R 649

Technical Report

STRUCTURAL RESPONSE OF UNSTIFFENED TOROIDAL SHELLS

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STRUCTURAL RESPONSE OF UNSTIFFENED TOROIDAL SHELLS

Technical Report R-649

51-006

by

W. J. Nordell, J. E. Crawford, and R. M. Beard

ABSTRACT

Seven model epoxy toroidal shells were tested, and the results were compared with those from analytical solutions. The toroidal shells had a mean radius about the axis of revolution of 6 inches, a mean tube radius of 2 inches, and a mean shell thickness of 0.086 inch. The static elastic strain response of the epoxy models was in satisfactory agreement with that computed using a finite element analysis for axisymmetric shells. Critical buckling pressures for the models were approximately 85% of the analytical prediction, which was based on the mean dimensions.

INTRODUCTION

The toroidal shell has been considered for use as an underwater structure because it has potential advantages over other shapes. As with the sphere, the torus' double curvature gives it a structural advantage. For example, the critical elastic buckling pressure of a torus is greater than that of a cylinder having a similar tube radius and shell thickness. The spherical shell is superior to the toroidal shell from a structural viewpoint, but the toroidal shell has a greater potential for better interior space utilization, especially in the case of manned underwater structures.[1]

In order to fully evaluate and utilize these potentials, it is necessary to study the stress response, buckling modes, and ultimate load capacity of these shells with supports, openings, and various geometrical imperfections. The ultimate goal of such studies is to develop a capability for predicting failure modes and the conditions which cause them, so that suitable design specifications can be established.

In this report, results are presented from a preliminary series of tests on epoxy toroidal shell models having the same geometry. The results are compared to predictions obtained from elastic analyses. The static stress—strain response was computed using a finite element computer program, and the critical buckling pressure was determined using an elastic stability theory for axisymmetric shells by Flügge and Sobel.[2]

In the following section, the finite element analysis of the structural response of toroidal shells is discussed. Then, the experimental program is outlined, followed by a comparison of the experimental and predicted results. This is followed by findings and conclusions. A detailed description of the model fabrication is contained in the Appendix.

ANALYSIS

Stress Response

The toroidal shell is an axisymmetric shell; its generator axis and the meridional and hoop directions are shown in Figure 1. The linear membrane theory can be used to define the elastic stress response of toroidal shells of uniform thickness subjected to uniform static internal or external pressure.

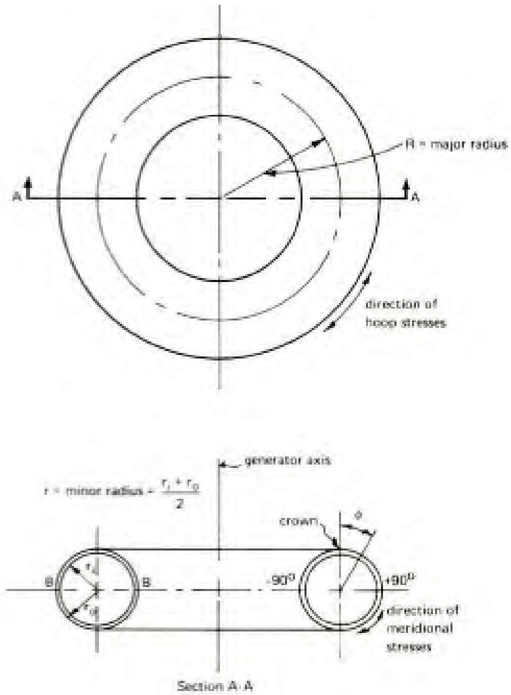

Figure 1. Nomenclature for a circular toroidal shell.

However, the stresses obtained using this theory result in incompatible displacements at the crown. Analytical solutions which satisfy the compatibility condition at the crown are of such complexity that it is difficult to apply them to practical problems over a wide range of shell parameters. Tsui and Massard[3] have discussed this problem and past efforts by others. They surmounted this difficulty by transforming the governing differential equations to remove the condition of incompatible displacements at the crown and by using a finite difference technique to solve the transformed equations.

In the present study, the finite element method was used to predict the stress response because this method could be easily extended to include certain geometrical complexities, such as windows, hatches, nonuniform shell thickness, and laminated construction. Furthermore, axisymmetric shell finite element computer codes were available which could be specialized to the case of a torus.

A computer analysis of toroidal shells was accomplished using a finite element computer code developed by Professor S. B. Dong.[4] The code may be used to analyze the linear elastic behavior of axisymmetric shells of arbitrary shape by idealizing the structure as a finite number of conical frustums. For example, Figure 2 shows an idealization of a toroidal shell; for clarity only a few elements are used.

Any type of static axisymmetric or asymmetric loading can be applied to the shell. In addition, the elements can be assigned different thicknesses to simulate a variable thickness shell or they can have anisotropic material properties. The conical element in this code uses linear in-plane and cubic out-of-plane displacement functions which yield solutions comparable with the classical Kirchhoff—Love shell-bending theory. At each node circle, displacements and strains and interior and exterior surface stresses are calculated. Dong's code was modified to allow for computer generation of the input data for the torus and to provide graphical output of stresses or strains, and displacements as shown in Figures 3 and 4. The stress and displacement data in Figures 3 and 4 are for an unstiffened toroidal shell having overall dimensions similar to those of the underwater station described in Reference 1.

