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COUPLED LINE FILTERS

by

Nobuji Saito

Translated by Akio Matsumoto

Research Report PIBMRI-1048-62

for

Rome Air Development Center
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Air Force Systems Command
United States Air Force
Griffiss Air Force Base, New York

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55 Johnson Street
Brooklyn 1, New York

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Foreword
Abstract
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FOREWORD

**The work reported herein was sponsored by Rome Air Development Center,
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ABSTRACT

This paper describes the method of designing distributed networks consisting chiefly of distributed coupled lines.

Coaxial filters are the representative ones of distributed filters. They are of practical use because of their simplicity of construction and design procedures. On the other hand, it is sometimes difficult to obtain those of desired characters, owing to many restrictions. Coupled line filters came into use to supplement these drawbacks and displayed their merits as narrow band filters. As they grew familiar, varieties of networks were found, and have now significant uses in microwave bands as strip-line filters.

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Introductory

I. 1 On ingredients

This paper describes the method of designing distributed networks consisting chiefly of distributed coupled lines.

Coaxial filters are the representative ones of distributed filters. They are of practical use because of their simplicity of construction and design procedures. On the other hand it is sometimes difficult to obtain those of desired characters, owing to many restrictions. Coupled line filters came into use to supplement these drawbacks and displayed their merits as narrow band filters. As they grew familiar, varieties of networks were found, and have now significant uses in microwave bands as strip-line filters.

In early stages of theoretical treatments of distributed networks, the elementary method was to replace lumped elements by corresponding distributed elements. The treatment has shown a great progress, since P.I. Richards introduced a parameter $p = j \tan (2\pi fl/c)$ which corresponds to $p = j 2\pi f$ in lumped networks. This was done by making all line elements of equal length. In this paper also, the technique has been used to systematize, even though partly, the theoretical treatments of coupled-line filters.

This paper may be roughly divided into three parts. Chapters 2 and 3 treat with those networks consisting mainly of distributed coupled lines combined with one or two coaxial elements. The characters of the networks are studied with reference to equivalent networks, and it is aimed to have a whole view of the characteristics of coupled-line networks. Chapters 4 and 5 describe methods of obtaining coupled-line networks through transformation, using equivalence relations in preceding two chapters, from the networks designed for lumped constants. Chapters 6, 7, 8 and 9 treat the extension of coax extraction, proposed by Richards, into extraction of coupled two-wire lines, and explain that this procedure will be a method of synthesis in coupled-line networks. Finally some experiments are shown in addition, to see the validity of theoretical calculations.

Studies on coupled-line networks were carried out by Hirota¹, Moriwaki², Uchida³, Nagai and Sato⁴. This paper has its basis on the earlier studies due to these authors, and has been developed by the writer.

Following sections describe certain items that will be of reference in

reading this paper.

I. 2 Frequency transformation into distributed networks

In distributed networks, the frequency f comes into expression in a form $\tan(2\pi fl/c)$ or $\tanh(\alpha + j2\pi fl/c)$, because lines are used as elements. Sometimes cosines or sines may appear, but they may be transformed into tangents. Therefore, the characteristics of distributed networks are periodic functions in f and will repeat with frequency. If the lengths (l) of the line element are scattered, the periods of repetition vary from element to element, and the characteristics of the whole network will have a complicated periodicity. As a result, it will be very difficult to compute the characteristics or to have representations in equivalent networks. From this view, Richards⁵ and Matsumoto⁶ proposed that

(a) lengths of elements should be all equal

(b) a frequency parameter

$$p = j \tan \frac{2\pi f}{c} l$$

or

$$p = \tanh(\alpha + j\beta)l = \tanh\left(\alpha + j \frac{2\pi f}{c} l\right)$$

should be used.

As a consequence of condition (a), the periodicity becomes so simple that the period is 2π in $2\pi fl/c$. On the other hand, condition (b) enables the correspondence of $2\pi f$ and $\tan(2\pi fl/c)$, and one can easily consider a network of lumped constants to be that of distributed ones by merely replacing $p = j2\pi f$ by p .

This proposal has made the theoretical treatments of distributed networks very easy, and caused an abrupt progress of distributed network theory.

The same is in the treatment of coupled line filters and the frequency transformation is used throughout this paper. In making actual networks, one need not always use elements of equal lengths. The characteristics for unequal lengths of elements can, to a certain extent, be interpreted from those for equal lengths.

I. 3 Coaxial filters

Since the proposal of the frequency transformation concerning theoretical treatment of coaxial networks, developments were made, chiefly by those of our country⁷⁻¹¹. The outline will be presented here for reference.

The theoretical developments have been made under the conditions

- (1) all elements are of equal length, and the frequency transformation is used, as described above.
- (2) In order to avoid multiple coaxial structures, series elements will not be used, shunt and cascade connections will be used instead. Thus the three kinds of elements are of most significance:
 - (a) inductance Wp
 - (b) capacitance W/p
 - (c) unit coax

Examples of actual networks are:

- (i) bar network
- (ii) simple open (or short) branch network
- (iii) tree-and-branch network
- (iv) loop network (tree-and-branch network in a broader sense)

One can naturally assume parallel or cascade connections of any networks above mentioned.

I. 4 Richards' key theorem

Richards' key theorem and coax extraction will be mentioned here, since they have some concern with the extraction of coupled two-wire lines.

Richards' theorem says

"If Y is a positive real function, then

$$Y_1 = Y(p_1) \frac{pY(p) - p_1 Y(p_1)}{pY(p_1) - p_1 Y(p)} \quad p_1 > 0$$

is also a positive real function.

Let $p_1 = 1$, in particular, then one has

"If Y is a positive real function, then

$$Y_1 = Y(1) \frac{pY(1) - Y(p)}{pY(p) - Y(1)}$$

is also a positive real function. Here Y_1 is the remainder after extraction of a unit coax of characteristic admittance $Y(1)$ from Y , and it is proved in this theorem that this Y_1 is a positive real function (to be two-terminal admittance).

I. 5 Construction of lines

A coaxial filter is made of a combination of coaxial lines, but theoretically it may be made of open two-wire lines. In short, it is a combination of single-phase lines. But open lines are apt to interfere with other systems of lines through radiation, etc., and consequently coaxial elements are used in filters.

A coupled two-wire line may be shielded or open, and also may be of any shape. Theoretically this does not matter, but shielded lines should be used in practice. In this paper, a coupled two-wire line is represented by two wires over the ground.

CHAPTER I. Equations of transmission in lines of parallel wires

This chapter described equations of transmission of distributed coupled lines consisting the major part of coupled-line filters. Various manners of representation have been made by various authors.^{1, 2, 3, 12, 13} Here equations are presented, with coupling coefficients and symmetry coefficients, that would be of use in describing the characters of coupled-line filters.

1.1. Equations of transmission in multi-wire lines.

Take any point χ on a line of parallel wires stretched over the ground, let the effective values of voltages and currents be $[V]$ and $[I]$ respectively, and assume the direction of χ as shown in the figure, then the following equations will hold between the voltages and currents:

$$-\frac{d}{d\chi} [V] = [Z][I] \quad (1.1.a)$$

$$-\frac{d}{d\chi} [I] = [Y][V] \quad (1.1.b)$$

where $[Z]$ and $[Y]$ are impedances and admittances of the line per unit length.

If the line is lossless, one may use the inductance $[L]$ and the capacitance $[C]$ per unit length in Eq. (1), so that

$$-\frac{d}{d\chi} [V] = j\omega [L][I] \quad (1.2.a)$$

$$-\frac{d}{d\chi} [I] = j\omega [C][V] \quad (1.2.b)$$

Differentiate Eq. (1.2.a) in χ and put it into Eq. (1.2.b), then one has

$$\frac{d^2}{d\chi^2} [V] = \omega^2 [L][C][V] \quad (1.3)$$

There is a relation

$$[L][C] = \frac{1}{v_o^2} [1] \quad (1.4)$$

among the velocity v_o , of propagation of the electromagnetic wave, $[L]$, and $[C]$, so that Eq. (1.3) can also be written

$$-\frac{d^2}{d\chi^2} [V] = \frac{\omega^2}{v_o^2} [V] \quad (1.5)$$

Here define β by

$$\omega / v_o = \beta \quad (1.6)$$

which is called the phase constant of the line.

The solution of Eq. (1.5) is in general given by

$$V = [a e^{j\beta x} + b e^{-j\beta x}] \quad (1.7)$$

where a and b are constants of integration to be determined from boundary conditions.

Combining the above equation and Eq. (1.2.b), one obtains

$$- \frac{d}{dx} [V] = -j\beta [a e^{j\beta x} - b e^{-j\beta x}] = j\omega [C][I] \quad (1.8)$$

which yields currents, with notice to the direction of x ,

$$[I] = \frac{1}{v_o} [L]^{-1} [a e^{j\beta x} - b e^{-j\beta x}] \quad (1.9)$$

Eqs. (1.7) and (1.9) are the fundamental equations for the voltages and currents.

To determine the constants a and b of integration, let the voltages and currents at $x=0$ be $[V_o]$ and $[I_o]$ respectively, then from Eqs. (1.7) and (1.9), one has the relations

$$[a+b] = [V_o], [a-b] = v_o [L][I_o] \quad (1.10.a)$$

which lead to

$$[2a] = [V_o] + v_o [L][I_o], [2b] = [V_o] - v_o [L][I_o] \quad (1.10.b)$$

and finally to

$$\begin{aligned} [V] &= \frac{1}{2} \left\{ [V_o] + v_o [L][I_o] \right\} e^{j\beta x} + \frac{1}{2} \left\{ [V_o] - v_o [L][I_o] \right\} e^{-j\beta x} \\ &= [V_o] \cos \beta x + j v_o [L][I_o] \sin \beta x \end{aligned} \quad (1.11.a)$$

$$[I] = [I_o] \cos \beta x + j \frac{1}{v_o} [L]^{-1} [V_o] \sin \beta x \quad (1.11.b)$$

1.2. Line constants of multi-wire lines

In the equations (1.11) of transmission, primary constants $v_o [L]$ of the line appear in place of the characteristic impedance W of a coaxial line. Rewrite $v_o [L]$

by Eq. (1.4), then one has

$$v_o [L] = \sqrt{[C]^{-1} [L]^{-1}} \cdot [L] = [C]^{-1} [L] = [W] \quad (1.12)$$

Here new line constants $[W]$ should be defined which will be called the characteristic impedance of a multi-wire line. Making use of this quantity, Eq. (1.11) will read

$$[V] = [V_o] \cos \beta x + [W][I_o] j \sin \beta x \quad (1.13.a)$$

$$[I] = [I_o] \cos \beta x + [W]^{-1} [V_o] j \sin \beta x \quad (1.13.b)$$

This form of equations of transmission will be used hereafter. Entries of $[W]$ can be written

$$[W] = \begin{bmatrix} W_{11} & W_{12} \dots & W_{1n} \\ W_{21} & W_{22} \dots & W_{2n} \\ \vdots & \vdots & \vdots \\ W_{n1} & W_{n2} & W_{nn} \end{bmatrix} \quad (1.14)$$

The law of reciprocity should hold in the establishment of Eq. (1.1), so that $[L]$ and $[C]$ should be symmetrical matrices, and consequently $[W]$ should also be a symmetrical matrix. That is,

$$W_{ij} = W_{ji}$$

and

$$[W] = \begin{bmatrix} W_{11} & W_{12} \dots & W_{1n} \\ W_{12} & W_{22} \dots & W_{2n} \\ \vdots & \vdots & \vdots \\ W_{1n} & W_{2n} & W_{nn} \end{bmatrix} \quad (1.15)$$

Rewrite this, for the sake of convenience in practical use, in the form

$$[W] = W_{11} \begin{bmatrix} 1 & W_{12}/W_{11} \dots W_{1n}/W_{11} \\ W_{12}/W_{11} & W_{22}/W_{11} \dots W_{2n}/W_{11} \\ \vdots & \vdots & \vdots \\ W_{1n}/W_{11} & W_{2n}/W_{11} \dots W_{nn}/W_{11} \end{bmatrix} = W_{11} \begin{bmatrix} 1 & k_{12} \dots k_{1n} \\ k_{12} & d_2 \dots k_{2n} \\ \vdots & \vdots & \vdots \\ k_{1n} & k_{2n} \dots d_n \end{bmatrix} \quad (1.16)$$

Here are introduced d_i and k_{ij} which will be called symmetry coefficients and coupling coefficients respectively. W_{11} is the self characteristic impedance of the first wire.

In the following sections are obtained equations of transmission using these line constants for 2-wire lines and 3-wire lines.

1.3. A parallel 2-wire line.

(A general 2-wire line) Define voltages and currents of a parallel 2-wire line as shown in Fig. 2. Here one has

$$\begin{aligned} [V] &= \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, [V_o] = \begin{bmatrix} V_{10} \\ V_{20} \end{bmatrix} \\ [I] &= \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}, [I_o] = \begin{bmatrix} I_{10} \\ I_{20} \end{bmatrix} \\ [W] &= W_{11} \begin{bmatrix} 1 & k_{12} \\ k_{12} & d_2 \end{bmatrix} \end{aligned} \quad (1.17)$$

which will yield, upon substituting into Eq. (1.13),

$$\left. \begin{aligned} V_1 &= V_{10} \cos \beta x + W_{11} (I_{10} + k_{12} I_{20}) j \sin \beta l \\ V_2 &= V_{20} \cos \beta x + W_{11} (k_{12} I_{10} + d_2 I_{20}) j \sin \beta l \\ I_1 &= I_{10} \cos \beta x + \frac{1}{W_{11} (d_2 - k_{12}^2)} (d_2 V_{10} - k_{12} V_{20}) j \sin \beta l \\ I_2 &= I_{20} \cos \beta + \frac{1}{W_{11} (d_2 - k_{12}^2)} (-k_{12} V_{10} + V_{20}) j \sin \beta l \end{aligned} \right\} \quad (1.18)$$

These are the fundamental equations of transmission in 2-wire lines.

(Case of Symmetry) If the line is symmetrical with respect to the ground, $W_{11} = W_{22}$, so that

$$d_2 = W_{22} / W_{11} = 1 \quad (1.19)$$

and therefore the fundamental equations can be written

$$\begin{aligned}
V_1 &= V_{10} \cos \beta l + W_{11}(I_{10} + k_{12} I_{20}) j \sin \beta l \\
V_2 &= V_{20} \cos \beta l + W_{11}(k_{12} I_{10} + I_{20}) j \sin \beta l \\
I_1 &= I_{10} \cos \beta l + \frac{1}{W_{11}(1-k_{12}^2)} (V_{10} - k_{12} V_{20}) j \sin \beta l \\
I_2 &= I_{20} \cos \beta l + \frac{1}{W_{11}(1-k_{12}^2)} (-k_{12} V_{10} + V_{20}) j \sin \beta l
\end{aligned} \quad (1.20)$$

(NOTE) W_{11} , d_2 and k_{12} have following relations with the balance and unbalance characteristic impedances (W_b and W_u) and the degree of symmetry δ :

$$\begin{aligned}
\delta &= \frac{1-k_{12}}{d_2-k_{12}} \\
W_b &= W_{11} \{(1-k_{12}) + (d_2-k_{12})\}, \quad W_u = \frac{W_{11}(d_2-k_{12}^2)}{(1-k_{12} + (d_2-k_{12}))}
\end{aligned} \quad (1.21)$$

These quantities are encountered with in the line equations, as line constants, obtained by the method of symmetrical coordinates.

In other works, upon substituting these relations into Eq. (1.18), one will obtain a result perfectly coincident with that obtained from the method of symmetrical coordinates.

Rewriting the above relations, one has

$$\begin{aligned}
W_{11} &= W_u + W_b \delta^2 / (1 + \delta)^2 \\
W_{11} k_{12} &= W_u - W_b \delta / (1 + \delta)^2, \quad W_{11} d_2 = W_u + W_b / (1 + \delta)^2
\end{aligned} \quad (1.22)$$

so that

$$W_{11}(1-k_{12}) = W_b \delta / (1+\delta), \quad W_{11}(d_2-k_{12}) = W_b / (1+\delta) \quad (1.23)$$

Since W_b , δ and $W_{11} \geq 0$, $W_u \geq W_b \delta / (1 + \delta)^2$, one has the restrictions

$$1, d_2 \geq k_{12} \geq 0 \quad (1.24)$$

1.4. A parallel 3-wire line

(A general 3-wire line) In a parallel 3-wire line, one has similarly,

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} V_{10} \\ V_{20} \\ V_{30} \end{bmatrix} \cos \beta l + W_{11} \begin{bmatrix} 1 & k_{12} & k_{13} \\ k_{12} & d_2 & k_{13} \\ k_{13} & k_{23} & d_3 \end{bmatrix} \begin{bmatrix} I_{10} \\ I_{20} \\ I_{30} \end{bmatrix} + j \sin \beta l \quad (1.25)$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} I_{10} \\ I_{20} \\ I_{30} \end{bmatrix} \cos \beta l + \frac{1}{|W|} \begin{bmatrix} |1| & -|k_{12}| & |k_{13}| \\ -|k_{12}| & |d_2| & -|k_{23}| \\ |k_{13}| & -|k_{23}| & |d_3| \end{bmatrix} \begin{bmatrix} V_{10} \\ V_{20} \\ V_{30} \end{bmatrix} + j \sin \beta l$$

$$|W| = W_{11} \begin{bmatrix} 1 & k_{12} & k_{13} \\ k_{12} & d_2 & k_{23} \\ k_{13} & k_{23} & d_3 \end{bmatrix} [1] = \begin{bmatrix} d_2 & k_{23} \\ k_{23} & d_3 \end{bmatrix}$$

where $|k|$ means a minor determinant.

(Case of symmetry) If the line is symmetrical with respect to the ground, one has

$$W_{11} = W_{22} = W_{33} \quad (1.26)$$

so that

$$d_2 = d_3 = 1 \quad (1.27)$$

Furthermore, if the wires are symmetrical to one another, one has

$$\begin{aligned} W_{12} &= W_{13} = W_{23} \\ k_{12} &= k_{13} = k_{23} \quad (\equiv k_0) \end{aligned} \quad (1.28)$$

so that

$$[W] = W_{11} \begin{bmatrix} 1 & k_0 & k_0 \\ k_0 & 1 & k_0 \\ k_0 & k_0 & 1 \end{bmatrix}$$

$$[W]^{-1} = \frac{1+k_0}{W_{11}(1+2k_0)(1-k_0)} \begin{bmatrix} 1 & \frac{-k_0}{1+k_0} & \frac{-k_0}{1+k_0} \\ \frac{-k_0}{1+k_0} & 1 & \frac{-k_0}{1+k_0} \\ \frac{-k_0}{1+k_0} & \frac{-k_0}{1+k_0} & 1 \end{bmatrix} \quad (1.29)$$

These relations will be used in Eq. (1.25).

(Case of symmetry with respect to wire 2) If wire 1 and wire 3 are symmetrical with respect to the ground as well as to wire 2, one has

$$\begin{aligned} d_3 &= 1 \\ k_{12} &= k_{23} \end{aligned} \quad (1.30)$$

so that

$$\begin{aligned} [W] &= W_{11} \begin{bmatrix} 1 & k_{12} & k_{13} \\ k_{12} & d_2 & k_{12} \\ k_{13} & k_{12} & 1 \end{bmatrix} \\ [W]^{-1} &= \frac{1}{W_{11}\Delta} \begin{bmatrix} d_2 - k_{12}^2 & -k_{12}(1-k_{13}) & k_{12}^2 - d_2 k_{13} \\ -k_{12}(1-k_{13}) & 1 - k_{13}^2 & -k_{12}(1-k_{13}) \\ k_{12}^2 - d_2 k_{13} & -k_{12}(1-k_{13}) & d_2 - k_{12}^2 \end{bmatrix} \\ \Delta &= (1-k_{13}) \{ d_2(1+k_{13}) - 2k_{13}^2 \} \end{aligned} \quad (1.31)$$

Take in particular a value of d_2 :

$$d_2 = (1 + k_{13}) / 2 \quad (1.32)$$

then one has

$$\begin{aligned} [W] &= W_{11} \begin{bmatrix} 1 & k_{12} & k_{13} \\ k_{12} & (1+k_{13})/2 & k_{12} \\ k_{13} & k_{12} & 1 \end{bmatrix} \\ [W]^{-1} &= \frac{1}{W_{11}\Delta'} \begin{bmatrix} \frac{1+k_{13}}{2} - k_{12}^2 & -k_{12}(1-k_{13}) & k_{12}^2 - k_{13} \frac{1+k_{13}}{2} \\ -k_{12}(1-k_{13}) & 1 - k_{13}^2 & -k_{12}(1-k_{13}) \\ k_{12}^2 - k_{13} \frac{1+k_{13}}{2} & -k_{12}(1-k_{13}) & \frac{1+k_{13}}{2} - k_{12}^2 \end{bmatrix} \\ \Delta' &= (1-k_{13}) \left\{ \frac{(1+k_{13})^2}{2} - 2k_{12}^2 \right\} \end{aligned} \quad (1.33)$$

These relations will be useful in reducing the calculation to that of 2-wire lines, as described later.

For a larger number of wires, one can similarly make calculations by the use of the equations (1.13) of transmission, but the examples will be omitted because of no direct necessity.

CHAPTER 2. Simple networks made of coupled two-wire lines.

Here will be mentioned those networks consisting of a set of a coupled two-wire line and simple impedances, as simplest examples of coupled-line filters, along with their equivalent networks and the relations with coaxial filters.

2.1. Simple networks made of coupled two-wire lines and their four terminal constants.

A 2-wire line over a ground has, so to say 4 pairs of terminals, if one takes any terminal and the ground to make up a pair of terminals. Taking any two pairs of terminals as input and output terminals and treating the remaining pairs of terminals in a certain way, one has a four-terminal network. Networks of various properties will come out depending on the treatment of the terminals, but here only the cases will be considered where the terminals are open circuited, connected to the ground directly or through simple impedances. They are shown in Tables 2.1 ~ 2.5. L's and C's in the tables show impedances $jW \tan \beta l$ and $-jW \cot \beta l$ respectively. No citations are made to those networks having complicated impedances or parallel impedances at the input or output terminals.

The four-terminal constants of these networks may be obtained by substituting end conditions into the fundamental equations of transmission of the two-wire line, described in section 1.3. For the sake of simplicity, the line is assumed to be symmetrical, and Eq. (1.20) has the following form

$$\begin{aligned}
 V_1 &= V_{10} \cos \beta l + W_0 (I_{10} + k I_{20}) j \sin \beta l \\
 V_2 &= V_{20} \cos \beta l + W_0 (k I_{10} + I_{20}) j \sin \beta l \\
 I_1 &= I_{10} \cos \beta l + \frac{1}{W_0 (1-k^2)} (V_{10} - k V_{20}) j \sin \beta l \\
 I_2 &= I_{20} \cos \beta l + \frac{1}{W_0 (1-k^2)} (-k V_{10} + V_{20}) j \sin \beta l
 \end{aligned} \tag{2.1}$$

The notations A*P, A*A, L*P, H*P, B*E and B*P in the tables signify the types of the networks of all-pass, all-attenuating, low-pass, high-pass, band-stop and band-pass.

Those networks marked with ✕ and ✕ ✕ have already been reported^{1 2 13} by other authors, but they are cited because they are necessary to obtain equivalent networks in the following section.

W_U and W_B in tables 2.4 and 2.5 are given by Eq. (1.21), and since the line is assumed to be symmetrical, they are in the relation

$$W_B = 2W_0 (1 - k), \quad W_U = \frac{1}{2} W_0 (1 + k) \quad (2.2)$$

2.2. Equivalent circuits

Equivalent circuits are shown in Tables 2.6~2.10, which are obtained from the four-terminal constants described in the preceding section. In the tables, the left columns show coupled-line networks, the middle columns the equivalent circuits given in lumped constants, and the right columns the equivalent networks made of coaxial lines (not always of convenient forms for realization).

The frequency parameter p' in the equivalent lumped networks stands for

$$p' = j \tan (\beta l/2)$$

and is related to p by

$$p = \frac{2p'}{1+p'^2}$$

This parameter comes in when cascade coaxial lines (unit coax) are represented by lumped constants, and may seem to be unfavorable; but it will not appear in the overall network characters.

One may have a rough aspect of the properties of the networks through their equivalent circuits. Their characteristics will be studied in the following sections. Since those networks marked with asterisks have already been reported in detail, only others will be studied.

2.3. Low-pass type networks

The networks (b) and (f) in Table 2.6 are of the same property, and are low-pass filters. The equivalent network is like Fig. 2.2, through which the characteristics will be studied. This is one of the simplest networks among coaxial filters, and is reported in many articles, so that only the network constants will be referred to here.

Image impedances (Z_{o1} , Z_{o2}) and the transfer constant θ_o are given by

$$Z_{o1} = W_1 \sqrt{\frac{\omega_1^2}{1 + \omega_1^2}} \frac{1}{\sqrt{1 - \Omega^2}}$$

$$Z_{o2} = W_1 \sqrt{\frac{\omega_1^2}{1 + \omega_1^2}} \sqrt{1 - \Omega^2}$$

$$\tanh \theta_o = \sqrt{1 + \omega_1^2} \frac{j\Omega}{\sqrt{1 - \Omega^2}}$$

where

$$\omega_1^2 = W_2/W_1, \quad -\Omega^2 = p^2/\omega_1^2$$

In the networks (b) and (f) used in table 2.6, those values should be used that are shown in the following table. These networks have

$$Z_{o1} Z_{o2} = W_1^2 \frac{\omega_1^2}{1 + \omega_1^2} = R^2$$

and are "constant-k" type filters. ω_1 is the cutoff, and there are no attenuation poles. The characteristics are shown in Fig. 2.3, with respect to Ω .

It can be easily seen from the equivalent circuits that the networks (d) in table 2.6 and (a) in table 2.9 have also the same character. These are of two-element structures. On the other hand, the networks (b) and (f) in Table 2.6 are of 3-element structures and therefore the formers are degenerate filters each with one superfluous element. As for low-pass filters, therefore, the latter two networks are more significant than the formers.

2.4. Band-stop type networks

In contrast to the networks that are insignificant because of degeneracy described in the foregoing section, those networks of (c) and (e) in Table 2.6 act as 3-element networks, and have band-stop characteristics. They are significant since they can have narrow band characteristics, by making the coupling coefficient smaller. Let the image parameters be given by

$$Z_{o1} = \frac{\omega_{-1}}{\omega_o} \frac{p^2 + \omega_o^2}{\sqrt{(p^2 + \omega_{-1}^2)(p^2 + \omega_1^2)}}$$

$$Z_{o2} = \frac{\omega_o}{\omega_{-1}} \sqrt{\frac{p^2 + \omega_{-1}^2}{p^2 + \omega_1^2}}$$

$$\tanh \theta_o = \frac{\omega_{-1}}{\omega_o} p \sqrt{\frac{p^2 + \omega_1^2}{p^2 + \omega_{-1}^2}}$$

then their relations to line constants are shown in the following table.

In the two filters above, if one makes k smaller, one will have $\omega_{-1} \approx \omega_o \approx \omega_1$, and can make the stop band narrower. It is easy in practice to make the coupling coefficient smaller, because it is only needed to increase the separation of the wires, and a narrow band-stop filter will result easily. Details of experiments will be described in Chapter 10, section 1.

The network (b) in Table 2.9 is also of band-stop type. Its equations of characteristics are:

$$Z_{o1} = \sqrt{W_o W \frac{1+k}{1-k} \frac{\omega_2^2}{\omega_1^2 \omega_3^2} \frac{(p^2 + \omega_1^2)(p^2 + \omega_3^2)}{(p^2 + \omega_2^2)(p^2 + \omega_4^2)}}$$

$$Z_{o2} = \sqrt{W_o W \frac{1+k}{1-k} \frac{\omega_1^2}{\omega_2^2 \omega_3^2} \frac{(p^2 + \omega_2^2)(p^2 + \omega_3^2)}{(p^2 + \omega_1^2)(p^2 + \omega_4^2)}}$$

$$\tanh \theta_o = \sqrt{\frac{W_o(1-k)}{W(1+k)} \frac{\omega_1^2 \omega_2^2}{\omega_3^2} \frac{(p^2 + \omega_3^2)(p^2 + \omega_4^2)}{(p^2 + \omega_1^2)(p^2 + \omega_2^2)}} p^2$$

$$\omega_1^2 = \frac{1}{(1-k) \left\{ \frac{1}{1+k} + \frac{2W_o}{W} \right\}}, \quad \omega_2^2 = \frac{W}{W_o}$$

$$\omega_3^2 = \frac{W}{W_o(1-k)}, \quad \omega_4^2 = \frac{1}{1-k} \left\{ (1+k) + \frac{2W}{W_o} \right\}$$

This is a network of BE-4.

If, in particular,

$$\omega_1 = \omega_2 \text{ or } \omega_3 = \omega_4$$

it will be of BE-2. The condition thereof is to determine W such that satisfies

$$\frac{W}{W_0} = \frac{1+k}{1-k} (2k-1) \text{ or } \frac{1+k}{2k-1}$$

The characteristic is alike to that of the coaxial network shown in Fig. 10.2. It is not suited for narrow band requirements.

2.5. Derived-M type high-pass filters

The networks (c) and (e), Table 2.7, have forms of inverted-L type high-pass filters with single phase lines attached, as seen from the equivalent circuits. One may consider that they have the characteristics of HP-3, but under certain conditions they become HP-1. The conditions for HP-1 and the characteristics are shown in Table 2.13. Fig. 2.5 is an example of the frequency characteristics of the resulting network.

The networks (b) and (f), Table 2.8, are combinations of derived-M type high-pass filters and transformers. Their equivalent circuits and expressions of the characteristics are shown in Table 2.14. These networks are of special interest.

2.6. Band-pass filters

The networks (b) and (f), Table 2.7, have forms consisting of all-attenuation networks cascaded with all-pass single phase lines, as seen from the equivalent circuits. The expressions for their characteristics are given in the table below. The cutoff frequencies ω_1 and ω_2 will approach to each other as the coupler coefficient k tends to zero, and a narrow band will result. There are no attenuation poles. The circuits at the bottom of the table 2.15 are also equivalent to those at the top.

The networks (c) and (e), Table 2.8, have also band-pass characteristics, and their equivalent circuits can be given as a combination of an L-type network and a transformer. The constants are as given in the following table (Table 2.16). As in the preceding case, ω_1 will tend to ω_2 as k tends to zero, and a narrow band will come out. One will obtain the simplest 3-element band-pass filters, as shown in the bottom part of the table, if the condition $W = W_0 k$ (in the network (c), table 2.8) or $Wk = W_0 (1-k^2)$ (in the network (e)) is introduced in particular. Fig. 2.6 shows an example of the band-pass characteristics obtained. The network (c), Table 2.9 is also a BP-2, and its characteristics are given in the following equations

$$Z_{o1} = \sqrt{W_o W \frac{W_o + W(1-k^2)}{W_o + W} \frac{1-k}{1+k} \omega_1^2 \frac{p^2 + \omega_2^2}{p^2 + \omega_1^2} p^2}$$

$$Z_{o2} = \sqrt{W_o W \left(1 + \frac{W_o}{W}\right) \left\{1 + \frac{W_o(1-k^2)}{W}\right\} \frac{1+k}{1-k} \omega_1^2 \frac{p^2}{(p^2 + \omega_1^2)(p^2 + \omega_2^2)}}$$

$$\tanh \theta_o = \sqrt{\frac{W_o(1+k)}{W(1-k)} \frac{W_o + W(1-k^2)}{W+W_o} \frac{1}{\omega_1^2} \frac{p^2 + \omega_1^2}{p^2 + \omega_2^2}}$$

$$\omega_1^2 = \frac{W_o(1+k)}{2W + W_o(1-k)}$$

$$\omega_2^2 = \frac{1+k}{1-k} \left\{1 + 2 \frac{W_o(1-k)}{W}\right\}$$

This network, like coaxial networks, is not suited to meet narrow band requirements.

2.7. Equivalent circuits for those networks with unsymmetrical 2-wire lines.

In preceeding sections, those networks were examined, that are built up with a symmetrical 2-wire line and one more element. A 2-wire line can also be unsymmetrical, and the equivalent circuits will be obtained for the networks with a 2-wire line and an element. Ozaki and Ishii¹⁹ obtained some special cases of these networks, with expressions of odd or even transfer impedances, but here the expressions will be given in coupling coefficients and symmetry coefficients for more general cases, because of necessity in the following chapters.

