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PERVASIVE DAMPING AND STABILITY WITH APPLICATION TO A GRAVITATIONALLY STABILIZED SATELLITE AUGMENTED BY A CONSTANT SPEED ROTOR

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Prepared by R. K. [illegible]
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FOREWORD

This report is published by The Aerospace Corporation, El Segundo, California, under Air Force Con'ract No. F04701-69-C-0066.

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ABSTRACT

The efficacy of a particular satellite design cannot be guaranteed without first examining the problem of pervasiveness of damping. Two recent contributions in the field of motion stability analysis to the general problem of identifying whether a system is pervasively damped are discussed. If the damping is indeed pervasive, then a corollary to the Thomson-Tait-Chetaev stability theorem can be used to establish conditions for which the system is asymptotically stable. These results are illustrated in terms of a gravitationally stabilized vehicle that has been augmented by means of constant speed rotors.

1. INTRODUCTION

This report considers the stability of a gravitationally stabilized satellite that has been augmented by means of a constant speed rotor.

The essential features of the strong form of the Thomson-Tait-Chetaev theorem are reviewed, and its relevance to problems in attitude dynamics is discussed. The notion of pervasive damping is treated, along with a detailed exposition of two recent papers in the literature that specifically treat the problem of identifying systems that are pervasively damped.

The derivation of the linearized differential equations of small angle motion about equilibrium can be found in Ref. 1. Generically, they are of the form

$$M\ddot{x} + G\dot{x} + D\dot{x} + Kx = 0 \qquad (1)$$

where (as was defined in Ref. 1)

- x is a state vector of displacements from equilibrium
- M is a symmetric positive definite inertia matrix
- G is a skew-symmetric matrix arising from gyroscopic forces
- D is a symmetric matrix arising from damping in the system
- K is a symmetric matrix

The strong form of the Thomson-Tait-Chetaev theorem can immediately be deduced from a standard application involving a Lyapunov function. The function V is said to be a Lyapunov function if the following conditions are fulfilled:

a. $V(x,t)$ is defined in Ω for all $t > 0$ where Ω is a set, defined as Ω: $\|x\| < A$ and $t \geq 0$, over which the existence and unicity theorem holds for the system of differential equations.

b. $V(0,t) = 0$ for $t \geq 0$

c. $V(x, t)$ dominates a certain $\omega(x)$ for all x in Ω and all $t \geq 0$

d. $V(x, t) \leq 0$ in Ω

The system under consideration is

$$\ddot{x} = f(x, \dot{x}, t)$$

The interpretation of x in the original definition of a Lyapunov function, however, can be broadened to include this special system. Consider the Lyapunov function

$$V = 1/2\dot{x}^T M\dot{x} + 1/2x^T Kx \quad (2)$$

The time rate of change of V is

$$\frac{dV}{dt} = \dot{x}^T M\ddot{x} + \dot{x}^T Kx$$

If Eq. (1) is premultiplied by $\dot{x}^T$, then

$$\dot{x}^T M\ddot{x} + \dot{x}^T D\dot{x} + \dot{x}^T G\dot{x} + \dot{x}^T Kx = 0$$

Since G was assumed to be skew-symmetric, the following identity can be obtained:

$$\dot{x}^T M\ddot{x} + \dot{x}^T Kx = -\dot{x}^T D\dot{x}$$

The time rate of change of the Lyapunov function is given by

$$\frac{dV}{dt} = -\dot{x}^T D\dot{x} \quad (3)$$

The properties of the matrices that are necessary to insure that the equilibrium solution is asymptotically stable can immediately be deduced from an application of a theorem originated by Lyapunov.

Under the conditions of the definition of a Lyapunov function, it can be further established that there exists a continuous, non-decreasing scalar function $\beta(x)$ such that $\beta(0) = 0$ and, for all t,

$$V(x,t) \leq \beta(\|x\|)$$

and $V(x,t)$ is strictly decreasing for all solutions Ω except the null solution, then the equilibrium solution is asymtotically stable and any other solutions in Ω converge to the equilibrium solution. Upon application of the theorem we find that: (1) if M, K, and D are positive definite, V is positive definite, $\dot{V}$ is negative definite, and asymptotic stability is assured; and, (2) due to an instability theorem originally due to Lyapunov, if M or K is negative definite or sign variable and D is positive definite, V is not positive definite, $\dot{V}$ is negative definite, and the equilibrium solution is unstable. Thus, for a positive definite damping matrix, the stability or instability of the equilibrium solution can be inferred from the inertia and stiffness matrix alone. Since the inertia matrix is guaranteed to be positive definite, asymptotic stability or instability can be determined solely from the stiffness matrix alone. Collectively, these results constitute the "strong-form" of the Thomson-Tait-Chetaev theorem.

