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REVIEW OF THE PROPERTIES OF SNOW AND ICE

by

UNIVERSITY OF MINNESOTA
INSTITUTE OF TECHNOLOGY
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REVIEW OF THE PROPERTIES OF SNOW AND ICE

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PREFACE

This report was prepared in the Engineering Experiment Station of the University of Minnesota by a research group under the direction of Dr. Richard C. Jordan. The work was sponsored by the Snow, Ice and Permafrost Research Establishment, U.S. Corps of Engineers. It is difficult to assess the individual contribution in a joint effort such as this; a number of persons have collaborated on some portions of the work. The various project members have benefited by the association and discussions with the two project members with extensive experience in snow research, Dr. Henri Bader and Mr. B. Lyle Hansen. The project members and the areas of their major contribution to this report are: Henri Bader, Chapter IV; Carl S. Benson, Chapters VIII, IX and X; Paul P. Bey, Chapter VII; Robert H. Doherty, Chapters I and II; Roger G. Johnson, Chapter III; Jerome A. Joseph, Chapter II; Richard J. Goldstein, Chapters I, V and VI; B. Lyle Hansen, Chapters V and VI; Stanley W. Rasmussen, Chapter III; and Daniel C. Schiavone, Chapter I. Acknowledgment is also made of the services of Marvin Sandgren for the preparation of the figures and to Maxine Mandt for assistance in assembling and proofreading as well as typing the manuscript.

Homer T. Mantis

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TABLE OF CONTENTS

LIST OF FIGURES

PAGE

INTRODUCTION

The purpose of this review of the properties of snow and ice is to provide the necessary background for the organization of a research program and to summarize the information on these properties for the use of workers in the field of snow and ice research. Any compilation of the properties in all phases of water substance is overshadowed by the masterly work of N. E. Dorsey, Properties of Ordinary Water Substance. The level of competence attained by Dorsey is not claimed for this work; most of the material reported here is that published after Dorsey's compilation and the greatest emphasis laid on the mechanical properties of snow and ice.

The report has been divided into chapters which are based on a somewhat arbitrary grouping of similar properties. Several general subjects, The Heat Economy of the Snow Pack and Ice Formation in Open Water also are presented in single chapters. The bibliography has been arranged by subject and appears at the end of each chapter. A number of abstracts of pertinent literature have been appended. An asterisk before the author's name in the bibliography indicates that an abstract of this paper appears in the Appendix.

Chapter I

MECHANICAL PROPERTIES OF SNOW AND ICE

1. General Concepts

Before discussing the specific question of the mechanical properties of snow and ice, it may be useful to review some of the fundamental concepts and the terminology employed in the subsequent sections of this report. The aim of this portion of the report is to present a brief summary of the important factors encountered when discussing the mechanical properties of materials.

Materials can generally be classified as belonging to one of three main groups according to their behavior under stress, namely: 1) elastic, 2) plastic and 3) viscous. In some cases overlapping of the main groups occurs and sub-dividing such as plasto-elastic or visco-elastic becomes necessary.[1]

An elastic material is one which deforms when subjected to an external load but returns to its original condition with the removal of the load. Hence an elastic deformation is one which disappears with its promoting stress. Elastic materials are usually studied with the aid of stress-strain diagrams as shown in Fig. 1. The solid line represents the linear relationship between stress and strain, which is very nearly satisfied by many materials if the stress is less than a certain limiting value. A tensile or compressive stress (σ) is related to the unit deformation (ϵ) by means of Young's modulus (E),

$$E = \frac{\sigma}{\epsilon} \tag{1}$$

In the case of a shearing stress (τ) we have a similar expression,

$$G = \frac{\tau}{\gamma} \tag{1a}$$

in which γ represents the unit shearing deformation and G, the counterpart of Young's modulus, is referred to as either the shear modulus or modulus of rigidity. Both equations (1) and (1a) are known as Hooke's law.

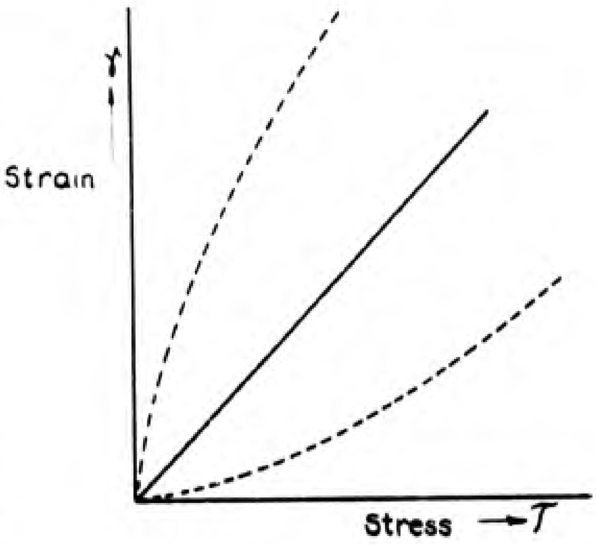

Figure 1. Stress-strain diagram for an elastic material

An elastic body suffers a deformation not only in the direction of the applied load, but also perpendicular to it. The ratio of the deformations is constant for a given material and is known as Poisson's ratio (μ). It can be shown[2] that Young's modulus and the shear modulus are related by the equation

$$G = \frac{E}{2(1+\mu)} \tag{2}$$

The usual procedure is to determine one of the modulii and Poisson's ratio experimentally and from this data calculate the other modulus. Surprisingly most materials have a Poisson ratio of approximately 0.30 and, therefore, $\frac{E}{G} \cong 2.5$. Another factor that is often determined is K, the bulk modulus, $\equiv V\frac{dP}{dV}$ where V = volume and P = pressure. It is related, for elastic deformation, to the other elastic constants by the formula, $E = 3K(1 - 2\mu)$.

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[1] For a detailed classification and defining diagrams see British Rheologists' Club, "Systematik der elastischen und plastischen Deformation," Nature, 149 (1942) 702.

[2] Timoshenko, S., 1940: Strength of Materials. Van Nostrand Co. p. 57.

A viscous material is one in which a constant stress, however small, causes a permanent deformation which increases continuously with time. The behavior of a viscous material is characterized by Fig. 2, in which t represents time and the other symbols are as previously described. The solid line represents the linear relationships associated with the so-called Newtonian Fluids--those which satisfy the equation

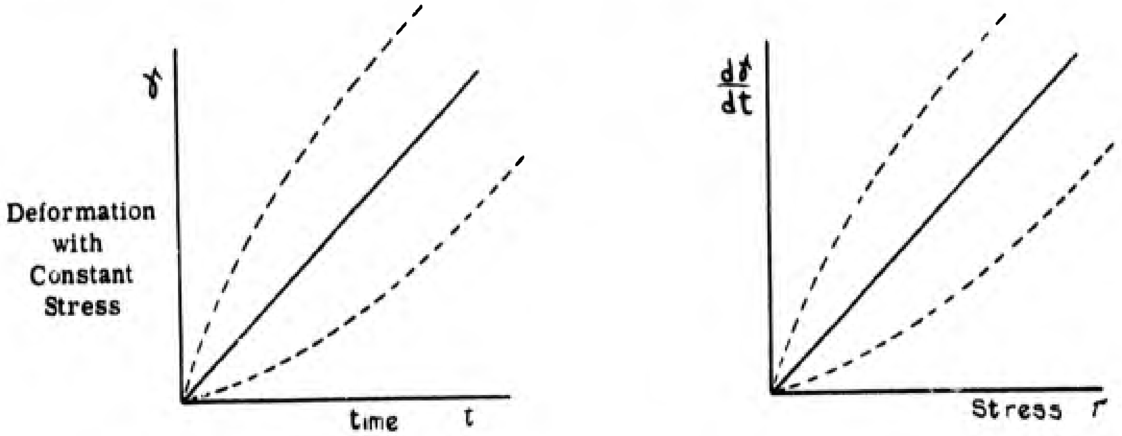

Figure 2. Deformation-time diagram for a viscous material

$$\tau = \eta \frac{dv}{dz} \tag{3}$$

in which $\frac{dv}{dz}$ is the velocity gradient perpendicular to the direction of flow and η a proportionality constant referred to as the coefficient of viscosity. Equation (3) assumes the shearing stress between adjacent layers of the fluid to be proportional to the velocity gradient; such a flow is referred to as viscous flow. Equation (3) is often written in the form

$$\tau = \eta \frac{d\gamma}{dt} \tag{3a}$$

As defined in either of equations (3) or (3a), η is known as the absolute viscosity. In the absolute cgs system of units it has the dimensions of dynes sec cm^{-2} or gm sec^{-1} cm^{-1} known as the poise. Probably the most common unit, however, is the centipoise--1/100 of a poise.

Although the term plastic material is very common and widely used, it is very difficult to define. Probably the best and simplest definition is the one usually associated with Maxwell and is as follows,[3] "If the form of a body is found to be permanently altered when the stress exceeds a certain value, the body is said to be soft or plastic and the state of the body when the alteration is just going to take place is called the limit of perfect elasticity." The behavior of such a material is shown in Fig. 3, the solid line depicting what is usually referred to as a Bingham plastic. It should be noted that a Bingham plastic differs from a Newtonian fluid by the requirement of exceeding the limiting stress f_B. Accordingly, Bingham defines the relationship in equation form

$$\tau - f_B = \frac{d\gamma}{dt} \tag{4}$$

By differentiating equation (1a) with respect to time we obtain

$$\frac{d\tau}{dt} = G \frac{d\gamma}{dt} \tag{5}$$

If a material simultaneously exhibits elastic and plastic properties it may be possible to combine the

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[3] Bingham, E. C., 1916: An Investigation of the Laws of Plastic Flow. Scientific Papers of the Bureau of Standards, No. 278.

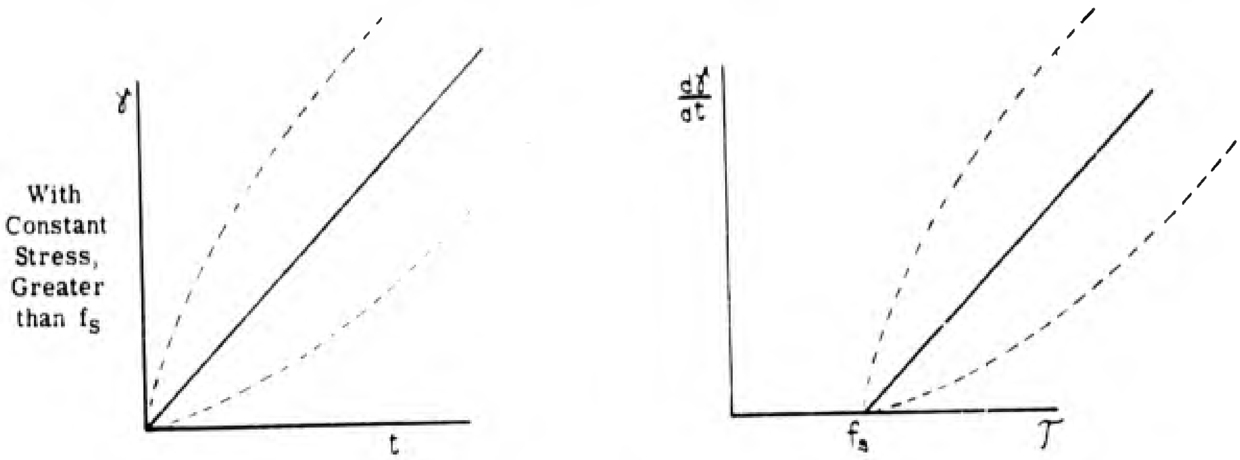

Figure 3. Deformation-time diagram for a plastic material

two preceeding equations to discover its behavior. Maxwell[4] does this by introducing a new constant λ, the time of relaxation, such that

$$\frac{d\tau}{dt} + \frac{\tau}{\lambda} = G\frac{d\gamma}{dt} \tag{6}$$

In analyzing equation (6), the deformation can be considered to be composed of two terms, $\frac{d\tau}{dt}$ and $\frac{\tau}{\lambda}$, representing the elastic and plastic contributions respectively.

The meaning of λ becomes clear if we consider the case for which the deformation is held constant, i.e., $\frac{d\gamma}{dt} = 0$. By integration, we obtain from equation (6)

$$\tau = \tau_0 e^{-\frac{t}{\lambda}} \tag{7}$$

where τ_0 is the stress at time $t = 0$. Hence we see that the time of relaxation (λ) is the time required for the stress to decrease to $\frac{1}{e}$ of its original value under constant deformation. From equation (7) it will be noted that, for a perfectly elastic body, $\lambda \longrightarrow \infty$, and therefore the value of λ may be considered as a measure of the elastic and plastic components of a deformation. For a very large time of relaxation the material is predominantly elastic, whereas a very small value indicates plastic behavior.

Correspondingly, by considering a constant stress, $\frac{d\tau}{dt} = 0$, we obtain from (6),

$$G\frac{d\gamma}{dt} = -\frac{\tau}{\lambda}$$

or

$$\tau = G\lambda\frac{d\gamma}{dt} \tag{8}$$

By comparison with equation (3a) we see that equation (8) defines a viscous material and consequently

$$\eta = G\lambda$$

or

$$\lambda = \frac{\eta}{G} \tag{9}$$

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[4] Houwink, R., 1937: Elasticity, Plasticity, and Structure of Matter. Cambridge Press, Cambridge, 376 pp.

From this we see that the measurement of the time of relaxation can be useful in determining the viscosity of a material. However, attempts to verify Maxwell's original hypothesis, equation (6), have not been too successful and consequently the validity of equation (9) must remain questionable.

Umstätter[5] has drawn attention to the influence of time on the viscosity recorded for systems in which the flows caused certain structural changes which are influenced by relaxation (relieving of internal stresses with time through plastic flow). Starting with equation (6), for constant velocity of deformation he derived the formula

$$\eta_t = \eta_\infty + (\eta_0 - \eta_\infty)\, e^{-\frac{t}{\lambda}} \tag{10}$$

where η_t = viscosity at time t
η_0 = viscosity at $t = 0$
η_∞ = viscosity at $t = \infty$

Weinberg[6] starting from equation (7) and taking into account the limit of elasticity, according to Schwedoff's theory of relaxation, derived an expression for the viscosity in terms of rate of deformation which checked well with his previous experiments on ice. He obtained for a constant ϕ

$$\eta_\phi = \eta_\infty + \frac{F_0}{\phi} \tag{11}$$

where F_0 = limit of elasticity
ϕ = angular velocity of shearing
η_∞ = viscosity for infinitely great velocity of shearing
η_ϕ = viscosity for velocity of shearing

Experimental data on various plastic substances often leads to the empirical equations[7]

$$\frac{d\gamma}{dt} = \frac{1}{\eta^*}\, \tau^n \tag{12}$$

or

$$\frac{d\gamma}{dt} = \frac{1}{\eta^*}\, (\tau - f)^n \tag{13}$$

where η^* does not have the same dimensions as viscosity since $n \neq 1$.

Flow that is described by equation (12) is called quasi-viscous or non-Newtonian flow and that following equation (13) is described as being quasi-plastic.

2. Elastic Constants

Theory: Since Young's modulus, Poisson's ratio, and the shear modulus are closely related and often determined simultaneously, we shall accordingly treat them as a group.

Young's modulus and the shear modulus are the proportionality constants defined in Hooke's law, namely

$$E = \frac{\sigma}{\epsilon} \tag{1}$$

and

$$G = \frac{\tau}{\gamma} \tag{2}$$

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[5] Umstätter, H., 1935: Zustandsänderungen zäher Systeme, IX, Viskosität und Elastizität, Kolloid. Zeitschrift., 70:174-180.

[6] Weinberg, Boris, 1938: Mechanical properties of ice, Association Internationale d'Hydrologie Scientifique, Bulletin No. 23, 509-536.

[7] Houwink, op. cit.

in which

E = Young's modulus

σ = Tensile or compressive stress

ϵ = Unit deformation (elongation or compression)

G = Shear modulus or rigidity

τ = Shear stress

γ = Unit deformation in shear

Since E and G are considered as constants, Hooke's law assumes a linear stress-strain relationship. Equations (1) and (2) apply only to an elastic material stressed below the elastic limit and assume a complete return of the material to its original condition upon removal of the stress.

Poisson's ratio is defined as the ratio of the unit deformations perpendicular to and parallel to the applied load, or

$$\mu = \frac{\epsilon_x}{\epsilon_z} = \frac{\epsilon_y}{\epsilon_z} \tag{3}$$

in which

μ = Poisson's ratio

ϵ_z = Unit deformation in "z" direction, assumed parallel to load

$\epsilon_x = \epsilon_y$ = Unit deformations in "x" and "y" directions

It can be shown that E, G and μ are related by the equation

$$G = \frac{E}{2(1 + \mu)} \tag{4}$$

Therefore, if two of the three variables are known, the third is easily calculated.

Reported values: Wide variations are found in the values for the elastic constants for ice, obtained by static methods (usually by the bending of beams), and they are considered unreliable. Dynamic methods have been used by several investigators; their results show good agreement and are usually accepted as the most accurate. A summary of the data is presented in Table I, page 6. More complete tables can be found in the original papers.

Evaluation and interpretation of measured values: Most of the static determinations were made prior to Dorsey's work, those listed in Table I were taken mainly from his book. Variations greater than one order of magnitude are easily found. Dorsey mentions that the static values have been interpreted as indicating a dependency of E on the direction of the optic axis. This dependency is not established but rather contradicted by sonic determinations.

The work of Boyle and Sproule was among the earliest dynamic (or sonic) work to determine the elastic constants of ice. Their results are considered as very reliable, Dorsey going so far as to say, "The most accurate values are those of Boyle and Sproule." In their work the velocity of propagation of longitudinal waves was determined, and Young's modulus calculated from the equation

$$V = \sqrt{\frac{E}{\rho}} \tag{5}$$

where V = Velocity of propagation of longitudinal waves

ρ = Density

As noted in Table I, a slight variation in E with temperature was obtained. The authors admitted the lack of temperature control was a handicap, measurements being made on outdoor samples at the reported temperatures. A time lag in the value of E with temperature was noted and mentioned as probably deserving more work. In attempting to determine the effects of orientation, the exact direction of the optic axis was not determined; it was assumed the optic axes were perpendicular to the surface of freezing which is the usual case. The data indicate that orientation has a very small effect upon Young's modulus.

TABLE I

Elastic Constants of Ice

Investigator and Year	Method	Orientation (Direction of Optic Axis)	Temperature $-^\circ$ C	$\frac{E}{10^{10}}$ dynes cm^{-2}	$\frac{G}{10^{10}}$ dynes cm^{-2}	μ
*Young (1820?)	Pitch of tone of a prism			2.0		
Bevan (1826)	Static	θ = 90° (a)		(c) 4.90		
*Frankenheim (?)				5.3		
*Trowbridge (?)				7.5		
*Reusch (?)	Pitch of tone of a prism		below 0	2.4		
Moseley (1871)	Static			(c) 9.31		
*Fabian (1877)	Extension		0	1.7		
Frühling (?)	Bending		? to 0	0.4 - 2.5		
Hess (1902)	Static	Parallel to length	0 to 1	1.77		
		Parallel to width	0 to 1	3.76		
		Parallel to load	1 to 5	2.49		
		Parallel to width	1 to 5	4.10		
Weinberg (1907)	Static	Parallel to length	0		1.00	
			5		1.70	0.38
Beardsley (1908)				1.3 - 2.5		
Koch (1913)	Static	θ = 0°	6.5 to 7.8	(c) 5.97		
		θ = 90°	6.5 to 7.8	(c) 6.10		
Koch (1914)	Static	θ = 0°		(c) 11.00	2.72	
		θ = 90°		(c) 9.39	2.94	
Matsuyama (1920)	Static	Parallel to length	3.5	(c) 1.81		
		Parallel to load	3.5	(c) 0.59		
		Parallel to width	3.5	(c) 0.90		
Krayger (1921)			3 to 1	5.1		
Hargis (1922)	Torsional pendulum	Perpendicular to length			2.82	

TABLE I (continued)

Elastic Constants of Ice

Investigator and Year	Method	Orientation (Direction of Optic Axis)	Temperature -° C	$\frac{E}{10^{10}}$ dynes cm^{-2}	$\frac{G}{10^{10}}$ dynes cm^{-2}	μ
Pineghin (1923, 1924)	Bending and Longitudinal compression		12 to 3	0.3 - 7.8		
Sokolov (1926)	Bending		6	2.5		
Brockamp and Mothes (1930)	Velocity of explosive waves		about 0	6.8	2.5	(b) 0.36
Boyle and Sproule (1931)	Sonic (Longitudinal vibrations)		9	9.29		
			10	9.48		
			30	10.20		
			35	10.90		
			Between 30 and 60	11.50		
	Sonic	Parallel to length	20	9.12		
			26	9.67		
			33	9.75		
		Perpendicular to length	20	9.20		
			26	9.85		
			33	10.00		
		Perpendicular to length but in third rectilinear direction	20	9.06		
			26	9.37		
			33	9.40		
		45% to length	20	8.65		
			26	8.92		
			33	8.92		
Ewing, Crary and Thorne (1934)	Sonic		5 to 15	9.17	3.36	(b) 0.365
Kornfeld and Shestikhin (1942)	Torsional waves	Parallel to length	.1		1.14	
			1.74		1.31 - 1.09	
			2.6		1.34 - 1.23	
			5.1		1.43 - 1.27	
			7.5		1.44	
			10.6		1.43 - 1.36	
			21		1.43	
			25.6		1.43	
			32.6		1.44	
			185		1.68	

TABLE I (continued)

Elastic Constants of Ice

Investigator and Year	Method	Orientation (Direction of Optic Axis)	Temperature -° C	$\frac{E}{10^{10}}$ dynes cm^{-2}	$\frac{G}{10^{10}}$ dynes cm^{-2}	μ
Northwood (1947)	Sonic		15	9.8	3.68	(b) 0.33
Soils and Frost Effects Laboratory (1949)	Vibration of beams	Perpendicular to length	0	8.09		
		Parallel to length	0	9.36		
		Perpendicular to length	9.5	9.48		
		Parallel to length	9.5	9.00		
		Snow				
Soils and Frost Effects Laboratory (1949)	Vibration of beams	Density				
		0.60	0	0.00828		
		0.61	0	0.00606 - 0.0138		
		0.58	0	0.0159 - 0.0348		
		0.61	11.1	2.86 - 3.14		

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* From Weinberg (1936)

(a) θ = Angle between length of specimen and the normal to the surface of freezing, thus for $\theta = 0°$, optic axis is probably parallel to the length.

(b) Value calculated from other two reported values.

(c) Sheet ice.

Ewing, et al., (1934) also studied the velocity of wave propagation, using both longitudinal and torsional waves. For the latter case, equation (5) must be modified to

$$V_t = \sqrt{\frac{G}{\rho}} \tag{6}$$

where V_t is the velocity of propagation of torsional waves. Equation (4) was applied to calculate μ after E and G had been determined. Tests on four samples again showed orientation of optic axis to be unimportant.

The most accurate work is probably that of Northwood (1947), in which the velocity of wave propagation was again employed. In addition to longitudinal waves, he measured the velocity of propagation (V_e) of waves through an infinite medium, for which

$$V_e = \sqrt{\frac{E}{\rho}\,\frac{(1-\mu)}{(1+\mu)(1-2\mu)}} \tag{7}$$

and the velocity (V_r) of transverse waves for which

$$V_r = M\sqrt{\frac{E}{\rho}\,\frac{1}{2(1+\mu)}} \tag{8}$$

in which $M = f(\mu)$. The samples were cut from large blocks of ice in a number of different directions and no great variations were noted in the results.

Kornfeld and Shestikhin (1942) determined the shear modulus as a function of temperature by means of torsional waves. Their samples were obtained by freezing water in aluminum foil tubes, therefore it may be assumed the optic axes were perpendicular to the length of the specimen. A plot of their data revealed a nearly linear relationship of G to temperature below -20° C, above which the shear modulus decreases rapidly. The linear relationship was found to fit the equation

$$G = (1.42 - .00165t)10^{10} \tag{9}$$

where t = temperature °C. This temperature dependency is of course a small effect and for temperatures below -1 or -2° C, G is found, from their data, to be essentially constant for engineering purposes.

A different dynamical method was used by the Soils and Frost Effects Laboratory (1949). There the resonant frequency of a simply supported beam of ice was determined, and Young's modulus computed from the formula

$$E = \frac{4\pi^2 f^2 l^4 \rho}{k^2 m^4 g} \tag{10}$$

in which

f = Frequency, cycles $\sec^{-1}$

l = Length

k = Radius of gyration of cross section

m = Constant = 4.73

ρ = Density, lb in^{-3}

Hargis (1922) determined the period of torsional oscillations of an ice pendulum and used the equation

$$K\ddot{\theta} + L\dot{\theta} + M\theta = 0 \tag{11}$$

with

$$L = \eta\,\frac{\pi r^4}{2l} \qquad M = G\,\frac{\pi r^4}{2l}$$

where

θ = Angular displacement between top and bottom of rod

η = Viscosity of ice

r = Radius of sample

l = Length of sample

to calculate the coefficient of viscosity and shear modulus. The temperature at which the tests were run was not mentioned.

Hess (1940) obtained empirical relationships for E, G, and μ in terms of temperature (in range of 0 to -9° C) and pressure using data obtained (static determinations) by several experimenters. He obtained

$$E\ (p, t) = E(t) + (-5{,}925t + 987t^2 - 18.6t^3 + 0.52t^4)\ \ln p$$

$$\mu\ (p, t) = \mu\ (t) + (0.0346t - 0.0001t^2)\ \ln p$$

where

$$E\ (t) = 19{,}280 - 5{,}925t + 449t^2 - 0.8t^3$$

$$\mu\ (t) = 0.3875 + 0.0191t \qquad \ln \equiv \text{logarithm to the Naperian base}$$

also at atmospheric pressure $G\ (t) = 6{,}948 - 2{,}231t + 192t^2 - 2.93t^3$

$$G\ (p, t) = \frac{E\ (p, t)}{2\,[1 + \mu\ (p, t)]}$$

p = pressure in atmospheres

t = temperature °C E and G are in kg cm^{-2}

The data used to obtain these relations (based on several experiments mentioned previously in this report) is highly questionable and therefore the final equations are open to grave doubts.

Hess also claims that E obtained statically must be multiplied by a factor ϵ (between 1.5 and 3) for it to agree with sonic measurements. His reasoning is based on the effect of internal friction in the ice.

Soils Laboratory (1949) tested beams of compacted snow as well as ice. Results on the snow samples were not considered too reliable since a weak response was obtained to the applied vibrations, and the resulting difficulty and uncertainty in selecting the particular natural frequency of the beam itself from the natural frequencies of various parts of the apparatus and from harmonic frequencies. The large difference in values at 0° C and -11° C is possibly due to internal melting at the higher temperature.

Yosida, et al., (1948) obtained Young's modulus from static tests on pillars of snow. For snow densities between 0.2 and 0.35 gm cm^{-3} they obtained

Temp -1 to -3° C $E = (-1.9 + 17\rho) \times 10^4$ gr-wt cm^{-2}

Temp -5 to -13° C $E = (-11 + 60\rho) \times 10^4$ gr-wt cm^{-2}

3. Viscosity.

Ice is generally accepted as being a plastic material, and consequently should be defined according to Maxwell's law

$$\tau - f_s = \eta \frac{d\gamma}{dt} \tag{1}$$

in which, using absolute cgs units

τ = shearing stress--dynes cm^{-2}

f_s = elastic limit--dynes cm^{-2}

η = viscosity--dynes sec cm^{-2} (poise)

$\frac{d\gamma}{dt}$ = rate of deformation

Since the elastic limit of ice is extremely small, in much of the reported work, the threshold stress, f_s, is neglected, in which case equation (1) becomes

$$\tau = \eta \frac{d\gamma}{dt} \tag{2}$$

the equation of viscous flow.[8]

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[8] Although this approximation may be valid for determining the magnitude of η, Perutz (Journal of Glaciology 1(2):47), and others find that the flow of glaciers can be explained only by the plastic behavior of ice, and that f_s cannot be neglected.

Reported Values: A summary of most of the available information is presented in Table II, below. Although more complete tables can be found in the original articles, Table II should be sufficient since several values are listed for each observer to show any predominant trends. Other data, usually of the same order of magnitude, were omitted due to lack of information concerning the test conditions.

Dorsey (1940; p. 453) defines viscosity by using equation (1) and refers to the viscosity as defined in equation (2) as the "effective viscosity" η_e. Following this notation we have

$$\tau = \eta_e \frac{d\gamma}{dt}$$

TABLE II

Viscosity of Ice

Investigator	Year	Method	Orientation(1)	Temperature -° C	$\frac{\eta}{10^{13}}$ poise	
Hess(2)	1902	Bending of beams	River Ice			(Some time after application of load)
			Optic axis parallel to length		0.10	0.12
			Axis parallel to width		0.10	0.12
			Axis parallel to depth			0.17
Weinberg(3)	1907	Torsion of cylinder	Optic axis parallel to cylinder axis	0	60.0	
				5	76.0	
				10	97.0	
		Rate of shear of 10^{-8} radians sec^{-1}	River Ice	15	134.0	
				0	83.8	
				5	104.4	
				10	178.0	
			Glacier Ice	15	475.0	
Deeley (using McConnell's expt'l. values)	1908	Bending of beams		1.7 to 15.3	0.04 to 1.34	
Deeley	1908	Study of glacier movement			3 to 300	
Hargis	1922	Torsional pendulum			3.7 to 6.21	
Höppler(4)	1941	Höppler Konsistometer	Axis perpendicular to flow			
			τ = 63 dynes cm^{-2}	1	0.0017	
			τ = 9.5	1	0.355	
			τ = 63	5	0.039	
			τ = 26	5	2.390	
			τ = 63	10	0.53	
			τ = 47	10	2.40	
			τ = 63	15	0.78	
Kobeko and others	1945	Deformation of natural ice cover	Optic axis probably parallel to load	0	0.005 to 2(5)	
				0	8 to 2200(6)	
			Lake ice in situ			
Lavrov	1947	Bending of beams	Load perpendicular to optic axis	3	260 ± 12%	
				13	410 ± 16%	
			Artificial ice	23	690 ± 20%	

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(1) The orientation of the optic axis was determined directly only by McConnell and Lavrov.

(2) Hess observed large variations in η with load and time after application of load.

(3) Weinberg found large differences in η for different rates of shear.

(4) Höppler reported a variation of η with the shearing stress (see values).

(5) Assuming no bending.

(6) Applying Maxwell's theory.

and from equation (1)

$$\eta_e = \eta + \frac{f_s}{\frac{d\gamma}{dt}}$$

Dorsey lists the work prior to 1937, some of which is summarized below.

Weinberg (1907) working with cylinders of ice clamped at one end and subjected to a torque at the free end deduced the following formula

$$\eta_e = \eta_0 \left(a - \frac{b}{t}\right)^{-t} + \frac{c}{\psi} \qquad (3)$$

where η_0 = effective viscosity (η_e) at 0° C and $\psi = \infty$

t = temperature ° C

ψ = rate of shear--radians sec^{-1}

a, b, c = constants

Working with both river ice and glacier ice he found:

	River Ice (Planes of slipping perpendicular to optic axis)	Glacier Ice
η_0	9.5×10^{12} poise	3.8×10^{12} poise
a	1.12	1.32
b	0.54° C	0.65° C
c	5.0×10^{5} dyne radians cm^{-2}	8.0×10^{4} dyne radians cm^{-2}

Hess (1902) studied the bending of beams made of ice and calculated the viscosity by using the formula

$$\eta_e = \frac{M}{v} \qquad (4)$$

$$M = \frac{lP}{4hb}$$

in which P = load

l = length of beam

h = depth of beam

b = width of beam

$v = \frac{l\psi}{2}$ = velocity of depression of midpoint

ψ = rate of shear, radians sec^{-1}

Hess observed that under moderate loads η_e increased with time, the rate of increase being constant after five minutes.

Equation (4) assumes a uniform shearing stress throughout the beams cross-section. Since this is not true for an elastic material, it appears doubtful in the case of a plastic material and, consequently, the values obtained using this method appear questionable.

McConnell and McConnell and Kidd (1888) attempted to determine the viscosity of ice from the bending of beams in the same manner as Hess. The experiments were very crude and they did not carry out the necessary calculations for computing the viscosity. Deeley (1908) computed the viscosity from McConnell's data and obtained values of the same order of magnitude as Hess (10^{10} poise) for the optic axis parallel to the length, but he found values for the optic axis perpendicular to the length to be 10^{12} poise. Dorsey was unable to duplicate the values Deeley obtained from McConnell's data. Deeley also calculated viscosity from movement of glaciers.

Hargis (1922) used a torsional pendulum and applied the equation

$$K\ddot{\theta} + L\dot{\theta} + M\theta = 0 \qquad (5)$$

with $L = \eta \frac{\pi r^4}{2l}$ $\qquad M = G \frac{\pi r^4}{2l}$

r = rod radius

l = rod length

G = modulus of shear

θ = angular displacement between top and bottom of rod

From measurements of the period and the damping, he was able to compute both the viscosity and shear modulus of ice. His samples were obtained by freezing ice rods in a brass tube, and although not mentioned it seems appropriate to assume the ice crystals were formed with their optic axes radial. The temperature at which the tests were run was not mentioned. He also found the viscosity to vary with the time that the stress was acting.

In 1941 Höppler attempted to determine the viscosity of ice by using the so-called Höppler-Konsistometer. The procedure consisted of carefully freezing a sample of ice in a small cylinder and then forcing a ball downward through the sample, resulting in the ice flowing upward through the annulus between the ball and the cylinder. The crystal orientation and the temperature of the samples were very carefully controlled, and each sample was subjected to a standardizing test to insure a reproducible sample. The rate of travel of the ball was used as a measure of the flow velocity which was plotted against the compressive stress. The curves thus obtained appear very similar to the theoretical $\frac{d\gamma}{dt}$ - τ curves for a plastic material.

Höppler reported his results in terms of a so-called quasi-viscosity, which he in turn converted to viscosity in centipoises by employing a conversion factor. This factor is not mentioned in his article, but is evidently based on previous work with his konsistometer. It was not possible to duplicate all his curves from his reported data, and a careful examination revealed a great deal of smoothing of the data. It should be noted that his data give viscosity as a function of the shearing stress as well as of the temperatures and magnitude of load. His results varied from 10^9 to 10^{13} poise depending upon temperature, time after application of load, and magnitude of load. In view of the procedure, amount of subjective analysis, and large deviation from other reported results, the absolute values reported by Höppler would seem to be open to doubt.

Kobeko, Shishkin, Marey and Ivanova (1945) measured the viscosity of ice using a natural ice cover deformed by various weights. (The loading is therefore presumably parallel to the optic axis.) The viscosity was calculated using equation (4) and also by considering the bending and applying Maxwell's theory of plastic deformation. The results varied greatly with thickness of ice, load and area over which the load was applied. The large variation in results is possibly due to internal melting since the tests were carried out with the ice at 0° C. Viscosity was found (upon examination of deformation-time curves by this author) to be essentially constant with time.

Lavrov (1947) obtained the viscosity of artificial ice in a cold room. Using samples which had homogeneous crystal structure, he measured the deformations under a central load. He found the viscosity of ice by applying Maxwell's theory, a long period of time having elapsed between application of load and final measurements. The experiments were conducted at temperatures of -3, -13 and -23° C. Both Kobeko and Lavrov have actually employed an extension of Maxwell's theory by converting formulae applicable to elastic deformation to viscous deformation equations. The original stress-strain relations are equivalent to the formulae for viscous flow if one substitutes the gradient of the velocity for the gradiant of the deformation and replaces the expression $\frac{E}{2(1 + \mu)}$ where E is Young's modulus and μ is Poisson's ratio, by the viscosity.[9] μ is 1/2 for an incompressible fluid and hence the viscosity, $\eta = \frac{E}{3}$. The validity of these substitutions into the elastic deformation formulae is questionable. The value for μ of ice is known (see preceding section) to be less than 0.5 and the assumption (which is necessary to justify the substitution), that ice strictly behaves as a viscous material is open

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[9] Joos, G., 1934: Theoretical Physics. Hafner Pub. Co., New York, p. 205.

to some question. Superficially, however, the assumptions appear as well justified as those made by previous investigators.

Bucher (1948) studied the viscosity of snow by means of compressing snow samples and measuring the rate of compression and the applied load. He modified the basic equation (2) by introducing a constant "c" to account for the compressive stress in place of the usual shearing stress. Replacing $\frac{d\gamma}{dt}$ by $\frac{dl}{l\,dt}$ and τ by σ he obtains

$$\frac{dl}{l} = \frac{\sigma dt}{c\eta} \qquad (6)$$

in which

l = length of sample

σ = compressive stress

Upon integration equation (6) becomes

$$\eta = \frac{\sigma t}{c \ln\left(\frac{l_0}{l}\right)} \qquad \text{where } l = l_0 \text{ at time, } t = 0 \qquad (7)$$

from which Bucher calculated the viscosity of snow. His values lay between 10^8 poise for new snow and 10^{13} poise for old snow. The effects of density, grain size, and temperature were studied, and all were found to influence the viscosity. The influences of the mentioned variables are shown in Fig. 4.

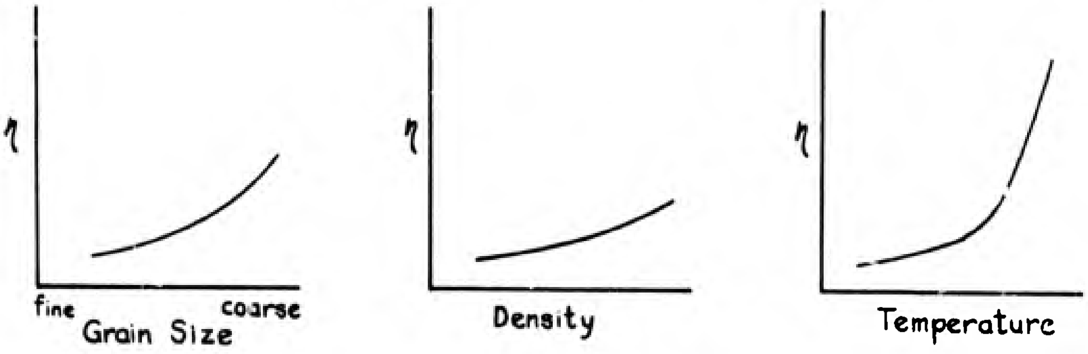

Figure 4. Effects of grain size, density and temperature on viscosity of snow

Bucher assumed the constant c to have a value of one (he claims it should lie between 2 and 3 from theoretical calculations) when computing his results. Because of the numerous assumptions it is felt the trends indicated by his work are of greater importance than his reported values.

4. Friction

Theory: The resistance experienced in the process of sliding one surface over another, which is generally termed friction, is not a simple phenomenon but probably arises through the action of several different mechanisms. Palmer (1949), in a review of the subject, suggests at least four possible mechanisms for producing this resistance: (1) The meshing of the irregularities of the two surfaces as assumed by Coulomb; (2) Molecular forces--which is supported by the observation of marked changes in frictional forces with even mono-molecular adsorbed gas films and also Bowden's result which showed that the force is proportional to the area of contact; (3) A ploughing action when materials of different hardness are brought together; and (4) Separation of electrostatic charges, i.e., directly related to the phenomenon of triboelectricity, as proposed by Shaw (1930).

Since the causes of the frictional forces are so complex, it is not surprising to find considerable variation in the theories postulated to explain the low coefficient of friction of ice and snow. The number of studies of the friction of ice and snow are rather limited, but all theories attribute the low coefficient of friction to a layer of lubricant between the sliding material and the snow or ice surface. The earliest theory, and probably the least acceptable in view of the present information, was the pressure melting of snow and ice, advanced by Reynolds (1901). According to Reynolds' theory the pressure of the bearing surface melted a thin layer of water which then lubricated the running surface.

Bowden (1939b) theorized that a liquid film was produced by the frictional heat developed when a material slides over an ice or snow surface. He further pointed out that the area of contact between the bearing surface and the snow or ice was much less than the total area of the sliding surface. Bowden's calculations showed that the frictional heat would cause a greater melting than that due to pressure, over any probable area of contact. Either of the above theories agree with the observed facts that the coefficient of friction increases both as the load increases and as the temperature lowers for it requires more heat or more pressure to melt snow or ice when it is colder and both the frictional heat and the pressure will increase with the load.

According to the frictional melting theory the coefficient of friction should be higher for materials having a high thermal conductivity since some of the heat would be conducted away from the snow surface, and therefore not melt the snow. Bowden found that brass, a material possessing a high thermal conductivity, had a higher coefficient of friction on ice than did wood. Recently, however, very fast skis have been made from magnesium, which also possesses a high thermal conductivity. This as well as other considerations led McConica (1948, 1950) to postulate the theory of a vapor film as a lubricant. For example, the so-called "dry slipperiness" of graphite has been shown quite conclusively to be due to a vapor film (Savage, 1948). The coefficient of friction for graphite was measured as a function of the water vapor pressure, and the coefficient was found to be very high in a vacuum. Bowden (1938, 1939a, 1944) has also shown that for metals, the oxide coatings decrease the coefficient of friction and that the coefficient of friction is very high for perfectly clean out-gassed metals. The increase in friction as the temperature decreases would also be in accordance with McConica's vapor film theory, since the vapor pressure also decreases with the temperature. McConica's experiments show, in fact, that the coefficient of friction does not simply decrease with the temperature, but that there is a maximum in the friction temperature curve. Maximum values, called critical values by McConica, were found to exist in the three curves of coefficient of friction versus (1) load, (2) temperature, and (3) velocity (see Figure 6 for the load friction curve). The existence of these maxima had evidently not been reported before, although Bucher (Figure 9) has shown the coefficient of friction to increase with an increase in load on wet snow at low velocities. The increase in the coefficient of friction with the load, for low loadings, is explained as due to the increasing rupture of the vapor film as the load increases until at the critical load value or maximum the friction is caused by dry sliding. For loadings beyond this point, if the surface contact area remains constant, the heat (frictional heat?) developed at the contact points would raise the partial pressure of water vapor at the interface.

When sliding occurs on wet snow or ice, the coefficient of friction is usually higher than for snow or ice at temperatures just below the freezing point. This fact would be explained, in the case of the frictional melting theory, as being caused by the viscous drag of water upon the sliding surface. The explanation according to the vapor film theory is a drowning out of the vapor film.

The resistance to sliding on snow contains factors other than just the friction between the sliding surface and the snow. Saito (1949) divides the resistance to the sliding of a sled on snow as follows:

1. Base friction--friction between sled runner and snow surface
2. Side resistance (author believed it was of subordinate importance)
3. Head resistance

The head resistance will show large fluctuations of values according to snow conditions, hence most snow friction work is difficult to interpret.

In addition to the lubrication by a vapor or liquid film, the theory that there may be somewhat of a ball bearing action of the snow particles has also been advanced (Seligman, 1943). The fact that skiing is much faster on older snow that has undergone some metamorphosis (i.e., lost the spicules and interlacing effect), is proposed as evidence in support of this theory. This theory also receives some support from experiments performed by Nakaya (1936), who studied the movement of the snow using soot as an indicator. A slit was cut in the snow transverse to the direction of motion, and filled with soot, and it was thus shown that the snow itself flows along under the ski or sled. If the ball bearing action is a factor in the frictional sliding on snow, the entire frictional lubrication would no doubt be a combination of this effect and also the effect of a lubricant film.

In summary then, the frictional melting theory and the vapor film theory of lubrication both explain the general variation of the sliding friction of ice at least qualitatively. According to the frictional melting theory one would anticipate that magnesium would have a relatively high coefficient of friction. However, experimentation (McConica, 1950) has shown magnesium has as low as or lower coefficient of friction on ice and snow than bakelite at all loadings and temperatures. Bakelite on the other hand was shown to be the best material of many tested to be used for airplane skis (Klein, 1947).

The frictional melting theory cannot explain why magnesium should have a low coefficient of friction, whereas the vapor film theory explains the effect as being due to the affinity for water vapor of the chemical films ordinarily formed on magnesium. If in the vapor theory, the heat produced by loadings beyond the critical loadings is frictional heat, there is little difference between the two theories in this range other than the importance of the thermal conductivity. There may be a possibility that the full theoretical explanation is a combination of the two theories.

Data and Discussion: It has been shown by many experimenters that the static coefficient of friction on snow and ice is greater than the dynamic coefficient of friction[10] (Bowden, 1939b; Klein, 1947; Arnold-Alabieff, 1938; Conant, 1949). This fact is in general agreement with all of the theories mentioned. Morphy (1913) using an inclined plane has reported that the static coefficient of friction does not vary continuously with load but was 0.36 for loads from 5.68 gm cm^{-2} to 14.52 gm cm^{-2} but dropped to 0.17 for loads from 15.10 gm cm^{-2} to 24.52 gm cm^{-2}.

Table III gives maximum and minimum values for both static and dynamic coefficient of friction as reported by Klein (1947) in his work on aircraft skis. Figure 5 shows the variation of the coefficient of sliding friction (μ_s) with temperature, as reported by Bowden (1939b), with water vapor saturation pressure curve added by McConica to show the inverse relationship between the friction and the vapor pressure. Figure 6 illustrates the variation in the coefficient of friction with load as reported by McConica. The maximum in the curve is the critical loading. Bucher (1946) measured the coefficient of friction of snow on snow and he found it to be a function of snow type, water content, temperature, normal load, and velocity (Figures 7 and 8). Table IV and Figure 9 show the coefficient of friction on wet snow of other material as given by Bucher. The measurements of the coefficient of friction of snow and ice are difficult to reproduce, since there are such a large number of variables, especially for snow, a great many measurements must be taken to provide a representative average.

TABLE III

Coefficients of Friction on Snow

(after Klein, 1947)

	μ_s		μ_a	
Material of Ski Surface	Min	Max	Min	Max
1/16 in thick beeswax (scooped smooth)	.029	.288	.092	.808
16 g. brass	.122	.428	.226	.977
16 g. monel metal	.103	.167	.197	.847
22 g. stainless steel	.128	.322	.056	.992
Bakelite varnish	.072	.211	.336	.631
American white ash treated with raw linseed oil	.069	.215	.420	.811
1/8 in thick bakelite, grade F14-2, fabric base	.064	.223	.227	.620
1/8 in thick bakelite, grade F15-1, fabric base with graphite incorporated in surface	.068	.162	.145	.605

μ_s = Coefficient of sliding friction

μ_a = Coefficient of adhesion or coefficient of static friction

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[10] Exception: G. J. Klein found, for three materials on hard packed snow, the minimum static coefficient of friction was less than the minimum dynamic coefficient of friction.

