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DYNAMIC PROCESSES IN LIQUID-FUEL ROCKET ENGINES

(CHAPTERS 1 and 2)

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ABSTRACT

The book shows the derivation and analysis of equations describing liquid and gas motion in liquid-fuel rocket engine assemblies under the influence of forces acting upon them. This study allows engine parameter changes with time to be determined. Starting, operating and switching-off periods are examined. The book may serve as a text book for students of advanced courses, engineers and scientific workers involved in the study of liquid-fuel rocket engines.

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DYNAMIC PROCESSES IN LIQUID-FUEL ROCKET ENGINES

Ye. K. Moshkin

Introduction

This book is devoted to an examination of dynamic processes in liquid-fuel rocket engines, i.e., those processes which must be described through the use of dynamic equations. These equations characterize the movement of operating bodies (fuel components, the products of vaporization and combustion), and separate assemblies and systems (the rotating parts of turbo-driven systems, the moving parts of valves, and the like), under the influence of forces acting upon them. The utilization of the suggested methods of investigation should help to improve the organization of the intermediate processes in the design and development of an engine.

This book utilizes material intended for a wide circle of readers, published in native and foreign literature, and also the results of several investigations conducted by the author. In order to avoid unnecessary repetition, widely known methods of investigation and theoretical principles have not been included by the author in the contents of this book. Therefore, it is recommended that for best comprehension the material of this book should be used in conjunction with several books published earlier by other authors (see the bibliography at the end of the book).

In order to present in three small chapters, insofar as possible, a general picture of the relationships between processes, it is necessary to omit a series of problems. At the same time, with the purpose of establishing links between separate equations, several relationships have had to be stated in an abbreviated form. In this connection, we have been forced to decline to generalize and to analyze the views of various authors, since this would have led to a significant increase in the size of the book.

The first chapter is devoted to an examination of dynamic processes which occur in the combustion chamber. A description of processes is given and the most important stages of operation are examined: the starting routine, operation, transient processes, and switching-off of the engine. The derivation and analysis of the basic equation of the combustion chamber are presented. A great deal of attention is devoted to equations describing internal chamber processes. All the equations characterize the movement of a two-phase fuel -- the combustion flow consisting of atomized components and gaseous products.

Partial differential equations permit the investigation of the combustion stream element in time, and during variations within the internal cavity of the chamber. The intake of gaseous products upon vaporization of the liquid fuel is defined in the course of the derivation of the equation of the law of the conservation of matter, with the aid of the divergence of the vector of relative component consumption. Combustion stream motion is characterized by Euler's equations.

The general energy equation of the combustion stream, the equation of the conservation of mechanical energy, the first law of thermodynamics, and the energy equation of a motionless gas isolated from its environment, are examined in detail. Equations of the law of the conservation of momentum have permitted the determination of the combustion stream velocity, the dissipation of mechanical energy and the derivation of a generalized thrust formula. At the end of the chapter, the problem of the gasification of fuel components and the changes in the relationships between components with respect to time and with respect to chamber coordinates are examined.

The second chapter is devoted to the feed system. In contrast to the first chapter, where a series of questions were examined from theoretical positions, basically the engineering method of design is discussed in the second chapter. The structure of the book has been chosen not only from a desire to show various approaches to the calculation and investigation of the processes, but also in connection with regular generalities of certain processes which occur both in the combustion chamber and in the assemblies of the feed system.

The theoretical principles stated in the first chapter may be applied to the investigation of the feed system, and the engineering methods examined in the second chapter may be applied to the study of the combustion chamber.

First, the derivation of the equation of the motion of an elastic fluid in a deformed tube including a changing volume is examined. The wave equation obtained permits us to solve with a sufficiently high degree of accuracy many complex problems, which at the present time have a great deal of practical significance.

Notwithstanding the fact that this equation allows for both fluid compressibility and pipe deformation both in radial and in axial directions, and for the presence of an input or an output of fluid, it also proves to be very useful in engineering calculations insofar as proper tolerance acceptance is concerned.

In an analysis of particular cases of the application of the equation examined, the engineering equation for a hydraulic circuit is derived, and this permits the establishment of the energy relationship between processes of separate assemblies of the feed system.

A small part of one of the sections is devoted to a determination of hydraulic losses in circulating channels. The method of determining hydraulic resistance has not been stated in detail, since a great deal of work has been devoted to this question, and it has been described in native publications. The definition of mass forces in circulating channels of an arbitrary form has been examined. The operating formulas obtained are very convenient for the study of fuel component dynamics.

Further, a detailed derivation of the equations characterizing the filling of the hydraulic systems is presented. The influence of counter-pressure and of the gasification of components having a low boiling point are considered. An investigation of the process of filling lines permits solving a series of very important questions concerning the conditions of fuel input into the combustion chamber before ignition and during the initial combustion period.

At the end of the derivation and after several investigations related to fluid motion in circulating parts of the feed system, operating conditions in the pressure system are examined. The derivation of two equations based on the fundamental laws of the conservation of energy and mass is suggested, for a conditionally-accepted "universal" system. These equations are employed in the design of a feed system with a gas, powder, or a heated accumulator.

The sixth section of Chapter II is devoted to the derivation of the equation of a centrifugal pump pressure characteristic. The derivation is given in an engineering form. The equation, which takes into consideration the action of mass forces and fluid leakage, is suitable for design and investigations.

An examination of the processes occurring in the assemblies of the feed system power drive -- in the turbine and in the generator -- are outside the scope of this book. However, in order to carry out a complex engine evaluation it is necessary to know the equation used to determine turbine power, which is also examined in the seventh paragraph.

The eighth paragraph of Chapter II is devoted to the study of the order of the application of the equations obtained in investigating operating conditions of the feed system. Conditions are examined in which, during transient processes, the velocity of change in the composition of products of combustion equals zero ($\dot{K} = 0$).

In section 9, Chapter II, an approximate method for determining thermal losses in tanks is examined.

In Chapter II all of the basic sources of energy leading to fuel component motion, and the most important forms of resistance opposing liquid motion, are examined. Insofar as the equation of the law of conservation of energy in the form of a balance of pressures is concerned, the basic relative forces imparting motion to the fluid are: surplus pressure produced by pumps, the pressure in the tanks, and the pressure produced by external mass forces.

The basic relative forces of opposition are combined from: hydraulic losses in hydraulic circuit elements, the pressure force caused by the action of internal mass forces, the pressure expended on the increase of fluid kinetic energy, and the pressure in the combustion chamber whose characteristic variations are examined in Chapter I.

The third chapter is devoted to an examination of the interrelationship between processes occurring within the engine. The graphic-analytic method of calculation gives a general presentation of the relationship between processes. Here the relationship between basic parameters during engine starting and running operations is established. Although the graphic-analytic method does not take mass forces into consideration, it does give a sufficiently complete and graphic presentation of the non-steady state, and of the nature of the relationships during parameter changes within a wide range.

The graphic-analytic method is employed in carrying out engineering designs for an engine, in calculating the adjustment process and the actions of external factors during quality evaluation of a projected engine, and for preliminary evaluation of the operating stability of separate assemblies and of the engine as a whole. The graphic-analytic method of calculation is defined more accurately in relation to the design of the engine under investigation. The size of the graphs presented are established with consideration for the accuracy required. The graphic-analytic method may be successfully applied in engineering practice.

The third section of Chapter III is devoted to an investigation of the relationship of chamber parameters and of the hydraulic circuits. First, the derivation of a calculated equation is presented, which permits an investigation of a wide range of questions. The results of investigations of the starting and operating conditions are noted. With the aim of obtaining a more graphic presentation of basic parameter relationships, the solution to the equation is examined without considering the delay parameter.

A good deal of attention is devoted to investigation of the interrelationships between average parameter values and dynamic processes in liquid-fuel rocket engines through the use of the method of small finite deviations. The investigation of processes is based on a system of linearized equations. Dynamic equations differ from static equations through the presence of terms comprising time derivatives of the deviations of parameters from their nominal values. It is convenient to preface an investigation of an engine with the aid of differential equations with an investigation employing static equations. The method of linearization and the order of the determination of equation coefficients are discussed.

For engines with loaded tanks and for engines equipped with a turbine pump assembly, a system of algebraic equations has been formulated which serves as an aid in engine adjustment for a calculated (or a specified) condition, and it also aids in the definition of the action of external factors and in conducting an analysis of manufacturing accuracy. After a thorough investigation of the system of algebraic (static) equations, terms consisting of derivatives of variables are added to them. The dynamic equations obtained are employed in the investigation of transient processes during engine operation.

The last section is devoted to an examination of the possibilities of employing automatic computer technology in the investigation of liquid-fuel rocket engines. As examples, several problems are solved on electronic models and digital calculators.

In working on this book, our attention has first of all been directed toward the clarification of the interrelationships between processes; we have attempted, within the limits of the tolerances accepted, to mathematically describe more exactly these processes, and to point out a method of analysis and of further development of the equations obtained.

The author considers it a pleasant duty to express his gratitude to the Doctors of Technical Sciences, Professors I. I. Kulagin and V. A. Makhin; to the Doctor of Technical Sciences, I. I. Ivanoc; Professor L. S. Dushkin; to the Candidates of Technical Sciences, B. F. Glikman, V. D. Kurpatenkov, V. V. Pilipenko, D. A. Shushko, and V. D. Khar'kov; and to Engineers A. G. Grigor'yev, B. Ye. Moshkin, and V. A. Zadontsev for a careful examination of various sections of the book and for valuable advice.

The author requests that all comments concerning this book be sent to the address: Moscow, I-51, Petrovka, 24, "Mashinostroyeniye" Press.

Conventional Symbols

x--Linear dimension

L;l--Length

d--Diameter

R;r--Radius

F--area

S--Surface

V--Volume

M;m;Y--Mass

G--Mass rate of flow per unit time (per second)

t--Instantaneous time

τ--Combustion time delay

ϵ--Gas residence time

W;u--Gas velocity

C--Liquid propellant velocity

P--Force, thrust

p--Pressure

Δp--Hydraulic losses

ρ_g--Gas density

ρ_l--Liquid density

K--Propellant reactant ratio

T--Temperature

θ--Momentum

Principal Indexes

t--Tank, in the tank

w--Mass forces

j--External forces

c--Combustion chamber

p--Pump

ex--External

1--Oxidizer

2--Fuel

i--Equation index number

k--Parameter index number

0--Initial value

e--Emitted quantity; discharge

l--Liquid

g--Gaseous

sp--Powder; solid propellant

a--At the nozzle tip

dr--Drop

The International System of Units, SI, is used in the book.

CHAPTER I

THE DYNAMICS OF THE COMBUSTION CHAMBER

§ 1. The Dynamics of Intrachamber Processes

If we exclude secondary factors in our examination, then we may consider that intrachamber processes are generated at the moment of the exit of fuel components from the injector. For the chamber, the following are typical: starting (sustained condition), operation, transient processes during the operating condition, and switching-off of the engine.

During the course of the entire period of engine operation, intrachamber processes to a certain degree are caused by the action of mass forces, the parameters of which are constantly changing. If we adhere to a chronological order, then our examination of the dynamic processes must begin with the starting condition. However, more uniform dynamic processes occur during the operating condition, therefore, in conformity with these conditions it is convenient to begin the study here [3, 21].

1. Operation (sustained condition)

The engine has started. From a power point of view, the operating condition is the main mode. Here the engine operates for the longest period of time. The operational time limit depends on a series of factors, including the specific thrust value. The higher the specific thrust, the longer the engine may operate.

If we direct our attention to average values of the parameters, unchanging with respect to time, then for each point in the volume of the chamber the dynamics of the processes appear in such a manner that the velocity of the motion of fluid and gas masses will change mainly with respect to the length of the combustion chamber and the nozzle; forces arising mainly as a result of expansion of the products of combustion act on each elementary particle of the gas or liquid burning fuel within the chamber. Due to the presence of wave processes, the parameters of every point of the chamber volume are constantly changing, and low frequency and high frequency oscillations are observed. During the decrease in pressure noted, a low frequency stability threshold appears -- a sharp, almost step-wise increase in the amplitude of oscillations in chamber pressure. The study of the operating condition is complicated by the fact that the frequency and amplitude of oscillations undergo continuous changes with respect to complex, insufficiently studied principles of oscillatory processes.

Thus, during the operating conditions, the dynamic processes lead to parameter changes with respect to both chamber coordinates and time. Therefore, in order to study dynamic processes even under "sustained" conditions, it is necessary to include a system of partial differential equations. The processes which occur during the operating condition are characterized by relatively small parameter changes. On this basis, it is possible to apply equations with small deviations, and to subject several differential equations to linearization. This permits the study of processes which occur under operating conditions in greater detail than those during the transient condition.

2. The Starting Condition

Parameter changes during the starting condition will have a much more complex nature than during the operating condition. During the starting procedure, the average pressure in the chamber changes from a certain small pressure before the start to a calculated or nominal value. The starting process is complicated by the fact that it is usually organized on a step by step basis: during the processes several commands are carried out.

During the starting process, significant accelerations in the motion of fuel components and the products of combustion are observed. These processes most actively occur during the initial period, from the moment of fuel arrival in the chamber until ignition, during ignition and the development of combustion, and until the pressure in the chamber reaches approximately 40-70%. After this the pressure in the chamber increases somewhat more slowly. Sometimes uneven changes in pressure are observed in this area. The reasons for the possible temporary decreases in pressure will be explained later.

A change in chamber pressure occurs as a result of the action of geometric factors, as a result of the presence of wave processes, of a change in the condition of delivery of fuel component to the chamber (here one must consider a change in the general mass of the fuel, and a change in the relationship between components and a derivative of this relationship with respect to time), and of the quantity of fuel located in the chamber in liquid form.

The sensitivity of the delay period to pressure and the speed of change of component relationships [25, 28] are very important in the organization of processes during the starting operation. High values of the derivative of the delay period with respect to chamber pressure leads to a double change in the sign of the pressure derivative with respect to time during the starting operation, which may promote the formation of detonation effects.

During an increase in the speed of change of the relationship between components, the process of fuel burning is accelerated, which promotes the formation of low-probability detonation effects or of unusual high frequency [25] oscillations.

3. The Transient Processes During Operation

The average parameter values of the combustion chamber during operation are not constant. They change as a result of changes in engine thrust with altitude, as a result of the intensity of the effects of external forces, and as a result of changes in the operating mode of separate engine assemblies, as well as changes in the physical-chemical composition of fuel components, and in the incoming commands from the control system [30].

A change in parameters during transient processes of engine operation occurs slowly, or, if rapidly, then by relatively small amounts. In the study of transient processes this permits, just as in the study of operating processes, the employment of equations with small deviations.

4. Switching Off the Engine

After the main fuel valves are switched off, the delivery of fuel components to the chamber is significantly reduced. Small amounts of fuel enter the chamber as a result of displacement in the operating parts of the valves, deformation of hydraulic lines and other factors. In an evaluation of the conditions of the delivery of the last portions of fuel into the chamber, it is important to consider the effects of mass forces. As a result of the sharp pressure decrease, the quantity of liquid fuel in the chamber decreases rapidly and additional feeding of the chamber is effected by combustion products. This phenomenon is explained by the influence of the pressure on the value of the delay period of fuel burnup. During the course of the process described, changes in the thermodynamic parameters are often observed, both as a result of a pressure change and as a result of a change in the relationship between fuel components.

After almost complete pressure decrease in the chamber, a new, relatively small pressure increase in the chamber is often observed, lasting for tenths of a second, or occasionally for several seconds [3, 21]. The indicated phenomenon is explained by the entrance into the chamber of burn-up vapor components from the chamber cooling channel.

The switch off period, including the time of fuel afterburning, is called the aftereffect period. It is characterized not only by the average value of the pressure pulse, but also by its dispersion.

5. Operation of the Engine after Switching Off

If a repeated start-up or the utilization of the engine as a brake is foreseen, then after switching off, the operation of temporary closing down is performed. Then the engine is prepared for a new, repeated operation. During this time the pressure in the accumulators is normalized, fuel is transferred to the tanks, and automatic equipment elements are prepared for the execution of new commands.

6. External Characteristics of Dynamic Processes

The nature of intrachamber processes is judged in accordance with their external manifestations. The basic source of information is the oscilloscope, which shows changes with respect to time of many parameters, including thrust, fuel component expenditures, and pressure in the central part of the chamber. An oscillogram or a group of oscillograms or a set, where the nature of the changes in the basic parameters are recorded during the course of the entire operating period of the combustion chamber, may be termed the external chamber characteristic. Figure 1 shows one of the oscillograms obtained during a test of an experimental prototype model of a liquid-fuel rocket engine. In order to obtain a sufficiently accurate representation of the operation of the chamber, the recording of the parameters must be accomplished through the use of a modern quick-response apparatus possessing high resolution.

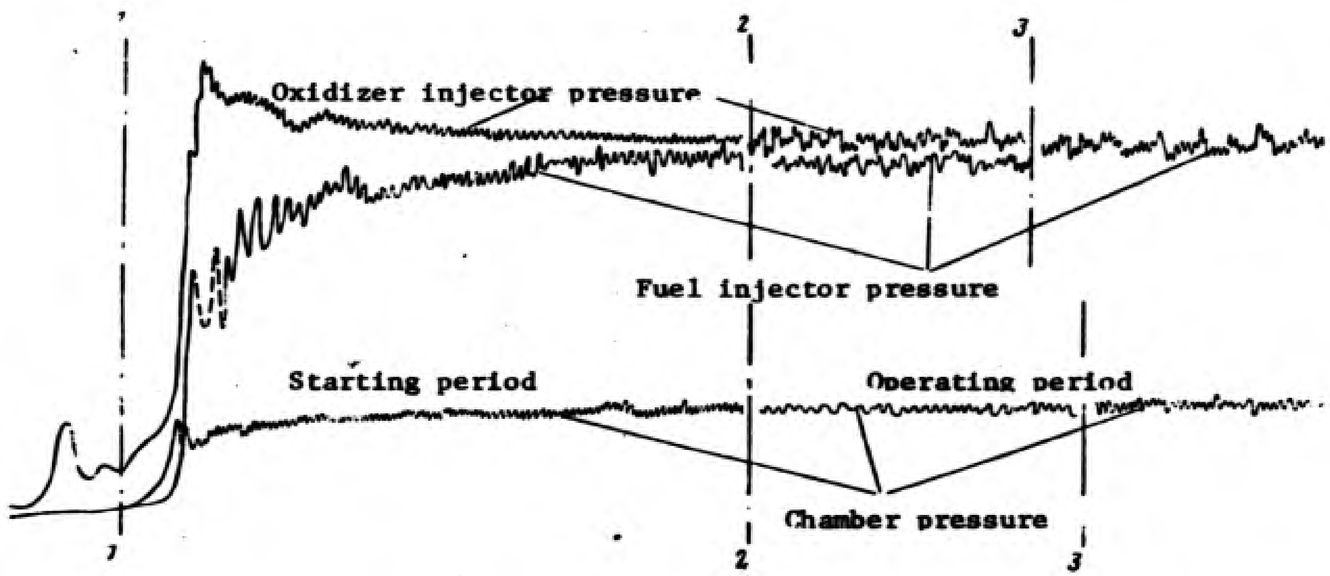

Figure 1. Record (oscillogram) of changes in chamber and injector pressures during starting and operating periods.

In examining the oscillograms, our attention is first of all directed to the presence of three periods of engine operation, which we have already discussed: the starting period, the operating period, and the switching off. Often, instead of the term "operating", we use the term "sustained condition". However this term, which assumes an average of varying parameter values, is considered to be arbitrary. In fact, various changes in their instantaneous values are revealed in the oscillograms. They occur during the starting period, during long, uninterrupted operation of the engine, and during the switching-off period.

The nature of the change in the parameters with respect to time, especially chamber pressure, is exceptionally complex. We shall examine, for example, several oscillograms. Figure 1 represents a record of the change in pressure at a certain point in the chamber and the pressure in front of the injectors, as obtained during engine tests on a stand. Before fuel ignition (left of the line 1 - 1) a temporary increase in pressure is established preceding the oxidizer injectors, caused by liquid movement through the hydraulic channels.

Before starting, a sudden pressure increase preceding the oxidizer injectors is noted. After this, some decrease in pressure in the chamber and high-amplitude progressively-damped pressure oscillations preceding the fuel injectors are observed. It is usually difficult to establish a clear line between the end of the starting function and the beginning of the operating function. On the oscillogram which we have examined, after line 1 - 1 chamber pressure slowly increases to a nominal value; rated pressure is gradually established in proportion to the movement toward 2 - 2. The curves included between the lines 2 - 2 and 3 - 3 represent an engine function during the operating mode in the first seconds after starting, and the curves to the right of the line 3 - 3, correspond approximately to the one hundredth second of engine operation.

Figure 2 represents examples of oscillograms obtained during a test of a small experimental liquid-fuel rocket engine. Depending on starting conditions, the various curves (A, B, C, D), were obtained. During operating conditions (curves E, F, G, H), low-amplitude high-frequency oscillations are observed, superimposed on low-frequency oscillations. Sawtooth oscillations (I, K) and oscillations, the amplitude of which periodically changes with time (J, L), are recorded. A record made with the aid of more accurate devices (M, O, P, Q, R, T) permitted a more precise determination of the nature of the oscillations.

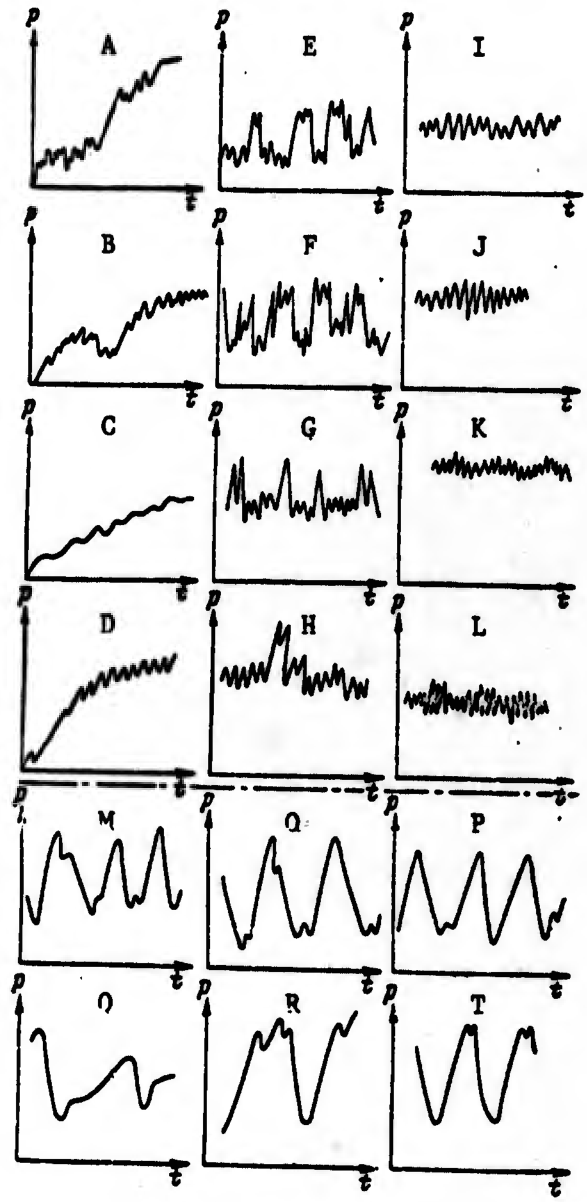

Figure 2. Oscillograms obtained during a liquid fuel rocket engine test. Pressure changes against time are shown.

We shall examine the processes occurring within the engine in accordance with average parameter values, i.e., without considering rapid changes or oscillations. Now it is necessary to distinguish between a "slow" and a "rapid" start. If 95% of the nominal thrust is achieved in 1 - 3 seconds, then the starting process is considered to be slow. Additional feeding of the chamber by fuel, accumulated during the starting period, exerts no noticeable influence on the nature of parameter changes with respect to time (Figure 3A).

During a rapid start, when tenths of a second are sufficient to reach 95% of the thrust, it is frequently possible to detect a segment of abrupt change in the indication of the derivative in the form of so-called "peaks" (Figure 3B), or "teeth" (Figure 3C) on the curve of the change in chamber pressure.

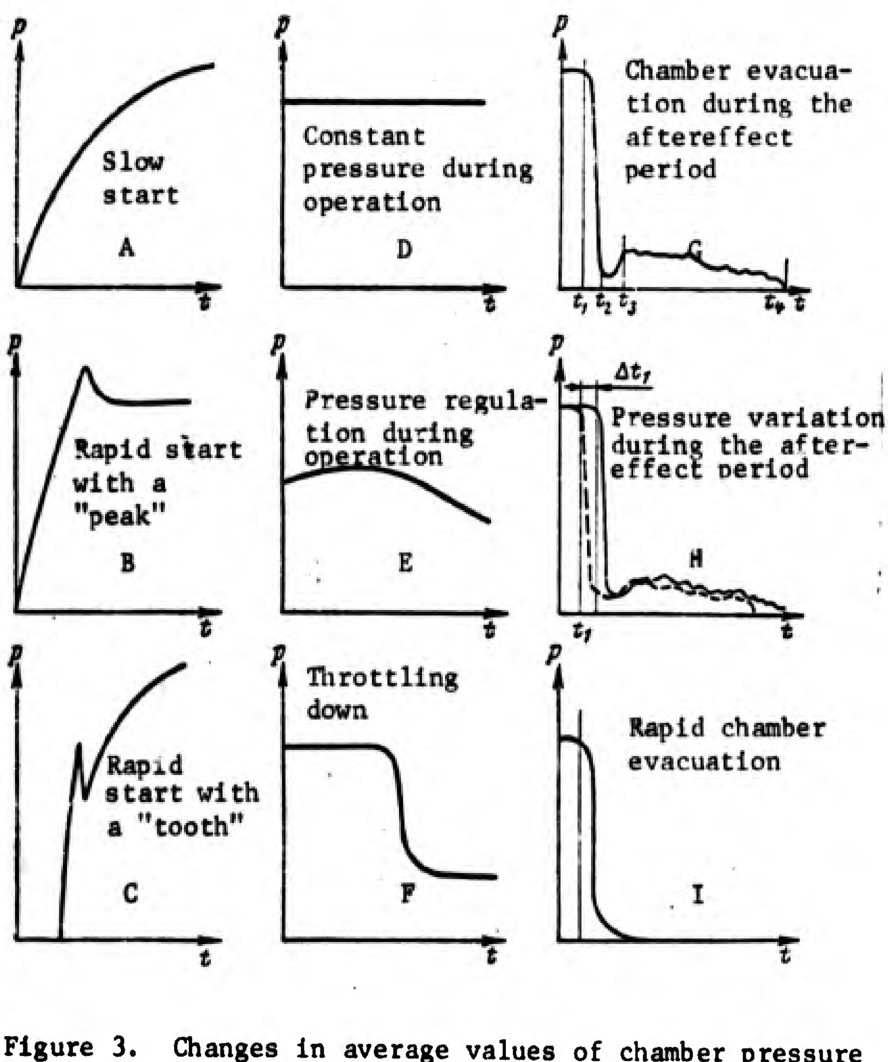

Figure 3. Changes in average values of chamber pressure versus time.

After the starting period, engine operation may proceed according to various routines. In some cases the average chamber pressure is maintained at a constant rate (Figure 3D); in others, the pressure changes with respect to time (Figure 3E) under the influence of external factors or even in compliance with incoming commands from the control or regulation system. If stage switching is carried out, then the average pressure changes comparatively rapidly (Figure 3F).

The switching off of the engine, as has already been pointed out, is accompanied by the curves of a series of distinctive processes. Oscillograms help to study the aftereffect period. Time t_1 corresponds to the delay in the operation of the fuel cut-off valves (Figure 3G). During the short interval of time $t_2 - t_1$, the main bulk of the gases from the internal chamber cavity are discarded. Initial warm-up of the internal side is observed until time t_3. During the relatively long period of time, equal to $t_4 - t_3$, burning of the component boiling away in the circulating channel of the cooling system continues within the combustion chamber. It is apparent from Figure 3G, that the burning is of an undefined nature. The curve p(t) represented in Figure 3G is called the aftereffect curve, and the area defined by it is called the afterpulse.

Due to a lack of definition in the process examined, the afterpulse is characterized by a variation in its values. Various values of the pulse may be recorded not only for a test of several engines of one and the same series, but also for a repeated test of one and the same model. The presence of a variation in impulse values leads to a decrease in the accuracy of rocket flight with respect to distance. The variation results from many causes.

Figure 3H reveals a displacement of the curve p(t) in a repeated engine test as a result of the variation in the moment of closure of the fuel valves: during the second test the valve was closed Δt_1 seconds later than in the first test. With the aim of decreasing the significance of the afterpulse and its variation, the accuracy of the operation of fuel cut-off valves is increased, and measures are taken to exclude fuel vapors from the internal cavity of the chamber. Thus chamber evacuation will occur rapidly and with sufficient uniformity (Figure 3I).

7. Several Reasons for the Formation of Oscillations

The investigation of oscillations begins with the study of oscillograms on which pressure, temperature, gas velocity and other parameters must be recorded with the aid of quick-response, sensitive devices. The form and nature of the oscillations may be quite different, as has already been illustrated in Figures 1 and 2. However, after an examination of many oscillograms it is possible to distinguish separately the characteristic forms of oscillations.

"Shock" low-frequency oscillations are often observed (Figure 4A); in some cases high-frequency oscillations are superimposed on these (Figure 4B). Low-frequency oscillations may be "sawtooth" (Figure 4C), or close to sinusoidal (Figure 4E). High-frequency oscillations (Figure 4D, F) may also be superimposed on these types of oscillations.

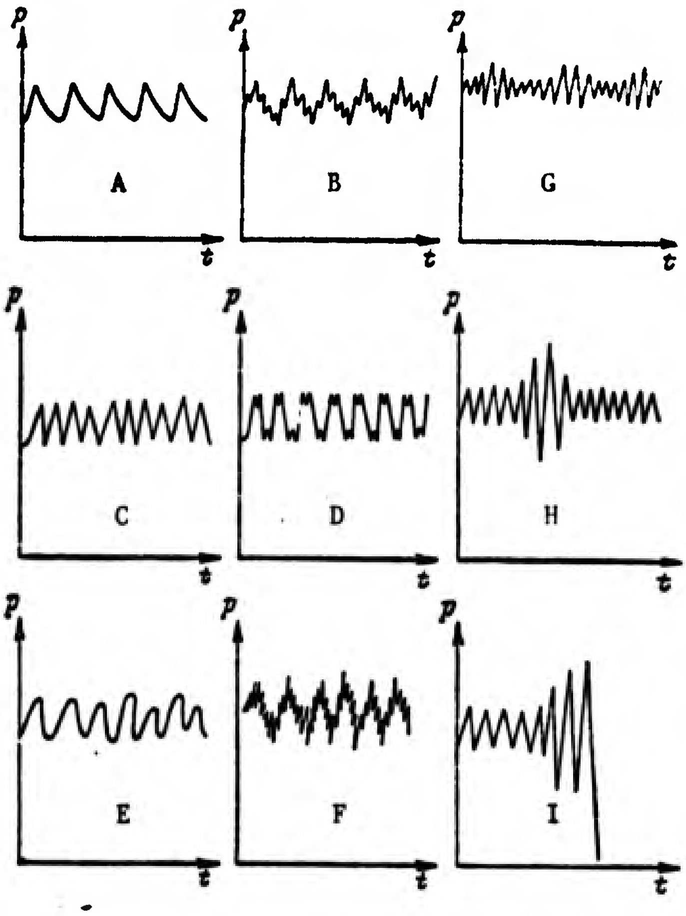

Figure 4. Various types of pressure oscillations in the combustion chamber.

More complex types of oscillations are also encountered; non-uniform and non-recurrent figures appear, and continuously occurring pulsations (Figure 4G), caused by resonant phenomena, occur both in the chamber and in feed-system subassemblies as well. In some cases, during the course of the entire period of an engine test, the pulsations appear only one time. If the amplitude of the oscillations is relatively low (Figure 4H), the preceding condition is reestablished. Excessively powerful oscillations, caused by resonance (Figure 4I), lead to engine destruction.

There are several types of low-frequency oscillations. In addition to oscillations generated as a result of the connection between the chamber and the hydraulic circuits, pressure oscillations, caused only by intrachamber processes, are also observed. The nature of the oscillations depends on the effects of pressure on the combustion delay period, on the inertia of the hydraulic circuits, on changes in the reactant ratio with respect to time, on the absolute value of the chamber pressure, and on other factors.

Let us examine oscillations generated as a result of the existing relationships between the chamber and the feed system. We shall assume that the reactant ratio is constant and that the combustion delay is equal to zero. Thus, the simplest case of the relationship between the chamber and the feed system is examined. Under the influence of randomly appearing external influences, the chamber pressure increases from nominal value p_0 to some new value p_1 (Figure 5). Since gas discharge from the nozzle, characterized by line 1, will be greater than the fuel intake into the chamber, which is defined by line 2, then chamber pressure will begin to decrease.

After the effect of the perturbation factor, the pressure change in the chamber with respect to time may be exponential or oscillatory. If the factors damping the system prove to be sufficiently intensive, then the movement will be aperiodic with an exponential change in parameters. In an opposite case, oscillations will be generated. Since the moving fluid and the combustion products are inert during a reduction in pressure, point a, which corresponds to the established cycle, will be "passed", and the pressure will decrease to value p_2 (curve 1). If damping occurs, then the pressure p_2 will be less than p_1. Now the fuel intake will be greater than the discharge of gases. This leads to a new increase in pressure. Thus, low-frequency damped oscillations will be observed in the system.

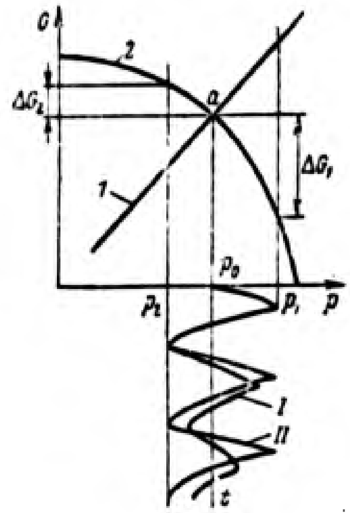

Figure 5. Schematic diagram of oscillations generated by virtue of the relationship between the parameters of the chamber and the hydraulic circuit.

In fact, the fuel burns with a delay. Let us examine the instant when the chamber presssure decreases to value p_2 (see Figure 5). The intake of fuel into the chamber is now at a value ΔG_2 greater than the nominal value, which is defined by point a. An additional quantity of fuel does not burn at the instant corresponding to pressure p_2, but with a delay. The system receives additional excitation, leading to an increase in amplitude (curve 2) in comparison with the amplitude that would be obtained if the oscillations were damped. When a pressure close to p_1 is achieved, the fuel input into the chamber, as shown in Figure 5, is less than the nominal amount. However, considering the delay, the effect obtained from the combustion of a quantity of fuel reduced by ΔG_1 appears with a delay when the pressure is less than p_0; this also supports the generation of oscillations. If deviations in consumptions ΔG_1 and ΔG_2 are not equal, then the nature of the pressure increase in the oscillatory process will differ from the nature of the pressure decrease.

It is known that the delay period decreases with an increase in pressure. This relationship exerts an influence on the nature of the oscillations. Finally, under the influence of all of the factors examined, during the period of oscillations the mass intake into the chamber is equal to the mass expenditure, just as the energy intake is equal to its expenditure, by virtue of which the amplitude of the low-frequency oscillations are maintained constant during engine operation.

Low-frequency oscillations may also occur without the presence of perturbation and damping factors from the feed system. Let us assume that the feed system delivers ingredients into the combustion chamber uniformly and with a constant relationship between these components. It is necessary to note, however, that when intrachamber perturbations are present, they are partially transmitted to the hydraulic circuit. They may be detected in the circulating part of the injectors and, sometimes in the head, before the entrance into the injector. Intrachamber instability arises as a consequence of periodically recurring feeding of the chamber, caused by the influence of pressure on the value of the delay period.

Let us assume that the chamber operates with constant pressure p_c, which corresponds to a certain constant value of the delay period τ. Therefore, a constant quantity of liquid is located in the chamber, prepared for combustion. When the pressure increases, period τ decreases, which leads to a decrease in the quantity of liquid in the chamber, and therefore to an increase in the input of gaseous products. However, formerly, the transition from the liquid phase to the gaseous occurs with a delay. With a pressure decrease, the reverse phenomenon is observed, i.e., an increase in τ, and an increase in the quantity of the liquid phase.

In the foregoing cases, feeding of the chamber with fuel occurred under the influence of mass forces, moving the liquid, and as a result of the influence of pressure on the delay. Intrachamber instability is explained only by the latter factor. Therefore the power of the intrachamber oscillations differs from the power of oscillations caused by the relation between the chamber and the hydraulic circuits of the feed system.

The entire mass of gases in the chamber takes part in the generation of low-frequency oscillations. During the investigation, gas parameters are considered to be unchanging with respect to chamber coordinates. Under this assumption, the study of low-frequency oscillations includes differential equations to the zero degree. In fact, gas velocity and the pressure change in accordance with the length and the section of the chamber, and even counter-currents are observed in the area of injector placement. Preparation of the fuel for combustion and fuel combustion occur in such a manner that the intensity of gas generation changes with respect to time and differs at various points within the chamber. Combustion products, having formed in various places of the chamber, intersect the critical nozzle section for only certain time intervals. A definite amount of time passes from the instant of the beginning of combustion of a certain amount of fuel to the instant of detection of the greatest pressure increase. The starting instant of combustion does not coincide with the instant of the discharge of those gases which are formed during combustion of that portion of the fuel which feeds the chamber during the oscillatory process. The indicated shifts in time are considered to be a preface to the "chamber relaxation time", which determines the delay in the influence of the combustion velocity on intrachamber processes. As a result of deformation in pipes and the compressibility of liquid, a time displacement in the injection velocity is observed. This displacement is considered a preface to the "feed system relaxation time".

The presence of the factors examined indicates the necessity for studying the low-frequency oscillations with the aid of equations which take into consideration the parameter changes in the length of the chamber.

The behavior, with respect to time, of the reactant ratio K exerts a definite influence on the nature of the oscillations. If $K = \text{const}$, then the product RT, which characterizes efficiency, is practically independent of time. If the reactant ratio changes with time, then the derivative of RT with respect to t appears, which has an output dimension. Thus the intrachamber processes are characterized not only by parameter K, but also by parameter $\dot{K}$, which represents a derivative of K with respect to t.

The parameter $\dot{K}$ differs fundamentally from K. The reactant ratio characterizes the composition of combustion products (with a specified fuel and with specified combustion conditions), but $\dot{K}$ defines the velocity of the changes in composition of combustion products. As is known, the combustion velocity depends on the reactant ratio. If $\dot{K}$ equals zero, then with the given pressure in the chamber, the combustion velocity is constant; if

$\dot{K}$ is non-zero, then accelerated combustion will continue. The accelerated gas formation causes an accelerated movement of gas masses. If the value $\dot{K}$ is sufficiently large, then due to gas movement acceleration, a noticeable localized pressure increase or a pressure increase in the entire chamber volume may occur.

With reduced engine throttle, i.e., with reduced pressure in the chamber, the so-called threshold of stability is observed, which is developed due to the fact that with a pressure decrease in the chamber at a definite moment in time, comparatively rapid and sharp increases in the amplitude of oscillations occur [12]. At least two factors contribute to the formation of the threshold of stability. When the engine is throttled down, a decrease in hydraulic losses at the injectors occurs, due to a decrease in propellant consumption. Even if the period $\tau = 0$, then, as has been pointed out earlier, the expo... ...al nature of the attentuation of the pressure, generated under the influ... of an external disturbance, becomes periodic. With a decrease in hydraulic losses the quality of atomization worsens, and therefore the value of the combustion lag period increases. Thus an increase in τ and a decrease in the stability of engine operation as a whole also leads to a sudden increase i the amplitude of pressure oscillations.

In addition to low frequency oscillations, high frequency oscillations are also observed in the chamber, which owe their origin to wave processes. If the frequency of oscillations is sufficiently high, the wavelength will be commensurate with the length or the diameter of the combustion chamber. Under these conditions one may not consider the pressure to be uniformly distributed throughout the entire chamber volume. An individual pressure value, continuously changing with time, will be observed at each point of the chamber volume. If in an investigation of low-frequency oscillations we were interested in the lag period τ, then it would be necessary to also know the distribution of burning drops throughout the chamber volume.

The study of low-frequency oscillations may be effected in some cases by using ordinary differential equations, but for the investigation of high-frequency oscillations partial differential equations must be included, insofar as the process takes place in time and in space. In Chapter I partial differential equations will be examined which characterize intrachamber processes. These equations may also be used to a certain extent in the study of high-frequency oscillations. However, this is a large independent problem which exceeds the limits of this book.

High-frequency oscillations may be described by the following highly simplified model. Let us imagine a standing half-wave distributed along the axis of the chamber. Let us assume that in the area of the head, the pressure will continuously change with time. During the period in which the lowest pressure is predominant in the upper part of the chamber, conditions will be produced suitable for the delivery of an additional portion of propellant to the chamber. The propellant particles delivered, burning with a delay, will support the oscillating condition. The nature of the oscillations is also defined by the distinctive distribution of the burning propellant throughout the chamber volume.

Another type of high-frequency oscillations is caused by the presence of longitudinal pressure waves. Let us assume that due to accelerated burning in the upper part of the chamber, localized increases in pressure have developed. A single wave pulse, in the process of its movement along the axis of the chamber, is cut off from the walls of the nozzle entrance, and is directed upward toward the head. In intersecting the burning front it accelerates there a transformation leading to an intensification of the burning, and to the appearance of a new pulse. The process examined, depending on a combination of parameters, may become attenuated with time, or it may develop and lead even to the destruction of the chamber, or as more often happens it may become stabilized and continue at a constant amplitude.

The third type of high-frequency oscillations is to a certain extent analogous to that already examined, but it is supported by transverse (radial) waves, which develop mainly in the upper part of the chamber.

In an investigation of intrachamber processes, fundamental frequency oscillations are most clearly revealed. In a majority of cases the remaining frequencies are not multiples of the fundamental frequency.

8. On the Systems of Equations Describing Intrachamber Processes

The conditions of engine operation examined reveal that the number and type of equations are determined by the problems of the investigation. If the starting function, the operating function, or the switching off function is examined without considering the oscillatory processes, then we direct our attention to ordinary non-dimensional differential equations. In the study of low-frequency oscillations, ordinary differential equations are used, but one of them contains a delay parameter.

In addition to differential equations, algebraic equations are included, but particular solutions are also shown in tabular or graph form.

In the study of parameter changes not only in time, but in space as well, partial differential equations are used.

The system of equations must be closed. In its formulation, the orientation must be toward the possibilities for solution available to the investigator; however the choice of equations and the methods of solution must not be detrimental to the accuracy of the results obtained. Simultaneously with the formulation of the system, adequate boundary conditions are required for determining the solutions.

A large quantity of experimental data, the analysis of statistical material, and the results of independent investigations permit the refinement of design equations. With each instance, practice makes new and increasingly more stringent demands on the results of theoretical investigations, which occasionally are too modest with respect to practical recommendations. This is explained by the fact that in a majority of cases, exceptionally complex intrachamber processes are studied with the aid of simplified equations. As a result of the neglect of many factors, which seem at first to be secondary, the equations employed in investigations do not reflect the true picture, and naturally theoretical investigation could not lead to the desired results. Therefore the refinement (and not simplification) of equations must be combined with the development of new methods of mathematical analysis.

The equations discussed in the foregoing paragraphs, notwithstanding their relative complexity, approximately reflect the nature of the processes. They contain a whole series of limitations; the equations do not consider all possible forms of heat exchange, vortical and circulating phenomena .. not examined, the influence of friction forces is insufficiently detailed, fuel flame equations have been simplified, and so on. In organizing an investigation these equations may not be used without a preliminary critical analysis.

The processes which occur in engine assemblies are so complex and distinctive that in the organization of new investigations, the employment of resultant equations, developed by other authors, is completely inadmissible. It is possible to formulate the required system of differential and algebraic equations only after a thorough study of all of the peculiarities in the relationships between the processes.

While organizing an investigation, we must direct our attention to fundamental objective laws, but in this connection we must also carefully study other equations and factors typical of specified concrete problems.

§ 2. Derivation of the Law of Conservation of Mass for the Combustion Chamber

It has already been noted that propellant entering the chamber burns with a certain time delay. Before the engine is started, the pressure in the chamber is equal to the pressure of the surrounding medium, that during which time delay $t = -\tau_0$. In deriving the combustion chamber equation, time is computed from the time of combustion initiation.

1. Entrance of Propellant into the Chamber

Time $t = -\tau_0$ corresponds to the initiation of propellant delivery into the chamber. In the course of τ_0 seconds, a definite amount of liquid propellant has already accumulated in the chamber (Figure 6).