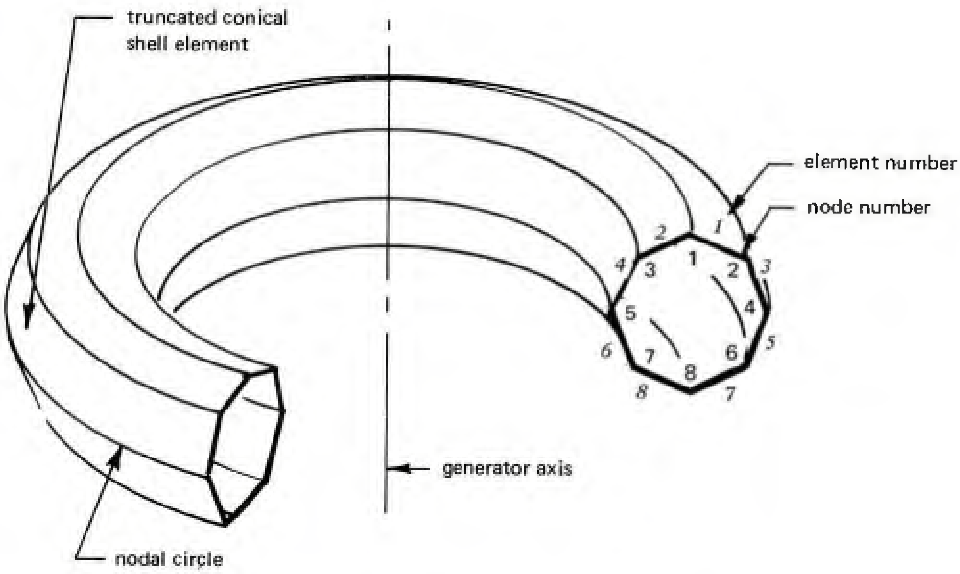

Figure 2. Example of finite element idealization of toroidal shell (eight elements).

Stability

Flügge and Sobel[2] have investigated the elastic stability of general axisymmetric shells subjected to arbitrary loads, and have applied the governing equations to the particular case of a toroidal shell loaded by uniform external pressure. Numerical results for materials with Poisson's ratio (ν) = 0.3 are presented in graphical form in Reference 2. The critical buckling mode is asymmetric with respect to the generator axis, and either symmetric or nonsymmetric with respect to the midplane of the toroid, plane B-B in Figure 1. Their test results compared well with the theoretical predictions (within 10%). However, the toroids tested by them had geometries defined by the ratios **R**/r = 8 and r/**h** = 70 – 80. (Parameters defined below.) Flügge and Sobel[2] noted that the predictions may not be as satisfactory for smaller values of **R**/r, say 3, which was the value used in the present study.

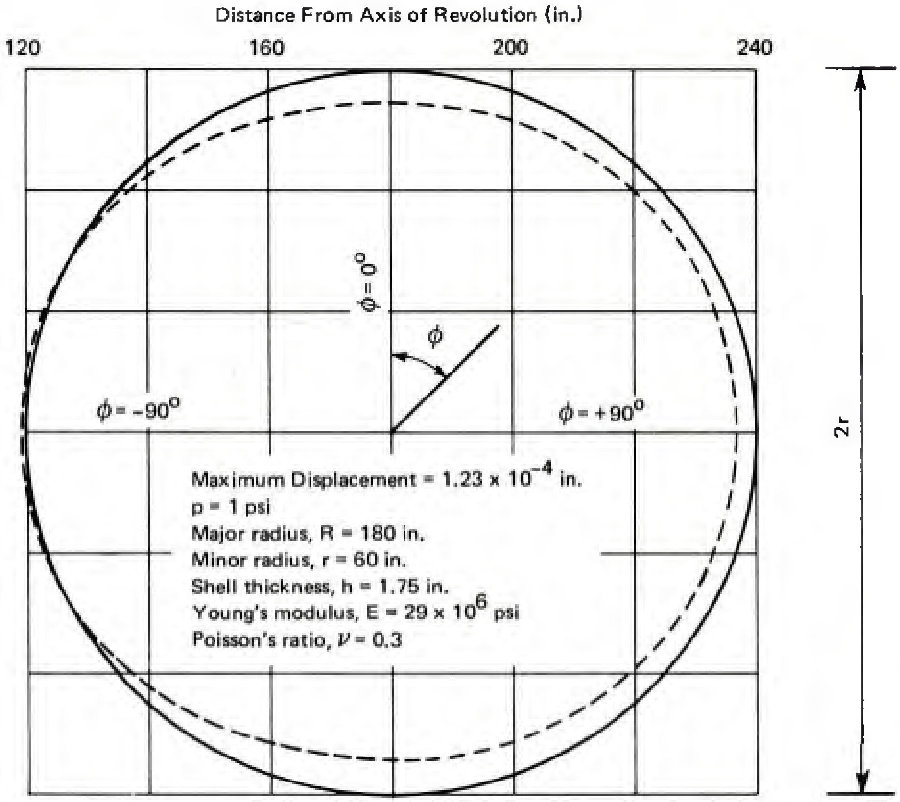

Figure 3. Radial deflections of a toroidal shell.

In Reference 2, the critical pressure, P_{cr}, of a toroidal shell was expressed as

$$P_{cr} = C \frac{E}{(r/h)^2} \tag{1}$$

where nondimensional parameter $C = f(R/r, r/h, \nu)$

E = elastic modulus

r = minor radius

h = shell thickness

R = major radius

For the epoxy shells tested (for which $R/r = 3$, $r/h = 25$, and $\nu = 0.36$), the critical pressure is[5]

$$P_{cr} = 0.03785 \frac{E}{(r/h)^2} \qquad (2)$$

EXPERIMENTAL PROGRAM

Description of Models

Seven structural models (T1 through T7) were cast of epoxy with a major radius of 6 inches, a minor radius of 2 inches, and a mean shell thickness of 0.086 inch; the measured thicknesses are given in Table 1. The epoxy was an epichlorohydrin bisphenol A-type resin (Shell Epon 815) cured with aminoethylpiperazine (AEP). Application of an elevated temperature post cure produced a compressive modulus which increased slightly with age, as shown in Figure 5. The mean value of Poisson's ratio, ν, was 0.365.