In the absence of damping, it is possible to have a stable equilibrium solution, but examination of the eigenvalues of K alone do not lead to all possibilities for stability [2]*. In general, other possibilities exist in which the G matrix plays a role. Systems that rely upon the gyroscopic matrix for stabilization will be unstable upon the introduction of the appropriate damping into the system.**

* Numbers in brackets refer to references.

** Appropriate damping, at least at this juncture, means damping such that a positive definite damping matrix exists.

The assumption of a positive definite damping matrix is too restrictive. In the design of attitude control systems, one strives not for a positive definite damping matrix, but rather for damping that affects the entire system, so that any deviation from the nominal motion elicits energy dissipation. Systems for which energy dissipation exists for any motion of the system are said to be pervasively damped. The criteria of pervasive damping for systems whose primary mechanism for the dissipation of energy is via viscous damping are

$$-\dot{x}^T D\dot{x} < 0 \qquad \text{for} \qquad \phi(t, x_0, t_0)$$

and

$$-\dot{x}^T D\dot{x} = 0 \qquad \text{for} \qquad x = 0$$

where $\phi(t, x_0, t_0)$ represents a possible motion of the system. This definition does not necessarily entail the imposition of a positive definite damping matrix. At this juncture, we are in a position to state a corollory to the "strong-form" of the Thomson-Tait-Chetaev theorem. If we can construct linearized differential equations of small angle motion about equilibrium in the form suggested in Eq. (1). with a positive definite inertia matrix, a gyroscopic matrix, a symmetric matrix K, and a damping matrix derivable from a Raleigh dissipation function that is not necessarily positive definite, and if further the damping is pervasive, then the equilibrium solution is asymptotically stable if the matrix K is positive definite.

These results serve to motivate some basic ideas encapsuled in a theorem that is of paramount utility in motion stability analyses. The theorem was originally advanced by R. Pringle [3] and subsequently refined by P. W. Likins [4] and by P. W. Likins and D. L. Mingori [5].

Theorem

A holonomic mechanical system with the Lagrangian not explicitly dependent upon time and with damping both solely generalized and pervasive in the noncyclic generalized coordinates is: (1) asymptotically stable in these coordinates if the dynamic potential energy is positive definite, and (2) unstable in these coordinates if the dynamic potential energy P can take on negative values arbitrarily close to the origin in the coordinate space.

In essence, the Hamiltonian is advanced as an appropriate Lyapunov function. The Hamiltonian, which is represented in terms of the potential energy and homogeneous forms determined from the kinetic energy, is given as

$$\begin{aligned} H &= T_2 + V - T_0 \\ &= T_2 + P \end{aligned} \tag{4}$$

Since the homogeneous form T_2 can be guaranteed to be positive definite, the positive definiteness of the Hamiltonian is established solely by examination of the dynamic potential energy P.

The link between the two separate approaches relies upon the observation that the Lyapunov function in Eq. (2) is a quadratic approximation of the Hamiltonian [4]. In addition, the inner product $x^T Kx$ is identified as a first approximation to the dynamic potential energy. The Hamiltonian (or its approximation) is positive definite if the dynamic potential energy (or its approximation) is positive definite.

2. PERVASIVE DAMPING

In most practical applications, the damping matrix of Eq. (1) is not positive definite. We have seen that this provides no real impediment to the determination of whether an equilibrium solution is asymptotically stable or unstable, providing the damping is pervasive. Determination of whether a system is pervasively damped or not was at best uncertain prior to the recent publication of two articles that specifically deal with this problem. R. E. Roberson establishes a constructive method, using the concepts of graph theory, for determining from the structure of the dynamical equations alone if a linear system is pervasively damped [6]. G. M. Connell subsequently advanced an alternate method [7]. Essentially, one determines a manifold of solutions for which there exists no damping, and proceeds to show that the manifold of solutions do not, in fact, satisfy the differential equations. Thus, any possible system motions elicit energy dissipation. These two alternative methods of determining whether a dynamic system is pervasively damped will be elaborated upon in more detail.

