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STATISTICAL DETECTION THEORY OF THRESHOLD VISUAL PERFORMANCE

H. A. Ory

PREPARED FOR:
UNITED STATES AIR FORCE PROJECT RAND

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PREFACE

This study is part of Rand's continuing effort to apply fundamental scientific knowledge to problems of military interest. It was stimulated by questions concerning the relationships between vision and various electro-optical sensors, as pertinent to reconnaissance, pattern recognition, and image display requirements. In this Memorandum a statistical detection theory of visual perception is developed and shown to be consistent with results of visual threshold experiments over a wide range of brightness and target parameters.

The results should be broadly useful to those concerned with applications involving visual perception, especially in relation to reconnaissance, pattern recognition, and image display. The results should also be of fundamental importance to those concerned with the psychophysics and physiology of vision.

SUMMARY

A statistical detection theory of threshold visual performance is developed in which the neural excitation noise results from random fluctuations in both target and background luminance and the decision criterion is assumed to be programmed. Detection occurs if the number of excitations resulting from observation of the fluctuating target and background luminance exceeds the criterion. The functional form of the programmed decision criterion is interpreted in terms of optimization of performance within the information-acquisition and processing limitations of the visual system. Extension of the retinal image, which results from ocular aberrations in foveal vision and from retinal neural organization in parafoveal vision, is shown to influence the threshold dependence on target size. An analytical expression is obtained for the quantitative description of threshold luminance as a function of the mean and standard deviation of total luminance and of target size and duration.

The theoretical expression resulting from the statistical detection model is compared with the extensive Tiffany experimental data on visual thresholds. An excellent quantitative description of the data is obtained for both foveal and parafoveal vision over a wide range of background luminance and target parameters, which is largely the result of introducing a programmed decision criterion. Foveal luminance thresholds are obtained within 14 percent rms relative error and parafoveal luminance thresholds within 9 percent rms relative error. Analysis of the experimental data demonstrates the importance of considering noise from total luminance fluctuations, both target and background, and the effect of retinal image extension, points which have hitherto been ignored. The effect of natural variation in pupil diameter, on the other hand, is shown to be negligible. A quantitative correspondence is drawn between the model description and the approximate description in terms of de Vries-Rose and Weber-Fechner regimes.

The results of the comparison with experimental data are used to support the validity of the features of the assumed model, and some of its limitations are pointed out. Suggestions are made for the design of threshold experiments to test the model further.

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I. INTRODUCTION

Because most of our observations of the world are obtained through use of the visual sense, there is a natural scientific interest in the capabilities, mechanisms, and underlying principles of visual perception. At present a disparity exists between the large amount of available specific empirical information and the meager theoretical understanding of the principles that govern the operation of the visual system. For example, a considerable collection of data exists on the visual threshold detection performance of the eye. The data include measurements of threshold luminance and contrast for the detection of targets over a wide range of size, duration, and background luminance. The measurements are obtained from a variety of experimental arrangements, some of which were designed to test the essential features of theoretical models proposed to account for results already observed. Although a number of such models have been proposed, none provides an adequate description of visual performance over a broad range of experimental parameters or even accounts for all the qualitative features of the experimental results.

The possible influence of the quantum nature of light and the statistical fluctuations in the number of absorbed photons in determining the fundamental performance of the eye was pointed out by Rose(1) and by de Vries.(2) This conjecture led to statistical theories that were derived on the basis of photon fluctuation limitations. However, the theories indicated a threshold luminance increment that varied as the square root of the background luminance and as the inverse square root of the target area and duration, thus providing a valid description of visual threshold performance over only a limited range of background and target parameters.

Nevertheless, many features of the experimental data are consistent with a statistical description; some of these are encompassed by models proposed by Tanner and Swets(3,4) and by Blackwell.(5) These theories, which are adapted from statistical decision theory, are similar in assuming that the excitation responses to target and background luminances are additive and that the number of excitations is randomly variable because of fluctuations in background luminance only. Detection is assumed to occur if the number of excitations exceeds a criterion that differs from the mean background excitation by an amount proportional to the standard deviation of background luminance, or noise. The theories differ in that Tanner and Swets assume a relatively low criterion which is frequently exceeded by background fluctuations (giving rise to bona fide false detections), while Blackwell assumes a relatively high criterion which is seldom exceeded by background fluctuations (false detections are attributed to spurious observer responses without a sensory basis). Neither theory provides a satisfactory quantitative description of detection threshold as a function of background luminance and of target size and duration. In addition, the Tanner and Swets theory predicts a dependence of detection probability on target luminance increment that disagrees with that observed experimentally for large false detection rates, while the Blackwell theory leads to a shape of operating characteristic curve, relating probabilities of detection and false detection, that disagrees with the shape that is derived from experimental data.

The statistical detection theory of visual threshold performance presented here also derives from statistical decision theory, but has the following distinguishing characteristics:

1. The decision criterion is assumed to be variable and programmed as a function of the noise to optimize performance within the limitations of the visual system.

2. The noise is attributed to quantum fluctuations in the total number of excitations, including those from both target and background luminance.

3. Retinal image extension resulting from both ocular aberrations and retinal neural organization is taken into account.

The model is shown to be consistent with the details of the experimental data, and a quantitative description of threshold target luminance as a function of background luminance and of target size and duration is obtained over a wide range of experimental parameters.

II. THE STATISTICAL DETECTION MODEL

Visual threshold detection performance is considered to depend fundamentally on the number of primary neural excitations, E, that are produced in the retina in proportion to the incident luminous energy, as well as on the fluctuations in E. The number of excitations within a given test area resulting from a randomly fluctuating background luminance has a probability density P(E/B) and standard deviation σ_B about a mean E_B. In the presence of additional target luminance within the test area, the incremental number of excitations E_S is additive to give a probability density P(E/T) and standard deviation σ_T about a mean $E_T = E_B + E_S$. Thus the signal is E_S and the noise is given by the appropriate σ. Detection is achieved if a particular realization E exceeds the detection criterion E_c. Figure 1 illustrates these relationships. Hence the detection probability is

$$P_d = \int_{E_c}^{\infty} P(E/T)dE$$

and the false detection probability is

$$P_f = \int_{E_c}^{\infty} P(E/B)dE$$

Although not necessary, it is convenient for clarity of presentation to limit consideration to incoherent luminance produced by thermal sources without amplification. The fluctuations in excitation arise from quantum fluctuations in the number of photons absorbed by the retinal receptors. The probability density function is Poisson, but the number of excitations is almost always sufficiently large that a satisfactory description is given by the more convenient gaussian form. Then

$$P_d = \frac{1}{\sigma_T\sqrt{2\pi}} \int_{E_c}^{\infty} \exp\left[-\frac{(E - E_T)^2}{2\sigma_T^2}\right] dE \qquad (1)$$

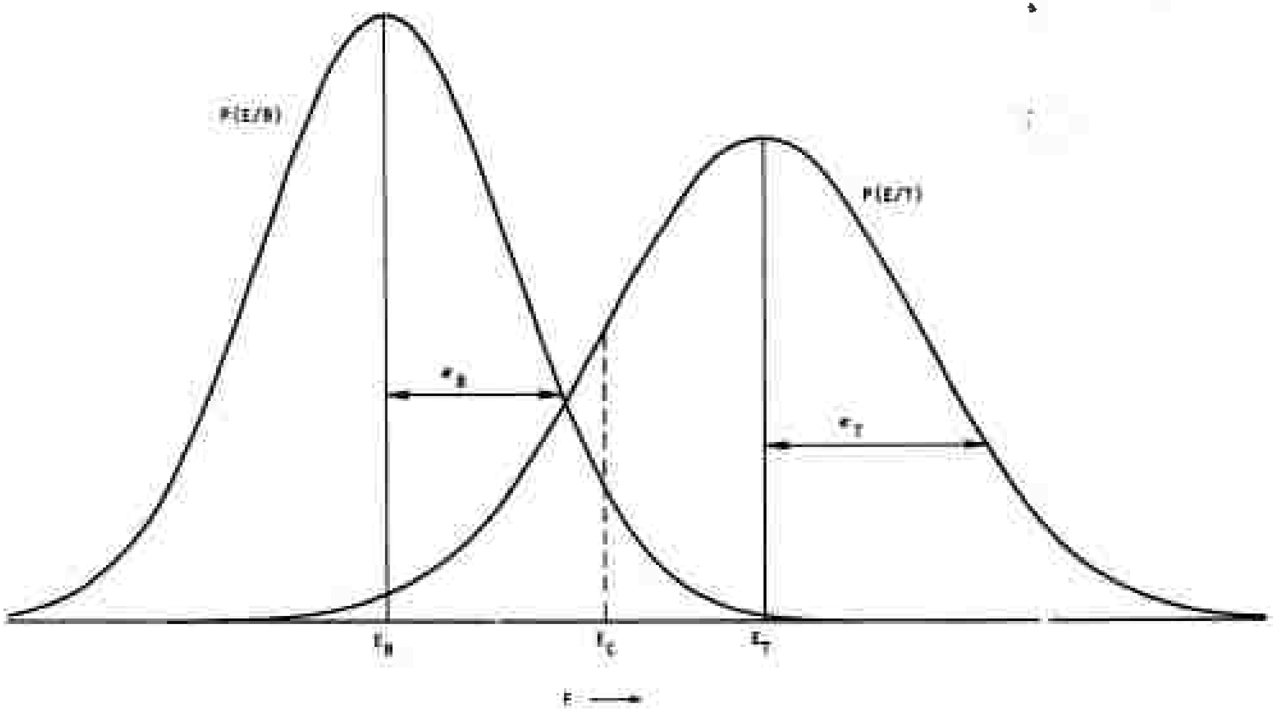

Fig. 1--Probability density functions of excitation resulting from background luminance only, P(E/B), and from both background and incremental target luminance, P(E/T)

and

$$P_f = \frac{1}{\sigma_B\sqrt{2\pi}} \int_{E_c}^{\infty} \exp\left[-\frac{(E - E_B)^2}{2\sigma_B^2}\right] dE \qquad (2)$$

With the substitution $v = (E - E_T)/\sigma_T$ and $w = (E - E_B)/\sigma_B$, Eqs. (1) and (2) can be rewritten as

$$P_d = \int_{\frac{E_c - E_T}{\sigma_T}}^{\infty} \exp(-v^2/2)\, dv \qquad (3)$$

and

$$P_f = \int_{\frac{E_c - E_B}{\sigma_B}}^{\infty} \exp(-w^2/2)\, dw \qquad (4)$$

which shows that P_d is the function only of $(E_c - E_T)/\sigma_T$, and P_f only of $(E_c - E_B)/\sigma_B$.