At arbitrary time t, Y mass units of propellant will have entered the chamber. The nature of the accumulation Y(t) with respect to time is approximately as follows. At first, in τ_0 sec, while the pressure in the chamber is equal to the pressure of the surrounding medium or to some other nominal value, the propellant input into the chamber will depend on the method of operation of the feed system. If the feed system pressure is constant, then the input per second will be practically constant, and the propellant accumulation is represented by line 0a.

During the process of an increase in chamber pressure, which begins after the moment of combustion initiation, the chamber resistance will increase. In this connection a change in input pressure is often noted due to a change in the number of revolutions of the turbine shaft, and it often happens that the propellant input per second into the chamber decreases somewhat at first, and then increases to a nominal value. The resultant quantity of propellant which has entered the chamber is characterized by the curve abc. At point b, which corresponds to minimum consumption, a change in the sign of the derivative $\dot{Y}$ occurs. After the engine starts, the propellant input to the chamber on the segment c'd' does not change with respect to time; therefore the function Y(t) is presented as the straight line cd.

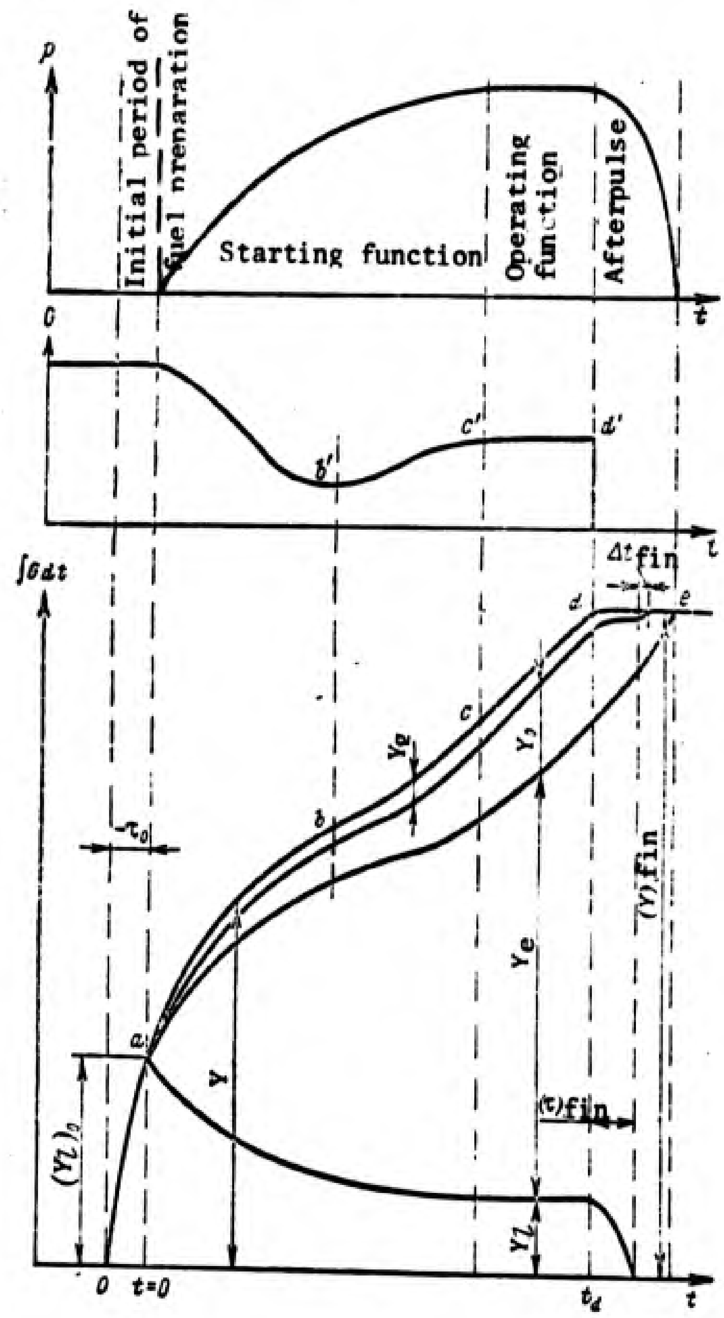

Figure 6. The nature of the change of certain parameters and their time integral values.

Switching off of the engine occurs at time t_d, and propellant delivery to the chamber ceases. Then the value Y_{fin} remains fixed for all times. For a certain interval, this is present as de.

2. Preparation for Burning and Burning of the Liquid Propellant

At first, the propellant constituents which are atomized by the injectors (during the time delay) are warmed, deformed, and divided; some vaporization and diffusion is observed. Then the period of intensive vaporization occurs, and the propellant constituents change from a liquid to a gaseous state. Vaporization is accompanied by diffusion, and at high pressures, exceeding a critical value, it exerts a decisive influence on the mass exchange process. As a result of the reactions which occur between vaporization products, combustion products are formed. We call the mixture of liquid propellant with inter-reacting combustion products the burning propellant or the burning stream. Let us examine the nature of a change with respect to time of a quantity of liquid propellant within the chamber. By the time of initiation of burning (t = 0), the quantity of still unvaporized liquid propellant is equivalent to $(Y_l)_0$ (See Figure 6). The value of the time delay decreases in proportion to the increase in pressure, which leads to a decrease in the quantity of liquid propellant Y_l in the chamber in a preparatory stage for burning. Burn-up of the remainder of liquid propellant begins at the moment the engine is switched off, and is completed during the period equal to $(\tau)_{fin}$ sec.

In addition to the liquid propellant in the chamber in the stage of preparation for burning, burning liquid propellant is also present. Here we have in mind that part of liquid propellant Y_g, which has been transformed into a gaseous state as a result of vaporization. After the initiation of combustion, the quanity of liquid burning (vaporized) propellant, located within the chamber, will constantly change. This quantity depends on the intensity of propellant delivery to the chamber, on the rate of preparation of the liquid propellant for burning, and on the rate of vaporization.

After the engine is switched off, during time interval τ_{fin} all of the propellant which was in the preparatory stage for burning will be consumed, and the entire burning process will cease Δt_{fin} sec later. This time is not long since it consists of hundredths or thousandths of a second, but it must be considered in a study of the aftereffect.

3. A Change in the Quantity of Gaseous Products

The quantity of vaporization products will depend mainly on the rate of vaporization and the mutual speed of response of the gaseous products, i.e., on the burning rate which occurs mainly in the gaseous phase.

The total quantity of vaporization products and combustion products located within the chamber and designated by Y_0 will increase to agree with the equation of state in proportion to the increase in chamber pressure. During the engine operating function, Y_0 = const, but after the engine is switched off the quantity will decrease rapidly.

4. The Emission of Combustion Products

During engine operation the emission of combustion products from the nozzle is observed. The longer the time interval involved, the higher the value Y_e. The nature of the change $Y_e(t)$ depends on many factors, including the change in the values examined above with respect to time. After the engine is stopped, the total quantity of emitted combustion products will equal the total quantity of propellant delivered to and burned within the chamber for the entire period of engine operation.

5. Derivation of the Equation

The law of the conservation of mass for the chamber is expressed as follows:

$$Y - Y_l - Y_g - Y_0 - Y_e = 0. \tag{1.1}$$

In this equation any total quantity Y_i supplied to or accumulated within the chamber or a product emitted is an integrated value, in which

$$Y_i = \int_0^t G_i\,dt. \tag{1.2}$$

Differentiating (1.1) with respect to t, we obtain:

$$\dot{Y} - \dot{Y}_l - \dot{Y}_g - \dot{Y}_0 - \dot{Y}_e = 0. \tag{1.3}$$

Let us define the terms of this equation.

An aggregate quantity of fuel introduced into the chamber at time t, considering that fuel delivery began at time $(-\tau_0)$, comprises

$$Y = \int_{-\tau_0}^{t} G_\Sigma \, dt, \tag{1.4}$$

where $G_\Sigma = G_1 + G_2$, the mass flow per second of oxidizer and fuel through the exit areas of the injectors.

Differentiating with respect to t, we find:

$$\dot{Y} = G_\Sigma = G_1 + G_2. \tag{1.5}$$

The reactant ratio is defined by the value

$$K = \frac{G_1}{G_2}. \tag{1.6}$$

The derivative $\dot{K}$ changes with respect to time even during the operating function.

$$\dot{K} = \frac{d}{dt}\left(\frac{G_1}{G_2}\right). \tag{1.7}$$

The quantity of fuel in the chamber in a liquid state, and in the stage of preparation, equals:

$$Y_l = \int_{t-\tau}^{t} G_\Sigma \, dt. \tag{1.8}$$

Therefore,

$$\dot{Y}_l = G_\Sigma - (G_\Sigma)_{-\tau} \cdot (1 - \dot{\tau}). \tag{1.9}$$

Here the index $(-\tau)$ reveals that the flow quantities $(G_\Sigma) = (G_1 + G_2)$ must be introduced into the calculation referred to the time $(t - \tau)$.

In the initial starting period, when $\tau = \tau_0 = \text{const}$, the aggregate quantity of liquid propellant accumulated within the chamber is

$$(Y_l)_0 = \int_{-\tau_0}^{0} G_\Sigma \, dt. \tag{1.10}$$

If $(G_2) = \text{const}$, then

$$(Y_l)_0 = \dot{G}_\Sigma \tau_0. \tag{1.11}$$

The equation for the first two terms (1.3) is expressed thusly:

$$\dot{Y} - \dot{Y}_l = (G_\Sigma)_{-\tau} \cdot (1 - \dot{\tau}). \qquad (1.12)$$

Let us determine the quantity of burning liquid propellant. A detailed study of propellant combustion within the chamber is outside the limits of this book. Much work by native and foreign authors has been devoted to intrachamber combustion of liquid fuel rocket engines. Propellant vaporization conditions within the chamber are separately examined in § 8 of this Chapter. Let us assume that the process of vaporization is described by the equation introducted in [12]:

$$\frac{dS_i}{dt} = -4\pi\Psi, \qquad (1.13)$$

where S_i is the burning surface;

Ψ is the burn-up parameter.

An investigation of the burning stream reveals that the shape of the burning drops differs from the spherical; frequently they are lenticular. In the process of flying and burning the drops are deformed -- their shape changes and their division into small parts occurs. For a drop of random shape

$$S_i = \xi_s^{-1} \cdot 4\pi R^2, \qquad (1.14)$$

where ξ_s is a coefficient which takes into consideration the shape of the drop, i.e.,

$$\xi_s = \left(\frac{R}{r_s}\right)^2; \qquad (1.15)$$

r_s is the radius of a spherical drop, the surface of which does not differ in value from the surface of the drop observed;

R is the instantaneous value of a certain characteristic drop dimension.

Utilizing (1.13) and (1.14), we find:

$$R = \left(R_0^2 - \int_0^t \xi_s \Psi \, dt\right)^{0,5}, \qquad (1.16)$$

where R_0 is the initial value of the characteristic dimension.

Parameter Ψ depends on the nature of the course of intrachamber processes, the properties of propellant constituents, the conditions and quality of atomization, the reactant ratio and so on. Sometimes in carrying out a first approximation, Ψ is considered a constant. In this case in place of (1.16), we have:

$$R = (R_0^2 - \xi_s \Psi t)^{0,5}. \tag{1.17}$$

We determine the area of the burning surface and the quantity of the burning propellant through various methods. Often we direct our attention to several characteristic drop dimensions or use the continuous law for the probability distribution of the appearance of various values of drop dimensions. Let us assume that characteristic drop dimensions m have been chosen, and the number of drops of each dimension equals n_i; the area of the burning surface will equal:

$$S = 4\pi\xi_s^{-1} \sum_1^m n_i R_i^2. \tag{1.18}$$

If Ψ = const, then

$$S = 4\pi \sum_1^m n_i \left(\xi_s^{-1} R_{0i}^2 - \Psi t\right). \tag{1.19}$$

Even if $\Psi \neq$ const, then

$$S = 4\pi \sum_1^m n_i \left(\xi_s^{-1} R_{0i}^2 - \int_0^t \Psi \, dt\right). \tag{1.20}$$

If all of the drops are identical and when t = 0 and radius R = R_0, then the law of mass change of one drop with respect to time is expressed as:

$$Y_{dr} = \frac{4}{3} \pi \varrho_l \xi_Y^{-1} \left(R_0^2 - \int_0^t \xi_s \Psi \, dt\right)^{3/2}. \tag{1.21}$$

If $\xi_s \Psi$ = const, then

$$Y_{dr} = \frac{4}{3} \pi \varrho_l \xi_Y^{-1} (R_0^2 - \xi_s \Psi t)^{3/2}, \tag{1.22}$$

where ξ_Y is a coefficient which takes into consideration the shape of the drop.

Since

$$Y_{dr} = \frac{4}{3}\pi\xi_Y{}'\varrho_l R^3, \tag{1.23}$$

then

$$\xi_Y = \left(\frac{R}{r_Y}\right)^3.$$

where r_Y is the radius of a spherical drop, the mass of which is equal to the mass of the drop examined.

The coefficients ξ_s and ξ_Y are time functions, since, as has already been noted, the shape of the drop in the process of its flight is continuously changing. However, due to the exceptional complexity involved in the studv of the nature of the deformation of burning drops, these coefficients are accepted as constant, and average values are assigned to them. In carrying out an approximation, the number of drops within the chamber in the burning stage may be determined from the formula

$$n = \frac{G_\Sigma t_0}{\overline{Y}_{dr\ 0}}, \tag{1.24}$$

where $\overline{Y}_{dr\ 0}$ is the average initial mass value of a burning drop.

The total burning time t_0 is easily determined from the expression (1.16), if R = 0 is assumed. Then we obtain:

$$R_0^2 = \int_0^{t_0} \xi_s \Psi\, dt. \tag{1.25}$$

In determining t_0, which is the upper limit of the integration, it is necessary to know the function $\xi_s(t) \cdot \Psi(t)$ and to find the integral (1.25).

If $\xi_s\Psi$ = const, then

$$t_0 = \frac{R_0^2}{\xi_s \Psi}. \tag{1.26}$$

If $(\xi_s\Psi) \neq$ const, then with given R_0 and in the absence of further division, the average arithmetical value is

$$\overline{Y}_{dr} = \frac{4}{3}\pi\xi_Y{}'\varrho_l t_0^{-1} \int_0^{t_0}\left(R_0^2 - \int_0^t \xi_s \Psi\, dt\right)^{3/2} dt. \tag{1.27}$$

If $(\xi_s\Psi)$ = const, then, using (1.26), we find:

$$\overline{Y}_{dr} = \frac{4}{3}\pi\xi_s\xi_Y^{-1}\Psi\rho_l R_0^{-2}\int_0^{t_0}(R_0^2-\xi_s\Psi t)^{3/2}dt. \qquad (1.28)$$

After integration, t_0 is replaced in accordance with (1.26) and after transformations, we obtain:

$$\overline{Y}_{dr} = \frac{8}{15}\pi\xi_Y^{-1}\rho_l R_0^3. \qquad (1.29)$$

If m initial values R_{0i} are examined, and, just as before, if their quantities n_i are known, then for $\xi_s\Psi$ = const the total quantity of burning liquid propellant comprises

$$\overline{Y}_g = \frac{8}{15}\xi_Y^{-1}\pi\rho_l\sum_1^m n_i R_{0i}^3. \qquad (1.30)$$

If we direct our attention to one typical dimension, then in accordance with the formulas (1.23), (1.24), (1.26) and (1.30), we find

$$\overline{Y}_g = \frac{2R_0^2}{5\xi_s\Psi}G_\Sigma. \qquad (1.31)$$

Therefore,

$$\dot{Y}_g = \frac{2R_0^2}{5\xi_s\Psi}\dot{G}_\Sigma. \qquad (1.32)$$

The quantity of gas in the chamber, consisting of vaporization and combustion products, is determined in accordance with the equation of state:

$$Y_{vap} + Y_{co} = Y_0 = \frac{p_c V}{RT} \qquad (1.33)$$

where V is the unoccupied chamber volume.

Designating the constant volume of the chamber, determined in accordance with the geometric dimensions (from a diagram), by the symbol V_0, we obtain:

$$V = V_0 - \frac{Y_l + Y_g}{\rho_l}. \qquad (1.34)$$

Employing (1.8), we find:

$$V = V_0 - \frac{1}{\varrho_l}\left[\int_{t-\tau}^{t} G_\Sigma\, dt + Y_g\right]. \qquad (1.35)$$

Differentiation with respect to t gives:

$$\dot{V} = -\frac{1}{\varrho_l}\,[G_\Sigma - (G_\Sigma)_{-\tau}(1-\dot{\tau}) + \dot{Y}_g]. \qquad (1.36)$$

If the product RT is a time function, then in differentiating (1.33) with respect to t, we find:

$$\dot{Y}_0 = \frac{V}{RT}\dot{p}_c + \frac{p_c}{RT}\dot{V} - \frac{p_c V}{(RT)^2}\frac{d}{dt}(RT). \qquad (1.37)$$

If the time of the flow of intrachamber processes is commensurate with combustion time, then it is necessary to consider changes in gas composition caused by an increase in the completeness of combustion. In this connection, according to the results of calculation, burning of the gaseous mixture may be given as [2]:

$$RT = RT(t). \qquad (1.38)$$

During the time under consideration, a change in the reactant ratio may occur, and consequently a change in gas composition may take place. The circumstance indicated is the second reason for a change in gas efficiency. Thus,

$$RT = RT(K,\ t). \qquad (1.39)$$

If the product RT, which characterizes gas efficiency, changes due to a change in the reactant ratio, then

$$\frac{d}{dt}(RT) = \frac{\partial(RT)}{\partial K}\dot{K}. \qquad (1.40)$$

The partial derivative

$$\frac{\partial(RT)}{\partial K} = f(K) \qquad (1.41)$$

may be computed for any propellant, if the functional graph is known (1.39).

Thus in employing equation (1.37), it is also necessary to include equation (1.39) and (1.40).

In order to determine the last term of the equation (1.1), we shall employ the equation

$$Y_e = \int_0^t G_{cr} dt, \tag{1.42}$$

where G_{cr} is the mass flow of combustion products from the nozzle. In differentiating with respect to t, we find:

$$\dot{Y}_e = G_{cr}. \tag{1.43}$$

As is known (see [7]),

$$G_{cr} = aF_{cr} (RT)^{-0.5} \cdot p_c, \tag{1.44}$$

where F_{cr} is the area of the exit nozzle throat; and a is a polytropic exponential function, expressed as

$$a = \left(\frac{2}{n+1}\right)^{\frac{1}{n-1}} \cdot \left(2\frac{n}{n+1}\right)^{0,5}. \tag{1.45}$$

The specific pressure pulse

$$\beta = \frac{(RT)^{0,5}}{a} \tag{1.46}$$

is a ratio of the pressure to the flow rate of the combustion products through a unit of area of the exit nozzle throat. It depends on propellant constituent properties, on the reactant ratio, and on combustion completeness. The function a reflects little change with the change in the index n and usually a ≈ 2.

Now in accordance with (1.44) and (1.46) we determine:

$$G_{cr} = \frac{F_{cr}}{\beta} p_c. \tag{1.47}$$

For further transformations it is important to note that gas residence time is:

$$\tau = \frac{1}{F_{cr}} \int_0^{l_{cr}} \frac{\beta F}{RT} dx, \tag{1.48}$$

where l_{cr} is the distance from the head to the exit nozzle throat;

F is the instantaneous value of the cross-sectional area of the chamber.

Directing our attention to average values of the parameters, we find that [7, 46]:

$$\varepsilon \approx \beta \frac{V}{F_{cr} RT}. \tag{1.49}$$

It is convenient to employ equation (1.48) in those cases when the oscillations are studied under the assumption that the parameters are changing with time and with respect to the length of the combustion chamber.

Having utilized the expressions (1.12), (1.32), (1.37), (1.47), and (1.49), the equation of conservation of mass for the chamber after transformation is expressed as follows:

$$\varepsilon \dot{p}_c + p_c - \frac{\beta}{F_{cr}} (G_\Sigma)_{-\tau} (1 - \dot{\tau}) + \frac{\beta}{F_{cr}} \left[\frac{p_c}{RT} \dot{V} + \dot{Y}_g + \frac{p_c V}{(RT)^2} \frac{d}{dt} (RT) \right] = 0. \tag{1.50}$$

The equation (1.50) is solved together with the equations which characterize the change in flow rates G_i and in the chamber parameters (ε, β, τ and so on) with respect to time.

§ 3. Application of the Combustion Chamber Equation in the Solution of Several Problems

The fundamental equation (1.50), which characterizes the law of the conservation of mass and supplementary combustion chamber equations which have aided in the determination of ε, β, τ, V, ξ_s, Ψ and RT, are employed successfully in the calculation and investigation of many processes, occurring within the chamber, notwithstanding the fact that in the process of their derivation certain assumptions were made.

In § 1 a description of processes involving two categories was given: those processes which are studied through the employment of average parameter values, and those processes which are characterized by instantaneous parameter values. Included among the processes of the first category are the filling of the chamber with propellant up to the time of combustion initiation, the starting function, the operating function, and switching off the engine. Here the oscillations are not considered and therefore the influence of the delay parameter is disregarded. The processes of the second category include pressure oscillations within the chamber and propellant consumption. These investigations are carried out with the aid of differential equations with a delay parameter.

1. Filling of the Chamber with Propellant before Ignition

After the command to begin the delivery of propellant constituents to the chamber· the accumulation of liquid propellant in the internal cavity is observed as a result of force exerted during the induction period. The time lag assumes a definite and constant value, depending on chamber pressure. It is usually considered [28], that

$$\tau = \frac{\tau_1}{p_c^n}, \qquad (1.51)$$

where τ_1 is the value of the time lag which corresponds to normal pressure. It depends on the properties of the propellant constituents, on the reactant ratio, and on conditions of propellant injection into the chamber. The time lag up until the beginning of the moment of ignition is

$$\tau_0 = \frac{\tau_1}{p_0^n}, \qquad (1.52)$$

where p_0 is the chamber pressure before the start.

Provided that during the entire period under consideration pressure p_0 does not change, then τ_0 = const. Before the beginning of combustion fuel will accumulate in the internal cavity of the chamber, the quantity of which equals:

$$(Y_l)_0 = \int_{-\tau_0}^{0} G_\Sigma dt; \quad G_\Sigma = G_1 + G_2. \qquad (1.53)$$

If $(G_1 + G_2)$ = const, then

$$(Y_l)_0 = G_\Sigma \tau_0 = G_\Sigma \frac{\tau_1}{p_0^n}. \qquad (1.54)$$

The method of determination of the flow rate $(G_1 + G_2)$ is examined in Chapters II and III.

2. Starting of the Combustion Chamber

1. In the case of a rapid start, as has already been noted, besides considering the fuel entering from the feed system, it is also necessary to consider additional feeding which the chamber obtains as the result of burn-up of part of the liquid propellant located in the chamber in a state prepared for combustion. A decrease in the liquid phase is caused by a decrease in τ with an increase in pressure.

In order to determine τ, it is necessary to use formula (1.51). If it is possible, it is recommended that the influence of the drop in pressure at the injectors be considered. Good agreement between experimental and calculated data is observed in the case when in expression (1.51) it is understood that the pressure changes as a result of a change in propellant consumption (while maintaining K = const), and not the result of the change in F_{cr}. We note that every given combustion chamber requires an experimental determination of τ_1. In place of formula (1.51), we may record:

$$\tau=\tau_1 (aF_{cr})^n (RT)^{-0,5n} (G_\Sigma)^{-n}. \qquad (1.55)$$

The quantity of liquid propellant in the chamber, both in a state of preparation and in the burning state, comprises:

$$Y_l + Y_g = \int_{t-\tau}^{t} G_\Sigma dt + Y_g. \qquad (1.56)$$

If we consider $G_\Sigma = (G_1 + G_2) = \text{const}$, then

$$Y_l + Y_g = G_\Sigma \frac{\tau_1}{p^n_{av}} + Y_g. \qquad (1.57)$$

If $G_\Sigma = (G_1 + G_2) = \text{const}$ and in this connection the propellant flow rate before combustion is equal to the flow rate during the burning period, then the quantity of propellant consumed for additional feeding during the period of increased pressure from p_0 to p_c taking into consideration (1.31), will equal:

$$\Delta Y = \left[\tau_1 \left(\frac{1}{p_0^n} - \frac{1}{p_c^n}\right) - \frac{2R_0^2}{5\xi_s \Psi}\right] G_\Sigma. \qquad (1.58)$$

The equation for the determination of the propellant input flow rate, which arises as a result in the change in τ, is obtained by means of differentiating the expression (1.58); it is apparent that

$$G_{add} = n\tau_1 G_\Sigma \frac{\dot{p}_c}{p_c^{n+1}}. \tag{1.59}$$

If after the initiation of combustion a change in propellant input to the chamber from the feed system is observed, then

$$\Delta Y = (G_\Sigma)_0 \frac{\tau_0}{p_0^n} - \int_{t-\tau}^{t} G_\Sigma dt - Y_g. \tag{1.60}$$

In order to determine the additional feeding it is necessary to differentiate the expression (1.60). In this connection we obtain:

$$G_{add} = G_\Sigma + (G_\Sigma)_{-\tau}(1-\dot{\tau}) - \dot{Y}_g. \tag{1.61}$$

It is necessary to keep in mind that $\tau = \tau(p_c)$.

2. The equation derived in the preceding paragraph is a reference for the investigation of the combustion chamber's starting function:

$$\varepsilon\dot{p}_c + p_c - \beta F_{cr}^{-1}(G_\Sigma)_{-\tau}(1-\dot{\tau}) + \beta F_{cr}^{-1}\left[\frac{p_c}{RT}\dot{V} + \dot{Y}_g + \frac{p_c V}{(RT)^2}\frac{d}{dt}(RT)\right] = 0. \tag{1.62}$$

Before we begin the calculation, we shall evaluate separately the influence of the derivatives of the last term in the expression (1.62) on the resultant solution expected. If the last member of the equation may be disregarded, then the equation will be simplified and will assume the following form:

$$\varepsilon\dot{p}_c + p_c - \beta F_{cr}^{-1}(G_\Sigma)_{-\tau}(1-\dot{\tau}) = 0. \tag{1.63}$$

3. In order to plot a curve of the starting function without considering the oscillations (Figure 7b), it is necessary to employ the equation (1.63), but to remove the sign $(-\tau)$, while maintaining in agreement with (1.8) and (1.9) the derivative $\dot{\tau}$. Employing the equation (1.51), we obtain:

$$\dot{\tau} = -n\tau_1 \frac{\dot{p}_c}{p_c^{n+1}}. \tag{1.64}$$

Therefore, in place of (1.63) we have:

$$\varepsilon\dot{p}_c + p_c - \beta F_{cr}^{-1} G_\Sigma \left(1 + n\tau_1 \frac{\dot{p}_c}{p_c^{n+1}}\right) = 0. \qquad (1.65)$$

It is not difficult to see that the equation (1.65) contains the term

$$p_{add} = n\tau_1 \beta F_{cr}^{-1} G_\Sigma \frac{\dot{p}_c}{p_c^{n+1}}. \qquad (1.66)$$

In agreement with (1.59), it represents a pressure increase as a result of the additional feeding. The equation (1.65) reveals that the influence of the additional feeding is especially noticeable for high values of the derivative $\dot{p}_c$. Therefore in the case of a slow start, the last component in the equation (1.65), i.e., the derivative

$$\beta F_{cr}^{-1} G_\Sigma \dot{\tau},$$

may be disregarded. If the starting function occurs fairly rapidly, then the effect of the additional feeding is more apparent at the beginning of the period, i.e., when chamber pressures are low, than at the end of the starting function, when pressures are high. In the use of (1.65), it is necessary to calculate ε according to formula (1.49), and to calculate, according to formula (1.46), the specific pressure pulse β and to determine the values F_{cr}, n and τ_1; we note that equation (1.65) comprises three variables: p_c, G_1 and G_2.

If the constant reactant ratio

$$K = \frac{G_1}{G_2} = \text{const}$$

is provided in the transient state, then only one additional equation is included. At the beginning of the time period when t = 0, the chamber pressure $p_c = p_0$. However the flow G_Σ at this time usually differs from zero.

4. Pressure oscillations during the starting function (see Figure 7a) are investigated with the aid of equation (1.63). Since the chamber pressure during the starting process changes within wide limits, the time lag ($-\tau$) must be considered as a variable. The solution of the differential equation (1.63) with a variable delay parameter presents a considerable amount of difficulty. Figure 7c shows the time base of oscillations -- pressure variations during the starting function from the corresponding p(t), obtained without considering oscillations. In order to plot the time base from the solution to equation

(1.62) or (1.63) it is necessary to deduct the solution to equation (1.65).

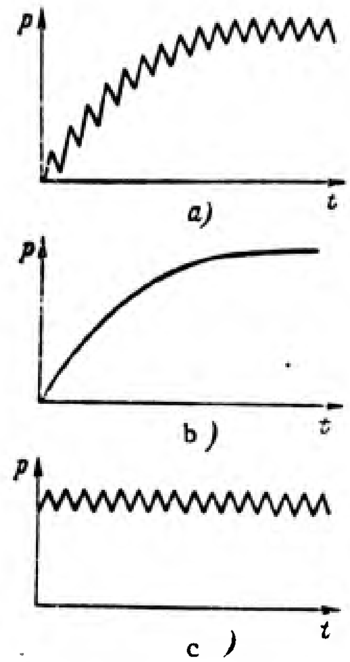

Figure 7. Time base of oscillations during the engine starting function.

3. <u>The Operating Function</u>

After starting (sustained condition), the average chamber pressure is maintained constant, or deviates from the nominal pressure under the influence of external factors, or changes to correspond with commands issuing from the control system. During the operating function, pressure oscillations and propellant consumption are observed.

1. If the pressure shows practically no change with respect to time, then all of the derivatives in the expression (1.50) becomes zero, and we proceed to the well known algebraic equation:

$$p_c - \beta F_{cr}^{-1} G_{\Sigma} = 0. \qquad (1.67)$$

The calculations are preceded by a determination of the value of the pressure pulse from the formula (1.46) or from experimental data. The thermodynamic parameters are determined from the results of thermodynamic calculations. It is necessary to point out that for a given propellant, the value of the specific pressure pulse is practically independent of chamber pressure, but changes substantially with a change in the reactant ratio. However during pressure changes due to control over the feed system components, the reactant ratio and therefore the specific pressure pulse may change. This relation will be considered in Chapter III.

2. The equation (1.67) is employed in an analysis of small deviations in chamber parameters. We shall rewrite expression (1.67) as follows:

$$p_c = \beta F_{cr}^{-1}(G_1+G_2). \tag{1.68}$$

The expression of the total functional differential is written in the following form:

$$dp_c = \frac{\partial p_c}{\partial \beta}d\beta + \frac{\partial p_c}{\partial F_{cr}}dF_{cr} + \frac{\partial p_c}{\partial G_1}dG_1 + \frac{\partial p_c}{\partial G_2}dG_2. \tag{1.69}$$

Passing on to small finite deviations, we find:

$$\Delta p_c = \frac{\partial p_c}{\partial \beta}\Delta\beta + \frac{\partial p_c}{\partial F_{cr}}\Delta F_{cr} + \frac{\partial p_c}{\partial G_1}\Delta G_1 + \frac{\partial p_c}{\partial G_2}\Delta G_2. \tag{1.70}$$

In solving problems with small finite deviations, the values for the partial derivatives are calculated for several given parameter values. Thus the values of the partial derivatives are presented in the form of numbers. The indicated expression is written as follows:

$$a_i = \left(\frac{\partial p_c}{\partial x_i}\right)_{\substack{p_c = p_0, \\ G_1 = G_{10}, \\ G_2 = G_{20}, \\ \dots}} \tag{1.71}$$

Therefore the number a_i is calculated for values $x_i = x_{i0}$. Usually the composite index in equation (1.71) is replaced by one symbol or another, for example, by an asterisk. The equation for the determination of partial derivatives is expressed using the usual rules of differentiation. In the example considered

$$a_1 = \left(\frac{\partial p_c}{\partial \beta}\right)_* = \left(\frac{G_\Sigma}{F_{cr}}\right)_*; \tag{1.72}$$

$$a_2 = \left(\frac{\partial p_c}{\partial F_{cr}}\right)_* = -(\beta F_{cr}^{-2} G_\Sigma)_*; \tag{1.73}$$

$$a_3 = \left(\frac{\partial p_c}{\partial G_1}\right)_* = (\beta F_{cr}^{-1})_*; \tag{1.74}$$

$$a_4=\left(\frac{\partial p_c}{\partial G_2}\right)_0=(\beta F_{cr}^{-1})_0. \qquad (1.75)$$

The calculated equation finally assumes the following form:

$$\Delta p_c=a_1\Delta\beta+a_2\Delta F_{cr}+a_3\Delta G_1+a_4\Delta G_2. \qquad (1.76)$$

3. In the case of a comparatively slow change in chamber parameters during the operating function, the derivatives appearing in the last term of equation (1.50) may be disregarded. The equation assumes the following form:

$$\varepsilon\dot{p}_c+p_c-\beta F_{cr}^{-1}G_\Sigma=0. \qquad (1.77)$$

In order to solve the problem it is necessary to know the nature of the change G_Σ (t), which depends on the operating conditions of the feed and control systems. At the initial moment, before the feed system exerts any influence on the chamber, quantities p_c, G_1 and G_2 show nominal values.

If we consider a change in the quantity of liquid propellant, located in the chamber in a stage prepared for combustion, then in place of equation (1.77), we write:

$$\varepsilon\dot{p}_c+p_c-\beta F_{cr}^{-1}G_\Sigma(1-\dot{\tau})=0. \qquad (1.78)$$

Employing equation (1.64), we find that:

$$\varepsilon\dot{p}_c+p_c-\beta F_{cr}^{-1}G_\Sigma\left(1-n\tau_1\frac{\dot{p}_c}{p_c^{n+1}}\right)=0. \qquad (1.79)$$

4. If small pressure deviations and flow rates are examined during the operating function, then it is convenient while disregarding τ to employ a linear homogeneous equation obtained from (1.78) by means of substituting

$$x = G_\Sigma - G_{\Sigma_0},$$

$$y = p_c - p_0;$$

we obtain:

$$\varepsilon\dot{y} + y - \beta F^{-1} x = 0. \qquad (1.80)$$

Here $G_\Sigma = G_1 + G_2$;

G_{Σ_0} is the calculated flow rate value;

p_0 is the calculated pressure.

The equations (1.79) and (1.80) each have three variables. Therefore, for a solution, the hydraulic circuit equations are included. If K = const, then, just as in the former case, one additional equation is sufficient.

5. Pressure oscillations during the operating function occur with a constant value of average pressure. It is therefore convenient to conduct an investigation of oscillations with equations with small deviations. The delay parameter is considered, but in some cases it may be considered as a constant. Let us consider the following variations in notation of the combustion chamber equation. In case it is necessary to obtain possibly a more accurate solution, with relatively large oscillation amplitudes and the high sensitivity of the time delay to pressure change, we must consider the differential equation with a variable time delay value in the form:

$$\varepsilon\dot{p}_c + p_c - \beta F_{cr}^{-1}(G_\Sigma)_{-\tau\neq\mathrm{const}}(1-\dot{\tau}) = 0. \qquad (1.81)$$

Less accurate results will be obtained if we employ the equation in the form:

$$\varepsilon\dot{p}_c + p_c - \beta F_{cr}^{-1}(G_\Sigma)_{-\tau=\mathrm{const}}(1-\dot{\tau}) = 0. \qquad (1.81a)$$

It is easier to solve equation (1.81a) than equation (1.81). It is valid for relatively low amplitudes of oscillations and for low sensitivity τ to pressure change. At low amplitudes, i.e., in those cases when small pressure changes occur, a determination of τ may be made by using a linear approximation in the form:

$$\tau = a - bp_c.$$

Here the values a and b depend on the average pressure p_0. If in the oscillatory process the time derivative of the pressure is small, then for the calculation we may recommend the formula

$$\varepsilon\dot{p}_c + p_c - \beta F_{cr}^{-1}(G_\Sigma)_{-\tau=\mathrm{const}} = 0. \qquad (1.82)$$

Thus depending on the nature of the oscillations, on the pressure change, and on the properties of the propellant constituents, assumptions are made in relation to the derivative ($\dot{\tau}$) and the propellant lag ($-\tau$) at the combustion chamber.

The statements in relation to the assumptions discussed may be tabulated.

Table of Recommendations

Process characteristic	Derivative ($\dot{\tau}$)	Propellant Lag ($-\tau$)
The pressure changes within wide limits	Exerts great influence in the range of low pressures	Pressure influence on the value of the time delay must be considered
The average pressure shows little change	The linear dependence of τ on p_c may be employed	The linear dependence may be employed
High-amplitude oscillations	A linear dependence may not be employed	It is desirable to consider the pressure effect
Low-amplitude oscillations during the operating function	The linear dependence may be employed	τ = const may be assumed
Sinusoidal oscillations. The value $\dot{p}_c$ is small	The effect of $\dot{\tau}$ may be disregarded	-
Shock oscillations. High values of $\dot{p}_c$	Acceleration of $\dot{\tau}$ may not be disregarded	-
τ is relatively insensitive to pressure change	The effect of $\dot{\tau}$ may be disregarded	τ = const may be assumed

It must be noted however, that resorting to the use of assumptions must be done very carefully. In the course of investigation it is necessary to compare a series of solutions obtained through the use of assumptions with solutions obtained while considering $\dot{\tau}$ and when $(-\tau) \neq$ const.

Deviations in parameters and in their derivatives from calculated values are designated as follows:

$$\left.\begin{aligned} x_1 &= G_1 - G_{10}; \\ \dot{x}_1 &= \dot{G}_1; \\ x_2 &= G_2 - G_{20}; \\ \dot{x}_2 &= \dot{G}_2; \\ y &= p_c - p_{c0}; \\ \dot{y} &= \dot{p}_c. \end{aligned}\right\} \tag{1.83}$$

Employing (1.83) in place of the equation (1.81) we have:

$$\varepsilon\dot{y} + p_0 + y - \beta F_{cr}^{-1}(G_{\Sigma 0})_{-\tau} - \beta F_{cr}^{-1}(x_1 + x_2)_{-\tau} + \beta F_{cr}^{-1}(G_{\Sigma 0})_{-\tau}\dot{\tau} + \beta F_{cr}^{-1}(x_1 + x_2)_{-\tau}\dot{\tau} = 0. \tag{1.84}$$

Flow values $G_{10} + G_{20} = G_{\Sigma 0}$ do not change with time, therefore the sign $(-\tau)$ in the fourth and sixth terms loses its meaning.

Under statistical conditions [3, 4]

$$p_0 - \beta F_{cr}^{-1} G_{\Sigma 0} = 0. \tag{1.85}$$

Subtracting (1.85) from (1.84), we obtain:

$$\varepsilon\dot{y} + y - \beta F_{cr}^{-1}(x_1 - x_2)_{-\tau}(1 - \dot{\tau}) + \beta F_{cr}^{-1} G_{\Sigma 0}\dot{\tau} = 0. \tag{1.86}$$

Although we are examining the behavior of the deviation values from their nominal values p_0, $G_{\Sigma 0}$, the absolute quantity of liquid propellant located in the chamber also exerts an influence on the nature of the process. Therefore in the equation (1.86), there has been included not only the term

$$\beta F_{cr}^{-1}(x_1 + x_2)_{-\tau}\dot{\tau},$$

but also the term

$$\beta F_{cr}^{-1}(G_{10} + G_{20})\dot{\tau}.$$

Their sum, which equals

$$\beta F_{cr}^{-1}(G_1 + G_2)_{-\tau}\dot{\tau},$$

characterizes the effect of additional feeding of the chamber as a result of a constantly changing quantity of propellant, located in the internal cavity of the chamber in the stage prepared for combustion. Chamber supply conditions from the feed system are characterized by the term

$$\beta F_{cr}^{-1}(x_1+x_2)_{-\tau}.$$

Equation (1.86) contains four variables: x_1, x_2, y and τ. Therefore in solving the problem it is necessary to include the additional equation (1.64) and two hydraulic circuit equations, with small deviations. These equations will be discussed below.

It is sometimes convenient to express equation (1.64) with small deviations. Let

$$z = \tau - \tau_0, \tag{1.87}$$

where τ_0 is some constant value of the ignition time delay.

Employing equations (1.64), (1.83) and (1.87), we find that:

$$\dot{z} = n\tau_1 \frac{\dot{y}}{(p_{c0}+y)^{n+1}}. \tag{1.88}$$

Taking into consideration that y is negligibly small in comparison to p_0, it is completely permissible to state:

$$\dot{z} = n\tau_1 \frac{\dot{y}}{p_{c0}^{n+1}}. \tag{1.89}$$

Now equation (1.86) takes the following form:

$$\varepsilon\dot{y} + y - \frac{\beta}{F_{cr}}(x_1+x_2)_{-\tau} + \frac{\beta}{F_{cr}}[G_{\Sigma 0}+(x_1+x_2)_{-\tau}]\frac{n\tau_1}{p_{c0}^{n+1}}\dot{y} = 0. \tag{1.90}$$

It must be kept in mind that τ for the oxidizer (τ_1) may differ from τ for the fuel (τ_2); thus in place of $(x_1 + x_2)_{-\tau}$ it is better to state:

$$(x_1)_{-\tau_1} + (x_2)_{-\tau_2}.$$

With a constant time delay value the equation may be simplified to:

$$\varepsilon\dot{y} + y - \frac{\beta}{F_{cr}}(x_1+x_2)_{-\tau=\mathrm{const}} = 0. \tag{1.90a}$$

4. Influence of the $\dot{K}$ Parameter on the Dynamic Processes

As has already been pointed out, accelerated combustion is observed during a rapid change in the reactant ratio.

Let the nature of the combustion velocity dependence on the reactant ratio and on pressure be expressed in the form:

$$U = U(K, p_c). \tag{1.91}$$

It is apparent that

$$\dot{U} = \frac{\partial U}{\partial K}\dot{K} + \frac{\partial U}{\partial p_c}\dot{p}_c.$$

Having taking the partial derivatives from (1.91), we obtain:

$$\frac{\partial U}{\partial K} = \frac{\partial}{\partial K}[U(K, p_c)]; \tag{1.92}$$

$$\frac{\partial U}{\partial p_c} = \frac{\partial}{\partial p_c}[U(K, p_c)]. \tag{1.92a}$$

During the operating function when propellant flow oscillations are present, the rate of change of propellant composition in the chamber may reach high values.

Let

$$G_1 = \bar{G}_1 + \bar{A}_1 e^{-j(\omega_1 t + \varphi_1)};$$
$$G_2 = \bar{G}_2 + \bar{A}_2 e^{-j(\omega_2 t + \varphi_2)},$$

where $\bar{G}_1$, $\bar{G}_2$ are nominal flow rates;

ϕ_1, ϕ_2 are phase shifts;

ω_i is frequency of oscillation.

After differentiation we find that:

$$\dot{G}_1 = -j\omega_1 \bar{A}_1 e^{-j(\omega_1 t + \varphi_1)};$$
$$\dot{G}_2 = -j\omega_2 \bar{A}_2 e^{-j(\omega_2 t + \varphi_2)}.$$

Since

$$\dot{K} = G_2^{-1}(\dot{G}_1 - K\dot{G}_2),$$

then for $\omega_1 = \omega_2 = \omega$, after a series of transformations, we find that:

$$\dot{K} = \frac{j\omega}{\bar{G}_2 e^{j\omega t} + \bar{A}_2 e^{-j\varphi_2}} \times \left[-\bar{A}_1 e^{-j\varphi_1} + \frac{\bar{G}_1 e^{j\omega t} + \bar{A}_1 e^{-j\varphi_1}}{\bar{G}_2 e^{j\omega t} + \bar{A}_2 e^{-j\varphi_2}} \cdot \bar{A}_2 e^{-j\varphi_2}\right]. \tag{1.93}$$

Equation (1.93) is employed for the investigation of dynamic processes occurring during the operating function.

Thus when perturbations are evident in the hydraulic circuits a continuous change in $\dot{K}(t)$ occurs. Not only $\dot{K}$, but also the nature and the principle of change in $\dot{K}$ with time determines the dynamic processes. The nature of the change $\dot{K}(t)$ depends on the nominal values of the propellant flow rate $\overline{G}$, the reactant ratio K, phase shifts ϕ_i, the frequency of oscillation ω_i and on amplitudes $\overline{A}_i$. In the case of sinusoidal oscillations

$$\dot{K}=\frac{j\omega}{\overline{G}_2\cos\omega t+j\overline{G}_2\sin\omega t+\overline{A}_2\cos\varphi_2-j\overline{A}_2\sin\varphi_2}\times$$
$$\times\Big[-\overline{A}_1\cos\varphi_1+j\overline{A}_1\sin\varphi_1+$$
$$+\frac{\overline{G}_1\cos\omega t+j\overline{G}_1\sin\omega t+\overline{A}_2\cos\varphi_1-j\overline{A}_1\sin\varphi_1}{\overline{G}_2\cos\omega t+j\overline{G}_2\sin\omega t+\overline{A}_2\cos\varphi_2-j\overline{A}_2\sin\varphi_2}\times$$
$$\times(\overline{A}_2\cos\varphi_2-j\overline{A}_2\sin\varphi_2)\Big]. \tag{1.93a}$$

In order that $\dot{K}=0$ during the operating function, it is required that $\frac{\overline{A}_1}{\overline{A}_2}=K$, $\omega_1=\omega_2$ and $\phi_1=\phi_2$. The forces resulting from the presence $\dot{K}(t)$ and, as a consequence, from combustion acceleration $\dot{U}(t)$ mav, as is known, lead to the appearance of oscillatory processes and detonation phenomena. An example employing the equation (1.93) in the investigation of dynamic processes is shown at the end of Chapter III.

