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The Electric and Magnetic Fields Near a Buried Long Horizontal Line Source

PETER R. BANNISTER
E/M Systems Planning Branch
Radio Communication Systems Division

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Navy Underwater Sound Laboratory
Fort Trumbull, New London, Connecticut 06320

ABSTRACT

Expressions for the electric and magnetic fields produced by a buried horizontal line source are derived for the special case of the quasi-static range in which the measurement distance is much less than an earth skin depth. It is shown that the resulting expressions are nearly identical to those produced by an elevated horizontal line source.

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THE ELECTRIC AND MAGNETIC FIELDS NEAR A BURIED LONG HORIZONTAL LINE SOURCE

INTRODUCTION

The fields of a horizontal line source located near the surface of a plane, conducting, homogeneous earth have been considered by Carson,[1] Price,[2] Wait,[3-6] von Aulock,[7] Vanyan,[8] Sunde,[9] and Bannister,[10-12] among others. These fields are utilized in the induction methods of geophysical prospecting that are discussed in considerable detail by Keller and Frischknecht[13] and Vanyan.[8] They are also used in determining the coupling between power lines and nearby antennas.

It is the purpose of this report to derive expressions for the magnetic and electric field components produced by a horizontal line source for a special portion of the quasi-static range, where the measurement distance is not only much less than a free-space wavelength but is also much less than an earth skin depth.

Choosing a rectangular coordinate system (x, y, z), we find the ground is defined by the space $z < 0$. The wire, located at height (or depth) $z = h$, is parallel to the x axis. For frequencies less than 30 kHz and for measurement distances much less than a free-space wavelength (λ_{air}), the displacement currents in the air and in the ground may be neglected.

Three specific cases of antenna configuration are examined. In case A, both the horizontal line source and the receiving antennas are elevated; in case B, both antennas are buried; and in case C, the horizontal line source is buried and the receiving antennas are elevated.

DERIVATION OF THE MAGNETIC FIELD COMPONENTS

CASE A — BOTH ANTENNAS ABOVE GROUND

For case A, where both antennas are above ground (which has been considered previously by Bannister[10]), when there is a uniform current I in the wire of radius a, the electric field has only an x component, which is given by (Carson,[1] Wait[3,5,6])

$$E_x \approx -\frac{i\omega\mu I}{2\pi}\left[\ln\left(\frac{R_1}{R_0}\right) + P\right] , \qquad (1)$$

and

$$P = 2 \int_0^{\infty} \frac{e^{-\lambda(h+z)}}{\lambda + u} \cos \lambda y d\lambda = P_+ + P_- \quad , \qquad (2)$$

where

$$P_{\pm} = \int_0^{\infty} \frac{e^{-\lambda(z+h\pm iy)}}{\lambda + u} d\lambda \quad ,$$

$$u = (\lambda^2 + \gamma^2)^{1/2} \quad ,$$

$$\gamma \approx (i\omega\mu\sigma)^{1/2} \quad ,$$

$R_0^2 = y^2 + (h-z)^2$ (see Fig. 1), and

$$R_1^2 = y^2 + (h+z)^2 \quad .$$

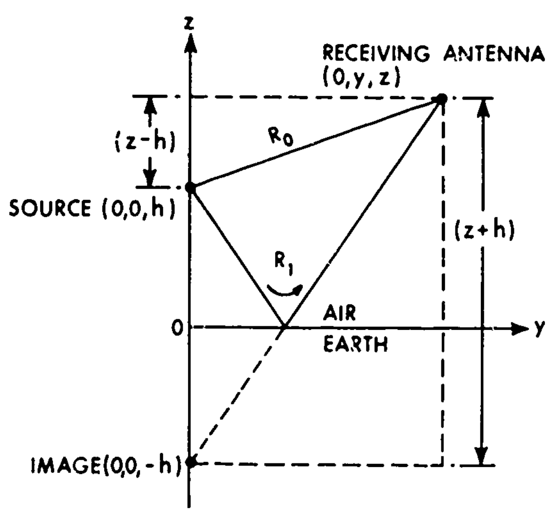

Fig. 1. Both Antennas Above Ground

Since $H_y = -(1/i\omega\mu)(\partial E_x/\partial z)$ and $H_z = (1/i\omega\mu)(\partial E_x/\partial y)$,