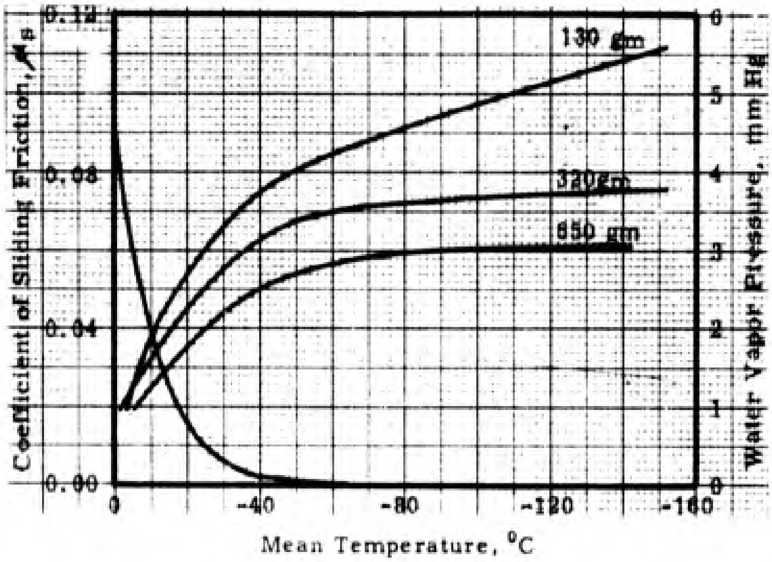


Figure 5. Variation of the coefficient of sliding friction with temperature (reported by Bowden) as compared to the water vapor pressure curve (added by McConica, 1948)

TABLE IV

Coefficients of Dynamic Friction μ_s

(after Bucher and Roch, 1946)

Material	Air Temperature °C	Velocity v in m sec^{-1}			
		v = 0.18	v = 0.40	v = 0.70	v = 1.25
Glass	0	---	---	---	---
		0.015	0.012		
	+8.5	0.015	0.012	---	---
Aluminum	0	0.046	0.041	0.040	0.040
		0.043	0.041	0.039	0.043
	+8.5	0.039	0.041	0.038	0.046
Iron	0	0.189	0.124	0.088	0.075
		0.148	0.121	0.050	0.043
	+8.5	0.107	0.118	0.012	0.012
Wood lacquered	0	0.140	0.119	0.111	0.071
		0.163	0.138	0.116	0.099
		0.186	0.157	0.121	0.127
Wood unlacquered	0	---	0.069	0.071	0.071
		0.183	0.165	0.124	0.128
	+8.5	0.183	0.164	0.158	0.185

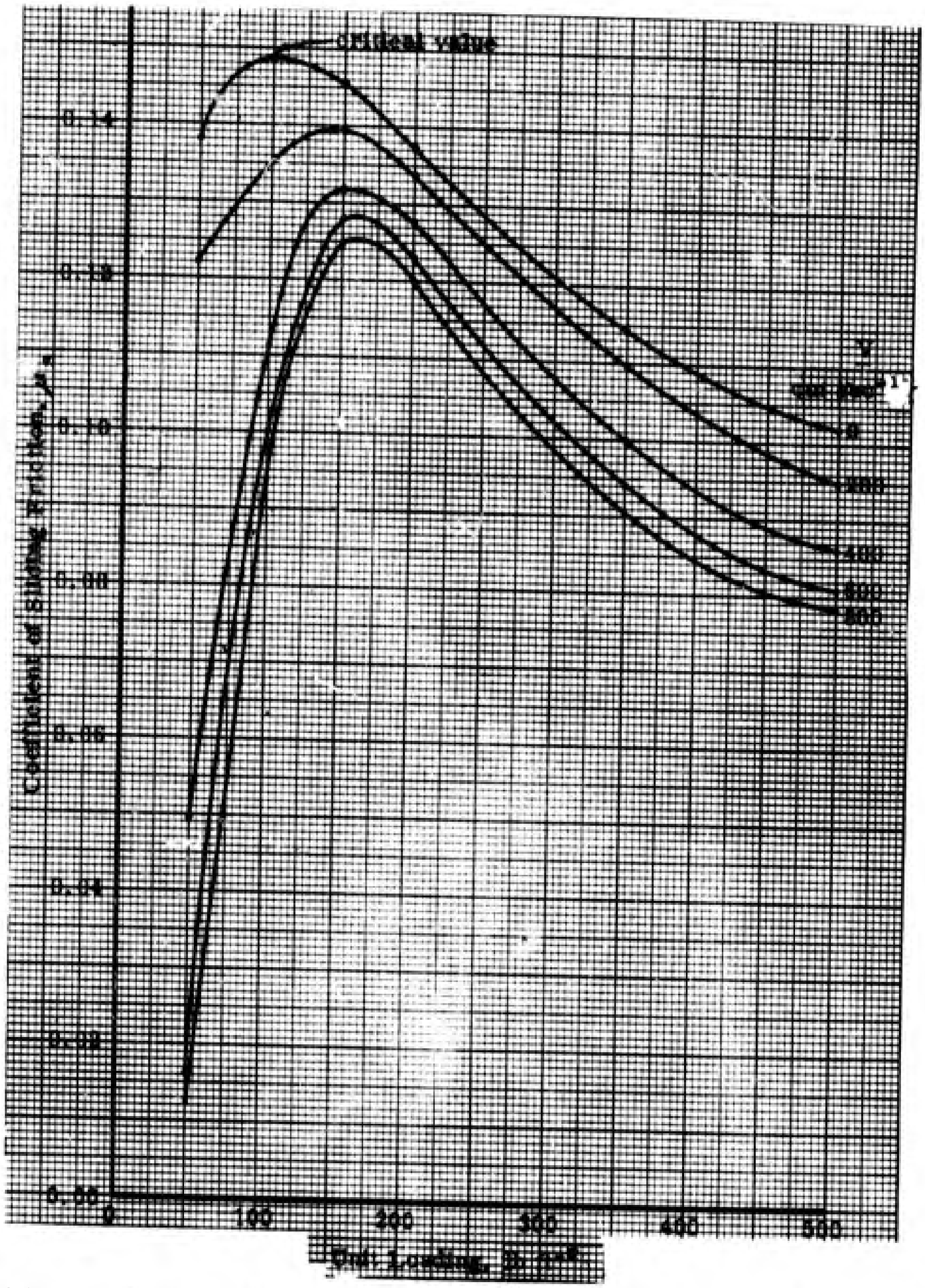


Figure 6. Variation in coefficient of sliding friction with unit loading at -15° C for polished bakelite on smooth ice as reported by McConica (1950)

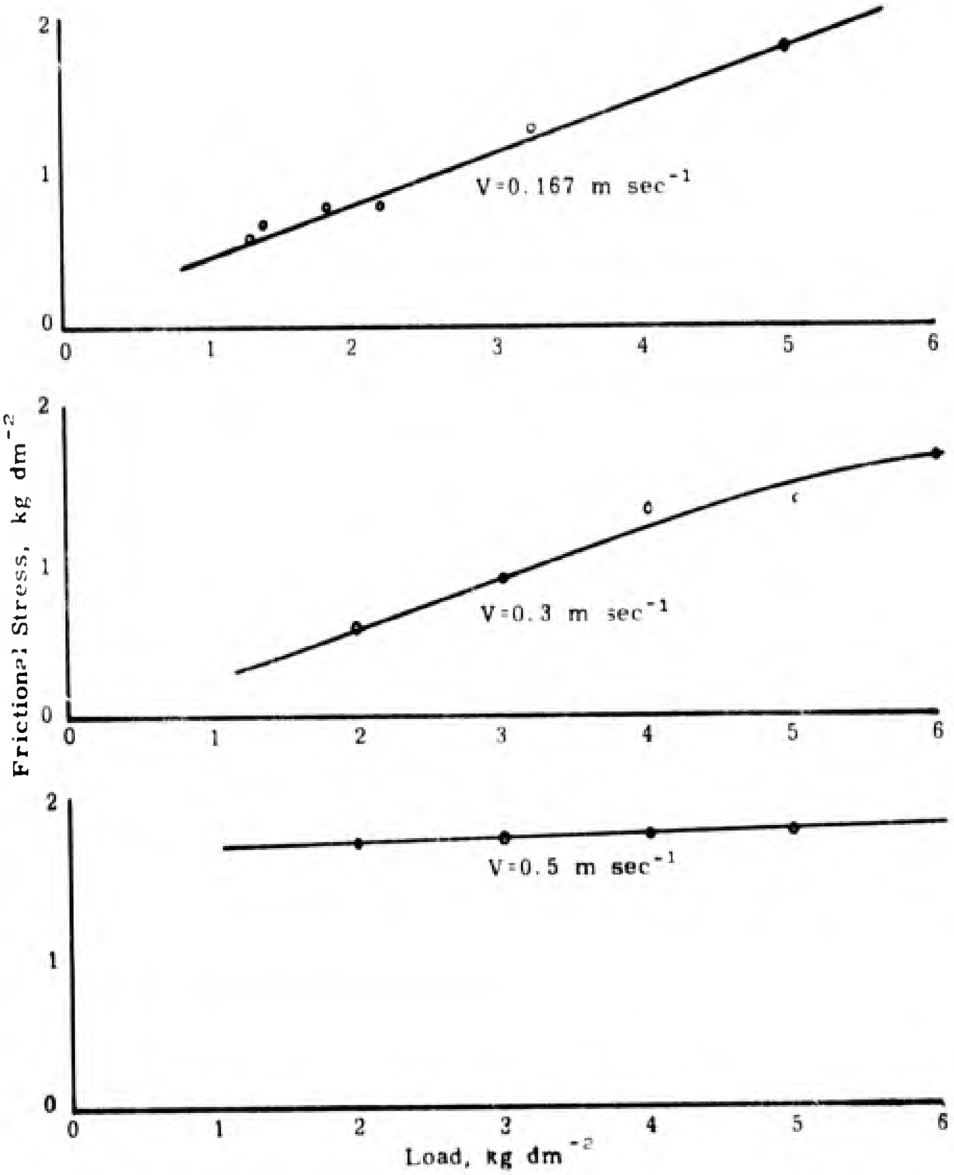


Figure 7. Frictional stress of snow on snow for varying loads at constant velocities as reported by Bucher and Roch (1946)

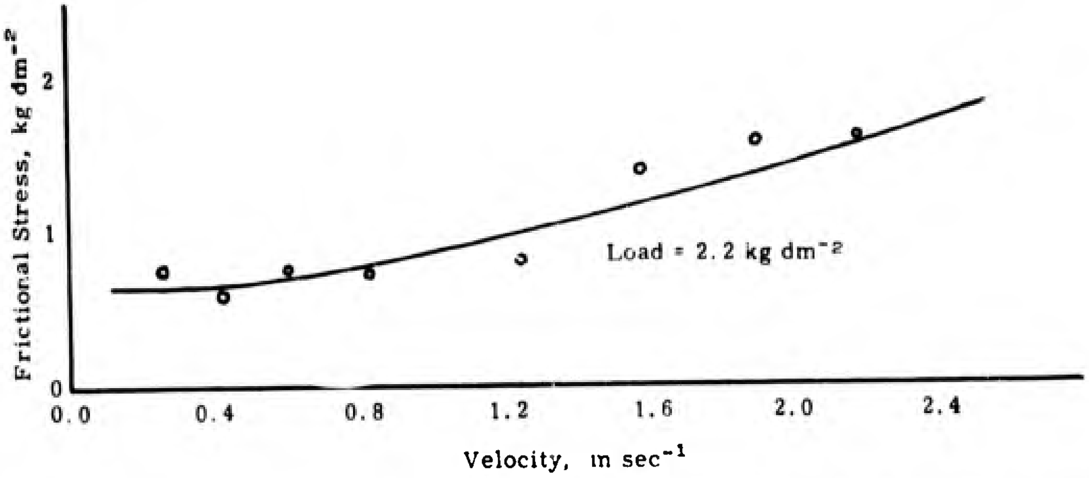
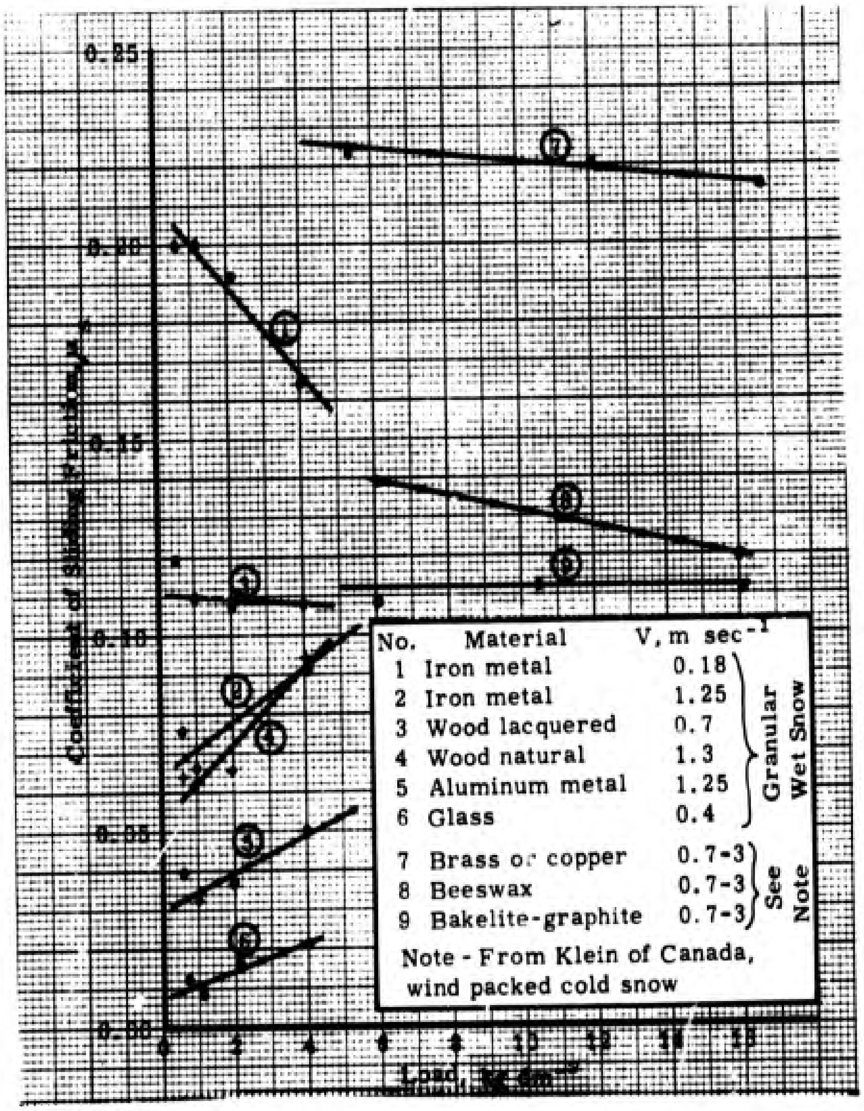


Figure 8. Frictional stress of snow on snow for varying velocities at constant load as reported by Bucher and Roch (1946)



No.	Material	V, m sec⁻¹	
1	Iron metal	0.18	Granular Wet Snow
2	Iron metal	1.25	
3	Wood lacquered	0.7	
4	Wood natural	1.3	
5	Aluminum metal	1.25	
6	Glass	0.4	
7	Brass or copper	0.7-3	See Note
8	Beeswax	0.7-3	
9	Bakelite-graphite	0.7-3	

Note - From Klein of Canada, wind packed cold snow

Figure 9. Coefficient of sliding friction as a function of the load as reported by Bucher and Roch (1946)

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Chapter II

ULTIMATE STRENGTHS OF ICE AND SNOW

The ultimate strengths of a material are those values of the imposed stress under which structural failure will occur. The stresses can, in general, be resolved into components of shear and of tension or compression. If it is assumed that the body is an isotropic crystal aggregate and that, therefore, there are no preferred planes at which fracture will occur, then the tensile strength will be equal to the shear strength. The compressive strength of a substance will generally be higher, however, than either the shear or the tensile strengths. Ultimate strengths determined by flexure or bending will thus provide a measure of the tensile or shear strength since failure will normally occur at the bottom surface which is under tension.

1. Ultimate Strengths of Ice

Data and Discussion: Reported values for the ultimate strengths of ice show a wide variation which cannot readily be explained. The ultimate strengths are known to depend on a number of variables, but the values obtained for this variation differ greatly between observers so that no precise relationship can be established. A dependency of the crushing strength on the direction of the crystal axis is suggested by some workers, but Strong (1948) points out that the average of all values obtained where the load was applied parallel to the crystal axis is 625 p.s.i., and for the values where the load was applied perpendicular it was 607 p.s.i. He further states that this 1.5 percent variation is probably smaller than the error introduced by ignoring the possibility of other controlling factors. Also most of the workers assumed that the principal axis of each crystal was normal to the freezing surface and did not check on the orientation by optical methods.

It is frequently claimed that the ultimate strength increases as the temperature decreases although some workers were unable to detect any temperature dependence. Horeth and Wilson (1948) deduced the following formula relating the strength of ice in flexure to the temperature:

$$\text{Ultimate strength} = 240 - 1.7 \times {}^{\circ}\text{F} - 0.01 \times ({}^{\circ}\text{F})^2 \text{ p.s.i.}$$

This formula can be accepted only as a very crude approximation, however, when the amount of variation between readings is considered. The ultimate strength of ice is generally considered to be less when the ice contains any impurities such as air, salt or organic materials, a result which might have been anticipated without experimentation. The strengths are probably also dependent upon the rate of loading, age of the ice, and structure of the ice, but experimental data on the variation of the strengths with these parameters is lacking. A tabulation of the results of several authors is included to illustrate the observed range and the average values of the ultimate strengths (Table I).

2. Ultimate Strengths of Snow

Theory: If one assumes an isotropic crystal aggregate of unchanging structure and texture, (according to Haefeli, 1938) one can relate the forces in a plane by the Coulomb friction equation

$$S = C + \sigma \tan \phi$$

where S is the shear strength; σ is the normal stress; ϕ is the angle of internal friction; and C is the shear strength at zero normal stress.

At the boundary of the crystal aggregate, the resultant stress τ forms an angle ρ with the normal stress σ as shown in Figure 10. This angle is designated as the apparent angle of internal friction.

In the case of snow, the shear strength is a function of at least three variables; namely, pressure, time, and temperature. For a constant loading period (the time interval over which S - σ curve is determined) and temperature, the shear strength S_s of snow as a function of the normal stress is shown by the curve in Figure 11.

The shear strength first decreases with increasing load as the crystal structure is being destroyed. As the normal stress increases, the shear strength reaches a minimum (point B) and then increases

TABLE I

Ultimate Strengths of Ice in Pounds per Square Inch

Experimenter	Approximate Average	Variation	Comments
	CRUSHING TESTS		
Barnes (1914)	363	356-370	Few tests
Bell (1911)	625	590-659	Few tests
Brown (Strong 1948)	600	300-811	
Romanowicz (Strong 1948)	590	568-626	
Koechlin (Strong 1948)	312		
Finlayson (1927)	1400	1800-1050	Few tests
Vitman (Strong 1948)	400	227-650	
Bezonov (Weinberg 1938)	600	219-907	
Pineghin (Weinberg 1938)	400	296-508	
Ice Station, Russia (Weinberg 1938)	420	272-587	
Weinberg (1939)	440	General mean of 1456 determinations compiled by Weinberg	
	TENSILE STRENGTH		
Hess (Strong 1948)	107	100-114	
Finlayson (1927)	120	103-136	
Romanowicz (Strong 1948)	250	210-260	
Pineghin (Weinberg 1938)	150	102-182	
Vasenko (Weinberg 1938)	90		
Bucher	95	36.9-270	
	FLEXURE OR BENDING TESTS ON ICE		
Brown (Strong 1948)	195	155-239	
Wilson and Horeth (1948)	220	180-256	
Weinberg (1938)	229	General mean of many values reported	
Weinberg (1939)	244	General mean of 882 values reported	
Neronov	80	38-113	Warm melting ice
	SHEAR STRENGTH		
Pineghin (Weinberg 1938)	100	88-117	
Finlayson (1927)	110	91-115	
Wilson and Horeth (1948)	95	75-115	

rapidly due to the increasing density of the snow. At point D, the snow has been compressed to the density of ice and thus reaches a maximum shear strength. Beyond this point, pressure melting takes place and the shear strength decreases.

Since the snow structure is being changed as the load is increased, a different snow exists at each applied load. Consequently, if the load is increased to a value of σ_1, and then released, the cohesion of the snow will be C_1 and not C as shown in Figure 11.

If the failure under shear is due solely to the breaking of the crystal bonds, then the shear strength is theoretically equal to the tensile strength at zero normal stress.

Discussion of results: Bucher (1948) has published an example of the tensile strength for one snow measured using a centrifugal apparatus (Table II). The results show a variation of 35 per cent which may be due in part to the inhomogeneity of the snow and the small size of the individual sample.

TABLE II

Tensile Strength of Snow (Bucher 1948)

Number	Density kg m^{-3}	T kg dm^{-2}
1	405.5	67.5
2	394.6	55.5
3	392.8	74.5
4	389.0	64.5
5	385.0	71.5
6	372.4	69.5
7	443.0	73.6
8	420.8	100.0
9	392.0	32.6
10	401.7	86.5
11	403.8	73.0
12	399.0	51.5
13	435.6	64.0
14	425.5	55.0
15	421.2	41.0
16	434.0	72.0
17	418.3	69.5
18	414.0	77.0
19	426.8	67.9
20	427.6	56.0
21	418.0	69.5
22	413.0	54.0
23	419.0	35.0

TABLE III

Shear Strength of Snow (Haefeli 1938)
Kg dm^{-2}

	Normal Stress σ in kg dm^{-2}							
	0	1	2	3	4	6	8	10
1 Series	0.68	1.13	1.44	2.34	2.49	4.12	4.95	6.31
2 Series	0.68	1.06	1.81	1.93	2.89	4.46	4.95	5.90
3 Series	0.68	1.21	1.50	2.27	2.50	3.44	5.25	5.78
4 Series	0.68	1.13	1.62	1.97	2.42	3.86	5.15	6.28
5 Series	0.76	1.07	1.36	1.80	2.68	3.25	4.84	6.35
Average	0.70	1.12	1.55	2.08	2.60	3.83	5.03	6.12
Cohesion C_1	0.70	0.85	0.98	1.00	1.05	1.22	1.27	1.37

Shear strengths as measured by Haefeli (1938) are given in Table III. The data conform in general to the curve of Figure 11 and the results by Mark (1942).

The effect of age on cohesion as measured by Haefeli (1938) is given in Table IV, which shows that cohesion increases with time.

The results of tensile strength and zero normal load shear strength tests by the University of Minnesota are shown in Table V. It is seen that the average tensile and shear strengths are approximately equal, which is according to theory. Again we note the high variation of values obtained in the test.

In Figure 12 are shown actual shear strength curves for various types of snow as measured by Haefeli (1938). Curve (a) is the curve for a new snow. Curve (b) is the curve for a slightly wind-packed new snow almost twice as heavy as (a). The larger dip in the beginning of curve (a) as compared to curve (b) indicates that the latter offered a greater resistance to the collapsing of the structure than the former.

TABLE IV

Variation of Cohesion with Age (Haefeli 1938)

Test Series	1	2	3	4
Density kg m^{-3}	142	151	151	150
Permeability cm sec^{-1}	70	72	70	72
Age, Days	1/4	1	3	7
Cohesion C kg dm^{-2}	1.0	1.7	1.9	3.3

TABLE V

Tensile Strength and Shear Strength of Snow at Zero Normal Load
University of Minnesota

Test No.	Density γ_s kg m^{-3}	Permeability cm sec^{-1}	Tensile Strength T_s kg dm^{-2}	Shear Strength S_s kg dm^{-2}
1	408	71	32	
2	408	90	94	
3	422	72		39
4	406	72		50
5	408	91	36	
6	396	94	55	
7	432	75		60
8	434	55	72	
9	418	81	87	
10	384	112	58	
11	456	66	61	
12	394	79	37	
Average			59	50

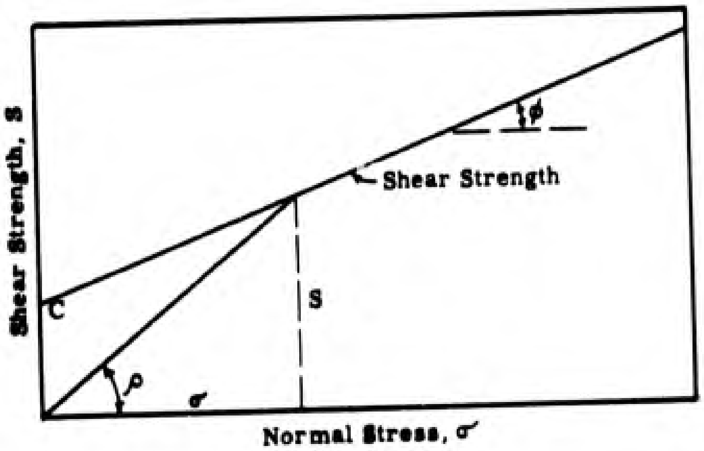


Figure 10. Ideal shear resistance as a function of normal stress

Curves (c) and (d) are graphs for an old coarse-grained snow called "Schwimm Schnee" and a disaggregated old snow, respectively. In the case of the former, the angle of apparent internal friction is seen to approach a minimum value with increasing normal stress; whereas, in the case of the latter the angle continuously decreases. This is an important factor in avalanche work. Curve (c) is also characteristic of a coarse-grained sand with a small cohesion value.

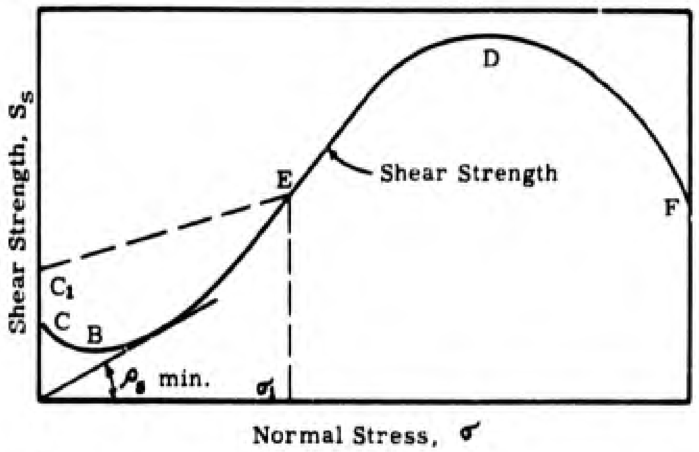
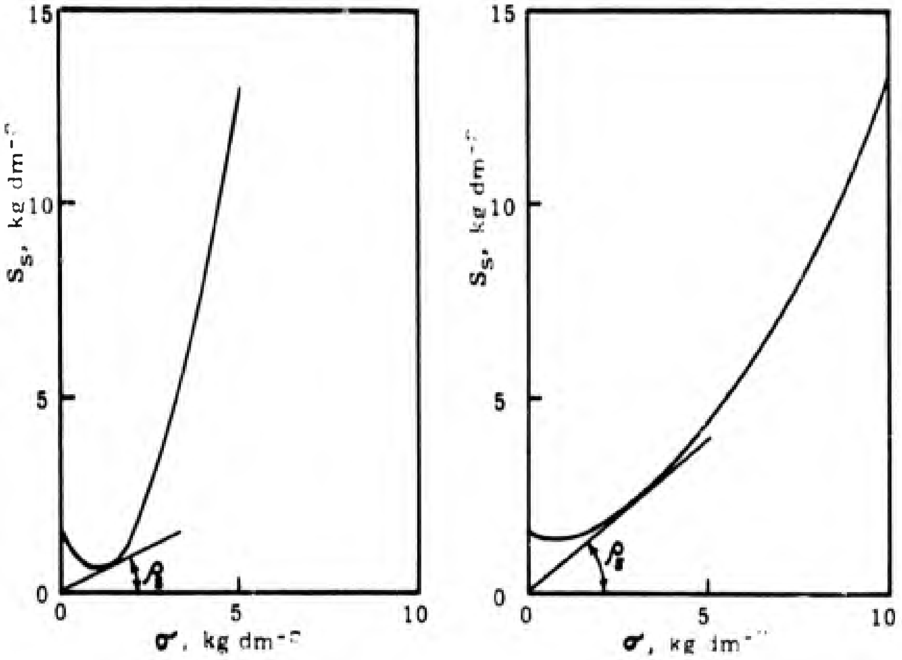
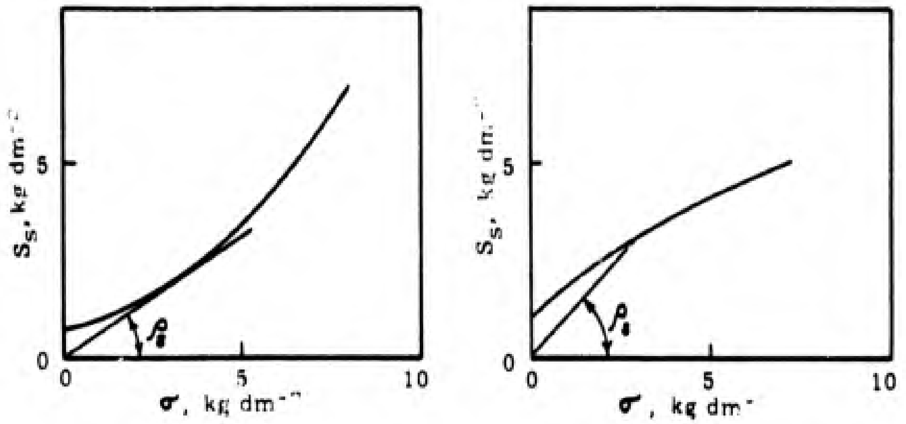


Figure 11. Shear resistance as a function of normal stress for snow



(a) New Snow

(b) New Snow - Slightly Wind Packed



(c) "Schwim Schnee" (Coarse-grained)

(d) Disaggregated Old Snow

Figure 12. Actual shear strength curves for snow

3. Bearing Strength of an Ice Sheet: An Example of the Practical Application of the Properties of Ice

Introduction: As may be inferred from the previous sections of this report on the mechanical properties and ultimate strengths, the behavior of ice under stress is complex and difficult to predict. It is not too early, however, to anticipate that further difficulties will be encountered in utilizing even accurate information on properties for such practical purposes as determining the bearing capacity of a floating ice sheet. The floating ice sheet does not present a conventional engineering problem although a theoretical treatment has been made by Hertz[1] in 1895 and generalized more recently by Schleicher (1926). The theory has recently been reexamined and put into engineering form in a compilation of Russian articles on "Ice Crossings" appearing in a translation published by the U.S. Corps of Engineers.[2]

The assumptions and the basis of the classical theory are reviewed in this section of the report. Some of the results obtained by the Russians are presented here as an illustrative example of the practical utilization of the theory, observation and the known properties of the material.

It is not possible to evaluate the final results of the theory without field tests. The values of the properties assumed in the derivations are consistent or at least not in contradiction with those appearing in the review in the preceding sections.

Theory: The theory of the deformation of a floating ice sheet, developed by Hertz,[3] treats ice as an elastic medium. The extension of this theory to evaluate the case of the limiting stress, therefore, assumes elastic behavior up to the yield point. This assumption seems to conflict with the reported values of the elastic limit of 7 to 14 pounds per square inch as compared to the tensile strength of over 200 lb per square inch. Some justification for this assumption, according to Lagutin and Shul'man, lies in the fact that it is only the limit of proportionality which has been reported and that the elastic limit must be larger. The rate of loading and period of load bearing is important; the apparent elastic reaction of ice to impact is cited in this connection.

As pointed out by Schleicher (1926), the buoyant force exerted on a unit area of the ice sheet is proportional to the amount of water displaced, i.e., the magnitude of the reaction is proportional to the depression of the ice sheet. The mathematical expression for the buoyant force, q, will therefore be identical with reaction of an elastic foundation

$$q = \gamma w$$

where w is the vertical displacement and γ is the density of water (for an elastic support, the elastic modulus).

Schleicher (loc.cit., p. 82) considers the case of a circularly symmetric load at the center of a circular elastic plate. The differential equation for the vertical displacement of the middle plane of the plate is:[4]

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}\right)\left(\frac{d^2w}{dr^2} + \frac{1}{r}\frac{dw}{dr}\right) = \frac{p_a - w\gamma}{D} \qquad (1)$$

where w is the vertical displacement; r is the radial distance; p_a is the unit load and D is the flexural rigidity of the plate. D is defined as follows:

$$D = \frac{Eh^3m^2}{12(m^2 - 1)}$$

where E is Young's modulus for the plate, h is the thickness and m the inverse of Poisson's ratio. Equation (1) is usually simplified before solution by the following substitutions:

$$L^4 = \frac{\gamma}{D}$$

$$\bar{w} = w - \frac{p_a}{\gamma}$$

and

$$\xi = \frac{r}{L}$$

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[1] Hertz, H., 1885: Gesammelte Werke. Barth, Leipzig, 288-294.

[2] Soils, Foundation and Frost Effects Laboratory, 1950: Translations.

[3] The theory has been applied by Bernshtein to the practical calculations of the load carrying capacity during ice crossings, according to A. P. Shul'man.

[4] See for example, S. Timoshenko, 1940: Theory of Plates and Shells. McGraw-Hill, p. 275.

The equation to be solved therefore appears:

$$\nabla_{\xi}^{4}\bar{w} + \bar{w} = 0 \qquad (2)$$

where ∇^2 is the Laplacian operator.

The solution to this equation has been presented in a very general form by Schleicher as

$$w(\xi) = \frac{p_a}{\gamma} + K\sum_{i=1}^{4} C_i Z_i\ (\xi) \qquad (3)$$

The four functions $Z_i(\xi)$ are defined by Schleicher and their values also tabulated; they are simple combinations of Bessel's or Hankel's functions of zero order. The terms p_a and K depend only on the magnitude of the load. The constants C_i will depend upon the position of the load and the boundary conditions. For the case of the plate of infinite dimensions, the C_i terms will be a function of $\frac{a}{L}$ alone, (where a is the radius of the applied load).

To investigate the maximum permissible load the value of the maximum stress σ_{max} is determined. The maximum stress will of course be a function of the load and one can readily solve for maximum permissible load by substituting the measured value of the ultimate strength (200 lb in^{-2}) for σ_{max}. One actually first determines the maximum bending moment, M_{max}, which will occur at the center of the plate:

$$M_{max} = L^4\left(\frac{1}{r}\frac{dw(0)}{dr} + \frac{1}{m}\frac{d^2w(0)}{dr^2}\right),$$

and the value of w(0) taken from (3), after appropriate boundary conditions have been applied. σ_{max} is determined from the relation

$$\sigma_{max} = M_{max}\frac{6}{h^2}$$

In summation the assumptions employed are:

1. The ice behaves as an elastic body
2. The ice sheet is of infinite horizontal extent.
3. The bending follow Kirchhoff's hypothesis (the planes normal to the neutral axis of the plate before tension, remain plane and normal during tension.)
4. The load is evenly distributed over a circular area.
5. The reaction of the water is proportional to the deformation.

Even though Schleicher has greatly simplified the evaluation of the solution by tabulating the functions $Z_i(\xi)$, the evaluation of a single case requires considerable effort. Shul'man has greatly simplified the computation by obtaining a simple empirical formula designed to very nearly duplicate the theoretical results. He finds:

$$p = 0.375R\ (h^2 + 7.8a\ E^{-1/4}\ h^{5/4})$$

where R is the tensile strength, p is the permissible load, applied on the area of radius a. The simplified formula differs from the precise formula by less than 10% when $0.66 < \frac{a}{L} < 1.0$, while in the range $0.1 < \frac{a}{L} < 0.65$, the result is practically coincident. It may be noted that the accuracy of the simplified formula fulfills practical demands for all ordinary types of crossings. In the case of crossings with superstructure (that is, roads built above the ice surface), the formula may be revised as follows:

$$p = 4aRE^{-1/4}\ h^{5/4}$$

for the range $0.65 < \frac{a}{L} < 1.2$ which again includes all cases of practical interest.

The Hertz-Bernshtein formula may be in error if applied under certain conditions due to the assumptions made in the derivation. The assumption of limitless horizontal dimensions would require a radius of approximately 30 to 60 meters for the formula to apply; therefore there may be errors in its use close to shores and close to cracks in the ice sheet. Also, due to the fact that it is based on the assumption of an elastic medium, it will not be valid for stationary or very slowly moving loads.

A number of other studies have provided formulas of the general form

$$p = Ah^2$$

where A is considered a constant. However, both observations and theoretical considerations by Korunov, suggest that A should vary with the thickness and therefore one constant cannot be used over a wide range of ice thickness.

Zubov has corrected the original formula as follows:

$$p = AKMSh^2$$

where A remained the same constant, but K denoted a temperature correction, M a correction for different areas of loading, and S a correction for salinity.

Zvalinskii has derived a formula based on the assumptions that the displacements u and v of a thin plate are of the type $u = u_0(x) + u_1(x)y$; $v = v_0(x)$ where x and y are the horizontal rectangular coordinates, rather than the assumption of pure bending and Kirchoff's postulate. He claims that by varying his formula he can consider all cases ranging from pure displacement and no bending to that of pure bending and no displacement. His results are also difficult to apply in engineering work and the character of his formula indicates that if it were in usable form it would still be much more complex to work with than even the Hertz-Bernshtein formula.

The crossing of an ice sheet has been definitely shown to produce a vibration or wave in the ice and also in the water under the ice. A formula derived by K. E. Ivanov, P. P. Kobeko and A. R. Shul'man gives the velocity of propagation of this wave as:[5]

$$v_k = \sqrt{gH\left(1 - \frac{\pi^4 Eh^3}{24\lambda^4}\right)}$$

where λ is wave length; H is depth of the water basin; E is modulus of elasticity; g is gravitational constant; and h is thickness of the ice layer. This velocity usually occurs around 30 to 40 km hr^{-1}, and if the load were travelling at that rate a resonance might be set up that would break the ice sheet. The loads should travel at either a slower rate or a faster rate, but it was also explained that loads travelling at faster rates caused waves which would be superimposed on one another and thus cause ice fatigue much sooner than the vehicles travelling at a slower rate.

Data and Discussion: A table has been prepared and reported by A. R. Shul'man and M. M. Kazanskii which takes into account many variables that are important in ice crossings. The table is calculated for the load moving at a rate up to 20 km hr^{-1} and is calculated from the basic formulas mentioned earlier which are

$$p_{max} = 0.375R\,(h^2 + 7.8aE^{-1/4}h^{5/4})$$

for the natural ice crossings and

$$p_{max} = 4RaE^{-1/4}h^{5/4}$$

for crossings which have a strong superstructure where p_{max} is the maximum allowable load which can proceed over the ice; R is the allowable stress; E is the modulus of ice elasticity; h is the thickness of the ice; a is the equivalent radius of the load distribution, which is

$$a = \sqrt{\frac{bd}{\pi}}$$

where b is the width of the load base, and d is the length of the load base. The value R is the ultimate strength tested in flexure (σ) divided by the safety factor. The safety factor is determined mainly by the amount of risk that can be taken in making the crossing. The various constants such as E and σ are determined on the basis of past experience with ice crossings with information obtained by the study of samples in laboratories serving as a check on the data derived.

Certain formulas are conjectured to consider the effect of temperature on the ice crossings. The value of σ and also of E are dependent on the temperature, but only the variation in the value σ was considered, as it was assumed that the change in E would have negligible effect since it is used in the formula only to the minus 1/4 power. Since in the reaction of the ice cover to bending, it is the lower layers which are most affected, the temperature changes in the lower layers must be considered. This

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[5] No derivation of this formula could be found in any of the articles that reported it.

temperature change is much less than the air temperature change since the water below the ice is always at approximately the same temperature. When there is a snow covering on the ice the decrease in air temperature has no effect at all, but when the ice is clear the following temperature correction factor has been derived by the authors as probably the best approximation:

$$K = \frac{T + 100}{100}$$

where T is the temperature below 0° C averaged over a period of 3 days. It is pointed out that the correction is not worth while unless T is colder than -12° C since this would only be a 10% correction. A formula is also included for positive temperature of the form: $K = (1 - .05\tau)$ where τ is the number of thaw days since water first appeared on the ice. This formula is obviously a very crude approximation and should be used only in case of definite need. The best recommendation would be to stop using the ice when the thawing process starts.

The salinity of the ice is definitely known to effect the strength of the ice. Since a great deal of data was not available the authors followed the method suggested by N. N. Zubov which is

$$p_s = p_o \frac{1}{(1 + 0.02s_i)^2}$$

where s_i is the salinity of the salt water (parts per thousand) from which the ice was formed and p_o is the load carrying capacity of the same thickness of fresh water ice. Further designating

$$s = \frac{1}{(1 + 0.02s_i)^2}$$

we have a correction factor s by which we can multiply the original formula.

The determination of σ and thus R is obtained mainly from data from past experience of ice crossings as stated above. The following relation is used

$$R = \frac{\sigma}{VN}$$

where V is a constant coefficient of the safety factor which takes into account the possible variation in thickness over the ice sheet. The values obtained for the case of loads on wheels

$$R = \frac{120}{N}$$

where V is 2.1, and for loads on caterpillar vehicles

$$R = \frac{146}{N}$$

where V is 1.7. The units are in metric tons per meter². The authors evidently obtained a value of 350 p.s.i. for the maximum ultimate bending strength of ice. This value is considerably larger than the average value obtained in laboratory tests (about 220 p.s.i.). This variation seems to indicate that the laboratory values are not too well related to those obtained in field experience. The N in this formula is still the safety factor required for the type of crossing. The authors have divided the crossings into three groups according to safety factors.

1. Crossing at the limit of safety with a special risk to be used only in emergency
2. A normal crossing where twice the load could be held up
3. Crossing with lowered safety factor intermediate between the two types described above.

The values for N are given in the table. The condition of the ice as to cracks is also taken into account in a portion of the table and the corresponding values for N are given. The final formulas used for calculating the values given in the table are

$$p_{max} = \frac{45}{N}(h^2 + 0.27ah^{5/4})\ Ks \quad \text{for loads on wheeled vehicles}$$

$$p_{max} = \frac{58}{N}(h^2 + 0.27ah^{5/4})\ Ks \quad \text{for loads on caterpillar type vehicles}$$

The simplified formulas given in the table are meant only to be used for field calculations where the table of values doesn't apply. The formulas are given in such a way that any inaccuracies due to using a simplified formula may tend to increase the safety factor. The formula for the time that a load may be stopped can be considered only as a first approximation. It is explained quite clearly in the tables. The distance between loads can be calculated from the consideration that the positive bending of an ice cover is as follows:

$$r = 2.17h^{3/4}E^{1/4} = 61.5h^{3/4}$$

The movement of one load is assumed not to affect the movement of another if they are at a distance.

$$l = 2r = 123h^{3/4}$$

The following simplified formula with a sufficient safety factor is suggested for use rather than the one above.

$$l = 300h.$$

TABLE VI

The Minimal Thicknesses of Ice in Cm, Necessary for the Passage of Specific Loads over Fresh Water Ice at Air Tempertures Between -1° and -12°C.

Type of Load	Weight in tons	Ice thickness in cm at various values of the coefficient N. N = 1	N = 1.2	N = 1.4	N = 1.6	N = 1.9
A single ski trooper		3	4		5	
Air sleigh, with four skis	1.5	7	9	12	14	16
Light car	2.7	16	17	19	21	23
5 ton truck	10	32	36	39	42	47
Loaded caterpillar vehicles, weighing up to	20	41	46	51	55	61
Loaded caterpillar vehicles, weighing up to	65	77	87	95	108	113

TABLE VII

Coefficient of Safety Factor and of Amount of Cracks N.

Character of crossing	Condition of ice Level ice without cracks	Dry cracks not through up to 30 cm wide	Wet cracks through up to 5 cm wide
Crossing at the limit of safety factor with special risk	1.0	1.2	1.6
Crossing with the lowered safety factor	1.2	1.4	1.9
Normal crossing	1.6	---	---

TABLE VIII

Simplified Formulas for Calculations

For wheeled, loaded vehicles on nonreinforced ice		$p_m = \frac{100}{N} h^2$ K. S.
For caterpillar type loaded vehicles on nonreinforced ice	up to 18 t.	$p_m = \frac{125}{N} h^2$ K. S.
	Above 18 t.	$p_m = \frac{115}{N} h^2$ K. S.

where p_m is the permissible load in tons, and

h is the actual, smallest ice thickness (not counting the snow formations).

The permissible time for loadhalting on ice:

$$\tau = \frac{(p_m - p)^2}{p_m p} (\theta + 1)^3$$

where τ is the permissible time for loadhalting on ice (in hours),

p is the weight of the load standing on ice,

p_m is the permissible load for ice of a given thickness,

θ is determined from the following table:

TABLE IX

Value of θ	Specific Occasions
0	1. The halting of a transport on roads not cleared of snow temperature, or if the ice is covered by water. 2. Basic long-range constructions, cribwork, flooring, etc. 3. The halting of loads on cleared or partially cleared ice at a temperature of less than -5° C.
1	1. The halting of a transport on roads cleared of snow at a temperature less than -5° C. 2. The halting of a transport on roads partially cleared of snow at a temperature less than -10° C.
2	1. The halting of a transport on roads cleared of snow at a temperature less than -10° C. 2. The halting of a transport on roads partially cleared of snow at a temperature less than -15° C.
3	1. A short halt of a transport on roads cleared of snow at a temperature less than -15° C.

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Chapter III

ELECTRICAL PROPERTIES OF SNOW AND ICE

1. Dielectric Constants and Dielectric Absorption

Definitions: The dielectric constant of a material is commonly defined as the ratio of the electric field strength in a vacuum to the electric field strength within the material, for the same distribution of charge. The dielectric constant, ϵ , is most frequently measured by observations of the capacitance and is defined by the equation

$$\epsilon = \frac{C}{C_0} \tag{1}$$

where C is the capacity of a condenser immersed in the material whose dielectric constant is to be measured, and C_0 the capacity of the condenser in a vacuum. If an alternating potential is applied to the plates of the condenser a power dissipation is usually observed within the dielectric material. Equation (1) has therefore been generalized so that the apparent dielectric constant, ϵ , is considered to be a complex quantity and is written

$$\epsilon = \epsilon' - i\epsilon'' \tag{2}$$

That is, ϵ consists of a real part ϵ' generally called the dielectric constant and an in[illegible] quantity ϵ'' associated with the dissipation.

The dissipation of energy within a condenser upon which a sinusoidal voltage is impressed will also mean that the current no longer leads the voltage by an angle of 90° but by an angle of $(90^0 - \phi)$. The angle ϕ is termed the phase defect. By our definition of ϵ,

$$\tan\phi = \frac{\epsilon''}{\epsilon'} . \tag{3}$$

A qualitative and, to a limited extent, quantitative concept of the dielectric behavior may be obtained by reference to the dipole theory according to Debye (1924). It may be assumed that a more or less rigid dipole is attached to the molecules which, under the influence of an electric field, will tend to align themselves with the field. Such rotation and alignment of the molecules will be resisted by forces of a viscous nature. Thermal agitation of the molecules will continuously disrupt this alignment.

A rotation of the molecules or the polarization of the dielectric is accompanied by a dissipation of energy, called by Dorsey, the dielectric absorption to distinguish it from losses associated with "true" conductivity. For pure water or ice the conductivity is very nearly zero so that almost all dissipation of energy must be due to dielectric absorption.

The Debye theory successfully predicts certain aspects of the behavior of the dielectric constant under an alternating potential. At high frequencies (optical frequencies) the dipoles will not have time to become oriented hence the dielectric constant will approach a constant value $\epsilon \rightarrow \epsilon_0$, ϵ_0 = dielectric constant at optical frequencies. Also as the frequency decreases $\epsilon \rightarrow \epsilon_1$, ϵ_1 = dielectric constant for static conditions. The so-called Drude-Debye relations express the complex quantity ϵ in terms of these two constants (for the given substance) ϵ_1 and ϵ_0 and the time of relaxation τ (defined as the time required for the moments of the molecules to revert to a random distribution). For convenience, define a quantity, α, a constant (for substance at constant temperature), such that

$$\alpha f = \omega\tau \left(\frac{\epsilon_1 + 2}{\epsilon_0 + 2}\right) \tag{4}$$

where the imposed frequency f and the angular velocity ω are related as follows

$$f = \frac{\omega}{2\pi}$$

The Drude-Debye relations are then

$$\epsilon' = \frac{\epsilon_1 + \epsilon_0 \alpha^2 f^2}{1 + \alpha^2 f^2} \tag{5}$$

and

$$\epsilon'' = \frac{(\epsilon_1 - \epsilon_0)\alpha f}{1 + \alpha^2 f^2} \tag{6}$$

Also by a formal generalization of the relation from Maxwell's theory which states that the dielectric constant is equal to the square of the index of refraction (n) we write

$$\epsilon = \epsilon' - i\epsilon'' = n^2(1 - ik)^2 , \tag{7}$$

the dielectric constant is equal to the square of the generalized index of refraction $n^2(1 - ik)^2$ where k is called the absorption index. The source of this relation may be seen by writing, according to the wave theory, the equation for the propagation of the electrical vector in an absorbing medium. For a plane polarized wave of unit amplitude travelling in the x direction, we have the expression

$$e^{i\omega\left[t - nx\left(\frac{1 - ik}{c}\right)\right]} = e^{-\frac{\omega knx}{c}} e^{i\omega\left(t - \frac{nx}{c}\right)}$$

where c is the velocity of light in a vacuum. The meaning of the absorption index is also apparent here; it is related to the decrease in amplitude of the electromagnetic wave. In travelling a distance $x = \lambda_0$, where λ_0 is the wave length of the incident radiation, the amplitude will decrease by the factor $e^{2\pi nk}$.

A number of interrelations of the above expressions may be useful in a discussion of the dielectric constant. The transition frequency f_S is defined such that,

$$\alpha f_S = 1$$

or

$$\epsilon' = \frac{\epsilon_1 + \epsilon_0}{2}$$

The phase defect, ϕ, may be expressed in terms of the other parameters, from (4) and (5) and (6)

$$\tan\phi = \frac{\epsilon''}{\epsilon'} = \frac{(\epsilon_1 - \epsilon_0)\alpha f}{\epsilon_1 + \epsilon_0 \alpha^2 f^2} \tag{8}$$

or from (7) since

$$\epsilon' = n^2(1 - k^2) \text{ and } \epsilon'' = 2n^2 k ,$$

$$\tan\phi = \frac{2k}{1 - k^2} \text{ or, by trigonometry, } \tan\frac{\phi}{2} = k$$

Another frequency of interest is the frequency for which $\tan\phi$ is a maximum. From (8) this will be

$$f_{(\tan\phi = \max)} = \frac{1}{\alpha}\sqrt{\frac{\epsilon_1}{\epsilon_0}}$$

and also at this frequency

$$\epsilon' = \frac{2\epsilon_1\epsilon_0}{\epsilon_1 + \epsilon_0}$$

and

$$(\tan\phi)_{\max} = \frac{\epsilon_1 - \epsilon_0}{2\sqrt{\epsilon_0\epsilon_1}}$$

These equations were derived, as already noted, employing rather oversimplified models, but they apply very well in the case of ice. The extent of applicability will be mentioned later.

The orientation of a molecular dipole in ice probably does not involve actual rotation of an H_2O molecule. For a simple explanation of this phenomenon using Pauling's model for the ice lattice, the reader is referred to a paper by Latimer (1949). Neutron diffraction study (Wollan, 1949) supports the Pauling model for ice structure as against other models, including the free rotator.

At sufficiently low temperatures "jumping" of H atoms (Born, 1946) will not be possible, and ice will act like other kinds of solids. Work on nuclear relaxation in ice lends some support to this assumption (Pake and Gutowski, 1948 and Bloembergen, 1948).

It has been found (Bloembergen) that the nuclear relaxation time in ice is proportional to τ down to -40° C. Unfortunately, no good measurements of τ are available at low temperatures where the nuclear relaxation time becomes anomalous.

The calculation of molecular polarization from dielectric constants is a valid procedure for vapors and most liquids, and has been discussed extensively in the case of water (Debye, 1945; Onsager, 1936; Frank, 1936; and Latimer, 1949). Bauer (1944, 1946) has published the most recent extensions of the dipole theory as applied to ice. The theory is not yet developed sufficiently to permit accurate calculations either for water or for ice. A satisfactory theory will undoubtedly include consideration of other concepts such as the hydrogen bond (Faraday Society, 1946; Onsager, 1939; Latimer, 1949; Lonsdale, 1949).

Data: The results of Smyth and Hitchcock (1932) still appear to be the most generally consistent ones. Since 1938, three full-scale studies have been made. Averbukh and Kosman (1949), working with a d.c. apparatus at high field intensities (2 kv cm^{-1}) observed a further polarization of ice (or space charge) and a variation in the apparent dielectric constant with the charging period of the condenser. For short charging periods (10^{-3} sec) they found the value of ϵ to be constant and independent of the direction of the current and field intensity, and equal to 77 (at -4° C), which agrees well with $\epsilon_1 = 74.6$ (at -5° C) from Smyth and Hitchcock. For longer charging periods the dielectric constant increased and was highly dependent on the current direction.

Eder (1947) studied dielectric properties of ice from 0 to -50° C, and from 50 cps to 3×10^9 cps. His work, however, is open to some objection because his data are insufficiently reported, and because it is not apparent what precautions were taken to avoid internal heating by dissipation of heat in the sample.

The results of Lamb (1946), extended and corrected by Lamb and Turney (1949), appear to be the best recent ones. They contain the only basic study of ice at radar frequencies.

Discussion: Lamb's data for t = -5° C agree with those of Smyth and Hitchcock fairly well. They fit formulae (5) and (6) within experimental accuracy. The constants are taken as follows:

ϵ_1 = 74.6 (from Smyth-Hitchcock)

ϵ_0 = 3.05 (later corrected to 3.18 [1949])

α = 1.4×10^{-4} sec

The formula for $\tan\phi$ breaks down completely at frequencies of 10^{10} cps or higher. Presumably some other process such as atomic polarization is causing an additional dielectric loss. It is not possible at present to make a correction for true conductivity.

Lamb's results on the variation of $\tan\phi$ with temperature from -1° to -50° C at a frequency of 10^{10} cps can be represented within about 5 percent by the empirical equation

$$\tan\phi = 0.00105 \times (-t)^{-0.53}$$

where t is degrees Centigrade. (See Tables I and II for numerical results.)

It will be noted that the index of refraction calculated at centimeter wave lengths is 1.78, much different from the optical index (1.31).

The dielectric constants of ice are known to vary greatly with amount and kind of impurities. Few systematic studies have been made of the effect of impurities in ice (Smyth and Hitchcock, 1932; see also Dorsey, p. 496), so the properties of natural ice will not be exactly predictable from the data on the pure substance. This will be especially true of sea ice.

The great difference between the properties of water and ice has been used to determine the freezing point of water in soil or other substances (Shaw 1938, 1942 and Alexander, 1937), and to measure the free water content of snow.[1] Yosida, et al. (1949), has reported measurements of the dielectric constant of snow varying the frequency from 10^2 sec^{-1} to 10^6 sec^{-1}. His results indicate that the dielectric constant of snow may not simply be derived from the dielectric constant for air and ice. Applying a theory according to Wiener (1912) he writes for ϵ_s, the dielectric constant for snow at one frequency,

$$\frac{\epsilon_s - 1}{\epsilon_s + U} = r\frac{\epsilon_i - 1}{\epsilon_i + U}$$

where ϵ_i is the dielectric constant for ice; r is the ratio of density of snow to that of ice; and U is the so-called form number depending on crystal form. The form number was found to vary in the range of from 4.0 to 19.6 for different snows. The visual characteristics and the value of U do not seem to be well related, however, and hence the value of U appears unpredictable.

The dielectric properties of ice also make it possible to distinguish between water droplets and ice particles in a cloud. Recently several theoretical and experimental studies have been made on the intensity of radar echoes from water- and ice-clouds, as a function of particle size (Hooper and Kippax, 1950; Austin and Bemis, 1950; Ryde, 1946).

Radar detection of floating ice (U.S. Coast Guard, 1947) also depends partly on dielectric differences between ice and water, and is probably often complicated by melt water on the ice.

TABLE I

Loss Factor of Ice at 10^{10} cps

read from a graph given by Lamb (1946)

$-t^0$ C	$10^4 \tan\phi$	$-t^0$ C	$10^4 \tan\phi$
0.75	10.5	10	3.2
1.0	10.0	11.5	3.0
1.4	9.0	15	2.55
1.7	8.0	20	2.05
2.5	6.5	21.5	2.0
2.75	6.0	25	1.8
3.6	5.0	30	1.65
5.0	4.45	35	1.55
6	4.0	40	1.5
7.5	3.7	45	1.4

Experimental values fall on this curve within 0.1 - 0.2 x 10^{-4}.

TABLE II

Loss Factor of Ice at 2.4 x 10^{10} cps

read from a graph given by Lamb and Turney (1949)

$-t^0$ C	$10^4 \tan\phi$	$-t^0$ C	$10^4 \tan\phi$
5	2.30	60	1.15
10	2.00	70	1.05
15	1.83	80	0.95
20	1.70	90	0.88
25	1.60	100	0.78
30	1.51	120	0.60
35	1.45	140	0.44
40	1.37	160	0.29
50	1.25	180	0.14

Experimental values follow this curve with maximum error of about 0.2 x 10^{-4}.

2. Electrical Conductivity

We will define the "true" conductivity, as that which might be measured in a d.c. experiment. This is contrasted with the equivalent conductivity σ_e calculated from dielectric loss, which can be found from the formula (after converting to units of $[\text{ohm-cm}]^{-1}$).

$$\sigma_e = \frac{f\epsilon''}{1.8 \times 10^{12}} = \frac{f\epsilon' \tan\phi}{1.8 \times 10^{12}} \tag{1}$$

as given by Dorsey (1940, p. 506). By the use of equation (6) of the preceeding section, we see that

$$1.8 \times 10^{12}\, \sigma_e = \frac{(\epsilon_1 - \epsilon_0)\, \alpha f^2}{1 + \alpha^2 f^2} \tag{2}$$

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[1] B. L. Hansen, verbal communication regarding unpublished work of Cooperative Snow Investigations.

Or, using equation (8) (loc. cit.),

$$1.8 \times 10^{12}\,\sigma_e = \frac{\epsilon'(\epsilon_1 - \epsilon_0)\,\alpha f^2}{\epsilon_1 + \epsilon_0 \alpha^2 f^2}$$

At very high frequencies, σ_e becomes constant:

$$1.8 \times 10^{12}(\sigma_e)_\infty = \frac{(\epsilon_1 - \epsilon_0)}{\alpha}$$

σ_e is given in (ohm-cm)$^{-1}$ All the other symbols have been defined in the section on dielectric constants.

Equation (4) theoretically gives a simple relation for discussing values of α, and has been used for this purpose by Murphy (1950).

In the following discussion, we shall assume that the observed conductivity has been corrected for dielectric loss.

The true conductivity of dielectrics is further obscured by what we may call "surface effects." Some of these effects are listed as follows:

1. Surface conductivity caused by condensation or adsorption of atmospheric water. This effect should disappear when experiments are performed in air which has been previously cooled to the temperature of the ice.
2. Polarization of external surfaces. If ions migrating to a surface under the action of a field cannot be removed, a back emf will be set up. This effect may be reduced by use of electrodes whose surfaces can be continually renewed (e.g., liquid mercury), or alternating currents may be used.
3. Polarization of internal surfaces, such as cracks, gas bubbles or intergranular boundaries. The amount of gas absorbed or entrained in a sample may have a profound influence on its electrical properties (Dorsey, p. 496). It is also possible that conduction across inter-crystalline boundaries is accompanied by polarization. If so, the conductivity of a sample of ice will depend on its grain size.

Assuming that the surface effects have been accounted for, consider the "true" conductivity in ice. The current must be carried by electrons, by ions, or as a special case by vacancies in the lattice. Since the ice crystal is molecular, and since its structure is open so that foreign ions can occupy interstitial positions, we expect that vacancies in the lattice will not contribute to the conductivity.[2]

Unless the ice has been damaged by X-Rays or corpuscular radiation, we expect that very few free electrons will be present. In that case, electronic conduction will also be negligible.