In an investigation of the influence of K on engine operating conditions, it is necessary to remember that $\dot{K}$ changes with time due to the operating characteristics of the feed system already considered. K changes along the chamber section due to irregularity in propellant atomization and mixing, and along the length of the chamber due to non-uniformity in propellant vaporization or diffusion. Thus

$$\dot{K}=\frac{\partial K}{\partial t}+\overline{W}\operatorname{grad}K. \tag{1.94}$$

5. Concerning the Random Nature of Certain Dynamic Processes

We shall distinguish between simple and complex chemical reactions. A simple reaction occurs in one step and fully corresponds to the original chemical formula. A complex reaction is characterized by the presence of a series of successively and simultaneously occurring steps. If an active center develops within a complex reaction, which generates a new or similar complex reaction, then it is called a chain reaction. Active centers are chemically saturated particles, for example, atomic hydrogen, atomic oxygen, or the radical OH, which develop in the combustion chamber as a result of dissociation in great quantities as the temperature increases and the pressure decreases. The generation of the chain reaction is more probable when the propellant heat capacity is higher.

The reaction rate in the initial period is

$$U=U_0\nu \exp\left|-\frac{t}{t_a}\right|,$$

where ν is the average chain length;

t_a is the lifetime of the active center.

Dampening of the chain reaction occurs after termination of the generation of active centers, during which

$$U=U_0\nu \exp\left|-\frac{t}{t_a}\right|.$$

The average chain length is

$$\nu = \alpha(1 - \varepsilon\alpha)^{-1},$$

where ε is the average number of active centers in one link of the chain;

α is the probability of chain continuation.

The lifetime of the active center is

$$t_a=(v_1+v_2)^{-1}\cdot(1-\varepsilon\alpha)^{-1};$$

v_i is the number of interrelated steps of the i-th particle per unit of time, which is defined as the work of the velocity constant of a corresponding elementary process on the concentration of molecules reacting with the particles discussed (molecules, atoms and radicals). An exponential increase in reaction rate is typical for a branched chain reaction.

For sufficiently large values v_i, ε and α, the rate U for a short time interval becomes so large that the reaction acquires the nature of an explosion.

In the kinetics of chain reactions a large role is played by catalysts and the initiating action of surfaces. Chain reactions are very sensitive to very small quantities of catalysts, which may accelerate or decelerate the process. The active center, having been produced on the surface (for example, on the chamber wall), may shift into the combustion space. There are indications concerning the catalytic effect of silver, to the extent that the generation of radicals on a catalytic surface is thermodynamically more advantageous than their generation in the process of dissociation. In order to decrease the probability of the generation of chain reactions, it is recommended that an inhibitor be introduced into the reactant mixture. The addition of an inhibitor to the propellant mixture does not lead to positive results. In particular, nitric acid is recommended as an inhibitor. In order to decrease the probability of the generation of chain explosions it is recommended that the surface of the vessel (chamber) be treated with potassium chloride, potassium tetraborate or some other products.

The shape and dimensions of the combustion chamber exert an influence on the nature of the increase in the reaction rate. Through a series of simplified assumptions, we obtained the average chain length for a cylinder

$$\nu=(10q)^{-1}d^2l_{pa}^{-2}.$$

For a spherical combustion chamber

$$\nu=(20q)^{-1}d^2l_{pa}^{-2},$$

where d is the vessel (chamber) diameter;

l_{pa} is the mean free path;

q is the probability factor.

Thus the generation of unusual phenomena in a spherical chamber for the reasons considered is less likely. It is important to note that quantity ν, and therefore rate U increase in proportion to an increase in the chamber diameter. Therefore low-probability unusual phenomena are not encountered in small chambers. The probability of chamber destruction from chain explosions increases with an increase in the size of the chamber. There are published indications that during a steady-state reaction, an increase in temperature, pressure or a change in the composition of the mixture may initiate a sharp increase in the reaction rate, terminating with a chain explosion (see [5, 25, 26, 32, 39, 44]).

6. Switching Off the Engine

The fall in pressure in the combustion chamber is a result of: the discharge of combustion products accumulated within the chamber; the discharge of products formed as a result of the burn-up of liquid propellant located within the chamber at the time of switch-off; the discharge of products formed as a result of the burning of additional portions of propellant constituents, entering the chamber after issuance of the command to switch off the engine.

During operation on the gaseous components and with no additional portions of propellant entering the chamber, the chamber evacuation equation may be obtained from the equation (1.77), if we substitute $(G_\Sigma) = (G_1 + G_2) = 0$. Here the chamber equation may be expressed as follows:

$$\varepsilon \dot{p}_c + p_c = 0. \tag{1.95}$$

At the time the engine is switched off the chamber pressure equals p_0. Therefore

$$p_c = p_0 \exp\left|-\frac{t}{\varepsilon}\right|. \tag{1.95a}$$

If liquid propellant is used in the engine, after the hydraulic mainlines are disconnected a quantity of propellant remains in the chamber in a liquid phase. Propellant transition from the liquid to the gaseous state as a result of a change in τ is determined in accordance with the equation (1.9). If during the last τ sec the propellant flow rate remains constant, then the equation (1.9) assumes the following form

$$\dot{Y}_l = (G_\Sigma)_0 \dot{\tau}.$$

Employing (1.64), we may state the chamber equation in the following form:

$$\varepsilon \dot{p}_c + p_c + \beta F_{cr}^{-1} n \tau_1 G_{\Sigma 0} \frac{\dot{p}_c}{p_c^{n+1}} = 0. \tag{1.96}$$

In carrying out engineering calculations of the switching off process, we may assume that n = 1. Here

$$\varepsilon \dot{p}_c p_c^2 + p_c^3 + \beta F_{cr}^{-1} \tau_1 G_{\Sigma 0} \dot{p}_c = 0. \quad (1.96a)$$

When the engine is switched off, delivery to the chamber of supplementary amounts of propellant is observed. When the main or cutoff valve to the combustion chamber is closed, during time interval t_I the following amount of fuel will be delivered to the chamber:

$$Y_I = \int_0^{t_I} G_{\Sigma I}\, dt, \quad (1.97)$$

the nature of the change $G_1(t)$ may differ from the nature of change $G_2(t)$. The burning of this portion of propellant will be characterized by specific pulse β_I. As a result of deformation of structural elements during a pressure drop, during time interval t_{II} the propellant will be delivered to the chamber in a quantity given by

$$Y_{II} = \int_0^{t_{II}} G_{\Sigma II}\, dt. \quad (1.97a)$$

It cannot be excluded that $G_1(t)$ will differ significantly from $G_2(t)$; the burning will be characterized by the specific pulse β_{II}.

Soon after the command to switch off, propellant spray from both portions of fuel is observed. If t_I and t_{II} coincide and are equal or almost equal, then we may consider that $\beta_I = \beta_{II}$. During the course of a certain time interval commensurate with t_I and t_{II}, boiling of propellant constituents may begin in the circulating part of the cooling system. As a result gaseous products enter the combustion chamber in quantity

$$Y_{III} = \int_0^{t_{III}} (G_t)_{III}\, dt. \quad (1.97b)$$

Time t_{III} is usually greater than time t_I. The change $G_1(t)$ has the nature of an ill defined process. The integrands (1.97), (1.97a) and (1.97b) are defined by experimental means or with the aid of supplementary calculations. The chamber equation may now be stated as follows:

$$\varepsilon \dot{p}_c p_c^2 + p_c^3 - \beta F_{cr}^{-1} \tau_1 G_{\Sigma 0} \dot{p}_c - p_c^2 F_{cr}^{-1} \sum \beta_i G_{\Sigma i} = 0, \quad (1.98)$$

where

$$\sum \beta_i G_{\Sigma i} = \beta_I (G_1 + G_2)_I + \beta_{II} (G_1 + G_2)_{II} + \beta_{III} (G_1 + G_2)_{III}.$$

In calculating a first approximation, the burn-up of propellant which had previously accumulated in the chamber is disregarded, and the flow rates of new portions of propellant are considered to be constants. In solving the chamber equation, we obtain

$$p_c = \frac{\sum \beta_i G_{\Sigma i}}{F_{cr}} \left(1 - \exp\left|-\frac{t}{\epsilon}\right|\right) + p_0 \exp\left|-\frac{t}{\epsilon}\right|. \qquad (1.98a)$$

In plotting a graph of the function (1.98), we must set time intervals for the delivery of separate portions of propellant constituent, through the use of experimental or calculated data. In an evaluation of the organization of the switching-off process, it has already been noted that a great deal of significance is attached to the afterpulse pressure

$$I_p = \int_0^{t_k} p_c dt \qquad (1.99)$$

and dispersion ΔI_p, which is characterized both by engine uniformity and by the results of separate tests. If equation (1.98a) is used in a determination of the pressure, then the afterpulse may be calculated from the formula:

$$I_p = \sum_i [\beta_i G_{\Sigma i} t_i] F_{cr}^{-1} + \epsilon_I [\beta_I (G_1 + G_2)_I F_{cr}^{-1} - p_0] \exp\left|-\frac{t_I}{\epsilon_I}\right| +$$
$$+ \epsilon_{II} [\beta_{II} (G_1 + G_2)_{II} F_{cr}^{-1} - p_0] \exp\left|-\frac{t_{II}}{\epsilon_{II}}\right| +$$
$$+ \epsilon_{III} [\beta_{III} (G_1 + G_2)_{III} F_{cr}^{-1} - p_0] \exp\left|-\frac{t_{III}}{\epsilon_{III}}\right|.$$

The approach considered for the determination of the afterpulse is approximate.

7. Examples

Employing approximate relationships and assigning a definite nature for the change in propellant consumption with time, we plot curves of the starting process, of the pressure change during the operating function, and a curve of the pressure drop during the removal of the propellant feed.

Example 1.

Plot a graph of the pressure time variations in the chamber during the starting function.

Given:

Specific pressure pulse $\beta = 2600 \frac{n}{kg} \cdot sec.$

The area of the exit nozzle throat $F_{cr} = 0.0319\ m^2$.

Propellant flow rate during the operating function $G_\Sigma = 122.8$ kg/sec.

Gas residence time in the chamber $\varepsilon = 0.02$ sec.

During the calculation we assume the period of the time delay $\tau = 0$.

Solution.

In order to plot the graph of the function $p_c(t)$ we shall use the combustion chamber equation in the form:

$$\varepsilon \dot{p}_c + p_c - \frac{\beta}{F_{cr}} G_\Sigma = 0. \qquad (a)$$

If we consider a sufficiently general case when the functions $\varepsilon(t)$ and $G_\Sigma(t)$ change with time during the starting function, the original equation may be stated as follows:

$$\frac{dp_c}{dt} + \frac{p_c}{\varepsilon(t)} = \frac{p_0}{\varepsilon(t)} G_\Sigma(t) G_\Sigma^{-1}. \qquad (b)$$

The solution of the linear differential equation (b) has the following form:

$$P_c = C \cdot \exp\left|-\int \frac{dt}{\varepsilon(t)}\right| + \exp\left|-\int \frac{dt}{\varepsilon(t)}\right| \cdot \frac{p_0}{G_\Sigma} \int \frac{G_\Sigma(t)}{\varepsilon(t)} \times \exp\left|\int \frac{dt}{\varepsilon(t)}\right| dt, \qquad (c)$$

where C is a constant integration.

During an exponential change in propellant consumption:

$$P_c = C \cdot \exp\left|-\int \frac{dt}{\varepsilon(t)}\right| + \exp\left|-\int \frac{dt}{\varepsilon(t)}\right| \cdot p_0 \times \int \frac{1 - \exp\left|-\frac{t}{\varepsilon}\right|}{\varepsilon(t)} \exp\left|\int \frac{dt}{\varepsilon(t)}\right| dt. \qquad (d)$$

We shall assume that the gas residence time in the chamber during the period of the starting function is a constant and is $\varepsilon_0 = 0.02$ sec.

Here in place of the equation (d) we have:

$$p_c = C\cdot\exp\left|-\frac{t}{\varepsilon_0}\right| + \exp\left|-\frac{t}{\varepsilon_0}\right|\cdot\frac{p_0}{\varepsilon_0}\left[\varepsilon_0\exp\left|\frac{t}{\varepsilon_0}\right| - \frac{\exp\left|\frac{t}{\varepsilon_0}\right|\cdot\exp\left|-\frac{t}{\alpha}\right|}{\varepsilon_0^{-1}-\alpha^{-1}}\right],$$

or

$$p_c = C\cdot\exp\left|-\frac{t}{\varepsilon_0}\right| + p_0\left(1-\frac{\exp\left|-\frac{t}{\alpha}\right|}{1-\frac{\varepsilon_0}{\alpha}}\right). \quad \text{(e)}$$

In order to determine the constant integration we shall assume that p = 0 when t = 0. According to (e) we find that:

$$C = \frac{\varepsilon_0}{\alpha-\varepsilon_0}p_0.$$

Substituting the value of the constant obtained in (e), after transformations we obtain

$$p_c = p_0\left(\frac{\varepsilon_0}{\alpha-\varepsilon_0}\exp\left|-\frac{t}{\varepsilon_0}\right| + 1 - \frac{\alpha}{\alpha-\varepsilon_0}\exp\left|-\frac{t}{\alpha}\right|\right). \quad \text{(f)}$$

The condition α = 0 corresponds to constant consumption. From (f) we find that:

$$p_c = p_0\left(1-\exp\left|-\frac{t}{\varepsilon_0}\right|\right). \quad \text{(g)}$$

Let us examine equation (g). Substituting given values, we find that:

$$p_0 = \frac{\beta}{F_{cr}}G_\Sigma = \frac{2600}{0{,}0319}\cdot 122{,}8\cdot 10^{-6} \approx 10{,}0 \text{ Mn/m}^2;$$

$$p_c = 10{,}0\left(1-\exp\left|-\frac{t}{0{,}02}\right|\right) \text{ Mn/m}^2.$$

Calculated results are tabulated below:

t, sec.	0,002	0,004	0,006	0,008	0,010	0,016	0,020	0,040	0,060
p_c Mn/m^2	0,952	1,813	2,592	3,297	3,935	5,507	6,321	8,647	9,502

If fuel consumption is exponential then we must use formula (f).

Let us show the results of calculation for a case when α = 0.01.

Operation number	Parameters	t, sec.				
		0,01	0,02	0,04	0,06	0,08
1	$\alpha-\varepsilon$	−0,01	−0,01	−0,01	−0,01	−0,01
2	$\varepsilon(\alpha-\varepsilon)^{-1}$	−2,0	−2,0	−2,0	−2,0	−2,0
3	$\exp\left\|-\frac{t}{\varepsilon}\right\|$	0,6065	0,3679	0,1353	0,0498	0,0183
4	(2)·(3)	−1,213	−0,7365	−0,2706	−0,0996	−0,0366
5	$\alpha(\alpha-\varepsilon)^{-1}$	−1,0	−1,0	−1,0	−1,0	−1,0
6	$\exp\left\|-\frac{t}{\alpha}\right\|$	0,3679	0,1353	0,01832	0,0025	0,0003
7	(5)·(6)	−0,3679	−0,1353	−0,0183	−0,0025	−0,0003
8	(4)−(7)	−0,8451	−0,6003	−0,2523	−0,0971	−0,0363
9	1+(8)	0,1549	0,3997	0,7477	0,9029	0,9637
10	p_c, Mn/m²	1,549	3,997	7,477	9,029	9,637

Let us plot from the results of our calculations a graph of the function $p_c(t)$, which characterizes the chamber starting function. Figure 8 shows curves of the increase in chamber pressure for various values α.

Calculation reveals that for exponential fuel delivery, the chamber pressure increases more slowly than when G_Σ = const. The time duration of the starting function increases.

In the investigation of the processes occurring within the combustion chamber, simplified relationships of the change of propellant consumption with time were accepted. Therefore the results obtained with the aid of materials cited in § 2 must be regarded as approximate. Fairly good agreement of calculated data with experimental data is obtained during a comparison of the solution of the engine equation system as a whole with the results of processing of accurately recorded experimental data.

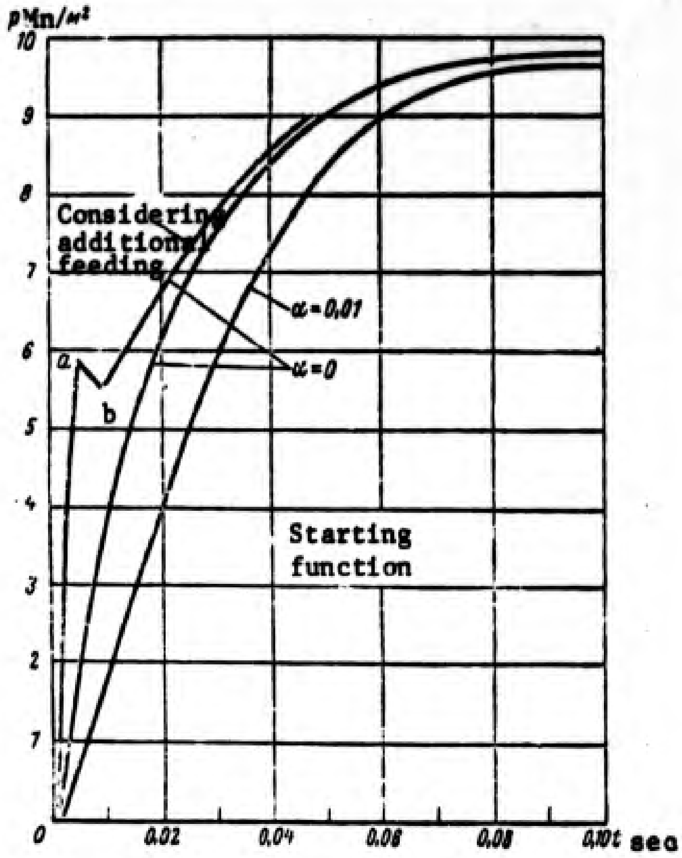

Figure 8. Calculated changes in chamber pressure.

Example 2.

Make an approximate evaluation of the influence of additional feeding of the combustion chamber by propellant, due to a decrease in the time delay during the engine starting function.

Given:

$$\tau_1 = 0{,}2 \text{ sec};$$
$$n = 1;$$
$$G_2 = 122{,}8 \text{ kg/sec}.$$

Solution.

In order to obtain a sufficiently accurate result, it is necessary to use equation (1.65). An approximate evaluation may be made with the aid of equation (1.59) in the following manner. In the previous example a graph of function p(t) was plotted while disregarding additional feeding. We must determine p(t) from the graph or from calculations. According to formula (1.59) it is easy to calculate the additional feeding

$$G_{add}(t) = n\tau_1 G_\Sigma \frac{\dot{p}_c}{p_c^{n+1}}.$$

Since in the previous example the calculations were made in accordance with a formula, in agreement with which the chamber pressure depended linearly on propellant consumption, for a recalculation of the pressure we shall use the formula

$$\frac{p_*}{p_c} \approx \frac{G_\Sigma + G_{add}}{G_\Sigma},$$

where p_* is the pressure, taking into consideration additional feeding;

p_c is the pressure, disregarding additional feeding.

The calculation is tabulated below.

t, sec	0,005	0,01	0,02	0,04	more than 0,04
p_c, Mn/м²	2,2	3,9	6,3	8,6	—
$\dot{p}_c$ Mn/м²·sec	395	300	140	75	—
$\frac{G_{add}}{G_\Sigma}$	1,63	0,394	0,071	0,02	Insignificant
p_*, Mn/м²	5,8	5,5	6,8	8,8	$p_* \approx p_c$

Let us plot a graph of the function $p_*(t)$. The calculation reveals (see Figure 8) that in a rapidly occurring chamber starting process, in the course of several time intervals (in the region ab) a change in the sign of the derivative $\dot{p}$ is observed twice. In the initial starting period (line 0a) the pressure increases most rapidly. A continuation of the starting interval, after point b, occurs more slowly with continuously decreasing values $\dot{p}_c$.

<u>Example 3</u>.

During the engine operating function a decrease in the value of rocket flight velocity was detected. With the aim of correcting the error, the control system issued a command to increase the propellant flow rate.

Determine the nature of the change in chamber pressure with time, if it is known that the propellant flow rate increased by 1 kg/sec. The values β, F_{cr}, ε are identical with those in the preceding example.

Solution.

In order to plot a graph of the function y(t) we must use the equation

$$\varepsilon\dot{y}+y-\frac{\beta}{F_{cr}}x_0=0,$$

where x_0 = 1 kg/sec.

The solution when x_0 = const has the following form:

$$y=\frac{\beta}{F_{cr}}x_0\left(1-\exp\left|-\frac{t}{\varepsilon}\right|\right).$$

Remember that $y = p_c - p_0$; $x_0 = G_{\Sigma_n} - G_{\Sigma_{ca}}$,

where G_{Σ_n} is the new flow rate value;

$G_{\Sigma_{ca}}$ is the calculated value.

Substituting numerical values we find that:

$$y=8,119\cdot10^4\left(1-\exp\left|-\frac{t}{0,02}\right|\right)\frac{n}{m^2}.$$

The calculation is tabulated below.

t, sec.	0,01	0,02	0,04	0,06	0,08
$\frac{t}{\varepsilon}$	0,5	1,0	2,0	3,0	4,0
$\exp\left\|-\frac{t}{\varepsilon}\right\|$	0,6065	0,3679	0,1353	0,0498	0,0183
$1-\exp\left\|-\frac{t}{\varepsilon}\right\|$	0,3935	0,6321	0,8647	0,9502	0,9977
$y\cdot10^{-4}$ n/m²	3,20	5,13	7,02	7,71	8,05

Example 4.

Determine the pressure variation in the chamber from the calculated pressure with a 1% change in parameters which make up equation (1.67).

Solution.

We shall use equation (1.70). In carrying out transformations (1.72) to (1.75) with the aid of (1.68), we obtain the reference equation:

$$\frac{\Delta p_c}{p_c} = \frac{\Delta \beta}{\beta} - \frac{\Delta F_{cr}}{F_{cr}} + \frac{\Delta (G_1 + G_2)}{(G_1 + G_2)}.$$

Thus with an increase of 1% in specific pulse β and in flow rates $(G_1 + G_2)$, chamber pressure will also increase by 1%. With a 1% increase in the exit nozzle throat area, chamber pressure decreases by 1%.

Example 5.

Plot a curve of the change in chamber pressure after the engine is switched off.

Given:

The pressure at the moment the engine is switched off $p_c = 10$ Mn/m^2.

Exit nozzle throat area $F_{cr} = 0.0319$ m^2.

The specific pressure pulse in the initial switching-off period β = = 2552 n/kg · sec. Gas residence time in the chamber during the initial switching-off period is ε = 0.02 sec.

Simultaneously with the issuance of the command to switch off, and as a result of movement of the working parts of the valves and deformation of structural components, propellant begins to enter the combustion chamber. Let us conditionally accept that the flow rate is a constant and equals 5.0 kg/sec. The duration of propellant delivery is 0.06 sec. In 0.04 sec, after termination of delivery of the indicated portion of propellant, gaseous products from the circulating channel of the cooling system begin to enter the combustion chamber. The flow rate is conditionally considered to be constant and is equal to 10 kg/sec. The specific pressure pulse β = 1276 n/kg · sec. Duration of delivery of the portion in question is 0.08 sec.

Solution.

The formula (1.98a) corresponds to the condition of the problem. Substituting numerical values we find that:

$$p_c = 0{,}4\left(1 - \exp\left|-\frac{t}{0{,}02}\right|\right) + 10\exp\left|-\frac{t}{0{,}02}\right| \frac{\text{Mn}}{\text{м}^2}.$$

In order to calculate the process associated with the additional feeding of the chamber by gaseous products, again we must use formula (1.98a). Since the specific pulse decreases by a factor of two, the gas residence time in the chamber increases by approximately a factor of two. This is explained by the fact that changes in β and ε are due mainly to a change in the product RT. The specific pressure pulse is directly proportional to the quantity $(RT)^{0.5}$, but gas residence time is inversely proportional to it.

$$\beta = \frac{(RT)^{0,5}}{aF_{cr}} = 1276\frac{\text{n}}{\text{kg}}\cdot\text{sec};$$

$$\varepsilon = \frac{\beta V}{RT} = \frac{V}{aF_{cr}(RT)^{0,5}} = 0{,}04\ \text{sec}.$$

The calculation is tabulated below.

t, sec.	0	0,01	0,02	0,04	0,06
$\frac{t}{\varepsilon}$	0	0,5	1	2	3
$\exp\left\|-\frac{t}{\varepsilon}\right\|$	1	0,6065	0,3679	0,1353	0,04979
$1-\exp\left\|-\frac{t}{\varepsilon}\right\|$	0	0,3935	0,6321	0,8647	0,9502
$0{,}4\left(1-\exp\left\|-\frac{t}{\varepsilon}\right\|\right)$	0	0,1574	0,2528	0,3459	0,3801
$10\exp\left\|-\frac{t}{\varepsilon}\right\|$	10	6,065	3,679	1,353	0,4979
p_c, Mn/м^2	10,0	6,2224	3,9318	1,6989	0,8779

The subsequent process in the course of 0.04 sec occurs without additional feeding. The calculation is made in accordance with the formula

$$p_c = 0{,}8779\exp\left|-\frac{t-0{,}06}{0{,}02}\right|.$$

The calculation is tabulated below.

t, sec.	0,06	0,08	0,10
$t-0,06$	0	0,02	0,04
$\frac{t-0,06}{0,02}$	0	1	2
$\exp\left\|-\frac{t-0,06}{0,02}\right\|$	1	0,3679	0,1353
p_c, Mn/м²	0,8779	0,3230	0,1188

For subsequent time intervals the reference equation is stated as follows

$$p_c = 0,4\left(1-\exp\left|-\frac{t-0,1}{0,04}\right|\right) + 0,1188\exp\left|-\frac{t-0,1}{0,04}\right|.$$

The calculation is tabulated below.

t, sec.	0,10	0,12	0,14	0,16	0,18
$t-0,1$	0	0,02	0,04	0,06	0,08
$\frac{t-0,1}{0,04}$	0	0,5	1	1,5	2
$\exp\left\|-\frac{t-0,1}{0,04}\right\|$	1	0,6065	0,3679	0,2231	0,1353
$1-\exp\left\|-\frac{t-0,1}{0,04}\right\|$	0	0,3935	0,6321	0,7769	0,8647
$0,4\left(1-\exp\left\|-\frac{t-0,1}{0,04}\right\|\right)$	0	0,1574	0,2528	0,3108	0,3459
$0,1188\exp\left\|-\frac{t-0,1}{0,04}\right\|$	0,1188	0,0722	0,0440	0,0262	0,0167
p_c, Mn/м²	0,1188	0,2296	0,2968	0,3370	0,3626

The last stage -- chamber evacuation -- is considered in accordance with the formula

$$p_c = 0{,}3626 \exp\left|-\frac{t-0{,}18}{0{,}04}\right|.$$

The calculation is tabulated below.

t, sec.	0,18	0,20	0,22	0,24
$t-0{,}18$	0	0,02	0,04	0,06
$\frac{t-0{,}18}{0{,}04}$	0	0,5	1	1,5
$\exp\left\|\frac{t-0{,}18}{0{,}04}\right\|$	1	0,6065	0,3679	0,2231
p_c, Mn/m^2	0,3626	0,2374	0,1332	0,0830

Let us plot a graph of the function $p_c(t)$ from the results of our calculations for the engine switching-off period (the afterpulse) in linear and logrithmic scales (Figures 9a and b).

§ 4. Partial Differential Equation of the Conservation of Mass for the Combustion Stream

This and subsequent paragraphs of Chapter I are devoted to an examination of several problems in the theory of the parametric fields of intrachamber processes.

In the combustion chamber, especially in the region near the head, complex mass and energy exchanges take place between propellant constituents which enter the chamber, their vaporization products, and a gaseous mixture formed as a result of the burning process. Let us examine the volume element abcdefjh, shown in Figure 10. During the burning of liquid propellant, this element will be filled partially with liquid and partially with gas. Individual propellant drops with various dimensions move in a certain direction and will have axial, radial, and tangential component velocities, the values of which depend on the location of the volume element considered and on time. Due to vaporization of the drops or diffusion of the liquid in gas, the liquid flow, velocity, density and other gas parameters in the element will change with respect to the element coordinates, and will also depend on time and on element location.

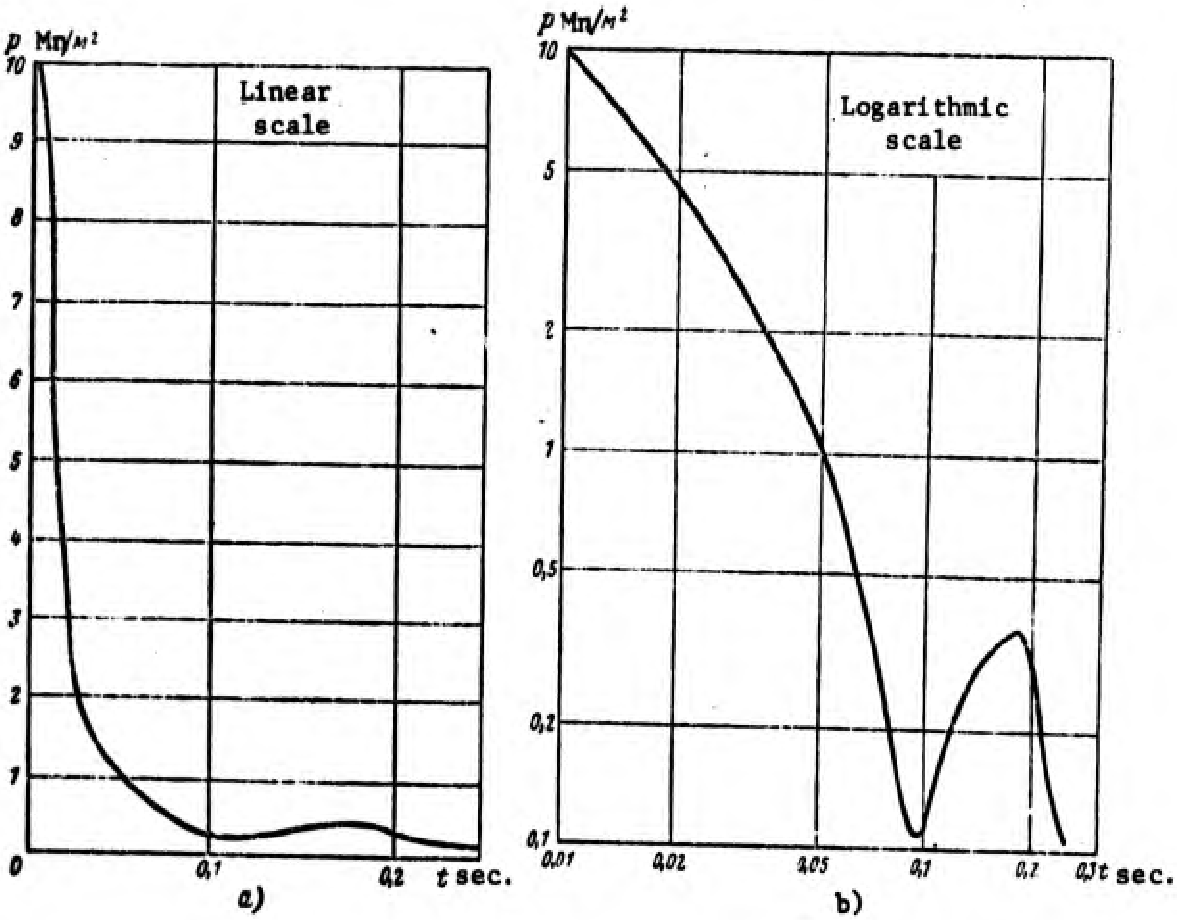

Figure 9. The calculated change in chamber pressure during the afterpulse period.

The largest quantity of fluid will be in those elements which are located close to the head. If preparation for burning of propellant constituents is observed in this region of the chamber, then the size of the drops change very little with respect to element coordinates. At a slightly greater distance from the head, where intensive vaporization occurs, the size of the drops is rapidly reduced. If the element is located at a sufficiently large distance from the head, close to the nozzle, then its volume will be essentially filled with gas. As a result of reactions occurring in the gaseous phase, the gas composition may change constantly.

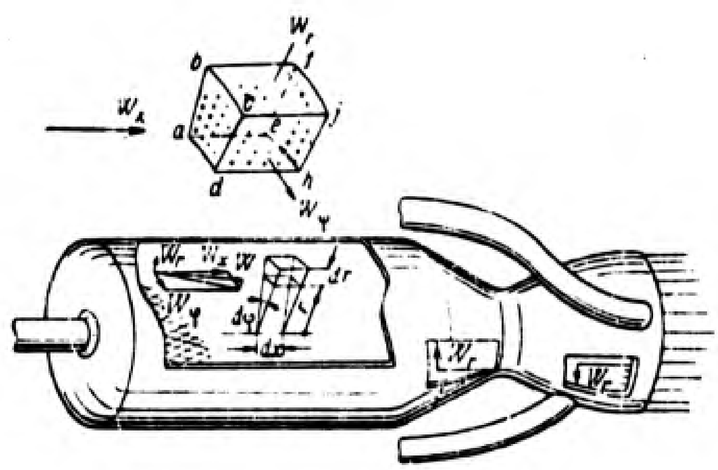

Figure 10. Volume element in the combustion stream.

Thus an attempt to describe mathematically the complex process -- the movement of the combustion stream -- may lead to positive results if we direct our attention to partial differential equations. However, the equations cited below are approximations, since in their derivation many peculiarities of the interaction between fluid and gaseous products in a two-phase flow were not considered.

1. Gas Flow through the Element

The area abcd is equal to drr $d\phi$. The fluid passing through the same area has the total sectional area equal to $S_x$drr $d\phi$. The side section abcd, through which the gas passes, will equal

$$dF_x = (1 - S_x)drr\ d\phi. \qquad (1.100)$$

The ratio of the sections which are filled with liquid and gas will equal

$$\frac{S_x}{1 - S_x}.$$

Therefore S_x is a coefficient which characterizes the section occupied by fluid in the limited area abcd.

With a lack of fluid $S_x = 0$; if the entire section is filled with fluid, then $S_x = 1$.

In an examination of the existing process, in which the movement of drops of finite size occurs, it will be necessary to direct our attention to the finite area of the element abcd, which equals $\Delta rr\Delta\phi$, and the area $S_x\Delta rr\Delta\phi$ occupied by fluid. However, the transition to a conditional plan of calculation through elemental areas does not introduce errors, if in the assignment of limits the mass flow rate per unit time (per second) through the cross-sectional area of the element will equal

$$d\dot{m}_x = \varrho W_x (1 - S_x) drr\, d\varphi, \qquad (1.101)$$

where m_x is the gas mass, moving in the direction of the x axis.

On the element dx of the path, a change in the flow of gas, which moves within the element in the direction W_x, comprises:

$$d\dot{m}_x - d\dot{m}_{x+dx} = \frac{\partial}{\partial x}[\varrho W_x (1 - S_x) drr\, d\varphi]\, dx. \qquad (1.102)$$

The quantity drr $d\phi$ is not a function of x; the area $S_x drr\, d\phi$, which is occupied by the burning propellant while in a liquid phase, decreases due to propellant depletion in the direction W_x. The area occupied by the fluid on the side efjh, differs from area

$$d\sigma_x = S_x drr\, d\phi$$

on side abcd by the value $\partial S_x/\partial x \cdot dx\, drr\, d\phi$.

Thus in place of equation (1.102) we may state:

$$d\dot{m}_x - d\dot{m}_{x+dx} = \left[\frac{\partial}{\partial x}(\varrho W_x) - \frac{\partial}{\partial x}(\varrho W_x S_x)\right] dx\, drr\, d\varphi. \qquad (1.103)$$

The first term of the right side of the equation (1.103) characterizes the change in the mass flow rate of the gas, flowing through an element of constant area; the second characterizes a decrease in the difference of the mass rate of the flow observed, due to a restriction within the transfer section of the element. Due to the burning of propellant, flow restriction by liquid is not identical for the various sections distributed along the x axis. In the section abcd it is determined by the quantity

$$d\sigma_x = S_x drr\, d\phi,$$

and in section efjh, by value

$$d\sigma_{x+dx}=\left(S_x-\frac{\partial S_x}{\partial x}dx\right)dr\,r\,d\varphi. \qquad (1.104)$$

If there is no propellant burn-up along the length of the element, then in place of the second term in equation (1.103) we must write:

$$d\dot{m}_{x+dx\,(S_x\ \text{const})}=S_x\frac{\partial}{\partial x}(\varrho W_x)dx\,dr\,r\,d\varphi. \qquad (1.105)$$

Thus as a result of an increase in the gas transfer section, due to burning of the propellant, the difference under consideration is increased to the quantity

$$d\dot{m}_{x+dx}=\varrho W_x\frac{\partial}{\partial x}S_x\,dx\,dr\,r\,d\varphi. \qquad (1.106)$$

Due to burning accompanied by a decrease in the mass of the liquid phase:

$$\frac{\partial}{\partial x}S_x<0. \qquad (1.107)$$

The direction W_x is fundamental; the velocity W_x constantly increases along the length of the chamber and the nozzle, although speed fluctuations and even negative values of W_x in the area of the head of the chamber are possible, due to localized counter-flows.

If we consider the gas movement only in the x direction, then the problem is unidimensional. In conducting accurate investigations we consider W_r and W_ϕ. The radial component W_r is generated under the influence of geometric factors, due to the presence of radial velocity components of particles of fuel injected into the chamber, owing to flow turbulence and the presence of counter-flows, as a result of interaction between individual propellant and gas particles and for a whole series of other reasons. If the perturbation factors enumerated did not exist, then the velocity component W_r in the central section of the chamber would equal zero. In the entry section of the nozzle it is negative, i.e., it is directed toward the axis of the chamber (see Figure 10), and in the exit section it is positive.

At the time under consideration let $W_r > 0$. The unit mass gas flow through the side aehd will equal:

$$d\dot{m}_r = \rho W_r (1 - S_r)\, dxr\, d\phi. \qquad (1.108)$$

On a segment of path dr, a change in the gas flow moving in r direction comprises:

$$d\dot{m}_r - d\dot{m}_{r+dr} = \left[\frac{1}{r}\frac{\partial}{\partial r}(\varrho W_r r) - \frac{1}{r}\frac{\partial}{\partial r}(\varrho W_r S_r r)\right] dx\, dr r\, d\varphi. \qquad (1.109)$$

The tangential velocity component W_ϕ is mainly dependent upon the rotary exit of propellant constituents from swirl of other types of centrifugal injectors.

On a segment of path r $d\phi$, a change in gas flow moving in W_ϕ direction, will equal:

$$d\dot{m}_\varphi - d\dot{m}_{\varphi+d\varphi} = \left[\frac{\partial}{r\partial\varphi}(\varrho W_\varphi) - \frac{\partial}{r\, d\varphi}(\varrho W_\varphi S_\varphi)\right] dV, \qquad (1.110)$$

where

$$dV = dxdrr\, d\phi.$$

The equation for the determination of the total change in gas flow within the element is expressed as follows:

$$\left[\frac{\partial}{\partial x}(\varrho W_x) + \frac{1}{r}\frac{\partial}{\partial r}(\varrho W_r r) + \frac{\partial}{r\partial\varphi}(\varrho W_\varphi)\right] dV -$$
$$- \left[\frac{\partial}{\partial x}(\varrho W_x S_x) + \frac{1}{r}\frac{\partial}{\partial r}(\varrho W_r S_r r) + \frac{1}{r}\frac{\partial}{\partial\varphi}(\varrho W_\varphi S_\varphi)\right] \times$$
$$\times dV = [\operatorname{div}(\varrho\overline{W}) - \operatorname{div}(\varrho\overline{W}S)]\, dV. \qquad (1.111)$$

2. Functions Characterizing the Transition from the Liquid Phase to the Gaseous Phase

Propellant flow through the ring located perpendicular to the x axis in possessing width dr, is:

$$dG_x = \frac{\partial}{\partial r} G_x dr, \qquad (1.112)$$

where G_x is the mass rate of propellant flow per unit time (per second), which corresponds to the side abcd.

Due to nonuniformity in the distribution of propellant along the chamber section, to a change in the area of the chamber section with respect to its length, and to propellant burn-up in the direction of the x axis, derivative $\partial/\partial r\ G_x = f(x, r)$. If on the ring examined the sector is isolated by an angle $d\phi$, then a unit flow of

$$dG_x = \frac{\partial^2}{\partial r r\,\partial\varphi} G_x\, dr\, r\, d\varphi. \tag{1.113}$$

will pass through it.

The supply of gas in the element equals

$$dG_{x+dx} - dG_x = \frac{\partial}{\partial x}\left(\frac{\partial^2}{\partial r r\,\partial\varphi} G_x\, dr\, r\, d\varphi\right) dx = \frac{\partial^3 G_x}{\partial x\,\partial r r\,\partial\varphi} dV. \tag{1.114}$$

Let us examine the process which is flowing in the direction of the r axis. The flow rate element is:

$$dG_r = \frac{\partial^2}{\partial x r\,\partial\varphi} G_r\, dx\, r\, d\varphi. \tag{1.115}$$

The supply of gas in the element is

$$dG_{r+dr} - dG_r = \frac{1}{r}\,\frac{\partial}{\partial r}\,\frac{(r\partial^2 G_r)}{\partial x r\,\partial\varphi} dV. \tag{1.116}$$

The unit flow in W_ϕ direction is

$$dG_\varphi = \frac{\partial^2 G_\varphi}{\partial x\,\partial r} dx\, dr, \tag{1.117}$$

and the supply of gas is

$$dG_{\varphi+d\varphi} - dG_\varphi = \frac{1}{r}\,\frac{\partial}{\partial\varphi}\left(\frac{\partial^2 G_\varphi}{\partial x\,\partial r}\right) dV. \tag{1.118}$$

The x derivative in the formula (1.114) characterizes propellant burn-up along the length, but derivatives r and ϕ determine unit flow quantity. When propellant is burning, the x-derivative is non-vanishing. When burning is not present (for example in the area of the injectors where preparation of the fuel for burning occurs in the course of the induction period or during a cold passage through the chamber), the x derivative equals zero. We may similarly consider processes flowing in W_r and W_ϕ directions. We shall be limited by an examination of (1.115), which characterizes r direction. Here the unit flow is determined by the x and ϕ derivatives, and the burn-up process by the r derivative.

The full supply of gas to the element resulting from propellant burning will be:

$$\Omega\, dV = \varrho_l \frac{\partial}{\partial t}\chi\, dV + \left[\frac{\partial^3 G_x}{\partial x\, \partial rr\, \partial\varphi} + \frac{1}{r}\frac{\partial}{\partial r}\frac{(r\partial^2 G_r)}{\partial xr\, \partial\varphi} + \frac{1}{r}\frac{\partial}{\partial\varphi}\left(\frac{\partial^2 G_\varphi}{\partial x\, \partial r}\right)\right] dV. \quad (1.119)$$

The first component in equation (1.119) characterizes local (not resulting from a change in coordinates) gas delivery to the element and χ is determined according to (1.151).

