USAAVLABS TECHNICAL REPORT 68-61

DESCRIPTION OF A HELICOPTER ROTOR NOISE COMPUTER PROGRAM

By

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U. S. ARMY AVIATION MATERIEL LABORATORIES
FORT EUSTIS, VIRGINIA

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DESCRIPTION OF A HELICOPTER ROTOR NOISE COMPUTER PROGRAM

Final Report

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By

J. B. Ollerhead and R. B. Taylor

Prepared by

Wyle Laboratories
Huntsville, Alabama

for

U. S. ARMY AVIATION MATERIEL LABORATORIES
FORT EUSTIS, VIRGINIA

SUMMARY

This report contains a comprehensive description of a computer program developed for the numerical evaluation of the helicopter noise equations derived in USAAVLABS TR 68-60, "Studies of Helicopter Rotor Noise." It is completely self-contained in that the program details are described, starting from two basic acoustic equations and covering methods by which these equations are applied to the rotor noise problem. Program flow diagrams and a complete listing are presented together with input instructions and sample inputs and outputs. The program is written in FORTRAN IV for the CDC 3300 Computer.

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1.0 INTRODUCTION

The rotor sound field is calculated according to the general acoustic equation for aerodynamic forces in motion which is derived in Section 3.3 of USAAVLABS TR 68-60:

$$p(t) - p_0 = \left[\frac{(x_i - y_i)}{4\pi(1-M_s)^2 c s^2} \left\{ \frac{\partial F_i}{\partial t} + \frac{F_i}{1 - M_s} \frac{\partial M_s}{\partial t} \right\} \right] \quad (1)$$

This equation is written in tensor notation, where the i denotes summation over the three component directions, and the brackets denote evaluation at the appropriate retarded time $t' = t - s/c$ which is the time at which the source generated the sound reaching the observer at time t.

$p(t) - p$	is the instantaneous acoustic pressure.
x_i	are the coordinates of the observer.
y_i	are the coordinates of the source.
M_s	is the component of the source Mach number in the direction of the observer.

$$= \frac{(x_i - y_i)}{s} \frac{1}{c} \frac{dy_i}{dt}$$

s	is the distance between source and observer.
F_i	are the components of the aerodynamic force.
c	is the speed of sound.

In addition, the near field pressure fluctuations are calculated according to Lowson[1] as follows:

$$p'(t) - p_0 = \left[\frac{1}{4\pi(1-M_s)^2 s^2} \left\{ \frac{F_i(x_i - y_i)}{s} \frac{(1 - M^2)}{(1 - M_s)} - F_i M_i \right\} \right] \quad (2)$$

---

1. Lowson, M.V., "The Sound Field for Singularities in Motion", Proceedings of the Royal Society, Volume A 286, pp. 559-572 (1965). (Equation 18).

where M is the source Mach number $= \sqrt{\frac{\dot{y}_1^2 + \dot{y}_2^2 + \dot{y}_3^2}{c}}$

and M_i is the component of M in the i-direction.

In all cases the sum of Equations (1) and (2) is calculated, but as shown in Section 6 of USAAVLABS TR 68-60 the component due to (2) is negligible at any significant distance from the rotor.

Equations (1) and (2) are solved by a numerical method which has been programmed for digital computation. This solution is exact to the extent that no approximations are made. The accuracy of the solution is only limited by that with which the real loads and motions experienced by the rotor dynamic system can be represented by the model used. The sound field for the simulated system is calculated accurately at any point in the near or far field.

The sound generated by a rotor blade in motion is the result of the distributed aerodynamic pressure acting over its entire surface. Rotational noise, which is the subject of this study, is defined as that component of the sound field which is directly attributable to the lift and drag forces acting on the blade. Strictly, the entire spanwise and chordwise distributions of these components should be taken into account, but for the sake of numerical expediency it is necessary to simulate the actual distributions by a discrete set of point loads. The implications of doing so are fully discussed in Section 5 of USAAVLABS TR 68-60. The spanwise loading distributions are divided into a number of segments, each of which is represented by two single force components, lift and drag. This model is illustrated in Figure 1. The point of application of each force pair then becomes an acoustic source which generates sound according to Equations (1) and (2). The F_i in those equations are the forces acting upon the air and are therefore opposed to the lift and drag forces acting upon the blade.

If an arbitrary set of orthogonal axes X, Y, Z are defined, Equations (1) and (2) can be combined. Using also a more convenient notation,

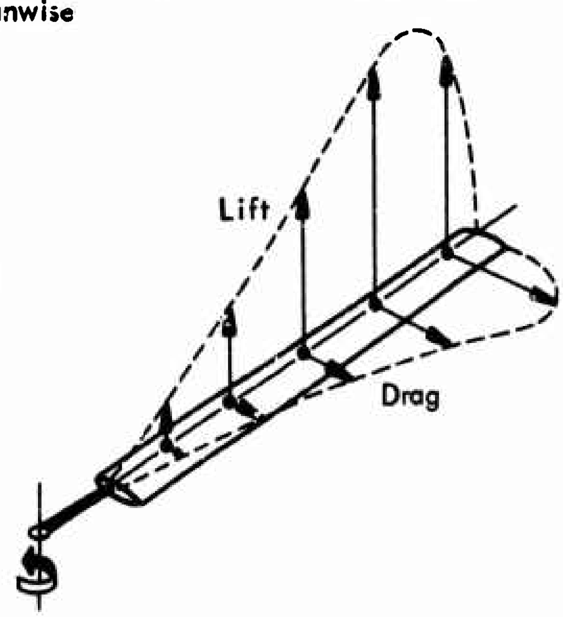


Figure 1. Discrete Representation of Aerodynamic Loading.

$$\Delta p(t) = \frac{-1}{4\pi(1-M_s)^2 c s^2}\left[\bar{x}\left\{\dot{F}_X + F_X\left(\frac{\dot{M}_s + \frac{c}{s}(1-M^2)}{1-M_s} - \frac{\dot{X}}{x}\right)\right\} + \bar{y}\left\{\dot{F}_Y + F_Y\left(\frac{\dot{M}_s + \frac{c}{s}(1-M^2)}{1-M_s} - \frac{\dot{Y}}{y}\right)\right\} + \bar{z}\left\{\dot{F}_Z + F_Z\left(\frac{\dot{M}_s + \frac{c}{s}(1-M^2)}{1-M_s} - \frac{\dot{Z}}{z}\right)\right\}\right] \quad (3)$$

$\Delta p(t)$ is the observed sound pressure at time t due to the aerodynamic forces acting at any particular point on the blade whose components in the X, Y and Z directions, at the retarded time $t - \frac{s}{c}$, were F_X, F_Y and F_Z. The dot denotes differentiation with respect to time. The quantities $\bar{x}$, $\bar{y}$ and $\bar{z}$ are the coordinates of the observer, relative to the source in the X, Y and Z directions. The negative sign accounts for the use of the blade loads which are equal and opposite to the forces which act on the air.

It can be seen from this equation that the force generates sound in two ways: through its fluctuation in time and its fluctuations in position which cause its accelerations toward the observer $(\dot{M}_s)$. The term $(1 - M_s)$ in the various denominators is essentially a Doppler effect which amplifies the sound radiated in the direction of motion. Dipole sound (the term in $\dot{F}$) is amplified by the factor $(1 - M_s)^{-2}$ and quadrupole sound (the accelerative term in $F \cdot \dot{M}_s$) by $(1 - M_s)^{-3}$. Thus the second term becomes increasingly important as the velocity in the direction of the observer increases.

The elements of the method of computing the sound field of a complete rotor are as follows:

(1) Define the geometry, blade loading and resulting blade motions of the rotor as a function of time.

(2) Define the position, attitude and velocity of the rotor with respect to the observer at the time t .

(3) Calculate the orientation and magnitude of each of the elemental blade airloads at its appropriate retarded time.

(4) Calculate and integrate the observed sound pressures due to all the s[illegible]s.

(5) Repeat operations (2) through (4) for a series of successive time int[illegible]s t to construct a time history of the acoustic pressure amplitude and harmonically analyze this to obtain its frequency spectrum.

A computer program has been written to perform these operations taking account of the following variables:

(a) Number of rotors.

(b) Number of blades.

(c) Rotor diameter.

(d) Helicopter position in space with regard to a fixed observer including:

1. Rotor hub vertical displacement.

2. Rotor hub horizontal displacement.

3. Rotor shaft inclination.

(e) Rotor hub motion including:

1. Linear velocity.

2. Roll angular velocity.

3. Pitch angular velocity.

(f) Rotor blade tip speed.

(g) Articulated blade motions.

(h) Blade motion resulting from flapping bending, edgewise bending, and twist about the elastic axis.

(i) Phase relationships between rotor angular positions for multirotor vehicles.

(j) Time dependent blade loading for at least 20 radial blade segments. The blade loading includes loading normal to the hub plane, radial loading in the hub plane, and tangential loading in the hub plane. These loadings are treated as loads at a chordwise point on the blade element with direction determined by orientation of the blade element in space. Variation in disc loading is considered as a function of variation in blade loading with the number of blades and blade chord held constant.

This program, code-named HERON 1, is described in detail in Sections 4 and 5. The axis transformations and outlines of the computational steps are discussed in the following sections.

2.0 COORDINATE SYSTEMS AND TRANSFORMATIONS

2.1 Rotor Axes x',y',z'

Since blade motions are measured relative to the rotor shaft it is desirable, for the direct utilization of experimental data, to specify all blade responses, including flapping, lagging and elastic deformations with respect of a set of "shaft axes".

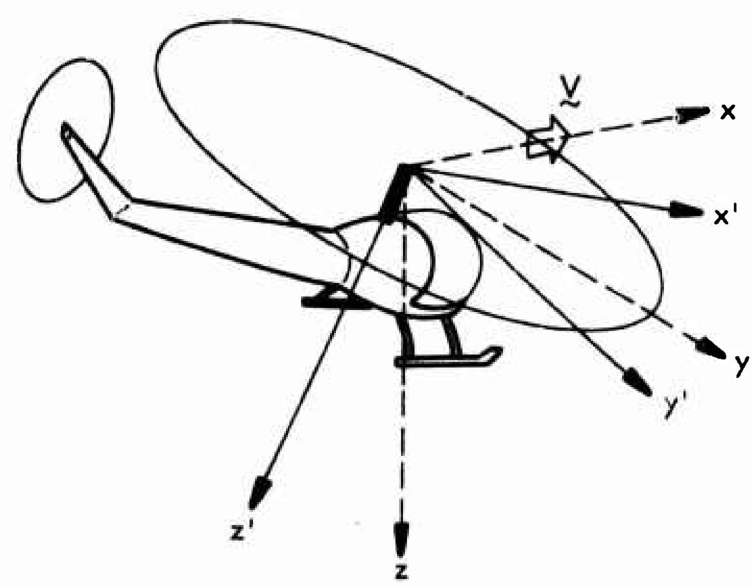


Figure 2. Shaft Axes (x',y',z') and Flight Path Coordinates (x,y,z).

Accordingly, a set of right-handed orthogonal axes x', y', z' are chosen with the z' axis coincident with the rotor shaft and positive downward (Figure 2). The longitudinal axis x' lies in the vertical plane through the aircraft velocity vector $\underset{\sim}{V}$.

2.2 Flight Path Axes x,y,z

This system is introduced merely to simplify the transformations between the x',y',z' and X, Y, Z systems. The origin of this system is coincident with that of the rotor axes. However, the x y plane is horizontal with the x axis in the vertical $\underset{\sim}{V}$–x' plane. The aircraft rotation is specified with respect to this system in the following sequence (Figure 3).

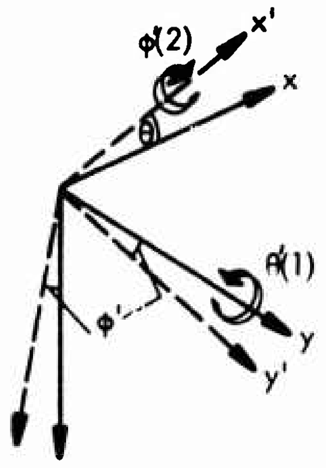


Figure 3. Pitch and Roll Rotations.

(1) The pitch angle θ' is measured clockwise about the positive y axis, rotating x to x'.

(2) The roll angle ϕ' is then measured clockwise about the x' axis, rotating y to y' and z to z'.

(Vehicle angular displacements are measured in pitch and roll only, since rotor yaw is simply a shift in azimuth reference.)

The shaft inclinations to the vertical z axis, measured in the x z and y z planes, are θ' and ϕ' so that (Figure 4) $\theta = \theta'$ and $\tan \phi = \cos \theta \tan \phi'$.

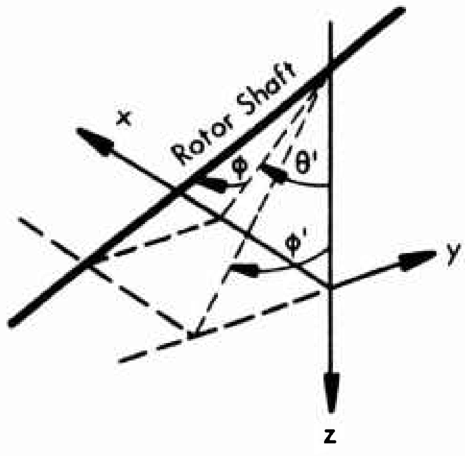


Figure 4. Shaft Inclinations.

The transformations between shaft and flight path axes are:

$$x = x' \cos\theta + y' \sin\phi \sin\theta + z' \cos\phi \sin\theta$$

$$y = y' \cos\phi - z' \sin\phi \qquad (4)$$

$$z = x' \sin\theta + y' \sin\phi \cos\theta + z' \cos\phi \cos\theta .$$

2.3 Fixed Axes X, Y, Z

This coordinate system is used to define the aircraft and observer positions in space. The axes are fixed in space with arbitrary origin, and XY is the horizontal ground plane. Z is measured positive vertically upwards.

The X Y and x y planes are thus parallel, and the angle between the x and X axes is defined as χ .

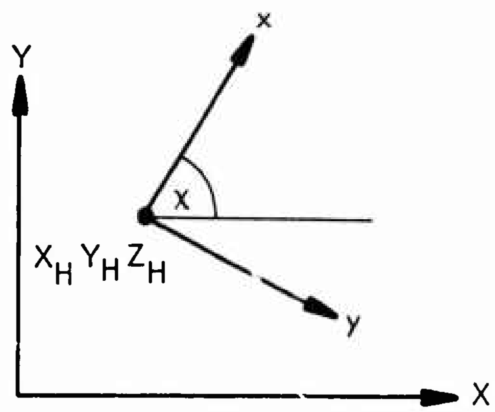


Figure 5. Fixed Axes (Plan View).

Thus, if the rotor hub coordinates are X_H, Y_H, Z_H, any point in the X, Y, Z system is defined by

$$X = X_H + x \cos\chi + y \sin\chi$$

$$Y = Y_H + x \sin\chi - y \cos\chi$$

$$Z = Z_H - z \qquad (5)$$

The coordinates of an observer at X_0, Y_0, Z_0, with respect to the source at x, y, z, are

$$\bar{x} = X_0 - X$$

$$\bar{y} = Y_0 - Y$$

$$\bar{z} = Z_0 - Z \tag{6}$$

The distance s between source and observer is

$$s = (\bar{x}^2 + \bar{y}^2 + \bar{z}^2)^{\frac{1}{2}} \tag{7}$$

2.4 Blade Element Displacements

The x', y', z' coordinates of each specified blade loading point are calculated as a function of azimuth angle, flapping and lagging angles (where applicable), and normal and in-plane elastic displacements. The rotor azimuth angle is measured from the negative x' axis, clockwise about the positive z' axis, $\psi = \Omega t$.

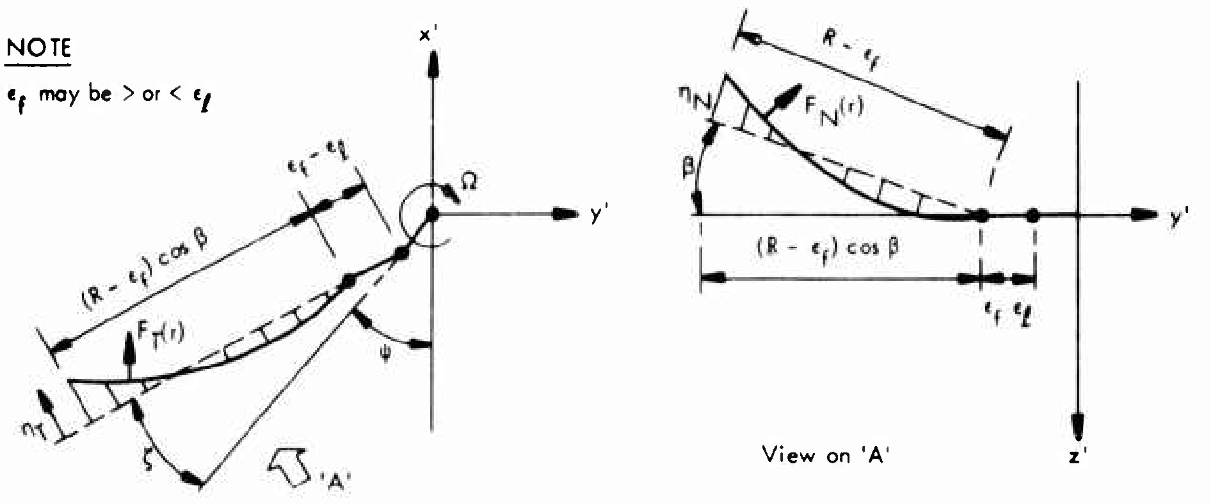


Figure 6. Blade Element Displacements in Rotor Axes.

Referring to Figure 6,

ϵ_f is the flap hinge offset from the shaft centerline.

ϵ_ℓ is the lag hinge offset.

ζ is the lag angle projection on the x' y' plane (positive in the direction of rotation).

β is the flap angle between the undeformed blade axis and the x'y' plane, positive in the negative z' direction.

$\eta_T(r)$ is the elastic displacement of blade station r parallel to the x' y' plane, normal to the undeformed blade axis and positive in the direction of rotation.

$\eta_N(r)$ is the elastic displacement normal to the undeformed blade axis, in the plane containing the z' axis and positive in the negative z' direction.

These coordinates thus take account of all possible motion of the blade axis upon which the point aerodynamic loads are assumed to act.

If the flapping hinge is outboard of the lagging hinge, the coordinates of the radial station r are:

$$
\begin{aligned}
x' &= -\epsilon_\ell \cos\psi - \left\{(r-\epsilon_f)\cos\beta + (\epsilon_f - \epsilon_\ell)\right\}\cos(\psi+\zeta) \\
&\quad + \eta_T \sin(\psi+\zeta) + \eta_N \sin\beta\cos(\psi+\zeta) \\
y' &= -\epsilon_\ell \sin\psi - \left\{(r-\epsilon_f)\cos\beta + (\epsilon_f - \epsilon_\ell)\right\}\sin(\psi+\zeta) \\
&\quad + \eta_T \cos(\psi+\zeta) + \eta_N \sin\beta\sin(\psi+\zeta) \\
z' &= -(r-\epsilon_f)\sin\beta - \eta_N \cos\beta \qquad (8a)
\end{aligned}
$$

If the flapping hinge is inboard of the lagging hinge, we have

$$x' = -(\epsilon_f + \epsilon_\ell \cos\beta)\cos\psi - (r - \epsilon_\ell)\cos\beta\cos(\psi + \zeta)$$

$$+ \eta_T \sin(\psi + \zeta) + \eta_N \sin\beta\cos(\psi + \zeta)$$

$$y' = -(\epsilon_f + \epsilon_\ell \cos\beta)\sin\psi - (r - \epsilon_\ell)\cos\beta\sin(\psi + \zeta)$$

$$- \eta_T \cos(\psi + \zeta) + \eta_N \sin\beta\sin(\psi + \zeta)$$

$$z' = -(r - \epsilon_f)\sin\beta - \eta_N \cos\beta \tag{8b}$$

It is now possible, using Equations (4), (5) and (8), to find the fixed coordinates X, Y, Z of any blade station, defined in convenient rotor coordinates as a function of time, and subsequently, using Equations (6) and (7), the displacement components $\bar{x}$, $\bar{y}$, $\bar{z}$ and s of the sound Equation (3).