$$H_y \approx \frac{I}{2\pi}\left\{\frac{(h+z)}{R_1^2} + \frac{(h-z)}{R_0^2} - 2\int_0^\infty \frac{\lambda e^{-\lambda(h+z)}}{u+\lambda}\cos\lambda y d\lambda\right\} \tag{3}$$

and

$$H_z \approx -\frac{I}{2\pi}\left\{\frac{y}{R_1^2} - \frac{y}{R_0^2} - 2\int_0^\infty \frac{\lambda e^{-\lambda(h+z)}}{u+\lambda}\sin\lambda y d\lambda\right\} . \tag{4}$$

When R_1 is much less than an earth skin depth $\delta(\approx\sqrt{2/\omega\mu\sigma})$, it is permissible to let u equal λ in the exact integral expressions. Therefore,

$$H_y \approx \frac{I}{2\pi}\left\{\frac{(h+z)}{R_1^2} + \frac{(h-z)}{R_0^2} - \int_0^\infty e^{-\lambda(h+z)}\cos\lambda y d\lambda\right\} \tag{5}$$

and

$$H_z \approx -\frac{I}{2\pi}\left\{\frac{y}{R_1^2} - \frac{y}{R_0^2} - \int_0^\infty e^{-\lambda(h+z)}\sin\lambda y d\lambda\right\} . \tag{6}$$

From Erdélyi[14]

$$\int_0^\infty e^{-\lambda(h+z)}\cos\lambda y d\lambda = \frac{(h+z)}{R_1^2} \tag{7}$$

and

$$\int_0^\infty e^{-\lambda(h+z)} \sin \lambda y d\lambda = \frac{y}{R_1^2} . \tag{8}$$

Thus,

$$H_y \approx \frac{I(h-z)}{2\pi R_0^2} \tag{9}$$

and

$$H_z \approx \frac{Iy}{2\pi R_0^2} , \tag{10}$$

which agree with Bannister's[10] earlier results that were derived in a different manner.

CASE B — BOTH ANTENNAS BURIED

For case B, where both antennas are buried, when $h \geq |z| \geq 0$ (see Fig. 2)

$$E_x \approx -\frac{i\omega\mu I}{2\pi} \left\{ \ln\left(\frac{R_1}{R_0}\right) + 2 \int_0^\infty \frac{e^{-u(h-z)}}{u+\lambda} \cos \lambda y d\lambda \right\} . \tag{11}$$

In the above equation,

$$R_0^2 = y^2 + (h+z)^2 = y^2 + (h-|z|)^2$$

and

$$R_1^2 = y^2 + (h-z)^2 = y^2 + (h+|z|)^2 .$$

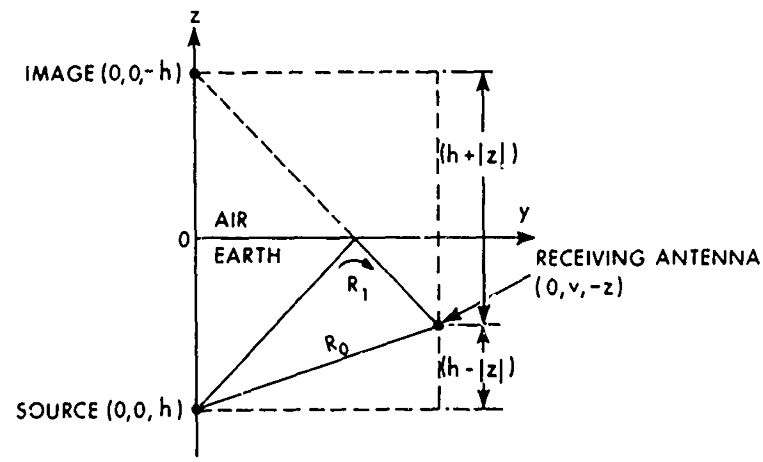

Fig. 2. Both Antennas Buried

When the same procedure used in case A is followed,

$$
\begin{aligned}
H_y &\approx \frac{I}{2\pi}\left\{-\frac{(h-z)}{R_1^2}-\frac{(h+z)}{R_0^2}+2\int_0^\infty \frac{ue^{-u(h-z)}}{u+\lambda}\cos\ \lambda y d\lambda\right\} \\
&\approx \frac{I}{2\pi}\left\{-\frac{(h-z)}{R_1^2}-\frac{(h+z)}{R_0^2}+\int_0^\infty e^{-\lambda(h-z)}\cos\ \lambda y d\lambda\right\}.
\end{aligned}
\qquad (12)
$$