The fabrication of models required casting half sections and mating two such halves to form the final structure. The casting procedure involved de-airing the resin for 24 hours prior to mixing with the curing agent and for 15 minutes afterwards. The epoxy was then poured into the female half of the two-piece aluminum mold (Figure 6). Next, the male half was set in place and the mold temperature raised to $90^{\circ}C$ over a period of 2 hours. Then, the mold was allowed to cool slowly for approximately 20 hours before the half torus was removed. Excess epoxy was ground away with silicon carbide on a glass plate; this process provided planar edges as shown in Figure 7, suitable for mating with another half torus. Details of the casting procedure are presented in the Appendix.

The following dimensions of the toroidal models obtained from measurements of the models and the mold indicate the degree of precision attained in model fabrication.

$h = 0.086$ in. $\pm$ 0.003 in. $\qquad$ $r_i = 1.954$ in. $\pm$ 0.005 in.

$r_o = 2.040$ in. $\pm$ 0.003 in. $\qquad$ $R = 6.000$ in. $\pm$ 0.001 in.

Instrumentation

Each model was instrumented with foil-type electrical resistance strain gages having a 1/8 x 1/8-inch grid. Use of electrical resistance strain gages on epoxy can cause problems in evaluating strain measurements, because of the low thermal conductivity of the epoxy. Heating of the gage can cause drift in the measuring circuit and localized heating of the epoxy directly under the gage can change the mechanical properties of the epoxy. To minimize this problem, a cold active gage and a cold compensating gage were similtaneously switched into the circuit to obtain a strain reading.[6]

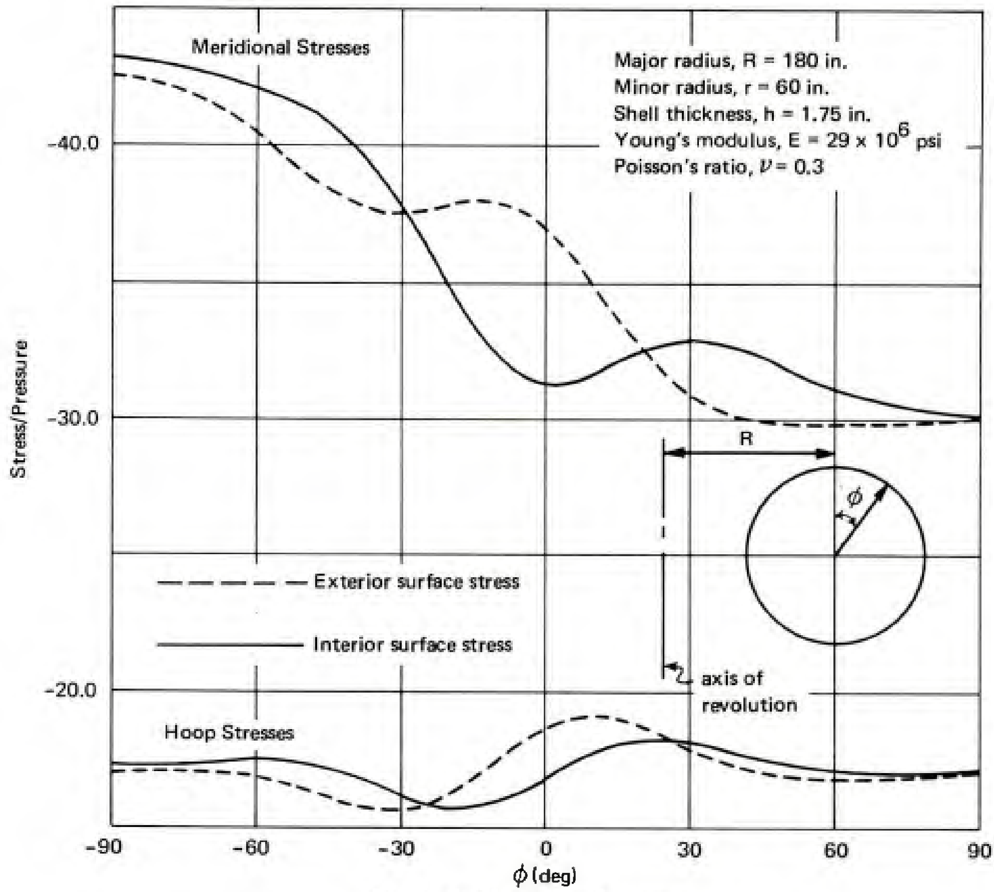

Figure 4. Normalized stresses.

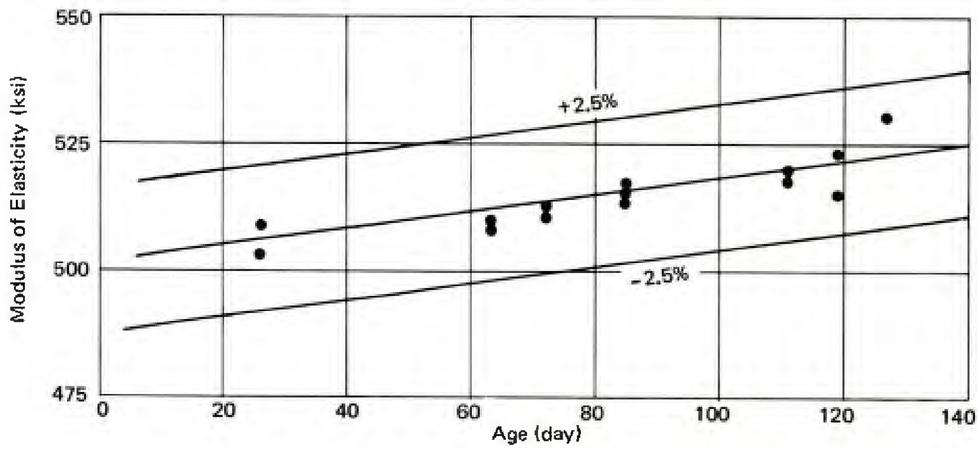

Figure 5. Variation of elastic modulus of cast epoxy with age.

