs
vs. Use Relationships

1. $D = 0$

2. $D = \left(\frac{j}{1-j} \right) U$

3. $D = \left(\frac{pt^m}{t - pt^m} \right) U$

TABLE 2

FORMULAE FOR CALCULATING EQUIPMENT LIFE EXPECTANCY (e) BY
CATEGORIES OF VARIOUS COST/USE RELATIONSHIPS

Category	Combinations of Cost/Use Relationships Described in Table 1	Formulae e =
I	A1, B1, C1, D1 B2 B3	∞ or as long as required
II	A2, B1, C1, D1	$\sqrt{\frac{U-b}{c_2}}$
III	A2, B2, C1, D1	$\sqrt{\frac{U}{c_2}} - \frac{1}{q}$
IV	A2, B3, C1, D1	$\sqrt{\frac{(U-b)(1+k)}{qc_2}} - \frac{t_b}{q}$ Provided $e < t_b$. If $e > t_b$, use $e = \sqrt{\frac{U-b}{c_2}}$
V	A2, B1, C1, D2	$\sqrt{\frac{U-(1-j)b}{(1-j)c_2}}$
VI	A3, B1, C1, D1	$\left(\frac{U-b}{(n-1)c_2} \right)^{\frac{1}{n}}$
VII	A3, B1, C1, D2	$\left(\frac{U-(1-j)b}{(1-j)(n-1)c_2} \right)^{\frac{1}{n}}$
VIII	All other combinations with C1.	e must be calculated for each case.
IX	Any combination with C2.	• Same as the corresponding combination with C1 to determine e_1. • Using $\frac{Y}{e_1}$ as a basis, determine e_2 and e_3. • Calculate and compare C_Y when $e = e_1, e_2$, and e_3. Note: See procedure described in par. 3, Appendix III, p. 6.

would otherwise prevail, calculate the equipment life expectancy if the equipment were to be used indefinitely.

Then, e_1 = the equipment life expectancy calculated under the assumption that the equipment will be used indefinitely.

Now, determine the number of units of equipment required to satisfy a specific use requirement (y) by the equation:

$$N_1 = \frac{y}{e_1}$$

If N_1 is a whole number, then the equipment life expectancy (e) for limited use equipment is equal to e_1 . If N_1 is not a whole number, determine N_2 and N_3

where

$$N_2 = \text{nearest whole number } < N_1; \text{ and}$$

$$N_3 = \text{nearest whole number } > N_1.$$

Now, calculate e_2 and e_3 respectively from the following equations:

$$e_2 = \frac{y}{N_2}$$

$$e_3 = \frac{y}{N_3}$$

The next step is to calculate and compare the total cost (C_y) from the general cost equation (Eq. 5) for each value of (e), that is, ($e = e_1; e_2; \text{ and } e_3$).

Let C_{y_1} = the total cost when $e = e_1$,

C_{y_2} = the total cost when $e = e_2$, and

C_{y_3} = the total cost when $e = e_3$.

Since e_2 and e_3 result in all units of equipment being used equally, the following general equations may be used to calculate C_{y_2} and C_{y_3} :

$$C_{y_2} = \frac{y}{e_2} \left[U - f_1(t) + f_2(t) + f_3(t) \right] \text{ when } t = e_2; \text{ and}$$

$$C_{y_3} = \frac{y}{e_3} \left[U - f_1(t) + f_2(t) + f_3(t) \right] \text{ when } t = e_3.$$

Since (e_1) does not result in all units of equipment being used equally, it should be assumed that all units except the last one will be used equally, and the last one will be used for whatever use requirement remains. Therefore:

$$\begin{aligned} C_{y_1} = N_2 & \left[U - f_1(t_1) + f_2(t_1) + f_3(t_1) \right] \\ & + \left[U - f_1(t_2) + f_2(t_2) + f_3(t_2) \right] \\ & \text{when } t_1 = e_1 \text{ and } t_2 = y - N_2 e_1. \end{aligned}$$

Finally, the equipment life expectancy (e) for an equipment which has limited use requirements is equal to either e_1 , e_2 , or e_3 , whichever results in the lowest total cost (C_y).