Now, one can use the fundamental equations of transmission Eq. (1.18) in case of an unsymmetrical 2-wire line; using W_o , k , and d in place of W_{11} , k_{12} and d_2 , the equations read

$$V_1 = V_{10} \cos \beta l + W_o (I_{10} + k I_{20}) j \sin \beta l$$

$$V_2 = V_{20} \cos \beta l + W_o (k I_{10} + d I_{20}) j \sin \beta l$$

$$I_1 = I_{10} \cos \beta l + \frac{1}{W_o(d-k^2)} (d V_{10} - k V_{20}) j \sin \beta l$$

$$I_2 = I_{20} \cos \beta l + \frac{1}{W_o(d-k^2)} (-k V_{10} + V_{20}) j \sin \beta l$$

One has only to put conditions of termination into these equations, determine 4-terminal constants, and find out equivalent circuits. The equivalent circuits obtained are shown in Tables 2.17 ~ 2.19, along with network constants. Those equivalent networks in Tables 2.6 ~ 2.9 are nothing but those in Tables 2.17~2.19 with the condition, $d = 1$, of symmetry of the line.

2.8. Transformations from coaxial networks into coupled line networks.

Coaxial networks are of the most significance among distributed networks, and a good deal of study has been reported on the coaxial networks. They are practical because of the simplicity of the construction of the lines. On the other hand, certain inherent restrictions do not allow one to obtain networks of versatile characteristics. Some trials were made by Ozaki and Ishii, to adopt a coupled line element, in order to make supplements to the defects of coaxial networks. Their report is excellent.²⁰ Some of their networks are shown in Table 2.20, in terms of W_0 , d and k , to meet with this paper. The boxes in the left column of the table represent unit coaxial lines. Those two networks at the lower half of the table have transformers, and if the transformation ratio be taken 1:1, one would have $k = 1$ or d , and consequently $\delta = 0$ or ∞ , and the coupled line should take a double coaxial configuration.

Ozaki and Ishii made no comments on the transformation of tree-and-branch networks with attenuation poles. One can have a transformation into coupled line networks, even in the presence of attenuation poles, through the use of two transformations described below. In the two networks in Table 2.21, W_0 and W'_0 are of like magnitude, but k^2 can be made as small as one wishes. Therefore there is a possibility of easier realization in the form of coupled lines, even if the impedances of the resonant lines are of extreme magnitudes in case of coaxial networks. These transformations are useful in making attenuation poles.

2.9. Brune Sections

Lastly, a unit loop will be considered. It consists of 4 coaxial lines in loop to realize a Brune section with a negative inductance. Its equivalent circuit is, as in the figure, a cascade of a Brune section and a unit coax U .

Take

$$Z = W_c/p$$

in the network (g), Table 2.18, then one has a network shown in table 2.22, which is the one under discussion. Taking off the ideal transformer, one will find that the equivalent network will be the same as that of the unit loop. Conditions of equivalence are also shown in the table.

Example of a network having Brune sections.

Take a numerical example given by Ikeno in his report, and here will be shown the transformation into coupled line configuration.

Ikeno gave an effective transfer coefficient S such that

$$S\bar{S} = 1 + \frac{2000 p^4 (p^2 + 0.87)^2}{(p^2 + 2.25)^2 (p^2 + 16)^2}$$

The realized network has negative capacitances as in Fig. 2.10(a). Transforming these negative capacitances into series inductances by Kuroda's method¹⁷, one will have Brune sections. By further transformation, unit loops will come out and the final network takes the form Fig. 2.10(c).

Brune sections in Fig. 2.10(b) may be substituted by coupled two-wire lines, using the relations in Table 2.22, resulting a network Fig. 2.11.

The transmission characteristics have been reproduced from the original in the reference (7).

The tree-and-branch type coaxial networks can be transformed into coupled-line type ones by means of combinations of the transformations Table 2.20, 2.21 and 2.22. As regards to the construction itself, the coupled-line type ones may be more complicated than the others, but may sometimes be made up with elements of easier realization.

As stated above, it has been shown in this chapter how the equivalent networks can be obtained, and how the action of line elements, especially the coupling, take part in the characteristics of networks. Novel networks have been studied of their properties in detail, some of which may be of interest. The equivalent networks are represented in coaxial networks, so that one can also transform various coaxial filters into coupled-line type filters, if he has transformations from coaxial into coupled-line networks.

CHAPTER 3. Simple symmetrical networks made from three-wire lines.

Take a 3-wire line, which has one more wire than a 2-wire line, a different network will be obtained as compared with that having a 2-wire line. Only the case of symmetrical networks will be studied, for the sake of simplicity, because the structure of a 3-wire line is complicated.

3.1. Equations of a coupled 3-wire line

As described in chapter 1, the equations of transmission of a coupled 3-wire line are given in Eq. (1.13):

$$\begin{cases} [V] = [V_0] \cos \beta l + [W][I_0] j \sin \beta l \\ [I] = [I_0] \cos \beta l + [W]^{-1}[V_0] j \sin \beta l \end{cases} \quad (3.1)$$

where $[W]$ is the characteristic impedance of the 3-wire line. Only the case of symmetry is under consideration, so that one may assume a structure in which wire 1 and wire 3 are in symmetry with the ground with wire 2. Thus Eq. (1.32) comes into use:

$$\begin{aligned} [W] &= W_{11} \begin{bmatrix} 1 & k_1 & k_2 \\ k_1 & \frac{1+k_2}{2} & k_1 \\ k_2 & k_1 & 1 \end{bmatrix} \\ [W]^{-1} &= \frac{1}{W_{11}\Delta} \begin{bmatrix} \frac{1+k_2}{2} - k_1^2 & -k_1(1-k_2) & k_1^2 - k_2 \frac{1+k_2}{2} \\ -k_1(1-k_2) & 1 - k_2^2 & -k_1(1-k_2) \\ k_1^2 - k_2 \frac{1+k_2}{2} & -k_1(1-k_2) & \frac{1+k_2}{2} - k_1^2 \end{bmatrix} \\ \Delta &= (1 - k_2) \left\{ \frac{(1+k_2)^2}{2} - 2k_1^2 \right\} \end{aligned} \quad (3.2)$$

Equations of the network will be obtained from the above equations by putting terminal conditions of the line ends into the latter.

3.2. Lattice representation of symmetrical networks.

In treating a symmetrical network, it is more profitable to study the matter on the equivalent symmetrical lattice network.

The fundamental equations of a symmetrical 4 terminal network (A, B, C, A), are

$$\begin{aligned} V_1' &= A V_2' + B I_2' \\ I_1' &= C V_2' + A I_2' \end{aligned} \quad (3.3)$$

which may be rewritten

$$\frac{V_1' + V_2'}{I_1' - I_2'} = \frac{1}{C} (A + 1), \quad \frac{V_1' - V_2'}{I_1' + I_2'} = \frac{1}{C} (A - 1) \quad (3.4)$$

These impedances give just the elements of the symmetrical lattice network. In the reverse way, if the voltages and currents of the lattice network is known, one can obtain the elements (Z_a and Z_b) of the equivalent symmetrical lattice network; that is:

$$Z_a = \frac{1}{C} (A + 1) = \frac{V_1' + V_2'}{I_1' - I_2'}, \quad Z_b = \frac{1}{C} (A - 1) = \frac{V_1' - V_2'}{I_1' + I_2'} \quad (3.5)$$

3.3. Z_a and Z_b of symmetrical networks made from 3-wire lines.

Make a symmetrical network, with input and output terminals on wires 1 and 3 on the same end of the line as shown in the figure, then one can obtain Z_a and Z_b , elements of the equivalent lattice circuit, in the way described in the foregoing section.

Substitute voltages and currents of Eq. (3.1) into Eq. (3.5) (pay attention to the directions of currents), one will obtain

$$\begin{aligned} Z_a &= \frac{V_1 - V_3}{I_1 - I_3} \\ &= \frac{W_{11}(1 - k_2)p + (V_{10} - V_{30}) / (I_{10} - I_{30})}{1 + \frac{p}{W_{11}(1 - k_2)} \frac{V_{10} - V_{30}}{I_{10} - I_{30}}} \end{aligned} \quad (3.6a)$$

In this expression, $(V_{10} - V_{30}) / (I_{10} - I_{30})$ is determined by the treatment of the line end. Assume that the treatment is symmetrical with respect to the ground as well as to wire 2, and write $(V_{10} - V_{30}) / (I_{10} - I_{30}) = Z_{ao}$, then one has

$$Z_a = \frac{W_{11}(1 - k_2)p + Z_{ao}}{1 + \frac{Z_{ao} \cdot p}{W_{11}(1 - k_2)}} \quad (3.6b)$$

which is equal to the input impedance of a line with a characteristic impedance $W_{11}(1 - k_2)$ whose far end is terminated in Z_{ao} .

Now, an expression for Z_b will be obtained. Eq. (3.1) may be written as

$$\begin{aligned}
\frac{V_1 + V_3}{2} &= \frac{V_{10} + V_{30}}{2} \cos \beta \ell + W_{11} \left\{ \frac{1+k_2}{2} (I_{10} + I_{30}) + k_1 I_{20} \right\} j \sin \beta \ell \\
V_2 &= V_{20} \cos \beta \ell + W_{11} \left\{ k_1 (I_{10} + I_{30}) + \frac{1+k_2}{2} I_{20} \right\} j \sin \beta \ell \\
I_1 + I_3 &= (I_{10} + I_{30}) \cos \beta \ell + \frac{1}{W_{11} \Delta} \left\{ (1-k_2^2) \frac{V_{10} + V_{30}}{2} + 2k_1 (1-k_2) V_{20} \right\} j \sin \beta \ell \\
I_2 &= I_{20} \cos \beta \ell + \frac{1}{W_{11} \Delta} \left\{ -k_1 (1-k_2) \frac{V_{10} + V_{30}}{2} + (1-k_2^2) V_{20} \right\} j \sin \beta \ell \\
\Delta &= (1-k_2) \left\{ \frac{(1+k_2)^2}{2} - 2k_1^2 \right\}
\end{aligned} \tag{3.7}$$

With the substitutions

$$\begin{aligned}
\frac{V_1 + V_3}{2} &= V_b, \quad \frac{V_{10} + V_{30}}{2} = V_{bo} \\
I_1 + I_3 &= I_b, \quad I_{10} + I_{30} = I_{bo} \\
W_{11} \frac{1+k_2}{2} &= W_o, \quad \frac{2k_1}{1+k_2} = k
\end{aligned} \tag{3.8}$$

Eq. (3.7) may be rewritten:

$$\begin{aligned}
V_b &= V_{bo} \cos \beta \ell + W_o (I_{bo} + k I_{2o}) j \sin \beta \ell \\
V_2 &= V_{2o} \cos \beta \ell + W_o (k I_{bo} + I_{2o}) j \sin \beta \ell \\
I_b &= I_{bo} \cos \beta \ell + \frac{1}{W_o (1-k^2)} (V_{bo} - k V_{2o}) j \sin \beta \ell \\
I_2 &= I_{2o} \cos \beta \ell + \frac{1}{W_o (1-k^2)} (-k V_{bo} + V_{2o}) j \sin \beta \ell
\end{aligned} \tag{3.9}$$

This is identical with the equations of transmission of a symmetrical line having a self characteristic impedance W_o and a coupling factor k . Therefore one can apply the same technique of calculations to Z_b as that for networks with coupled 2-wire lines, described in the preceding

chapter. Here, Z_b is obtained as:

$$Z_b = \frac{V_1 + V_3}{I_1 + I_3} = 2 \frac{V_b}{I_b}$$

TABLES 3.1 and 3.2 show Z_a and Z_b of symmetrical networks consisting mainly of a 3-wire line, combined with another element. These values were obtained as described above.

3.4. 3-element networks

Various networks will be obtained by choosing Z in the networks shown in TABLES 3.1 and 3.2. Here the simplest cases will be considered, Z means an opencircuit or a ground-connection. One may naturally expect 3-element networks, but some may be degenerate. If the both ends of wire 2 are open-circuited or ground-connected at the same time, the wire becomes degenerate; such ones are excluded. The following TABLE shows networks thus formed.

Networks (4) and (5) of this TABLE are very alike to networks (b) and (c) of TABLE 2.10, and will be identical if one takes $k = 0$ ($k_1 = 0$). Networks (6), (7) and (8) are degenerate, being 2-element networks, and do not have advantages of a 3-wire line. The network (2) will be examined in the next paragraph, and the networks (1) and (3) in the next to next.

3.5. Low-pass filters

The network (2) in TABLE 3.3 is, as can be seen from the equivalent circuit, a low-pass one, with a constant $-k$ image impedance and a derived $-m$ attenuation. At this point, it has the same property as the network (b) in TABLE 2.10. The symmetry coefficient, in TABLE 3.3, takes a particular value of Eq. (1.31). Let it be d_{12} , in order to keep generality, then the equivalent network should be corrected as in Fig. 3.6, and its image parameters are obtained as follows:

$$\begin{aligned} Z_o = \sqrt{Z_a Z_b} &= W_1 \sqrt{\frac{1 - k_2}{d_{12}}} \left\{ d_{12} (1 + k_2) - 2k_1^2 \right\} \sqrt{1 + \frac{2k_1^2}{d_{12} (1 + k_2) - 2k_1^2} p^2} \\ &= R \sqrt{1 + \frac{p^2}{2}} \\ \tanh \frac{\theta_o}{2} &= \sqrt{\frac{Z_a}{Z_b}} = \sqrt{\frac{d_{12} (1 - k_2)}{2k_1^2}} \sqrt{\frac{2k_1^2 p^2 / d_{12} (1 + k_2) - 2k_1^2}{1 + 2k_1^2 p^2 / d_{12} (1 + k_2) - 2k_1^2}} = \sqrt{\frac{1}{m} \frac{p^2 / 2}{1 + p^2 / 2}} \end{aligned}$$

$$R^2 = W_{11}^2 \frac{1 - k_2^2}{d_{12}} \{ d_{12}(1 + k_2) - 2k_1^2 \}$$

$$\frac{2}{1} = \frac{d_{12}(1 + k_2) - 2k_1^2}{2k_1^2}$$

$$m^2 = \frac{2k_1^2}{d_{12}(1 - k_2)}$$

The advantage of this network is that w_1 can be made greater easily by making k_1 smaller.

3.6. High-pass filters (I)

The network (3) of TABLE 3.3 is one of high-pass type. Generalize the symmetry factor as d_{12} and the equivalent circuit goes into that shown in Fig. 3.7. Its image parameters will be obtained from the equivalent circuit as:

$$Z_o = W_{11} \sqrt{(1 - k_2^2) \frac{\Delta'}{2k_1^2} \frac{p^2/w_1^2}{1 + p^2/w_1^2}}$$

$$\tanh \frac{\theta_o}{2} = \sqrt{\frac{1 - k_2^2}{1 + k_2^2} \left(1 + \frac{p^2}{\omega_1^2}\right)}$$

$$\Delta' = d_{12}(1 + k_2) - 2k_1^2$$

$$\omega_1^2 = \Delta'/2k_1^2$$

The factor $(1 - k_2^2)/(k_1 + k_2)$ in the expression for $\tanh(\theta_o/2)$ is smaller than 1, so that there are no attenuation poles. If one transforms the equivalent circuit Fig. 3.7, it will go into a coaxial line with a shunt admittance on either end, which is similar to a symmetrical connection of two networks TABLE 2.7(g) in cascade; from this fact one can be sure that he has not attenuation poles. The network can have a greater ω_1 by making k_1 smaller, in contrast with the networks TABLE 3.5(5) or TABLE 2.10(e); this feature may be convenient to obtain a narrow band characteristic.

3.7. High-pass networks (II)

The network Table 3.3 (1) will have an equivalent circuit Fig. 3.8(b), with a general symmetry coefficient d_{12} . This network has a significance of having series capacitances, so that its characteristic is a high-pass one. The image parameters will be obtained from the equivalent circuit as follows:

$$Z_o = W_{11} \sqrt{\frac{(1-k_2)^2 k_1^2}{d_{12}}} \sqrt{\frac{1 + p^2/\omega_1^2}{p^2/\omega_1^2}}$$

$$\tanh \frac{\theta_o}{2} = \sqrt{\frac{d_{12} (1-k_2)}{\Delta'}} \sqrt{\frac{1}{1 + p^2/\omega_1^2}}$$

$$\Delta' = d_{12} (1 + k_2) - 2 k_1^2$$

$$\omega_1^2 = \Delta' / 2 k_1^2$$

If $d_{12} (1-k_2) / \Delta' < 1$ in the expression for $\tanh (\theta_o/2)$, there will be attenuation poles. This may be also understood from the fact that the equivalent network can be transformed into a series derived-m type one as shown in Fig. 3.8(c).

If in particular, there is a relation

$$d_{12} k_2 = k_1^2,$$

the network will become a constant-k type one, which is equivalent to the network (b) made of coupled 2-wire lines. Its image parameters are

$$Z_o = W_{11} \sqrt{2 k_2 (1 - k_2)} \sqrt{\frac{1 + p^2/\omega_1^2}{p^2/\omega_1^2}}$$

$$\tanh \frac{\theta_o}{2} = \frac{1}{\sqrt{1 + p^2/\omega_1^2}}$$

$$\omega_1^2 = (1 - 2k_2) / 2 k_2$$

Fig. 3.10 shows some examples of its frequency characteristics.

3.8. 4-element networks

In the networks Table 3.1 and 3.2, Z_a is of one element, and Z_b depends upon the choice of Z . If one takes Wp or W/p as Z , then Z_b can be of 3 element, and the whole network will be a lattice network of 4 controlling elements, so that one may obtain a filter of a band-stop type or a band-pass type.

One may choose Z in any way; those are desirable that give higher network grades, while those are of no use that give degenerate networks. Those networks in the following Tables are 4-element ones made with attention above.

3.9. Band-stop networks

The band-stop networks, Table 3.4, will be explained with an example deductible from the network, Table 3.1(h). If one takes a capacitance as Z in the network Table 3.1(h), then Z_b will be only a capacitance, and the whole network becomes an all-pass. For this reason, an inductance Wp will be used as Z . The network will then be a band-stop filter. Here, since

$$Z_a = W_{11}(1 - k_2)p, \quad Z_b = \frac{2W_o}{p} \frac{\frac{W_o(1 - k^2)}{W} + p^2}{\frac{W_o}{W} + p^2}$$

the image parameters are obtained as

$$\begin{aligned} Z_o &= \sqrt{Z_a Z_b} \\ &= \sqrt{2W_{11}(1 - k_2)W_o(1 - k^2) \frac{1 + p^2 W/W_o(1 - k^2)}{1 + p^2 W/W_o}} \\ \tanh(\theta_o/2) &= \sqrt{Z_a/Z_b} \\ &= \frac{p}{2} \sqrt{\frac{2W_{11}(1 - k_2)}{W_o(1 - k^2)} \frac{1 + p^2 W/W_o}{1 + p^2 W/W_o(1 - k^2)}} \end{aligned}$$

As one can see from the above expressions, it is Z_b that plays a major role directly in the characteristics. Cutoff frequencies ω_1 and ω_2 are given by the equations

$$\omega_1^2 = W_o/W, \quad \omega_2^2 = W_o(1 - k^2)/W$$

and the attenuation poles are such that satisfy

$$Z_a = Z_b$$

or

$$W_{11}(1 - k_2)p = \frac{2W_o}{p} \frac{p^2 + W_o(1 - k^2)/W}{p^2 + W_o/W}$$

The poles of attenuation correspond to the state of the balance of a bridge network. The networks from the top to the third in Table 3.4 are suitable for narrow bands but the other two are not.

3.10. Band-pass filters^{22,23}

Band-pass networks will be examined, with the network Table 3.1(d) as an example. Take W_p as Z , then Z_a and Z_b becomes:

$$Z_a = W_{11} (1 - k_2) \frac{1}{p}, \quad Z_b = \frac{2 W_o}{p} \frac{p^2 + W_o (1 - k^2)/W}{p^2 + W_o/W}$$

The only difference from the preceding paragraph is that Z_a is here capacitive. Image parameters are obtained, in like manners,

$$Z_o = \frac{R}{p} \sqrt{\frac{1 + p^2/\omega_1^2}{1 + p^2/\omega_2^2}}$$

$$\tanh \frac{\theta_o}{2} = \sqrt{M \frac{1 + p^2/\omega_2^2}{1 + p^2/\omega_1^2}}$$

$$\omega_1^2 = W_o (1 - k^2)/W$$

$$\omega_2^2 = W_o/W$$

$$R^2 = W_{11}^2 (1 - k_2^2) \left\{ 1 - \left(\frac{2 k_1}{1 + k_2} \right)^2 \right\}$$

$$M^2 = \frac{1 - k_2}{(1 + k_2) (1 - k^2)}$$

The width of the pass band is

$$\omega_2^2 - \omega_1^2 = W_o k^2/W$$

and is dependent on the coupling coefficient k ($= 2 k_1 / (1 + k_2)$), so that it may be made easily small by making k_1 small. Attenuation poles will be produced at such frequencies

$$p_{\infty}^2 = \frac{1 - M^2}{\frac{M^2}{\omega_2^2} - \frac{1}{\omega_1^2}} = - \frac{W_o (1 - k^2)}{W} \frac{1 - k_2 - (1 + k_2) (1 - k^2)}{(1 - k_2) - (1 + k_2) (1 - k^2)^2}$$

that satisfy $\tanh(\theta_0/2) = 1$. That is, one has attenuation poles if $M^2 < 1$ or if $M^2 > \omega_2^2/\omega_1^2$

Those 4-element networks from Table 3.1(a) and (b) have almost the same properties. Their parameters are shown in Table 3.5.

Networks formed from Table 3.2(j), (k) and (l) are also band-pass filters, suited for narrow bands. They are in an antimetric relation to those of Table 3.5, as shown in Table 3.6.

Networks to be formed from Table 3.2(m), (n), (o) and (p) are band-pass ones but they are difficult to have narrow bands. One may adopt them as wide band ones. For example, the band width of the network from (m) is

$$\omega_2^2 - \omega_1^2 = \frac{W_0(1+k)}{2W(1-k)} \left(1 + k + 2 \frac{W}{W_0}\right)$$

and cannot be made zero. On the contrary, let $k \rightarrow 1$, then the network tends to a low-pass one. Table 3.7 shows their network parameters.

CHAPTER 4. Ladder-type networks

In previous chapters, properties of simple networks consisting mainly of coupled two-wire lines or coupled three-wire lines have been treated on the basis of their equivalent circuits. The properties of coupled line networks have thus been revealed to some extent, and at the same time, there are included many networks that will be of practical use in their original forms. Some of them are examined in Chapter 10 in more detail, accompanied by experimental results.

One might consider about increasing the numbers of wires in coupled lines to obtain networks of higher grades. But the structures of the lines will be complicated rendering the manufacturing very difficult. Moreover, it is also a difficult problem to investigate the relations between the line constants and the dimensions of line structures. Narrow band networks, one of the advantageous types of coupled line networks, need high Q elements, while it may be difficult to obtain high Q elements with lines of many wires. From these reasons, lines of too many wires will not be used except for very special aims. In this paper, therefore, lines of four or more wires will not be considered.

To have networks of higher degrees, combinations of networks from 2-wire lines and 3-wire lines will be considered. The manners of combination of networks may be duplexing or cascading, just as in lumped network techniques. This chapter and the following give design procedures to obtain coupled line networks, transformed from networks designed in lumped parameters, through the use of equivalence

relations described in previous chapters.

Chapters 6 - 9 describe network synthesis by the extraction of coupled 2-wire lines, corresponding to the synthesis of coaxial networks by the extraction of unit coaxials.

This chapter describes ladder networks, which are the most basic in the lumped domain.

4.1. L-type networks²⁵

The basic network of a ladder structure is an L-type network. In the L-type network, Fig. 4.1, the realization of the series element is of the severest problem. One can use a double coaxial structure to realize a series element. Otherwise, one can realize Z_1 accompanied by Z_2 . If Z_2 is capacitive, the degree of Z_1 must be smaller than or equal to that of Z_2 .

In coaxial filters, endeavors have been made in design procedures to avoid the use of double coaxial structures. In coupled line filters, double coaxial lines may be considered as a special case ($k = 1$ or $k = d$)* of a coupled 2-wire line. Table 4.1 shows examples of simple L-type network, in which the parts $d=k$ are double coaxial.

If one dislikes a double coaxial structure, as in the situation of coaxial filters, one should consider ladder networks with T-type structure as a basis, as described in the following paragraphs.

4.2. Low-pass ladder networks

If one chooses Wc/p as Z in Table 2.19, then he will have a fundamental low-pass network. Its equivalent network is T-type, with a low-pass characteristic. The shunt arm at the middle is generally a resonant one, but it can also be made capacitive by a proper choice of conditions; it is very flexible as a basic section.

*The specific feature of an L-type network is in $Y_{11}(p) = Y_{12}(p)$ because $Y_{11}(p) = 1/Z_1$, $Y_{22}(p) = (1/Z_1) + (1/Z_2)$, $Y_{12}(p) = 1/Z_1 = Y_{11}(p)$. The line constants of the line to be extracted are $d = k = Y_{11}(1)/Y_{22}(1) = Y_{12}(1)/Y_{22}(1)$, according to Chapter 6. The remaining network has

$$Y_{11}'(p) = Y_{12}'(p) = Y_{11}(1) \left\{ Y_{11}(p) - p Y_{11}(1) \right\} / \left\{ Y_{11}(1) - p Y_{11}(p) \right\}$$

which again is an L-type network. But the relation, $d = k$, which comes from $Y_{11} = Y_{12}$, leads to $d = \infty$, according to Eq.(1.21); this means the coaxial structure. In other words, an L-type network needs a double coaxial structure, which is made up of a double coaxial combination of bar-type networks of $Y_{11} = 1/Z_1$ and $Z_{22} = Z_2$.

First design a reference lumped low-pass network (with notice on $p = j \tan 2 \pi fl/c$), and then divide it into T -sections in an appropriate way, and next transform each section into a coupled line network with the use of equivalence given in Table 4.2.

[Example 1] LPF of Wagner character, $n = 6$.

An LPF of Wagner character, $n = 6$, is given in a ladder structure²⁶ as shown in Fig. 4.3(a). Divide it as in (b) and transform each section into a coupled line one, then the whole network will be shown in (c). (Refer to Fig. 9.5, $n = 6$, Chapter 9).

[Example 2] LPF of non-polar Tchebycheff character, $n = 5$.

The maximum loss in the pass band is specified to be 3 db. Then one has

$$\epsilon^{2a} = 1 + \frac{1}{2} + \frac{1}{2} \cos (10 \cos^{-1} \omega)$$

and

$$A = 9.1749 p^4 + 8.7649 p^2 + 1$$

$$C = \frac{p}{r} (32 p^4 + 42.306 p^2 + 11.486)$$

The input impedance, for 1 ohm termination, is

$$\begin{aligned} Z_{in} &= \frac{A}{C} = \frac{9.1749 p^4 + 8.7649 p^2 + 1}{p (32 p^4 + 42.306 p^2 + 11.486)} \\ &= \frac{1}{3.4878 p + \frac{1}{0.7607 p + \frac{1}{4.4999 p + \frac{1}{0.7607 p + \frac{1}{3.4878 p}}}}} \end{aligned}$$

and the network is expressed in a ladder structure as in the figure. It may be transformed into a coupled line structure, as in the previous example. (As to the characteristics, refer to Fig. 7.5).

[Example 3] A ladder network of Tchebycheff character with attenuation poles.

A network with attenuation poles will be obtained if one cascades two series derived- m LPF, as shown in Fig. 4.5. (Reports are made on shunt derived- m networks in reference (28)). Design the image impedance of each section to match each other, and let M of derivation be m_1 and m_2 respectively, then the equivalent network will be a symmetrical lattice with Z_a and Z_b given by:

$$\left. \begin{aligned} Z_a &= \frac{R (m_1 + m_2) \frac{P}{\omega_1} \left(\frac{P^2}{\omega_1^2} + 1 \right)}{(1 + m_1 m_2) \frac{P}{\omega_1} + 1} \\ Z_b &= \frac{R \left\{ (1 + m_1 m_2) \frac{P^2}{\omega_1^2} + 1 \right\}}{(m_1 + m_2) \frac{P}{\omega_1}} \end{aligned} \right\} \quad (4.1)$$

where ω_1 is the cutoff frequency and R the nominal impedance.

Now, the effective attenuation α of a symmetrical network is

$$\alpha = 10 \log_{10}(1+E^2) \left\{ 1 - \frac{\left(1 - \frac{Z_a Z_b}{R_T}\right)^2}{\left(\frac{Z_b}{R_T} - \frac{Z_b}{R_T}\right)^2} \right\} \quad (4.2)$$

where R_T is the resistance of the generator and of the load. Using the values Eq. (4.1), one will have

$$E = \frac{(m_1 + m_2) (1 + m_1 m_2) \frac{P}{\omega_1} \left\{ \frac{P^2}{\omega_1^2} + \frac{1}{1 + m_1 m_2} \right\} \left\{ \frac{P^2}{\omega_1^2} + \left(1 - \frac{R_T^2}{R^2}\right) \right\}}{\frac{R_T}{R} (1 - m_1^2) (1 - m_2^2) \left\{ \frac{P^2}{\omega_1^2} + \frac{1}{1 - m_1^2} \right\} \left\{ \frac{P^2}{\omega_1^2} + \frac{1}{1 - m_2^2} \right\}} \quad (4.3)$$

Here a new frequency parameter P will be introduced:

$$P = \frac{P}{\omega_0} = \begin{cases} j \frac{1}{k} & \text{at the upper limit of guaranteed pass band} \\ j k & \text{at the lower limit of guaranteed pass band} \end{cases} \quad (4.4)$$

With this P , the expression of E becomes

$$E = \frac{(m_1 + m_2) (1 + m_1 m_2) P \left\{ P^2 + \frac{\omega_1^2 / \omega_o^2}{1 + m_1 m_2} \right\} \left\{ P^2 + \frac{\omega_1^2}{\omega_o^2} \left(1 - \frac{R_T^2}{R^2} \right) \right\}}{\frac{R_T}{R} \left(\frac{\omega_1}{\omega_o} \right)^5 \frac{(1 - m_1^2) (1 - m_2^2)}{(\omega_1^2 / \omega_o^2)^2} \left\{ P^2 + \frac{\omega_1^2 / \omega_o^2}{1 - m_1^2} \right\} \left\{ P^2 + \frac{\omega_1^2 / \omega_o^2}{1 - m_2^2} \right\}}$$

(4.5)

$$= H \frac{P (P^2 + a_2^2) (P^2 + a_4^2)}{a_2^2 a_4^2 (P^2 + \frac{1}{a_2^2}) (P^2 + \frac{1}{a_4^2})}$$

if one have the relations

$$a_2 = \frac{\omega_1}{\omega_o} \sqrt{1 - \frac{R_T^2}{R^2}}, \quad a_4 = \frac{\omega_1}{\omega_o} \frac{1}{\sqrt{1 + m_1 m_2}}$$

(4.6)

$$\frac{1}{a_2} = \frac{\omega_1}{\omega_o} \frac{1}{\sqrt{1 - m_2^2}}, \quad \frac{1}{a_4} = \frac{\omega_1}{\omega_o} \frac{1}{\sqrt{1 - m_1^2}}$$

$$H = \frac{(m_1 + m_2) (1 + m_1 m_2)}{\frac{R_T}{R} \left(\frac{\omega_1}{\omega_o} \right)^5}$$

(4.7)

A Tchebycheff characteristic will be obtained if a_2 and a_4 are given to be

$$a_2 = k \operatorname{sn} \left(\frac{2K}{5}, k^2 \right), \quad a_4 = k \operatorname{sn} \left(\frac{4K}{5}, k^2 \right)$$

(4.8)

Solve Eq. (4.6), one will obtain

$$\left(\frac{\omega_1^2}{\omega_o^2} \right)^2 = \frac{2a_4^2 - a_4^4 (a_2^2 + a_4^2)}{1 - a_2^2 a_4^6}, \quad m_1^2 = 1 - \left(\frac{\omega_1}{\omega_o} a_4 \right)^2$$

(4.9)

$$\left(\frac{R_T}{R} \right)^2 = 1 - (a_2 / \frac{\omega_1}{\omega_o})^2, \quad m_2^2 = 1 - \left(\frac{\omega_1}{\omega_o} a_2 \right)^2$$

Actual numerical values will be computed. Let the guaranteed pass band $0 - 0.8207$, then one has

$$k = 0.8207, \omega_0 = 1, a_2^2 = 0.2796, a_4^2 = 0.6251$$

which yield

$$\omega_1^2 = \frac{2 \times 0.6251 \times 0.6251^2 (0.2796 + 0.6251)}{1 - 0.2796 \times 0.6251^3} = 0.9624$$

$$m_1^2 = 1 - 0.6251 \times \omega_1^2 = 0.3996$$

$$m_2^2 = 1 - 0.2796 \times \omega_1^2 = 0.7315$$

$$\left(\frac{R}{R_T}\right)^2 = \frac{1}{1 - 0.2796/\omega_1^2} = 1.4095$$

The final values of the parameters are obtained to be $\omega_1 = 0.9810$, $m_1 = 0.6321$, $m_2 = 0.8552$, $R/R_T = 1.187$. The network will be like the figure, for the value $R_T = 1$. The network, obtained by the transformation Table 4.2, is also shown therein. Its attenuation characteristics is shown on the next page. (Fig. 4.7)

4.3. High-pass lattice networks

The network Table 4.3 will be examined, as a high-pass basic network. Its equivalent circuit will be given to be a T-type one, from equations obtained by putting end conditions into Eq. (1.25). As in the low-pass network, the arm in the middle is resonant, and can be an inductance according to conditions.