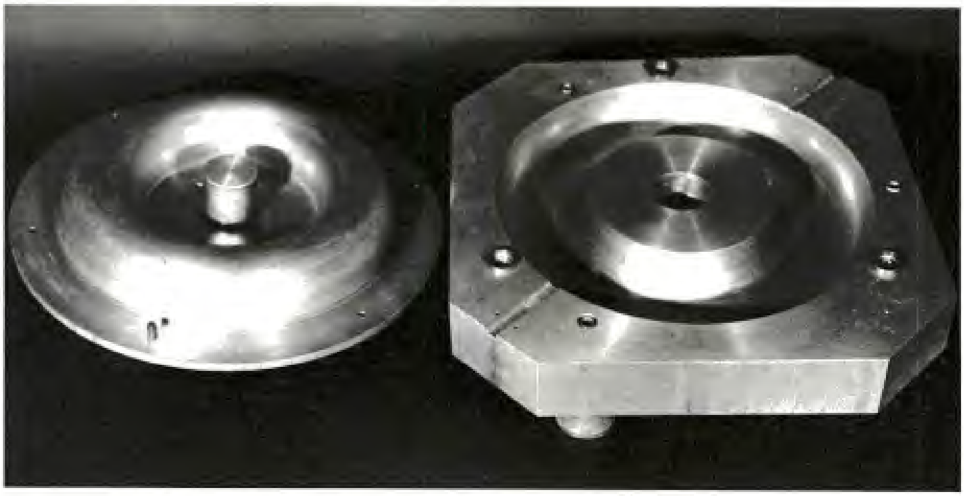

Figure 6. Casting mold, male and female halves.

Figure 7. One half of model toroidal shell.

Table 1. Shell Thickness of Structural Models

ϕ^a (deg)	Shell Thickness (in.) of Model Numbers—							Average
	T1	T2	T3	T4	T5	T6	T7	
0	0.085	0.085	0.088	0.085	0.087	0.085	0.085	0.086
22½	0.084	0.084	0.086	0.084	0.085	0.083	0.083	0.084
45	0.084	0.086	0.087	0.084	0.085	0.085	0.084	0.085
67½	0.085	0.086	0.087	0.084	0.085	0.085	0.085	0.085
90	0.085	0.085	0.086	0.083	0.085	0.084	0.083	0.084
112½	0.089	0.087	0.085	0.084	0.085	0.087	0.084	0.086
135	0.088	0.086	0.084	0.083	0.085	0.086	0.084	0.085
157½	0.086	0.085	0.083	0.083	0.084	0.085	0.083	0.084
180	0.087	0.087	0.086	0.084	0.084	0.085	0.084	0.085
-157½	0.087	0.086	0.087	0.086	0.087	0.086	0.086	0.087
-135	0.088	0.086	0.089	0.087	0.086	0.087	0.087	0.087
-112½	0.087	0.086	0.088	0.088	0.088	0.087	0.088	0.087
-90	0.085	0.085	0.086	0.086	0.086	0.086	0.085	0.086
-67½	0.089	0 087	0.087	0.088	0.089	0.088	0.088	0.088
-45	0.089	0.088	0.088	0.088	0.089	0.087	0.089	0.088
-22½	0.088	0.086	0.087	0.086	0.089	0.086	0.089	0.087
Average	0.0866	0.0859	0.0865	0.0852	0.0862	0.0858	0.0854	0.086

a See Figure 1.

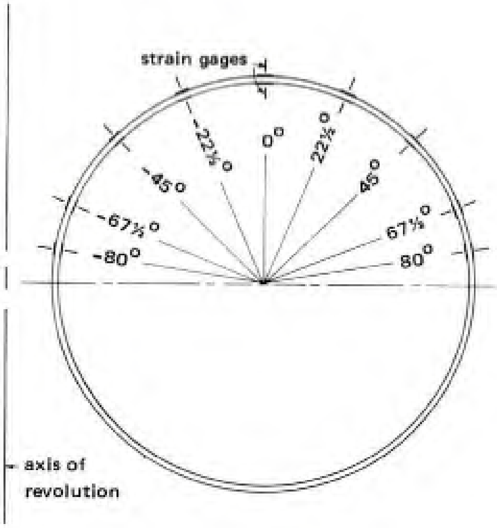

Figure 8. Strain gage locations for strain response of models T4 and T6.

Two different strain gage layouts were used. One was used to provide data for a detailed study of the strain response of the shell (Figure 8) and the other layout was used to determine the buckled shape (Figure 9). Strain was taken with both an SR-4 Type M Strain Indicator and a Datran Digital Strain Indicator Model A 111 D coupled with a Datran Printer Control Unit, Model E 150.

Test Procedure

The models were subjected to external hydrostatic pressure applied directly from a municipal water system via a hose. A dial gage was used to measure the pressure in the tank. Elastic strain response was studied at 5-psi increments up to 25 psi, data being recorded at the highest pressure first and then incrementally to no load. Another method used to study strain response was that of raising the pressure to the desired level, recording the data, and then removing the pressure to permit recovery of any strain resulting from nonlinear behavior. Then, the model was subjected to the next pressure level. As shown in Figure 10, the maximum stresses developed in the models were small relative to the ultimate strength of the epoxy material, and consequently the nonlinear stress–strain response was negligible.

