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DEPARTMENT OF THE ARMY
Fort Detrick
Frederick, Maryland

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The Preferred Size of Balls for Ball Mills.

by V. A. Olovskiy - candidate of technical sciences.

Gornyy Zhurnal. 122: 1: 30-33, 1948.

In the article of K. A. Razumov, "The rationed feeding of mills with balls *(1," there was given an analysis of those important reserves which modern technique makes available for the exploitation of ball mills.

In connection with this, it is expedient to call the attention of the millers to some supplements and correctives to the original positions - and, subsequently, to the conclusions of the article - that will contribute to the further development of a theory for the rationing of the ball loading of mills, and will facilitate the application of the theoretical conclusions in practice.

As a result of his critical evaluation of formulas that were proposed by American researchers for the selection of ball diameter, Razumov came to the conclusion that not one of them meets the requirements of practice.

In order to approximate as closely as possible the practical conditions of the work of mills in concentration plants, Razumov proposed a new formula which determines the relation between the optimun diameter of the balls D, on the one hand, and the diameter d of the largest pieces of ore in the feeding of the mill, on the other. In general, this formula is represented by the equation $D = kd^n$; and in a particular case, with a calculation of the balls used in practical grinding $D_{max} = 100$ mm, $D_{min} = 12.5$ mm - by the equation

$$D = 28 \sqrt[3]{d}.$$

Thus, by Razumov's formula the diameters of the balls must be proportional to the cube root of the coarseness of the ore. However, the conclusions of the author must not be considered completely inflexible. Actually: on what is based the belief that a ball diameter of D = 100 mm is the preferred size for ore of a coarseness of 45.5 mm? It is also just as unconvincing to consider D = 12.5 mm as the preferred size for such comparatively small original ore as 0.087 mm (170 mesh). There is a basis to assume that for such fine ore it is more expedient to use the smallest balls (with a diameter of 6 mm or even 4 mm).

It is also evident that if in the equation $D = kd^n$ one should substitute different values for the maximum sizes of the balls, then in the final formula the coefficient before the radical will not be equal to 28 and the radical itself will receive a different power. For example, with $D_{max} = 125$ mm and $D_{min} = 10$ mm we have $D = 26 \sqrt[5]{d^2}$.

*(1 - Gornyy Zhurnal, 1947, No 3

It is apparent in this way one can receive as many formulas as are desired; all of them will be empirical, unsupported by any kind of conclusive data, because the given limits of D_{max} and D_{min} are arbitrary.

It is necessary to make one more important observation concerning the formulas of Stark, Coghill, Bond and Razumov. They all calculate the coarseness of the original feeding and do not take into consideration a second extremely essential factor which determines the preferred size of the balls - the fineness of the finished product. In connection with this there arises the question: are the balls which are recommended by the formulas still preferred even in the case where the original ore in question is to be reduced to some different coarseness.

The answer can only be negative: it is well known that the coarser the material to be ground the larger the balls it is necessary to utilize (and vice-versa).

The rational correlation which determines the preferred diameter of the balls must be not only substantiated by the proper experimental data, but also the initial coarseness of the feeding d and the fineness of the prepared product d_k must be taken into consideration.

Such a formula was found by us on the basis of works which were completed by cand. of technical science, B. N. Dubrovin, and published in his thesis, "The role of the crushing medium in the work of ball mills." (1947).

In these works the experiments of fine pulverizing were conducted on quartz (the characteristic of which was in all cases rectilinear) in laboratory mills, 490 x 230 and 300 x 200 mm. The running speed of the mills amounted to 84 % of critical; the coefficient of admission by the balls was selected as equalling $\varphi \approx 45$ %; and the admission of ore comprised 12 % of the volume of the mills. In this manner the pulverization system approached the optimum.

(Fig 1)

Wrought steel balls of four sizes were tested: with diameters of 62, 48, 25 and 10 mm.

The mills worked in 'open cycle' with periodic material loading. Batches of the quartz were pulverized for different time periods: 30, 60, 90, 120 min. and so on. After each experiment the product of the grinding was analyzed on screens of the Taylor standard scale; the finished product was conditionally considered to be that which left a 10 % residue on the assigned screen d_k.