One should design a HPF in a ladder network, divide into appropriate T sections to be cascaded, and transform each section into coupled line structure.

[Example 1] HPF of Wagner character, $n = 6$.

A frequency transformation $p \rightarrow 1/p$, applied to the network Fig. 4.3 (b), will produce a HPF of Wagner character, $n = 6$, as shown in Fig. 4.8 (a). One have only to transform each section into a coupled line structure.

[Example 2] HPF of Tchebycheff character without poles.

The LPF in Fig. 4.4 will be transformed into a HPF of Tchebycheff character without poles, if a frequency transformation $p \rightarrow 1/p$ is applied. As in the previous examples, one have only to reform the network in coupled line structure, by the use of the relations given in Table 4.3. One can also realize it into a combination of two-wire lines without using a 3-wire line, if he makes use of the equivalence relations in Table 4.4.

[Example 3] HPF of Tchebycheff character with poles.

Apply a frequency transformation $p \rightarrow 1/p$ to the network Fig. 4.6, and utilize transformations Table 4.3, the network Fig. 4.10 will result.

4.4. Band-stop and band-pass filters.

Network parameters of BEF or BPF may be obtained by applying a frequency transformation

$$p = \frac{\Delta p}{p^2 + \omega_0^2}$$

Δ : bandwidth ($\omega_2 - \omega_1$)

ω_0 : center frequency ($\sqrt{\omega_1 \omega_2}$)

to those of LPF or HPF. The structures of line elements will be changed as shown in Table 4.5. For instance, one will obtain a configuration Fig. 4.11, if he looks for a BEF with an application of the frequency transformation to the LPF in Fig. 4.4. Obtain a BPF from the HPF in Fig. 4.9, a configuration Fig. 4.12 will come out.

The relations, among the elements of LPF or HPF before transformation and those of BEF or BPF after transformation, are complicated, and omitted here.

CHAPTER 5. Narrow band filters.

In the preceding chapter, there is described how to realize primitive and basic ladder networks in the form of coupled line type ones. The method adopts T-networks as basic, rather than L-networks, and enables one to avoid the use of double coaxial structures.

One may come to a difficulty that the values of the elements may not happen to lead to easy construction in an actual network design. It is very probable to come to the necessity of line elements with extremely large or small values, if the designer assumes an extremely high or low cutoff frequencies, or extremely narrow pass band; such elements will bring him to distress how to make. One can make shielded line elements with characteristic impedances only from 10 to 200 ohms, so that he has much more restrictions than in the design of lumped networks. Even in lumped networks special design techniques are used in narrow band filters. Similarly, in distributed networks one should have special design techniques in narrow band filters. In lumped networks, there is a technique of coupled resonant circuits, which suggests the use of coupled resonant lines of quarter or half wavelengths.

In this chapter, the theory starts from ladder networks and comes to a result with coupled resonant lines. From the coupling point of view, one will notice that the structure of a coupled line network is very flexible. This is an advantage of coupled line networks.

5.1. An example of a narrow band filter.

Here will be explained a BPF of Wagner character, $n = 3$, as an example of a narrow band filter. Fig. 5.1 shows its lumped representation; as one takes the bandwidth $\Delta = \omega_2 - \omega_1$ extremely small, the value of the series element will become very large while that of the shunt element very small. One must have a different procedure in mind, because the values of the line elements will be of extreme ones if the network is designed in the way described in the preceding chapter. One can obtain realizable values as described below.

First insert ideal transformers so that the factors Δ in the series and the shunt element will disappear. The insertion of such ideal transformers should not affect the transmission characteristics of the network. The network goes into that shown in Fig. 5.2, where Δ is related only to the transformation ratios of the ideal transformers.

One has to make up this network in a coupled line type; but since an ideal transformer can only be realized if it is accompanied by some elements on both sides, one may connect lines in cascade without changing the amplitude characteristic of the network. The amount of the phase shift of the network increases inevitably by the amount contributed by the coaxial elements added. The network is shown in Fig. 5.3.

Divide the network by the broken lines into five portions; each portion may be transformed into coupled line type ones through the equivalence relations given in 5 App. 2.3 and 2.2, and the whole network will take the form Fig. 5.4. This may also take the form Fig. 5.5 or 5.6 if one applies the equivalence relations of 5 App. 3.1 or 3.2. It will be noted that the center frequency ω_0 is taken 1 in the networks Fig. 5.4 and 5.6.

The above procedure may be summarized as follows:

- (1) Add 1 ohm coaxial lines to the input and output terminals.
- (2) Insert ideal transformers, so that Δ is related only to the ratios of transformation.
- (3) Transform each portion into coupled line type.

5.2. Narrow band filters with coaxial lines added.

Examples will be given on BPF, of Wagner characteristics, with coaxial lines added to the input and output terminals, in the same manner as described in the preceding paragraph.

[Case $n = 1$]

A network of Wagner characteristic, $n = 1$, of a series resonant type, shown in Fig. 5.7a, may be transformed into one shown in Fig. 5.7e, if one adds 1 ohm coaxial lines to both ends (b), insert ideal transformers (c), and apply transformations 5 App. 2.3 and 5 App. 1. It may also be transformed into one in Fig. 5.7f (30) or 5.7g, by the use of the relations given in 5 App. 2.1 or 2.5.

In the network Fig. 5.7b, one may invert the phase at the output terminal without changing its amplitude characteristics (Fig. 5.8a). Represent it in a lattice network (b), and one may also have the network⁽²²⁾ Fig. 5.8c, with a coupled 3-wire line, by the use of the relations given in 5 App. 3.3.

One may also start from a parallel resonant circuit Fig. 5.9a, instead of a series resonant one, Fig. 5.7a. First, add 1 ohm coaxial lines (b), insert ideal transformers (c), transform by the relations 5 App. 2.4 (d), retransform (e), and finally one will obtain the network (f) or (g)⁽²³⁾, by the relations in 5 App. 3.1 or 3.2.

[Case $n = 2$]

BPF of Wagner character, $n = 2$, has the form Fig. 5.10a. With this as a basic network, one will obtain the network (d) or (e) in the figure.

[Case $n = 3$]

If one starts from the network, $n = 3$, in Fig. 5.1, he will get to the networks Fig. 5.4, 5.6, as described previously. Start from the network Fig. 5.11a, then the networks (c) and (d) in the figure will be obtained.

As an alternative method, one may also have the network Fig. 5.12(f), if he moves the coaxial lines.

[Case $n = 4$]

One may obtain various network, depending on the manner of transformations. Some are shown in Fig. 5.13.

One can have networks in almost the same manner for greater values of n . For a value of n , there may be a variety of networks, from which one may choose those with fittest structures. No further comment will be made on this point.

[Examination of the expressions of the characteristics]

The change of the expressions, of the characteristics, will be examined, that will be brought about by adding coaxial lines. The square amplitude function (y^2) of the basic network, that contains no superfluous coaxial lines, is given

$$y^2 = |S(p)|^2 = 4 \left[1 + \delta^2 \{ \varphi(p) \}^2 \right]$$

where $\varphi(p)$ is the original function, $S(p)$ the inverse transmission function, and δ the constant of deviation. Also the input and output resistances are taken to be both 1 ohm, and the minimum attenuation to be 2. To increase the number of circuit elements without changing y^2 , one should multiply the numerator and the denominator, of the characteristic function $\{ \varphi(p) \}^2$, by the same factor. Multiply them with $(p^2 - 1)^2$, then

$$y^2 = 4 \left[1 + \delta^2 \{ \varphi(p) \}^2 \frac{(p^2 - 1)^2}{(p^2 - 1)^2} \right] \quad (5.2)$$

and the inverse transmission coefficient $S'(p)$ becomes

$$S'(p) = S(p) \frac{(p^2 + 1)^2}{p^2 - 1} \quad (5.3)$$

This function has a degree 2 higher than that of the original inverse transmission function $S(p)$, corresponding to 2 more circuit elements; y^2 itself is not changed, of course.

The calculation will be shown on an example of Wagner characteristic, $n = 3$, mentioned above. In Wagner case,

$$\{ \varphi(p) \}^2 = \left\{ - \left(\frac{p^2 + 1}{\Delta p} \right)^2 \right\}^n \quad (5.4)$$

and one has, taking $\delta = 1$,

$$y^2 = 4 \left[1 + \left\{ - \left(\frac{p^2 + 1}{\Delta p} \right)^2 \right\}^n \frac{(p^2 - 1)^2}{(p^2 - 1)^2} \right] \quad (5.5)$$

where Δ gives the bandwidth:

$$\Delta = \omega_2 - \omega_1 \sqrt{\omega_1 \omega_2} = \omega_0 = 1 \quad (5.6)$$

The center frequency ω_0 is here taken to be 1. The inverse transmission function is, for Wagner character $n = 3$,

$$S(p) = 2 \left\{ \left(\frac{p^2 + 1}{\Delta p} \right)^3 + 2 \left(\frac{p^2 + 1}{\Delta p} \right)^2 + 2 \left(\frac{p^2 + 1}{\Delta p} \right) + 1 \right\} \quad (5.7)$$

and the inverse transmission function $S'(p)$, specified by Eq. (5.3), is obtained, from Eq. (5.7),

$$S'(p) = 2 \left\{ \left(\frac{p^2 + 1}{\Delta p} \right)^3 + 2 \left(\frac{p^2 + 1}{\Delta p} \right)^2 + 2 \left(\frac{p^2 + 1}{\Delta p} \right) + 1 \right\} \frac{(p+1)^2}{(p^2-1)} \quad (5.8)$$

The function $\varphi(p)$ is:

$$\varphi(p) = 2 \left(\frac{p^2 + 1}{\Delta p} \right)^3 \quad (5.9)$$

Separate $S'(p)$ and $\varphi(p)$ into odd and even parts:

$$S'(p) = H_1 + p H_2 \quad (5.10)$$

$$\varphi(p) = G_1 + p G_2 \quad (5.11)$$

where H_1 and G_1 are even parts, and $p H_2$ and $p G_2$ are odd parts. Network parameters A , B , C , D can be obtained:

$$\begin{aligned} A = D &= \frac{1}{2} H_1 \\ &= \frac{\left\{ 2 \left(\frac{p^2 + 1}{\Delta p} \right)^2 + 1 \right\} (p^2 + 1) + \frac{p^2 + 1}{\Delta p} \left\{ \left(\frac{p^2 + 1}{\Delta p} \right)^2 + 2 \right\} 2p}{1 - p^2} \end{aligned} \quad (5.12a)$$

$$\begin{aligned} B &= \frac{p}{2} (H_2 + G_2) \\ &= \frac{\left\{ 2 \left(\frac{p^2 + 1}{\Delta p} \right)^2 + 1 \right\} 2p + 2 \left(\frac{p^2 + 1}{\Delta p} \right)^3 + 2 \frac{p^2 + 1}{\Delta p} (p^2 + 1)}{1 - p^2} \end{aligned} \quad (5.12b)$$

$$\text{or } = \frac{\left\{ 2 \left(\frac{p^2+1}{\Delta p} \right)^2 + 1 \right\} 2p + 2p^2 \left(\frac{p^2+1}{\Delta p} \right)^3 + 2 \frac{p^2+1}{\Delta p} (p^2+1)}{1-p^2} \quad (5.12c)$$

$$C = \frac{p}{2} (H_2 \pm G_2) \quad (5.12d)$$

These expressions will give the network Fig. 5.3 or Fig. 5.11b.

The same applies to other values of n .

5.3. Networks obtainable as cascade of lattice sections.

In preceding sections, examples were shown to make realization easier by adding coaxial lines to both terminals. Other manners may also be taken into consideration, in adding surplus elements. For instance, add surplus L and C to a network Fig. 5.14a of Wagner character $n = 1$, one will have a network (b)⁽³¹⁾. Notify the elements as shown in the figure, then the network parameters are obtained:

$$A = \left(1 + \frac{L_1}{L_2} \right) + p^2 L_1 C_2 \quad (5.13a)$$

$$B = \frac{L_1}{C_1} \frac{p^2 C_2 L_2 + 1}{p L_2} + \frac{p^2 C_1 L_1 + 1}{p C_1} \quad (5.13b)$$

$$C = \frac{p^2 C_2 L_2 + 1}{p L_2} \quad (5.13c)$$

$$D = \frac{p^2 L_2 (C_1 + C_2) + 1}{p^2 C_1 L_2} \quad (5.13d)$$

This network cannot be symmetrical. Assume it is antimetrical; then, since $B = C$, it is necessary that

$$L_1 + L_2 = C_1, \quad L_1 C_1 = L_2 C_2 \quad (5.14)$$

Also the square amplitude function y^2 becomes

$$\begin{aligned} y^2 &= 4 \left[1 + \frac{1}{4} (A - D)^2 \right] \\ &= 4 \left[1 + \left\{ \frac{L_1 L_2 C_1 C_2 p^4 - 1}{2 C_1 L_2 p^2} \right\}^2 \right] \\ &= 4 \left[1 + \left\{ \frac{(L_1 C_1 p^2 - 1)(L_1 C_1 p^2 + 1)}{2 C_1 L_2 p^2} \right\}^2 \right] \end{aligned} \quad (5.15)$$

If one assumes the condition

$$L_1 C_1 = L_2 C_2 = 1 \quad (= 1/\omega_o^2) \quad (5.16)$$

then he has

$$y^2 = 4 \left[1 + \left\{ \frac{(p^2 - 1)(p^2 + 1)}{2 C_1 L_2 p^2} \right\}^2 \right] \quad (5.17)$$

Let the frequencies, at which the effective attenuation becomes 3 db, be ω_1 and ω_2 . These are the frequencies where the magnitude of the characteristic function becomes unity. Thus, putting ω_1 and ω_2 into the above expression, one has

$$\frac{\omega_1^4 - 1}{2 C_1 L_2 \omega_1^2} = 1, \quad \frac{\omega_2^4 - 1}{2 C_1 L_2 \omega_2^2} = -1 \quad (5.18)$$

which yields

$$\begin{aligned} 2 C_1 L_2 &= \frac{1}{\omega_1^2} - \omega_1^2 = \omega_2^2 - \frac{1}{\omega_2^2} \\ &= \omega_2^2 - \omega_1^2 = (\omega_2 - \omega_1)(\omega_2 + \omega_1) \end{aligned} \quad (5.19a)$$

$$\omega_1^2 \omega_2^2 = 1 \quad (= \omega_o^4) \quad (5.19b)$$

Denote $\omega_2 - \omega_1 = \Delta$, $\omega_2 + \omega_1 = K$, then

$$y^2 = \left[1 + \left\{ \frac{(p^2 + 1)}{\Delta p} - \frac{(p^2 - 1)}{K p} \right\}^2 \right] \quad (5.20)$$

This is evidently different from the Wagner character already described, and also different from those of 3-element networks described later.

Next, one may determine the values of the network elements of Fig. 5.14 (-), so that the characteristic follows Eq. (5.20), by the use of conditions, Eqs. (5.14), (5.16) and (5.19). They are:

$$L_1 = \frac{1}{\sqrt{1 + \Delta K/2}} \quad (5.21a)$$

$$L_2 = \frac{\Delta K/2}{\sqrt{1 + \Delta K/2}} \quad (5.21b)$$

$$C_1 = \sqrt{1 + \Delta K/2} \quad (5.21c)$$

$$C_2 = \frac{\sqrt{1 + \Delta K/2}}{\Delta K/2} \quad (5.21d)$$

as shown in Fig. 5.15 (a). This network will, by transformations, go through (b) to (c), which is a cascade of a symmetrical lattice network of L only and another symmetrical lattice network of C only. This is the specific feature of this network, and it can be easily realized into a coupled line network by means of the transformations 5 App. 2.1 or 2.2. The result is shown in (d).

The above discussion started from a Wagner network of $n = 1$; the reasoning will apply if one starts from that of $n = 2$. Add two L's to a Wagner network Fig. 5.16(a) transform the resulting network (b) by appropriate transformations, and finally a network (d) will be obtained which is a cascade connection of symmetrical lattice networks. The same will happen, if one adds C's instead of L's, and the result will be such that the lattice of L in (d) should be replaced by that of C, and the lattice of C in that of L. One has no change in the procedure starting from Wagner networks of greater n , and the destiny is always a cascade of symmetrical lattice networks.

The characteristic functions of this kind of networks can be made to have the forms:

$$\text{case } n = 1 \quad \left\{ \frac{(p^2 + 1)}{\Delta p} \frac{(p^2 - 1)}{k p} \right\}^2 \quad (5.22a)$$

$$\text{case } n = 2 \quad - \left\{ \frac{(p^2 + 1)^2}{(\Delta p)^2} \frac{(p^2 - 1)}{k p} \right\}^2 \quad (5.22b)$$

$$\text{case } n = 3 \quad \left\{ \frac{(p^2 + 1)^2}{(\Delta p)^3} \frac{(p^2 - 1)}{k p} \right\}^2 \quad (5.22c)$$

- - - - -

$$\text{case } n = n \quad (-1)^{n-1} \left\{ \left(\frac{p^2 + 1}{\Delta p} \right)^n \frac{p^2 - 1}{k p} \right\}^2 \quad (5.22d)$$

One can obtain network parameters from these characteristic functions; here one has to factorize algebraic expressions of higher degrees. Instead, one can better determine element values, by choosing conditions that the amplitude characteristics may be represented by the equations (5.22), because the network configurations are already known.

[Add elements to a network $n = 2$]

If one starts from a network of Wagner characteristic $n = 2$, the network Fig. 5.17 will come out. Its elements can be chosen as shown in the figure, because it must be a symmetrical network. The network parameters are:

$$A = \frac{(L_2 + L_1)^2 (C_1 + C_2)}{(L_2 - L_1)^2 (C_1 - C_2)} \left[\frac{4/p^2}{(C_1 + C_2)(L_1 + L_2)} + 1 + \frac{4L_1L_2}{(L_1 + L_2)^2} + \frac{4L_1L_2C_1C_2p^2}{(L_1 + L_2)(C_1 + C_2)} \right] \quad (5.24a)$$

$$B = \frac{(L_2 + L_1)^2 (C_1 + C_2)}{(L_2 - L_1)^2 (C_1 - C_2)} \left[\frac{2/p}{C_1 + C_2} + \frac{4L_1L_2p}{L_1 + L_2} + \frac{8(L_1L_2)^2 C_1C_2p^3}{(L_1 + L_2)^2 (C_1 + C_2)} \right] \quad (5.24b)$$

$$C = \frac{(L_2 + L_1)^2 (C_1 + C_2)}{(L_2 - L_1)^2 (C_1 - C_2)} \left[\frac{8/p^3}{(L_1 + L_2)^2 (C_1 + C_2)} + \frac{4/p}{L_1 + L_2} + \frac{2C_1C_2}{C_1 + C_2} \right] \quad (5.24c)$$

$$D = A \quad (5.24d)$$

Since $A - D = 0$, the square amplitude function is related to $(B - C)$,

$$y^2 = 4 \left[1 - \frac{1}{4} (B - C)^2 \right] \quad (5.25)$$

Obtain B - C:

$$B - C = \frac{(L_2 + L_1)^2 (C_1 + C_2)}{(L_2 - L_1)^2 (C_1 - C_2)} \left[\frac{8 \left\{ (L_1 L_2)^2 C_1 C_2 p^3 - \frac{1}{p^3} \right\}}{(L_1 + L_2)^2 (C_1 + C_2)} + \frac{4 \left(p L_1 L_2 - \frac{1}{p} \right)}{L_1 + L_2} + \frac{2 \left(\frac{1}{p} - p C_1 C_2 \right)}{C_1 + C_2} \right] \quad (5.26)$$

The relation

$$L_1 L_2 = C_1 C_2 = 1 \quad (5.27)$$

is necessary, in order that B - C has a factor $p^2 - 1$. Put this relation into (5.26), then

$$\begin{aligned} B - C &= \frac{(L_1 + L_2)^2 (C_1 + C_2)}{(L_2 - L_1)^2 (C_1 - C_2)} \left\{ \frac{8 (p^4 + p^2 + 1)}{p^3 (L_1 + L_2)^2 (C_1 + C_2)} + \frac{4/p}{L_1 + L_2} - \frac{2/p}{C_1 + C_2} \right\} (p^2 - 1) \\ &= \frac{8 (p^2 - 1)}{(L_2 - L_1)^2 (C_1 - C_2) p^3} \left[p^4 + \left\{ 1 + \frac{(L_1 + L_2) (C_1 + C_2)}{2} - \frac{(L_1 + L_2)^2}{4} \right\} p^2 + 1 \right] \end{aligned} \quad (5.28)$$

and one has the factor $(p^2 - 1)$ as he should. Moreover one must have the factor $(p^2 + 1)^2$ in order that B - C would have the form Eq. (5.22); that is, the expression, inside the brackets on the right hand side of Eq. (5.28), must be equal to $(p^2 + 1)^2$.

Thus, it is necessary that

$$1 + \frac{(L_1 + L_2) (C_1 + C_2)}{2} - \frac{(L_1 + L_2)^2}{4} = 2 \quad (5.29)$$

Further, the coefficient of the right hand side of Eq. (5.28) must satisfy, comparing with Eq. (5.22),

$$\frac{2 (L_2 - L_1)^2 (C_1 - C_2)}{8} = \Delta^2 K \quad (5.30)$$

If these are the case, y^2 takes the form of Eq. (5.22);

$$y^2 = 4 \left[1 - \left\{ \frac{(p^2 + 1)^2}{(\Delta p)^2} \frac{(p^2 - 1)^2}{k p} \right\}^2 \right]$$

From the conditions (5.27), (5.29) and (5.30), the network elements are obtained:

$$\frac{(C_1^2 - 1)^2}{C_1} = \Delta^2 K, \quad C_2 = \frac{1}{C_1} \quad (5.31 \text{ a, b})$$

$$L_1 = C_1 \pm \sqrt{C_1^2 - 1}, \quad L_2 = \frac{1}{L_1} \quad (5.31 \text{ c, d})$$

To give the network in a coupled line type, one can apply transformations 5 App. 2.2, and the network Fig. 5.18 will be obtained.

The same may be done with adding C's to a Wagner network $n = 2$. Here one has to replace pL by $1/pC'$ by pL' , and he has

$$B - C = \frac{(C_1' + C_2')^2 (L_1' + L_2')}{(C_1' - C_2')^2 (L_2' - L_1')} \left[\frac{8 (1 - p^6 C_1'^2 C_2'^2 L_1' L_2')}{p^3 (C_1' + C_2')^2 (L_1' + L_2')} \right. \\ \left. + \frac{4 (1 - p^2 C_1' C_2')}{p (C_1' + C_2')} + \frac{2 (p^2 L_1' L_2' - 1)}{p (L_1' + L_2')} \right] \quad (5.32 \text{ a})$$

Under the condition $L_1' L_2' = C_1' C_2' = 1$, it goes down to

$$B - C = \frac{8 (1 - p^2)}{(C_1' + C_2')^2 (L_2' - L_1')} \left[p^4 + p^2 \left\{ 1 + \frac{(C_1' + C_2') (L_1' + L_2')}{2} \right. \right. \\ \left. \left. - \frac{(C_1' + C_2')^2}{4} \right\} + 1 \right] \quad (5.32 \text{ b})$$

To have the form Eq. (5.22), it is necessary that

$$1 + \frac{(C_1' + C_2')(L_1' + L_2')}{2} - \frac{(C_1' + C_2')^2}{4} = 2 \quad (5.33a)$$

$$\frac{(C_1' - C_2')^2 (L_2' - L_1')}{4} = \Delta^2 K \quad (5.33b)$$

From these relations, the elements are determined:

$$\frac{(L_2'^2 - 1)^2}{L_2'} = \Delta^2 K, \quad L_1' = \frac{1}{L_2'} \quad (5.34 a, b)$$

$$C_2' = L_2' \pm \sqrt{L_2'^2 - 1}, \quad C_1' = \frac{1}{C_2'} \quad (5.34 c, d)$$

These values have, in comparison with those in Eq. (5.31), the same forms with the correspondences $L_2' \rightarrow C_1$, $L_1' \rightarrow C_2$, $C_2' \rightarrow L_1$, $C_1' \rightarrow L_2$. Numerical values of Eq. (5.34) may be obtained from Fig. 5.19 with the respective correspondence. In the coupled line type network, one has to make correspondences $W_L' \rightarrow W_c$, $W_c' \rightarrow W_L$, $k_c' \rightarrow k_L$.

[Add elements to a network $n = 3$]

Add excess L 's to a Wagner network $n = 3$, a network Fig. 5.21 will be obtained whose network parameters are:

$$A = \frac{(L_2 + L_1)(C_1 + C_2)(L_2' + L_1')(C_1' + C_2')}{(L_2 - L_1)(C_1 - C_2)(L_2' - L_1')(C_1' - C_2')} \left[\left\{ 1 + \frac{4p^2 L_1 L_2 C_1 C_2}{(L_1 + L_2)(C_1 + C_2)} \right\} \left\{ 1 + \frac{4p^2 L_1' L_2' C_1' C_2'}{(L_1' + L_2')(C_1' + C_2')} \right\} \right. \\ \left. + 4 \left\{ \frac{1/p}{C_1 + C_2} + \frac{p L_1 L_2}{L_1 + L_2} \right\} \left\{ \frac{1/p}{L_1' + L_2'} + \frac{p C_1' C_2'}{C_1' + C_2'} \right\} \right]$$

$$B = \frac{(L_2 + L_1)(C_1 + C_2)(L_2' + L_1')(C_1' + C_2')}{(L_2 - L_1)(C_1 - C_2)(L_2' - L_1')(C_1' - C_2')} \left[2 \left\{ 1 + \frac{4p^2 L_1 L_2 C_1 C_2}{(L_1 + L_2)(C_1 + C_2)} \right\} \left\{ \frac{1/p}{C_1' + C_2'} + \frac{p L_1' L_2'}{L_1' + L_2'} \right\} \right. \\ \left. + 2 \left\{ \frac{1/p}{C_1 + C_2} + \frac{p L_1 L_2}{L_1 + L_2} \right\} \left\{ 1 + \frac{4/p^2}{(L_1' + L_2')(C_1' + C_2')} \right\} \right]$$

$$\begin{aligned}
C &= \frac{(L_2+L_1)(C_1+C_2)(L_2'+L_1')(C_1'+C_2')}{(L_2-L_1)(C_1-C_2)(L_2'-L_1')(C_1'-C_2')} \left[2 \left\{ \frac{1/p}{L_1+L_2} + \frac{p C_1 C_2}{C_1+C_2} \right\} \left\{ 1 + \frac{4 p^2 L_1' L_2' C_1' C_2'}{(L_1'+L_2')(C_1'+C_2')} \right\} \right. \\
&\quad \left. + 2 \left\{ 1 + \frac{4/p^2}{(L_1+L_2)(C_1+C_2)} \right\} \left\{ \frac{1/p}{L_1'+L_2'} + \frac{p C_1' C_2'}{C_1'+C_2'} \right\} \right] \\
D &= \frac{(L_2+L_1)(C_1+C_2)(L_2'+L_1')(C_1'+C_2')}{(L_2-L_1)(C_1-C_2)(L_2'-L_1')(C_1'-C_2')} \left[4 \left\{ \frac{1/p}{L_1+L_2} + \frac{p C_1 C_2}{C_1+C_2} \right\} \left\{ \frac{1/p}{C_1'+C_2'} + \frac{p L_1' L_2'}{L_1'+L_2'} \right\} \right. \\
&\quad \left. + \left\{ 1 + \frac{4/p^2}{(L_1+L_2)(C_1+C_2)} \right\} \left\{ 1 + \frac{4/p^2}{(L_1'+L_2')(C_1'+C_2')} \right\} \right] \quad (5.35)
\end{aligned}$$

If $B = C$, as an antimetric one, the 4 conditions,

$$L_1 L_2 = C_1 C_2, \quad L_1' L_2' = C_1' C_2' \quad (5.36 \text{ a, b})$$

$$L_1 + L_2 = C_1' + C_2', \quad L_1' + L_2' = C_1 + C_2 \quad (5.36 \text{ c, d})$$

are necessary. Rewrite A and D under these conditions,

$$\begin{aligned}
A &= \frac{(L_2+L_1)(C_1+C_2)(L_2'+L_1')(C_1'+C_2')}{(L_2-L_1)(C_1-C_2)(L_2'-L_1')(C_1'-C_2')} \left[\left\{ 1 + \frac{4 p^2 L_1 L_2 C_1' C_2'}{(L_1+L_2)(C_1+C_2)} \right\}^2 + 4 \frac{1/p}{C_1+C_2} + \frac{p L_1 L_2}{L_1+L_2} \right] \\
D &= \frac{(L_2+L_1)^2 (C_1+C_2)^2}{(L_2-L_1)(C_1-C_2)(L_2'-L_1')(C_1'-C_2')} \left[\left\{ 1 + \frac{4/p^2}{(L_1+L_2)(C_1+C_2)} \right\}^2 + 4 \left\{ \frac{1/p}{L_1+L_2} + \frac{p C_1 C_2}{C_1+C_2} \right\}^2 \right] \quad (5.37 \text{ a, b})
\end{aligned}$$

Add one more condition

$$L_1 L_2 = C_1 C_2 = 1 \quad (5.38)$$

then $A - D$ will have a factor $(p^2 - 1)$ as follows:

$$A - D = \frac{16 (p^2 - 1) (p^2 + 1)}{(L_2 - L_1)(C_1 - C_2)(L_2' - L_1')(C_1' - C_2')} \left[p^4 + \frac{(C_1 + C_2)(L_1 + L_2)}{4} \left\{ 2 + (C_1 + C_2)^2 - (L_1 + L_2)^2 \right\} p^2 + 1 \right] \quad (5.39)$$

In order that the expression inside the brackets is identical to $(p^2 + 1)^2$, it is necessary that:

$$\frac{(C_1 + C_2)(L_1 + L_2)}{4} \left\{ 2 + (C_1 + C_2)^2 - (L_1 + L_2)^2 \right\} = 2 \quad (5.40)$$

Moreover, if the coefficient has the relation

$$\frac{(L_2 - L_1)(C_1 - C_2)(L_2' - L_1')(C_1' - C_2')}{8} = \Delta^3 K \quad (5.41)$$

then the characteristic function will take the form Eq. (5.22). From the 6 conditions in Eqs. (5.38), (5.40), (5.41) and (5.36), the values of the elements are determined by solving the relations:

$$C_1 = L_2', \quad C_2 = L_1', \quad L_1 = C_2', \quad L_2 = C_1' \quad (5.42 \text{ a,b,c,d})$$

$$(C_1 + C_2)^2 = \frac{8 \Delta^3 K + 4 \{ (L_1 + L_2)^2 - 4 \}}{(L_1 + L_2)^2 - 4} \quad (5.42 \text{ e})$$

$$(C_1 + C_2)^3 (L_1 + L_2) + (C_1 + C_2)(L_1 + L_2) \{ 2 - (L_1 + L_2)^2 \} - 8 = 0 \quad (5.42 \text{ f})$$

Fig. 5.22 shows the relations between Δ and $\Delta^3 K$, between Δ and values of the elements.

The same procedure applies to greater values of n .

[Note] The attenuation character of this network differs from those of Wagner networks in the point that the former has the factor $(p^2 - 1)/K p$ in surplus. A new frequency variable may be defined

$$p' = \frac{p^2 + 1}{\Delta p}$$

The usual frequency transformation. Then the surplus factor is transformed into

$$\frac{p^2 - 1}{Kp} = \frac{p^2 + 1 - 2}{\Delta p} \frac{\Delta}{K} = \left\{ p' - \frac{2}{\Delta p} \right\} \frac{\Delta}{K}$$

$$p = \frac{1}{2} \left\{ \Delta p' \pm \sqrt{(\Delta p')^2 - 4} \right\}$$

Δ is very small, if one is concerned with a narrow band. So long as $\Delta p' \ll 2$, one has

$$p \approx j1, \quad K = \omega_1 + \omega_2 \approx 2\omega_0 = 2$$

$$\frac{p^2 - 1}{Kp} \approx \frac{-1}{j}$$

Therefore the characteristic function approximates $\{-(p')^2\}^n$, and coincides with that of a Wagner network. That is, the attenuation character in the neighborhood of the center frequency is like a Wagner one, if Δ is small.