From Eqs. (7) and (12)

$$
H_y \approx -\frac{I(h+z)}{2\pi R_0^2} = -\frac{I(h-|z|)}{2\pi R_0^2} \quad . \qquad (13)
$$

Similarly,

$$
\begin{aligned}
H_z &\approx -\frac{I}{2\pi}\left\{\frac{y}{R_1^2}-\frac{y}{R_0^2}-2\int_0^\infty \frac{\lambda e^{-u(h-z)}}{u+\lambda}\sin\ \lambda y d\lambda\right\} \\
&\approx -\frac{I}{2\pi}\left\{\frac{y}{R_1^2}-\frac{y}{R_0^2}-\int_0^\infty e^{-\lambda(h-z)}\sin\ \lambda y d\lambda\right\} \quad .
\end{aligned}
\qquad (14)
$$

From Eqs. (8) and (14)

$$H_z \approx \frac{Iy}{2\pi R_0^2} . \tag{15}$$

When $h = 0$, Eqs. (13) and (15) reduce to von Aulock's results.[7]

CASE C — LINE SOURCE BURIED, RECEIVING ANTENNA ELEVATED

For case C, where the line source is buried and the receiving antenna is elevated (see Fig. 3),

$$E_x \approx -\frac{i\omega\mu I}{\pi} \int_0^\infty \frac{e^{-(uh+\lambda z)}}{u + \lambda} \cos \lambda y d\lambda \quad . \tag{16}$$

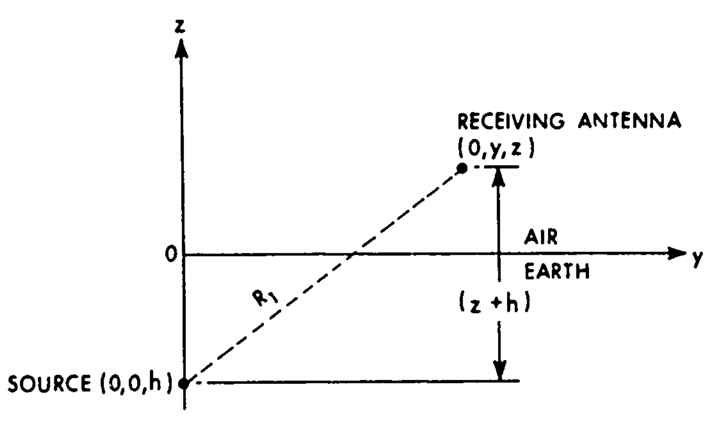

Fig. 3. Line Source Buried, Receiving Antenna Elevated

Again, when the same procedure used in case A is followed,

$$H_y \approx -\frac{I}{\pi} \int_0^\infty \frac{\lambda e^{-(uh+\lambda z)}}{u+\lambda} \cos \lambda y d\lambda$$

$$\approx -\frac{I}{2\pi} \int_0^\infty e^{-\lambda(h+z)} \cos \lambda y d\lambda \quad . \tag{17}$$

From Eqs. (7) and (17)

$$H_y \approx -\frac{I(h+z)}{2\pi R_1^2} \; . \tag{18}$$

Similarly,

$$H_z \approx \frac{I}{\pi} \int_0^\infty \frac{\lambda e^{-(uh+\lambda z)}}{u+\lambda} \sin \lambda y d\lambda$$

$$\approx \frac{I}{2\pi} \int_0^\infty e^{-\lambda(h+z)} \sin \lambda y d\lambda \quad . \tag{19}$$

From Eqs. (8) and (19)

$$H_z \approx \frac{Iy}{2\pi R_1^2} \; . \tag{20}$$

DERIVATION OF THE ELECTRIC FIELD COMPONENTS

CASE A — BOTH ANTENNAS ABOVE GROUND

The electric field component for case A, where both antennas are above ground (see Fig. 1), may be expressed as

$$E_x \approx -\frac{i\omega\mu I}{2\pi}\left[\ln\left(\frac{R_1}{R_0}\right) + P\right] . \tag{1}$$

In the above equation

$$P = 2\int_0^{\infty} \frac{e^{-\lambda(h+z)}}{\lambda + u} \cos \lambda y d\lambda . \tag{2}$$

If we let $u = \lambda$ in Eq. (2), the integral diverges. Therefore, we must solve the integral as it stands and then let $|\gamma R_1|$ become small.