In figure 1 is presented a graphic description of the results of one series of the experiments. In this series the original coarseness of the mill's feeding (d) and the sizes of the balls (D) were changed. The end fineness of of the material remained constant in all cases: d = 74 microns, that is, the pulverization was reduced as far as a 10 % residue on a 200 mesh screen. Along the upper broken line (fig 1) it is possible to establish that for the reduction of an original material of a 25-0 mm size to an end fineness of

less than
90 % ~~xi/xx~~ 200 mesh by balls of D = 62 mm, the pulverizing time amounted to T = 120 min (point A, fig 1); for balls of D = 48 min - 100-103 min (point A'). After passing through a certain minimum (point B), the pulverization time again rises: thus, for the balls with a diameter of D = 25 mm we have T = 120 min. Upon transfer to the smallest balls the pulverization time is sharply increased. The other broken lines on figure 1, which correspond to the coarseness of the original feeding, 10-0 mm, 5-0 mm, etc, are analogous.

A similar type of diagram was also made according to the data of the experiments of all the other series, in which the finished end product was material of 90 % less than 150 mesh, then 100 mesh, 65 mesh and 48 mesh. Analogous results of the formations were received in all cases. All curves on the diagrams have almost an identical appearance. If one moves along the curves from right to left, that is, from the largest balls in the direction of the smaller, then in all cases it is possible to establish that the initial (right) sides of the curves are very near to parallel straight lines; all the curves approach their minimum, after which, with a further diminution of the diameter of the balls, the left, rising sides of all the curves "strive for infinity" that is, for each ball diameter there exicts a certain lower limit at which the necessary reduction time T becomes extremely great and incommensurable with the duration of a normal experiment. The corresponding diameter of the ball - the smallest theoretically possible - can be referred to as critical. For example, on fig 1, for the dotted curve relating to the original feeding of 10-0 mm, the size of the ball D = 9 mm evidently will be critical; having loaded a mill, which has balls of the critical diameter of 9 mm, from batches of quartz with an original coarseness of 10-0 mm, it will be impossible to receive a finished product of 90 % less than 200 mesh, even with an extremely great continuance of pulverization.

The existence of critical diameters represents one of the characteristic peculiarities of all pulverization curves. More important is the existence of a certain minimum on each curve. Thus, for example, for the upper curve (see fig 1) the theoretical minimum for the pulverization time is determined by means of an interpolation in $T_0 \approx 38$ min (point B) that is in correspondence with the ball diameter $D_0 \approx 44$ mm; for the second curve (10-0 mm) the minimal time amounts to approximately $T_0 \approx 69$ min, and the corresponding ball diameter equals $D_0 \approx 27$ mm (point C) etc. The diameters of the balls, with which the least pulverization time T_0 is achieved, are the preferred, or optimum, sizes and are designated for the future through D_0. Evidently they correspond to the maximum output of the laboratory mill, because its productivity in a periodic cycle is inversely proportional to the reduction time.

Joining all the minimum points of the dotted curves on fig 1 gives a curve of minimum time FCB. This curve is at the same time the curve for the optimum ball diameters, because it graphically presents the function $T_0 = f(D_0)$. Along this curve it is possible to find the preferred size of a ball in the interjacent, uninvestigated cases.

Making analogous constructions according to all the series of the experiments, we receive for the mill undergoing test as many minimal time curves $T_0 = f(D_0)$ as there are finished products.

On fig 2, which relates to the 490 x 230 mm mill, there are five curves of minimal time drawn with solid lines, which correspond to the five finished products: 200, 150, 65 and 48 mesh; of these, the upper curve is transferred directly from fig 1, where it was depicted by the solid line FCB; the others were made in the same manner.