The propellant mass flow rate through the element with area abcd is

$$dG_x = \frac{\partial^2 G_x}{\partial rr\, \partial\varphi} drr\, d\varphi = \varrho W_x\, drr\, d\varphi. \quad (1.120)$$

Gas delivery to the element is

$$dG_{x+dx} - dG_x = \frac{\partial^3 G_x}{\partial x\, \partial rr\, \partial\varphi} dV = \frac{\partial}{\partial x}(\varrho W_x) dV. \quad (1.121)$$

The propellant mass flow rate through any section F, which corresponds to area abcd is

$$G_x = \int_F \varrho W_x dF, \quad (1.122)$$

and gas delivery on path dx comprises

$$G_{x+dx} - G_x = \frac{\partial}{\partial x}\int_F \varrho W_x dF \cdot dx. \quad (1.123)$$

3. Calculation of Specific Consumption Rates S_1

We shall find the specific mass consumption rates

$$q_x = \frac{\partial^2 G_x}{\partial rr\, \partial\varphi} = \frac{dG_x}{dF_x}; \quad (1.124)$$

$$q_r = \frac{\partial^2 G_r}{\partial xr\, \partial\varphi} = \frac{dG_r}{dF_r}; \quad (1.125)$$

$$q_\varphi = \frac{\partial^2 G_\varphi}{\partial x\, \partial r} = \frac{dG_\varphi}{dF_\varphi}. \quad (1.126)$$

Now we may introduce the equation for Ω while utilizing the divergence of vector $\bar{q}$:

$$\Omega = \varrho_T \frac{\partial}{\partial t} \chi + \operatorname{div} \bar{q} = \frac{\partial}{\partial x} q_x + \frac{1}{r} \frac{\partial}{\partial r}(r q_r) + \frac{\partial}{r \partial \varphi} q_\varphi + \varrho_T \frac{\partial}{\partial t} \chi. \tag{1.127}$$

Let us examine the gas flow from evaporating particles of a propellant constituent. In the spherical system of coordinates with a condition of flow symmetry with respect to the origin, a continuity condition over distance r is noted as follows:

$$W = \frac{\int dG}{4 \pi r^2 \varrho}. \tag{1.128}$$

The velocity potential is designated by ϕ. In the spherical system of coordinates

$$W = W_r = \frac{\partial \varphi}{\partial r}, \tag{1.129}$$

therefore,

$$d\varphi = \frac{dr}{4 \pi r^2 \varrho} \int dG. \tag{1.130}$$

The required velocity potential is

$$\varphi = -\frac{\int dG}{4 \pi r \varrho}. \tag{1.131}$$

Substituting

$$q = \frac{\int dG}{4 \pi r^2},$$

we find that:

$$\varphi = -q \frac{r}{\varrho}. \tag{1.132}$$

Let us examine one of the possible methods of experimentally determining the derivatives of the propellant flow rate. The height per unit time of liquid in test tubes which collect the liquid (Figure 11) is computed according to the results of passage through the head.

From the results of processing experimental data, it is necessary to establish that

$$H = H(r). \tag{1.133}$$

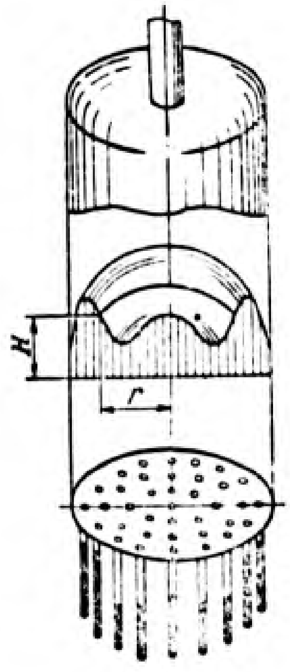

Figure 11. Diagram of an apparatus for determining the distribution of the liquid flow rate of the chamber section.

The mass flow rate will equal:

$$G_x = 2\pi\rho_l \int_0^r rH\,dr; \tag{1.134}$$

therefore,

$$\frac{dG_x}{dr} = 2\pi\rho_l rH, \tag{1.135}$$

where ρ_l is fuel density. In passing from unidimensional to three-dimensional flow, in place of the usual derivatives we shall have partial derivatives; in our case:

$$\frac{\partial G_x}{\partial r} = 2\pi\rho_l rH. \tag{1.136}$$

Similarly, we may compute the value of other partial derivatives, which enter into equation (1.119) from the results of processed experimental data.

According to the equation of continuity

$$S_x = \frac{1}{\varrho_l C_x} \frac{\partial^2 G_x}{\partial r r \partial \varphi}, \qquad (1.137)$$

or

$$S_x = \frac{q_x}{\varrho_l C_x}, \qquad (1.138)$$

where C_x is the propellant velocity component.

In order to determine the derivatives we have:

$$\frac{\partial S_x}{\partial x} = \frac{1}{\varrho_l} \frac{\partial}{\partial x} \left(\frac{1}{C_x} \frac{\partial^2 G_x}{\partial r r \partial \varphi} \right), \qquad (1.139)$$

or

$$\frac{\partial S_x}{\partial x} = \frac{1}{\varrho_l} \frac{\partial}{\partial x} \left(\frac{q_x}{C_x} \right). \qquad (1.140)$$

Similar to equation (1.138), we find that:

$$S_r = \frac{q_r}{\varrho_l C_r}; \qquad (1.141)$$

$$S_\varphi = \frac{q_\varphi}{\varrho_l C_\varphi}. \qquad (1.142)$$

The values S_x, S_r and S_ϕ may be approximately determined by means of calculation of the total area of the imprint of the drops obtained upon atomization of the liquid by the head injectors.

4. The Equation of the Conservation of Mass (The Equation of Continuity)

After transformations the equation (1.111) assumes the following form:

$$\left[\frac{\partial (\varrho W_x)}{\partial x} + \frac{1}{r} \frac{\partial (\varrho W_r r)}{\partial r} + \frac{\partial (\varrho W_\varphi)}{r \partial \varphi} \right] - \frac{1}{\varrho_l} \left[q_x \frac{\partial \left(\varrho \frac{W_x}{C_x} \right)}{\partial x} + q_r \frac{1}{r} \frac{\partial \left(\varrho \frac{W_r}{C_r} \right)}{\partial r} + \right.$$
$$\left. + q_\varphi \frac{\partial \left(\varrho \frac{W_\varphi}{C_\varphi} \right)}{r \partial \varphi} \right] + \frac{1}{\varrho_l} \left[\varrho \frac{W_x}{C_x} \frac{\partial q_x}{\partial x} + \varrho \frac{W_r}{C_r} \frac{1}{r} \frac{\partial (r q_r)}{\partial r} + \varrho \frac{W_\varphi}{C_\varphi} \frac{\partial q_\varphi}{r \partial \varphi} \right] =$$
$$= \operatorname{div} (\varrho \overline{W}) - \operatorname{div} (\varrho \overline{W} S). \qquad (1.143)$$

The expression in the first set of brackets characterizes a change in the gas mass without considering constraint. Expression in the second set of brackets reflects a decrease in the mass change, generated as a result of gas flow constraint in the length of the channel. The expression in the third set of brackets reflects a reduction in this effect along the element length. It must be kept in mind that the derivatives, in the case of fuel burn-up in directions corresponding to the axes, will be negative.

$$\frac{\partial}{\partial x} q_x < 0; \tag{1.144}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(q_r r) < 0; \tag{1.145}$$

$$\frac{\partial}{r\partial\varphi} q_\varphi < 0. \tag{1.146}$$

Thus equation (1.143) characterizes mass change occurring along the element, but equation (1.119) characterizes a mass change of combustion products occurring within the element. As a result of the liquid mass change, a gas mass change will be observed within the element.

According to the law of the conservation of mass, the change in the quantity of gas in the element is defined as a local time change in density and relative volume per unit time:

$$d\dot{m}_t - d\dot{m}_{t+dt} = \frac{\partial}{\partial t}[\varrho(1-\chi)]\,dV, \tag{1.147}$$

where χdV is liquid volume in the element.

This volume may be determined for a case of variable area values of the fluid on the sides, using

$$\chi - d\chi = \left[\left(S_x - \frac{1}{2}\frac{\partial S_x}{\partial x}dx\right)\left(S_r - \frac{1}{2}\frac{\partial S_r}{\partial r}dr\right)\times\left(S_\varphi - \frac{1}{2}\frac{\partial S_\varphi}{r\partial\varphi}rd\varphi\right)\right]^{0,5}. \tag{1.148}$$

Having calculated the area values and their derivatives, we obtain:

$$\chi - d\chi = \varrho_l^{-3/2}\left[\frac{q_x}{C_x} - \frac{1}{2}\frac{\partial}{\partial x}\left(\frac{q_x}{C_x}\right)dx\right]^{0,5}\times$$
$$\times\left[\frac{q_r}{C_r} - \frac{1}{2}\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{q_r}{C_r}r\right)dr\right]^{0,5}\times$$
$$\times\left[\frac{q_\varphi}{C_\varphi} - \frac{1}{2}\frac{\partial}{r\partial\varphi}\left(\frac{q_\varphi}{C_\varphi}\right)rd\varphi\right]^{0,5}. \tag{1.149}$$

If we may disregard a change in the instantaneous values of the fluid areas in the element, then equation (1.149) is simplified and assumes the following form:

$$\chi = Q_l^{-3/2}\left[\frac{q_x}{C_x}\frac{q_r}{C_r}\frac{q_\varphi}{C_\varphi}\right]^{0,5}. \qquad (1.150)$$

Subtracting term by term (1.149) from (1.150), after transformations and integration we obtain:

$$\chi = Q_l^{-3/2}\frac{1}{4}\left\{\int\left[\frac{q_x\partial}{C_x\partial x}\left(\frac{q_x}{C_x}\right)dx + \frac{q_r\partial}{C_r\partial r}\left(\frac{q_r}{C_r}\right)dr + \frac{q_\varphi}{C_\varphi}\frac{\partial}{\partial\varphi}\left(\frac{q_\varphi}{C_\varphi}\right)d\varphi\right]\right\}^{0,5}. \qquad (1.151)$$

The equation (1.150) reveals that the area of the sides and the liquid volume in the element depends on the ratios of the flows through the element to the corresponding velocities. If the flow and velocity equal zero, then the fluid volume in the element may be non-vanishing (Figure 12).

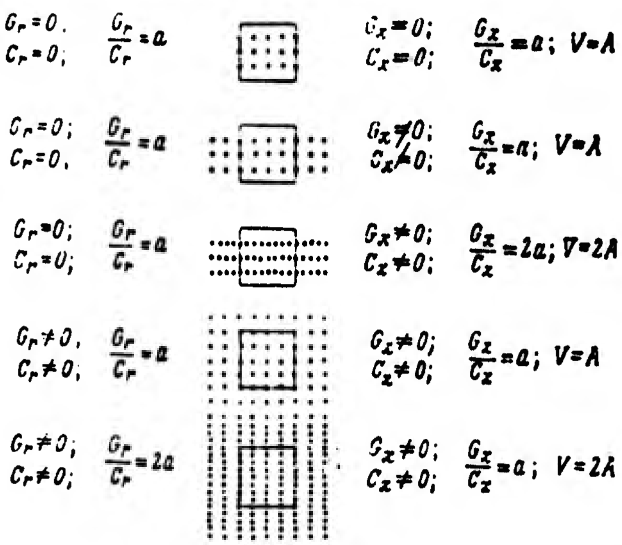

Figure 12. Explanation of equations (1.152) and (1.153).

We note that since

$$\frac{\partial G_l}{\partial F_l} = q_l, \qquad (1.152)$$

the following relation then occurs:

$$\frac{\partial G_l}{\partial F_l}\frac{1}{C_l} = \frac{q_l}{C_l}. \qquad (1.153)$$

Therefore the ratio examined characterizes the gravimetric density.

The equation (1.147) is rewritten as follows:

$$d\dot{m}_t - d\dot{m}_{t+dt} = \frac{\partial}{\partial t}\left\{\varrho\left[1 - \varrho_l^{-\frac{3}{2}}\left(\frac{q_x}{C_x}\,\frac{q_r}{C_r}\,\frac{q_\varphi}{C_\varphi}\right)^{0,5}\right]\right\} dV. \qquad (1.154)$$

In the most general case, when it is necessary to consider a change in instantaneous fluid areas within the element (which is especially important during rapid burning), in formulating equation (1.154), we must direct our attention not to equation (1.150), but to (1.151).

In the case of burning drops of the propellant constituents

$$\frac{\partial}{\partial t}[\varrho(1-\chi)] = \frac{\partial\varrho}{\partial t}(1-\chi) - \varrho\frac{\partial\chi}{\partial t}. \qquad (1.155)$$

During preparation of the propellant for burning, χ = const; therefore,

$$\frac{\partial}{\partial t}[\varrho(1-\chi)] = \frac{\partial\varrho}{\partial t}(1-\chi), \qquad (1.156)$$

Utilizing equations (1.143), (1.119) and (1.147), we shall write the equation of the conservation of mass for the gas in the element of the combustion stream, formulating it thusly: a change in the density and volume of the gas within the element per unit of time is equal to a change in the gas mass, moving through the element, and to the gas mass which forms during the period of the same unit of time as a result of burn-up of propellant which is located within the element.

$$\frac{\partial}{\partial t}[\varrho(1-\chi)] + \left[\frac{\partial(\varrho W_x)}{\partial x} + \frac{1}{r}\frac{\partial(\varrho W_r r)}{\partial r} + \frac{\partial(\varrho W_\varphi)}{r\partial\varphi}\right] - $$
$$- \frac{1}{\varrho_l}\left[q_x\frac{\partial}{\partial x}\left(\varrho\frac{W_x}{C_x}\right) + q_r\frac{1}{r}\frac{\partial}{\partial r}\left(\varrho\frac{W_r}{C_r}r\right) + \right.$$
$$\left. + q_\varphi\frac{\partial}{r\partial\varphi}\left(\varrho\frac{W_\varphi}{C_\varphi}\right)\right] - \frac{\varrho}{\varrho_l}\left[\frac{W_x}{C_x}\frac{\partial}{\partial x}q_x + \right.$$
$$\left. + \frac{W_r}{C_r}\frac{1}{r}\frac{\partial(q_r r)}{\partial r} + \frac{W_\varphi}{C_\varphi}\frac{\partial}{r\partial\varphi}q_\varphi\right] = \Omega. \qquad (1.157)$$

5. Range of Application of Equation (1.157)

The presence of functions characterizing and reflecting gas flow constraints, and functions defining the supply of gas as a result of propellant burn-up, is a characteristic of the equation of the conservation of mass for the combustion stream which distinguishes it from the analogous equation for the gas flow. The constraint is due to the presence in the stream of the liquid phase, which is characterized by the specific mass flow rates q_x, q_r, and q_ϕ. Since the mass flow quantity changes in the process of liquid movement, the derivatives $\frac{\partial}{\partial x} q_x$, $\frac{1}{r}\frac{\partial}{\partial r}(q_r r)$, $\frac{\partial}{r\partial\varphi}(q_\varphi)$, are considered, which characterize the burn-up intensity. Constraint is considered along with two functions. The first of these is expressed as follows:

$$\Psi_1 = 1 - \chi. \tag{1.158}$$

In expanded form, and in agreement with equation (1.151), the first function is written as follows:

$$\Psi_1 = 1 - \rho_l^{-3/2} \frac{1}{4} \left\{ \int \left[\frac{q_x \partial}{C_x \partial x} \left(\frac{q_x}{C_x} \right) dx + \frac{q_r \partial}{C_r \partial r} \left(\frac{q_r}{C_r} \right) dr + \frac{q_\varphi}{C_\varphi} \frac{\partial}{r\partial\varphi} \left(\frac{q_\varphi}{C_\varphi} \right) r d\varphi \right] \right\}^{0,5}. \tag{1.159}$$

If the transition of the liquid phase to the gaseous phase is not observed, then in agreement with equation (1.150) we shall have:

$$\Psi_1 = 1 - \rho_l^{-3/2} \left[\frac{q_x}{C_x} \frac{q_r}{C_r} \frac{q_\varphi}{C_\varphi} \right]^{0,5}. \tag{1.160}$$

It is convenient to record the equation for the second function in the form of a sum:

$$\Psi_2 = \frac{1}{\rho_l} \left[q_x \frac{\partial}{\partial x} \left(\rho \frac{W_x}{C_x} \right) + q_r \frac{1}{r} \frac{\partial}{\partial r} \left(\rho \frac{W_r}{C_r} r \right) + q_\varphi \frac{\partial}{r\partial\varphi} \left(\rho \frac{W_\varphi}{C_\varphi} \right) \right] + \frac{\rho}{\rho_l} \left[\frac{W_x}{C_x} \frac{\partial}{\partial x} q_x + \frac{W_r}{C_r} \frac{1}{r} \frac{\partial}{\partial r} (q_r r) + \frac{W_\varphi}{C_\varphi} \frac{\partial}{r\partial\varphi} q_\varphi \right]. \tag{1.161}$$

The first term takes into consideration the presence of the liquid phase, and the second, its change according to the coordinate axes.

The equation (1.157) may be used for an investigation of processes in separate sections of the combustion chamber. In the highest section of the chamber, near the injectors, the propellant is prepared for combustion. Here, if ρ_l = const, then

$$\varrho \chi \frac{\partial}{\partial t} \chi + \operatorname{div} \bar{q} = 0. \quad (1.162)$$

Equation (1.157) is written as follows:

$$\frac{\partial}{\partial t} [\varrho(1-\chi)] + \operatorname{div} (\varrho \overline{W}) =$$
$$= \frac{1}{\varrho \chi} \left[q_x \frac{\partial}{\partial x} \left(\varrho \frac{W_x}{C_x} \right) + q_r \frac{\partial}{\partial r} \left(\varrho \frac{W_r}{C_r} r \right) + \right.$$
$$\left. + q_\varphi \frac{\partial}{r \partial \varphi} \left(\varrho \frac{W_\varphi}{C_\varphi} \right) \right]. \quad (1.163)$$

In the combustion area, where the liquid phase is present, equation (1.157) must be used. However, if it is possible to consider that here the process of gas formation is the main, predominant process over the channel processes, if this area is essentially constrained by the liquid phase and if an increase in gas density occurs as a result of burning, then the equation of the conservation of mass assumes the following form:

$$\frac{\partial}{\partial t} [\varrho(1-\chi)] = \Omega. \quad (1.164)$$

After the termination of gasification, but during burning of gaseous products, all functions characterizing constraints must be disgarded and the equation assumes the following form:

$$\frac{\partial \varrho}{\partial t} + \operatorname{div} (\varrho \overline{W}) = \Omega. \quad (1.165)$$

If conditions are created which support ρ = const, then in place of equation (1.165) we shall have:

$$\varrho \operatorname{div} \overline{W} = \Omega. \quad (1.166)$$

For a uniform flow, we obtain:

$$\varrho dW = dq. \quad (1.167)$$

After completion of all the chemical transformations, we come to the well known equation which characterizes gas movement:

$$\frac{\partial \varrho}{\partial t} + \operatorname{div} (\varrho \overline{W}) = 0. \quad (1.168)$$

§ 5. The Equation of Combustion Stream Motion [16, 29, 34, 55]

1. Derivation of the Equation

Let us examine the previous combustion stream element. The gaseous phase mass is

$$m_g=\int_V \varrho(1-\chi)dV. \quad (1.169)$$

The liquid phase mass is

$$m_l=\int_V \varrho_l\chi dV. \quad (1.170)$$

The total combustion stream mass is

$$m=\int_V [\varrho(1-\chi)+\varrho_l\chi]dV. \quad (1.171)$$

Let the total force acting upon the gas stream equal $\overline{P}$, and the total force setting the liquid in motion equal $\overline{P}_l$. As the result of the effects of these forces, gas velocity $\overline{W}$ and liquid velocity $\overline{C}$ may be different.

For a gas mass element, Newton's second law may be stated as follows:

$$\overline{P}=\int_V \varrho(1-\chi)\dot{\overline{W}}dV. \quad (1.172)$$

For the flow of liquid particles

$$\overline{P}_l=\int_V \varrho_l\chi\dot{\overline{C}}dV. \quad (1.173)$$

In carrying out a term by term summation for the entire combustion stream element, we obtain:

$$\overline{P}_\Sigma=\int_V [\varrho(1-\chi)\dot{\overline{W}}+\varrho_l\chi\dot{\overline{C}}]dV. \quad (1.174)$$

The forces setting the combustion stream in motion are composed of mass (volume) and surface forces. As a result of the effects of external forces, the velocity of the mass of the gas flow will change with time. If a rocket is in motion, in order to determine the mass force, we have:

$$\overline{F}=\int_V \varrho(\overline{g}+\overline{j})(1-\chi)dV, \quad (1.175)$$

where $\overline{g}$ is the gravity acceleration;

$\bar{j}$ is rocket flight acceleration.

For components in a coordinate system moving with the rocket, we obtain:

$$\bar{F}_x=\int_V \varrho(g_x+j_x)(1-\chi)\,dV=(g_x+j_x)\int_V \varrho(1-\chi)\,dV; \qquad (1.176)$$

$$\bar{F}_r=(g_r+j_r)\int_V \varrho(1-\chi)\,dV; \qquad (1.177)$$

$$\bar{F}_\varphi=(g_\varphi+j_\varphi)\int_V \varrho(1-\chi)\,dV. \qquad (1.178)$$

Mass forces are potential forces, if a coordinate and time function $\varepsilon(x, r, \phi, t)$ exists such that its partial derivatives with respect to the coordinates respectively equal:

$$\bar{F}_x=-\frac{\partial\varepsilon}{\partial x}; \qquad (1.179)$$

$$\bar{F}_r=-\frac{\partial\varepsilon}{\partial r}; \qquad (1.180)$$

$$\bar{F}_\varphi=-\frac{1}{r}\frac{\partial\varepsilon}{\partial\varphi}. \qquad (1.181)$$

Let us examine one-dimensional flow. The equation (1.179) is stated as follows:

$$d\varepsilon = -\bar{F}_x dx. \qquad (1.182)$$

We obtain through integration:

$$\varepsilon=-\int_0^l \bar{F}_x\,dx. \qquad (1.183)$$

Thus ε characterizes the potential energy stored within the system in the process of mechanical motion in the direction of the x axis.

Under the influence of a difference in the hydrodynamic pressure, the surface force appears.

For the x component we have:

$$\Delta P_x=\int_V (1-S_x)\frac{\partial p}{\partial x}\,dx\cdot dr\,r\,d\varphi=\int_V (1-S_x)\frac{\partial p}{\partial x}\,dV \qquad (1.184)$$

and similarly, for the other components we obtain:

$$\Delta P_r = \int_V (1 - S_r) \frac{\partial p}{\partial r} dV; \quad (1.185)$$

$$\Delta P_\varphi = \int_V (1 - S_\varphi) \frac{\partial p}{r \partial \varphi} dV. \quad (1.186)$$

Due to interaction between the gas flow and the liquid flow, ballistic forces are generated. A drop located in the space under consideration has a certain characteristic size R_0. As a result of combustion, this dimension changes with time.

The ballistic force acting on one drop is:

$$f_1 = \frac{1}{2} \rho c_x \sigma_1 (W - C)^2. \quad (1.187)$$

The ballistic coefficient c_x is specified by burning conditions and depends on the shape of the drop, on velocities W and C, on the parameters and composition of the propellant and the gas, on the quality of atomization and mixing of propellant, as well as on many other factors. Often the value of the ballistic coefficient is specified as a function of the Reynolds number Re and of the parameter D, which characterizes drop deformation. In certain specific cases we accept that

$$c_x \sim Re^{-0.5}. \quad (1.188)$$

The value of the ballistic coefficient changes with time, since it depends on the size, the shape and the relative velocity of the drop. Much work has been devoted to an investigation of the coefficient c_x, and in complete calculations the problem of determining the ballistic coefficient must be examined separately [1, 27, 41, 55].

In addition to ballistic forces, viscous friction forces also influence the drop. They produce satellite flows, the investigation of which is applicable to any flow movement, and is outside the scope of this book.

The cross-sectional area of any flying drop is

$$\sigma_1 = \pi R_1^2, \quad (1.189)$$

where R_1 is the nominal radius of the center section of the drop.

The drop volume is

$$v_1 = \frac{\pi}{3} \xi_1^{-1} R_1^3 = \frac{\pi}{3} r^3, \qquad (1.190)$$

where r is the radius of a spherical drop;

ξ_1 is a coefficient which normalizes the magnitudes r and R_1, namely

$$\xi_1 = \left(\frac{R_1}{r}\right)^3. \qquad (1.191)$$

The number of drops, relative to the volumn examined, equals

$$n = \int\limits_V \frac{\chi}{v_1} dV \approx \frac{\chi V}{v_1}. \qquad (1.192)$$

Thus the ballistic force per volume element under consideration is:

$$df = \frac{3}{2} \varrho c_x \frac{\xi_1}{n} \frac{\chi}{R} (W - C)^2 dV. \qquad (1.193)$$

Directing our attention to well known functions in hydromechanics and considering constraints of the fluid volume element, frictional coordinate forces of the viscous compressible liquid are established as [41]:

$$\tau_x = \int\limits_V \varrho (1 - \chi) \nu \left(\frac{1}{3} \frac{\partial}{\partial x} \operatorname{div} \overline{W} + \nabla^2 W_x\right) dV; \qquad (1.194)$$

$$\tau_r = \int\limits_V \varrho (1 - \chi) \nu \left(\frac{1}{3} \frac{\partial}{\partial r} \operatorname{div} \overline{W} + \nabla^2 W_r\right) dV; \qquad (1.195)$$

$$\tau_\varphi = \int\limits_V \varrho (1 - \chi) \nu \left(\frac{1}{3} \frac{\partial}{r \partial \varphi} \operatorname{div} \overline{W} + \nabla^2 W_\varphi\right) dV, \qquad (1.196)$$

and

$$\nabla^2 W_x = \frac{\partial^2 W_x}{\partial x^2} + \frac{\partial^2 W_x}{\partial r^2} + \frac{1}{r} \frac{\partial W_x}{\partial r} + \frac{1}{r^2} \frac{\partial^2 W_x}{\partial \varphi^2}; \qquad (1.197)$$

$$\nabla^2 W_r = \frac{\partial^2 W_r}{\partial x^2} + \frac{\partial^2 W_r}{\partial r^2} + \frac{1}{r} \frac{\partial W_r}{\partial r} - \frac{W_r}{r^2} + \frac{1}{r^2} \frac{\partial^2 W_r}{\partial \varphi^2} - \frac{2}{r^2} \frac{\partial W_\varphi}{\partial \varphi}; \qquad (1.198)$$

$$\nabla^2 W_\varphi = \frac{\partial^2 W_\varphi}{\partial x^2} + \frac{\partial^2 W_\varphi}{\partial r^2} + \frac{1}{r} \frac{\partial W_\varphi}{\partial r} - \frac{W_\varphi}{r^2} + \frac{1}{r^2} \frac{\partial^2 W_\varphi}{\partial \varphi^2} + \frac{2}{r^2} \frac{\partial W_r}{\partial \varphi}. \qquad (1.199)$$

The conditions of gas movement within the chamber are so complex and distinctive that equations (1.194) to (1.199) prove to be suitable only for calculations made as first approximations.

Now we may write the equation force projections for the total element:

$$P_x=\int_V \varrho\Big[(1-\chi)(g_x+j_x)-\frac{1}{\varrho}\frac{\partial p}{\partial x}(1-S_x)-\frac{3}{2}\xi_x c_x\frac{\chi}{nR}\times$$

$$\times(W_x-C_x)^2+\nu\Big(\frac{1}{3}\frac{\partial}{\partial x}\operatorname{div}\overline{W}+\nabla^2 W_x\Big)(1-\chi)\Big]\,dV; \qquad (1.200)$$

$$P_r=\int_V \varrho\Big[(1-\chi)(g_r+j_r)-\frac{1}{\varrho}\frac{\partial p}{\partial r}(1-S_r)-\frac{3}{2}\xi_r c_r\frac{\chi}{nR}\times$$

$$\times(W_r-C_r)^2+\nu\Big(\frac{1}{3}\frac{\partial}{\partial r}\operatorname{div}\overline{W}+\nabla^2 W_r\Big)(1-\chi)\Big]\,dV; \qquad (1.201)$$

$$P_\varphi=\int_V \varrho\Big[(1-\chi)(g_\varphi+j_\varphi)-\frac{1}{\varrho}\frac{\partial p}{r\partial\varphi}(1-S_\varphi)-\frac{3}{2}\xi_\varphi c_\varphi\frac{\chi}{nR}\times$$

$$\times(W_\varphi-C_\varphi)^2+\nu\Big(\frac{1}{3}\frac{\partial}{r\partial\varphi}\operatorname{div}\overline{W}+\nabla^2 W_\varphi\Big)(1-\chi)\Big]\,dV. \qquad (1.202)$$

Before conducting calculations, it is necessary to establish through experiment the dependence of the ballistic force on velocity differences for the propellant atomization and combustion conditions considered.

The differential equations of gas motion, projected on their corresponding axes, are written as follows:

$$\frac{\partial W_x}{\partial t}+\frac{\partial W_x}{\partial x}W_x+\frac{\partial W_x}{\partial r}W_r+\frac{\partial W_x}{\partial\varphi}\frac{W_\varphi}{r}=(g_x+j_x)-$$

$$-\frac{1}{\varrho}\frac{\partial p}{\partial x}\frac{1-S_x}{1-\chi}-\frac{3}{2}\xi_x c_x\frac{\chi}{1-\chi}\frac{(W_x-C_x)^2}{nR}+$$

$$+\nu\Big(\frac{1}{3}\frac{\partial}{\partial x}\operatorname{div}\overline{W}+\nabla^2 W_x\Big); \qquad (1.203)$$

$$\frac{\partial W_r}{\partial t}+\frac{\partial W_r}{\partial x}W_x+\frac{\partial W_r}{\partial r}W_r+\frac{\partial W_r}{\partial\varphi}\frac{W_\varphi}{r}-\frac{1}{r}W_\varphi^2=(g_r+j_r)-$$

$$-\frac{1}{\varrho}\frac{\partial p}{\partial r}\frac{1-S_r}{1-\chi}-\frac{3}{2}\xi_r c_r\frac{\chi}{1-\chi}\frac{(W_r-C_r)^2}{nR}+$$

$$+\nu\Big(\frac{1}{3}\frac{\partial}{\partial r}\operatorname{div}\overline{W}+\nabla^2 W_r\Big); \qquad (1.204)$$

$$\frac{\partial W_\varphi}{\partial t}+\frac{\partial W_\varphi}{\partial x}W_x+\frac{\partial W_\varphi}{\partial r}W_r+\frac{\partial W_\varphi}{\partial \varphi}\frac{W_\varphi}{r}+\frac{1}{r}W_rW_\varphi=(g_\varphi+j_\varphi)-$$

$$-\frac{1}{\varrho}\frac{\partial p}{r\partial\varphi}\frac{1-S_\varphi}{1-\chi}-\frac{3}{2}\xi_r c_\varphi\frac{\chi}{1-\chi}\frac{(W_\varphi-C_\varphi)^2}{nR}+$$

$$+\nu\left(\frac{1}{3}\frac{\partial}{r\partial\varphi}\operatorname{div}\overline{W}+\nabla^2W_\varphi\right). \quad (1.205)$$

2. Design Equations

At a certain distance from the head, where the liquid phase is lacking, motion equations assume the well known form:

$$\frac{\partial W_x}{\partial t}+\frac{\partial W_x}{\partial x}W_x+\frac{\partial W_r}{\partial r}W_r+\frac{\partial W_\varphi}{\partial \varphi}\frac{W_\varphi}{r}=(g_x+j_x)-$$

$$-\frac{1}{\varrho}\frac{\partial p}{\partial x}+\nu\left(\frac{1}{3}\frac{\partial}{\partial x}\operatorname{div}\overline{W}+\nabla^2W_x\right); \quad (1.206)$$

$$\frac{\partial W_r}{\partial t}+\frac{\partial W_r}{\partial x}W_x+\frac{\partial W_r}{\partial r}W_r+\frac{\partial W_\varphi}{\partial \varphi}\frac{W_\varphi}{r}-\frac{1}{r}W_\varphi^2=(g_r+j_r)-$$

$$-\frac{1}{\varrho}\frac{\partial p}{\partial r}+\nu\left(\frac{1}{3}\frac{\partial}{\partial r}\operatorname{div}\overline{W}+\nabla^2W_r\right); \quad (1.207)$$

$$\frac{\partial W_\varphi}{\partial t}+\frac{\partial W_\varphi}{\partial x}W_x+\frac{\partial W_\varphi}{\partial r}W_r+\frac{\partial W_\varphi}{\partial \varphi}\frac{W_\varphi}{r}+\frac{1}{r}W_rW_\varphi=(g_\varphi+j_\varphi)-$$

$$-\frac{1}{\varrho}\frac{\partial p}{r\partial\varphi}+\nu\left(\frac{1}{3}\frac{\partial}{r\partial\varphi}\operatorname{div}\overline{W}+\nabla^2W_\varphi\right). \quad (1.208)$$

Let us examine the liquid flow. The projections of the mass force elements satisfy:

$$d\overline{F}_{xl}=\varrho_l(g_x+j_x)\chi dV; \quad (1.209)$$

$$d\overline{F}_{rl}=\varrho_l(g_r+j_r)\chi dV; \quad (1.210)$$

$$d\overline{F}_{\phi l}=\varrho_l(g_\varphi+j_\varphi)\chi dV. \quad (1.211)$$

The projection differentials of the hydrodynamic pressures are expressed as:

$$dP_{xl}=\frac{\partial p}{\partial x}S_xdV; \quad (1.212)$$

$$dP_{rl}=\frac{\partial p}{\partial r}S_rdV; \quad (1.213)$$

$$dP_{\phi l}=\frac{\partial p}{r\partial\varphi}S_\varphi dV. \quad (1.214)$$

The ballistic forces are determined according to equation (1.193), but they are taken into consideration as part of a general motion equation with an inverse sign, in comparison to those cases in which they are determined in accordance with equations (1.203), (1.204) and (1.205).

The differential projection equations of the liquid phase motion are expressed as follows:

$$\frac{\partial C_x}{\partial t}+\frac{dC_x}{\partial x}C_x+\frac{dC_x}{\partial r}C_r+\frac{\partial C_x}{\partial \varphi}\frac{C_\varphi}{r}=(g_x+j_x)-$$
$$-\frac{1}{\varrho_l}\frac{\partial p}{\partial x}\;\frac{S_x}{\chi}+\frac{3}{2}\xi_x c_x \varrho\frac{(W_x-C_x)^2}{nR}; \qquad (1.215)$$

$$\frac{\partial C_r}{\partial t}+\frac{\partial C_r}{\partial x}C_x+\frac{\partial C_r}{\partial r}C_r+\frac{\partial C_r}{\partial \varphi}\frac{C_\varphi}{r}-\frac{1}{r}C_\varphi^2=(g_r+j_r)-$$
$$-\frac{1}{\varrho_l}\frac{\partial p}{\partial r}\;\frac{S_r}{\chi}+\frac{3}{2}\xi_r c_r \varrho\frac{(W_r-C_r)^2}{nR}; \qquad (1.216)$$

$$\frac{\partial C_\varphi}{\partial t}+\frac{\partial C_\varphi}{\partial x}C_x+\frac{\partial C_\varphi}{\partial r}C_r+\frac{\partial C_\varphi}{\partial \varphi}\frac{C_\varphi}{r}+\frac{1}{r}C_r C_\varphi=(g_\varphi+j_\varphi)-$$
$$-\frac{1}{\varrho_l}\frac{\partial p}{r\partial \varphi}\;\frac{S_\varphi}{\chi}+\frac{3}{2}\xi_\varphi c_\varphi \varrho\frac{(W_\varphi-C_\varphi)^2}{nR}. \qquad (1.217)$$

It must be kept in mind that the calculation in accordance with these equations is carried only up to the time of the complete transition of the liquid phase to the gaseous.

§ 6. The Equation of Conservation of Energy

The general (full) equation of the conservation of energy contains terms characterizing both mechanical energy and the energy of heat processes. The equation of the conservation of mechanical energy is obtained from Newton's second law by an examination of the movement of an ideal fluid. The equation of the conservation of energy of heat processes is obtained from the first law of thermodynamics. We obtain the general equation of the conservation of energy of the combustion flow from a combination of these two equations.

1. The Equation of the Conservation of Mechanical Energy

With the aim of clarifying the meanings of separate terms of the equation of the conservation of mechanical energy, let us examine the flow of an ideal gas, excluding any liquid particles. The fact is that when viscous friction forces are present, a part of the mechanical energy will be transformed into heat. This phenomenon is called dissipation of mechanical energy. Equations

(1.206), (1.207), and (1.208), which we shall accept as reference points, are expressed as follows [34, 41]:

$$\left.\begin{aligned}\dot{W}_x&=(g_x+j_x)-\frac{1}{\varrho}\frac{\partial p}{\partial x};\\ \dot{W}_r&=(g_r+j_r)-\frac{1}{\varrho}\frac{\partial p}{\partial r};\\ \dot{W}_\varphi&=(g_\varphi+j_\varphi)-\frac{1}{\varrho}\frac{\partial p}{r\partial\varphi}.\end{aligned}\right\} \tag{1.218}$$

Let us multiply the right and left sides of the equations by the corresponding velocity components. We obtain:

$$\left.\begin{aligned}\dot{W}_xW_x&=(g_x+j_x)W_x-\frac{1}{\varrho}\frac{\partial p}{\partial x}W_x;\\ \dot{W}_rW_r&=(g_r+j_r)W_r-\frac{1}{\varrho}\frac{\partial p}{\partial r}W_r;\\ \dot{W}_\varphi W_\varphi&=(g_\varphi+j_\varphi)W_\varphi-\frac{1}{\varrho}\frac{\partial p}{\partial\varphi}\frac{W_\varphi}{r}.\end{aligned}\right\} \tag{1.219}$$

After term by term addition, we find that:

$$\dot{W}_xW_x+\dot{W}_rW_r+\dot{W}_\varphi W_\varphi=\sum(g_i+j_i)W_i-\frac{1}{\varrho}\left(\frac{\partial p}{\partial x}W_x+\frac{\partial p}{\partial r}W_r+\frac{\partial p}{\partial\varphi}\frac{W_\varphi}{r}\right), \tag{1.220}$$

however,

$$\dot{W}_xW_x+\dot{W}_rW_r+\dot{W}_\varphi W_\varphi=\frac{1}{2}\frac{d}{dt}(W^2). \tag{1.221}$$

Utilizing the rules of product differentiation, we find that:

$$\begin{aligned}\frac{\partial p}{\partial x}W_x+\frac{\partial p}{\partial r}W_r+\frac{\partial p}{\partial\varphi}\frac{W_\varphi}{r}&=\frac{\partial}{\partial x}(pW_x)+\frac{\partial}{\partial r}(pW_r)+\\ &+\frac{\partial}{\partial\varphi}\frac{(pW_\varphi)}{r}-p\left(\frac{\partial W_x}{\partial x}+\frac{\partial W_r}{\partial r}+\frac{\partial W_\varphi}{r\partial\varphi}\right)=\\ &=\operatorname{div}(p\overline{W})-p\operatorname{div}\overline{W}.\end{aligned} \tag{1.222}$$

Therefore,

$$\frac{1}{2}\frac{d}{dt}(W^2)=\sum(g_i+j_i)W_i-\frac{1}{\varrho}[\operatorname{div}(p\overline{W})-p\operatorname{div}\overline{W}]. \tag{1.223}$$

Equation (1.223) is an energy equation, which takes into consideration the types of mechanical energy under observation.

The external mass volumetric forces that are potential in nature ($\bar{g}$) perform the following work in a unit of time:

$$-\sum \pi_i W_i = \frac{\partial \varepsilon}{\partial x} W_x + \frac{\partial \varepsilon}{\partial r} W_r + \frac{\partial \varepsilon}{\partial \varphi} \frac{W_\varphi}{r} \qquad (1.224)$$

or

$$\sum \pi_i W_i = -(\bar{W}, \text{grad}\, \varepsilon). \qquad (1.225)$$

If the forces under consideration are not potential and depend neither on the coordinates nor on time, then we shall specify the condition

$$\sum \pi_i W_i = \bar{\pi}_i \bar{W}_i = \text{K}. \qquad (1.226)$$

Finally, with forces which are not potential, but which depend on time ($\bar{j}$), we have

$$\sum \pi_i W_i = \bar{\pi}_i \bar{W}_i = \text{K}(t). \qquad (1.226a)$$

In this manner, we obtain in the most general form:

$$\sum \pi_i W_i = \text{K}(t) - \frac{d\varepsilon}{dt}; \qquad (1.227)$$

$$\frac{d\varepsilon}{dt} = (\bar{W}, \text{grad}\, \varepsilon).$$

Let us assume that the equation of rocket flight is under consideration. At each point on the trajectory, forces due to the presence of the gravitational field will have different values of the potential function, independent of time. Therefore, here

$$\sum \pi_i W_i = -(\bar{W}, \text{grad}\, \varepsilon).$$

In an examination of the chamber equation, if the coordinate system is "bound" to the engine, then the same forces may be considered to be independent of the location of the combustion stream element in the chamber. If the engine is in flight, then

$$\sum \pi_i W_i = K(t).$$

During test stand operation

$$\sum \pi_i W_i = K.$$

Let us return to equation (1.223). After multiplication by ρdV and integration, we obtain

$$\frac{1}{2}\frac{d}{dt}\int\limits_V \rho W^2 dV = \int\limits_V \rho \sum \pi_i W_i dV - \int\limits_V \mathrm{div}(p\overline{W})\, dV + \int\limits_V p \,\mathrm{div}\, \overline{W} dV. \tag{1.228}$$

According to Ostrogradskiy's formula [33][1]

$$\int\limits_V \mathrm{div}(p\overline{W})\, dV = \int\limits_F p(\overline{W}, \overline{n})\, dF, \tag{1.229}$$

where F is a surface limiting volume V;

$\overline{n}$ is the unit vector of an external normal to the surface F.

Thus the integral (1.229) represents the work per unit time of pressure forces applied to the surface F. The integrand p div $\overline{W}$ of the last integral in the equation (1.228) represents the work of pressure forces expended on the change of volume V per unit time.

Since the meaning of the terms of the equation of the conservation of mechanical energy is clear, we may approach the composition of the equation of the conservation of mechanical energy for the combustion stream. Let external volumetric forces influence the stream. In the most general case work, produced on the gas included within a certain volume V, is:

$$L_g = \int\limits_V \rho(1-\chi)[K(t)-\varepsilon]\, dV, \tag{1.230}$$

and work produced on liquid is:

$$L_l = \int\limits_V \rho_l \chi [K(t)-\varepsilon]\, dV, \tag{1.231}$$

where ε and K(t) are congruent functions.

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[1] See p. 186.

The work of the pressure forces, applied to the surface, and limiting the gas, is:

$$L_{Fg}=\int_F p(\overline{W},\overline{n})\,dF=\int_V (1-\chi)\operatorname{div}(p\overline{W})\,dV, \qquad (1.232)$$

and the work of the pressure forces applied to the surface and limiting the liquid is:

$$L_{F_l}=\int_V \chi\operatorname{div}(p\overline{C})\,dV. \qquad (1.233)$$

During the liquid phase the work of the pressure forces expended on a gas volume change is:

$$L_{V_g}=\int_V (1-\chi)\,p\operatorname{div}\overline{W}\,dV. \qquad (1.234)$$

As a result of work carried out on volume V the change in the kinetic energy of the gas equals:

$$L_{W_g}=\frac{1}{2}\frac{d}{dt}\int_V \varrho(1-\chi)\,W^2dV, \qquad (1.235)$$

and the change in the kinetic energy of the liquid is:

$$L_{W_l}=\frac{1}{2}\frac{d}{dt}\int_V \varrho_l\chi C^2dV. \qquad (1.236)$$

The equation for the conservation of mechanical energy of the combustion stream in a case when the liquid is incompressible is expressed as follows:

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\int_V \varrho(1-\chi)W^2dV+\frac{1}{2}\frac{d}{dt}\int_V \varrho_l\chi C^2dV=\\ =\int_V \varrho(1-\chi)[K(t)-\dot{\varepsilon}]\,dV+\int_V \rho_l\chi[K(t)-\dot{\varepsilon}]\,dV-\\ -\int_V (1-\chi)\operatorname{div}(p\overline{W})\,dV-\int_V \chi\operatorname{div}(p\overline{C})\,dV+\\ +\int_V (1-\chi)\,p\operatorname{div}\overline{W}\,dV.\end{aligned} \qquad (1.237)$$

During a combination of flows, the law of the conservation of mechanical energy is not satisfied; in this connection, the method of determining the heat release is considered in the second part of § 7.

2. The First Law of Thermodynamics

The aggregate quantity of heat obtained bv the volume element in a unit of time equals the change of the internal gas energy, of the heat content of the liquid, and of the work of gas expansion, all accomplished within the same time interval. Thus it is possible to formulate the first law of thermodynamics for a moving system of coordinates applicable to the combustion stream. The aggregate quantity of heat obtained by the element per unit time is

$$\frac{d}{dt}\int_V Q dV = \frac{d}{dt}\left[\int_V Q_g dV + \int_V Q_\lambda dV + \int_V Q_R dV\right]. \tag{1.238}$$

where Q_g is heat released during propellant burning;

Q_λ is heat supplied by means of thermal conductivity and through other thermal transfer processes;

Q_R is heat released due to the dissipation of mechanical energy.

Since during the combustion of a mass unit of fuel, Q_0 joules of heat are released and conversion of the liquid phase to the gaseous phase is characterized by div $\bar{q}$, then

$$\frac{d}{dt}\int_V Q_g dV = \int_V Q_0 \operatorname{div} \bar{q} dV. \tag{1.239}$$

The heat supply by means of thermal conductivity is [35]:

$$\frac{d}{dt}\int_V Q_\lambda dV = \frac{d}{dt}\int_V \xi\left[\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial r}\left(\lambda\frac{\partial T}{\partial r}\right) + \frac{1}{r\partial\varphi}\left(\lambda\frac{\partial T}{r\partial\varphi}\right) + \frac{1}{r}\lambda\frac{\partial T}{\partial r}\right] dV, \tag{1.240}$$

where ξ is a coefficient which considers the role of convection and radiant thermal transfer.