From Bannister[10]

$$P = \left\{-\frac{1}{(\beta_+)^2} - \frac{1}{(\beta_-)^2} + \frac{\pi}{2\beta_+}\left[H_1(\beta_+) - Y_1(\beta_+)\right] + \frac{\pi}{2\beta_-}\left[H_1(\beta_-) - Y_1(\beta_-)\right]\right\} , \tag{21}$$

where

$\beta_\pm = \gamma(z+h\pm iy)$,

$H_1(\beta)$ is the Struve function of order one, and

$Y_1(\beta)$ is the Bessel function of the second kind of order one.

When $|\gamma R_1| \ll 1$, Eq. (21) reduces to (Bannister[10])

$$P \approx \ln\left(\frac{2}{\gamma R_1}\right). \tag{22}$$

From Eqs. (1) and (22)

$$E_x \approx -\frac{i\omega\mu I}{2\pi} \ln\left(\frac{2}{\gamma R_0}\right). \tag{23}$$

CASE B — BOTH ANTENNAS BURIED

The electric field component for case B, where both antennas are buried (see Fig. 2), may be expressed as (when $h \geq |z| \geq 0$)

$$E_x \approx -\frac{i\omega\mu I}{2\pi} \left\{ \ln\left(\frac{R_1}{R_0}\right) + 2 \int_0^\infty \frac{e^{-u(h-z)}}{u+\lambda} \cos \lambda y d\lambda \right\}. \tag{11}$$

In the above equation,

$$R_0^2 = y^2 + (h+z)^2 = y^2 + (h-|z|)^2$$

and

$$R_1^2 = y^2 + (h-z)^2 = y^2 + (h+|z|)^2 .$$

The integral in Eq. (11) may be expressed as

$$I = 2 \int_0^\infty \frac{e^{-u(h-z)}}{u+\lambda} \cos \lambda y d\lambda = \frac{2}{\gamma^2} \int_0^\infty (u-\lambda) e^{-u(h-z)} \cos \lambda y d\lambda = \frac{2}{\gamma^2} (I_1 + I_2) . \tag{24}$$

When $|\gamma R_1| << 1$,

$$I_2 = - \int_0^\infty \lambda e^{-u(h-z)} \cos \lambda y d\lambda \approx - \int_0^\infty \lambda e^{-\lambda(h-z)} \cos \lambda y d\lambda$$

$$+ \frac{\gamma^2(h-z)}{2} \int_0^\infty e^{-\lambda(h-z)} \cos \lambda y d\lambda \; . \tag{25}$$

From Erdélyi[14]

$$- \int_0^\infty \lambda e^{-\lambda(h-z)} \cos \lambda y d\lambda = - \frac{(h-z)^2}{R_1^4} \left[1 - \frac{y^2}{(h-z)^2}\right]$$

$$= \frac{1}{R_1^2} \left[1 - \frac{2(h-z)^2}{R_1^2}\right] . \tag{26}$$

From Eqs. (7), (25), and (26)

$$I_2 \approx \frac{1}{R_1^2} \left[1 - \frac{2(h-z)^2}{R_1^2} + \frac{\gamma^2 (h-z)^2}{2}\right]. \tag{27}$$

Now

$$I_1 = \int_0^\infty u e^{-u(h-z)} \cos \lambda y d\lambda = - \frac{\partial}{\partial (h-z)} \int_0^\infty e^{-u(h-z)} \cos \lambda y d\lambda \; . \tag{28}$$

From Erdélyi[14]

$$\int_0^\infty e^{-u(h-z)} \cos \lambda y d\lambda = \frac{\gamma(h-z)}{R_1} K_1 (\gamma R_1) \quad , \tag{29}$$

where $K_1(\gamma R_1)$ is the modified Bessell function of the second kind and order one. Therefore,

$$I_1 \approx -\frac{\gamma}{R_1} \left\{ \left[1 - \frac{2(h-z)^2}{R_1^2} \right] K_1(\gamma R_1) - \frac{\gamma(h-z)^2}{R_1} K_0(\gamma R_1) \right\} \quad , \tag{30}$$

where $K_0(\gamma R_1)$ is the modified Bessel function of the second kind and order zero.