With a consideration of fig 1 and of what is analogous to it in fig 2, there are points (joined by dotted curves) corresponding to the same coarseness of the original feeding d. As a result there are on figure 2 two systems of intersecting curves with the help of which, as on a common nomogram, it is possible to determine the two values sought for T_0 and D_0 by the two given: the coarseness of the finished product d_k (which has a 10 % residue on a given screen d_k) and the size of the feeding d. For example, there are given: the initial coarseness of 10-0 mm (d = 10mm) and the fineness of the finished product: 90 % less than 150 mesh (d_k = 104 microns). In order to determine the minimal pulverization time T_0 and the preferred ball diameter D_0 let us take point A of fig 2 at the intersection of the dotted curve, which corresponds to the coarseness 10-0 mm, with the solid curve corresponding to the finished product for a 150 mesh. The known point A on the nomogram has the coordinates: T_0 = 54 min and D_0 = 30 mm. By these data are determined the size of the preferred balls under the given conditions, that is, a collection of single sized balls which will guarantee the greatest productivity of the laboratory ball mill being tested in pulverizing quartz of a given initial coarseness (with a rectilinear characteristic) to a given fineness.

(Fig 2)

Exactly the same construction (fig 3) can be made on the basis of the tests relating to the 300 x 200 mm mill. Principally fig. 3 differs in no way from the preceding one, as can easily be seen through comparison.

The nomograms of figures 2 and 3 can be used to establish the mathematical relationship between the diameter of the preferred ball D_0, on the one hand, and the initial coarseness of the ore d and the fineness of the finished product d_k, on the other. For this let us construct a semilogarithmic graph (fig 4) where the fineness of the finished product d_k (in microns or by the mesh) is plotted, with the help of the logarithmic scale, on the axes of the abscissae; and on the axes of the ordinates are drawn the diameters of the preferred balls D_0, which are taken from one or another of the nomograms.

Both nomograms (figs 2 and 3) with this give, within the limits of accuracy of the constructions, the same points on the diagram $D_0 = f(d_k)$, and therefore the semilogarithmic graph (fig 4) pertains to both the mills being tested. This, by the way, proves that the preferred ball diameter - at least in initial approximation - is independent of the diameter of the mill.

In the semilagarithmic graph in question, all of the curves $D_0 = f(d_k)$ are reduced to straight lines, whereupon the latter form a series of rays which descend to the single point A on the abscissas' axis. All of the rays have a finite length which is established on one side by a common terminal, on the other by the points of intersection of one of the rays with a certain limiting curve. The latter is drawn on fig 4 as a dotted line and represents that contour line at which each ray is intersected by the vertical $d_k \approx d$. Thus, for example, for the ray 2-0 mm (ray AB on fig 4) the limit is established by point B, because the abscissa of point B is equal to d_k = 2000 microns = 2 mm, that is, it coincides numerically with the upper limit of the feeding d = 2 mm to which the ray in question (AB) relates.

The presence of a contour curve on the graph shows that for any given coarseness of the feeding there is, theoretically speaking, an upper limit for the preferred diameter D_o. Thus, for the case in question (d = 2 mm) the maximum diameter of the balls should not exceed D_{max} = 22 mm, moreover, this abstract ball size can be "selected" only when the original product of 2-0 mm is also the finished product (d_k = d = 2 mm). If one has in mind to actually pulverize it to a smaller size, the diameter of the balls must be less than the limit; for example, in the reduction of a 2-0 mm product to a 48 mesh there is demanded D_o = 17 mm; to 200 mesh - D_o = 13 mm, etc.

(Fig 3)

As seen in fig 4 the point of juncture of all the straight-line rays, or point A, has the abscissa d_k = 1 micron. The log of this size (that is, of the unit) is equal to 0; in view of this it is possible to consider that with all other conditions being equal, the preferred diameter D_o of the balls is directly proportional to the log of the size of the finished product.

(Fig 4)

If, on the other hand, while considering some size of the finished product d_k as constant, one should transpose the characteristic points of nomograms 2 and 3 onto a new graph (fig 5) on which the axis of the abscissae gives the variable size of the feeding (d) and the ordinate axis gives the preferred ball diameter (D_o): then it appears that each curve $D_o = f(d)$ which corresponds to any single size of the finished product d_k = const (for example, the curve FCB, fig 1) is depicted here by a regular parabola of the $y^2 = 2rx$ type. Analogous parabolic curves are received upon transfer of all the other points from both nomograms onto the graph in question (see fig 5). From here it is easy to conclude that the preferred diameter of the balls must be proportional to the square root of the original feeding, that is $D_o = K\sqrt{d}$. (1)

The coefficient K can be determined from the semilogarithmic graph, on the basis of which $K = K_o \log d_k$.