If it is more convenient to describe the process of thermal conduction in the spherical system of coordinates, we obtain:

$$\frac{d}{dt}\int_V Q_\lambda dV = \frac{d}{dt}\int_V \xi\left[\frac{\partial}{\partial r}\left(\lambda\frac{\partial T}{\partial r}\right) + \frac{2}{r}\lambda\frac{\partial T}{\partial r} + \right.$$

$$\left. + \frac{1}{r^2}\frac{\partial}{\partial\psi}\left(\lambda\frac{\partial T}{\partial\psi}\right) + \frac{\cos\psi}{r^2\sin\psi}\lambda\frac{\partial T}{\partial\psi} + \frac{1}{r^2\sin^2\psi}\frac{\partial}{\partial\varphi}\left(\lambda\frac{\partial T}{\partial\varphi}\right)\right] dV. \tag{1.241}$$

Heat generated as a result of internal friction is:

$$\frac{d}{dt}\int_V Q_R dV = \frac{d}{dt}\left[\int_V \mu_g (1-\chi)\Phi_g dV + \int_V \mu_l \chi \Phi_l dV + \int_V \mu \Phi dV\right], \tag{1.242}$$

where μ_i is the friction coefficient;

Φ_i is the dissipation function.

The first integral of the right side of the equation characterizes the heat supply due to the ct of viscous friction forces. Part of the total work which is produced b ...nal and tangential components of frictional forces is transformed into mechanical energy, and part into heat. The latter is determined by the first integral of the right side of equation (1.242). The second integral on the right side of this equation characterizes the heat supply as a result of friction in the liquid part of the element, and the third term considers heat released during equilibrium of the velocity of the gas stream and the stream of liquid particles. In order to determine the dissipation function Φ_g, it is necessary to use the equation whose general form in Cartesian coordinates is expressed as follows [55]:

$$\Phi_g = 2\left[\left(\frac{\partial W_x}{\partial x}\right)^2 + \left(\frac{\partial W_y}{\partial y}\right)^2 + \left(\frac{\partial W_z}{\partial z}\right)^2\right] + \\ + \left(\frac{\partial W_y}{\partial x} + \frac{\partial W_x}{\partial y}\right)^2 + \left(\frac{\partial W_z}{\partial y} + \frac{\partial W_y}{\partial z}\right)^2 + \left(\frac{\partial W_x}{\partial z} + \frac{\partial W_z}{\partial x}\right)^2 - \\ - \frac{2}{3}\left(\frac{\partial W_x}{\partial x} + \frac{\partial W_y}{\partial y} + \frac{\partial W_z}{\partial z}\right)^2. \tag{1.243}$$

In order to determine the dissipation function Φ_l, it is necessary to use the same equation, substituting W_i by corresponding values of C_i. The dissipation of mechanical energy observed during the mixing of gas and liquid flows will be examined below.

The change in the internal energy of the gas and the liquid comprises:

$$\frac{d}{dt}\int_m u\,dm = \frac{d}{dt}\int_V \varrho (1-\chi)(c_v T) dV + \frac{d}{dt}\int_V \rho_l \chi (c T_l) dV. \tag{1.244}$$

The work of expansion, in agreement with (1.234), will be:

$$L_{V_g} = \int_V (1-\chi)\, p \operatorname{div} \overline{W} dV. \tag{1.245}$$

Now we may formulate the equation for the first law of thermodynamics in the following form:

$$\int_V Q_0\chi \operatorname{div} \bar{q} dV + \frac{d}{dt}\int_V \xi\left[\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial r}\left(\lambda\frac{\partial T}{\partial r}\right) + \frac{1}{r\partial\varphi}\left(\lambda\frac{\partial T}{r\partial\varphi}\right) + \right.$$

$$\left. + \frac{1}{r}\lambda\frac{\partial T}{\partial r}\right] dV + \frac{d}{dt}\left[\int_V \mu_g (1-\chi)\Phi_g \, dV + \right.$$

$$\left. + \int_V \mu_l \chi \Phi_l dV + \int_V \mu\Phi \, dV\right] = \frac{d}{dt}\int_V \varrho(1-\chi)(c_v T) dV +$$

$$+ \frac{d}{dt}\int_V \varrho_l \chi (cT_l) dV + \int_V (1-\chi) p \operatorname{div} \overline{W} dV. \qquad (1.246)$$

The equation (1.246) permits us to determine the work of expansion of gas produced due to the flow of thermal processes. Equation (1.237) reveals possible methods of the transformation of mechanical energy. Replacing the last term in equation (1.246) with the similar term from equation (1.237), we obtain:

$$\int_V Q_0\chi \operatorname{div} \bar{q} dV + \frac{d}{dt}\int_V \xi\left[\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial r}\left(\lambda\frac{\partial T}{\partial r}\right) + \frac{1}{r\partial\varphi}\left(\lambda\frac{\partial T}{r\partial\varphi}\right) + \right.$$

$$\left. + \frac{1}{r}\lambda\frac{\partial T}{\partial r}\right] dV + \int_V \varrho(1-\chi)[K(t) - \dot{\varepsilon}] dV + \int_V \varrho_l \chi [K(t) - \dot{\varepsilon}] dV =$$

$$= \int_V (1-\chi)\operatorname{div}(p\overline{W}) dV + \int_V \chi \operatorname{div}(p\overline{C}) dV +$$

$$+ \frac{1}{2}\frac{d}{dt}\int_V \varrho(1-\chi) W^2 dV + \frac{1}{2}\frac{d}{dt}\int_V \varrho_l \chi C^2 dV +$$

$$+ \frac{d}{dt}\int_V \varrho(1-\chi)(c_v T) dV + \frac{d}{dt}\int_V \varrho_l \chi (cT_l) dV -$$

$$- \frac{d}{dt}\left[\int_V \mu_r (1-\chi)\Phi_r \, dV + \int_V \mu_l \chi \Phi_l dV + \int_V \mu\Phi dV\right]. \qquad (1.247)$$

The general equation of the conservation of energy of the combustion stream (1.247) is read as follows: the quantity of heat obtained by the volume element of the combustion stream as a result of propellant burning and thermal transfer processes, plus the work produced on the element by external forces, equals the work of the pressure forces acting on the surface which limits the volume, the increase in the kinetic energy of the combustion stream, the increase in the internal energy of the mass of the combustion stream and the heat released due to the dissipation of mechanical energy.

3. A Determination of Discharge Velocity

Let us write the energy equation for an element which does not contain the liquid phase. We shall consider that the heat supply is lacking and that external forces are not acting upon the element. It is apparent that equation (1.247) will, in this case, assume the following form:

$$\frac{1}{2}\frac{d}{dt}\int_V \varrho W^2 dV + \frac{d}{dt}\int_V \varrho c_v T dV + \int_V \operatorname{div}(p\overline{W})\, dV = 0. \quad (1.248)$$

Let us examine the integrand of the last integral in equation (1.248). It is apparent that:

$$\operatorname{div}(p\overline{W}) = \left(\frac{\partial p}{\partial x} W_x + \frac{\partial p}{\partial r} W_r + \frac{\partial p}{\partial \varphi}\frac{W_\varphi}{r}\right) + p \operatorname{div}\overline{W}. \quad (1.249)$$

However,

$$\left(\frac{\partial p}{\partial x} W_x + \frac{\partial p}{\partial r} W_r + \frac{\partial p}{\partial \varphi}\frac{W_\varphi}{r}\right) = \dot{p} - \frac{\partial p}{\partial t}. \quad (1.250)$$

Employing the equation of continuity in the form

$$\dot{\varrho} + \varrho \operatorname{div}\overline{W} = 0, \quad (1.251)$$

we find that:

$$p \operatorname{div}\overline{W} = -\dot{\varrho}\frac{p}{\varrho}. \quad (1.252)$$

Let us add to the right side of equation (1.252) and subtract from it the term:

$$\varrho \frac{d}{dt}\left(\frac{p}{\varrho}\right).$$

According to the rules of partial differentiation, we have:

$$-\varrho \frac{d}{dt}\left(\frac{p}{\varrho}\right) = -\dot{p} + \frac{p}{\varrho}\dot{\varrho}. \quad (1.253)$$

Therefore,

$$p \operatorname{div}\overline{W} = -\dot{p} + \varrho \frac{d}{dt}\left(\frac{p}{\varrho}\right). \quad (1.254)$$

Keeping in mind equation (1.250), we find that:

$$\operatorname{div}(p\overline{W}) = \rho \frac{d}{dt}\left(\frac{p}{\rho}\right) - \frac{\partial p}{\partial t}. \tag{1.255}$$

Let us employ the equation of state in the following form:

$$p = \rho RT. \tag{1.256}$$

Therefore,

$$\frac{d}{dt}\left(\frac{p}{\rho}\right) = \frac{d}{dt}(RT). \tag{1.256a}$$

The equation (1.248) may be written as follows:

$$\frac{1}{2}\frac{d}{dt}\int_V \rho W^2 dV + \frac{d}{dt}\int_V \rho c_v T dV + \frac{d}{dt}\int_V \rho (RT) dV - \int_V \frac{\partial p}{\partial t} dV = 0. \tag{1.257}$$

Since

$$c_v + R = c_p, \tag{1.258}$$

then

$$\frac{1}{2}\frac{d}{dt}\int_V W^2 \rho dV + \frac{d}{dt}\int_V \rho c_p T dV - \int_V \frac{\partial p}{\partial t} dV = 0. \tag{1.259}$$

If the parameters of the integrands do not depend on the coordinates, then under conditions where $\partial p/\partial t = 0$, we find that:

$$d\left(\frac{W^2}{2}\right) + d(c_p T) = 0. \tag{1.260}$$

After integration and solving the problem, we obtain the following equation:

$$W = [2(c_p T)_0 - 2(c_p T) + W_0^2]^{0,5}. \tag{1.261}$$

We shall designate by the symbol σ the ratio of the heat capacity at temperature T to the heat capacity at T_0:

$$\sigma = \frac{c_p}{c_{p0}}$$

In this connection

$$W = [2c_{p0}(T_0 - \sigma T) + W_0^2]^{0,5}. \quad (1.262)$$

The equation obtained permits us to calculate the average value of the gas flow velocity during engine operation.

The equation

$$1 - \sigma \frac{T}{T_0} = \eta_t \quad (1.263)$$

represents thermal efficiency.

4. The Heating of Stationary Burning Propellant

If the gas is stationary (W = 0) and is bounded by the walls of the combustion chamber ($\mathrm{div}|\rho \overline{W}| = 0$), then the energy equation assumes the following form:

$$\int_V Q_0 \chi \,\mathrm{div}\, \overline{q} dV = \frac{d}{dt}\int_V \varrho(1-\chi)(c_v T)\, dV + \frac{d}{dt}\int_V \varrho_l \chi (cT_l)\, dV. \quad (1.264)$$

Thus during propellant burning within a closed constant volume, all of the heat is expended in increasing the internal energy of the combustion stream. In the process of burning, the liquid phase quantity decreases to zero, and all of the heat developed will be transformed into internal energy of gaseous products.

Let us examine the process of combustion stream heating in a certain closed but varying space. Maintaining the assumptions given above, and directing our attention to equation (1.246), we find that:

$$\int_V Q_0 \chi \,\mathrm{div}\, \overline{q} dV = \frac{d}{dt}\int_V \varrho(1-\chi)(c_v T)\, dV + \frac{d}{dt}\int_V \varrho_l \chi (cT_l)\, dV + \int_V (1-\chi)\, p \,\mathrm{div}\, \overline{W} dV. \quad (1.265)$$

Considering (1.254) and (1.258), we have:

$$\int_V Q_0 \chi \,\mathrm{div}\, \overline{q} dV = \frac{d}{dt}\int_V \varrho(1-\chi)(c_p T)\, dV + \frac{d}{dt}\int_V \varrho_l \chi (cT_l)\, dV - \int_V \dot{p} dV. \quad (1.266)$$

Here the noteworthy point is the last term, which takes into consideration the expansion characteristics for changing pressure. If the expansion occurs at a constant pressure, then:

$$\int_V Q_0\chi \operatorname{div} \bar{q} dV = \frac{d}{dt}\int_V \varrho(1-\chi)(c_pT)dV + \frac{d}{dt}\int_V \varrho_l\chi(cT_l)dV. \tag{1.267}$$

If the liquid phase is lacking, we have, in place of equation (1.266) the following:

$$\int_V Q_0 \operatorname{div} \bar{q} dV = \frac{d}{dt}\int_V \varrho(c_pT)dV - \int_V \dot{p}dV. \tag{1.268}$$

Here div $\bar{q}$ characterizes the heat supply as a result of the burning of gaseous propellant.

If p = const, we find from equation (1.268) that:

$$\int_V Q_0 \operatorname{div} \bar{q} dV = \frac{d}{dt}\int_V \varrho(c_pT)dV. \tag{1.269}$$

If the parameters of the integrand do not depend on the coordinates, the burnt propellant per unit of mass is

$$Q_0 = i = c_pT, \tag{1.270}$$

where T is the temperature increase produced by the combustion products.

5. The Adiabatic Flow

The energy equation permits us to investigate the flow without heat transfer, i.e., the adiabatic flow. The energy equation without considering dissipation is written as follows:

$$\begin{aligned}\int_V \dot{\varepsilon}\varrho(1-\chi)dV + \int_V \dot{\varepsilon}\varrho_l\chi dV = \\ = \int_V (1-\chi)\operatorname{div}(p\bar{W})dV + \int_V \chi \operatorname{div}(p\bar{C})dV + \\ + \frac{1}{2}\frac{d}{dt}\int_V \varrho(1-\chi)W^2dV + \frac{1}{2}\frac{d}{dt}\int_V \varrho_l\chi C^2dV + \\ + \frac{d}{dt}\int_V \varrho(1-\chi)(c_vT)dV + \frac{d}{dt}\int_V \varrho_l\chi(cT_l)dV.\end{aligned} \tag{1.271}$$

When the liquid phase is not present we have:

$$\int_V \varepsilon\varrho dV = \int_V \operatorname{div}(p\overline{W})\, dV + \frac{1}{2}\frac{d}{dt}\int_V \varrho W^2 dV + \frac{d}{dt}\int_V \varrho(c_v T)\, dV. \qquad (1.272)$$

If we employ equation (1.237), having written it for the gas flow with exclusion of liquid components, and solve it jointly with equation (1.272), then we obtain the following equation:

$$\frac{d}{dt}\int_V \varrho(c_v T)\, dV + \int_V p \operatorname{div} \overline{W} dV = 0. \qquad (1.273)$$

Employing equations (1.254), (1.256a) and (1.258), we find that:

$$\frac{d}{dt}\int_V \varrho(c_p T) dV - \int_V \dot{p} dV = 0. \qquad (1.274)$$

Let us divide the integrands by ρ. In those particular cases when volume V does not depend on time, and the parameters of the integrand are constant within the space under investigation, the equation (1.274) may be presented as:

$$d(c_p T) = \frac{dp}{\varrho}. \qquad (1.275)$$

Employing the equation of state, we find that:

$$\frac{d(c_p T)}{RT} = \frac{dp}{p}. \qquad (1.276)$$

Therefore,

$$\frac{c_p}{R}\frac{dT}{T} + \frac{dc_p}{R} = \frac{dp}{p}, \qquad (1.277)$$

or

$$\frac{k}{k-1}\frac{dT}{T} + d\left(\frac{k}{k+1}\right) = \frac{dp}{p}. \qquad (1.278)$$

The equation (1.278) describes the process, allowing for a possible change in the characteristic k. If we accept k = const, then we obtain the adiabatic curve equation in the form:

$$\frac{k}{k-1}\frac{dT}{T} = \frac{dp}{p}, \qquad (1.279)$$

Integrating and normalizing, we find that:

$$\frac{T}{T_{ex}}=\left(\frac{p}{p_{ex}}\right)^{\frac{k-1}{k}}. \qquad (1.280)$$

6. The Equation of State

Let us successively reduce the range of the possible applications of the equation of the conservation of energy.

If we disregard heat transfer and dissipation, on the basis of equation (1.247) for a case of burning of gaseous propellant, we may write:

$$\int_V Q_0 \operatorname{div}\bar{q}\,dV+\int_V \dot{\varepsilon}\varrho\,dV=\frac{d}{dt}\int_V \varrho(c_v T)\,dV+\int_V \operatorname{div}(p\bar{W})\,dV+\frac{1}{2}\frac{d}{dt}\int_V \varrho W^2 dV. \qquad (1.281)$$

Since it has already been shown that

$$\left.\begin{aligned}\operatorname{div}(p\bar{W})&=\dot{p}-\frac{\partial p}{\partial t}+p\operatorname{div}\bar{W};\\ p\operatorname{div}\bar{W}&=-\dot{p}+\varrho\frac{d}{dt}(RT),\end{aligned}\right\} \qquad (1.282)$$

therefore,

$$\operatorname{div}(p\bar{W})=\varrho\frac{d}{dt}(RT)-\frac{\partial p}{\partial t},$$

the energy equation for the gaseous burning propellant assumes the following form:

$$\int_V Q_0 \operatorname{div}\bar{q}\,dV+\int_V \dot{\varepsilon}\varrho\,dV=\frac{d}{dt}\int_V \varrho(c_p T)\,dV+\frac{1}{2}\frac{d}{dt}\int_V \varrho W^2 dV-\int_V \frac{\partial p}{\partial t}\,dV. \qquad (1.283)$$

After burning ceases, when heat transfer and external effects are not present, the energy equation assumes the following form:

$$\frac{d}{dt}\int_V \varrho(c_p T)\,dV+\frac{1}{2}\frac{d}{dt}\int_V \varrho W^2 dV-\int_V \frac{\partial p}{\partial t}\,dV=0. \qquad (1.284)$$

Since [see (1.323)]

$$\frac{d}{dt}\int_V \varrho(c_p T)\,dV=\int_V \varrho\frac{d}{dt}(c_p T)\,dV, \qquad (1.285)$$

$$\frac{d}{dt}\int\limits_V \varrho W^2 dV = \int\limits_V \varrho \frac{d}{dt} W^2 dV, \qquad (1.286)$$

Then in place of equation (1.284) we have:

$$\int\limits_V \varrho \frac{d}{dt}(c_p T)\,dV + \frac{1}{2}\int\limits_V \varrho \frac{d}{dt} W^2 dV - \int\limits_V \frac{\partial p}{\partial t}\,dV = 0. \qquad (1.287)$$

If the gas element is fixed, i.e., W = 0, then the energy equation (1.287) has the following form:

$$\int\limits_V \varrho \frac{d}{dt}(c_p T)\,dV - \int\limits_V \frac{\partial p}{\partial t}\,dV = 0, \qquad (1.287a)$$

and equation (1.281) is written as follows:

$$\int\limits_V \varrho \frac{d}{dt}(c_v T)\,dV + \int\limits_V \operatorname{div}(p\overline{W})\,dV = 0. \qquad (1.288)$$

Equalizing the left sides of (1.287a) and (1.288), we find that:

$$\int\limits_V \varrho \frac{d}{dt}(RT)\,dV = \int\limits_V \operatorname{div}(p\overline{W})\,dV + \int\limits_V \frac{\partial p}{\partial t}\,dV. \qquad (1.289)$$

Under steady conditions work is not expended on a volume change, therefore $p \operatorname{div} \overline{W} = 0$. In this connection, in place of (1.289) we obtain:

$$\int\limits_V \varrho \frac{d}{dt}(RT)\,dV = \int\limits_V \dot{p}\,dV. \qquad (1.290)$$

The equation (1.290) characterizes the relationship between the volume parameters under consideration at a given moment of time.

If the parameters which appear in the integrands do not depend on the coordinates, then in place of (1.290), we have:

$$\rho RT = p. \qquad (1.291)$$

§ 7. The Equation of the Conservation of Momentum for the Combustion Stream

1. The Determination of the Average Velocity of the Combustion Stream

At a certain arbitrary moment in time, the mass of the gas is located within the volume element dV, distributed within the internal chamber cavity, comprise:

$$dm_g = \varrho(1-\chi)dV. \tag{1.292}$$

At the same time and in the same volume element, the mass of the liquid is:

$$dm_l = \varrho_l \chi dV. \tag{1.293}$$

Combustion stream momentum $d\theta$ within the element is determined by the sum:

$$d\theta = \varrho(1-\chi)WdV + \varrho_l \chi CdV. \tag{1.294}$$

Combustion stream momentum occupying finite volume V at time t, will be:

$$\theta = \int_V \varrho(1-\chi)WdV + \int_V \varrho_l \chi CdV. \tag{1.295}$$

During integration it must be kept in mind that the integrand parameters may depend on x, r and ϕ. Since they also depend on time, at time t + dt the combustion stream momentum will be:

$$\theta + d\theta = \int_V \varrho(1-\chi)WdV + \int_V \varrho_l \chi CdV + d\int_V \varrho(1-\chi)WdV + d\int_V \varrho_l \chi CdV. \tag{1.296}$$

According to the law of the conservation of momentum, we must equalize the right sides of equations (1.295) and (1.296). The change in momentum comprises:

$$d\theta = d\int_V \varrho(1-\chi)WdV + d\int_V \varrho_l \chi CdV = 0. \tag{1.297}$$

The law of the conservation of momentum may be written in another form, if we examine two points in time: t_1 and t_2, which differ from each other by a finite time period. Denoting the parameters of the integrands by their corresponding indexes, we may write the equation of the conservation of momentum insofar as average values are concerned in the following manner:

$$\varrho_1(1-\chi_1)W_1 + \varrho_{l1}\chi_1 C_1 = \varrho_2(1-\chi_2)W_2 + \varrho_{l2}\chi_2 C_2. \tag{1.298}$$

For a finite volume we have

$$\int_V \rho_1(1-\chi_1)W_1dV+\int_V \rho_{l1}\chi_1C_1dV=\int_V \rho_2(1-\chi_2)W_2dV+\int_V \rho_{l2}\chi_2C_2dV. \quad (1.299)$$

If the liquid is incompressible, then

$$\rho_{l1}=\rho_{l2}=\text{const.}$$

If one or another parameter does not depend on the coordinates, then, just as for ρ_l in the case of an incompressible fluid, it must be taken out of the integral sign.

It is convenient to employ the equation of the conservation of momentum for a determination of gas flow velocity, corresponding to the termination of the propellant burning process. If ρ, χ, ρ_l, W, C, and ρ_0 are known, then the gas flow velocity U is determined in accordance with the following equations.

If the calculation is carried out with average values, then

$$\rho(1-\chi)W+\rho_l\chi C\approx\rho_0U. \quad (1.300)$$

Therefore,

$$U\approx\frac{\rho}{\rho_0}(1-\chi)W+\frac{\rho_l}{\rho_0}\chi C. \quad (1.301)$$

The equation (1.301) permits us to determine U for any point of the chamber volume according to the parameters calculated for any instant in time.

For finite volume

$$\int_V \rho(1-\chi)WdV+\int_V \rho_l\chi CdV=\int_V \rho_0UdV. \quad (1.302)$$

2. The Determination of Heat Released during Flow Mixing

During the mixing of moving flows, the law of the conservation of mechanical energy is not fulfilled. Let the mechanical energy of the combustion stream be presented only in the form of kinetic energy. The equation of the conservation of energy insofar as average values are concerned assumes the following form:

$$\rho(1-\chi)W^2dV+\rho_l\chi C^2dV=\rho_0U^2dV+2dQ_R. \quad (1.303)$$

In an examination of a finite volume we have:

$$\int_V \varrho(1-\chi)W^2dV + \int_V \varrho_l \chi C^2 dV = \int_V \varrho_0 U^2 dV + 2Q_R. \qquad (1.303a)$$

By inserting the velocity value from equation (1.301) into (1.303), we obtain:

$$dQ_R = \frac{1}{2}\left[\varrho(1-\chi)W^2 + \varrho_l \chi C^2 - \varrho_0\left(\frac{\varrho}{\varrho_0}(1-\chi)W + \frac{\varrho_l}{\varrho_0}\chi C\right)^2\right]dV. \qquad (1.304)$$

For a finite volume according to equations (1.301) and (1.303a), we obtain:

$$Q_R = \frac{1}{2}\int_V \left[\varrho(1-\chi)W^2 + \varrho_l \chi C^2 - \varrho_0\left(\frac{\varrho}{\varrho_0}(1-\chi)W + \frac{\varrho_l}{\varrho_0}\chi C\right)^2\right]dV. \qquad (1.305)$$

During the burning of gaseous propellant, ρ_l and ρ_0 must be replaced by the value ρ. Then equation (1.305) assumes the following form:

$$Q_R = \frac{1}{2}\varrho\int_V (1-\chi)\chi(W-C)^2 dV. \qquad (1.306)$$

Equation (1.306) reveals that during the initial period of combustion, when $\chi = 1$, just as at the end, when $\chi = 0$, dissipation of mechanical energy is not observed. The greatest heating effects occur when $\chi = 0.5$. If W and C are constants with respect to the volume, or are averages, then

$$Q_{R\,max} = 0{,}125\,\varrho(W-C)^2 V. \qquad (1.306a)$$

The change in combustion stream momentum with respect to time comprises:

$$\dot{\theta} = \frac{d}{dt}\int_V \varrho(1-\chi)W dV + \frac{d}{dt}\int_V \varrho_l \chi C dV. \qquad (1.307)$$

When only gas is present

$$\dot{\theta} = \frac{d}{dt}\int_V \varrho W dV = \frac{d}{dt}\int_m W dm. \qquad (1.308)$$

3. Reactive Force and Thrust Formulas

The leading vector of all the external mass forces applied to the combustion stream, which are included within the combustion chamber in the nozzle, comprise:

$$\bar{P}=\int_V \bar{X}\varrho dV, \qquad (1.309)$$

where $\bar{X}$ is the external force per unit of mass. The leading vector of surface forces applied to elementary flow particles distributed along the surface, and bounded by the walls of the combustion chamber and the nozzle, is determined by the integral [33, 34]:

$$\int_F \bar{p}dF=-\int_F p\bar{n}dF=-\int_V \text{grad}\, pd V, \qquad (1.310)$$

where p is the pressure modulus;

$\bar{n}$ is the unit vector of the external normal to the surface F.

The surface F, which limits volume V, may be presented in the form of the sum

$$F = F_c + F_a, \qquad (1.311)$$

where F_c is the combustion chamber and nozzle surface;

F_a is the surface of the exit area.

In place of (1.310) we may now write:

$$\int_F \bar{p}dF=\int_{F_c} \bar{p}dF+\int_{F_a} \bar{p}dF. \qquad (1.312)$$

The first integral in the right side of equation (1.312) represents a vector of pressure forces applied to stream particles distributed along the surface and bounded by the walls of the combustion chamber and by the nozzle.

The vector which is equal in value but opposite in direction represents reactive force:

$$\bar{R}=-\int_{F_c} \bar{p}dF. \qquad (1.313)$$

The equation (1.313) clarifies the meaning of reactive force. The second integral in the right side of equation (1.312) represents the vector of pressure forces on stream particles, located on the surface of the nozzle exit area. Now, following the theorem concerning the momentum change, we have for the combustion stream:

$$\frac{d}{dt}\int_V \varrho(1-\chi)WdV+\frac{d}{dt}\int_V \varrho\chi CdV=\left[\int_V \bar{X}\varrho dV+\int_{F_c}\bar{p}dF+\int_{F_a}\bar{p}dF\right]. \qquad (1.314)$$

Employing equations (1.313) and (1.314) we find the equation for the determination of the reactive force vector in the form:

$$-\bar{R}=\frac{d}{dt}\int_V \varrho(1-\chi)\bar{W}dV+\frac{d}{dt}\int_V \rho_l\chi\bar{C}dV-\int_V \bar{X}\varrho dV-\int_{F_a} \bar{p}dF. \tag{1.315}$$

If we do not take into consideration the effects of mass forces and consider that only gas is contained in the external chamber cavity, then the reactive force vector is expressed as follows:

$$-\bar{R}=\frac{d}{dt}\int_V \varrho\bar{W}dV-\int_{F_a} \bar{p}_a dF. \tag{1.316}$$

During an examination of the forces acting on the combustion chamber, the law of momentum change must be applied to two streams, which wash the sides of the chamber. The first of these -- the combustion stream -- we have already examined. The second is the gaseous medium which washes the chamber from outside. The surface force vector, applied to the surface of volume V, is:

$$\int_F \bar{p}_{ex}dF=\int_{F_c} \bar{p}_{ex}dF+\int_{F_a} \bar{p}_{ex}dF=0, \tag{1.317}$$

where p_{ex} is environmental pressure. The first integral in the right side of equation (1.317) represents an external pressure force vector, applied to the outer contour of the chamber and nozzle. The second integral reflects the effect of external pressure forces on the surface F_a, which limits the gas discharge flow.

Let us specify that:

$$\bar{P}_{ex}=\int_{F_c} \bar{p}_{ex}dF. \tag{1.318}$$

The direction of vector $\bar{P}_{ex}$ is opposite to that of the reactive force vector; therefore, the thrust

$$\bar{P}=\bar{R}+\bar{P}_{ex}. \tag{1.319}$$

Taking environmental effects into consideration, we obtain:

$$P=\bar{R}+\bar{P}_{ex}=-\frac{d}{dt}\int_V \varrho(1-\chi)\bar{W}dV-\frac{d}{dt}\int_V \varrho_l\chi\bar{C}dV+\int_V \bar{X}\varrho dV+\int_{F_a}\bar{p}_a dF-\int_{F_a}\bar{p}_{ex}dF. \quad (1.320)$$

The thrust vector equation (1.320) describes the interrelation of the combustion stream and the environment with the combustion chamber.

The time derivative with respect to the first integral of the right side of equation (1.320) characterizes the momentum change of the gaseous products. The second term considers a fluid momentum change. The third term reflects the effect of external forces, and the following two reflect the effect of forces from both the discharge flow and from the environment.

With both a conical and a shaped nozzle, the lines of flow intersecting the surface F_a are not parallel with the axis of the chamber in the nozzle. Uniform flow is observed only in the simplest cases.

If the liquid phase in the chamber is not considered, then we must write in place of the first term of the right side of equation (1.320):

$$\dot{\theta}=\frac{d}{dt}\int_V \varrho W dV. \quad (1.321)$$

This equation may be rewritten as follows:

$$\frac{d}{dt}\int_V \varrho W dV=\int_V \frac{d}{dt}\varrho W dV. \quad (1.322)$$

According to the law of conservation of mass,

$$\frac{d}{dt}(\varrho dV)=0;$$
$$\int W\frac{d}{dt}(\varrho dV)=0, \quad (1.323)$$

therefore,

$$\frac{d}{dt}\int_V \varrho W dV=\int_V \varrho\frac{d}{dt}W dV. \quad (1.324)$$

It is convenient to write the integral (1.321) in the following form:

$$\frac{d}{dt}\int_V \varrho W dV=\frac{\partial}{\partial t}\int_V \varrho W dV+\int_F \varrho W W_n dF. \quad (1.325)$$

The first integral of the right side of equation (1.325) may be rewritten as follows:

$$\frac{\partial}{\partial t}\int\limits_V \varrho W dV = \frac{\partial}{\partial t}\int\limits_0^{x_a}\left[\int\limits_{F(x)} \varrho W dF\right]dx = \frac{\partial}{\partial t}\int\limits_0^{x_a} G(t, x)\, dx, \qquad (1.326)$$

where x_a is the distance from the head to the exit area along the chamber axis.

If the parameter field is stationary, then

$$\frac{\partial}{\partial t}\int\limits_V \varrho W dV = 0. \qquad (1.327)$$

In this connection,

$$\frac{d}{dt}\int\limits_V \varrho W dV = \int\limits_F \varrho W W_n dF = G(t, x_a) W, \qquad (1.328)$$

where $G(t, x_a)$ is the gas flow through the nozzle exit area.

Thus if we do not consider the liquid phase and disregard the effect of external forces, then for the established condition in a case of substitution

$$\int\limits_{F_a} \vec{p}_a dF - \int\limits_{F_a} \vec{p}_{ex} dF = F_a (p_a - p_{ex})$$

the equation (1.320) assumes the following form:

$$P = G(t, x_a) W + F_a (p_a - p_{ex}). \qquad (1.329)$$

§ 8. Approximation Equations Characterizing Propellant Combustion

The preparation of propellant constituents for combustion begins even before their entrance into the internal cavity of the chamber, within the injector channels where the constituents are pre-warmed, and where their delivery and rotary velocities are increased. If centrifugal injectors are used, vaporization will be observed in the central part of the internal cavity of the chamber. The liquid enters the chamber in the form of conical sheets, which quickly collapse, form relatively large drops, are deformed and divided into smaller drops. Atomization is accompanied by the intermixing of propellant constituents, during which time the intrusion of smaller propellant constituent masses into larger masses is observed, drop mixing and partitioning occurs, all accompanied by a change in the velocity and direction of motion. The liquid entering the combustion chamber is subject to the action of heat fluxes. The heating is accomplished through radiant energy and convection fluxes. The latter results from a difference in the average velocities of liquid and gas, from the presence of large scale vortexes which accompany the inverse motion of gas masses in the head area (the counter flux phenomenon), and from gas flow turbulence in the immediate vicinity of the moving drops.

As a result of the intensive action of heat fluxes, the thin surface layer of a drop is quickly pre-warmed to the burning point, but the temperature of the central part of the drop does not increase at the same rate. Further, the heat flux energy is expended on fluid vaporization from the surface of the drop and on heating the entire mass of the drop to the boiling point. The vaporization products form a distinctive cloud near the drop, completely surrounding it. The chemical reaction between the vaporization products of the propellant constituents, i.e., combustion, occurs along the surface of the cloud. The combustion products formed and to a certain degree the vaporization products diffuse within the basic gas volume. If the chamber pressure exceeds a critical value, then mass transfer depends not on vaporization, but on the diffused liquid within the gas.

When a cloud is present, the nature of the heating of the drop is more complex. The flow of radiant energy, penetrating the cloud, reaches the surface of the drop and heats it. Convection fluxes transmit heat to the external surface of the cloud, which as a result of thermal conduction reaches the surface of the drop. In engineering calculations, which are directed toward conventional design, it is usually considered that convection heat transfer is caused by direct contact between combustion products and the drop surface.

In a combustion calculation it is necessary to consider non-uniform atomization. It depends on propellant constituent quality, on the condition of flow from separate injectors, and on the frequency and nature of the distribution of injectors in the head. Non-uniformity in the atomization of a single injector is characterized by a function of drop distribution with respect to drop size (by radius). Non-uniform propellant atomization by all head injectors is characterized by similar distribution functions for each of the propellant constituents, by a distribution function of each of the constituents by radius and in direction W_ϕ of the upper section of the chamber, taken at a certain distance from the head and as a result, by the distribution (also by radius and in direction W_ϕ) of the reactant ratio. Due to drop partitioning, the distribution pattern changes along the length of the chamber. In addition to non-uniform atomization, non-uniform gas formation is also examined. It is a consequence of non-uniformity in the course of the processes of atomization, preparation of propellant for burning, and combustion.

1. Heating and Vaporization of Separate Drops

Initial heating of the drop as a result of thermal conductivity from the cloud may be described by the well known equation of thermal conductivity, which in spherical coordinates assumes the following form [35]:

$$\frac{\partial}{\partial t}[r, T(r, t)] = a\frac{\partial^2}{\partial r^2}[rT(r, t)], \qquad (1.330)$$

where a is the thermal conductivity coefficient, and

$$a = \frac{\lambda}{c\varrho}.$$

The boundary conditions are written as follows:

$$T(r, 0) = T_{ex};$$
$$\frac{\partial}{\partial r}T(r, t) - \frac{q}{r} = 0;$$
$$\frac{\partial}{\partial r}T(r, 0) = 0;$$
$$T(0, t) \neq 0.$$

The solution and the analysis of equation (1.330) are well known [35].

In the study of the mass transfer through the cloud, the equation of the conservation of mass for the vaporizing drop is included. For each of the propellant constituents at a given point in time, the following relationship is true:

$$\dot{m}_l + \dot{m}_{cl} + \dot{m}_D = 0, \tag{1.331}$$

where m_l is the instantaneous value of the mass of liquid in a drop;

m_{cl} is the mass of the vaporization products in the cloud;

m_D is the mass of the vaporization products, diffused from the cloud into the surrounding medium.

The mass of the drop is

$$m_l = \frac{4}{3}\pi\rho_l r^3, \tag{1.332}$$

where r is the characteristic size -- the radius of the drop, the latter having an approximately spherical shape.

The derivative of this expression has the following form:

$$\dot{m}_l = 4\pi\rho_l r^2\dot{r}. \tag{1.333}$$

The mass of the vaporization products in the cloud is

$$m_{cl} = \frac{4}{3}\pi\rho_{cl}(R^3 - r^3), \tag{1.334}$$

where R is the characteristic size of the cloud;

ρ_{cl} is cloud density.

The derivative is

$$\dot{m}_{cl} = 4\pi\rho_{cl}(R^2\dot{R} - r^2\dot{r}). \tag{1.335}$$

According to the Fick law, the derivative of the diffused mass is

$$\dot{m}_D = -F_D D\nabla C, \tag{1.336}$$

or

$$\dot{m}_D = -4\pi R^2 D\nabla C, \tag{1.337}$$

where F_D is the cloud surface;

D is the diffusion coefficient;

C is the concentration of the diffused mass.

Substituting (1.333), (1.335) and (1.337) in (1.331), and after transformation we have:

$$(\varrho_l-\varrho_{c1}) r^2 \dot{r} + \varrho_{c1} R^2 \dot{R} - R^2 D \nabla C = 0. \qquad (1.338)$$

Since a new variable has been introduced into the equation -- the concentration, it is necessary to include the diffusion equation [32]:

$$\frac{\partial C}{\partial t} = -D\nabla^2 C. \qquad (1.339)$$

Vaporization and further pre-heating of the drops may be described with the aid of equations of the conservation of energy. We shall direct our attention to the conventionally accepted plan for convection heating. The heat element transmitted to the drop by the combustion products, is:

$$dQ_a = \alpha (T_c - T_s) F dt. \qquad (1.340)$$

The heating surface for a spherical drop is

$$F = 4\pi r^2. \qquad (1.341)$$

Under conditions of complex turbulent flow, which is characteristic for contemporary combustion chambers, the shape of the drops is variable. Sometimes they have a lenticular shape, they move forward with extensive lateral movement, they are deformed, change their shape, and are divided.

In order to determine the relative heat transfer coefficient from the combustion products to the drop, we shall employ the usual formula:

$$Nu = a' Re^n Pr^m. \qquad (1.342)$$

Now the equation (1.340) is written as follows:

$$dQ_a = 4\pi a Z (C - W)^n r^{1+n} dt, \qquad (1.343)$$

where

$$Z = \frac{\lambda}{\nu^n} Pr^m (T_c - T_s). \qquad (1.344)$$

Some authors [3] in place of (1.342) recommend the formula

$$Nu = B + a'' Re^n Pr^m,$$

where parameter B characterizes the heat transfer, independent of velocity difference (C - W).

The heat expended in pre-heating of the mass element from a certain initial value of temperature T_0 to temperature T_s is [35]:

$$dQ_c = c_l (T_s - T_0) 4\pi\varrho_l r^2 dr. \tag{1.345}$$

The following amount of heat will be consumed in vaporization of a mass element from the surface of the drop:

$$dQ_r = r^* 4\pi\varrho_l r^2 dr, \tag{1.346}$$

where r^* is the latent heat of vaporization. Employing the law of the conservation of energy, we find that:

$$aZ(C-W)^n r^{1+n} dt = -[c_l (T_s - T_0) + r^*]\varrho_l r^2 dr. \tag{1.347}$$

Taking the mean values of the parameters by the sign of the integral, after integration we arrive at an equation which characterizes the vaporization of an individual drop of the liquid component:

$$r = (r_0^{2-n} - \Psi t)^{\frac{1}{2-n}}, \tag{1.348}$$

where

$$\Psi = \frac{(2-n)aZ(C-W)^n}{\varrho_l [c_l (T_s - T_0) + r^*]}. \tag{1.349}$$

Under conditions when the determining factor is not vaporization, but diffusion, equation (1.349) cannot be employed. The time for complete vaporization of a drop is

$$t_0 = \frac{r_0^{2-n}}{\Psi}. \tag{1.350}$$

If the heat transfer does not depend on the Reynolds number, then n = 0; thus:

$$r = (r_0^2 - \Psi t)^{0,5}. \tag{1.351}$$

When forced convection is involved, if we adcept the value n = 0.5, recommended by several authors [12, 13], then we obtain:

$$r=(r_0^{3/2}-\Psi t)^{2/3}. \tag{1.352}$$

In this connection

$$m_l=\frac{4}{3}\pi\varrho_l(r_0^{3/2}-\Psi t)^2. \tag{1.353}$$

During the determination of the heat flux from the combustion products to a drop, it was accepted that the relative velocity C - W = Const. In fact, the velocity changes with time, i.e., (C - W) = f(t).

According to Newton's second law

$$m_l\dot{C} = P_w, \tag{1.354}$$

where P_w is ballistic force, and [13]:

$$P_w=c_x\pi r^2\varrho_0\frac{(C-W)^2}{2}. \tag{1.355}$$

The drag coefficient satisfies

$$c_x = f(Re) \tag{1.356}$$

and is determined experimentally.

Directing our attention to the mean value c_x, we obtain

$$\frac{dC}{(C-W)^2}=\frac{3}{8}\frac{\varrho_0}{\varrho_l}c_x\frac{dt}{(r_0^{2-n}-\Psi t)^{\frac{1}{2-n}}}. \tag{1.357}$$

If for part of the vaporization of a drop W = const, then

$$C=W+\left[\frac{1}{C_0-W}+\frac{3}{8}\frac{\varrho_0}{\varrho_l}\frac{c_x}{\Psi}\frac{2-n}{1-n}\times\left|r_0^{1-n}-(r_0^{2-n}-\Psi t)^{\frac{1-n}{2-n}}\right|\right]^{-1}. \tag{1.358}$$

Equation (1.358) reveals that $C = C_0$, if $t_0 = 0$. The greater t, the closer C is to W. However, for a determination of the finite-time value of C, it is necessary to solve the system of equations examined.

2. Equations Characterizing the Burn-Up of Propellant Constituents

A characteristic of propellant burn-up is the function

$$\varphi(t)=1-\frac{n_{fr}m_l}{n_0m_0} \tag{1.359}$$

where m_0 is the initial mass of the drop;

m_l is the instantaneous value of the mass of the drop;

n_0 is the relative (per second) quantity of drops possessing mass m_0;

n_{fr} is the instantaneous value of the drops with mass m_l, calculated upon consideration of their partitioning effect.