The number of excitations is determined as follows. If a Lambertian surface of luminance L (cd/m^2) and angular subtense θ (min) is viewed through a pupil of diameter d (mm) for a time t (sec), then $680\ \eta d^2 L t \theta^2$ excitations are produced, where η is the quantum efficiency of excitation. However, for a target in the presence of background luminance, in order to allow for extension of the retinal image (discussed further in Section III), an additional equivalent angular subtense θ_b is added to the angle θ_t subtended by the target. Then the number of excitations produced by target luminance ΔL is unchanged because in the extended image the apparent luminance $\Delta L'$ is reduced such that $\Delta L \theta_t^2 = \Delta L'(\theta_t + \theta_b)^2$, but more excitations are produced by background luminance over the larger retinal area than would have been produced within the unextended area. Taking the retinal image extension into account, the mean numbers of excitations are given by

$$E_B = ktL\left(\theta_t + \theta_b\right)^2 \quad (5)$$

and

$$E_T = kt\left[\Delta L\theta_t^2 + L\left(\theta_t + \theta_b\right)^2\right] \quad (6)$$

where $k = 680\ \eta d^2$. For luminance as considered here, the standard deviations are given by the square roots of the means, $\sigma_B = \sqrt{E_B}$ and $\sigma_T = \sqrt{E_T}$.

The decision criterion can be separated into two parts, $E_c = E_B + C$. Then C, the amount by which the criterion exceeds the mean background, is assumed to be a function of the noise; it also depends on observer motivational factors, such as values of correct decisions and costs of incorrect decisions, and on knowledge of a priori probabilities of signal occurrence. Such dependence is shown, for example, in the results reported by Swets et al.(4) In most reported experiments, including those used here for comparison, these observer factors are held constant; they are thus not treated explicitly here, but are absorbed into coefficients. The criterion increment C then depends effectively only on the noise.

The decision criterion increment is considered here to be under programmed control as a function of the noise, such that the visual system performance is optimized within the limitations of the information-acquisition and processing capabilities of the system. The implications of such control are discussed in Section IV in more detail. The programmed-control assumption is more complex than merely taking the criterion increment as proportional to the noise, as in previous theories. However, it leads to accurate quantitative description of the observed data and is closely analogous to the appropriate criterion control for optimization of the performance of limited electro-optical sensor systems. The form chosen here for the decision criterion increment is consistent with the analogy.

$$C = x + y\sigma_T + z\sigma_T^2 \quad (7)$$

The criterion control is accomplished through adjustment of the coefficients, which also include the influence of observer factors mentioned above. Insufficient information is available to specify the coefficients independently. Here they are to be determined phenomenologically in order to test whether the model can accurately describe the dependence of threshold target luminance on background luminance and target size and duration.

According to Eq. (3), P_d depends only on $(E_c - E_T)/\sigma_T$, so that for a given P_d,

$$\frac{(E_c - E_T)}{\sigma_T} = \frac{(C - E_S)}{\sigma_T} = K \tag{8}$$

where K is the particular constant that corresponds to the given P_d. Then the threshold excitation increment depends on C according to

$$E_S = C - K\sigma_T \tag{9}$$

and, from Eq. (7),

$$E_S = x + (y - K)\,\sigma_T + z\sigma_T^2 \tag{10}$$

Some characteristics of this relationship can be pointed out. Provision is made for a minimum value of C which corresponds to a finite limit on the detectable signal. At relatively low levels of noise, the term proportional to σ_T is dominant, detection is limited by quantum fluctuations in the number of absorbed photons, and a constant signal-to-noise ratio is maintained. (Note that this condition is not the same as Blackwell's suggestion(5) that the detection criterion is set to maintain a constant false detection rate; it can be seen from Eq. (4) that this would require C to be proportional to σ_B, not σ_T.) At relatively high levels of noise, the term proportional to σ_T^2 is dominant; for the type of luminance considered, which includes most common luminance, the number of luminance levels that can be distinguished visually reaches a limit, and a constant threshold contrast is maintained.

The relationships resulting from this model can be expressed directly in terms of the dependence of threshold target luminance on background luminance and on target size and duration. If the expressions for E_S and σ_T are substituted into Eq. (10) and the result is divided by k, then

$$\Delta L\theta_t^2 t = \alpha + \beta\left\{\left[\Delta L\theta_t^2 + L\left(\theta_t + \theta_b\right)^2\right]t\right\}^{\frac{1}{2}} + \gamma\left[\Delta L\theta_t^2 + L\left(\theta_t + \theta_b\right)^2\right]t \qquad (11)$$

where $\alpha = x/k$, $\beta = (y - K)/k^{\frac{1}{2}}$, and $\gamma = z$. It is noteworthy that only β depends on K, hence P_d, and that γ is independent of k. This expression is convenient for comparison with available experimental data and for evaluation of the coefficients.

III. COMPARISON WITH EXPERIMENTAL DATA

In order to determine whether the theoretical detection model presented in Section II provides an accurate description of threshold visual performance, Eq. (11) is compared here with the results of experimental measurements. The Tiffany data reported by Blackwell(6) are well known and widely used. They span several orders of magnitude of background luminance and threshold target luminance increment. The data from a 6-sec search of eight locations are more useful for quantitative evaluation than the extended time data in that a target duration of 0.75 sec can be taken as a reasonable approximation, although equally extensive data for more definitely controlled time would be preferable. Five target angular subtenses are included, ranging from 3.6 to 121 min of arc. The data are reproduced in Fig. 2, in which the Tiffany data points are shown and the illustrative lines are empirically fitted.

According to Blackwell,(6) the breaks in the curves correspond to a change by the observers from foveal to parafoveal vision near a background luminance of 3×10^{-3} cd/m^2, or slightly less than 10^{-3} fL, as was verified by interrogating the observers. Considering the experimental conditions and objectives, this could be interpreted equally well as a change from photopic to scotopic vision with an associated change in retinal position, as required for maximum sensitivity. On a physiological basis, foveal-photopic and parafoveal-scotopic vision involve different systems, and this difference is reflected in the data shown in Fig. 2. Therefore, the two sets of data are treated separately.

Actually, k includes a dependence on pupil diameter. This is a variable factor that is ignored in the analysis given below. (In principle, of course, it could be included explicitly.) However, consideration of the pupil diameter and retinal illuminance measurements reported by Bartleson(7) leads to the conclusion that the effective dependence on pupil diameter is small. Over the background luminance range of the parafoveal data, the pupil remains essentially fully dilated (with ranges from 7.8 to 8.0 mm diameter), so the use of the

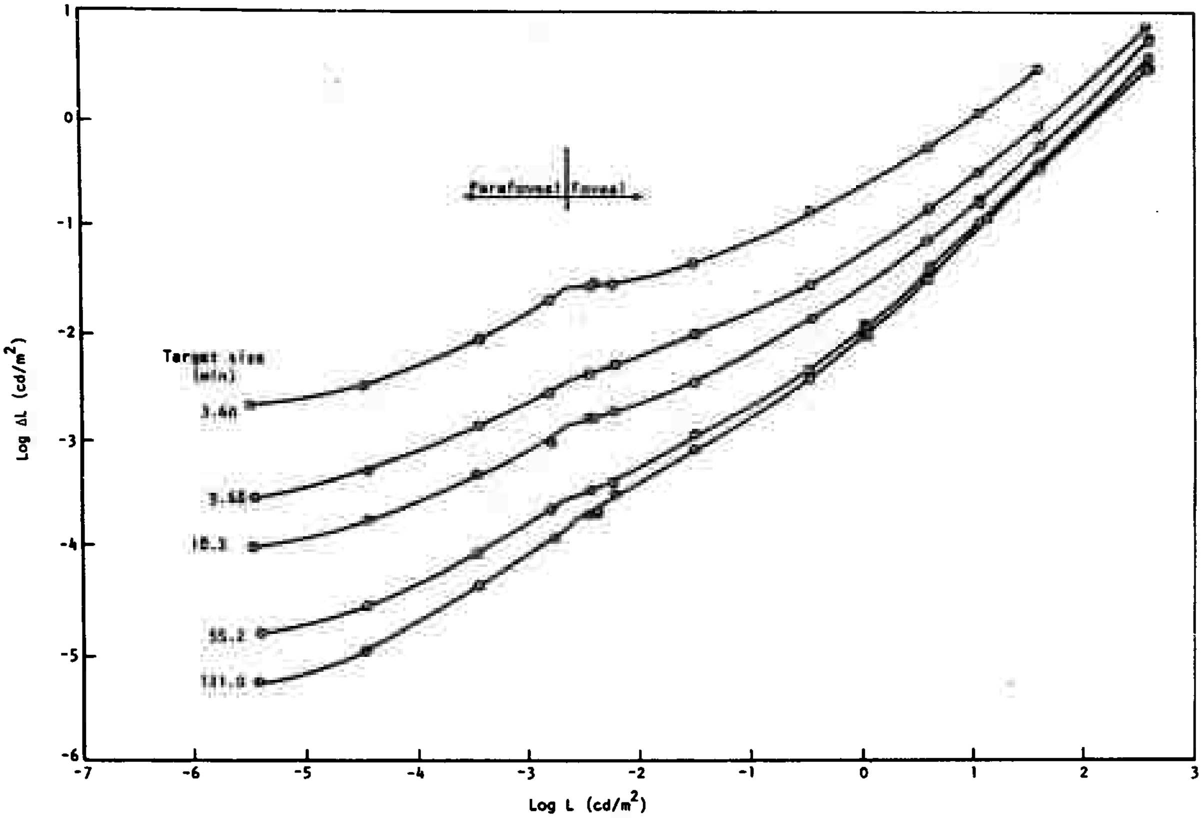

Fig. 2--The Tiffany experimental results for threshold target luminance, ΔL, as a function of background luminance, L, for targets subtending 3.60, 9.68, 18.2, 55.2, and 121.0 minutes of arc

constant k is valid. Although the pupil diameter does change greatly within the background luminance range of the foveal data, for background luminances large enough that the associated change in pupil diameter appreciably alters retinal illuminance (especially allowing for the Stiles-Crawford effect), the last term in Eq. (11) is dominant, and this term is independent of k. Thus, the neglect of variation in pupil diameter is valid over the entire range of the data.