Thus, it is finally possible to present the overall formula for the diameter of the preferred ball as follows:

$$D_o = K_o (\log d_k) \cdot \sqrt{d}. \qquad (2)$$

where D_o = the diameter of the preferred balls;
d = the upper limit of the feeding size;
d_k = the conditional size of the finished product.

The preceding formula (1), which was discovered for the optimum ball size, coincides with formulas which have been published in the literature: American researchers received a formula of the same type, $D = K\sqrt{d}$. However, these researchers do not functionally connect the importance of the coefficient of proportionality K with the size of the finished product. In this regard the subsequent formula (2), which was developed, has a principal advantage.

Transferring to the numerical values for the coefficients, let us indicate that on the basis of the described laboratory experiments the coefficient K_o in the formula (2) proved to be numerically equal to $K_o = 4.8$; consequently the formula (2) applicable to the laboratory mill gives the preferred ball diameter for pulverizing quartz (with a rectilinear characteristic):

$$D_o = K \sqrt{d} = 4.8 \, (\log d_k) \cdot \sqrt{d}. \qquad (3)$$

If one should substitute in the formula some other values for the size of the finished product d_k, then the common coefficient of proportionality K will prove to be within the limits of K = 9 (for the product of a 200 mesh) and K = 12 (for the product of a 48 mesh).

For comparison let us point out that Coghill and de Vanney found, in the pulverization of ores under laboratory conditions to a fineness of 200 mesh with the formula $D = K \sqrt{d}$, that for hornstone the coefficient K = 35, and for dolomite - K = 30. Stark, using the same formula, found that for clinker K = 13 (in a pulverization to a 5 % residue on a 170 mesh screen).

In the transfer from laboratory experiments to industrial conditions it is necessary to keep in mind the "rule of equality in pulverizing" and the necessity of "durability" in the work regime of the balls in a mill.

The laboratory experiments which are being described confirm the opinion of Coghill and de Vanney, that the preferred ball diameter is not dependent upon the size of the mills, but upon the fixed correlation between the ball diameter and the size of the pellets. To be convinced of this one should compare the nomograms (figs 2 and 3) which were made for two different mills, the diametrical ratio of which amounted to 490:300. Regardless of the significant difference of the mills' diameters the preferred sizes of the balls proved in both cases to practically correspond. It is possible that the transfer to mills of industrial sizes will necessitate the injection of some readjustment into the position stated above, however, for an initial approximation with an accuracy corresponding to the required accuracy of the calculations, it is possible to extend the results of the laboratory experiments to mills of industrial sizes. Thus, in principle, the formulas (1) and (2) retain their effectiveness for industrial mills, it being only necessary to interpolate a readjustment on the "durability of the reduction system".

As seen from fig 1 it is expedient to increase the ball diameter (which we will designate by D_m) somewhat over the optimum size D_o (which corresponds to the minimum time T_o) in order to avoid the danger of moving the system onto the rising left side of the curve. We will, by this, move to the right from the zone of "critical diameters." In view of the lack of appropriate experiments the selection of the increase over the optimum D_o, for the tentative diameter D_m, has proven to be difficult at present. It is necessary to assume that the small increase in diameter, for example, 15-25 % over D_o, is expedient because it insures against falling into the dangerous zone of the curve; and with this it will cause only a comparatively small decrease in the mill's productivity. As a first attempt let us settle on an allowance coefficient of 15-25 %; that is in the formula $D = K_o (\log d_k) \sqrt{d}$ let us use, instead of $K_o = 4.8$, the coefficient $K_1 = 1.15\text{-}1.25 \cdot K_o$. Then $K_1 =$ $= (1.15\text{-}1.25) \cdot 4.8 = 5.5$ to 6, and for practical calculations we have

$$D_m \leqslant 6 (\log d_k) \cdot \sqrt{d}, \qquad (4)$$

where the coefficient 6 is limiting (its lowering to 5.5 is desirable).