When $|\gamma R_1| << 1$,

$$I_1 \approx -\frac{\gamma}{R_1} \left\{ \left[1 - \frac{2(h-z)^2}{R_1^2} \right] \left[\frac{1}{\gamma R_1} - \frac{\gamma R_1}{2} \left\{ \ln\left(\frac{2}{\Delta \gamma R_1}\right) + \frac{1}{2} \right\} \right] - \frac{\gamma(h-z)^2}{R_1} \ln\left(\frac{2}{\Delta \gamma R_1}\right) \right\}, \tag{31}$$

where $\Delta = 1.7811$.

Since $\ln(1/\Delta) + 1/2 = -0.5773 + 1/2 = -0.0773 \approx 0$,

$$I_1 \approx -\frac{1}{R_1^2} \left[1 - \frac{2(h-z)^2}{R_1^2} \right] + \frac{\gamma^2}{2} \ln\left(\frac{2}{\gamma R_1}\right) - \frac{\gamma^2 (h-z)^2}{2R_1^2} \quad . \tag{32}$$

From Eqs. (24), (27), and (32)

$$I = \frac{2}{\gamma^2} (I_1 + I_2) \approx \ln\left(\frac{2}{\gamma R_1}\right) \tag{33}$$

and from Eqs. (11) and (33)

$$E_x \approx -\frac{i\omega\mu I}{2\pi} \ln\left(\frac{2}{\gamma R_0}\right) . \tag{34}$$

When h = 0, Eq. (34) reduces to von Aulock's result.[7]

CASE C – LINE SOURCE BURIED, RECEIVING ANTENNA ELEVATED

The electric field component for case C, where the line source is buried and the receiving antenna is elevated (see Fig. 3), may be expressed as

$$E_x \approx -\frac{i\omega\mu I}{\pi} \int_0^\infty \frac{e^{-(uh+\lambda z)}}{u+\lambda} \cos \lambda y d\lambda . \tag{16}$$

If $|uh| \ll 1$ and $z > h$,

$$e^{-uh} \approx 1 - uh + u^2 h^2/2 . \tag{35}$$

Inserting Eq. (35) into Eq. (16) yields

$$E_x \approx -\frac{i\omega\mu I}{\pi} \left\{ \int_0^\infty \frac{e^{-\lambda z} \cos \lambda y d\lambda}{u+\lambda} - h \int_0^\infty \frac{ue^{-\lambda z}}{u+\lambda} \cos \lambda y d\lambda + \frac{h^2}{2} \int_0^\infty \frac{u^2 e^{-\lambda z}}{u+\lambda} \cos \lambda y d\lambda \right\} \tag{36}$$

$$\approx -\frac{i\omega\mu I}{\pi} \left\{ \lim_{h \to 0} \left(\frac{P}{2}\right) - \frac{h}{2} \int_0^\infty e^{-\lambda z} \cos \lambda y d\lambda + \frac{h^2}{4} \int_0^\infty \lambda e^{-\lambda z} \cos \lambda y d\lambda \right\} .$$

From Eqs. (8), (22), and (26)

$$E_x \approx -\frac{i\omega\mu I}{2\pi}\left\{\ln\left(\frac{2}{\gamma R}\right) - \frac{hz}{R^2} - \frac{h^2}{2R^2}\left(1-\frac{2z^2}{R^2}\right)\right\}, \tag{37}$$

where

$R^2 = y^2 + z^2$ and

$z > h$.

