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A METHOD OF INCREASING THE ACCURACY AND SPEED OF A DIGITAL INTEGRATOR

by

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ABSTRACT

The possible ways of increasing the accuracy and speed of digital differential analyzers are examined. It is concluded that a conversion to multiple digit increments is expedient. The application of various numerical integration equations when operating with multiple digit increments is considered. The investigation was carried out in solving a second order differential equation based on a comparison of equations for rectangles, trapezoids and average rectangles. It is shown that the speed of a digital differential analyzer can be 10-100 times greater than the speed of the parallel-series type machine. The block diagram is given of a computer which can be used to obtain the algorithm described. 4 figures, 7 references. Translation of abstract].

A METHOD OF INCREASING THE ACCURACY AND SPEED OF A DIGITAL INTEGRATOR

V. V. Kokhanov and T. M. Andreyeva

A number of problems involved in the processing of telemetry data are frequently reduced to solving a system of differential equations. Both analog and digital computers can be used for this purpose. The advantage of analog computers is their relatively high operating speed, simplicity in programming and setting up of problems. Their disadvantages include their low calculation accuracy, bulkiness and limitations in the problems they can solve. Universal computers can solve a broad range of problems with high accuracy, but in the majority of cases they cannot operate in the natural time scale and require relatively complex programs. These factors led to the development of specialized computers, the so-called "digital models," whose category includes digital differential analyzers (TsDA). A number of papers have been written lately on digital models and the analysis of their accuracy and speed, and proposals have also been made for improving them [1, 2, 3]. However, this concerns for the most part digital models operating with single digit increments (TsDA). As we know, the basic defect of the TsDA is its low speed, which decreases in proportion to an increase in the accuracy of solution of problems. The operating speed of the TsDA can be increased by increasing its cyclic frequency, and also by using a parallel structure. However, even if we use a cyclic frequency of f_T = 10 mc and a series-parallel structure, the threshold frequency obtained for the TsDA is of the order of fractions of a cycle [3] and is given by $f_{max}=\frac{|\Delta x|}{2\pi T}$, where $|\Delta x|$ is an increment in the independent variable and T is the iteration time.

For a TsDA with a series-parallel structure,

$$T=m\cdot k\cdot\frac{1}{f_T},$$

where m is the number of integrators; and k is a coefficient taking into account the time required to sum numbers in the adders.

When $m=50$, $\kappa=2$, $|\Delta x|=2^{-18}$,

$T=100\cdot10^{-7}=10^{-5}$ sec., we have

$$f_{max}=\frac{2^{-18}}{10^{-5}}\cong 0{,}4 \text{ cps.}$$

The use of a TsDA with a strictly parallel structure increases the speed in the given case approximately fiftyfold, but requires a significant increase in equipment. The low speed of the TsDA limits its applications and ordinarily does not permit its utilization for operation in the natural time scale. In order to compensate for the error arising with a decrease in the number of steps of integration, some authors [1] have suggested using more precise integration formulas. But then, in spite of the increase in the amount of equipment, there is hardly any increase in the accuracy of solution. This is explained by the fact that although the systematic error decreases, the round-off error does not decrease and the total error changes by a small amount. In order to realize all of the advantages from utilization of more accurate integration formulas, it is advisable to convert to multidigit increments. Despite the fact that the time involved in one integration increases due to the operation of multiplying by the multidigit number, the overall speed increases. This was noted in a 1958 article by A. A. Kozharskiy [4], who proposed a series-type digital integrator with data storage on a magnetic drum and utilizing the iterative problem-solving process. However, the algorithm of such a computer is quite complicated, and the speed is low, even though it is higher than that of an ordinary TsDA using data storage on a magnetic drum. Some efforts to expand the potentialities of the TsDA have complicated its structure and logic of operation to such an extent that the machine lost its advantages over the universal computer. Therefore, in developing a specialized digital model with multidigit increments, special emphasis was placed on retaining simplicity of structure and logic of operation. The possibility of using various digital integration equations for operation with multidigit increments was explored.

The second-order differential equation $y'' + y = 0$ was solved with the following initial conditions: $t = 0$, $y = 0$, $y' = 2^7$ or $y = 2^7 \cdot \sin x$. Rectangular, trapezoidal and mean-rectangular rules were used.

As we know [3], the systematic error in one step of integration for rectangular and trapezoidal rules can be written in the form

$$|E_n| \leqslant \begin{cases} \frac{1}{2}(\Delta x)^2 \cdot \max |y''| & \text{— for the rectangular rule,} \\ (\Delta x)^2 \cdot \max |y''| & \text{— for the trapezoidal rule} \end{cases} \tag{1}$$

or for K steps,

$$E_{nK} \leqslant \begin{cases} \frac{1}{2}\frac{(x_{K+1}-x_1)^2}{K} \cdot y''(\xi) — & \text{for the rectangular rule,} \\ \frac{(x_{K+1}-x_1)^2}{K} \cdot y''(\xi) — & \text{for the trapezoidal rule.} \end{cases} \quad \xi \in (x_1, x_{K+1}). \tag{2}$$

The algorithm of the digital integrator employed in using rectangular and trapezoidal rules was the same one ordinarily used in a TsDA [5, 6].