The last formula can be recommended for determining the diameter of the balls (in the feeding of a mill which has single sized balls, that is, with a ball loading consisting of balls of some single diameter, but not of a mixture of them).

A graph, which illustrates formula 4, of the tentative ball diameters (D_m) is depicted in fig 5; it is in relation to the size of the original ore (d) and to the fineness of the finished product (d_k); it is for ores of medium hardness (such as quartz of hornstone), and can be used in practical calculations. For example, let the size of the initial ore d = 45.5 mm and the fineness of the finished produce - 90 % less than 200 mesh (d_k = 74 microns). Then by Stark's formula / $D = 13 \sqrt{d}$ / we have D = 88 mm, by Razumov's formula / $D = 28 \sqrt[3]{d}$ / we have D = 100 mm, by the formula for horn-stone of Coghill and de Vanney / $D = 35 \sqrt{d}$ / we have D = 80 mm and by our formula / $D_m \leqslant 6 (\log d_k) \sqrt{d}$ / we have D = 76 mm (in the latter case instead of computing it is also possible to use the graph, fig 5).

The difference of the figures given by these formulas is not great, but the figures which was found by the formula (4) is better substantiated because it is based on precise experimental data.

By the nature of the formula (4) it is necessary to note that it is applicable for the pulverization of a hard ore (such as quartz) with a rectilinear characteristic; it was brought out for the cases of feeding ball mills with single-sized balls and not a mixture; and it refers only to the mills with periodic loading or with a short duration of stay for the balls in the mill, as it does not take ball wear into consideration. If this last circumstance is taken into consideration then it will prove that in an industrial mill (where each ball gradually decreases from its initial size) the balls which wereselected by formula (4) will soon "depart from the optimum". It is necessary,therefore, to select their sizes with a consideration of the wear in industrial mills.

Should the size log d_k not be calculated at any time in the formula (4) it can be written in the shortened form:

$$D_m \leqslant K_m \sqrt{d}, \qquad (5)$$

where the coefficient K_m is considered as some size of the finished product: $K_m = 6(\log d_k)$. On all curves in fig 5 are presented the appropriate individual formulas.

From these data it is seen that to desregard the dependence of the diameter D_m on the fineness of the grist is improper, because the size d_k is essentially reflected in the size of the ball.

For the prevailing case in our concentration plants of pulverizing an initial ore with a limiting coarseness of d = 40 mm to 90 % less than 150 mesh (d_k = 104 microns) the largest ball size is determined by formula (5) as equalling:

$$D_m \leqslant K_m \sqrt{d} \leqslant 12.1 \cdot \sqrt{40} \leqslant 76 \text{ mm}.$$

To use larger balls in the given conditions, for example, D = 100 mm or D = 125 mm, is not expedient because this lowers the production of the mill by 20-30 %.

Conclusions

1. The formulas published in the literature for the selection of ball diameter do not take into consideration such an important factor as the fineness of the pulverization's finished product.

2. On the basis of our latest works, the following formula can be recommended for size selection:

$$D_m \leqslant 6 \,(\log d_k) \sqrt{d},$$

where D_m = the diameter of the single-sized balls, in mm.
d = the diameter of the largest chunks of ore in the feeding of the mill, in mm.
d_k = the fineness of the finished product, in microns.

With this the finished product is considered that which will leave a 10 % residue on a screen with meshes d_k. (in microns).