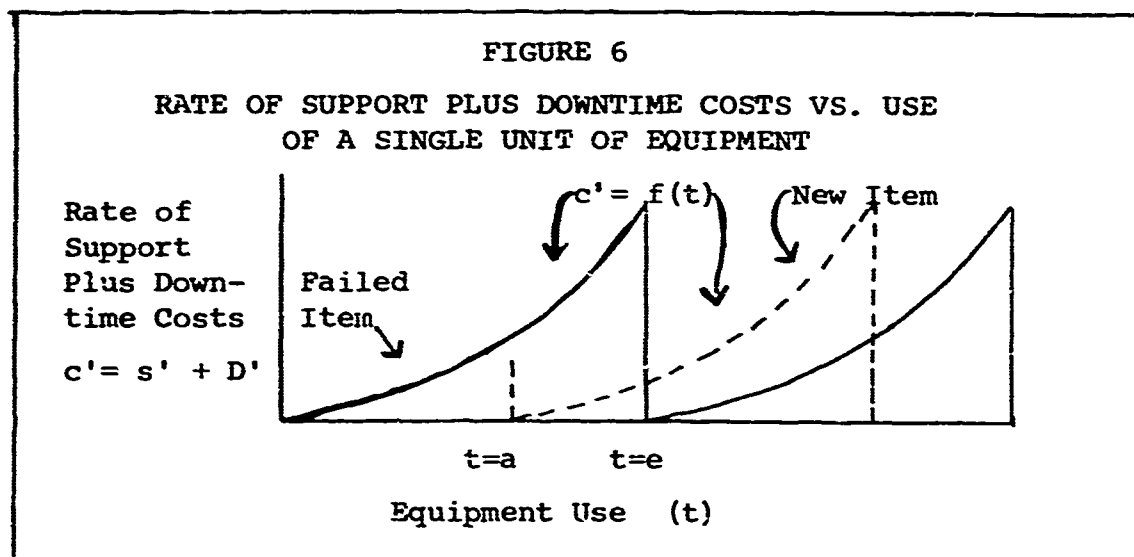
DEVELOPMENT OF MATHEMATICAL MODEL FOR
ESTABLISHING THE ECONOMIC REPAIR LIMIT

1. General Approach

The general approach is to (1) determine the anticipated cost of retaining the failed item for the duration of its normal life expectancy period, including the cost to repair the item at any given point during that period; (2) determine the anticipated cost of discarding the failed item and replacing it with a new one for the remaining period of normal life expectancy; and (3) by equating these two costs, determine the maximum amount that could be expended for a single repair at a given point during the normal life expectancy period.

Referring to Figure 6, suppose that the rate at which support plus downtime costs are incurred is expressed by the equation

$$c' = f(t) = s' + D'$$



downtime costs occur, respectively. Suppose further that the normal life expectancy occurs when $t = e$. Thus, when $t = e$, the item would normally be discarded and replaced by a new item which would result in decreasing the rate of costs (c') to the same value as when $t = 0$. These costs rates are depicted in Figure 6 by the solid lines. The cycle would normally be repeated for as long as the equipment is required.

Now, suppose that the equipment is required indefinitely and that a specific item of equipment fails during a specific life cycle when $t = a$, as shown in Figure 6. If the item is discarded at this point and replaced with a new item, the rate of costs (c') will be as indicated in Figure 6 by the dotted line. In examining the costs of repairing or discarding the failed item, let

C_R = the costs incurred over the remaining life cycle of the failed item, if the failed item is repaired at $t = a$; and

C_D = the costs allocated to the same period if the failed item is discarded at $t = a$ and replaced with a new and identical item.

In examining the repair costs (C_R) first, let

M_a = the costs to repair the failed item when $t = a$; and

V_e = the ultimate disposal value of the failed item or any identical item at the end of its normal life cycle (i.e., when $t = e$). Thus,

$$C_R = M_a + \int_a^e c' dt - V_e \quad \text{Eq. (1)}$$

where $\int_a^e c' dt$ represents the remaining support plus downtime

costs anticipated throughout the normal life of the failed item.

But $c' = s' + D'$, so that

$$C_R = M_a - V_e + \int_a^e s' dt + \int_a^e D' dt \quad \text{Eq. (2)}$$

Now examine the discard costs (C_D) and let

V_a = disposal value of the failed item in its unrepaired condition when $t = a$; and

U = the standard inventory price of a new item of equipment.

If the failed item is discarded and a new item of equipment is placed into service before the normal life expectancy has expired, three elements of costs should be considered: 1) the disposal value of the failed item; 2) the additional acquisition costs of a new item; and 3) that portion of the new item's support and downtime costs which should be allocated to the remaining period of the failed item's normal life expectancy.

The disposal value of the failed item should be determined on the basis of the salvage value of the item in its unrepaired condition at the time of failure. The disposal value/use relationship used to determine the normal life expectancy is generally based on the assumption that the equipment is in satisfactory operating condition at all times. Therefore, care should be taken not to apply the normal disposal value/use function as the only basis of determining the disposal value of the item in its unrepaired condition. Unless the actual value of the item in its unrepaired condition is known, it is best to assume that the disposal value of an unrepaired item at any given time of failure is equal to its minimum salvage value, (b) or $_{-}(0)$.