R. E. Roberson starts with the observation that the system of equations represented in Eq. (1) is isomorphic to a set of n point masses free to move in one dimension and interconnected by lumped, linear, passive, mechanical elements, such as springs, viscous dampers, and gyrators. The linear differential equation of motion for the point mass system in matrix form is

$$\begin{vmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{vmatrix} \left\{ \ddot{z} \right\} + \begin{vmatrix} 0 & g_{21} & g_{31} & g_{41} \\ -g_{21} & 0 & +g_{32} & +g_{42} \\ -g_{31} & -g_{32} & 0 & g_{43} \\ -g_{41} & -g_{42} & -g_{43} & 0 \end{vmatrix} \left\{ \dot{z} \right\}$$

$$+ \begin{vmatrix} d_1 + \sum_{j \neq i} d_{1j} & -d_{21} & -d_{31} & -d_{41} \\ -d_{21} & d_2 + \sum_{j \neq i} d_{2j} & -d_{32} & -d_{42} \\ -d_{31} & -d_{32} & d_3 + \sum_{j \neq i} d_{3j} & -d_{43} \\ -d_{41} & -d_{42} & -d_{43} & d_4 + \sum_{j \neq i} d_{4j} \end{vmatrix} \left\{ \dot{z} \right\}$$

$$
+\begin{vmatrix} k_1+\sum_{j\neq i}k_{1j} & -k_{12} & -k_{13} & -k_{14} \\ -k_{21} & k_2+\sum_{j\neq i}k_{2j} & -k_{23} & -k_{24} \\ -k_{31} & -k_{32} & k_3+\sum_{j\neq i}k_{3j} & -k_{34} \\ -k_{41} & -k_{42} & -k_{43} & k_4+\sum_{j\neq i}k_{4j} \end{vmatrix} \begin{Bmatrix} z \end{Bmatrix} = \begin{Bmatrix} 0 \end{Bmatrix} \quad (5)
$$

This can be conveniently accomplished by an orthogonal transformation of the original equations of motion. Defining the orthonormal transformation such that the M matrix is diagonalized, we have

$$M'\ddot{z} + D'\dot{z} + G\ \dot{z} + Kz = 0$$

where

$$z = Px$$

The identification can be made complete by selecting the masses of the isomorphic point mass systems equal to the diagonal elements of M'. The matrix sum $D' + G'$ is then rewritten as a sum of a symmetric and a skew-symmetric matrix. Next, the point mass isomorphic system is represented in a diagram. The most general representation for the conical case appears in Fig. 1. It is assumed that the configuration parameters are consistent with the general representation that appears in Fig. 1. An investigation in terms of a diagram is then made to determine whether the system is completely connected or whether it separates into several disconnected pieces. For the latter alternative, damping in one part of the system will not be experienced in another portion of the system. To facilitate this investigation, a connective matrix U is defined as follows. The connective matrix U has as many rows as there are point masses comprising the system, and it has as many columns as there are mechanical element connections between the point masses. If a mechanical element j is connected to a point