2.5 Blade Element Velocities and Accelerations

We also require the velocity and acceleration components defined in the sound Equation (3) as M, M_s and $\dot{M}_s$ which are the source Mach number and rate of change of Mach number in the direction of the observer. Resolving the source Mach number into its three components we have

$$M = \sqrt{M_X^2 + M_Y^2 + M_Z^2} \tag{9}$$

$$M_s = \frac{\bar{x}}{s} M_X + \frac{\bar{y}}{s} M_Y + \frac{\bar{z}}{s} M_Z \tag{10}$$

and

$$\dot{M}_s = \frac{\bar{x}}{s} \dot{M}_X + \frac{\bar{y}}{s} \dot{M}_Y + \frac{\bar{z}}{s} \dot{M}_Z \tag{11}$$

where M_X, M_Y, and M_Z are simply $\frac{\dot{X}}{c}$, $\frac{\dot{Y}}{c}$ and $\frac{\dot{Z}}{c}$ respectively; that is, the velocity components of the source, relative to the stationary air, expressed as a fraction of the atmospheric speed of sound. To obtain the first and second time derivatives of X, Y, and Z it is necessary to differentiate Equations (5), and subsequently Equations (4) and (8). From Equation (5),

$$
\begin{aligned}
\dot{X} &= \dot{X}_H + \dot{x}\cos\chi - x\dot{\chi}\sin\chi + \dot{y}\sin\chi + y\dot{\chi}\cos\chi \\
&= \dot{X}_H + (\dot{x} + y\dot{\chi})\cos\chi + (\dot{y} - x\dot{\chi})\sin\chi \\
\dot{Y} &= \dot{Y}_H + \dot{x}\sin\chi + x\dot{\chi}\cos\chi - \dot{y}\cos\chi + y\dot{\chi}\sin\chi \\
&= \dot{Y}_H - (\dot{y} - x\dot{\chi})\cos\chi + (\dot{x} + y\dot{\chi})\sin\chi \\
\dot{Z} &= \dot{Z}_H - \dot{z}
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
\ddot{X} &= \ddot{X}_H + (\ddot{x} + 2\dot{y}\dot{\chi} - x\dot{\chi}^2 + y\ddot{\chi})\cos\chi + (\ddot{y} - 2\dot{x}\dot{\chi} - y\dot{\chi}^2 - x\ddot{\chi})\sin\chi \\
\ddot{Y} &= \ddot{Y}_H - (\ddot{y} - 2\dot{x}\dot{\chi} - y\dot{\chi}^2 - x\ddot{\chi})\cos\chi + (\ddot{x} + 2\dot{y}\dot{\chi} - x\dot{\chi}^2 + y\ddot{\chi})\sin\chi \\
\ddot{Z} &= \ddot{Z}_H + \ddot{z}
\end{aligned}
\tag{13}
$$

and from Equation (3),

$$
\begin{aligned}
\dot{x} &= \dot{x}'\cos\theta - x'\dot{\theta}\sin\theta - (z'\dot{\phi} - \dot{y}')\sin\phi\sin\theta \\
&\quad + (\dot{z}' + y'\dot{\phi})\cos\phi\sin\theta + y'\dot{\theta}\sin\phi\cos\theta + z'\dot{\theta}\cos\phi\cos\theta \\
\dot{y} &= (\dot{y}' - z'\dot{\phi})\cos\phi - (y'\dot{\phi} + \dot{z})\sin\phi \\
\dot{z} &= z - \dot{x}'\sin\theta - x'\dot{\theta}\cos\theta + (\dot{y}' - z'\dot{\phi})\sin\phi\cos\theta \\
&\quad + (y'\dot{\phi} + \dot{z}')\cos\phi\cos\theta - y'\dot{\theta}\sin\phi\sin\theta - z'\dot{\theta}\cos\phi\sin\theta
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
\ddot{x} &= (\ddot{x}' - x'\dot{\theta}^2)\cos\theta - (2\dot{x}'\dot{\theta} + x'\ddot{\theta})\sin\theta \\
&\quad + (2y'\dot{\phi}\dot{\theta} + 2\dot{z}'\dot{\theta} + z'\ddot{\theta})\cos\phi\cos\theta \\
&\quad + (\ddot{y}' - y'(\dot{\phi}^2 + \dot{\theta}^2) - 2\dot{z}'\dot{\phi} - z'\ddot{\phi})\sin\phi\sin\theta \\
&\quad + (2\dot{y}'\ddot{\phi} + y'\ddot{\phi} + \ddot{z}' - z'(\dot{\phi}^2 + \dot{\theta}^2))\cos\phi\sin\theta \\
&\quad + (2\dot{y}'\dot{\theta} + y'\ddot{\theta} - 2z'\dot{\phi}\dot{\theta})\sin\phi\cos\theta
\end{aligned}
$$

$$\ddot{y} = (\ddot{y}' + y'\dot{\phi}^2 - z'\ddot{\phi}) \cos\phi - (\ddot{z}' - z'\dot{\phi}^2 + y'\ddot{\phi}) \sin\phi$$

$$\begin{aligned} \ddot{z} = & - (2\dot{x}'\dot{\theta} + x'\ddot{\theta}) \cos\theta - (\ddot{x}' - x'\dot{\theta}^2) \sin\theta \\ & + (2\dot{y}'\dot{\phi} + y'\ddot{\phi} + \ddot{z}' - z'(\dot{\phi}^2 + \dot{\theta}^2)) \cos\phi \sin\theta \\ & - (2\dot{y}'\dot{\theta} + y'\dot{\theta} - 2z\dot{\phi}\dot{\theta}) \sin\phi \sin\theta \\ & - (2y'\dot{\phi}\dot{\theta} + 2\dot{z}'\dot{\theta} + 2\dddot{\theta}) \cos\phi \sin\theta \\ & + (\ddot{y}' - y'(\dot{\phi}^2 + \dot{\theta}^2) - 2\dot{z}'\dot{\phi} - z'\ddot{\phi} - z'\ddot{\phi}) \sin\phi \cos\theta \end{aligned} \tag{15}$$

Expressions for the derivatives of x', y' and z' are not derived explicitly for reasons which will be explained in Section 3.0.

2.6 Aerodynamic Forces and Their Derivatives

The aerodynamic force components in the sound Equation (3) are derived from the components F_N and F_T which act directly on the blade. F_N is defined as the component of the aerodynamic load on the blade which acts normal to the blade axis (in its deformed condition) in the plane containing the shaft axis z'. It acts in the same sense as the lift on the blade. F_T is the tangential component, again acting normal to the blade axis but parallel to the $x'y'$ plane. Its sense is opposite to the blade drag force. Referring to Figure 6, we see that the blade force components in the x', y' and z' directions are:

$$\begin{aligned} F_{x'} &= F_T \sin(\psi + \zeta') + F_N \sin\beta' \cos(\psi + \zeta') \\ F_{y'} &= - F_T \cos(\psi + \zeta') + F_N \sin\beta' \sin(\psi + \zeta') \\ F_{z'} &= - F_N \cos\beta' \end{aligned} \tag{16}$$

where β' and ζ' are the blade slopes relative to the $x'y'$ plane and the radius at azimuth ψ respectively. That is,

$$\beta' = \beta + \tan^{-1}\left(\frac{d\eta_N}{dr}\right)$$

$$\zeta' = \zeta + \tan^{-1}\left(\frac{d\eta_T}{dr}\right) \qquad (17)$$

The transformations which are required to obtain the components F_X, F_Y and F_Z in the fixed coordinate system are identical to those used to convert x', y', z' to X, Y, Z (Equations (4) and (5)). The derivatives $\dot{F}_X$, $\dot{F}_Y$ and $\dot{F}_Z$ are also derived from the rotor axis values using similar transformations (Equations (12) through (15)), and again the derivatives in the rotor coordinate system $\dot{F}_{x'}$, $\dot{F}_{y'}$ and $\dot{F}_{z'}$ are not derived explicitly for reasons outlined in the following section.

3.0 METHOD OF SOLUTION BY DIGITAL COMPUTER

The technique for calculating the helicopter rotor sound field is best explained by a description of the computational steps of the program HERON 1. Figure 7 is an illustration of these basic steps in flow diagram form. The program is divided into two phases. The first processes the input data into a convenient form for storage and the second computes the sound pressure level at a preselected number of field points.

3.1 Phase I Initial Data Processing

For each rotor, the blade loading and motion data is read by the program either as a number of arrays, listing their values at a discrete number of points over the rotor disc, or as a series of Fourier coefficients, one set for each radial station. To reduce the volume of later computations, this initial phase of the program calculates and stores the displacements of each loading point with respect to the rotor axes x', y', z' according to Equation (7). The three force components $F_{x'}$, $F_{y'}$ and $F_{z'}$ are also using Equation (16). These calculations are performed, upon the input data in whichever form it comes, at a predetermined number of azimuth stations. The history of each of these six variables around the azimuth is then harmonically analyzed for every radial station, and the Fourier coefficients are stored. This operation is carried out for one blade only since it is assumed that all blades are loaded and respond in identical manners. From the stored coefficients it is possible for later elements of the program to interpolate for all required blade loading and motion variables at any arbitrary azimuth station.

3.2 Phase II Calculation of the Sound Field

Phase II of the program is repeated in its entirety for each required observer position, X_0, Y_0, Z_0.

3.2.1 "Observer Time" t

The sound field observed at the position X_0, Y_0, Z_0 is calculated as a time history of the acoustic pressure Δp at the "observer time" intervals 0, Δt, $2\Delta t$, ... T, where T is the fundamental period. The sound harmonic amplitudes are then obtained through a Fourier analysis of this time history.

The fundamental period T, to an observer moving with the aircraft, is the blade passage period $2\pi/\Omega B$. However, for a stationary observer this shifts by the factor $(1 - M_0)^{-1}$ where M_0 is the aircraft Mach number component in the direction of the observer.

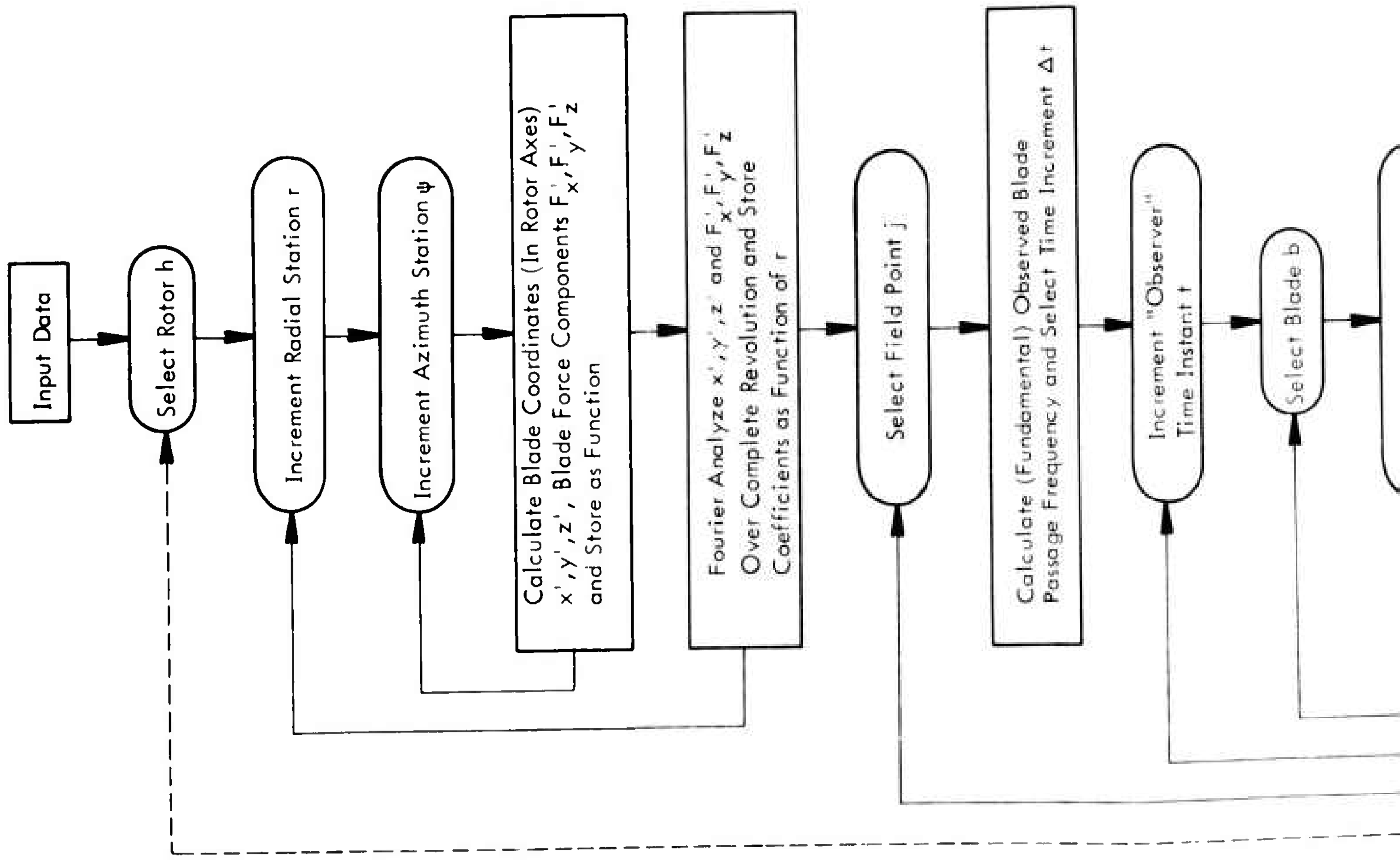
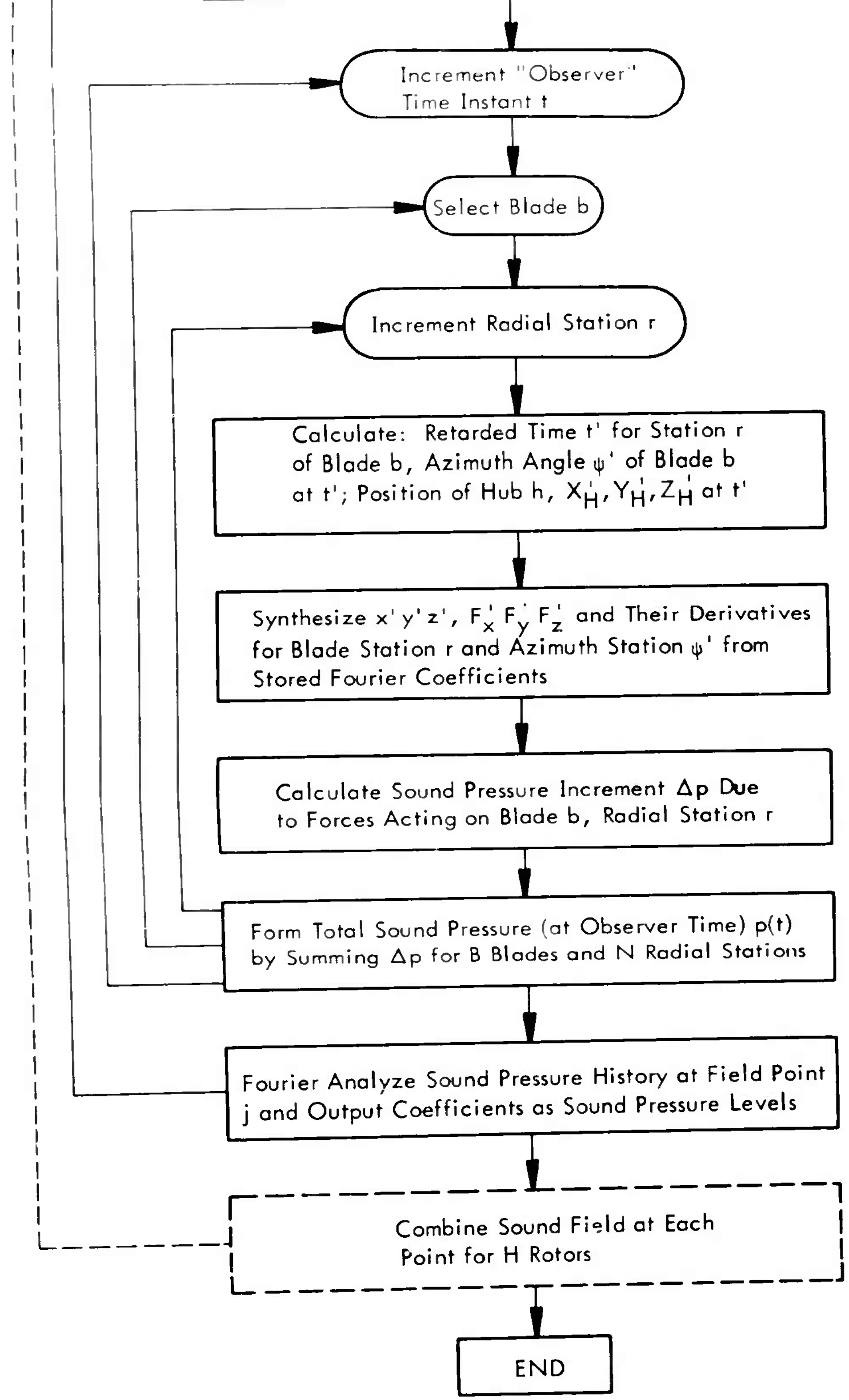


Figure 7. Block Schematic Flow Diagram For HERON 1.


Increment "Observer" Time Instant t
Select Blade b
Increment Radial Station r
Calculate: Retarded Time t' for Station r of Blade b, Azimuth Angle ψ' of Blade b at t'; Position of Hub h, X'_H, Y'_H, Z'_H at t'
Synthesize x' y' z', F'_x F'_y F'_z and Their Derivatives for Blade Station r and Azimuth Station ψ' from Stored Fourier Coefficients
Calculate Sound Pressure Increment Δp Due to Forces Acting on Blade b, Radial Station r
Form Total Sound Pressure (at Observer Time) p(t) by Summing Δp for B Blades and N Radial Stations
Fourier Analyze Sound Pressure History at Field Point j and Output Coefficients as Sound Pressure Levels
Combine Sound Field at Each Point for H Rotors
END

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One approach is to calculate r' e observed period which can then be divided into an appropriate number of intervals. However, because of the aircraft motion, a slight error is introduced by this procedure which results in the computation of something slightly less than a complete period. This error increases with sound harmonic number (which is equivalent to a reduction of wavelength).