The additional acquisition cost incurred in placing a new item into service in lieu of retaining the failed item is proportionate to the period of use remaining $(e-a)$ as the life cycle acquisition cost $(U - V_e)$ is to the normal life expectancy period (e) . Thus,

$$\text{Additional Acquisition Costs} = \left(\frac{U - V_e}{e} \right) (e-a)$$

It will be noted from Figure 6 that while the support and downtime costs of a new item between $t = a$ and $t = e$ are significantly less than such costs if the failed item is repaired and retained in service for the same period, these costs are incurred at a greater rate after the normal life expectancy period if the new item is placed into service at $t = a$. Since it cannot be anticipated how often any given item will be subjected to a repair/discard decision, the support and downtime costs associated with a discard decision at $t = a$ should be proportionate to the period of use remaining $(e-a)$ as the total life cycle support and downtime costs (c') are to the normal life expectancy (e) . Thus,

$$\begin{array}{l} \text{Support and downtime costs} \\ \text{associated with discard} \end{array} = \left(\int_0^e c' dt \right) \left(\frac{e-a}{e} \right)$$

Therefore, the costs associated with a discard decision at $t = a$ may be expressed as follows:

$$C_D = (U - V_e) \left(1 - \frac{a}{e} \right) - V_a + \left(1 - \frac{a}{e} \right) \int_0^e c' dt \quad \text{Eq. (3)}$$

But $c' = s' + D'$, so that

$$C_D = (U - V_e) \left(1 - \frac{a}{e}\right) - V_a + \left(1 - \frac{a}{e}\right) \int_0^e s' dt + \left(1 - \frac{a}{e}\right) \int_0^e D' dt \quad \text{Eq. (4)}$$

Let (ERL) = the economic repair limit. M_a from Equation (2) will equal (ERL) when $C_R = C_D$. Thus, substituting (ERL) for M_a and equating Eq. (2) and Eq. (4):

$$\begin{aligned} (\text{ERL}) = & (U - V_e) \left(1 - \frac{a}{e}\right) - V_a + \left(1 - \frac{a}{e}\right) \left[\int_0^e s' dt + \int_0^e D' dt \right] + \\ & V_e - \int_a^e s' dt - \int_a^e D' dt \end{aligned}$$

Let $r = \frac{a}{e}$, so that substituting and regrouping, the general equation for calculating the economic repair limit (ERL) at any equipment age (a) for a given type of equipment having a normal life expectancy (e) is:

$$(\text{ERL}) = (1-r) \left[U - V_e + \int_0^e s' dt + \int_0^e D' dt \right] - \int_a^e s' dt - \int_a^e D' dt + V_e - V_a \quad \text{Eq. (5)}$$

where U = the standard inventory price of a given type of equipment,

V_e = the normal disposal value of a single unit of equipment when the equipment has reached its normal life expectancy (e).

V_a = the disposal value of a particular item of equipment in its unrepaired condition at equipment age (a).

s' = the rate at which operating and maintenance costs are incurred expressed as a function of equipment use (t).

D' = the rate at which downtime for maintenance costs are incurred expressed as a function of equipment use (t);

and

r = the ratio of equipment age to equipment life.

4. Economic Repair Limit by Equipment Categories

Just as the formulae for calculating equipment life expectancy varies for different cost/use relationships, the formulae for calculating the economic repair limit also varies for different cost/use relationships. Table 1, Appendix III indicates certain cost/use relationships which might be considered in developing formulae for determining equipment life expectancy.¹ These same cost/use relationships will be considered in developing formulae for calculating the economic repair limit. It will be convenient, however, to describe the cost/use relationships of Table 1 in terms that are appropriate for application in the general equation for calculating the economic repair limit. Therefore, Table 3 is presented for this purpose.

Formulae for calculating the economic repair limit are developed in Appendix VI for each category of cost/use behavior pattern that lends itself to a relatively simple derivation. Table 4 summarizes the results and indicates those categories where no simple formula is possible.

¹These cost/use relationships are discussed in some detail in Appendix II.