Ionic impurities should increase the conductivity of ice greatly. We expect an abnormally large effect from the hydrogen ion, because the mechanism of conduction by H ions discussed by Stearn and Eyring (1937, pp. 119-120) applies to conduction in ice as well as in water. The equivalent conductance of other ions must depend on their size, charge and hydration. (Small univalent ions should move more readily through the lattice.) The conductance will also depend on the temperature and the thermal history of the sample. This last factor controls homogeneity or lack of it in impure ice samples (Whitman, 1926). The effect of an ionic salt must be enhanced at temperatures above its eutectic point with water, since ice freezing out of solution will leave the remaining liquid more concentrated and hence more strongly conducting.

The conductivity of snow should depend on the following factors:

1. The presence of melt water.
2. Amount and kind of impurities. This can be determined roughly from the conductivity of water obtained by melting the snow.
3. Density and porosity, because the resistance will obviously depend on the area and nature of contact between snow grains.
4. Temperature. Snow conductivity should vary with temperature in the same way as ice of similar composition.

Data and discussion: The true conductivity cannot be determined from most of the results given by Dorsey (pp. 508-509) since they include the equivalent conductivity due to dielectric losses. The older

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[2] See F. Seitz, 1940; pp. 550-551 for a discussion of lattice defects.

values determined by Dewar and Fleming[3] do not need dielectric corrections, but the work is not complete enough to be useful.

The high dielectric loss of ice observed by Lamb (1946) at 10^{10} cps is hardly explainable on the basis of d.c. conductivity. The equivalent conductivity at -5° C calculated from his data is about 8×10^{-6} $(\text{ohm-cm})^{-1}$, which is much too high to be due to d.c. effects.

The work of Weissman (1948) on frozen solutions is important as an experimental confirmation of the theory of conduction by H ions, but it was intended to be qualitative rather than quantitative. His results are given in Table III. They would be useful if it should ever become necessary to make semiconducting ice or snow.

TABLE III

Conductivity of Ice and Frozen Solutions

Weissman (1948)

a) Conductivity was not detected at liquid air temperature, $(\sigma < 2 \times 10^{-10})$.

b) Conductivity was not detected with any salts.

c)

Substance	Temperature -^{0} C	Conductivity (ohm-cm)
HCl, 0.06N	115	6×10^{-9}
	106	3×10^{-8}
	94	9×10^{-8}
LiOH, 0.08N	69	2×10^{-9}
	51	1.5×10^{-8}
	42	2×10^{-8}
HNO_3, 0.056N	84	3×10^{-9}
	70	1×10^{-8}

These results were reproducible only within a factor of 2-3; they also changed with time.

Roman (1938) found the resistivity of a snow pack to be 10^8 to 10^9 ohm-cm. The resistivity of natural snow has also been measured by Shimada (1941) using a d.c. potentiometer arrangement. The specific resistance was found to vary from 9×10^6 to 31×10^6 ohm-cm. This work may lead to a resistivity method for determining the thickness of snow beds, but it cannot be regarded at present as a standard procedure. The theory of resistivity methods is outline by Heiland (1940, pp. 707 et seq.), and more details are given by Tagg (1930) and Hummel (1931).

As might be inferred from the work of Weissman, the electrical conductivity of sea ice, a relatively complex frozen solution, is hardly predictable. Dichtel and Lundquist (1950) made measurements of the resistivity of sea ice and found it to increase with the decreasing temperature, but the actual values varied remarkedly from sample to sample.

The salt content of the sea ice samples was found to vary between 0.3% to 0.5%. However, streamers of ice of high salt content (approximately 1.4% salt), called brine pockets, were found throughout the ice, generally oriented vertically. Even higher salt content is found in the so-called brine flowers consisting of needlelike ice crystals coated with salt crystals. While these brine flowers apparently form at the surface they may be buried in the ice under subsequent snowfalls thus leaving pockets of extremely high salt content. Unfrozen brine is still found in the ice at -30° C and small temperature changes may cause this brine to reorient itself. These structural changes are not reversible and consequently the resistivity measurements for a given temperature are not reproducible. Salt concentration in the brine increased as the temperature lowers and the brine concentrates along the vertically oriented ice crystals. This causes the resistivity in the vertical direction to be about 10% of the resistivity in the horizontal direction. With samples containing no brine pockets, the resistivity was found to be from 0.003 to 0.020 megohm-cm at -12° C measured in the vertical direction. Horizontal measurements gave 0.020 to 0.50 megohm-cm at -7° C, 0.050 to 0.200 megohm-cm at -16° C and 0.5 to 3 megohm-cm at -24° C.

Dichtel and Lundquist also evaluated the Gish-Rooney system, a method for measuring the depth of ice by the resistivity between two current probes placed in the ice. Their investigations showed no indication of current travelling in the ice between the probes. They found the current flowed vertically

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[3] Reported by Dorsey, p. 509.

into the sea water and through the sea water to the point vertically under the other electrode. Because the brine pockets are quite irregular, Dichtel and Lundquist feel that a.c. measurements at high frequency are more practical for electrical instrumentation using ice as a conducting medium. Further discussion of these effects may be found in Dorsey (pp. 423-424).

3. Magnetic Properties

When ice is placed in a magnetic field, the field intensity within the sample will be less than in the surrounding space. This phenomenon, known as diamagnetism, is a universal property of all substances but is sometimes obscured when the molecule possesses a permanent magnetic moment (paramagnetic substances) which is free to rotate. According to the classical theory of Langevin (Stoner, 1948: 22-23), the diamagnetic effect arises by the precession of the electron orbits under the imposed magnetic field. This precession gives rise to a magnetic field in opposition to the imposed field.

Langevin has treated the case of electrons moving about a center of force and for atoms with zero initial moment he finds for χ, the atomic susceptibility:

$$\chi = - \frac{e^2}{4mc^2} \sum_n \overline{r_1^{\,2}}$$

where e is the charge, and m the mass of the electron; c is the velocity of light, and r_1 is the radius of the "resolved" orbit of the electron with plane perpendicular to the field; the summation in this expression extends over the n electrons in the atom. For a system of n orbits with random orientation, $\sum_n \overline{r_1^{\,2}} = 2/3 \sum_n \overline{r^2}$, where $\overline{r^2}$ is the mean square radius of the orbit and hence

$$\chi = - \frac{e^2}{6mc^2} \sum_n \overline{r_1^{\,2}},$$

The quantum-mechanical theory of magnetic susceptibilities, given by Van Vleck, yields the identical expression above. The quantum theory also provides further relationships between atomic structure and diamagnetism. The details of electron distribution, however, are often difficult to treat theoretically. $\sum \overline{r_1^{\,2}}$, for example, will be highly dependent on the outermost electrons whose positions are most difficult to predict.[4] For this reason magnetic studies are usually used to clarify electronic structure rather than the reverse.

It has been reported (Cabrera, 1941), that the susceptibility of many atoms or ions is essentially the same in many different compounds; this is especially notable for C, H and O atoms. The susceptibility of many organic molecules can therefore be predicted from their chemical constitution, though the prediction is rarely exact.

Since diamagnetism depends on electronic structure it is expected to be independent of temperature although a temperature dependency has been noted for ice (see below).

Reviews of the theory may be found in Stoner (1948: 22-41) and Selwood (1943: 31-33) and the number of experimental procedures for measurement of diamagnetism have been discussed by Bates (1939: 91-163).

Data and Discussion: Cabrera (1941) published a discussion of the diamagnetic properties of ice and water in 1941. The susceptibility of ice at 0° C is 12.05×10^{-6}. It falls linearly with decreasing temperature to -50° C, the decrease being given by $\frac{1}{\chi_m}\frac{d\chi}{dt} = 6.7 \times 10^{-4}$, where χ_m is the value at 0° C. Presumably, however, χ becomes independent of t somewhere below -50° C.

Cabrera interprets these results as due to a more or less continuous change of the lattice type of ice, as suggested by Bernal and Fowler (1933); the "tridymite" type of ice is supposed to have a smaller value for $(-\chi)$ than the "quartz" type. This suggestion should be treated with some reserve until more related information on the lattice structure and vibrations of ice is available.

Cabrera (1941) reports a marked difference in the magnetic properties of H_2O and D_2O. Experiments performed only with D_2O may therefore by misleading as to the structure of ordinary ice.

Lonsdale (1949a and 1949b) has not definitely found any magnetic anisotropy in ice. This is interpreted as meaning that the hydrogen bond does not play any significant role in its diamagnetism.

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[4] Possibly the method of Boys (1950) will be suitable for this prediction.

4. Piezoelectric Effects in Ice

Theory: The possibility of observing piezo or pyroelectric effects in any substance depends on its crystal symmetry. It is not possible at the present time to assign the ice crystal to a given system of symmetry without ambiguity; therefore one cannot say with certainty that piezo effects should or should not be observed.

Owston and Lonsdale (1948) say that the "mean" structure of ice must be that observed by Barnes (1939). Barnes asserted that the symmetry group of the ice lattice is either D_{3h} or D_{6h}. He assigned the D_{6h} symmetry to ice, but not on X-Ray considerations alone. Since his model has proved inadequate in detail (Owston and Lonsdale, 1948 and Wollan, et al., 1949), the assignment to D_{6h} is not final.

The distinction is important here, because the D_{6h} group has a center of symmetry and therefore cannot be piezoelectric, whereas the D_{3h} group can be piezoelectric but not pyroelectric (Cady, 1946, pp. 19-20, 700; Wooster, 1938, pp. 190-203; Hettich and Schleede, 1927). The behavior of silver iodide often provides a guide to the behavior of ice, but the analogy fails in this respect, for AgI is assigned C_{6v} symmetry and can therefore be both piezoelectric and pyroelectric (Cady, 1946, pp. 232-233).

As recently as 1937, Golovkov (1937) assigned ice to the C_{3v} symmetry group, and Rogers (1937) advocated C_{6v} symmetry. These two classes can show "true" pyroelectricity.

If the ice crystal is indeed polar (i.e., with no center of symmetry) it must undergo twinning on the basal plane with remarkable ease since no piezoelectric effects have been observed (Adams, 1930; Pauling, 1935; and Bernal and Fowler, 1933). The contradictory results obtained with AgI, topaz, and some other crystals may be due to twinning, so we do not need to assume that ice is unique in this respect (Cady, 1946, p. 709; Wooster, 1938, p. 230).

Data and Discussion: Apparently no experiments on piezoelectricity in ice have been made since 1927 when Hettich and Schleede obtained negative results. It is unlikely that piezoelectricity will be observed on any ice crystals except microscopic ones. Kundt's method (Cady, 1946, p. 703; Wooster, 1938, p. 224) is probably the simplest way to observe piezo effects, although it was intended for pyro effects only. The D_{3h} class, of course, does not exhibit "true" pyroelectricity, but quasi-pyroelectric effects are so common that Kundt's procedure should be feasible if the piezoelectric coefficient is not too small. The method must be adopted for use with a microscope.

A method devised by Martin (1931) for detecting pyroelectricity has been tried on ice (Bernal and Fowler, 1933) without success. A more gentle procedure may work better; if the ice crystal undergoes twinning as easily as we suppose, then the stresses set up by strong and sudden cooling might themselves cause twinning. In such a case an experiment designed to be especially sensitive would defeat its own purpose.

The elastic and dielectric constants of a piezoelectric crystal depend on the conditions of measurement. Cady has given a thorough discussion of the influence of piezoelectric effects on electrical and mechanical crystal properties,(pp. 260-283). No discussion is needed here because the effects, if they exist, are probably small.

Wooster (1938, p. 227) describes an experiment in which ice filaments appear on a strongly chilled quartz sphere which is then suspended in moist air. This may be some clue to structure because the matrix of piezoelectric coefficients of compression is the same for D_3 symmetry (quartz) and for D_{3h} symmetry (ice ?). Suppose an ice crystal formed at a very low temperature should fall into a layer of relatively warm moist air. Condensation on the surface would warm the crystal rapidly and might set up internal strains. If the crystal were polar, then we would expect ice filaments to grow from it very rapidly, as in the experiment with quartz. This is therefore a mechanism for starting the complicated hexagonal branching of snowflakes--a mechanism which does not require such calm air or such slow growth as Seligman's proposals would suggest (1936, pp. 36-39). Consideration of the piezoelectric constants in the D_{3h} system (Cady, 1946, pp. 192-193) shows that branching and growth of the crystal due to this mechanism must occur parallel to the basic plane.

The foregoing paragraph is offered only as a speculation which might merit further thought if ice should be found to have D_{3h} symmetry. The discussion does not apply if C_{3v} or C_{6v} symmetry should be found.[5]

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[5] The morphological evidence which points to a choice of symmetry classes C_{3v} or C_{6v}, is discussed in the chapter on Geometric Properties.

5. Triboelectric Effects

Theory: The term frictional electricity or triboelectricity is used to describe the phenomenon of the charging of non-metallic substances produced by rubbing, by impact or by contact, between these substances or metals. The phenomenon is not well understood, and as pointed out in a review of static electrification by Loeb (1945), undoubtedly several basic mechanisms combine to produce the effect. The charging is very sensitive to adsorbed liquid or gas films (Macky, 1928; Shaw, 1930; Debeau, 1944). Therefore, in the investigation of electrification of any natural surface one must consider the possibility of perhaps four phenomena: (Following Loeb, loc. cit.) (1) a rupture of an oriented double layer of ions in the liquid surface film; (2) the exchange of atomic or molecular ions to fill up holes or remove excess ions (which will be complicated by ionic surface films of gases); (3) the breaking of a polar chain of molecules where the surface film is an organic liquid; and (4) rupture of the crystal during pressure-induced polarization.

Data and Discussion: The triboelectric effects of snow have received considerable attention recently, and it has been shown conclusively that snow static in airplanes is due to electrification of the aircraft by impinging snow particles (Gunn, 1946; Hucke, 1939; Schaefer, 1947). As would be expected, the rate of charging of an airplane is highly dependent on its surface and speed. It is possible, indeed, to find surfaces for which the net charge transferred is very near zero (Gunn, 1946, pp. 171-2). Unfortunately, such surfaces usually retain their characteristics for only one flight, being completely changed in character by the thin films of oil or grease which accumulate during servicing.

If it is assumed that the charging is proportion to the kinetic energy impact, one may write:

The rate of charging of the plane = constant x $(\text{speed})^3$ since the number of particles struck is proportional to the speed. It appears however that the exponent actually depends upon the surface and the temperature and is observed to vary in the range of from 2 to 5 (Gunn, 1946).

The electrification of snow particles by dispersion as observed by Yosida (1944) may be due to the mechanism suggested by Kunkel (1950). He showed that the rupture of the crystal lattice would be expected, on statistical grounds, to result in a separation of charge with the greatest charges on the largest particles--a fact which corresponds to his observations with dispersed dust particles.

The electrical disturbances which sometimes occur in polar regions when snow is blowing (Simpson and Scrase, 1937) are probably due to the operation of one or several of the suggested mechanisms of static electrification. The separation of charge in thunderstorms may be quite a different effect (Workman and Reynolds, 1948).

6. Surface Potentials and Freezing Potentials

Theory: When water is in contact with a solid, the water will generally contain traces of ions which may be preferentially adsorbed by the surface. Ions of opposite sign will be attracted to the surface by electrostatic forces, and so an electrical double layer is formed. (Abramson, 1934, pp. 42-48, 100ff.). It is then possible to measure a potential difference, called the surface potential, between the solid phase and the solution.

Different complications arise if a solid is being precipitated from solution, for then the "adsorbed layer" is constantly being taken into the interior of the solid. If the adsorbed layer of ions is taken into an ionic crystal, the effect is called coprecipitation, as when NH_4^+ ions are carried down with $KClO_4$. If, however, the adsorbed layer of ions is taken into a molecular crystal, the crystal will become charged, a phenomenon called "Workman-Reynolds effect" after its co-discoverers (Schaefer, 1950).

The above explanation of the large potentials developed during the freezing of some aqueous solutions is subject to experimental tests, which will be mentioned in the next section.

The phenomenon of the electric double layer at the water-solid surface has found application in the electrokinetic stabilization of soils and the subject is reviewed here briefly because of its relationship to the phenomenon discussed in this section.

Consider a capillary tube of radius r whose inner surface is covered by such a double layer. If now an electric field of strength X is applied along the tube, a shearing stress will arise within the double layer. If the negative side of the layer is toward the liquid, then the liquid as a whole will move toward the anode with a uniform velocity V which is given by the Helmholtz equation:

$$V = \frac{\zeta_0 DX}{4\pi\eta} \qquad (1)$$

where D is the dielectric constant of the double layer, η is its viscosity, and ζ_0, is the potential difference between the stationary layer and the bulk of the liquid (Adam, 1938, pp. 42-46). Smoluchowski showed that the equation also applies to a generally distributed system of capillaries, such as occurs in soil, provided a) the solid material does not conduct electricity, and b) the thickness of the double layer is small compared to the radius of the tube. If the last condition is not met, further assumptions must be made about charge distribution in the double layer.

D and η are usually taken equal to the corresponding quantities in the bulk liquid, an approximation which is useful only in the absence of a better one (Bikerman). We emphasize that ζ_0 is not the electrochemical potential between the bulk solid and the bulk liquid (Adam, 1938, pp. 304-306; Abramson, 1934, pp. 122-125). It is the potential difference between the immobile layer adsorbed on the solid, and the body of the liquid.

The volume rate of flow through a capillary is

$$U = \pi r^2 V = \frac{r^2 \zeta_0 DX}{4 \eta} \tag{2}$$

As mentioned before, the equation breaks down if r is comparable to the thickness of the double layer. It also breaks down if the capillary is wide enough so that internal circulation is easily set up. There is therefore an optimum thickness of capillary for demonstrating electro-osmosis.

Theoretically a correction should be applied for conductivity of the solid phase, but this is difficult and is seldom done in practice (Abramson, 1934, pp. 121-122).

For further discussion of the theory, see Abramson's book (Abramson, 1934, pp. 100-135) and later papers (Robinson, 1946; Neale, 1946).

Data and Discussion: Lange (1941) has found the surface potential for water ice to be -0.10 v. Another interesting result was that hoar frost and ice exhibited a potential difference of -0.30 v at temperatures below -30° C. Recently Arabadzhi (1948) has found the water ice potential to be 0.19 v.

The Workman-Reynolds effect, which is the development of a potential difference between ice and aqueous solutions during freezing, involves voltages of an entirely different order of magnitude. Potential differences as high as 185 v have been noted, and charge separations up to 400,000 e.s.u. per gram of solution frozen (Workman and Reynolds, 1950). As yet, no discussion of the effect has appeared, except for Schaefer's confirmation of it (Schaefer, 1950).

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Chapter IV

GEOMETRIC PROPERTIES OF SNOW AND ICE

By geometric properties we mean those properties which pertain to shape and spacial distribution of particles in the aggregate.

1. Crystallography

There are several modifications of ice, but most of them are stable only at high pressures not occurring under natural conditions, nor in snow and ice engineering practice.

It is almost certain that a single modification of ice occurs in nature.

The symmetry of the ice crystal is not known with certainity. The oxygen crystal lattice is centro-symmetric (Owston and Lonsdale, 1948) but the position of the hydrogen ions which alone can cause hemimorphism, is doubtful. Theoretical considerations and neutron diffraction patterns (Wollan, et al., 1949) favor a centro-symmetric symmetry D_{6h} or D_{3h}, hexagonal or trigonal holohedral, but this is in conflict with morphological evidence, which definitely points to a hemimorphic main crystallographic axis, and the choice of symmetry classes C_{6v} or C_{3v}, hexagonal or trigonal hemimorphic.

Well-formed crystals are found in depth hoar. They are invariably of hemimorphic habit and show simple forms, the prism $(10\bar{1}1)$, the base (0001), and pyramids such as $(30\bar{3}4)$ and $(50\bar{5}4)$ (Bader, 1938). The axial ratio $c/a = 1.62$.

The morphology of snow crystals was investigated by Nakaya (1934-1938) and Schaefer (1942-1949). Dendritic forms dominate with prismatic and tabular crystals. The base plane (0001) is a frequent plane of twinning, which is only possible if the main crystallographic axis is polar.

Hoar crystals sublimated onto solid objects have complex shapes, plates and scrolls, but simple crystallographic forms (base, prism and pyramids). They have been well described by Seligman (1936).

The morphology of snow and hoar crystals depends on temperature and degree of supersaturation of the atmosphere during growth (Nakaya, et al., 1934-1938).

Observations of the shape of falling snow crystals may possibly permit conclusions on conditions of the atmosphere in which they were formed.

The grains of glacier ice show no crystallographic forms, but inclusions in the grains again point to a polar axis (Bader, 1950).

The plastic deformation of single crystals appears to be due to a single lattice slip plane (plane of translation), namely the base (0001).

2. Structure and Texture of Aggregates

Very little work has been done on the structure of snow and ice aggregates (Bader, 1938, de Quervain, 1945, 1948). Granular snow is a very loose aggregate consisting of a network of chains of grains rather than a heap of grains. Grain size distribution of granular snow can be determined by screening (Bader, 1938) or sedimentation in a liquid (de Quervain, 1948).

The structure of dense ice aggregates (lake and sea ice, glacier ice, ice masses in permafrost) is now being investigated by a number of men (Ahlmann, Bader, Black, Seligman, Sharp, Wilson), and publications on the subject are becoming more numerous (Ahlmann, 1949; Bader, 1938, 1950, 1951; Perutz, 1939; Renaud, 1949; Seligman, 1949, 1950).

3. Grain Size

In snow, grain size varies from 0.01 to 5 mm, in wet nevé from 1 to 3 mm, and in glacier ice from several mm to several dm. Grain size is an important descriptive parameter, but difficult to evaluate with sufficient accuracy.

4. Size of the Interphase Surface

This varies from a maximum in new snow (ice-air interphase area) to a minimum in coarse grained glacier ice (intergranular boundary area). The ratio of interphase area to mass is approximately 10^4 cm^2 gm^{-1} for fluffy new snow and 1 cm^2 gm^{-1} for very coarse grained glacier ice.

5. Porosity of Snow and Permeability to Air

The pore volume of snow is practically wholly communicating, i.e., the apparent porosity is equal to the absolute porosity. At high densities the pores begin to become separated, and at a density of approximately 0.6 (porosity 13% or 0.13) permeability to air becomes zero and the snow becomes ice by definition.

$$\text{Porosity } n = \frac{\gamma_e - \gamma_s}{\gamma_e} = 1 - 1.09\gamma_s$$

γ_e = density of ice = 0.917

γ_s = density of snow

Quantitative measurements of permeability to air (Bader, 1938) show that permeability is a most useful variable for the definition of snow type. The permeability, K, has been defined as follows:

$$K = \frac{QL\lambda_t}{\Delta hq\lambda_0}$$

where Q is air flow through specimen in $\text{cm}^3 \text{ sec}^{-1}$; L is the length in cm, and q the cross section in cm^2; Δh is the pressure fall across the specimen in cm of water; λ_t and λ_0 are the viscosity of air at temperature t and 0° C, respectively.

$Q/\Delta h$ is only constant for laminar flow, which is nearly realized at air velocities of a few centimeters per second.

The maximum value of K is around 700 for depth hoar, a relatively light very coarse grained snow.

If the permeability of a snow specimen is measured at different degrees of compaction, the following relation is found to be valid over a long range of densities.

$$\frac{K - an}{K} = \frac{n}{N}$$

Where n is the porosity.

N and a are constants which are characteristic of snow types. If n_0 is the porosity of the uncompacted snow, N/n_0 varies only between 1.03 and 1.08 for different snow types. N is therefore not a useful parameter. But a, which varies from 5 to 100, can be related to grain size of the snow. For snow of uniform grain size, (screened), the relation between a and d (grain diameter in mm) is linear.

$$d \cong 0.048a$$

This relation also holds for mixtures of grains of two different sizes in different proportions, in which d is the average grain size. It thus appears that for granular snow this parameter provides a measure of average grain size, but more work is necessary for confirmation.

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Chapter V

THERMAL PROPERTIES

1. Specific Heat and Heat of Fusion

The specific heat of a substance is defined by the equation

$$C \equiv \lim_{\Delta T \to 0} \frac{\Delta Q}{\Delta T}$$

where ΔQ is the heat added to a unit mass of the material to cause an increase in temperature, ΔT. In the results presented, C has the units cal $gm^{-1}\,{}^{0}C^{-1}$. The specific heat has a definite value only for a definite process (e.g., constant pressure addition of heat). The specific heat of a substance will depend not only on the process it undergoes, but also on the physical state (including phase, temperature, etc.) of the substance. The gain or loss of heat by solids can generally be considered to be a process at constant pressure and only one type of specific heat is defined, although its value will still be a function of the physical state. However, the variation of the specific heat with pressure is so small it is neglected for most purposes. This latter assumption may not be valid for the ice subjected to the enormous pressures at the bottom of glaciers.

The specific heat of snow is assumed to be the same as that for ice. The effect of the air in the snow is considered negligible due to its relatively small mass. Any changes in internal energy which might occur with changes in the crystal form in snow are also neglected.

A property closely related to the specific heat is the latent heat of fusion, L. This is defined as the quantity of heat that must be added at constant temperature to a unit weight of a material to produce a change from the solid phase to the liquid phase. The units of L in the values reported here are cal gm^{-1} unless otherwise noted.

Small amounts of impurities in a substance will normally have a negligible effect on the values of the specific heat. Near the melting point, however, even a very small amount of impurities may have a considerable bearing on the values of C. Dickinson and Osborne (1915) treated this point, making the assumption that the lowering of the freezing point is proportional to the amount of impurities for a small concentration of impurities.

Let ϵ = lowering of the initial freezing point
K = the concentration of impurities when sample is all melted
θ = any temperature below ϵ
m = amount of unfrozen solution per gram of entire material at temperature θ
L = latent heat of fusion of ice
C = true specific heat of pure ice at temperature θ
C_a = apparent specific heat of ice specimen at temperature θ
H_f = total heat of fusion of the portion unfrozen at temperature θ per gram of original material;
$H_f = mL$

The concentration of the unfrozen portion is equal to $\frac{K}{m}$ since the total amount of impurity is fixed.

Now $\theta \propto \frac{K}{m}$ and $\epsilon \propto K$ with the same proportionality constant.

$$\therefore m = \epsilon/\theta$$

and $H_f = mL = \frac{\epsilon}{\theta} L$

Upon differentiation with respect to θ one obtains

$$\frac{dH_f}{d\theta} = \frac{-\epsilon L}{\theta^2}$$

Now C_a is made up of (a) the specific heat of ice, (b) the specific heat of the water present, (c) the fusion of a part of the ice, and (d) the effect of dilution of the solution present. Let these be represented by $\frac{dH_i}{d\theta}$, $\frac{dH_w}{d\theta}$, $\frac{dH_f}{d\theta}$, and $\frac{dH_s}{d\theta}$, respectively

$$\frac{dH_i}{d\theta} = (1 - m)\, C = (1 - \frac{\epsilon}{\theta})\, C$$

$\frac{dH_w}{d\theta} = mC_w = \frac{\epsilon}{\theta} C_w$ where C_w is the specific heat of water

$$\therefore\ C_a = (1 - \frac{\epsilon}{\theta})\, C + \frac{\epsilon}{\theta} C_w - \frac{L\epsilon}{\theta^2} + \frac{dH_s}{d\theta} = C + (C_w - C)\frac{\epsilon}{\theta} - L\frac{\epsilon}{\theta^2} + \frac{dH_s}{d\theta}$$

$\frac{dH_s}{d\theta}$ may be neglected since we are only considering very dilute solutions, and since $L \gg (C_w - C)$, we can make the approximation $C_w - C \cong 1.01 - .50 = .51$.

$$C_a \cong C + 0.51\, \frac{\epsilon}{\theta} - L\, \frac{\epsilon}{\theta^2}$$

For small values of θ the term $0.51\frac{\epsilon}{\theta}$ is small compared to $L\frac{\epsilon}{\theta^2}$, and for large values of θ it is small compared to C. Neglecting that term we have

$$C_a \cong C - L\,\frac{\epsilon}{\theta^2}$$

It must be recalled that the above equation is valid only for small concentrations of impurities where the lowering of the freezing point is proportional to the concentration.

Data and Discussion: Almost all of the available experimental work on the specific heat and latent heat of fusion of ice was performed before 1938 and is included in Dorsey. A resume of some of the most important work will be presented here.

Probably the most accurate determinations were performed by Dickinson and Osborne (1915). An aneroid calorimeter (employing the thermal conductivity of copper to distribute heat supplied electrically) was used in the experiments. Four samples of ice of high but yet different degrees of purity were studied from -40° C to 0° C. Their results confirmed the effect of included impurities as presented above. For their purest ice sample they found

$$C_a = 0.5057 + 0.001863\, T - \frac{0.004}{T^2}\,,\ \mathrm{cal}_{20}\ {}^{\circ}\mathrm{C}^{-1}\ \mathrm{gm}^{-1}$$

$$T = \text{temperature } -{}^{\circ}\mathrm{C}$$

In the same series of experiments they found L corrected for the effect of impurities to be 79.76 cal_{20} °C gm^{-1}. In 1913, they had obtained a value of 79.74 cal_{20} °C gm^{-1} for L.

Osborne (1939) corrected the above-mentioned value of L using new values for the heat capacity of water. The corrected value is

$$L = 333.5 \pm 0.2 \text{ joule gm}^{-1}$$

$$= 79.77 \pm 0.05\ \mathrm{cal}_{20^{\circ}}\ \mathrm{gm}^{-1}$$

Dickinson and Osborne (1915) report Nernst's (1910) equation for the specific heat as

$$C = 0.4706 + 0.001533T - \frac{0.78}{T},\ \mathrm{cal}\ {}^{\circ}\mathrm{C}^{-1}\ \mathrm{gm}^{-1}$$

$$T = \text{temperature } {}^{\circ}\mathrm{C}$$

Barnes and Maas (1930) using an adiabatic calorimeter equipped with a radiation thermal obtained the latent heat of fusion of ice at 0° C and the specific heat of ice from -80° to 0° C. They found no effect due to impurities even though they took no great precaution in the preparation of their samples. They found

$$C = 0.40107 + 0.0063032T_a - 0.000009828T_a^2$$

where C is in cal $^{\circ}\mathrm{C}^{-1}$ gm^{-1} and T_a is the temperature in degrees absolute. They found

$$L = 79.40 \text{ cal gm}^{-1}.$$

As mentioned above, Dickinson's and Osborne's values are considered to be the most reliable. (For the data of other investigators, see Dorsey).

2. Thermal Conductivity

The thermal conductivity of a material, k, is defined by the equation

$$\vec{J} = -k \text{ grad } T$$

where $\vec{J}$ is the flow of thermal energy (thermal flux) at a point in the medium and T is the temperature in the medium. The units of k as presented in this report are cal $°C^{-1}$ cm^{-1} sec^{-1}, unless otherwise noted.

No values for the thermal conductivity of ice have been found that are not contained in Dorsey (1940). Some of the important values, taken from this work, are given in Table I.

TABLE I

Thermal Conductivity of Ice 10^{-4} cal $°C^{-1}$ cm^{-1} sec^{-1}

t - °C	Jakob and Erk	Van Dusen (compilation)
0	53.5	50.0
10	55.5	50.9
20	58.1	51.6
30	60.9	52.6
40	63.6	53.1
50	66.4	54.2

Dorsey (p. 481) has also noted that the thermal conductivity along the optic axis is probably greater than in the direction perpendicular to the axis.

Although a great number of investigations of the thermal conductivity of snow have been made, there is little agreement among the results. Most of the experimenters considered the density of the snow to be the only variable. Though the density undoubtedly is an important factor, it would seem that other variables such as crystal structure and orientation would also have an effect on the conductivity. These other factors would be difficult to measure and would probably vary in the course of the experiment.

Abels[1] (1894) was one of the first to undertake a fairly thorough study of the thermal conductivity of snow. For $0.14 < \rho < 0.34$ he found

$$k = 0.0068\, \rho^2$$

$$\rho = \text{snow density in gm cm}^{-3}$$

Jansson (1901) obtained k by comparing the thermal conductivity of water. He let heat flow through a series of alternate sections of snow and water and from the temperature measured at the various interfaces, he was able to obtain values of k relative to the thermal conductivity of water. He derived the following expression relating k and ρ in the range of density, $0.08 < \rho < 0.5$.

$$k = 0.00005 + 0.0019\rho + 0.006\, \rho^4$$

The equation was adjusted so that k would take on the value for the conductivity of air as $\rho \rightarrow 0$ and the conductivity of ice as $\rho \rightarrow 0.5$.

Devaux (1933) measured the thermal conductivity of snow utilizing three techniques.

1. Measurement of the change with time, of the slope of the temperature vs. depth curve for a natural snow cover.[2]
2. Measurement of the radial temperature gradient and the energy output of a heated cylinder placed in the snow bed.
3. Measurement of the radial temperature gradient and the energy output of a heated sphere placed in the snow bed.

From the applicable equation for heat flow in each case, the thermal conductivity is obtained. The three methods gave very similar results and Devaux states that to a first approximation k is a function only of the density. He found

$$k = (0.7 + 70\rho^2) \times 10^{-5} \text{ cal } °C^{-1} \text{ cm}^{-1} \text{ sec}^{-1} \text{ for } 0.1 < \rho < 0.6$$

$$\rho \text{ in gm cm}^{-3}$$

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[1] From Jansson and others. Original paper was not available.

[2] This method gives the thermal diffusivity (see following section) directly, from which k may be calculated.

Yosida and Iwai give values of k determined for snow samples of uniform density. The snow samples were placed in a container and the time measured for the sample to attain a uniform temperature, after the temperature of the top and bottom of the sample was changed. Using this data, the thermal diffusivity (see later section) can be obtained directly. The conductivity was calculated from the diffusivity assuming a specific heat for snow of 0.4807. The large differences in conductivity for the same sample (see table below) were believed to be due to evaporation and a change in crystal structure (observed under a microscope). Some of their average results are given in Table II.

Ingersoll and Koepp (1924) found k from the heat flow through a sample of uniform density. From the measurements of the thermal diffusivity they found k (assuming a specific heat of 0.5) to be 0.0011 cal $^{\circ}C^{-1}$ cm^{-1} sec^{-1} for snow with a density of 0.54.

Dorsey (1940) reports that Ingersoll and Zobel (1913) found

$$k = 1.3 \text{ milliwatt } {}^{\circ}C^{-1}\, cm^{-1} = 3.1 \times 10^{-4} \text{ gm-cal } {}^{\circ}C^{-1}\, cm^{-1}\, sec^{-1}, \text{ for } \rho = 0.50 \text{ gm cm}^{-3}$$

and Kuroda (1930) obtained

$$k = 2.1 \text{ milliwatt } {}^{\circ}C^{-1}\, cm^{-1} = 5.0 \times 10^{-4} \text{ gm-cal } {}^{\circ}C^{-1}\, cm^{-1}\, sec^{-1}, \text{ for } \rho = 0.125 \text{ gm cm}^{-3}.$$

The results of a number of earlier Japanese investigations of the thermal conductivity of natural snow is given in Table III.

TABLE II

Thermal Conductivity of Snow of Uniform Density (Yosida and Iwai) in Cal $^{\circ}C^{-1}$ cm^{-1} sec^{-1} x 10^{-4}

ρ = gm cm^{-3}	k = 10^{-4} conductivity k	k = 10^{-4} k*
0.40	13.2	4.9
0.39	6.4	6.4
0.28	2.49	4.31
0.14	1.52	5.39
0.085	1.60	1.73
0.072	4.2	2.2

*These values were obtained for the same sample, but for a rise in temperature instead of a temperature drop. The second values were also obtained some time after the first (supposedly after a crystal change).

According to Schofield and Hall (1927), Okada, Abe, and Yomada (1908) found (after conversion of units)

ρ gm cm^{-3}	k, gm-cal $^{\circ}C^{-1}$ cm^{-1} sec^{-1}
0.24	3.99 x 10^{-4}
0.25	4.49 x 10^{-4}
0.27	3.20 x 10^{-4}

The Cooperative Snow Investigation (1946) calculated k from the diurnal temperature variation at different depths in a natural snow cover. Using the standard Fourier expression they found values of k. The equation

$$k = (22.7\rho - 0.46) \times 10^{-4}$$

fitted their data best. The least squares line passing through the origin was

$$k = 18.3\rho \times 10^{-4}.$$

TABLE III

Thermal Conductivity of Natural Snow (Yosida)*

k = cal x 10^{-4} cm^{-1} sec^{-1} $^{\circ}C^{-1}$	ρ = gm cm^{-3}	Investigator
2.8	0.18	Okada
4.5	0.24	Okada
1.9 - 4.5	0.212 - 0.250	Okada, Abe, Yomada
2.1 - 3.2	0.179 - 0.271	Okada, Abe, Yomada
4.22	0.324	Fukada
0.48	0.112	Kanaya

*Yosida lists these values as being compiled by Taguchi (1940) Snow, p. 53.

The density ranged from 0.045 to 0.178. Both equations are plotted in Figure 13.

Rikhter (1945) reports Niederdorfer's values for k of different types of snow. The values are 0.0003 for fresh snow, 0.0004 for frozen snow, and 0.0008 for moist snow, all in cal $^\circ C^{-1}$ cm^{-1} sec^{-1}. The classification of snow by these visual characteristics does not define the conductivity as well as the density, as may be seen by comparison with the results already tabulated, hence Niederdorfer's results have little value.

Many other experimenters have reported values for the thermal conductivity, but without specifying the density or any other property of snow. It was not thought necessary to include any of those values.

The temperature of snow has not been specified in any of the preceding determinations of k. Most of the work was carried out over a fairly large range of temperatures with few of the experimenters giving definite values for the temperature. Most of the work was carried out above -30°C.

Van Dusen (1929) inferred, from the data then available, that between 0° and -30° C

$$k = (2.1 + 42\rho + 216\rho^3) \times 10^{-4} \text{ watts } {}^\circ C^{-1} \text{ cm}^{-1}$$

or

$$k = (0.50 + 10.0\rho + 52\rho^3) \times 10^{-4} \text{ gm-cal } {}^\circ C^{-1} \text{ cm}^{-1} \text{ sec}^{-1}$$

A plot of thermal conductivity vs. density for most of the results described above is given in Figure 13 to show the variation in values obtained by the different researchers. All of the empirical equations

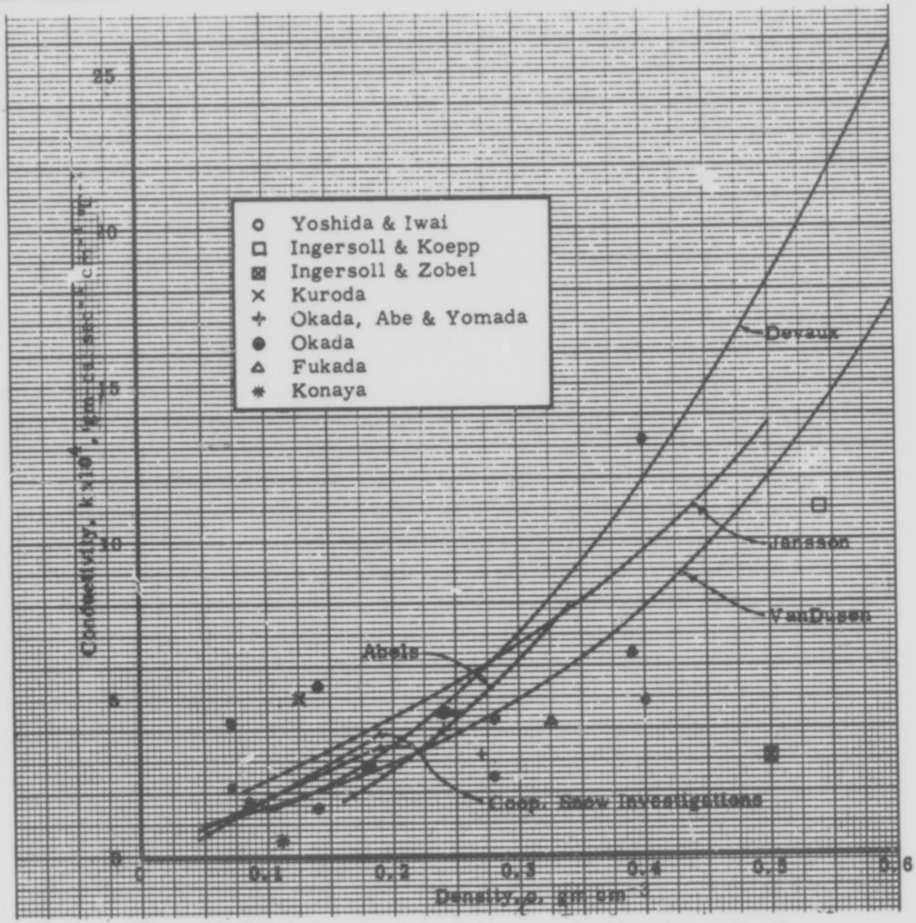

Figure 13. Thermal conductivity of snow as a function of snow density

(except Van Dusen's, for which no range was specified) are plotted only in the range of densities that were observed in the original experiment.[3]

3. Thermal Diffusivity

The thermal diffusivity, a^2, is defined by the relation

$$a^2 \equiv \frac{k}{C\rho}$$

where k is the thermal conductivity, C the specific heat, and ρ the density. In this report a^2 has the units $cm^2\ sec^{-1}$.

The diffusivity is important since it enters directly into the differential equation of thermal conduction.

Since the values of C are accurately known (in comparison to the accuracy of k) and the density of the snow is given for all values of k presented previously, only a simple calculation is required to obtain a^2.

Many of the experiments performed to obtain k (as noted in the preceding section) actually determine the thermal diffusivity directly since the heat flow equation is used. Only the results of the experiments from which a^2 was obtained directly are presented here in Table IV. Those that found a^2 directly, but did not give the value of C used, are not included. Description of the actual experiments may be found in the section on thermal conductivity.

TABLE IV

Thermal Diffusivity of Snow
$cm^2\ sec^{-1}$

ρ	a^2	a^{2*}	(see note in section on k)
0.40	6.9	2.5	Yosida and Iwai (1946)
0.39	3.9	3.9	
0.28	1.85	3.22	
0.14	1.86	6.62	
0.085	3.90	4.22	
0.072	12.0	6.3	
0.54	0.0041		Ingersoll and Koepp (1924)
0.19	0.0025		Abels (1893) from Schofield and Hall (1927)
0.33	0.0046		

It should be noted that the results of Yosida and Iwai show a wider variation of values than those of some of the other investigators; and should the need arise to use the diffusivity, it is suggested that the value of a^2 be determined from the mean of the values of k.

Van Dusen (1929) concluded from the data available (almost all work previous to that date) that between 0^0 and -30^0 C,

$$a^2 = (\rho^{-1} + 20 + 103\rho^2) \times 10^{-4},\ cm^2\ sec^{-1}.$$

4. Thermal Coefficient of Expansion

The coefficient of thermal expansion, β, of a substance is defined as

$$\beta \equiv \left(\frac{\partial v}{\partial T}\right)_p$$

where v is the specific volume of the material and T and p are its temperature and pressure.

The thermal coefficient of linear expansion, α, is defined by the equation

$$\alpha \equiv \left(\frac{\partial L}{\partial T}\right)_p$$

where L is the unit length of the specimen.

If a substance is isotropic, which is not true for ice,

$$\beta = 3\alpha$$

As for the specific heat of ice, the effect of impurities should not be neglected. The observed values of β and α are actually the results of (a) the change in volume of the ice and (b) the change in volume attending the melting or freezing that is required to adjust the concentration of the impure solution entrapped among the ice crystals. No values of β and α for ice have been found which are not in Dorsey (1940). For this reason only a few of the most important values are presented here.

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[3] The thermal conductivity near the surface of a natural snow layer may be much higher than the value of k presented here due to the effect of convection of the air in the snow (see section on Heat Economy of the Snow Pack).

Dorsey believes that the only satisfactory series of determination of α for ice is that of Jakob and Erk (1928). Jakob and Erk believed that the axes of the constituent crystals in the rods studied had a pronounced radial component. Their results are presented in Table V.

TABLE V

Thermal Coefficient of Linear Expansion of Ice
(Jakob and Erk)

T, - °C	α, 10^{-6} °C^{-1}	T, - °C	α, 10^{-6} °C^{-1}
0	52.7	50	45.6
10	51.7	60	43.7
20	50.5	70	41.5
30	49.0	80	39.2
40	47.4	90	36.7

Values of the coefficient of thermal expansion are given in Table VI.

TABLE VI

Coefficient of Thermal Expansion of Ice
(β compiled by Dorsey)

Observer	Year	β 10^{-6} °C^{-1}	Range of T - °C
Brunner	1845	122	0.8 to 19.5
Plucker and Geissler	1852	155	0 to 24
Pettersson	1883	170	below 3
Zakrzewski	1892	77	0.7 to 4.7
Vincent	1902	152	0.4 to 10
3 α from Jakob and Erk		153	0 to 20
For other values see Dorsey			

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Chapter VI

RADIATION PROPERTIES OF SNOW AND ICE

1. General Concepts

When electromagnetic radiation strikes a medium there are three general modes of disposal of the energy. It may be reflected, absorbed or transmitted through the medium. For convenience in the subsequent discussion we will express this relationship by the following equation:

$$I = R + A + D \tag{1}$$

where I is the energy of incident radiation; R is the energy reflected; A is the energy absorbed; and D is the energy transmitted.

The quantity A will be the essential factor affecting such problems as the heat economy of and the temperature distribution in the snow pack. It is almost impossible to measure the absorbed radiation directly and instead I, R and D are measured, and A is computed from equation (1). For a thick snow pack $D \rightarrow 0$ and only I and R need be determined.

The measurement of the radiation reflected from a snow surface also entails considerable difficulty. Quite frequently it is assumed that the snow surface behaves as a perfectly diffuse radiator and follows Lambert's cosine law. In which case the intensity of the reflected radiation is proportional to the cosine of the angle between reflected ray and a normal to the surface and is independent of the direction of the incident radiation. For diffuse reflection then, one need measure the intensity in one direction only to determine the total reflected radiation intensity (assuming a cosine response of the measuring instrument).

The absorption of radiation by a homogeneous body will follow the Bouguer-Lambert law which may be written:

$$I_\lambda = I_{\lambda_0} e^{-\nu z}$$

where I_λ is the intensity of radiation (in the z direction) for the spectral region λ to $\lambda + d\lambda$ at the point z below the surface; I_{λ_0} is the intensity of non-reflected radiation in the interval λ to $\lambda + d\lambda$ at $z = 0$; and ν is termed the absorption coefficient, in this report in units of cm^{-1}. The absorption coefficient normally varies with wave length, hence the Bouguer-Lambert law holds strictly only for a small wave length band.

A body will also emit electromagnetic radiation, the intensity of which will depend upon the temperature. The capacity to emit radiation is usually characterized by comparison with the most efficient radiator, a black body. The ratio of the intensity of radiation emitted to that emitted by a black body at the same temperature is generally termed the <u>emissivity</u>, ϵ, and is defined by the equation

$$\epsilon = \frac{J}{\sigma T^4}$$

where
J = the intensity of radiation emitted (e.g., watts cm^{-2})
σ = Stefan constant = 5.735×10^{-12} watts cm^{-2} $^0K^{-4}$
T = absolute temperature ^0K

The emissivity will also, in general, be a function of the temperature.

2. Reflection from Snow

The total reflecting power of a surface is often expressed as the albedo, α, which is defined by the equation

$$\alpha = \frac{R}{I} \tag{2}$$

where I is the total incident radiation flux and R the total reflected radiation flux. Then from equation (1) we may write for a thick snow layer, for which $D \to 0$,

$$A = (1 - \alpha) I \tag{3}$$

The reflecting power, however, will generally vary with the wave length of the incident radiation, the angle of incidence of the incoming radiation, character of the surface, etc. It is more precise to define the reflecting power, r, for a definite wave length such that

$$r = \frac{\text{Reflected energy in range } (\lambda \text{ to } \lambda + d\lambda)}{\text{Incident energy in range } (\lambda \text{ to } \lambda + d\lambda)}$$

where λ is the wave length of the radiation. The albedo, therefore, may be defined

$$\alpha = \frac{\int_0^\infty r\, I_\lambda\, d\lambda}{\int_0^\infty I_\lambda\, d\lambda}$$

If r is a constant for all wave lengths, $\alpha = r$. Unfortunately, the number of measurements of r are limited and the reported values show considerable variation.

The passage of radiation through snow differs from the passage of radiation through a continuous medium in that internal reflection takes place. Thus r, and of course α also, will depend on the condition of the snow below as well as at the surface. It will also depend somewhat on the snow depth as long as $D \neq 0$. The description of the snow in the earlier investigations of r is often too general (old, new, frozen, etc.), to provide satisfactory correlation between snow type and reflective power.

Conformity to Lambert's cosine law: A major defect of many of the investigations of the albedo and the reflective power is to assume that the snow surface is a perfectly diffuse reflector and therefore that Lambert's law would hold. Although there seems to be general agreement among investigators that a fresh snow surface does obey Lambert's law, old snow may easily be observed to have a great deal of specular reflection. When the snow surface is not a perfectly diffuse reflector, the readings of the radiation instrument will depend on the height of the sun, the angle between the directions in which the radiation instrument is aimed and the normal to the snow surface, and the angle between the instrument and the sun's rays. Thus to get the true value of α or r, it would be necessary to take a whole series of readings to describe the variation in intensity over the enclosing hemisphere.

Devaux (1933) using a photometer, found that new snow very closely approximates Lambert's cosine law. For old and frozen snow he found a great deal of specular reflection.

Eckel and Thams (1939) using a pyranometer found that the height of the sun had no effect on their albedo measurements of snow (presumably all kinds) and therefore the snow surface acts as a completely diffuse reflector. They did observe specular reflection for a crusted snow.

Trainor (1947) studied the observance to Lambert's cosine law for new, old and old melting snow. Taking wedge photographs he measured the intensity of reflected radiation at different angles to the surface and in different positions relative to the sun. Although he concludes that all three types of snow surfaces closely approximate a Lambert diffusing surface, it would seem, from the data presented, that this is far from true for the old and melting snow surfaces. Specular reflection and large variations for different angles to the surface were observed by Trainor for old and melting snow.

According to Rikhter (1945), Kalitin found the illumination of a horizontal snow surface by dispersed light from a cloudless sky. His results are shown in Table I.

The values of illumination are only relative, but their large variation would seem to indicate that a snow surface does not follow Lambert's cosine law too closely.

B. L. Hansen[1] described experiments which showed that new snow closely obeys Lambert's cosine law while other types do not. He also states that a close approximation to the true values for r and α may be obtained by aiming the radiation instrument at an angle of 90° to the sun's rays and 45° to the snow surface.

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[1] In a private communication describing studies undertaken by the Cooperative Snow Investigations.

TABLE I

Illumination of Horizontal Snow Surface after Kalitin (Rikhter, 1945)

Altitude of Snow Above Horizon in Degrees	Relative Illumination of Snow Cover, per cent
0	1.6
5	4.0
10	5.8
15	7.2
20	8.5
25	9.6
30	10.8
35	11.9
40	12.9
45	14.0
50	14.9

TABLE II

Reflecting Power for Freshly Fallen Snow after Hulbert (1928)

Range of λ (microns)	r (Relative Values Only)
0.3-0.4	35
0.4-0.8	40
0.8-2.6	15
2.6-7	18
7 -	26

TABLE III

Reflecting Power for a Melting Snow Layer of Large Grains after Devaux (1935)

λ (microns)	r
1.6	0.67
0.8	0.61
0.9	0.55
1.0	0.34
1.1	0.4
1.2	0.18
1.3	0.21
1.4	0.08
1.5	0.01

Reflecting power: Hulbert (1928) obtained relative values of r for different wave lengths using a thermocouple as the radiation measuring instrument and various absorbing screens to control the range of wave lengths reaching the thermocouple, (r should be low when emissivity is high, i.e., for $\lambda >$ about 6 microns, see next section).

Devaux (1933) using Wratten filters, found the névé on glaciers to reflect more red (0.61-0.67μ) than violet (about 0.54μ) radiation.

Devaux (1935) measured r using a spectroscope to separate the different wave lengths and a thermopile to measure the intensity of radiation. The radiation which was assumed to be diffusely reflected, was compared to that from a known quartz diffuser.

According to Rikhter (1945), Kalitin (1938) found that 95 per cent of the energy in the blue to bright red spectrum is reflected as compared to 91 per cent in the ultra-violet and 88 per cent in the dark red. Rikhter also reports that wet snow, unlike dry snow, is more reflecting in the dark red than in the ultra-violet spectrum and, as the snow melts, the reflection of ultra-violet rays declines even further.

Sauberer (1938) measured r with a photovoltaic cell. Various filters were inserted directly in front of the cell to control the wave length of radiation reaching the instrument. Sauberer's values are summarized in Table IV.

Eckel and Thams (1939) state that Götz found little difference in the albedo of snow for different spectral regions.

Suga (1941) found the reflecting power of snow by using a spectrograph and measuring the density on the photographic plate. His results are tabulated in Table V.

Trainor (1947) found the reflective power by comparison with $MgCO_3$. He separated the wave lengths with a spectrometer and determined the intensity from wedge photographs. His values for r for old and somewhat granular snow layer (of only 2 inches thickness), in the wave length range of from 0.42 to 0.64 microns, is given in Table VI.