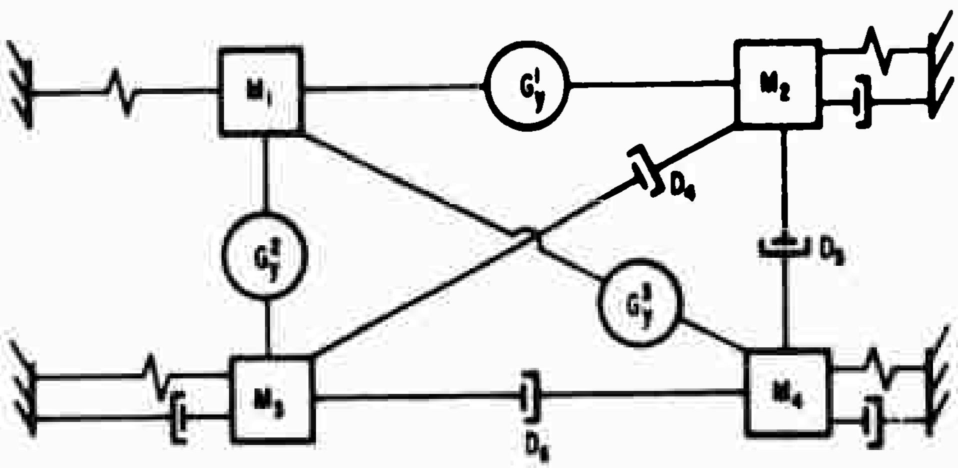

Figure 1. Isomorphic System -
Most General Conical Case

mass i, then a 1 is entered in that matrix element (i, j) corresponding to the ith point mass and the jth connecting element. In this way, a connection matrix can be evaluated. For the case examined here, the matrix U is

$$U = \begin{vmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{vmatrix}$$

If the rows and columns of U can be rearranged in any way such that it takes the form

$$U^* = \begin{bmatrix} U_1^* & 0 & 0 \\ 0 & U_2^* & 0 \\ 0 & 0 & U_3^* \end{bmatrix}$$

that is, of a diagonal partitioned matrix with zero off-diagonal blocks, then the system contains as many separate parts as there are submatrices on the diagonal of the partitioned matrix. Roberson develops an algorithm that is beneficial for assessing the number of partitions in the event that the system is extremely complex. The existence of separated subsystems assures that the damping is non-pervasive. Roberson observed that a sufficient condition for instability is that the columns of the damping and stiffness matrices are linearly dependent. If a motion exists such that there is no dissipation of energy, the coordinates Z_i must all remain equal. Moreover, any coordinate corresponding to a particle connected by a damper to inertial space must remain zero. Denote a matrix V consisting of those columns of U associated with a dissipative element. The matrix V is subjected to the same analysis that the matrix U was subjected to. For the case examined, we find

$$V = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Let T_α denote those point mass elements associated with the αth subgraph of the matrix V. The set T is comprised of exactly one element from each set T_α. For the case considered here, we define

$$T: M_2$$

$$T_1: M_2, M_3, M_4$$

Define also a set Z comprised of the union of the zero rows of matrix V and set T. The set Z_0 denotes those indices corresponding to a particle connected by a damper to inertial space. Thus, Z and Z_0 are comprised of the elements

$$Z: M_1, M_2$$

$$Z_0: M_2, M_3, M_4$$

The complete motion is described by a new set of coordinates $\omega_1 \ldots \omega_r$ smaller in number than the Z_i and specifically equal to the number of elements in set Z diminished by the number of elements in Z_0.

The transformation

$$Z = Q\omega$$

where Q is an $n \times r$ matrix constricted algorithmically as follows. If $i \epsilon Z_0$, the ith row of Q is a row of zeros. If $i \notin (T_\alpha U Z_0)$, enter a digit 1 in the ith row, αth column of Q and fill the remainder of the row with zeros. Otherwise, if $i \epsilon (T_\alpha U Z_0)$, enter a digit 1 in the ith row at the first column that does not already contain a digit 1 in some other row. This specific process has the effect of assigning label numbers to the ω_i which describe the motion of the α part of the matrix V. The remaining point mass indices on ω_i comprise those masses which are neither a part of V nor are fixed in position with respect to inertial space. The resulting matrix is

$$Q = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

One can observe from Fig. 1 that in order for there to be no dissipation of energy, point masses 2, 3, and 4 cannot move relative to each other, and further, there can be no motion of the subpart comprised of the point masses 2, 3, and 4. When transformed, the equations of motion become