In the case of mean rectangles, the following algorithm was used (see Figure 1):

$$y_{1,n-1} \cdot 2\Delta x = \Delta s_1, \quad y_{2,n-2} + \Delta s_1 = y_{2,n}, \tag{3}$$

$$y_2' = y_1, \quad 2\Delta x = x_n - x_{n-2}, \tag{4}$$

where $y_{1,n-1}$ is the value of the integrand of the first integrator in the n − 1 step; and $y_{2,n-2}$ is the value of the integrand of the second integrator in the n − 2 step.

Solving the system of equations (3), we obtain

$$y_{2,n} = y_{2,n-2} + y_{1,n-1} \cdot 2\Delta x.$$

Expressing $y_{1,n-1}$ through $y_{1,n-2}$ using Taylor's formula and taking into account Equation (4), we obtain

$$y_{2,n} = y_{2,n-2} + y'_{2,n-2} \cdot 2\Delta x + y''_{2,n-2} \cdot 2\Delta x^2 + y'''_{2,n-2} \cdot \Delta x^3 + \ldots \tag{5}$$

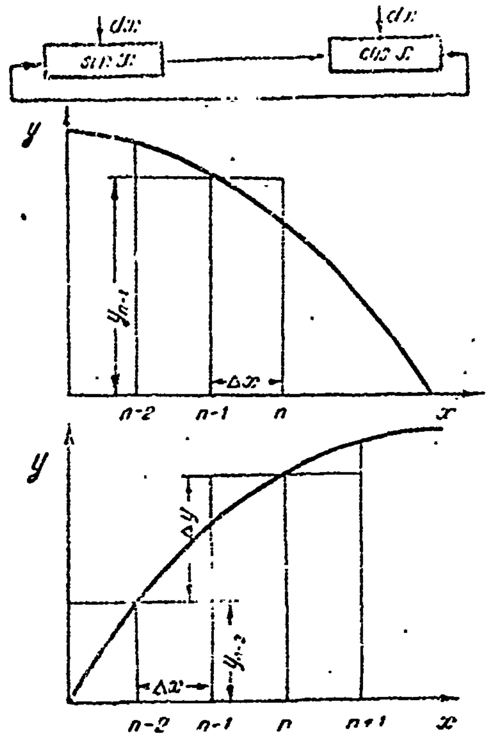

Figure 1.

We obtain the true value of $y_{2,n}$ through $y_{2,n-2}$ from Taylor's formula:

$$y_{2,n(\text{tr})} = y_{2,n-2} + y'_{2,n-2}\cdot 2\Delta x + 2\Delta x^2\cdot y''_{2,n-2} + \frac{4}{3}\Delta x^3\cdot y'''_{2,n-2} + \ldots \quad (6)$$

Comparing (4) and (6), we obtain the error in calculating $y_{2,n}$ for one binary step in integration ($2\Delta x$):

$$|E| = \frac{1}{3}\Delta x^3\cdot y'''_{2,n-2} = \frac{1}{3}\Delta x^3 y''_{1,n-2}. \quad (7)$$

Hence we can calculate the total error for K steps of integration:

$$E_{nK} \leqslant \frac{1}{6}\cdot\frac{(x_{K+1}-x_1)^3}{K^2}\cdot y''(\xi), \quad \xi\in(x_1, x_{K+1}). \quad (8)$$

For our problem

$$E_{nK} \leqslant \begin{cases} \left(\frac{\pi}{2}\right)^3 \cdot \frac{1}{K} \cdot 128 \cdot 0{,}636 - \text{for the trapezoidal rule,} \\ \left(\frac{\pi}{2}\right)^3 \cdot \frac{1}{2 \cdot K} \cdot 128 \cdot 0{,}636 - \text{for the rectangular rule,} \\ \frac{1}{6} \cdot \left(\frac{\pi}{2}\right)^3 \cdot \frac{1}{K^2} \cdot 128 \cdot 0{,}636 - \text{for the mean-rectangular rule,} \end{cases} \tag{9}$$

$$y''(\xi) = \frac{2}{\pi} = 0{,}636; \quad x_{K+1} - x_1 = \frac{\pi}{2}.$$

Figure 2 presents true error curves for the integration equations: rectangles (Curves II), trapezoids (Curves III) and mean rectangles (Curves I) with a varying number of integration steps and using multidigit increments (Curves I', II", III' are given for single-digit increments).