The alternative which has been adopted in the program is to calculate the time history at a point which "moves" with the same velocity as the rotor hub but which has the average position X_0, Y_0, Z_0 during the period T. The only modification required for the final result is to correct the observed frequency for the Doppler shift factor $(1 - M_0)^{-1}$. The quotation marks are used because at each time increment the sound pressure is still calculated at a stationary point which has simply moved the same distance as the rotor since the previous time increment. All relative velocity effects in the acoustic calculation are retained.

For each observer time t, the sound pressure Δp is calculated at the effective observer positions:

$$X'_0 = X_0 + \dot{X}_H \left(t - \frac{T}{2}\right)$$

$$Y'_0 = Y_0 + \dot{Y}_H \left(t - \frac{T}{2}\right)$$

$$Z'_0 = Z_0 + \dot{Z}_H \left(t - \frac{T}{2}\right) \qquad (18)$$

To calculate the Mach number component M_0, and hence the Doppler shift correction, it is first necessary to calculate the sound propagation time τ_0 as follows:

The position of the rotor hub at time t is X_H, Y_H, Z_H and the distance between the hub and the observer at X_0, Y_0, Z_0, according to Equation (6), is

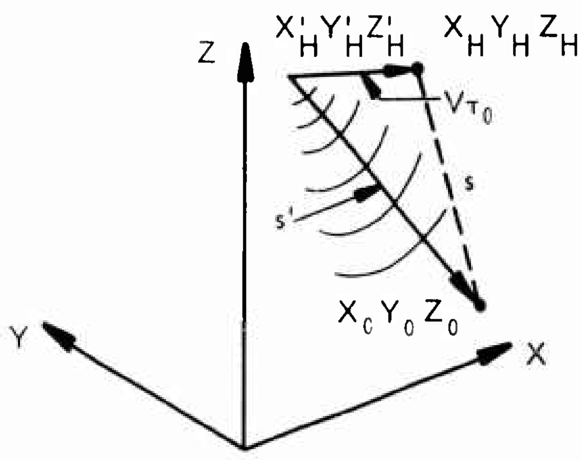


Figure 8. Velocity Diagram for Sound Radiation by Moving Aircraft.

$$s = (\bar{x}_0^2 + \bar{y}_0^2 + \bar{z}_0^2)^{\frac{1}{2}}$$

However, the sound arriving at the observer at time t was generated at the retarded time $t - \tau_0$ when the hub was at X'_H, Y'_H, Z'_H. The corresponding velocity triangle is shown in Figure 8. The resultant aircraft velocity is

$$V = \left(\dot{X}_H^2 + \dot{Y}_H^2 + \dot{Z}_H^2\right)^{\frac{1}{2}} \tag{19}$$

and the propagation time τ_0 is given by

$$\tau_0 = -\frac{(\bar{x}_0 \dot{X}_H + \bar{y}_0 \dot{Y}_H + \bar{z}_0 \dot{Z}_H) + \sqrt{(\bar{x}_0 \dot{X}_H + \bar{y}_0 \dot{Y}_H + z_0 \dot{Z}_H)^2 + s^2 (c^2 - V^2)}}{c^2 - V^2} \tag{20}$$

From the propagation time the coordinates at the instant of sound emission can be calculated. The component hub to observer separations are $\bar{x}'_0$, $\bar{y}'_0$ and $\bar{z}'_0$ where

$$\begin{aligned} \bar{x}'_0 &= \bar{x}_0 + \dot{X}_H \tau_0 \\ \bar{y}'_0 &= \bar{y}_0 + \dot{Y}_H \tau_0 \\ \bar{z}'_0 &= \bar{z}_0 + \dot{Z}_H \tau_0 \end{aligned} \tag{21}$$

Hence, the required Mach number component is

$$M_0 = \frac{\bar{x}'_0 \dot{X}_H + \bar{y}'_0 \dot{Y}_H + \bar{z}'_0 \dot{Z}_H}{c^2 \tau_0} \tag{22}$$

3.2.2 Retarded Time

The retarded time of each blade loading point has to be calculated independently for each observer time instant. This requires solution of the transcendental equation

$$c\tau = \sqrt{\bar{x}^2(t-\tau) + \bar{y}^2(t-\tau) + \bar{z}^2(t-\tau)} \quad (23)$$

This equation can only be solved by iteration, starting with some realistically chosen value of τ and successively substituting newly calculated values into the right-hand side until the solution converges. The right-hand side of Equation (23) is in fact a very lengthy expression which includes Equations (4), (5), (6) and (8). The following procedure is followed to minimize the computational time involved.

For the most inboard blade loading point, the rotor hub retarded time $(t - \tau_0)$ is used as a starting value. The RHS of Equation (23) is calculated using the first harmonic Fourier coefficients of x', y' and z'. As an example,

$$\psi = \psi(t - \tau)$$

$$x' = a_{x'_0}(r) + a_{x'_1}(r)\cos\psi + b_{x'_1}(r)\sin\psi$$

The values of y' and z' are calculated similarly and the necessary transformations applied to compute $\bar{x}, \bar{y}$ and $\bar{z}$. These are substituted into Equation (23) to yield a second approximation to the retarded time. The iteration proceeds but a successively greater number of harmonics is admitted in the calculation of the x', y', z' coordinates in successive iterations. By the time convergence is reached, the pre-determined limiting number of harmonics is admitted. It is found that this method considerably reduces the volume of computation (through avoiding a large number of Fourier summations) without increasing the number of iterative cycles to convergence.

The calculations for the next radial station use the final value of τ from the first radial station, and so on. Experience shows that an average of four or five iterations is required for a convergent solution with an error of less than 10^{-5} seconds.

3.2.3 Blade Loads and Motions

The Fourier representation of the load and motion distributions enables the required displacements, velocities, accelerations, forces and rate of change of forces to be computed with a minimum amount of effort. From the retarded time $t - \tau$, the relevant blade azimuth angle ψ is obtained and the motion and load variables in the x', y', z' system are synthesized as shown in the following example:

$$\psi = \psi(t - \tau)$$

$$x' = a_{x'_0} + \sum_{k=1}^{K} \left\{ a_{x'_k} \cos k\psi + b_{x'_k} \sin k\psi \right\}$$

$$x' = -\Omega \sum_{k=1}^{K} k \left\{ a_{x'_k} \sin k\psi - b_{x'_k} \cos b\psi \right\}$$

$$x' = -\Omega^2 \sum_{k=1}^{K} k^2 \left\{ a_{x'_k} \cos k\psi + b_{x'_k} \sin k\psi \right\}$$

The transformations listed in Section 2.1.2 are then applied to derive all the necessary components of the sound Equation (3).

3.2.4 The Sound Field

The observed sound pressure increment generated by each loading point is calculated according to Equations (1) and (2). The increments due to all loading points on all blades are then summed to give the total pressure increment at time t. The entire process is repeated for successive time instants until a history is obtained for one complete (blade passage) period. The final step is to Fourier analyze this time history into its sine and cosine component harmonics (a_n and b_n). Each harmonic sound pressure level is then expressed in decibels by performing the transformation

$$SPL_n = \left(10 \log \sqrt{a_n^2 + b_n^2} + 124.6 \right) \text{ dB re } .0002 \ \mu\text{Bar}$$

3.2.5 Multiple Rotors

In its present form the program computes the sound harmonics at each field point for each rotor and outputs the results independently. If the rotors have the same fundamental blade passage frequency then the sound harmonics due to the individual rotors reinforce each other and the in-phase and out-of-phase components are simply added together separately. If the rotors have different fundamental frequencies (for example the main and tail rotors of a "single rotor" helicopter), then the individual harmonics do not interfere and the sound spectra are merely superimposed. The final block in Figure 7 is drawn with a broken line since the program does not perform this function automatically. However, sufficient information is output (namely the frequency, cosine and sine harmonic amplitude in lb/ft^2 and sound pressure level of each harmonic) to enable the necessary summations to be performed rapidly by hand.

4.0 PROGRAM FLOW CHARTS, EQUATIONS AND LISTING

The computer program HERON 1 is written in FORTRAN IV for the Control Data Corporation's CDC 3300 utilizing the SCOPE operat[illegible] system. The minimum hardware configuration required is as follows:

1. 3300 series computer with 32,000-word core storage.
2. Card reader.
3. Line printer.
4. One scratch tape unit (LUN2)

A complete description of the program is contained in this section which includes the following items:

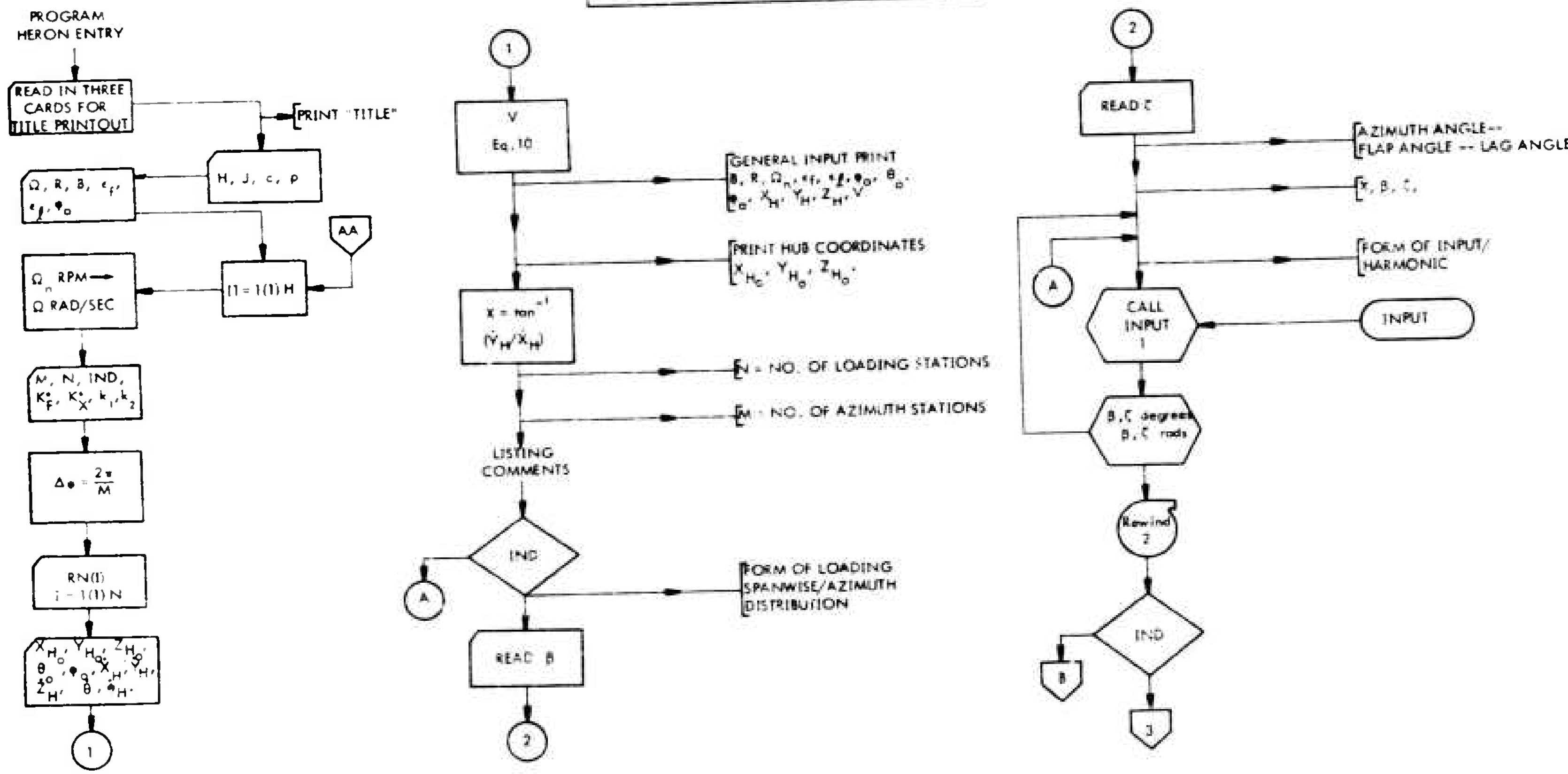
4.1 A detailed flow chart for the program. The noted equation numbers correspond to list 4.2.

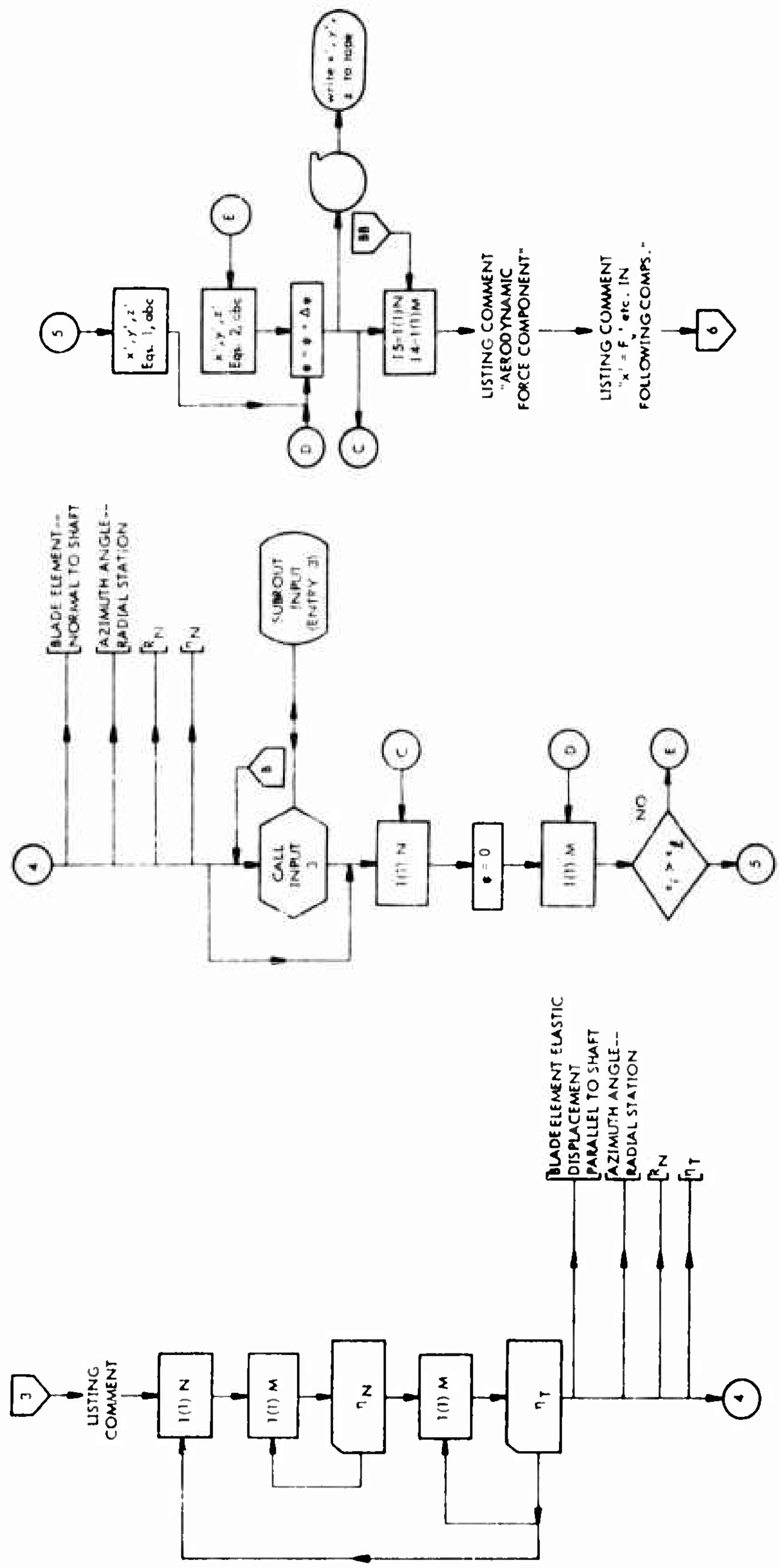
4.2 A complete list of programmed equations.

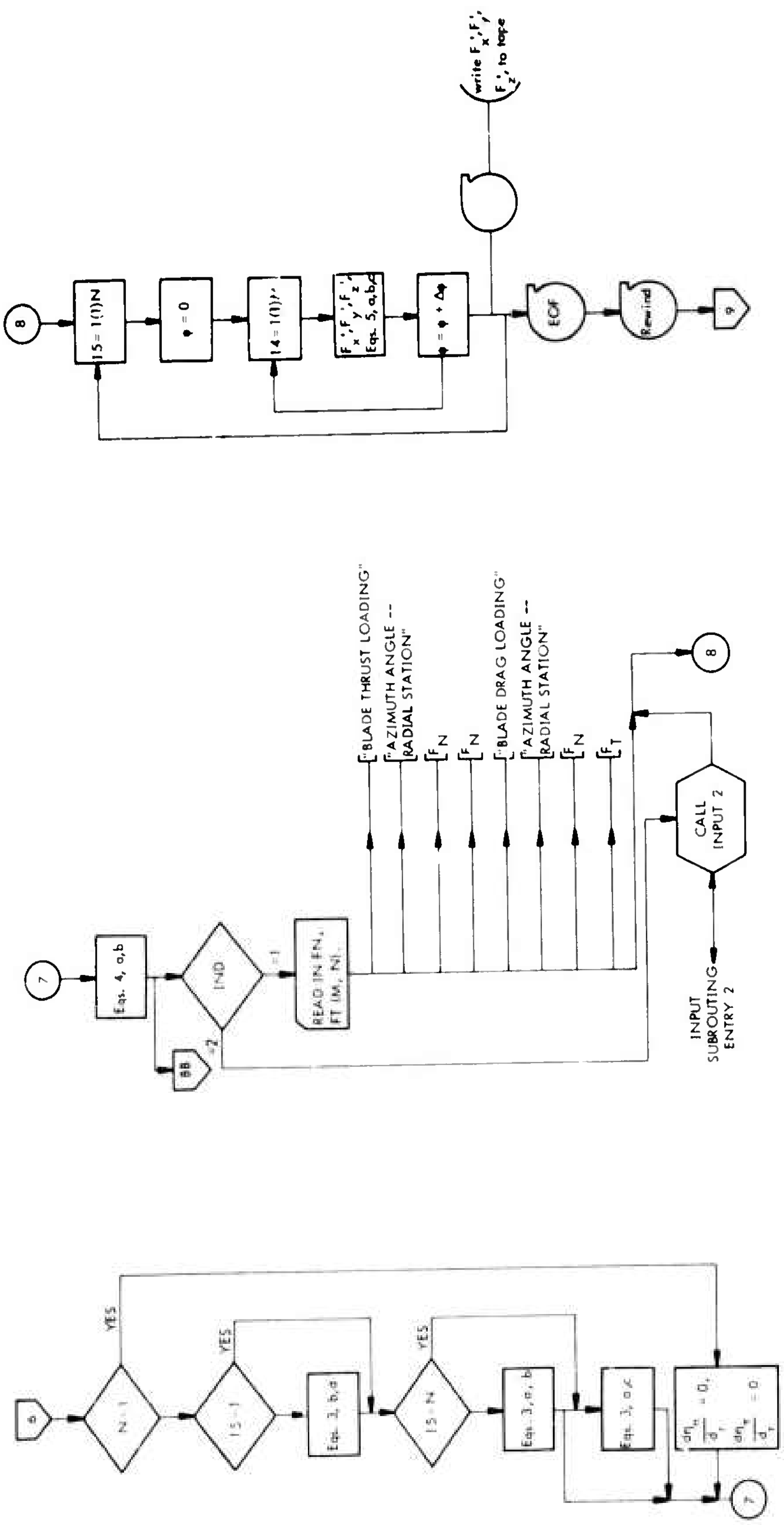
4.3 A program listing.