For an exact computation, one has

$$p = j \tan \left(\frac{2\pi f}{c} l \right), \quad \omega_1 = \tan \left(\frac{2\pi f_1}{c} l \right), \quad \omega_2 = \tan \left(\frac{2\pi f_2}{c} l \right)$$

with which one obtains

$$\frac{p^2 - 1}{Kp} = \frac{j^2 \tan^2 \frac{2\pi f}{c} l - 1}{\left(\tan \frac{2\pi f_2}{c} l - \tan \frac{2\pi f_1}{c} l \right) j \tan \frac{2\pi f}{c} l} = j \frac{\sin \left(\frac{2\pi f_1}{c} l \right)}{\sin \left(\frac{2\pi f}{c} l \right)}$$

and consequently the characteristic function becomes

$$\left\{ \frac{\tan \left(\frac{2\pi f_1}{c} l \right) \sin \left(\frac{2\pi f_1}{c} l \right)}{\tan \left(\frac{2\pi f}{c} l \right) \sin \left(\frac{2\pi f}{c} l \right)} \right\}^2$$

Then one has only to assume f_1 to make computations.

5.4. 3-element band-pass filters

The basic circuits of band-pass filters, called 3-element type ones, are shown in Fig. 5.23; they are capacitance-coupled or inductance-coupled, and are widely used in intermediate frequency amplifiers and the like, as lumped networks. Design methods are given in references (32) and (33) with the use of approximate formulas near the center frequency. Some techniques are also reported in reference (20) to have coupled line filters. Here will be presented those derived by the procedures described in preceding paragraphs.

Filters of this type have unsymmetrical attenuation characteristic with respect to the center frequency; networks (a) and (b) have greater attenuation in the upper frequency side, while networks (c) and (d) have greater attenuation in the lower frequency side. As to the values of elements, those of series arms in networks (a) and (c) are inversely proportional to the bandwidth Δ , and will be very large if Δ is small; on the other hand, elements of the shunt arms in networks (b) and (d) are in the opposite relation, being small with small Δ . Other elements are directly or inversely proportional to $K = \omega_1 + \omega_2$, and do not change much with the variation of bandwidths. To have a narrow band, one can insert transformers whose ratios are related to Δ , so that he has elements related only to K . Thus one need not add any new elements, to the contrary to those networks described in the previous paragraph.

In Fig. 5.24 are shown coupled line networks derived from 6 such sections in cascade. As to the element values, one can refer to the literature. Here are shown only the derivation of coupled line networks and their configurations.

5. App. 1. Kuroda's Theorem¹⁷

The equivalence of the two networks below is given by Kuroda, under the conditions

$$a = \frac{c d}{c + d}, \quad b = \frac{c^2}{c + d}$$

$$c = a + b, \quad d = \frac{a(a + b)}{b}$$

5. App. 2. Equivalent circuits of networks with 2-wire lines [App. 2.1] From Table 2.18 (e),

$$W = a = \frac{c n^2 + e}{n^2}, \quad d = \frac{b}{a} + m^2 = \frac{e n^2}{c n^2 + e}, \quad k = m = \frac{e n}{c n^2 + e}$$

$$a = \frac{cn^2 + e}{n^2} = W, \quad b = \frac{cen^2}{cn^2 + e} = W(d - k^2), \quad m = \frac{en}{cn^2 + e} = k$$

$$c = \frac{ab}{am^2 + b} = W(1 - \frac{k^2}{d}), \quad e = am^2 + b = Wd, \quad n = \frac{am^2 + b}{am} = \frac{d}{k}$$

In case of symmetry:

$$W = \frac{1}{2} \left(\frac{1}{c_1} + \frac{1}{c_2} \right), \quad k = \frac{C_1 - C_2}{C_1 + C_2}, \quad d = 1$$

[App. 2.2] From TABLE 2.19

$$W = a = \frac{cn^2 + e}{n^2}, \quad d = \frac{b}{a} + m^2 = \frac{en^2}{cn^2 + e}, \quad k = m = \frac{en}{cn^2 + e}$$

$$a = W = \frac{cn^2 + e}{n^2}, \quad b = W(d - k^2) = \frac{ecn^2}{cn^2 + e}, \quad m = k = \frac{en}{cn^2 + e}$$

$$c = W(1 - \frac{k^2}{d}) = \frac{ab}{am^2 + b}, \quad e = Wd = am^2 + b, \quad n = \frac{d}{k} = \frac{am^2 + b}{am}$$

In case of symmetry

$$W = \frac{1}{2} (L_1 + L_2), \quad k = \frac{L_2 - L_1}{L_2 + L_1}, \quad d = 1$$

[App. 2.3] From TABLE 2.17 (c):

$$W = a = \frac{en^2 + c}{n^2}, \quad d = \frac{b}{a} + m^2 = \frac{cn^2}{en^2 + c}, \quad k = m = \frac{cn}{en^2 + c}$$

$$a = \frac{en^2}{n^2} = W, \quad b = \frac{cen^2}{en^2 + c}, \quad m = k = \frac{nc}{en^2 + c}$$

$$c = am^2 + b, \quad e = \frac{ab}{am^2 + b}, \quad n = \frac{am^2 + b}{am}$$

[App. 2.4] From TABLE 2.17(d):

$$W = \frac{am^2 + b}{m^2} = e, \quad d = \frac{bm^2}{am^2 + b} = \frac{c + en^2}{e}, \quad k = \frac{bm}{am^2 + b} = n$$

$$a = W(1 - \frac{k^2}{d}) = \frac{ce}{c + en^2}, \quad b = Wd = c + en^2, \quad m = \frac{d}{k} = \frac{c + en^2}{en}$$

$$c = W(d-k^2) = \frac{abm^2}{am^2+b}, e = W = \frac{am^2+b}{m^2}, n = k = \frac{bm}{am^2+b}$$

[App. 2.5] From TABLE 2.17(c):

$$W = a + cn^2, d = \frac{c}{a + cn^2}, k = \frac{cn}{a + cn^2}$$

$$W_c = b, a = W(1 - \frac{k^2}{d}), b = W_c$$

$$c = Wd, n = \frac{k}{d}$$

5 App. 3. Equivalence Relations between networks of coupled 3-wire lines.

[App. 3.1]

$$W_{11} = \frac{W^2(d-k^2) - WW'}{Wd + W'}, W_{22} = d_2 W_{11} = \frac{WdW'}{Wd + W'}$$

$$W_{33} = d_3 W_{11} = \frac{WdW'd + W^{12}(d' - k^{12})}{Wd + W'}$$

$$W_{11}k_{12} = \frac{WkW'}{Wd + W'}, W_{11}k_{13} = \frac{WkW'k'}{Wd + W'}$$

$$W_{11}k_{23} = \frac{WdW'd'}{Wd + W'}$$

[App. 3.2] The two networks (a) and (b) are assumed to be symmetrical.

$$W_{11} = W_{11}', k_{13} = k_{13}'$$

$$\frac{d_2}{W_3} = \frac{d_2'}{W_3'} + \frac{2k_{12}'^2}{W_{11}'a'}$$

$$\frac{k_{12}^2}{d_2^2} = \frac{k_{12}'^2 \{W_3' (1+k_{13}') + W_{11}'a'\}}{W_3' 2k_{12}'^2 + W_{11}'d_{12}'a'}$$

$$a' = -\{(1+k_{13}')d_2' - 2k_{12}'^2\}$$

$$\frac{k_{12}'^2}{d_2'^2} = \frac{W_{11}'k_{12}'^2}{W_3' + W_{11}'d_2'} W_{11}' + \frac{W_3' (1+k_{13}')}{d_2' (1+k_{13}') - 2k_{12}'^2}$$

$$\frac{W_3'}{d_2'} = \frac{W_3}{W_3 + W_{11}d_2} \left\{ W_{11} + \frac{W_3(1+k_{13})}{d_2(1+k_{13}) - 2k_{12}^2} \right\}$$

[App. 3.3] The network (a) is assumed to be symmetrical.

$$W_{11} = \frac{1}{2} (a + b), \quad k_{13} = \frac{b - a}{b + a}$$

$$\frac{k_{12}^2}{d_2} = \frac{b^2}{(a+bX_0+c)}, \quad \frac{W_3}{d^2} = \frac{e}{2} \frac{a+b}{b+c}$$

$$a = W_{11}(1 - k_{13}), \quad b = W_{11}(1 + k_{13})$$

$$c = W_{11}(1 + k_{13}) \left\{ \frac{d_2(1 + k_{13})}{2k_{12}^2} - 1 \right\}$$

$$d = W_{11}(1 + k_{13})^2 \frac{1}{2k_{12}^2}$$

CHAPTER 6. Extraction of a two-wire line (34)

Extension of Richards' key theorem to 4-terminal networks

In Chapters 4 and 5 are described ladder network building procedures, connecting sections of simpler structures, in a manner developed by Zobel. This is because it needs complicated line constructions if one tries to obtain networks of higher degrees by means of using coupled lines of increased number of wires. This will meet most requirements in practice. Synthesis of networks is not necessarily unique, and some other procedures may also come into consideration. Even in lumped networks, one may consider Cauer's lattice networks along with Zobel's ladder networks.

In coaxial filters, it is usual to use combinations of shunt elements and cascade elements (unit coaxials). The same is applicable to coupled line networks. This chapter and the following chapters describe synthesis of networks by combinations of shunt and cascade elements of coupled 2-wire lines. Shunt elements have no problems. The treatment of cascade elements is more or less complicated and needs a detailed examination. This chapter describes the extraction process of cascade lines, and proves that Richards' key theorem can be extended to multiterminal networks.

6.1 Equations of transmission in a coupled 2-wire line

Equations of transmission will be again cited. Among voltages and currents of a parallel 2-wire line, set up arbitrarily over the ground, hold the following relations. Here it is assumed that the line is lossless.

$$\left. \begin{aligned} V_1 &= V_{10} \cos \beta l + W_0 (I_{10} + k I_{20}) j \sin \beta l \\ V_2 &= V_{20} \cos \beta l + W_0 (k I_{10} + d I_{20}) j \sin \beta l \\ I_1 &= I_{10} \cos \beta l - \frac{1}{W_0 (d - k^2)} (d V_{10} - k V_{20}) j \sin \beta l \\ I_2 &= I_{20} \cos \beta l + \frac{1}{W_0 (d - k^2)} (-k V_{10} + V_{20}) j \sin \beta l \end{aligned} \right\} \quad (1-18)$$

where l is the length of the line, $\beta (= 2 \pi f / c)$ the phase constant of the line, W_0 the self characteristic impedance of the first line, d the symmetry coefficient of the second wire to the first, k the coupling coefficient between two wires. These parameters are determined by the cross-sectional structure of the line, and $d = 1$ in a symmetrical line.

6.2 Cascade adding of a coupled 2-wire line

The network parameters (Y_{11} , Y_{22} , Y_{12}) will be obtained, of the network Fig. 6.2, which is made up of any four-terminal network (whose admittance parameters

are Y_{11}' , Y_{22}' , Y_{12}') in cascade with a coupled two-wire line (with line constants W_0 , d , k) in front.

Set the voltages and the directions of currents as shown in the figure, one has the relations, from Eq. (1.18),

$$\begin{cases} [V] = [V_0] \cos \beta l + [W] [I_0] j \sin \beta l \\ [I] = [I_0] \cos \beta l + [W]^{-1} [V_0] j \sin \beta l \end{cases} \quad (1-18)$$

$[V_0]$ and $[I_0]$ are related by

$$[I_0] = [y'] [V_0], \quad [y'] = \begin{bmatrix} Y_{11}' & -Y_{12}' \\ Y_{12}' & Y_{22}' \end{bmatrix} \quad (6.1)$$

Put this into Eq. (1.18), then one has

$$[I] = ([y'] + [W] p) [1] + [W] [y'] p^{-1} [V] \quad (6.2)$$

Represent this relation by

$$[I] = [y] [V] \quad (6.3)$$

Then $[y]$ will be represented

$$\begin{aligned} [y] &= \begin{bmatrix} Y_{11} & -Y_{12} \\ -Y_{12} & Y_{22} \end{bmatrix} \\ &= [y'] + [W]^{-1} p [1] + [W] [y'] p^{-1} \end{aligned} \quad (6.4)$$

This is the admittance parameter of the new network thus made up. In 4-terminals networks, the entries of $[y]$ are as follows:

$$\begin{aligned} Y_{11}(p) &= \frac{1}{\nabla} \cdot \left\{ \left\{ Y_{11}' + \frac{dp}{W_0(d-k^2)} \right\} \left\{ 1 + W_0 p (d Y_{22}' - k Y_{12}') \right\} \right. \\ &\quad \left. - \left\{ Y_{12}' + \frac{kp}{W_0(d-k^2)} \right\} (d Y_{12}' - k Y_{11}') W_0 p \right\} \\ Y_{22}(p) &= \frac{1}{\nabla} \cdot \left\{ \left\{ Y_{22}' + \frac{p}{W_0(d-k^2)} \right\} \left\{ 1 + W_0 p (Y_{11}' - k Y_{12}') \right\} \right. \\ &\quad \left. - \left\{ Y_{12}' + \frac{kp}{W_0(d-k^2)} \right\} (Y_{12}' - k Y_{22}') W_0 p \right\} \\ Y_{12}(p) &= \frac{1}{\nabla} \cdot \left\{ - \left\{ Y_{12}' + \frac{kp}{W_0(d-k^2)} \right\} \left\{ 1 + W_0 p (Y_{11}' - k Y_{12}') \right\} \right. \end{aligned}$$

$$\begin{aligned}
 & + \left\{ Y_{11}' + \frac{dp}{W_o(d-k^2)} \right\} (Y_{12}' - k Y_{22}') W_o p \Big] \\
 \nabla' = & \left\{ 1 + W_o p (Y_{11}' - k Y_{12}') \right\} \left\{ 1 + W_o p (d Y_{22}' - k Y_{12}') \right\} \\
 & - W_o^2 p^2 (Y_{12}' - k Y_{22}') (d Y_{12}' - k Y_{11}') \quad (6.5)
 \end{aligned}$$

Here it will be noticed that upon putting $p = 1$ into Eq. (6.4), it will yield

$$\left[\bar{y}(1) \right] = \left[\left[\bar{y}'(1) \right] + \left[W \right]^{-1} \right] \left[(1) + \left[\bar{W} \right] \left[\bar{y}'(1) \right] \right]^{-1} = \left[W \right]^{-1} \quad (6.6a)$$

and a relation

$$\left[\bar{y}(1) \right] = \begin{bmatrix} Y_{11}(1) & -Y_{12}(1) \\ -Y_{12}(1) & Y_{22}(1) \end{bmatrix} = \frac{1}{W_o(1-k^2)} \begin{bmatrix} d-k & \\ -k & 1 \end{bmatrix} \quad (6.6b)$$

which is independent on $\left[\bar{y}' \right]$

6.3 Extraction of a coupled 2-wire line

Now, in contrary to the previous paragraph, an extraction of a coupled 2-wire line will be considered from a given four-terminal network (Y_{11}, Y_{22}, Y_{12}) .

It has been shown that, in the network $\left[\bar{y}(p) \right]$ composed of an arbitrary network $\left[\bar{y}'(p) \right]$ and a coupled 2-wire line of characteristic impedance $\left[W \right]$ in cascade, the relation Eq. (6.6) holds which is independent on $\left[\bar{y}'(p) \right]$. Therefore, if $\left[\bar{y}(p) \right]$ is given, one can obtain $\left[W \right]$ of the coupled 2-wire line, putting $p = 1$.

$$\begin{aligned}
 W_o &= \frac{Y_{11}(1)}{Y_{11}(1) Y_{22}(1) - Y_{12}^2(1)} = Z_{11}(1) \\
 d &= \frac{Y_{11}(1)}{Y_{22}(1)} = \frac{Z_{22}(1)}{Z_{11}(1)} \\
 k &= \frac{Y_{12}(1)}{Y_{22}(1)} = \frac{Z_{12}(1)}{Z_{11}(1)}
 \end{aligned} \quad (6.7)$$

The remaining network $\left[\bar{y}'(p) \right]$ after extraction of the coupled 2-wire line may be obtained

$$\left[\bar{y}'(p) \right] = \left[(1) - \left[\bar{y}(p) \right] \left[W \right] p \right]^{-1} \left[\bar{y}(p) - \left[W \right]^{-1} p \right] \quad (6.8)$$

by solving (6.4). Entries of this expression can be written in precise as follows:

$$\begin{aligned}
 Y_{11}'(p) &= \frac{1}{\nabla} \left\{ 1 - W_o p (Y_{11} - k Y_{12}) \right\} \left\{ Y_{11} - \frac{dp}{W_o(d-k^2)} \right\} \\
 &\quad + W_o p (k Y_{11} - d Y_{12}) \left\{ Y_{12} - \frac{kp}{W_o(d-k^2)} \right\} \\
 Y_{22}'(p) &= \frac{1}{\nabla} \left\{ 1 - W_o p (d Y_{22} - k Y_{12}) \right\} \left\{ Y_{22} - \frac{p}{W_o(d-k^2)} \right\} \\
 &\quad + W_o p (k Y_{22} - Y_{12}) \left\{ Y_{12} - \frac{kp}{W_o(d-k^2)} \right\} \\
 Y_{12}'(p) &= \nabla \left\{ 1 - W_o p (Y_{11} - k Y_{12}) \right\} \left\{ Y_{12} - \frac{kp}{W_o(d-k^2)} \right\} \\
 &\quad + W_o p (k Y_{11} - d Y_{12}) \left\{ Y_{22} - \frac{kp}{W_o(d-k^2)} \right\}
 \end{aligned} \tag{6.9}$$

Rewrite the given network $[y(p)]$ and the remaining network $[y'(p)]$ as

$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12} & Y_{22} \end{bmatrix}, \quad [Y'] = \begin{bmatrix} Y_{11}' & Y_{12}' \\ Y_{12}' & Y_{22}' \end{bmatrix} \tag{6.10}$$

then Eq. (6.8) will go into

$$[Y'(p)] = \left[[1] - [Y(p)] [Y(1)]^{-1} p \right]^{-1} \left[[Y(p)] - [Y(1)] p \right] \tag{6.11}$$

Look back the procedure; $Y(p)$ is given, and one extracts a coupled 2-wire line, with line constants W given by Eq. (6.7), from the given network, then the network parameters $[Y'(p)]$ of the remaining network will be given by Eq. (6.11).

This is nothing but an extension of the Richards' technique, into coupled 2-wire lines, which originally enables the extraction of unit coaxials from given 2-terminal admittances.

6.4 Positive realness of $[Y'(p)]$

Here the problem is whether the remaining parameters $[Y']$ would have the property of those of networks or not. That is, it is the significant point whether $[Y']$ is a positive real matrix or not. If this is not confirmed, one can not be sure if he can proceed with the synthesis of the network.

In order to keep generality, the following term will first be proved.

Theorem: If $[Y(p)]$ is a positive real matrix, and p , is any positive number, then

$$[Y'(p)] = \left[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} p - [Y(p)] [Y(p_1)] p \right]^{-1} \left[\begin{bmatrix} Y(p) & 0 \\ 0 & Y(p_1) \end{bmatrix} p - \begin{bmatrix} Y(p) & 0 \\ 0 & Y(p_1) \end{bmatrix} p \right] \quad (6.12)$$

is also a positive real matrix.

This is the general form of Richards' key theorem extended into multiterminal networks. But the proof will be carried out for a 4-terminal network, as an example. Write the entries of Eq. (6.12) in precise,

$$\begin{aligned} Y_{11}' &= \frac{1}{\Delta} \left[Y_{11} p_1^2 - p^2 \right] - \frac{Z_{22} p}{Z_{11} Z_{22} - Z_{12}^2} \left\{ p_1 - p (Z_{11} Y_{11} + Z_{22} Y_{22} + 2 Z_{12} Y_{12}) \right\} \\ &\quad - Z_{22} p_1 p (Y_{11} Y_{22} - Y_{12}^2) \\ Y_{22}' &= \frac{1}{\Delta} \left[Y_{22} p_1^2 - p^2 \right] - \frac{Z_{11} p}{Z_{11} Z_{22} - Z_{12}^2} \left\{ p_1 - p (Z_{11} Y_{11} + Z_{22} Y_{22} + 2 Z_{12} Y_{12}) \right\} \\ &\quad - Z_{11} p_1 p (Y_{11} Y_{22} - Y_{12}^2) \\ Y_{12}' &= \frac{1}{\Delta} \left[Y_{12} (p_1^2 - p^2) + \frac{Z_{12} p}{Z_{11} Z_{22} - Z_{12}^2} p_1 - p (Z_{11} Y_{11} + Z_{22} Y_{22} + 2 Z_{12} Y_{12}) \right. \\ &\quad \left. + Z_{12} p_1 p (Y_{11} Y_{22} - Y_{12}^2) \right] \end{aligned} \quad (6.13)$$

where Z 's are the elements of $[Y(p_1)]^{-1}$.

5.4.2 Segregation of $[Z]$

Segregate $[Z]$, in the manner that Richards took to proving his theorem on two-terminal networks by the use of positive realness of functions. The only difference is that here matrices are under consideration, rather than functions themselves.

Let any entry of the matrix $[Z]$ be represented as:

$$\begin{aligned} Z &= Z^{(a)} + Z^{(b)} \\ Z^{(a)} &= \frac{p p Z - p_1 Z}{p^2 - p_1^2}, \quad Z^{(b)} = \frac{p_1 (p Z - p_1 Z)}{p^2 - p_1^2} \\ z &= Z(p_1), \quad p_1 > 0 \end{aligned} \quad (6.14)$$

Then $[Z]$ can be segregated into two parts:

$$\begin{aligned}
 [Z] &= \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_{11}^{(a)} & Z_{12}^{(a)} \\ Z_{12}^{(a)} & Z_{22}^{(a)} \end{bmatrix} + \begin{bmatrix} Z_{11}^{(b)} & Z_{12}^{(b)} \\ Z_{12}^{(b)} & Z_{22}^{(b)} \end{bmatrix} \\
 &= \begin{bmatrix} \frac{p(pZ_{11} - p_1 Z_{11})}{p^2 - p_1^2} & \frac{p(pZ_{12} - p_1 Z_{12})}{p^2 - p_1^2} \\ \frac{p(pZ_{12} - p_1 Z_{12})}{p^2 - p_1^2} & \frac{p(pZ_{22} - p_1 Z_{22})}{p^2 - p_1^2} \end{bmatrix} + \begin{bmatrix} \frac{p_1(pZ_{11} - p_1 Z_{11})}{p^2 - p_1^2} & \frac{p_1(pZ_{12} - p_1 Z_{12})}{p^2 - p_1^2} \\ \frac{p_1(pZ_{12} - p_1 Z_{12})}{p^2 - p_1^2} & \frac{p_1(pZ_{22} - p_1 Z_{22})}{p^2 - p_1^2} \end{bmatrix} \quad (6.15)
 \end{aligned}$$

If the positive realness of these $[Z^{(a)}]$ and $[Z^{(b)}]$ can be proved, $[Z]$, the series connection of the two, will also be understood to be positive real.

5.4.2 Positive realness of $[Z^{(a)}]$ and $[Z^{(b)}]$

Since $[Z^{(a)}]$ and $[Z^{(b)}]$ are symmetrical matrices, they can be proved to be positive real if the quadratic forms

$$\left. \begin{aligned}
 \text{Zin}^{(a)} &= t_1^2 Z_{11}^{(a)} + 2t_1 t_2 Z_{12}^{(a)} + t_2^2 Z_{22}^{(a)} \\
 \text{Zin}^{(b)} &= t_1^2 Z_{11}^{(b)} + 2t_1 t_2 Z_{12}^{(b)} + t_2^2 Z_{22}^{(b)}
 \end{aligned} \right\} \quad (6.16)$$

are positive real functions or any real coefficients (t_1, t_2). First, take the part of $\text{Zin}^{(a)}$ on the imaginary axis, it can be transformed as:

$$\begin{aligned}
 \text{Re Zin}^{(a)}(j\omega_1) &= \text{Re}(t_1^2 Z_{11}^{(a)} + 2t_1 t_2 Z_{12}^{(a)} + t_2^2 Z_{22}^{(a)}) \\
 &= t_1^2 \text{Re } Z_{11}^{(a)} + 2t_1 t_2 \text{Re } Z_{12}^{(a)} + t_2^2 \text{Re } Z_{22}^{(a)} \\
 &= \frac{t_2^2}{\text{Re } Z_{11}^{(a)}} \left[\left\{ \frac{t_1}{t_2} \text{Re } Z_{11}^{(a)} + \text{Re } Z_{12}^{(a)} \right\}^2 - \left\{ (\text{Re } Z_{12}^{(a)})^2 - \text{Re } Z_{11}^{(a)} \text{Re } Z_{22}^{(a)} \right\} \right] \quad (6.17)
 \end{aligned}$$

Therefore $\text{Re Zin}^{(a)}(j\omega_1)$ can be ≥ 0 if

$$\text{Re } Z_{11}^{(a)}(j\omega_1) \text{Re } Z_{22}^{(a)}(j\omega_1) - \left\{ \text{Re } Z_{12}^{(a)}(j\omega) \right\}^2 \geq 0 \quad (6.18)$$

holds. Put eq. (6.14) in the above expression, then it becomes

$$\begin{aligned}
& \operatorname{Re} \frac{p(pZ_{11} - p_1 Z_{11})}{p^2 - p_1^2} - \operatorname{Re} \frac{p(pZ_{22} - p_1 Z_{22})}{p^2 - p_1^2} - \left\{ \operatorname{Re} \frac{p(pZ_{12} - p_1 Z_{12})}{p^2 - p_1^2} \right\}^2 \\
&= \frac{-\omega_1^2 \operatorname{Re} Z_{11}(j\omega_1)}{-\omega_1^2 + p_1^2} - \frac{-\omega_1^2 \operatorname{Re} Z_{22}(j\omega_1)}{-\omega_1^2 + p_1^2} - \left\{ \frac{-\omega_1^2 \operatorname{Re} Z_{12}(j\omega_1)}{-\omega_1^2 + p_1^2} \right\}^2 \\
&= \frac{\omega_1^4}{(\omega_1^2 + p_1^2)^2} \left[\operatorname{Re} Z_{11}(j\omega_1) - \left\{ \operatorname{Re} Z_{12}(j\omega_1) \right\}^2 \right] \quad (6.19)
\end{aligned}$$

which is ≥ 0 from the condition that $[Z]$ is a positive real matrix. Thus it has been shown that $\operatorname{Re} Z_{in}^{(a)}(j\omega_1) \geq 0$. Similarly, as to $[Z^{(b)}]$ also, one has

$$\begin{aligned}
& \operatorname{Re} \frac{p_1(pZ_{11} - p_1 Z_{11})}{p^2 - p_1^2} - \operatorname{Re} \frac{p_1(pZ_{22} - p_1 Z_{22})}{p^2 - p_1^2} - \left\{ \operatorname{Re} \frac{p_1(pZ_{12} - p_1 Z_{12})}{p^2 - p_1^2} \right\}^2 \\
&= \frac{-p_1^2 \operatorname{Re} Z_{11}(j\omega_1)}{-(\omega_1^2 + p_1^2)} - \frac{-p_1^2 \operatorname{Re} Z_{22}(j\omega_1)}{-\omega_1^2 + p_1^2} - \left\{ \frac{-p_1^2 \operatorname{Re} Z_{12}(j\omega_1)}{-\omega_1^2 + p_1^2} \right\}^2 \\
&= \frac{p_1^4}{(\omega_1^2 + p_1^2)^2} \left[\operatorname{Re} Z_{11}(j\omega_1) \operatorname{Re} Z_{22}(j\omega_1) - \left\{ \operatorname{Re} Z_{12}(j\omega_1) \right\}^2 \right] \geq 0 \quad (6.20)
\end{aligned}$$

so that $\operatorname{Re} Z_{in}^{(b)}(j\omega_1) \geq 0$ holds.

Next, poles of $Z^{(a)}$, $Z^{(b)}$ will be such points that $p = \pm p_1$ or the poles of Z , but the point $p = p_1$ cannot be a pole because the factor $p - p_1$ appears in the denominator as well as in the numerator. Again, since Z is an element of a positive real matrix, it has no poles on the right half p -plane. Therefore $Z^{(a)}$ and $Z^{(b)}$ are analytical on the right half p -plane, and consequently the quadratic forms $Z_{in}^{(a)}$, $Z_{in}^{(b)}$ are also analytical on the right half p -plane. Suppose $Z^{(a)}$ and $Z^{(b)}$ have a pole at a finite point $p = j\omega_1$ on the imaginary axis, then the pole will coincide with that of Z , with the pertaining residue

$$= \frac{\omega_1^2}{(\omega_1^2 + p_1^2)} a \quad (6.21)$$

which is positive. Here a is the residue of Z at the pole $p = j\omega_1$, and is positive. one has

$$a_{11}a_{11} - a_{12}^2 \geq 0 \quad (6.22)$$

when either Z_{11} or Z_{22} has a pole, or even when Z_{11} , Z_{22} and Z_{12} all have a pole. Therefore the inequalities

$$\begin{aligned} \operatorname{Re} Z_{11}^{(a)}(j\omega_1) \cdot \operatorname{Re} Z_{22}^{(a)}(j\omega_1) - [\operatorname{Re} Z_{12}^{(a)}(j\omega_1)]^2 &\geq 0 \\ \operatorname{Re} Z_{11}^{(b)}(j\omega_1) \cdot \operatorname{Re} Z_{22}^{(b)}(j\omega_1) - [\operatorname{Re} Z_{12}^{(b)}(j\omega_1)]^2 &\geq 0 \end{aligned} \quad (6.23)$$

always hold, so that residues of $Z^{(a)}$, $Z^{(b)}$ at their poles are always positive.

From the above, it was proved that $[Z^{(a)}]$ and $[Z^{(b)}]$ are positive real matrices, because $Z^{(a)}$ and $Z^{(b)}$ satisfy the conditions to be positive real functions. The necessary and sufficient conditions, referred here, that a real rational function $W(p)$ is a positive real function are (i) its poles on the imaginary axis are simple and the residues are positive, (ii) $\operatorname{Re} W(j\omega) \geq 0$ on the imaginary axis, and is analytical on the right half plane.