If $h > z$, then

$$e^{-\lambda z} \approx 1 - \lambda z + \frac{\lambda^2 z^2}{2}. \tag{38}$$

Inserting Eq. (38) into Eq. (16) results in

$$E_x \approx -\frac{i\omega\mu I}{\pi}\left\{\int_0^\infty \frac{e^{-uh}}{u+\lambda}\cos\lambda y d\lambda - z\int_0^\infty \frac{\lambda e^{-uh}}{u+\lambda}\cos\lambda y d\lambda + \frac{z^2}{2}\int_0^\infty \frac{\lambda^2 e^{-uh}}{u+\lambda}\cos\lambda y d\lambda\right\}. \tag{39}$$

When $|\gamma R'| \ll 1\left(R' = \sqrt{y^2+h^2}\right)$, Eqs. (24) and (33) may be utilized to yield

$$\int_0^\infty \frac{e^{-uh}}{u+\lambda}\cos\lambda y d\lambda \approx \ln\left(\frac{2}{\gamma R'}\right). \tag{40}$$

From Eq. (8)

$$\int_0^\infty \frac{\lambda e^{-uh}}{u+\lambda} \cos \lambda y d\lambda \approx \frac{1}{2} \int_0^\infty e^{-\lambda h} \cos \lambda y d\lambda = \frac{h}{2(R')^2} \tag{41}$$

and from Eq. (26)

$$\int_0^\infty \frac{\lambda^2 e^{-uh}}{u+\lambda} \cos \lambda y d\lambda \approx \frac{1}{2} \int_0^\infty \lambda e^{-\lambda h} \cos \lambda y d\lambda$$

$$= \frac{1}{2(R')^2} \left[1 - \frac{2h^2}{(R')^2} \right] . \tag{42}$$

Therefore,

$$E_x \approx - \frac{i\omega\mu I}{2\pi} \left\{ \ln \left(\frac{2}{\gamma R'} \right) - \frac{hz}{(R')^2} - \frac{z^2}{2(R')^2} \left[1 - \frac{2h^2}{(R')^2} \right] \right\} , \tag{43}$$

where

$(R')^2 = y^2 + h^2$ and

$h > z$.

When the magnetic field components were derived for case C (Eqs. (18) and (20)), the restriction $h > z$ or $z > h$ was not required. However, the electric field component formulas seem to require these restrictions. The formula for the electric field component is

$$E_x \approx - \frac{i\omega\mu I}{\pi} \int_0^\infty \frac{e^{-(uh+\lambda z)}}{u+\lambda} \cos \lambda y d\lambda . \tag{16}$$

If we let $u = \lambda$ in just the exponential term exp (−uh), Eq. (16) reduces to

$$E_x \approx -\frac{i\omega\mu I}{\pi} \int_0^\infty \frac{e^{-\lambda(h+z)}}{u+\lambda} \cos \lambda y d\lambda = -\frac{i\omega\mu I}{\pi}\left(\frac{P}{2}\right) . \tag{44}$$

From Eqs. (21) and (44)

$$E_x \approx -\frac{i\omega\mu I}{2\pi}\left\{-\frac{1}{(\beta_+)^2} - \frac{1}{(\beta_-)^2} + \frac{\pi}{2\beta_+}\left[H_1(\beta_+) - Y_1(\beta_+)\right] + \frac{\pi}{2\beta_-}\left[H_1(\beta_-) - Y_1(\beta_-)\right]\right\}. \tag{45}$$

Note that Eq. (45) does not require the restriction $|\gamma R_1| \ll 1$. The only restriction on this equation is that $|\gamma h|$ must be $\ll 1$. If $|\gamma R_1| \ll 1$, Eq. (45) reduces to

$$E_x \approx -\frac{i\omega\mu I}{2\pi} \ln\left(\frac{2}{\gamma R_1}\right) . \tag{46}$$

The approach used in deriving Eq. (46) may not be fully justified. However, the correct magnetic field component equations (Eqs. (18) and (20)) may be obtained by differentiating Eq. (46). Furthermore, when $h > z$, Eq. (46) reduces to Eq. (43) and when $z > h$, Eq. (46) reduces to Eq. (37). Therefore, it appears that Eq. (46) is valid to at least a first-order approximation.

SUMMARY

The electric and magnetic field components produced by a long horizontal line source have been derived for the special case of the quasi-static range in which the measurement distance is much less than an earth skin depth.