4.4 A list of program symbols and definitions (Table I).

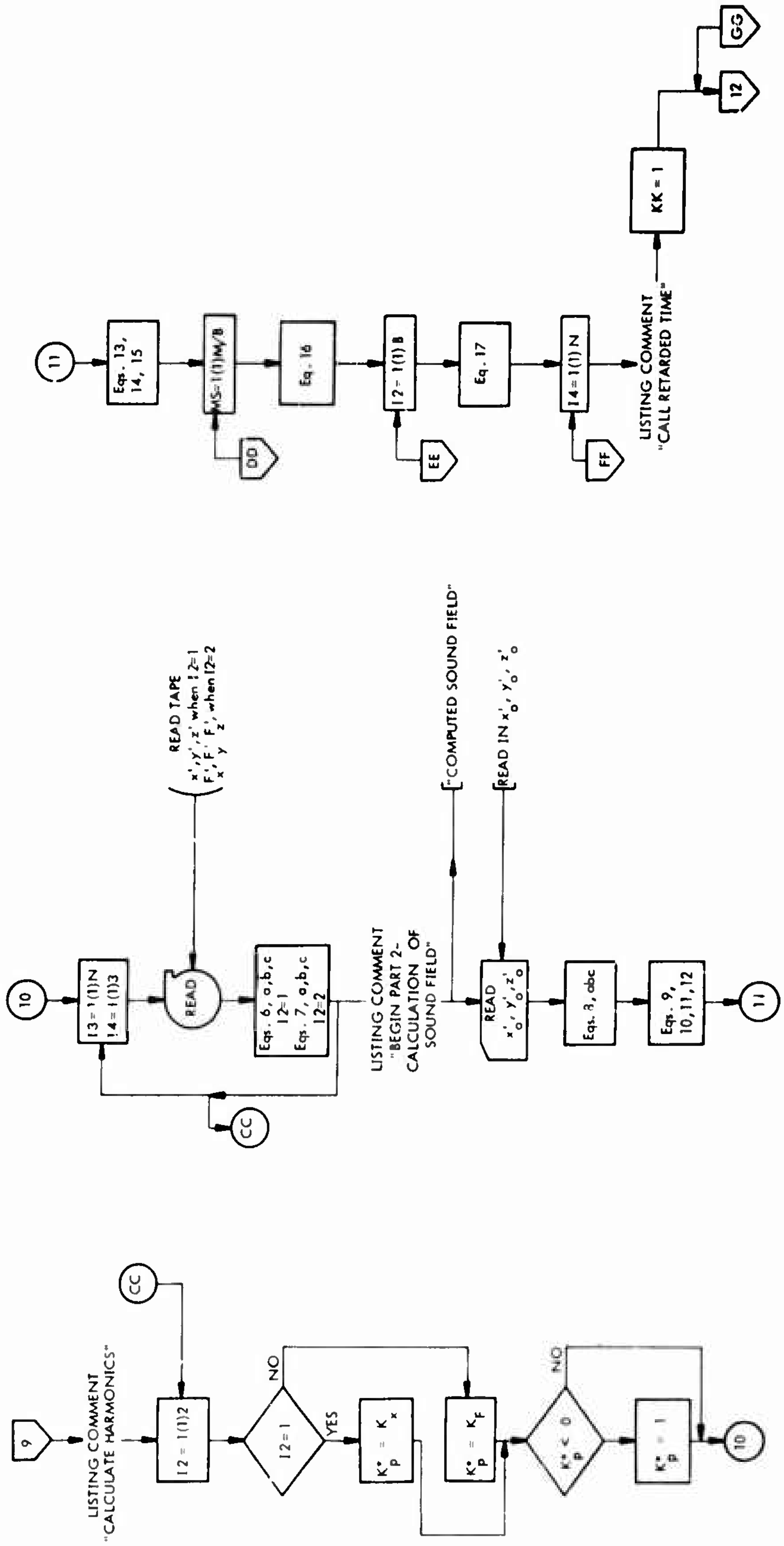
DETAILED FLOW CHART FOR PROGRAM HERON I



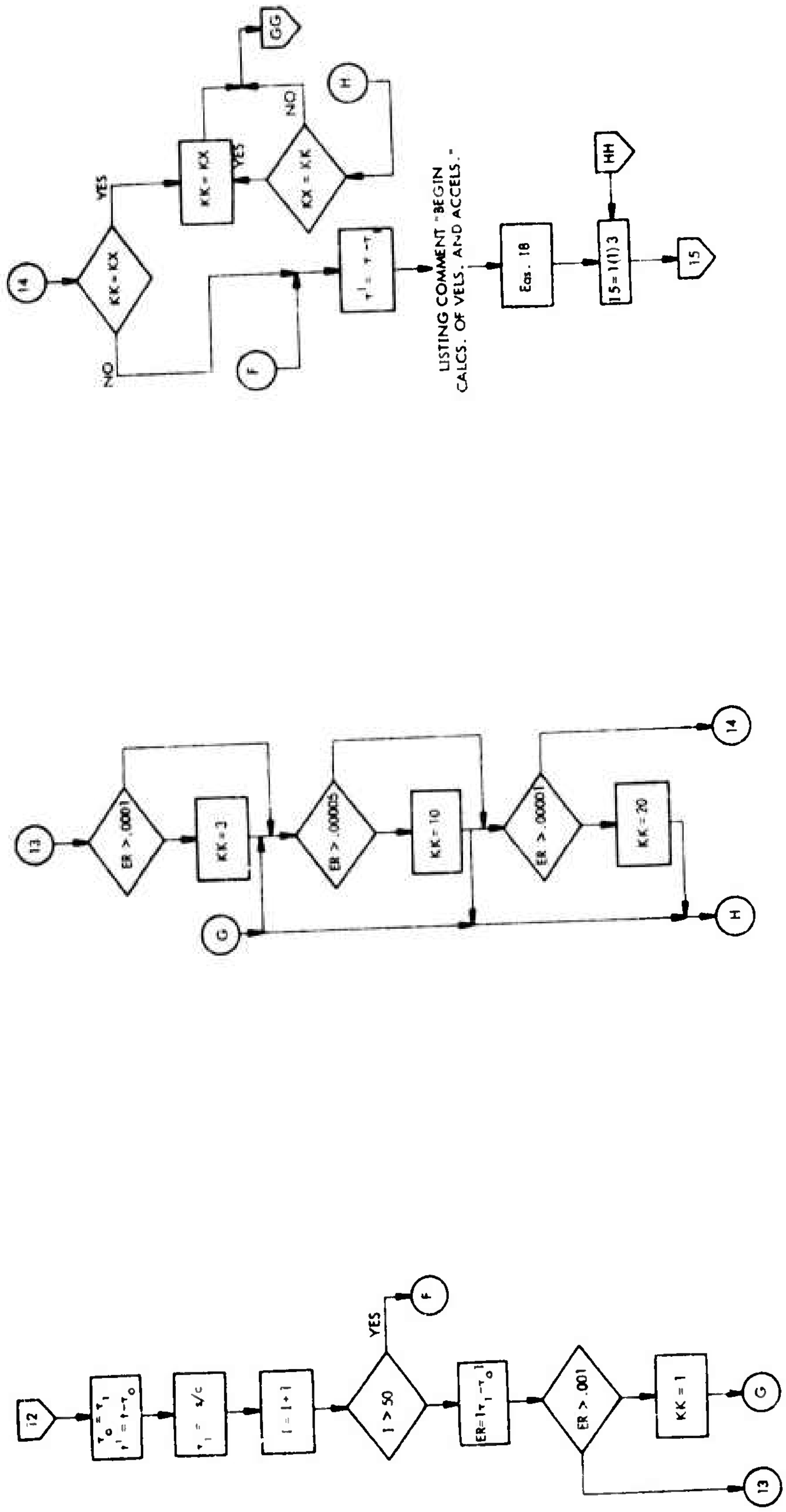

LISTING COMMENT
BLADE ELEMENT ELASTIC DISPLACEMENT PARALLEL TO SHAFT
AZIMUTH ANGLE-- RADIAL STATION
BLADE ELEMENT-- NORMAL TO SHAFT
AZIMUTH ANGLE-- RADIAL STATION
CALL INPUT 3
SUBROUT INPUT (ENTRY 3)
NO
LISTING COMMENT "AERODYNAMIC FORCE COMPONENT"
LISTING COMMENT "x' = F_x' etc. IN FOLLOWING COMPS."

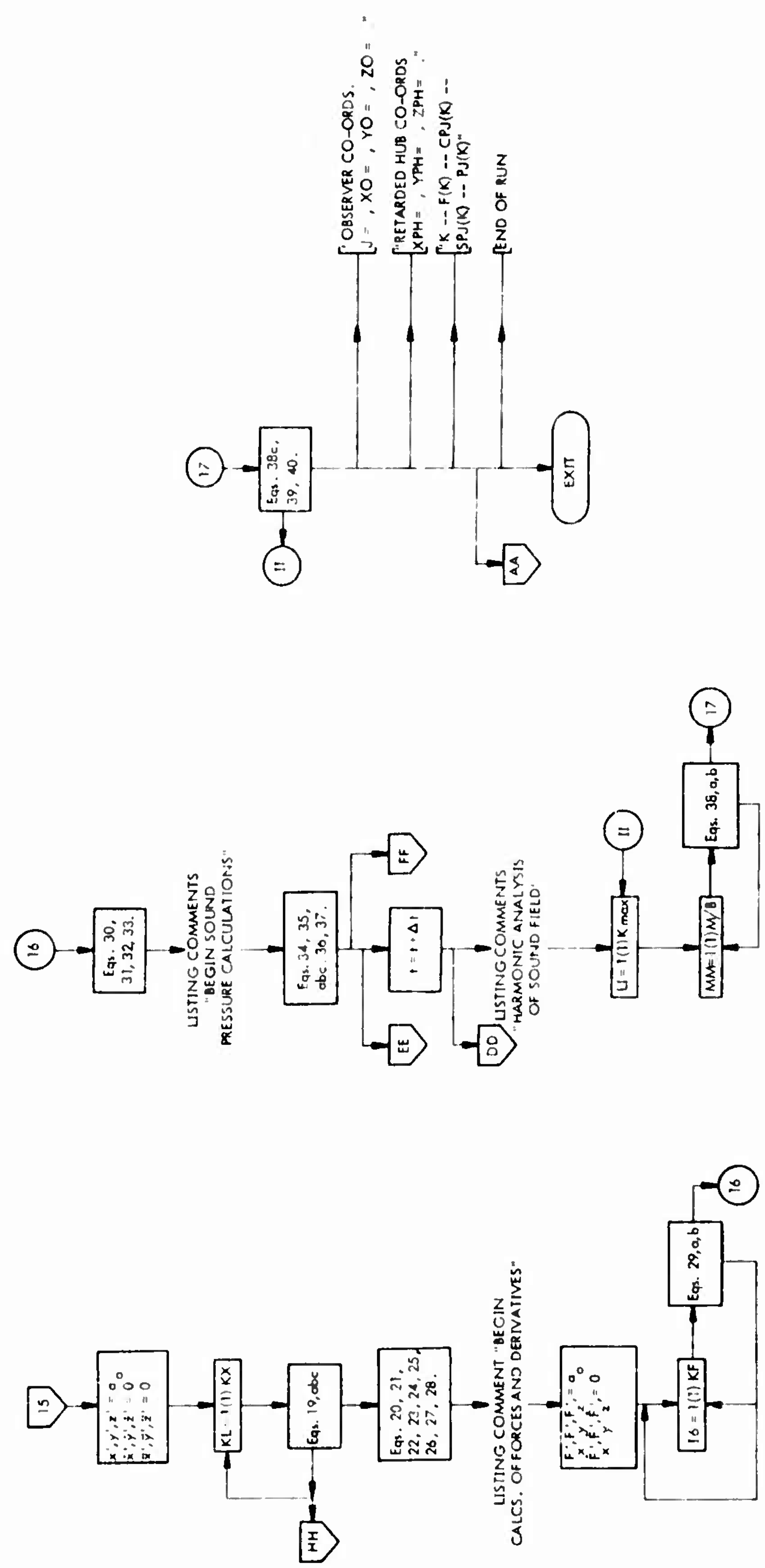


6
N = 1
YES
I5 = 1
Eqs. 3, b, d
I5 = N
Eqs. 3, a, b
Eqs. 3, a, c
7
Eqs. 4, a, b
IND
=1
=2
8B
READ IN FN, FT (M, N)
"BLADE THRUST LOADING"
"AZIMUTH ANGLE -- RADIAL STATION"
"BLADE DRAG LOADING"
CALL INPUT 2
INPUT SUBROUTING ENTRY 2
8
I5 = 1(1)N
φ = 0
I4 = 1(1)ν
Eqs. 5, a, b, c
φ = φ + Δφ
EOF
Rewind
9
write F′x, F′y, F′z, to tape



LISTING COMMENT "CALCULATE HARMONICS"
I2 = 1(1)2
I2 = 1
YES
NO
CC
10
I3 = 1(1)N
I4 = 1(1)3
READ
READ TAPE
Eqs. 6, a,b,c
I2=1
Eqs. 7, a,b,c
I2=2
LISTING COMMENT "BEGIN PART 2- CALCULATION OF SOUND FIELD"
"COMPUTED SOUND FIELD"
Eqs. 8, abc
Eqs. 9, 10, 11, 12
11
Eqs. 13, 14, 15
Eq. 16
Eq. 17
DD
EE
FF
LISTING COMMENT "CALL RETARDED TIME"
KK = 1
12
GG


14
KK = KX
YES
NO
KK = KX
KX = KK
GG
H
F
LISTING COMMENT "BEGIN CALCS. OF VELS. AND ACCELS."
Eqs. 18
I5 = 1(1)3
HH
15
13
ER > .0001
KK = 2
ER > .00005
KK = 10
ER > .00001
KK = 20
G
12
I > 50
ER > .001
KK = 1


15
KL = 1(1) KX
Eqs. 19,abc
HH
Eqs. 20, 21, 22, 23, 24, 25, 26, 27, 28.
LISTING COMMENT "BEGIN CALCS. OF FORCES AND DERIVATIVES"
I6 = 1(1) KF
Eqs. 29,a,b
16
16
Eqs. 30, 31, 32, 33.
LISTING COMMENTS "BEGIN SOUND PRESSURE CALCULATIONS"
Eqs. 34, 35, abc 36, 37.
t = t + Δt
EE
FF
DD
LISTING COMMENTS "HARMONIC ANALYSIS OF SOUND FIELD"
LI = 1(1) K max
11
MM = 1(1) M/B
Eqs. 38,a,b
17
17
Eqs. 38c, 39, 40.
11
"OBSERVER CO-ORDS. J = , XO = , YO = , ZO = "
"RETARDED HUB CO-ORDS XPH = , YPH = , ZPH = "
"K -- F(K) -- CPJ(K) -- SPJ(K) -- PJ(K)"
END OF RUN
EXIT
AA

List of Equations Computed by Program HERON 1

Equation No.

1 a) $$x' = -\epsilon_\ell \cos\psi - \left\{(r - \epsilon_f)\cos\beta + (\epsilon_f - \epsilon_\ell)\right\}\cos(\psi + \zeta) + \eta_T \sin(\psi + \zeta) + \eta_N \sin\beta\cos(\psi + \zeta)$$

b) $$y' = -\epsilon_\ell \sin\psi - \left\{(r - \epsilon_f)\cos\beta + (\epsilon_f - \epsilon_\ell)\right\}\sin(\psi + \zeta) - \eta_T \cos(\psi + \zeta) + \eta_N \sin\beta\sin(\psi + \zeta)$$

c) $$z' = -(r - \epsilon_f)\sin\beta - \eta_N\cos\beta.$$

2 a) $$x' = -(\epsilon_f + \epsilon_\ell\cos\beta)\cos\psi - (r + \epsilon_\ell)\cos\beta\cos(\psi + \zeta) + \eta_T\sin(\psi + \zeta) + \eta_N\sin\beta\cos(\psi + \zeta)$$

b) $$y' = -(\epsilon_f + \epsilon_\ell\cos\beta)\sin\psi - (r + \epsilon_\ell)\cos\beta\sin(\psi + \zeta) - \eta_T\cos(\psi + \zeta) + \eta_N\sin\beta\sin(\psi + \zeta)$$

c) $$z' = -(r - \epsilon_f)\sin\beta - \eta_N\cos\beta.$$

3 a) $$d\eta_N/dr = \left(\eta_N(n+1) - \eta_N(n-1)\right)/\left(r(n+1) - r(n-1)\right)$$

b) $$d\eta_N/dr = \left(\eta_N(2) - \eta_N(1)\right)/\left(r(2) - r(1)\right)$$

c) $$d\eta_T/dr = \left(\eta_T(n+1) - \eta_T(n-1)\right)/\left(r(n+1) - r(n-1)\right)$$

d) $$d\eta_T\,dr = \left(\eta_T(2) - \eta_T(1)\right)/\left(r(2) - r(1)\right)$$

e) $$d\eta_N/dr = \left(\eta_N(n) - \eta_N(n-1)\right)/\left(r(n) - r(n-1)\right)$$

f) $$d\eta_T/dr = \left(\eta_T(n) - \eta_T(n-1)\right)/\left(r(n) - r(n-1)\right)$$

4 a) $\beta' = \beta + \tan^{-1}(d\eta_N/dr)$

b) $\zeta' = \zeta' + \tan^{-1}(d\eta_T/dr)$

5 a) $F_{x'} = T \sin(\psi + \zeta') + F_N \sin\beta' \cos(\psi + \zeta')$

b) $F_{y'} = - F_T \cos(\psi + \zeta') + F_N \sin\beta' \sin(\psi + \zeta')$

c) $F_{z'} = - F_N \cos\beta'$.