Albedo: Unless otherwise specified the albedo results were obtained for a natural snow cover. The spectral range of the intensity measuring instrument is given whenever this data was contained in the original paper.

Nutting, Jones, and Elliott (1914) used a photometer to measure the relative brightness of two planes, one of which receives all of its illumination from the other.

TABLE IV

Reflecting Power of Snow
Sauberer (1938)

λ (microns)	r
0.380	0.72
0.435	0.74
0.475	0.73
0.525	0.76
0.590	0.76
0.630	0.75
0.660	0.76
0.700	0.75
0.735	0.74
0.760	0.72

TABLE V

Reflecting Power of Snow
Suga (1941)

λ (microns)	r
0.75 - 0.50	1.0
0.50 - 0.40	1.0
0.40 - 0.30	0.9

TABLE VI

Reflecting Power, Old Snow
after Trainor (1947)

λ (microns)	r
0.64	0.805
0.62	0.698
0.60	0.670
0.58	0.685
0.56	0.725
0.54	0.698
0.52	0.712
0.50	0.712
0.48	0.712
0.46	0.725
0.44	0.757
0.42	0.757

Abbot and Aldrich (1916) using a pyranometer, found the albedo of new fallen snow to be 0.70.

Ångström (1925) used a pyranometer enclosed in glass and therefore the wave lengths greater than 4μ did not reach the instrument. He obtained 0.695 for the albedo of an old snow surface and 0.81 for α of a fresh and "exceptionally white and smooth" snow surface.

The first extensive series of measurements of the albedo of snow were carried out by Kalitin (1930 and 1931). He observed α to vary from a minimum of 0.52 for melting snow to a maximum of 0.88 for a dense top layer. He found α decreased with thawing or over a period of time when no snow falls and increases with the fall of fresh snow. A table of some of his results, including albedo measurements, is contained in the following section on absorption.

Devaux (1933) using a photometer concluded that the albedo of fresh snow in the visible spectrum was greater than 0.8 and probably greater than 0.9. From indirect considerations he obtained a value of 0.95 for the albedo in the visible spectrum of fresh snow. For névé he found the albedo in the visible spectrum to be about 0.6. Using an actinometer (sensitive to radiation up to λ = 3 or 4μ) he found an albedo of about 0.7 for fresh snow and about 0.5 for névé.

Olsson (1936) measured the albedo of a snow surface with both a pyranometer and a photoelectric cell. The results, using the pryanometer (sensitive for $0.3 < \lambda < 3$ to 4μ), of a large number of observations are presented in Table VIII. The mean value of his measurements with a photoelectric cell (sensitive for $0.3\mu < \lambda < 0.6$ to 0.8μ) is 0.86 for wet and frozen snow when the sky is clouded. This is about 20 per cent greater than the results obtained using a pyranometer and suggest a variation in α for different spectral regions.

Rikhter (1945) states that Chernigovskii (1935, 1936, 1939) found that up to 94 per cent of the solar energy reaching the surface at Uedinenie Island (in the Arctic) was reflected back into space. Rikhter gives a value of 70 - 95 per cent for α of newly-fallen snow and 30 - 50 per cent for old settled snow. He also says there is considerably less reflection from wet snow than from dry and that the appearance of dust and soot, that had been scattered throughout the snow, at the surface greatly reduce α.

Eckel and Thams (1939) carried out a very extensive series of observations of the albedo over two winters. Their values range from 0.45 to 0.88. The minimum value was obtained during a spring thaw when there was considerable soot on the snow and the albedo of 0.88 was obtained right after a snow fall. Eckel and Thams found that the albedo of a snow surface would increase following new snow, reformation of crystals through surface freezing, and formation of surface hoar; and would decrease following compaction, recrystallization if accompanied by coarsening of the crystals, and thawing of the surface. An average daily albedo greater than 0.80 seldom occurred in the post-winter. They believed that the albedo of pure snow (not contaminated by soot or other impurities) would always be greater than 0.60. They compiled, for comparative purposes, the following reported values of the albedo for new snow.

Some of these values were presented and discussed earlier in this text.

Mörikofer (1939) describes some of the work performed by Eckel and Thams. The results of their work are reported in Table X.

TABLE VII

Albedo of Different Snow Surfaces after Nutting, Jones and Elliott (1914)

Type of Snow	Albedo
Fine, dry, drifted	0.847
Natural surface, fresh	0.702
Old, beady	0.678

TABLE VIII

Albedo of Snow after Olsson (1936)

Sky Condition	Albedo		
	New Snow on Surface	Frozen Snow Surface	Melted Snow Surface
Clear	0.81	0.75	0.59
Cloudy	0.75	0.65	0.65

TABLE IX

Results of Different Snow Albedo Investigations after Eckel and Thams (1939)

Observer	α	Method of Measuring
Kalitin	0.80	Calorimetric
Luneland	0.81	Calorimetric
Olsson	0.75-0.81	Calorimetric
Sauberer	0.73-0.84	Photoelectric
Dorno	0.74	Photometric
Götz	1.00	Photochemical
Eckel and Thams	0.77-0.88	Calorimetric

TABLE X

Albedo of Various Snow Surfaces after Mörikofer (1939)

Type of Snow	α
New snow	0.8 - 0.9
Settled dry snow	0.6 - 0.8
Melting snow	down to 0.5

3. Absorption by Snow

The absorption of radiation by snow is of prime importance to the heat economy of snow cover; not only the total amount of energy but the energy absorbed at every depth in a snow layer is required. This latter quantity is of greater importance for snow than for other media due to snow's low thermal conductivity.

If a snow layer were a homogeneously diffusing medium one would expect any radiation flux in it to follow the Bouguer-Lambert law (see above) and fall off exponentially in intensity. As has been pointed out (Devaux, Trainor and others), radiation will be subject to internal reflections within the snow. This will effectively lengthen the path of any radiation in the snow and thus increase the possibility of absorption.

Even if the Bouguer-Lambert law held for a homogeneous snow pack, one could not expect the law to hold rigorously for a natural snow cover, where the density and crystal structure vary greatly with depth. Thus measurements made in a snow pack have real significance only if the snow between the different radiation measuring instruments is uniform. This precaution was taken by few of the investigators.

Another important factor which will tend to vitiate any results obtained is the introduction of a relatively large cavity into the snow when the radiation instrument is placed therein. This may cause, due to internal reflection, the radiation intensity at the instrument to be greater than it would be if no cavity had been formed in the snow.[2] The only simple procedures that avoid this effect are either placing the radiation indicator in the ground or taking the observations of snow removed from its natural environment where the instrument can be placed behind a snow sample. The former modus operandi is very inflexible since the height and uniformity of the snow are quite uncontrollable and inhomogeneity (as pointed out above) presents great difficulties. The latter method, although the preparation and support of the sample might prove troublesome, has great advantages in that a sample of definite thickness, and definite and homogeneous structure could be studied. Only one (Trainor) of the investigators studied used this latter method and he did not have sufficient time to obtain any definite results.

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[2] See Benford, 1946: Radiation in a diffusing medium. Journal of the Optical Society of America, 36:524-554.

Few of the investigators have examined the absorption characteristics of the snow for different wave lengths. As with water, one would expect sharp maxima and minima at certain wave lengths in addition to a gradual variation with wave length Most experiments have neglected these possibilities and sought a total or average absorption effect.

As in most of the other properties, the classification of snow for which ν (from Bouguer-Lambert law, $I = I_0e^{-\nu z}$) and other absorption effects were measured is either too broad or nonexistent. Any characteristics of the snow studied or selectivity by the radiation instruments used, that were given by the original researcher, are presented here.

Most of the investigators have assumed that the radiation intensity falls off exponentially with depth and therefore the absorption coefficient, ν, from the equation, $I = I_0e^{-\nu z}$ has usually been presented. This property may be used to compare the results of the different experiments.

Sauberer (1938) used a photoelectric cell to measure the radiation transmitted through a snow cover and various filters to control the wave length falling on the cell. He obtained the following results[3] for a new snow of 0.13 gm cm^{-3} density. He found that wet snow transmitted more of the blue and violet light and absorbed a large percentage of the radiation at 0.80μ. This latter effect was attributed to the water in the snow.

TABLE XI

Spectral Distribution of Radiation Transmitted through New Snow (Sauberer 1938)

λ (microns)	Relative Intensity
0.380	79
0.435	86
0.475	94
0.525	99
0.590	100
0.630	99
0.660	99
0.700	96
0.735	96
0.760	95

Rikhter (1945) presents the following table obtained by Kalitin (1940) for the spectral transparency of snow. The type of snow was not specified.

Kalitin (1931) measured the radiation passing through a natural snow cover by placing a pyranometer in the ground before any snow had fallen and taking readings, mainly when the sky was overcast, over a four-month period. Simultaneous measurements of the incoming radiation and albedo were taken.

Eckel and Thams (1939) computed ν from the results of Kalitin's (1933) observations. They found, assuming the Bouguer-Lambert law to hold, $\nu = 0.197$ cm^{-1} for dry snow and $\nu = 0.446$ cm^{-1} for wet snow. Kalitin used a pyranometer in these experiments also.

TABLE XII

Spectral Transparency of Snow
(Kalitin 1940 from Rikhter 1945)

λ (microns)	% radiation passing through 1 cm of snow	ν* cm^{-1}
0.322	12	0.21
0.322	14	0.20
0.346	14	0.20
0.366	15	0.18
0.392	16	0.18
0.416	17	0.18
0.438	19	0.17
0.448	20	0.16

*Has been computed by compiler assuming $I = I_0e^{-\nu z}$

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[3] The results are again dependent on the response of the radiation measuring instruments to the angle of incidence of the radiation.

TABLE XIII

Radiation Transmitted by a Natural Snow Cover (Kalitin 1931)

Snow depth--cm	Albedo	% of Non-Reflected Radiation Reaching Ground	ν* cm^{-1}	Description of Snow
2.5	0.76	81.2	0.083	
2.5	0.76	90.9	0.038	
5	0.79	42.1	0.174	snow surface loose
5	0.79	45.2	0.160	snow surface loose
6	0.80	25.0	0.23	top of snow loose
8	0.82	31.2	0.15	top of snow loose
14	0.81	9.9	0.17	top of snow loose
14	0.81	6.7	0.19	top of snow loose
15	0.78	5.6	0.19	snow surface uneven and dense
15	0.78	6.3	0.18	
13	0.82	6.9	0.21	top of snow loose
13	0.82	6.5	0.21	top of snow loose
14	0.81	5.6	0.21	top of snow freshly fallen
14	0.82	4.2	0.23	
14	0.88	8.2	0.18	top of snow dense
14	0.88	6.1	0.20	
27	0.70	2.3	0.14	
52	0.84	2.3	0.073	snow surface packed after drifting
52	0.84	2.0	0.075	snow surface packed after drifting
52	0.86	2.2	0.[illegible]	snow surface dense and covered with hoar frost
52	0.86	2.7	0.070	snow surface dense and covered with hoar frost
62	0.86	0.6	0.083	top of pack is fresh fallen snow
62	0.86	0.3	0.094	top of pack is fresh fallen snow
54	0.83	1.14	0.083	top of pack is fresh fallen snow
54	0.83	1.12	0.083	top of pack is fresh fallen snow
47	0.72	0.57	0.11	snow surface uneven
47	0.72	0.54	0.11	snow surface uneven
42	0.62	0.37	0.13	snow surface thawed and granular
42	0.62	0.51	0.13	snow surface thawed and granular
27	0.52	0.93	0.17	snow surface thawed and granular
27	0.52	1.27	0.16	snow surface thawed and granular
24	0.53	2.22	0.16	snow surface thawed and granular
24	0.53	2.43	0.16	snow surface thawed and granular

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*The values of ν have been computed by the compiler assuming the relation, $I = I_0e^{-\nu x}$ holds true. Even though the snow pack is inhomogeneous causing the absolute values of ν to have no real value, they may be useful for comparative purposes.

Devaux (1933) found that the radiation in a snow pack falls off exponentially. He obtained

$$I = I_0\ (10)^{-0.1z}$$

where z is in cm. Therefore $I = I_0 e^{-2.3z}$ or $\nu = 0.23\ \text{cm}^{-1}$ for fresh snow.

Idumi (1936) measured the penetration of radiation through a natural snow cover. He used a Dorno Graukeil dark wedge photometer. From his average results (for different types of snow) one obtains[4] a value for ν of $0.124\ \text{cm}^{-1}$.

Olsson (1936) placed a photoelectric cell and pyranometer at various depths in a snow cover. Assuming an exponential falling off of intensity he obtained the absorption coefficient. No characteristics of the snow were given.

Olsson--Absorption Coefficient, ν,

Photoelectric cell $(0.3 < \lambda < 0.6$ to $0.8\mu)$

Range (cm^{-1})	Mean (cm^{-1})
0.067-0.087	0.074

Pyranometer $(0.3 < \lambda < 3$ to $4\mu)$

Range (cm^{-1})	Mean (cm^{-1})
0.07-0.16	0.114

Sauberer (1938) using a photovoltaic cell found the absorption coefficient to be $0.150\ \text{cm}^{-1}$ for new snow.

Eckel and Thams (1939) used a step wedge photometer to measure the radiation in snow. Assuming an exponential decrease in the intensity, they found (after many observations), an average value of 0.083 for the absorption coefficient of dry snow. For wet snow, they found $\nu = 0.136\ \text{cm}^{-1}$. The value for dry snow was obtained after extensive observation while ν for wet snow was obtained only with a few observations.

Kalitin's (1939) results as reported by Rikhter (1945) are given in Table XIV.

Rikhter says the translucence of wet snow is considerably less than tnat of dry. If 20% of the energy striking the surface will pass through a 10 cm thickness of dry snow, only 2.4% will traverse the same thickness of wet.

Kudo (1941) determined the absorption coefficient of snow by placing a selenium photo-cell at various depths in a snow cover. He found ν to vary from 0.10 to $0.25\ \text{cm}^{-1}$ for white settled snow and from 0.20 to $0.35\ \text{cm}^{-1}$ for packed granular snow. He found no radiation pentrated to a depth of 50 cm in a natural snow cover.

Hand and Lundquist (1942) studied the penetration of solar radiation into a snow cover using an Eppley pyrheliometer placed at various depths in the snow. They found that "the amount of transmission varies in proportion to the average quality of snow and is greatly influenced by the factors of snow and intermediate ice layers." (See Table XV.)

Gerdel (1948) used a specially designed pyrheliometer to study the penetration of radiation into a snow pack. The instrument was placed at various depths in a snow cover and the radiation intensity at these depths and the incident intensity were measured. If the snow density was uniform between two depths at which readings were taken, the absorption coefficient for snow of the particular density could be obtained. Assuming the Bouguer-Lambert law to hold for a snow layer of uniform density, he obtained

Density %	26.1	32.3	39.7	44.8
ν cm^{-1}	0.280	0.184	0.106	0.106

For further data see the original report.

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[4] Several workers have stated that Idumi found $\nu = 0.117\ \text{cm}^{-1}$. The difference (fortunately small in magnitude) is due to lack of understanding of Idumi's definition of "transmissibility," which is not defined as $(1 - \nu)$ as has been reported.

TABLE XIV

Radiation Transmitted by a Natural Snow Cover
(Kalitin 1939 from Rikhter 1945)

Thickness of Snow Cover in cm	% of Radiant Energy Transmitted; Neglecting Reflection	ν^* cm^{-1}	% of Light Transmitted, Neglecting Reflection	ν^* cm^{-1}
0	100		100	
		0.035		0.16
3	90			
5			45	
		0.21		0.26
10	20		12	
		0.26		0.25
15	5.5		3.5	
	and 5.7	0.11		0.47
20	3.2		0.34	
		0.066		0.27
25	2.3		0.09	
		0.060		0.22
30	1.7		0.03	
		0.026		0.36
35	1.5		0.005	
		0.045		
40	1.2			
		0.036		
45	1.0			
		0.024		
60	0.7			

TABLE XV

Solar Radiation Transmitted by a Snow Cover
(Hand and Lundquist 1942)

Snow Depth in	% of Incident Radiation Transmitted	ν^* cm^{-1}	
4	0		Fine granular snow. 1/8" ice layer at a depth of 1-3/4".
		0.54	
2-1/2	1.0		
		0.54	
1	0.8		
7	0		Uniform, fine granular compact snow, average density (% of water in a given volume) of 45%, from 8-16" below original surface. No ice or slush layers.
		0.091	
5-3/4	1.2		
		0.31	
4	1.8		
		0.27	
2-1/8	7.8		
		0.22	
1	16.9		
1/2	22.3		

*Has been computed by compiler assuming $I = I_0 e^{-\nu z}$

4. Emissivity of Snow

The emissivity, ϵ, as explained previously, is a measure of how closely a substance approximates a black body as a thermal radiator. Since it is extremely difficult to directly measure the temperature of a snow surface, the only available measurements are for melting snow, where the temperature would be 0° C.

Falckenberg (1928) measured the emissivity of melting snow using an apparatus designed to act like a black body. This device had a thermopile incorporated in it and could be placed over the snow. ϵ may be determined from readings of the radiation falling on the thermopile for different temperatures of the "black body." He found ϵ to be greater than 0.995, indicating that melting snow acts very much like a black body.

B. L. Hansen[5] states that measurements carried out by the Cooperative Snow Investigations indicate that radiation from melting snow closely approximates that from a black body.

5. Radiation Properties of Ice

Kalitin (1935) measured the transmissibility of ice to radiation of different wave lengths. He used a spectrograph in his observations. His results are shown in Table XVI.

TABLE XVI

Percentage of Incident Radiation Passing Through Ice
Kalitin (1935)

Thickness of Ice--cm	λ (microns) 0.332	0.346	0.366	0.392	0.416	0.438	0.446
10 (clear)	97	96	99	99	98	99	98
10 (brown and bubbles)	60	59	59	60	63	62	60
107 (clear)	46	46	51	52	54	52	55

Sauberer (1938) measured the reflectivity of ice for different wave lengths using a photo-cell and various filters.

TABLE XVII

Reflectivity of Ice for Different Wave Lengths
Sauberer (1938)

λ (microns)	r
0.380	0.44
0.435	0.46
0.475	0.48
0.525	0.50
0.596	0.50
0.630	0.48
0.660	0.50
0.700	0.48
0.735	0.47
0.760	0.43

TABLE XVIII

Transmissibility of Ice at Different Wave Lengths
Sauberer (1938)

Clear Ice 25 mm thick λ (microns)	Per Cent Transmitted
0.380	84
0.435	86
0.525	86
0.590	85
0.630	87
0.660	84
0.700	87
0.735	87
0.760	86

Sauberer (1938 and 1950) also measured the spectral variation in the transmission of radiation through ice. His results are given in Tables XVIII and XIX.

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[5] Private communication pertaining to work carried out by Cooperative Snow Investigations.

TABLE XIX

Transmissibility of Clear Ice for Different Wave Lengths
Sauberer (1950)

λ (microns)	Percentage Transmitted Less Reflected Per-Meter Thickness	ν^* (cm^{-1})
0.313	90.0	0.0011
0.350	95.0	0.0005
0.400	96.0	0.0004
0.450	95.0	0.0005
0.500	92.0	0.0008
0.550	88.0	0.0013
0.600	81.5	0.0020
0.650	71.0	0.0034
0.700	55.0	0.0060
0.750	34.5	0.011
0.800	17.0	0.018
	1 cm Thick	
0.8	97	0.030
0.9	91	0.094
1.0	73	0.32
1.1	85	0.16
1.2	22	1.5
1.3	19	1.7

*Computed by compiler.

The above measurements are the only ones found that are not contained in Dorsey (1940). For other values, the reader is referred to that work.

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Chapter VII

THE HEAT ECONOMY OF THE SNOW PACK

The study of snow properties reveals that many of the mechanical properties of snow are highly temperature dependent. The subject of the heat economy of the snow pack has, therefore, been reviewed so as to lead to a better understanding of the mechanism of temperature change within the snow pack and the energy exchange between the atmosphere and the snow pack. While a number of studies of the energy exchange are available, it is believed that the most complete and satisfactory theoretical treatment of the problem has been made by Reuter (1948). The review of Reuter's study constitutes the major portion of this paper: the basic assumptions in this study are taken directly from Reuter; a different method of solution is employed so that the generality of the solution is established; and applications of the theory and conclusions are re-examined in the light of more recent information.

The principal contributions of Reuter's work are the introduction of a more complete equation for the energy flux and the utilization of more realistic boundary conditions. The effect of the permeability of snow to short wave radiation is considered for the first time. Many of the difficulties inherent in the selection of proper boundary conditions are avoided by setting the boundary conditions on the heat flux rather than the temperature.

EQUATION FOR THE ENERGY FLUX

An expression for the flux of energy within the snow pack, under condition of exposure to solar radiation, must account for the flux of radiant energy as well as thermal energy since the snow is permeable to short-wave radiation. From the conservation of energy, the flux of energy through the boundaries of a system is equal to the energy change within the system, which in this case for thermal and radiant energy becomes:

$$\iint k\nabla T \cdot d\vec{s} - \iint \vec{i} \cdot d\vec{s} = \frac{\partial}{\partial t} \iiint c\rho T d\tau$$

where k = thermal conductivity

c = specific heat

ρ = density

T = temperature

$d\vec{s}$ = element of area

$\vec{i}$ = intensity of the short-wave radiation

$d\tau$ = element of volume

t = time

To a first approximation, a snow cover may be regarded as a homogeneous mass of infinite horizontal extent. It is also assumed that the absorption of short-wave radiation may be expressed by means of the exponential law, $i = i_0 e^{-\nu z}$, where z is the vertical coordinate measured downward from the snow surface, i the intensity (energy flux) at the depth z, i_0 the intensity of the short-wave radiation absorbed at the snow surface and ν the absorption coefficient. The following differential equation for the temperature distribution in the snow pack may then be deduced from the above expression for the energy flux by the application of the theorem of Gauss:

$$\frac{\partial T}{\partial t} = a^2 \frac{\partial^2 T}{\partial z^2} + \frac{i_0 \nu}{c\rho} e^{-\nu z} \qquad (1)$$

where $a^2 = \frac{k}{c\rho}$ = diffusivity. This equation will be recognized as the conventional expression for unidimensional heat flow but with an additional term, $\frac{i_0}{c\rho}\, e^{-\nu z}$, to account for the absorption of short-wave radiation.

It may be well to note the restrictive nature of the assumptions; horizontal homogeniety is presumably a good assumption but Trainor (1947), reviewing the work of early investigators, has pointed out that the law of absorption is not strictly an exponential one and further that the absorption does depend to a slight extent on the wave length of the incident radiation.

1. Initial and Boundary Conditions

To obtain a solution of (1), certain initial and boundary values must be imposed. The choice of the initial value, the distribution of temperature at $t = 0$, will not have a marked affect on the solution where one is interested in the temperature distribution after a sufficiently long time. For reasons of simplicity and convenience, the initial condition is assumed to be an isothermal snow pack at zero degrees Centigrade. On the other hand, the boundary conditions, the specified function of time which the solution or its derivative must assume at some point in the region under consideration, will be very important in determining a realistic solution. The choice of boundary conditions is therefore discussed in some detail.

In most solutions to the heat flow equation, boundary values are imposed on the temperature. However, measurement of the surface temperature of a body exposed to solar radiation is technically quite difficult. Nor is it plausible to use the free air temperature as a substitution for the surface temperature since extremely wide differences in air temperature are observed within a few feet of the snow surface. It would therefore seem logical to explore the possibilities of the use of the derivative as a boundary condition. The derivative, of course, is proportional to the heat flux,

$$\text{Heat flux} = -k\frac{\partial T}{\partial z}$$

There are several ways in which heat transfer can take place from the snow surface to the atmosphere; namely, conduction to or from the snow, turbulent convection in the air immediately above the snow surface, condensation or evaporation on the snow surface, and radiation to or from the snow. Of these modes of energy transfer, radiation is generally believed to be the most important.[1] At temperatures near zero degrees Centigrade, it is found that the net outgoing long-wave radiation, the difference between the radiation from the snow and radiation from the atmosphere to the snow, is about 0.1 cal $cm^{-2}\ min^{-1}$ (Wexler, 1936; p. 126), and that this value does not change greatly with a change in the surface temperature. It would, therefore, appear reasonable as a first approximation to assume a constant radiation loss at the surface for the boundary condition.

The problem is then the determination of the solution of (1) under the following conditions:

$$t = 0,\ z > 0\ :\ T = 0 \ \text{(or, more generally, } T = T_a\text{, a constant)}$$

$$t > 0,\ z = 0\ :\ k\frac{\partial T}{\partial z} = R \tag{2}$$

where it is assumed that the total heat flux from the snow surface, $k\left(\frac{\partial T}{\partial z}\right)_{z\,=\,0}$, is equal to a constant R, the net outgoing long-wave radiation from the snow. (Actually the boundary condition may be quite generally interpreted as assuming the flux through the surface is equal to a constant no matter what the mode of transfer from the boundary, whether by conduction, convection or radiation.)

2. Solution

In order to simplify, make the transformation

$$Y = T + qe^{-\nu z}$$

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[1] A frequent but hardly justifiable assumption. See for example, W. T. Wilson, 1941: An outline of the thermodynamics of snow melt, Transactions of the American Geophysical Union, Part I B, p. 194.

where $q = \frac{i_0}{k\nu}$, equation (1) then becomes

$$\frac{\partial Y}{\partial t} = a^2 \frac{\partial^2 Y}{\partial z^2} \tag{3}$$

The conditions equivalent to (2) then become

$$\begin{aligned} t = 0,\ z > 0 &: Y = q\, e^{-\nu z} \\ t > 0,\ z = 0 &: \left(\frac{\partial Y}{\partial z}\right)_{z = 0} = \frac{R - i_0}{k} = -\frac{B}{k} \end{aligned} \tag{4}$$

where, for convenience, $(R - i_0)$ is written as $-B$.
Let $y\ (z, s)$ represent the Laplace transform of the solution of (3) with (4). Then (3) reads

$$\frac{d^2y}{dz^2} - \frac{s}{a^2}\, y = -\frac{q}{a^2} e^{-\nu z}$$

which has the solution (the initial condition has already been introduced)

$$y = c_1(s)\, e^{\frac{\sqrt{s}}{a} z} + c_2(s)\, e^{-\frac{\sqrt{s}}{a}} + \frac{q}{s - a^2\nu^2}\, e^{-\nu z}$$

If one further stipulates that the solution must remain finite as $z \to \infty$, then $c_1(s) \equiv 0$.
The boundary condition (4) may now be introduced,

$$\frac{\partial y}{\partial z}(s, o) = -\frac{c_2(s)\sqrt{s}}{a} - \frac{q\nu}{s - a^2\nu^2} = -\frac{B}{ks}$$

The function $c_2\ (s)$ may be found by rearranging terms

$$c_2(s) = \frac{B}{ks}\, \frac{a}{\sqrt{s}} - \frac{q\nu a}{\sqrt{s}\,(s - a^2\nu^2)}$$

which substituted in the solution yields

$$y = \left(\frac{B}{ks} \cdot \frac{a}{\sqrt{s}} - \frac{q\nu a}{\sqrt{s}\,(s - a^2\nu^2)}\right) e^{-\frac{\sqrt{s}}{a} z} + \frac{q}{s - a^2\nu^2}\, e^{-\nu z}$$

The solution may now be expressed in terms of the original variables by consulting a table of Laplace transforms, from which is found

$$\begin{aligned} Y = \frac{q}{2} \Bigg(& e^{a^2\nu^2 t - \nu z} \left[\operatorname{erfc}\left(a\nu\sqrt{t} - \frac{z}{2a\sqrt{t}}\right)\right] \\ & + e^{a^2\nu^2 t + \nu z} \operatorname{erfc}\left(a\nu\sqrt{t} + \frac{z}{2a\sqrt{t}}\right)\Bigg) \\ & + \frac{2\,B\sqrt{t}}{k\sqrt{\pi}}\, e^{-\frac{z^2}{4a^2t}} - \frac{zB}{k} \operatorname{erfc}\left(\frac{z}{2a\sqrt{t}}\right) \end{aligned}$$

Upon substitution for the original variable, the expression for the temperature will be

$$T = \frac{q}{2}\left(e^{a^2\nu^2 t - \nu z}\left[\operatorname{erfc}\left(a\nu\sqrt{t} - \frac{z}{2a\sqrt{t}}\right)\right] + e^{a^2\nu^2 t + \nu z} \operatorname{erfc}\left(a\nu\sqrt{t} + \frac{z}{2a\sqrt{t}}\right)\right) + \left(\frac{2Ba\sqrt{t}}{k\sqrt{\pi}} e^{-\frac{z^2}{4a^2t}} - \frac{zB}{k}\operatorname{erfc}\left(\frac{z}{2a\sqrt{t}}\right)\right) - qe^{-\nu z} \tag{5}$$

Expression (5) may be greatly simplified if one restricts consideration to the solution only when t is large. Then, since the complementary error function may be written for large argument x

$$\operatorname{erfc} x = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-\beta^2} d\beta \sim \frac{e^{-x^2}}{x\sqrt{\pi}}$$

by the assumption that t is large, equation (5) reduces to

$$T = q\frac{1}{\sqrt{\pi}} e^{-\frac{z^2}{4a^2t}}\left[\frac{a\nu\sqrt{t}}{a^2\nu^2t - \frac{z^2}{4a^2t}}\right] + \frac{2Ba\sqrt{t}}{k\sqrt{\pi}} e^{-\frac{z^2}{4a^2t}} - \frac{zB}{k}\operatorname{erfc}\left(\frac{z}{2a\sqrt{t}}\right) - qe^{-\nu z} \tag{6}$$

More specifically if one assumes $a^2\nu^2t >> \frac{z^2}{4a^2t}$, equation (6) further simplifies

$$T = \frac{q}{\sqrt{\pi}} \frac{e^{-\frac{z^2}{4a^2t}}}{a\nu\sqrt{t}} + \frac{2Ba\sqrt{t}}{k\sqrt{\pi}} e^{-\frac{z^2}{4a^2t}} - \frac{Bz}{k}\operatorname{erfc}\left(\frac{z}{2a\sqrt{t}}\right) - qe^{-\nu z} \tag{7}$$

3. Position of Maximum Temperature

The position of the maximum will be found by differentiating the expression for the temperature (7) with respect to z and setting it equal to zero. One thus obtains an expression for z_0, the distance below the surface at which the maximum temperature occurs.

$$\frac{-i_0 z_0}{2a^3\nu^2\sqrt{\pi t}\; t} e^{-\frac{z_0^2}{4a^2t}} - B + B\operatorname{erf}\left(\frac{z_0}{2a\sqrt{t}}\right) + i_0 e^{-\nu z_0} = 0$$

For large t, the first member of the last expression may be neglected and the following relation is obtained:

$$(i_0 - R)\operatorname{erf}\frac{z_0}{2a\sqrt{t}} = i_0 - R - i_0 e^{-\nu z_0}$$

where the original variables (i_0 - R) are substituted for -B. The depth of the point of maximum temperature will thus depend on the difference (i_0 - R). Consider the effect upon z_0 of the various possible values of this term:

Case 1. $i_0 = 0$

$$\operatorname{erf}\left(\frac{z_0}{2a\sqrt{t}}\right) = 1$$

i.e., z_0 must be very large in this case.

Case 2. $i_0 = R$

$$R\, e^{-\nu z_0} = 0$$

z_0 is large for this case also.

Case 3. $R > i_0$

$$\operatorname{erf}\left(\frac{z_0}{2a\sqrt{t}}\right) = 1 - \frac{i_0}{R - i_0}\, e^{-\nu z_0}$$

Since $\operatorname{erf}\left(\frac{z_0}{2a\sqrt{t}}\right) \leq 1$, again the maximum occurs when $e^{-\nu z_0} = 0$, i.e., for z_0 large.

Hence we find that z_0 is very large for $0 \leq i_0 \leq R$

Case 4. $R < i_0$

$$\operatorname{erf}\left(\frac{z_0}{2a\sqrt{t}}\right) = 1 - \frac{i_0}{i_0 - R}\, e^{-\nu z_0}$$

After a sufficiently long time,

$$\frac{i_0}{i_0 - R} = e^{\nu z_0}$$

Hence,

$$z_0 = \frac{1}{\nu} \ln \frac{i_0}{i_0 - R} \qquad (8)$$

To evaluate z_0 for case (4) of $R < i_0$, values for i_0, ν, and R must be assumed. A few of the values of these snow properties which have been reported in the literature are given for reference in Table I.

TABLE I. Reported Values of Snow Properties

ν, the absorption coefficient for short-wave radiation		
	0.23	Devaux (1933)
	0.083	Eckel and Thams (1939)
	0.150	Sauberer (1938)
Albedo		
	60 - 90%	Eckel and Thams (1939)
	73 - 85%	Sauberer (1938)
R, the net outgoing long-wave radiation		
	0.08 - 0.11 cal cm^{-2} min^{-1}	Reuter (1948)

The value of z_0 will be determined for the following case: a value for ν of 0.15 cm^{-1}; R of 0.08 - 0.11 cal cm^{-2} min^{-1}; albedo of 75 - 85%; and the intensity solar radiation at the surface of 1 cal cm^{-2} min^{-1}. The range of values for i_0 is fixed by the last two assumptions so that $0.25 \leq i_0 \leq 0.15$ cal cm^{-2} min^{-1}.

Equation (8) with the above assumptions gives the position of maximum temperature to be 3-4 cm below the snow surface. This compares favorably with observations by Niederdorfer (1933) which indicates a maximum temperature at 1 to 2 cm depth. (All temperature measurements made near the surface must be viewed with some suspicion, however, because of the difficulty of excluding radiation from the temperature measuring element.)

4. Magnitude of the Maximum Temperature

From equations (7) and (8), one finds that

$$T_{z_0} - T_0 = \frac{1}{k\nu}\left(R - B \ln \frac{i_0}{B}\right) \tag{9}$$

where

T_{z_0} = temperature of the snow at the point of maximum, a distance z_0 from the surface

T_0 = temperature of the snow at the surface.

Now Keränen (1929) has observed for a snow density of the order of 0.16 gm cm^{-3}, that the temperature difference between the surface temperature and maximum temperature amounts to around 2°. For comparison, equation (9) is applied with the value of k calculated from a formula after Abels' (1892), $k = .0068\rho^2$. Using, in this formula, a value for the density of 0.16 gm cm^{-3}, the temperature difference is found to be 15° C. Agreement between formula (9) and observation can only be obtained by assuming that the product $k\nu$ should be larger or the bracketed quantity, $\left(R - B \ln \frac{i_0}{B}\right)$, is smaller. One can readily show that the latter quantity will be smaller only if R, (the net radiation flux or more generally, the net outward energy flux from the snow surface), is smaller. During the day when the convective flux would be presumed to be upwards, the net heat flux would probably be larger than the value of R assumed here.

One is led, therefore, to examine the assumed magnitude of the term $k\nu$. The observed values of ν, the absorption coefficient, as noted in Table I, do not show the magnitude of the variation required to explain this effect so that it is probable the lack of agreement is due to the value assumed for k.

To obtain agreement between the formula and observation, it would be necessary to assume a value for k about seven times larger than has been reported. It is unlikely that the reported values of conductivity are in error to this extent. However, since this effect is observed near the surface in snow of low density, it is quite reasonable to assume that the effective conductivity, which would be due to convective transfer within the uppermost layers of the snow pack, is seven times larger than the conductivity obtained by experiment. This phenomenon will be, of course, very important in any study of the heat economy.

5. The Solution After a Single Period of Nocturnal Cooling Followed by a Period of Solar Heating.

Suppose that, before solar radiation commenced, the nocturnal radiation process had continued for a time t_0. Then, as the solution of the heat equation for the nocturnal radiation process where $i_0 = 0$, we obtain from equation (5),

$$T = Ta - \frac{2Ra}{k\sqrt{\pi}} \sqrt{t_0}\, e^{-\frac{z^2}{4a^2 t_0}} + \frac{zR}{k} \operatorname{erfc}\left(\frac{z}{2a\sqrt{t_0}}\right) \tag{10}$$

where Ta, a constant, is the assumed initial temperature. Therefore, the solution of the heat equation for nocturnal cooling for a time t_0 followed by an exposure to solar radiation for a time t_1 is:

$$T = Ta + A_R + A_{i_0} + E \tag{11}$$

where

$$A_R = -\frac{2Ra}{k\sqrt{\pi}} \sqrt{t_0 + t_1}\, e^{-\frac{z^2}{4a^2(t_1 + t_0)}} + \frac{zR}{k} \operatorname{erfc}\left(\frac{z}{2a\sqrt{t_0 + t_1}}\right)$$

$$A_{i_0} = \frac{2i_0 a\sqrt{t_1}}{k\sqrt{\pi}}\, e^{-\frac{z^2}{4a^2 t_1}} - \frac{z i_0}{k}\,\mathrm{erfc}\left(\frac{z}{2a\sqrt{t_1}}\right)$$

$$E = \frac{i_0}{2k\nu}\left(e^{a^2\nu^2 t_1 - \nu z}\,\mathrm{erfc}\left(a\nu\sqrt{t_1} - \frac{z}{2a\sqrt{t_1}}\right) + e^{a^2\nu^2 t_1 + \nu z}\,\mathrm{erfc}\left(a\nu\sqrt{t_1} + \frac{z}{2a\sqrt{t_1}}\right)\right) - \frac{i_0}{k\nu}\, e^{-\nu z}$$

The temperature distribution after 12 hours of cooling as determined from equation (10) by Reuter is shown in Figure 14, along with the temperature distribution given by equation (11) with $t_0 = 12$ hours and $t_1 = 6$ hours. Two sets of temperature curves are plotted in the figure: the solid curve is plotted for an assumed value of k of 7 times the value given by Abels' formula, and the dotted curve for an assumed value of k of 20 times Abels' value. The remaining constants are assumed to be:

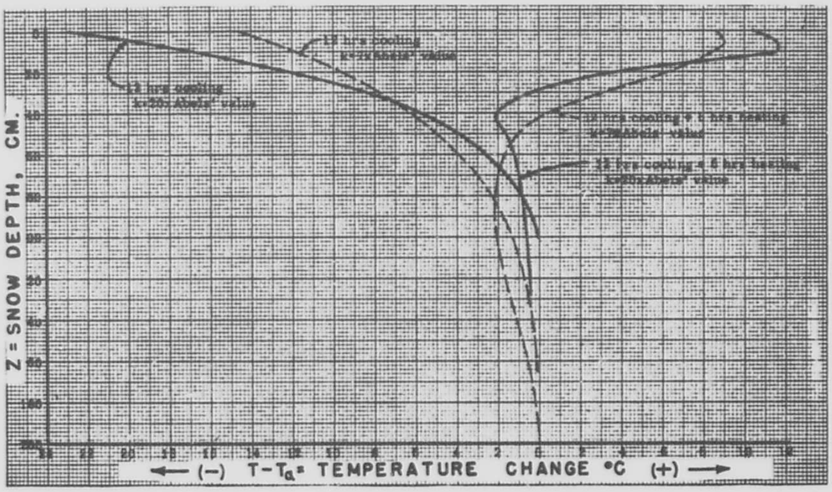

Figure 14. Temperature distribution in snow pack as computed by Reuter (1948)

$\rho = 0.2$ gm cm^{-3}

R = 0.08 cal cm^{-2} min^{-1}

Incident intensity of solar radiation = 1 cal cm^{-2} min^{-1}
Albedo = 75%

The point of intersection of the temperature curve representing 12 hours of cooling with the curve representing 12 hours of cooling plus 6 hours of solar radiation gives the approximate depth to which heating from the solar radiation penetrates. In both cases this occurs at about the same depth, namely 80 cm. However, Keränen (1929) has noted that the diurnal temperature wave only penetrates to 35 cm. Furthermore, Niederdorfer (1933) found the amplitude of the diurnal temperature wave to be only 2.5° C at 20 cm, while in Figure 14, the amplitude at 40 cm must be at least 2.5° C. The temperature maximum, as in the approximation discussed in the previous section, is found to be at a depth of a few centimeters. Therefore, although the assumption of a thermal conductivity larger than that given by Abels' formula explains the temperature distribution in the uppermost layer, this large value of the conductivity cannot hold for the lower layers.

EXTENSION OF THE THEORY FOR PERIODIC EXPOSURE TO RADIATION

We now consider a radiation process varying with time. Any such variation with time can be represented by means of a Fourier Series of sine terms and, as a first approximation, by the first two terms of the series, thus,

$$i_0 = i_1 + i_2 \sin Nt \tag{12}$$

where $N = \frac{2\pi}{\tau} = 7.269 \times 10^{-5}\ \text{sec}^{-1}$ for $\tau = 24$ hours for a diurnal period

$N = \frac{2\pi}{\tau} = 1.991 \times 10^{-7}\ \text{sec}^{-1}$ for $\tau = 1$ year for an annual period

In place of equation (1), we now obtain as the differential equation for the temperature distribution

$$\frac{\partial T}{\partial t} = a^2 \frac{\partial^2 T}{\partial t^2} + \frac{i_1 \nu}{\rho c} e^{-\nu z} + \frac{i_1 \nu}{\rho c} e^{-\nu z} \sin Nt \tag{13}$$

with condition

$$\begin{aligned} t &= 0, \quad z = 0, \quad T = 0 \\ t &> 0, \quad z = 0, \quad k\left(\frac{\partial T}{\partial t}\right)_{z=0} = R \end{aligned} \tag{14}$$

For the complete solution of (13) with (14) (see Appendix), we have,

$$T = T_1 + v_1 + v_2 + we^{-\nu z} \tag{15}$$

where

$$w = -M \sin(Nt + \phi), \text{ a function of time alone}$$

$$v_2 = \frac{a\nu M}{\sqrt{N}} e^{-\frac{z}{a}\sqrt{\frac{N}{2}}} \sin\left(Nt + \phi - \frac{\pi}{4} - \frac{z}{a}\sqrt{\frac{N}{2}}\right)$$

$$v_1 = -\frac{w(0)}{2}\left(e^{a^2\nu^2 t - \nu z} \operatorname{erfc}\left(a\nu\sqrt{t} - \frac{z}{2a\sqrt{t}}\right) - e^{a^2\nu^2 t + \nu z} \operatorname{erfc}\left(a\nu\sqrt{t} + \frac{z}{2a\sqrt{t}}\right)\right)$$

where T_1 is given by equation (7) with i_0 replaced by i_1

$$M = \frac{i_2 \nu}{\rho c} \frac{1}{\sqrt{N^2 + a^4\nu^4}}$$

$$\tan\phi = \frac{N}{a^2\nu^2}$$

After a sufficiently long time, $v_1 = 0$ and

$$T = T_1 - M e^{-\nu z} \sin(Nt + \phi) + \frac{a\nu M}{\sqrt{N}} e^{-\frac{z}{a}\sqrt{\frac{N}{2}}} \sin\left(Nt + \phi - \frac{\pi}{4} - \frac{z}{a}\sqrt{\frac{N}{2}}\right) \tag{16}$$

The first member, T_1, given by equation (7), represents the temperature distribution about which the periodic variation takes place. With the simplification $\frac{z}{2a\sqrt{t}} \to 0$, it follows from (7) that

$$T_1 = Ta + \frac{2a}{k\sqrt{\pi}}(i_1 - R)\sqrt{t} - \frac{z(i_1 - R)}{k} - \frac{i_1}{k\nu} e^{-\nu z} \tag{17}$$

where, again, the initial condition is $T = Ta$ rather than $T = 0$.

Insufficient data are available for testing the validity of equations (16) and (17) for snow; however, the equations may be modified so as to apply to a barren surface of the earth for which observations are available. By allowing ν to approach infinity and setting $z = 0$, (16) becomes a suitable approximation for the expression for the temperature at the earth's surface,

$$T = Ta + \frac{2a(i_1 - R)}{k\sqrt{\pi}} \sqrt{t} + \frac{i_1}{\sqrt{Nk\rho c}} \sin\left(Nt - \frac{\pi}{4}\right) \tag{18}$$

which is known as Poisson's equation.

Using values from tables of Baur and Phillips (1934) for the various constants in equation (18) (40 - 50° N lat.), Reuter obtains a half-amplitude of 53° C for the annual period and the phase lag is of course $\frac{\pi}{4}$ (equivalent to 1 1/2 months for an annual period). He compares this with a half-amplitude of 37° C and a phase lag of 1 1/2 months obtained experimentally at Tiflis (Hann-Süring, 1937). Reuter notes that other modes of transfer other than radiation are probably important and should be included in the term R so that the coefficient of the term in $\sqrt{t}$ may be negative, in which case this term would have the effect of decreasing the predicted magnitude for the amplitude. Revision of the solution using more realistic and complete imposed boundary values would be indicated.

It would appear, however, that the theory has been extended beyond the limits imposed by the original assumptions; since no great local warming or cooling is observed over a period of a year, the net upward energy flux must generally balance the absorbed incoming radiation. It is of some interest that, for snow, the net incoming radiation balances the net outgoing radiation, $i_1 \rightarrow R$, so that the troublesome term in $\sqrt{t}$ does not appear in the solution.

There are several important differences in the results given by the complete equation for a snow pack (16) as compared to the simplified form according to Poisson's equation (18). The phase lag is actually $\frac{\pi}{4} - \phi$ and ϕ will be of a magnitude comparable to $\frac{\pi}{4}$ for a diurnal period in snow. Further, the amplitude, M, of the standing wave is of comparable magnitude to the amplitude of the travelling wave.

CONCLUSIONS

The comparison of the theory with observations on the location of the maximum temperature, amplitude of the diurnal temperature wave, and the penetration of the diurnal temperature wave suggest that convection in the upper few centimeters of the snow is several times more effective in transporting the heat than is conduction. Any calculations of the energy balance between the snow and the atmosphere, therefore, cannot be based on any simple expression for the heat flow in the snow pack. The results of the theory for the phase lag of the diurnal temperature wave are very interesting but it does not appear that the solution is complete and actual observations are so far unavailable.

The work of other investigators has indicated that in general the net outgoing radiation is not the most important mode of energy flux from the snow surface. There is, therefore, a requirement for better simultaneous measurements of the temperature (particularly near the surface), evaporation or condensation, incident radiation and albedo, before the theory can be verified.

APPENDIX TO THE HEAT ECONOMY OF THE SNOW PACK

SOLUTION OF THE HEAT EQUATION FOR A PERIODIC RADIATION

The differential equation is:

$$\frac{\partial T}{\partial t} = a^2 \frac{\partial^2 T}{\partial t^2} + \frac{i_1 \nu}{\rho c} e^{-\nu z} + \frac{i_1 \nu}{\rho c} e^{-\nu z} \sin Nt \qquad (13)$$

The initial and boundary conditions are:

$$\begin{aligned} &t = 0, \quad z > 0, \quad T = 0 \\ &t > 0, \quad z = 0, \quad k\left(\frac{\partial T}{\partial t}\right)_{z=0} = R \end{aligned} \qquad (14)$$

To solve equation (14) with (15), we set

$T = T_1 + T_2$ and obtain two differential equations.

$$\frac{\partial T_1}{\partial t} = a^2 \frac{\partial^2 T_1}{\partial z^2} + \frac{i_1 \nu}{\rho c} e^{-\nu z} \qquad (A1)$$

with boundary conditions:

$$\begin{aligned} &t = 0, \quad z > 0, \quad T = 0 \\ &t > 0, \quad z = 0, \quad k\left(\frac{\partial T_1}{\partial z}\right)_{z=0} = R \end{aligned} \qquad (A2)$$

The solution of this, however, has already been given by equation (6) or (7)
For T_2, we obtain the equation

$$\frac{\partial T_2}{\partial t} = a^2 \frac{\partial^2 T_2}{\partial z^2} + \frac{i_2 \nu}{\rho c} e^{-\nu z} \sin Nt \qquad (A3)$$

with boundary conditions:

$$\begin{aligned} &t = 0, \quad z > 0, \quad T_2 = 0 \\ &t > 0, \quad z = 0, \quad k\left(\frac{\partial T_2}{\partial z}\right)_{z=0} = 0 \end{aligned} \qquad (A4)$$

To solve (A3) with (A4), set

$$T_2 = v + w e^{-\nu z} \qquad (A5)$$

where v is a function of z and t while $w = w(t)$ is a function of t alone, so that one obtains

$$\frac{dw}{dt} = a^2 \nu^2 w + \frac{i_2 \nu}{\rho c} \sin Nt \qquad (A6)$$

and

$$\frac{\partial v}{\partial t} = a^2 \frac{\partial^2 v}{\partial z^2} \qquad (A7)$$

with boundary conditions.

$$t = 0, \quad z > 0, \quad v = -w(o)e^{-\nu z}$$

$$t > 0, \quad z = 0, \quad \left(\frac{\partial v}{\partial z}\right)_{z=0} = \nu w\,(t) \qquad \text{(A8)}$$

The solution of (A6) is obtained directly by a suitable choice of integration constants:

$$w = -M \sin\,(Nt + \phi) \qquad \text{(A9)}$$

where

$$M = \frac{i_2 \nu}{\rho c}\,\frac{1}{\sqrt{N^2 + a^4\nu^4}}$$

and

$$\tan\phi = \frac{N}{a^2\nu^2}$$

To solve (A7), set $v = v_1 + v_2$ in such a way that

$$\frac{\partial v_1}{\partial t} = a^2\,\frac{\partial^2 v_1}{\partial z^2} \qquad \text{(A10)}$$

with

$$t = 0, \quad z > 0, \quad v_1 = -w(0)\,e^{-\nu z}$$

$$t > 0, \quad z = 0, \quad v_1 = 0 \qquad \text{(A11)}$$

and

$$\frac{\partial v_2}{\partial t} = a^2\,\frac{\partial^2 v_2}{\partial z^2} \qquad \text{(A12)}$$

with

$$t = 0, \quad z > 0, \quad v_2 = 0$$

$$t > 0, \quad z = 0, \quad \left(\frac{\partial v_2}{\partial z}\right)_{z=0} = \nu w(t) \qquad \text{(A13)}$$

The solution of (A10) follows from the general solution of the heat conduction equation with the assumption that the surface temperature is held constant at zero degrees (see, for example, Churchill, Modern Operational Mathematics in Engineering).

$$v_1 = -\frac{w\,(0)}{2}\left(e^{a^2\nu^2 t - \nu z}\,\operatorname{erfc}\left(a\nu\sqrt{t} - \frac{z}{2a\sqrt{t}}\right) - e^{a^2\nu^2 t + \nu z}\,\operatorname{erfc}\left(a\nu\sqrt{t} + \frac{z}{2a\sqrt{t}}\right)\right) \qquad (28)$$

To solve (A12), set

$$\frac{\partial v_2}{\partial z} = u \qquad \text{(A14)}$$

and first solve the following equation for u

$$\frac{\partial u}{\partial t} = a^2\,\frac{\partial^2 u}{\partial z^2} \qquad \text{(A15)}$$

with

$$t = 0, \quad z > 0, \quad u = 0$$

$$t > 0, \quad z = 0, \quad u = \nu w\,(t) \qquad \text{(A16)}$$

and then determine v_2 as the integral of u with respect to z,

$$v_2 = -\int_z^\infty u\,dz \qquad \text{(A17)}$$

(A15) with (A16) may also be solved readily by operational methods giving

$$u = -\nu M\left[\sin(Nt+\phi)\,e^{-\frac{z}{a}\sqrt{\frac{N}{2}}}\cos\frac{z}{a}\sqrt{\frac{N}{2}} - \cos(Nt+\phi)\,e^{-\frac{z}{a}\sqrt{\frac{N}{2}}}\sin\frac{z}{a}\sqrt{\frac{N}{2}}\right] \qquad \text{(A18)}$$

Inserting (A18) in (A17),

$$v_2 = \frac{a\nu M}{\sqrt{N}}\,e^{-\frac{z}{a}\sqrt{\frac{N}{2}}}\sin\left(Nt+\phi-\frac{\pi}{4}-\frac{z}{a}\sqrt{\frac{N}{2}}\right) \qquad \text{(A19)}$$

The complete solution then appears as follows:

$$T = T_1 + v_1 + v_2 + w\,e^{-\nu z} \qquad \text{(A20)}$$

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Chapter VIII

PHASE RELATIONS

1. The Freezing Point of Water

The melting point, freezing point, or ice point is defined as an equilibrium condition between water and ice; the two existing together in perfect equilibrium when neither phase grows at the expense of the other. The temperature at this equilibrium point is called the freezing point. The terms "freezing point," "melting point," and "ice point" are used interchangeably. For water, the freezing point is at 0° C when under a pressure of 1 atmosphere. The definition given by Dorsey (1940, p. 603) is as follows: "By definition, the melting point of ice-I (ordinary ice) in contact with water saturated with air at a pressure of one normal atmosphere, but otherwise pure, the entire system being subjected to a uniform hydrostatic pressure of one normal atmosphere, is 0° C. This is often called the normal melting point of ice; also, the ice point."

Although the freezing point varies with pressure, the variation is much less than the variation of the boiling point with pressure. As an example, a pressure of 100 atmospheres lowers the freezing point of water 1° C but it raises the boiling point to 330° C. A quantitative idea of the variation of the freezing point with pressure is given in Table I (Dorsey, 1940, p. 603).