$$mQ\ddot{\omega} + gQ\dot{\omega} + (k + K)\,Q\omega = 0$$

Substituting the trial solution

$$\omega = \exp(\lambda t)\omega_0$$

we obtain

$$W\omega_0 = 0$$

where

$$W = \left[\lambda^2 m + \lambda g + (k + K)\right]Q$$

A necessary and sufficient condition that a non-trivial solution exists is that the rank of W is not maximum. The rank can be checked algorithmically as follows. Choose any r rows of W and determine the λ values that make its determinant vanish. Having λ equal to one of these r values is a necessary condition for rank less than r. There remain $n!/r!(n-r)! - 1$ possible determinants, each to be evaluated r times. If one encounters a non-zero determinant at any point in the evaluation sequence, it is guaranteed that no non-trivial solution exists. If the rank of W is not maximum, then there exists a system motion for which there is no dissipation of energy, and then the system is not pervasively damped. For the case considered here,

$$W = \lambda^2 \begin{pmatrix} M \\ 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ g_1 \\ g_2 \\ g_3 \end{pmatrix} + \begin{pmatrix} K \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

If g_1, g_2, and $g_3 = 0$, λ is identically zero. Letting, $\lambda = 0$ in the first equation, ω will be of maximum rank if $K \neq 0$. Therefore, one might conclude the system will be pervasively damped for those systems for which the isomorphic representation is as described in Fig. 1. Or, solving for λ^2 from the first equation ($K \neq 0$) and substituting the values of λ^2 into the equations

$$\lambda^2 g_1 \neq 0$$

$$\lambda^2 g_2 \neq 0$$

$$\lambda^2 g_3 \neq 0$$

we can verify that W is of maximum rank if g_1, g_2, or g_3 is not equal to zero.

An alternate approach to the determination of whether a system is pervasively damped is given presently [7]. This method was suggested by G. M. Connell. The power dissipation function, or the $\dot{V}$ in Eq. (3), is given as

$$F(\dot{x}) = -\dot{x}^T D \dot{x}$$

where the damping matrix for the conical and hyperbolic case is of the form

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & D \end{bmatrix}$$

We wish to determine those solutions for which there is no dissipation of energy. A necessary and sufficient condition for there to be no energy dissipation is that the control body be fixed relative to the main body. This condition is represented by specifying that the state variable component x_4 be some arbitrary constant. There exist eight possible conjectured system motions for which there is no dissipation of energy, and they are

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ c_4 \end{bmatrix}, \quad \begin{bmatrix} x_1 \\ x_2 \\ c_3 \\ c_4 \end{bmatrix}, \quad \begin{bmatrix} x_1 \\ c_2 \\ x_3 \\ c_4 \end{bmatrix}, \quad \begin{bmatrix} x_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix},$$

$$\begin{bmatrix} c_1 \\ x_2 \\ x_3 \\ c_4 \end{bmatrix}, \quad \begin{bmatrix} c_1 \\ x_2 \\ c_3 \\ c_4 \end{bmatrix}, \quad \begin{bmatrix} c_1 \\ c_2 \\ x_3 \\ c_4 \end{bmatrix}, \quad \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}$$

where c_i represents a state variable that is considered to be some arbitrary constant other than zero, and x_i denotes a state variable that is not considered constant.

If this manifold of solutions for which there exists no damping does not satisfy the differential equations, then the damping is pervasive. Consider the solution vector

$$x^* = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ c_4 \end{bmatrix}$$

Substituting the solution back into the differential equations of motion, we have

$$A'\ddot{\xi}^* + G'\dot{\xi}^* + K'\xi^* = C' \tag{6}$$

where

$$\xi^* = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

and the matrices A', G', and K' are $[n \times (n - m)]$.

The essence of the following amounts to the determination of those conditions for which the only possible solution of equations is the null solution. Thus, the manifold of solutions for which there exists no damping is not satisfied by the differential equations. In pursuit of these objectives, consider the homogeneous part of Eq. (6). Assuming the trial solution

$$\xi^* = \exp(\lambda t)\xi_0^*$$

and substituting into the homogeneous equation we find

$$W\xi^* = 0$$

where

$$W = \lambda^2 A' + \lambda G' + K'$$

In total, there exist

$$\frac{n!}{(n - m)!m!}$$

different sets of λ's obtained by selecting m rows of W and finding those values of λ for which the determinant vanishes. A necessary and sufficient condition for a non-trivial solution to the homogeneous equation to exist is that the rank of W be less than maximum. If the rank of any one of the possible W's is maximum, then the only solution that exists is the trivial one. On the other hand, if after the evaluation of the different W's, it is determined that no non-zero determinants exist, the rank would not be maximum and thus one is guaranteed that a non-trivial solution, other than the equilibrium solution, exists. Thus, damping would not be pervasive in this instance. The particular solution of Eq. (6) is found from

$$K' \xi^* = C'$$

since C' is constant.