6 a) $a_{0_n}(\omega) = \frac{1}{M} \sum_{m=1}^{M} \omega$

b) $a_{k_n}(\omega) = \frac{2}{M} \sum \omega \cos\left((m - 1) \dot{x}\, 2\pi k/M\right)$

c) $b_{k_n}(\omega) = \frac{2}{M} \sum \omega \sin\left((m - 1) \cdot 2\pi k/M\right)$

7 a) $a_{0_n}(F_i) = \frac{1}{M} \sum_{m=1}^{M} F_i$

b) $a_{k_n}(F_i) = \frac{2}{M} \sum F_i \cos\left((m - 1) \cdot 2\pi k/M\right)$

c) $b_{k_n}(F_i) = \frac{2}{M} \sum F_i \sin\left((m - 1) \cdot 2\pi k/M\right)$

8 a) $\bar{x}_H = X_H - X'_0$

b) $\bar{y}_H = Y_H - Y'_0$

c) $\bar{z}_H = Z_H - Z'_0$

9 $S_H = \sqrt{\bar{x}_H^2 + \bar{y}_H^2 + \bar{z}_H^2}$

10 $$V = \sqrt{\dot{X}_H^2 + \dot{Y}_H^2 + \dot{Z}_H^2}$$

11 $$\tau'_0 = \left\{ -(\bar{x}_H \dot{X}_H + \bar{y}_H \cdot \dot{Y}_H + \bar{z}_H \dot{Z}_H) + ((\bar{x}_H \dot{X}_H + \bar{y}_H \dot{Y}_H + \bar{z}_H \dot{Z}_H)^2 + S_H^2 (a_0^2 - V^2))^{\frac{1}{2}} \right\} \Big/ (a_0^2 - V^2)$$

12 a) $$\bar{x}'_H = \bar{x}_H + \dot{X}_H \tau'_0$$

b) $$\bar{y}'_H = \bar{y}_H + \dot{Y}_H \tau'_0$$

c) $$\bar{z}'_H = \bar{z}_H + \dot{Z}_H \tau'_0$$

13 $$M_0 = (\bar{x}'_H \dot{X}_H + \bar{y}'_H \dot{Y}_H + \bar{z}'_H \dot{Z}_H) / a_0^2 \tau'_0$$

14 a) $$X'_H = X_H - \dot{X}_H \tau'_0$$

b) $$Y'_H = Y_H - \dot{Y}_H \tau'_0$$

c) $$Z'_H = Z_H - \dot{Z}_H \tau'_0$$

15 a) $$T = 2\pi / \Omega B$$

b) $$\Delta T = 2\pi / \Omega M$$

16 a) $$X_0 = X'_0(j) + \dot{X}_H \left(t - \frac{T}{2} \right)$$ $t = 0\ (\Delta T)\ T - \Delta T$

b) $$Y_0 = Y'_0(j) + \dot{Y}_H \left(t - \frac{T}{2} \right)$$

c) $$Z_0 = Z'_0(j) + \dot{Z}_H \left(t - \frac{T}{2} \right)$$

17 $$\psi_0 = \psi_0 + \frac{2\pi}{B} \cdot b$$ $b = 1\ (1)\ B$

18 a) $\psi = \psi_0 + \Omega t'$

b) $X_H = X_{H_0} + \dot{X}_H t'$

c) $Y_H = Y_{H_0} + \dot{Y}_H t'$

d) $Z_H = Z_{H_0} + \dot{Y}_H t'$

e) $\theta = \theta_0 + \dot{\theta} t'$

f) $\phi = \phi_0 + \dot{\phi} t'$

g) $\phi^{-1} = \tan^{-1} (\cos\theta \tan\phi)$

h) $\dot{\phi}' = \cos^2\phi' (\dot{\phi} \cos\theta \sec^2\phi - \dot{\theta} \sin\theta \tan\phi)$

19 a) $\omega' = \omega' + a_{k_n}(\omega') \cos k\psi + b_{k_n}(\omega') \sin k\psi$

b) $\dot{\omega}' = \dot{\omega}' - k\Omega \left[a_{k_n}(\omega') \sin k\psi - b_{k_n}(\omega') \cos k\psi \right]$

c) $\ddot{\omega}' = \ddot{\omega}' - k^2 \Omega^2 \left[a_{k_n}(\omega') \cos k\psi + b_{k_n}(\omega') \sin k\psi \right]$

20 a) $x = x' \cos\theta + y' \sin\phi' \sin\theta + z' \cos\phi' \sin\theta$

b) $y = y' \cos\phi' - z' \cos\phi'$

c) $z = - x' \sin\theta + y' \sin\phi' \cos\theta + z' \cos\phi' \cos\theta$

21 a) $\dot{x} = \dot{x}' \cos\theta - x'\dot{\theta} \sin\theta - (z'\dot{\phi}' - \dot{y}') \sin\phi' \sin\theta$

$+ (\dot{z}' + y'\dot{\phi}') \cos\phi' \sin\theta + y'\dot{\theta} \sin\phi' \cos\theta$

$+ z'\dot{\theta} \cos\phi' \cos\theta$

b) $\dot{y} = (\dot{y}' - z'\dot{\phi}') \cos\phi' - (y'\dot{\phi}' + \dot{z}') \sin\phi'$

c) $\dot{z} = -\dot{x}'\sin\theta - x'\dot{\theta}\cos\theta + (\dot{y}' - z'\dot{\phi}')\sin\phi'\cos\theta$

$\quad + (y'\dot{\phi}' + \dot{z}')\cos\phi'\cos\theta - y'\dot{\theta}'\sin\phi'\sin\theta - z'\dot{\theta}'\cos\phi'\sin\theta$

22 a) $\ddot{x} = (\ddot{x}' - x'\dot{\theta}^2)\cos\theta - 2\dot{x}'\dot{\theta}\sin\theta + (2y'\dot{\phi}'\dot{\theta} + 2\dot{z}'\dot{\theta})\cos\phi'\cos\theta$

$\quad + (\ddot{y}' - y'(\dot{\phi}'^2 + \dot{\theta}^2) - 2\dot{z}'\dot{\phi}')\sin\phi'\sin\theta + (2\dot{y}'\dot{\phi}' + \ddot{z}' - z'(\dot{\phi}'^2$

$\quad + \dot{\theta}^2))\cos\phi'\sin\theta + (2\dot{y}'\theta - 2z'\dot{\phi}'\dot{\theta})\sin\phi'\cos\theta$

b) $\ddot{y} = (\ddot{y}' + y'\dot{\phi}'^2)\cos\phi' - (\ddot{z}' - z'\dot{\phi}^2)\sin\phi'$

c) $\ddot{z} = -2\dot{x}'\dot{\theta}\cos\theta - (\ddot{x}' - x\dot{\theta}^2)\sin\theta + (2\dot{y}'\dot{\phi}' + \ddot{z}'$

$\quad - z'(\dot{\phi}'^2 + \dot{\theta}^2))\cos\phi'\sin\theta - (2\dot{y}'\dot{\theta} - 2z'\dot{\phi}'\dot{\theta})\sin\phi'\sin\theta$

$\quad - (2y'\dot{\phi}'\dot{\theta} + 2\dot{z}'\dot{\theta})\cos\phi'\sin\theta + (\ddot{y}' - y'(\dot{\phi}'^2 + \theta^2)$

$\quad - 2\ddot{z}'\dot{\phi}')\sin\phi'\cos\theta.$

23 a) $\bar{x} = X_0 - X_H - x\cos\chi - y\sin\chi$

b) $\bar{y} = Y_0 - Y_H - x\sin\chi - y\cos\chi$

c) $\bar{z} = Z_0 - Z_H + z.$

24 a) $\dot{X} = \dot{X}_H + \dot{x}\cos\chi + \dot{y}\sin\chi$

b) $\dot{Y} = \dot{Y}_H + \dot{x}\sin\chi - \dot{y}\cos\chi$

c) $\dot{Z} = \dot{Z}_H - \dot{z}$

25 a) $\ddot{X} = \ddot{x}\cos\chi + \ddot{y}\sin\chi$

b) $\ddot{Y} = -\ddot{y}\cos\chi + \ddot{x}\sin\chi$

c) $\ddot{Z} = \ddot{z}.$

26 $S = \sqrt{\bar{x}^2 + \bar{y}^2 + \bar{z}^2}$

27 $$M_s = (\bar{x}\dot{X} + \bar{y}\dot{Y} + \bar{z}\dot{Z})/S\, a_0$$

28 $$\dot{M}_s = (\bar{x}\ddot{X} + \bar{y}\ddot{Y} + \bar{z}\ddot{Z})/S\, a_0$$

29 a) $$F_i = a_0(F_i) + a_{k_n}(F_i)\cos k\psi + b_{k_n}(F_i)\sin k\psi$$

b) $$\dot{F}_i = -k\Omega\ a_{k_n}(F_i)\sin k\psi - b_{k_n}(F_i)\cos k\psi$$

30 a) $$F_x = F_{x'}\cos\theta + F_{y'}\sin\phi'\sin\theta + F_{z'}\cos\phi'\sin\theta$$

b) $$F_y = F_{y'}\cos\phi' - F_{z'}\sin\phi'$$

c) $$F_z = -F_{x'}\sin\theta + F_{y'}\sin\phi'\cos\theta + F_{z'}\cos\phi'\cos\theta$$

31 a) $$\dot{F}_x = \dot{F}_{x'}\cos\theta - F_{x'}\dot{\theta}\sin\theta - (F_{z'}\dot{\phi}' - \dot{F}_{y'})\sin\phi'\sin\theta + (\dot{F}_{z'} + F_{y'}\dot{\phi}')\cos\phi'\sin\theta + F_{y'}\dot{\theta}\sin\phi'\cos\theta + F_{z'}\dot{\theta}\cos\phi\cos\theta$$

b) $$\dot{F}_y = (\dot{F}_{y'} - F_{z'}\dot{\phi}')\cos\phi' - (F_{y'}\dot{\phi}' + \dot{F}_{z'})\sin\phi'$$

c) $$\dot{F}_z = -\dot{F}_{x'}\sin\theta - F_{x'}\dot{\theta}\cos\theta + (\dot{F}_{y'} - F_{z'}\dot{\phi}')\sin\phi'\cos\theta + (F_{y'}\dot{\phi}' + \dot{F}_{z'})\cos\phi'\cos\theta - F_{y'}\dot{\theta}\sin\phi'\sin\theta - F_{z'}\dot{\theta}\cos\phi'\sin\theta$$

32 a) $$F_X = F_x\cos\chi + F_y\sin\chi$$

b) $$F_Y = F_x\sin\chi - F_y\cos\chi$$

c) $$F_Z = -F_z$$

33 a) $$\dot{F}_X = \dot{F}_x\cos\chi + \dot{F}_y\sin\chi$$

b) $$\dot{F}_Y = \dot{F}_x\sin\chi - \dot{F}_y\cos\chi$$

c) $$\dot{F}_Z = -\dot{F}_z\,.$$

34 $$X_T = \dot{M}_s + a_0 (1 - M^2)/s$$

35 a) $$X_{\dot{M}_s} = X_T - \dot{X}/\bar{x}$$

b) $$X_{\dot{M}_s} = X_T - \dot{Y}/\bar{y}$$

c) $$Z_{\dot{M}_s} = X_T - \dot{Z}/\bar{z}$$

36 $$\Delta p = - \left\{ \bar{x} \left(\dot{F}_X + F_X \cdot X_{\dot{M}_s} \right) + \bar{y} \left(\dot{F}_Y + F_Y \cdot Y_{\dot{M}_s} \right) + \bar{z} \left(\dot{F}_Z + F_Z Z_{\dot{M}_s} \right) \right\} \Big/ \left\{ 4\pi (1 - M_s)^2 a_0 S^2 \right\}$$

37 $$p_j(t) = p_j(t) + \Delta p$$

38 a) $$C_{p_{jk}} = 2B \left\{ \sum_{m=1}^{M/B} p_j(m) \cos (m-1) k \cdot 2\pi B/M \right\} \Big/ M$$

b) $$S_{p_{jk}} = 2B \left\{ \sum p_j (m) \sin (m-1) k \; 2\pi B/M \right\} \Big/ M$$