TABLE I

Variation of the Freezing Point with Pressure

Pressure P in atm	Freezing Point t °C
0.00603	+0.0099
1	0.0
590	-5.0
1090	-10.0
1540	-15.0
1910	-20.0

2. Vapor Pressure

A system of water and its vapor will be in equilibrium when the number of water molecules entering the gas phase is equal to the number entering the liquid phase. The pressure of the water vapor over the liquid water is a function of the temperature and the convexity of the liquid surface.

It is entirely independent of the volume of the system. A change in volume will merely cause a change in the relative volumes of the gas and liquid phase; i.e., some of the water will evaporate or some of the vapor will condense. Thus, for a given system, the vapor pressure is a function of temperature alone. For each temperature there exists a definite vapor pressure for the water-water vapor system. These equilibrium points for water and water vapor are plotted as curve OA in Figure 15.

At temperatures of 0° C and below, ice may also exist in equilibrium with the vapor phase. Changes between the solid and gas phase take place without passing through the liquid phase; i.e., evaporation and sublimation. Ice-I (ordinary ice) is the only ice that can exist in equilibrium with the vapor phase.

Below 0° C, the vapor pressure of ice is less than the vapor pressure of water at the same temperature. Both vapor pressures increase with a rise in temperature, but the vapor pressure of ice rises more rapidly than that of water, and at 0° C the two vapor pressures are equal and all three phases may co-exist in equilibrium.

It is of interest to note that the freezing point may be defined in terms of vapor pressure. The ice point of water is the temperature at which the ice and the water have the same vapor pressure and therefore remain indefinitely in equilibrium, without change in the relative amounts of the two phases.

Methods for calculating the vapor pressures of water and of ice at temperatures below 0° C are given by E. W. Washburn (1924). In this article there is a tabular presentation of the vapor pressures of ice from -90° C to 0° C, and of the vapor pressures of water from -16° C to 0° C. These tables were prepared for, and are presented in, the International Critical Tables, 3:210-213 (1929).

A quantitative idea of the difference in vapor pressure for water and ice at temperature of 0° C and below can be obtained from the following table prepared from values in the International Critical Tables, 3:211 (1929).

TABLE II

Vapor Pressures of Water and Ice Below 0° C

Temperature -° C	Pressure in mm of Hg		
	Water	Ice	Differences
0	4.579	4.579	0.0
2	3.956	3.880	0.076
5	3.163	3.013	0.150
8	2.514	2.326	0.188
10	2.149	1.950	0.189

A more recent paper on the vapor of ice was published by Goff (1942). He states that the values in the International Critical Tables have been accepted as sufficiently accurate, except possibly for applications to low temperature humidity. "Recently, however, important revisions in the fundamental data on which Washburn based his calculations have appeared, making it desirable to recalculate the vapor pressure and extend the range to lower temperatures."

These calculations are based on the Clausius-Clapeyron relation, as are the earlier calculations by Washburn. The method of calculation is described in Goff's paper and the vapor pressure of ice is arranged in a table. The values are given in °F and in. Hg. (The values in the International Tables are given in the metric system.) As an example of the magnitude of the new versus the older results, the value in the International Tables for -40° C is 3.8031×10^{-3}, while Goff's value for -40° C is 3.7929×10^{-3} in. Hg, a difference of about 1 percent.

3. Triple Point of Water

The triple point for water is the point at which the solid, liquid, and vapor are able to co-exist in equilibrium. (See Figure 15). This pressure-temperature diagram shows the solid-liquid-vapor equilibrium curves. The curve OA shows the variation with temperature of the vapor pressure of liquid water. The curve OB shows the variation of the vapor pressure of ice with temperature. These curves represent the equilibrium conditions for the water-vapor and ice-vapor systems respectively. These two curves meet at O and at this point the three phases, ice, water, and vapor, exist in equilibrium. This is the triple point.

The curve OC shows the equilibrium temperatures and pressures for ice and water. The variation of the freezing point with pressure has already been mentioned. Water and bismuth are the only substances known which expand on crystallization; their freezing point is lowered by an increase in pressure. The curves OC, OB, and OA must intersect at the same point, the only point where all three phases are in equilibrium with each other.

The temperature of the triple point is not exactly the same as the freezing point, because the freezing point, as defined, is measured under atmospheric pressure (760 mm) while the triple point is measured under the pressure of the vapor alone (about 4.6 mm). Thus the temperature at the triple point is +0.0075° C.

OA' is the curve for supercooled water, the difference between the vapor pressure of ice and supercooled water may readily be seen in the diagram.

The effect of dissolved substances in water is to lower the vapor pressure of the solution. The vapor pressure of the ice will not be affected. Upon lowering the temperature of a solution, ice, not a solid solution, crystallizes out, the vapor pressure curve of the solution will intercept the vapor pressure curve of the ice at lower temperature than the freezing point for pure water. Therefore, the freezing point of a solution of water and a dissolved substance will be lower than the freezing point of pure water. In Figure 15 the dashed curve starting at O' represents the vapor pressure of a water solution. O' is the new melting point.

For low concentrations of non-electrolytes, this depression of the freezing point is proportional to the molal concentration of the dissolved substance. The amount of depression can be calculated by Raoult's law when dealing with dilute, solutions of non-electrolytes. This amount of depression for water is 1.86° C (for 1 mole of dissolved substance). Raoult's law and freezing point measurements are commonly used in organic chemistry for determining molecular weights of substances.

Dorsey (1940, p. 605) presents a table of melting points for aqueous solutions of certain gases.

At very high pressures, over 2,000 atmospheres, the ordinary melting point gives way to a more complicated set of equilibrium points. There are several different kinds of ice--the high pressure forms of ice, and the ordinary ice (ice-I). All of the high pressure forms of ice have a density greater than that of water. Ice-I, of course, is less dense than water; it expands on freezing and floats. Bridgman (1912, 1935, 1937), and Tammann (1900, 1913, 1914) (references cited by Dorsey, 1940, p. 395) have done a great deal of work on the high pressure forms of ice.

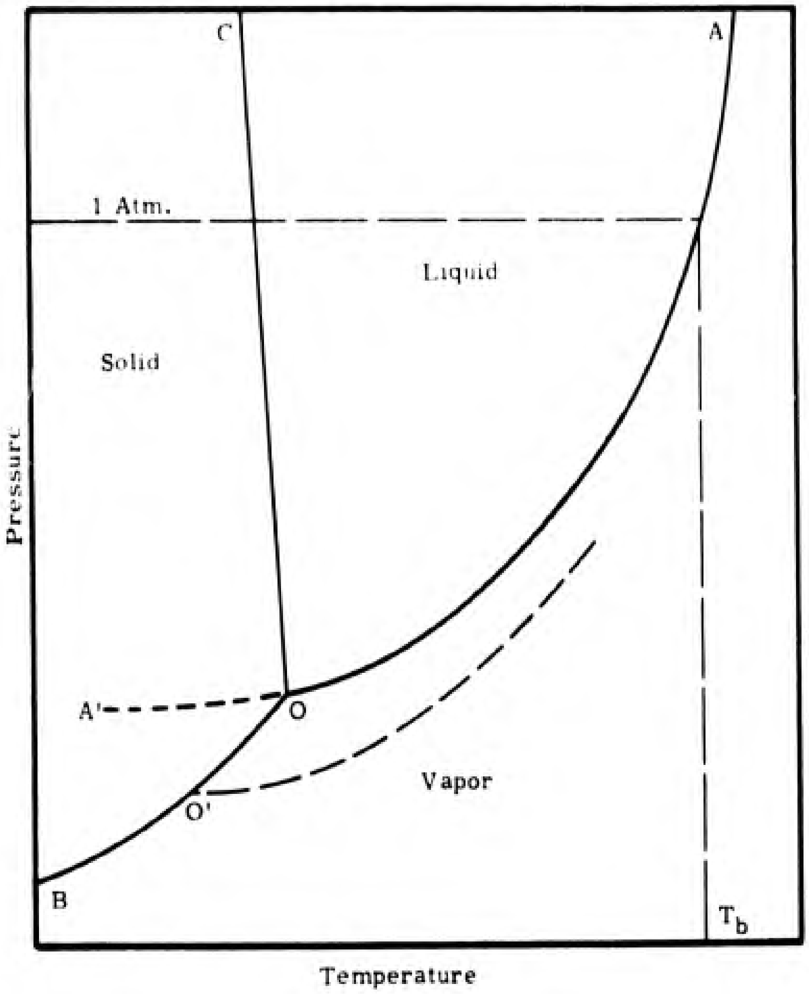

Figure 15. Phase diagram for water

4. Latent Heat of Fusion for Ice

On adding heat to a substance, we are adding to its internal energy. If we consider ice below 0° C and add heat at a constant rate, there will be a steady rise in the internal energy. This is measured by measuring the rise in temperature. The temperature will increase at a constant rate until we reach 0° C. At 0° C there is a great amount of heat added with no change in the temperature. However, there is a change in the internal energy--the heat energy that is being added goes into breaking down the crystalline form of the ice and converting it into water. This heat that is absorbed per unit mass during the change of phase is called the latent heat of fusion.

The accurate measurement of the latent heat of fusion for water has been undertaken by the National Bureau of Standards. Osborne (1939) has published a revision of earlier work performed by himself and others. The original references are cited in Osborne's paper. Methods are given for determining the heat of fusion and the changes in the older values are pointed out.

The revised figure for the latent heat of fusion of ice, obtained by weighing the results of four independent series of measurements and expressed in terms of the present calorimetric unit of energy, is

$$333.5 \text{ international joules gram}^{-1}$$

with an estimated uncertainty of 0.2 int. joules gram^{-1}. It is because of the transfer of latent heat that the ice point is easily reproduced.

5. Reproducibility of the Ice Point

At the freezing point the temperature remains constant even though there may be some heat transferred to or from the mixture. This heat energy is utilized in the change of phase process which is taking place. Thus, the freezing point of water is reproducible and it is possible to maintain a constant temperature for an extended length of time by the use of such equilibrium points.

The reproducibility of the ice point has been questioned by some investigators. White (1934) states that it is rather easy to get consistent results with a precision of 0.002° C, but it is considerably more difficult when accuracy of 0.0001° C is desired, an accuracy regarded as impossible by some workers.

White was able to obtain the ice point at atmospheric pressure for extended periods (at least one day) with an accuracy of 0.0001° C, using an ice bath made with commercial ice. The apparatus included a means for the ice to be continually washed with pure water (chilled to prevent any melting). This removed any impurities from the surface of the ice, and the accuracy was found to depend solely on the purity of the water.

Thomas (1934) also came to the conclusion that the freezing point is reproducible to 0.0001° C (using commercial ice) if corrections are applied for barometric pressure. Triple point measurements were also made. Thomas found the average difference between the triple point and the ice bath to be 0.0097° C.

White found the triple point measurements to involve more experimental difficulties. The impurities were not removed as in his ice bath work. The triple point is convenient and reliable for a short time, but the "ice cell" or "ice bath" method is better for continuous or extended work.

6. Regelation

When two pieces of ice are brought into contact at 0° C they will freeze together. This cementation will take place in air, in a vacuum, or in water. Very little pressure is required for the pieces to become firmly cemented. Faraday (in 1850) was the first to report this phenomenon for which the term "regelation" was applied by Tyndall.

The most generally accepted explanation of this phenomonon is the one proposed by the Thomson Bros. (Reference cited by Dorsey, 1940: Thomson, J., 1859, 1913; and W. Thomson, 1858.) They ascribed it to melting caused by stress and then subsequent freezing after the stress is relieved. There is sufficient stress to cause this melting whenever two pieces of ice are brought together. This may be in the form of capillary action when the pieces are in air or in a vacuum or due to the currents set up if the pieces are in water.

Blocks of ice floating in water may become cemented together in this way and form long chains. Also, tiny particles of ice will form clusters. The physical properties of ice that has been cemented together by regelation are described by Dorsey (1940, p. 413) as follows.

"The completeness with which two blocks of ice will freeze together increases with the pressure and its duration, and depends upon the relative orientations of the crystals of the two blocks. If the orientations are exactly the same in the two blocks, then they freeze together completely, the plane of union differing in no respect from any other parallel plane in either block. If the principal axes are parallel, but the subordinate axes of the crystals in one block are not parallel to those of the crystals in the other, the blocks freeze together so completely that the plane of separation cannot be detected by polariscopic observations, but can be by the production of Tyndall s flowers of ice, which show the difference in the orientations of the crystals in the two blocks. When tested by compression, the block yields first in that plane if the temperature of the room is over 0° C, but the initial yield bears no relation to that plane if the temperature is below 0° C. If the principal axes of the crystals of one block are perpendicular to those of the other, the union may again be invisible, but its strength is less than in the preceding case."

Weinberg (1936) has suggested that regelation plays an important role in the metamorphism of a snow cover. However, a pressure of several hundred atmospheres is required to produce melting at only a few degrees below zero. Therefore regelation could hardly be important in dry snow. Metamorphism of the snow cover at temperatures appreciably below 0° C must occur through the vapor phase by a more rapid sublimation at the sharper edges and points of the snow crystals and condensation on the flatter surfaces (Bader and Niggli, et al., 1939).

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Chapter IX

SUPERCOOLING

1. General Concepts

There is a difference in the freezing or melting point and the point at which freezing or crystallization begins. Although the freezing point of water is at 0° C, water does not freeze at 0° C.[1] This apparent paradox arises because the freezing point is a matter of definition. The freezing point of water is defined as an equilibrium condition between ice and water, but before we can have this equilibrium, both phases must be present. Consider an example. If we start with pure water at room temperature and cool it down to 0° C, we observe that it does not freeze. We continue to cool the water below 0° C; it is now called "supercooled water" because it is in the liquid state below its normal freezing point. When we reach a certain temperature, (which may be considerably below the freezing point), crystallization will begin. At the instant crystallization commences, there will be a rapid rise in temperature, to 0° C; any further freezing or melting will take place at 0° C. There will be no further change in temperature until one of the phases disappears.

The temperature at which crystallization begins has been called the "spontaneous freezing point" (t_{sf}) by Dorsey (1938). The difference between the spontaneous freezing point, or point of first crystallization, and the true freezing point (also melting point or ice point) should be clear, the latter is an equilibrium point between two phases while the other is not.

We have seen that the temperature at which the first crystallization or freezing occurs may vary; that is, water can be supercooled below its normal equilibrium freezing point. However, there is no evidence available that ice can be superheated, that is, raised above its melting point and remain solid. Therefore, the point at which ice melts is a constant, whereas the initial point of freezing for water is not. For this reason it is probably better to use the term "melting point" or "ice point" instead of "freezing point" when describing the equilibrium condition between water and ice.

2. Proposed Theories

An early theory for the behavior of supercooled solutions was proposed by Ostwald. Since then, numerous theories have been proposed which can be grouped under two general headings (according to Dorsey, 1948): the homogeneous and heterogeneous theories. Dorsey (1948) has extensively reviewed his own work on supercooling and the theories and experimental results of earlier investigations. The following discussion of the phenomenon of supercooling is taken largely from this source.

Repeated experimental evidence from many sources indicates that the freezing begins at certain discrete points within the water. These discrete points must represent singularities of one kind or another which exist just before the freezing starts. The number of these points is small, as Dorsey says (1948, p. 296), "almost vanishingly small, as compared with the number of molecules in the melt". The theories proposed to explain the freezing process differ in their assumptions regarding the nature of these singularities.

(a) Homogeneous Theory

In this type of theory the freezing is considered to be initiated at homogeneous singularities by certain aggregates of water molecules called "ice molecules". The simple molecule H_2O is believed to exist only in water vapor.

H. T. Barnes (1906), for example, considers water to be a solution of dihydrol $(H_2O)_2$ and trihydrol $(H_2O)_3$ molecules which become saturated with the trihydrol at the freezing point and the ice precipitates out of solution. Sutherland (1900) gives some account of the trihydrol molecules; Bernal and Fowler (1933) have also reported on the molecular structure of water and ice.

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[1] The story of the development of the Fahrenheit temperature scale is an interesting historical illustration of the difficulty of making water freeze at the "freezing point." Fahrenheit had originally wanted the freezing point of water as the zero point of the temperature scale but decided that this was not a constant. He used instead, a salt solution which had a readily reproducible freezing point.

These ice molecules are the homogeneous singularities in the melt and are believed to arise fortuitously and solely from the thermal agitation of the molecules. The ratio of these molecules over the ordinary water molecules would be expected to be small at high temperatures, but to increase as the temperature decreases. This ratio is assumed to have a fixed value for every temperature when equilibrium exists. One major deviation among the proponents of this theory is the estimate of the amount of time required for equilibrium to occur after a change in temperature. Some think that the equilibrium is established almost instantly while others claim that it requires a finite amount of time, perhaps hours or even days.

The ice molecules tend to cohere and grow with a finite velocity which depends on the temperature. At the freezing point the size of the molecular aggregates have grown to such a degree that they can annex the adjacent water molecules and convert them into ice molecules. Once this process has begun, equilibrium is rapidly established, and the process may go in either direction as heat is added to or taken from the system.

The homogeneous theory was proposed and developed by de Coppet (reference cited by Dorsey, 1948) in the period from 1872-1907, from considerations of the kinetic theory of matter and has enjoyed wide acceptance. It is the only theory of freezing that has been proposed, based on purely theoretical grounds, and until recently the only completely described theory.

(b) Heterogeneous Theory

The other types of theories may be broadly classed as the heterogeneous theory. According to this type of theory, the freezing process is initiated at heterogeneous singularities, i.e., foreign particles in the water. These particles may be on the container walls, or "motes" (Dorsey's term) freely floating within the water. These foreign particles serve as nuclei about which ice is built up until it forms a crystal and the freezing progresses outwardly from this point.

The action of the foreign particles in bringing about freezing at the heterogeneous singularities has not been established. The various ways in which the particles may be effective in producing freezing are listed by Dorsey as:

1. A purely mechanical action. It has been suggested that the foreign particles slow down the motion of the molecules sufficiently to enable them to orient themselves as required for crystallization.
2. The particle or spot acts as a base upon which the molecules of the melt are laid down in a crystalline array.
3. The occurrence of freezing depends on the action of the adsorbed layer covering the foreign particle or spot. Richards (1932) suggested a "crystalline adsorbate," which he has regarded as being composed of crystals of the melt.

Dorsey (1948) gives a rather complete summary of the various pre-existing ideas, and notes that although there are considerable areas of disagreement among proponents of the heterogeneous theory, the basic ideas have been employed for many years.

(c) Ostwald's Metastable Limit

Among early prominent hypotheses of the phenomenon of supercooling and its stability were those of Ostwald (reference cited by Young, 1910). He introduced the concept of the metastable limit, believing that when a liquid is cooled below its normal melting point, it will be in an unstable condition, the degree of unstability depending upon the supercooling. He suggested that the unstable field was composed of two parts:

1. The metastable field or state which exists at low degrees of supercooling--in this field the only way to cause crystallization is by the introduction of a crystalline part of the solid phase. (Without that "seeding" it cannot spontaneously crystallize no matter what shaking, stirring, jarring, etc., takes place.)
2. Upon further supercooling--the liquid will transfer abruptly into the labile state; in this condition it can be crystallized by mechanical disturbance. The point at which the abrupt transfer from the metastable to the labile fields occurs is known as the metastable limit.

Young (1910) has pointed out that there is no experimental evidence for dividing the supercooled range for water into two parts. In a series of experiments by Young and Van Sicklen (1913) the conclusion was drawn that there is no metastable limit for water unless it lies within 0.02° C of the true melting point.

(d) Dorsey's Theory of Freezing

After reviewing his own extended experimental work on the freezing of supercooled water and the results of others, Dorsey (1948) came to the conclusion that both the existing theories of freezing required modification. The homogeneous theory failed quite completely in its application to his work, while the heterogeneous theory served rather well in explaining most of the observations. There are certain points, however, that cannot be adequately explained by either theory. Dorsey's (1948) theory combines elements of both the homogeneous and heterogeneous theories and is the first well-defined theory involving heterogeneous singularities.

Dorsey has attributed much of the difficulty in interpreting previous work on supercooling to the use of inappropriate and confusing nomenclature for the descriptions of the various phenomena. He has therefore, introduced a number of new terms, very carefully defining their usage in each case. Dorsey's terminology has been employed in this portion of the report.

Singularities--a singularity is defined as follows: "Each of the singular points at which freezing is spontaneously initiated in a supercooled melt will be called a singularity." Two distinct kinds of singularities are recognized and are distinguished by adjectives, 1) homogeneous singularities, and 2) heterogeneous singularities.

"Embryos of crystals, or more briefly, embryos--an embryo of a crystal is any structural aggregation of the molecules of the melt that maintains its identity as an individual, distinct from the ambient melt, for an interval that is long as compared with the mean time between consecutive molecular collisions..." The embryo retains its identity, although it may continually exchange molecules with the melt.

The stability of an embryo depends on its size. An embryo of a given kind can be in equilibrium at a given temperature only if it is of a particular size. "That size will be called the critical size... Embryos larger than the critical size will mature into macroscopic crystals; those smaller than the critical size will decay...Hence, at any given temperature the initiation of a viable (one capable of growing) embryo requires the existence of an embryo of at least critical size."

Dorsey distinguishes between a simple embryo (one composed of molecules of the melt only) and a complex embryo (one with a foreign particle as a center and molecules of the melt adhering to it.) The initial structure of the very small embryo is not known, but it is though to undergo a progressive (not necessarily continuous) change toward the structure of a true macroscopic crystal as it grows.

Dorsey's theory does not demand that the singularities be of any special kind. The function of the heterogeneous singularities is to act only as a gathering spot for the molecules of the melt. These molecular layers which are adsorbed on the heterogeneous singularities can be torn loose by various mechanical effects. The disruption of the adsorbed layer provides a cluster of molecules of low velocity which will be able to orient themselves in crystal form. This is the basic idea of the new theory.

Because Dorsey has taken great care to state his theory precisely, his own statement is given here (1948, p. 310).

"It is generally agreed that the molecules of a liquid that are immediately adjacent to a foreign body are packed quite closely together, are bound quite strongly to the body, and have a preferred orientation with reference to the interface between the body and the liquid. In successively more distant layers, the packing becomes less close, the binding weaker, and the orientation less complete, until presently the status characteristic of the liquid in bulk and far from foreign bodies is attained.

"For simplicity, this entire group of layers,...is called the adsorbed layer of liquid.

"If the adsorbed layer can be so torn that a sufficient number of loosened molecules remain in one another's field of force for such a time and with such a degree of freedom that they can reorient themselves and become bound together in the manner characteristic of an embryo that can persist and grow at the existing temperature, they will do so.

"This new theory assumes (1) that the adsorbed layer can be so torn by various means, including the impact of free molecules of the melt, and (2) that both the spontaneous and the mechanically initiated freezing of a supercooled melt may be, and at least at the higher temperatures are, initiated solely in that manner.

"If the adsorbed layer can be so damaged by molecular impacts as to give rise to viable embryos, then the spontaneous freezing of a supercooled melt can obviously be initiated in that manner. And if the layer can be so damaged by molecular impact, it can surely be so damaged by other means, including gross mechanical ones."

Dorsey considers the following effects on the formation of a viable embryo:

1. The spread of the impinging molecule
2. The convexity of the interface

The outer layers, of the adsorbed molecules on the mote, are less strongly bound to the mote than the inner layers. It is assumed that, at a certain distance from the mote, these outer layers are capable of disruption by molecular impact.

(1) Effect of Speed of Impinging Molecule

If the speed of the striking molecule is very small, there will be very little breaking up of the adsorbed laye . On the other hand, if the speed is very great, there may be a great disruption of the adsorbed layer. However, as the speed and disruption increases, so does the scattering effect of the molecules. Consequently there will be one particular speed and direction of approach which will give rise to the largest possible embryo from the molecular bombardment of a particular point. This largest embryo will be called the <u>characteristic embryo</u> and the corresponding speed is the <u>characteristic speed</u>.

(2) Effect of Convexity of Interface

A molecule can escape from small drops easier than it can from large drops--also it should be easier to disrupt an adsorbed layer with a high convexity than one with a low convexity. For simplicity the motes which gather the molecules are regarded as spheres.

"It seems obvious that for small spheres both the characteristic speed and the size of the characteristic embryo will increase with the size of the sphere; and that for an infinitely large sphere, the characteristic speed will be very great, but the characteristic embryo will surely be small. That is, as the size of the sphere decreases from a very large value, the characteristic speed decreases monotonously, but the size of the characteristic embryo passes through a maximum." Thus, there is one intermediate size of sphere (mote) which will give the largest possible embryo. Motes larger or smaller than that particular size will give rise to a smaller embryo. That particular size of mote, which gives rise to the maximum size embryo, is called the <u>transition size</u>.

Motes that are larger than the transition size are called <u>major motes</u>. Motes that are smaller than the transition size are called <u>minor motes</u>. That maximum size embryo which arises from the transition size mote is called the <u>embryo proper</u>. The temperature at which the <u>embryo proper</u> is in equilibrium with the melt is the "proper temperature," t_p.

The temperature at which an embryo is in equilibrium with its melt depends solely on the size of the embryo. As the size of the embryo increases, the equilibrium temperature also increases. The mote that gives rise to the largest embryo will be the controlling mote. The melt cannot be cooled below the temperature at which the controlling mote can initiate freezing, nor can it freeze at a higher temperature.

3. <u>Applications of the Theories to Observation</u>

(a) <u>Frequency of Occurrence</u>

It has not generally been recognized that supercooling normally occurs whenever water freezes. Young (1910) states, "The phenomenon of supercooling of liquids is apparently of universal occurrence. It rarely, if ever, happens that a liquid cannot be cooled some distance below its melting point without solidifying."

Dorsey mentions that in all of his experiments he never found the spontaneous freezing point for water to be as high as 0° C.

Supercooling has been measured in all types of water bodies from laboratory specimens to lakes and rapidly flowing rivers. Such results are difficult to explain by the homogeneous theory but are readily accounted for by the heterogeneous theory and by Dorsey's theory.

(b) <u>Reproducibility of the Degree of Supercooling</u>

Dorsey performed experiments with water from various sources. The samples were contained in glass bulbs with a volume of about 8 cm³. Upon repeated freezing of a sample even over long periods of time, it was observed that the spontaneous freezing point remained constant. Table I illustrates the reproducibility of the spontaneous freezing point for several samples. It may also be noticed that

TABLE I

Reproducibility of the Observed Temperatures of Spontaneous Freezing
Dorsey (1948, p. 257)

C38 Brook		P10 Vac. Dist.		CIII Vac. Dist.		C12 Dist.		C5 Dist.	
Date	-t	Date	-t	Date	-t	Date	-t	Date	-t
1943	10.8	1943	16.1	1943	17.2	1936	6.8	1943	11.6
Apr. 9	10.3	Apr. 14	16.0	Apr. 12	17.1	Dec. 22	6.3	Jl. 6	12.0
	10.5		16.2		17.4		6.0		12.0
	10.1		16.1		17.7		6.1		12.0
	10.3		12.9[a]		17.0		6.6		12.1
	10.6		16.8		17.6		6.5	Jl. 9	12.0
	10.7		16.9		17.8		6.8		12.1
	10.4	Apr. 15	16.2		17.2		6.9	Jl. 16	12.2
	10.5		16.6		17.6		6.3		12.1
	10.8		16.5	Apr. 13	17.8		6.9		12.2
	10.6		16.2		17.7		6.8		
			16.2		17.6				
			16.0		17.6	1937	7.0		
			16.2		17.6	Jan. 5	6.7		
			16.0		17.3		6.4		
					17.4				
					17.2				

C23 Snow		C40 Pool		P25 Dist.		P2 Conductivity		C10 Conductivity	
Date	-t	Date	-t	Date	-t	Date	-t	Date	-t
1937	5.8	1937	6.0	1943	9.7	1937	13.4	1937	16.0
Feb. 17	6.0	Mar. 4	6.0	Jl. 5	9.8	Jan. 19	13.3	Feb. 25	16.0
	6.0	Mar. 6	6.9[b]		9.8	Jan. 28	13.8		16.0
	6.0		6.0	Jl. 6	9.6		13.6	Mar. 10	16.0
	7.5[b]	Mar. 8	6.0		10.0		13.5		15.9
	6.0		6.0		9.5	Feb. 24	13.4	Apr. 14	16.0
Feb. 24	6.1	Mar. 22	5.6	Jl. 9	9.6		13.0[c]		16.0
	6.1		6.0		9.9		13.1		
Mar. 9	6.0	Apr. 23	6.0		9.7	Mar. 10	13.2		
	5.9		6.0	Jl. 16	9.9		13.2		
	6.0				10.1				
					9.9				

(The water was observed to freeze spontaneously when the temperature of the bath was t° C. No value of t observed during any of these intervals has been omitted from the table.)

Figures such as C38, P10, etc., refer to specific samples. The notations below the numbers indicate the source of the water.

[a] P10. Cannot account for this high reading.

[b] C23, C40. The temperature of the bath when the bulb was placed in it was below the t_{sf} of the bulb.

[c] P2. The water froze within 2-1/2 minutes after the bulb was placed in the -13.0° C bath.

different samples freeze at quite different temperatures. It was also observed in Dorsey's work that nominally identical specimens had different spontaneous freezing points. Even if samples were drawn from the same source they would often freeze at different temperatures. Dorsey's theory, or the heterogeneous theory, explains this phenomenon as follows: the difference is due to the presence of a more effective mote in one sample than in the other. Work performed by Smith-Johannsen (1948) also shows a very remarkable reproducibility of the spontaneous freezing point for a given sample.

These observations can find no explanation in the pure homogeneous theory. However, the heterogeneous theories including Dorsey's account for this behavior quite readily.

(c) The Effect of Purity on the Degree of Supercooling

Barnes (1906) has stated that water may be supercooled to -6° C with caution. However, below this temperature, the instability becomes so great that extreme precautions must be taken to obtain further cooling. Many recent text books claim that only a few degrees of supercooling are possible and that quiescence is necessary. However, there have been published accounts, as long ago as 1837 (Despretz, 1837, 1839; reference cited by Dorsey, 1948) describing experiments in which water was cooled to -20° C. Also, in 1879 Pictet found that a stoppered flask half full of water at -19° C could be violently shaken withou causing the water to freeze (Oltramare, 1879; reference cited by Dorsey).

Rau (1944; reference cited by Frank, 1946) claimed to have supercooled water to -72° C by a series of repeated freezings. Frank (1946) has attempted to interpret Rau's results on the basis of molecular structure. Recently, however, an attempt by Cwilong (1947) to reproduce these results failed.

The most recent work on supercooling is that of Smith-Johannsen (1948) and that of Dorsey (1948). Smith-Johannsen employed a very elaborate experimental technique in which the observation of the first crystallization was facilitated by the use of polaroid lenses. He consistently obtained, for very pure water, a spontaneous freezing point in the vicinity of -20° C. He went so far as to say, "The normal temperature at which water freezes is, therefore, in the absence of any known foreign nucleating materials, very close to -20° C, and not as is commonly believed at 0° C."

Most of Dorsey's results for the spontaneous freezing point fell within the range of -13 to -18° C, some were as high as -5° or -6° C, and some were as low as -22° C. Dorsey's samples varied from pure to polluted water.

The explanation of Barnes's results may be that his sample was exposed to the open atmosphere and the possibility of foreign motes or other nucleation was great. According to Dorsey this nucleation would be much more effective in initiating the freezing than agitation.

Devik (1942) has reported supercooling in open waters which amounted to a few hundredths, tenths, or even 1° C.

Quiescence apparently is not a necessary requirement for supercooling. We quote Dorsey (1940, p. 639): "In no case was any precaution taken to keep the water quiescent; in fact, the supercooled water can be poured with impunity over the entire interior of the bulb. Violent splashing will cause the supercooled water to freeze, but sharp rapping of the exterior of the bulb is without effect. On the other hand, a very gentle wiping of the glass water interface will cause freezing at a temperature well above that at which the specimen freezes spontaneously." These observations will receive further consideration below.

(d) The Stability of Supercooled Water

Supercooled water has been regarded as a very unstable state. We shall now consider the various actions which have been supposed important for the initiation of freezing. An attempt will be made to demonstrate the stability of supercooled water under various conditions.

(1) Rate and duration of cooling:

In Dorsey's experiments the water was protected from the atmosphere by sealing it in glass bulbs. The samples were placed in cooling baths of alcohol--the temperature of the cooling bath could be varied at will. It could be held fairly constant for extended periods or it could be changed at a controlled rate. It was observed that the spontaneous freezing point of a sample was independent of the rate and duration of cooling.

The rate of cooling of the bath--and the sample within--was varied from periods of hours to periods of less than ten minutes, with no effect on the temperature at which the sample froze. The point at which the sample froze was found to depend on the temperature reached and not the manner of reaching it.

Richards (1932) carried out some interesting experiments on the relationship of the duration of cooling and crystallization. He used two organic materials which had freezing points of 42° and 48° C. These were heated to 130° for several hours and then placed in a cooling bath at 25° C. They were kept at 25° C for several months and none of the samples crystallized.

Dorsey (1948, p. 262-263) investigated the effect of prolonged chilling of water. He made many experimental trials--the most striking example was that of keeping water at temperatures of between -8.0° and -10.3° for a period of 312 days without any freezing. These results are perhaps the most damaging evidence presented in opposition to the homogeneous theory.

In connection with the prolonged chilling of water there is the question of what Dorsey calls the "rate of nucleation." According to the homogeneous type of theory there should be a certain number of homogeneous singularities forming at every temperature. Below the freezing point these homogeneous singularities should form viable embryos and act as starting points for crystallization. Tammann (1897) attempted to measure this rate of nucleation--his results however have experimental difficulties which invalidate them, (Mikhnevick and Browko, 1938; references cited by Dorsey, 1948).

Richards[2] attempted to measure the "crystallization frequency" (f). A large number of tubes containing samples of the substances investigated were prepared and set in a freezing bath. The total number of tubes is N_0. If after a time, "t," there remains a number of tubes "N_t" which are unfrozen, they are related to the total number of tubes by the following relation, according to Richards:

$$N_t = N_0 e^{-ft}$$

However, as has been stated above, none of the tubes crystallized even after several months. Richards concluded that "f" for these substances was very small. Actually it was zero.

The conclusions drawn from the above experiments are that the freezing of a sample of water is independent of: (1) the rate of cooling and (2) the length of time it is kept below the freezing point. It is dependent on the temperature that is reached by the sample and not the method of reaching it.

(2) Effect of time:

It has been shown earlier that the spontaneous freezing point is reproducible upon repeated freezings over an extended period of time. However, sometimes there is a shift of the spontaneous freezing point as time goes on. The shifting of the spontaneous freezing point may be according to one of two types.

It may be an irregular almost chaotic appearing shift in the spontaneous freezing point. However, it is noticed that there are certain temperatures which are repeated time and again although other values may occur in between these repeated ones. These repeated temperatures are called preferred temperatures by Dorsey and they are illustrated by the curve P 10 in Figure 16. The dots are temperatures at which the sample froze spontaneously. The dots are arranged from left to right in chronological order of the determinations. The dashed connecting line indicates an interruption of experiments such as pre-heating, opening bulb, prolonged cooling, or over a month between trials. The solid connecting lines indicate that the experiments were carried out continuously.

A greater number of preferred temperatures was observed in samples that were relatively free of motes, (pure water).

Only Dorsey's theory has an explanation for these preferred temperatures which he attributes to the passage of major motes thru the transition size. It is still difficult to understand why these oscillations would not converge since presumably the change in size of the motes would be in only one direction. He is also unable to explain all points in his curves as is indicated by question marks in curve P 10 (Figure 16).

The shift may be a continuous falling of the spontaneous freezing point. This is called secular variation by Dorsey and is illustrated in Figure 16 by curve C48. This steady drop in the t_{sf} can be explained on the basis of heterogeneous singularities which tend to dissolve as time goes on.

(3) Effect of preheating:

It has been recognized for some time that preheating of a melt to a high temperature will facilitate greater supercooling. Dorsey found that if the t_{sf} of a sample was at or below a preferred temperature in the -11° C to -15° C range, the preheating had little effect on it. However, if the t_{sf} was higher than that temperature range, it would be reduced by preheating.

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[2] Loc. cit.

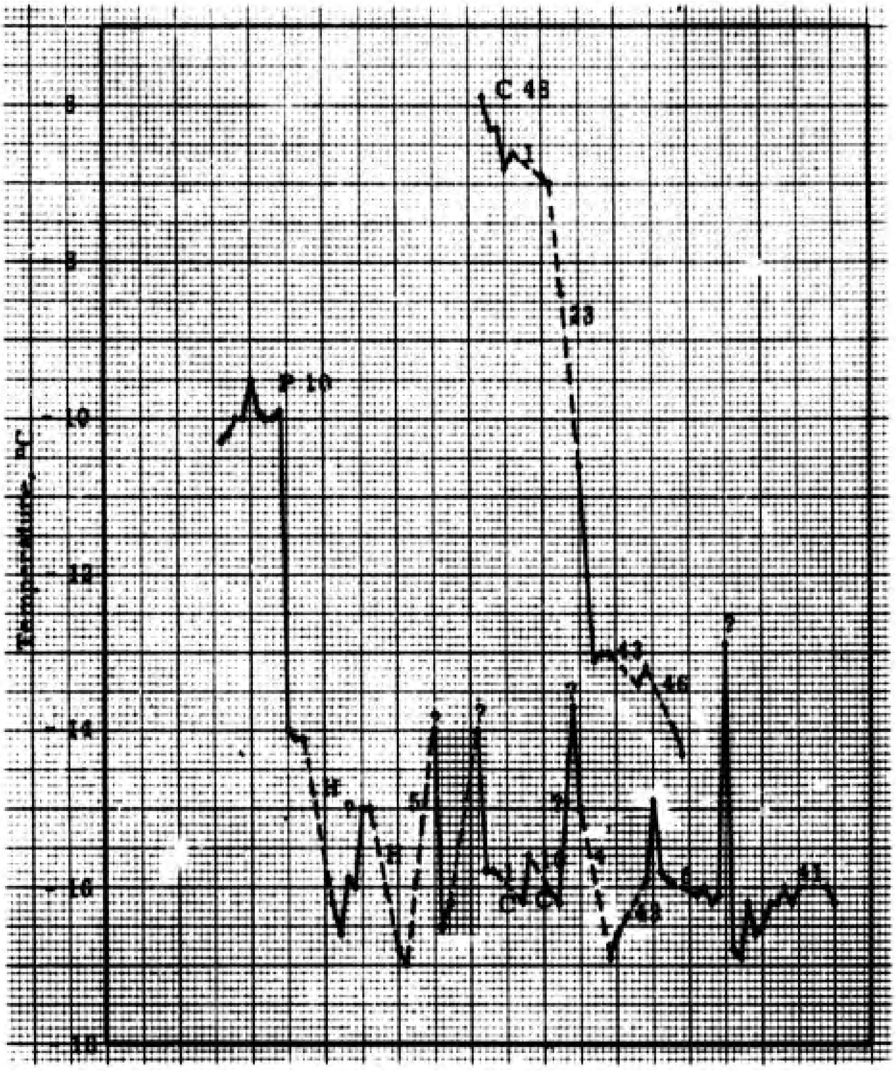

Figure 16. Secular variation and preferred temperature of spontaneous freezing point of water (Dorsey, 1948)

This phenomenon has been explained as due to the dissolving of motes within the water. It is comparable to a speeding up of the monotonic fall in t_{sf} (secular variation) sometimes observed as the specimen ages.

(4) Effect of filtration:

Richards, Kirkpatrick and Hutz (1936), and many others cited by Dorsey (1948), have observed that filtration of water will increase the degree of supercooling attained. This is also true of centrifuging of water. These results are obviously due to the removal of foreign particles.

(5) Effect of volume:

It has frequently been suggested that a small volume is necessary for supercooling. To a certain extent the volume has an effect. Very small droplets of water may be supercooled to a lower temperature with consistent results than can large samples. Heverly (1949) found that small droplets of water could be supercooled into the -35° to -30° C range easily. He found that the smaller droplets are capable of higher supercooling than larger ones.

There is a transition temperature below which water droplets in clouds will transform spontaneously into ice crystals. This temperature has been measured by Schaefer (1948); his latest figure is -39° C ± 0.1° C.

Dorsey worked with volumes of about 8 cm^3 which were supercooled consistently without any difficulty. Dorsey found that the volume of samples had no effect on the freezing point. On comparing two samples of different volumes either the larger or the smaller, (whichever had the most effective mote) froze at the higher temperature.

Large bodies of water such as lakes and rivers are also known to become supercooled before the freezing process takes place (Devik, 1942, 1944, 1949; Altberg, 1936). Thus, large volumes do not prohibit supercooling.

(6) Effect of Mechanical Disturbances:

Pouring: Dorsey found that pouring had no effect on bringing about freezing. A supercooled sample of water could be "poured with impunity" over the entire inner surface of the bulb without freezing. On the other hand a slight pouring--merely a movement of the meniscus would cause freezing. The water must have made contact with some foreign particle or even an ice crystal that could initiate the freezing.

Splashing: Violent splashing would cause the supercooled water to freeze. If the water were greatly supercooled less violent splashing was required than for slightly supercooled samples.

Supersonic Waves: Supersonic waves seemed to have the same effect as splashing on the initiation of freezing (Richards, 1932). If a water sample is exposed to supersonic irradiation before it is cooled the effect is to increase the degree of supercooling (Smith-Johannsen, 1948). This, in all probability is very similar to the effect of strong preheating.

Impact: A great amount of work on the effectiveness of kinetic impact on the freezing of supercooled water has been carried out by Young and his co-workers (Young, 1910; Young and Cross, 1911; Young and Van Sicklen, 1913). They showed that there is no reason to divide the supercooled range of water into two parts as suggested by Ostwald. Young's experiments were carried out with the impact parts under water. He and Van Sicklen devised a hammer and anvil apparatus by which they could compare the amount of impact necessary to produce freezing with the degree of supercooling. They found a large impact necessary to cause freezing in slightly supercooled water, and a slight impact necessary to freeze highly supercooled water. They do not say how many trials they had to make at each temperature before the freezing occurred.

Dorsey has pointed out that other factors may have entered to cause the freezing. Since the impact parts were in the water there could have been: (1) Chipping of the parts; (2) Cavitation caused by the withdrawal of the hammer; (3) Movement of the meniscus (which may bring the water into contact with a crystal or an effective mote).

From Dorsey's work (Dorsey, 1948, p. 288), "It was found that a much supercooled bulb of water could be struck violently with a similarly cooled rod while immersed in the chilling bath without causing the water to freeze. The same result was obtained with water cooled in an open test tube, and a bulb containing water that froze spontaneously at -16.3° C could be rapped sharply at -16.0° C without the water's freezing". Other experiments gave similar evidence that pure impact had little if any effect.

Rubbing: Although impact has little effect, a very slight rubbing of the glass-water interface will produce instant freezing. This was also reported by Young but he considered the rubbing friction as a series of small impacts. On the other hand, Dorsey considers the impact as having very little significance and the rubbing to be the controlling factor. Rubbing, in any event, is observed to be very effective. Walton and Judd (1914) utilized rubbing of the glass tube with a wire to initiate crystallization in their determinations of the velocity of crystallization.

Again these phenomena can only be explained by Dorsey's theory. The splashing and rubbing, and also the supersonic irradiation, are of such a nature that they can tear the adsorbed molecular layer on the motes. The negative results obtained with pouring and pure impact are to be expected in this theory since they have no tearing influence on the adsorbed layers.

(e) Initiation of Freezing by Seeding

It is a well known fact that freezing may be initiated in a supercooled mass of water by the introduction of a piece of solid ice. Altberg investigated the effect of other materials as "seeds" for initiating freezing (Altberg, 1938; Altberg and Lavrov, 1939 and 1940). Quartz sand, which was not cooled, had a very striking effect when thrown into supercooled water. The crystallization was almost instantaneous and gave the impression of an "ice explosion". Altberg also investigated the effect of compressed snow "seeds" thrown into very slightly supercooled water (-0.4° to -0.7° C). They produced ice nuclei which are called "primary nuclei" because they, in turn produce "secondary nuclei" and so on until the whole mass is at 0° C. Schaefer (1948 and 1950), Heverley (1949), Cwilong (1947), Vonnegut (1947) and Smith-Johannsen (1948), have reported on the production of ice crystals from clouds of supercooled water droplets. There are two ways to cause ice crystals to form in these clouds: (1) by cooling some portion of the cloud to a temperature below -39° C; (2) by the introduction of foreign substances which will initiate crystallization (called "sublimation nuclei").

An efficient way of accomplishing cooling to -39° C is to drop solid CO_2 (dry ice) into the cloud. One of the most effective substances for initiating crystallization by the second method was found to be silver iodide (Vonnegut, 1947).

Graphite was found to be a much more effective nucleus for crystallization in water than silver iodide. However, it does not act as a "sublimation nucleus" while silver iodide is a very good sublimation nucleus. Thus the conclusion is drawn that there is a difference between "sublimation nuclei" and "freezing nuclei" and that there is a different mechanism in each case (Smith-Johannsen, 1948).

(f) Velocity of Crystallization

The measurement of the velocity of crystallization is accomplished by observing the time it takes for crystallization progress along a horizontal tube filled with supercooled water. It has been observed that the velocity of crystallization increases steadily with the degree of supercooling.

The earliest measurements of the velocity of crystallization were made by Turmlirz in 1894. He worked in a range of supercooling which varied from -0.74° C to -4.60° C. He was unable to measure the velocity at lower temperature because spontaneous freezing set in.

More recently Walton and Judd (1914) have measured the velocity of crystallization for water. They were able to work at lower temperature than Turmlirz could obtain. However, they did not observe a maximum velocity of crystallization. The lowest temperature at which they made measurements was -9.0° C, and their results suggest the maximum velocity may be at about -15.0° C. Spontaneous crystallization interfered with the results below this temperature.

They found that the thickness of the tube used in these determinations is an important factor in determining the speed of crystallization. This fact should be helpful in future work. Spontaneous freezing occurred much more frequently in thin-walled tubes. This is attributed to the more rapid and uneven cooling of the liquid. Table II shows the dependency of the velocity of crystallization of the degree of supercooling and the thickness of the tube walls.

TABLE II

Velocity of Crystallization of Supercooled Water
Dorsey (1940; p. 645)

Walton and Judd (1914)

d = 7 mm; w = 2.5 mm		d = 11 mm; w = 1.5 mm		d = 3.5 mm; w = 3.25 mm	
-t° C	v	-t° C	v	-t° C	v
2.00	5.27				
3.61	8.07				
4.67	11.9				
5.86	17.8				
6.18	19.1	6.17	39.2	6.10	24.0
7.10	44.4			6.60	25.2
7.50	51.3	7.65	63.0	7.58	32.2
8.19	69.2				
8.38	85.5			8.58	39.1
9.07	114.0	9.22	91.6		

Turmlirz (1894)		Hartmann (1914)		Tammann and Büchner (1935)	
d = 18 mm; no data for calculating w		d = 2-3 mm; no data for calculating w		d = 1.2 mm; w = 0.8 mm	
-t° C	v	-t° C	v	-t° C	t
0.74	0.37	0.5	2.3		
1.12	1.44	0.8	3.3		
1.40	2.20	1.0	4.0		
1.54	2.76	1.5	6.0		
1.62	2.92	1.9	8.0		
2.00	3.32	1.9	8.5		
2.40	4.49	2.0	9.7		
2.54	5.24	2.0	9.7		
2.67	5.58	3.5	20.0	3.2	11.1
2.71	5.77	3.5	20.3	4.2	18.2
2.90	7.06	5.0	29.2	4.2	23.8
3.20	7.47	5.0	20.2	6.2	30.5
3.49	10.23	7.0	46.2	7.2	41.3
3.64	11.28	7.0	46.7	8.3	52.0
4.14	16.93			9.3	55.0
4.20	18.15			10.3	61.2
4.60	22.07			11.3	70.3
				12.3	84.0
				13.4	96.8

v_{mm} sec^{-1} = linear velocity at which freezing progresses along a glass tube
t° C = temperature of the water and the cooling bath which contains the tube at the time of freezing
d = internal diameter of glass tube in mm
w = wall thickness of tube in mm

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Chapter X

ICE FORMATION IN OPEN WATER

We shall be concerned here with the process that takes place in the initiation of freezing in open waters. It is possible to consider this natural ice formation under two subheadings: (1) Ice formation in lakes, ponds, and other quiet waters; (2) Ice formation in rapidly flowing, turbulent water.

1. Ice Formation in Quiet Waters

Olaf Devik (1942), measured the surface temperature of water, during freezing by a radiation thermocouple. It was observed that the surface layers are supercooled whenever ice forms. There must be a temperature gradient, from the ice to the water, in order to carry away the latent heat of fusion. Thus, as ice advances across a pond, or any other body of water, there will be a slightly supercooled layer of water preceding it.

Devik emphasizes that the first crystallization will take place at distinct starting places (the initiating "motes"). In open water, such as a pond or lake, there are many various sized particles suspended in the water, and infinite source material on the shore line. These suspended particles, together with rocks and other beach material and nucleation from the atmosphere, are very efficient in starting the crystallization. Thus, the degree of supercooling in open waters will never be as great as that for laboratory specimens which are contained in glass bulbs, and protected from the atmosphere. However, this amount of supercooling is measurable. Devik measured supercooling which varied from a few hundredths of a degree Centrigrade to as low as -1.2° C.

After the ice phase has appeared, it will grow from the distinct starting periods in various forms, crystal beams, leaves, feathers, stars, etc. These branching forms leave open spaces of water between the ice structures. These open spaces are supercooled; the degree of supercooling is approximately proportional to the area of the open space. As the ice progressively grows and the water spaces disappear, the supercooling decreases to zero.

The type of observations obtained by Devik is illustrated by Figure 17, which gives the reading of the galvanometer for the thermopile during the freezing process. This is a case where water just above the freezing point is exposed to the atmosphere at temperatures considerably below freezing. The temperature (galvanometer reading) falls rapidly and levels off at a broad minimum below zero. Then, as the ice formation begins the temperature rises, and when it approaches zero a thin ice sheet has formed over the water.

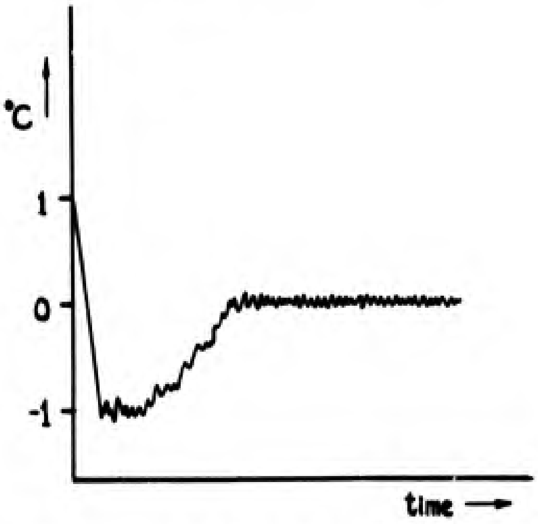

Figure 17. Water surface temperature (Photographic registering curve) (Devik, 1942)

2. Ice Formation in Rapidly-Flowing, Turbulent Water

The freezing of rapidly-flowing, turbulent rivers presents quite a different problem from that of freezing in still waters of slowly flowing streams. A phenomenon occurring in rapidly running water is the formation of under-water ice. This formation of underwater ice has been a curiosity with little or no attention until recently. With the use of water power in hydro-electric plants it was found that underwater forms of ice do indeed exist and can be very troublesome.

The terms frazil ice and anchor ice have been applied and widely used in discussing underwater formations of ice (Timonoff, 1936).

Frazil ice forms in supercooled running water, the degree of supercooling may be very slight. The structure of frazil ice has been described as small, disc-like plates of ice by Barnes (1906) and this observation has been confirmed by Altberg (1936). The frazil ice particles are very small, fine crystals and discs of ice which are distributed throughout a turbulent flowing stream. The frazil particles also cluster together, by adhesion and regelation, to form curtain-like, spongy masses which move along under water with the current.

Anchor ice forms on the bottom of the rivers and on objects under water. Anchor ice is firmly attached to the bottom and grows outward.

The theory proposed by Barnes (1906) on the formation of anchor ice has since been challenged by Altberg (1923, 1936). Barnes's theory called for radiation from the bottom of river beds at night which would cool off the river bed to the point where anchor ice would form, and grow continuously from the bottom outward.

Altberg carried out experiments in which he produced anchor ice under laboratory conditions which completely eliminated any possibility of radiation from the bottom. Barnes finally was persuaded that his point of view had to be revised (Altberg, 1936, p. 377).

Altberg does not make any distinction between frazil and anchor ice, but classes them together as underwater ice.[1] This underwater ice will form when the water temperature falls below 0° C.

It was found that, due to turbulence, the water temperature remains fairly constant throughout the river from top to bottom. The more turbulent the water is, the closer is the temperature at any point to the mean temperature (Altberg, 1923).

Measurements showed that the supercooling exists in these streams for long periods--hours and even days. Also, the water can remain in a supercooled condition even in the presence of the solid phase. It is this supercooling and the rapid transfer of heat that causes the formation of underwater ice.

There is an infinite number of starting points for ice in turbulent rivers. Motes suspended in the water, together with the turbulent splashing and striking on rocks, will easily suffice to initiate viable embryos, (according to Dorsey, 1948). The anchor ice forming on the bottom will grow at the sharper points faster than at the smooth points. The irregularities will be accentuated because there will be more heat conducted away from a sharp irregularity than from a flat or rounded area.

Thus, we see that the supercooling phenomenon in rapid flowing rivers is of lasting importance. In quiet waters it plays an important but transient role.