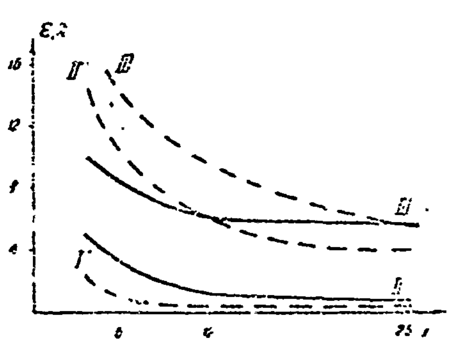

Figure 2.

The following table contains the results of error calculations, using Equations (9), for a varying number of steps K using multidigit increments. For the sake of comparison, the true errors obtained using desk calculators are also shown.

No. of steps, k	202		25		12		6		3	
Error, %	Calculated	True	Calculated	True	Calculated	True	Calculated	True	Calculated	True
Method of rectangles	0.4	0.4	3.2	3.1	6.4	6.2	12.8	12.1	-	-
Method of trapezoids	0.8	0.8	6.4	6.2	12.8	8.5	25.6	20.3	-	-
Method of mean rectangles	-	-	0.016	0.05	0.07	0.26	0.3	0.78	1.2	1.1

In a number of cases for the mean rectangular rule, the true error is obtained below the calculated error which is explained by the presence of the round-off.

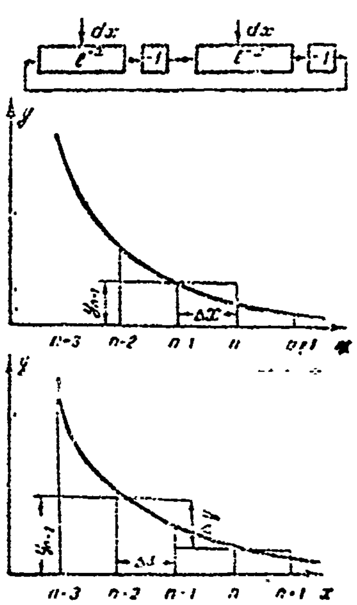

Figure 3.

As can be seen from the curves, the trapezoidal rule yields the maximum error and the mean triangular rule yields the smallest error. An error determination similar to the one above was made in calculating $y = e^{-x}$ functions, using one (method of rectangles) and two integrators (method of mean rectangles) (see Figure 3). The errors satisfy Equations (2) and (8). Thus the small error obtained using the mean rectangular rule makes it possible in many cases to reduce the calculation time in using multidigit increments. Since the algorithm in this case is quite simple, for a digital integrator it is possible to use only one parallel adder, on which the following operation is performed initially:

$$y_{n-2} + \Delta y_1 + \Delta y_2 + \ldots + \Delta y_l = y_n,$$

and then

$$s_{0n-2} + y_n \cdot 2\Delta x = s_{0n} + \Delta s_{n-1}.$$

Despite the fact that multiplication by the multidigit increment 2Δx is performed, there is a significant increase in speed. Thus, with m = 50 and k = 40:

$$|\Delta x| = 2^{-7} \text{ and } f_T = 10^6 \text{ cps},$$
$$T = 2000 \cdot 10^{-6} = 2 \cdot 10^{-3} \text{ sec},$$
$$f_{max} = \frac{2^{-7}}{2{,}0 \cdot 10^{-3}} \cong 4 \text{ cps},$$

i.e., the speed obtained is ten times greater than that of the previous example involving a TsDA with a series-parallel structure with single-digit increments, despite the decrease in cyclic frequency. The use of $f_T = 10^7$ cps increases the speed a hundredfold.

Figure 4 is a block diagram of a computer in which the aforementioned algorithm can be realized. The magnetic memory device (MOZU) for storage of the parameters y and s can be the same as in the ordinary digital differential analyzers described in the literature [7]. A parallel-type adder is typical. The MOZU for storage of parameter Δs is used as described in the literature [5].

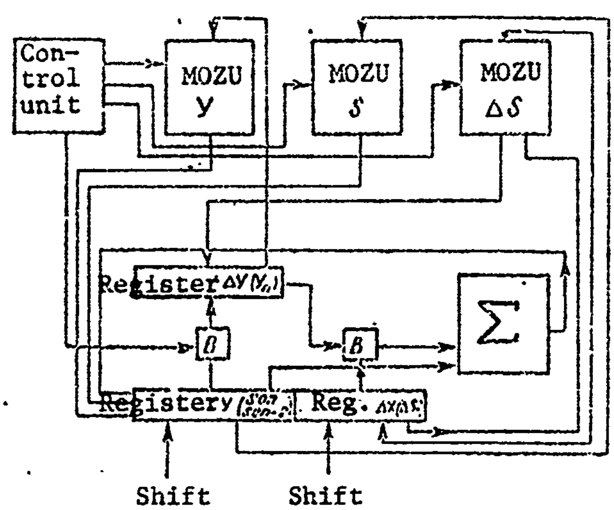

Figure 4.

However, multidigit increments are read out instead of single digit increments, and the program is set up by means of strings.

Thus, using the aforementioned computer, a considerable increase in speed and accuracy of solution can be achieved with a relatively slight increase in equipment.

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