TABLE 3
CATEGORIES OF VARIOUS COST/USE RELATIONSHIPS
CONSIDERED IN DEVELOPING FORMULAE FOR
ESTABLISHING ECONOMIC REPAIR LIMITS

CATEGORY/DESCRIPTION

A. Support Cost vs. Use Relationships

1. $s' = c_1$
2. $s' = c_1 + 2c_2t$
3. $s' = c_1 + c_2t^{(n-1)}$

B. Disposal Value vs. Use Relationships

1. $v_e = b$
2. $v_e = \frac{U}{1 + qe}$
3. $v_e = \frac{(U-b)(t_b - e)}{t_b + qe} + b$
4. $v_e = \frac{U - b}{1 + qe^k} + b$

C. Equipment Obsolescence

1. Use of like equipment indefinitely
2. Use of like equipment limited

D. Equipment Downtime for Maintenance Costs vs. Use Relationships

1. $D' = 0$
2. $D' = \left(\frac{j}{1-j} \right) \left(\frac{U}{e} \right)$
3. $D' = \left(\frac{mpU}{e - pe^m} \right) t^{(m-1)}$

TABLE 4

FORMULAE FOR CALCULATING THE ECONOMIC REPAIR LIMIT (ERL) AT
VARIOUS RATIOS OF EQUIPMENT AGE TO EQUIPMENT LIFE (r)
BY CATEGORIES OF VARIOUS COSTS/USE RELATIONSHIPS

Category	Combinations of Cost/Use Relationships Described in Table 3	Formulae (ERL) =
I	A1, B1, C1, D1 B2 B3	$U - V_a$
II	A2, B1, C1, D1	$(1-r)^2 (U-b) + b - V_a$
III	A2, B2, C1, D1	$(1-r)^2 U + r(3-2r) \left(\frac{\sqrt{c_2 U}}{q} \right) - r(1-r) \left(\frac{c_2}{q^2} \right) - V_a$
IV	A2, B3, C1, D1	$(1-r) (U - V_e - c_2 e^2 r) + V_e - V_a$ NOTE: $V_e = \frac{(U-b)(t_b - e)}{t_b + qe} + b$; see Table 2, Appendix III for value of (e)
V	A2, B1, C1, D2	$(1-r) (U-b) - (1-r)(r) \left(\frac{U}{1-j} - b \right) + b - V_a$
VI	A3, B1, C1, D1	$(1-r) (U-b) \left(1 - \frac{r-r^n}{(n-1)(1-r)} \right) + b - V_a$
VII	A3, B1, C1, D2	$(1-r) (U-b) \left(1 - \frac{r-r^n}{(1-j)(n-1)(1-r)} \right)$ $- \frac{jb(r-r^n)}{(1-j)(n-1)} + b - V_a$
VIII	All other combinations with C1	(ERL) must be calculated for each case
IX	Any combination with C2	(ERL) must be calculated for each case

CALCULATION OF EQUIPMENT LIFE EXPECTANCY (e)
UNDER SPECIFIED CONDITIONS

CATEGORY: A1, B1, C1, D1

GENERAL EQUATION: $C_y = N [U - V + D + S]$

CONDITIONS: $N = \frac{Y}{e}$

$$V = b$$

$$S = c_1 e$$

$$D = 0$$

CALCULATION: $C_y = \frac{Y}{e} [U - b + c_1 e]$ where $U > b$

$$\frac{dC_y}{de} = - \frac{Y(U - b)}{e^2}$$

Let $\frac{dC_y}{de} = 0$ to find value of e when C_y is minimum

$$\text{Thus, } \frac{Y(U - b)}{e^2} = 0$$

$$e = \infty$$

CATEGORY: A1, B2, C1, D1

GENERAL EQUATION: $C_y = N \left[U - V + D + S \right]$

CONDITIONS: $N = \frac{Y}{e}$

$$V = \frac{U}{1 + qe}$$

$$S = c_1 e$$

$$D = 0$$

CALCULATION:

$$C_y = \frac{Y}{e} \left[U - \frac{U}{1 + qe} + c_1 e \right]$$

$$C_y = \frac{YqU}{1 + qe} + Yc_1$$

$$\frac{dC_y}{de} = - \frac{Yq^2U}{(1 + qe)^2}$$

Let $\frac{dC_y}{de} = 0$ to find value of e when C_y is minimum

Thus, $\frac{Yq^2U}{(1 + qe)^2} = 0$

$$e = \infty$$

CATEGORY: A1, B3, C1, D1

GENERAL EQUATION: $C_y = N[U - V + D + S]$

CONDITIONS: $N = \frac{Y}{e}$

$$V = \frac{(U - b)(t_b - e)}{t_b + qe} + b$$

$$S = c_1 e$$

$$D = 0$$

CALCULATION:

$$C_y = \frac{Y}{e} \left[U - \frac{(U - b)(t_b - e)}{t_b + qe} - b + c_1 e \right]$$

where $U > b$ and $q > 0$

$$C_y = \frac{Y(U - b)(1 + q)}{t_b + qe} + Yc_1$$

$$\frac{dC_y}{de} = - \frac{Yq(U - b)(1 + q)}{(t_b + qe)^2}$$

Let $\frac{dC_y}{de} = 0$ to find value of e when C_y is minimum

Thus, $\frac{Yq(U - b)(1 + q)}{(t_b + qe)^2} = 0$

$$e = \infty$$

CATEGORY: A2, B1, C1, D1

GENERAL EQUATION: $C_y = N[U - V + D + S]$

CONDITIONS:

$$N = \frac{Y}{e}$$

$$V = b$$

$$S = c_1 e + c_2 e^2$$

$$D = 0$$

CALCULATION:

$$C_y = \frac{Y}{e} \left[U - b + c_1 e + c_2 e^2 \right] \text{ where } U > b \text{ and } c_2 > 0$$

$$\frac{dC_y}{de} = - \frac{Y(U - b)}{e^2} + Yc_2$$

Let $\frac{dC_y}{de} = 0$ to find value of e when C_y is minimum

$$\text{Thus, } e^2 = \frac{U - b}{c_2}$$

$$e = \sqrt{\frac{U - b}{c_2}}$$

CATEGORY: A2, B2, C1, D1

GENERAL EQUATION: $C_y = N \left[U - V + D + S \right]$

CONDITIONS:

$$N = \frac{Y}{e}$$

$$V = \frac{U}{1 + qe}$$

$$S = c_1 e + c_2 e^2$$

$$D = 0$$

CALCULATION:

$$C_y = \frac{Y}{e} \left[U - \frac{U}{1 + qe} + c_1 e + c_2 e^2 \right]$$

$$C_y = \frac{YqU}{1 + qe} + Yc_1 + Yc_2 e$$

$$\frac{dC_y}{de} = -\frac{Yq^2 U}{(1 + qe)^2} + Yc_2$$

Let $\frac{dC_y}{de} = 0$ to find value of e when C_y is minimum

$$\text{Thus, } (1 + qe)^2 = \frac{q^2 U}{c_2}$$

$$e = \sqrt{\frac{U}{c_2}} - \frac{1}{q}$$

CATEGORY: A2, B3, C1, D1

GENERAL EQUATION: $C_y = N \left[U - V + D + S \right]$

CONDITIONS: $N = \frac{Y}{e}$

$$V = \frac{(U - b)(t_b - e)}{t_b + qe} + b$$

$$S = c_1 e + c_2 e^2$$

$$D = 0$$

CALCULATION:

$$C_y = \frac{Y}{e} \left[U - \frac{(U - b)(t_b - e)}{t_b + qe} - b + c_1 e + c_2 e^2 \right]$$

$$C_y = \frac{Y(U - b)(1 + q)}{t_b + qe} + Yc_1 + Yc_2 e$$

$$\frac{dC_y}{de} = - \frac{Yq(U - b)(1 + q)}{(t_b + qe)^2} + Yc_2$$

Let $\frac{dC_y}{de} = 0$ to find value of e when C_y is minimum

$$\text{Thus, } (t_b + qe)^2 = \frac{q(U - b)(1 + q)}{c_2}$$

$$e = \sqrt{\frac{(U - b)(1 + q)}{qc_2}} - \frac{t_b}{q}$$

CATEGORY: A2, B1, C1, D2

GENERAL EQUATION: $C_y = N [U - V + D + S]$

CONDITIONS: $N = \frac{Y}{e}$

$$V = b$$

$$S = c_1 e + c_2 e^2$$

$$D = \left(\frac{j}{1-j} \right) U$$

CALCULATION:

$$C_y = \frac{Y}{e} \left[U - b + \left(\frac{j}{1-j} \right) U + c_1 e + c_2 e^2 \right]$$

$$C_y = \frac{Y \left(\frac{U}{1-j} - b \right)}{e} + Y c_1 + Y c_2 e$$

$$\frac{dC_y}{de} = - \frac{Y \left(\frac{U}{1-j} - b \right)}{e^2} + Y c_2$$

Let $\frac{dC_y}{de} = 0$ to find value of e when C_y is minimum

$$\text{Thus, } e = \sqrt{\frac{\frac{U}{1-j} - b}{c_2}}$$

$$e = \sqrt{\frac{U - (1-j)b}{(1-j)c_2}}$$

CATEGORY: A3, B1, C1, D1

GENERAL EQUATION: $C_y = N \left[U - V + D + S \right]$

CONDITIONS: $N = \frac{Y}{e}$

$V = b$

$S = c_1 e + c_2 e^n$

$D = 0$

CALCULATION: $C_y = \frac{Y}{e} \left[U - b + c_1 e + c_2 e^n \right]$

$$\frac{dC_y}{de} = - \frac{Y(U-b)}{e^2} + (n-1)Yc_2 e^{n-2}$$

Let $\frac{dC_y}{de} = 0$ to find value of e when C_y is minimum

Thus, $e^n = \frac{U - b}{(n-1)c_2}$

$$e = \left(\frac{U - b}{(n-1)c_2} \right)^{\frac{1}{n}}$$

CATEGORY: A3, B1, C1, D2

GENERAL EQUATION: $C_y = N \left[U - V + D + S \right]$

CONDITIONS: $N = \frac{Y}{e}$

$V = b$

$S = c_1 e + c_2 e^n$

$D = \left(\frac{j}{1-j} \right) U$

CALCULATION:

$C_y = \frac{Y}{e} \left[U - b + \frac{jU}{1-j} + c_1 e + c_2 e^n \right]$

$C_y = \frac{Y \left(\frac{U}{1-j} - b \right)}{e} + Yc_1 + Yc_2 e^{n-1}$

$\frac{dC_y}{de} = - \frac{Y \left(\frac{U}{1-j} - b \right)}{e^2} + (n-1)Yc_2 e^{n-2}$

Let $\frac{dC_y}{de} = 0$ to find value of e when C_y is minimum

Thus, $e^n = \frac{\frac{U}{1-j} - b}{(n-1)c_2}$

$e = \left(\frac{\frac{U}{1-j} - b}{(1-j)(n-1)c_2} \right)^{\frac{1}{n}}$

APPENDIX VI

CALCULATION OF ECONOMIC REPAIR LIMIT UNDER SPECIFIED CONDITIONS

CATEGORY: A1, B1, C1, D1
B2
B3

GENERAL EQUATION:

$$(ERL) = (1-r) \left[(U-V_e) + \int_0^e S' dt + \int_0^e D' dt \right] - \int_a^e S' dt - \int_a^e D' dt + V_e - V_a$$

CONDITIONS:

$$S' = c_1$$

$$D' = 0$$

$$V_e = b; \frac{U}{1+qe} : \frac{(U-b)(t_b-e)}{t_b + qe} + b$$

$$(ERL) = (1-r) \left[(U-V_e) + \int_0^e c_1 dt \right] - \int_a^e c_1 dt + V_e - V_a$$

$$(ERL) = (1-r) \left[(U-V_e) + c_1 e \right] - c_1 e + c_1 a + V_e - V_a$$

$$(ERL) = (1-r) \left[(U-V_e) + c_1 e \right] - c_1 e \left(1 - \frac{a}{e}\right) + V_e - V_a$$

But $\frac{a}{e} = r$, so that

$$(ERL) = (1-r) (U-V_e) + V_e - V_a$$

Put $r = \frac{a}{e}$ and e for this category = ∞ from Table 2, Appendix III

Therefore, $r = 0$ for any value of a and thus

$$(ERL) = U - V_a$$

CATEGORY: A2, B1, C1, D1

GENERAL EQUATION:

$$(ERL) = (1-r) \left[(U-V_e) + \int_0^e S' dt + \int_0^e D' dt \right] - \int_a^e S' dt - \int_a^e D' dt + V_e - V_a$$

CONDITIONS:

$$S' = c_1 + 2c_2 t$$

$$D' = 0$$

$$V_e = b$$

$$(ERL) = (1-r) \left[(U-V_e) + \int_0^e (c_1 + 2c_2 t) dt \right] - \int_a^e (c_1 + 2c_2 t) dt + V_e - V_a$$

$$(ERL) = (1-r) \left[U-V_e + c_1 e + c_2 e^2 \right] - \left[c_1 (e-a) + c_2 (e^2 - a^2) \right] + V_e - V_a$$

$$(ERL) = (1-r) \left[U-V_e + c_1 e + c_2 e^2 \right] - \left[c_1 e(1-r) + c_2 e^2(1-r^2) \right] + V_e - V_a$$

$$(ERL) = (1-r) \left[U-V_e - c_2 e^2 r \right] + V_e - V_a$$

But $e^2 = \frac{U-b}{c_2}$ from Table 2, Appendix III; and $V_e = b$, so that

$$(ERL) = (1-r) \left[U-b - (U-b)r \right] + b-V_a$$

$$(ERL) = (U-b)(1-r)^2 + b-V_a$$

CATEGORY: A2, B2, C1, D1,

GENERAL EQUATION:

$$(ERL) = (1-r) \left[U - V_e + \int_0^e S' dt + \int_0^e D' dt \right] - \int_a^e S' dt - \int_a^e D' dt + V_e - V_a$$