3. In order to accelerate the calculation it is possible to use the shortened (individual) formula:

$$D_m \leqslant K_m \sqrt{d},$$

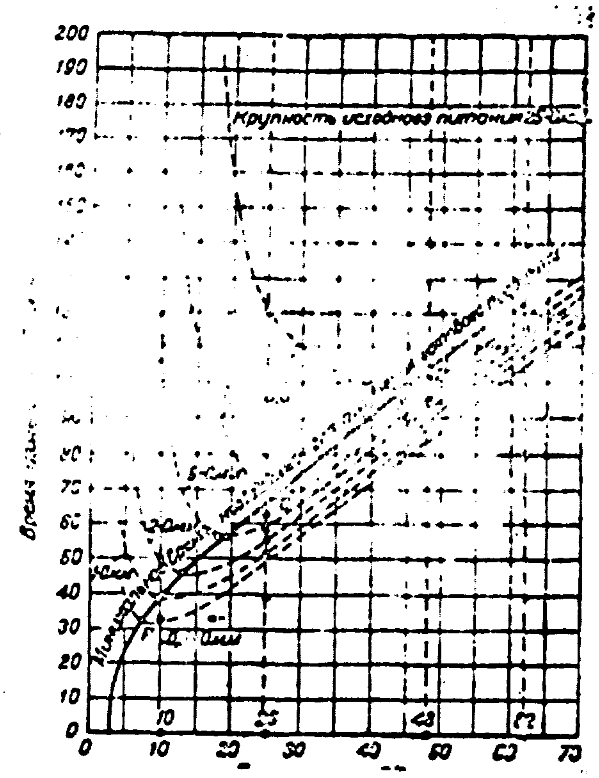
where the coefficient $K_m = 6(\log d_k)$ is taken from fig 5 according to the fineness of the finished product d_k.

4. The cited formulas are applicable to the case of feeding a mill with only single-sized balls and not a mixture, and moreover - only to short periods of pulverization, that is, without consideration for ball wear.

In the event of work in ball mixtures ("rationed" loading) it is expedient to select balls which are also recommended by the cited formula (but in accordance with results of rationing experiments).

The correction for wear must be introduced on the basis of some theory of ball wear in continuously running mills, however, the basic formula retains its applicability even in this event.

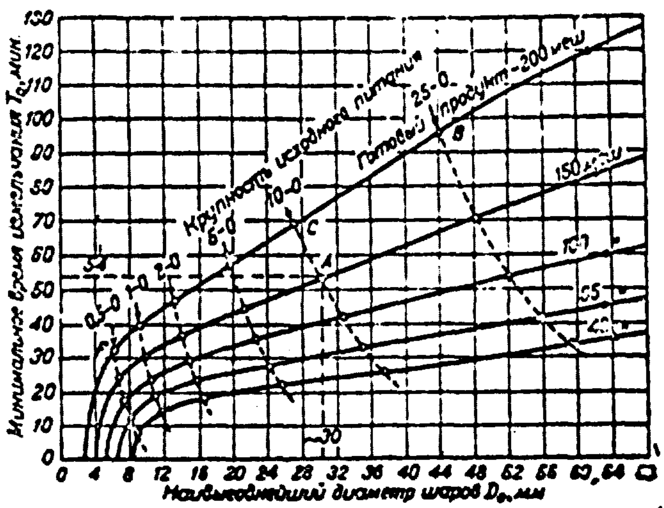
T (Pulverization time in min)



D (Ball diameter in mm)

Fig 1. Results of pulverizing quartz to a 10 % residue on a 74 micron screen (200 mesh) in a laboratory ball mill 490 x 230 mm (according to the experiments of B. N. Dubrovin).

The solid line represents the minimal time required to produce the finished product.
The dotted lines represent the coarseness of the original feedings.