The buckled shape was determined by increasing the pressure at a rate of approximately 30 psi/min until the pressure suddenly leveled off. Then, the ingress of water was stopped, thereby holding the model in a deflected shape, and strain measurements were taken. The models were unloaded and after a period of 20 minutes at zero pressure were reloaded to destruction. The pressure in all cases reached the same maximum value attained in the previous load cycle, although a slight pressure fall off (< 1.5 psi) preceded fracture of the model. The models failed in a similar manner, elastic instability, followed by complete fracturing of the epoxy model. The models were reconstructed from the fragments in order to determine the crack pattern.

COMPARISON OF EXPERIMENTAL AND COMPUTED RESULTS

Stress Response

The finite element computer program with 240 elements was used to predict the strain response of epoxy toroidal shells T4 and T6. The measured values of shell thickness for these two models at the location of the strain gages (Figure 8 and Table 1) were considered in the analysis by specifying the thickness of individual elements. The elastic modulus was assumed to be the same in the meridional and hoop directions and to be equivalent to that measured in the uniaxial compressive coupon tests (Figure 5). For models T4 and T6, an elastic modulus of 507,000 psi was used.

The measured and computed strains for these two specimens are shown in Figures 11 through 14. The measured and computed meridional strains were in satisfactory agreement (within 10%) with a few exceptions. One notable exception is the data point in Figure 14 representing the measured internal strain at $\phi = +80$ degrees, Figure 1, which is located near the outer circumferential joint. The magnitude of the strain is about 30% less than the computed value. In addition, the external meridional strain magnitude at this location is greater than expected, indicating the possibility that the joint or other localized effect influenced the strains at this section.

The agreement between the measured and computed hoop strains is less satisfactory than that for the meridional strains. However, the magnitude of the difference between measured and computed strains is essentially the same for the hoop and meridional directions. The lesser magnitude of the hoop strains causes the percentage deviation to be greater. The trend of

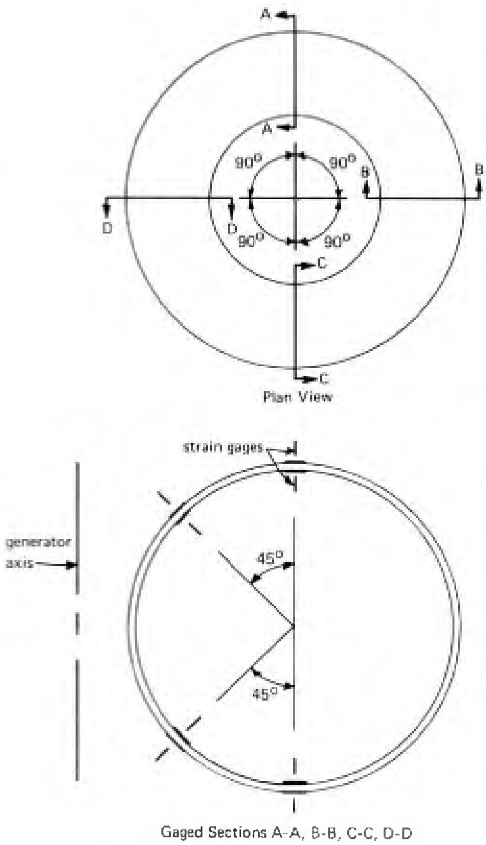

Figure 9. Strain gage locations for buckling mode, models T1, T2, T3, T5, and T7.

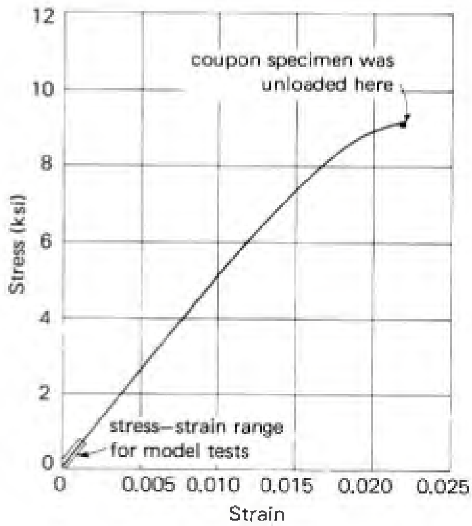

Figure 10. Stress versus strain for cast epoxy material.

the measured meridional and hoop strains corresponds to that for the computed values.

The finite element code provides the classical linear elastic solution for the toroidal shell and therefore, the results show how well the model response corresponded to classical theory. The local deviations of measured strains from the computed values could indicate some model imperfections, particularly around the outer circumferential joint. These imperfections affected the strain field locally, and could have a significant overall effect on the critical pressure. The general agreement between the measured and computed results shows that the model fabrication and the material properties (from coupon tests) were satisfactory; this justifies the use of these material property values in the stability analysis.

Stability

The critical buckling mode of toroidal shells has two circumferential waves and one of two radial mode shapes (A or B in Figure 15).[2] Mode A is symmetrical about the midplane of the cross section, and mode B is nonsymmetrical about this plane. For the geometry under study here, modes A and B are about equally likely; that is, the critical pressure is essentially the same for both modes. The buckled shape determined from the experimental strain data is also shown in Figure 15 and is similar to the nonsymmetric mode (mode B), although the agreement is not exact in the lower half of the section shown.

Post-fracture analysis of the models indicated that the critical deformations occurred in the crown region. A continuous circumferential crack occurred within a 10-degree region of each crown in all models, indicating that this was the critical region of deformation.