c) $$P_{jk} = \sqrt{C^2_{p_{jk}} + S^2_{p_{jk}}}$$

39 $$P_{jk} = 20 \log_{10} P_{jk} + 124.58.$$

40 $$f_k = k \Omega B / 2\pi .$$

```
SEQUENCE,     16
OPERATOR

      PROGRAM HERON 1
$
      COMMON K1,K2,KSF,KSX,DPSI,M,N
      COMMON BET(72),ZET(72),FN(72,12),FT(72,12) ,DMD(72)   ,D(22)
      COMMON PSIO,K,     C,OM        ,AM
      COMMON XO,YO,ZO,XDH,YDH,ZDH
      COMMON XHO,YHO,ZHO,THO,THD,PHIO,PHID
      COMMON AO(72    ),AK(36,72),BK(36,72),F(3),FD(3)
      COMMON XD(72),                          YD(72),ZD(72),WP(3),WDP(3)
      COMMON WDDP(3)
      COMMON BETP(72,12),ZETP(72,12)
      DIMENSION CARD(30),RN(12)
      DIMENSION FF(72),CPJ(72),SPJ(72),PJ(72),ALPHAJ(72)
     C,PJK(72)
       EQUIVALENCE (TAUO,TAUO),(XD,CXD),(YD,CYD),(ZD,CZD),(D(1),RN(1))
      EQUIVALENCE (CYDH,YDH),(CZDH,ZDH),(BET,FF),(ZET,CPJ)
$
      PI=3.1415926536
      WRITE(61,1)
      READ(60,2)CARD
      WRITE(61,3)CARD
      READ(60,1027)IH,JCON,C,RHO                                     READ 1
      DO 800 I1=1,IH
      READ(60,1028)OMN,R,BB,EPF,EPL,PSIO                             READ 2
      OM=OMN*6.283185307/60.
      READ(60,1000)M,N       ,IND, KSF,KSX   ,K1,K2                  READ
      KX=MAX1(KSX,K1,K2)+1
       KF=KSF+1
      AM=XM=FLOATF(M)
      ZZ=2./XM
      DT   =6.283185307/(OM*AM)
      TT   =6.283185307/(OM*BB)
      DPSI=6.283185307/XM
      TT=TT-DT
      MPT=M-1
      DMD(1)=0.
      DDSI=360./XM
      DO 405 J=1,MPT
 405  DMD(J+1)=DMD(J)+DDSI
      READ (60,1028) (D(I),I=1,N)                                    READ 4
      DO 10 I=1,N
   10 RN(I)=R*D(I)
      READ(60,1028)XH,YH,ZH,TH,PHI,XDH,YDH,ZDH,THD,PHID              READ 5
      IF(XDH.EQ.YDH.AND.YDH.EQ.ZDH.AND.ZDH.EQ.0.)1083,1084
1083  V=0.
      GO TO 1085
1084  V=SQRT(XDH*XDH+ YDH*YDH+ZDH*ZDH)
1085  CONTINUE
      I9=IFIX(BB)
      WRITE(61,4)I9,R,OMN,EPF,EPL,PSIO,TH,PHI,XDH,YDH,ZDH,V
      WRITE(61,5)XH,YH,ZH
      WRITE(61,6)
      SINPD=SINF(PHID)
      COSPD=COS(PHID)
      XHO=XH
      YHO=YH
      ZHO=ZH
      THO=TH
      PHIO=PHI
```

```
      CHI=ATAN(YDH/XDH)
      IF(XDH.EQ.0.)11,12
 11   IF(YDH.EQ.0.)14,15
 14   CHI=0.
      GO TO 12
 15   CHI=PI/2.
 12   COSCHI=COS(CHI)
      SINCHI=SIN(CHI)
      WRITE(61,9)N
      WRITE(61,61)M
C
C     N IS NUMBER OF RADIAL STATIONS
C     M IS THE NUMBER OF AZIMUTH STATIONS
C     X, Y, Z IS THE POSITION OF OBSERVER   * * * * * * * * * *
C
C*    BET IS THE FLAPPING ANGLE,                                   *********
C*    ZET IS THE LAG ANGLE,                                        *********
C     BETA AND ZETA ARE INPUT IN DEGREES,
C
      GO TO (16,18) IND
   16 CONTINUE
      WRITE(61,7)
      READ(60,1028)(BET(I),I=1,M)                                  READ 6
      READ(60,1028)(ZET(I),I=1,M)                                  READ 7
      WRITE(61,67)
      WRITE(61,68)(DMD(I),BET(I),ZET(I),I=1,M)
      GO TO 20
   18 CONTINUE
      WRITE(61,8)
      CALL INPUT (1)
   20 CONTINUE
      DO 25 I3=1,M
      BET(I3)=BET(I3)*0.01745329
   25 ZET(I3)=ZET(I3)*0.01745329
      REWIND 2
      GO TO (30,40) IND
   30 DO 35 J=1,N
C
C     THE FOLLOWING STATEMENTS INPUT THE ETAN AND ETAT ARRAYS INTO THE FN AND
C     FT STORAGE LOCATIONS, THESE SAME LOCATIONS WILL LATER BE USED FOR THE FN
C     AND FT ARRAYS,
C
      READ (60,1028) (FN(I,J),I=1,M)                               READ 8
   35 READ (60,1028) (FT(I,J),I=1,M)                               READ 9
      WRITE(61,69)
      WRITE(61,63)
      WRITE(61,64)(RN(I),I=1,N)
      DO 111 J=1,M
      WRITE(61,65)((DMD(J),(FT(J,I),I=1,N)      ))
 111  WRITE(61,1091)
      WRITE(61,71)
      WRITE(61,63)
      WRITE(61,64)(RN(I),I=1,N)
      DO 112 J=1,M
      WRITE(61,65)((DMD(J),(FN(J,I),I=1,N)      ))
 112  WRITE(61,1091)
      GO TO 42
   40 CALL INPUT (3)
   42 DO 70 I5=1,N
      PSI=0.0
      DO 60 I4=1,M
```

```
      COSB=COS(BET(I4))
      COSPZ=COS(PSI+ZET(I4))
      SINPZ=SIN(PSI+ZET(I4))
      SINB=SIN(BET(I4))
      COSP=COSF(PSI)
      SINP=SINF(PSI)
C
C*    RELATIVE COORDINATE OF BLADE STATION,                          *********
C
      IF(EPF.GT.EPL)54,58
 54   T=(RN(I5)-EPF)*COSB+EPF-EPL
      XD(I4)=-EPL*COSP-T*COSPZ+FN(I4,I5)*SINB*COSPZ
     C+FT(I4,I5)*SINPZ
      YD(I4)=-EPL*SINP-T*SINPZ-FT(I4,I5)*COSPZ+FN(I4,I5)*SINB*SINPZ
      ZD(I4)=-(RN(I5)-EPF)*SINB-FN(I4,I5)*COSB
      GO TO 60
 58   XD(I4)=-(EPF+EPL*COSB)*COSP-(RN(I5)+EPL)*COSB*COSPZ+FT(I4,I5)*
     1SINPZ+FN(I4,I5)*SINB*COSPZ
      YD(I4)=-(EPF+EPL*COSB)*SINP-(RN(I5)+EPL)*COSB*SINPZ-FT(I4,I5)*COSP
     1Z+FN(I4,I5)*SINB*SINPZ
      ZD(I4)=-(RN(I5)-EPF)*SINB-FN(I4,I5)*COSB
 60   PSI=PSI+DPSI
      WRITE (2,1003) (XD(I4),I4=1,M)
      WRITE (2,1003) (YD(I4),I4=1,M)
   70 WRITE (2,1003) (ZD(I4),I4=1,M)
      DO 93 I5=1,N
      DO 93 I4=1,M
C
C*    AERODYNAMICS FORCE COMPONENT,     * * * * * * * * * * * *      *********
C
C
C     XD=FXD , YD=FYD , ZD=FZD IN THE FOLLOWING COMPUTATIONS,
C
      IF (N.EQ.1) 91,83
   83 IF(I5.EQ.1)84,88
 84   DNDR=(FN(I4,2)-FN(I4,1))/(RN(2)-RN(1))
      DTDR=(FT(I4,2)-FT(I4,1))/(RN(2)-RN(1))
      GO TO 92
 88   IF(I5.EQ.N)89,90
 89   DNDR=(FN(I4,I5)-FN(I4,I5-1))/(RN(I5)-RN(I5-1))
      DTDR=(FT(I4,I5)-FT(I4,I5-1))/(RN(I5)-RN(I5-1))
      GO TO 92
 90   DNDR=(FN(I4,I5+1)-FN(I4,I5-1))/(RN(I5+1)-RN(I5-1))
      DTDR=(FT (I4,I5+1)-FT(I4,I5-1))/(RN(I5+1)-RN(I5-1))
      GO TO 92
   91 DNDR=DTDR=0.
   92 BETP(I4,I5)=BET(I4)+ATAN(DNDR)
   93 ZETP(I4,I5)=ZET(I4)+ATAN(DTDR)
      GO TO (94,96) IND
   94 DO 95 J=1,N
      READ (60,1028) (FN(I,J),I=1,M)
   95 READ (60,1028) (FT(I,J),I=1,M)
      WRITE(61,62)
      WRITE(61,63)
      WRITE(61,64)(RN(I),I=1,N)
      DO1112 J=1,M
      WRITE(61,1091)
 1112 WRITE(61,65)((DND(J),(FN(J,I),I=1,N)       ))
      WRITE(61,66)
      WRITE(61,63)
      WRITE(61,64)(RN(I),I=1,N)
      DO1111 J=1,M
```

```
      WRITE(61,1091)
1111  WRITE(61,65)((DMD(J),(FT(J,I),I=1,N)       ))
      GO TO 97
   96 CALL INPUT (2)
   97 DO 160 I5=1,N
      PSI=0.
      DO 100 I4=1,M
      ZE=ZETP(I4,I5)
      BE=BETP(I4,I5)
      XD(I4)=FT(I4,I5)*SIN(PSI+ZE   )+FN(I4,I5)*SIN(BE   )*COS(PSI+ZE   )
      YD(I4)=-FT(I4,I5)*COS(PSI+ZE   )+FN(I4,I5)*SIN(BE   )*SIN(PSI+ZE   )
      ZD(I4)=-FN(I4,I5)*COS(BE   )
  100 PSI=PSI+DPSI
      WRITE(2,1003)(XD(I4),I4=1,M)
      WRITE(2,1003)(YD(I4),I4=1,M)
      WRITE(2,1003)(ZD(I4),I4=1,M)
  160 CONTINUE
      END FILE 2
      REWIND 2
C
C*    CALCULATE HARMONICS    * * * * * * * * * * * * * * * *         *******
C
      I7=0
      DO 190 I2=1,2
      GO TO (165,166) I2
  165 KSP=KX
      GO TO 167
  166 KSP=KF
  167 IF (KSP.LE.0) 168,169
  168 KSP=1
  169 DO 190 I3=1,N
      DO 190 I4=1,3
      I7=I7+1
      READ (2,1003) (BET(I5),I5=1,M)
      SUM1=0.
      DO 172 I9=1,M
  172 SUM1=SUM1+BET(I9)
      A0(I7)=SUM1/XM
      DO 190 I6=1,KSP
      XX=DPSI*FLOATF(I6)
      SUM2=SUM3=X=0.
      DO 180 I8=1,M
      SUM2=SUM2+BET(I8)*COS(X*XX)
      SUM3=SUM3+BET(I8)*SIN(X*XX)
C     WRITE(61,1044)SUM2,SUM3
  180 X=X+1.
      AK(I6,I7)=SUM2*ZZ
      BK(I6,I7)=SUM3*ZZ
  190 CONTINUE
C
C****** ********  BEGIN PART 2 --- CALC. OF SOUND FIELD *************
C
      I3=IFIX(BB)
      WRITE(61,78)
      DO 700 I11=1,JCON
      I7=0
      READ (60,1016)XOP,YOP,ZOP
      SXBH=-XOP+XHO
      SYBH=-YOP+YH O
      SZBH=-ZOP+ZHO
      IF(SXBH.EQ.SYBH.AND.SYBH.EQ.SZBH.AND.SZBH.EQ.0.)1073,1072
1073  SH=0.
```

```
      GO TO 1075
1072  SH=SQRT(SXBH*SXBH+SYBH*SYBH+SZBH*SZBH)
1075  QQ= SXBH*XDH+SYBH*YDH+SZBH*ZDH
      QT=C*C-V*V
      QS=QQ*QQ+SH*SH*QT
      IF(QS,EQ,0,)1071,1092
1071  TAUOP=-QQ/QT
      GO TO 1069
1092   TAUOP=(-QQ+SQRT(QS))/QT
1069  CONTINUE
      XPH=XHO-XDH*TAUOP
      YPH=YHO-YDH*TAUOP
      ZPH=ZHO-ZDH*TAUOP
      SXBPH=SXBH+XDH*TAUOP
      SYBPH=SYBH+YDH*TAUOP
      SZBPH=SZBH+ZDH*TAUOP
      BHO=SXBPH*XDH+SYBPH*YDH+SZBPH*ZDH
      BHO=BHO/C/C/TAUOP
      TT=6,2831853/(OM*BB)
      DT=6,2831853/(OM*AM)
      T=0,
      MB=M/IFIX(BB)
      DO 400 MS=1,MB
      I7=I7+1
      XO=XOP+XDH*(T-TT/2,)
      YO=YOP+YDH*(T-TT/2,)
      ZO=ZOP+ZDH*(T-TT/2,)
      PJ(I7)=0,
      DO 350 I2=1,I3
      PNEW=PSIO+6,28318[illegible]*FLOATF(I2-1)/BB
      TAU1=TAUOP
      II=0
      JJ=3*N
      DO 330 I4=1,N
C
C*    [illegible]TE RETARDED TIME        * * * * * * * * * * * *               *******
C
      [illegible]=0
      KK=1
  210 TAUC=TAU1
      TP=T-TAU0
      TAU1=SF(TP,KK,I4,PNEW,CHI)/C
      I=I+1
      IF (I,GT,50) 260,220
  220 ER=ABSF(TA[illegible]1-TAU0)
      IF (ER,GT,,001) 225,228
  225 KK=1
      GO TO 250
  228 IF (ER,GT,,0001) 230,232
  230 KK=3
      GO TO 250
  232 IF (ER,GT,,00005) 234,236
  234 KK=10
      GO TO 250
  236 IF (ER,GT,,00001) 238,242
  238 KK=20
      GO TO 250
  242 IF (KK,EQ,KX) 260,254
  250 IF(KX-KK)251,210,210
  251 KK=KX$ GO TO 210
  254 KK=KX$ GO TO 210
260   CONTINUE
```

```
      TP=T-TAU1
C
C     BEGIN CALCULATION OF VELOCITIES AND ACCELERATIONS,
C
      PSI=PNEW+OM*TP
      XH=XHO+XDH*TP
      YH=YHO+YDH*TP
      ZH=ZHO+ZDH*TP
      TH=THO+THD*TP
      PHI=PHIO+PHID*TP
      COST=COSF(TH)
      SINT=SINF(TH)
      TANP=TANF(PHI)
      PHIP=ATAN(COST*TANP)
      SINPHIP=SIN(PHIP)
      COSPHIP=COS(PHIP)
      PHIDP=COSPHIP*COSPHIP*(PHID*COST/(COSF(PHI)**2)-THD*SINT*TANP)
 1045 FORMAT(6X, 6HPSIX8=,12E10,5)
      DO 265 I5=1,3
      II=II+1
      WP(I5)=AO(II)
      XK=1,
      WDP(I5)=WDDP(I5)=0,
      DO 263 KL=1,KX
      ARG=XK*PSI
      COSKP=COS(ARG)
      SINKP=SIN(ARG)
      WP(I5)=WP(I5)+AK(KL,II)*COSKP+BK(KL,II)*'   .P
      WDP(I5)=WDP(I5)-XK*OM*(AK(KL,II)*SINKP-BK(KL,II)    *COSKP)
      WDDP(I5)=WDDP(I5)-XK*XK*OM*OM*(AK(KL,II)*COSKP+BK(KL,II)*SINKP)
  263 XK=XK+1,
  265 CONTINUE
C
      THD2=THD*THD
      P2=PHIDP*PHIDP
      X=WP(1)*COST+WP(2)*SINPHIP*SINT+WP(3)*COSPHIP*SINT
      Y=WP(2)*COSPHIP-WP(3)*SINPHIP
      Z=-WP(1)*SINT+WP(2)*SINPHIP*COST+WP(3)*COSPHIP*COST
C
       XD=WDP(1)*COST-WP(1)*THD*SINT+(WP(3)*PHIDP+WDP(2))*SINPHIP*SINT+
     1(WDP(3)+WP(2)*PHIDP)*COSPHIP*SINT+WP(2)*THD*SINPHIP*COST+WP(3)*
     2THD*COSPHIP*COST
       YD=(WDP(2)-WP(3)*PHIDP)*COSPHIP-(WP(2)*PHIDP+WDP(3))*SINPHIP
       ZD=-WDP(1)*SINT-WP(1)*THD*COST+(WDP(2)-WP(3)*PHIDP)*SINPHIP*COST+
     1(WP(2)*PHIDP+WDP(3))*COSPHIP*COST-WP(2)*THD*SINPHIP*SINT-WP(3)*
     2COSPHIP*SINT
C
       XDD=(WDDP(1)-WP(1)*THD2    )*COST-2,*WDP(1)*THD*SINT+(2,*WP(2)*
     1PHIDP*THD+2,*WDP(3)*THD)*COSPHIP*COST+(WDDP(2)-WP(2)*(P2+
     2THD2    )-2,*WDP(3)*PHIDP)*SINPHIP*SINT+(2,*WDP(2)*PHIDP+WDDP(3)-
     3WP(3)*(P2    +THD2             ))*COSPHIP*SINT+(2,*WDP(2)*THD-2,*WP (3)
     4*PHIDP*THD)*SINPHIP*COST
       YDD=(WDDP(2)+WP(2)*P2             )*COSPHIP-(WDDP(3)-WP(3)*P2)*SINPHIP
       ZDD=- 2,*WDP(1)*THD*COST-(WDDP(1)-WP(1)*THD2    )*SINT+(2,*WDP(2)
     1*PHIDP+WDDP(3)-WP(3)*(P2+THD2               ))*COSPHIP*SINT-(2,*WDP(2
     2)*THD-2,*WP(3)*PHIDP*THD)*SINPHIP*SINT-(WP(2)*PHIDP+WDP(3))*COSPHI
     3P*SINT*2,*THD+(WDDP(2)-WP(2)*(P2+THD2               )-2,*WDP(3)*PHIDP
     4)*SINPHIP*COST
C
       XBAR=XO-XH-X*COSCHI-Y*SINCHI
       YBAR=YO-YH-X*SINCHI+Y*COSCHI
```

```
      ZBAR=ZO-ZH+Z
C
      CXD=XDH+XD*COSCHI+YD*SINCHI
      CYD= YDH+XD*SINCHI-YD*COSCHI
      CZD= ZDH-ZD
C
      CXDD=XDD*COSCHI+YDD*SINCHI
      CYDD=-YDD*COSCHI+XDD*SINCHI
      CZDD=-ZDD
C
      SS=SQRT(XBAR*XBAR+YBAR*YBAR+ZBAR*ZBAR)
C
      XMAS=(CXD*CXD+CYD*CYD+CZD*CZD)/C/C
      XMS=(XBAR*CXD+YBAR*CYD+ZBAR*CZD)/(SS*C)
      XMDS=(XBAR*CXDD+YBAR*CYDD+ZBAR*CZDD)/(SS*C)
C
C     BEGIN CALCULATION OF FORCES AND DERIVATIVES,
C
      DO 275 I5=1,3
      JJ=JJ+1
      F(I5)=AO(JJ)
      FD(I5)=0,
      XK=1,
      DO 270 I6=1,KF
      COSK=COSF(XK*PSI)
      SINK=SINF(XK*PSI)
      F(I5)=F(I5)+AK(I6,JJ)*COSK+BK(I6,JJ)*SINK
      FD(I5)=FD(I5)-OM*XK*(AK(I6,JJ)*SINK-BK(I6,JJ)*COSK)
 270  XK=XK+1,
 275  CONTINUE
C
      FX=F(1)*COST+F(2)*SINPHIP*SINT+F(3)*COSPHIP*SINT
      FY=F(2)*COSPHIP-F(3)*SINPHIP
      FZ=-F(1)*SINT+F(2)*SINPHIP*COST+F(3)*COSPHIP*COST
C
      FDX=FD(1)*COST-F(1)*THD*SINT-(F(3)*PHIDP-FD(2))*SINPHIP *SINT+
     1    (FD(3)+F(2)*PHIDP)*COSPHIP*SINT+F(2)*THD*SINPHIP*
     2    COST+F(3)*COSPHIP*COST*THD
      FDY=(FD(2)-F(3)*PHIDP)*COSPHIP-(F(2)*PHIDP+FD(3))*SINPHIP
      FDZ=-FD(1)*SINT-F(1)*THD*COST+(FD(2)-F(3)*PHIDP)*SINPHIP*COST+
     1(F(2)*PHIDP+FD(3))*COSPHIP*COST-F(2)*THD*SINPHIP*SINT-F(3)*COSPHIP
     2*SINT*THD
C
      FX=FX*COSCHI+FY*SINCHI
      FY=FX*SINCHI-FY*COSCHI
      FZ=-FZ
C
      FDX=FDX*COSCHI+FDY*SINCHI
      FDY=FDX*SINCHI-FDY*COSCHI
      FDZ=-FDZ
C
C     BEGIN SOUND PRESSURE CALCULATIONS,
      XT=XMDS+C*(1,-XMAS)/SS
      XMXS=XT-XD/XBAR
      XMYS=XT-YD/YBAR
      XMZS=XT-ZD/ZBAR
C
      TEMP=1,-XMS
      TEMP2=TEMP*TEMP
      TMP=-1,/(4,*PI*TEMP2*C*SS*SS)
      DP=(XBAR*(FDX+FX*XMXS/TEMP)+YBAR*(FDY+FY*XMYS/TEMP)+ZBAR*(FDZ+FZ*
     1XMZS/TEMP))*TMP
```

```
      PJ(I7)=PJ(I7)+DP
  330 CONTINUE
  350 CONTINUE
      T=T+DT
  400 CONTINUE
C
C     HARMONIC ANALYSIS OF SOUND FIELD,
C
      F1=2,*BB/XM
      F2=2,*PI*BB/XM
      F3=CM*BB/(PI+PI)
      F4=20,*,43429448
      XMB=XM/BB
      BM=XMB/2,
      BM2=FLOAT(MB/2)
      IF (BM,GT,BM2) 605,610
  605 KMAX=M/(2*IFIX(BB))
      GO TO 615
  610 KMAX=M/(2*IFIX(BB))+1
  615 XL=0,
      DO 680 L1=1,KMAX
      XL=XL+1,
      SUM1=SUM2=X=0,
      DO 660 MM=1,MB
      SUM1=SUM1+PJ(MM)*COS(X*F2*XL)
      SUM2=SUM2+PJ(MM)*SIN(X*XL*F2)
 664    FORMAT(2X,2I2,9(3X,E10,5))
  660 X=X+1,
      CPJ(L1)=F1*SUM1
      SPJ(L1)=F1*SUM2
      ALPHAJ(L1)=ATAN(SPJ(L1)/CPJ(L1))
      IF(CPJ(L1),EQ,0,)661,662
 661  ALPHAJ(L1)=PI/2,
 662   SPJ(L1)=ABSF(SPJ(L1))
       CPJ(L1)=ABSF(CPJ(L1))
      PJK(L1)=SQRT(CPJ(L1)*CPJ(L1)+SPJ(L1)*SPJ(L1))
      PJK(L1)=F4*ALOG(PJK(L1))+124,58
      FF(L1)=XL*F3  /(1,+BMO)
  680 CONTINUE
      WRITE (61,1030)
      WRITE (61,1032) II1,XOP,YOP,ZOP
       WRITE(61,1029)XPH,YPH,ZPH
      WRITE (61,1030)
      WRITE (61,1033)
      WRITE(61,79)
      DO 700 L1=1,KMAX
  700 WRITE (61,1034) L1,FF(L1),CPJ(L1),SPJ(L1),PJK(L1)
      WRITE (61,1030)
  800 CONTINUE
      WRITE (59,1002)
      WRITE (61,1030)
      WRITE (61,1002)
1     FORMAT(1H1,60X,16HPROGRAM ,HERON 1)
2     FORMAT(        10A8)
3     FORMAT(3(28X,10A8/))
6     FORMAT(1H0,28X,63HBLADE LOADING AND MOTION DATA (PARALLEL AND NORM
     1AL TO THE SHAFT//)
4     FORMAT(1H ////,28X,22H GEOMETRY OF ROTOR H ,,I10  ,11X
     1 ,6HBLADES/,1H ,50X,E10,3,3X,3HFT,,4X,6HRADIUS/,1H ,50X,E10,3,3X,
     26HR,P,M,/,1H ,50X,E10,3,3X,3HFT,,4X,17HFLAP HINGE OFFSET/,1H ,50X,
     3E10,0,3X,3HFT,,4X,17HDRAG HINGE OFFSET/,1H ,50X,E10,3,2X,7HDEGREES
     4,14H AZIMUTH PHASE/,1H ,50X,E10,3,9H  DEGREES,27H REARWARD SHAFT I
```

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     5NCLINATION/,1H ,50XE10,3,9H  DEGREES,31H SHAFT INCLINATION TO STAR
     6BOARD/,50X,E10,3,2X,6HFT/SEC,25H  VELOCITY IN X-DIRECTION/,1H ,50X
     6,E10,3,2X,6HFT/SEC,25H  VELOCTIY IN Y-DIRECTION/,1H ,50X,E10,3,2X,
     86HFT/SEC,25H  VELOCITY IN Z-DIRECTION/,1H ,50X,E10,3,2X,6HFT/SEC,2
     90H  RESULTANT VELOCITY/)
 5    FORMAT(1H ,50X,E10,3,3X,3HFT,,18H    X HUB ORDINATE/,1H ,50X,E10,3
     1,3X,3HFT,,18H    Y HUB ORDINATE/,1H ,50X,F10,3,3X,3HFT,,18H    Z H
     2UB ORDINATE)
 9    FORMAT(1H ,28X,27HNUMBER OF LOADING STATIONS=,I6//)
 61   FORMAT(1H ,28X,37HNUMBER OF AZIMUTH INTEGRATION POINTS=,I6//)
 7    FORMAT(1H ,28X,49HFORM OF INPUT, SPANWISE/AZIMUTHWISE DISTRUBUTION
     1S//)
 8    FORMAT(1H ,28X,23HFORM OF INPUT, HARMONIC//)
 67   FORMAT(1H ,//28X,13HAZIMUTH ANGLE,7X,10HFLAP ANGLE,10X, 9HLAG ANGL
     1E//)
 68   FORMAT(1H ,28X,E10,2,10X,F7,3,13X,F7,3)
 69   FORMAT(1H ,///28X,57HBLADE ELEMENT ELASTIC DISPLACEMENTS PARALLEL
     1TO THE SHAFT//)
 63   FORMAT(1H , 2X,13HAZIMUTH ANGLE,30X,14HRADIAL STATION//)
 64   FORMAT(1H ,15X,10F10,3)
 65   FORMAT(1H , 5X, F10,3    ,10E10,3)
 71   FORMAT(1H ,///28X,57HBLADE ELEMENT ELASTIC DISPLACEMENTS NORMAL
     1TO THE SHAFT//)
 1091 FORMAT(/)
 62   FORMAT(1H1 ,28X,26HBLADE THRUST LOADING (LBS)//)
 66   FORMAT(1H ,///28X,18HBLADE DRAG LOADING//)
 78   FORMAT(1H ,///48X,20HCOMPUTED SOUND FIELD//)
 79   FORMAT(1H ,28X, 8HHARMONIC,6X, 9HFREQUENCY,8X,14HIN PHASE PRES,,5X
     1,18HOUT OF PHASE PRES,,3X,20HSOUND PRESSURE LEVEL/,30X,3HNO,,31X,
     28H(F,S,F,),10X,8H(P,S,F,),9X,28H(DB, RE, .0002 DYNES/SQ,CM,))
 1029 FORMAT(1H ,///28X,25HRETARDED HUB CO-ORDINATES,10X,4HXPH=,E16,8,1X,4HYH
     1,4HYPH=,E16,8,1X,4HZPH=,E16,8)
 1032 FORMAT(1H ,///28X,21HOBSERVER CO-ORDINATES, 4X,
     U          2HJ=,I4,5X,3HXO=,E16,8,2X,3HYO=,E16,8,2X,3HZO=,E16,8)
 1000 FORMAT(20I4)
 1002 FORMAT (2X,10HEND OF RUN)
 1003 FORMAT( 8E16,8)
 1016 FORMAT (8F10,0)
 1027 FORMAT(2I4,2X,6F10,0)
 1028 FORMAT(8F10,0)
 1030 FORMAT (/)
 1034 FORMAT(30X,I4,4E20,8)
 1033 FORMAT(32X,1HK,13X,4HF(K),13X,6HCPJ(K),15X,6HSPJ(K),14X,5HPJ(K))
      END