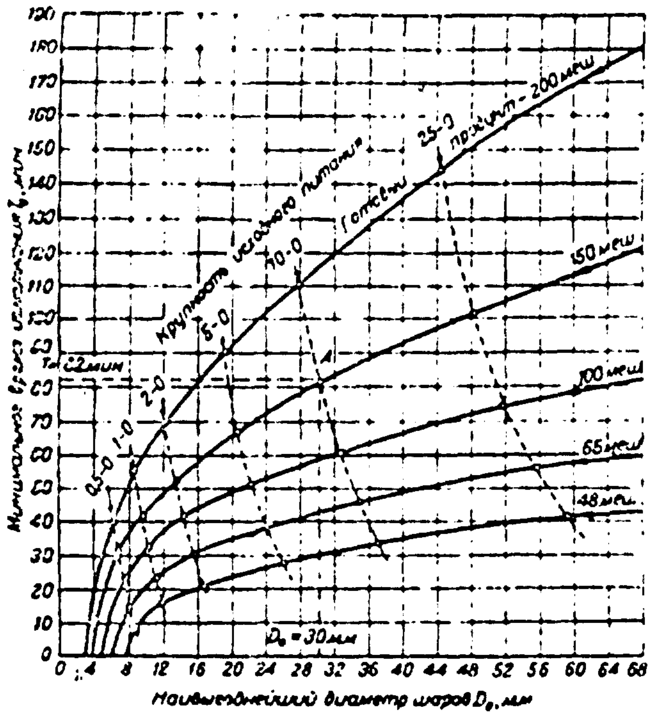


T_o (minimal pulverizing time in min)

D_o (the preferred ball diameter in mm)

Fig 2. A Nomogram for the computation of a 490 x 230 mm mill in the pulverization of quartz.

Dotted lines represent the coarseness of the original material.
Solid lines represent the finished product.

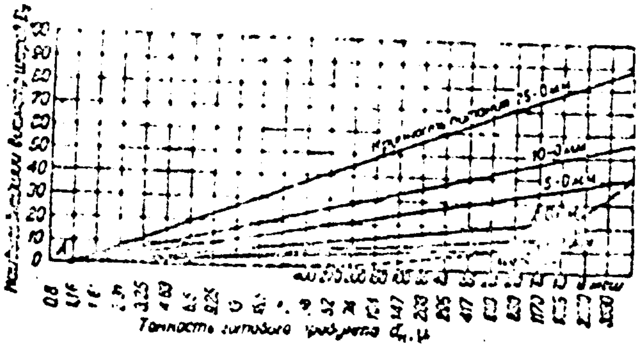


T (minimal pulverizing time in mln)

D_0 (preferred ball diameter in mm)

Fig 3. A nomogram for the computation of a 300 x 200 mm mill.

D_o (preferred ball diameter)



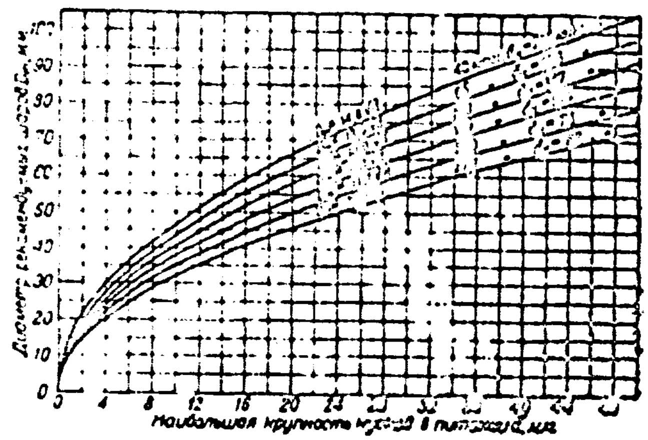
d_k (fineness of the finished product in microns)

Fig 4. The relation of the preferred ball diameter D_o to the fineness of the prepared product d_k with different feeding coarseness d.

Solid lines represent the coarseness of the original feeding.

Dotted line is the limiting curve.

D_m (diameter of recommended balls in mm)



d (the largest chunks of ore in the feeding in mm)

Fig 5. The relation of the limiting diameters of the recommended balls to the coarseness of the feeding and the fineness of the finished product.

Reading from top to bottom:

$D_m \leqslant 14.8 \sqrt{d}$	48 mesh	d_k = 295 microns.
$D_m \leqslant 13.9 \sqrt{d}$	65 "	" " 208 "
$D_m \leqslant 13.0 \sqrt{d}$	100 "	" " 147 "
$D_m \leqslant 12.1 \sqrt{d}$	150 "	" " 104 "
$D_m \leqslant 11.2 \sqrt{d}$	200 "	" " 74 "
$D_m \leqslant 10.3 \sqrt{d}$	270 "	" " 52 "