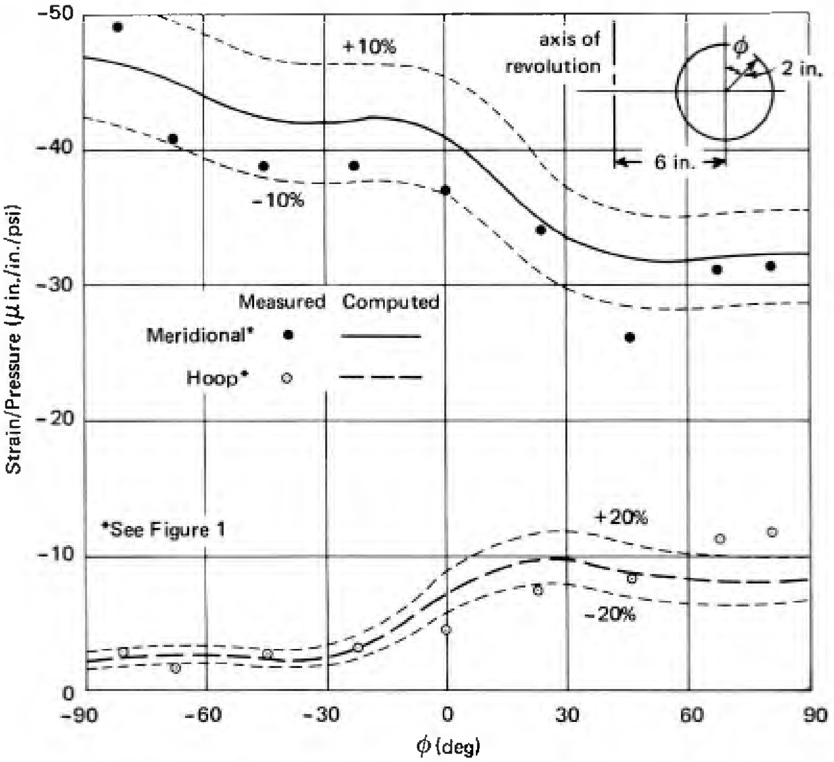

Figure 11. Strain distribution on external surface, model T4.

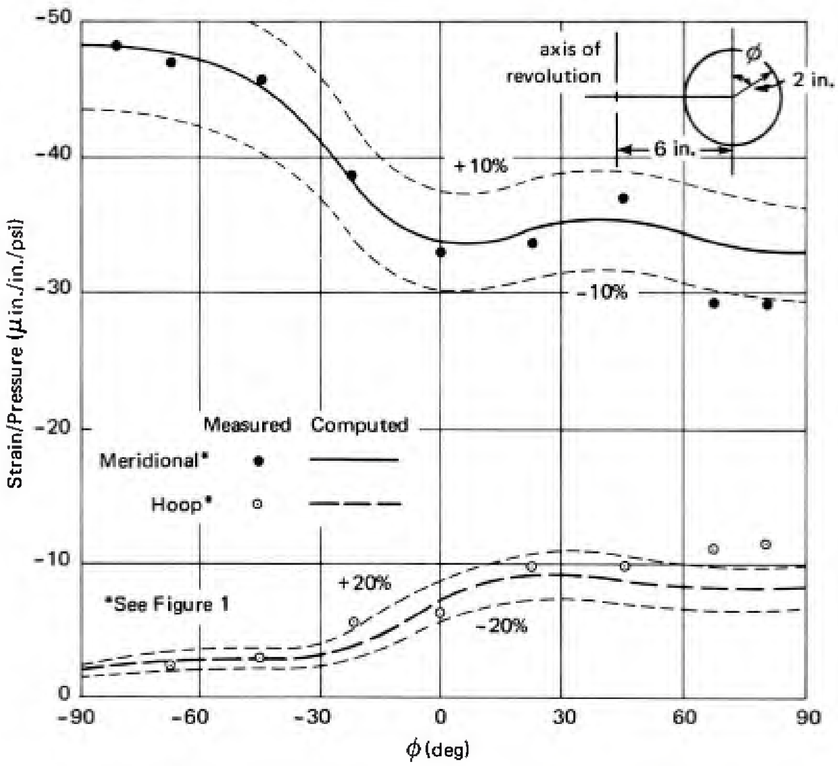

Figure 12. Strain distribution on internal surface, model T4.

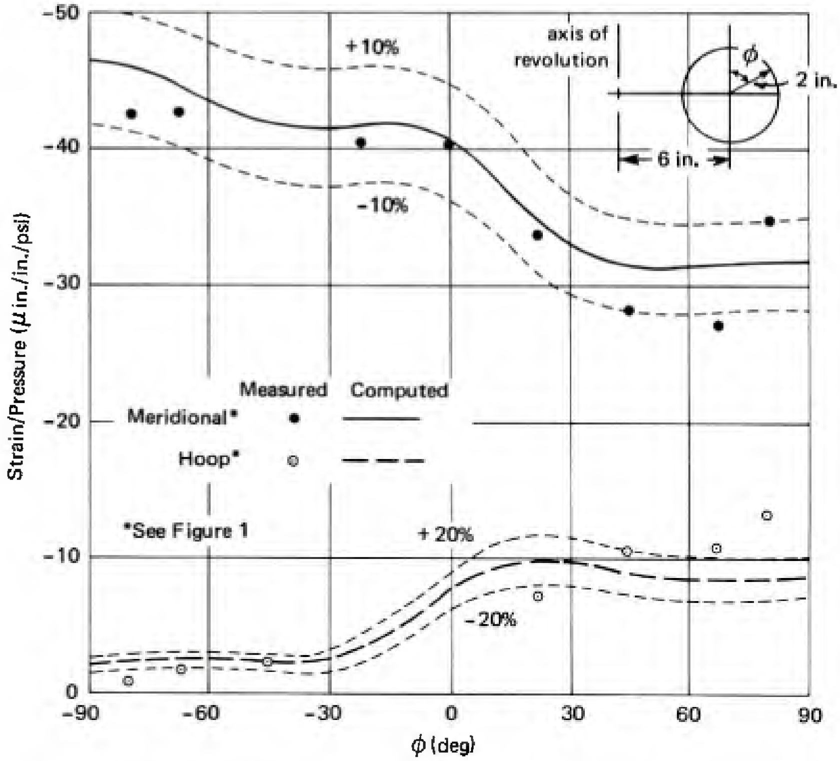

Figure 13. Strain distribution on external surface, model T6.