CONDITIONS: $S' = c_1 + 2c_2 t$

$$D' = 0$$

$$V_e = \frac{U}{1+qe}$$

$$(ERL) = (1-r) \left[U - V_e + c_1 e + c_2 e^2 \right] - \left[c_1 (e-a) + c_2 (e^2 - a^2) \right] + V_e - V_a$$

$$(ERL) = (1-r) \left[U - V_e - c_2 e^2 r \right] + V_e - V_a \quad \text{Eq. (1)}$$

$$V_e = \frac{U}{1+qe} \text{ for this category of equipment, and}$$

$$e = \sqrt{\frac{U}{c_2}} - \frac{1}{q} \text{ for this category (from Table 2, Appendix III)}$$

Thus,

$$V_e = \frac{U}{q \sqrt{\frac{U}{c_2}}} = \frac{\sqrt{c_2 U}}{q}$$

$$e^2 = \frac{U}{c_2} - \frac{2}{q} \sqrt{\frac{U}{c_2}} + \frac{1}{q^2}$$

Substituting for V_e and e^2 in Eq(1)

$$(ERL) = (1-r) \left[U - \frac{\sqrt{c_2 U}}{q} - c_2 r \left(\frac{U}{c_2} - \frac{2}{q} \sqrt{\frac{U}{c_2}} + \frac{1}{q^2} \right) \right] + \frac{\sqrt{c_2 U}}{q} - V_a$$

(cont'd)

$$(ERL) = (1-r) \left[U - \frac{\sqrt{c_2 U}}{q} - rU + 2r \frac{\sqrt{c_2 U}}{q} - r \frac{c_2}{q^2} \right] + \frac{\sqrt{c_2 U}}{q} - v_a$$

$$(ERL) = (1-r) \left[U(1-r) - \left(\frac{\sqrt{c_2 U}}{q} \right) (1-2r) - r \frac{c_2}{q^2} \right] + \frac{\sqrt{c_2 U}}{q} - v_a$$

$$(ERL) = (1-r)^2 U - (1-r)(1-2r) \left(\frac{\sqrt{c_2 U}}{q} \right) - r(1-r) \frac{c_2}{q^2} + \frac{\sqrt{c_2 U}}{q} - v_a$$

$$(ERL) = (1-r)^2 U + (3r-2r^2) \left(\frac{\sqrt{c_2 U}}{q} \right) - r(1-r) \frac{c_2}{q^2} - v_a$$

$$(ERL) = (1-r)^2 U + r(3-2r) \left(\frac{\sqrt{c_2 U}}{q} \right) - r(1-r) \frac{c_2}{q^2} - v_a$$

CATEGORY: A2, B3, C1, D1

GENERAL EQUATION:

$$(ERL) = (1-r) \left[U - V_e + \int_0^e S' dt + \int_0^e D' dt \right] - \int_a^e S' dt - \int_a^e D' dt + V_e - V_a$$

CONDITIONS:

$$S' = c_1 + 2c_2 t$$

$$D' = 0$$

$$V_e = \frac{(U-b)(t_b - e)}{t_b + qe} + b$$

$$(ERL) = (1-r) \left[U - V_e + c_1 e + c_2 e^2 \right] - \left[c_1 (e-a) + c_2 (e^2 - a^2) \right] + V_e - V_a$$

$$(ERL) = (1-r) \left[U - V_e + c_1 e + c_2 e^2 \right] - c_1 e \left(1 - \frac{a}{e} \right) - c_2 e^2 \left(1 - \frac{a^2}{e^2} \right) + V_e - V_a$$

But $\frac{a}{e} = r$, so that

$$(ERL) = (1-r) \left[U - V_e - c_2 e^2 r \right] + V_e - V_a$$

CATEGORY: A2, B1, C1, D2

GENERAL EQUATION:

$$(ERL) = (1-r) \left[U - V_e + \int_0^e S' dt + \int_0^e D' dt \right] - \int_a^e S' dt - \int_a^e D' dt + V_e - V_a$$

CONDITIONS:

$$S' = c_1 + 2c_2 t$$

$$D' = \left(\frac{j}{1-j} \right) \left(\frac{U}{e} \right)$$

$$V_e = b$$

$$(ERL) = (1-r) \left[U - b + \int_0^e (c_1 + 2c_2 t) dt + \frac{jU}{(1-j)e} \int_0^e dt \right] - \int_a^e (c_1 + 2c_2 t) dt - \frac{jU}{(1-j)e} \int_a^e dt + b - V_a$$

$$(ERL) = (1-r) \left[U - b + c_1 e + c_2 e^2 + \frac{jU}{1-j} \right] - \left[c_1 (e-a) + c_2 (e^2 - a^2) \right] - \frac{jU}{1-j} + \frac{jUa}{(1-j)e} + b - V_a$$

$$(ERL) = (1-r) \left[U - b + c_1 e + c_2 e^2 + \frac{jU}{1-j} \right] - c_1 e \left(1 - \frac{a}{e} \right) - c_2 e^2 \left(1 - \frac{a^2}{e^2} \right) - \frac{jU}{1-j} \left(1 - \frac{a}{e} \right) + b - V_a$$