When both the line source and the receiving antennas are elevated (Bannister[10]),

$$E_x \approx -\frac{i\omega\mu I}{2\pi} \ln\left(\frac{2}{\gamma R_0}\right), \tag{23}$$

$$H_y \approx \frac{I(h-z)}{2\pi R_0^2} , \qquad (9)$$

and

$$H_z \approx \frac{Iy}{2\pi R_0^2} , \qquad (10)$$

where

$$R_0^2 = y^2 + (h-z)^2 .$$

When both antennas are buried,

$$E_x \approx -\frac{i\omega\mu I}{2\pi} \ln\left(\frac{2}{\gamma R_0}\right) , \qquad (34)$$

$$H_y \approx -\frac{I(h-|z|)}{2\pi R_0^2} , \qquad (13)$$

and

$$H_z \approx \frac{Iy}{2\pi R_0^2} , \qquad (15)$$

where

$$R_0^2 = y^2 + (h-|z|)^2 .$$

When the line source is buried and the receiving antenna is elevated

$$E_x \approx -\frac{i\omega\mu I}{2\pi} \ln\left(\frac{2}{\gamma R_1}\right) , \qquad (46)$$

$$H_y \approx -\frac{I(h+z)}{2\pi R_1^2}, \qquad (18)$$

and

$$H_z \approx \frac{Iy}{2\pi R_1^2}, \qquad (20)$$

where

$$R_1^2 = y^2 + (h+z)^2 .$$

From the equations in this summary, it is noted that the field component expressions are essentially identical for the three cases considered. In fact, the magnetic field equations are consistent with the elementary results obtained from Ampere's law. The results may be explained by the fact that for measurement distances very close to the line source ($R_1 << \delta$), the primary field is dominant. The air-earth interface has a negligible effect to a first-order approximation.

REFERENCES

1. J. R. Carson, "Wave Propagation in Overhead Wires with Ground Return," Bell Systems Technical Journal, vol. 5, no. 4, October 1926, pp. 539-554.

2. A. T. Price, "Electromagnetic Induction in a Semi-Infinite Conductor with a Plane Boundary," Quarterly Journal of Mechanics and Applied Mathematics, vol. 3, part 4, 1950, pp. 385-410.

3. J. R. Wait, "Fields of a Line Source of Current Over a Stratified Conductor," Applied Scientific Research, sec. B3, 1953, pp. 279-292.

4. J. R. Wait, "Transmission and Reflection of Electromagnetic Waves in the Presence of Stratified Media," Journal of Research NBS, vol. 61, no. 3, 1958, pp. 205-232.

5. J. R. Wait, "On the Impedance of a Long Wire Suspended Over the Ground," Proceedings of the IRE, vol. 49, no. 10, 1961, p. 1576.

6. J. R. Wait, Electromagnetic Waves in Stratified Media, Pergamon Press, Oxford, England, 1962.

7. W. von Aulock, The Electromagnetic Field of an Infinite Cable in Sea Water, Bureau of Ships Minesweeping Branch Technical Report No. 106, 1 September 1953.

8. L. L. Vanyan, Electromagnetic Depth Soundings, Consultants Bureau, New York (translated from the Russian by G. V. Keller), 1967.

9. E. D. Sunde, Earth Conduction Effects in Transmission Systems, D. Van Nostrand Company, Inc., New York, 1949.

10. P. R. Bannister, "The Electric and Magnetic Fields Near a Long Horizontal Line Source Above the Ground," Radio Science, vol. 3 (New Series), no. 2, February 1968.

11. P. R. Bannister, "Electromagnetic Fields Within a Stratified Earth Produced by a Long Horizontal Line Source," Radio Science (New Series), vol. 3, no. 4, April 1968.

12. P. R. Bannister, The Quasi-Near Fields of a Long Horizontal Line Source, USL Report No. 918, 17 May 1968.

13. G. V. Keller and F. C. Frischknecht, Electrical Methods in Geophysical Prospecting, Pergamon Press, Oxford, England, 1966.

14. A. Erdélyi, editor, Tables of Integral Transforms, McGraw-Hill Book Co., Inc., vol. I, New York, 1954.

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13 ABSTRACT

Expressions for the electric and magnetic fields produced by a buried horizontal line source are derived for the special case of the quasi-static range in which the measurement distance is much less than an earth skin depth. It is shown that the resulting expressions are nearly identical to those produced by an elevated horizontal line source.

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