Consider the augmented $[n \times (n - m + 1)]$ matrix determined by combining the (n - m) columns of the matrix K' with the column matrix C'. A necessary and sufficient condition for a non-trivial solution to exist is that the rank of the matrix K' is equal to the rank of the augmented matrix $(K' \vdots C')$. If it is determined that the rank of both matrices is equal, then a non-trivial solution exists.

Having established the methodology, we will proceed to apply it to the hyperbolic case. The equations of motion for the specialized hyperbolic case were generically of the form

$$
\begin{vmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & m_4 \\ 0 & 0 & m_3 & 0 \\ 0 & m_4 & 0 & m_4 \end{vmatrix} \{\ddot{x}\} + \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d \end{vmatrix} \{\dot{x}\}
$$

$$
+ \begin{vmatrix} 0 & 0 & -g_1 & 0 \\ 0 & 0 & -g_2 & 0 \\ g_1 & g_2 & 0 & -g_3 \\ 0 & 0 & g_3 & 0 \end{vmatrix} \{\dot{x}\} + \begin{vmatrix} a & b & 0 & e \\ b & c & 0 & f \\ 0 & 0 & d & 0 \\ e & f & 0 & g \end{vmatrix} \{x\} = \{0\}
$$

Consider the possible solution

$$
x^* = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ c_4 \end{bmatrix}
$$

from the set of eight possible solutions for which there exists no damping. First, let us examine the particular solution

$$
K'\xi^* = C'
$$

where

$$
\xi^* = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}
$$

For the hyperbolic case and the possible solution under examination

$$K' = \begin{bmatrix} a & b & 0 \\ b & c & 0 \\ 0 & 0 & d \\ e & f & 0 \end{bmatrix}, \quad C' = \begin{bmatrix} K_2 e \\ K_2 f \\ 0 \\ K_2 g \end{bmatrix}$$

where K_2 is an arbitrary constant. If for this particular solution one can assume that the K' and the augmented matrices are of maximum rank, then the trivial solution is the only one possible. Since elemental operations on a matrix, such as multiplication of one column by a constant, do not change the rank of the matrix, one can see that the augmented matrix is of maximum rank if the stiffness matrix K is of the maximum rank. This is assured if the stability inequalities are satisfied. The K' matrix is also guaranteed to be of maximum rank if three of the stability inequalities are satisfied. The conclusion that the only solution is the trivial one depends upon the examination of the homogeneous solution.

The W matrix for the hyperbolic case and the particular solution examined is

$$W = \lambda^2 \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \\ 0 & m_4 & 0 \end{bmatrix} + \lambda \begin{bmatrix} 0 & 0 & -g_1 \\ 0 & 0 & -g_2 \\ g_1 & g_2 & 0 \\ 0 & 0 & g_3 \end{bmatrix} + \begin{bmatrix} a & b & 0 \\ b & c & 0 \\ 0 & 0 & d \\ e & f & 0 \end{bmatrix}$$

In total, there exist four λ sets obtained from the solutions of the four possible determinants. A necessary and sufficient condition for a non-trivial solution to exist is that the rank of W, for all solutions of λ obtained from the four different λ sets, be less than maximum.

In the event that the rank of W is maximum for any solution λ, the only solution that exists is the trivial one. The four equations obtained by equating these four possible determinants to zero are

$$\lambda^6(m_1m_2m_3) + \lambda^4(am_2m_3 + cm_1m_3 + dm_1m_2 + g_1^2m_2 + g_2^2m_1)$$