5.4.3 Positive realness of $[Y']$

Since $[Z^{(b)}]$ has been proved to be positive real, its inverse matrix $[Z^{(b)}]^{-1}$ must also be positive real. Rewrite $[Z^{(b)}]^{-1}$ as follows:

$$\begin{aligned} [Z^{(b)}]^{-1} &= \begin{bmatrix} \frac{p_1 (p z_{11} - p_1 z_{11})}{p^2 - p_1^2} & \frac{p_1 (p z_{12} - p_1 z_{12})}{p^2 - p_1^2} \\ \frac{p_1 (p z_{12} - p_1 z_{12})}{p^2 - p_1^2} & \frac{p_1 (p z_{22} - p_1 z_{22})}{p^2 - p_1^2} \end{bmatrix}^{-1} \\ &= \frac{(p^2 - p_1^2)/p_1}{(p z_{11} - p_1 z_{11})(p z_{22} - p_1 z_{22}) - (p z_{12} - p_1 z_{12})^2} \begin{bmatrix} p z_{22} - p_1 z_{22} & -(p z_{12} - p_1 z_{12}) \\ -(p z_{12} - p_1 z_{12}) & p z_{11} - p_1 z_{11} \end{bmatrix} \\ &= \frac{1}{z_{11} z_{22} - z_{12}^2} \begin{bmatrix} z_{22} & -z_{12} \\ -z_{12} & z_{11} \end{bmatrix} \frac{p}{p_1} + [Y'(p)] \end{aligned} \quad (6.24)$$

Also, since the entries of $[Z^{(b)}]$ have a zero point at $p = \infty$, $[Z^{(b)}]^{-1}$ has a pole at $p = \infty$. Let its residue matrix be $[Y_\infty]$; it belongs to a positive real function:

$$\begin{aligned} Y_{11\infty} &= \frac{(z_{22} - p_1 a_{22\infty}) / p_1}{(z_{11} - p_1 a_{11\infty})(z_{22} - p_1 a_{22\infty}) - (z_{12} - p_1 a_{12\infty})^2} \geq 0 \\ Y_{22\infty} &= \frac{(z_{11} - p_1 a_{11\infty}) / p_1}{(z_{11} - p_1 a_{11\infty})(z_{22} - p_1 a_{22\infty}) - (z_{12} - p_1 a_{12\infty})^2} \geq 0 \end{aligned} \quad (6.25)$$

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$$Y_{11\infty} Y_{22\infty} - Y_{12\infty}^2 = \frac{1/p_1^2}{(z_{11} - p_1 a_{11\infty})(z_{22} - p_1 a_{22\infty}) - (z_{12} - p_1 a_{12\infty})^2} \geq 0$$

Here ∞ denotes residues of Z at $p = \infty$. Thus

$$z_{22} - p_1 a_{22\infty} \geq 0 \quad (6.26)$$

$$z_{11} - p_1 a_{11\infty} \geq 0$$

hold.

$$\begin{aligned} Y_{11\infty} &= \frac{(z_{22} - p_1 a_{22\infty})/p_1}{(z_{11} - p_1 a_{11\infty})(z_{22} - p_1 a_{22\infty}) - (z_{12} - p_1 a_{12\infty})^2} \geq 0 \\ Y_{22\infty} &= \frac{(z_{11} - p_1 a_{11\infty})/p_1}{(z_{11} - p_1 a_{11\infty})(z_{22} - p_1 a_{22\infty}) - (z_{12} - p_1 a_{12\infty})^2} \geq 0 \\ Y_{11\infty} Y_{22\infty} - Y_{12\infty}^2 &= \frac{1/p_1^2}{(z_{11} - p_1 a_{11\infty})(z_{22} - p_1 a_{22\infty}) - (z_{12} - p_1 a_{12\infty})^2} \geq 0 \end{aligned} \quad (6.25)$$

$$Y_{11\infty} = \frac{1/p_1}{\frac{z_{11} z_{22} - z_{12}^2}{z_{22}} - p_1 \frac{a_{11\infty} a_{22\infty} - a_{12\infty}^2}{a_{22\infty}} - \frac{p_1 z_{22} (a_{12\infty} - z_{12} a_{22\infty} / z_{22})^2}{a_{22\infty} (z_{22} - p_1 a_{22\infty})}} \quad (6.27)$$

The second term in the denominator in the above expression is positive as it is the determinant of the residues at the pole of Z , and the third term is positive owing to the relation (6.26). Therefore

$$Y_{11\infty} > \frac{1}{p_1 \frac{z_{11} z_{22} - z_{12}^2}{z_{22}}} = \frac{1}{p_1} \quad Y_{11}(p_1) > 0 \quad (6.28)$$

holds. Similar one has

$$Y_{22\infty} > \frac{z_{11}}{p_1 (z_{11} z_{22} - z_{12}^2)} = \frac{1}{p} \quad Y_{22}(p_1) > 0 \quad (6.29)$$

Separate the pole $p = \infty$ from $[Z^{(b)}]^{-1}$:

$$[Z^{(b)}]^{-1} = [Y_{\infty}] p + [Z_{\infty}^{(b)}]^{-1} \quad (6.30)$$

then $[Z_{\infty}^{(b)}]^{-1}$ must be a positive real matrix because it is the remainder of a positive real matrix after separating the pole at $p = \infty$.

From Eq. (6.24) and above relations, one has

$$\begin{aligned}
 [Y'] &= [Z_{\infty}^{(b)}]^{-1} + \left\{ [Y_{\infty}] - \frac{1}{p_1} [Y \ p_1] \right\} p \\
 &= [Z_{\infty}^{(b)}]^{-1} + \begin{bmatrix} Y_{11\infty} - \frac{1}{p_1} Y_{11}(p_1) & Y_{1\infty} - \frac{1}{p_1} Y_{12}(p_1) \\ Y_{12\infty} - \frac{1}{p_1} Y_{12}(p_1) & Y_{22\infty} - \frac{1}{p_1} Y_{22}(p_1) \end{bmatrix} p
 \end{aligned} \quad (6.31)$$

As to the entries, one has, from (6.25), (6.28) and (6.29),

$$\begin{aligned}
 Y_{11\infty} - \frac{1}{p_1} Y_{11}(p_1) &\geq 0 \\
 Y_{22\infty} - \frac{1}{p_1} Y_{22}(p_1) &\geq 0 \\
 \left\{ Y_{11\infty} - \frac{1}{p_1} Y_{11}(p_1) \right\} \left\{ Y_{22\infty} - \frac{1}{p_1} Y_{22}(p_1) \right\} - \left\{ Y_{12\infty} - \frac{1}{p_1} Y_{12}(p_1) \right\}^2 &\geq 0 \\
 &= \frac{a_{11\infty} a_{22\infty} - a_{12\infty}^2}{(z_{11} z_{22} - z_{12}^2) \{ (z_{11} - p_1 a_{11\infty}) (z_{22} - p_1 a_{22\infty}) - (z_{12} - p_1 a_{12\infty})^2 \}} \geq 0
 \end{aligned} \quad (6.32)$$

Thus one can see that $\left\{ [Y_{\infty}] - \frac{1}{p_1} [Y \ p_1] \right\} p$ is a positive real matrix. It turns out that $[Y']$ must naturally be positive real, because it is given as a sum of $[Z_{\infty}^{(b)}]^{-1}$ and $\left\{ [Y_{\infty}] - \frac{1}{p} [Y \ p_1] \right\} p_1$.

One can trace the proof also by using $[Z^{(a)}]$.

Although the explanation has been made on a four terminal network as an example one can in general prove the positive realness of $[Y']$ in case of an n-terminal-pair networks, the only difference being the complexity of the computation with the matrix n lines and n columns.

6.5 Physical meanings

If one solves Eq. (6.12) with respect to $[Y(p)]$, he obtains $[Y(p)] = [p [Y(p_1)] + p_1 [Y(p)] + [p_1 [1] + p [Y(p_1)]]^{-1} Y'(p)]^{-1}$

$$= \left[[Y(p_1)]^{-1} \frac{p_1}{p} + [Y(p_1)]^{-1} [Y'(p)] [Y(p_1)]^{-1} \right]^{-1} + [Y'(p)]^{-1} + [Y(p_1)]^{-1} \frac{p}{p_1} \quad (6.33)$$

The term $[Y(p_1)]^{-1} [Y'(p)] [Y(p_1)]^{-1}$ can easily be shown to be positive real, because $[Y(p_1)]$ and $[Y'(p)]$ are positive real matrices. All matrices on the right hand side of the above equation are positive real, so that one can give the network in the form Fig. 6.5(a).

Also obtain $[Z(p)]$,

$$[Z(p)] = \left[[Y(p_1)] \frac{p}{p_1} [Y'(p)]^{-1} + \left[[Y(p_1)] [Y'(p)] [Y(p_1)] + Y(p_1) \frac{p_1}{p} \right]^{-1} \right] \quad (6.34)$$

This may be represented in the network Fig. 6.5(b).

It seems as if $[Y'(p)]$ has a degree two higher than that of $[Y(p)]$, from Eq. (6.13), but the numerator and the denominator have a common factor $(p-p_1)^2$, which can be eliminated, so that they are of the same degree. In case of pure reactances, one can take off a factor $(p+p_1)^2$, and will have a degree two lower.

As Richards applied his theorem to extract a unit coaxial, the extended theorem may be applied to the extraction of a coupled multi-wire line. That is the case $p_1 = 1$, as explained in paragraph 6.3. But one must pay notice to the restrictions of a coupled two-wire line. The proof, stated in the preceding paragraph, holds, and the restrictions on $[Y(l)]$ are, from the positive realness,

$$Y_{11}(l) \geq 0, \quad Y_{22}(l) \geq 0, \quad Y_{11}(l) Y_{22}(l) - Y_{12}^2(l) \geq 0 \quad (6.35)$$

whereas one has the restrictions

$$1, d > k > 0 \quad (6.36)$$

if he is going to make the coupled 2-wire line in a shielded 2-wire line. Expression (6.35) becomes

$$W_0 \geq 0, \quad d \geq 0, \quad d - k^2 \geq 0 \quad (6.37)$$

One should note that the restrictions (6.36) are severer than these.

CHAPTER 7. Design of symmetrical networks⁽³⁸⁾⁽³⁹⁾

If one extracts a two-wire line in the way described in the preceding chapter, the degree of the network parameters will be lowered by 2 for every one extraction of a line. The remaining network preserves its property as a physical one, so that one can again extract a coupled 2-wire line. Repeat the process, the degree of the network will come down, finally to zero, and the synthesis will be completed. From this reason, the extraction of coupled two-wire lines can be a significant procedure of networks by using the procedure along with the methods of synthesis, say, extraction of unit coaxials or taking out shunt elements.

This chapter describes the synthesis of symmetrical networks.

7.1. Extraction of two-wire lines in symmetrical networks.

Let the network parameters $[Y]$ be given at first. If it is symmetrical, one has

$$Y_{11} = Y_{22} \quad (7.1)$$

Therefore the line constants, of the coupled two-wire line to be extracted, are, with the use of Eq. (6.7),

$$\left. \begin{aligned} W_o &= \frac{Y_{11}(1)}{Y_{11}^2(1) - Y_{12}^2(1)} = Z_{11}(1), d = 1, \\ k &= \frac{Y_{12}(1)}{Y_{11}(1)} = -\frac{Z_{12}(1)}{Z_{11}(1)} \end{aligned} \right\} \quad (7.2)$$

The network parameters of the remaining network are, from Eq. (6.9),

$$\left. \begin{aligned} Y_{11}(p) &= Y_{22}'(p) \\ &= \frac{Y_{11}(1-p^2) - \frac{p}{W_o(1-k^2)} \{ 1 - 2W_o p(Y_{11} - kY_{12}) \} - W_o p(Y_{11}^2 - Y_{12}^2)}{1 - 2W_o p(Y_{11} - kY_{12}) + W_o^2 (1-k^2) p^2 (Y_{11}^2 - Y_{12}^2)} \\ Y_{12}'(p) &= \frac{Y_{12}(1-p^2) - \frac{kp}{W_o}}{1 - 2W_o p(Y_{11} - kY_{12}) + W_o^2 (1-k^2) p^2 (Y_{11}^2 - Y_{12}^2)} \end{aligned} \right\} \quad (7.3)$$

$[Y']$ is already proved to be positive real, so that one can go on the synthesis by repeating the extraction of 2-wire lines.

7.2. Method of treating the matter in symmetrical lattice networks

When the network is symmetrical, one may represent it in an equivalent symmetrical lattice network, which makes understanding easier and calculations simpler, than to extract a coupled two-wire line as described in the preceding paragraph.

Let the lattice elements of a symmetrical network be Z_a and Z_b , they have the following relations with network parameters:

$$\left. \begin{aligned} Z_a(p) &= \frac{A(p) - 1}{C(p)} = \frac{1}{Y_{11}(p) + Y_{12}(p)} \\ Z_b(p) &= \frac{A(p) + 1}{C(p)} = \frac{1}{Y_{11}(p) - Y_{12}(p)} \end{aligned} \right\} \quad (7.4)$$

Put $p = 1$,

$$\left. \begin{aligned} Z_a(1) &= \frac{1}{Y_{11}(1) + Y_{12}(1)} = W_o(1 - k) \\ Z_b(1) &= \frac{1}{Y_{11}(1) - Y_{12}(1)} = W_o(1 + k) \end{aligned} \right\} \quad (7.5)$$

which yield line parameters:

$$\left. \begin{aligned} W_o &= \frac{1}{2} \{ Z_a(1) + Z_b(1) \} \\ k &= \frac{Z_b(1) - Z_a(1)}{Z_b(1) + Z_a(1)} \end{aligned} \right\} \quad (7.6)$$

Take out a unit coaxial, of characteristic impedance $W_o(1-k)$, from $Z_a(p)$ by Richards' process, the remaining two-terminal network $Z'_a(p)$ will be

$$\begin{aligned} Z'_a(p) &= Z_a(1) \frac{Z_a(p) - p Z_a(1)}{Z_a(1) - p Z_a(p)} \\ &= \frac{1 - W_o(1-k)p(Y_{11} + Y_{12})}{(Y_{11} + Y_{12}) - p/W_o(1-k)} \sim \frac{1 - W_o(1+k)p(Y_{11} - Y_{12})}{1 - W_o(1+k)p(Y_{11} - Y_{12})} \\ &= \frac{1 - 2W_o p(Y_{11} - kY_{12}) + W_o^2(1-k^2)p^2(Y_{11}^2 - Y_{12}^2)}{Y_{11}(1 + \frac{1+k}{1-k}p^2) + Y_{12}(1 - \frac{1+k}{1-k}p^2) - \frac{p(1+k)}{W_o(1-k)} - W_o(1+k)p(Y_{11}^2 - Y_{12}^2)} \\ &= \frac{1}{Y'_{11}(p) + Y'_{12}(p)} \end{aligned} \quad (7.7)$$

Similarly, extract a unit coaxial $W_o(1+k)$ from $Z_b(p)$, then the remainder will be

$$\begin{aligned} Z_b'(p) &= Z_b(l) \frac{Z_b(p) - pZ_b(l)}{Z_b(l) - pZ_b(l)} \\ &= \frac{1}{Y_{11}'(p) - Y_{12}'(p)} \end{aligned} \quad (7.8)$$

These values of $Z_a'(p)$ and $Z_b'(p)$ are identical with those element values that constitute a symmetrical lattice network representing the remaining network $[Y'(p)]$ after the extraction of a two-wire line from $[Y(p)]$. This fact means that the operation of extracting a coupled 2-wire line ($W, d = 1, k$) from a symmetrical network $[Y(p)]$ can be replaced by the operation of extracting coaxial elements $W_o(1-k)$ and $W_o(1+k)$ from the elements of the equivalent lattice network.

It is simpler to treat the matter in a lattice network and facilitates understanding.

7.3. Design examples (26)(27)(40-43)

The design procedure of symmetrical networks will go on as follows:

- (1) Give network parameters ($A, B, C; Y_{11}, Y_{12}$; or Z_{11}, Z_{12})
- (2) Obtain elements Z_a, Z_b of the equivalent lattice network
- (3) Realize Z_a and Z_b in bar-type by Richards' method (Eqs. (7.7), (7.8). TABLE 7.1)
- (4) Determine the line constants of the coupled two-wire line from the corresponding line elements of Z_a and Z_b . (Eq. (7.6))

[Example 1] Design with Q-functions.

Assume the given conditions that the deviations of input and output terminal resistances should be within 5% in the range $0 \sim 0.8207$ of the frequency ω ($\omega = \tan \beta l$), and that the attenuation (α_0) should be greater than 4 nepers in the frequency range $1.2185 \sim \infty$. Here one should take the Q-functions

$$\frac{Z_o}{R} = \sqrt{\frac{0.665}{1.57}} \frac{p^2 + 1.57}{\sqrt{p^2 + 1}}, \quad \tanh \frac{\theta_o}{2} = \frac{P}{\sqrt{1.57 \times 0.665}} \frac{\sqrt{p^2 + 1}}{p^2 + 0.637}$$

from which Z_a and Z_b are determined:

$$Z_a = Z_o \tanh \frac{\theta}{2} = R \frac{p(p^2 + 1.57)}{1.57(p^2 + 0.637)}$$

$$Z_b = Z_o \coth \frac{\theta}{2} = R 0.665 \frac{(p^2 + 0.637)(p^2 + 1.57)}{p(p^2 + 1)}$$

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The bar-type networks will be obtained as shown in Fig. 7.3, referring to the second line of TABLE 7.1(a) and the third line of TABLE 7.1(b). That is:

$$W_{a1} = R, W_{a2} = 0.285 R, W_{a3} = 0.285 R, W_{a4} = 0$$

$$W_{b1} = 1.4R, W_{b2} = 1.355 R, W_{b3} = 19.15 R, W_{b4} = 19.15 R$$

From these values, one can determine the line constants of the coupled 2-wire lines:

$$W_{o1} = \frac{1}{2} (W_{a1} + W_{b1}) = \frac{1}{2} (1 + 1.4) R = 1.2 R$$

$$k_1 = \frac{W_{b1} - W_{a1}}{W_{b1} + W_{a1}} = \frac{1.4 - 1}{1.4 + 1} = 0.167$$

$$W_{o2} = 0.82R, W_{o3} = 9.72R, W_{o4} = 9.58R$$

$$k_2 = 0.652, k_3 = 0.97, k_4 = 1$$

In Fig. 7.3 are shown the network structure and the frequency characteristics of Z_o and α . ($\theta_o = \alpha_o + j\beta_o$).

[Example 2] A network of Wagner character.

The effective attenuation α of a network of Wagner character $n = 5$ is

$$e^{2\alpha} = 1 - p^{10}$$

and the network parameters are

$$A = (1 + \sqrt{5}) p^4 + (3 + \sqrt{5}) p^2 + 1$$

$$B = Rp \left\{ (3 + \sqrt{5}) p^2 + (1 + \sqrt{5}) \right\}$$

Thus the arms of the equivalent lattice are

$$Z_a = \frac{\beta}{A-1} = \frac{1+\sqrt{5}}{2} \frac{p}{p^2-1}$$

$$Z_b = \frac{\beta}{A+1} = \frac{\frac{1+\sqrt{5}}{2} p^2 + 1}{p(p^2 + \frac{1+\sqrt{5}}{2})}$$

Represent these into bar-type by TABLE 7.1, one has Z_a and Z_b in Fig. 7.4, and after converting the network into coupled line type, he will have the desired one. The structure and the characteristics are shown in Fig. 7.4.

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Example 3 A network of Tchebycheff characteristic with no attenuation poles.

The attenuation for $n = 5$ is given by

$$\epsilon^{2a} = 1 + \frac{1}{2} + \frac{1}{2} \cos(10 \cos^{-1} w)$$

where the maximum attenuation in the pass band is taken to be 3db. Network parameters are obtained as

$$A = 9.1749 p^4 + 8.7649 p^2 + 1$$

$$C = \frac{P}{R} (32 p^4 + 42.306 p^2 + 11.486)$$

from which one can determine

$$Z_a = R \frac{0.7607 p}{2.6533 p^2 + 1}, \quad Z_b = R \frac{7.6179 p^2 + 1}{p (26.346 p^2 + 7.308)}$$

One can develop these Z_a and Z_b into bar-type ones and deduce the coupled line network as shown in Fig. 7.5, with $R = 1$ ohm. In the figure is also shown its characteristic.

Example 4 Tchebycheff characteristic with attenuation poles.

Let the maximum attenuation be 0.5 db in the guaranteed pass band $\omega = 0$ 0.8207, and $n = 5$. Then one has

$$\epsilon^{2\psi} = 1 + \psi(p) \psi(-p)$$

$$\psi(p) = 10.512 p \frac{(p^2 + 0.6251)(p^2 + 0.2796)}{(0.6251 p^2 + 1)(0.2796 p^2 + 1)}$$

which yield

$$A = \frac{9.278 p^\psi + 8.107 p^2 + 1}{0.1748 p^\psi + 0.9046 p^2 + 1}$$

$$B = \frac{Rp (5.151 p^2 + 2.605)}{0.1748 p^2 + 0.9046 p^2 + 1}$$

From these values one can determine:

$$Z_a = \frac{5.151 p}{9.453 (p^2 + 0.3516)}$$

$$Z_b = \frac{5.151 p^2 + 2.605}{p(9.103 p^2 + 7.202)}$$

The network and its characteristic are shown in Fig. 7.6

CHAPTER 8. Design of antimetrical networks

The antimetrical networks are the most frequently used ones, next to symmetrical ones, and here will be examined their design procedures.

8.1 Extraction of coupled two-wire lines in antimetrical networks

The definition of an antimetrical network is, in terms of the four-terminal parameters, $B/C = R^2$ (R is constant). Rewrite this in Y -parameters, one obtains

$$Y_{11} Y_{22} - Y_{12}^2 = \frac{AD-1}{B^2} = \frac{C}{B} = \frac{1}{R^2} \quad (8.1)$$

If one extracts a line from a network satisfying this condition, he has, from a network satisfying this condition, he has, from Eqs. (6.6) and (6.7):

$$\begin{aligned} \frac{W_0}{R} &= \frac{1}{R} \frac{Y_{22}(l)}{Y_{11}(l) Y_{22}(l) - Y_{12}^2(l)} = R Y_{22}(l) = \frac{R}{W_0(d-\kappa^2)} \\ d &= \frac{Y_{11}(l)}{Y_{22}(l)}, \quad \kappa = \frac{Y_{12}(l)}{Y_{22}(l)} \end{aligned} \quad (8.2)$$

for line constants. As for the remaining circuit, one has, from Eq. (6.9):

$$\begin{aligned} Y_{11}'(p) &= \frac{1}{\nabla} \left[Y_{11}(1-p^2) - \frac{W_0 d}{R^2} p \left\{ 2 - W_0 p (Y_{11} + d Y_{22} - 2\kappa Y_{12}) \right\} \right] \\ Y_{22}'(p) &= \frac{1}{\nabla} \left[Y_{22}(1-p^2) - \frac{W_0}{R^2} p \left\{ 2 - W_0 p (Y_{11} + d Y_{22} - 2\kappa Y_{12}) \right\} \right] \\ Y_{22}'(p) &= \frac{1}{\nabla} \left[Y_{12}(1-p^2) - \frac{W_0 \kappa}{R^2} p \left\{ 2 - W_0 p (Y_{11} + d Y_{22} - 2\kappa Y_{12}) \right\} \right] \\ \nabla &= (1+p^2) - W_0 p (Y_{11} + Y_{22} - 2\kappa Y_{12}) \end{aligned} \quad (8.3)$$

8.2 The antimetry of $[Y']$ network

It is evident from Eq. (7.3) that a symmetrical network ($Y_{11} = Y_{22}$) remains symmetrical ($Y_{11}' = Y_{22}'$) after an extraction of a coupled two-wire line ($d = 1$).

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Also an L - type network ($Y_{11} = Y_{12}$) remains to be an L - type one ($Y_{11}' = Y_{12}'$) after an extraction of a coupled two-wire line ($d = \kappa$).

Let it be examined whether $[Y']$ remains antimetrical or not in case of an antimetrical network. Using the values Eq. (8.3), one has

$$\begin{aligned}
 & Y_{11}' Y_{22}' - Y_{12}'^2 \\
 &= \frac{\left\{ \begin{aligned} & \left[Y_{11} (1-p^2) - \frac{W_0}{R^2} d p \{ 2 - W_0 p (Y_{11} + dY_{22} - 2\kappa Y_{12}) \} \right] \\ & \times \left[Y_{22} (1-p^2) - \frac{W_0}{R^2} p \{ 2 - W_0 p (Y_{11} + dY_{22} - 2\kappa Y_{12}) \} \right] \\ & - \left[-Y_{12} (1-p^2) - \frac{W_0}{R^2} \kappa p \{ 2 - W_0 p (Y_{11} + dY_{22} - 2\kappa Y_{12}) \} \right]^2 \end{aligned} \right\}}{\left\{ \begin{aligned} & (1-p^2)^2 (Y_{11} Y_{22} - Y_{12}^2) + 4W_0^2 (d - \kappa^2) p^2 / R^4 \\ & + (Y_{11} + dY_{22} - 2\kappa Y_{12})^2 \{ W_0^4 (d - \kappa^2) p^4 / R^4 + W_0^2 p^2 (1-p^2) / R^2 \} \\ & - (Y_{11} + dY_{22} - 2\kappa Y_{12}) \{ 4W_0^3 (d - \kappa^2) p^3 / R^4 + 2W_0 p (1-p^2) / R^2 \} \end{aligned} \right\}} \\
 &= \frac{\{ (1+p^2) - 2W_0 p (1+p^2) (Y_{11} + dY_{22} - 2\kappa Y_{12}) + W_0^2 / 0^2 (Y_{11} + dY_{22} - 2\kappa Y_{12}) \}}{\left\{ \begin{aligned} & (1+p^2)^2 - 2W_0 p (1+p^2) (Y_{11} + dY_{22} - 2\kappa Y_{12}) + W_0^2 p^2 (Y_{11} + dY_{22} - 2\kappa Y_{12})^2 \\ & + \frac{W_0^2}{R^2} p^2 (Y_{11} + dY_{22} - 2\kappa Y_{12})^2 \end{aligned} \right\}} \\
 &= \frac{1}{R^2}
 \end{aligned} \tag{8.4}$$

Thus it is proved that $[Y']$ is also antimetrical. that is, extract a coupled two-wire line $[Y(1)]$ from an antimetrical network, then the remaining network $[Y']$

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is also antimetrical.

One may state in general, not only in antimetrical networks, that the extraction of a coupled 2 - wire line does not change the property of the network but preserves it.

8.3 Design examples (26) (27) (40~44)

[Example 1] Wager network, $n = 4$

The effective attenuation is, for $n = 4$,

$$\epsilon^2 = 1 + p^8$$

and the network parameters will be obtained, with input and output resistances of 1 ohm,

$$A = 2p^4 + (2 + \sqrt{2}) p^2 + 1$$

$$B = C = \sqrt{2} \sqrt{2 + \sqrt{2}} (p^2 + 1) p$$

$$D = (2 + \sqrt{2}) p^2 + 1$$

convert these into Y parameters:

$$Y_{11}(p) = \frac{D}{B} = \frac{(2 + \sqrt{2}) p^2 + 1}{\sqrt{2} \sqrt{2 + \sqrt{2}} (p^2 + 1) p}$$

$$Y_{22}(p) = \frac{A}{B} = \frac{2p^4 + (2 + \sqrt{2}) p^2 + 1}{\sqrt{2} \sqrt{2 + \sqrt{2}} (p^2 + 1) p}$$

$$Y_{12}(p) = \frac{1}{B} = \frac{1}{\sqrt{2} \sqrt{2 + \sqrt{2}} (p^2 + 1) p}$$

Put here $p = 1$, then from Eq. (8.2) one obtains

$$W_0 = Y_{22}(1) = 1.2274$$

$$d = Y_{11}(1) / Y_{22}(1) = 0.6882$$

$$\kappa = Y_{12}(1) / Y_{22}(1) = 0.1559$$

As for the remaining network, one has, from Eq. (8.3),

$$Y_{11}'(p) = \frac{2W_0^2 d^2 p^2 + 1}{(\sqrt{2} \sqrt{2 + \sqrt{2}} - 2W_0 d) p}$$

$$Y_{22}'(p) = \frac{2W_0^2 \kappa d p^2 + 1}{(\sqrt{2} \sqrt{2 + \sqrt{2}} - 2W_0 d) p}$$

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$$Y_{12}'(p) = \frac{2W_0^2 \kappa d p^2 + 1}{(\sqrt{2}\sqrt{2+\sqrt{2}} + \sqrt{2} - 2W_0 d)p}$$

which give line constants of the second coupled two-wire line by putting $p = 1$:

$$W_0' = Y_{22}'(1) = 1.1617$$

$$d' = Y_{11}'(1) / Y_{22}'(1) = 2.2612$$

$$\kappa' = Y_{12}'(1) / Y_{22}'(1) = 1.2329$$

and the remaining network becomes $Y_{11}'' = Y_{22}'' = Y_{12}'' = \infty$.

Thus one would obtain the network structure as shown in Fig. 8.1

Here is a problem $\kappa' > 1$, and this can not be realized by a 2-wire line over ground, owing to the restriction (6.36). The relation $\kappa' > 1$ (or $\kappa' > d'$) has a lumped equivalent of Brune section and no contrivance has been made to realize itself alone by a coupled line over ground; another structure should be taken into consideration. First, take out the shunt capacitance component from $Y_{22}(p)$:

$$Y_{22}(p) = \frac{2p^4 + (2 + \sqrt{2})p^2 + 1}{\sqrt{2}\sqrt{2+\sqrt{2}}(p^2 + 1)p} = \frac{2p}{\sqrt{2}\sqrt{2+\sqrt{2}}} + \frac{\sqrt{2}p^2 + 1}{\sqrt{2}\sqrt{2+\sqrt{2}}(p^2 + 1)p} = Y_c + Y_{22}(p)$$

and extract a coupled two-wire line from Y_{11} , Y_{22} , Y_{12} , the line constants will be,

with $p = 1$,

$$W_0 = Y_{22}(1) / \{ Y_{11}(1) Y_{22}(1) - Y_{12}^2(1) \}$$

$$= \sqrt{2 + \sqrt{2}} / \sqrt{2} = (1 + \sqrt{2}) / \sqrt{2 + \sqrt{2}}$$

$$d = Y_{11}(1) / Y_{22}(1) = (3 + \sqrt{2}) / (1 + \sqrt{2})$$

$$\kappa = Y_{12}(1) / Y_{22}(1) = 1 / (1 + \sqrt{2})$$

$[Y']$ of the remaining network becomes ∞ , from which one can obtain nothing. Take Z parameters instead, one has

$$Z_{11}(p) = \frac{Y_{22}}{Y_{11}Y_{22} - Y_{12}^2} = \frac{\sqrt{2}p^2 + 1}{\sqrt{2 + \sqrt{2}}p}$$

$$Z_{22}(p) = \frac{Y_{11}}{Y_{11}Y_{22} - Y_{12}^2} = \frac{(2 + \sqrt{2})p^2 + 1}{\sqrt{2 + \sqrt{2}}p}$$

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$$Z_{12}(p) = \frac{-Y_{12}}{Y_{11}Y_{22} - Y_{12}^2} = -\frac{1}{\sqrt{2+\sqrt{2}} p}$$

Z' of the remaining network * becomes

$$Z_{11}'(p) = Z_{22}'(p) = Z_{12}'(p) = 2/\sqrt{2+\sqrt{2}} p$$

which yields, with $p = 1$,

$$Z'(1) = 2/\sqrt{2+\sqrt{2}}$$

The network will take the form Fig. 8.3

[Example 2] Tchebycheff network without attenuation poles.

Let the permissible amplitude deviation be 3db, then, for $n = 4$, one has

$$\epsilon^{2a} = 1 + \frac{1}{2} + \frac{1}{2} \cos \left\{ 4 \cos^{-1} \frac{2\omega^2 + (\sqrt{2}-1)^2 - 1}{(\sqrt{2}-1)^2 + 1} \right\}$$

The network parameters for $R = 1$ will be found:

$$A = 7.8196 p^4 + 8.8241 p^2 + 1$$

$$D = 2.3910 p^2 + 1$$

$$B = C = p (4.3101 p^2 + 3.3489)$$

* Eq. (6.9), in Z parameters, goes into

$$\left. \begin{aligned} Z_{11}'(p) &= \frac{\left\{ \begin{aligned} &Z_{11}(1-p^2) - Wop \left\{ 1 - \frac{P}{Wo(d-\kappa^2)} (Z_{11}d + Z_{22} + 2\kappa Z_{12}) \right\} \\ &-\frac{P}{Wo(d-\kappa^2)} (Z_{11}Z_{22} - Z_{12}^2) \end{aligned} \right\}}{1 - \frac{P}{Wo(d-\kappa^2)} (Z_{11}d + Z_{22} + 2\kappa Z_{12}) + \frac{P^2}{Wo^2(d-\kappa^2)} (Z_{11}Z_{22} - Z_{12}^2)} \\ -Z_{12}'(p) &= \frac{\left\{ \begin{aligned} &Z_{22}(1-p^2) - Wop \left\{ 1 - \frac{P}{Wo(d-\kappa^2)} (Z_{11}d + Z_{22} + 2\kappa Z_{12}) \right\} \\ &-\frac{\kappa p}{Wo(d-\kappa^2)} (Z_{11}Z_{22} - Z_{12}^2) \end{aligned} \right\}}{\text{denominator of } Z_{11}'(p)} \end{aligned} \right\} \quad (8.5)$$

$$-Z_{12}'(p) = \frac{\left\{ \begin{array}{l} -Z_{12}(1-p^2) - W_0 \kappa p \left\{ 1 - \frac{1}{W_0(d-\kappa^2)} (Z_{11}d + Z_{22} + 2\kappa Z_{12}) \right\} \\ - \frac{\kappa p}{W_0(d-\kappa^2)} (Z_{11}Z_{22} - Z_{12}^2) \end{array} \right\}}{\text{denominator of } Z_{11}'(p)} \quad (8.5)$$

Thus Y parameters are obtained:

$$Y_{11}(p) = \frac{D}{B} = \frac{2.3916 p^2 + 1}{p (4.3101 p^2 + 3.3489)}$$

$$Y_{22}(p) = \frac{A}{B} = \frac{7.8196 p^4 + 8.8241 p^2 + 1}{p (4.3101 p^2 + 3.3489)}$$

$$Y_{12}(p) = \frac{1}{B} = \frac{1}{p (4.3101 p^2 + 3.3489)}$$

If one would try to extract a coupled two-wire line directly from the network, he will again come to a Brune Section. Decompose $Y_{22}(p)$:

$$Y_{22}(p) = 1.8026 p + \frac{2.7874 p^2 + 1}{p (4.3101 p^2 + 3.3489)} = Y_c + Y_{22}(p)$$

Extraction of a coupled two-wire from Y_{11} , Y_{22} , Y_{12} yields

$$W_0 = 2.4493, \quad d = 0.8953, \quad \kappa = 0.2640, \quad W' = 1.1492$$

The capacitance Y_c is formed by a line of $Y_c(1) = 1/W_c = 1.8026$

Thus the network may be realized as shown in Fig. 8.4

[Example 3] A Tchebycheff network with attenuation poles.