      SUBROUTINE INPUT (II)
      COMMON K1,K2,KSF,KSX,DPSI,M,N
      COMMON BET(72),ZET(72),FN(72,12),FT(72,12) ,DMD(72)   ,D(22)
      DIMENSION ZRO2(12),CFN(12,12),ZRO3(12),CFT(12,12),E(12),SFN(12,12)
      DIMENSION ZRO1(1),CBET(72),GGG(1),SBET(72),HHH(1),SFT(12,12)
C
C     CALL PARAMETER OF 1 RETURNS BETA AND ZETA ARRAYS ONLY,
C     CALL PARAMETER OF 2 RETURNS  FN AND FT ARRAYS ONLY,
C     CALL PARAMETER OF 3 RETURNS ETAN AND ETAT ARRAYS ONLY,
C
      WRITE (61,1505)
      I=0
 77   FORMAT(1H ,28X,34HBLADE  LAGGING HARMONICS (DEGREES))
      GO TO (50,110,120) II
   50 CONTINUE
C
C     COMPUTE BETA AND ZETA ARRAYS,
```

```
C
      READ (60,1028) (CBET(KK),KK=I,K1)
      XJ=1.
 74   FORMAT(1H ,32X,1HA,I2,7X,  E10.3)
 76   FORMAT(1H ,28X,34HBLADE FLAPPING HARMONICS (DEGREES))
        WRITE(61,76)
      WRITE(61,74)(J,CBET(J),J=I,K1)
      READ (60,1028) (SBET(KK),KK=1,K1)
 84   FORMAT(1H ,32X,1HB,I2,7X,  E10.3)
      WRITE(61,84)(J,SBET(J),J=1,K1)
      XJ=2.
      X=-1.
      DO 60 I1=1,M
      X=X+1.
      PSI=X*DPSI
      BET(I1)=CBET(I)
      Y=0.
      DO 60 I2=1,K1
      Y=Y+1.
      YK=PSI*Y
   60 BET(I1)=BET(I1)+CBET(I2)*COS(YK)+SBET(I2)*SIN(YK)
C
C     COMPLETES COMPUTATION OF BETA ARRAY.
C
 75   FORMAT(1H ,28X,31HBLADE DRAG LOAD HARMONICS (LBS)///)
 85   FORMAT(1H ,28X,33HBLADE THRUST LOAD HARMONICS (LBS)///)
      READ (60,1028) (CBET(J),J=I,K2)
      XJ=3.
        WRITE(61,77)
      WRITE(61,74)(J,CBET(J),J=I,K2)
      READ (60,1028) (SBET(J),J=1,K2)
      XJ=4.
      WRITE(61,84)(J,SBET(J),J=1,K2)
      X=-1.
      DO 70 I3=1,M
      X=X+1.
      PSI=X*DPSI
      ZET(I3)=CBET(I)
      Y=0.
      DO 70 KK =1,K2
      Y=Y+1.
      YK=Y*PSI
   70 ZET(I3)=ZET(I3)+CBET(KK)*COS(YK)+SBET(KK)*SIN(YK)
 79   FORMAT(1H ,32X,1HA,I2,7X,10E10.3)
 78   FORMAT(1H ,32X,1HB,I2,7X,10E10.3)
 72   FORMAT(1H ,28X,14HRADIAL STATION,F10.3)
C
C     COMPLETES COMPUTATION OF ZETA ARRAY.
C
      GO TO 1200
  110 KSTP=KSF
      KF=1
      GO TO 125
  120 KSTP=KSX
      KF=2
  125 DO 130 NN=1,N
      READ (60,1028) (CFN(NN,J),J=I,KSTP)
      XJ=5.
      READ (60,1028) (SFN(NN,J),J=1,KSTP)
      XJ=6.
      READ (60,1028) (CFT(NN,J),J=I,KSTP)
```

```
      XJ=7.
      READ (60,1028) (SFT(NN,J),J=1,KSTP)
      XJ=8.
130      CONTINUE
 69   FORMAT(1H ,///28X,57HBLADE ELEMENT ELASTIC DISPLACEMENTS PARALLEL
     1TO THE SHAFT//)
 71   FORMAT(1H ,///28X,57HBLADE ELEMENT ELASTIC DISPLACEMENTS NORMAL
     1TO THE SHAFT//)
      IF( KF,EQ,1    )101,502
 101  WRITE(61,85)
      GO TO 503
 502  WRITE(61,71)
 503  WRITE(61,72)( D(K),K=1,N)
      DO 104 J=1,KSTP
 104  WRITE(61,79)(J,CFN(NN,J),NN=1,N)
      DO 105 J=1,KSTP
 105  WRITE(61,78)(J,SFN(NN,J),NN=1,N)
      IF( KF,EQ,1    )102,504
 102  WRITE(61,75)
      GO TO 505
 504  WRITE(61,69)
 505  WRITE(61,72)( D(K),K=1,N)
      DO 107 J=1,KSTP
 107  WRITE(61,79)(J,CFT(NN,J),NN=1,N)
      DO 108 J=1,KSTP
 108  WRITE(61,78)(J,SFT(NN,J),NN=1,N)
 126    CONTINUE
      DO 140 NN=1,N
      X=0.
      DO 140 MM=1,M
      PSI=DPSI*X
      X=X+1.
      FN(MM,NN)=CFN(NN,1)
      FT(MM,NN)=CFT(NN,1)
      Y=0.
      DO 140 I4=1,KSTP
      Y=Y+1.
      ARG=Y*PSI
      SINARG=SIN(ARG)
      COSARG=COS(ARG)
      FN(MM,NN)=FN(MM,NN)+CFN(NN,I4)*COSARG+SFN(NN,I4)*SINARG
  140 FT(MM,NN)=FT(MM,NN)+CFT(NN ,I4)*COSARG+SFT(NN,I4)*SINARG
C
C     COMPLETES FN AND FT OR ETAN AND ETAT CALCULATIONS.
C
 1028 FORMAT (8F10,0)
 1200 RETURN
 1500 FORMAT (10X,7E16,8)
 1505 FORMAT (/)
      END

      FUNCTION SF(TP,KK,NUM,PNEW,CHI)
C
      COMMON K1,K2,KSF,KSY,DPSI,M,N
      COMMON BET(72),ZET(72),FN(72,12),FT(72,12) ,DMD(72)   ,D(22)
      COMMON PSI),K,     C,OM        ,AM
      COMMON XO,YO,ZO,XDH,YDH,ZDH
      COMMON XHO,YHO,ZHO,THO,THD,PHIO,PHID
      COMMON AO(72    ),AK(36,72),BK(36,72),F(3),FD(3)
      DIMENSION W(3)
C*    CALCULATED SOUND SOURCE TO OBSERVER DISTANCE,                 *********
C
```

```
C
      PSI=PNEW+OM*TP
      XH=XHO+XDH*TP
      YH=YHO+YDH*TP
      ZH=ZHO+ZDH*TP
      TH=THO+THD*TP
      PHI=PHIO+PHID*TP
      PHIP=ATAN(COSF(TH)*TANF(PHI))
      DO 100 I1=1,3
      II=3*(NUM-1)+I1
      WP=AO(II)
      XK=1.
      DO 80 I2=1,KK
  500 FORMAT (5X,8HSUB DATA,5X,7E14.6)
      WP=WP+AK(I2,II)*COSF(XK*PSI)+BK(I2,II)*SIN(XK*PSI)
 80   XK=XK+1.
 100  W(I1)=WP
      COST=COSF(TH)
      SINPHIP=SINF(PHIP)
      SINT=SINF(TH)
      COSPHIP=COS(PHIP)
      SINP=SINF(CHI)
      COSP=COSF(CHI)
      X=W(1)*COST+W(2)*SINPHIP*SINT+W(3)*COSPHIP*SINT
      Y=W(2)*COSPHIP-W(3) *SINPHIP
      Z=-W(1)*SINT+W(2)*SINPHIP*COST+W(3)*COSPHIP*COST
      W(1)=XO-XH-X*COSP+Y*SINP
      W(2)=YO-YH-X*SINP+Y*COSP
      W(3)=ZO-ZH+Z
      SF=SQRT(W(1)*W(1)+W(2)*W(2)+W(3)*W(3))
      RETURN
      END
```

TABLE I. MAJOR FORTRAN SYMBOLS USED IN PROGRAM HERON 1

(Other Symbols Which Appear are Generally Used for Local Calculations and have Self-Evident Meanings)

PROGRAM SYMBOLS	ALGEBRAIC SYMBOLS	DEFINITION
IH	H	Number of rotors
JCON	J	Number of observer positions
C	c, a_0	Atmospheric speed of sound (ft/sec)
RHO	ρ_0	Atmospheric density (slugs/ft^3)
OHM	Ω_N	Rotor speed (rpm)
R	R	Rotor radius (ft)
BB	B	Number of blades
EPF	ϵ_f	Flapping hinge offset (ft)
EPL	ϵ_ℓ	Lagging hinge offset (ft)
PSIO	Ψ_0	Azimuth reference angle (radians)
OM	Ω	Angular velocity of rotor (radians/sec)
M	M	Number of azimuth stations
N	N	Number of radial stations
IND	IND	Indicator: If IND = 1, loading/motion read as span-wise/azimuthwise distributions
–	–	If IND = 2, loading/motion read as Fourier coefficients
KSF	K_f^*	Number of force harmonics read
KSX	K_x^*	Number of displacement harmonics read

TABLE I - Continued		
PROGRAM SYMBOLS	ALGEBRAIC SYMBOLS	DEFINITION
K1	K_1	Number of flapping angle harmonics read
K2	K_2	Number of lagging angle harmonics read
KX	K_x	Number of displacement harmonics calculated
KF	K_f	Number of force harmonics calculated
DT	Δt	Time increment between sound pressure values in final time history (sec)
TT	T	Fundamental blade passage period (sec)
DPSI	$\Delta\psi$	Azimuth increment (radians)
D(I)	x(n)	i^{th} radial station (nondimensional)
R(I)	r(n)	Radial distance of i^{th} station - R · D(I)
XHO	X_{H_0}	Hub coordinates relative to fixed ground axes (ft) at time $t = 0$
YHO	Y_{H_0}	
ZHO	Z_{H_0}	
THO	θ_0	Pitch angle relative to aircraft trajectory coordinates (radians) at time $t = 0$
PHIO	ϕ_0	Roll angle relative to aircraft trajectory coordinates (radians) at time $t = 0$
XDH	$\dot{X}_H$	Hub velocity components in fixed coordinate directions (ft/sec)
YDH	$\dot{Y}_H$	
ZDH	$\dot{Z}_H$	

TABLE I - Continued		
PROGRAM SYMBOLS	ALGEBRAIC SYMBOLS	DEFINITION
THD	$\dot{\theta}$	Pitch and roll rates (radians/sec)
PHID	$\dot{\phi}$	
V	V	Resultant hub velocity (ft/sec)
CHI	χ	Azimuth angle between x and X axes (radians)
XH	X_H	Rotor hub coordinates at retarded time t' (ft)
YH	Y_H	
ZH	Z_H	
BET	β	Flap angle (radians)
ZET	ζ	Lag angle (radians)
FN	F_N or η_N	Normal section loading (lb) or elastic displacement (ft)
FT	F_T or η_T	In-plane section loading (lb) or elastic displacement (ft)
XD	x'	Coordinates in shaft axis system (ft). [Also used for blade section force components in same system (lb)]
YD	y'	
ZD	z'	
DNDR	$d\eta_N/dr$	Local blade slope with respect to plane of rotation (radians)
DTDR	$d\eta_T/dr$	Local blade slope with respect to nominal blade azimuth (radians)
AO	a_0	Fourier coefficients of blade section force and displacement with respect to rotor axes (ft)
AK	a_k	
BK	b_k	

TABLE I - Continued

PROGRAM SYMBOLS	ALGEBRAIC SYMBOLS	DEFINITION
XOP	$X'_0(j)$	Nominal observer position in fixed coordinate system (ft)
YOP	$Y'_0(j)$	
ZOP	$Z'_0(j)$	
SXBH	$\bar{x}_H$	Position of rotor hub relative to observer at time t in fixed coordinate system (ft)
SYBH	$\bar{y}_H$	
SZBH	$\bar{z}_H$	
SH	S_H	Distance between hub and observer at time t (ft)
TAUOP	τ'_0	Time of propagation of sound from the rotor hub which reaches observer at time t (sec)
XPH	X'_H	Hub coordinates in fixed axis system at retarded time t' (ft)
YPH	Y'_H	
ZPH	Z'_H	
SXBPH	$\bar{x}'_H$	Position of hub relative to observer at retarded time t' (ft)
SYBPH	$\bar{y}'_H$	
SZBPH	$\bar{z}'_H$	
BMO	M_0	Hub Mach number component in direction of observer
T	t	"Observer" time t
XO	X_0	Coordinates of "moving observer" at time t
YO	Y_0	
ZO	Z_0	

TABLE I - Continued

PROGRAM SYMBOLS	ALGEBRAIC SYMBOLS	DEFINITION
PNEW	ψ_0	Reference azimuth angle for particular blade
TAU 1	τ_1	Sound propagation time
TP	t'	Retarded time $t' = t - \tau$
PSI	ψ	Blade azimuth angle (nominal) at retarded time t'
XH	X_H	Hub coordinates (in fixed axes) at retarded time t'
YH	Y_H	
ZH	Z_H	
TH	θ	Pitch angle at retarded time (radians)
PHI	ϕ	Roll angle at retarded time (radians)
PHIP	ϕ'	Roll angle at retarded time (relative to vehicle) (radians)
PHIDP	$\dot{\phi}'$	Roll rate at retarded time (relative to vehicle)(radians/sec)
WP(I)	w'	Coordinates of blade station (rotor axes)(ft) ($w = x, y$ or z)
WDP	$\dot{w}'$	Blade element velocity components (ft/sec)
WDDP	$\ddot{w}'$	Blade element acceleration components (ft/sec^2)
X	x	Aircraft flight path coordinates (x is flight azimuth direction) (ft)
Y	y	
Z	z	
XD	$\dot{x}$	Aircraft flight path velocity components (ft/sec)
YD	$\dot{y}$	

TABLE I - Continued

PROGRAM SYMBOLS	ALGEBRAIC SYMBOLS	DEFINITION
ZD	$\dot{z}$	Aircraft flight path velocity component (ft/sec)
XDD	$\ddot{x}$	
YDD	$\ddot{y}$	Aircraft flight path acceleration components (ft/sec^2)
ZDD	$\ddot{z}$	
XBAR	$\bar{x}$	
YBAR	$\bar{y}$	Coordinates of blade element relative to observer at retarded time (ft)
ZBAR	$\bar{z}$	
CXD	$\dot{X}$	
CYD	$\dot{Y}$	Velocity components of blade element at retarded time (fixed axes)(ft/sec)
CZD	$\dot{Z}$	
CXDD	$\ddot{X}$	
CYDD	$\ddot{Y}$	Acceleration components of blade element at retarded time (ft/sec^2)
CZDD	$\ddot{Z}$	
SS	S	Distance traveled by sound (ft)
XMAS	M	Square of absolute Mach number of blade element
XMS	M_s	Mach number of element toward observer
XMDS	$\dot{M}_s$	Rate of change of Mach number toward observer
F(I)	F_i	Aerodynamic force component relative to rotor axes (lb)
FD(I)	$\dot{F}_i$	Rate of change of aerodynamic force component relative to rotor axes (lb/sec)

TABLE I - Continued

PROGRAM SYMBOLS	ALGEBRAIC SYMBOLS	DEFINITION
FX	F_x	Aerodynamic force components relative to aircraft flight path axes (lb)
FY	F_y	
FZ	F_z	
FDX	$\dot{F}_x$	Rate of change force components relative to aircraft flight path axes (lb/sec)
FDY	$\dot{F}_y$	
FDZ	$\dot{F}_z$	
FX	F_X	Forces and rate of change of forces relative to fixed axes (lb, lb/sec)
FY	F_Y	
FZ	F_Z	
FDX	$\dot{F}_X$	
FDY	$\dot{F}_Y$	
FDZ	$\dot{F}_Z$	
DP	Δ_p	Acoustic pressure increment due to forces and motions of blade element (lb/ft²)
PJ(I)	$p_j(t)$	Total acoustic pressure at time t (lb/ft²)
CPJ(I)	C_{pjk}	In-phase harmonic pressure component (lb/ft²)
SPJ(I)	S_{pjk}	Out-of-phase harmonic pressure component (lb/ft²)
PJK(I)	P_{jk}	Harmonic pressure amplitude (lb/ft²)
[illegible]	f_k	Frequency (Hz)

5.0 PROGRAM INPUT/OUTPUT

Table II describes the preparation of the input data cards for HERON 1. This is followed by an example comprising a complete set of data written on coding forms. In conclusion, the computer output obtained for the example is presented.

TABLE II. PROGRAM HERON 1. DATA INPUT

CARD NO.*	DESCRIPTION	SYMBOL	UNITS	FORMAT	CARD COL.
i, ii, iii	TITLE CARDS			10A8	1-80
1	No. of rotors	H	-	I4	1-4
1	No. of field points	J	-	I4	4-8
1	Speed of sound	C	ft/sec	F10.0	11-20
1	Atmospheric density	ρ_0	$slugs/ft^3$	F10.0	21-30
2	Rotor rpm****	Ω_N	rpm	F10.0	1-10
2	Rotor radians	R	ft	F10.0	11-20
2	No. of blades	B	-	F10.0	21-30
2	Flapping hinge offset	ϵ_f	ft	F10.0	31-40
2	Lagging hinge offset	ϵ_ℓ	ft	F10.0	41-50
2	Reference azimuth	ψ_0	rad	F10.0	51-60
3	No. of azimuth stations****	M	-	I4	1-4
3	No. of radial stations****	N	-	I4	5-8
3	Input format indicator****	IND	-	I4	9-12
3	No. of loading harmonics	K_f	-	I4	13-16
3	No. of displacement harmonics	K_x	-	I4	17-20
3	No. of flapping harmonics	K_1	-	I4	21-24
3	No. of lagging harmonics	K_2	-	I4	25-28

TABLE II - Continued

CARD NO.*	DESCRIPTION	SYMBOL	UNITS	FORMAT	CARD COL.
4+	Radial stations (r/R)	x(n)	-	F10.0	1-80
5	Hub coordinates * * * *	X_H	ft	F10.0	1-10
5	At time t = 0	Y_H	ft	F10.0	11-20
5		Z_H	ft	F10.0	21-30
5	Shaft * * * * longitudinal	θ_0	rad	F10.0	31-40
5	Inclination lateral	ϕ_0	rad	F10.0	41-50
5		$\dot{X}_H$	ft	F10.0	51-60
5	Hub velocity components* * * *	$\dot{Y}_H$	ft	F10.0	61-70
5		$\dot{Z}_H$	ft	F10.0	71-80
6	Pitch rate* * * *	$\dot{\theta}$	rad/sec	F10.0	1-10
6	Roll rate* * * *	$\dot{\phi}$	rad/sec	F10.0	11-20
IF IND = 1 - Time History Input					
7+	Flapping angle at successive azimuth stations (m = 1(1)M)	B(m)	deg	F10.0	1-10 etc.
8+	Lagging angle at successive azimuth stations (m = 1(1)M)	ζ(m)	deg	F10.0	1-10 etc.

TABLE II - Continued

CARD NO.*	DESCRIPTION	SYMBOL	UNITS	FORMAT	CARD COL.
9+	Normal elastic displacement at successive radial stations (n = 1(1)N) and successive azimuth stations (m = 1(1)M)	$\eta_N(m,n)$	ft	F10.0	1-10 etc.
10+	Tangential elastic displacement at successive radial stations (n = 1(1)N) and successive azimuth stations (m = 1(1)M)	$\eta_T(m,n)$	ft	F10.0	1-10 etc.
11+	Normal blade load at successive radial stations (n = 1(1)N) and successive azimuth stations (m = 1(1)M)	$F_N(m,n)$	lb	F10.0	1-10 etc.
12+	Tangential blade load at successive radial stations (n = 1(1)N) and successive azimuth stations (m = 1(1)M)	$F_T(m,n)$	lb	F10.0	1-10 etc.
IF IND = 2 - Harmonic Input					
7a+	Flapping angle cosine harmonics for $k = 0(1)K_1$	a(k)	deg	F10.0	1-10 etc.
7b+	Flapping angle sine harmonics for $k = 1(1)K_1$ **	b(k)	deg	F10.0	1-10 etc.

TABLE II - Continued

CARD NO.*	DESCRIPTION	SYMBOL	UNITS	FORMAT	CARD COL.
8a+	Lagging angle cosine harmonics for $k = 0(1)K_1$	a(k)	deg	F10.0	1-10 etc.
8b+	Lagging angle sine harmonics for $k = 1(1)K_1$	b(k)	deg	F10.0	1-10 etc.
9a+ ***	Normal elastic displacement cosine harmonics for $k = 0(1)K_X$ } Repeated for successive radial stations $n = 1(1)N$	a(n,k)	ft	F10.0	1-10 etc.
9b+	Normal elastic displacement sine harmonics for $k = 1(1)K_X$ } Repeated for successive radial stations $n = 1(1)N$	b(n,k)	ft	F10.0	1-10 etc.
9c+	Tangential elastic displacement cosine harmonics for $k = 0(1)K_X$ } Repeated for successive radial stations $n = 1(1)N$	a(n,k)	ft	F10.0	1-10 etc.
9c+	Tangential elastic displacement sine harmonics for $k = 1(1)K_X$ } Repeated for successive radial stations $n = 1(1)N$	b(n,k)	ft	F10.0	1-10 etc.

TABLE II - Continued

CARD NO *	DESCRIPTION	SYMBOL	UNITS	FORMAT	CARD COL.
10a+ **	Normal blade force cosine harmonics for $k = 0(1)K_F$ (Repeated for successive radial stations $n = 1(1)N$)	a(n,k)	lb	F10.0	1-10 etc.
10b+ **	Normal blade force sine harmonics for $k = 1(1)K_F$ (Repeated for successive radial stations $n = 1(1)N$)	b(n,k)	lb	F10.0	1-10 etc.
10c+ **	Tangential blade force cosine harmonics for $k = 0(1)K_F$ (Repeated for successive radial stations $n = 1(1)N$)	a(n,k)	lb	F10.0	1-10 etc.
10d+ **	Tangential blade force sine harmonics for $k = 1(1)K_F$ (Repeated for successive radial stations $n = 1(1)N$)	b(n,k)	lb	F10.0	1-10 etc.
11 +	Observer coordinates for $j = 1(1)J$	X_0 (j)	ft	F10.0	1-10
11 +		Y_0 (j)	ft	F10.0	11-20
11 +		Z_0 (j)	ft	F10.0	21-30

* Where a card number is followed by a + sign, additional cards are permissible if required to accommodate total data.

** When the number of harmonics (K) is specified as zero, there are no sine components and relevant cards are omitted.

TABLE II - Continued

* * *	For $n = 0$, all necessary (a) cards are punched, followed by all (b), (c) and (d) cards, for $n = 1$. These are followed by complete sets for $n = 2, 3 \ldots . N$.
* * * *	These parameters are determined as follows: -
M	Number of azimuth intervals. This controls the number of sound harmonics which will be calculated (= integral part of M/2B if M/2B is nonintegral, or = (M/2B) - 1 if M/2B is integral). With the present storage limits, M must not exceed 72. When the loading and motion data is read in time history form, M defines the number of azimuthal stations for which values must be specified.
N	Number of radial stations must not exceed 12.
IND	Input format indicator. When IND = 1, all loading and motion data must be specified as azimuthal/radial distributions. When IND = 2, data is input as a set of harmonic coefficients for each radial station.
$X_H Y_H Z_H$ $X_0 Y_0 Z_0$ Ω_N	In determining hub and observer coordinates, it should be remembered that the positive direction of rotor rotation (Ω_N positive) is clockwise (viewed from above) that the Z axis is vertically upwards, and that the Y axis is rotated 90° anticlockwise from the X axis.
$\theta_0, \dot{\theta}_0$	The shaft longitudinal inclination and rate of pitch are positive in the nose-up sense.
$\phi_0, \dot{\phi}_0$	The shaft lateral inclination and rate of roll are positive in a starboard-down sense.

COMPUTER CODING FORM

NAME ______ CUSTOMER ______ JOB NO. ______

DATE ______ PAGE 1 OF 5

i	CASE H.1.51							
ii	H-34 IN LEVEL FLIGHT AT 0 KTS			DATA FROM NASA TM. X-952 TABLE 8.				
iii								
1	1 11	1117.	.00213					
2	217.	28.	.4	1.	1.			
3	36 7	2 10 0	10 0					
4	.25	.4	.55	.75	.85	.9	.95	
5	0.	250.	200.	-.0157	-.0157	67.5	0.	0.
6	0.	0.						
7a	3.397	.316	.014	.15	.077	.006	.038	.014
	.026	.006	.018					
7b	-.672	.087	-.065	.055	-.049	.024	.008	.000
	-.004	.002						
8a	-7.							
9a	0.							
	0.							
	0.							
	0.							
	0.							
	0.							

COMPUTER CODING FORM

NAME ______ CUSTOMER ______ JOB NO. ______

DATE ______ PAGE 2 OF 5

	0.							
9b	0.							
	0.							
	0.							
	0.							
	0.							
	0.							
	0.							
10a	147.1	-13.08	27.31	-.8865	5.983	6.483	-3.157	-5.540
	-7.213	6.760	6.150					
10b	81.15	1.995	21.33	6.593	-2.547	-5.65	.7755	3.713
	-2.216	-1.441						
10c								
10d								
	333.4	3.024	56.90	17.49	-2.117	2.52	4.94	-12.04
	-3.325	7.860	-4.232					
	95.81	5.340	1.057	-11.79	-9.223	-14.26	-1.764	1.263
	1.764	-1.26						

COMPUTER CODING FORM

NAME ____________ CUSTOMER ____________ JOB NO. ________

DATE ____________ PAGE 3 OF 5

566.0	26.52	95.55	31.04	7.877	.05875	.1763	12.00
-4.528	-15.70	10.05					
108.5	10.17	1.350	-12.88	-15.82	-16.82	-9.935	4.528
-5.232	-9.230						
808.3	13.81	129.3	66.37	10.58	-24.82	17.74	12.70
-19.25	-3.628	15.67					
-3.73	12.25	-9.878	-14.11	-32.86	-13.36	6.552	-6.300
5.846	.3528						

COMPUTER CODING FORM

NAME ______________________ CUSTOMER ______________ JOB NO. ________

DATE ______________________ PAGE 4 OF 5

454.3	-17.79	64.49	77.29	.8820	-18.93	1.94	4.385
3.377	3.780	5.468					
-46.12	-2.041	-22.65	-20.21	-10.48	-21.04	2.293	10.21
-10.23	-5.645						
301.4	-6.166	23.40	68.76	11.42	-24.01	-1.243	11.04
1.2016	-3.562	2.47					
-47.53	-6.854	-5.158	-14.70	-17.91	-7.342	3.226	1.764
1.176	-1.781						
308.1	6.359	-5.678	66.93	37.62	-29.06	-29.67	10.20
20.90	1.428	-11.09					
-40.00	-14.03	-13.68	1.714	-17.39	-20.26	2.520	14.72
6.787	-9.610						

COMPUTER CODING FORM

NAME ________ CUSTOMER ________ JOB NO. ________

DATE ________ PAGE 6 OF 5

-250.	250.	0.
-200.	250.	0.
-150.	250.	0.
-100.	250.	0.
-50.	250.	0.
0.	250.	0.
50.	250.	0.
100.	250.	0.
150.	250.	0.
200.	250.	0.
250.	250.	0.

11 →

SAMPLE OUTPUT

The following pages give the program output for the example case presented previously. This output is fairly self-explanatory, although the following points are worthy of mention.

The results for the first two field points only (11 were requested) are included for the sake of brevity.

It should be remembered that the program calculates the sound field observed when the helicopter rotor is positioned at the nominal position X_H, Y_H, Z_H. This sound was actually generated by the rotor when it was in some other position (at the retarded time). This position is denoted in each case by the "retarded hub coordinates."

Four sound harmonics are calculated since M was specified as 36 so that $(M/2B) = 4.5$. The Doppler shift effect can be noted in the slightly different frequencies observed at the two positions.