In order to treat properly the dependence on target size, it is necessary to take into account that the effective area of the retinal image is not simply that indicated by geometrical optics. The retinal image is extended over a larger area by aberrations (of primary importance in the case of foveal vision) and by retinal neural structure (of primary importance in the case of parafoveal vision). The particular cases are discussed in more detail below.

FOVEAL-PHOTOPIC VISION

Foveal vision involves cone receptors, and the retinal neural structure provides separate neural connections to the optic nerve trunk for each individual cone receptor. The width associated with a receptor is small compared with the line-spread function of the retinal image. Hence the equivalent angular subtense θ_b by which the retinal image of a given source is extended results primarily from the line-spread function (for a finite source). Although this does not affect the number of excitations produced by the target luminance, the number of excitations produced by background luminance increases with retinal image size, including the equivalent angular subtense, as indicated in Eqs. (5) and (6). No measurements of the line-spread functions of the Tiffany observers are available. As a basis for obtaining an estimate useful in comparing the theoretical model with experimental results, the line-spread function reported by Westheimer and Campbell$^{(8)}$ for a living eye in best focus under favorable conditions was used. Their line-spread function gives the retinal illuminance as a function of angle φ in minutes from the center of the distribution as $f(\varphi) = \exp(-0.7\varphi)$. The full angular subtense between 1/e points is 2.8 min. For purposes of analysis of the Tiffany data, this value is increased by half to include the slight additional spread due to receptor width and to allow for the unfavorable focusing conditions of the Tiffany experiments. This gives $\theta_b = 4.2$ min for foveal vision, which is the value used here in evaluating the constants in Eq. (11). This approximation is considered adequate because θ_b is essentially a correction to target size, and the results are thus sensitive to uncertainty in θ_b only to a secondary degree.

Figures 3 through 6 illustrate the validity of various model assumptions and the quantitative agreement provided by the model presented here with the Tiffany foveal data for all target sizes. In Fig. 3 $\Delta L\theta_t^2 t$, which is proportional to the number of excitations produced by target luminance (or signal), is plotted logarithmically against $(L\theta_t^2 t)^{\frac{1}{2}}$, which would be proportional to the noise if it arose only from background luminance, as has been assumed in previous theories$^{(1-5)}$ and if there were no extension of the retinal image. The points are scattered, and those for different target sizes follow different courses, so that the effect of target size is not properly accounted for. There is abrupt curvature at low excitation levels and an apparent independence of signal from the noise. In Fig. 4, $\Delta L\theta_t^2 t$ is plotted logarithmically against $[L(\theta_t + \theta_b)^2 t]^{\frac{1}{2}}$, which is similarly proportional only to background noise but accounts for retinal image extension. Although the abrupt curvature remains, the points for different target sizes follow similar courses and the scattering of points is much reduced. Thus the effect of target size is reasonably well accommodated. In Fig. 5, $\Delta L\theta_t^2 t$ is plotted logarithmically against $[(\Delta L + L)\ \theta_t^2 t]^{\frac{1}{2}}$, which would be proportional to the noise assumed in the model presented here if there were no retinal image extension. The abrupt curvature is removed, indicating that the noise is more accurately described, but the points for different target sizes follow different courses, indicating that target size is not properly taken into account. In Fig. 6, $\Delta L\theta_t^2 t$ is plotted logarithmically against $\{[\Delta L\theta_t^2 + L(\theta_t + \theta_b)^2]t\}^{\frac{1}{2}}$, which is proportional to the noise assumed in the model presented here, including the effect of retinal image extension. The abrupt curvature is removed, the scattering of points is sharply reduced, and the points for different target sizes follow the same course.

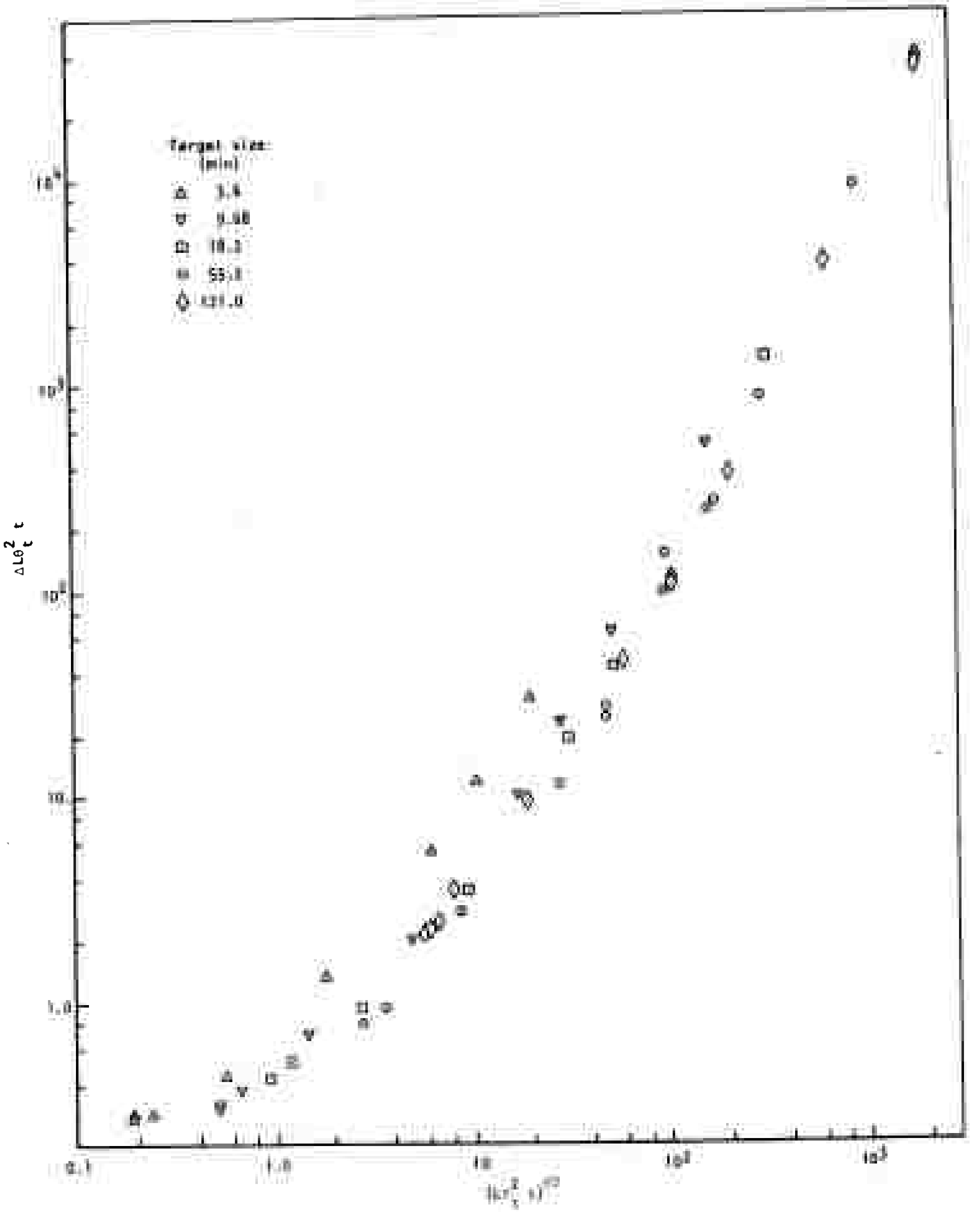

Fig. 3--The dependence of foveal threshold target luminance on noise that is assumed to arise from fluctuations in the number of excitations produced by background luminance only, and assuming no retinal image extension