Let the maximum attenuation in the guaranteed pass band $p = j\omega$ 0.819036 be 0.5 db, the Y parameters for Tchebycheff character with poles, $n = 4$, will be given as follows:

$$Y_{11}(p) = \frac{0.71079 p^2 + 0.21409}{p (1.19230 p^2 + 0.68950)}$$

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$$Y_{22}(p) = \frac{2p^4 + 1.72762 p^2 + 0.21409}{p(1.19230 p^2 + 0.68950)}$$

$$Y_{12}(p) = \frac{0.10883 p^2 + 0.21409}{p(1.19230 p^2 + 0.68950)}$$

Proceeding in a similar way, as in the previous examples, one has

$$Y_{22}(p) = 1.67743 p + \frac{0.57083 p^2 + 0.21409}{p(1.19230 p^2 + 0.68950)} = Y_c + y_{22}(p)$$

Extract a coupled two-wire line from Y_{11} , Y_{22} , Y_{12} :

$$W_0 = 2.37592, \quad d = 1.17831, \quad \kappa = 0.41140$$

$$W' = 1.00228, \quad W_c = 1/1.67743$$

The network structure and the characteristic are shown in Fig. 8.5

[Example 4] Case $n = 6$

Y Parameters of a Wagner network, $n = 6$, are :

$$Y_{11}(p) = \frac{7.4641 p^4 + 7.4641 p^2 + 1}{p(3.8637 p^4 + 9.1416 p^2 + 3.8637)}$$

$$Y_{22}(p) = \frac{2p^6 + 7.4641 p^4 + 7.4641 p^2 + 1}{\text{denominator of } Y_{11}(p)}$$

$$Y_{12}(p) = \frac{1}{\text{denominator of } Y_{11}(p)}$$

First decompose $Y_{22}(p)$:

$$Y_{22}(p) = 0.51764 p + \frac{2.7320 p^4 + 5.46 p^2 + 1}{\text{denominator of } Y_{11}} = Y_c + y_{22}(p)$$

The coupled two-wire line to be extracted from Y_{11} , Y_{22} , Y_{12} should have, from Eq. (6.7),

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$$W_0 = \frac{y_{22}(l)}{Y_{11}(l) y_{22}(l) - Y_{12}^2(l)} = 1.0664$$

$$d = \frac{Y_{11}(l)}{y_{22}(l)} = 1.7322, \quad \kappa = \frac{Y_{12}(l)}{y_{22}(l)} = 0.1087$$

$[Y']$ of the remaining network will be

$$Y_{11}'(p) = \frac{0.9880 p^2 + 0.84607}{p} = 0.8880 p + \frac{0.1 p^2 + 0.84607}{p} = Y_{c11}' + y_{11}'$$

$$Y_{22}'(p) = \frac{1.9039 p^2 + 0.84607}{p} = 1.8039 p + \frac{0.1 p^2 + 0.84607}{p} = Y_{c22}' + y_{22}'$$

$$Y_{12}'(p) = \frac{0.4579 p^2 + 0.84607}{p} = 0.5579 p + \frac{-0.1 p^2 + 0.84607}{p} = Y_{c12}' + y_{12}'$$

The first terms of the right hand sides of the above expressions make a coupled two-wire line open-circuited at the other end:

$$W_{oc} = \frac{Y_{c22}'(l)}{Y_{c11}'(l) Y_{c22}'(l) - Y_{c12}'(l)^2} = 1.3977$$

$$d_c = \frac{Y_{c11}'(l)}{Y_{c22}'(l)} = 0.4923, \quad \kappa_c = \frac{Y_{c12}'(l)}{Y_{c22}'(l)} = 0.3093$$

The second terms y_{11}' , y_{22}' , y_{12}' correspond to a coupled two-wire line short-circuited at the other end:

$$W_{sc} = \frac{y_{22}'(l)}{y_{11}'(l) y_{22}'(l) - y_{12}'(l)^2} = 2.7955$$

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$$d = \frac{y_{11}'(1)}{y_{22}'(1)} = 1, \quad \kappa' = \frac{y_{12}'(1)}{y_{22}'(1)} = 0.7886$$

The network configuration is depicted in Fig. 8.6

Chapter 9. Frequency transformations

Synthesis of networks with prescribed characteristic functions are chiefly treated on symmetrical and antisymmetrical networks, so that Chapters 7 and 8 will suffice the matter. Other networks may also be synthesized but will not be discussed here.

In Chapters 7 and 8, for the sake of simplicity of investigations, only L. P. F.'s were discussed with cutoff frequency $\omega_1 = 1$. Almost the same configurations will give L. P. F.'s with different cutoff frequencies and also B. E. F., as will be examined here.

9.1. The reference network and frequency transformation.

The L. P. F. for $\omega_1 = 1$ takes the form Fig. 9.1 as described in the examples, Chapters 7 and 8. To have the required frequency transformation, one should use, just as in lumped networks,

$$\left. \begin{array}{l} \text{in L. P. F.} \quad p \rightarrow p/\omega \\ \text{in B. E. F.} \quad p \rightarrow \left\{ \frac{\omega_0}{\Delta} \left(\frac{p}{\omega_0} + \frac{\omega_0}{p} \right) \right\}^{-1} \\ \Delta = \omega_2 - \omega_1, \quad \omega_0 = \sqrt{\omega_1 \omega_2} \end{array} \right\} \quad (9.1)$$

Here ω_1, ω_2 are cutoff frequencies,

$$\omega_1 = \tan(2\pi \text{fil}/c) \quad (9.2)$$

One have to synthesize the network by the use of network parameters applying the new frequency variable Eq. (9.1) to those of the reference network (L. P. F. with $\omega_1 = 1$). One should not apply frequency transformations to each element, what could be done in lumped networks. The frequency transformation should be applied to network parameters. This is due to the fact that there appear unit coaxials

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or coupled two-wire lines in cascade, that are used to construct the network.

9.2. Frequency transformations of low pass filters

Frequency transformations of low pass filters will be illustrated on Wagner networks for example. The frequency transformation is

$$p \rightarrow p/\omega_1, \quad \omega_1 \text{ is the cutoff frequency}$$

Symmetrical networks

Wagner characteristic functions will yield parameters of symmetrical networks as given in TABLE 9.1 (a). Obtain the elements Z_a and Z_b of the equivalent symmetrical lattice:

$$\frac{Z_a}{R} = \frac{A-1}{C}, \quad \frac{Z_b}{R} = \frac{A+1}{C} \quad (9.3)$$

which are shown in TABLE 9.1 (b). Apply frequency transformation $p \rightarrow p/\omega_1$ to these values of the reference network, TABLE 9.1 (c) will be obtained. Make bar-type networks, from these values, by Richards' method (TABLE 7.1), TABLE 9.1 (d) comes out. Line constants ω_{a1} and ω_{b1} of each element in the bar-type networks have values shown in the TABLE; determine coupled two-wire lines respective ω_{a1} and ω_{b1} ; the network will take the form Fig. 9.2. ω_1 and κ_1 in this figure can be obtained putting the values of ω_{a1} and ω_{b1} given in TABLE 9.1 (d). ω_1 and κ_1 are dependent on the cutoff frequency ω_1 , as their expressions contain ω_1 . The Wagner attenuation characteristics are shown in Fig. 9.3.

[Antimetric networks]

Here the frequency transformation will be applied to Y parameters.

The parameters of the reference networks (L. P. F. with $\omega_1 = 1$) of antimetric Wagner characteristics are given as shown in TABLE 9.2 (a), and consequently their Y parameters are obtained as shown in TABLE 9.2 (b). An application of the frequency transformation $p \rightarrow p/\omega_1$ to these parameters gives their values as shown in TABLE 9.2 (c). It is taken that R , defined by the condition of antimetry $B/C = R^2$, is unity.

(Case $n = 2$)

From TABLE 9.2 (c), one has, for $n = 2$,

$$Y_{11}(p) = \frac{1}{\frac{\sqrt{2}}{\omega_1} p}, \quad Y_{22}(p) = \frac{2 \frac{p^2}{\omega_1^2} + 1}{\frac{\sqrt{2}}{\omega_1} p}, \quad Y_{12}(p) = \frac{1}{\frac{\sqrt{2}}{\omega_1} p}$$

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As for the coupled two-wire line, putting $p = 1$, one has

$$\omega_0 = Y_{22}(1) = \frac{\sqrt{2}}{\omega_1} - \frac{\omega_1}{\sqrt{2}}, \quad d = \frac{Y_{11}(1)}{Y_{22}(1)} = \frac{\omega_1^2}{2 + \omega_1^2}$$

$$\kappa = \frac{Y_{12}(1)}{Y_{22}(1)} = \frac{\omega_1^2}{2 + \omega_1^2}$$

where the relation $d = \kappa$ requires a double coaxial structure. As to the remaining network,

$$Y_{11}' = Y_{22}' = Y_{12}' = \infty$$

In Z parameters

$$Z_{11}'(0) = Z_{22}'(0) = Z_{12}'(0) = \infty$$

Therefore the network structure will be shown in Fig. 9.4.

(Case $n = 4$)

From TABLE 9.2(c), one has, for $n = 4$,

$$Y_{11}(p) = \frac{(2 + \sqrt{2}) \frac{p^2}{\omega_1^2} + 1}{\sqrt{2} \sqrt{2 + \sqrt{2}} \frac{p}{\omega_1} \left(\frac{p^2}{\omega_1^2} + 1 \right)}$$

$$Y_{22}(p) = \frac{2 \frac{p^4}{\omega_1^4} + (2 + \sqrt{2}) \frac{p^2}{\omega_1^2} + 1}{\sqrt{2} \sqrt{2 + \sqrt{2}} \frac{p}{\omega_1} \left(\frac{p^2}{\omega_1^2} + 1 \right)}$$

$$Y_{12}(p) = \frac{1}{\sqrt{2} \sqrt{2 + \sqrt{2}} \frac{p}{\omega_1} \left(\frac{p^2}{\omega_1^2} + 1 \right)}$$

Decompose Y_{22} :

$$Y_{22}(p) = \frac{\sqrt{2}}{\sqrt{2} \sqrt{2}} \frac{p}{\omega_1} + \frac{\sqrt{2} \frac{p^2}{\omega_1^2} + 1}{\sqrt{2} \sqrt{2 + \frac{p^2}{\omega_1^2}} \frac{p}{\omega_1} \frac{p^2}{\omega_1^2} + 1}$$

$$= Y_c + y_{22}(p)$$

The line constant of Y_c , the shunt capacitance, is

$$\omega_c = \frac{1}{Y_c(l)} = \frac{\sqrt{2 + \sqrt{2}}}{\sqrt{2}} \omega_1$$

As for the coupled two-wire line, to be extracted from Y_{11} , Y_{22} , Y_{12} , one obtains

$$\omega_0 = \frac{y_{22}(l)}{Y_{11}(l) Y_{22}(l) - Y_{12}^2(l)} = \frac{\sqrt{2} + \omega_1^2}{\sqrt{2 + \sqrt{2}} \omega_1}$$

$$d = \frac{Y_{11}(l)}{y_{22}(l)} = \frac{(2 + \sqrt{2}) + \omega_1^2}{\sqrt{2} + \omega_1^2}$$

$$\kappa = \frac{Y_{12}(l)}{y_{22}(l)} = \frac{\omega_1^2}{\sqrt{2} + \omega_1^2}$$

The remaining network is an open-circuited line with

$$\omega' = \frac{\omega_1 (1 + \omega_1^2)}{\sqrt{2} + \sqrt{2}}$$

The network structure is shown in Fig. 9.5.

The Case $n = 6$ may be treated likewise as already shown in Chapter 8 [Example 4], but will not be cited here to avoid complication. Antimetrical L. P. F. have, in general, the same form as the reference one, only the line constants are different for different ω_1 . Curves of attenuation are shown in Fig. 9.6

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9.3 Frequency transformations of band stop filters.

In band stop filters, one should make use of the frequency transformation

$$p \rightarrow \left\{ \frac{\omega_o}{\Delta} \left(\frac{p}{\omega_o} + \frac{\omega_o}{p} \right) \right\}^{-1}$$

$$\Delta = \omega_2 - \omega_1, \quad \omega_o = \sqrt{\omega_1 \omega_2}$$

ω_1 and ω_2 are the cutoff frequencies. Here again those of Wagner characters will be taken for examples.

[Symmetrical networks]

An application of the above frequency transformation to Z_a and Z_b of TABLE 9.1(b) gives values shown in TABLE 9.3(a). Their bar-type representations, due to TABLE 7.1, will be obtained as shown in TABLE 9.3(b); $W_{a'}$ and $W_{b'}$ will yield coupled two-wire lines as shown in Fig. 9.7. Symmetrical B. E. F. can be obtained generally in this form.

[Antimetrical networks]

The matter will be explained on Wagner B. E. F., $n = 4$, as an example.

Network parameters of the reference L. P. F., $n = 4$, are given, from TABLE 9.2(b):

$$Y_{11}(p) = \frac{(2 + \sqrt{2}) p^2 + 1}{\sqrt{2} \sqrt{2 + \sqrt{2}} (p^2 + 1) p}$$

$$Y_{12}(p) = \frac{1}{\sqrt{2} \sqrt{2 + \sqrt{2}} (p^2 + 1) p}$$

$$Y_{22}(p) = \frac{2p}{\sqrt{2} \sqrt{2 + \sqrt{2}} (p^2 + 1)} + \frac{\sqrt{2} p^2 + 1}{\sqrt{2} \sqrt{2 + \sqrt{2}} (p^2 + 1) p} = Y_c + y_{22}(p)$$

The frequency transformation $p \rightarrow \Delta p / (p^2 + \omega_o^2)$ will be applied herein; first one has:

$$Y_c(p) = \frac{\sqrt{2}}{\sqrt{2 + \sqrt{2}}} \frac{\Delta p}{(p^2 + \omega_o^2)}$$

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This consists of two lines W_{c1} and W_{c2} , of which

$$Y_c(l) = \frac{1}{W_{c1}} = \frac{\sqrt{2} \Delta}{\sqrt{2+\sqrt{2}} (1 + \omega_o^2)}$$

Applying Richards' procedure to $Y_c(p)$:

$$Y_c'(p) = \frac{1}{W_{c1}} \frac{Y_c(p) - p/W_{c1}}{1/W_{c1} - pY_c(p)} = \frac{p}{W_{c1}\omega_o^2}$$

$$Y_{c1}'(l) = \frac{1}{W_{c2}} = \frac{1}{W_{c1}\omega_o^2} = \frac{\sqrt{2} \Delta}{\sqrt{2+\sqrt{2}} \omega_o^2 (1 + \omega_o^2)}$$

Rewrite the Y parameters in Z parameters:

$$Z_{11}(p) = \frac{\sqrt{2} p^2 + 1}{\sqrt{2+\sqrt{2}} p}, \quad Z_{12}(p) = - \frac{1}{\sqrt{2+\sqrt{2}} p}$$

$$Z_{22}(p) = \frac{(2+\sqrt{2}) p^2 + 1}{\sqrt{2+\sqrt{2}} p}$$

Apply the frequency transformation:

$$Z_{11}(p) = \frac{(p^2 + \omega_o^2)^2 + \sqrt{2} \Delta^2 p^2}{\sqrt{2+\sqrt{2}} \Delta p (p^2 + \omega_o^2)}$$

$$Z_{22}(p) = \frac{(p^2 + \omega_o^2)^2 + (2+\sqrt{2}) \Delta^2 p^2}{\sqrt{2+\sqrt{2}} \Delta p (p^2 + \omega_o^2)}$$

$$Z_{12}(p) = \frac{-(p^2 + \omega_o^2)^2}{\sqrt{2+\sqrt{2}} \Delta p (p^2 + \omega_o^2)}$$

Put $p = 1$, and obtain the coupled two-wire line to be extracted:

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$$W_o = Z_{11}(i) = \frac{(1 + \omega_o^2)^2 + \sqrt{2} \Delta^2}{\sqrt{2 + \sqrt{2}} \Delta (1 + \omega_o^2)}$$

$$W_{od} = Z_{22}(l) = \frac{(1 + \omega_o^2)^2 + (2 + \sqrt{2}) \Delta^2}{\sqrt{2 + \sqrt{2}} \Delta (1 + \omega_o^2)}$$

$$W_{ok} = -Z_{12}(l) = \frac{1 + \omega_o^2}{\sqrt{2 + \sqrt{2}} \Delta}$$

The remaining part of the network has parameters, by Eq. (8.5),

$$Z_{11}'(p) = \frac{\left[(\sqrt{2} - 1)p^4 \Delta^2 (1 + \omega_o^2) + \left\{ \sqrt{2} \Delta^4 \omega_o^2 + \Delta^2 (1 + \omega_o^2) (2\omega_o^4 + \sqrt{2} \omega_o^2) \right\} \right. \\ \left. + \omega_o^4 (1 + \omega_o^2)^3 \right] p^2 + \omega_o^6 (1 + \omega_o^2) \{ \Delta^2 + (1 + \omega_o^2)^2 \}}{\sqrt{2 + \sqrt{2}} \omega_o^2 (1 + \omega_o^2) \Delta p \{ (1 + \omega_o^2) p^2 + \omega_o^2 (1 + \omega_o^2) + \Delta^2 \omega_o^2 \}}$$

$$Z_{22}'(p) = \frac{\left[(1 + \sqrt{2}) \Delta^2 (1 + \omega_o^2) p^4 + \left\{ (2 + \sqrt{2}) \Delta^4 \omega_o^2 + \Delta^2 (1 + \omega_o^2) \{ 2\omega_o^4 + (\sqrt{2} + 2) \omega_o^2 \} \right\} \right. \\ \left. + \omega_o^4 (1 + \omega_o^2)^3 \right] p^2 + \omega_o^6 (1 + \omega_o^2) \{ \Delta^2 + (1 + \omega_o^2)^2 \}}{\text{denominator of } Z_{11}'(p)}$$

$$Z_{12}'(p) = \frac{\left[-\Delta^2 (1 + \omega_o^2) p^4 + \left\{ \Delta^2 (1 + \omega_o^2) 2\omega_o^4 + \omega_o^4 (1 + \omega_o^2)^3 \right\} p^2 \right. \\ \left. + \omega_o^6 (1 + \omega_o^2) \{ \Delta^2 + (1 + \omega_o^2)^2 \} \right]}{\text{denominator of } Z_{11}'(p)}$$

Consequently the line constants of the coupled line to be extracted are

$$W' = Z_{11}'(l) = \frac{\sqrt{2} \Delta^2 \{ \omega_o^2 \Delta^2 + (1 + \omega_o^2)^2 \} + (1 + \omega_o^2) \Delta^2 (\omega_o^6 + 2\omega_o^4 - 1) + \omega_o^4 (1 + \omega_o^2)^4}{\sqrt{2 + \sqrt{2}} \omega_o^2 (1 + \omega_o^2) \Delta \{ (1 + \omega_o^2)^2 + \Delta^2 \omega_o^2 \}}$$

$$W'd' = Z_{22}'(l) = \frac{(2 + \sqrt{2}) \Delta^2 \{ \omega_o^2 \Delta^2 + (1 + \omega_o^2)^2 \} + (1 + \omega_o^2) \Delta^2 (\omega_o^6 + 2\omega_o^4 - 1) + \omega_o^4 (1 + \omega_o^2)^4}{\text{denominator of } W'}$$

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$$W'K' = -Z'_{12}(1) = \frac{(1 + \omega_o^2) \Delta^2 (\omega_o^6 + 2\omega_o^4 - 1) + \omega_o^4 (1 + \omega_o^2)^4}{\text{denominator of } W'}$$

The remainder $Z''(p)$ is

$$Z''_{11}(p) = \frac{\Delta^4 + \Delta^2 (1 + \omega_o^2)^3 + \omega_o^2 (1 + \omega_o^2)^4}{\sqrt{2 + \sqrt{2}} \omega_o^4 (1 + \omega_o^2) \Delta^3 \{ \Delta^4 + 2\Delta^2 (1 + \omega_o^2)^2 + (1 + \omega_o^2)^3 (1 + \omega_o^4) \}}$$

$$\times \frac{(1 + \omega_o^2) \{ \Delta^4 \omega_o^2 + \Delta^2 (1 + \omega_o^2) \omega_o^2 (1 + 2\omega_o^2) + 1 + \omega_o^4 (1 + \omega_o^2)^3 \} p^2 + \{ \Delta^2 \omega_o^2 + (1 + \omega_o^2)^2 \}}{p \{ \omega_o^2 \Delta^2 + (1 + \omega_o^2)^2 \}}$$

$$= Z''_{22}(p) = -Z''_{12}(p)$$

The coupled two-wire line to be extracted next has

$$W'' = Z''_{11}(1)$$

$$\begin{aligned} & \{ \Delta^4 + \Delta^2 (1 + \omega_o^2)^3 + \omega_o^2 (1 + \omega_o^2)^4 \} [(1 + \omega_o^2) \{ \Delta^4 \omega_o^2 (1 + \omega_o^2) (1 + 2\omega_o^2) \\ & + \omega_o^4 (1 + \omega_o^2)^3 + 1 \} + \{ \Delta^2 \omega_o^2 + (1 + \omega_o^2)^2 \}] \\ & = \frac{\sqrt{2 + \sqrt{2}} \omega_o^4 (1 + \omega_o^2) \Delta^3 \{ \Delta^4 + 2\Delta^2 (1 + \omega_o^2)^2 + (1 + \omega_o^2)^3 (1 + \omega_o^2 + \omega_o^4) \} \{ \omega_o^2 \Delta^2 + (1 + \omega_o^2)^2 \}}{\end{aligned}$$

$$d'' = \kappa'' = 1$$

The last element is, by Richards' procedure,

$$W''' = W'' \omega_o^2$$

Thus B. E. F., $n = 4$, has the structure Fig. 9.9.

One can go on the same way for $n \geq 6$, but will be omitted here.

[Case $\omega_o = 1$]

The frequency transformation that has been used in band stop filters is

$$\left\{ \frac{\omega_o}{\Delta} \left(\frac{p}{\omega_o} + \frac{\omega_o}{p} \right) \right\}^{-1}$$

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where $\Delta = \omega_2 - \omega_1$, $\omega_0^2 = \omega_1 \omega_2$. In case $\omega_0 = 1$, one has

$$\left\{ \frac{1}{\Delta} \left(p + \frac{1}{p} \right) \right\}^{-1} = \Delta \left(p + \frac{1}{p} \right)^{-1} = \frac{(\omega_2 - \omega_1) p}{1 + p^2} = \frac{1 - \omega_1^2}{\omega_1} \frac{p}{1 + p^2}$$

Put

$$p = j \tan \beta l, \quad \omega_1 = \tan \beta_1 l$$

then

$$\frac{1 - \tan^2 \beta_1 l}{\tan \beta_1 l} \cdot \frac{j \tan \beta l}{1 + (j \tan \beta l)^2} = j \frac{\tan 2\beta l}{\tan 2\beta_1 l}$$

This value will be represented as p'/ω_1' , if one put $j \tan 2\beta l = p'$, $\tan 2\beta_1 l = \omega_1'$; that is of the same for as p/ω_1 of the frequency variable of L. P. F. The only difference is that the length of the elements are twice as long ($2l$). Thus one can treat a B. E. F. as an L. P. F. when $\omega_0 = 1$.

9.4 Numerical examples of line constants.

It should be examined what values of line constants will come out constituting networks designed by the method above, on an example of L. P. F., $n = 5$.

The line constants of L. P. F. of Wagner character $n = 5$, as given by TABLE 9.1(d) and Fig. 9.2, are

$$W_1 = \frac{1}{2} \left\{ \frac{\frac{1+\sqrt{5}}{2} \omega_1}{1 + \omega_1^2} + \frac{\omega_1 \left(\omega_1 + \frac{1+\sqrt{5}}{2} \right)}{1 + \frac{1+\sqrt{5}}{2} \omega_1^2} \right\}$$

$$W_2 = \frac{1}{2} \left\{ \frac{\frac{1+\sqrt{2}}{2}}{\omega_1(1 + \omega_1^2)} + \frac{\left(\omega_1^2 + \frac{1+\sqrt{5}}{2} \right) \left(\frac{1+\sqrt{5}}{2} \omega_1 \phi + 2\omega_1^2 + \frac{1+\sqrt{5}}{2} \right)}{\frac{1+\sqrt{5}}{2} \omega_1 \left(1 + \frac{1+\sqrt{5}}{2} \omega_1^2 \right)} \right\}$$

$$W_3 = \frac{1}{2} \omega_1 \left\{ \omega_1 \phi + \frac{\phi}{1+5} \omega_1^2 + 1 \right\}$$

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$$\kappa_1 = \frac{1}{2W_1} \left\{ \frac{\omega_1 \left(\omega_1^2 + \frac{1+\sqrt{5}}{2} \right)}{1 + \frac{1+\sqrt{5}}{2} \omega_1^2} - \frac{\frac{1+\sqrt{5}}{2} \omega_1}{1 + \omega_1^2} \right\}$$

$$\kappa_2 = \frac{1}{2W_2} \left\{ \frac{\left(\omega_1^2 + \frac{1+\sqrt{5}}{2} \right) \left(\frac{1+\sqrt{5}}{2} \omega_1^4 + 2\omega_1^2 + \frac{1+\sqrt{5}}{2} \right) \frac{1+\sqrt{5}}{2}}{\frac{1+\sqrt{5}}{2} \omega_1 \left(1 + \frac{1+\sqrt{5}}{2} \omega_1^2 \right) \omega_1 (1 + \omega_1^2)} \right\}$$

$$\kappa_3 = 1$$

Fig. 9.10 shows the variations of these values with respect to ω_1 . One should notice that the characteristic impedance of a shielded two-wire line can be made easily in the range from 10 to 200 ohms, but very difficult to be below or above the range. Assume the nominal impedance R , of the filters cited in the examples, be 50 ohms, then the range $W/R = 0.2 \sim 4$ is realizable without difficulty; this corresponds to $\tan^{-1} \omega_1 = 21 \sim 53^\circ$ from Fig. 9.10. If the given requirements go outside the above range, one cannot make line elements owing to the need of too thick or too thin conductors. Therefore it is necessary to examine the numerical values of the characteristic impedances in the actual design of filters; there may occur some cases where one can not build a network, even though the design may be theoretically possible. In this sense, one should consider several network configurations of the same characteristic, compare line constants and structures, and choose the best fit one for the purpose.

Fig. 9.11 and Fig. 9.12 show networks with the same characteristics as that of Fig. 9.10, but with different configurations. The network Fig. 9.11 is a parallel connection of two coupled two-wire lines which represent the first and second terms of the partial fraction expansions:

$$\frac{R}{Z_a} = \frac{2p}{(1+\sqrt{5}) \omega_1} + \frac{1}{\frac{1+\sqrt{5}}{2} \omega_1} p$$

$$\frac{R}{Z_b} = \frac{2p}{(1+\sqrt{5})\omega_1} + \frac{\frac{p}{\omega_1}}{1 + \frac{1+\sqrt{5}}{2} \frac{p^2}{\omega_1^2}}$$

given from TABLE 9.1(c). In the network Fig. 9.11, the range of easy building is $\tan^{-1} \omega_1 = 24^\circ 60'$, being wider than the preceding example.

The network Fig. 9.12 is the one that realizes the second terms of the above in a 3-wire line structure; here the range is $\tan^{-1} \omega_1 = 24^\circ 68'$, being even broader than the preceding ones.

As seen from the three examples cited, the range of practical realizability differs with the structure adopted, so that a designer should take various network structures into consideration.

Those with bandwidths especially narrow or wide can be made rather simply by other design procedures, as shown in Chapter 5.

9.5 Additional remarks

As described above, LPF and BEF have been shown to be deduceable easily from reference L. P. F. by frequency transformations. The problem, whether H. P. F. and B. P. F. can or cannot be deduced in the same manner, is difficult, and is left open to the future. The first point of the problem is in obtaining a series capacitance, which will need the use of a three-wire line (for example, the network TABLE 4.3). The next is that a negative coupling coefficient would appear (if one executes the extraction of two-wire lines in a simple manner), which cannot be realized by a two-wire line over the ground (including 2-core cables), and perhaps lines of more wire or 4-wire lines without ground will be needed. This situation is very troublesome from practical point of view, and will not be attacked any further. The author has his expectations on the ladder networks, Chapter 4 and narrow band filters, Chapter 5.

Thus one will notice that the design procedure based on the extraction of coupled two-wire lines is not almighty, but is only a means of network synthesis. The proof was made in general in Chapter 6, but if the line is restricted to be a coupled two-wire line over ground, the necessary condition of realizability is determined by the inequality (6.36). This can be understood because the coupled two-wire line is a network with a common return. One can synthesize at least 4 kinds of networks for one and the same characteristic in lumped networks. In coupled line networks, however, one can have only two variations in most cases. For example take the case of

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designing a symmetrical netowrk as in Chapter 7. If the grade of Z_b is less than that of Z_a , one can not realize the network unless he takes degenerate elements. One may have other restrications or disadvantages, but the proposal will be estimated in its value as presenting a method.

CHAPTER 10. Experimental examples

This chapter collects experiments made on coupled line type filters. The aims of the experiments are:

- (1) To check the principle of designing coupled line type filters, if it is correct or not.
- (2) To examine whether various network connections (compositing, duplexing, multi-terminal connections, etc.), carried out in lumped networks, is also valid or not in distributed networks.
- (3) Relations between line constants and line construction.
- (4) The effect of line resistances to network characteristics.
- (5) Problem of combinations with lumped elements.

Here are presented 6 experimental examples:

10.1. Band stop filter: This represents "simple networks made of coupled two-wire lines", and assures that the analysis of Chap. 2 is not mistaken. Also the effect of line resistance is examined in case of a narrow band.

10.2. Capacitance coupled narrow bandpass filter: This examines the analysis of "simple symmetrical networks made of 3-wire lines" Chap. 3, and checks also the "narrow band filters", Chap. 5. There are descriptions on line resistances, dimensions of line structure, and temperature dependence.

10.3. Two frequency separator: It is shown that, with the use of the above two networks, the numerical results calculated in the same manner are also valid for 6-terminal connections.

10.4. Inductance coupled bandpass filter: This is an example of Chap. 3 and Chap. 5, just as in 10.2, but here the point of significance is in the composite connection, and the combination with a lumped capacitance element.

10.5. Antimetrical narrow bandpass filter: This is an example of Chap. 5, antimetrical (special) networks.

10.6. Wagner lowpass filters: These represent the design procedures with the extraction of coupled two-wire lines, Chap. 6-9, and experiments are made on grades $n = 3$ and 5.

10.1 Bandstop filter ⁽¹⁸⁾

The top network of TABLE 2.12 is a simplest bandstop filter. It is easy to make the pass band narrower, and is useful to suppress the transmission of a particular single frequency.

(Image parameters) - Let

$$\omega_1 = 1 / \omega_{-1}, \omega_0 = 1$$

$$\theta = \Omega \omega_0 \epsilon i \quad (10.1)$$

With these values, and from TABLE 2.12, one has the image parameters

$$\left. \begin{aligned} Z_{o1} &= W_o \frac{\omega_{-1} (\omega^2 - 1)}{\sqrt{(\omega^2 - \omega_{-1}^2) (\omega^2 - 1/\omega_{-1}^2)}} \\ Z_{o2} &= W_o \frac{1}{\omega_{-1}} \sqrt{\frac{\omega^2 - \omega_{-1}^2}{\omega^2 - 1/\omega_{-1}^2}} \\ \tanh \theta_o &= j \omega \omega_{-1} \sqrt{\frac{\omega^2 - 1/\omega_{-1}^2}{\omega^2 - \omega_{-1}^2}} \end{aligned} \right\} \quad (10.2)$$

where, from the given conditions,

$$\omega_{-1} = \sqrt{1 - k^2}, \quad W = W_o (1 - k^2) \quad (10.3)$$

Specify W_o and ω_{-1} , then the line constants W_o , k and W will be determined from the above three equations. Fig. 10.1 shows the frequency characteristics of its image parameters.