The continually moving supercooled water is an excellent medium for carrying away the latent heat of crystallization for anchor ice. Also, there is an extension of growth by accumulation and adhesion (due to regelation) of fine frazil particles of ice always found in the supercooled water. Experiments on the "seeding" of supercooled water with small "seeds" of compressed snow were carried out by Altberg and Lavrov (1940). They employed no mechanical disturbance but introduction of these "seeds" into slightly supercooled water (-0.4° C to -0.7° C) had interesting results. A primary seed yields nuclei of the "first order," these in turn produce "second order nuclei" which produce "third order nuclei," etc.

"On increasing the extent of supercooling of water such reproduction of nuclei acquires an explosive character and, on turbulent agitation of water, as in the case of swift river currents, the whole stream may become filled with germinal ice particles to such an extent as to make a dense gruel-like mixture of liquid phase and ice particles." This explanation of frazil ice is dependent upon the turbulent mixing found to exist in swift currents.

The tendency for the irregularities to be accentuated often causes anchor ice to dam up the stream. There are ice dams formed when the turbulence and angularities are great, as at the base of rapids. Sometimes the river beds are partly filled up with this underwater ice and there will be a flooding of the banks (Stalke, 1936).

This may build up a stair case made up of a series of ice dams with a little still water trapped behind them. This still water quickly forms a surface ice sheet over itself. (Devik 1948, p. 364).

It has been mentioned that the particles of frazil ice appear as small, flat, rounded discs. There have been excellent photos taken of these discs and other modifications of the early crystal forms of ice in water, (Altberg, 1938; Altberg and Lavrov, 1939; Schaefer, 1950).

There has been some speculation on the buoyancy of frazil ice particles. Since even when the stream flow is not extremely turbulent, frazil particles remain below the surface. As water becomes more "even flowing," more and more of them are found floating on top. Barnes (1917) explained this

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[1] Altberg's work was started as an investigation of ice formation on water intake pipes in 1914 when the whole bottom of the River Neva was found to be covered by a thick (one meter) crust of porous anchor ice. This left the whole city of Leningrad without water. Many measurements were made on the temperatures of river waters in the Neva and also several rivers in eastern and north-central Russia. (Angara, Amur, Enisey, etc.)

in terms of the viscosity and a consideration of Stokes Law. There is no reason to believe that any ice formed under atmospheric pressure and normal conditions is denser than liquid water. The viscous drag on an object is a surface effect, whereas the gravitational force is a volume effect. When the diameter is decreased, the gravitational force (depending on volume) will decrease faster than the viscous force (depending on surface).

Frazil and anchor ice present a serious problem to hydroelectric plants. An accumulation of this underwater ice on the grates and working parts can put a large plant out of operation in a short time. Some measures must be taken to prevent the formation of this underwater ice before it can do the damage.

Several ideas are presented by Devik (1942, 1944) on methods of preventing underwater ice formation in water-power installations. In many Scandinavian plants there has been employed an artificial heating of the grids which are placed in front of the inlet to the turbines. This often works if the grid is kept just slightly warmer than the water. Sometimes this is not sufficient. Since the formation of frazil and anchor ice depends on both turbulence and supercooling, attempts are made to cut down on these two prerequisites.

The entry to the inlet gates and turbines should be made as large and as streamlined as possible to prevent turbulence. It also helps to place logs, chained together, across the channel. This cuts down on the circulation and it helps form a surface ice sheet which may start growing upstream. Apparently the insulating effect is sufficient to prevent underwater ice formation under a good surface ice sheet.

In some cases it may be desirable to prevent water from freezing over a certain area, such as sorting bins in saw mills. In the winter of 1940 experiments were carried out in Finland with the object of keeping minor areas of water from being covered with ice. This was done by utilizing the thermal reserve of the lower layers of water by pumping compressed air below the surface. When rising to the surface, these bubbles carry up warm water which melts the ice.

The water in the lower layers need not be very warm, a few °C will be very efficient for melting purposes. There is quite a thermal gradient in the water of quiet ponds. At a depth of 17 meters the temperature may rise from 0° C to as much as 4° C. The original paper has a very clear account of the experimental work, a summary of which is in the abstract published with this report (Kaitera, 1948).

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APPENDIX

Abstracts of the Literature Prepared in Conjunction With the Compilation of Data

Abbot, C. G. and L. B. Aldrich, 1916:
The pyranometer--an instrument for measuring sky radiation
Smithsonian Miscellaneous Collections, 66(7), May

Discussed the design and testing of an instrument (pyranometer) for measuring solar and sky radiation by day and terrestial radiation by night. Two forms of the instrument are described.

Among the preliminary measurements was one determining the albedo of new fallen snow. They found this value to be 0.70.

Adams, John M., 1930:
The polar properties of single crystals of ice
Proceedings of the Royal Society, (London), A128:588

Many small ice crystals have been grown and photographed. Adams concludes that ice crystals are polar, but show a strong tendency toward twinning in the (0001) plane.

(Author's Abstract) Photomicrographs of monocrystalline specimens of ice show some individuals which, though having the simple external form of a short right hexagonal prism terminated by basal planes, give evidence of polar structure. This evidence consists in the appearance of a pit at one end only of the C axis, when the crystal is brought into an atmosphere favorable to disintegration. Two other types of disintegration are observed in crystals of the same external form, one type characterized by a pit at each end of the C axis, the other by a cavity at the middle of the C axis. These two types are explained as due to the two possible modes of twinning of a polar crystal on the basal plane. In view of the conclusions of Barnes, the polarity of the ice crystal is ascribed to the asymetric location of the hydrogen ions in the lattice with respect to the basal plane.

Altberg, W. J., 1923:
On the cause of the formation of ice at the bottom of rivers and lakes
Quarterly Journal of the Royal Met. Society, 49(205):54-60.

The work was started in an investigation of ice formation on water intake pipes and on the bottom of the River Neva. Measurements were made of the temperature in the river. Due to turbulence the temperature remains fairly even throughout the river from top to bottom. The more turbulent the water is, the closer is the temperature at any point to the mean temperature.

The measurements show that supercooling exists in open waters for long periods--hours and even days.

The water remained in a supercooled condition even in the presence of the solid phase.

Water invariably must be slightly supercooled before freezing, irrespective of whether it is in contact with ice.

During the freezing of natural water bodies, a maximum degree of supercooling is observed. Altberg divides the supercooling period in two at this maximum. Some crystallization occurs in the first period; in the second period, the rate of crystallization had increased so that a warming is observed due to the addition of latent heat. The degree of supercooling decreases to negligible amount during the second half of the supercooling period.

It is also shown that water can be supercooled while being cooled rapidly and stirred.

Barnes's theory of anchor ice formation by radiation from the bottom is shown to be incorrect.

Altberg advances the theory that anchor ice formations are started by frazil ice accumulation and adhesion (due to regelation) on solid objects. The heat liberated during the formation and growth of the anchor ice is carried away by the moving supercooled water.

Altberg, W. J., 1936:
20 years of work in the domain of underwater ice formation (1915-1935)
International Assn. of Scientific Hydrology, Bulletin No. 23, pp. 373-407.

This paper contains a review of the author's work for the last 20 years. It is the first real work on ice formation in rapidly flowing rivers, also the first quantitative reliable work on natural outdoor ice formation. A very good discussion of frazil ice and anchor ice is given. Altberg has conducted experiments, and gives theoretical considerations, (on the opaqueness of water to heat radiation) which completely disprove Barnes's theory of anchor ice formation by radiation from the bottom of a stream bed.

The fundamental identity of anchor and frazil ice is established; they are grouped together as underwater ice.

Four fundamental problems are stated; (1) the problem of the causes of ice formation at the bottom of rivers; (2) to determine the nature of ice elements suspended in water; (3) to mark the methods of quantitative determination of the phenomenon and to put them into practice; (4) to work out a means of forecasting this phenomenon. A fifth test was added later; (5) to work out the principles and effective means of ice protection, as well as those of control.

There are many very good and some very unusual photographs in the text. Barnes's work on "ice discs" as the first forms of underwater ice is supported with several photos.

The problems stated are all discussed and examples are given of their practical applications. There is a good summary at the end of the paper.

Altberg, W. J., 1938:
Crystallization nuclei in water
Acta Physicochimica, URSS., 8:677-678.

A brief report on some of the author's observations. The fully developed crystallization nuclei of water constitute regular disc of transparent ice a few millimeters in diameter. There is a photograph of the discs. The author states that according as water is supercooled, both the number of nuclei and their rate of growth is increased. The studies were made under conditions that reduced the rate of growth to hundredths and even thousandths of the usual values quoted in the literature. He does not describe the conditions.

The discs develop into regular hexahedral plates and then into sixrayed stars, similar to snow flakes. There are drawings and photos that illustrate the shapes.

Altberg, W. J. and W. Lavrov, 1939:
Experiments on the Crystallization of Water. II
Acta. Phys. Chim. 11(2):287-290.

These investigations refute the idea of the impossibility of nucleation inside water. Experiments were made with slightly supercooled water (from -1° C. to -4° C.). By striking the bottom of the vessel containing the supercooled water crystallization was started.

Other methods of stimulating crystallization were discovered: rubbing the inner walls of the container with a glass rod, and throwing non-cooled sand into the water.

"Quartz sand thrown into water at -4° C. and below produced nearly instantaneous crystallization, without any stirring;" it gives the impression of an ice explosion.

There are five photographs in the text showing crystallization, progressing from a starting point caused by different mechanical processes. Also a photo of ice stars grown in supercooled water.

Previous work by Young and Van Sicklen (Jour. Am. Chem. Soc. 35:1067, 1913) has shown that crystallization can be caused by mechanical means. The authors of this report are investigating the different characteristics of crystalliation caused by various methods.

In general the type of crystallization which is caused by different mechanical methods varies and the different types are discussed and illustrated.

Altberg, W. J. and W. Lavrov, 1940:
Experiments on the crystallization of water. III.
Acta Physicochimica, 13:725-729.

The paper describes experiments on the crystallization of water with "seeding." There is no mechanical disturbance but "seeds" of compressed snow are introduced in water that is slightly supercooled

(-0.4° to -0.7° C). A primary seed yields nuclei of the first order, these in turn produce secondary nuclei which produce third order nuclei, etc.

"On increasing the extent of supercooling of water such reproduction of nuclei acquires an explosive character and, on turbulent agitation of water, as is the case in swift river currents, the whole stream may become filled with germinal ice particles to such an extent as to make a dense gruel-like mixture of liquid phase and ice particles." This is an explanation of the formation of frazil ice and shows how it can choke up turbine gratings in hydroelectric plants in a short time.

The authors quote Weinberg as stating that hoar frost particles may be a starting point for crystallization. Weinberg tries to show that Young and Von Sicklen's work on impact crystallization may have been due to hoar frost germation; however, hoar frost does not form over the entire temperature range of Young and Von Sicklen's experimental work.

Angstrom, Anders, 1919:
On the radiation and temperature of snow and the convection of the air at its surface
Arkiv für Matematik Astronomi ock Fysik de l'Academie royale de Suede, 13(21):1-18.

Discussed heat exchange between snow cover and atmosphere. Data used was taken during the arctic night when effect of solar radiation and variation in ground temperature is small.

The snow surface was assumed to radiate like a black body. The effective radiation (R) which is the difference between the outgoing temperature radiation of the snow surface and the incoming temperature radiation from the atmosphere was measured as the net radiation against the sky and space. Various air temperatures on and above the snow surface were measured.

The effective radiation was found to be a function of the air temperature above the snow in the form

$$R \cong 0.03\,\Delta t$$

where Δt is difference in temperature between snow surface and air 0.6 meters above surface in °C. R is in cal/cm^2 per min. Since the above equation represents heat equilibrium of the surface and other factors are small, the amount of heat transferring from the air to the surface by convection is $0.03\,\Delta t$.

Determination of cloudiness from radiation measurements is discussed.

Angstrom, Anders, 1925:
The albedo of various surfaces of ground
Geografiska Annaler, 7:323-342.

Used a pyranometer enclosed in glass so that long wave lengths ($\lambda > 4\mu$) radiated by the ground would not enter into readings. This also eliminates effects due to solar radiation of long wave length. Since this radiation (greater than 4μ) is a very small percentage of the total solar radiation, the elimination of it was thought to have little effect on final results.

The apparatus was calibrated against a pyreheliometer and the non-selectivity of the pyranometer to different wave lengths was checked.

Measured albedo of dry ground, wet ground, ground covered by various kinds of vegetation, and smooth and rough water surfaces. Made some observations of a biological character including radiation transmission of various leaves and radiation in the shade of trees.

Found the albedo of an old snow surface (several days old and very white) to be 0.695 and the albedo of a fresh snow surface (exceptionally white and smooth) to be 0.81.

Arnold-Alabieff, V. I., 1938:
The external friction of ice
Assn. International Scientific Hydrology, Bull. 23, pp. 563-570.

A very simple and seemingly inaccurate method was used to measure the coefficient of friction of ice on smooth steel and painted steel. Several measurements made on (Neva) fresh water ice, (Baltic Sea) de-salted ice, and polar salt-water ice. The measured frictional stress shows large variations. The article deals mainly with the problems of ice-breaking ships.

Atkins, W. R. C., 1947:
Electrical conductivity of river, rain, and snow water
Nature, 159:674

The conductivity of two water samples melted from snow was compared with that for river water and rain water. The range of values follows:

Source	Conductivity $(\text{ohm-cm})^{-1}$ at 0^{0} C.	Remarks
tap water	$25\text{-}28 \times 10^{-6}$	Plymouth: 1923-4, 1945
river water, Ceylon	$10\text{-}20 \times 10^{-6}$	3.1-3.5 ppm. of silica also present
rain (two samples)	$24{,}28 \times 10^{-6}$	"clean snow"
snow (Downderry)	44×10^{-6}	"Taken from middle of snow blanket...
snow (Plymouth)	72×10^{-6}	surprisingly sooty..."

Averbukh, R. E. and M. S. Kosman, 1949:
O Poliarizatsii l'Da (The Polarization of Ice)
Zhurnal Eksperimental 'noi i Theoreticheskoi Fiziki (Journal of Experimental and Theoretical Physics--translation prepared by Stefannson Library), 19(11):971-972.

Averbukh and Kosman measured the dielectric constant of ice using a ballistic method. They impressed a single D. C. pulse across the plates of an ice condenser. The pulses used were of different time lengths and voltages. Field strengths greater than one K V per cm. caused a space charge formation with a time of relaxation of 10^{-1} sec. at -9^{0} C. This space charge causes an increase in the effective, formerly non-directional, dielectric constant to about three times its former value with the current in the forward direction and to about 30 times with the current in the reverse direction. This directional effect becomes less pronounced at lower temperatures.

Barnes, H. T., 1917:
The bouyancy of frazil crystals in water
Trans. Royal Society of Canada, 11(3rd series):61.

The author shows that there is no need to assume that the density of frazil ice is greater than water. There is no evidence to show that any ice formed under atmospheric conditions is denser than liquid water.

The viscous drag on an object is a surface effect whereas the gravitational force is a volume effect. When the diameter is decreased, the gravitational force (depending on volume) will decrease faster than the surface effect (due to viscosity).

There is a sample calculation and the basic relations between Stokes law and gravitational forces are presented.

Barnes, H. T., 1925:
Colloidal water and ice
Colloid Symposium Monograph III, pp. 103-111

X-ray determinations of the structure of ice give inconsistent data. They do not conform with theory nor with other measurements. This is due to the difference in the manner of formation of ice. Since water is colloidal there will be some change in structure under different conditions of temperature and manner of formation.

Barnes describes the different types of fogs and mists and the relation between them and the types of molecular groups which act as a nuclei for their formation.

The gradual increase in the size of the molecular groups gives all grades from the fine haze in the atmosphere to the white fogs and the gradual alteration to the thick yellow and black fogs.

There is a discussion of the color of ice and water, frazil ice, anchor ice, mist and fog and some of the effects of these phenomena.

Barnes, H. T., 1926:
Colloid forms of water and ice
Colloid Chem. (J. Alexander Edition), 1:435-444

The formation of fog and mist is generally believed to be a result of water vapor molecules precipitating onto basic nuclei, i.e., particles which form a center for the condensation. Dust particles do not seem to account entirely for the copius and often repeated nucleation in a foggy atmosphere, especially in the winter. It is probably that water molecules coalesce to form colloidal size particles which can act as a nuclei.

Water is a highly associated liquid. "Hydrol" is the basic single water molecule H_2O; it is the form of a vapor particle. Dihydrol is the water molecule and Trihydrol is the ice molecule.

Ice molecules (Trihydrol) are dissolved in water--the concentration of dissolved ice molecules increases with a temperature decrease. At the freezing point the solution is saturated and precipitation occurs.

The beginning of the true colloidal form of water is from the dihydrol which merges into trihydrol at temperatures below 60° C. to the final ice molecule. "It is devoid of crystal form" and remains invisible until just before the freezing point, as the proportion of trihydrol becomes large compared to the number of hydrol molecules, it affects the rays of light reflected from the interior depths of the water, producing a slight change of hue. One who is experienced in observing can tell when the water is at the freezing point by the slight color change.

Iceberg ice--has no crystal structure or cleavage, it is pressure ice, formed by compaction of snow called névé--contains more dissolved air than crystalline ice--has a lower s.g. Glacier ice has a vivid blue tone. Glacier streams and lakes are always the bluest and are deepest in shade when they are the coldest.

Lake and river ice--the slowness of formation causes large crystals--when formed rapidly from slightly supercooled water the crystals become fine and colloidal.

Frazil ice or needle ice--a flocculation phenomenon of a colloidal solution. It occurs in open, flowing water when at the freezing point. Fine needle-like crystals and disc-like particles. It may form as long streamers, spongy masses of curtain-like clusters that float along below the surface of the water; it can catch on objects and build up to considerable size.

As long as the temperature stays above 0° C no frazil can form. The smallest drop below 0° C can cause its formation. In industry, power plants, etc., frazil ice can be eliminated by maintaining the temperature of the water just above 0° C.

Anchor ice--forms on the bottom of rivers and streams. It is of loose structure formed of large crystalline plates and is easily distinguished from frazil ice or snow blown into the water. It forms at night--with bright sun it rises and floats away in the daytime.

Mist and fog--water does not lose its trihydrol until it is close to the boiling point, therefore, all evaporation at ordinary temperatures releases a reasonable amount of ice molecules into the atmosphere. During cold weather these trihydrol molecules aggregate and precipitate as hoar frost.

Snow crystals and ice molecules exist in the upper atmosphere. Sundogs and moondogs are related to ice particles.

Halos are produced by colloidal particles of ice in the atmosphere. White fogs over the sea--colloidal water particles. White fleecy cloud-like forms in the winter with ice particles. Hoar frost with heavy percentage of ice crystals. Irritating to the lungs. "White death."

Barnes describes several types of fogs including the heavy black fogs seen in very cold weather on the St. Lawrence river.

There are some photos by Bentley of crystalline forms.

Ice itself presents colloidal properties at or near its freezing point. This is the main reason for the difficulty in X-ray work on ice structure. It varies with the manner and temperature of formation.

Bauer, Edmond and Daniel Massignon, 1946:
Theory of the crystalline field in solid and liquid polar dielectrics
Faraday Society: A General Discussion on Dielectrics (Transactions), 42A:12-15

The "crystalline field" idea is introduced, which is a modification of the "internal field" concept. The energy of a molecular dipole is considered to be a function of the polar angle θ, but the axial field is considered symmetric. In general, the energy $\underline{W}$ of the dipole will be a minimum for various values of θ, and there will be a potential barrier between two successive minima. The vibrations of the dipole within a small angle about a minimum are considered to be those which contribute to infrared and Ramon spectra; vibrations between equilibrium positions have to do with dielectric dispersion.

For ice, a calculation of the Ramon frequency (experimentally observed at about 600 cm^{-1}) from dielectric constants gives 360 cm^{-1}, which is only qualitative. A calculation based on relaxation time gives 450-500 cm^{-1}. These calculations are partly based on a modified Onsager theory.

A difficulty is mentioned in calculating the "reaction field."

Bell, A. E., 1948:
Theory of skating
Nature, 161:391-392

The amount of ice displaced by the runner of an ice skate cannot be explained by frictional melting or pressure melting. A standard skate tilted at 20° running on ice at 0° C. is examined. It was found that .03 gm. of ice were displaced per length of 4.28 inches. The heat required to melt this ice is 2.4 cal., but from the equation

$$Q = \frac{M_g \mu k}{J} L$$

the heat obtained would be 0.363 cal.

The theory is proposed that the large displacement is caused by the plastic flow of solid ice.

Bell, George G., 1911:
Results of experiments on strength of ice
Maine Society of Civil Engineers, 1:41-46

The previously reported values for the tensile and crushing strength of ice are discussed. Tensile strength tests were made on samples of ice that were frozen in cement molds and also on samples cut from ice blocks. The average tensile strength was found to be 120 p.s.i. The results obtained for the crushing strength of ice were 590 p.s.i. parallel to the crystal axis and 654 p.s.i. perpendicular to the crystal axis.

In the adhesion tests it was found that the ice frozen to the cement blocks would develop the full crushing strength, irrespective of the line of ice thrust to the surface of the concrete, as long as the temperature of the ice is below the freezing point.

Assuming 400 p.s.i. as the average crushing strength of ice, the greatest thrust per linear foot that an ice sheet can exert is 2400 t where t is the thickness in inches. The author assumes that the greatest thrust occurs when the ice is about one to one and a half feet thick. The writer therefore believes that a dam exposed to the full pressure of ice should be designed to resist from 43,000 to 48,000 lb. per linear foot.

Bentley, W. A., 1923:
Studies of snow, ice and frost crystals
National Geographical Magazine, 43:103-112

This is a popularized article which contains some general information on crystals and 7 very good plates.

Snow crystals which grow rapidly tend to assume branching forms; slower growth of crystals tends to give solid forms.

Dew is one of the most common forms of water formation, though jointly with hoar frost, it is one of the least in regards to quantity. Both dew and hoar frost are nighttime phenomena because only then is the air quiet enough for their gathering on objects.

Snow formation occurs usually, if not invariably, within all storms and in all climes; its formation is confined to upper parts of storm clouds in torrid zones or in the summer in temperate zones. In summer it precipitates as snow only on high mountains.

Bentley, W. A. and W. J. Humphreys, 1931:
Snow Crystals (McGraw-Hill Book Co.)

Large collection of photographs of snow flakes with a method of classification; has strongly influenced Nakaya's classification and is somewhat similar to the recent proposed international classification. Humphreys has added a discussion of the variation of snow flake form as a function of the meteorological conditions. The simpler needle forms are associated with low temperatures and humidities while the more elaborate fluffy types with higher temperatures and humidities.

Bergman, P. G., 1943:
The formation of centers of condensation in supercooled phases
Phys. Review, 63(2):465.

This article is an abstract of a paper presented to the American Phys. Soc. in New York, April 2, 1943. In the investigation of the spontaneous formation of a more stable phase in the interior of a super-saturated or supercooled phase, it is usually assumed that a count of the centers of condensation which become visible per unit time furnishes us directly with the rate at which new centers are being formed. This assumption is objectionable, because new centers will not necessarily continue to grow, but may redissolve before they become visible. In fact, it is well known that for every degree of supercooling, there exists a "critical" size below which the droplet is more likely to dissolve than to grow.

To overcome this difficulty, the author has considered the change to which an arbitrary size distribution of droplets or crystals is subject in the course of time. If the number of droplets containing between μ and $\mu + d\mu$ moles at the time T is Z $(\mu\ t)$ $d\mu$ then Z satisfies the equation

$$\frac{\partial Z}{\partial t} = \frac{1}{N^2} \frac{\partial}{\partial \mu}\left(v(\mu)\left[\frac{\partial Z}{\partial \mu} + \frac{Z}{kT}\frac{d f(\mu)}{d\mu}\right]\right)$$

where v (μ) is the rate at which droplets of size μ annex or drop one molecule, and f (μ) is the free energy per droplet. It is proposed to discuss this equation with the help of Heaviside calculus.

The idea of a "critical size" in freezing embryo is used as one of the main assumptions in Dorsey's theory of freezing.

Born, Max, 1946:
Elastic constants of ice
Nature, 158:830

A hypothesis that diffuse X-ray bands in ice, and a broad Raman band (3000-3600 $cm.^{-1}$) can be explained by "finite jumps" of hydrogen atoms from the neighborhood of one oxygen atom to another, is proposed. Elastic constants of ice are given. (See AHA Penny, Proc. Cambridge Phil. Soc., 44(267):423.

Bowden, F. P., 1944:
The physics of rubbing surfaces
Royal Society of New South Wales, Journal and Proceedings, 77-78:187-219.

The real area of contact of rubbing surfaces is much less than the apparent area of contact, even for highly polished surfaces. The real area of contact is essentially independent of the degree of polishing as can be seen from the formulae $\mathcal{A}$ = const. $W^{2/3}$ for elastic deformation and $\mathcal{A}$ = const. $W^{1/2}$ for plastic deformation, where $\mathcal{A}$ is area of contact and W is the load. Experimental determinations of the area of contact of two steel cylinders, as measured by the electrical conductance method varied from 0.0013 cm^2 for a 1 Kg load to 0.042 cm^2 for a 500 Kg load. The experimental determinations of the variation of area of contact with increase in load shows the deformation to be mainly plastic for most metals.

Experimental determinations of the surface temperature during frictional sliding show the temperature will rise to a sufficient value to melt the material with the lower melting point. After the melting point temperature has been reached there will be a flow of the melted materials. This explains the fact that in a polishing procedure the polishing material must have a higher melting point than the material being polished.

Frictional melting is advanced as the theory of sliding on ice and snow. This theory is shown to be more consistent with experimental results than the pressure melting theory.

The surface damage of sliding metals is explained by the fact that the points of contact of sliding metals are momentarily welded together and then torn apart again. The action consists of a harder metal gouging the softer metal or the softer rubbing off onto the surface of the harder.

The force necessary to slide one metal over another is a combination of two components: a ploughing of one metal through the other, and a shearing of the surfaces. The ploughing force is high for soft metals, but the shearing force is low; whereas, for hard metals the converse is true. Low ploughing force as well as low shearing force can be obtained by coating the surface of a hard metal with a film of the soft metal, but various difficulties arise, such as breakdown and wear of the films. Lead seems to be the best material to use for a surface film, although a copper lead alloy may sometimes be used to achieve the same results. Surface films of any type will almost always reduce the friction of rubbing materials.

Bowden, F. P. and T. P. Hughes, 1939:
The mechanism of sliding on ice and snow
Proceedings of the Royal Society of London, Series A (172):280-297.

A turntable was constructed with a sliding arm in such a fashion that both the material on the table and that on the contact arm could be changed. The apparatus could be evacuated and the load and speed could be varied.

The friction of ice on ice was studied and it was found that the coefficient of friction decreased for an increase in load and that the coefficient increased for a decrease in temperature. The coefficient of sliding friction was found to increase at very low speeds, but was independent of the speed at higher speeds.

For friction of other materials on ice and the above relationships were true and it was further found that the coefficient of friction was greater for good thermal conductors than for poor ones.

On the basis of the experimental results obtained the theory was proposed that the lubrication provided on ice and snow is caused by a layer of water melted by the friction, rather than by pressure considerations. Several results are pointed out which are explained much better by the frictional melting theory than by the pressure melting theory.

Bowden, F. P. and T. P. Hughes, 1939:
The friction of clean metals and the influence of adsorbed gases. The temperature coefficient of friction.
Proceedings of the Royal Society of London, Series A, 173:263-279.

The coefficient of friction of outgassed metals is much higher than for metals cleaned in air. Metals cleaned in air have transient adsorbed oxide layers that reduce the coefficient of friction. For example, μ for nickle on tungsten cleaned in air was found to be 0.3, for the same combination after outgassing $\mu = 6.0$. Upon admission of a small quantity of oxygen the coefficient was decreased, but if hydrogen or nitrogen were admitted no change was observed. The temperature influence on most of the outgassed metals was, an increase in temperature caused a decrease in the coefficient of friction.

Bowden, F. P. and L. Leben, 1938:
The nature of sliding and the analysis of friction
Proceedings of the Royal Society of London, Series A, 169:371-391.

An apparatus was set up for measuring the coefficient of friction of metal on metal. The metals were thoroughly cleaned in air, and the coefficient of friction of various metals were determined.

The friction of metal on metal was found to be of a slip and grab nature rather than a smooth sliding action. The rubbing metal would gouge, or rub off on, the other metal depending on which was the harder. This action was explained by small areas of contact and local heat at these areas great enough to cause welding action. The measurement of the temperature showed large momentary increases at the instant of slipping.

Three types of sliding are defined: high melting metal sliding on low melting metal (motion is in short jerks); low melting metal sliding on high melting metal (motion in long jerks); surfaces of the same metal (friction is much higher and has large fluctuations).

Boyle, R. W. and D. C. Sproule, 1931:
Velocity of longitudinal vibrations in solid rods with special reference to the elasticity of ice
Canadian Journal of Research, 5:601.

The article reports an experimental research to determine the effect of radius to length ratio on the propagation of longitudinal waves. It was found that for small values of kr/L (k = 1, 2, 3 -- depending on type of vibration) Rayleigh's formula gave very accurate results, but when kr/L exceeds a certain figure, depending on the material, the formula is no longer reliable.

The research was extended to determine Young's modulus (E) for ice. A slight temperature dependency was noted, E varying from 9.29×10^{10} dynes/cm^2 at -9^0 C. to 10.9×10^{10} dynes/cm^2 at -35^0 C. Inability to control temperature cited as a handicap because a time lag in E behind the temperature was observed; the lag appeared to be independent of the size of rod. Results indicate that crystal orientation has very little effect on Young's modulus: rods cut with their long axis parallel and perp. to the optic axis gave practically the same results.

Bradish, Brain and McFarlane, 1947:
Vacuum sublimation of ice in bulk
Nature, 159:28-29.

Construction of a freeze drier for biological fluids is described. The drying process proceeds in two steps: (a) "recession from the product surface of the horizontal plane of separation between upper partially dry product and lower frozen product;" (b) a much slower evaporation of the residual ice (water) from the partially dry solid.

An average effective condensation coefficient (accomodation coefficient) is calculated; this has little direct relationship with the true coefficient for an ice surface.

Brown, E. and G. C. Clarke, 1932:
Ice thrust in connection with hydro-electric plant design
Engineering Journal, Jan.:18-25.

The investigation described was intended to ascertain the pressure exerted by ice in expanding as its temperature rises, the information being desired for the purpose of estimating the allowance to be made for the possible pressure exerted by an ice sheet on the crest of a dam.

A cube of ice, held in a testing machine, was allowed to rise gradually in temperature. The thrust, due to the combined effect of the expansion and flow of ice, exerted against the loading blocks was measured.

The temperature was controlled by varying an amount of dry ice near the specimen and the value of the temperature was obtained from four thermometers close to the test piece. The ice was free to expand sideways.

A final curve of rate of temperature rise vs. rate of pressure rise was obtained from the data taken. They apparently assumed that this curve was independent of the initial temperature and pressure although other information indicates that the curve should depend on these two quantities. An attempt to do the experiment when imposing lateral restraint on the specimen failed.

Bucher, Edwin, 1948:
Beitrag zu den theoretischen Grundlagen des Lawinenverbaus, (Kummerly and Frey: Bern)
English translation by the Army Map Service, 137 pp.

Chapter I: (1) Distinction between various snow types; (2) The relationship of snows to elastic and plastic substances. The concept of relaxation is explained and the relaxation time is estimated; (3) Plastic properties of snow. The plasticity and viscosity of snow are discussed and determined theoretically; (4) The property of solidity of snow. The experimental values for the tensile strength or cohesion of snow are reported and explained.

Chapter II: Statics and dynamics of snow cover. (1) Development of snow cover. The development of the snow cover as to cohesion properties is explained. Examples of snow profiles for several winters are examined for possible avalanche conditions; (2) General relationship between the distribution of stress and creep. The stresses within the snow body are analysed and the theoretical conditions for snow creep or avalanche are determined; (3) Gliding on the base. The stress and shear zones are studied and the resulting creep explained; (4) Avalanche formation. The types of failure resulting in avalanches are explained; and the types of avalanches that occur are given.

Chapter III: The avalanche defense structure. (1) General measures; The simple defense methods are described; (2) Chief requirements for defense construction; (3) Action of various types of construction. All of the various types of defense structures are described, (4) Evaluation of the snow pressure acting on skeleton construction.

Bucher, E. and A. Roch, 1946:
Reibungs- und Packungswiderstände bei raschen Schneebewegungen
Mitteilungen aus den eidg. Institut für Schnee- und Lawinenforschung. (unpublished).

Since snow is a loose aggregate the force necessary to pull a ski or runner over the snow consists of sliding frictional resistance and resistance of compaction, neglecting the resistance of the point.

A mold of 10 dm^2 was packed with snow and then pulled over a 10 meter path of snow. The frictional force was measured with springs and was measured as a function of load and velocity. After several runs on the same surface the friction due to compaction becomes negligible. The coefficient of friction of snow on snow was found to be a function of snow type, water content, temperature, normal load, and velocity.

Other materials were measured on the same compact course of wet snow. The smallest coefficient was found to be snow on glass at 0.012, the other materials in order were aluminum, iron, varnished wood and unvarnished wood, the coefficient for unvarnished wood being 0.185.

The coefficient of packing resistance is derived mathematically in the second half of the article.

Cabrera, B., 1941:
Diamagnetism and molecular structure
Journal de Chimie Physique, 38:1.

In a short historical sketch the contributions to diamagnetic theory of Wever, Langevin, and Pascal are mentioned. Pascal's additivity law is

$$\chi_M = \sum a \chi_A + \lambda$$

where χ_M is molecular susceptibility, χ_A is the susceptibility of a constituent atom, a is the number of such atoms in the molecule, and λ is a constant correction for a given type of molecule. Cabrera prefers to keep the strict additivity law $\chi_M = \sum a \chi_A$, changing the value of χ_A as occasion may demand.

The temperature dependence of χ for water and ice is discussed in terms of different "structures" for water. Thus the "% of tridymite type of structure" is calculated above and below the melting point of ice. (Cf. Bernal and Fowler, J. Chem. Phys., 1:515 [1934].)

The values of χ for C, H, CH_3 and OH groups are discussed, with applications to normal alcohols and other organic compounds. It is emphasized that liquids must always be compared to liquids, gases to gases, etc.

Coast Guard, U.S.A., 1947:
Radar detection of floating ice
U.S. Coast Guard, Bulletin No. 32, International Ice Observation...in the North Altantic...", pp. 178-188.

Experiments were carried out on the detection of floating ice by shipborne and by air-borne radar. A general description of the conditions of uses is followed by a semiquantitative discussion of results.

For ship-borne radar, a 10 cm wave length set gives a better echo and more certainty of detection, though a 3 cm set has a larger range. Neither set is entirely satisfactory, as choppy seas cause much interference and echoes are weaker from icebergs than from other objects. The minimum size of berg detection depends on the sea roughness. The sets are better at finding leads in field ice.

Air-borne radar is more satisfactory, though skill in operation is demanded as well as experience in interpretation. It is not generally possible to distinguish ice from other floating objects such as boats. The edge of pack ice can usually be found even if in quite choppy seas.

Conant, F. S., J. L. Dum, and C. M. Cox, 1949:
Frictional properties of tread-type compounds on ice
Industrial and Engineering Chemistry, 41:120-26.

(Author's Abstract) Apparatus and techniques for the measurement of coefficients of dynamic and static friction of tread-type stocks on ice are described. Both coefficients are shown to increase with increased age of the ice or with a decrease in temperature. An increase in vertical loading pressure decreased the dynamic value more than it did the static. In general, little relationship was found between coefficients of an ice and hardness, stiffness, or hysteresis. The coefficients are shown, however, to be sensitive to type and amount of softener or black. In such cases the softer stock usually gave higher values. Hevea, acid-emulsion butadiene styrene, polyisoprene, and GR-S-Hevea blends gave higher coefficients of friction than did any of the other polymers studied. In some cases ice pickup is shown to decrease the coefficient of dynamic friction. Correlation with road test results shows the laboratory test to be valid. It is also shown that the coefficients of both dynamic and static friction must be high to insure superior tire performance on ice.

Cooperative Snow Investigations, 1946:
Heat transmission constants of snow
Technical Report No. 3, Processing and Analysis Unit

The thermal conductivity k, was calculated by means of the equation

$$k = \frac{\pi c \rho x^2}{T\left(\log_e \frac{2\theta_0}{R}\right)} \qquad (1)$$

where:
x = depth between surfaces at which temperature is observed.
T = period = 86,400 sec.
$2\theta_0$ = temperature change at upper surface
R = temperature change at the lower surface
ρ = density

Thermohms were installed in the snow pack. To eliminate the effect of solar radiation, a canvas shade was constructed over the test area.

Using Poisson's temperature equation with assumed sinusoidal variation of heat transfer, equation (1) results.

Curve fitting of the data gave:

$$k = .00183\rho \quad \text{(assuming } k = 0\text{, when } \rho = 0\text{).}$$

Cwilong, B. M., 1947:
Observations of the incidence of supercooled water in expansion chambers and on cooled solid surfaces
Journal of Glaciology, 1(2):54.

Condensation of water on various cooled surfaces was studied, both in the presence and absence of air. Effects of various surfaces were studied as a function also of surface temperature and dew point of the air. An attempt was made to repeat the experiments of Rau (see F. C. Frank, Nature, 157:267 [1946],) which only served to discredit Rau's work.

Deeley, R. M., 1908:
The viscosity of ice
Proceedings of the Royal Society of London, pp. 250-9.

Article is concerned with two items, namely: (1) determining viscosity of glacier ice and (2) interpretation of McConnel's (McConnel, J. C., On the plasticity of an ice crystal, Proceedings of the Royal Society of London, 1891, p. 323) data and calculation of viscosity of ice from same. Determinations for the viscosity of glacier ice made on assumption that slip on bottom of glacier is about 15 percent of surface velocity. Reported viscosities for glacier ice range from 3.27×10^{13} to 2.922×10^{14} Poise. Many assumptions are made concerning McConnel's work and, therefore, reported values undoubtedly erroneous. Values range from 3.32×10^{10} to 1.34×10^{12} poise.

Devaux, Joseph, 1933:
L'Economie radio-thermique des champs de neige et des glaciers
Annales de Physique, 20:5-67.

An experimental and theoretical study of the temperature and thermal conductivity of snow and the radiation properties of snow and glaciers.

The temperature of snow is found with thermometers and thermocouples and important characteristics of the diurnal variation and the influence of season, relief, etc., are discussed.

The thermal conductivity of snow is measured over a range of densities using three different techniques with fairly consistent results. As a first approximation the conductivity is only dependent on the snow density.

Using an actinometer (a pyrheliometer by any other name) and a photometer the albedo and absorption factors of snow and glaciers are measured for solar radiation. The radiant energy in glaciers is found to decrease exponentially with depth. The internal albedo of a snow layer and the coefficient of absorption of ice are also obtained.

Devaux, Joseph, 1935:
Etude de l'albedo de la neige dans le spectre infra-rouge
Comptes Rendus, 200:80-82.

Measured the albedo of snow using a spectroscope to separate out the different wave lengths and a vacuum thermopile to measure the radiation intensity at the various wave lengths examined.

The radiation diffusely reflected by the snow was compared to the radiation from a known quartz diffuser. (The exact method of comparison is not explained in the report nor the way in which the quartz diffuser is used.)

For a melting snow layer of large grains at about mid-day, the results for different wave lengths are:

λ (microns)	0.6	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
Albedo	0.67	0.61	0.55	0.34	0.4	0.18	0.21	0.08	0.01

The minimums at 1.0 μ and 1.2 μ correspond closely to the absorption bands of water and ice.

Devik, Olaf, 1942:
Supercooling and ice formation in open waters
Geofysiske Publikasjoner, 13(8):1-10.

The supercooling of water plays a transient, but important part in static ice formation in still water, in so far as it necessarily occurs when the freezing process of the surface layer is being started. It is of lasting importance, however, when the process of dynamic ice formation takes place in running water. The present paper concerns measurements of the supercooling in both cases, the chief available material concerning static ice formation, comprising records of the surface temperature by means of the outgoing radiation, observed by Moll thermopile combined with photographic registering. The instrumental equipment is described, and examples are represented of surface temperature diagrams related to static ice formation. Those records so far represent a documentation of a supercooling of the surface layer of open water descending below -1° C. It is emphasized that the state of supercooling is a stable one, the formation of ice crystals being dependent upon the existence of nuclei in the water, or of solid boundary surfaces from which crystallization will start and the liberated heat of crystallization will flow. The process of thermal conduction necessarily needs time, and thus a state of supercooling may be retained outside the crystal boundary surface, which passes more or less slowly into the adjacent medium as the freezing process goes on.

It is shown that the portions of water between the branching ice elements, during freezing, are actually supercooled. It was found that the degree of supercooling will be approximately proportional to the open fraction. Supercooling of the open areas of water occurs even when in contact with ice.

The formation of ice in open water which is at rest or moving slowly, will normally be preceded by a supercooling of the water surface layer.

Devik, Olaf, 1944:
Ice formation in lakes and rivers
Geographical Journal, 103(5):193.

This paper is a review of earlier work done by Devik, which has also been reported elsewhere. The use of the Moll thermopile in measuring surface temperature is described. Some examples on how meteorological conditions affect the amount of heat lost from surface water are given. The formation of frazil and anchor ice is discussed together with the formation of ice dams in streams while freezing. Some examples are given of how frazil and anchor ice can affect hydrologic power plants.

Devik, Olaf, 1949:
Freezing water and supercooling
Journal of Glaciology, 1(6):307-309.

(Author's Abstract) The supercooling of water in nature occurs more frequently than is generally known. It always takes place when still water is in process of being covered with the first ice needles. It is of even greater importance when ice formation takes place in running and turbulent water, in which case it represents the necessary conditions for the formation of frazil and anchor ice. The supercooling may be observed by a sensitive thermometer when the bulb is heated before being immersed in the water. The degree of supercooling of still water may be more than 1° C. in a thin surface layer as the author has demonstrated by measuring the temperature by a recording thermopile exposed to radiation from the surface. It is emphasized that the state of supercooling in water is a stable one, the formation of ice being dependent upon the existence of nuclei or solid boundary surfaces from which crystallization will start and from which liberated heat of solidification will flow. This process of thermal conduction needs time.

The first measurements of supercooling in rivers were made by W. Altberg in the Neva during the years of 1916-1917-1921. He measured conditions of supercooling which generally amounted to some hundredths of a degree Centigrade, but in particular cases even to -0.1° C. This was consistent through the whole section of the river and even at the bottom. He observed the formation of small ice crystals within the water and on objects under water.

Dichtel, William J. and George A. Lundquist, 1949:
An investigation into the physical and electrical characteristics of sea ice
Paper presented before the Alaskan Conference, Washington, D.C., November, 1950. Unpublished.

The results of measurements of the resistivity of sea ice are reported. The resistivity was found to be dependent on temperature, salinity, and structure. Streamers of ice, (brine pockets), having a high salt content and low resistivity were found distributed irregularly throughout the ice, but oriented mainly in a vertical direction. The sea ice consisting of vertically oriented ice crystals with salt and brine distributed along their axis gave a much lower resistance in the vertical direction than the horizontal direction. So that even samples without brine pockets varied in resistivity from 3 k/ohm. cm. to 3 mg/ohm cm. The authors suggest that high frequency A. C. will give more constant measurements of resistivity.

Dickinson, H. C. and N. S. Osborne, 1915:
Specific heat and heat of fusion of ice
Bulletin of the Bureau of Standards, 12(1):49-82.

(Author's Abstract) By means of a calorimeter of the aneroid type--that is, employing the thermal conductivity of copper instead of the convectivity of a stirred liquid to distribute heat supplied electrically--the specific heat and heat of fusion of ice were determined.

The specific heat was determined upon four samples of ice, all of a high but yet of different degrees of purity.

A variation in the specific heat of ice is found which depends upon the degree of purity.

It is found that at a given temperature θ, between -40° and -2°, for the purest ice experimented on, the specific heat in the 20° calories per gram per degree is represented within the limit of experimental accuracy by the equation

$$\delta = 0.5057 + 0.001863\,\theta$$

and that from -2° to -0.05° the specific heat does not exceed the value given by the above equation by more than $0.004/\theta^2$

The specific heat of impure ice at any temperature θ above -40° is greater than that of pure ice by $\underline{1}\ L/\theta^2$ where L is the heat of fusion and $\underline{1}$ the initial freezing point.

The value found for the heat of fusion of ice is 79.76—20° calories per gram, which is within 1/4000 of the value previously determined at the Bureau by a different method employing a stirred water calorimeter.

Dorsey, H. G., 1921:
Peculiar ice formations
Physical Review, 18(2):162-164.

The author gives an example of an unusual freezing phenomenon--water in an ordinary dishpan, which was placed on a chair on an open porch, froze to form a thin irregular column of ice near the side of the pan, which was next to the back of the chair.

There are four pictures which show this column of ice standing right up in the pan. Attempts to reproduce the phenomenon by freezing water in the same pan under similar conditions were unsuccessful.

The cause of such unusual formation is not known.

Dorsey, N. Ernest, 1938:
Supercooling and freezing of water
Journal of Research of the National Bureau of Standards, vol. 20, Research Paper 1105.

(Author's Abstract) This is a strictly preliminary report, rendered necessary by a temporary suspension of the work. No doubt attaches to the observations reported, but the setting of limits within which they are valid, and the determining of the most satisfactory way in which to explain them, should await further work. Contrary to current opinions, it has been found that the extreme supercooling of water (at least down to -21° C.) is not dependent upon either extreme quiescence or upon the use of a minute volume, and that motes in the water exert a strong non-specific effect. For each specimen there is a definite temperature (the spontaneous-freezing-point), characteristic of the specimen, at which ice suddenly appears, at which the water freezes spontaneously. The well-known fact that a preheating of the water may facilitate its supercooling has been found to depend upon a lasting change so produced in the water--a change that persists at room temperatures for at least 6 months, and that seems to be permanent. By sedimentation, the spontaneous-freezing-points of the extreme layers of a column of water become different, that of the bottom ones becoming higher than that of the top ones. All the observations are consistent with the idea that the spontaneous-freezing-point of a specimen is determined, within undetermined limits, by the size of the largest mote that it contains; the smaller the mote, the lower the spontaneous-freezing-point. The effect of agitation has been considered, and three distinct types of freezing have been recognized.

"As before, it is hard to see how a change in the mere number of motes per unit of volume can affect the spontaneous-freezing-point. And that it actually does not, seems to be indicated by the fact that the observed spontaneous-freezing-point of a mixture is the same as that of the higher-freezing constituent; e.g., one containing 10 percent of water freezing at -7° C, and 90 percent of water freezing at -15° C. froze at -7° C."

Dorsey, N. Ernest, 1948:
The freezing of supercooled water
Trans. American Philosophical Society, N. S. 38(3):247-327.

Much of the data accumulated on the supercooling and the freezing of supercooled water is conflicting, and in this paper Dorsey starts by reviewing, as completely as possible, the results of the experimental work and from this review to establish logical conclusions.

There is a brief review of the two existing theories on freezing of supercooled water: the homogeneous and the heterogeneous theory. Dorsey shows how the homogeneous theory fails to explain the phenomenon in all of its aspects while the heterogeneous theory fits far better than the observed data, but not completely.

Dorsey puts forth a new theory of freezing which explains all of the observations made in this 12 years of work on the subject. His theory actually contains both the homogeneous and the heterogeneous theory. Many of the ideas and results are contained in a preliminary report on this investigation which was published 10 years before the final report. (See N. E. Dorsey, 1938: Supercooling and freezing of water, Journal of Research of the National Bureau of Standards, 20:799-808.

Briefly (A) The homogeneous theory states that freezing is initiated by molecular aggregates of H_2O (trihydrol) which are supposed to exist in water even at temperatures far above the freezing point. The number of such molecules in proportion to the water molecules (dihydrol) increases as the temperature is lowered. There is a fixed number of these molecules corresponding to each temperature when equilibrium exists. Freezing will begin when the molecular aggregates reach a size that enables them to appropriate the adjacent water molecules.

(B) The heterogeneous theory advocates freezing which is initiated by foreign solids, either on the walls of the vessel or small motes suspended in the water. This theory was advanced to correspond with the experimental evidence that could not be interpreted by the homogeneous theory.

Dorsey's theory, while similar to the heterogeneous theory, combines some points of the homogeneous theory also. Freezing is initiated by motes in the liquid or on the walls of the containing vessel.

These motes serve as a gathering place for the molecules; it is well known that molecules will adhere to small particles, dust, etc.

These molecules are held on to the mote by adsorptive forces.

Molecular collisions can disrupt this layer and there will be a change of molecules knocked loose and having a very low velocity; there will be a strong tendency for these to unite in the lowest energy form; they will form embryos which can grow into crystals.

Dorsey explains how the temperature is the deciding factor in determining whether embryos and crystals can form. His theory appears to be in accordance with the observed data accumulated in his work.

Eder, F. X., 1947:
Dielectric loss of ice
Funk und Ton, No 1:21-29
(National Physical Laboratory abstract)

Measurements of the dielectric constant and loss factor were made between 0° C. and -50° C. for frequencies from 50 cps to 3000 mc. With increases of frequency, the dielectric constant falls rapidly from its initial value of about 80 to a value of near 2. This low value is reached at about 10 kc. for a temperature of -50° C. and at about 100 kc. for -3° C.

The loss factor (tan δ) curves all have a high maximum, which occurs at a lower frequency with decrease of temperature. For -3° C. the maximum value is about 2.4 and occurs near 40 kc. while for -50° C. the maximum is about 2.1 near 1500 c.p.s. For very high frequencies, both the dielectric constant and the loss factor show little variation with frequency.

Comparison of the results with Debye's dipole theory of fluid dielectrics show very good agreement when account is taken of the effect of crystallization, which makes the internal friction, and consequently the molecular relaxation time, appreciably greater for ice than for water.

The results are discussed briefly with regard to (a) the effect of icing on the radiation from aerials and on carrier telephony on overhead wires, and (b) electromagnetic wave propagation through ice clouds.

Eriksson, B. E., 1942:
Meteorological records and the ablation on the Froya Glacier in relation to radiation and meteorological conditions
Geografiska Annaler, 24(Part IV):23-50.

Presentation of certain summaries of the meteorological records of the expedition together with an attempt to evaluate the terms in the energy equation

$$80\,H = aI - R + Q + 600\,h$$

where H = ablation in cm; I = incoming rad. in cal $cm^{-2} min^{-1}$; R = radiation from snow; (1-a) 100 = albedo; h = evaporation; and Q = convection. I measured with Robetsch actinograph; H measured; Q and h computed by Sverdrup's methods from vertical temperature and humidity gradients. (aI - R) computed from the energy equation (a assumed known).

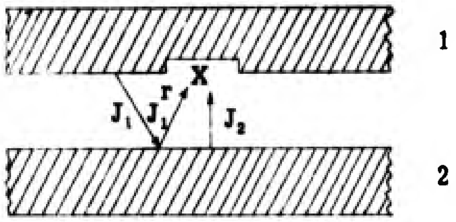
Found in clear weather there is less ablation than in cloudy weather. Convection is the greatest source of heat for ablation. Evaporation is only a small part of the ablation.

Ewing, M., A. P. Crary and A. M. Thorne, Jr., 1934:
Propagation of elastic waves in ice
Physics, 5:165-168

Elastic constants of ice determined by producing longitudinal and torsional vibrations in rod of ice and measuring velocity of propagation of waves. Four rods used for tests; results supposedly show crystal orientation unimportant. No mention of temperature control other than measurements made in range from -5° C. to -15° C., for which $E = 9.17 \times 10^{10}$ dynes cm^{-2} and $\mu = 0.365$ (Poisson's ratio). Actual measurements of velocity of longitudinal waves were made at Saylors Lake and Lehigh Canal and checked values calculated using given values for E and μ.

Falckenberg, 1928:
Absorptionskonstanten einiger meteorologisch wichtiger Körper für infrarote Wellen
Meteorologische Zeitschrift 45:334-337.

An attempt was made to see how closely certain substance temperature radiation approached that of a black body. Only radiation with $\lambda < 10\mu$ was considered.



The basic theory of the experiments can be explained with reference to the figure at left. "1" is a black body, and "2" is the substance which we wish to check. 'X" is a series of thermocouples (thermopile) to measure the radiation intensity

J (radiation intensity) = acT^4

where a = absorptivity = 1 for black body
c = constant
T = absolute temperature

A, the reading of the galvanometer attached to X, is proportional to the total radiation at X

Thus $A \propto J_2 + J_1 r - J_1 = acT_2^4 + rcT_1^4 - cT_1^4$

r = % of reflected energy from "2". r + a = 1 if "2" is opaque.

J_1 = radiation intensity from "1" = radiation intensity from thermopile
J_2 = radiation intensity from "2" which is at same temperature

Therefore $A \propto acT_2^4 - (1 - r)\, cT_1^4$ or $A \propto -ac\,(T_1^4 - T_2^4)$

The value of "a" can be found taking galvanometer readings at different temperatures.

The actual apparatus used was similar in principle to that shown above and could be placed over a bed of snow or other material. For melting snow "a" was found to be over 0.995, thus closely approximating the radiation properties of a black body.

Fink, Donald G., 1937:
Advances against snow static
Aviation, 36(8):30-31.