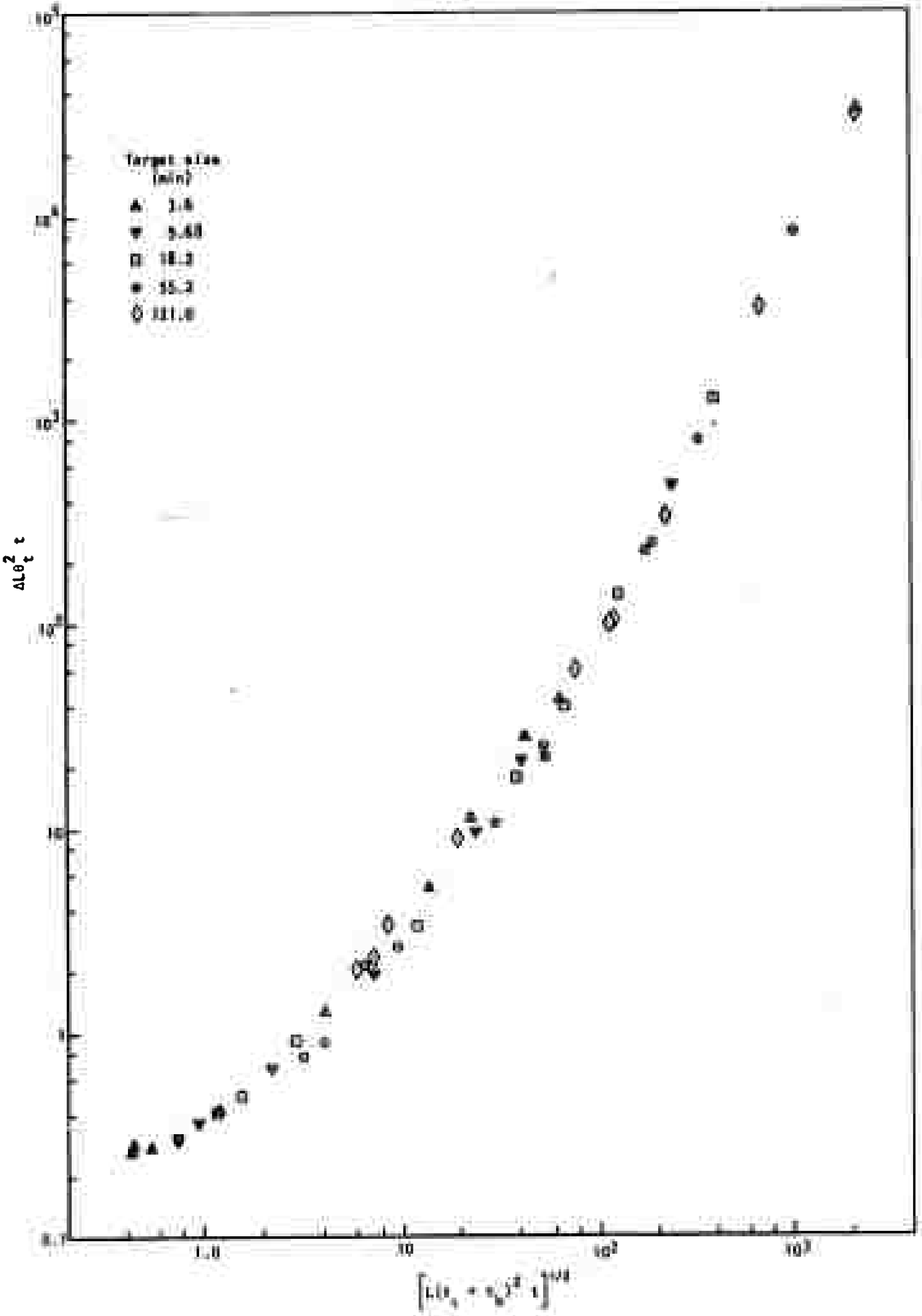

Fig. 4--The dependence of foveal threshold target luminance on noise that is assumed to arise from fluctuations in the number of excitations produced by background luminance only, and accounting for retinal image extension

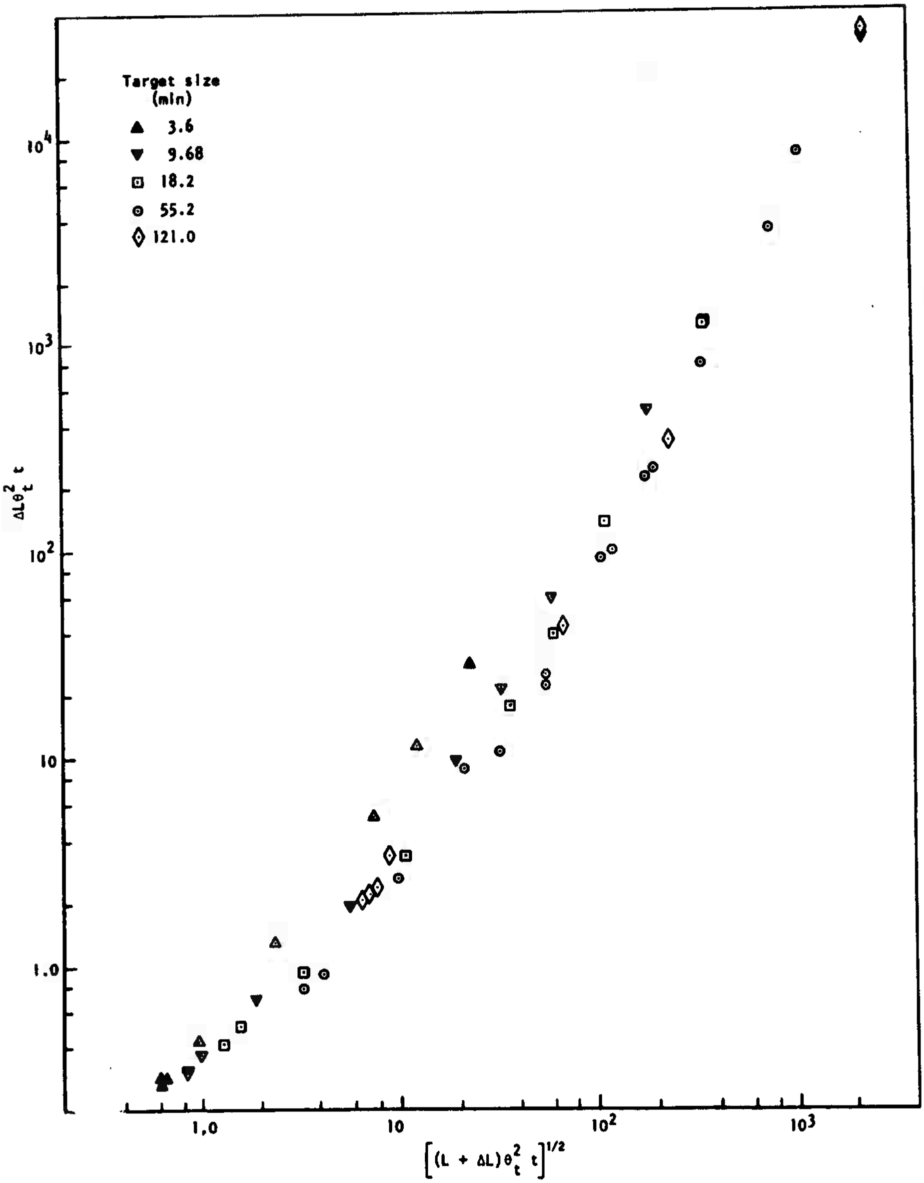

Fig. 5--The dependence of foveal threshold target luminance on noise that is assumed to arise from fluctuations in the number of excitations produced by total luminance, including both target and background, and assuming no retinal image extension

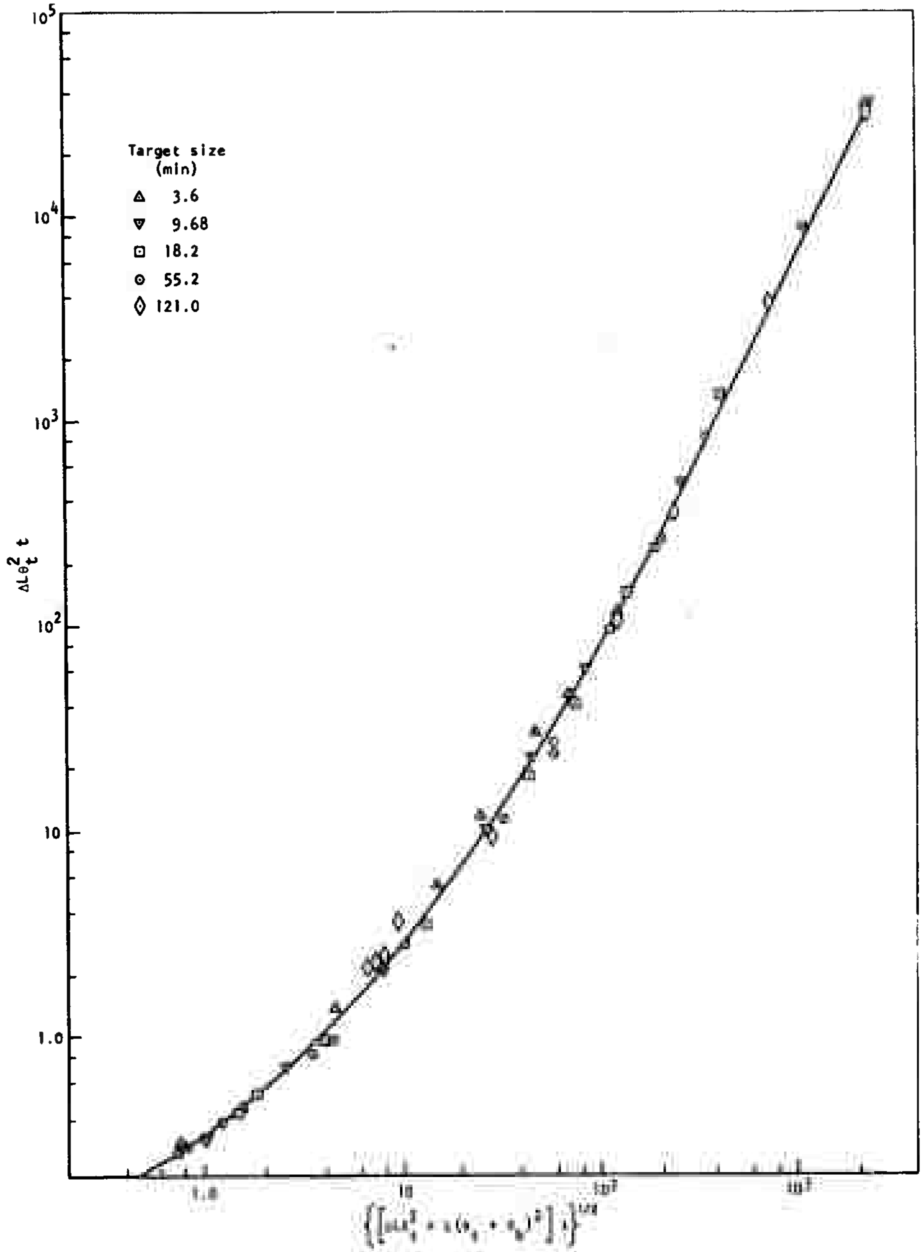

Fig.6--The dependence of foveal threshold target luminance on noise that is assumed to arise from fluctuations in the number of excitations produced by total luminance, including both target and background, and accounting for retinal image extension. The line is given by Eq. (11), with α = 0.116, β = 0.228, and γ = 0.00665