It is apparent that

$$m_l = \frac{4}{3}\pi \varrho_l r^3. \qquad (1.360)$$

The equation of the burn-up curve of a unitary propellant, when the masses of all drops at the initial point in time are equal, and without considering partitioning is written as follows:

$$\varphi(t) = 1 - \frac{(r_0^{2-n} - \Psi t)^{\frac{3}{2-n}}}{r_0^3}. \qquad (1.361)$$

For a dual-component propellant, under the same assumptions, we obtain

$$\varphi(t) = 1 - \frac{\varrho_{1l}(r_{10}^{2-n_1} - \Psi_1 t)^{\frac{3}{2-n_1}} + \varrho_{2l}(r_{20}^{2-n_2} - \Psi_2 t)^{\frac{3}{2-n_2}}}{\varrho_{1l} r_{10}^3 - \varrho_{2l} r_{20}^3}. \qquad (1.362)$$

The reactant ratio of propellant in the gaseous phase may change with time, therefore,

$$K(t) = \frac{m_{10} - m_{1l}}{m_{20} - m_{2l}}, \qquad (1.363)$$

or

$$K(t) = \frac{\varrho_{1l}}{\varrho_{2l}} \frac{\left[r_{10}^3 - (r_{10}^{2-n_1} - \Psi_1 t)^{\frac{3}{2-n_1}}\right]}{\left[r_{20}^3 - (r_{20}^{2-n_2} - \Psi_2 t)^{\frac{3}{2-n_2}}\right]}. \qquad (1.364)$$

As a result of the investigation of the atomization of propellant constituents, we have shown several characteristic drop dimensions. If the relative number of drops possessing mass m_i is designated by the symbol n_i, then the mass flow at the initial point in time will be:

$$G = \sum n_{i0} m_{i0}. \qquad (1.365)$$

At any time

$$G=\sum n_i m_i. \tag{1.366}$$

or

$$G=\frac{4}{3}\pi\varrho_l\sum n_i\left(r_{i0}^{2-n}-\Psi t\right)^{\frac{3}{2-n}}. \tag{1.367}$$

The equation of the burn-up curve for a unitary propellant is written as follows:

$$\varphi(t)=1-\frac{\sum n_i\left(r_{i0}^{2-n_i}-\Psi_i t\right)^{\frac{3}{2-n_i}}}{\sum n_{i0}m_{i0}}. \tag{1.368}$$

For a dual-component propellant we have:

$$\varphi(t)=\frac{\varrho_{l1}\sum n_{i1}\left(r_{10}^{2-n_1}-\Psi_1 t\right)^{\frac{3}{2-n_1}}+\varrho_{l2}\sum n_{i2}\left(r_{20}^{2-n_2}-\Psi_2 t\right)^{\frac{3}{2-n_2}}}{\varrho_{l1}\sum n_{i10}r_{i10}^3+\varrho_{l2}\sum n_{i20}r_{i20}^3}. \tag{1.369}$$

The expression for the determination of the instantaneous value of the propellant reactant ratio assumes the following form:

$$K(t)=\frac{\varrho_{l1}}{\varrho_{l2}}\frac{\sum n_{i10}r_{i10}^3-\sum n_{i1}\left(r_{10}^{2-n_1}-\Psi_1 t\right)^{\frac{3}{2-n_1}}}{\sum n_{i20}r_{i20}^3-\sum n_{i2}\left(r_{20}^{2-n_2}-\Psi_2 t\right)^{\frac{3}{2-n_2}}}. \tag{1.370}$$

If the size of the drop of the liquid propellant constituent changes from m_{min} to m_{max}, then the number of drops

$$n_0=\int_{m_{0\,min}}^{m_{0\,max}} f(m_0)\,dm_0. \tag{1.371}$$

For any continuous range of distribution, even when drops possessing mass $0 < m_0 < m_{min}$ and $m_{max} < m < \infty$ are not present, the number of drops may be determined by the integral

$$n_0\approx\int_0^\infty f(m_0)\,dm_0. \tag{1.372}$$

The amount of propellant constituent in a liquid state at an arbitrary point in time is:

$$G_l\approx\int_0^\infty f(m_0)\,m(t)\,dm_0. \tag{1.373}$$

Directing our attention to the vaporization principle accepted above, we obtain:

$$Q_l \approx \int_0^\infty f(m_0)\frac{4}{3}\pi\varrho_l(r_0^{2-n}-\Psi t)^{3/2-n}dm_0. \tag{1.374}$$

For a unitary propellant, the equation of the burn-up curve assumes the following form:

$$\varphi(t)=1-\frac{\int_0^\infty f(m_0)\left(r_0^{2-n}-\Psi t\right)^{\frac{3}{2-n}}dm_0}{\int_0^\infty f(m_0)\,m_0dm_0}. \tag{1.375}$$

For a dual-component propellant:

$$\varphi(t)=1-\left[\varrho_{l1}\int_0^\infty f_1(m_{10})(r_{10}^{2-n_1}-\Psi_1 t)^{\frac{3}{2-n_1}}dm_{10}+\right.$$

$$\left.+\varrho_{l2}\int_0^\infty f_2(m_{20})(r_{20}^{2-n_1}-\Psi_2 t)^{\frac{3}{2-n_1}}dm_{20}\right]\cdot\left[\varrho_{l1}\times\right.$$

$$\left.\times\int_0^\infty f_1(m_{10})\,m_{10}dm_{10}+\varrho_{l2}\int_0^\infty f_2(m_{20})\,m_{20}dm_{20}\right]^{-1}. \tag{1.376}$$

The expression for the determination of the instantaneous value of the propellant reactant ratio is written in the following manner:

$$K(t)=\frac{\varrho_{l1}}{\varrho_{l2}}\cdot\frac{\int_0^\infty f_1(m_{10})\,m_{10}dm_{10}-\int_0^\infty f_1(m_{10})\left(r_{10}^{2-n_1}-\Psi_1 t\right)^{\frac{3}{2-n_1}}dm_{10}}{\int_0^\infty f_2(m_{20})\,m_{20}dm_{20}-\int_0^\infty f_2(m_{20})\left(r_{20}^{2-n_1}-\Psi_2 t\right)^{\frac{3}{2-n_1}}dm_{20}}. \tag{1.377}$$

It must be noted that in plotting $\phi(t)$ and $K(t)$, it is necessary to employ well developed and widely published methods, and to consider the distribution of propellant constituents along the section of the chamber.

§ 9. The Wave Equation

We are not always successful in solving a system consisting of several complex equations. Therefore it is necessary to direct our attention to simplified equations. As an example we shall examine the wave equation. This equation usually links the following variables: sound pressure p, density ρ

and velocity potential ψ, the latter being differentiated to find the velocity components:

$$W_x = -\frac{\partial\psi}{\partial x}; \quad (1.378)$$

$$W_r = -\frac{\partial\psi}{\partial r}; \quad (1.379)$$

$$W_\varphi = -\frac{\partial\psi}{r\partial\varphi}. \quad (1.380)$$

In contrast to the cylindrical system accepted previously, the wave equation is written in the Cartesian system of coordinates. In order to determine the five variables p, ρ, W_x, W_r and W_ϕ, we employ the five equations: the equation of state, three equations of Newton's second law, and the equation of continuity.

The equation of state is often employed in the following form [27, 34]:

$$\frac{\partial p}{\partial \varrho} = c^2. \quad (1.381)$$

The equation of Newton's second law is written without considering the liquid phase, the effects of external forces and viscous friction forces. In this case we obtain:

$$\varrho \dot{W}_x = -\frac{\partial p}{\partial x}; \quad (1.382)$$

$$\varrho \dot{W}_y = -\frac{\partial p}{\partial y}; \quad (1.383)$$

$$\varrho \dot{W}_z = -\frac{\partial p}{\partial z}. \quad (1.384)$$

Further, stipulating the presence of small amplitudes only, we replace ρ with the value ρ_0 = const. The equation of Newton's second law contains the local accelerations

$$\frac{\partial W_i}{\partial t} = \dot{W}_i - \overline{W}_i \operatorname{grad} W_i \quad (1.385)$$

and the transfer acceleration

$$\overline{W}_i \operatorname{grad} W_i = \frac{\partial W_i}{\partial x} W_x + \frac{\partial W_i}{\partial y} W_y + \frac{\partial W_i}{\partial z} W_z. \quad (1.386)$$

Local acceleration produces an external field of forces which varies with time, and the transfer acceleration is due to the fact that in a time interval dt the gas element changes its position in the space the velocity field is characterized differently. We shall further consider that the transfer acceleration may be disregarded. Therefore we have [47]:

$$\varrho_0 \frac{\partial W_x}{\partial t} + \frac{\partial p}{\partial x} = 0; \tag{1.387}$$

$$\varrho_0 \frac{\partial W_y}{\partial t} + \frac{\partial p}{\partial y} = 0; \tag{1.388}$$

$$\varrho_0 \frac{\partial W_z}{\partial t} + \frac{\partial p}{\partial z} = 0. \tag{1.389}$$

We must point out the fact that in equations (1.387), (1.388) and (1.389) a series of very important factors have not been considered, which may, under operating conditions of liquid-fuel rocket engines, cause important changes in the nature of the oscillatory processes.

The equation of continuity is written as follows:

$$\frac{1}{\varrho_0} \frac{\partial \varrho}{\partial t} + \operatorname{div} \overline{W} = 0. \tag{1.390}$$

It is evident that in the same manner equation (1.390) represents the equation of the conservation of mass (the equation of continuity), written in the same simplified form. The system of reference equations has the following form:

$$\left.\begin{aligned} &\frac{\partial p}{\partial \varrho} - c^2 = 0; \\ &\varrho_0 \frac{\partial W_x}{\partial t} + \frac{\partial p}{\partial x} = 0; \\ &\varrho_0 \frac{\partial W_y}{\partial t} + \frac{\partial p}{\partial y} = 0; \\ &\varrho_0 \frac{\partial W_z}{\partial t} + \frac{\partial p}{\partial z} = 0; \\ &\frac{1}{\varrho_0} \frac{\partial \varrho}{\partial t} + \operatorname{div} \overline{W} = 0. \end{aligned}\right\} \tag{1.391}$$

Replacing the pressure in three of Newton's equations with the density through the use of the first equation of the system (1.391), we obtain the new system:

$$\left.\begin{aligned}\varrho_0\frac{\partial W_x}{\partial t}+c^2\frac{\partial\varrho}{\partial x}=0;\\ \varrho_0\frac{\partial W_y}{\partial t}+c^2\frac{\partial\varrho}{\partial y}=0;\\ \varrho_0\frac{\partial W_z}{\partial t}+c^2\frac{\partial\varrho}{\partial z}=0;\\ \frac{\partial\varrho}{\partial t}+\varrho_0\operatorname{div}\overline{W}=0.\end{aligned}\right\} \quad (1.392)$$

Let us multiply the right and left sides of the equations of the system (1.392) by the operators in the following manner: the first equation, by the operator $\partial/\partial x$, the second equation, by the operator $\partial/\partial y$, the third equation, by the operator $\partial/\partial z$, and the fourth, by the operator $\partial/\partial t$. Now we obtain the system:

$$\left.\begin{aligned}\varrho_0\frac{\partial^2 W_x}{\partial t\partial x}+c^2\frac{\partial^2\varrho}{\partial x^2}=0;\\ \varrho_0\frac{\partial^2 W_y}{\partial t\partial y}+c^2\frac{\partial^2\varrho}{\partial y^2}=0;\\ \varrho_0\frac{\partial^2 W_z}{\partial t\partial z}+c^2\frac{\partial^2\varrho}{\partial z^2}=0;\\ \frac{\partial^2\varrho}{\partial t^2}+\varrho_0\left(\frac{\partial^2 W_x}{\partial t\partial x}+\frac{\partial^2 W_y}{\partial t\partial y}+\frac{\partial^2 W_z}{\partial t\partial z}\right)=0.\end{aligned}\right\} \quad (1.393)$$

Combining term by term the first three equations and simplifying with the aid of the fourth term

$$\varrho_0\left(\frac{\partial^2 W_x}{\partial t\partial x}+\frac{\partial^2 W_y}{\partial t\partial y}+\frac{\partial^2 W_z}{\partial t\partial z}\right)=\varrho_0\frac{\partial}{\partial t}(\operatorname{div}\overline{W}), \quad (1.394)$$

we obtain the wave equation for the density:

$$\frac{1}{c^2}\frac{\partial^2\varrho}{\partial t^2}-\nabla^2\varrho=0, \quad (1.395)$$

where

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}. \quad (1.396)$$

The wave equation for the sound pressure has the following form:

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}-\nabla^2 p=0. \qquad (1.397)$$

The wave equation for the velocity potential is

$$\frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2}-\nabla^2 \psi=0. \qquad (1.398)$$

In cylindrical coordinates, the wave equation for the velocity potential is written as follows:

$$\frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2}-\frac{\partial^2 \psi}{\partial x^2}-\frac{\partial^2 \psi}{\partial r^2}-\frac{1}{r}\frac{\partial \psi}{\partial r}-\frac{1}{r^2}\frac{\partial^2 \psi}{\partial \varphi^2}=0. \qquad (1.399)$$

The presence of a large quantity of literature devoted to the analysis and to the solution of wave equations frees the author from the necessity of setting forth well known methods. In the paragraph under consideration two extreme cases are examined: a system of nine equations and the wave equation, obtained by means of elementary transformations of five equations written in simplified form.

Practice makes the most diverse demands on the researcher, including the necessity for formulating an effective system of equations. Differential equations of the system describe the relationship between time and spatial parameter changes of the intrachamber process.

It is necessary to know the boundary conditions in order to solve the differential equation system, i.e., to find the parameter values at any moment in time. The boundary conditions represent a combination of initial and limiting conditions. Parameter distributions within a body at an initial point in time are called initial conditions. For the example under investigation, the parameter distribution condition in a general form is written as follows:

$$\left.\begin{aligned} p(x, r, \varphi, 0)&=f_1(x, r, \varphi);\\ T(x, r, \varphi, 0)&=f_2(x, r, \varphi);\\ \varrho(x, r, \varphi, 0)&=f_3(x, r, \varphi);\\ W(x, r, \varphi, 0)&=f_4(x, r, \varphi);\\ C(x, r, \varphi, 0)&=f_5(x, r, \varphi).\end{aligned}\right\} \qquad (1.400)$$

The conditions in (1.400) are set according to the results of the processing of experimental data, on the basis of preliminary calculations, or on the basis of other considerations. It is necessary to note that the determination of initial conditions is occasionally linked with considerable difficulties. Two methods of stating the initial conditions have achieved widespread acceptance. In those cases when it is possible, uniform distribution of parameters are given at the initial instant in time, for example:

$$p(x, r, \phi, 0) = p_0 = \text{const}. \tag{1.401}$$

If the solution to the static problem is available, then the initial conditions are given using the static condition. Let us assume that switching off of the engine is being examined. At an initial instant in time, corresponding to chamber operation during the operating function, the nature of the parameter distribution along the length of the chamber (along x) is known from the solution to the algebraic equations.

Further, we know the geometry of the combustion chamber, and the conditions of the interaction between the parameters of the intrachamber process and those of the chamber enclosure, including the head of the nozzle, are set. In other words, conditions on the boundary, or boundary conditions, are set. These conditions are set in the form of the distribution of parameters or their derivatives along the internal surface as a function of time:

$$\left.\begin{aligned} p(t) &= \Phi_1(t); \\ T(t) &= \Phi_2(t); \\ \varrho(t) &= \Phi_3(t); \\ W(t) &= \Phi_4(t); \\ C(t) &= \Phi_5(t). \end{aligned}\right\} \tag{1.402}$$

The system of differential equations, under a condition when the geometry of the chamber is known, and when initial and boundary conditions are given, is solved to its conclusion, i.e., the function of the distribution of parameters along the chamber volume for any instant in time is determined [47].

CHAPTER II

THE DYNAMICS OF THE FEED SYSTEM

§ 1. The External Characteristics of the Fead System

Just as in Chapter I, which was devoted to dynamic processes occurring within the combustion chamber, the study of the internal properties of sub-assemblies of the feed system more completely characterizing their specific features, is carried out by means of an investigation of the change in their basic parameters with respect to time.

An external characteristic of the feed system represents a record of the change in basic parameters with respect to time. The external characteristic is obtained by means of oscillographs of the outputs of measuring devices during the testing process.

1. Types of Feed Systems

Two basic types of feed systems are known. In literature they have several different designations. The first type is called the system with loaded tanks, or the gas pressure system (which is not always accurate), or the pressurized propellant feed system. The second type is called the system with unloaded tanks, or the turbopump assembly (TNA) system, since in contemporary engines other power devices are not utilized.

In the system with loaded tanks (the pressurized system), propellant constituents are expelled from the tanks into the chamber by the pressure of gases which enter the upper tank cavity from an accumulator. Accumulator types are gas, powder, or liquid, sometimes called hot. Pressure in the propellant constituent tanks will be higher than in the chamber. Therefore the thickness of the walls and the weight of the tanks with such a feed system are usually quite significant. The gas accumulator, which represents a high pressure cylinder, is heavier than the powder chamber with a load. The liquid accumulator is lighter than the powder accumulator, but is significantly more complex in construction. However, the weight of the entire system is determined not by the accumulators, but by the tanks.

An investigation of a feed system with loaded tanks revealed that their relative weight is practically independent of engine thrust and increases insignificantly with an increase in chamber pressure. Feed systems with loaded tanks are employed most often in stationary installations or in low-thrust engines.

Pressure accumulators (mainly liquid) are employed not only to force liquids into the combustion chamber. They are also used in pressurization systems to provide for the normal movement of liquids from the tanks to the pumps, and sometimes to improve the stability characteristics of unloaded tanks.

The pressure within the internal cavities of feed systems with unloaded tanks is not great and changes only by several atmospheres. This is explained by the circumstance that the relative weight of the tanks for this system is practically independent of both chamber pressure and engine thrust. Investigation has revealed that the weight qualities of engines equipped with the unloaded tank system improve in proportion to the increase in engine thrust and rocket dimensions. It proved to be the case that engine weight qualities also improved somewhat in proportion to an increase in chamber pressure, up to specified values. Therefore the feed system with unloaded tanks has occupied a lasting position in contemporary rocket construction.

For the time being the turbopump assembly is employed for propellant transfer from tanks to the chamber, but other pressure systems are possible, i.e., injectors, cyclic pumps, electromagnetic pumps, electric motors with pumps, and so on. We shall study the advisability of their employment.

The turbine obtains its working medium from the generator, which may work both on the basic propellant constituents, or through the employment of a special working medium. In the majority of cases, the generators of contemporary liquid-fuel rocket engines employ the main propellant constituents. The products exhausted in the turbine are thrown off through supplementary nozzles or are burned up in the main or in a supplementary chamber. Designs are encountered in which the turbine feed is obtained not from the main propellant directly, but by means of a power take-off of part of the heat energy from the combustion chamber. Investigations have revealed that the employment of the combustion chamber as an intermediate link in the process of generating the working medium for the turbine leads to construction complexity and to an increase in engine weight.

2. Feed System Subassemblies

The feed system includes pumps, pressure system elements, the hydraulic lines, automatic equipment elements, and the tanks.

In contemporary liquid-fuel rocket engine, pumps are the basic assemblies which transmit energy from the turbine to the moving liquid. Their characteristics determine the entire operating condition of the feed system. The impellers of the oxidizer and fuel pumps are usually located on the main shaft or are closely coupled to each other through a reducing gear train. The presence of such a connection leads to the fact that the parameters of the oxidizer circuit prove to be dependent on the parameters of the fuel circuit. A deviation in pump parameters from their nominal values exerts a noticeable effect in distorting the engine operating function.

The pressurization system transmits energy from the accumulator to the moving fluid. This energy is less than that produced by the turbine, and it is usually applied to the segment from the tanks to the input section of the pump impeller. In this manner the pumps and the pressurization systems prove to be sources of energy. The combustion chamber and the hydraulic lines, including the hydraulic circuits and automatic system elements, are energy consumers.

In order to determine the required turbine power, given the chamber pressure and knowing the propellant flow rate, we must determine the total hydraulic losses for each propellant constituent; then, considering the pressure produced by the pressurization system, we may determine the required pump pressures. We find the required turbine power from the pump characteristics.

In this manner, we must study first the hydraulic line calculations, then determine the output parameters of the pressurization system, and finally choose the required pump characteristics.

3. The External Characteristic of the Feed System

The starting function of the feed system is more complex than that of the combustion chamber. It is composed of a series of successively accomplished operations, each of which may be described by its own equation system, and which is solved within the specified limits of a change in parameters. Each successive operation is begun after a full completion of the former operation is achieved, which is characterized by the presence of stationary values in the parameters values. If the operation is studied with the aid of equations solved by a computer, then the transition from one operation to another, i.e., from one equation system to another, is conducted in conformity with the results of the calculation.

A single starting sequence for the liquid-fuel rocket engine does not exist.

Let us examine, as an example, one of the possible variations (Figure 13). At first, with the aid of the pressurization system, the pressure rises in the tank (0 - 1). During this time, the main valves of the hydraulic lines are closed. If we direct our attention to the gas pressure accumulators, the process is described by equations which characterize the gas flow from the accumulator to the tanks and the filling of the lines from the tank to the main valves with propellant.

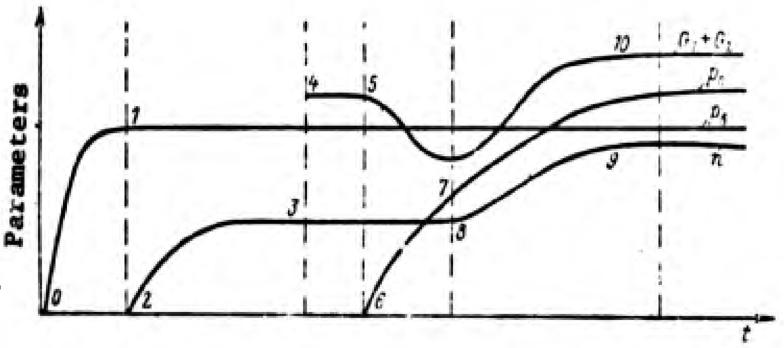

Figure 13. One of the possible variations of a change in engine parameters during the starting function.

Let us assume that the next operation is the unwinding of the turbopump assembly shaft to a certain number of revolutions having an intermediate value (2 - 3). During this operation, the main valves remain closed. This unwinding occurs as a result of the fact that the generator or the poder starter begins to operate (if the generator begins to act later and is fed by main propellant constituents, which enter the generator after entering the TNA pumps).

When the turbopump assembly shaft attains a certain number of revolutions, the main valves partially open. As a result, propellant constituents fill the pipe lines connecting the main valves with the engine head. We note that the main valves may not open simultaneously. The discharge of propellant (4 - 5) into the chamber, ignition, and a partial increase in chamber pressure (6 - 7) begin.

Then the full opening of the main valves occurs; the conditions of their opening may differ for each. The turbine is shifted to feed from the main propellant constituents, after which propellant consumption in the main chamber attains nominal values (10).

In attainment of the operating condition, almost all the engine sub-assemblies play a part. The processes occurring within the engine are described here by the most complete equation system.

§ 2. Hydraulic Circuit Equations

The hydraulic circuit is one of the elements of the feed system which exerts a significant influence on the operating condition of the engine as a whole. The majority of contemporary engines contain two circuits: the oxidizer and the fuel circuits. The processes occurring within the hydraulic circuits are exceptionally complex. In order to study these processes we must include equations which describe them with the degree of accuracy required for the solution of a specific problem.

1. The Movement of an Elastic Fluid in a Deformable Pipe Line with Supplementary Variable Volume

Let us examine the unstable movement of a compressible liquid through a pipe line, which under the influence of static and dynamic loads is deformed in both the radial and also the axial directions. Let us assume that a supplementary volume, filled with liquid, is connected to a section of pipe line under examination (Figure 14). The equation of the conservation of mass may be written as follows:

$$Y_1 - Y_2 - Y_\rho - Y_r - Y_x - Y_v = 0, \qquad (2.1)$$

where Y_1 is the quantity of liquid which entered the hydraulic system under consideration by time t;

Y_2 is the quantity of liquid which was discharged from the system by the same time;

Y_ρ is the quantity of liquid which accumulated in the system during the same time interval as a result of an increase in the liquid density;

Y_r is the quantity of liquid which accumulated in the system during the same time interval due to an increase in the pipe line radius;

Y_x is the same, but due to pipe line stretch;

Y_v is the quantity of liquid which entered the supplementary volume during time t.

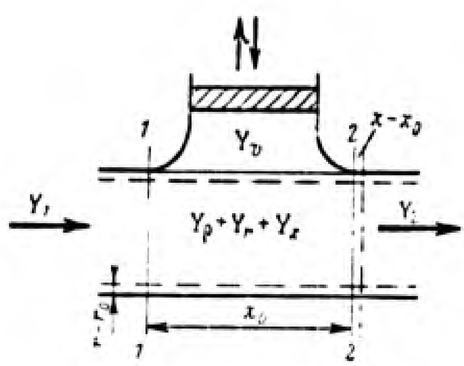

Figure 14. Diagram of an elastic pipeline with supplementary volume.

Differentiating with respect to t, we find that:

$$\dot{Y}_1-\dot{Y}_2-\dot{Y}_\rho-\dot{Y}_r-\dot{Y}_x-\dot{Y}_v=0. \qquad (2.2)$$

We note that in the problems examined it was assumed that by the initial point in time, theentire internal cavity of the hydraulic system had already been filled with liquid.

The liquid flow through the first (input) section will equal

$$\dot{Y}_1=G_1=F\varrho W_x. \qquad (2.3)$$

The liquid flow through the second (exit) section will equal:

$$\dot{Y}_2=G_2=F\varrho W_x+F\frac{\partial(\varrho W_x)}{\partial x}x_0. \qquad (2.3a)$$

The change in liquid quantity between sections 1 and 2 as a result of a density change with time is:

$$\dot{Y}_{\rho}=\frac{\partial \varrho}{\partial t} F x_0. \tag{2.4}$$

The change in liquid quantity due to an increase in the pipe line radius will equal:

$$\dot{Y}_{r}=\varrho \dot{F} x_0. \tag{2.5}$$

The change in liquid quantity due to pipe line stretch is determined according to the formula:

$$\dot{Y}_{x}=\varrho F \frac{\partial x_0}{\partial t}. \tag{2.6}$$

The change in liquid quantity in the supplementary volume will equal:

$$\dot{Y}_{v}=\varrho \frac{\partial V_0}{\partial t}, \tag{2.7}$$

where V_0 is the supplementary volume.

Now equation (2.2) assumes the following form:

$$F \frac{\partial (\varrho W_x)}{\partial x}+\frac{\partial \varrho}{\partial t} F+\varrho \dot{F}+\frac{\varrho F}{x_0} \frac{\partial x_0}{\partial t}+\frac{\varrho}{x_0} \frac{\partial V_0}{\partial t}=0, \tag{2.8}$$

or

$$\frac{\dot{\varrho}}{\varrho}+\frac{\partial W_x}{\partial x}+\frac{\dot{F}}{F}+\frac{1}{x_0} \frac{\partial x_0}{\partial t}+\frac{1}{V} \frac{\partial V_0}{\partial t}. \tag{2.8a}$$

Equations (2.8) and (2.8a) represent equations of a moving compressible fluid in the case of a change in the geometric dimensions of the channel if the pipe line is not deformed, and if supplementary volume V_0 is lacking then in place of (2.8) we obtain the equation of continuity in the usual form:

$$\frac{\dot{\varrho}}{\varrho}+\frac{\partial W_x}{\partial x}=0. \tag{2.9}$$

Let us examine the most general case. Let the moving liquid fill certain arbitrary volume V, which is limited by surface S, such that volume V will change with time under the influence of factors of one type or another. The equation of continuity in cylindrical coordinates is written as follows:

$$\frac{\dot{\varrho}}{\varrho}+\left[\frac{\partial W_x}{\partial x}+\frac{1}{r}\frac{\partial (W_r r)}{\partial r}+\frac{1}{r}\frac{\partial W_\varphi}{\partial \varphi}\right]+\frac{\dot{V}}{V}=0. \qquad (2.10)$$

For a cylindrical pipe line with no supplementary volume, for a case when $W_\phi = 0$, in place of (2.10) we have:

$$\frac{\dot{\varrho}}{\varrho}+\left[\frac{\partial W_x}{\partial x}+\frac{1}{r}\frac{\partial (W_r r)}{\partial r}\right]+\frac{\dot{F}}{F}+\frac{1}{x_0}\frac{\partial x_0}{\partial t}=0. \qquad (2.11)$$

In the future this equation will be examined in detail, since it may be employed in the solution of many problems having practical significance. If we consider only radial deformation, then the equation of continuity is written as follows:

$$\frac{\dot{\varrho}}{\varrho}+\left[\frac{\partial W_x}{\partial x}+\frac{1}{r}\frac{\partial (W_r r)}{\partial r}\right]+\frac{\dot{F}}{F}=0. \qquad (2.12)$$

We note that during an examination of the processes which occur in the vicinity of a certain steady state condition characterized by the constant density value ρ_0, we may write the value $\dot{\rho}/\rho_0$ in equations (2.8a) - (2.12) in place of ratio $\dot{\rho}/\rho$. For an examination of uniform flow, we shall assume that $W_r = 0$ in equations (2.11) and (2.12).

Let us carry out the derivation of the equation which characterizes the condition of pipe line deformation. In a majority of cases it appears due to the effect of static and dynamic loads. We may put the proportionality condition between elementary changes in pressure and density in the form [19]:

$$\frac{dp}{E_l}=\frac{d\varrho}{\varrho_0}, \qquad (2.13)$$

where E_l is the modulus of the fluid, proportional to the pressure derivative with respect to density,

$$E_l=\frac{dp}{d\varrho_l}. \qquad (2.14)$$

For an evaluation of the pipe line deformation, we employ the law of elastic deformations:

$$P = \varepsilon E F, \qquad (2.15)$$

where P is the force stretching the pipe line;

E is the modulus of elasticity of the pipe line material;

F is the area of the section;

ε is the relative stretch.

Force P is determined in accordance with the value of the static pressure and according to the kinetic energy difference. In direction W_x on section x_0

$$P_x=\left[p+\varrho W_x\int_0^{x_0}\frac{\partial W_x}{\partial x}dx-\frac{\varrho}{2}\left(\int_0^{x_0}\frac{\partial W_x}{\partial x}dx\right)^2\right]F_x. \tag{2.16}$$

In direction W_r on section r_0:

$$P_r=\left[p+\varrho W_r\int_0^{r_0}\frac{\partial W_r}{\partial r}dr-\frac{\varrho}{2}\left(\int_0^{r_0}\frac{\partial W_r}{\partial r}dr\right)^2\right]F_r \tag{2.16a}$$

With the aim of simplifying the form of the equations, for forces P_x and P_r will be further defined under the assumption of full flow stagnation. Under such an assumption, the force which may stretch the annular segment of the pipe line in a radial direction on segment x_0 is defined according to the equation:

$$P_r=\left(p+\varrho\frac{W_r^2}{2}\right)2r_0x_0; \tag{2.17}$$

the area of the section being

$$F_r = 2x_0\delta, \tag{2.18}$$

where δ is the pipe line thickness.
The relative stretch is

$$\varepsilon_r=\frac{r-r_0}{r_0}. \tag{2.19}$$

The law of elastic deformations (2.15) is expressed as follows:

$$\left(p+\varrho\frac{W_r^2}{2}\right)2r_0x_0=2Ex_0\delta\frac{r-r_0}{r_0}, \tag{2.20}$$

or

$$p+\varrho\frac{W_r^2}{2}=E\delta\frac{r-r_0}{r_0^2}. \quad (2.20a)$$

Differentiating, we obtain after transformations:

$$\dot{r}=\frac{r_0^2}{E\delta}\left(\dot{p}+\dot{\varrho}\frac{W_r^2}{2}+\varrho W_r\dot{W}_r\right). \quad (2.21)$$

If $W_r = 0$, or if the influence of the dynamic radially directed loads may be disregarded, then

$$\dot{r}=\frac{r_0^2}{E\delta}\dot{p}. \quad (2.21a)$$

The force which may stretch the pipe line in the axial direction equals:

$$P_x=\left(p+\varrho\frac{W_x^2}{2}\right)\pi r_0^2; \quad (2.22)$$

the area of the section being

$$F_x = 2\pi r_0\delta. \quad (2.23)$$

The relative stretching is

$$\varepsilon_x=\frac{x-x_0}{x_0}.$$

The law of elastic deformations is expressed as follows:

$$\left(p+\varrho\frac{W_x^2}{2}\right)\pi r_0^2=2\pi E r_0\delta\frac{x-x_0}{x_0}, \quad (2.24)$$

or

$$p+\varrho\frac{W_x^2}{2}=2E\frac{\delta}{r_0}\,\frac{x-x_0}{x_0}. \quad (2.24a)$$

Differentating, after transformations we obtain:

$$\dot{x}=\frac{r_0x_0}{2E\delta}\left(\dot{p}+\dot{\varrho}\frac{W_x^2}{2}+\varrho W_x\dot{W}_x\right). \quad (2.25)$$

If the influence of the dynamic load may be disregarded, then

$$\dot{x}=\frac{r_0 x_0}{2E\delta}\dot{p}. \qquad (2.25a)$$

In agreement with equation (2.13)

$$\dot{\varrho}=\frac{\varrho_0}{E_l}\dot{p}; \qquad (2.26)$$

Equations (2.21) and (2.25) correspondingly are written as follows:

$$\dot{r}=\frac{r_0^2}{E\delta}\left[\left(1+\frac{\varrho_0}{E_l}\frac{W_r^2}{2}\right)\dot{p}+\varrho W_r\dot{W}_r\right]; \qquad (2.27)$$

$$\dot{x}=\frac{r_0 x_0}{2E\delta}\left[\left(1+\frac{\varrho_0}{E_l}\frac{W_x^2}{2}\right)\dot{p}+\varrho W_x\dot{W}_x\right]. \qquad (2.28)$$

In place of equation (2.27) we may write:

$$\frac{\dot{F}}{F}=\frac{2r_0}{E\delta}\left[\left(1+\frac{\varrho_0}{E_l}\frac{W_r^2}{2}\right)\dot{p}+\varrho W_r\dot{W}_r\right]. \qquad (2.27a)$$

Let us carry out a transformation of equation (2.11). Employing equation (2.27a) and (2.28), we obtain:

$$\frac{\dot{p}}{E_l}+\left[\frac{\partial W_x}{\partial_x}+\frac{1}{r}\frac{\partial(W_r r)}{\partial r}\right]+\frac{2r_0}{E\delta}\left[\left(1+\frac{\varrho_0}{E_l}\frac{W_r^2}{2}\right)\dot{p}+\right.$$
$$\left.+\varrho W_r\dot{W}_r\right]+\frac{r_0}{2E\delta}\left[\left(1+\frac{\varrho}{E_l}\frac{W_x^2}{2}\right)\dot{p}+\varrho W_x\dot{W}_x\right]=0, \qquad (2.29)$$

or

$$\dot{p}\left[\frac{1}{E_l}+\frac{5}{2}\frac{r_0}{E\delta}+\frac{r_0}{E\delta}\frac{\varrho_0}{E_l}\left(W_r^2+\frac{W_x^2}{2}\right)\right]+$$
$$+\left[\frac{\partial W_x}{\partial x}+\frac{1}{r}\frac{\partial(W_r r)}{\partial r}+\frac{r_0}{E\delta}\varrho\left(2W_r\dot{W}_r+\frac{1}{2}W_x\dot{W}_x\right)\right]. \qquad (2.29a)$$

If from the results of supplementary investigations it is clear that, in the vicinity of the factor $\dot{p}$, the velocities W_r and W_x may be assumed to equal their average values, then formula (2.29a) may be rewritten as follows:

$$\dot{p}+\lambda^2\left[\frac{\partial W_x}{\partial x}+\frac{1}{r}\frac{\partial(W_r r)}{\partial r}+\frac{r_0}{E\delta}\varrho\left(2W_r\dot{W}_r+\frac{1}{2}W_x\dot{W}_x\right)\right]=0, \qquad (2.30)$$

where

$$\lambda=\left[\frac{1}{E_l}+\frac{5}{2}\frac{r_0}{E\delta}+\frac{r_0}{E\delta}\frac{\varrho_0}{E_l}\left(W_{r0}^2+\frac{W_{x0}^2}{2}\right)\right]^{-1/2}. \qquad (2.31)$$

If $W_r = 0$, then

$$\dot{p}+\lambda^2\left(\frac{\partial W_x}{\partial x}+\frac{1}{2}W_x\dot{W}_x\right)=0, \qquad (2.30a)$$

$$\lambda=\left(\frac{1}{E_l}+\frac{5}{2}\frac{r_0}{E\delta}\right)^{-1/2}. \qquad (2.31a)$$

In the case of incomplete flow stagnation, in the derivation of formula (2.30) we must employ formulas (2.16a) and (2.16) for a determination of forces P_r and P_x, rather than formulas (2.17) and (2.22).

In order to solve the problem of the movement of an elastic fluid in a deformable pipe line, since the formula (2.30) contains three variables, namely p, W_x and W_r, we must include two more equations of Newton's second law. These equations are examined in detail in ordinary and specialized courses in hydrodynamics. In the choice of equations, we must consider both the possibility and also the necessity for calculating the effects of viscous friction forces.

In a solution of a uniform problem without considering linear deformation, liquid viscosity, and dynamic loads, the reference equations are simplified and are reduced to the known system [19]:

$$\left.\begin{aligned}&\dot{W}+\frac{1}{\varrho}\frac{\partial p}{\partial x}=0;\\&p+\lambda^2\frac{\partial W}{\partial x}=0;\\&\lambda^{-2}=\left(\frac{1}{E_l}+2\frac{r_0}{E\delta}\right).\end{aligned}\right\} \qquad (2.32)$$

2. An Approximate Solution for the Hydraulic Circuit Equation

The equation for Newton's second law for a uniform flow while taking into account the effects of external forces and the viscous friction forces is written as follows:

$$\dot{W}_x - X + \frac{1}{\varrho}\frac{\partial p}{\partial x} - \nu\nabla^2 W = 0. \tag{2.33}$$

We shall consider that the process is examined for a certain segment of length δx, and that during the calculation of this segment we may direct our attention to average parameter values. Therefore, in place of equation (2.33), we may write:

$$\dot{W}\delta x = X\delta x - \frac{1}{\varrho_l}\delta p + \nu\nabla^2 W\delta x. \tag{2.34}$$

In writing equation (2.34), the pressure drop ratio of the chosen line segment appeared in place of the pressure partial derivative with respect to the coordinate.

For an entire hydraulic circuit having a length L, where

$$L = \sum \delta x, \tag{2.35}$$

we find that:

$$\varrho_l \sum \dot{W}\delta x = \varrho_l \sum X\delta x - (p_1 - p_2) + \varrho_l \sum (\nu\nabla^2 W\delta x). \tag{2.36}$$

Equation (2.36) represents a pressure balance along the circuit; the combined hydraulic losses

$$\varrho_l \sum (\nu\nabla^2 W\delta x) = \sum \Delta p_i = \Delta p \tag{2.37}$$

may be calculated through approximation or determined by experiment.

The extreme pressure drop is determined in accordance with circuit construction; if the hydraulic circuit includes the pump, the tank and the combustion chamber, then under static conditions

$$-(p_1 - p_2) = p_t + p_p - p - \Delta p_h, \tag{2.38}$$

where p_t is tank pressure;

p_p is pressure produced by the pump;

p is combustion chamber pressure;

Δp_h is pressure expended on liquid kinetic energy change along the circuit length.

External forces acting on the system produce an external force pressure in the circuit [29]:

$$p_j = \varrho_l \sum X \delta x. \qquad (2.39)$$

Mass forces lead to a pressure change [29]:

$$p_w = \varrho_l \sum \dot{W} \delta x. \qquad (2.40)$$

The balance of pressures may now be written as follows:

$$p_t + p_p - p = -p_j + \Delta p + p_w + \Delta p_h = \\ = \varrho_l \left[-\sum X \delta x + \sum (\nu \nabla^2 W \delta x) + \sum \dot{W} \delta x + \frac{1}{2} (W_2^2 - W_1^2) \delta x \right]. \qquad (2.41)$$

The derivation of the equation examined is not unique; the equation of the conservation of energy in the form:

$$\sum dL_{ai} = \sum dL_{ci}, \qquad (2.42)$$

may serve as a basis for the derivation of the equation of the balance of pressures.

Where L_{ai} is elementary work which is accomplished on the moving liquid;

L_{ci} is the elementary work of resistive forces.

The elementary work is

$$dL_i = PdS, \qquad (2.43)$$

where P is the force;

S is the path.

Since

$$W = \frac{dS}{dt}, \qquad (2.44)$$

in place of equation (2.43) we have:

$$dL = PW \cdot dt. \tag{2.45}$$

If the area of the section of the circuit channel under consideration is designated by the symbol F, then

$$dL - pFW\, dt, \tag{2.46}$$

where p is the corresponding pressure.

For an incompressible liquid and a non-deformable pipe line we have, in accordance with the equation of continuity [27]:

$$\rho_l FW = G, \tag{2.47}$$

where G is the mass liquid flow rate per second. Since one and the same quantity of liquid passes through any section of the circuit at a given point in time, the deformation coefficient is

$$A = \frac{G}{\rho_l} = FW. \tag{2.48}$$

It must be kept in mind that A, which does not change in accordance with the circuit length (within the framework of the assumptions accepted earlier), may certainly change with time: in all dynamic processes the coefficient of deformation undergoes a change due to a change in the liquid flow rate with respect to time. Now the energy equation (2.42) is written as follows:

$$\sum(Ap)_{al}\, dt = \sum(Ap)_{cl}\, dt. \tag{2.49}$$

It is apparent that equation (2.49) for A = const may be presented in the form of a balance of pressures:

$$\sum p_a = \sum p_c. \tag{2.50}$$

If liquid or pipe line deformation exists, then the quantity A becomes a variable and in place of equation (2.50) we must write:

$$\sum(Ap)_a = \sum(Ap)_c. \tag{2.51}$$

The solution of (2.51) is possible with supplementary equations in which functions are included to characterize a change in liquid density and a change in geometric pipe line dimensions due to pressure.

3. Pressure Losses in Elements of the Hydraulic Circuit

In agreement with equation (2.37), hydraulic losses in an element are:

$$\Delta p_i = \varrho_l \nu \nabla^2 W \delta x. \quad (2.52)$$

It is known that for any element of the lines, the pressure losses are proportional to the kinetic energy:

$$\Delta p_i = \varrho_l \varepsilon_i \frac{W_i^2}{2}. \quad (2.53)$$

The coefficient of resistance is

$$\varepsilon_i = \lambda_i \frac{l_i}{d_{ei}}, \quad (2.54)$$

where λ is the coefficient of friction;

d_e is the equivalent diameter,

and in this connection

$$d_{ei} = 4 \frac{F_i}{\Pi_i}. \quad (2.55)$$

Here F_i is the cross-sectional area of the channel; Π_i is the effective perimeter. It represents the perimeter which "plays a part" in the process under consideration. For a circular pipe, for example, $\Pi = \pi d$; for an annular section, $\Pi = 2\pi d$.

It is known that [27]:

$$\lambda = \lambda(Re); \quad (2.56)$$

For smooth pipes in a laminar flow [41]:

$$\lambda = \frac{64}{Re}; \quad (2.57)$$

For smooth pipes in a turbulent flow [41]:

$$\lambda = \frac{0.3164}{Re^{0.25}}. \quad (2.58)$$

In a turbulent flow with a square-law dependence of hydraulic losses on the flow rate, the coefficient of friction is practically independent of the Re number, but increases significantly with an increase in relative roughness, and [41]:

$$\lambda=\left(1,74+2\log\left|\frac{r}{\Delta}\right|\right)^{-2}, \tag{2.59}$$

where Δ is the average height of the irregularities of the flow channel.

The laws of friction have their ranges of influence: equation (2.57) is correct up to $\log|Re| = 3.3$; equation (2.58) is limited by $\log|Re| = 3.7$ and $\log|Re| = 4.8$; equation (2.59) for sufficiently smooth pipes is effective from $\log|Re| = 5.8$.

Employing the equation of continuity

$$\rho_l FW = G, \tag{2.60}$$

we obtain

$$Re=\frac{Gd_e}{\varrho_l \nu F}; \tag{2.61}$$

$$\Delta p_i=\varepsilon_i\frac{G^2}{2\varrho_l F_i^2}. \tag{2.62}$$

In a laminar flow we shall consider that:

$$\varepsilon = \frac{\text{const}}{G} \tag{2.63}$$

and in a turbulent flow with a square-law variation:

$$\varepsilon\approx\text{const}, \tag{2.64}$$

i.e., ε shows little dependence on the Re number.

Thus in a laminar flow

$$\Delta p_i = a_{la_i} G \tag{2.65}$$

where

$$a_{la_i}=\frac{(\text{const})\ la}{2\varrho_l F_i^2}. \tag{2.66}$$

In a turbulent flow with a square-law variation (log Re > 5.8)

$$\Delta p_i = a_{sq\ i} G^2, \tag{2.67}$$

where

$$a_{sq\ i} = \frac{(\text{const})_{sq\ i}}{2\varrho_l F_i^2} \tag{2.68}$$

For the entire hydraulic circuit

$$\Delta p = \xi \sum a_i G^n \approx a_\Delta G^2 = \frac{a'_\Delta}{2\varrho_l} G^2. \tag{2.69}$$

Coefficient ξ in equation (2.69) takes into account the influence of perturbations, which are generated in local resistances, on the value of the hydraulic losses. The exponent $n \approx 1$ in a laminar flow, $n \approx 1.75$ in a turbulent flow, $n = 2$ in a turbulent flow with a square-law variation, and $n = 1.8 - 2.2$ for a complex composite channel.

The pressure expended in a change of liquid kinetic energy is:

$$\Delta p_h = \varrho_l \frac{W_{in}^2 - W_t^2}{2} = \frac{1}{2\varrho_l} \left(\frac{1}{F_{in}^2} - \frac{1}{F_t^2}\right) G^2. \tag{2.70}$$

In processing experimental data $n = 2$ is often accepted in equation (2.69); the sum of equations (2.69) and (2.70) for the entire hydraulic circuit is written as follows:

$$\Delta p + \Delta p_h = \frac{1}{2\varrho_l} \left[a'_\Delta + \left(\frac{1}{F_{in}^2} - \frac{1}{F_t^2}\right)\right] G^2 = a G^2. \tag{2.71}$$

Let us examine the hydraulic channel in an arbitrary form. Let the cross-sectional area change along the length of the channel so that $F = F(x)$.