This is a popular account of experiments conducted by United Air Lines, Bell Labs, Bendix Corporation, and university scientists. Evidence is against the idea that simple charge transfer accounts for the high charge accumulated on planes. It appears that the accumulated charge is removed in flight by discharge from trailing edges.

It is suggested that a long wire trailing from a plane will remove the discharge to a safe distance from the plane's radio. Shielding of antennas is also mentioned.

Frank, F. C., 1946:
Molecular structure of deeply supercooled water
Nature, (157):267

This article describes the extreme supercooling of water done by Rau at Prof. Regener's laboratory. He succeeded in supercooling water to -72° C., and made some remarkable observations. He worked with droplets up to a few millimeters in diameter which were supported on highly polished surfaces of chromium.

The crystallization took place at a reproducible temperature with a reproducible pattern from a single nucleus. After repeated freezing and thawing this nucleus loses its activity and a lower temperature of freezing about a new nucleus was observed. This nucleus in turn becomes inactive after a while and a further degree of supercooling is possible. When he got the freezing point down to -72° C. they invariably froze forming many nuclei.

The resulting crystals are regular (tetrahedra or cubes). They melt again at -70° C.

Below -72° C. it is possible to show a direct condensation of an ice fog in the air. (Same cubic modification m.p. = -70° C.)

The water largely loses its surface tension below the range -60° C. to -50° C. The droplets flatten below -50° C. and commonly spread to a film by -70° C. On warming, droplets separate again at -60° C. and gradually resume their form.

Observations were made on two other modifications of ice m.p. -40° C. and -55° C. The -40° C. modification had the form of hexagonal prisms. The -55° C. modification had no visible form. The lowest temperature form, i.e., the -72° C. modification, may correspond to Bridgman's highest pressure form Ice VI.

The striking fact of the lowering of surface tension is treated in the article. The high surface tension at ordinary temperatures is due to the hydrogen bond. This bond is supposed to change to a more "introverted" form at the lower temperature.

On physico-chemical considerations the structure of this deeply supercooled water appears to be made up of six water molecules, with the six oxygen atoms at the corners of a regular octahedron.

A prediction from this theory which might be tested is that the dielectric constant of supercooled water should pass through a maximum near -40° C. and go to low values at -60° C.

Further research is needed on water below -20° C.

The droplets were kept from fresh air which would introduce new foreign bodies to act as nuclei.

The foreign bodies which ordinarily prevent the supercooling of water to any great extent, can be rendered innocuous by the prolonged influence of liquid water in a saturated atmosphere, especially, it seems, with repeated freezing and thawing.

Finlayson, John N., 1927:
Tests on the shearing strengths of ice
The Canadian Engineer, 53:101-103.

The orientation of the crystal axis is believed to be important in determining the shearing strength of ice. The orientation is established by focusing a beam of sunlight on the interior, and noting the strata of air bubbles.

Motonori Matsuyama's paper is discussed and, according to his calculation of the modulus of rigidity, it is determined that the shear values of ice would not be more than 30 p.s.i.

Ice samples 3" x 3" in section were held in a cast-iron shearing tool and loaded in a 30,000 lb. Riehle testing machine. The shear values obtained are quite variable. The author asserts this to be expected in such a material as ice. The values obtained in a range from -10° F. to +30° F. do not seem to show any dependence on change in temperature. The average values of the shear strength of ice are 114 p.s.i. perpendicular to the crystal plane and 98 p.s.i. parallel to the crystal plane.

A few tests were made to obtain the shear strength of artificial ice. It was found to be about 80% of the strength of river ice. Also the compressive strength of river ice at 11° F. was found to be 1800 p.s.i. parallel to the crystal plane and 1,050 p.s.i. perpendicular to the crystal plane. In a few experiments made upon molded briquettes of ice, it was found that the average tensile strength of water ice was 136 p.s.i. and that of ice formed of snow and water mixed was 103 p.s.i.

Frost Effects Laboratory, 1949:
Appendix A, mathematical studies of thermal properties of soils
Addendum No. 1, 1945-1947 to report on Frost Investigation 1944-1945. New England Division, Corps of Engineers, U.S. Army, Boston, Mass., mimeographed.

Appendix A presents the mathematical solutions to 17 problems treating the thermal conditions in a semi-infinite, homogeneous, isotropic soil mass due to variations in surface air temperature. The problems deal with the determination of the thermal diffusivity, depth of frost and melt penetration, effect of radiation and surface film, and the effect of an insulating layer. A series of formulae were developed to predict the depth of frost penetration at airfields where frost observations were made. These mathematical studies were principally performed by Dr. L. A. Pipes, Harvard University. Sample calculations are presented.

Gerdel, R. W., 1948:
Penetration of radiation into the snow pack
Transactions of the American Geophysical Union, 29(3):366-374.

The design of a very sensitive pyrheliometer, constructed by the author for measurement of the quantity of radiation transmitted through a snow pack, is described. All the data was obtained from a natural snow cover, using the instrument previously mentioned.

It was found that the depth to which solar radiation was transmitted increased as snow density increased. Absorption coefficients (assuming exponential decrease in intensity of radiation for constant density snow) were obtained for snow between 20-40% density. The presence of ice and slush in the snow surface resulted in a decreased albedo and increase in the amount of radiation absorbed. However, there was approximately 50% reduction in the amount of radiation transmitted through the ice and slush layer to lower horizons in the snow.

Using the relation $I = I_0e^{-kd}$ for a constant density snow pack where I = intensity of the radiation at a depth of "d" centimeters and I_0 = intensity at the surface, he obtained the following results:

Density - %	28.1	32.3	39.7	44.8
Absorption Coefficient - k -cm^{-1}	0.280	0.184	0.106	

Ginnings, Defoe C. and Robert J. Corruccini, 1947:
An improved ice calorimeter--the determination of its calibration factor and the density of ice at 0° C.
National Bureau of Standards J. of Research, 38:583-92.

The Bunsen ice calorimeter depends on the contraction of a closed ice-water system when heat is added to melt some of the ice. A determination of the calibration constant of such a calorimeter to 0.1% involves a determination of density of ice to 0.01%.

The calorimeter is described and calibration procedure is given from the calibration factor the density of ice at 0° C. is calculated to be 0.91671 ± 0.00005 gr/m. The density appears to be constant at this temperature.

Goff, John A., 1942:
The vapor pressure of ice from 32 to -280° F.
Heating, Piping, Air Conditioning, 14:121-4.

The Clausius--Clapeyron equation $\frac{dp_s}{dT} = \frac{1}{T}\frac{h_g - h_i}{v_g - v_i}$ **

is used to calculate vapor pressure in inches of mercury, over the temperature range given. The calculations of h_i are based on specific heat data by Giauque and Stout corrected for thermal expansion of ice. The relation $p_s v_g = Rt - A_{ww}$ is used to find v_g, where A_{ww} is the second virial coefficient and seems to be found from the equation of Keyes. v_i is shown to be negligible. The calculation of h_g is based on the band spectrum of H_2O, following Gordon (Jour. Chem. Phys., 2:65). The reviewer is unable to determine the extent of validity of Gordon's equations, but they appear to be valid within this temperature range. A further correction h_g is introduced by Goff. Accuracy of about 0.05% is claimed for the upper part of the range; error at the lowest temperature may be a few parts per thousand.

** T = absolute temperature
p_s = vapor pressure
h_g = specific enthalpy of vapor
h_i = specific enthalpy of solid
v_g = specific volume of vapor
v_i = specific volume of solid

Gunn, Ross, 1947:
In-flight icing of highly electrified aircraft
Journal of the Aeronautical Sciences, 14:527.

Tests were made of the possibility that a highly electrified airplane will not ice up as readily as a neutral one. Three identical rods were mounted on a B-25 and charged to potentials of +8000 v, 0, -8000 v, negative. No difference could be detected in the ice layers on the three rods. The experiment was repeated with the same results.

It is concluded that high voltages on airplanes will not prevent icing.

Hand, Irving, F. and Roy E. Lundquist, 1941
Preliminary study of radiation penetration through snow
Proceedings of the Central Snow Conference (Michigan State College), 1:42.

This study of transmission of radiation through snow was made using two Eppley total radiation pyrheliometers. Tables are given for two cases (1) Uniform fresh snow. (2) Slush and ice layers formed in sample. The measurements were made under conditions normally leading to runoff.

The progressive variation of transmission was studied and note is made of the change in composition of the snow with variation in the transmission.

The correlation of the factors of decrease in both albedo and transmission as melting progresses should provide a method of estimating the coefficient of absorption of snow.

Hand, Irving F. and R. E. Lundquist, 1942:
Observations of radiation penetration through snow
Monthly Weather Review, 70(2):23-25.

Solar radiation incident on a snow surface was measured with a ten junction Eppley pyrheliometer while the radiation present at various depths in the snow was found using a 50 junction Eppley pyrheliometer.

Data was taken for a uniform, fine granular compact snow and for snow with slush and ice in it. Snow density (%of water per unit value of snow) and snow quality were measured.

An attempt to determine the effect of extraneous matter on snow surfaces was made by measuring absorption of snow that was covered by various paint pigments.

(This report covers the same experiments and data as "Preliminary Study of Radiation Penetration Through Snow," Central Snow Conference Proceedings, 1:42-44, but is slightly more detailed.)

Hargis, C. D., 1922:
The viscosity and rigidity of ice
Physical Review, 19(Series 2):526-27.

The coefficient of viscosity and the modulus of rigidity for ice were determined by means of a tortional pendulum. The equation of motion used is

$$K\ddot{\theta} + L\dot{\theta} + M\theta = 0 \qquad (1)$$

in which

$$L = n\frac{\pi r^4}{2l} \qquad M = G\frac{\pi r^4}{2l}$$

r = radius of rod
l = length
n = coef. of viscosity
n = G = shear modulus

By solving (1) and measuring the period of oscillation n and G can be calculated. No mention of temperature control or ice formation given. Values reported are n = 3.8×10^9 and 6.33×10^9. (No units given) for periods of 0.286 sec. and 0.468 sec. respectively. (Dorsey reports Hargis' results as 3.7×10^{12} and 6.21×10^{12} poise for periods of 0.286 sec. and 0.448 sec. respectively.)

Hess, Hans, 1940-41:
Über die elastizitäts Konstanten des Eises (On the elastic constants of ice)
Zeitschrift für Gletscherkunde, 27:1-19.

Using several experimentally (static method) determined values of E at different temperatures, a functional relationship for E in terms of temperature was obtained. Using equation $E = 3K(1 - 2\sigma)$ and obtaining $K = (\frac{dV}{dp} + \frac{1}{v})$ from Bridgeman's measurements on ice, the dependence of σ (Poisson's ratio) on temperature is obtained. The functional relation between G (the rigidity) and temperature is obtained from equation

$$G = \frac{E}{2(1 + \sigma)}$$

Using the relations for E and σ that were previously determined, he obtained:

$$K(t) = 28.600 - 3.910t$$
$$E(t) = 19,280 - 5,295t + 449t^2 - 0.8t^3$$
$$G(t) = 6,948 - 2.231t + 192t^2 - 2.93t^3$$
$$\sigma(t) = 0.3875 + 0.019t$$

K, E and G in kg/cm² t = temperature in °C.

The dependence of E and σ upon pressure was also determined resulting in:

$$E(t, p) = E(t) + (-5925t + 987t^2 - 18.6t^3 + 0.52t^4) \log_e p$$

$$\sigma(t, p) = \sigma(t) + (0.0346t - 0.0001t^2) \log_e p$$

p = pressure in atmospheres.

The results obtained are only valid for the range 0 to -9° C, since no experimental data outside this range was used for the original relations.

The resultant equations were applied to the study of the depths of glacier ice. Data obtained in the glacier ice study indicated that the value of E obtained from static measurements must be multiplied by a factor ϵ (ϵ is between 1.5 and 3) to be able to use E in the normal equations for the velocities of sonic waves in ice.

Smaller depths for the Greenland inland ice were obtained, than the calculations hitherto made. This supports the view that the laws of movement which apply to an alpine glacier, whose tip blows on a slightly inclined plane and which receives a strong moving impulse from the steeper firn area above, apply also to that gigantic mass of ice.

Heverley, J. Ross, 1949:
Supercooling and crystallization
Transactions AGU, 30:205-210

The supercooling of water droplets was investigated under conditions which bracket those found in nature. The spontaneous freezing point of the droplets was measured as a function of size; it was found to be independent of the rate of cooling and independent of the source of water, unless the water had been seeded with Ag I. The maximum temperature at which water vapor will condense to snow crystals was found (-28° C). An investigation was made of the ratio of water droplets to snow crystals at lower temperatures. It is suggested that some current theories of precipitation need revision.

Höppler, F., 1941:
Die Plastizität des Eises
Kolloid Zeitschrift, 97:154-60.

By employing the so-called Höppler-Konsistometer, the plasticity of ice was studied. Great care was taken in the preparation of the sample, and each sample was subjected to a preliminary test to insure obtaining reproducible results. The temperature of the sample was carefully controlled during each test so that temperature effects were accurately determined. Tests consisted of forcing a ball through a cylindrical sample of ice, which in turn flowed through an annulus between the ball and container wall. Curves of flow-velocity vs. shearing stress were plotted, which appear very similar to the usual $d\gamma/dt$ vs. τ curves used to define a plastic solid. Results reported in terms of so-called quasiviscosity (NQ) and evidently converted to centipoises by means of a conversion factor, which, however, was not mentioned.

The quasiviscosity of ice is a function of the temperature and of the shearing stress, reported values lying between 1.72×10^{11} and 2.13×10^{15} for temperatures from -1° to -30° C, and shearing stresses from 9.5 to 63 kg/cm^2.

Huggins, Maurice L., 1936:
Hydrogen bridges in ice and liquid water
Journal of Physical Chemistry, 40:723-731.

Discussion of the hydrogen bond. Following Pauling, Huggins proposes a dynamic randomness of orientation of H_2O molecules in ice, and shows how the dielectric constant of ice may be qualitatively accounted for. There follows a discussion of the hydrogen bond with relation to conductivity of pure water and mobility of hydrogen ions.

The present interest of the paper is due to a calculation of the potential energy of the H atom between two oxygens, as related to the internuclear distance of the oxygens. From this it is possible to calculate the magnitude of thermal 0-0 vibrations which will permit the H atom to go from one oxygen to a neighboring one.

Ingersoll, L. R. and O. A. Koepp, 1924:
Thermal diffusivity and conductivity of some soil materials
Physical Review, 24:92-93.

Direct measurements of the thermal diffusivity of several soil materials are quoted. Each substance was packed in a rectangular box of sheet copper. After the sample had reached a uniform temperature, the temperature at two opposite faces (the other faces being insulated) was suddenly changed.

The temperature at the center of the sample was measured with a resistance thermometer at different times. With this data and the equation for heat flow, the diffusivity is obtained. Snow, of 0.54 density was found to have a diffusivity of 0.0041. Assuming a specific heat of 0.50, the conductivity is found to be 0.0011.

Ivanov, K. Ye., P. P. Kobeko and A. R. Shullman, 1945
Deformation of an ice cover under moving loads
Physico-Technical Institute, Academy of Sciences, Leningrad, U.S.S.R.

Report contains only main results of work done by authors in their study of the deformation of ice under moving loads.

Various loads (automobiles, internal-combustion locomotives, etc.), moving at a variety of speeds on a natural ice cover were used and showed:

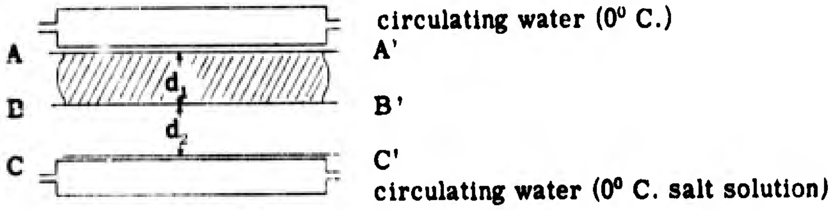
(1) Elastic deformation with loads moving at speeds of 5 to 15 km/hr. (The shock spread through the ice with the speed of movement of the vehicles and vanished with practically the same speed. The magnitude of depression in the case of slow movement was 1/2 to 2/3 as great as with static tests

(2) A wave vibration which was observed as far as 100 meters from the route for speeds above 20 km/hr. (The wave length and the velocity of the wave were measured and the velocity was found to be independent of the speed and magnitude of the load. The waves were definitely not sonic ones in the ice, and are believed to be hydrodynamic waves. A formula was derived with ice waves as a function of the coefficient of elasticity, the thickness of the ice cover and the depth of the pond. Using the formula the elasticity, E, was found to be 4,000 kg/cm² from their set of measurements. Velocity of wave was 35 km/hr and the wave length was 150 meters, both of these being measured directly. When resonance occurs (i.e., velocity of load equals velocity of wave) amplitude large enough to cause the ice to break may develop.)

The temperature of the air and ice was -6° C. Several practical rules for the movement of vehicles along an ice route are given.

Jansson, Martin, 1901-2
Über die Wärmeleitungsfähigkeit des Schnees
Ofversigt Af Kgl. Vetenskahsakademiens, Förhandliger

The heat conductivity of snow is measured by means of the apparatus shown:



A, B, and C are copper plates at distances d_1, and d_2 apart respectively. A', B', and C' are thermocouple leads. A glass plate was inserted between plates A and B as a comparison substance having a conductivity k_2, whereas the unknown with a heat conductivity k_2 is placed between BC. For a steady-state flow,

$$\frac{k_1}{k_2} = \frac{d_1}{d_2}\,\frac{t_2 - t_3}{t_1 - t_2} \qquad (1)$$

k_2 was determined by making measurements with a substance of known heat conductivity (H_2O was chosen) between plates B and C. Since the volume between the plates B and C was known, the densities could be measured by weighing the samples.

Jansson fits a curve of the form

$$k = a + b\rho + c\rho^2 + d\rho^3 + e\rho^4$$

where he assumes snow to be a mixture of ice and air. Thus he takes as a limiting value for $\rho = 0$, k = heat conductivity of air, and for $\rho = .5$, k = the value of conductivity for ice. He finally obtains,

$$k = .00005 + .0019\rho + .006\rho^4 \text{ cgs}$$

Johnson, John C., 1950:
Measurement of the surface temperature of evaporating water drops
Journal of Applied Physics, 21(1):22-23

Theoretical derivation of rate of evaporation of water drops in still air is:

$$a^2 = a_0^2 - \frac{8k}{\rho}(\rho_0 - \rho)t$$

or

$$\frac{dM}{dt} = k2\pi a \, (\rho_0 - \rho)$$

a = drop diameter at time t and a_0 at time t = 0 ($\rho_0 = \rho$) difference in vapor density in air and at surface of drop

k = diffusivity

ρ_0 is a function of T_0 the surface temperature. Fuchs derived for T_0, from expression for heat flow at surface

$$\frac{dQ}{dt} = -2\pi K \, (T - T_0) \, a \qquad K = \text{thermal conductivity of air}$$

$\therefore T - T_0 = (\frac{kL}{K}) \, (\rho_0 - \rho)$ and since $T_0 = f\,(\rho_0)$, can solve by "cut and trial." T_0 was measured by thermocouples of varying mass, and value of T_0 extrapolated. Found to agree with Fuchs theory.

Kaitera, Pentti, 1948:
Keeping water from freezing by means of compressed air
Association Internationale d'Hydrologie Scientifique Assemblee Generale d'Oslo, pp. 390-398.

In the winter of 1940 experiments were carried out in Finland with the object of keeping minor areas of water from being covered with ice. This was done by utilizing the thermal reserve of lower layers of water by pumping compressed air below the surface. When rising to the surface, these bubbles carry up warm water which melts the ice.

The amount of ice melted by this method depends on several factors: (a) the temperature of the water at the level of the air discharge nozzle; (b) the depth at which the nozzle is placed; (c) the number of nozzles; (d) the thickness of the ice cover; (e) the volume of compressed air per unit time.

There is a strong temperature gradient in the surface layers of the water. Right below the ice the temperature is at 0° C.; this rises to as much as 4° C. at 17 or 18 meters depth.

If there was more than one nozzle at the point of discharge the melting effect was increased. This served to spread out the air and bring up more water.

In general, the deeper the nozzle the greater the melting. For nozzles below twenty meters this effect is presumably due to the greater volume of water set in motion.

The greater volume of compressed air caused more rapid movement and more melting.

If the air was trapped below the ice, it cut down the melting effect by forming an insulating layer which kept the warm water from coming into contact with the ice. This difficulty was overcome by drilling holes in the ice to allow for the escape of the air.

This method of preventing ice formation was put into practical use in Finland. Saw mills on the banks of Finnish lakes have kept their sorting bins open and free of ice during entire winters although the air temperature was well below 0°.

Kalitin, N. N., 1930:
The measurements of the albedo of a snow cover
Monthly Weather Review, 58:59-61.

Measured albedo of ground surface (usually, but not always, snow covered) over a continuous period of 2 1/2 months in the winter and spring of 1929.

Data was obtained using a pyranometer designed by Angstrom* and a loop galvanometer; method of calibration of instrument was not given by author.

Observed the albedo of the snow cover to decrease with thawing or over a period of time when no fresh snow had fallen. The albedo increased after the fall of fresh snow. The maximum value of the albedo was 87%.

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*A. Angstrom, "A new instrument for measuring sky radiation," Monthly Weather Review, 1919, no. 4.

Kalitin, N. N., 1935:
Quelques donnees sur la transparence de la glace pour la radiation ultra-violette du soleil.
Comptes Rendus (Doklady) de l'Academie des Sciences de l'URSS, v. 4(9) n. 3(72):145-146.

The transparency of ice to ultra-violet radiation was studied with the aid of a spectrograph.

Radiation from the sun was first directly analyzed in a spectrograph, and then using the same sheet of film and the same exposure time, the radiation of the sun passing through different blocks of ice was analyzed. The spectrograph was set to study only wave lengths in the ultraviolet. Relative values of intensity were measured from the film with the aid of a microphotometer.

Two samples of ice (one clear and one brown with many small air bubbles) of 10 cm thickness and one sample (clear ice) 107 cm thick were studied. All samples were pond ice. The following results were obtained:

Thickness	Percentage of Incident Radiation Passing Through Ice Wave Lengths in Angstroms						
cm	3320	3460	3660	3920	4160	4380	4460
10 (clear)	97	96	99	99	98	99	98
10 (brown and bubbles)	60	59	59	60	63	62	60
107	46	46	51	52	54	52	55

Klein, G. J., 1938:
The snow performance of aircraft skis
National Research Council of Canada, No. 722, mimeographed

A very detailed description is given of the ski dynamometer, the towing vehicle, and other equipment used. The skis were constructed so that the following variables could be tested: (1) the ratio of length to breadth; (2) the shape of the ski profile; (3) the shape of the ski cross-section; (4) the material in contact with the snow; (5) either rigid or flexible construction. Also the skis were tested under variable loads, velocity, snow conditions, and point of application of the load.

Both sliding resistance and adhesion tests were performed on all the different types of skis. Most of the adhesion tests were made at five-minute intervals. The order of importance of the variables in the sliding resistance tests were found to be: (1) the condition of the snow; (2) the material in contact with the snow; (3) the ratio of length to breadth of the ski; (4) the point of application of the load; (5) the fineness of the bow of the ski.

It was finally concluded that a material that tends to resist wetting with water would probably be of the best type for the ski surface; that scratches on the ski bottom impairs the performance of the skis; that the unit loading should be about double that previously used; and that the bow should be finer than the bows on the aircraft skis that were being used.

Two abstracts are appended; "Investigations into the Effect of Weather Conditions on the Friction of Sleigh Runners on Snow," by Gliddon, 1922, and "Investigation on Ski Friction," by Söderberg, 1932.

Klein, G. J., 1946:
Method of measuring the significant characteristics of a snow-cover
Report No. NM-192, Division of Mechanical Engineering, National Research Council of Canada, Ottawa.

(Author's Abstract) Snow is a material the properties of which vary considerably. The conditions and features of the snow which determine its properties are discussed, and standardized methods and instruments for measuring these characteristics are described. The principal applications of the measments are: (a) snow-cover surveys for obtaining data on snow conditions in different localities and (b) performance trials of winter equipment in which it is desirable to define the snow conditions that occur during the trials. The report includes a snow nomenclature and general information on snow which may be helpful to observers.

Topics covered include purpose of the tests, basis on which the tests have been chosen, instruments, grain-form density scale, test procedure, general instructions, general notes on snow, nomenclature of snow. Figures of instruments are given.

Klein, George J., 1947:
The snow characteristics of aircraft skis
National Research Council of Canada, Aeronautical Report AR-2, Division of Mechanical Engineering, Ottawa, 17 pp.

Skis of many different designs and materials were tested for sliding and adhesion resistance. The ski dynamometer equipped with instruments for measuring velocity and resistance, and with a variable load from 375 to 2000 lb/ski was used in the measurements.

μ_p = coefficient of resistance due to parking
μ_s - coefficient of sliding resistance
$\mu_f = \mu_s - \mu_p$ = coefficient of resistance due to friction
μ_a = coefficient of adhesion.

Since μ_s and μ_p can be measured directly, μ_f can be calculated and μ_a can be measured directly.

It was found that both material and design had a great effect on both dynamic and static friction; also the condition of the snow and temperature caused variations. The sliding tests were made at approximately 2.5 and 10 feet/sec, and the adhesion tests were all made at 5 minute intervals.

The theory is proposed that contact between ski and snow takes place at a large number of small areas, and that the lubrication is provided by frictional melting of the snow. Various observed facts were given to substantiate the theory.

The best all-around material found for low dynamic and static friction and for best wearing properties was bakelite. The aspect ratio of length to width was recommended to be at least 6; and the angle of the bow was recommended to be between 20° and 25°. It was found that the best results were obtained at high unit loading between 400 and 500 lb/sq. ft.

The article contains very good tables and graphs of variation of dynamic and static friction with various materials and limit loadings. Both the maximum and minimum values are included.

Kobeko, P. P., N. I. Shishkin, (F. I. Marey) and N. S. Ivanova, 1945:
Breakdown and carrying capacity of ice
Physico-Technical Institute, Leningrad, Academy of Sciences, USSR

A load was applied to a natural ice cover through plates of different shapes and areas, and values of the deformation of the ice at breakdown were observed.

Perceived radial cracks emanating from position of load after sufficient load had been applied. These cracks do not indicate that breakdown will occur immediately thereafter and the ice still retains capacity to carry more load. Eventually, as more load was applied, circular cracks around the load formed and the breakdown occurred simultaneously.

The breakdown load was bound to depend essentially on the thickness of the ice and the empirical relationship: $P = \alpha h^2$ was found to be quite good for values of thickness of 0.15 to 40 cm. P = breakdown load in kg, h = thickness of ice in cm, $\alpha \cong 20$. The effect of variation of area was apparently not considered in the equation even though it was considerably larger (~3600 times) for the 40 m ice than for the thinner ice.

Experiments were conducted over a wide range of temperatures (0° C to -20° C) with concurring results in all cases, indicating that atmospheric temperature (the ice temperature was presumably assumed to be the same as the atmospheric temperature) has negligible effect on breakdown load within these limits.

When the area over which the loads were distributed was increased 200 times, the breakdown load increased only two fold. Breakdown with elastic deformation, for a given ice thickness, was observed to occur with a distinct value of deformation which is seemingly the same as that for plastic deformation. With prolonged loads, the carrying capacity of the ice was determined by the combined total of elastic and plastic deformation.

Kobeko, P. P., N. E. Shishkin, F. I. Marey and N. S. Ivanova, 1945:
Plastic deformation and viscosity of ice
Physico-Technical Institute, Leningrad, Academy of Sciences, USSR

Studied plastic deformation using a natural ice cover of large area. Used lake ice formed in ice holes up to 8 meters in diameter. Experiments were made at 0° C. Temperature was controlled by covering surface of ice with 2 or 3 cm thick layer of dry snow. Ice was subjected to concentrated load acting through a plate which rested on the ice. Measurements of deformation were taken at different distances from the load.

When short-time small loads were applied, the ice deformed elastically and deformation was simultaneous with application of load. When loads were applied over a long period of time, plastic, as well as elastic, deformation was observed. The elastic deformation was small (0 to 5% of total) compared to the plastic which was allowed to increase till breakdown. Readings were taken at different times beginning with the application of the load.

Viscosity was calculated two ways: (1) Assuming only shear displacement (i.e., neglecting bending in plate of ice) they found viscosities of 5.5 x 10^9 to 2 x 10^{12} poise depending on load, area over which it is applied and thickness of ice. (2) Taking into account the bending and applying Maxwell's theory on plastic deformation, they obtained viscosities of 2.8 x 10^{12} to 22 x 10^{14} poise.

The rate of plastic deformation was found to be essentially constant with a constant prolonged load and eventually led, even with small loads to breakdown.

The breakdown occurred, regardless of load, when plastic deformation reached a certain value depending on thickness of ice. Shear stress of less than 0.2 kg cm^2 did not cause breakdown except when applied over very long periods. This was taken as the maximum allowable stress for long time loads.

Kornfield, M. and P. Shestikhin, 1942:
Elasticity of a crystal as dependent on temperature
Comptes Rendus (Doklady) de l'Academie des Sciences de l'URSS, 36(2):52-56.

The rigidity modulus (shear modulus G) for ice was determined as a function of temperature by measuring the velocity of propagation of torsional elastic waves. Ice samples were prepared by freezing water in aluminum foil tubes. Tests were also run on impure ice obtained by adding various impurities to pure distilled water; results practically coincide with those for pure ice. Plot of data shows G as linear function of temperature below approximately -25° C, above which G decreases rapidly.

Lamb, J., 1946:
Measurements of the dielectric properties of ice
Transactions of the Faraday Society, 42A:238.

The dielectric properties of ice were investigated (a) at a frequency of 10^{10} cps. over the temperature range 0° to -40° C, and (b) at a temperature of -5° C. over a frequency range 8 x 10^3 to 1.25 x 10^5 cps. Apparatus and procedure are given. Results are given in graphical form. Only one maximum is found in the curve for loss factor vs. frequency, but there is a discrepancy between the factor at 10^5 and that at 10^{10} cps. No satisfactory explanation is advanced for this anomaly.

Lamb, J. and A. Turney, 1949:
The dielectric properties of ice at 1.25 cm wavelengths
Proceedings of the Physical Society (London) 62B:272.

This is an extension and correction of earlier work (Lamb, Trans. Faraday Soc., 42A:239). The permittivity is 3.18, independent of temperature from 0° to -195° C. The loss factor is slightly less for a given temperature at 1.25 cm than at 3 cm, but the temperature dependence is similar at the two wavelengths.

No mention is made of crystal orientation.

Landsberg, H. 1940:
The use of solar energy for the melting of ice
Bulletin of American Meteorological Society, 21(3):102-107.

A thin cover of finely powdered coal was spread over a snow surface and on blocks of ice. This resulted in an appreciable increase in radiation absorption with a consequent increase in melting rate.

The possibility of decreasing the albedo of a glacier surface by putting a layer of dark particles on it was discussed. The resulting decrease of cold glacier winds would tend to stunt glacier growth, decrease harmful effect of cold winds on vegetation, and increase the growing season.

Langbein, Walter B., 1948:
Computing soil temperatures
Transactions of the American Geophysical Union, 30(4):543-547.

This paper develops a method for computing temperatures at a point in soil from surface temperatures. The method is adaptable to the irregular fluctuations in temperature ordinarily found in nature as well as to ideal cases. The temperature at a point in soil is expressed as a weighted function of the antecedent temperatures at the surface. An analogy is made with the unit hydrograph, a familiar tool in hydrologic practice.

A curve of soil temperatures shows comparison between computed and observed values.

Langmuir, Irving, 1950:
Control of precipitation from cumulus clouds by various seeding techniques
Science, 112:35-41

A discussion is given of the formation of cumulus clouds in dry regions. An explanation is given for presence or absence of rainfall, in terms of "condensation nuclei." In view of the theory, conditions and seeding procedure are given for producing more rain or for dissipating clouds without rain.

Seeding experiments in New Mexico are described. An equation for the rainfall to be expected at various distances from the smoke generator is developed. The results are compared with the equation. Langmuir estimates the probability that the observed rainfall would have occurred without seeding to be of the order of $1:10^6$.

Latimer, W. M., 1949:
The dielectric constants of hydrogen-bonded substances
Chemical Reviews, 44:59

A discussion of "atomic polarization" caused by the proton in the hydrogen bond is followed by a discussion of the hydrogen bond in ice. A mechanism is suggested for orienting the dipoles in ice which depends on the jumping of hydrogen atoms rather than upon rotation of water molecules. 15 refs.

Lavrov, V. V., 1947:
The temperature dependence of ice viscosity
Journal of Technical Physics, 17(9):1027-1034.

Studied viscosity of ice by observing the deflection of a centrally loaded bar of ice. The ice samples were first checked optically and two samples, which were sufficiently uniform in structure, were used as standards.

It was found that the phenomenon of relaxation holds for ice as well as other crystals. Thus each of the specimens could be used for several tests.

Samples which showed anomalies in the speed of plastic deformation had sharply defined heterogeneity of structure, the speed of plastic deformation being much larger for these samples than for the uniform ones and usually increasing with time till fracture. The rate of deformation of samples with fairly uniform crystalline structure remained practically constant after five or six hours.

An anomalous change in the rate of plastic deformation of specimens of sufficiently regular crystalline structure occurred when they were subjected to a sharp change in temperature. This phenomenon should be of great interest to the theory of plasticity.

The viscosity of ice was calculated from the formula for elastic deformation of a centrally loaded bar, freely suspended on its supports, using substitutions of rate of plastic deformation for the elastic deformation and the viscosity for 1/3 Young's Modulus in accordance with Maxwell's theory.

For the standard samples, when the rate of plastic deformation was constant the viscosity at -3° C was 2.6×10^{14} poise (within a margin of error of 12%); at -13° C it was 4.1×10^{14} poise (within a margin of error of 16%); and at -23° C it was 6.9×10^{14} poise (within a margin of error of 20%).

Light, Phillip, 1943:
Boundary-layer problems involved in snow melt
New York Academy of Sciences, 44:55-68.

The various sources of energy available for ablation are discussed. Emphasis is placed on the eddy convection of heat and moisture (latent heat). The magnitude of the observed heat exchange predicted by Sverdrup's formula is compared with that computed from ablation and radiation measurements of the Norwegian Swedish Expedition of 1934, and fair agreement is found. Sverdrup's formula applied to ablation in a basin, however, required a reduction by a factor of 0.65 be applied to the predicted values. This factor is termed the basin coefficient.

Lonsdale, Kathleen and P. G. Owston, 1946:
X-ray examination of hail
Nature, 157:479.

Sections 1-2 mm thick of three hailstones were examined. Two hailstones showed a single crystal, the other showed several crystals. The stones were accidentally allowed to melt before very careful observations could be made.

Lonsdale, Kathleen, 1948:
Statistical structure of ice and ammonium fluoride
Nature, 158:582.

The diffuse Laue patterns of ice cannot be explained in terms of any normal lattice vibrations. It is presumed that they are due to some abnormal "vibration" of hydrogen atoms. (See Max Born, Nature, 158:830). It is pointed out that the isomorphous crystal of NH_4F also shows such patterns.

Lonsdale, Kathleen, 1949:
Diamagnetic susceptibility and anisotropy of ice
Nature, 164:101.

Measurements were made of the diamagnetic anisotropy of ice crystals. The anisotropy found was no more than 0.06×10^{-6} for $X_a - X_c$ (c.g.s.-e.m.u./mole). The suggestion is made therefore that the diamagnetic susceptibility of ice is largely a property of the H_2O molecule.

Accepting the value of Cabrera and Fahlenbrach (Naturwiss., 22:417, 1934), for X_m, this gives:

$$\begin{aligned} X_c &= -12.68 \times 10^{-6} \\ X_a &= -12.62 \times 10^{-6} \end{aligned} \qquad \text{c.g.s. - e.m.u./mole}$$

The temperature was -5 to -15° C.

Lonsdale, Kathleen, 1949:
The study of anisotropy in crystals
Bulletin de la Societe Chimique de France, p. D (Documentation) 476.

Methods for studying molecular crystals are outlined. These include x-ray data, diamagnetic and optical anisotropy measurements, and studies of compressibility and thermal expansion. Special attention is given to ice. The properties discussed are crystal polarity, lattice structure, dielectric behavior, neutron diffraction, and thermal vibration is indicated by diffuse streaks in Laue photographs. An explanation is given for the creep of ice in directions parallel to the basic plane.

Martin, A. E., 1947:
Hydrogen bonding in ice (Reply to M. Born)
Nature, 159:403

Two low-frequency Raman bands, which Born believes to be derivable from elastic constants of ice, are also found in modified form in water. These have been described as due to molecular effects (not crystal-elastic effects) in ice. The very broad Raman band (3000-3600 cm^{-1} in ice is ascribed to an O-H vibration modified by hydrogen bonding.

Matsuyama, Motonori, 1920:
On some physical properties of ice
Journal of Geology, 28:607-631.

(Author's abstract) In some specimens of ice a sharply defined elastic limit was noted, though in other cases it was not so clearly shown.

The modulus of rigidity of ice when the crystals are perpendicular to the axis of the test piece, is very small compared to that of metals, and is about 2×10^9 c.g.s. There is a slight indication that it is greater when the shearing is parallel to the base of the consitituent crystals than when it is perpendicular.

The Young's modulus is also very small compared to that of metals. It is largest when the crystals are parallel to the length of the test piece, and has the numerical value about 20×10^9 c.g.s.

Elastic fatigue was marked after repeated torsion. On account of the fact that it was often necessary to use certain bars in successive experiments during which they suffered from different amounts of fatigue, it was difficult to compare the results bearing on ice bars with crystal parallel and perpendicular to the length of the test piece. Still there were some indications that beyond the limit of elasticity the former orientation was stronger than the latter against torsion. In the case of bending experiments, this was clearly shown.

The torsional deformations both by constant and by increasing forces were observed and the result is shown by curves, though no mathematical conclusions were made. The recovery curves showed that the observation was approaching the stage where no recovery would take place after removal of the force.

When an ice bar with crystals parallel to its length was bent, the bent portion showed the change of optical character, the extinction swinging around the curve. In each crystal when the best specimen consisted of parallel crystals horizontally across it, parallel straight lines were observed.

These facts seem to show that gliding planes parallel to the base of each crystal are not the controlling factor. But instead, adjustments along the contact surfaces of adjacent crystals and perhaps the development of planes of weakness in the constituent crystals parallel to their long axis seems more effective in the process of deformation.

McConica, T. H., III, 1950:
Sliding on ice and snow
Report to the Research and Development Branch, Office of the Quartermaster General, U.S. Army, American Ski Company, (mimeographed).

A report on the friction of various materials on snow and ice. A turntable was used to control the velocity and surfaces to be studied at various temperatures. A variety of surfaces were studied at various speeds, unit loadings, and temperatures and several graphs are included to show the results.

A very interesting theory of lubrication by an adsorbed transient layer of water "vapor," rather than a liquid film caused by friction or pressure is proposed. It was stated that this theory is in agreement with the "dry slipperiness" of graphite. Several observations are included to support the theory.

An explanation is proposed for rather large variations of the coefficient of friction with velocity, on successive runs under the same condition.

The resistance to sliding is discussed with reference to snow metamorphoses and mechanical properties of the snow masses. The mechanical resistance to sliding is discussed, and the design factors of skis are very thoroughly discussed.

Ski bases and waxes are reviewed and analyzed, with several suggestions for future developments being offered.

McConnell, J. C., 1891:
On the plasticity of an ice crystal
Proceedings of the Royal Society of London, 43:323.

Describes a series of crude experiments on the bending of a loaded beam cut from a sheet of ice. Theory from which viscosity could be supposedly calculated is included. No calculations for viscosity carried out because author considered them useless since the viscosity depends on temperature, load, and history of ice bar. Deeley (R. M. Deeley, 1908: The viscosity of ice, Proceedings of the Royal Society of London, 250-9.) calculated and reported values of 3.32×10^{10} to 1.34×10^{12} poise.

Miyamoto, Susumu, 1933:
A theory of the rate of sublimation
Transactions of the Faraday Society, 29:794-797. (Extension of previous paper on p. 789 on the rate of sublimation of a solid into a liquid)

A theory of the rate of sublimation is proposed, based upon the assumptions, that among the molecules which make up the solid surface, only those molecules can escape whose energies of vibration resolved perpendicularly to the interface is greater than ϵ_0, and that among the molecules in the gas phase, only those which collide with the interface with perpendicular velocities greater than a threshold value u_0, can condense on the surface. From the Boltzman energy distribution the rate of sublimation can be determined as a function of ϵ_0 and u_0.

Monfore, G. E. and F. W. Taylor, 1949:
The problem of an expanding ice sheet
Proceedings of the Western Snow Conference, Reno, Nevada. Meeting, April 15, 16, and 17, 1948, issued at Fort Collins, Colorado.

The U.S. Bureau of Reclamation is conducting experimental investigations to determine adequate allowances for ice thrust in designing dams. Ice pressures are determined experimentally by pressure gages of two types; an indentor gage in which the impression of a steel ball is measured to get the maximum load, and a resistance gage in which a coil of wire is stretched to change the resistance as the load changes. Ice pressures resulting from controlled rates of temperature rise have been determined in the laboratory. Temperature rise in an ice-sheet resulting from daily changes in meteorological conditions are calculated considering: (1) exposure to air at a constant temperature; (2) exposure to air whose temperature is varying; (3) absorption of solar energy at the upper surface; (4) absorption of part of that portion of solar energy transmitted through the ice. Conditions in an ice-sheet covered with snow and in an ice-sheet free of snow were compared with surprising differences due to the insulation effect of snow. Curves show good agreement of computed and observed values. Authors state a need for more work on absorptivity of ice and check on pressure-temperature relations for ice. Ice-sheet temperatures were measured at Eleven Mile Canyon Dam in Colorado.

Mörikofer, W., 1939:
The physical properties of the snow cover
Association Internationale d'Hydrologie Scientifique, Reunion de Washington, Commission on Snow, General Papers, Report of the Scientific Results of Snow-Researches at Davos, (unpublished)

Summarizes the results of observations on the snow cover carried out at Davos from 1936-1939.

Mean values and variation of the density and total weight of snow cover are given for different climatological conditions.

The thermal economy is discussed briefly and general heat balance equations are presented. The variation in temperature along a snow profile is discussed with respect to the heat balance equations. The terms cold content and total cold content are defined.

The reflecting power and transmitting power of the snow cover are spoken of in some detail.

The results given for the albedo and absorption coefficient are from Thams' work. (Thams, Chr., 1938: Ueber die Strahlungseigenschaften der Schneedecke, Gerlands Beiträge zur Geophysik, 53:371-388.)

Morphy, Henry, 1913:
The influence of pressure on the surface friction of ice
Philosophical Magazine, 25(6th series):133.

A small aluminum sleigh was made to slide on a flat surface of ice, which could be tilted to any desirable angle. The coefficient of static friction was measured as a function of the load by tilting the surface until the sleigh started to slide. The temperature was held essentially constant. It was found that the static coefficient of friction remained constant at .36 ± .01 while loads were increased from 5.68 grams to 14.31 grams. The coefficient then dropped to .17 ± .01 for loads from 15.10 gm to 24.52 gm. Throughout the experiment the temperature remained approximately -5.63° C.

Murphy, E. J., 1950:
Alternating current conduction in ice
Physical Review, 79:396.

The equivalent conductivity of a dielectric at very high frequencies is denoted 0_p. Murphy asserts (no data given) that 0_p varies with temperature: $0_p = 631e^{-11500/RT}$ (ohms--cm)$^{-1}$ in the range 0 to -35° C. (E. J. M. says that a more complete formula will be given in the future.)

From this formula are calculated (a) the distance between the two equilibrium positions for the H atom between two oxygen atoms in ice, (b) the energy of the hydrogen bond in ice. Calculation (a) gives 0.39×10^{-8} cm instead of 1.6×10^{-8} cm. Calculation (b) gives 5.75 k cal/mole, as compared to 5.8 k cal/mole derived from measurements of the heat of sublimation.

Nakaya, U. and T. Iizima, 1934:
Physical investigations on snow. Part I, Snow crystals observed in 1933 at Sapporo and some relations with meteorological conditions.
Journal of the Faculty of Science, The Hokkaido Imperial University, Series II, 1(5):149-162, 4 plates.

Observations of snowflakes during one winter, classified according to the system used by Humphreys; of the possible 17 types, 11 were found. Some weather maps accompany study but the complicated topography and the complicated synoptic situations which usually form in the sea of Japan make the correlation with weather conditions extremely difficult.

Nakaya, U. and K. Hasikura, 1934:
Part II, Classification and explanation of snow crystals observed in the winter of 1933-34 at Mt.Tokati and at Sapporo.
Journal of the Faculty of Science, Hokkaido Imperial University, Series II 1(6).163-180, 14 plates

A modification of Bentley's snowflakes classification is presented. Photographs of examples of each class are presented.

Nakaya, U. and T. Terada, 1934:
On the electrical nature of snow particles
Journal of the Faculty of Science, Hokkaido Imperial University, Series II, 1(6):181-190, one plate.

The electrical charge on snow particles was determined by allowing the snowflakes to fall between oppositely charged plates and the deflection recorded photographically. An excess of negatively charged particles was noted but in the case of water droplets being attached, an excess of positive charge was noted. No essential difference in charge as a function of snowflakes size was noted as had been reported by other investigators.

Nakaya, U. and T. Terada, Jr., 1935:
Simultaneous observations of the mass, falling velocity and form of individual snow crystals.
Journal of Faculty of Science, the Hokkaido Imperial University, Series II, 1(7):191-200, 3 plates.

(Author's abstract) A simple method suitable for the observations of the mass, the falling velocity and the form of a snow crystal within a short time interval is described. The snow crystals are classified into six kinds and for each of them an empirical formula is obtained, representing the relation between the mass and the dimensions of the crystal. The density of graupel is found to be constant for various sizes of the samples, being 0.125. The thickness of plane dendritic crystals is more or less the same for all crystals and is about 0.01 mm. The falling velocity of a plane dendritic crystal is constant, being independent of the dimension. A similar fact is also observed in the cases of spatial dendritic crystals and powder snow. The velocities are also measured for graupel and needles, in which cases they are found to increase with the dimensions of the sample.

Nakaya, U., 1935:
On the correspondence of snow and rime crystals.
Journal of Faculty of Science, The Hokkaido Imperial University, Series II, 1(7):201-205, 3 plates

A number of microphotos of crystalline rime structure are compared with photos of natural snow crystals to show a general correspondence of crystal forms.

Nakaya, U. and I Sato, 1935:
On the artificial production of frost crystals, with reference to the mechanism of formation of snow crystals.
Journal of Faculty of Science, The Hokkaido Imperial University, Series II, 1(7):206-214, 3 plates.

Frost crystals of varying form were produced by varying the supersaturation of the water vapor above the surface at which sublimation occurs. Practically all types of natural snowflakes forms were reproduced in the artificial frost.

Nakaya, U., Y. Sekido and M. Tada, 1936:
Notes on irregular snow crystals and snow pellets
Journal of Faculty of Science, The Hokkaido Imperial University, Series II, 1(8):215-226, 7 plates.

Irregular and asymetric crystals are attributed to the various effects of the freezing nucleus; attachment of two nuclei which break apart, attachment of a second nuclei to the growing snowflake cause a distortion of the snowflake.

Snowflakes in the early graupel stage are also studied. A separate classification of graupel-like snow is proposed for snowflakes which have only a relatively few cloud particles attached.

Nakaya, U. and Y. Sekido, 1936:
General classification of snow crystals and their frequency of occurrence.
Journal of Faculty of Science, The Hokkaido Imperial University, Series II, 1(9):243-264, 6 plates.

(Author's abstract) A general classification of snow crystals is proposed which includes all types of snow observable in our climate. Some rare crystals that are not known in the literature have been observed and included in the list of this general classification. The results of observations on the frequency of occurrence of every type of crystal are described. It was seldom found that a snowfall consisted of only one or two types of crystal, and usually many types of crystal, sometimes almost all types were found in one snowfall, being intermixed with each other. The probable size of each of the types of crystal was measured and it was found that the crystals with dendritic branches were always largest, the columnar and plate forms were smallest, the intermediate forms going between the two extremities. Relations between the form of crystal and the meteorological elements were studied. No simple relation was found, except a rule that needle crystals were associated with the warmer weather.

Nakaya, U., M. Tada, Y. Sekido and T. Takano, 1936:
The physics of skiing, the preliminary and general survey
Journal of Faculty of Science, The Hokkaido Imperial University, Series II, 1(9):265-287, 5 plates.

(Author's abstract) Part I. The physical nature of ground snow is discussed. The transformation of snow crystals into ice grains by sublimation is studied under a microscope and microphotographs showing the process of transformation are reproduced. It is found that the stratigraphic structure of ground snow can be made visible by a blazing fire. This phenomenon is due to the varying ability to absorb thawed water found in the different strata of snow.

Part II. The mode of snow-compression by ski-running is studied by the blazing fire method which makes the stratigraphic structure of snow visible. This blazing fire method is found to be very convenient for studying the distribution of stress in snow. The flow of snow accompanied by the motion of the ski is examined, using soot as an indicator.

Part III. Supplementary experiments on the mechanism of ski-running are described. The position of the sliding plane of the ski is made visible by using red ochre pasted with alcohol to the tail end of the ski. The existence of a mobile layer in the snow is found by using the soot method.

Nakaya, U., I Sato and Y. Sekido; 1938:
Preliminary experiments on the artificial production of snow crystals
Journal of Faculty of Science, The Hokkaido Imperial University, Series II, 2(1):1-11, 4 plates.

(Author's abstract) An apparatus was designed, in which water vapour was brought up by natural convection to a cooled space where it condensed into frost crystals. The effect of sublimation was discussed in the case of these artificial frost crystals. A similar experiment was carried out in a cold chamber which can be cooled to -50° C. Conditions for giving a plate or dendritic crystal was examined by taking microphotographs of successive stages during its formation.

Artificial snow crystals were produced in a similar manner, by suspending the germ of the crystal with a fine rabbit's hair. Almost every sort of snow crystal observable in nature was produced in the laboratory. The relation between the rate of growth and the form of the crystal was examined. Similar artificial snow crystals were made with heavy water. They were almost the same in form as those made with ordinary water.

Nakaya, U., Y. Toda and S. Maruyama, 1938:
Further experiment on the artificial production of snow crystals
Journal of Faculty of Science, The Hokkaido Imperial University, Series II, 2(1):13-57, 10 plates

(Author's abstract) The artificial production of snow crystals was carried on in the cold chamber laboratory by using improved apparatus. The results of experiments carried out with the room temperature between -20° C and -30° C showed that the form of crystal is determined by the rate of growth of the crystal. The mean rate of growth was found to be 4.6 mm/hour for fern-like crystals, 1.3 mm/hour for sectors and plate and 0.5 mm/hour for side planes and irregular form. The favourable conditions for the development of crystal in a dendritic form was studied. The similar experiments were carried out with the room temperature at nearly -40° C or below, and it was found that the form of crystal was quite different from that obtained in the foregoing case.

Nakaya, U., M. Hanazima and K. Dezuno, 1939:
Experimental researches on window hoar crystals, a general survey.
Journal of the Faculty of Science, Hokkaido Imperial University, Series II, 3(1):1-13, 5 plates.

(Author's abstract) The difference in the origin and structure of window hoar and window rime is discussed. An apparatus for making window hoar crystals in laboratory is designed and the crystals grown on the glass surface are examined under various conditions of the surface. The relation between the cleanliness of glass surface and the form of hoars is investigated for three cases: that is (1) the glass plate thoroughly cleaned is dried in a natural course, (2) dried in a desiccator containing P_2O_5, (3) rubbed gently with cotton. It was found that the tendency of the water molecule to combine with glass exerts a strong influence on the form and structure of hoar crystals. This hypothesis was confirmed by the experiments of making the hoars on the glass surface coated with a very thin film of paraffin wax, alcohol or another organic compound. The hoars developed on a new cleavage-face of mica showed a star-like pattern and it was made clear that the form of surface hoar is strongly influenced also by the action of the atomic force of the base solid.

Northwood, T. D., 1947:
Sonic determination of the elastic properties of ice
Canadian Journal of Research, A25:88-95.

Young's modulus and Poisson's ratio for ice were determined by measuring the velocities of various types of elastic waves. Ice samples were obtained from Ottawa River or manufactured ice. Assumes samples consisted of several crystals and hence isotropic. Samples cut from ice blocks in a number of different directions gave no great variations. Temperature controlled in a test chamber with most measurements made at -15° C, for which $E = 9.8 \times 10^{10}$ dyne cm^{-2} and $\mu = 0.33$ (Poisson's ratio).

Nutting, P. G., L. A. Jones and F. A. Elliott; 1914:
Tests of some possible reflecting power standards
Transactions of Illuminating Engineering Society, 9:593-597.

This paper deals with determinations of the absolute reflecting power of various reproducible, diffusely reflecting surfaces with which the reflecting powers of other surfaces may readily be compared.

The reflecting power is defined as the relative brightness of two infinite parallel planes, one of which receives all its illumination from the other. The reflecting power of the surface illuminated is its brightness relative to that of the surface illuminating it. Since actual planes used are finite an attempt to reduce end effects is made.

Results relevant to SIPRE research are given below.