$$+ \lambda^2(acm_3 + am_2d + cdm_1 - 2bg_1g_2 + g_1^2c - b^2m_3 + ag_2^2)$$

$$+ (acd - b^2d) = 0 \tag{7}$$

$$\lambda^4(g_3m_1m_2 + g_2m_1m_4) + \lambda^2(g_3am_2 + g_3cm_1 - g_1m_4b$$

$$+ g_1em_2 + g_2am_4 + g_2fm_1) + (ag_3c - bg_2e - g_1fb$$

$$+ g_1ec - b^2g_3 + g_2af) = 0 \tag{8}$$

$$\lambda^4(em_2m_3 - g_2g_1m_4 - g_1g_3m_2 - bm_3m_4)$$

$$+ \lambda^2(bg_2g_3 + cm_3e + cdm_2 - g_1g_2f + g_2^2e$$

$$- cg_1g_3 - bfm_3 - bdm_4) + (ecd - bfd) = 0 \tag{9}$$

$$\lambda^6(-m_1m_4m_3) + \lambda^4(g_2g_3m_1 - g_1^2m_4 - m_3am_4 - fm_3m_1$$

$$- dm_1m_4) + \lambda^2(ag_2g_3 + bem_3 - g_1^2f + g_1g_2e$$

$$- bg_1g_3 - afm_3 - adm_4 - fdm_1) + (bde - daf) = 0 \tag{10}$$

Solutions for λ identically equal to zero have been specifically excluded. The particular degenerate case is considered later. For a non-trivial solution to exist, all the determinants would have to be identically zero for all solutions of λ. Thus, Eq. (7) would have to be satisfied by the solution to Eqs. (8), (9), and (10), and similarly, for the remaining equations.

It can be observed immediately that Eqs. (8) and (9) will be satisfied by the roots to Eqs. (7) and (10), only if the latter two equations have repeated roots. Further, it will be observed that the roots of each equation must span the space of roots corresponding to the other equations in order for all determinants to vanish identically, and that W will be less than full rank. The examination of seven additional possible system motions for which there exists no damping remains. One additional case will be elaborated in the text, then a table that summarizes calculations associated with the remaining possible motions will be presented.

Consider the possible solution

$$x^* = \begin{bmatrix} x_1 \\ c_2 \\ x_3 \\ c_4 \end{bmatrix}$$

from the remaining set of possible motions for which there exists no damping. The matrices K′ and C′ become

$$K' = \begin{bmatrix} a & 0 \\ b & 0 \\ 0 & d \\ e & 0 \end{bmatrix} \qquad c' = \begin{bmatrix} bk_1 + ek_2 \\ ck_1 + fk_2 \\ 0 \\ fk_1 + gk_2 \end{bmatrix}$$

where K_1 and K_2 are arbitrary constants. The matrix K′ is guaranteed to be of full rank if the stability inequalities are satisfied. Specifically,

$$a > 0 \qquad \text{and} \qquad d > 0$$

The augmented matrix is

$$\left[K' \vdots C\right] = \begin{bmatrix} a & 0 & bk_1 + ek_2 \\ b & 0 & ck_1 + fk_2 \\ 0 & d & 0 \\ e & 0 & fk_1 + gk_2 \end{bmatrix}$$

The rank of the augmented matrix can be guaranteed to be maximum if one of the four possible determinants obtained by alternately deleting a row is non-zero. One of the determinants was identically zero since the matrix contained a column of zeros. The three remaining determinants are

$$k_1(af - eb) + k_2(ag - e^2) \overset{?}{=} 0 \tag{11}$$

$$k_1 d(b^2 - ac) + k_2 d(eb - af) \overset{?}{=} 0 \tag{12}$$

$$k_1 d(bf - ec) + k_2 d(bg - ef) \overset{?}{=} 0 \tag{13}$$

Consider a solution to Eqs. (11) and (12) for the constants k_1 and k_2.

$$\begin{bmatrix} af - eb & ag - e^2 \\ d(b^2 - ac) & d(eb - af) \end{bmatrix} \begin{Bmatrix} k_1 \\ k_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