(Effective attenuation) Let the input and output terminating resistances be R , then the effective attenuation α is given from

$$\epsilon^{2\alpha} = 1 + \frac{1}{4} \left\{ (A - D)^2 - \frac{\beta}{R} - CR \right\}^2 \quad (10.4)$$

Putting the relations

$$\left. \begin{aligned} A &= \sqrt{\frac{Z_{o1}}{Z_{o2}}} \cosh \theta_o, \quad \beta = \sqrt{Z_{o1} Z_{o2}} \sinh \theta_o \\ C &= \frac{1}{\sqrt{Z_{o1} Z_{o2}}} \sinh \theta_o, \quad D = \sqrt{\frac{Z_{o2}}{Z_{o1}}} \cosh \theta_o \end{aligned} \right\} \quad (10.5)$$

into Eq. (10.4), one obtains

$$\epsilon^{2\alpha} = 1 + \frac{1}{4} \left(\sqrt{\frac{Z_{o1}}{Z_o}} - \sqrt{\frac{Z_{o2}}{Z_o}} \right)^2 \cosh^2 \theta_o$$

$$\left(\frac{\sqrt{Z_{o1}Z_{o2}}}{R} - \frac{R}{Z_{o1}Z_{o2}} \right)^2 \sinh^2 \theta_o \left\{ \right. \\ \left. = 1 + \frac{1}{4} \left\{ \frac{\omega_{-1}^4 (\omega_{-1}^2 - 1)^2}{\omega_{-1}^4 (\omega^2 - 1)^2 (\omega^2 + 1)} + \frac{\omega^2 \left\{ (\omega^2 - 1) - \left(\frac{R}{W_o} \right)^2 (\omega^2 - 1 / \omega_{-1}^2) \right\}^2}{\left(\frac{R}{W_o} \right)^2 (\omega^2 - 1)^2 (\omega^2 + 1)} \right\} \right\} \quad (10.6)$$

Assume here

$$R / W_o = 1 \quad (10.7)$$

then the above expression reduces to

$$\epsilon^{2\alpha} = 1 + \frac{1}{4} \frac{\omega^2 (\omega_{-1}^2 - 1)^2 (\omega^2 - \omega_{-1}^2)}{\omega_{-1}^4 (\omega^2 - 1)^2 (\omega^2 + 1)} \quad (10.8)$$

(Equivalent network) The equivalent network has the form of a single phase line of characteristic impedance W_o with a resonant circuit in shunt at the input terminals. As a specific feature of the coupled line type network, the shunt impedance has k^2 in its denominator, and grows larger with smaller k . The cutoff frequency $\omega_{-1} = \sqrt{1-k^2}$ will in the meanwhile, draw nearer to 1, and the passband will be narrower. If one builds up the network, for the same requirements, only with coaxial lines, the bandwidth cannot be made so narrow, because only up to 200 ohms of $2W_o (1-k^2)$ is practicable as the shunt line. This corresponds to k down to 0.6. In the coupled line type, k may be made almost zero, so that it is advantageous for narrower bands. (Effect of line resistances) If the line elements have resistances, the characteristics will be affected, the most at the attenuation pole. As seen from the equivalent network, the shunt impedance at the input terminal will go down to zero at the attenuation pole and the transmission zero occurs, but if some resistance exists, the impedance cannot go perfectly to zero, only its reactance part can be zero, leaving some resistance part. In the state of Fig. 10.3, the output will be

$$P = \frac{E^2}{W_o \left(2 + \frac{W_o}{Y} \right)^2} \quad (10.9)$$

whereas the output for a matched load would be

$$P_o = E^2 / 4 W_o \quad (10.10)$$

Hence the attenuation is given by

$$[a]_{\omega = \omega_o} = 10 \log_{10} \frac{P}{P_o} = 10 \log_{10} \left(1 + \frac{W_o}{2Y} \right)^2 \quad (10.11)$$

The shunt impedance is

$$\frac{W_0(1-k^2)}{k^2} (j\omega \frac{1}{j\omega}) \quad (10.12)$$

Replace $j\omega$ by p , and further,

$$p = \tanh \gamma l = \tanh (\alpha' - j\beta)l$$

α' is the attenuation constant of the line. If this is small, then

$$p = \frac{\tanh \alpha' l - j \tan \beta l}{1 + \tanh \alpha' l j \tan \beta l} \approx \frac{\alpha' l + j \tan \beta l}{1 + \alpha' l j \tan \beta l} \\ \approx (\alpha' l - j\omega) / (1 - \alpha' l j\omega) \approx -l(1 - \omega^2) + j\omega \quad (10.13)$$

Use this value instead of $j\omega$ in Eq. (10.12), then the shunt impedance will be

$$\frac{W_0(1-k^2)}{k^2} \left\{ \alpha' l (1 - \omega^2) + j\omega \frac{1}{\alpha' l (1 - \omega^2) - j\omega} \right\} \quad (10.14)$$

Put herein $\omega = \omega_0 = 1$, the frequency of attenuation pole, then the resistance part γ becomes

$$\gamma \approx \frac{W_0(1-k^2)}{k^2} \quad 4\alpha' l = W_0 \frac{\omega_0^2 - 1}{1 - \omega_0^2} \quad 4\alpha' l \quad (10.15)$$

This value of γ becomes larger according as α' is the larger and $1 - \omega_0^2$ is the smaller (the passband is narrower).

α' depends upon the dimensions, structure and materials of the line; suppose the line is of a coaxial structure, and the loss is due only to resistance dissipation, then α' may be approximately obtained from the formula:

$$\alpha' = \frac{1}{2W} \frac{f_0 (M_c)}{10} \left\{ \frac{1}{\sqrt{\sigma_1}} + \frac{2}{D} + \frac{1}{\sqrt{\sigma_2}} + \frac{2}{d_1} \right\} \\ = \frac{f_0 (MC)}{436 \log_{10} (D/d_1)} \left\{ \frac{1}{2D\sqrt{\sigma_1}} + \frac{1}{2d_1\sqrt{\sigma_2}} \right\} \quad (10.16)$$

where σ_1 and σ_2 are the conductivities of the outer and the inner conductors, D and d_1 are the radii of the outer and the inner conductors. Lastly, one has

$$[\alpha]_{\omega = \omega_0} = 20 \log_{10} \left(1 + \frac{1 - \omega_0^2}{\omega_0^2 - 1} \right) \frac{1}{8\alpha' l} \quad (10.17)$$

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(Design example) The requirements on design are taken to be: center frequency $f_0 = 184$ Mc, cutoff frequency $f_1 = 182.8$ Mc, nominal impedance $r = W_0 = 57$ ohms, attenuation peak $(\alpha) \omega = \omega_0 > 20$ dB.

From

$$\omega_0 = \tan \frac{2\pi f_0 l}{c} \quad l = 1$$

one has the length of the elements

$$l = \frac{c}{2\pi f_0} \tan^{-1} = \frac{3 \times 10^{10} \times \pi / 4}{2\pi \times 184 \times 10^6} = 2.4 \text{ cm}$$

The values of ω at the cutoffs are

$$\omega_{-1} = \tan \left(\frac{2\pi \times 182.8 \times 10^6}{3 \times 10^{10}} \times 20.4 \right) = 0.99$$

$$\omega_2 = 1 / \omega_{-1} = 1.01$$

Consequently, from Eq. (10.3)

$$k = \sqrt{1 - \omega_{-1}^2} = 0.1411$$

The values W_0 and k may be expressed, in terms of the geometrical dimensions of the 2-core cable, as follows:

$$\left. \begin{aligned} W_0 &= 138 \log_{10} (D/d_1) \\ k &= \log \frac{D}{d_1} \\ d_2 &= \frac{D^2 - h^2}{D^2} d_1 \end{aligned} \right\} \quad (10.18)$$

where the coupling between the two conductors is assumed to be small. Put the values of W_0 and k , already determined, into these relations, one obtains

$$D/d_1 = 2.59, D/h = 1.14, D/d_2 = 14.3$$

Let the outer conductor be of brass and the inner of copper.

$$\sigma_1 = 1.5 \times 10^7 \text{ v/m}, \quad \sigma_2 = 6.0 \times 10^7 \text{ v/m} \quad (10.19)$$

then $2D > 38$ mm in order to meet $[a]_{\omega = \omega_0} > 20$ dB.

Fig. 10.5 shows the network structure made for trial. For the so made network, it is estimated that

$$\alpha' = 7.8 \times 10^{-4} \text{ nep/m}$$

$$\alpha \omega = \omega_0 = 23.8 \text{ dB}$$

Fig. 10.6 shows the measured and the calculated values of α .

10.2 Capacitance-coupled narrow bandpass filters.

The networks in Paragraph 3.10 and Fig. 5.8 become bandpass filters of capacitance-coupled type. They are suited for narrow bands, and will be examined in details.

(Expressions of the characteristics) Those values may be used that are given in 3.10, but the case $d_{12}=1$ will be considered in particular. Putting

$$[I_0] = 0, V_2 / -I_2 = W_p \quad (10.20)$$

into equations (1.13) of a multi-wire line, one has

$$\left. \begin{aligned} [V] &= [V_0] \cos \beta l \\ [I] &= [W]^{-1} [V_0] j \sin \beta l \end{aligned} \right\} \quad (10.21)$$

which yields

$$[V] = [W] [I] \frac{1}{p} \quad (10.22)$$

Define the characteristic impedance $[W]$ as:

$$[W] = W_{11} \begin{bmatrix} 1 & k_1 & k_2 \\ k_1 & 1 & k_1 \\ k_2 & k_1 & 1 \end{bmatrix} \quad (10.23)$$

then Z_a and Z_b of the equivalent lattice are, from Eq. (3.5),

$$\begin{aligned} Z_a &= \frac{V_1 - V_3}{I_1 - I_3} = W_{11} (1 - k_2) \frac{1}{p} \\ Z_b &= \frac{V_1 + V_3}{I_1 + I_3} = W_{11} \frac{W_{11}(1 + k_2 - 2k_1^2) + W(1 + k_2)p^2}{(W_{11} + Wp^2)p} \end{aligned} \quad (10.24)$$

Put, for simplicity,

$$W = W_{11} \quad (10.25)$$

and obtain the image parameters:

$$\begin{aligned}
 Z_o = \sqrt{Z_a Z_b} &= \frac{W_{11} \sqrt{1-k^2}}{p} \sqrt{\frac{p^2 (1 - \frac{2k_1^2}{1+k_2})}{p^2 - 1}} \\
 \tanh \frac{\theta_o}{2} &= \sqrt{\frac{Z_a}{Z_b}} = \sqrt{\frac{1-k_2}{1+k_2}} \sqrt{\frac{p^2 - 1}{p^2 (1 - \frac{2k_1^2}{1+k_2})}}
 \end{aligned} \quad (10.26)$$

The cutoff frequencies W_1 and W_2 are determined from these equations,

$$W_1^2 = 1 - \frac{2k_1^2}{1+k_2}, \quad W_2 = 1 \quad (10.27)$$

The effective attenuation is given, with the condition of symmetry in Eq. (10.4),

$$\epsilon^{2a} = 1 - \frac{1}{4} \left(\frac{\beta}{R} - CR \right)^2 \quad (10.28)$$

Substitute herein

$$B = \frac{2Z_a Z_b}{Z_b - Z_a}, \quad C = \frac{2}{Z_b - Z_a} \quad (10.29)$$

then one obtains

$$\begin{aligned}
 \epsilon^{2d} &= 1 - \left\{ \frac{Z_a Z_b - R^2}{(Z_b - Z_a)R} \right\}^2 \\
 &= 1 - \left[\frac{W_{11}(1-k_2^2) \left[\frac{R^2}{W_{11}^2(1-k_2^2)} p^4 - \left\{ 1 - \frac{R^2}{W_{11}^2(1-k_2^2)} \right\} p^2 - (1 - \frac{2k_1^2}{1+k_2}) \right]}{R_{2p} \{ k_{2p}^2 + (k_2 - k_1^2) \}} \right]^2
 \end{aligned} \quad (10.30)$$

If one takes, as the value of R ,

$$R = W_{11} \sqrt{1-k_2^2} \quad (10.31)$$

then he has

$$2a = 1 - (1 - k_2^2) \left[\frac{p^4 - (1 - \frac{2k_1^2}{1 - k_2})}{2p\{k_2 p^2 (k_2 - k_1^2)\}} \right]^2 \quad (10.32)$$

From this equation, the frequency W_0 , at which $a = 0$, is obtained

$$W_0^2 = \sqrt{1 - \frac{2k_1^2}{1 - k_2}} \quad (10.33)$$

The frequency W_∞ , of $a = \infty$, is given by

$$W_\infty^2 = 1 - \frac{k_1^2}{k_2} \quad (10.34)$$

(Approximate equations near the center frequency)

In case of a very narrow filter, the cutoff frequencies ω_1 , ω_2 and the center frequency are all very near to 1, so that k , must be very small:

$$1 \gg k_1^2 \quad (10.35)$$

In order that the attenuation pole W_∞ does not come close to the center frequency, k_1^2 / k_2 should have an adequate magnitude. Therefore it is necessary that

$$1 \gg k_2 \quad (10.36)$$

Consider a frequency

$$\omega = 1 + \Delta\omega \quad (10.37)$$

where $\Delta\omega$ is very small, of the order of k_1^2 or k_2 .

Here one has

$$p = j(1 + \Delta\omega), \quad p^2 = -(1 + \Delta\omega)^2 \approx -(1 + 2\Delta\omega) \quad (10.38)$$

with which the equations may be rewritten.

Eq. (10.26) becomes

$$Z_0 = \frac{W_{11}}{j} \sqrt{\frac{\Delta\omega k_1^2}{\Delta\omega}}, \quad \tanh \frac{\theta_0}{2} = \sqrt{\frac{\Delta\omega}{\Delta\omega k_1^2}} \quad (10.39)$$

and (10.32) becomes

$$\epsilon 2a = 1 + \left(\frac{k_1^2}{k_2} \frac{2\Delta\omega^2}{k_1^2} \right) \quad (10.40)$$

The values of $\Delta\omega$ corresponding to the cutoff and center frequencies are

$$\Delta\omega_1 = -k_1^2, \Delta\omega_2 = 0, \Delta\omega_0 = -k_1^2/2 \quad (10.41)$$

so that the bandwidth is

$$\Delta\omega_2 - \Delta\omega_1 = k_1^2 \quad (10.42)$$

and the width from the center frequency to the cutoff frequencies are

$$\Delta\omega_2 - \Delta\omega_0 = k_1^2/2, \Delta\omega_0 - \Delta\omega_1 = k_1^2/2 \quad (10.43)$$

The bandwidth may be considered also as $\pm k_1^2/2$.

Bring the origin of the frequency to the center frequency, and put

$$\Delta\omega' = \Delta\omega + k_1^2/2, \Delta\omega' / \frac{k_1^2}{2} \equiv \Omega \quad (10.44)$$

then one has

$$\begin{aligned} Z_0 &= W_{11} \sqrt{-\frac{\Delta\omega' - k_1^2/2}{\Delta\omega' - k_1^2/2}} = W_{11} \sqrt{-\frac{\Omega - 1}{\Omega - 1}} \\ \tanh \frac{\theta_0}{2} &= \sqrt{\frac{\Delta\omega' - k_1^2/2}{\Delta\omega' - k_1^2/2}} = \sqrt{\frac{\Omega - 1}{\Omega - 1}} \end{aligned} \quad (10.45)$$

$$\frac{2\alpha}{\epsilon} = 1 - \left(\frac{\Delta\omega'}{k_1^2/2} \right) = 1 + \Omega^2$$

These have the forms of Q' functions.

Fig. 10.8 and 10.9 show the characteristics with respect to Ω .

(Effect of line resistance) In order to examine the effect of line resistance on the network characteristics, let the propagation constant be $\gamma_1 = \alpha_1' + j\beta$, and make distinctions of the lines, then one can rewrite Eq. (10.22) as

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} W \\ 1 \end{bmatrix} \begin{bmatrix} I_1 \coth \gamma_1 l \\ -I_2 W_{11} \coth \gamma_2 l \end{bmatrix} \quad (10.46)$$

and consequently Z_a and Z_b becomes

$$\begin{aligned}
 Z_a &= W_{11} (1 - k_2) \coth \gamma_1 l \\
 Z_b &= W_{11} (1 + k_2) \coth \gamma_1 l - \frac{W_{11} 2k_1^2 \coth^2 \gamma_2 l}{\coth \gamma_2 l + \tanh \gamma_2 l}
 \end{aligned} \quad (10.47)$$

Here it is taken $\gamma_1 = \gamma_3$ because the circuit is symmetrical. Substitute these values in Eq. (10.30), the effective attenuation will be

$$\begin{aligned}
 \epsilon^{2\alpha} &= 1 + N \left(\frac{Z_a Z_b - R^2}{R (Z_b - Z_a)} \right) \\
 &= 1 + N \left(\sqrt{1 - k_2^2} \coth \gamma_1 l \frac{(\coth \gamma_1 l - \tanh \gamma_1 l) (\coth \gamma_2 l - \tanh \gamma_2 l) - \frac{2k_1}{1+k_2} \coth^2 \gamma_2 l}{2k_2 \coth \gamma_1 l (\coth \gamma_2 l + \tanh \gamma_2 l) - 2k_1^2 \coth^2 \gamma_2 l} \right) \quad (10.48)
 \end{aligned}$$

where N is the symbol of Norm, meaning the square of the magnitude inside the parenthesis. The value herein is taken that of Eq. (10.31).

Expand $\tanh \gamma l$ and $\coth \gamma l$ and neglect $(a' l)^2$ and higher terms.

$$\begin{aligned}
 \tanh (a' + j\beta) l &= \frac{\tanh a' l + j \tan \beta l}{1 - \tanh a' l \cdot j \tan \beta l} \approx \frac{a' l + p}{1 + a' l p} \\
 &\approx (a' l + p) (1 - a' l p) \approx p + a' l (1 - p^2) \\
 \coth (a' + j\beta) l &= \frac{1}{\tanh (a' + j\beta) l} \approx \frac{1 + a' l p}{a' l + p} \\
 &\approx (1 + a' l p) \frac{1}{p} (1 - a' l \frac{1}{p}) \approx \frac{1}{p} + a' l \frac{p^2 - 1}{p^2} \\
 \coth \gamma l - \tanh \gamma l &= p \frac{1}{p} - a' l \frac{(p^2 - 1)^2}{p^2} \\
 \coth \gamma l + \tanh \gamma l &\approx p + \frac{1}{p} + a' l \frac{p^4 - 1}{p^2} \\
 \coth^2 \gamma l &\approx \frac{1}{p^2} + 2a' l \frac{p^2 - 1}{p^3}
 \end{aligned} \quad (10.49)$$

Confine the examination only around the center frequency, then one may use Eq. (10.37), so that

$$\begin{aligned}
 \frac{1+p^2}{p} &\approx \frac{1-(1+2\Delta\omega)}{j(1+\Delta\omega)} \approx j2\Delta\omega \\
 \frac{1-p^2}{p} &\approx \frac{1+(1+2\Delta\omega)}{j(1+\Delta\omega)} \approx \frac{2}{j} \\
 \frac{1}{p^2} &\approx -\frac{1}{1+2\Delta\omega} \approx -(1-2\Delta\omega)
 \end{aligned}
 \tag{10.50}$$

With these values, one has

$$\begin{aligned}
 \coth \gamma l + \tanh \gamma l &\approx j2\Delta\omega + 4\alpha' l \\
 \coth \gamma l - \tanh \gamma l &\approx -j2 - 4\alpha' l \Delta\omega \approx -j2 \\
 \coth^2 \gamma l &\approx -(1-2\Delta\omega) - 4\alpha' l (1+2\Delta\omega) \approx -1
 \end{aligned}
 \tag{10.51}$$

Substitute these relations in Eq. (10.48), and making use of the condition $1 \gg k_2$, one will finally obtain

$$\begin{aligned}
 \epsilon^{2d} &= 1 + N \left(\sqrt{1} \frac{1}{j} - \frac{-j2(j2\Delta\omega + 4\alpha' l)}{2k_2 j(j2\Delta\omega - 4\alpha' l)} - \frac{\frac{2k_1^2}{1}(-1)}{2k_1^2(-1)} \right) \\
 &\approx 1 + N \left(\frac{4\Delta\omega - j8\alpha' l + 2k_1^2}{2k_2(j2\Delta\omega + 4\alpha' l) + j2k_1^2} \right) \\
 &\approx 1 + N \left(\frac{2\Delta\omega + k_1^2 - j4\alpha' l}{jk_1^2} \right) \\
 &= 1 + \left(\frac{\Delta\omega + \frac{k_1^2}{2}}{k_1^2/2} \right)^2 + \left(\frac{4\alpha' l}{k_1^2} \right)^2 \\
 &= 1 + \Omega^2 + \left(\frac{4\alpha' l}{\Delta\omega_1} \right)^2
 \end{aligned}
 \tag{10.52}$$

The third term of this equation is due to the line resistance, and is related to $\alpha' l$ as well as to $\Delta\omega_1$. Therefore this term grows larger as the loss of the second (resonant) wire is larger, or as $\Delta\omega_1$ (bandwidth) is smaller; even at the center frequency, a certain loss occurs

$$\left[\epsilon^{2d} \right]_{\omega = \omega_0} = 1 + \left(\frac{4\alpha' l}{\Delta\omega_1} \right)^2
 \tag{10.53}$$

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(Trial construction and experiments)

The given conditions for the design are:

- (1) center frequency f_o : 150 MC
- (2) nominal impedance R: 75 ohms
- (3) bandwidth $f_2 - f_1$: ± 45 kc

From the condition (1) of the center frequency, one has, from Eq. (10.37) and (10.41),

$$\omega_o = \tan(2\pi f_o l / c) = 1 + \Delta \omega_o = 1 - k_1^2 / 2 \approx 1$$

and consequently the length of the elements is determined:

$$l = \frac{c}{2\pi f_o} \tan^{-1} \omega_o \approx \frac{c}{2\pi f_o} \tan^{-1} 1 = \frac{c}{2\pi f_o} \frac{\pi}{4}$$

$$= \frac{3 \times 10^{10}}{8 \times 150 \times 10^6} = 25 \text{ cm}$$

As to the condition (2), one has, from Eq. (10.31),

$$R = W_{11} \sqrt{1 - k_2^2} \approx W_{11} = 75 \text{ ohms}$$

The condition (3) will now be taken into consideration. Eq. (10.42) may be rewritten:

$$\Delta \omega_2 - \Delta \omega_1 = (1 + \Delta \omega_2) - (1 + \Delta \omega_1) = \omega_2 - \omega_1$$

$$= \tan \frac{2\pi f_2}{c} l - \tan \frac{2\pi f_1}{c} l = k_1^2$$

On the other hand

$$\omega_i = 1 + \Delta \omega_i$$

$$= \tan \frac{2\pi f_i}{c} l = \tan \frac{2\pi f_o}{c} \frac{f_i}{f_o} l \approx \tan \frac{\pi}{4} \frac{f_o + \Delta f_i}{f_o} l$$

$$= \tan \frac{\pi}{4} \left(1 + \frac{\Delta f_i}{f_o} \right) = \frac{\tan \frac{\pi}{4} \tan \frac{\pi}{4} \frac{\Delta f_i}{f_o}}{1 - \tan \frac{\pi}{4} \tan \frac{\pi}{4} \frac{\Delta f_i}{f_o}} \approx \frac{1 + \frac{\pi}{4} \frac{\Delta f_i}{f_o}}{1 - \frac{\pi}{4} \frac{\Delta f_i}{f_o}}$$

$$\approx \left(1 + \frac{\pi}{4} \frac{\Delta f_i}{f_o} \right)^2 \approx 1 + \frac{\pi}{2} \frac{\Delta f_i}{f_o} \quad (10.54)$$

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From these relations, one obtains

$$\Delta \omega_2 - \Delta \omega_1 = \frac{\pi}{2} \frac{1}{f_0} (\Delta f_2 - \Delta f_1) = k_1^2$$

in which the bandwidth $\Delta f_2 - \Delta f_1 = 2 \times 45$ kc. Thus k_1 is determined from

$$k_1^2 = \frac{\pi}{2} \frac{2 \times 45 \times 10^3}{150 \times 10^6} = 3 \pi 10^{-4}$$

Since it is assumed that $k_2 \approx 0$ in the 3-wire line, the structure Fig. 10.11 (a) will be adopted, which would easily have very small k_2 . On the grounds that $k_2 \approx 0$, one may compute k_1 from the two-wire line in the figure, with the use of Eq. (10.18), resulting in

$$d_2 / D = 0.0102, h / D = 0.963, d_1 / D = 0.2865$$

Take

$$D = 150 \text{ mm}$$

Then the other dimensions should be

$$d_1 = 42.98 \text{ mm}, h = 144.36 \text{ mm}, d_2 = 3.174 \text{ mm}$$

As to the loss at the center frequency, one has, from Eqs. (10.16) and (10.19),

$$\alpha_{21} = 7.97 \times 10^{-4}$$

Putting this value into Eq. (10.53), $[a]_{\omega = \omega_0}$ is calculated to be

$$[a]_{\omega = \omega_0} = 2.80 \text{ dB}$$

The filter was constructed; Fig. 10.12 shows the structure, Fig. 10.13 the insertion loss characteristic, Fig. 10.14 the effect of the fine adjusting screw to the center frequency, Fig. 10.15 the change of the center frequency due to the temperature of the whole network.

The calculated and the measured values are in good accord, suggesting the correctness of the principle of the design.

10.3 A two-frequency wave separator

A wave separator was constructed with two narrow B.P.F. of the preceding paragraph. This experiment has been made to show that it is possible to calculate 6-terminal networks in distributed elements just as in lumped networks.

(Principle) Prepare two filters of the preceding paragraph, one with the center frequency at f_1 , and the other at f_2 , and connect them in parallel at the input. The output terminals are separated. Two frequencies f_1 and f_2 coming to the input simultaneously may be taken out separately from the output terminals. An additional shunt susceptance is connected at the input terminal so that one filter does not disturb the other. This idea resembles that of the wave separator with x-termination.

(6-terminal connection) Refer to Fig. 10.16. The two component networks have Y parameters:

$$[Y_2] = \begin{bmatrix} Y_{11'} & Y_{12} \\ Y_{12} & Y_{22} \end{bmatrix}, \quad [Y_3] = \begin{bmatrix} Y_{11''} & Y_{13} \\ Y_{13} & Y_{33} \end{bmatrix} \quad (10.55)$$

Connect them in parallel at the input terminals. Then the resulting 6-terminal network will have the Y parameters

$$[Y] = \begin{bmatrix} Y_{11'} + Y_{11''} & Y_{12} & Y_{13} \\ Y_{12} & Y_{22} & 0 \\ Y_{13} & 0 & Y_{33} \end{bmatrix} \quad (10.56)$$

Connect a correcting admittance Y_0 in parallel to the input terminals of this network, then the relation between the voltages and the currents will be

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_0 + Y_{11'} + Y_{11''} & Y_{12} & Y_{13} \\ Y_{12} & Y_{22} & 0 \\ Y_{13} & 0 & Y_{33} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \quad (10.57)$$

Connect a conductance G to each of the output terminals 2-2' and 3-3', one has

$$-E_2 G = I_2, \quad -E_3 G = I_3 \quad (10.58)$$

and consequently he can obtain the input admittance:

$$Y_{in} = \frac{I_1}{E_1} = Y_0 + (Y_{11'} - \frac{Y_{12}^2}{G + Y_{22}}) + (Y_{11''} - \frac{Y_{13}^2}{G + Y_{33}}) \quad (10.59)$$

The effective transfer constant will here be defined by the ratio of each of the separator output I_n to the current I_o that would flow into a matched load. $1/G$ from the same generator with an internal resistance $1/G$. Thus, from

$$I_n = \frac{E G^2}{1 + Y_{in}} \frac{Y_{in}}{G + Y_{in}} \quad \left. \vphantom{\frac{E G^2}{1 + Y_{in}}} \right\} \quad (10.60)$$

$$I_o = EG/2$$

one obtains

$$\begin{aligned} Y_{in} &= a_{in} + j\beta_{in} = \ln(I_o / I_n) \\ &= \ln \frac{G + Y_{in}}{G} + \ln \frac{G + Y_{nn}}{Y_{in}} \end{aligned} \quad (10.61)$$

(Input admittance) The Y parameters of the partial networks may be obtained from the image parameters of the capacitance-coupled BPF of the preceding paragraph, as given in Eq. (10.45); Thus from

$$\begin{aligned} Z_a &= Z_o \tanh \frac{\theta_o}{2}, \quad Z_b = Z_o \coth \frac{\theta_o}{2} \\ A &= \frac{Z_b + Z_a}{Z_b - Z_a}, \quad \beta = \frac{2Z_a Z_b}{Z_b - Z_a} \end{aligned} \quad (10.62)$$

one obtains

$$\begin{aligned} y_{11} &= y_{22} = \frac{A}{\beta} = \frac{j\Omega}{W_{11}(1+\Omega)} \\ y_{12} &= y_{21} = \frac{1}{\beta} = \frac{a}{W_{11}(1+\Omega)} \end{aligned} \quad (10.63)$$

If one makes two such BPF with center frequencies f_1 and f_2 respectively, he should have a certain technique to admit the use of a common frequency, because they have different Ω 's. Assume the difference of the center frequencies be small and take

$$f_o = (f_2 - f_1) / 2 \quad (10.64)$$

as the origin of the frequency, and assume also the unit of frequency to be

$$\Omega' = \frac{\Delta f}{\Delta f_1} = \frac{\Delta f}{\Delta f_2} = \frac{\Delta f}{\Delta f_o} \quad (10.65)$$

Represent also the distance from f_0 to f_1 and f_2 by

$$a = (f_2 - f_1) / 2 \Delta f_0 \quad (10.66)$$

Then the parameter Ω in each BPF may be replaced by

$$\Omega = \Omega' \pm a \quad (10.67)$$

and one can use the same frequency parameter Ω' common to both B. P. F. 's

The input admittances Y_{1in} and Y_{2in} of the two B. P. F. 's are now

$$\left. \begin{aligned} Y_{1in} &= Y_{11'} - \frac{Y_{22}^2}{G + Y_{22}} = jG \frac{2\Omega'^2 - j}{(1 + \Omega')^2 + \Omega'^2} \\ &= jG_1 \frac{2(\Omega' + a)^2 - j}{1 + 2(\Omega' + a) + 2(\Omega' + a)^2} \\ Y_{2in} &= Y_{11''} - \frac{Y_{13}^2}{G + Y_{33}} \\ &= jG \frac{2(\Omega' - a)^2 - j}{1 + 2(\Omega' - a) + 2(\Omega' - a)^2} \end{aligned} \right\} \quad (10.68)$$

where

$$W_{11} = 1 / G$$

(Correcting admittance) If two such B. P. F. 's are connected in parallel at the input terminals, the resultant input impedance is

$$\begin{aligned} Y_{in}' &= Y_{1in} + Y_{2in} \\ &= jG \frac{2(\Omega' + a)^2 - j}{1 + 2(\Omega' + a) + 2(\Omega' + a)^2} + jG \frac{2(\Omega' - a)^2 - j}{1 + 2(\Omega' - a) + 2(\Omega' - a)^2} \end{aligned} \quad (10.69)$$

Fig. 10.19 shows the dependence of Y_{1in} , Y_{2in} and Y_{in}' on Ω' .