Type of snow	Average Reflecting Power
Fine, dry drifted	84.7
Natural surface, fresh	70.2
Old, beady	67.8

Appreciable differences were found in the reflecting power for polarization in, and perpendicular to, the plane of observation with the former being greater by 4 to 14 %.

Olsson, Hilding, 1936:
Radiation measurements on Isachsen's Plateau
Geografiska Annaler, 18:225-244.

The measurements of the radiation conditions made by the Norwegian-Swedish Arctic Expedition during its two month stay on Isachsen's Plateau are presented in this paper.

The values for the duration of sunshine, based on ocular observation, are presented in tabular form.

Values for total radiation from the sun and sky as measured with an actinograph are given. The actinograph was calibrated against two different radiation instruments. The effects of clouds and fog on the total radiation is discussed and the results obtained are compared with those of other experiments.

The albedo of the snow was measured with an Angstrom pyranometer. The results are summarized in the following table:

Mean Albedo (%)

	New snow on the surface	Frozen snow surface	Melting snow surface
Clear sky	81	75	59
Cloudy sky	75	65	65

Measurements of the albedo made with a photo-electric cell for frozen and melting snow in cloudy weather resulted in an average albedo of 85 % or about 20 % higher than the pyranometer measurements.

The radiation absorption of a natural snow cover was studied with the aid of a photo-electric cell and a pyranometer. The effects of snow density and type of snow could not be found. The radiation was found to fall off exponentially with depth in the snow cover. The value of k (absorption coefficient) was found to be 0.074 from pyranometer measurements at 5 and 10 cm below the surface, and 0.114 from the photo-electric cell measurements which were taken down to a depth of 1.3 meters.

The difference in results for albedo and radiation absorption from the two instruments may be partially accounted for by the difference in the wave length covered. (This would indicate that the reflecting power and absorption coefficient of snow was strongly affected by the wave length of the radiation falling upon it.) The portion of the spectrum covered by the pyranometer extends from 0.3μ up to 3 or 4μ while the photo cell covered only 0.3μ up to 0.6 to 0.8μ.

Olsson, Hilding, 1936:
Sunshine and radiation, Mount Nordenskiold, Spitzbergen
Geografiska Annaler, 18:93-118.

The results of observations made during the period August 1, 1932 to September 1, 1932, are presented here. The ground was apparently always snow covered.

The distribution of sunshine and the average cloudiness are presented in tabular and graphical form. Little discussion is appended to this data.

The effective outgoing radiation is measured (only when the sun is beneath the horizon) with an Angstrom compensation pyrgeometer. The values obtained are compared to the results of other experimenters and the relation of radiation to cloudiness is discussed.

Using the relations first expounded by Angstrom, the interchange of heat between the snow surface and the atmosphere is discussed using the observed data.

Osborne, Nathan S., 1939:
Heat of fusion of ice. A revision
Journal of Research of the National Bureau of Standards, 23:643-6.

The heat of fusion of ice has been recalculated from data published by the Bureau of Standards in 1914 and 1916. (See Harper and Osborne, Bull. Bur. Stds., 10:235 (1914) 5209; Dickinson and Osborne, Bull. Bur. Stds., 12:49, (1915-16) 5248).

New values for the heat capacity of water are used (Osborne, Stimson, and Ginnings, J. Research Nat'l Bur. Stds., 23: 197 (1939), R. P. 1228). The result is expressed in international joules:

$$h_f = 333.5 \pm 0.2 \text{ joules gm}^{-1}$$

Owston, P. G. and Kathleen Lonsdale, 1948:
The crystalline structure of ice
Journal of Glaciology, 1(3):118.

This is a general review of proposed structures for ice, and a discussion of possible crystal forms. Mention is made of the unexplained diffuse spots and streaks observed in X-ray photographs with ice. (See Owston, P. G. Acta Crystallographica, 2:222, 1949).

There is also a discussion of "unusual forms of ice," with special criticism of the work of Seiyakov.

Owston, P. G., 1949:
Diffuse scattering of X-rays by ice
Acta Crystallographica, 2:222.

The diffuse scattering is more carefully examined than before, and the general features of the scattering are given. The existing theory is inadequate to explain the observations, and needs extension to include the effects of shorter wave length vibrations.

The amplitudes of vibration of the molecules are of the order of 0.4 A. at -5° C. The strong local disturbances which cause diffuse streaks could include movements of the hydrogen atoms between neighboring oxygen atoms.

Penrod, E. B., 1950:
Measurement of the thermal diffusivity of a soil by the use of a heat pump
Journal of Applied Physics, 21(5):425-427.

Heat exchanger Pipe 56' long assumed infinite line source. Equation for heat conduction is

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right)$$

T = temperature
t = time
α = diffusivity

Computed by finite differences:

$$\frac{\dfrac{\Delta T}{\Delta t}}{\dfrac{\Delta^2 T}{\Delta r^2} + \dfrac{1}{r}\left(\dfrac{\Delta T}{\Delta r}\right)}$$

Thermocouples were placed in the soil at each 1/2 foot and temperature-time records obtained.

Found $\alpha = 0.019\ \text{ft}^2\ \text{hr}^{-1}$ for one soil sample.

Peterson, H. C., 1948:
Results of the solar radiation project of the Ronne Antarctic Expedition
Technical Report No. 3, Office of Naval Research, Department of the Navy, Washington, D.C.

Total horizontal incidence, sky and solar radiation were measured at 68° 12' S from October 1947 to February 1948 using a modified Epply pyrheliometer. Measured values of the total radiation are higher than those given for similar arctic latitudes and this discrepancy is discussed. Besides instrumental error this difference must be due to either (1) a smaller atmosphere transmission coefficient in the Antarctic (2) diffused radiation from the snow being scattered back toward the earth. Although investigation is inconclusive, the latter effect was believed to be the most important. The percentage decrease in solar radiation by clouds was also found to be less than that reported for temperate latitudes.

Pfalzner, P. M., 1940:
The friction of heated sleigh runners on ice
Canadian Journal of Research, 25(Sec. F):192-195.

The turntable method of measuring friction was used, and a miniature sleigh with electrically heated runners and various loadings was brought in contact with ice on the turntable.

The saving in work was calculated to be $2\pi \times 16/12 \times N \times F$ lb min^{-1}, where the diameter on the ice was 16 inches, R was the number of turns per minute and F was the difference in force measured. It was found that under the most efficient conditions, the saving in work expended in pulling the load was only 10% of the electrical power input at 20° F and 20% at -10° F. The maximum decrease in the work was also approximately 10% at 20° F and 20% at -10° F.

The author thinks that the results are contradictory to the frictional melting of ice, because, if the crucial factor in low friction is a film of water, experience in other connections shows that direct heating is more economical than frictional heating. The temperature of the runners was measured and it was found to be 32° F when in contact with the ice although the temperature was as high as 200° F when the runners were not in contact.

Plyler, E. K., 1926:
The growth of ice crystals
Journal of Geology, 34:58-64.

Present theories of the growth of ice crystals are briefly reviewed. The assumption of Wright and Priestly that ice contains some molecules which move around in the crystals is discussed. An account is given of the method of finding the infrared absorption bands of ice. It was found that the thin region between the ice crystals is highly absorbing. The boundary region between the ice crystals has an absorption coefficient many times that of the ice in the crystals. This fact is used to explain crystal growth in a solid piece of ice. This theory can be applied to ice in the natural state if proper allowance is made for the existing conditions. The author gives evidence that shows the theory of Wright and Priestly, concerning two types of molecules, to be unnecessary. There are two photos of ice crystals and boundaries in the paper.

Pope, N. K., 1949:
Thermal scattering of X-rays by a close-packed hexagonal lattice
Acta Crystallographica, 2:325.

An attempt is made to calculate the diffuse X-ray patterns which appear in Laue photographs of hexagonal crystals. The "extra spots" are surrounded by a weak background which exhibits considerable fine structure. The streaks observed by Lonsdale (K. Lonsdale, Nature, 158:582; see also P. G. Owston, Acta Cryst., 2:222) are not accounted for by this theory. (See M. Born, Nature, 158:830.)

Prager, W., 1948:
Theory of plastic flow versus theory of plastic deformation
Journal of Applied Physics, 19:540-3.

Discusses typical theories of plastic flow and plastic deformation. Shows that both theories (pl. flow and pl. def.) predict the same, and correct, result when either loading or unloading occurs.

Introduces the concept of a neutral change of stress which is a condition of neither loading nor unloading (e.g., loading when stress components are varied in such a manner that principal stresses preserve their magnitudes while principal directions rotate with respect to the material.)

Thus a neutral stress can be considered as a limiting case of both loading and unloading, and stress-strain relations for both loading and unloading should predict similar results when applied to this case, one obtains different results for the two cases (loading and unloading).

This casts severe doubts on the validity of the theory of plastic deformation.

Formulae are not complete (materials are assumed incompressible, and all viscosity effects are neglected, i.e., stress-strain relation assumed independent of the speed of deformation). Gives references where these effects are considered.

Reuter, H., 1948:
Ueber die Theorie des Wärmehaushaltes einer Schneedecke
Archiv für Meteorologie, Geophysik, and Bioklimatologie I, 1(Series A):62-92.

In Chapter I, a mathematical formula (1) for nocturnal cooling of the snow surface is set down. The effect of snow density, austausch coefficient and surface radiation is considered. Chapter II deals with the temperature variation with depth in a snow pack. The heat-flow problem is solved for a constant surface out-radiation and absorbed solar radiation, the absorbed radiation assumed to follow the exponential law. Calculations of the distance below the surface to the maximum temperature are made. Calculated distance, as to order of magnitude, agrees with observations, but the corresponding calculated temperature difference is several times the observed values. The author explains this as convection heat exchange in the upper layers. In Chapter III, diurnal and annual periodic variations of the temperature of the surface are calculated, assuming a sinusoidal solar radiation of the form, $i_0 = i_1 + i_2 \sin Nt$. ($N = 2\pi/T$ where T is the period). The annual calculations give good results, but calculations based on the diurnal period are not consistent with observations.

(1) Reuter, H., 1947: Zur Theorie der nächtlichen Abkühlung der bodennahen Schicht und Ausbildung-der Bodeninversion. S. Ber. Akad. Wiss. Wien, Abt. IIa 155, Heft 7 u. 8, 333 ff.

Reynolds, Osborne, 1901:
On the slipperiness of ice
Scientific Papers, 2:734-738.

Pressure melting of ice to cause a film of water is proposed as the lubrication causing the slipperiness of ice. At 31° F a man of 140 lb would have to have a skate bearing of 1.4/10 sq. in, at 22° F the bearing must be reduced to 1.4/100 sq. in. The observations used to substantiate the theory are the fact that a man slips on ice easier at 31 1/2° F than he does at 27° F. Also, that the ice skater experiences no resistance skating at any speed at 32° F, but on harder ice he must move quickly, or the skates seize. This is explained by the fact that the ice melts under the edge, but owing to the small area of lubricating surface, the lubricant is squeezed out rapidly, thus destroying the lubrication below certain speeds.

Richards, W. T., 1932:
The persistence and development of crystal nuclei above the melting point
Journal Am. Chem. Soc., 54:479-495.

The author considers various theories which account for crystallization in supercooled liquids and how preheating affects the crystallization. None of the theories are able to explain the phenomenon completely. The crystal nuclei are foreign particles carrying an adsorbed layer of the melt. This layer is assumed to be actual crystals of the melt.

Consideration is given to Tammann's work on "nuclei number." Dorsey calls this the rate of nucleation. Tammann defines a quantity called the crystallization frequency "f".

A large number of tubes containing samples of the substance investigated are prepared and set in a freezing bath.

The number of tubes is N_0.

If after a time "t," there remains a number of tubes "N_t," which remain unfrozen, they are related to the total number of tubes by this relation according to Richards.

$$N_t = N_0 e^{-ft}$$

However, Richards made experiments with some organic materials which had a freezing point of 42° C and 48° C. These were heated to 130° C for several hours and then placed in a cooling bath at 25° C and kept at 25° C for several months and none of the tubes crystallized. Richards concluded that "f" for these substances was "small." Actually it was zero.

This is in accord with Dorsey's theory in which there is no crystallization frequency ($f = 0$). The freezing temperature is determined by the largest effective mote present.

The work is concerned with different types of crystallization centers consisting of adsorbed layers of the crystal form on particles. These are supposed to be stable above the melting point.

The effect of supersonic waves is also investigated, which seem to have the same effect as shaking. Slight shaking when the substance was "deactivated" (the number, size, and effectiveness of the crystal adsorbates was lowered) had little effect, with more active crystal adsorbates. (Dorsey would call these "motes"). The shaking would produce freezing. Severe shaking would have a much greater effect on producing crystallization.

Richards, Kirkpatrick and Hutz, 1936:
Further observations concerning the crystallization of undercooled liquids
Journ. Am. Chem. Soc., 7(58):2243-2248.

The liquids investigated in this report are salol, benzophenol and benzene. The results are applicable to studies on water with slight limitations. Their experimental work definitely points toward the heterogeneous theory of freezing. The work is briefly described and in the discussion at the end of the paper the results are viewed in the light of theoretical considerations.

It was observed that very fine filtration and over-heating would increase the amount of supercooling possible; also, a liquid could be held in the supercooled state for a long period of time. Dorsey's theory of freezing is based, to some degree, on the results of the experimental work described in this paper.

Roper, Edwin, 1938:
The Ice-point as a standard of reference
Journal of the American Chemical Society, 60:866-869

An ordinary ice bath gives a temperature reproducible to 0.004°; this work describes an ice bath reproducible to 0.001°. This is done by washing the ice with cold (0.0 ± 0.1° C) distilled water to remove electrolytes from the surface. If the ice is melting rapidly, the washing must be repeated often, as impurities are exposed by melting. There is a brief discussion on air saturation of the wash water. It is shown that the presence of visible liquid in the ice bath is not necessary.

Rose, Edwin, 1946:
Thrust exerted by expanding ice sheet
Proceedings of American Society of Civil Engineers, 72(5):571-585.

A study and analysis conducted to obtain a rational procedure for estimating the magnitude of ice pressures on dams and other hydraulic structures, and a review of available material on the subject are presented in this paper.

Using experimentally determined (Brown E., and G. C. Clarke: Ice thrust in connection with hydro-electric plant design, The Engineering Journal, January, 1932, 18-25) ice temperature-pressure relationships, an analysis is made of ice thrust for different thicknesses of ice and rates of air temperature rise, and taking into account the effect of solar radiation. The effects of conditions of restraint around the ice sheet is estimated. Thickness and buckling of ice sheets, extreme temperature variations occurring in nature, and field measurements of ice temperatures and pressures are discussed. An analysis is made for comparison of measured and computed pressures. Curves are given to provide a guide for the reduction in reservoir water elevation allowing for the ice-thrust effect on existing dams subject to ice pressure but not designed for it.

It is concluded that the maximum design loads often used, 40 to 50 kips per lin. ft. (kip = 1,000 lb) are too extreme and are not realized even under the most severe conditions. The results (presented in graphical form) indicate maximum probable ice thrusts for continental U.S. to range from 5 to 20 kips per lin. ft.

Saito, R., 1949:
Physics of fallen snow
The Geophysical Magazine (Japan), 19(1-2)

Snow metamorphis was observed in a closed vessel and in natural snow cover. The two separate observations seemed to obey the same fundamental rules. Snow metamorphis was divided into three stages, the first of 5 days, the second of about 1 month, and the third for as long as the temperature stayed below 0° C. Many good photographs on various stages of snow metamorphis are given.

Diameter enlargement during metamorphis are given by $d = 0.86 - 0.83e^{-0.039t}$ where d is in mm and t is in days. Whereas the density variation of the snow below 0° C is $\rho = 0.34 - 0.25e^{-0.057t}$ where ρ is in gm cm^{-3} and t is in days. The metamorphis is divided into two groups dry and wet snow metamorphis; the density variation for wet snow metamorphis being much more intense.

Compression patterns differ greatly according to changes in temperature and density of snow masses. A comprehensive report on the variation of thickness, weight and density of snow masses, as a function of the time, is included.

Various formulae are proposed and results are given on "the ablation of snow." Experiments performed on the subsidence of snow surfaces and on snow hardness are explained, and results obtained are included. A way to artificially harden snow is discussed.

The article states that the coefficient of friction is independent of the base area of a sled, and that the static friction is usually 0.08 to 0.14, whereas the dynamic friction is usually 0.06 to 0.10. The resistance of a sled is divided into base friction, side friction and head friction.

Sauberer, Franz, 1938:
Versuche über spektrale Messungen der Strahlungseigenschaften von Schnee und Eis mit Photoelementen
Meteorologische Zeitschrift, Band 55, Heft 7, Juli

The author used a photovoltaic cell mounted in a container covered with an opal glass. Various filters were inserted between the ground glass and cell. To measure the albedo, the instrument is held in a horizontal plane, first with the opal glass toward the sky and then toward the snow or ice. By properly calibrating the instrument, the readings of the galvanometer may be used to measure the albedo. Since the sun and sky do not represent a diffuse source, corrections had to be made for various altitudes of the sun.

Using the expression, $I = I_0e^{-Kd}$ values of the absorption coefficient were determined for various wave lengths.

Snow Albedo

in Å	3800	4350	4750	5250	5900	6300	6600	7000	7350	7600
%	72	74	73	76	76	76	76	75	74	72
Ice										
%	44	46	48	50	50	48	50	48	47	43

Absorption Coefficient: Snow K = 0.150cm^{-1}

Ice 25 mm. thick

in Å	3800	4350	5250	5900	6300	6600	7000	7350	7600
%	84	86	86	85	87	84	87	87	86

Saxton, J. A., 1950:
Reflection coefficient of snow and ice at V. H. F.
Wireless Engineer, 27(316):17-25.

(Author's abstract) A review is given of the nature and composition of snow, and of the experimental knowledge of the dielectric properties of ice at very high radio frequencies. From this information an estimate had been made of the dielectric properties of snow. The presence of a layer of ice or snow on the earth's surface will produce a modification in the resultant reflection coefficient of the surface as a consequence of multiple reflections within the layer. A general formula is given in the paper by means of which the resultant reflection coefficient for plane waves in the presence of such a layer may be calculated. The formula is in terms of several basic parameters which have been determined for frequencies of 30, 300, 3,000 and 30,000 Mc/s. To illustrate the effect of these layers, the resultant reflection coefficients of a stratum of ice on sea water and one of snow on land have been calculated for frequencies of 300 and 3,000 Mc/s at angles of incidence of 0° (normal incidence), 45° and 80°, and for radiation polarized with the electric vector either perpendicular to or in the plane of incidence. It thus appears, for example, that the vertical-coverage diagram of a very high frequency radio transmitter may be appreciably modified by the presence of layers of ice or snow in depths likely to occur in practice.

Savage, Robert, H., 1948:
Graphite lubrication
Journal of Applied Physics, 14(1):1-10.

The slipperiness of graphite is explained as being due to adsorption films of water or other condensable vapors. Experiments were performed in vacuum, in air, and in the presence of other condensable vapors. The coefficient of friction in a vacuum is high, but where as little water vapor as 3 mm of pressure is admitted, the coefficient of friction decreases very rapidly.

The powder carbon produced when the experiment is performed in a vacuum absorbs hydrogen gas at a much higher rate than might be expected.

The friction of graphite on graphite at very low pressures of water vapor and high velocities is higher than for graphite on some metals. This fact is explained by the low thermal conductivity of graphite and the greater evaporation of the water in this case.

Schaefer, Vincent J., 1946:
The production of ice crystals in a cloud of supercooled water-droplets
Science, 104(2707):457.

Experiments are performed on a supercooled cloud formed by introducing moist air into a small commercial freezing unit. There is a temperature difference throughout the cloud: room temperature 27° C, temperature of air in center of cloud -15° C, at bottom of cloud -20° C, and -10° C at the top. During more than 100 experiments the water-droplet clouds never developed into ice-crystal clouds except upon cooling a portion of the cloud below -39° C or upon the introduction of certain foreign nuclei (silver iodide is the most effective).

Many substances were added to the supercooled cloud in an attempt to form ice crystals. The only one that was effective was solid CO_2--dry ice--within 10 seconds the supercooled cloud was completely converted to a cloud of ice crystals.

There are two excellent figures in the paper showing the crystals formed and also comparing them with snow and cirrus cloud crystals. The crystals formed were small.

"Besides using dry ice as a source of ice nuclei, a rod cooled in liquid air and passed rapidly through the supercooled cloud leaves a trail containing great numbers of submicroscopic nuclei which, due to microturbulence, spread through the cloud, causing it to dry up as the ice crystals grow."

Also a copper rod at -35° C will cause myriads of ice nuclei to form spontaneously from a cloud at -12° C. Replicas have been made of these nuclei; they are found to have dimensions of less than 1μ. "Some of these tiny crystals show the trigonal symmetry of crystalline ice and are thin, triangular prisms."

These methods of producing ice crystals depend on the lowering of the temperature below a certain point. Ice crystals may also be formed in a supercooled cloud by the introduction of crystals of a foreign substance, silver iodide was found to work very well.

Schaefer, Vincent J., 1947:
Properties of particles of snow and the electrical effects they produce in storms
Transactions of the American Geophysical Union, 28:587-614.

Measurements were made of the electrical field in the atmosphere over various periods of time, especially during storms. Apparatus is drawn and described. Results are discussed in relation to (a) passage of "fronts," (b) types of solid precipitation. Two general types of storm activity are recognized. The first is an air-to-earth current whose sign changes at frequent intervals; the second is a smaller current from sky to earth with positive sign.

The apparatus of Nakaya and Tarada was used for quantitative determination of the charge on individual snowflakes, together with a study of the type of flake observed. It is shown that the charge carried on snowflakes is negligible compared to that acquired when flakes are broken on impact.

The fragmentation of snowflakes on impact was studied, and the charge imparted to the impacting surface was measured. The charge was found to vary greatly, depending on surface coating. The results are related to the question of snow static in aircraft.

Finally, a procedure is given for studying the droplet size in clouds, fog, etc.

Schaefer, Vincent J., 1948:
The production of clouds containing supercooled water droplets or ice crystals under laboratory conditions
Bulletin of the American Meteorological Society, 29:175-182.

The basic laboratory experiment in which a supercooled cloud may be seeded and converted to ice crystals is described in considerable detail. Such information is given as: type and presentation of the cold chamber, light sources which are effective for different purposes, methods to follow in forming supercooled clouds, and procedures to follow in seeding them.

The transition temperature at which ice crystals form spontaneously is given as -39° C ± 0.1° C. as the result of a recent study. A simple replica technique for preserving the exact structure of the crystals in plastic is described.

The supercooled cloud may be transformed to ice crystals in two ways, either by chilling a small region of the cloud below -39° C or by introducing foreign particles of a specific crystalline form into the supercooled cloud. Dry ice introduced to the cloud is a very good way of cooling it down locally to the point where crystallization will commence. Silver iodide has been found to be a very efficient substance for the nucleation of ice formation. This work is described by Vonnegut in "The nucleation of ice formation by silver iodide," Journ. Applied Physics, 18(7):593-595; 1947.

Schaefer, Vincent, J., 1950
The formation of frazil and anchor ice in cold water
Transactions of the American Geophysical Union, 21(6):885-893.

(Author's abstract) Observations are presented concerning the formation of frazil and anchor ice under natural conditions and several methods are described for studying their physical properties. Photomicrographs show that the stable form of frazil ice is a thin, free-floating, rounded disk and that accumulations of these simple forms followed by regelation probably account for most of the underwater structures forming in bulk water supercooling to about -0.01° C. Frazil ice particles tend to gather on the upstream side of underwater objects with low adhesion between the collector and the ice mass.

Anchor ice forms as thin crystalline sheets firmly attached to underwater objects. Those observed were unlike frazil ice in appearance and general characteristics. A method is described for inducing the formation of frazil ice on a large scale by seeding supercooled water with small fragments of solid CO$_2$. Practical applications are suggested to determine whether this method may have economic importance wherever frazil ice causes trouble to hydroplants and similar installations in flowing streams which develop supercooling.

The experimental work in this report is very good but the theoretical interpretation of anchor ice according to Barnes's theory has been challenged by Altberg. (See: 20 years of work in the domain of underwater ice formation, Ass. Int. d'Hydrologie Sci. Bull. N. 23 (1936) p. 373-402.) Altberg is of the opinion that there is no basic difference between anchor ice and frazil ice. He classifies them as the same phenomenon which he terms underwater ice.

Schnurman, R., 1940:
Angle of repose of snow on solids
Nature, 145(2):553-554.

Angle of repose of loose snow and compressed snow measured at approximately 20° C. Compression by hand with no measurements reported. Coefficient of static friction also reported. Data summarized in following table:

	Angle of repose		Coefficient of static friction	
Base	**Loose**	**Compressed**	**Loose**	**Compressed**
Bakelite	16°	3°	0.29	0.05
Glass	19	3	0.34	0.05
Ferrotype	16	7	0.29	0.12
Steel	> 90	35		0.70

Seligman, G., and Frank Debenham, 1943:
Friction of snow surfaces
Polar Record, 4(25)2-11

Part I: Friction on Ski. A summary of work done by various people along with field observations made by the author. The condition of snow as a definite factor in resistance to skiing is discussed thoroughly. Observed skiing conditions are correlated with observed weather and snow conditions. The possibility of the snow particles having a ball bearing action between ski and snow is suggested, and the fact that skiing is more difficult in snow with long spicules or interlaced snow is given as possible evidence of the theory.

Part II: Friction on Sledge Runners. Also more or less a summary of observations made by men on polar expeditions. The observations seemed to show that the friction of sledge runners was least between 0° F and 15° F, and that the friction increased if the temperature increased above or decreased below these values; the friction becoming very great at -40° F or near melting. Most expeditions highly favored wooden runners over metal runners as having lower friction, although one expedition on salty ice favored metal runners. The mudding of sledge runners by Eskimos is discussed and an attempt is made to explain the reason for it.

Seligman, G., 1947:
Extrusion flow in glaciers
Journal of Glaciology, 1(1)

The author discusses the work of Dr. R. Streiff-Becker, who made yearly measurements of snow accumulation on the Claridenfirn (Canton Glarus). From the collected data Streiff-Becker concludes that the surface speed of the glacier is not sufficient to remove the yearly accumulation and therefore the flow must be faster below the surface. Extrusion flow hypothesis is presented and discussed. A lengthy discussion in which several glaciologists comment on Streiff-Becker's work is included.

Sharp, Robert P., 1947:
Suitability of ice for aircraft landings
Transactions of the American Geophysical Union, 28(1):111-119.

(Author's abstract) The thickness of ice required for aircraft landings on skis can be calculated from the following formulae:

$$S_r = (15/4)\ W_t\ ;\ S_l = (27/8)\ W_t\ ;\ S_s = (27/4)\ W_t$$

where S_r is the thickness in inches for river ice, S_l for lake ice, and S_s for old sea ice. W_t is the weight of the plane in tons. These formulas allow for the static weight of the plane and its dynamic impact at the time of landing. The thicknesses calculated are for ice formed at or below 16° F. For ice formed at higher temperatures the thicknesses must be increased by about 25%. Young sea ice is weaker than old sea ice and must be about three times as thick as river ice. Planes on wheels require 20% greater thickness than calculated from the above formulae. Ice is stronger at lower temperatures, and its strength increases about four times between 23° F and -76° F. Salt, air bubbles, included vegetation, cracks, and heavy snow cover all make ice weaker. Ice also experiences elastic fatigue under constant heavy use and must be rested frequently.

Freeze up usually takes place many days or even several weeks after the mean air temperature falls below freezing. After the water temperature reaches freezing, the rate of formation and growth of ice may be predicted from a curve of degree days of frost or calculated by formula.

Salinity retards freezing by lowering both the freezing point and the maximum density temperature. Currents, waves, and snow cover also retard the growth of ice, but under proper conditions wind can be an asset. Ice continues to increase in thickness well beyond the period of minimum air temperature. Thereafter, it maintains its maximum thickness for a short time before showing a rapid decrease just prior to breakup. Ice deteriorates considerably in spring and its strength decreases more rapidly than its thickness. This is particularly true of lake ice which "candles." Salty ice honeycombs extensively.

The surface of most frozen fresh water bodies is usually suitable for landings, but sea ice is often rough and broken. However, many successful landings and takeoffs have been made from the ice pack, and a suitable place can frequently be found. Firm, wind-packed snow can have a bearing strength of 100 to 200 p.s.i., but wind-swept surfaces are often rough owing to drifts and sastrugi.

Shaw, P. E., 1930:
Frictional electricity
Philosophical Magazine, 9(7th series):577.

Some recent results in frictional electricity are discussed: (1) discharging an electrified body by passing it through a flame alters the character of the surface. Better results are obtained with the ionized air surrounding the flame; (2) A glass rod with a clean surface is shown to rub negative with respect to silk. Most glass rods do the opposite because of a film of organic material on the glass; (3) Frictional effects when two nominally identical substances are rubbed together are shown to be due to surface differences induced by rubbing. A strain-and-recovery mechanism is proposed for the surfaces involved; (4) Impact of surfaces can also generate "frictional" effects, but if two insulators are struck sharply together by a glancing blow, they acquire a net negative charge. A net positive charge must be imparted to the air; (5) Similarly, powders become charged when blown from a vessel by air;

(6) Films of water on surfaces usually make the surface negative when rubbed. Alkalies do also, possibly by formation of OH^-. Acid films are more likely to be positive, possibly by formation of H^+.

The explanation of triboelectric effects by contact potentials is critized. The equation

$$Q = \frac{a}{b}(1-e^{-bw}) \quad \begin{array}{l}(Q = \text{charge developed; } w = \text{total work done;}\\ a,b = \text{constants of surface)}\end{array}$$

is considered to be too simple to explain many effects.

Smith-Johannsen, Robert, 1948:
Some experiments in the freezing of water
Science, 108:652-654.

A brief review of some previous works by other investigators is given. Work has been done on the spontaneous freezing of liquid water in the bulk and also on supercooled water droplet clouds. Vonnegut has investigated the effect of a number of foreign particles which will turn a water droplet cloud into an ice crystal cloud at temperatures higher than the critical temperature. The "critical temperature" referred to is the one established by Schaefer as -38.9° C. The most effective substance in causing ice-crystals to form in a water-droplet cloud was found to be silver iodide.

In the present paper experimental work was done to determine the crystallization temperature of pure water and the effect of various materials on it. The experimental work is clearly described and results given. The presence of ice in the water was observed by a combination of polaroids.

Graphite was found to be a much more effective nucleus for crystallization in water than silver iodide. However, it does not act as a sublimation nucleus while silver iodide is a very good sublimation nucleus. Thus the conclusion is drawn that there is a difference between "sublimation" nuclei and "freezing" nuclei and that there is a different mechanism in each case.

A table, showing the effect of eleven materials on the spontaneous freezing point of water, is included. (The water used was very pure and dust free.)

Ultrasonic irradiation of water before freezing will lower the spontaneous freezing point considerably. The average temperature of crystallization after ultrasonic irradiation was found to be -30.0° C.

The average crystallization temperature of pure water--in these experiments--was -19.2° C with a maximum of -18.0° C and a minimum of -20.5° C.

Soils, Foundation and Frost Effects Laboratory, 1949:
Appendix to report of investigations dated June, 1949: Investigation of snow compaction methods conducted for Engineer Research and Development Laboratories, fiscal year 1949: Tables, drawings and photographs; descriptions of specialized procedures; plan of test. Corps of Engineers, U.S. Army, New England Division, Boston, Massachusetts.

The modulus of elasticity (E) for compacted snow and ice were determined by measuring the resonant frequencies of test beams. The method proved quite satisfactory for ice, but difficulties were encountered in the case of snow samples, especially at or near the freezing temperature. The ice samples were tested under two different crystal orientations; with the optic axis parallel to the load and perpendicular to both the load and the length. Results in the two cases differed slightly, the variation being less at lower temperatures. However, more important than the variation is the fact that they were not consistent; the parallel orientation gave lower values at 33° F but gave higher values at 15° F. Reported values (approximately 7.0×10^{10} dynes/cm^2 are slightly lower than the values obtained by other investigators using sonic methods (10.0×10^{10} dynes/cm^2).

Stakle, Peter, 1936:
Frazil and anchor ice in the rivers of Latvia
International Assn. of Scientific Hydrology, Bull. No. 23, 351-366.

Ice conditions on the three largest rivers of Latvia (the Daugava, the Tielupe, and the Venta, catchment areas of which are 83000 km^2, 17814 km^2, and 11817 km^2, respectively) are considered in this report; also several smaller rivers are considered.

Frazil and anchor ice form every year in these rivers. After frost has occurred and the river temperature drops a little below 0° C, anchor and frazil ice begin to form. Photographs in the text illustrate the formation of anchor ice in the same positions in successive years. Anchor ice may stay in the same position all winter and grow until it forms an ice island.

When anchor ice breaks loose from the bottom it joins the frazil ice and surface ice--all together composing drift ice. The effects of drift ice on the river banks and artificial barriers such as dams and bridges is discussed.

The break up and ice jamming every spring is discussed together with methods for its control.

Data on ice movements have been recorded since the year 1530. The average date (average of 110 years) of freezing up of the Daugava at Riga is the 24th of November and of the breaking up (average of 286 years) the 5th of April.

Stern, W., 1930:
Ueber Grundlagen, Methodik, and Bisherige Ergebnisse elektrodynamischer Dickenmessung von Gletschereis.
Zeitschrift für Gletscherkunde, 18:24-42.

Four procedures for measuring the thickness of surface covers are discussed briefly. Conditions are described under which they might be applicable to depth measurements on ice. The "capacity" method is given special attention, since that was the one actually tried. Measurements are described as made on the Hochvernagtferner during 1927 and 1928, using two different sets of apparatus. Results of the measurements are given (depth ranging from 8 to 23 meters on the tongere, to 38 meters farther back).

Sverdrup, H. U., 1935:
Scientific results of the Norwegian-Swedish Spitzbergen expedition in 1934, Part III: The temperature of the firn on Isachsen's Plateau, and general conclusions regarding the temperature of the glaciers on West-Spitzbergen.
Geografiska Annaler, 17(1-2): 54-88.

This paper is divided into eleven sections; (1) Introduction; (2) Instruments and methods of observation; (3) Observations; (4) Time changes of temperature at given depths; (5) Variation of temperature with depth; (6) The rise of temperature of the firn in summer; (7) The average density of the firn; (8) Cooling of the firn in winter; (9) The formation of the ice layers and the irregular rise of the temperature in summer; (10) Drainage of the firn fields; (11) Temperature of the snow melt during freezing.

Observations were made at nine stations plus the main station. Data is presented in tables and curves. Formulae used to check observed values with computed values are given.

Warming of the firn in the summer is attributed to heat given up by melt water trickling through. Formation of ice layers is explained. Ice layers affect water percolation. Cold winter temperatures do not penetrate beyond 10 meters. Below this depth temperature 0° C. Frost penetration at night is less than heat conduction calculations show because of heat given up when melt water freezes.

Swift, H. W., 1926:
Determination of the modulus of elasticity by dynamical methods
Philosophical Magazine, 2(series 7):351-368.

Theoretical derivation of periods of vibration for various types of beams. Using derived equations and knowing period of vibration Young's modulus E or flexural rigidity EI can be determined. Experimental results verify theory and derived formulae.

Tarr, R. S. and von Engeln, 1914:
Experimental studies of ice with reference to glacier structure and motion
Zeitschrift für Gletscherkunde, 82-107.

Series of tests was run to determine the crushing strength and flow characteristics of pond and glacier ice. No attempt was made to control temperature or rate of application of load, and measurements appear to be at best approximate.

Tests showed that when a cylinder of ice was subjected to compressive load and left overnight; the crystal orientation changed by 90 degrees. Crystals were originally oriented with optic axis parallel to cylinder's axis, but after the load was removed the optic axes were perpendicular to cylinder's axis. Crushing strength of pond ice reported as 1000 psi and 350 psi for loads respectively parallel to perpendicular to optic axis and temperatures between 18° and 20° F.

Thomas, James L., 1934:
Reproducibility of the ice point
Journal Research National Bureau of Standards, 12:323.

Measurements were made by comparing the temperatures of different ice water baths. Temperature measurements were made by resistance measurements and triple point measurements. The average difference in temperature between the triple point and the ice bath was 0.0097° C.

The conclusion of the work was that the ice point is reproducible to 0.0001° C if corrections are applied for barometric pressure. This can be done even when preparing the ice bath with commercial ice.

Timonoff, V. E., 1936:
On the establishment of a working hypothesis of ice phenomena in lakes and rivers
International Assn. of Scientific Hydrology, Bull. No. 23, 367-372.

The author cites the need for a theory covering all types of ice phenomenon in nature and mentions the practical importance in hydro-electric power plants, etc. The previous observations are scattered and incomplete. The establishment of such a broad theory can only be possible through a systematic study of the ice phenomenon in the laboratory and in the field.

The principles involved are listed and briefly discussed. A distinction is made between dynamic and static ice formation and the differences are discussed.

Frazil ice, anchor ice, supercooling, heat transfer from bed rock, water, atmosphere and the occurrence of crystals are all discussed.

The author has also outlined a general plan for the construction of a crytechnical laboratory.

Trainor, Lynne E. H., 1947:
The spectral reflection and absorption of radiation by snow
M. A. Thesis in the Physics Department, University of Saskatchewan, Saskatoon, Saskatchewan.

A description of a cavity radiator and spectrometer for measurements of reflectivity and absorption of snow in the infra red is given. No data, however, is given for such measurements.

Albedo measurements were made in the visible region, using a constant deviation Hilger spectrometer and making wedge photographs of alternate reflections from a two-centimeter snow sample and $MgCO_3$ diffuser. With a similar arrangement, the relative coefficient of absorption was measured.

Ubbelohde, A. R., 1946:
Metastable forms of ice produced by supercooled water
Nature, 157:625.

Frank's reference to a "low-temperature form of ice" (Nature, 157:267) may lead to confusion; a better term would be "metastable form of ice."

Vadilo, P. S., 1939:
Methods for the investigation of the structure of ice
Comptes Rendus (Doklady) de l'Academie des Sciences de l'URSS

Two methods are given. (1) A slab of pond ice is exposed to damp air above 0° C; the air is then cooled below 0° C. The hoar crystals which form on the ice are oriented according to the orientation of the crystal on which they formed. By examining these crystals the grain structure and orientation of the slab may be ascertained, even if the grain size is very small. (2) Graph paper is pressed on the (bottom) side of the ice sheet, and the plain side is rubbed with a black pencil. In this way the area of the individual grains is easily determined by counting squares on graph paper. (Technique No. 2 has been used by Seligman, Bader, et al.)

Walton, J. H., Jr. and R. C. Judd, 1914:
The velocity of the crystallization of undercooled water
Journal of Phys. Chem., 18:722-728.

Very little work has been done on the velocity of crystallization of water. Turmlirz in 1894 measured this velocity with the degree of supercooling ranging from -0.74° C to -4.60°. The results of his work are listed in a table at the beginning of this paper. They show a steady increase in the velocity of crystallization with the increase in the amount of supercooling.

The present paper is concerned with the various factors which affect the velocity of crystallization. The experimental work is described--the freezing is done in a long tube and the linear velocity of crystallization is measured. The freezing is initiated in one end of the tube by scratching it with a piece of wire--this freezing phenomenon and other remarks on freezing in the article are readily explained by Dorsey's theory of freezing.

The lowest temperature that it was able to work out measurements for was -9.07° C. At lower temperatures spontaneous freezing took place. The maximum velocity, however, should correspond to a lower temperature than -09.0° C as can be seen by the curves in the paper.

The thickness of the wall of the tube used in these determinations is an important factor in determining the speed of crystallization. This fact should be helpful in future work. Spontaneous crystallization occurred much more frequently in thin-walled tubes. This is attributed to the more rapid and uneven cooling of the liquid.

Washburn and Smith, 1933:
Isotopic fractionation of water by distillation and by adsorption
Journ. Chem. Physics, 1:426.

The boiling point of water is known to increase with higher concentration of the heavier isotopes of H_2 and O_2. This letter to the editor is concerned with the separation of these isotopes by distillation, taking advantage of the differences in boiling points. In a paper by Wahl and Urey (Journal of Chemical Physics, 3:411) the vapor pressures of these different concentrations of isotopes is sought.

The lighter isotopic molecules have the higher vapor pressures. "Distillation fractionation is possible and should find practical application in combination with electrolysis fractionation." The amount of adsorption by various material is different for different isotopic composition also.

Weinberg, B., 1936:
Mechanical properties of ice
Transactions of the International Committee of Snow and Glaciers; International Association of Scientific Hydrology, Bulletin No. 23, Riga.

The behavior of solids subjected to external forces is divided into three consecutive groups, namely: (1) Elastic stage; (2) Plastic stage; (3) Brittle or breaking stage. The usual definitions and mathematical equations are used to define each stage.

A search of the existing literature was evidently made, and data compiled for the various properties. Indications are that the limit of elasticity is very low, of the order of 1.0 kg/cm². On this basis Weinberg states that most research was probably carried out beyond elastic limit and hence reported values of Young's modulus are not true values. However, most values reported are of the same order of magnitude (10.0×10^{10} dyne/cm²) as those obtained from sonic determinations. Article includes great quantity of data, most of which is quite old.

Weissman, S. I., 1948:
Mechanism of conductance by hydrogen and hydroxyl ions in aqueous solutions
Nature, 161:241-242

Direct-current measurements were made of the conductivity of electrolyte solutions frozen very rapidly to liquid air temperature and then warmed to temperatures from -120° C for measurement. The limit of sensitivity was 2×10^{-10} (ohm-cm)$^{-1}$. Results could be reproduced only within a factor of 2-3, probably because of internal cracking and variations in the concentration of solute.

Results: (a) Conductivity was not detectable in any case at liquid air temperatures; (b) Conductivity was not detectable with any salts.

Substance	Temperature	Conductivity
0.06 N HCl	-115° C	6×10^{-9} (ohm-cm)$^{-1}$
	-106	3×10^{-8}
	- 94	9×10^{-8}
0.08 N LiOH	- 69	2×10^{-9}
	- 51	1.5×10^{-8}
	- 42	2×10^{-8}
0.056 N HNO_3	- 84	3×10^{-9}
	- 70	1×10^{-8}

Calculation of activation energy from these results gives 6-10 k cal/mole, as compared to ~ 8 k cal/mole for the activation energy of "rotation" of a water molecule in ice calculated from dielectric data.

White, W. P., 1934:
Freezing points and triple points with water
Journ. Am. Chem. Soc., 56:20.

This paper is concerned with the very accurate determination of the ice point. It is rather easy to get consistent results with a precision of 0.002° C. but it is considerably more difficult when the accuracy of 0.0001° C is desired. Such accuracy, in fact, has been deemed impossible.

Two methods of getting the ice point are described and compared. First, the "cold cell" or ice bath was used. The ice and water are kept as pure as possible by keeping the ice washed with very pure distilled water and excluding any external melting effects.

An ice point at atmospheric pressure, constant to 0.0001° C for a day at least, can be made with good commercial ice. By washing the completed bath with water, (chilled water to prevent any melting) the impurities in the ice are completely removed from the surfaces, and then the accuracy of the bath depends solely on the purity of the water. A device was employed which enabled water to be run through the bath without changing the ice, this kept the ice surface clean.

Triple point determinations were also made but they involved more experimental trouble. The impurities were not removable as in the "ice bath" work. The triple point is convenient and reliable for a short time, but the "ice cell" or "ice bath" method is better for continuous or extended work. The triple point apparatus is excellent, though it requires more attention and work than the cold cell.

Whitman, W. G., 1926:
Elimination of salt from seawater ice
American Journal of Science. 11(62-5th series):126-132.

A mechanism is proposed for the elimination of salt from solid ice. If drops of liquid brine are embedded in ice where there is a temperature gradient, the liquid drop will work its way toward the warmer surface, becoming progressively more dilute. The brine will finally be eliminated. This mechanism is inactive below -6° F. (-21° C). The action is explained in terms of the phase diagram H_2O-NaCl, and the explanation is supported by experiments on ice containing NaCl.

Wilson, J. T. and J. M. Horeth, 1948:
Bending and shear tests on lake ice
Transactions AGU. 29(6)

Tests were performed on samples of lake ice and on samples frozen in the laboratory. The tests on lake ice were carried out on the deck of a coast guard cutter on Lake Michigan. The laboratory samples were so prepared as to have the same crystal orientation as the lake ice. All tests were carried out with the load applied parallel to the optic axis. Temperature was controlled in the laboratory to within a few degrees. Rate of application of load not uniform, but applied so that all tests were completed in from one to three minutes. Bending tests made over a much longer period indicate strength increases with decrease in temperature, average strength varying from 180 psi to 256 psi at temperatures of 32° F and -9° F. Results of tests carried out at the same temperature varied widely, for example from 126 to 266 psi at 32° F. Results of shearing tests indicates temperature not a factor. The average strength reported is approximately 100 psi, again results varied widely for a given temperature, ranging from 41 to 161 psi at 32° F.

Wilson, W. T., 1941:
An outline of the thermodynamics of snow melt
Transactions of the AGU, Part I, 182-195.

Estimations of melt water per 1/2 day due to various effects are made. An outline of the topics discussed is as follows:

A. Theoretical melting at a point. (1) Heat required to bring snow to the melting temperature. (2) Rate of heat transfer through the snow. (3) Heat diffusivity of snow. (4) Sources of heat. (5) Heat losses. (6) Temperature of the snow surface.

B. Sources of moisture made available as the snow melts. (1) Melting frost in the soil. (2) Release of liquid water in the snow.

C. The effect of mechanical forces on the thermodynamics of snow melt.

D. Areal significance of melting rates. (1) Trajectory of air over snow. (2) Effects of variation in elevation. (3) Areal distribution of snow. (4) Exposure of the snow.

E. Example.

A graphical method is given for forecasting the average snow melt in inches for given wind velocity and wet and dry bulb temperature measurements.

Wollan, E. O., W. L. Davidson and C. H. Shull, 1949:
Neutron diffraction study of the structure of ice
Physical Review, 75:1348.

Ice made from D_2O was ground and studied in a neutron diffraction apparatus. The calculated patterns obtained from four different proposed structures were compared with experimental results. Pauling's proposed structure (L. Pauling, J. Chem. Soc., 57:2680) agrees most nearly with the observed diffraction. The models by Barnes and by Bernal-Fowler are definitely in disagreement with experiment. The temperature was -90° C.

Workman, E. J. and S. E. Reynolds, 1948:
Thunderstorm electricity
Signal Corps Research, File #15842-PH-46-91 (SCEL)

General properties of thunderstorms are described, especially electrical properties. Previously reported properties of ice and water are believed inadequate to account for all phases of thunderstorm electricity. Previous studies by Workman et. al., on charge distribution in thunderstorms are reviewed in some detail, and a list of "basic physical specifications" for thunderstorms is given.

After many perliminary studies, including repetitions of several classical experiments, two types of equipment were set up to determine "freezing potentials" and the separation of charge induced by freezing. The p.d. between water and ice which occurs during freezing was found to vary from -185 to +70 volts, depending on the nature and concentration of the solvent, with charge separations up to 4.4×10^5 e.s.u. per cc of water frozen. The results are discussed in terms of a crystal growth process in which ions are preferentially absorbed into the crystal lattice. This mechanism is supported by some rough chemical tests on the frozen and unfrozen parts of the solution.

It is suggested that the effect may have applications in chemical analysis.

A mechanism for charge separation in thunderstorms due to this effect is considered, with regard to some of the properties mentioned earlier. Attempts to support the hypothetical process by refreezing rain water were unsuccessful; this is probably because chemical changes took place during the first freezing.

Construction and operation of a "generating voltmeter type electrometer" are described in an appendix.

Note: A more concise version of this paper is presented in the Physical Review, 78:254 (May 1, 1950). The suggestion is made there that thunderstorm electricity should be capable of great modification by the use of ammonia.

Yosida, J. and Hiroshi Iwai, 1946:
Thermal conductivity of ground snow
Seppyo, 8:48-53.

A snow sample of uniform density was placed in an air-tight cylindrical container. The temperature at the top and bottom surfaces could be rapidly changed, using an ethylene-glycol solution as a heat sink. The average temperature of the snow was measured using an instrument similar to a gas-expansion thermometer. Air, which permeates the snow, was used as the working fluid of this instrument. The equations for heat flow in the snow are given and from the temperature of the snow at different times, after change of top and bottom surface temperatures, the diffusivity of the snow could be found directly. The thermal conductivity was found from the diffusivity, density and specific heat (which was assumed to be .4807 cal/gm).

The results of several other experimenters are reproduced.

Different values of diffusivity for the same sample were obtained when surfaces were warmed (-1° C) after being cooled to -6° C. In general increased with density, but effects of crystal shape and general state of snow were believed to be important factors. Evaporation from the snow crystal was believed to materially effect the heat transfer.

Young, S. W., 1910:
Mechanical stimulus to crystallization in supercooled liquids
Journal American Chemical Society, 33:148-162.

The effects of stirring, friction, and impact on the freezing of supercooled water are investigated. The experimental work is described clearly and completely. It is stated that "the phenomenon of the supercooling of liquids is apparently of universal occurrence. It rarely, if ever, happens that a liquid cannot be cooled some distance below its melting point without solidifying."

Ostwald introduced the idea of the metastable limit. Young shows by his experiments that Ostwald's metastable limit need not exist. Ostwald's theory can be summed up in a few words.

When a liquid is cooled below its normal melting point it is in an unstable condition. The greater the degree of supercooling the greater is the unstability.

This unstable field is then broken into two parts. (1) The metastable field or state--in this field the only way to cause it to crystallize is by the introduction of a crystalline part of the solid phase. (Without the "seeding" it cannot spontaneously crystallize no matter what shaking, stirring, jarring, etc., takes place.) (2) Upon further supercooling--the liquid will transfer abruptly into the labile state; in this condition it can be crystallized by mechanical disturbance.

Experiments were performed in which the water was disturbed by stirring, sliding friction and bubbling gas, by rotational friction and also by impact.

The freezing in the stirring and rubbing experiments always occurred at the point of rubbing contact.

The results of the rotational friction experiments are interpreted, by Young, as caused by a succession of impacts of the two sliding surfaces.

Young concludes that no matter how slightly supercooled a sample of water may be one can always cause it to crystallize by impact. The more extreme the supercooling becomes, the smaller is the impact necessary. (Dorsey believes that there may be some other factors entering into Young's experiments such as movement of the meniscus, which might introduce additional motes.)

The results are plotted on a curve to show that "there seems to be no justification for dividing the supercooled field into metastable and labile fields..."

Young, S. W. and R. J. Cross, 1911:
The mechanical stimulus to crystallization, II
Journ. Am. Chem. Soc., 33:1375-1388

In this paper the authors investigate the effects on crystallization caused by foreign particles in water. Two possible effects of foreign particles are proposed: (1) That the effect is entirely a kinetic one--all stimulus to crystallization is of a mechanical nature; (2) That it is at least in part of some other nature, not at present definable.

In this investigation the consequence of the variation of the following factors are examined, assuming a purely kinetic effect: (1) Effect of mass--the larger the mass of the particles the less supercooling is to be expected; (2) Effect of the number of particles--the greater the number of particles the greater is the probability of impact and, consequently, less supercooling will take place. When few particles are present the values of supercooling are expected to be more erratic. After a fairly large number of particles are present the results for the supercooling observed will be more nearly constant and will be independent of increases in number (within considerable limits); (3) Effect of viscosity--the viscosity of the liquids was also investigated. The greater the viscosity the more slowly particles would move and the rate of the impact and severity of impact were less. The viscosity increase will effect smaller particles more than the larger ones because of their relatively greater surface area.

Experimental work: Quartz particles of varying sizes were prepared. Known numbers of a known size were placed in water and agitated by a stream of gas within the tube. By varying the size and number of particles present a series of results were obtained which seem to confirm the ideas stated above. Experiments on increasing the viscosity by adding gelatin also supported the above ideas.

Young, S. W. and W. J. Van Sicklen, 1913:
The mechanical stimulus of crystallization
Journ. Am. Chem. Soc., 35:1067-1078.

Experimental apparatus and results are carefully described. The apparatus consisted of a hammer which dropped on an anvil, the hammer was controlled by a sleeve of metal and dropped from a variable height.

All of the experiments in this article are concerned with crystallization due to impact. The impact value is determined by the mass of the hammer and the distance through which it falls.

When starting with polished surfaces (on the hammer and anvil) a rather large impact value was needed to start the crystallization. Upon repeating the procedure, it was found that a smaller impact value would start the crystallization than was required in the first case. This was called an "increase in the sensitiveness of the impact parts." This increase continued for a considerable number of impacts and then remained fairly constant for some time. After a while, the parts began to show fluctuating values and "became wholly unreliable."

The authors give more evidence on this increase in sensitiveness and offer some explanations for it.

In all cases the increased sensitiveness could be destroyed by repolishing.

An investigation is made to determine the increasing of hardness of the impact parts, and what effect this has on the sensitiveness.

The degree of supercooling necessary before crystallization can be forced by impact is investigated. It is found that very slight supercooling is necessary. (less than -0.02)

The relation between energy of impact and supercooling: The authors have tried to find a quantitative relation for this. Curves have been drawn up which show a definite relation between the amount of supercooling and the energy of the impact. Curves are made for two hammers--one is made of stellite, an alloy of cobalt and chromium, the other is made of steel. The curves are entirely similar except that they are slightly displaced. The impact values for stellite are slightly higher than those for steel at the same temperature.