```
                                 PROGRAM ,HERON 1
                                 CASE  H.1.51.
H-34 IN LEVEL FLIGHT AT 40 KTS.       DATA FROM NASA TM. X-952 TABLE 8.



GEOMETRY OF ROTOR      1          4           BLADES
                        2,800E 01   FT,      RADIUS
                        2,170E 02   R,P,M,
                        1,000E 00   FT,      FLAP HINGE OFFSET
                        1,000E 00   FT,      DRAG HINGE OFFSET
                                0  DEGREES AZIMUTH PHASE
                       -1,570E-02  DEGREES REARWARD SHAFT INCLINATION
                       -1,570E-02  DEGREES SHAFT INCLINATION TO STARBOARD
                       6,750E 01  FT/SEC  VELOCITY IN X-DIRECTION
                                0  FT/SEC  VELOCTIY IN Y-DIRECTION
                                0  FT/SEC  VELOCITY IN Z-DIRECTION
                        6,750E 01  FT/SEC  RESULTANT VELOCITY

                                0   FT,     X HUB ORDINATE
                                0   FT,     Y HUB ORDINATE
                        2,000E 02   FT,     Z HUB ORDINATE

BLADE LOADING AND MOTION DATA (PARALLEL AND NORMAL TO THE SHAFT


NUMBER OF LOADING STATIONS=      7


NUMBER OF AZIMUTH INTEGRATION POINTS=     36


FORM OF INPUT. HARMONIC



BLADE FLAPPING HARMONICS (DEGREES)
    A 0         3,307E 00
    A 1         3,160E-01
    A 2         1,400E-02
    A 3         1,500E-01
    A 4         7,700E-02
    A 5         6,000E-03
    A 6         3,800E-02
    A 7         1,400E-02
    A 8         2,600E-02
    A 9         6,000E-03
    A10         1,800E-02
    B 1        -6,720E-01
    B 2         8,700E-02
    B 3        -6,500E-02
    B 4         5,500E-02
    B 5        -4,900E-02
    B 6         2,400E-02
    B 7         6,000E-03
    B 8                 0
    B 9        -4,000E-03
    B10         2,000E-03


BLADE  LAGGING HARMONICS (DEGREES)
    A 0        -7,000E 00
```

BLADE ELEMENT ELASTIC DISPLACEMENTS NORMAL TO THE SHAFT

RADIAL STATION	7,000	11,200	15,400	21,000	23,800	25,200	26,600
A 0	0	0	0	0	0	0	0

BLADE ELEMENT ELASTIC DISPLACEMENTS PARALLEL TO THE SHAFT

RADIAL STATION	7,000	11,200	15,400	21,000	23,800	25,200	26,600
A 0	0	0	0	0	0	0	0

BLADE THRUST LOAD HARMONICS (LBS)

RADIAL STATION	7,000	11,200	15,400	21,000	23,800	25,200	26,600
A 0	1.471E 02	3,334E 02	5,660E 02	8,083E 02	4,543E 02	3,014E 02	3,081E 02
A 1	-1,308E 01	3,024E 00	2,652E 01	1,381E 01	-1,779E 01	-6,166E 00	6,350E 00
A 2	2,731E 01	5,690E 01	9,555E 01	1,293E 02	6,449E 01	2,340E 01	-5,678E 00
A 3	-8,665E-01	1,749E 01	3,104E 01	6,637E 01	7,729E 01	6,876E 01	6,693E 01
A 4	5,983E 00	-2,117E 00	7,877E 00	1,056E 01	8,820E-01	1,142E 01	3,762E 01
A 5	6,483E 00	2,520E 00	5,875E-02	-2,182E 01	-1,893E 01	-2,401E 01	-2,906E 01
A 6	-3,157E 00	4,940E 00	1,763E-01	1,774E 01	1,940E 00	-1,243E 00	-2,967E 01
A 7	-5,540E 00	-1,204E 01	1,200E 01	1,270E 01	4,385E 00	1,104E 01	1,020E 01
A 8	-7,313E 00	-3,325E 00	-4,528E 00	-1,925E 01	3,377E 00	2,016E-01	2,090E 01
A 9	6,760E 00	7,860E 00	-1,570E 01	-3,628E 00	3,780E-01	-3,562E 00	1,428E 00
A10	_6,150E 00	-4,232E 00	1,005E 01	1,567E 01	5,468E 00	2,470E 00	-1,109E 01
B 1	8,115E 01	9,581E 01	1,085E 02	-3,730E 00	-4,612E 01	-4,753E 01	-4,000E 01
B 2	1,995E 00	5,340E 00	1,017E 01	1,225E 01	-2,041E 00	-6,854E 00	-1,403E 01
B 3	2,133E 01	1,057E 00	1,350E 00	-9,878E 00	-2,265E 01	-5,158E 00	-1,368E 01
B 4	6,593E 00	-1,179E 01	-1,288E 01	-1,411E 01	-2,021E 01	-1,470E 01	1,714E 00
B 5	-2,547E 00	-9,223E 00	-1,582E 01	-3,286E 01	-1,048E 01	-1,791E 01	-1,739E 01
B 6	-5,650E 00	-1,426E 01	-1,682E 01	-1,336E 01	-2,104E 01	-7,342E 00	-2,026E 01
B 7	7,755E-01	-1,764E 00	-9,935E 00	6,552E 00	2,293E 00	3,226E 00	2,520E 00
B 8	3,713E 00	1,253E 00	4,528E 00	-6,300E 00	1,021E 01	1,764E 00	1,472E 01
B 9	-2,216E 00	1,764E 00	-5,232E 00	5,846E 00	-1,023E 01	1,176E 00	6,787E 00
B10	-1,441E 00	-1,260E 00	-9,230E 00	3,528E-01	-5,645E 00	-1,781E 00	-9,610E 00

BLADE DRAG LOAD HARMONICS (LBS)

RADIAL STATION	7,000	11,200	15,400	21,000	23,800	25,200	26,600
A 0	0	0	0	0	0	0	0
A 1	0	0	0	0	0	0	0
A 2	0	0	0	0	0	0	0
A 3	0	0	0	0	0	0	0
A 4	0	0	0	0	0	0	0
A 5	0	0	0	0	0	0	0
A 6	0	0	0	0	0	0	0
A 7	0	0	0	0	0	0	0
A 8	0	0	0	0	0	0	0
A 9	0	0	0	0	0	0	0
A10	0	0	0	0	0	0	0
B 1	0	0	0	0	0	0	0
B 2	0	0	0	0	0	0	0
B 3	0	0	0	0	0	0	0
B 4	0	0	0	0	0	0	0
B 5	0	0	0	0	0	0	0
B 6	0	0	0	0	0	0	0
B 7	0	0	0	0	0	0	0
B 8	0	0	0	0	0	0	0
B 9	0	0	0	0	0	0	0
B10	0	0	0	0	0	0	0

COMPUTED SOUND FIELD

OBSERVER CO-ORDINATES J= 1 XO= -2,50000000E 02 YO= 2,50000000E 02 ZO= 0

RETARDED HUB CO-ORDINATES XPH= -2,36923922E 01 YPH= 0 ZPH= 2,00000000E 02

K HARMONIC NO.	F(K) FREQUENCY	CPJ(K) IN PHASE PRES, (P.S.F.)	SPJ(K) OUT OF PHASE PRES. (P.S.F.)	PJ(K) SOUND PRESSURE LEVEL (DB. RE. .0002 DYNES/SQ.CM.)
1	1,51038167E 01	2,42557947E-03	1,04885518E-02	8,52205780E 01
2	3,02076334E 01	7,25207500E-03	2,87507291E-03	8,24232223E 01
3	4,53114501E 01	1,25524132E-03	1,00197493E-03	6,86954979E 01
4	6,04152669E 01	1,08763223E-03	8,97429345E-04	6,75648703E 01

OBSERVER CO-ORDINATES J= 2 XO= -2,00000000E 02 YO= 2,50000000E 02 ZO= 0

RETARDED HUB CO-ORDINATES XPH= -2,21322116E 01 YPH= 0 ZPH= 2,00000000E 02

K HARMONIC NO.	F(K) FREQUENCY	CPJ(K) IN PHASE PRES, (P.S.F.)	SPJ(K) OUT OF PHASE PRES. (P.S.F.)	PJ(K) SOUND PRESSURE LEVEL (DB. RE. .0002 DYNES/SQ.CM.)
1	1,50170596E 01	1,17483472E-02	7,00402784E-03	8,73002754E 01
2	3,00341192E 01	4,77944402E-03	7,01753579E-03	8,31586838E 01
3	4,50511787E 01	3,58252364E-05	8,15807666E-04	6,28201229E 01
4	6,00682383E 01	4,91181960E-04	1,62177906E-03	6,91609781E 01

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13. ABSTRACT

AD-684 374

This report contains a comprehensive description of a computer program developed for the numerical evaluation of the helicopter noise equations derived in USAAVLABS TR 68-60, "Studies of Helicopter Rotor Noise." It is completely self-contained in that the program details are described, starting from two basic acoustic equations and covering methods by which these equations are applied to the rotor noise problem. Program flow diagrams and a complete listing are presented together with input instructions and sample inputs and outputs. The program is written in FORTRAN IV for the CDC 3300 Computer.

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