As seen from the figure, the input admittance is not a pure conductance at frequencies f_1 ($\Omega' = -a$) and f_2 ($\Omega' = a$), but has a superfluous susceptance. The values are, putting $\Omega' = \pm a$ into the above expression,

$$Y_{in}' \quad \Omega' = \pm a = G \left(1 \pm \frac{1}{4a + 8a^2} \right) + jG \frac{8a^2}{1 \pm 4a + 8a^2} \quad (10.70)$$

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If a is large, one has

$$[Y_{in}]_{\Omega'} = \frac{1}{a} \cong G + jG \frac{8a^2}{1 + 4a + 8a^2} \quad (10.71)$$

The correcting admittance should act to cancel the susceptance of the above expression. It might be ideal, if it could cancel the susceptance over the whole passband. This being difficult, one considers good cancellations only at f_1 and f_2 , which requires

$$\left. \begin{aligned} [Y_o]_{\Omega'} &= -a = -jG \frac{8a^2}{1 - 4a + 8a^2} \\ [Y_o]_{\Omega'} &= a = jG \frac{8a^2}{1 + 4a + 8a^2} \end{aligned} \right\} \quad (10.72)$$

There are an infinite number of functions that satisfy these conditions, but the simplest one will be good for practical use. For example one may take

$$Y_o = -jG \frac{8a^2}{(1 + 8a^2) + 4\Omega'} \quad (10.73)$$

(Correcting network) Let it be tried to approximate the above function by a coaxial line, short-circuited at the other end, with a characteristic impedance W and length l , as shown in Fig. (10.20). The input admittance of this line is

$$Y_c = -j \frac{1}{W} \tan \frac{2\pi f l}{c} \quad (10.74)$$

Take

$$l = c / m f_o \quad (10.75)$$

with an arbitrary constant m . Then Y_c will be

$$\left. \begin{aligned} Y_c &\cong -j \frac{1}{W} \frac{1}{T + (1 + T^2) \frac{2\pi}{m} \Delta\Omega'_o \Omega'} \\ T &= \tan \frac{2\pi}{m}, \quad \Delta\Omega'_o = \Delta f_o / f_o \end{aligned} \right\} \quad (10.76)$$

approximately. Compare this with Eq. (10.73), they will have the same form if

$$\begin{aligned} WT &= \frac{1}{G} \left(1 + \frac{1}{8a^2} \right) \\ W(1 + T^2) \frac{2\pi}{m} \Delta\Omega'_o &= \frac{1}{2a^2} \end{aligned} \quad (10.77)$$

If the specific bandwidth $\Delta\Omega'$ is fairly small, then T must be large; Y_c is near resonance, leading to delicate adjustment, with additional problem of Q . On the other hand, a complicated correcting network is not desirable from practical point of view. A certain amount of correcting effect may be expected if one chooses

$$W = 1/G, \quad m = 8 \quad (10.78)$$

so that $Y_c \approx -jG$ near f_0 . The effective transfer constants, taking this Y_c as Y_0 , will be

$$\begin{aligned} a_{12} = \ln \frac{1}{2} \left\{ 1 + \frac{1 + 2j(\Omega' + a)^2}{1 + 2(\Omega' + a) + 2(\Omega' + a)^2} + \frac{1 + 2j(\Omega' - a)^2}{1 + 2(\Omega' - a) + 2(\Omega' - a)^2} - j \right\} \\ + \ln \left[(\Omega' + a) - j \left\{ 1 + (\Omega' + a) \right\} \right] \end{aligned} \quad (10.79a)$$

Y_{13} = the first term of Y_{12}

$$+ \ln \left[(\Omega' - a) - j \left\{ 1 + (\Omega' - a) \right\} \right] \quad (10.79b)$$

Fig. 10.21 shows the variation of a_{12} for various values of a ; some loss occurs at $\Omega' = -a$ owing to imperfect correction, and the smaller the value of a , the larger the loss. Fig. 10.22 shows the loss with respect to a .

(Experimental results) Fig. 10.23 shows the wave separator made up of two B. P. F.'s with the specifications.

- (1) center frequencies (f_1 and f_2) 400 and 421 MC
- (2) Bandwidth (Δf_0) 2 Mc
- (3) Nominal impedance ($1/G$) 75 ohms

The correcting network has characteristic impedance $W = 1/G = 75$ ohms, length $l = \lambda_0 / 8.3$ cm. The short-circuiting strip is adjustable. Fig. 10.24 shows the measured data of the attenuation, in good accord with calculations from Eq. (10.79). The loss due to incomplete correction at the center frequency, cannot be distinguished from the loss due to resistance of the B. P. F.'s themselves.

The above result shows that the lumped network theory can also be applied to distributed networks.

10.4 Inductance-coupled bandpass filters

Let us consider inductance coupled bandpass filters such as given in the top line of TABLE 3.6 or Fig. 5.9(g). The attenuation characteristics of these network are the same as capacitance-coupled ones; but one can make the axial length of the network shorter if one realizes the impedance, connected to the second conductor, by a

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capacitor of counterfacing circular plates; this technique is preferable in lower frequencies. Here it was confirmed that a distributed line and a lumped element can be combined to make up a network and also that cascading of two sections is permissible.

(Representations of the characteristics)

The relations among the voltages and currents of the network in the figure are

$$\left. \begin{aligned} \begin{bmatrix} V \\ I \\ V_o \end{bmatrix} &= \begin{bmatrix} V_o \\ I_o \end{bmatrix} \cos \beta l + \begin{bmatrix} W \\ W^{-1} \end{bmatrix} \begin{bmatrix} I_o \\ V_o \end{bmatrix} j \sin \beta l \\ V_o &= 0 \end{aligned} \right\} \quad (10.80)$$

$$V_2 = -I_2 \frac{1}{j 2 \pi f C}$$

which yield

$$\left. \begin{aligned} [V] &= [W] [I] p, \quad V_2 = -I_2 / 2 \pi f C \\ \text{where} \quad [W] &= W_{11} \begin{vmatrix} 1 & k_1 & k_2 \\ k_1 & d & k_1 \\ k_2 & k_1 & 1 \end{vmatrix} \end{aligned} \right\} \quad (10.81)$$

Elements Z_a and Z_b of the equivalent lattice are obtained:

$$\left. \begin{aligned} Z_a &= \frac{V_1 - V_3}{I_1 - I_3} = W_{11} (1 - k_2) p \\ Z_b &= \frac{V_1 + V_3}{I_1 + I_3} = \frac{W_{11}^2 \left\{ d(1 + k_2) - 2k_1^2 \right\} p^2 + W_{11}(1 + k_2)p / j 2 \pi f C}{d W_{11} p + 1 / j 2 \pi f C} \end{aligned} \right\} \quad (10.82)$$

and consequently the image parameters also:

$$Z_o = \sqrt{Z_a Z_b} = W_o \sqrt{1 - k_2^2} \frac{\sqrt{\left(d - \frac{2k_1^2}{1 + k_2} \right) p + \frac{1}{W_{11} j 2 \pi f C}}}{\sqrt{d p + \frac{1}{W_{11} j 2 \pi f C}}} p^2 \quad (10.83a)$$

$$\tanh \frac{\theta_c}{2} = \sqrt{\frac{Z_a}{Z_b}} = \frac{(1 - k_2)}{(1 + k_2)} \frac{\sqrt{d p + \frac{1}{W_{11} j 2 \pi f C}}}{\sqrt{\left(d - \frac{2k_1^2}{1 + k_2} \right) p + \frac{1}{W_{11} j 2 \pi f C}}} \quad (10.83b)$$

The cutoff frequencies ω_1 and ω_2 are those frequencies that satisfy

$$\begin{aligned}\omega_1: dp + 1/W_{11} j 2\pi f C &= 0 \\ \omega_2: (d - \frac{2k_1^2}{1+k_2}) p + \frac{1}{W_{11} j 2\pi f C} &= 0\end{aligned}\quad (10.84)$$

and the center frequency ω_0 is such that satisfies

$$\omega_0: (d - k_1^2) p + \frac{1}{W_{11} j 2\pi f C} = 0 \quad (10.85)$$

The image impedance at ω_0 is

$$[Z_0]_{\omega = \omega_0} = W_{11} (1 - k_2) \omega_0 \quad (10.86)$$

Let this value be R , then the effective attenuation is given by

$$\begin{aligned}2a &= 1 - \left\{ \frac{Z_a Z_b - R^2}{R(Z_b - Z_a)} \right\}^2 \\ &= 1 - \left[\frac{\{(1+k_2)p^2 - (1-k_2)\omega_0^2\} (dp + 1/W_{11} j 2\pi f C) - 2k_1^2 p^3}{\omega_0 2p \{(dp + 1/W_{11} j 2\pi f C) k_2 - k_1^2 p\}} \right]^2\end{aligned}\quad (10.87)$$

(Approximate equations of characteristics)

For the purpose of easy examination near the center frequency, let the deviation of the frequency f from the center frequency f_0 be Δf . Then

$$\begin{aligned}f &= f_0 \frac{f}{f_0} = f_0 \frac{f_0 + \Delta f}{f_0} = f_0 (1 + \frac{\Delta f}{f_0}) \\ &\approx f_0 (1 + \Delta F), \quad \Delta F = \Delta f / f_0\end{aligned}\quad (10.88)$$

and only the region of small ΔF will be considered. One has the approximation:

$$\begin{aligned}p &= j\omega = j \tan \frac{2\pi f l}{c} = j \tan \frac{2\pi f_0 l}{c} (1 + \Delta F) \\ &= j\omega_0 (1 + \theta_0 \frac{1 + \theta_0}{\omega_0} \Delta F), \quad \theta_0 = \frac{2\pi f_0 l}{c}\end{aligned}\quad (10.89)$$

Eq. (10.85), which gives the center frequency, will then be

$$(d - k_1^2) \omega_0 - 1/W_{11} 2\pi f_0 C = 0 \quad (10.90)$$

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ω_o or f_o should satisfy this equation. Similarly, ΔF corresponding to the cutoff frequencies are

$$\left. \begin{aligned} \Delta F_1 &= \frac{k_1^2}{(d-k_1^2) + \frac{d\theta_o}{\omega_o}(1+\omega_o^2)} \\ \Delta F_2 &= \frac{k_1^2(1-k_2)/(1+k_2)}{(d-k_1^2) + (d - \frac{2k_1^2}{1+k_2})\theta_o \frac{1+\omega_o^2}{\omega_o}} \end{aligned} \right\} \quad (10.91)$$

Suppose ΔF_1 and ΔF_2 are small so that

$$1 \gg k_2, d \gg k_1^2 \quad (10.92)$$

then

$$\Delta F_1 \approx -\Delta F_2 \approx \frac{-k_1^2/d}{1 + \theta_o \frac{\omega_o^2 + 1}{\omega_o}} \quad (10.93)$$

The expression Eq. (10.87) for the effective attenuation becomes

$$\begin{aligned} 2\alpha &\approx 1 + \left[\frac{-d(1+K)\Delta F}{\{d(1+K)k_2 - 2k_1^2 K\}\Delta F - k_1^2} \right]^2 \\ &\approx 1 + \left\{ -\frac{d(1+K)}{k_1^2} \Delta F \right\}^2 \\ &= 1 + \left(\frac{\Delta F}{\Delta F_1} \right)^2, \quad K = \theta_o \frac{1+\omega_o^2}{\omega_o} \end{aligned} \quad (10.94)$$

This is the same as Wagner character.

(Two sections in cascade) First obtain the effective attenuation for the cascade of two same symmetrical networks. As shown in the figure, two identical symmetrical sections (A, B, C, A) are connected in cascade to form a new network (A', B', C', A'). The effective attenuation of the resulting network will be obtained from

$$2\alpha = 1 - \frac{1}{4} \left(\frac{B'}{R} - C' R \right)^2 \quad (10.95)$$

in which

$$B' = 2 A B, \quad C' = 2 A C$$

$$A = \frac{Z_a + Z_b}{Z_b - Z_a} \quad \beta = \frac{2 Z_a Z_b}{Z_b - Z_a} \quad , \quad C = \frac{2}{Z_b - Z_a} \quad (10.96)$$

Thus

$$\begin{aligned} \epsilon^{2\alpha} &= 1 - \frac{1}{4} \left(\frac{B}{R} - CR \right)^2 (2A)^2 \\ &= 1 - \left\{ \frac{Z_a Z_b - R^2}{R(Z_b - Z_a)} \right\}^2 \left\{ 2 \frac{Z_a + Z_b}{Z_b - Z_a} \right\}^2 \\ &= 1 - \frac{\left[\left\{ (1 + k_2) p^2 - (1 - k_2) \omega_o \right\} \cdot \left\{ dp + \frac{1}{W_{11} j 2\pi f C} \right\} - 2 k_1^2 p^3 \right]^2}{\omega_1^2 p^2 (dk_2 - k_1^2) p + \frac{k_2}{W_{11} j 2\pi f C}} \\ &\quad \times \frac{\left\{ (d - k_1^2) p + \frac{1}{W_{11} j 2\pi f C} \right\}^2}{\left\{ (dk_2 - k_1^2) p + \frac{k_2}{W_{11} j 2\pi f C} \right\}^2} \quad (10.97) \end{aligned}$$

An approximate formula will be obtained, as before,

$$\begin{aligned} \epsilon^{2\alpha} &= 1 + \left[\frac{-d(1+K)\Delta F}{\{d(1+K)k_2 - 2k_1^2 K\} \Delta F - k_1^2} \right]^2 \left[\frac{2d(1+K)\Delta F}{\{d(1+K)k_2 - k_1^2 K\} \Delta F - k_1^2} \right]^2 \\ &\approx 1 + 4 \left\{ \frac{d(1+K)\Delta F}{k_1^2} \right\}^4 = 1 + 4 \left(\frac{\Delta F}{\Delta F_1} \right)^4 \quad (10.98) \end{aligned}$$

This is a Wagner characteristic of $n = 2$. (Design procedure) Here the design procedure goes as follows in narrow band requirements:

- (i) Center frequency (f_o)
- (ii) Nominal impedance (R)
- (iii) Pass bandwidth (Δf_1)
- (iv) Length of coupled portion (l)

are assumed to be given.

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(1) From the conditions (i) and (iv), one determines $\omega_o = \tan \frac{2\pi f_o}{c} l$

$$\theta_o = 2\pi f_o l / c \quad \text{from Eq. (10.89)}$$

$$K = \theta_o \frac{1 + \omega_o^2}{\omega_o} \quad \text{from Eq. (10.94)}$$

(2) From the condition (ii), one has

$$R = [Z_o] \omega = \omega_o = W_{11} (1 - k_2) \omega_o \approx W_{11} \omega_o \quad \text{from Eq. (10.86)}$$

from which

$$W_{11} = R / \omega_o$$

may be obtained.

(3) Since the second conductor is a resonant line, one may take $dW_{11} = 75 \text{ ohms}$ so as to have the best Q, thus

$$d = 75 / W_{11}$$

(4) An equation

$$(d - k_1^2) \omega_o - 1/W_{11} 2\pi f_o C \approx d \omega_o - 1/W_{11} 2\pi f_o C = 0$$

must be satisfied at the center frequency, as obtained from Eq. (10.90). One can determine the value of C from this relation,

$$C = \frac{1}{d W_{11} 2\pi f_o \omega_o}$$

(5) From the condition (iii) and the equations (10.88) and (10.93), one has the relation

$$\Delta F_1 = \frac{\Delta f_1}{f_o} = \frac{-k_1^2}{d(1+k)}$$

which yields

$$k_1^2 = - \frac{\Delta f_1}{f_o} (1+K) d$$

(6) The relations between the line constants W_{11} , d , k , and the line structure can be given approximately by the following expressions, under the assumption that the coupling is small:

$$\left. \begin{aligned} W_{11} &= 138 \log_{10} \frac{D^2 h^2}{D d_2} \\ W_{11} k_1 &= 138 \log_{10} \frac{D}{h} \\ W_{11} d &= 138 \log_{10} \frac{D}{d_1} \end{aligned} \right\} \quad (10.99)$$

One can determine D/d_1 , D/d_2 and D/h from these relations. Thus, if one specifies any one of D , d_1 , d_2 , h , then all the others will be determined. Usually one specifies D or d_1 from the considerations on Q . k_2 is obtained from

$$W_{11} k_2 = 138 \log_{10} \frac{D^2 + h^2}{2 D h} \quad (10.100)$$

(An example of preparation) A filter was made with dimensions shown in Fig. 10.29, which has the center frequency near 70 MC. The properties of this network will be examined. It is made that

$D = 5.1$ cm, $d_1 = 1.475$ cm, $d_2 = 0.175$ cm, $h = 3.65$ cm
so that the line constants will be

$$W_{11} = 15.54, k_1 = 0.1257, k_2 = 0.01970, d = 0.46604$$

take $f_o = 70$ MC, then along with $l = 35$ cm, one has

$$\omega_o = \tan \frac{2 \pi f_o}{c} l = \tan \frac{2 \pi \cdot 70 \text{ MC}}{3 \times 10^{10}} \times 35 = 0.56347$$

$$\theta_o = \frac{2 \pi f_o}{c} l = 0.51313$$

$$K = \theta_o \frac{1 + \omega_o^2}{\omega_o} = 1.19979$$

and consequently

$$\Delta f_1 = \frac{-k_1^2 f_o}{d(1+k)} = \frac{-0.1257^2 \times 70 \times 10^6}{0.46604(1+1.19979)} = 1.0789 \text{ MC}$$

Therefore the values of α , calculated with the use of Eq. (10.94) will be as shown by the broken line in Fig. 10.30.

The value of C should be

$$C = \frac{1}{(d - k_1^2) \omega_o W_{11} 2 \pi f_o} = 56.174 \text{ p h}$$

The good agreement between the measured and the calculated values in Fig. 10.30 means the applicability of a combination of lumped and distributed elements, as well as that of lumped network theory to cascade connections.

10.5. Antimetrical bandpass filters⁽³¹⁾

An experiment was made on the antimetrical BPF in Fig. 5.15. This is a simplest example of a B.P.F. formed by the cascade of symmetrical lattice networks, one of L only, and the other of C only.

(Network characteristics). The configuration of the network is like that in Fig. 10.31, and the attenuation is given, from Eq. (5.20) as follows:

$$\epsilon^{2\alpha} = 1 + \left\{ \frac{p^2 + 1}{\Delta p} - \frac{p^2 - 1}{K p} \right\}^2 \quad \left. \begin{array}{l} \Delta = \omega_2 - \omega_1, \quad K = \omega_2 + \omega_1, \quad \omega_1 \omega_2 = 1 \end{array} \right\} \quad (10.101)$$

where ω_1 and ω_2 are such frequencies that give 3 db attenuation. The line constants are

$$W_0 = \sqrt{1 - 1 \frac{\Delta K}{2}}, \quad k = \sqrt{\frac{\Delta K/2}{1 + \Delta K/2}}, \quad d = 1 \quad (10.102)$$

(Approximate expression for the characteristic)

An approximate expression will be obtained that will be convenient to examine the characteristics near the center frequency, when the band is narrow.

Let the center frequency be f_0 , so that

$$[p^2]_{f_0} = -1$$

Consequently

$$\omega_0 = \tan^{-1} \frac{2\pi f_0}{c} l = 1, \quad \frac{2\pi f_0}{c} l = \frac{\pi}{4}$$

Use the notation

$$\omega = 1 + (\Delta\omega) \approx 1 + \frac{\pi}{2} \left(\frac{\Delta f}{f_0} \right)$$

then, one has the approximation

$$\Delta = \omega_2 - \omega_1 = \omega_2 - \frac{1}{\omega_2} = \pi \left(\frac{\Delta f_2}{f_0} \right)$$

$$K = \omega_2 + \omega_1 \approx 2$$

Hence the line constants should be

$$W_0 \approx 1, \quad k^2 \approx \pi \frac{\Delta f_2}{f_0}, \quad d = 1$$

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As mentioned (NOTE), 5.3, $(p^2 - 1)/K P \approx j$, when Δ is small, so that

$$\frac{2\alpha}{\epsilon} \approx 1 + \left(\frac{p^2 + 1}{\Delta p} j \right)^2 \approx 1 + \left\{ \frac{\pi \Delta f / f_o}{\pi \Delta f_2 / f_o} \right\}^2 = 1 + \left(\frac{\Delta f}{\Delta f_2} \right)^2$$

This approximates a Wagner character, $n = 1$.

The loss due to line resistance may be obtained, in the same manner as in Eq. (10.53) for capacitance coupled B. P. F.,

$$2\alpha_{\Delta f = 0} = 1 + \left(\frac{4a'l}{\Delta} \right)^2 = 1 + \left(\frac{4a'l}{\pi \Delta f_2 / f} \right)^2 = 1 + \left(\frac{4a'l}{k_1} \right)^2$$

(Trial construction) The design conditions are:

- (i) center frequency (f_o) 150 MC
- (ii) nominal impedance (R) 120 ohms
- (iii) bandwidth ($2 \Delta f_2$) $\pm$ 320 KC

(1) From $f_o = 150$ MC, one has

$$\omega_o = \tan \frac{2\pi f l}{c} \approx 1, \quad l = 25 \text{ cm}$$

(2) From the bandwidth $\pm$ 320 KC, one has

$$k^2 = \pi \Delta f_2 / f = \pi \times 320 \text{ KC} / 150 \text{ MC} = 0.670 \times 10^{-2}$$

$$k = 0.0819$$

(3) The line dimensions will be found from

$$W_o = 1.38 \log_{10} \frac{1 - (h/D)^2}{d/D}, \quad W_o k = 138 \log_{10} \frac{1 + (h/D)^2}{2h/D}$$

with $R = W_o = 120$ ohms and the above value of k . Take $2D = 34$ mm, then one obtains $h = 9.32$ mm, $2d = 3.19$ mm. The loss at the center frequency is calculated to be 0.25 dB with the outer conductor of copper and the inner of brass, while the measured value is 1 db. This discrepancy may be attributed to the increase of resistance due to bending as well as to the disturbance of the electromagnetic field.

10.6. Lowpass filters with Wagner characteristics.

The L. P. F. of Wagner characteristics of $n = 3$ or 5, TABLE 9.1, Fig. 9.2, may be realized as in Fig. 10.35, with cutoff frequency $\omega_1 = 1$.

To obtain line dimensions from line constants, one may use the following expressions, which have corrections due to proximity effect. The following Fig. 10.36(b) is a chart of numerical values obtained from the expressions.

$$W_o = 30 \cosh^{-1} \left\{ \frac{2h_1^2}{\gamma_1^2} - 1 \right\} = 30 \cosh^{-1} \left\{ \frac{2h_1^2}{\gamma_2^2} - 1 \right\},$$

$$W_{ok} = 30 \cosh^{-1} \frac{(h_1 + h_2)^2 - \gamma_1^2 - \gamma_2^2}{2\gamma_1\gamma_2} = 30 \cosh^{-1} \frac{(h_2 - h_1)^2 - \gamma_1^2 - \gamma_2^2}{2\gamma_1\gamma_2}$$

$$h_1 = \frac{1 - (\frac{b}{a})^2 + (\frac{d}{a})^2}{1 - (\frac{b}{a})^2 - (\frac{d}{a})^2}, \quad h_2 = \frac{1 - (\frac{b}{a})^2 + (\frac{d}{a})^2}{(1 + \frac{b}{a})^2 - (\frac{d}{a})^2}, \quad \gamma_1 = \frac{2 \frac{d}{a}}{(1 - \frac{b}{a})^2 + (\frac{d}{a})^2}, \quad \gamma_2 = \frac{2 \frac{d}{a}}{(1 + \frac{b}{a})^2 - (\frac{d}{a})^2}$$

Fig. 10.37 shows the experimental results made on a modified network of $n = 3$. The measured value of α is smaller than the calculated, which is considered to be due to stray capacitances arising from the disturbance of electromagnetic field at bends and at the input and output terminals. These points are the greatest disadvantage of this type of filters, one must pay good attention.

The attenuation peak moves if one shifts the short-circuit strip A, as could be expected from Sato's work⁽¹⁴⁾. The short-circuit strip β would not be necessary in theory, but without it the attenuation will go down steeply at some point. This suggests one that there exists some amount of coupling between the lines before and after the short-circuit strip A. Thus one has to use the short-circuit strip β so as to make the line from A to B act as a coaxial line.

Fig. 10.38 shows the experimental results on the network $n = 5$. The measured values of α is lower than the calculated, perhaps by the same reason as above. (b) in the figure shows various measured values during adjustments. (c) shows the dimensions of the network, which were calculated from the expressions mentioned. It will be convenient for adjustments if one makes the top length and the position of short-circuit adjustable. Measured values are close to the calculated, so that the Theory may be considered to be correct.

The aims have been almost attained, that was described in the beginning of this chapter, by the several examples of experiments described above. That is, the design principles on coupled line filters may be considered to be correct at large. It has also been confirmed that the network theory on lumped networks can be applied to distributed ones without any modifications, except the transformation of frequency.

Closing Remarks - Summary

Studies on coupled line filters have been presented, that have been made thus far. The ingredients will be summarized.

Chapter 1. Equations of transmission in parallel multi-wire lines: The transmission was studied in parallel multi-wire lines over ground. Three kinds of parameters, i. e., the self characteristic impedance, symmetry coefficients, and coupling coefficients were introduced to represent a characteristic impedance matrix, rendering it convenient to derive characteristics of coupled line networks. As examples, a two-wire line and a three-wire line have been cited, along with cases of particular constructions.

Chapter 2. Simple networks made of coupled two wire lines: 28 kinds of networks may be formed with a two-wire line, in which two of the four terminals are taken to be input and output terminals, and the other two may be open circuited or ground-connected or connected to the ground through another arbitrary element. Their network parameters have been obtained, and their equivalent network representations are also given in the form of combinations of coaxial elements. There are also given lumped equivalent representations obtained by the use of Richards' frequency transformation. Equivalent networks of coupled line networks have been given systematically, and hence the coupled line networks have become easy to understand. Among the 28 networks, the characteristics of 14 networks have been examined, whose properties were not yet known. Various kinds of networks have been obtained including L. P., B. E., derived-m H. P., B. P. etc. There are also tabulated equivalent networks of those made of unsymmetrical two-wire lines. Finally there are given the procedures of transformation of coaxial filters into coupled line filters; especially the transformation of a coaxial loop is explained in an example.

Chapter 3. Simple symmetrical networks made of three-wire lines: the networks with 3-wire lines are studied, but limited only to symmetrical ones, because of complexity in structure. There are treated 23 networks, with 2 terminals, out of six, are taken to be input and output, other 4 terminals open-circuited, ground-connected or connected to the ground through another element, all only for the case of symmetry. Of them expressions of characteristics are obtained for L. P. F. and H. P. F. with attenuation poles, 10 B. P. F.'s and 5 B. E. F.'s from their equivalent networks. It has been pointed out specifically that B. P. F.'s and B. E. F.'s of narrow bands can be easily made, which constitutes an advantage of coupled line filters.

Chapter 4. Ladder-type networks: With equivalence relations in the preceding two chapters in hand, there are presented transformations from lumped networks into

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coupled line networks. A procedure is shown where ladder networks, designed with lumped parameters, are divided into appropriate sections, each of which is transformed into coupled line type. The L type network may be built up with a double coaxial element, which is only a special case of a coupled two-wire line. Next, basic T-networks are treated, and in combination with shunt elements, ladder networks have been treated having Wager or Tchebycheff characteristics with or without attenuation poles. Likewise, the basic H. P. T-network has been realized in a 3-wire line, and ladder networks were cited on Wagner or Tchebycheff characteristics with or without attenuation poles. It is also pointed out that B. E. F.'s and B. P. F.'s can easily be deduced from L. P. F. and H. P. F., thus obtained, by means of frequency transformation.

Chapter 5. Narrow band filters: Narrow band filters can be made, if one makes good use of the coupling. First obtain a B. P. F. in lumped constants, and connect unit coaxials at the input and output terminals, where the coaxials elements are so chosen that they do not affect the amplitude characteristics. Divide the network into appropriate sections, transform each section into coupled-line type. Thus the bandwidth is controlled by the coupling coefficient, so that the bandwidth may be made narrow, by making the coupling small. Examples are shown for the combinations of Wagner networks $n = 1 \sim 4$ and unit coaxials. If one uses L or C as degenerate elements, in place of unit coaxials, one will have an alternate cascade of symmetrical lattice of L only and those of C only. Transform each lattice section into coupled-line type, and the resulting network will have a bandwidth dependent on the coupling coefficient. Values of elements are obtained for those derived from Wagner B. P. F., $n = 1 \sim 3$. An example is given on a 3-element type B. P. F. made in coupled-line type.

Chapter 6. Extraction of coupled two-wire lines: If one increases the number of wires in a line, the structure will become complicated and impractical, so that he should rather consider cascade connections of coupled two-wire lines of simpler structures. First obtain the network $[Y]$ resulting from the cascade connection of an arbitrary network $[Y']$ and an arbitrary coupled two-wire line. Next, decompose the given $[Y]$ into a cascade of a coupled two-wire line and a network $[Y']$. It has been proved that if $[Y]$ is positive real then $[Y']$ is also positive real and preserves the property of a network. The writer points out that the extraction of a coupled two-wire line is only an extension of the extraction of unit coaxial lines, proposed by Richards, to four terminal networks, and it can be applied to multi-terminal networks. Also a procedure is proposed on the decomposition of four-terminal networks.

Chapter 7. Design of symmetrical networks: A design procedure is described, that makes use of the extraction of coupled two-wire lines stated in the preceding chapter.

Under the condition of symmetry, the expressions for $[Y]$, $[Y']$ and line constants of the coupled two-wire line become simple, and the synthesis of the symmetrical network will be effected by the repetition of extracting coupled two-wire lines. If one treats the matter with equivalent lattice networks, the extraction of a coupled two-wire line may be replaced by the extraction of a coaxial line from each of two network arms; the latter is simpler in calculation and also makes the comprehension easier. Examples are given on the design by Q-functions, design of Wagner networks and Tchebycheff networks with or without attenuation poles.

Chapter 8. Design of antimetrical networks: With the use of the condition of antimetry, the expressions for the extraction of coupled two-wire lines becomes simpler. Again it has been proved that if $[Y]$ is antimetrical, the remaining network $[Y']$ is also antimetrical. As design examples, networks of Wagner or Tchebycheff characteristics, with or without attenuation poles, are cited. Attention should be paid to the point that if one repeats the extraction of two-wire lines, he may be encountered with a Brune section, which cannot be realized by a two-wire line over ground. In such cases one should consider another transformation. In this article, the problem is solved by the combination of taking out a shunt element and the extraction of a coupled two-wire line.

Chapter 9. Frequency transformations: In distributed networks, the frequency transformation can not be applied to each network elements, because there are cascade elements in the networks. The frequency transformation should be applied to the network parameters themselves. The transformed frequency may be of the same form except it is a tangent function. L.P.F.'s and B.E.F.'s can be synthesized from the transformed network parameters. Examples are shown on the design of Wagner networks. In cases of H.P.F. and B.P.F., simple extraction of coupled 2-wire lines will lead to negative coupling coefficients which still remains to be a problem. For comparison, an L.P.F. of $n = 5$ has been realized in 3 kinds of configurations, on each of which is shown the variation of line constants with the cutoff frequency.

Chapter 10. Experimental Examples: Here are cited various experiments made on coupled-line filters. (1) As an example of the network of a coupled two-wire line, Chap. 2, experimental results are described on a bandstop filter, along with its image parameters, effective attenuation, equivalent networks, effect of line resistance, and a design example. (2) A capacitance coupled B.P.F. is taken as an example for Chap. 3, and experimental data are described in detail along with expressions of characteristics, approximate expressions, approximate equivalent networks, effect of resistance. This deserves also as an example for Chap. 5. (3) Two capacitance-coupled B.P.F. were combined to form a wave separator, in like manner as in X-termination. This is to confirm the applicability of lumped network theory into distributed networks. There are described on the principle, six-terminal connection,

input admittance, correcting admittance and experimental data. (4) A cascade of two sections has been made, where each section consists of an inductance-coupled B. P. F. and a lumped capacitor. Here it is aimed to examine the problem of composite connection and also the validity of mixed use of lumped and distributed elements. Descriptions are made on the characteristics, approximate formulae, expressions for two sections in cascade, design procedure and experimental data. This deserves as an example for Chap. 3 and 5. (5) As an example of a narrow bandpass filter formed by the cascade of symmetrical lattice in Chap. 5, an experiment is described on the case $n = 1$, with network characteristics and approximate formulae. (6) An experiment was made on Wagner L. P. F. $n = 3$ and $n = 5$, as examples of Chap. 7 and Chap. 9.

The above experiments are all in good accord with theoretical calculations, so that one may consider the calculations, Chap. 2--9, are almost correct.

Thus here are arranged materials of study on coupled-line filters as systematically as possible, but the study is not yet completed and there remain many problems. With regard to synthesis, it has been proposed that the extraction of two-wire lines may be a significant process, but it is not a key that solves all problems but is only a method. One will notice that problems still remain open on the synthesis of H. P. F. and B. P. F. As a whole, the significance is placed on the network Theory; the practicability has only been mentioned at the description on experimental examples. It is also important to obtain the relations among line dimensions and line constants, but the description is here limited only within necessity, because it is a problem of the electromagnetic field. The writer would like to mention that the capacitance-coupled B. P. F. is used as a suppressing filter for the intermodulations of press car communication.

Acknowledgment - - - omitted.

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Abbridgements

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- PGCT - IECE - - - - - Meeting of the Professional Group on Circuit Theory, The Institute of Electrical Communication Engineers (Japan)
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CAPTIONS:

- Fig. 1. A parallel multi-wire line over ground
- Fig. 2. A parallel 2-wire line over ground
- Fig. 3. A symmetrical 2-wire line
- Fig. 4. A parallel 3-wire line over ground
- Fig. 5. A symmetrical 3-wire line
- Fig. 6. A 3-wire line symmetrical with respect to the second wire
- Fig. 2.1 A symmetrical 2-wire line over ground
- TABLE 2.1 Parameters of simple networks with a two-wire line (I)
- TABLE 2.2 The same (II)
- TABLE 2.3 The same (III)
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- TABLE 2.6 Equivalent circuits (I)
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———measured

-----calculated

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left top: input

left bottom: output

A, B: Short-circuit strip

--- calculated

—○— measured

—x— when strip A is farther from the terminals

—·— when strip A is nearer to the terminals

—△— when strip B is absent

Fig. 10.38 Wagner L. P. F., $n = 5$

(a) attenuation characteristic

--- calculated

—○— measured

(b) examples of characteristics happened to occur during adjustments

(c) structure of the trial-made network