The abrupt curvature at low excitation levels occurs in Figs. 3 and 4 because at low luminances ΔL becomes larger than L, and the noise becomes nearly proportional to $(\Delta L \theta_t^2 t)^{\frac{1}{2}}$, with negligible dependence on L. For a given θ_t and t, there is a minimum perceptible ΔL, which can be evaluated from Eq. (11), with L set equal to zero, as

$$\Delta L_{min} = \frac{\beta^2}{2(1 - \gamma)^2 \theta_t^2 t} \left[1 + \frac{2\alpha(1 - \gamma)}{\beta^2} + \sqrt{1 + \frac{4\alpha(1 - \gamma)}{\beta^2}} \right] = \frac{0.226}{\theta_t^2 t} \qquad (12)$$

Thus ΔL can approach zero only as θ_t or t becomes infinitely large, and neither possibility is physically reasonable. The apparent leveling off of the curves in Fig. 2 is not the approach to an absolute limit of sensitivity of receptors, but it is the approach of ΔL to a value limited by the noise, which becomes dominated by target luminance fluctuations rather than by background luminance fluctuations. If the values of the coefficients given in the next paragraph are inserted in Eq. (12), the quantum efficiency assumed to be 5 percent, and the effective pupil diameter estimated from Bartleson(7) ($d^2 = 30\ mm^2$), then the minimum number of absorbed photons for foveal (photopic) detection under the conditions of the Tiffany experiments is 230. By contrast, the absorption of a single photon is sufficient for production of a measurable neuroelectrical potential, which indicates that the limit is imposed by the mode of system operation in the presence of noise and not by the inherent sensitivity of system components.

The line shown in Fig. 6 is calculated from Eq. (11) with the coefficients determined from the data as $\alpha = 0.116$, $\beta = 0.228$, and $\gamma = 0.00665$. This equation, which results from the present statistical detection theory, then reproduces the experimental points for all target sizes over a background luminance range of more than five orders of magnitude and a target luminance range of more than four orders of magnitude within an rms relative error of less than 14 percent. The experimental data points themselves are reported by Blackwell(6) to have a scatter of 5 percent about each of the separate empirical curves for different target sizes.

The commonly used descriptions of de Vries-Rose and Weber-Fechner regimes can be related to Eq. (11) with coefficients as derived above. If the onset of the de Vries-Rose regime is taken to occur when the second term of Eq. (11) exceeds the first term, and the onset of the Weber-Fechner regime occurs when the third term exceeds the second, then for a detection probability of 50 percent the de Vries-Rose regime corresponds to values of $[\Delta L\theta_t^2 + L(\theta_t + \theta_b)^2]t$ between 0.252 and 1170 and higher values fall in the Weber-Fechner regime. In terms of Fig. 6, this corresponds to values of $\{[\Delta L\theta_t^2 + L(\theta_t + \theta_b)^2]t\}^{\frac{1}{2}}$ between 0.502 and 34.2. These regimes are not an entirely satisfactory means of description; the contribution from the third term is appreciable, and it introduces noticeable curvature, so that the de Vries-Rose regime is poorly defined for foveal vision. It can also be noted that the proper reference quantity is the noise in integrated luminance (proportional to noise in the number of excitations) and that simple background luminance is not an adequate basis for description.

The coefficient β depends on P_d. The Tiffany data are for $P_d = 0.5$, so that $K = 0$. For $P_d < 0.5$, $K > 0$, and β would be smaller than given above, so the de Vries-Rose regime would be less extensive. For $P_d > 0.5$, $K < 0$, and β would be greater than given above, so the de Vries-Rose regime would then be more extensive.

PARAFOVEAL-SCOTOPIC VISION

The retinal location used in obtaining the Tiffany parafoveal data is not given explicitly by Blackwell.(6) As mentioned earlier, the shift from foveal to parafoveal vision can also be interpreted as a shift from photopic to scotopic vision as required for optimum sensitivity. It is well recognized that such a shift occurs at the background luminance of about 3×10^{-3} cd/m^2, corresponding to the break in the curves of Fig. 2. It is consistent with this interpretation to identify the parafoveal retinal location used with the region of maximum rod density. The distribution of rods is given by Vilter(9) for a vertical cut through the retina, and by Osterberg(10) for a horizontal cut, including both nasal and temporal directions. The distribution of rods is not quite symmetrical about the fovea. The angular displacement of the maximum rod density from the fovea ranges from about 17 to 28 deg, depending on direction.

The retinal neural organization in the parafoveal region differs significantly from that in the fovea. Many receptors are connected to each optic nerve fiber through a single ganglion cell, and there are some lateral interconnections between receptors.(11) This structure is illustrated schematically in Fig. 7. The core receptor field of a given ganglion cell includes those receptors that are connected directly, while the periphery field includes receptors that are indirectly connected through lateral cells. Response within the core receptor field is homogeneous, and responses sum. Thus, the span of the ganglion core's receptive field contributes to the retinal image an additional equivalent angular subtense which exceeds that arising from aberrations. The size of the ganglion dendritic span increases with the distance from the fovea of a given cell. The distribution of core diameters as a function of retinal location for a primate retina is given by Hubel and Wiesel.(12) According to their data, the ganglion core diameter corresponding to the region of maximum rod density is about a third to a half of a degree, including the effect of aberrations. For analytical purposes, the additional equivalent angular subtense for the parafovea is taken here as 25 min of arc. Although the value is approximate, the sensitivity to this value is small, and a change by ±5 min produces no significant change in agreement with the experimental data.

Like the figures for the foveal data, Figs. 8 through 11 illustrate the model features by comparison with the Tiffany parafoveal data. In Fig. 8 $\Delta L\theta_t^2 t$, which is proportional to the signal, is plotted logarithmically against $(L\theta_t^2 t)^{\frac{1}{2}}$, which would be proportional to the noise if the target contribution and retinal image extension were not present. As for the foveal data, the points are scattered; those for different target sizes follow different courses, and there is abrupt curvature at low luminances. In Fig. 9, retinal image extension is taken into account, but all noise is attributed to background luminance fluctuations. This corrects the target size dependence, but the abrupt curvature remains for low luminances. In Fig. 10, noise

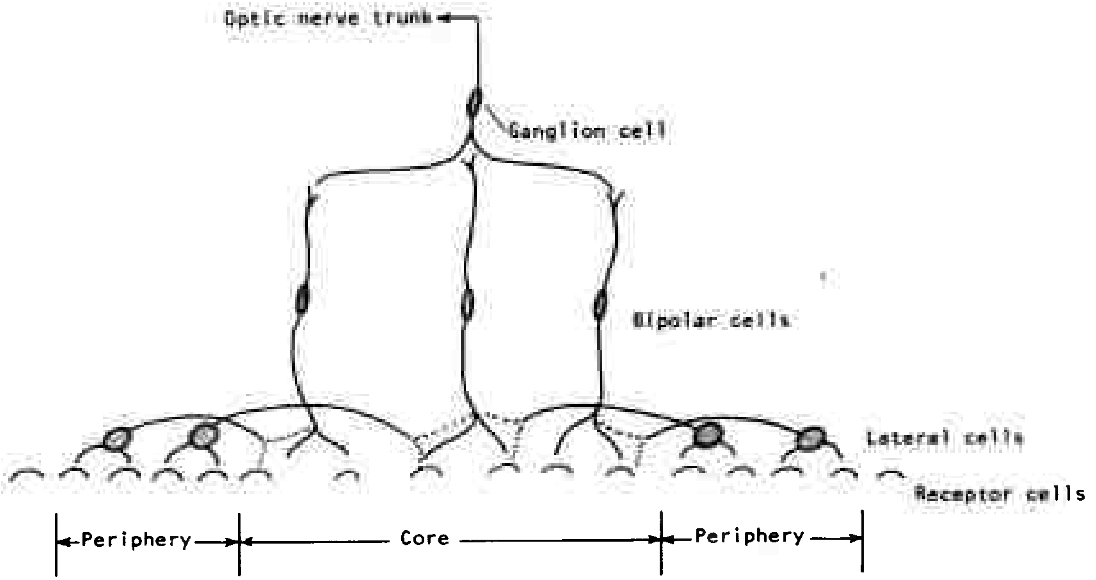

Fig. 7--Retinal neural organization associated with the ganglion cell receptive field. The core field includes receptor cells that are connected directly. The periphery field includes receptor cells that are connected indirectly through lateral cells.