On an elementary length segment, the liquid mass will equal:

$$dm = \rho_l F(x)\, dx. \tag{2.72}$$

The mass forces element is

$$dP = F(x)\, dp_w. \tag{2.73}$$

According to Newton's second law

$$dP = \dot{W}dm. \tag{2.74}$$

Employing the equation of continuity in order to determine accelerated fluid movement, we have:

$$\dot{W} = \frac{\dot{G}}{F(x)\varrho l}. \tag{2.75}$$

Substituting equations (2.72) and (2.75) in (2.73), we find, after transformations, the pressure element which is due to the influence of the mass force:

$$dp_{wl} = \dot{G}\frac{dx_l}{F(x)_l}.$$

On a segment of the length l, the pressure will be:

$$p_{wl} = \dot{G}\int_0^l \frac{dx_l}{F(x)_l}. \tag{2.76}$$

Let us specify that:

$$b_l = \int_0^l \frac{dx_l}{F(x)_l}. \tag{2.77}$$

Thus,

$$p_{wi} = b_i\dot{G}.$$

If we examine the complex hydraulic channel, which consists of a series of successively connected elements of any shape, then we shall find that the total pressure of the channel will be:

$$p_w = \dot{G}\sum\int_0^{l_l} \frac{dx_l}{F(x)_l}. \tag{2.78}$$

<u>Specific Cases</u>

1. For a channel segment of constant section (F = const) with length l:

$$p_w = lF^{-1}\dot{G}.$$

2. For a channel consisting of successively connected segments of constant section:

$$p_w = \dot{Q} \sum \frac{l_i}{F_i},$$

therefore,

$$b = \sum \frac{l_i}{F_i}.$$

3. For a pipe line segment with diameter d and length l:

$$p_w = \frac{4}{\pi} \frac{l}{d^2} \dot{Q}.$$

4. For a channel consisting of successively connected pipe line segments:

$$p_w = \frac{4}{\pi} \sum \frac{l_i}{d_i^2} \dot{Q}.$$

5. For a conical channel having radius r_1 at the input and radius r_2 at the output, it is apparent that

$$\tan \alpha = \frac{dr}{dx},$$

therefore,

$$p_w = \dot{Q} \int_{r_1}^{r_2} \frac{dr}{\pi \tan \alpha r^2}.$$

The solution has the following form:

$$p_w = \dot{Q} \frac{1}{\pi \tan \alpha} \left(\frac{1}{r_1} - \frac{1}{r_2} \right),$$

therefore,

$$b = \frac{1}{\pi \tan \alpha} \left(\frac{1}{r_1} - \frac{1}{r_2} \right).$$

6. For the cooling channel of a cylindrical chamber:

$$p_w = \dot{G}\,\frac{l}{2\pi r\Delta},$$

where Δ is the circulating channel width.

7. For the cooling channel of a conical chamber:

$$p_w = \dot{G}\int_{r_1}^{r_2}\frac{dr}{2\pi\tan\alpha\, r\Delta}$$

The solution has the following form:

$$p_w = \dot{G}\,(2\pi\tan\alpha\Delta)^{-1}\ln\left|\frac{r_2}{r_1}\right|,$$

therefore,

$$b = (2\pi\tan\alpha\Delta)^{-1}\ln\left|\frac{r_2}{r_1}\right|.$$

8. Parallel connection of channels. If any number of channels is connected in parallel, then the pressure drops at the chamber head, for example, will be equal for all channels. The pressure drops are presented in the form of the sum:

$$p_i = p_w + \Delta p + \Delta p_k,$$

therefore,

$$p_i = b_i\dot{G}_i + \frac{(1+\epsilon)G_i^2}{2\varrho_l F_i^2},$$

where G_i is the liquid flow rate through the channel under consideration. If n identical channels, connected in parallel, are involved, then

$$p_i = \frac{b_i\dot{G}}{n} + \frac{(1+\epsilon)G^2}{2n^2\varrho_l F_i^2}.$$

Thus, here

$$b = \frac{b_i}{n}.$$

Now equation (2.41) of the balance of pressures for the hydraulic circuit is written as follows:

$$p_t + p_p + p_j = p + aG^2 + b\dot{G}. \tag{2.79}$$

Chamber pressure p may be expressed with the aid of equations presented in Chapter I, and the terms of the left side of equation (2.79) will be considered below.

§ 3. Filling of the Hydraulic Lines

In preparing an engine for the start, a series of preliminary operations are carried out. Their content and sequence depend on the design under which the engine was manufactured, on the tactical-technical problem at the rocket start, and on many other factors. Several operations, however, are typical for the majority of contemporary liquid-fuel rocket engines. Among these are included: a preliminary pressure increase in the tanks, starting of the generator, shaft acceleration of the TNA, opening of the main valves of the hydraulic lines, and filling of the main lines with propellant constituents.

The operation of filling of the pipe lines depends on the nature of the pressure rise at the main valves, on the sequence of their opening, on the dimensions and the hydraulic resistances of the lines, and on the properties of the moving liquid.

The nature of fluid entrance into the combustion chamber is based on its dependence on the filling operation. It is limited by the values of the flow rates and component ratios at the moment of propellant entrance into the combustion chamber and at the moment of ignition, and by the nature of their change with respect to time.

After propellant ignition, liquid movement will depend not only on the feed system parameters but also on chamber parameters.

The interrelationship of processes occurring in subassemblies of the feed system and the chamber are examined in Chapter III. However, in order to solve similar problems, it is necessary to know the liquid behavior up to the moment of ignition. Here a change in flow rates with time depends fully on the parameters of the feed system.

Let us examine the movement of one of the propellant constituents from the main valve to the end of the injectors, assuming instantaneous opening of the valve. The equation of the balance of pressures is written as follows:

$$p=\Delta p+p_w+p_{co} \tag{2.80}$$

where p is the pressure at the valve;

Δp are hydraulic losses in the pipe line, which is filled with liquid;

$\Sigma F_i p_{wi}$ represents mass forces;

p_{co} is pressure generated due to gas contraction in the pipe line at the injectors.

The pressure at the valve depends on the value of the tank pressure, the pressure produced by the pump, and hydraulic losses and mass forces in the channel from the tank to the main valve; this pressure is a function of time t.

The hydraulic losses are

$$\Delta p = alG^2, \tag{2.81}$$

but here, in contrast to equation (2.71),

$$a=\frac{\lambda}{2\varrho_l d_e F^2}, \tag{2.82}$$

where d_e is the equivalent diameter, determined according to the formula

$$d_e = \frac{4F}{\Pi},$$

and Π is the perimeter of the channel section. The length of the channel filled with liquid, when the cross-sectional area F of the channel is constant, equals:

$$l=\frac{1}{\varrho_l F}\int_0^t G\,dt. \tag{2.83}$$

If F is a function of l, which occurs in an examination of the circulating part of the cooling jacket, then the determination of l must proceed from the relationship

$$\int_0^t F(l)\,dl = \frac{1}{\varrho_l}\int_0^t G\,dt. \qquad (2.84)$$

Thus,

$$\Delta p = \frac{a}{\varrho_l F} G^2 \int_0^t G\,dt. \qquad (2.85)$$

The pressure expended in overcoming mass forces in one element is:

$$p_w = \dot{G}\int_0^l \frac{dl}{F}. \qquad (2.86)$$

Taking equation (2.83) into account, we find that:

$$p_w = \dot{G}(\varrho_l F^2)^{-1}\int_0^t G\,dt. \qquad (2.87)$$

In order to determine p_{co} it is necessary to employ the equation of the conservation of mass:

$$\dot{Y}_m - \dot{Y}_0 - \dot{Y}_u = 0, \qquad (2.88)$$

where Y_m is the mass of the gas in the pipe line, formed as a result of propellant constituent gasification;

Y_0 is the mass of the gas located in the pipe line;

Y_u is the mass of the gas flowing through the injector into the chamber.

All these values pertain to an arbitrary point in time.

Gasification takes place during the movement of a propellant constituent with a low boiling point. We shall consider that the propellant constituent is at the boiling point and that gasification occurs as a result of an instantaneous transmission of heat from the channel walls.

Heat given off by the wall [37] equals:

$$q = \pi d l \delta \varrho_c c (T - T_s), \qquad (2.89)$$

where d is the channel diameter;

δ is the channel wall thickness;

c is the specific heat of the material;

T is the initial wall temperature;

T_s is the temperature of the propellant constituent equal to the boiling point.

Heat obtained by the propellant constituent [37] equals:

$$q = r^* Y_m, \tag{2.90}$$

where r^* is the latent heat of vaporization.

Thus

$$Y_m = \frac{1}{r^*} \pi d l \delta \varrho_c c (T - T_S); \tag{2.91}$$

$$\dot{Y}_m = \frac{1}{r^*} \pi d \delta \varrho_c c (T - T_S) W = \frac{4 \delta \varrho_c c (T - T_S)}{r^* d \varrho_l} G. \tag{2.92}$$

Employing the equation of state and considering that RT_s = const, we find that:

$$Y_0 = \frac{p_{co}}{\varrho R T_S} \int_0^t G\, dt. \tag{2.93}$$

Therefore,

$$\dot{Y}_0 = (\varrho R T_S)^{-1} \left(p_{co} G + \dot{p}_{co} \int_0^t G\, dt \right). \tag{2.94}$$

The derivative of the quantity of emitted gas in the case of small pressure drops at the injectors for a sub-critical discharge may be approximately determined as follows:

$$\dot{Y}_u = G = \mu F_{in} [\varrho (p_{co} - p_c)]^{0,5}, \tag{2.95}$$

where μ is the discharge coefficient;

F_{in} is the total area of the injectors;

p_c is the chamber pressure before starting.

Now equation (2.88) may be written as follows:

$$4 [r^* d \varrho_l]^{-1} (\delta \varrho_c c |T - T_S|) G - (\varrho R T_S)^{-1} \left(p_{co} G + \dot{p}_{co} \int_0^t G\, dt \right) - \mu F_{in} (\varrho |p_{co} - p_c|)^{0,5} = 0. \tag{2.96}$$

The initial equation (2.80) has the following form:

$$p - a(\varrho_l F)^{-1} G^2 \int_0^t G\,dt - (\varrho_l F^2)^{-1} \int_0^t G\,dt \cdot \dot{G} - p_{co} = 0. \qquad (2.97)$$

In order to determine G and p_{co}, it is necessary to solve simultaneously equations (2.96) and (2.97).

Let us introduce a new variable, which characterizes the rising quantity of propellant constituent:

$$Z = \int G\ dt; \qquad (2.98)$$

In this connection

$$\dot{Z} = G; \qquad (2.99)$$
$$\ddot{Z} = \dot{G}. \qquad (2.100)$$

The equation system for one propellant constituent is written as follows:

$$\left.\begin{array}{l} 4[r^{\bullet} d\,\varrho_l]^{-1}(\alpha\varrho_c c\,|T - T_s|)\,\dot{Z} - (\varrho R T_s)^{-1}(p_{co}\dot{Z} - \\ -\dot{p}_{co} Z) - \mu F_{in}(\varrho\,|p_{co} - p_c|)^{0{,}5} = 0; \\ p - (\varrho F)^{-1} a \dot{Z}^2 Z - (\varrho F^2)^{-1} \ddot{Z} Z - p_{co} = 0. \end{array}\right\} \qquad (2.101)$$

We note that in order to determine density we have:

$$\rho = \frac{p_{co}}{RT_s}. \qquad (2.102)$$

Let us determine the conditions which ensure that $\dot{K} = 0$ in the period from the time of opening the main valve to the time of propellant entrance into the chamber. We shall consider that $p_{co} \ll p$.

If $\dot{K} = 0$, then

$$K = \frac{Z_1}{Z_2} = \frac{\dot{Z}_1}{\dot{Z}_2} = \frac{\ddot{Z}_1}{\ddot{Z}_2}. \qquad (2.103)$$

Employing the second equation of the system (2.101) and applying it to both circuits, we find that $\dot{K} = 0$, if

$$\frac{a_2}{a_1}\,\frac{\varrho_1}{\varrho_2}\,\frac{F_1}{F_2}=K^3; \qquad (2.104)$$

$$\frac{\varrho_1}{\varrho_2}\,\frac{F_1^2}{F_2^2}=K^2; \qquad (2.105)$$

$$p_1=p_2. \qquad (2.106)$$

Such are the conditions which ensure that instantaneous values K are constant during the period of filling of the lines.

As has been noted earlier, the designer is interested in the nominal value of the reactant ratio at the time of ignition. It is completely clear that this condition is necessarily fulfilled if K corresponds to its nominal value in the course of filling the pipe line. However, the possibility of providing the required value K at the time of ignition and during a different condition of filling of the lines cannot be excluded. Here the system of valve opening exerts a great influence. If the necessity of considering the effect of the valve arises, then it is necessary to add to equation (2.80) still another term -- Δp_{v1}, which characterizes the value of the hydraulic resistance of the circulating part of the valve.

It is apparent that

$$\Delta p_{v1}=a_{v1}(t)G^2, \qquad (2.107)$$

where the hydraulic loss coefficient a_{v1} (t) is a time function and depends on the valve opening system.

§ 4. The Equation of the Conservation of Energy for the Pressurizing System

In the design and investigation of feed systems, it often proves desirable to know the change in parameters of the tank pressurizing system with time. Equations of the conservation of energy and mass are included in order to solve many problems, and the system consisting of the pressure accumulator and tank is examined. The equations of conservation of energy may be introduced in a general form, since it will be useful in studying the process of extracting liquid from the tank by means of any type of accumulator.

For the system presented in Figure 15, the law of the conservation of energy is written as follows [1]:

$$Q = U + L, \tag{2.108}$$

where Q is the quantity of heat supplied to the working medium (the gas) of the system per unit time;

U is the internal energy of the working medium;

L is the work accomplished by the gas.

Figure 15. Diagram of the tank pressurizing system.

Gas, entering the upper tank cavity from the accumulator, expands. If the liquid is motionless, then the gas accomplishes no work; the flow of gas from one closed vessel to another is considered according to throttling formulas. If expulsion of liquid from the tank occurs, then the work accomplished by the gas in the process of expansion (the work of expansion), will be expended on overcoming the resistive forces at the tank output and on increasing the kinetic energy of the flowing liquid.

If as a result of propellant combustion and heat processes in gas mixtures, heat Q_0 is released within the internal cavity of the accumulator, and heat transfer with the surrounding medium is characterized by the quantities Q_a for the accumulator and Q_t for the tank, then

$$Q = Q_0 + Q_a + Q_t. \quad (2.109)$$

Internal gas energy is determined by the sum

$$U = U_a + U_t, \quad (2.110)$$

where for gas included within the accumulator,

$$U_a = Y_a c_{va} T_a. \quad (2.111)$$

For accurate computations we assume that $c_v \neq$ const; here we find that:

$$\dot{U}_a = c_{va} \frac{d}{dt}(Y_a T_a) + Y_a T_a \frac{dc_{va}}{dt} = \xi_a c_{va} \frac{d}{dt}(Y_a T_a), \quad (2.112)$$

where ξ_a is a coefficient which takes into consideration the influence of a specific heat fluctuation.

We note that the product $Y_a T_a$ is in the right side of the equation under the differentiation sign, since in the process of operation of the system both gas quantity in the accumulator and gas temperature change with time.

For the gas included in the tank, we have:

$$\dot{U}_t = \xi_t c_{vt} \frac{d}{dt}(Y_t T_t). \quad (2.113)$$

Employing the equation of state, we obtain:

$$\dot{U} = \frac{1}{R}[c_{va}(V_a \dot{p}_a + \dot{V}_a p) + c_{vt}(V_t \dot{p}_t + \dot{V}_t p_t) + p_a V_a \dot{c}_{va} + p_t V_t \dot{c}_{vt}]. \quad (2.114)$$

The expansion derivative of the gas in the tank is expressed as follows:

$$\dot{L} = p_t \dot{V}_t. \quad (2.115)$$

We are reminded that [1]:

$$c_p - c_v = R; \quad (2.116)$$

$$\frac{c_p}{c_v} = k. \quad (2.117)$$

The equation of the conservation of energy may now be written as follows:

$$\frac{c_{va}}{c_{v6}}(V_a\dot{p}_a+p_a\dot{V}_a)+V_t\dot{p}_t+kp_t\dot{V}_t=\frac{R}{c_{vt}}(\dot{Q}_0+\dot{Q}_a+\dot{Q}_t)-p_aV_a\frac{\dot{c}_{va}}{c_{vt}}-p_tV_t\frac{\dot{c}_{vt}}{c_{vt}}. \quad (2.118)$$

1. The Energy Equation Expressed Through the Propellant Constituent Flow Rate

The free volume in the tank is

$$V_t = V_{t,in} + \frac{1}{\rho_l}\int_0^t G\,dt, \quad (2.119)$$

where $V_{t,in}$ is the initial free volume;

ρ_l is the liquid density;

G is the mass rate of liquid flow.

It is apparent that

$$\dot{V}_t = \frac{G}{\rho_l}. \quad (2.120)$$

The energy equation assumes the following form:

$$\varepsilon_c(V_a\dot{p}_a+p_a\dot{V}_a)+V_{t,in}\dot{p}_t+\dot{p}_t\frac{1}{\rho_l}\int_0^t G\,dt+k\frac{Gp_t}{\rho_l}=\xi\frac{R}{c_{vt}}(\dot{Q}_0+\dot{Q}_a+\dot{Q}_t), \quad (2.121)$$

where

$$\varepsilon_c=\frac{c_{va}}{c_{vt}};$$

$$\xi=1-\left[p_aV_a\frac{\dot{c}_{va}}{c_{vt}}+p_tV_t\frac{\dot{c}_{vt}}{c_{vt}}\right]\left[\frac{R}{c_{vt}}\dot{Q}\right]^{-1}.$$

2. The Energy Equation for the Gas Accumulator

Considering that for a system with a gas accumulator, thermal flows are directed from the surrounding medium into the system, we obtain:

$$\varepsilon_cV_a\dot{p}_a+V_{t,in}\dot{p}_t+\dot{p}_t\frac{1}{\rho_l}\int_0^t G\,dt+k\frac{Gp_t}{\rho_l}=\xi\frac{R}{c_{vt}}(\dot{Q}_a+\dot{Q}_t). \quad (2.122)$$

In some cases it is more convenient to write equation (2.122) with the polytropic exponent:

$$\varepsilon_c V_a \dot{p}_a + V_{t,in}\dot{p}_t + \dot{p}_t \frac{1}{\rho_l}\int_{t}^{t} G\,dt + n\frac{Gp_t}{\varrho_l} = 0, \quad (2.123)$$

and in this connection the polytropic exponent is:

$$n = k - \xi\frac{R}{c_{vt}}\,\frac{\dot{Q}_a + \dot{Q}_t}{Gp_t}\varrho_l. \quad (2.124)$$

It is apparent that $\dot{Q}_a$ and $\dot{Q}_t$ are time functions; therefore $n = f(t)$. In engineering calculations attention is often directed to a certain average value of the polytropic exponent for the process.

If heat transfer is not present, then

$$\dot{Q}_a = \dot{Q}_t = 0. \quad (2.125)$$

In this case

$$n = k + \frac{\varrho_l}{G}\left(\frac{p_a}{p_t}V_a\frac{\dot{c}_{va}}{c_{vt}} + V_t\frac{\dot{c}_{vt}}{c_{vt}}\right). \quad (2.126)$$

The condition (2.126) will also occur when heat transfer is present, but only if

$$\dot{Q}_a + \dot{Q}_t = 0. \quad (2.127)$$

During an isothermic process $n = 1$; in this connection

$$\xi\frac{R}{c_{vt}}\,\frac{\dot{Q}_a + \dot{Q}_t}{Gp_t}\varrho_l = k - 1. \quad (2.128)$$

as is known,

$$k - 1 = \frac{R}{d_{vt}}. \quad (2.129)$$

Therefore, in the isothermic process

$$\xi(\dot{Q}_a + \dot{Q}_t) = \frac{Gp_t}{\varrho_l}, \quad (2.130)$$

or

$$\xi(\dot{Q}_a + \dot{Q}_t) = p_t\dot{V}_t \quad (2.131)$$

Let us determine the required accumulator volume. Directing our attention to formula (2.118), we shall write the initial equation as follows:

$$\varepsilon_c V_a \dot{p}_a + V_t \dot{p}_t + k\frac{Gp_t}{\varrho_l} = \frac{R}{c_{vt}}(\dot{Q}_a + \dot{Q}_t) - \left(p_a V_a \frac{\dot{c}_{va}}{c_{vt}} + p_t V_t \frac{\dot{c}_{vt}}{c_{vt}}\right). \qquad (2.132)$$

With the aim of simplifying the mathematical means in carrying out approximations of accumulator pressure, sometimes we consider that

$$c_{va} = c_{vt} = c_v = \text{const}.$$

In this connection we obtain $\varepsilon_c = 1$ and $\xi = 1$. In this connection if the polytropic exponent is determined on the basis of the processing of experimental data, then error introduced by assumptions is excluded.

Let us examine a case when p_t = const. Stipulating that $\dot{c}_{va} = \dot{c}_{vt} = 0$, we find that:

$$V_a \dot{p}_a = -k\frac{Gp_t}{\varrho_l} + \frac{R}{c_{va}}(\dot{Q}_a + \dot{Q}_t). \qquad (2.133)$$

Integrating, we obtain

$$V_a \int_{p_{a\,in}}^{p_a} dp_a = -k\frac{p_t}{\varrho_l}\int_0^{t_n} G\,dt + \frac{R}{c_{va}}\int_0^{Q} d(Q_a + Q_t). \qquad (2.134)$$

The solution has the following form:

$$V_a = k\frac{V_t p_t}{p_{a\,in} - p_a} - \frac{R}{c_v}\frac{Q_a + Q_t}{p_{a\,in} - p_a}, \qquad (2.135)$$

or

$$V_a = \frac{nV_t p_t}{p_{a\,in} - p_a}; \qquad (2.136)$$

in this connection

$$n = k - \frac{R}{c_v}\frac{Q_a + Q_t}{p_t V_t}\varrho_l, \qquad (2.137)$$

or

$$n \approx k-(k-1)\frac{Q_a+Q_t}{p_t V_t}\varrho_l \qquad (1.138)$$

Thus one need not equip the accumulator and the tank with thermal insulation with the aim of decreasing accumulator volume. It is necessary to raise the initial pressure in the accumulator and to decrease the final pressure p_a. It is apparent that p_a will always be greater than p_t. We must employ reducers operating at a sub-critical condition with the aim of bringing the values p_a and p_t closer together. It is advantageous to employ a gas delivery system to the tank which consists of a valve controlled with the aid of a pressure relay.

The polytropic exponent in equation (2.136) is often determined experimentally. In the process of liquid displacement, the exponent n is initially, when t = 0, equal to the adiabatic exponent. Then it decreases rather rapidly and sharply, after which it begins to increase, and asymptotically approaches (according to the data of theoretical calculations) the value of the isothermic exponent, which is equal to one. In calculating the required accumulator volume for realizable systems we accept:

$$1.1 < n < 1.2. \qquad (2.139)$$

3. <u>The Energy Equation for a Solid-Reactant Accumulator</u>

In agreement with equation (2.118), and considering G = const, we have:

$$\varepsilon_c(V_a\dot{p}_a+p_a\dot{V}_a)+V_t\dot{p}_t+k\frac{Gp_t}{\varrho_l}=\xi\frac{R}{c_{vt}}(\dot{Q}_0-\dot{Q}_a-\dot{Q}_t). \qquad (2.140)$$

Here $\dot{Q}_a$ and $\dot{Q}_t$ have minus signs, since the thermal flows in the case examined are directed from the system to the surrounding medium.

The elementary quantity of heat per second released during charge combustion will equal:

$$\dot{Q}_0 = c_v T_1 \dot{Y}_{sp}, \qquad (2.141)$$

where T_1 is the combustion temperature;

Y_{sp} is the mass of the charge, and

$$Y_{sp} = \varrho_{sp} \int_0^l \varphi(l)\, dl = \varrho_{sp} \int_0^t \varphi(l)\, u\, dt; \qquad (2.142)$$

$$\dot{Y}_{sp} = \varrho_{sp} \varphi(l)\, u. \qquad (2.143)$$

For a cylindrical charge with burning from the end

$$\varphi(l) = \frac{\pi D^2}{4} = \text{const}; \qquad (2.144)$$

$$\dot{Y}_{sp} = \varrho_{sp} \frac{\pi D^2}{4} u. \qquad (2.145)$$

For the combustion velocity we shall assume the law [45]:

$$u = u_0 + ap^{\nu}. \qquad (2.146)$$

Now in place of equations (2.142) and (2.143) we have:

$$Y_{sp} = \varrho_{sp} \int \varphi(l)(u_0 + ap^{\nu})\, dt; \qquad (2.147)$$

$$\dot{Y}_{sp} = \varrho_{sp} \varphi(l)(u_0 + ap^{\nu}). \qquad (2.148)$$

Let us examine the equation:

$$\frac{R}{c_v} Q_0 = \frac{R}{c_v} c_v T_1 \dot{Y}_{sp} = RT_1 \dot{Y}_{sp} \qquad (2.149)$$

Considering that the force of the solid powder [45], i.e., the efficiency of the solid propellant gases, equals

$$f = RT_1, \qquad (2.150)$$

in place of equation (2.149) we find that:

$$\frac{R}{c_v} \dot{Q}_0 = f\dot{Y}_{sp} = kf_0\dot{Y}_{sp}, \qquad (2.151)$$

or

$$\frac{R}{c_v} \dot{Q}_0 = kf_0 \varrho_{sp} \varphi(l)(u_0 + ap^{\nu}), \qquad (2.152)$$

where f_0 is the cited solid propellant force.

The law of combustion velocity depends on the quantity of solid propellant, on combustion conditions and on other factors. Therefore equation (2.146) may be not employed in all cases. In carrying out calculations, the law of combustion velocity is chosen while considering the specific situation, and while directing our attention to the results of special investigations [13].

Equation (2.141) may also be written as follows:

$$\dot{Q}_0 = c_p T_a \dot{Y}_{sp} \qquad (2.153)$$

In place of equation (2.149) we obtain:

$$\frac{R}{c_v}\dot{Q}_0 = \frac{R}{c_v} c_p T_a \dot{Y}_{sp} = kRT_a \dot{Y}_{sp} \qquad (2.154)$$

Considering that

$$RT_a = f_0, \qquad (2.155)$$

we obtain, just as in equation (2.151):

$$\frac{R}{c_v}\dot{Q}_0 = kf_0 \dot{Y}_{sp}. \qquad (2.156)$$

The energy equation has the following form:

$$\varepsilon_c(V_a\dot{p}_a + p_a\dot{V}_a) + V_t\dot{p}_t + kp_t\dot{V}_t = \xi\left[kf_0\varrho_{sp}\varphi(l)(u_0 + ap^\nu) - \frac{R}{c_v}(\dot{Q}_a + \dot{Q}_t)\right]. \qquad (2.157)$$

Powder force f_0 is a reference value and a constant, whereas value RT_a may change with time. Therefore after the introduction of the value of the powder force into the calculation, the equation assumes a quasi-statistical character.

If p_t = const, then equation (2.157) is written as follows:

$$p_t dV_t = \eta\xi f_0 dY_{sp}, \qquad (2.158)$$

where coefficient η takes into consideration thermal losses and accumulator parameter changes;

$$\eta = 1 - \frac{\frac{R}{c_v}(\dot{Q}_a + \dot{Q}_t) + \frac{\varepsilon_c}{\xi}\frac{d}{dt}(p_a V_a)}{kf_0\dot{Y}_{sp}}. \qquad (2.159)$$

Integration of equation (2.158) is possible if η is established according to the results of the processing of experimental data or if the average value is computed and employed. In the latter case

$$p_t = \eta \frac{f_0 \gamma_{sp}}{V_t}. \quad (2.160)$$

4. The Energy Equation for a Hot (Liquid) Accumulator

If we accept $\dot{p}_a = \dot{V}_a = 0$, then the energy equation without considering the combustion time lag is written as follows:

$$V_t \dot{p}_t + k p_t \dot{V}_t = \xi \frac{R}{c_{v_t}} (\dot{Q}_0 - \dot{Q}_a - \dot{Q}_t). \quad (2.161)$$

The quantity of released heat [11] will equal:

$$Q_0 = \int_{\tau}^{t} (G_1 + G_2) c_p T_a \, dt, \quad (2.162)$$

where G_1 and G_2 are mass propellant constituent flow rates feeding the generator.

If G_1 = const, G_2 = const, then also $c_p T_a$ = const.

With a constant pressure in the tank, and considering that the thermal flow is directed from the system to the surrounding medium, we have:

$$k p_t \dot{V}_t \approx \xi \left[k (G_1 + G_2) R T_g - \frac{R}{c_{vt}} (\dot{Q}_a + \dot{Q}_t) \right]. \quad (2.163)$$

After transformations under the condition that $\dot{V}_t$ = const, we obtain:

$$p_t = \eta_t \frac{(G_1 + G_2) R T_a}{\dot{V}_t}, \quad (2.164)$$

where the coefficient taking thermal losses into account will equal:

$$\eta_t = \xi \left[1 - \frac{\dot{Q}_a + \dot{Q}_t}{c_p T_a (G_1 + G_2)} \right] \approx \xi \left[1 - \frac{\dot{Q}_a + \dot{Q}_t}{\dot{Q}_0} \right]. \quad (2.165)$$

Coefficient η_t is the coefficient of thermal efficiency, since it represents a ratio of effectively employed heat.

$$\dot{Q} = \dot{Q}_0 - \dot{Q}_a - \dot{Q}_t \quad (2.166)$$

to the general heat quantity released in the hot accumulator:

$$\dot{Q}_0=(G_1+G_2)c_pT_a, \quad (2.167)$$

where

$$\eta_t = \eta_t(t).$$

We note that in engineering practice r_t = const is often accepted for solid-reactant and hot accumulators; the assumption that $\dot{p}_a = \dot{V}_a = 0$ is acceptable only for a hot accumulator, but for the solid-reactant accumulator the admissibility of this assumption must be investigated in each separate case.

§ 5. The Equation of the Conservation of Mass for the Tank Pressurizing System

The law of the conservation of mass for the accumulator without considering ignition time lag is written in the following manner:

$$Y = Y_u + Y_a, \quad (2.168)$$

where Y is the propellant mass, which has entered the accumulator by time t;

Y_u is the mass of the products discharged by the same time;

Y_a is the mass of products, having accumulated within the accumulator.

The equation of the conservation of mass for the tank may be written as follows:

$$Y_u = Y_t, \quad (2.169)$$

where Y_t is the gas mass in the tank by time t.

The equation of the conservation of mass for the entire pressurizing system

$$\dot{Y} = \dot{Y}_a + \dot{Y}_t, \quad (2.170)$$

follows from equations (2.168) and (2.169).

Let us return to equation (2.168). In order to make further transformations we shall employ the equation of state; we find that:

$$\dot{Y}_a = \frac{1}{R_a T_a}\left[p_a \dot{V}_a + V_a \dot{p}_a - p_a V_a \frac{\frac{d}{dt}(R_a T_a)}{R_a T_a}\right]. \qquad (2.171)$$

The main mass of discharge gases is

$$Y_u = \int_0^t G\,dt, \qquad (2.172)$$

therefore [7],

$$\dot{Y}_u = \frac{F_{cr} a}{(R_a T_a)^{0,5}} p_a, \qquad (2.173)$$

where, just as formerly,

$$a = n^{0,5}\left(\frac{2}{n+1}\right)^{\frac{n+1}{2(n-1)}}. \qquad (2.174)$$

The derivative from the total flow is

$$\dot{Y} = G_1 + G_2, \qquad (2.175)$$

where G_1 and G_2 are mass flow rates for propellant constituents feeding the generator.

Substituting the values of the derivatives obtained in equation (2.168), we find that:

$$(G_1 + G_2) - \frac{F_{cr} a}{(R_a T_a)^{0,5}} p_a = \frac{\frac{d}{dt}(p_a V_a)}{R_a T_a} - \frac{p_a V_a}{R_a T_a}\,\frac{\frac{d}{dt}(R_a T_a)}{R_a T_a}. \qquad (2.176)$$

Now we employ equation (2.169) and the equation of state for the gas included within the tank; we find that:

$$\int_0^t G\,dt = \frac{p_t V_t}{R_t T_t}. \qquad (2.177)$$

If the liquid flow rate from the tank changes with time, then

$$V_t = V_{t,in} + \frac{1}{\varrho_l}\int_0^t G_l\,dt, \qquad (2.178)$$

where $V_{t,in}$ is the initial free volume in the tank;

G_l is the mass rate of flow per second of liquid from the tank;

ρ_l is the fluid density.

Therefore,

$$p_t = R_t T_t \frac{\int_0^t G\,dt}{V_{t,in} + \frac{1}{\varrho_l}\int_0^t G_l\,dt}. \tag{2.179}$$

Equation (2.179) establishes the connection between gas input into the tank and liquid flow from the tank.

1. The Equation of Mass for the Gas Accumulator

Let us employ equation (2.176). During engine operation the accumulator is not replenished with gas, therefore $(G_1 + G_2) = 0$. The accumulator volume V_a = const, therefore, when R_a = const,

$$F_{cr} a (R_a T_a)^{0,5} = -V_a \left(\frac{\dot{p}_a}{p_a} - \frac{\dot{T}_a}{T_a}\right). \tag{2.180}$$

If the process of gas flow is considered to be polytropic, then

$$\frac{\dot{T}_a}{T_a} = \frac{n-1}{n}\frac{\dot{p}}{p_a};\quad T_a = T_{a\,in}\left(\frac{p_a}{p_{a\,in}}\right)^{\frac{n-1}{n}}. \tag{2.181}$$

Now in place of (2.180) we may write:

$$F_{cr} a (R_a T_a)^{0,5} = -\frac{V_a}{n}\frac{\dot{p}}{p_a}. \tag{2.182}$$

After transformations:

$$\dot{p}_a = -Z p_a^{\frac{3n-1}{2n}}, \tag{2.183}$$

where

$$Z = n\frac{F_{cr} a (R_{a\,in} T_{a\,in})^{0.5}}{V_a p_{a\,in}^{\frac{n-1}{2n}}}. \tag{2.184}$$

Having separated the variables and integrating, we find that:

$$t=\frac{2V_a\left[\left(\frac{p_{a\,in}}{p_a}\right)^{\frac{n-1}{2n}}-1\right]}{(a-1)F_{cr}\,a\,(R_{a\,in}T_{a\,in})^{0.5}}. \tag{2.185}$$

Equation (2.185) permits us to plot a curve of pressure change versus time in the accumulator. Further, through equation (2.182) we may find temperature change T_a versus time, and from equation (2.173), in a case of super-critical discharge, we may establish the relationship between the gas flow rate and discharge time. In order to make such a calculation, graphs of functions $F_{cr}(t)$ and $n(t)$ are required, which are obtained through calculation or through the evaluation of experimental data.

2. The Equation of Mass for a Solid-Reactant Accumulator

Let us take advantage of equation (2.176). Considering that

$$f_0 = R_a T_a, \tag{2.186}$$

$$G_1 + G_2 = \dot{Y}_{sp}, \tag{2.187}$$

we find that:

$$f_0\dot{Y}_{sp} - F_{cr}af_0^{0.5}p_a = p_a\dot{V}_a + \frac{1}{n}V_a\dot{p}_a. \tag{2.188}$$

If $\dot{p}_a = \dot{V}_a = 0$, then

$$\dot{Y}_{sp} = \frac{F_{cr}a}{f_0^{0.5}}p_a. \tag{2.189}$$

Equation (2.160) permits us to determine the required weight of the charge, and equation (2.189) together with (2.143) and (2.146) make it possible to determine the required geometric dimensions of the charge, if pressure conditions have been chosen and if the powder characteristics are known.

§ 6. The Pressure Characteristic of a Centrifugal Pump

The centrifugal pump is one of the most important subassemblies of the feed system, which provides for movement of the propellant constituents through the hydraulic lines. The pump is driven from the turbine and provides the rated head (pressure) and the required flow rate at the rated number of revolutions. Upon deviation from the rated number of revolutions, changes in the head and in the flow rate occur. If the rated number of revolutions are maintained, but the head changes, then the flow rate will also change. In all of these cases, the power required by the pump and the efficiency will, as a rule, also change.

A change in the operating condition of the pump occurs for various reasons. All these reasons may be grouped in the following manner:

a) those that depend on the characteristics and peculiarities of turbine operation;

b) those that depend on the hydraulic line parameters and the pressurizing system;

c) those that depend on the parameters of the pump itself.

Pump parameters are chosen during design and are refined experimentally. Under conditions involving an unsteady operating condition, they change with time. Small deviations from rated values occur under the influence of external factors and also under the influence of production, technological and operating factors.

The study of an unstable engine operating condition and the influence of internal and external actions are possible in case the interrelationship between the power required by the pump, its efficiency, produced by the pump, and the liquid flow rate are known. This interrelationship is centrifugal pump characteristic.

The complexity of the mathematical equation of the characteristic depends on the theory accepted and the assumptions made, which, in their own turn, are determined by the aim of the investigation and the required accuracy of the calculation. In the paragraph below, one of the possible engineering methods is given which permits us to obtain a solution in the form of convenient engineering formulas.

During an unstable operating condition the law of the conservation of energy for a liquid in motion in the circulating part of the pump is written as follows:

$$dE=d(E_M+E_L+E_u+E_r+E_c+E_{hk}+E_{hc}+E_y), \qquad (2.190)$$

where dE is the elementary energy supplied from the drive to the pump shaft;

dE_M is a liquid energy change due to an angular momentum change in the circulating channel;

dF_L is a liquid energy change due to a change in the radial velocity component in the channel;

dE_u is the elementary energy expended in changing the angular velocity of rotation of the liquid in the circulating part of the impeller with time;

dE_r is the elementary energy expended in changing the radial velocity component of the liquid with time;

dE_c is the energy change due to the effect of mass forces in a spiral chamber;

dE_{hk} is an elementary change in energy which is expended in overcoming viscous forces in the circulating part of the impeller;

dE_{hc} is the same, in the circulating part of the spiral chamber;

dE_y is an energy change due to leaks.

1. The Head Created by the Pump

Let torque M be applied to the pump shaft; if the angular velocity of the shaft is ω, then the elementary energy transmitted to the pump is

$$dE = M\omega dt. \tag{2.191}$$

If all this energy is transformed into liquid energy, then

$$dE = H_t G dt, \tag{2.192}$$

where H_t is the theoretical head, produced by the pump;

G is the mass rate of flow of liquid per second.

Therefore,

$$H_t = \frac{M\omega}{G}. \tag{2.193}$$

The effective head

$$H = H_t \eta_p, \tag{2.194}$$

where η_p is the pump efficiency.

We must keep in mind that

$$\eta_p = \eta(H, G, \omega). \qquad (2.195)$$

In order to determine power, we find that:

$$N_t = H_t G = M\omega. \qquad (2.196)$$

If we take equation (2.194) into consideration, we obtain:

$$N = \frac{HG}{\eta_p}. \qquad (2.197)$$

The liquid head is computed from the static and the dynamic; the static head is

$$H_c = \frac{p}{\rho_l}, \qquad (2.198)$$

and the dynamic is:

$$H_{dy} = \frac{C^2}{2}. \qquad (2.199)$$

The sum (total) head is expressed as follows:

$$H = \frac{p}{\rho_l} + \frac{C^2}{2}, \qquad (2.200)$$

but the head increment at the pump is

$$\Delta H = \frac{p_2 - p_1}{\rho_l} + \frac{C_2^2 - C_1^2}{2}, \qquad (2.201)$$

where the indexes 1 and 2 characterize the input and output parameters.

2. The Liquid Energy Increment Generated as a Result of A Moment Change in the Channel

Let us examine the starting and operating functions but under the condition that the entire circulating part of the pump is filled with liquid and that cavitation is not occurring. The design diagram is shown in Figure 16.

Let us assume that the head develops only as a result of the reason under consideration; the condition (2.193) is written as follows:

$$H_M = \frac{M_2 - M_1}{G'}\omega. \qquad (2.202)$$

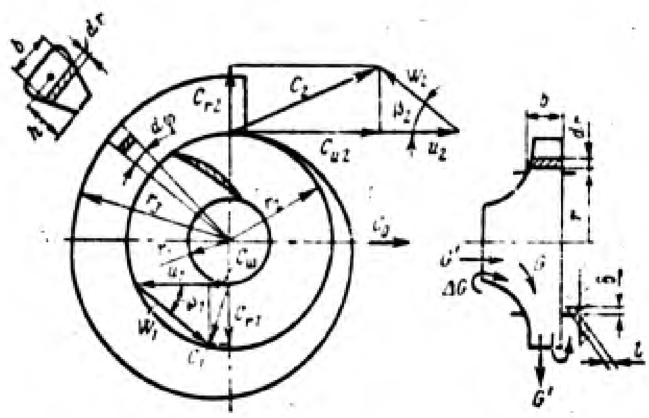

Figure 16. Diagram of the circulating part of a centrifugal pump.

At the output of the impeller

$$M_2 = G'C_{u2}r_2, \qquad (2.203)$$

where G' is the flow through the impeller;

C_{u2} is the tangential velocity component of the liquid at the output of the impeller.

At the input to the impeller

$$M_1 = G'C_{u1}r_1. \qquad (2.204)$$

Therefore,

$$H_{Mt} = (C_{u2}r_2 - C_{u1}r_1)\,\omega. \qquad (2.205)$$

By the velocity triangles we find that:

$$C_{u2} = U_2 - \frac{C_{r2}}{\tan\beta_2}; \qquad (2.206)$$

$$C_{u1} = U_1 - \frac{C_{r1}}{\tan\beta_1}. \qquad (2.207)$$

By the equation of continuity, we have:

$$C_{r1} = C_{r2}\frac{2r_2b_2k_2}{(r_1^2 - r_0^2)\,k_1}; \qquad (2.208)$$

$$C_{r2}=\frac{\varepsilon G}{2\pi r_2 b_2 \varrho_l k_2},\qquad (2.209)$$

where

$$\varepsilon = \frac{G'}{G};\qquad (2.210)$$

b_2 is the width of the circulating part of the impeller at the exit;

r_0 is the hub radius;

k_i is the coefficient of the decrease in section dimensions.

According to the moment equations we have:

$$U_1=\frac{r_1}{r_2}U_2.\qquad (2.211)$$

Effecting a substitution and transformation, we find that:

$$H_M=\left[(r_2^2-r_1^2)\,\omega^2-\frac{\varepsilon}{2\pi\varrho_l}\left(\frac{1}{b_2k_2\tan\beta_1}-\frac{2r_1}{(r_1^2-r_0^2)k_1\tan\beta_1}\right)\omega G\right]\eta_M.\qquad (2.212)$$

If the coefficients are calculated then equation (2.212) is presented as follows:

$$H_M=D_M\omega^2-D_M'\omega G.\qquad (2.213)$$

3. A Liquid Energy Change, Generated as a Result of a Change in the Radial Velocity Component in the Channel Length

As a result of a change in the radial component, a change in the kinetic energy of the flow is observed, thus the true head decrease will be:

$$H_{Lt}=\frac{C_{r2}^2-C_{r1}^2}{2}.\qquad (2.214)$$

Employing the equation of continuity, after transformations we obtain:

$$H_L=\frac{\varepsilon^2}{2(2\pi r_2 b_2 k_2 \varrho_l)^2}\left[1-\left(\frac{2r_2b_2k_2}{(r_1^2-r_0^2)\,k_1}\right)^2\right]G^2\eta_L,\qquad (2.215)$$

or

$$H_L = D_L G^2.\qquad (2.216)$$

4. Head Decrease as a Result of Energy Expenditure on a Change in the Angular Velocity of Fluid Rotation with Time

Under unsteady operating conditions, the energy expended in overcoming mass forces in rotary motion is expressed as follows:

$$E_t = \iint \omega_l \, dJ_l \, d\omega_l, \qquad (2.217)$$

where dJ_l is the moment of inertia of the liquid which is filling the ring with width dr.

The energy increment on this elementary path is

$$E_t = \iint G \, dH_u \, dt. \qquad (2.218)$$

In agreement with equations (2.217) and (2.218) we may write:

$$dH_u = \frac{\omega_l}{G} \dot{\omega}_l \eta_u \, dJ_l; \qquad (2.219)$$

since

$$dJ_l = r^2 dm, \qquad (2.220)$$

then

$$H_u = \frac{1}{G} \int \omega_l \dot{\omega}_l r^2 \eta_u \, dm. \qquad (2.221)$$

The elementary mass is:

$$dm = 2\pi r b k \varrho_l dr; \qquad (2.222)$$

the angular velocity for the liquid is:

$$\omega_l = \frac{C_u}{r}; \qquad (2.223)$$

the tangential component is:

$$C_u = \omega r - \frac{\iota G}{2\pi r b k \varrho_l \tan\beta}, \qquad (2.224)$$

where ω is the angular velocity of the impeller (but not the liquid).

The radial velocity component

$$C_r = \frac{dr}{dt} \qquad (2.225)$$

cannot change the moment balance, since C_r is perpendicular to C_u;

$$\dot{\omega}_l = \frac{\dot{C}_u}{r}; \tag{2.226}$$

$$\dot{\omega}_l = \dot{\omega} - \frac{\varepsilon \dot{G}}{2\pi r^2 b k \rho_l \tan\beta} \tag{2.227}$$

Finally, in place of equation (2.221), we obtain:

$$H_u = \frac{2\pi\rho_l}{G} \int_{r_1}^{r_2} \left(\omega - \frac{\varepsilon G}{2\pi r^2 b k \rho_l \tan\beta}\right) \times \left(\dot{\omega} - \frac{\varepsilon \dot{G}}{2\pi r^2 b k \rho_l \tan\beta}\right) r^3 b k \tau_u \, dr. \tag{2.228}$$

It must be kept in mind that b, k and β are functions of r. Considering that ω, G and derivatives do not depend on r, we write equation (2.228) as follows:

$$H_u = \left(D_u \frac{\omega}{G} + D_u'\right)\dot{\omega} + \left(D_u'' \frac{\omega}{G} + D_u'''\right)\dot{G}. \tag{2.229}$$

In order to determine D_u; D'_u; D''_u; D'''_u it is necessary to solve the integral (2.228). If the variables are given in functions of r, then the solution is most easily obtained through numerical integration.