But $r = \frac{a}{e}$ so that

$$(ERL) = (1-r) \left[U - b - c_2 e^2 r \right] + b - V_a$$

But $e^2 = \frac{U-(1-j)b}{(1-j)c_2}$ from Table 2, Appendix III so that

$$(ERL) = (1-r) (U-b) - (1-r) (r) \left(\frac{U}{1-j} - b \right) + b - V_a$$

CATEGORY: A3, B1, C1, D1

GENERAL EQUATION:

$$(ERL) = (1-r) \left[U - V_e + \int_0^e S' dt + \int_0^e D' dt \right] - \int_a^e S' dt - \int_a^e D' dt + V_e - V_a$$

CONDITIONS: $S' = c_1 + nc_2 t^{(n-1)}$

$$D' = 0$$

$$V_e = b$$

$$(ERL) = (1-r) \left[U - V_e + c_1 e + c_2 e^n \right] - \left[c_1 (e-a) + c_2 (e^n - a^n) \right] + V_e - V_a$$

$$(ERL) = (1-r) \left[U - V_e + c_1 e + c_2 e^n \right] - c_1 e \left(1 - \frac{a}{e} \right) - c_2 e^n \left(1 - \frac{a^n}{e^n} \right) + V_e - V_a$$

But $\frac{a}{e} = r$ so that

$$(ERL) = (1-r) \left[U - V_e - c_2 e^n r \left(\frac{1-r^{(n-1)}}{1-r} \right) \right] + V_e - V_a$$

But $V_e = b$ and $e^n = \frac{U-b}{(n-1)(c_2)}$ for this category from Table 2,

Appendix III. Thus,

$$(ERL) = (1-r) \left[U - b - \frac{(U-b)r}{n-1} \left(\frac{1-r^{(n-1)}}{1-r} \right) \right] + b - V_a$$

$$(ERL) = (1-r)(U-b) \left(1 - \frac{r-r^n}{(n-1)(1-r)} \right) + b - V_a$$

CATEGORY: A3, B1, C1, D2

GENERAL EQUATION:

$$(ERL) = (1-r) \left[U - v_e + \int_0^e s' dt + \int_0^e D' dt \right] - \int_a^e s' dt - \int_a^e D' dt + v_e - v_a$$

CONDITIONS: $s' = c_1 + nc_2 t^{(n-1)}$

$$D' = \left(\frac{j}{1-j} \right) \left(\frac{U}{e} \right)$$

$$v_e = b$$

$$(ERL) = (1-r) \left[U - v_e + c_1 e + c_2 e^n + \frac{jU}{1-j} \right] - \left[c_1 (e-a) + c_2 (e^n - a^n) \right] - \frac{jU(1-\frac{a}{e})}{1-j} + v_e - v_a$$

$$(ERL) = (1-r) \left[U - v_e + c_1 e + c_2 e^n + \frac{jU}{1-j} \right] - c_1 e(1-\frac{a}{e}) - c_2 e^n \left(1 - \frac{a^n}{e^n} \right) - \frac{jU}{1-j} \left(1 - \frac{a}{e} \right) + v_e - v_a$$

But $\frac{a}{e} = r$ so that,

$$(ERL) = (1-r) \left[U - v_e - c_2 e^n r \left(\frac{1-r^{(n-1)}}{1-r} \right) \right] + v_e - v_a$$

But $v_e = b$, and $e^n = \frac{U-(1-j)b}{(1-j)(n-1)c_2}$ for this category from Table 2,

Appendix III. Thus

$$(ERL) = (1-r) \left[U-b - \frac{(U-(1-j)b)(r)(1-r^{(n-1)})}{(1-j)(n-1)(1-r)} \right] + b - v_a$$

$$(ERL) = (1-r)(U-b) \left(1 - \frac{r-r^n}{(1-j)(n-1)(1-r)} \right) - \frac{jb(r-r^n)}{(1-j)(n-1)} + b - v_a$$

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13. ABSTRACT A methodology is developed for establishing repair expenditure limits during the normal service life of an individual unit of equipment. The methodology allows for the variation of repair expenditure limits with equipment use or age. Included is a decision logic network for item managers in establishing repair expenditure limits with respect to a given category of equipment, and for field repair personnel in applying such limits when an individual unit of equipment requires repair. The methodology includes a method of establishing equipment life expectancy and economic repair limits. Principal factors considered are standard inventory price of equipment, support costs, disposal value, obsolescence and equipment downtime.		

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