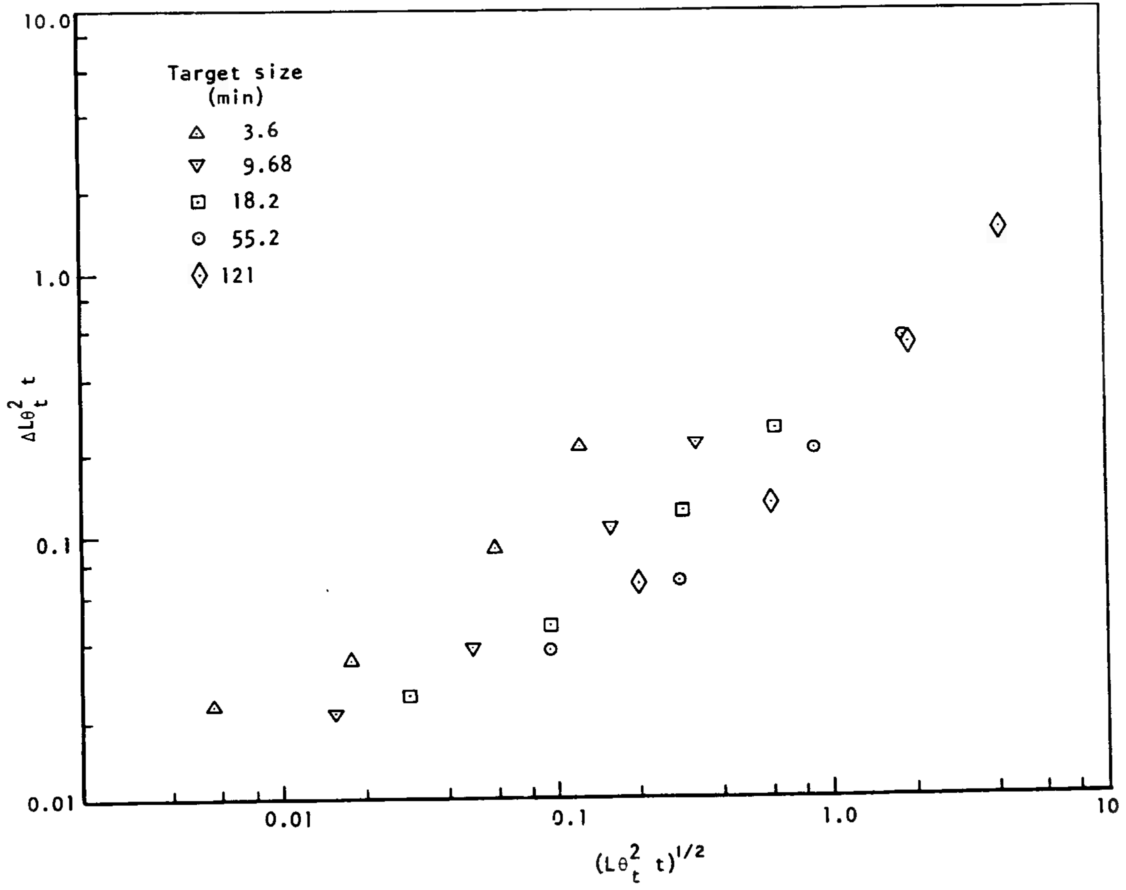

Fig. 8--The dependence of parafoveal threshold target luminance on noise that is assumed to arise from fluctuations in the number of excitations produced by background luminance only, and assuming no retinal image extension

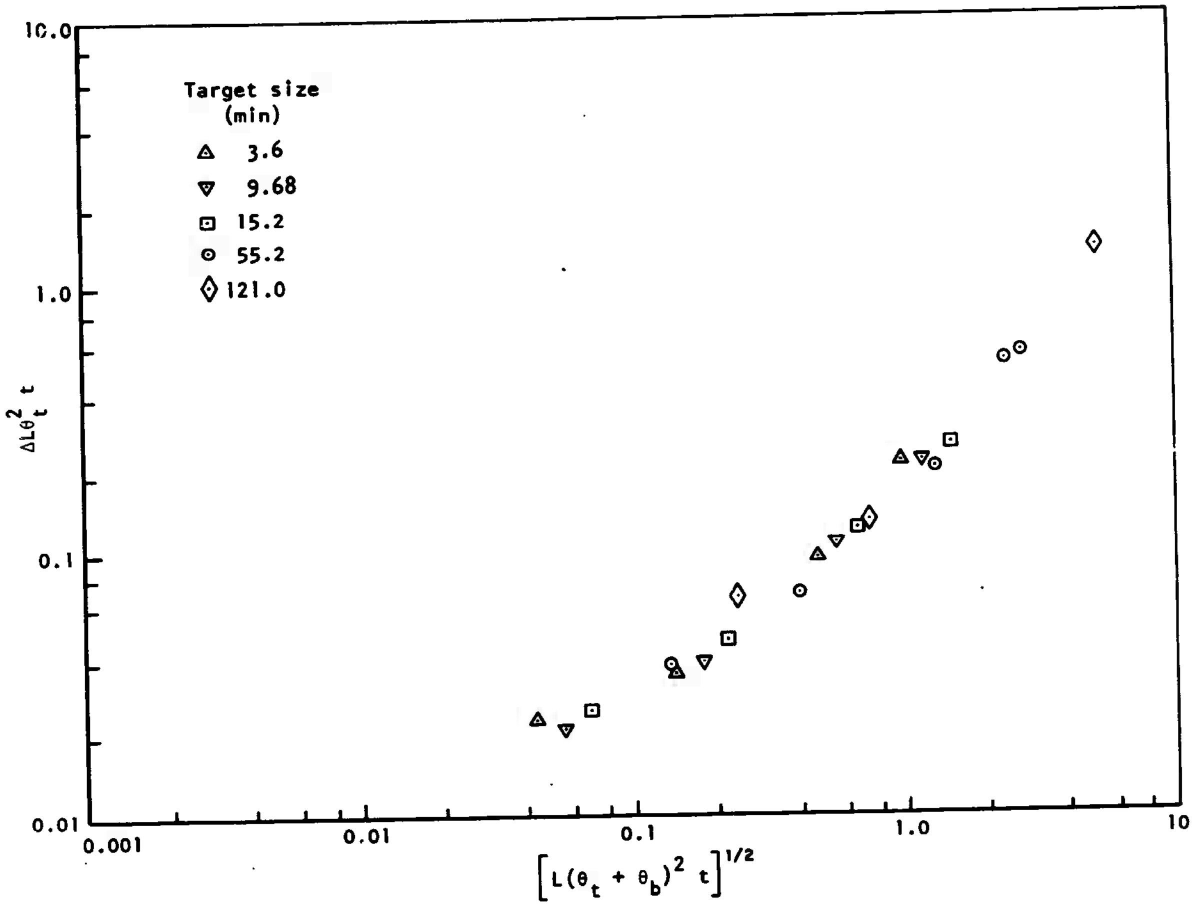

Fig. 9--The dependence of parafoveal threshold target luminance on noise that is assumed to arise from fluctuations in the number of excitations produced by background luminance only, and accounting for retinal image extension

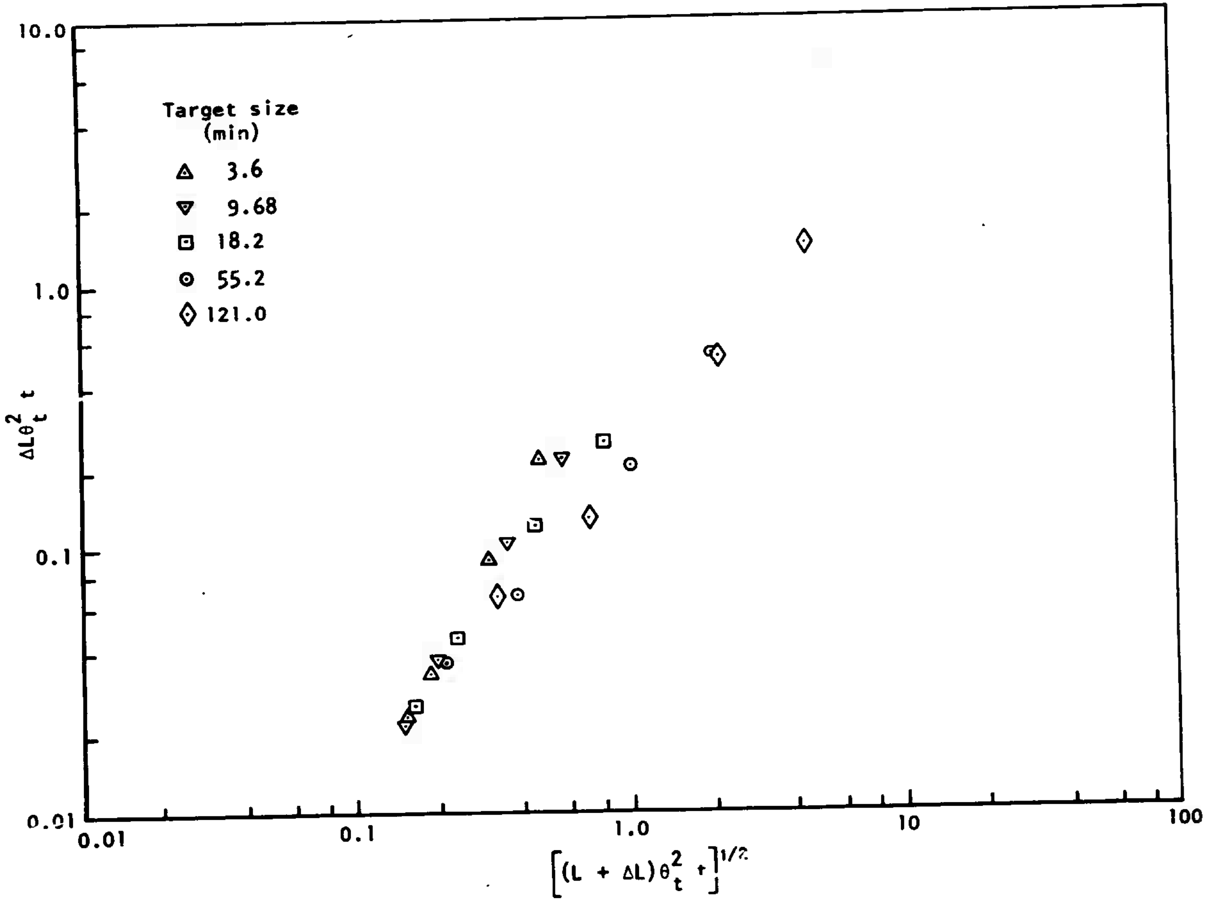

Fig. 10--The dependence of parafoveal threshold target luminance on noise that is assumed to arise from fluctuations in the number of excitations produced by total luminance, including both target and background, and assuming no retinal image extension