5. The Pressure Decrease in the Circulating Part of the Impeller Due to Energy Expenditure on the Change with Time of the Liquid Radial Velocity Component

The change in power -- a derivative of energy with time -- is:

$$N_r = \int \dot{G} \, dH_r. \tag{2.230}$$

From the other side this equation will equal:

$$N_r = \int C_r dP. \tag{2.231}$$

for a mass element we have

$$dP = \dot{C}_r dm. \tag{2.232}$$

Thus

$$dH_{rl} = \frac{C_r \dot{C}_r}{G} dm, \tag{2.233}$$

or

$$H_{rt}=\int \frac{C_r \dot{C}_r}{G}\, dm. \quad (2.233a)$$

Since

$$C_r = \frac{\varepsilon G}{2\pi r b k \varrho_l}, \quad (2.234)$$

then finally we have:

$$H_r = \frac{\varepsilon^2 \dot{G}}{2\pi \varrho_l} \int_{r_1}^{r_2} \eta_r \frac{dr}{rbk}. \quad (2.235)$$

If the integral solution is obtained, then equation (2.235) is written in the following form:

$$H_r = D_r \dot{G}. \quad (2.236)$$

6. The Pressure Decrease Due to the Effect of Mass Forces in a Spiral Chamber

Analogous to equation (2.233), after integration we have:

$$H_{ct}=\int \frac{C \dot{C}}{G_t}\, dm. \quad (2.237)$$

The elementary mass is expressed as follows:

$$dm = rb\varrho_l\, d\varphi\, dr. \quad (2.238)$$

Usually we accept that

$$G_t = \frac{\varphi}{2\pi} G; \quad (2.239)$$

$$C = \frac{r_2}{r} C_{u2}. \quad (2.240)$$

Just as formerly,

$$C_{u2} = \omega r_2 - \frac{\varepsilon G}{2\pi r_2 b_2 k_2 \varrho_l \tan\beta_2}; \quad (2.241)$$

Considering the propositions already stated, we find that

$$\dot{C}_{u2} = \dot{\omega} r_2 - \frac{\varepsilon \dot{G}}{2\pi r_2 b_2 k_2 \varrho_l \tan\beta_2}; \quad (2.242)$$

$$\dot{C} = \frac{r_2}{r} \dot{C}_{u2}. \quad (2.243)$$

Finally we obtain:

$$H_c=2\pi\frac{\varrho l}{G}\int_{\varphi}^{2\pi}\int_{r_2}^{r_3}\frac{1}{\varphi}\left(\frac{r_2}{r}\right)^2\left(\omega r_2-\frac{\varepsilon G}{2\pi r_2 b_2 k_2 \varrho_l \tan\beta_2}\right)\times\left(\dot{\omega} r_2-\frac{\varepsilon \dot{G}}{2\pi r_2 b_2 k_2 \varrho_l \tan\beta_2}\right)\eta_c r b\, d\varphi\, dr. \quad (2.244)$$

First it is necessary to take the internal integral and considering that r_3 is a function of angle ϕ, before executing the second integration we shall place

$$r_3=\frac{d_{30}}{2\pi}\varphi+r_2, \quad (2.245)$$

in the integrand. Where d_{30} is the diameter of the exit throat.

After completing the integration we have

$$H_c=\left(D_c\frac{\omega}{G}+D_c'\right)\dot{\omega}+\left(D_c''\frac{\omega}{G}+D_c'''\right)\dot{G}. \quad (2.246)$$

7. A Head Decrease Due to the Effects of Viscous Forces in the Circulating Part of the Impeller

As is known

$$dH_{kx}=\lambda_k\frac{W^2}{2d_e}dl, \quad (2.247)$$

where W is the relative velocity;

d_e is the equivalent dimension;

l is the length of the track washed by the fluid;

λ_k is the friction coefficient.

From the velocity triangle we have:

$$W=\frac{C_r}{\sin\beta}; \quad (2.248)$$

$$dl=\frac{dr}{\sin\beta}. \quad (2.249)$$

In agreement with equation (2.234)

$$C_r=\frac{\varepsilon G}{2\pi r b k \varrho_l}. \quad (2.250)$$

Since

$$d_e=4\frac{F}{\Pi}, \quad (2.251)$$

$$d_e = 4\,\frac{\pi r b k}{2\pi r + Z b}. \qquad (2.252)$$

Therefore,

$$H_{h\kappa} = \lambda_\kappa \frac{(\iota G)^2}{32\pi^2 \varrho_l^2} \int_{r_1}^{r_2} \eta_{h\kappa} \frac{2\pi r + Z b}{(r b k)^3 \sin^3 \beta}\, dr \qquad (2.253)$$

or

$$H_{h\kappa} = D_{h\kappa} G^2. \qquad (2.254)$$

8. A Head Decrease Due to the Effects of Viscous Forces in the Circulating Part of a Spiral Chamber

Analogous to equation (2.247) we may write:

$$dH_{hc} = \lambda_c \frac{C^2}{2 d_e}\, dl. \qquad (2.255)$$

From the equation of continuity we have:

$$C = \frac{G_l}{F_l \varrho_l}, \qquad (2.256)$$

and in this connection if

$$F_l = \frac{\varphi}{2\pi} F; \qquad (2.257)$$

$$G_l = \frac{\varphi}{2\pi} G, \qquad (2.258)$$

then

$$\frac{G_l}{F_l} = \text{const}. \qquad (2.259)$$

For an average line of the spiral

$$r = r_2 + a\varphi. \qquad (2.260)$$

Therefore,

$$dl = 2\pi a\, d\varphi. \qquad (2.261)$$

If we accept that

$$F_l = \pi r_c^2 = \frac{\varphi}{2\pi} F, \qquad (2.262)$$

then

$$r_c = \frac{1}{\pi}\left(\frac{\varphi}{2}F\right)^{0,5}. \qquad (2.263)$$

Accepting that

$$\Pi = 2\pi r_c = (2\varphi F)^{0,5}, \qquad (2.264)$$

we obtain

$$d_e = 2^{1,5}\frac{(\varphi F)^{0,5}}{\varphi_0} = \frac{(2\varphi F)^{0,5}}{\pi}, \qquad (2.265)$$

where F is the area of the circulating channel section of the spiral chamber, corresponding to $\phi_0 = 2\pi$.

After integration of equation (2.255), we obtain

$$H_{hc} = \frac{\pi^{2,5}\lambda_c a}{0,5}\cdot\frac{G^2}{F^{2,5}\varrho_l^2}\eta_{hc}. \qquad (2.266)$$

It is apparent that

$$H_{hc} = D_{hc}G^2. \qquad (2.267)$$

9. A Head Change Resulting from Leaks in the Area of the Operating Impeller

If the dimensions of the gap in the packing rings are l and δ, then the leakage through one seal, when $\sigma = 2\pi r\delta$ will be:

$$\Delta G_y = \mu\sigma(2\varrho_l H_p)^{0,5}; \qquad (2.268)$$

the coefficient of discharge is

$$\mu = \left(\frac{1}{\varepsilon} + \frac{4\delta}{\lambda l}\right)^{0,5}, \qquad (2.269)$$

where λ is the coefficient of friction for the seal channel;

ε is the coefficient of resistance for the channel, calculated without considering losses in the seal.

Under stable conditions, in agreement with equations (2.213), (2.216), (2.236), (2.254) and (2.267), and with no leakage, we have

$$H_p = D_м\omega^2 - D'_м\omega G + (D_L - D_{hк} - D_{hc})G^2. \quad (2.270)$$

Differentiating equation (2.270) and passing to small deviations when ω = const, we obtain:

$$\Delta H_{p1} = [-D'_м\omega + 2(D_L - D_{hк} - D_{hc})G]\Delta G. \quad (2.271)$$

Considering the presence of two leakage channels in determining the loss of head, we obtain:

$$\Delta H_p = H_{y1} = 2\left[2(D_L - D_{hк} - D_{hc}) - D'_м\frac{\omega}{G}\right]G\Delta G_y. \quad (2.272)$$

Keeping equation (2.268) in mind, we obtain:

$$H_y = 2\left[2(D_L - D_{hк} - D_{hc}) - D'_м\frac{\omega}{G}\right]\mu\sigma\eta_y(2\varrho_l)^{0.5}\times$$
$$\times[D_м\omega^2 - D'_м\omega G + (D_L - D_{hк} - D_{hc})G^2]^{0.5}. \quad (2.273)$$

Head H_y is incommensurably small in comparison to the full head produced by the impeller. Therefore for transformations of equation (2.273) it is permissible to accept the static equation of the centrifugal pump:

$$G = k_p\omega. \quad (2.274)$$

Now we obtain

$$H_y = 2\left[2(D_L - D_{hк} - D_{hc}) - \frac{D'_м}{k_p}\right]\mu\sigma\eta_y(2\varrho_l)^{0.5}\times$$
$$\times[D_м - k_p D'_м + (D_L - D_{hк} - D_{hc})k_p^2]^{0.5}\omega. \quad (2.275)$$

In place of equation (2.275) we may write:

$$H_y = D_y\omega. \quad (2.276)$$

10. Design Formulas

Under unstable operating conditions, in agreement with the functions already obtained, the head, produced by the pump, will be

$$H_{\text{p}} = D_{\text{м}}\omega^2 - D'_{\text{м}}\omega G - D_{L\,\text{кс}}G^2 - D_{\text{y}}\omega - - \left[D_{\text{и с}}\frac{\omega}{G} + D'_{\text{и с}}\right]\dot{\omega} - \left[D''_{\text{и с}}\frac{\omega}{G} + D''_{\text{и с}}\right]\dot{G}, \quad (2.277)$$

where

$$D_{L\,\text{кс}} = D_L - D_{h\,\text{к}} - D_{h\,\text{с}};$$
$$D'_{\text{и с}} = D'_{\text{и}} + D'_{\text{с}}.$$

A stable operating condition is characterized by the equation

$$\dot{\omega} = \dot{G} = 0.$$

In this connection the head will equal:

$$H_{\text{p}} = D_{\text{м}}\omega^2 - D'_{\text{м}}\omega G - D_{L\,\text{кс}}G^2 - D_{\text{y}}\omega. \quad (2.278)$$

Equations (2.277) and (2.278) will be employed in a simplified form in the next chapter.

In organizing the investigation, with the aim of simplifying the equation system, it is necessary to employ a separate calculation to evaluate the "specific weight" of individual terms in equations (2.277) and (2.278) and to exclude some of these from consideration.

§ 7. Determining Turbine Power

An examination of the processes occurring within the subassemblies of the power drive of the feed system, i.e., within the turbine and the generator is outside the scope of this book. However, in order to conduct a complex engine calculation, it is necessary to know the turbine equation which establishes the connection between the power (or the moment), developed by the turbine, on the one hand, and the generator agent flow rate and the angular velocity of shaft rotation, on the other.

In the circulating part of a one-stage turbine, let the mass element of the working medium dm be in motion with velocity C. Elementary momentum dθ will equal:

$$d\theta = dmC. \quad (2.279)$$

A change in element momentum is:

$$d^2\theta = d(dm \cdot C). \quad (2.280)$$

The elementary force moment dP will equal:

$$d^2I = dP\ dt. \quad (2.281)$$

Equalizing equations (2.280) and (2.281), we obtain

$$dPdt = d(dm \cdot C), \quad (2.282)$$

and it follows that:

$$dP = \frac{d}{dt}(dm \cdot C). \quad (2.283)$$

Since

$$\frac{dm}{dt} = G, \quad (2.284)$$

we find that:

$$dP = GdC + CdG. \quad (2.285)$$

For the entire circulating channel

$$P = \int GdC + \int CdG. \quad (2.286)$$

Equation (2.286) determines the force acting on the blades of the operating impeller. Circumferential force

$$P_u = -\int G\,d(C\cos\alpha) - \int C\cos\alpha\,dG, \quad (2.287)$$

will act in the direction of motion toward the blades. Where α is the angle which determines the direction of motion of the working medium in the channels between blades.

Axial force acting on the blades equals:

$$P_0 = -\int Gd(C\sin\alpha) - \int C\sin\alpha\,dG. \quad (2.288)$$

The minus signs in the right side of equations (2.287) and (2.288) are placed there because P_u and P_0 represent forces acting on the blades of the operating impeller, in contrast to force P in equation (2.286), which characterizes a pulse leading to a change in momentum of the working medium. The second integrals of the right sides of equations (2.286), (2.287) and (2.288) characterize a change in the corresponding forces under conditions when $G \neq$ const. During a constant flow of the working medium through the circulating channel of the turbine, the circumferential force

$$P_u = -G \int_{C_1 \cos \alpha_1}^{C_2 \cos \bar{\alpha}_2} d(C \cos \alpha). \qquad (2.289)$$

Thus,

$$P_u = G(C_1 \cos \alpha_1 + C_2 \cos \alpha_2). \qquad (2.290)$$

Here $\alpha_2 = \bar{\alpha}_2 - 180°$.

If the angular velocity of the operating impeller of the turbine is designated by the symbol ω, in order to determine turbine shaft power we have:

$$N_T = \frac{\bar{D}_T}{2} \omega P_u \eta, \qquad (2.291)$$

where $\bar{D}_T$ is the average diameter of the channels between blades;

η is an efficiency factor.

Substituting the value of the circumferential force acting on the blades of the operating impeller from equation (2.290) in (2.291), we find that:

$$N_T = \omega \frac{\bar{D}_T}{2} (C_1 \cos \alpha_1 + C_2 \cos \alpha_2) G \eta. \qquad (2.292)$$

From the velocity triangles at the nozzle output, it follows that

$$C_2 \cos \alpha_2 = w_2 \cos \beta_2 - \omega \frac{\bar{D}_T}{2}. \qquad (2.293)$$

The relative output velocity of the working medium from the circulating part of the channels between the blades equals:

$$w_2 = \psi (2H\rho - w_1^2)^{0.5}, \qquad (2.294)$$

where ψ is a coefficient accounting for losses in the channels between blades;

H is the heat differential at the turbine;

ρ is the degree of blade reaction;

ω_1 is the relative entrance velocity of the working medium in the circulating channel of the turbine.

From the velocity triangle we obtain

$$\omega_1 = \frac{C_1 \cos \alpha_1 - \omega \frac{\bar{D}_T}{2}}{\cos \beta_1}. \quad (2.295)$$

After the required transformations we find that:

$$N_T = \omega \frac{\bar{D}_T}{2} \left\{ C_1 \cos \alpha_1 + \psi \left[2H_Q - \left(\frac{C_1 \cos \alpha_1 - \omega \frac{\bar{D}_T}{2}}{\cos \beta_1} \right)^2 \right]^{0,5} \times \cos \beta_2 - \omega \frac{\bar{D}_T}{2} \right\} G\eta_i. \quad (2.296)$$

In some cases, with the aim of simplifying the mathematical equipment, we accept that

$$C_2 \cos \alpha_2 = C_1 \cos \alpha_1 - \omega \frac{\bar{D}_T}{2}. \quad (2.297)$$

In this connection the power equation assumes the following form:

$$N_T = \omega \frac{\bar{D}_T}{2} \left(2C_1 \cos \alpha_1 - \omega \frac{\bar{D}_T}{2} \right) G\eta_i. \quad (2.298)$$

If all parameters except ω and G are constants in equation (2.298), then for a determination of turbine power we may employ the following formula:

$$N_T = (A - B\omega)\omega G, \quad (2.299)$$

where

$$A = \bar{D}_T C_1 \cos \alpha_1 \eta_i; \quad (2.300)$$

$$B = \frac{\bar{D}_T^2}{4} \eta_i. \quad (2.301)$$

Discharge velocity from the turbine nozzle is calculated from the formula

$$C_1 = \varphi (2\Delta i + C_0^2)^{0,5}, \quad (2.302)$$

where ϕ is a coefficient which takes into consideration losses in the nozzle apparatus;

Δi is the differential in the turbine nozzle apparatus;

C_0 is the working medium velocity at the turbine nozzle approach.

The working medium flow rate under static conditions is:

$$G = \frac{\sum F_{cr} a}{(R_0 T_0)^{0,5}} p_0, \tag{2.303}$$

where $\sum F_{cr}$ is the total area of the critical sections of the turbine nozzle;

R_0, T_0, p_0 are working medium parameters at the entrance to the nozzle apparatus.

Engineering calculations are conducted, as a rule, according to formulas (2.299), (2.302) and (2.303). In an investigation of the operating function of the turbine, the following power formula is used, which has been obtained on the basis of the information stated above:

$$N_T = \omega \frac{\overline{D}_T}{2} \Bigg\{ \varphi (2\Delta i + C_0^2)^{0,5} \cos \alpha_1 + \\ + \psi \left[2H_Q - \left(\frac{\varphi (2\Delta i + C_0^2)^{0,5} \cos \alpha_1 - \omega \frac{\overline{D}_T}{2}}{\cos \beta_1} \right)^2 \right]^{0,5} \times \\ \times \cos \beta_2 - \omega \frac{\overline{D}_T}{2} \Bigg\} \frac{\sum F_{cr} a}{(R_0 T_0)^{0,5}} p_0 \eta \xi, \tag{2.304}$$

where ξ is a coefficient which takes into account the influence of the difference in the operating method of the turbine from the design condition. It must be kept in mind that η and ξ are functions of ω and G.

During conditions involving a variable flow rate, we must employ equation (2.287), which may now be written as follows:

$$N_T = -\frac{\overline{D}_T}{2} \left[\int \omega G \eta (\omega, G)\, d(C \cos \alpha) + \int \omega \eta (\omega, G)\, C \cos \alpha\, dG \right]. \tag{2.305}$$

In carrying out a complex engine calculation, the necessity of determining the turbine shaft moment appears, which during unstable operating conditions equals:

$$M_T = 2\pi(N_T\omega^{-1} - J_T\dot{\omega}), \qquad (2.306)$$

where J_T is the moment of inertia of the rotating part of the turbine.

§ 8. The Feed System Equations

At the beginning of this chapter one of the variations in engine starting was examined as an example. As has already been pointed out, a single standard succession of commands cannot exist. Therefore, in adhering to the order of the fulfillment of operations stated above, we shall write for each of these a system of design equations.

At first, the pressure in the propellant tanks increases. In order to investigate this phase of engine operation, we must include a series of equations, including the equation of the conservation of energy, the conservation of mass and a reducing equation. Depending on the design of the feed system and its operating conditions, the first two equations are simplified and made more accurate. In general form they are presented as equations (2.121) and (2.176). These equations may be rewritten as follows:

$$\frac{c_{va}}{c_{vt}}(V_a\dot{p}_a + p_a\dot{V}_a) + V_{t\,in}\dot{p}_t + p_t\frac{1}{\varrho_l}\times\int_0^t G\,dt + k\frac{Gp_t}{\varrho_l} = \frac{R}{c_{vt}}(\dot{Q}_0 \pm \dot{Q}_c \pm \dot{Q}_t); \qquad (2.307)$$

$$(G_1 + G_2) - \frac{F_{cr}a}{(R_aT_a)^{0.5}}p_a = \frac{\frac{d}{dt}(p_aV_a)}{R_aT_a} - \frac{p_aV_a}{R_aT_a}\cdot\frac{\frac{d}{dt}(R_aT_a)}{R_aT_a}. \qquad (2.308)$$

The number of equations required for an investigation of the pressure increase process in the tanks will depend on the design of the pressurizing system, on the operating conditions and on assumptions made. §§ 4 and 5 of this chapter were devoted to an analysis of the equations of the conservation of energy and mass. In solving the system of pressure equations, it is often necessary to include a reduction equation and a valve equation, which many authors have examined in detail (see, for example, [3, 21, 46]).

Let us assume that the second operation is the turning of the turbopump subassembly shaft to a certain value of angular velocity ω. During this operation the main valves of the hydraulic lines are open. In the process of preliminary turning, it is possible to obtain both nominal propellant circulation and a zero flow rate. If nominal circulation occurs, then the liquid energy delivered by the pumps is expended in overcoming hydraulic circuit losses of the nominal circulation and on the change of kinetic energy of the flows. Under conditions involving a closed nominal circulation, the influence of external forces may be disregarded. Keeping in mind equation (2.79), we may write the following for the hydraulic circuit:

$$\left.\begin{aligned} p_{p1} &= H_{p1}\varrho_1 = a_1 G_1^2 + b_1 \dot{G}_1; \\ p_{p2} &= H_{p2}\varrho_2 = a_2 G_2^2 + b_2 \dot{G}_2. \end{aligned}\right\} \quad (2.309)$$

In order to determine surplus pressure produced by the pump, we must employ equation (2.277). Removing the second degree elements, we obtain:

$$H_{pi} = D_{\text{м}\,i}\omega_i^2 - D'_{\text{м}\,i}\omega_i G_i - D_{L\,\text{кс}\,i} G_i^2 - D_{y\,i}\dot{\omega}_i - D^{\bullet}_{u\,c}\dot{\omega}. \quad (2.310)$$

Substituting the value $H_{p\,i}$ from equation (2.310) in equation (2.309), we now have two equations. We shall employ a turbo pump subassembly equation for the third equation. We shall write the moment balance as follows:

$$M_{\text{т}} = \Sigma M_{pi}, \quad (2.311)$$

where M_T is the turbine shaft moment under unstable operating conditions;

$M_{p\,i}$ is the moment discernible by the pump shaft (also under unstable conditions).

A determination of the turbine shaft moment was examined in § 7. If we direct our attention to equations (2.306) and (2.299), which corresponds to the assumptions discussed in § 7, we obtain:

$$M_{\text{т}} = [(A - B\omega)G - J_{\text{т}}\dot{\omega}]. \quad (2.312)$$

The moment, discerned by the pump shaft, will be:

$$Mp_i = \left[\frac{p_{pi}G_i}{\omega_i \eta_i \varrho_i} + J_{pi}\dot{\omega}_i\right]. \quad (2.313)$$

Employing (2.310), after inserting value $H_{p\,i}$, we have

$$M_{pi} = \left[\frac{(D_{\text{м}\,i}\omega_i^2 - D'_{\text{м}\,i}\omega_i G_i - D_{L\,\text{кс}\,i} G_i^2 - D_{y\,i}\omega - D^{\bullet}_{u\,c}\dot{\omega})\, G_i}{\omega_i \eta_i} + J_{pi}\dot{\omega}_i \right]. \qquad (2.314)$$

The turbo pump subassembly equation has the following form:

$$(A - B\omega)\,G - J_{\text{т}}\dot{\omega} - \sum J_{pi}\dot{\omega}_i = \sum \left[\left(\frac{D_{\text{м}\,i}\omega_i^2}{2\omega_i \eta_i} - \frac{D'_{\text{м}\,i}\omega_i G_i - D_{L\,\text{кс}\,i} G_i^2 - D_{y\,i}\omega - D^{\bullet}_{u\,c}\dot{\omega}}{\omega_i \eta_i} \right) G_i \right]. \qquad (2.315)$$

The sum

$$J_{pa}\dot{\omega} = J_{\text{т}}\dot{\omega} + \sum J_{pi}\dot{\omega}_i \qquad (2.316)$$

characterizes the moment expended on a change in angular velocity of the rotating part of the turbo pump installation.

It is apparent that

$$J_{pa} = J_{\text{т}} + \sum J_{pi}\frac{\dot{\omega}_i}{\dot{\omega}}. \qquad (2.317)$$

If the angular velocities of the turbine and pump shafts are equal, then:

$$J_{pa} = J_{\text{т}} + \sum J_{pi}. \qquad (2.318)$$

As has already been noted, turning of the turbo pumps subassembly is possible without circulation. Under these conditions we must write the following in place of equations (2.309) and (2.310):

$$H_{pi} = (D_{\text{м}\,i}\omega_i^2 - D_{y\,i}\omega_i - D^{\bullet}_{u\,c}\dot{\omega})\,\eta_{i*}, \qquad (2.319)$$

where η_* is the coefficient of agreement.

The turbo pump subassembly equation is written as follows:

$$(A - B\omega)\,G - J_{pa}\dot{\omega} = \sum \left[\left(\frac{D_{\text{м}\,i}\omega_i^2 - D_{y\,i}\omega_i - D^{\bullet}_{u\,c}\dot{\omega}}{\omega_i \eta_i} \right) \eta_{i*} G'_i \right]. \qquad (2.320)$$

In calculating from equation (2.320) it must be kept in mind that G'_i represents the flow rate through the circulating part of the impeller. If the flow rate to the pump under the conditions existing in the problem under consideration is equal to zero, then G'_i differs from zero. A decrease in efficiency is observed in proportion to a decrease in the flow rate through the pump. Therefore, during a complete cessation of liquid circulation through the pump due to the presence of extraneous circulation in the circulating part of the impeller and a sharp decrease in efficiency, the power required by the pump may prove to be significant.

In agreement with the design conventionally accepted, the next operation is the filling of the lines with propellant. This process has been examined in detail in § 3.

Further, the discharge of propellant constituents from the injectors begins. The hydraulic circuit equation may now be written as follows:

$$p_{t i}+p_{p i}+p_{ji}=p+a_i G_i^2+a_{i\,c1} G_i^2+b_i \dot{G}_i. \qquad (2.321)$$

The pressure change with time in the tank is determined from equations presented in §§ 4 and 5. Surplus pressure produced by the pump is computed in accordance with formulas cited in § 6.

Let us examine a method for determining the pressure produced by external forces. If the engine is operating on a test stand, then external forces are due only to the gravitational field. In this connection

$$P_{ji} = m_i g. \qquad (2.322)$$

In flight

$$P_{ji} = m_j (g + \dot{V}), \qquad (2.323)$$

where V is the velocity of the rocket's flight .

The liquid mass in an elementary segment of a pipe with an arbitrary shape will equal:

$$dm = F(x)\rho_l dx, \qquad (2,324)$$

and the pressure

$$p_{ji}=(g+\dot{V})_x \rho_l x_i. \qquad (2.325)$$

If the channel has a complex shape, then

$$p_j = (g + \dot{V})_x \rho_l \Sigma x_i. \qquad (3.326)$$

During an unstable operating condition the propellant reactant ratio may be maintained as a constant or it may change in accordance with the principle selected previously.

Let us examine the conditions for which $\dot{K} = 0$ during an unstable operating condition. Non-fulfillment of the conditions examined leads to the fact that the propellant reactant ratio K will change with time.

Since

$$K = \frac{G_1}{G_2}, \qquad (2.327)$$

then, under a condition when K = const,

$$\dot{G}_1 = K\dot{G}_2. \qquad (2.328)$$

The hydraulic circuit equations have the following form:

$$b_1 K\dot{G}_2 - D_1\omega^2 + D_1'\omega K G_2 - p_{t1} - a_1 K^2 G_2^2 + p + p_1 = 0; \qquad (2.329)$$

$$b_2\dot{G}_2 - D_2\omega^2 + D_2'\omega G_2 - p_{t2} - a_2 G_2^2 + p + p_2 = 0, \qquad (2.330)$$

where p_1 and p_2 consider the influence of the terms removed.

The condition K = const obtains if the coefficients in equations (2.329) and (2.330) are equal for the terms comprising G_2 or its derivative in identical degrees.

During a stable operating condition, the first terms of equations (2.329) and (2.330) equal zero, therefore conditions established by the given value K during stable operation will be:

$$D_1\omega^2 - D_2\omega^2 = p_{t2} - p_{t1}; \qquad (2.331)$$

$$D_2' = KD_1'; \qquad (2.323)$$

$$a_2 = K^2 a_1. \qquad (2.333)$$

If the operating condition is unstable, then the fourth condition appears:

$$b_2 = Kb_1. \qquad (2.334)$$

In designing an engine it is rather difficult to satisfy condition (2.333), since usually the hydraulic losses in the fuel circuit are greater than in the oxidizer circuit. It is possible to correct the situation, for example, by employing the oxidizer for cooling the generator chamber or by using the oxidizer in heat exchangers, or by cooling the main chamber with both propellant constituents.

It is possible to provide the required value K during unstable operating conditions in another manner. Let us stipulate that $\dot{G}_i = 0$ and solve the equation system as follows:

$$G_1 = \frac{-D_1'\omega + \left[(D_1'\omega)^2 + 4a_1(p_{t1} + D_1\omega^2 - p)\right]^{0.5}}{2a_1}; \tag{2.335}$$

$$G_2 = \frac{-D_2'\omega - \left[(D_2'\omega)^2 + 4a_2(p_{t2} + D_2\omega^2 - p)\right]^{0.5}}{2a_2}. \tag{2.336}$$

Let us divide both sides of equation (2.334) by K and then, by equalizing the less sides of the equation obtained and equation (2.336), we find that:

$$\frac{D_1'\omega - \left[(D_1'\omega)^2 + 4a_1(p_{t1} + D_1\omega^2 - p)\right]^{0.5}}{D_2'\omega - \left[(D_2'\omega)^2 + 4a_2(p_{t2} + D_2\omega^2 - p)\right]^{0.5}} = K^{-1}. \tag{2.337}$$

However, if under stable operating conditions the given value K is provided with the aid of equation (2.337), then under conditions of unstable operation a deviation from the nominal value of K may be detected. The fact that we have examined a method for establishing K = const under unstable operating conditions does not imply a requirement that K = const in all cases.

Cavitation may occur during the engine starting function, in the case of a sharp decrease in hydraulic line resistance at the pump or an increase in pump input resistance, under the influence of mass forces in the circulating part of the operating pump impellers. In this connection a significant decrease in propellant consumption and discharge of the turbine will be observed. The abnormal phenomenon described may be eliminated by increasing the cavitational qualities of pumps, by decreasing the values of mass coefficients b_i, by increasing the starting time, by a temporary pressure increase, and by other means.

§ 9. An Approximation Method for Determining Heat Losses and the Quantity of Vapor Condensing in the Tank

The investigation of tank pressurizing systems revealed that the heat transfer between gases entering the tank from the accumulator and with the surfaces limiting gas volume exert a significant influence on the process of liquid displacement. In addition to the change in heat balance considered in § 4 through the introduction of the derivatives $\dot{Q}_a$ and $\dot{Q}_t$, in some cases a decrease is observed in gas quantity in the tank cavity adjacent to the liquid surface, due to vapor condensation on the cold surfaces. The process of condensation will also continue on the liquid surface.

Let us consider heat transfer without considering vapor condensation. A detailed calculation may be carried out if information is present concerning the specific design of the tank, the pressurizing system, the engine in general, and given operating conditions. We shall examine a simplified problem with the aim of illustrating the approximation method.

A gas enters the upper tank cavity with temperature T higher than the temperature of the internal wall surface T_c and will heat the upper surface of the tank, the displaced liquid, the walls and the fixtures located in the tank. In the case of convectional heat flows alone, we have

$$dQ = [\alpha_b S_b(T - T_b) + \alpha_l S_l (T - T_l) + \alpha_c S_c (T - T_c) + \alpha_a S_a (T - T_a)]\,dt, \qquad (2.338)$$

where α_i is the heat transfer coefficient;

S_i is the heated surface.

As a result of heating, the tank wall receives heat

$$dQ_c = \xi_c \rho_c S_c \delta_c c_c dT_c, \qquad (2.339)$$

where ξ_c is a coefficient which takes into consideration the difference between the mean wall temperature and temperature T_c, the temperature of its internal surface;

δ_c is the tank wall thickness;

c_c is the specific heat of the tank wall material.

For the bottom and for the fittings we have:

$$dQ_b = \xi_b \rho_b S_b \delta_b c_b dT_b; \qquad (2.340)$$

$$dQ_a = \xi_a m_a c_a dT_a, \quad (2.341)$$

where m_a is the mass of the heated fittings.

The heat expended in heating the liquid equals:

$$dQ_l = \xi_l Y_l c_l dT_l. \quad (2.342)$$

Since the mass of the liquid Y_l is quite high, the liquid temperature increment usually proves to be insignificant.

A change in internal gas energy is observed as a result of the heat transfer. Considering that the gas quantity in the tank constantly changes, we find that

$$dU = d(Yc_vT), \quad (2.343)$$

where Y is the quantity of gas in the tank at a given moment;

c_v is the specific heat of the gas.

The thermal balance equation is written in the following form. First, the quantity of heat emitted by a heating surface is equal to the heat content of the heated masses, therefore

$$\alpha_b S_b (T - T_b) dt = \xi_b \rho_b S_b \delta_b c_b dT_b; \quad (2.344)$$

$$\alpha_l S_l (T - T_l) dt = \xi_l \rho_l S_l \delta_l c_l dT_l \quad (2.345)$$

$$\alpha_c S_c (T - T_c) dt = \xi_c \rho_c S_c \delta_c c_c dT_c; \quad (2.346)$$

$$\alpha_a S_a (T - T_a) dt = \xi_a m_a c_a dT_a. \quad (2.347)$$

Secondly, the heat quantity obtained by the heated masses is equal to a change in the internal gas energy:

$$\xi_b \rho_b S_b \delta_b c_b dT_b + \xi_l \rho_l S_l \delta_l c_l dT_l + \xi_c \rho_c S_c \delta_c \times \times c_c dT_c + \xi_a m_a c_a dT_a = d(Yc_vT). \quad (2.348)$$

The bottom surface

$$S_b = \varepsilon_b \pi r^2, \quad (2.349)$$

where ε_b is a coefficient which takes into consideration the variance in shape of the bottom from a plane;

r is the tank radius.

The liquid surface is

$$S_l = \varepsilon_l \pi r^2, \qquad (2.350)$$

where ε_l is a coefficient which takes into consideration the liquid surface increase as a result of its unevenness and the deviation in the vector of full flight acceleration from the direction corresponding to the rocket axis.

The volume of liquid expelled from the tank by time t, will equal:

$$V_t = \frac{1}{\varrho_l}\int_0^t G_l\,dt, \qquad (2.351)$$

where G_l is the mass liquid rate of flow per second from the tank.

The volume occupied by gases is:

$$V = V_{in} + \frac{1}{\varrho_l}\int_0^t G_l\,dt. \qquad (2.352)$$

The surface of the tank wall is

$$S_c = \varepsilon_c \frac{2\pi}{r}\left[V_{in} + \frac{1}{\varrho_l}\int_0^t G_l\,dt\right], \qquad (2.353)$$

where ε_c is a coefficient which takes into consideration the variance in the shape of the tank from the cylindrical;

V_{in} is the initial free volume of the tank.

The gas quantity in the tank at time t, without considering condensation, equals

$$Y = \frac{p_t V}{RT_t}, \qquad (2.354)$$

where p_t, T_t are gas pressure and temperature in the tank.

Let us examine one of the possible sequences in the calculation of heat transfer between the gas and the tank sides, without considering condensation on the walls. Let us analyze the tank along its length at separate angular sections, each with altitude Δl (Figure 17). The liquid flow rate from the tank will be considered constant. Here identical time intervals Δt correspond to the evacuation of each section. The calculation will be made according to average parameter values, assuming the intermittent displacement of liquid within the tank.

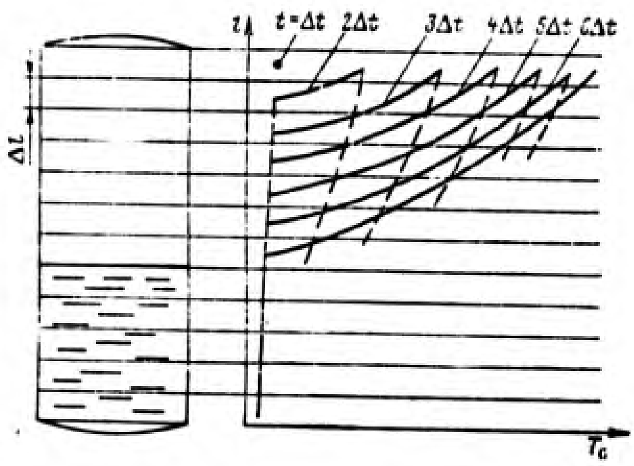

Figure 17. Diagram illustrating heat transfer in the tank.

After liquid displacement by value Δl the internal surface of the first annular section is exposed, which will interreact with the gas in the course of Δt seconds. Heating of the first ring in the course of the first time interval is characterized by the following equations.

The heat transmitted from the gas to the wall:

$$Q_0 = \alpha_c 2\pi r \Delta l (T_0 - T_{c0}) \Delta t, \tag{2.355}$$

where index "zero" denotes the beginning of the process.

In order to determine the temperature increment of the wall we have:

$$Q_0 = \xi_c \rho_c 2\pi r \Delta l \delta_c c_c \Delta T_c. \tag{2.356}$$

The wall temperature of the first ring by the end of the first time period is:

$$T_{c1} = T_{c0} + \Delta T. \tag{2.357}$$

The gas quantity in the tank at the time under consideration will equal:

$$Y_0 = \frac{p_0}{RT_0} (V_{in} + 2\pi r \Delta l). \tag{2.358}$$

The decrease in gas temperature in the tank is:

$$\Delta T_0 = \frac{Q_0}{Y_0 c_{v0}}. \qquad (2.359)$$

The new value of the gas temperature will be:

$$T_1 = T_0 - \Delta T_0. \qquad (2.360)$$

With this we shall conclude our calculation of heating of the first ring in the first time interval.

In the second interval of time the heating of two more rings is observed, however the initial temperature of the first upper ring T^{I}_{c1} is higher than the initial temperature T^{II}_{c0} of the second ring having just appeared from under the surface of the liquid.

The mass of the gas, having entered from the generator during time Δt, equals:

$$Y_{0-1} = \int_0^{\Delta t} G\,dt. \qquad (2.361)$$

The gas mass in the tank is:

$$Y_1 = Y_0 + \int_0^{\Delta t} G\,dt. \qquad (2.362)$$

The gas temperature in the tank at the beginning of the second heating interval, under an assumption concerning the constancy of the specific heat (since in the process under consideration the temperature change is not great) will equal:

$$T_{1(2)} = \frac{Y_0 T_1 + T_0 \int_0^{\Delta t} G\,dt}{Y_1}. \qquad (2.363)$$

The heat transmitted from the gases to the first ring are:

$$Q_1^{I} = \alpha_c 2\pi r \Delta l (T_{1(2)} - T_{c1}) \Delta t. \qquad (2.364)$$

The heat transmitted from the gas to the second ring will equal:

$$Q_1^{II} = \alpha_c 2\pi r \Delta l (T_{1(2)} - T_{c0}) \Delta t. \qquad (2.365)$$

The heat transmitted from the gas to the wall, Q_1 is equal to the sum:

$$Q_1 = Q_1^{I} + Q_1^{II}. \tag{2.366}$$

The decrease in gas temperature in the tank equals:

$$\Delta T_1 = \frac{Q_1^{I} + Q_1^{II}}{Y_1 c_{v1}}. \tag{2.367}$$

The new gas temperature value in the tank will be:

$$T_2 = T_1 - \Delta T_1. \tag{2.368}$$

The pressure in the tank will equal:

$$p_1 = \frac{RT_2}{V_{in} + 4\pi r \Delta l}. \tag{2.369}$$

In the third time interval we consider the heating of three rings, but the method remains analogous to that which we have examined above.

Let us examine several relationships which characterize the formation of condensate. The gas quantity in the tank at an arbitrary point in time is determined according to the formula:

$$Y = \left(\frac{p_t V}{RT_t}\right)_0 - \int_0^t G_{fin}\,dt, \tag{2.370}$$

where G_{fin} is the quantity of condensate which is formed per unit time.

The mass gas flow per second from the accumulator to the tank is:

$$Y = \int_0^t G\,dt. \tag{2.371}$$

In conducting experiments, if the operating condition of the accumulator is known, i.e., G(t), and the gas constant is R, then according to the results of measuring p_t and T_t, it is possible to determine the quantity of condensed vapors according to the formula:

$$-Y_{fin} = \frac{p_t V}{RT_t} - \int_0^t G\,dt. \tag{2.372}$$

Or, if we employ equation (2.352), then we obtain:

$$-Y_{fin} = \frac{p_t}{RT_t}\left(V_{in} + \frac{1}{\varrho l}\int_0^t G_l dt\right) - \int_0^t G\,dt. \qquad (2.373)$$

The mass input per second of condensate in a general case, when p_t, T_t, G_l and G are variables is:

$$-G_{fin} = \frac{1}{RT_t}\left(\dot{p}_t - p_t\frac{\dot{T}_t}{T_t}\right)\left(V_{in} + \frac{1}{\varrho l}\int_0^t G_l dt\right) + \frac{p_t}{RT_t}\frac{1}{\varrho_l}G_l - G. \qquad (2.374)$$

If p_t and T_t are constants, then

$$-G_{fin} = \frac{p_t}{RT_t}\frac{1}{\varrho_l}G_l - G. \qquad (2.375)$$

The appearance of condensate is possible in that part of the boundary layer where the gas temperature is less than the value T_S, calculated for a given tank pressure. The wall temperature increases in proportion to the distance from the surface of the liquid, and at distance l_S reaches the value $T_c = T_S$. Upon displacement from the center of the tank to the side, the gas temperature will decrease and at a certain distance δ_S from it, the value will be $T = T_S$. The farther (higher) from the surface of the liquid, the lower δ_S will be. At distance l_S, the value $\delta_S = 0$. Therefore, the gas volume within which condensation of vapors contained within the gas is occurring at a given point in time will equal:

$$V = 2\pi r\int_0^l \delta_S(l)\,dl. \qquad (2.376)$$

We note that calculation of the value l is made from the liquid surface. The thermal balance equation, with condensation present, is written in the following form:

$$d(Y_{fin}c_v T_t) = \varrho_c\delta c_c d(S_c T_c) - d(r^* Y_{fin}). \qquad (2.377)$$

This equation may be presented by the following two expressions:

$$dQ_{fin} = d(r^* Y_{fin}); \qquad (2.378)$$

$$dQ_{fin} = \varrho_c\delta c_c d(S_c T_c) - d(Y_{fin} c_v T_t). \qquad (2.379)$$

In equations (2.377) and (2.378), the concealed heat of condensation

r^* is introduced under the differential sign, taking into consideration a possible change in tank pressure. Keeping in mind equations (2.373), (2.374) and (2.353) and assuming that specific heat c_v = const, we obtain in place of (2.377):

$$-\frac{c_v}{R}\left(\dot{p}_t - p_t\frac{\dot{T}_t}{T_t}\right)\left(V_{in}+\frac{1}{\varrho l}\int_0^t G_l\,dt\right)-\frac{c_v}{R}\frac{p_t}{\varrho l}G_l+c_v G T_c -$$

$$-\frac{c_v}{R}\frac{p_t}{T_t}\left(V_{in}+\frac{1}{\varrho l}\int_0^t G_l\,dt\right)\dot{T}_t+\dot{T}_t\int_0^t G\,dt=\varrho_c\delta c_c T_c\varepsilon_c\frac{2\pi}{r}\frac{1}{\varrho l}G_l+$$

$$+\varrho_c\delta c_c\dot{T}_c\varepsilon\frac{2\pi}{r}\left(V_{in}+\frac{1}{\varrho l}\int_0^t G_l\,dt\right)-\frac{r^*}{RT_t}\left(\dot{p}_t-p_t\frac{\dot{T}_t}{T_t}\right)\times$$

$$\times\left(V_{in}+\frac{1}{\varrho l}\int_0^t G_l\,dt\right)-r^*\left(\frac{p_t}{RT_t}\frac{G_l}{\varrho l}-G\right)-$$

$$-\left[\frac{p_t}{RT_t}\left(V_{in}+\frac{1}{\varrho_l}\int_0^t G_l\,dt\right)-\int_0^t G\,dt\right]dr^*. \qquad (2.380)$$

If the calculation is carried out according to time periods Δt and utilizing the concepts concerning annular sectors, having a length Δl, then in carrying out engineering calculations we may stipulate $r^* = \dot{p}_t = \dot{T}_c = \dot{T}_t = 0$. In this connection in place of equation (2.380) we have:

$$-\frac{c_v}{R}p_t\frac{1}{\varrho_l}G_l+c_v G T_t=\varrho_c\delta c_c T_c\varepsilon_c\frac{2}{r}\frac{1}{\varrho l}G_l-r^*\left(\frac{p_t}{RT_t}\frac{G_l}{\varrho_l}-G\right). \qquad (2.381)$$

The first element on the left side of equation (2.381) characterizes a derivative of the total quantity of heat contained in the gases occupying the upper tank cavity. The second element characterizes a derivative of the heat quantity introduced by the working medium entering the tank from the accumulator. The first element on the right side of the equation represents a heat quantity derivative of the wall material. The last element equals a derivative of the quantity of heat consumed in condensation in the boundary layer.

Footnotes

1. To p. 82. Tr. note: Gauss divergence theorem.

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