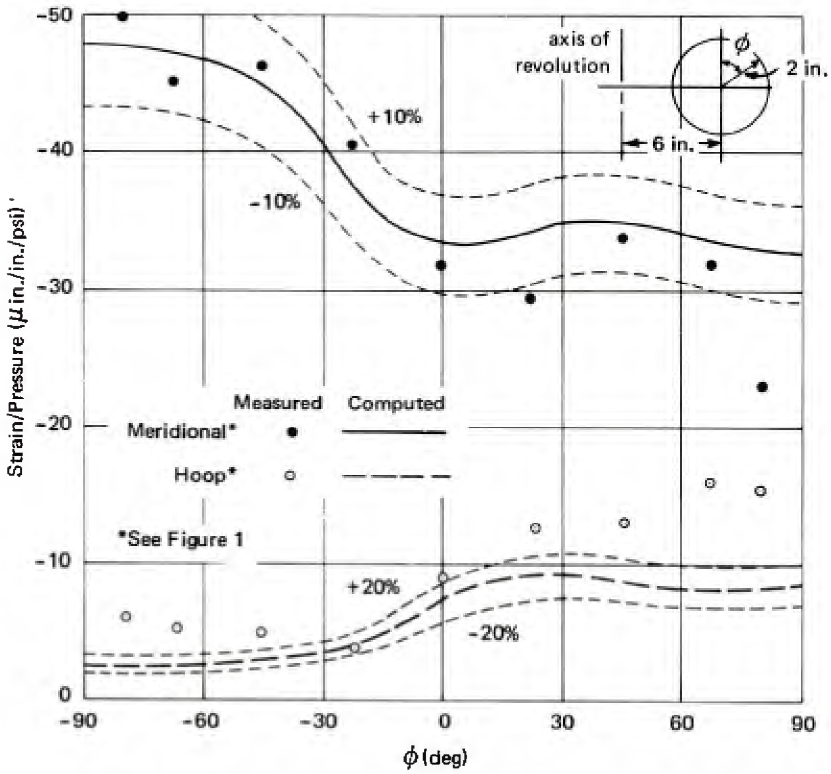

Figure 14. Strain distribution on internal surface, model T6.

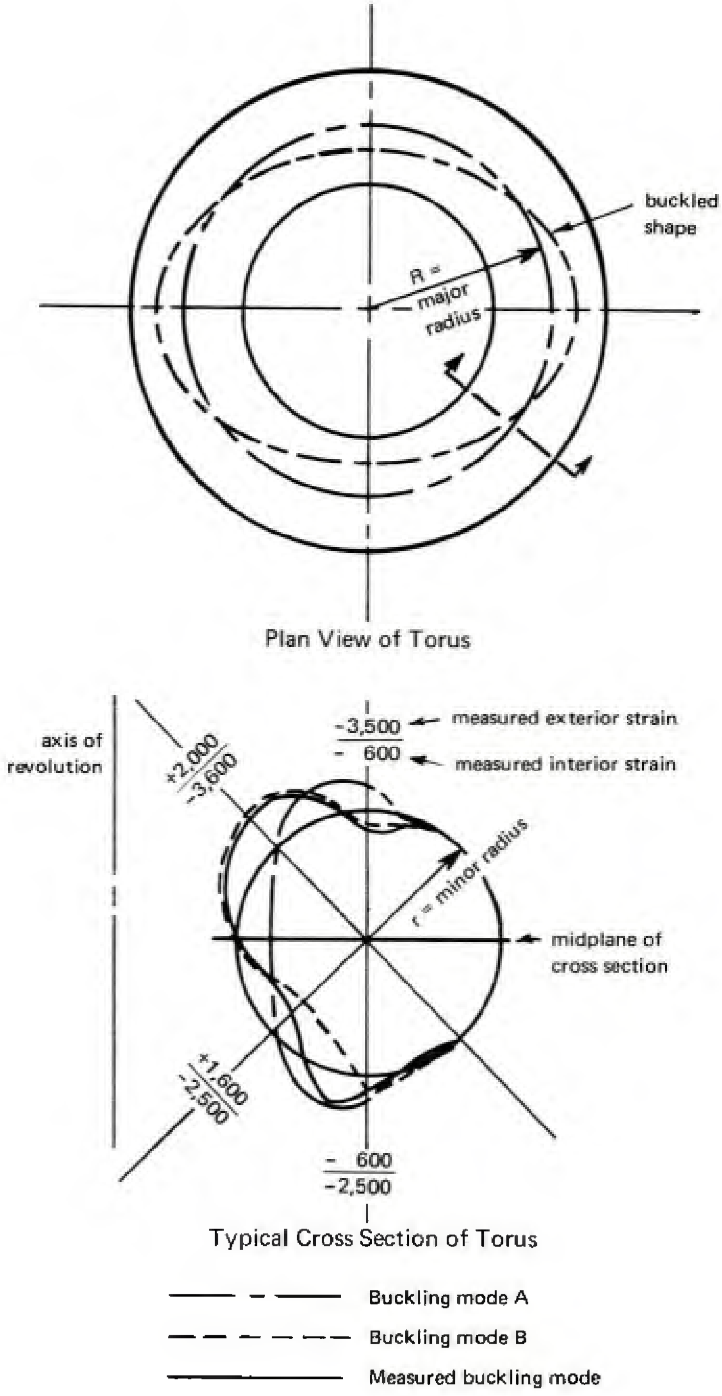

Figure 15. Buckling mode shapes of a circular toroidal shell.