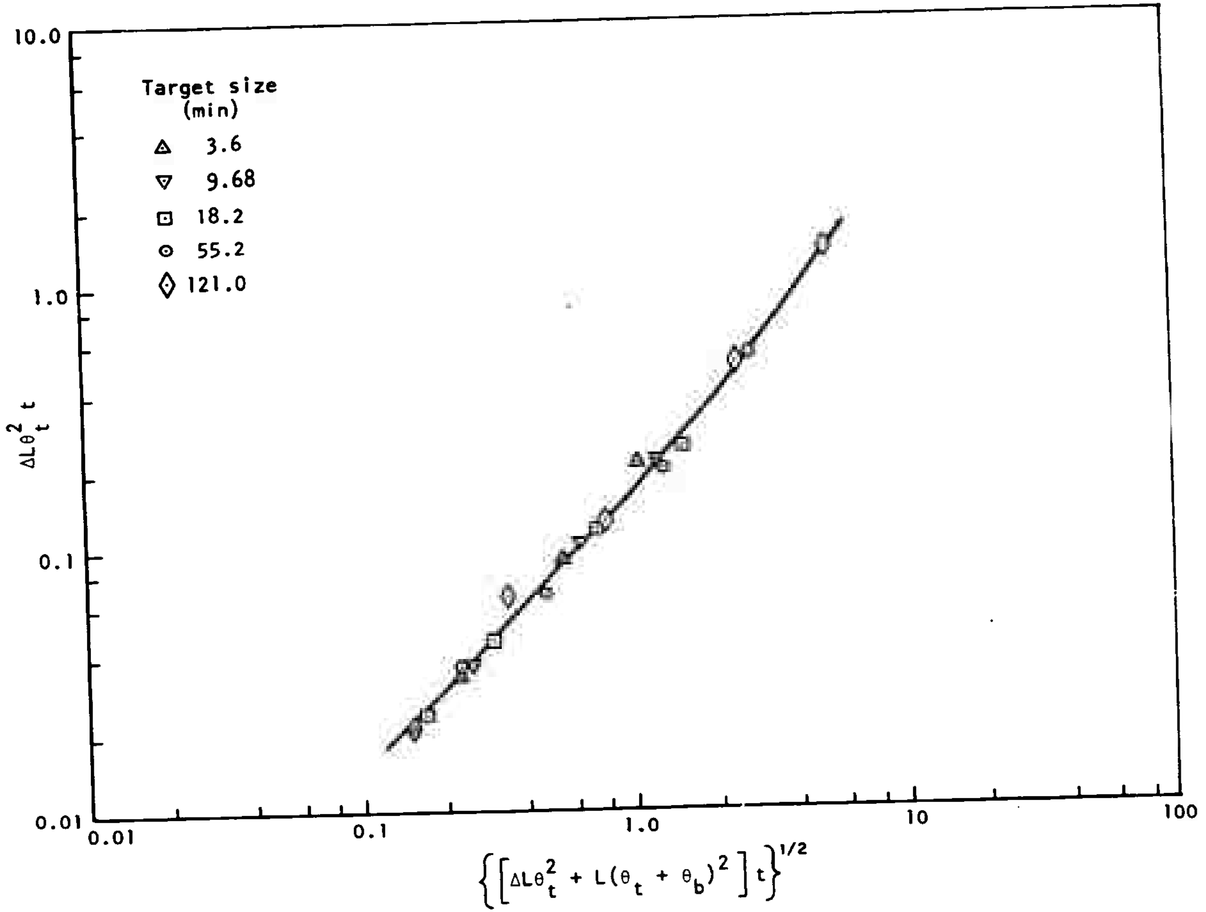

Fig. 11--The dependence of parafoveal threshold target luminance on noise that is assumed to arise from fluctuations in the number of excitations produced by total luminance, including both target and background, and accounting for retinal image extension. The line is given by Eq. (11), with $\alpha = 0$, $\beta = 0.142$, and $\gamma = 0.0183$

is attributed to total luminance fluctuations, but retinal image extension is not included. This removes the abrupt curvature, but the dependence on target size is not given correctly. In Fig. 11, $\Delta L\theta_t^2 t$ is plotted logarithmically against $\{[\Delta L\theta_t^2 + L(\theta_t + \theta_b)^2]t\}^{\frac{1}{2}}$, which is proportional to the noise according to the theory presented here, including the effect of extension of the retinal image. The abrupt curvature is removed and the data points follow closely along a single line.

The line shown in Fig. 11 is calculated from Eq. (11) with coefficients determined from the data as $\alpha = 0$, $\beta = 0.142$, and $\gamma = 0.0183$. The statistical detection theory equation then reproduces the experimental points for all target sizes over a background luminance range of three orders of magnitude and a target luminance range of more than three orders of magnitude within an rms relative error of 9 percent. The experimental data points themselves are reported by Blackwell(6) to have a scatter of 5 percent about each of the separate empirical curves for different target sizes.

It is noteworthy that the experimental points can be represented accurately with $\alpha = 0$, which implies that the lower limit on ΔL (or $\Delta L\theta_t^2 t$, which is proportional to the signal) is not an absolute one, and that any absolute limit associated with sensory component sensitivity is much smaller than the threshold values that have been measured. The lower limit in ΔL actually corresponds to a limit on signal-to-noise ratio. The leveling off of ΔL values with decreasing L, as shown in Figs. 2, 8, and 9, actually results from dominance of the noise by target luminance fluctuations. Even for a negligible absolute sensory threshold, there is a minimum detectable target luminance for a given target size and duration. The minimum can be determined from Eq. (11), using the parafoveal coefficients obtained above, as

$$\Delta L_{min} = \frac{\beta^2}{(1 - \gamma)^2 \, \theta_t^2 t} = \frac{0.021}{\theta_t^2 t} \tag{13}$$

The minimum detectable number of excitations is a constant for given observer factors and probability of detection, but varies with these factors. It should be reemphasized that, according to this analysis, it is incorrect to interpret the apparent leveling off of the ΔL value at low luminance as an approach to an absolute sensory threshold, and that previous determinations of absolute sensory threshold values based on such an interpretation are thus invalid. If values of the coefficients are inserted in Eq. (13), the quantum efficiency assumed to be 5 percent, and the effective pupil diameter estimated from Bartleson(7) ($d^2 = 30\ \text{mm}^2$), then the minimum number of absorbed photons for parafoveal (scotopic) detection under the conditions of the Tiffany experiments is 20. The limiting signal-to-noise ratio is then about 4.5. Again, this limit is imposed by the mode of system operation in the presence of noise, not by the inherent sensitivity of system components, which can respond to a single photon.

The de Vries-Rose and Weber-Fechner regimes are much better defined for the parafoveal than for the foveal data. The de Vries-Rose regime encompasses the parafoveal data for $[\Delta L\theta_t^2 + L(\theta_t + \theta_b)^2]t$ less than about 60, or for $\{[\Delta L\theta_t^2 + L(\theta_t + \theta_b)^2]t\}^{\frac{1}{2}}$ less than about 8, while larger values would correspond roughly to the Weber-Fechner regime. Considerations concerning the effect of P_d apply similarly to those already mentioned with respect to the foveal data.

EMPIRICAL LAWS

Empirical laws have been developed which attempt to summarize many experimental observations on the relations between target threshold luminance and target size and duration. These laws are rough approximations, and their ranges of validity are quite limited. Ricco's law states that threshold luminance is inversely proportional to target area; although originally intended to apply to small targets observed foveally, it also applies for sufficiently small targets (< 30 min) at 30 deg angular displacement from the fovea. Piper's law states that threshold luminance is inversely proportional to the square root of target size and applies to the parafoveal observation of relatively large targets, subtending from a few to a few tens of degrees. Besides these spatial laws, there are temporal laws such as Bloch's law, which states that for sufficiently short target durations threshold luminance is inversely proportional to target duration. For longer target durations, provided the target is not too large, Peiron's law states that threshold luminance is inversely proportional to the square root of target duration. Thus the empirical laws imply the equation

$$Lt^{p} \theta_{t}^{2q} = \text{const} \tag{14}$$

where definite values of p and q (appropriate to given conditions) are specified by the laws. Actually the exponents are not so well defined, but they span a range of values, so the laws are greatly simplified. The spatial and temporal laws are also coupled to some extent.

It can be seen from Eq. (11) that not only are the empirical laws included, but also the transitions between laws. Further, the actual dependence on target size and duration is more complicated than indicated by the approximate laws. Bloch's law should hold strictly only for foveal vision when the noise is sufficiently small that the constant term is dominant. As the noise increases and the second term assumes importance, Pieron's law is approached for foveal vision. For parafoveal vision, Pieron's law holds in the lower luminance portion of the Tiffany data (the same range as covered by the de Vries-Rose law if total luminance is considered). The dependence on target duration diminishes as total luminance increases and, over the range covered by the Weber-Fechner law, threshold luminance is independent of target duration.

The equivalent angular subtense that arises from retinal image extension and the noise that arises from target luminance excitation fluctuations complicate the dependence of threshold luminance on target area, except for foveal vision when the noise is sufficiently small that the constant term dominates. Then, corresponding to Bloch's temporal law, Ricco's spatial law obtains. As the noise increases and the second term becomes dominant for foveal vision, or when the corresponding first term dominates for parafoveal vision, the spatial law that holds depends on the relative importance of target noise and of image extension. When the noise is primarily from background luminance excitation fluctuations and the target is small compared with the retinal image extension, Ricco's law remains appropriate; for targets large compared with the retinal image extension, Piper's law holds. When the noise is primarily from target luminance excitation fluctuations, Piper's law holds. As the total luminance increases further and the last term becomes dominant, the dependence on target size diminishes and threshold luminance becomes independent of target size, except that Ricco's law still remains appropriate for sufficiently small and luminous targets (i.e., targets smaller than the retinal image extension, but which produce excitation fluctuations greater than background luminance excitation fluctuations). Thus, although the threshold dependences specified by the various empirical laws are contained within Eq. (11), a more detailed description is given by the equation, especially with respect to the important roles of target luminance excitation fluctuations and retinal image extension in determining threshold luminance.

IV. DISCUSSION

VALIDITY OF THE MODEL

As demonstrated in the previous section, the statistical detection model provides a consistent quantitative description of threshold visual performance for both foveal and parafoveal vision over a wide range of background luminance and target parameters. (Only a simple test of the dependence on target duration as compared with empirical laws is made, because only one definite observation time is included in the data used for detailed comparison.) The broad range of consistency between theory and experiment results largely from recognition of noise as the controlling parameter and the use of a decision criterion that is programmed as a function of the noise. The comparison with experimental data also shows the importance of fluctuation in the number of excitations produced by target luminance in contributing to the noise and the importance of retinal image extension, whether due to aberrations (as in the foveal case) or retinal neural organization (as in the parafoveal case). The noise is attributed to fluctuations in the number of excitations resulting from the luminance, including both background and target luminance. This provides a direct explanation of the apparent independence of ΔL at low background luminance, such that noise from fluctuations in the number of excitations produced by target luminance is dominant and noise due to fluctuations in background luminance excitations is relatively negligible. The additional equivalent angular subtense attributed to retinal image extension derives from reasonable physiological assumptions and provides an element that is essential to the quantitative description of the dependence of threshold luminance on target size. Thus, the validity of the model presented here is supported in detail by comparison with experimental results.

As described here, the decision criterion depends on the mean and standard deviation of the number of excitations produced by total luminance, including both target and background. This is not to imply the need to determine during the observation period the mean and standard deviation of the fluctuations in the number of excitations produced by the total luminance, which would not be logically consistent since only a sample of a random quantity is available during an observation period. Rather, for a given set of motivational factors and a priori knowledge, which influence the coefficients in Eq. (11), the equation provides a definite relationship between the (number of excitations produced by the) required threshold target luminance and the mean and standard deviation of background luminance (excitations), which are considered to be determined in the process of adaptation to the background luminance. Then, except for target size and duration, all quantities necessary to specify the decision criterion--and hence the required threshold target luminance--are available; target size and duration are included in a parametric sense. Within the spatial and temporal period of observations, a random sample of luminance is taken and matched against the decision criterion. The matching of integrated samples occurs continuously during the time that a target might be present (period of attention) over the array of possible target sizes and durations. This appears to be feasible for an array of receptors and memory functions, but no physiological mechanism is specified here.

LIMITATIONS OF THE MODEL

The development of the argument in Section II is based on the assumption of a linear relationship between the number of excitations and the luminous energy incident upon the retina. This is expected to be the case over a wide range of values, but clearly the condition can be violated at sufficiently high incident luminous energy. Aguilar and Stiles(13) report a saturation effect at high incident luminous energy, which they suggest can be explained by photochemical exhaustion or by refractory period limitations. Either of these possibilities would violate the linearity assumption. Another possibility is that the sharp rise in threshold that they attribute to saturation results from an additional control on the decision criterion, beyond those considered here. Also, the theory cannot accommodate unlimited extension of the observation period, since this would require consideration of finite limitations on the memory functions assumed necessary for matching over various possible target durations.

DECISION CRITERION CONTROL

The fundamental basis of the programmed control of the decision criterion is of some interest, and it can be interpreted in terms of the limitations of the information-acquisition and processing capabilities of the visual system. These differ somewhat in that the input information available is related to the number of inherently distinguishable levels of luminance deviation from background luminance, while processing is concerned with the number of levels of total luminance actually distinguished through operation of the visual system. Some complications are avoided by restricting attention to incoherent illumination. Also, the constant term in Eq. (7), which is significant only for the case of foveal vision, is ignored for the moment.

At relatively low luminance excitation noise, the decision criterion increment is dominated by the term in σ_T, and thus is programmed to maintain a constant signal-to-noise ratio, the minimum necessary within the constraints imposed by the observer factors. Then the visual system utilizes fully the available input information. This can be considered an input-limited regime, in which the full capability of the acquisition function is used but the capacity of the processing function is not used fully. That is, the number of inherently distinguishable levels of total luminance is smaller than the number of distinct levels that could be accommodated by the processor. In this regime, which corresponds to the de Vries-Rose regime, the rate of information acquisition is constant. However, the number of distinguishable levels of total luminance increases, or the distinguishable fraction of total luminance (contrast) decreases, as the square root of the total luminance. Thus demands on the processing function increase as luminance increases.

However, the processing function is finite and limited in capability, so that with increasing luminance an output-limited regime is approached as the capacity limit of the processor is approached. In this regime, which corresponds to the Weber-Fechner regime, the processing function of the visual system cannot fully utilize the available input information. The decision criterion increment is then dominated by the term in σ_T^2, and is thus programmed to maintain a constant ratio of threshold luminance increment to total luminance, corresponding to a constant number of distinguishable levels of total luminance. Thus the programmed decision criterion control can be interpreted as providing for optimal performance of the visual system within its limitations.

The constant term which is significant for foveal vision is not considered to be associated with an ultimate threshold, although it could arise from an internal noise source. It is controlling in the region in which there is a transition in maximum sensitivity between foveal (photopic) vision and parafoveal (scotopic) vision, and can perhaps best be considered as a mechanism which sharpens that transition.

DESIGN OF EXPERIMENTS

This analysis identifies a number of points that could be further tested and clarified by experiments specifically designed to consider them. The problem is to obtain comprehensive experimental results in which all the factors are taken into account. In the past they have all been studied, but not in an integrated fashion. Experiments such as were performed in obtaining the Tiffany data should be extended to include the considerations listed below.

Better experimental control of retinal illumination is desirable in a number of respects. Retinal image extension is a factor that exerts considerable influence on threshold luminance dependence on target size, but it is treated here in only an approximate manner. For foveal vision, arrangements that facilitate best focusing would be helpful, as would more detailed information on ocular aberrations of the observers. For parafoveal vision, fixation arrangements are needed that permit determination of the portion of the retina used in detection. This would permit testing more rigorously the assignment of an additional equivalent angular subtense to the ganglion dendritic span, which is a function of retinal location. Although the data discussed here depended little on pupil diameter, this variable can be controlled through the use of an artificial pupil.

The experimental data used here for detailed comparisons result from only one value of target duration, and that value is somewhat uncertain in that it is inferred from search conditions. Data obtained over a range of target duration would permit further testing of the statistical detection theory and would possibly disclose more structure in the time dependence. Control of target exposure time by means of shutters would permit separation of simple detection factors from those associated with search procedures. The exposure times should include those sufficiently long to determine possible effects of memory limitations.

The data reported from threshold experiments consist primarily of detection statistics. However, some aspects of statistical theories are best tested against false detection statistics, and collection of such data should be an experimental objective. Further exploration of the influence of a priori knowledge and observer motivational factors is also of interest.

V. CONCLUSIONS

A number of conclusions are indicated by the analysis presented here. Within the limitations imposed by some of the approximations necessary in treatment of the comparison data, they appear to be reasonably well established.

1. The statistical detection theory presented here is appropriate for the description of threshold visual performance, and is quantitatively consistent with the results of both foveal (photopic) and parafoveal (scotopic) visual threshold experiments over a wide range of background luminance and target parameters.

2. The decision criterion is programmed and is controlled for optimal performance within the information-acquisition and processing limitations of the visual system.

3. The excitation noise resulting from fluctuations in the number of excitations produced by both background and target luminance is a primary factor in determining visual performance.

4. Retinal image extension, which results primarily from ocular aberrations in foveal vision and the ganglion dendritic span in parafoveal vision, significantly affects the threshold dependence on target size, as described in Eq. (11).

5. Foveal visual thresholds are described quantitatively by Eq. (11) with $\alpha = 0.116$, $\beta = 0.228$, and $\gamma = 0.00665$; parafoveal thresholds are described quantitatively by Eq. (11) with $\alpha = 0$, $\beta = 0.142$, and $\gamma = 0.0183$.

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10. ABSTRACT

The development of a statistical detection model to provide an accurate, quantitative description of threshold visual performance over a wide range of background luminance and target parameters. This work investigates relationships that exist between visual performance and reconnaissance. A statistical theory is developed in which neural excitation noise results from random fluctuations in both target and background luminance, and decision criterion is assumed to be programmed. Detection is said to occur if the number of excitations resulting from observation of the fluctuating target and background luminance exceeds the criterion. The validity of the model is supported by comparison with experimental results using Tiffany data. Suggestions are made for the design of further experiments.

11. KEY WORDS

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