Predicitions of the critical pressure were obtained from Equation 1 using the following mean minor-radius-to-thickness ratio.

$$\frac{r}{h} = \frac{2.000}{0.086} = 23.3$$

The value of **C** (Equation 1) for **R/r** = 3, ν = 0.36, and the above value of **r/h** is 0.0394. This value was determined by extrapolation, using the value of **C** given in Equation 2 and the data presented in Reference 2.

Substituting these values for **C** and **r/h**, Equation 1 yields

$$P_{cr} = (7.25 \times 10^{-5})E \qquad (3)$$

Models T1, T2, and T3 were tested 20 days after casting, and the remaining models were tested 40 days after casting. The data in Figure 5 indicate that the elastic modulus changed less than 1% during this time span, so that the use of different elastic moduli for the models T1 through T3 and models T4 through T7 is not justified considering the precision of the experimental data. If the thickness is changed by 0.001 inch, the computed critical pressure changes by 3%. Consequently, an average value equal to 507,000 psi was used. Hence

$$P_{cr} = 36.8 \text{ psi} \qquad (4)$$

When this value is compared to the measured values in Table 2, the measured values are 11% to 17% less than the predicted value. The trend of this difference is similar to that observed for results from external pressure tests on spherical and cylindrical shells.[7, 8]

Test results on machined steel spherical shell models have shown that the critical pressure for spherical shells may vary by 30% or more.[8] This difference between experiment and theory has been charged primarily to deleterious effects of geometrical imperfections, although two other factors which have been attributed with having some effect are residual stresses and boundary conditions.[7, 8]

In the present study, bending stresses also may contribute to the discrepancy between experiment and theory. A more detailed analysis will be possible when the finite element computer program for the stability analysis of axisymmetric shells is published.* This program includes the effect of bending and will consider asymmetric buckling modes. With this computer program, it will be possible to evaluate the effect of geometrical imperfections and discontinuities and to determine the extent to which they affected the critical response.

Table 2. Measured and Predicted Collapse Pressures

Model	Measured Collapse Pressure (psi)	Ratio of Measured Pressure to Predicted Pressure[a]
T1	30.5	0.83
T2	31.0	0.84
T3	31.5	0.86
T4	32.0	0.87
T5	32.7	0.89
T6	31.0	0.84
T7	31.7	0.86
Average		0.856

[a] Predicted critical or collapse pressure = 36.8 psi

FINDINGS AND CONCLUSIONS

1. The experimental strain data and the finite element computer program output were in satisfactory agreement. The greatest discrepancies were those corresponding to data from external gages, from hoop gages, or from gages near the circumferential joint.

* Preliminary results from this program[9] compared closely with the results from Reference 2. For the measured geometrical and mechanical properties of the epoxy toroidal shells, a critical pressure of 32.9 psi was computed using the finite element program. This value of critical pressure is less than that given by Equation 4 and only slightly greater than the experimental values. The average value of the ratio of measured pressure to the value 32.9 psi is 0.96. This close agreement between the finite element analysis results and the experimental results may be due in part to the inclusion of bending and membrane stresses in the finite element program. Only membrane stresses were considered in the analysis in Reference 2.

2. Good repeatability of model structural response indicated that fabrication of similar models was achieved. Seven models were tested and the maximum deviation from the mean critical pressure was 3.8%.

3. The collapse pressure of the models was about 85% of that computed on the basis of the linear elastic theory in Reference 2.

Appendix

MODEL FABRICATION AND MATERIAL PROPERTIES

Casting epoxy has many inherent problems; the three most serious are caused by bubbles, shrinkage, and brittleness. In this study, a casting procedure was developed that reduced these problems, but at the price of being long and complex. To minimize deviations in physical properties it was mandatory to adhere strictly to the casting procedure, which was as follows:

1. The resin was heated and subjected to a vacuum of 24 inches of mercury for approximately 24 hours to remove air.
2. The curing agent was placed under a 24-inch vacuum for 30 minutes prior to mixing.
3. The mold was placed in an oven and heated to 40°C.
4. The resin and curing agent were mixed and placed under a 24-inch vacuum for 15 to 20 minutes to remove air induced by stirring.
5. The epoxy was poured into the female mold and the male mold was lowered on the spacer rings.
6. The mold was heated to 90°C over a period of 2 hours with the oven air temperature at 120°C.
7. The mold was allowed to cool overnight before the half torus was removed.

The physical properties of the epoxy were determined from uniaxial compression tests on cast flat sheets having approximately the same thickness as the toroidal shells. The casting and curing procedure for these sheets closely followed that of the toroids; the coupon mold was held in direct contact with the toroid mold. Table 3 is a summary of the physical properties as determined from these coupon tests.

Table 3. Elastic Properties of Epoxy Material

Coupon No.	Age at Test (day)	Young's Modulus, E (ksi)	Poisson's Ratio, ν
2-1[a]	26	508	0.36
2-2	26	498	0.36
4-1	63	509	0.37
4-2	63	507	0.37
3-1	72	511	0.36
3-3	72	511	0.36
5-1	85	498	0.36
5-2	85	515	0.37
6-1	85	514	0.36
6-2	85	514	0.36
4-1	111	519	0.36
4-2	111	518	0.37
3-1	119	523	0.36
3-3	119	515	0.36
2-3	125	532	0.37

[a] First digit identifies toroidal model which was cast from same epoxy mix as coupon; second digit is the coupon serial number.

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1. Underwater structures—unstiffened toroidal shells I. 51-006

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13. ABSTRACT

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