The solution for the two constants can be guaranteed to be other than a trivial one if the determinant is identically zero, that is

$$d(af - eb)^2 + d(ag - e^2)(b^2 - ac) \overset{?}{=} 0$$

If the stability inequalities are satisfied, then it can be verified that the determinant is not equal to zero, and the only solution is the trivial one. If k_1 and k_2 are equal to zero, then the homogeneous solution is the only one to be considered. Two additional cases remain. Consider a solution to Eqs. (12) and (13) for the constants k_1 and k_2:

$$\begin{bmatrix} d(b^2 - ac) & d(eb - af) \\ d(bf - ec) & d(bg - ef) \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Again, the solution for the two constants can be guaranteed to be other than a trivial one if the determinant is identically zero. Since d is guaranteed to be greater than zero,

$$(b^2 - ac)(bg - ef) - (eb - af)(bf - ec) \stackrel{?}{=} 0$$

or

$$b(acg + 2bfe - e^2c - b^2g - af^2) \stackrel{?}{=} 0$$

Since the term in parentheses is guaranteed to be greater than zero, the determinant is identically zero if b is equal to zero. So, if b is equal to zero, a solution for k_1 and k_2 exists other than the trivial one. In fact, if b is equal to zero, k_1 and k_2 are related by

$$k_1 = -\frac{f}{c} k_2$$

If this relation is substituted into Eq. (11), we find

$$- af^2 + cag - ce^2 \stackrel{?}{=} 0$$

If the relation above is identically zero, then we are guaranteed that the rank of the augmented matrix will be less than maximum.

Similarly, solving for k_1 and k_2 from Eqs. (11) and (13) we find that if e is equal to zero, and if further

$$(b^2 - ac)g + af^2 = 0$$

then the rank of the augmented matrix is not maximum. Now, if both e and b are identically equal to zero, Eqs. (11) and (12) become, since a is greater than zero,

$$k_1 f + k_2 g = 0$$

$$k_1 c + k_2 f = 0$$

and Eq. (13) vanishes identically. A solution other than the trivial one exists if the determinant is equal to zero. Thus, for the case when both b and e are equal to zero, if further

$$f^2 - eg = 0$$

then the rank of the augmented matrix is not maximum. Now, the homogeneous solution will be considered. The matrix W for the solution considered is

$$W = \lambda^2 \begin{bmatrix} m_1 & 0 \\ 0 & 0 \\ 0 & m_3 \\ 0 & 0 \end{bmatrix} + \lambda \begin{bmatrix} 0 & -g_1 \\ 0 & -g_2 \\ g_1 & 0 \\ 0 & g_3 \end{bmatrix} + \begin{bmatrix} a & 0 \\ b & 0 \\ 0 & d \\ e & 0 \end{bmatrix}$$

Consider the six possible determinants which have been equated to zero, as follows:

$$- g_2 m_1 \lambda^2 + g_1 b - g_2 a = 0 \tag{14}$$

$$m_1 m_3 \lambda^4 + (am_3 + m_1 d + g_1^2)\lambda^2 + ad = 0 \tag{15}$$

$$g_3 m_1 \lambda^2 + g_3 a + g_1 e = 0 \tag{16}$$

$$(bm_3 + g_1 g_2)\lambda^2 + bd = 0 \tag{17}$$

$$(bg_3 + g_2 e)\lambda = 0 \tag{18}$$

$$(g_1 g_3 - em_3)\lambda^2 - ed = 0 \tag{19}$$

If the solution to any one of these six possible equations is not satisfied by any of the remaining solutions, then the rank of the W matrix is maximum. Thus, the only system motion would be the trivial one. For example, if

$$bg_3 + g_2 e \neq 0$$

from Eq. (18), then the determinant is not equal to zero for any solution of λ other than a trivial one. The trivial solution is ruled out since it implies that one of the variables is constant, and those cases are specifically considered separately. The calculations associated with the remaining six possible motions as well as the calculations already presented in the text are enumerated in Table 1.

We are left with eight specialized system motions for which there is no damping. Additional conditions can be delineated such that damping exists only for the trivial solution. Damping exists for solutions

Table 1. Conditions for Pervasive Damping

Case	Homogeneous Solution	Particular Solution
I X_1, X_2, X_3, C_4	If all determinants are identically equal to zero for all λ, then the system is not pervasively damped. Equations (3-7) through (3-10).	The trivial solution is the only one possible if all of the stability inequalities are satisfied.
II X_1, X_2, C_3, C_4	$\lambda^4 m_1 m_4 + \lambda^2(am_4 + fm_1) + af - eb = 0$ $\lambda^4 m_1 m_2 + \lambda^2(am_2 + cm_1) + ac - b^2 = 0$ $\lambda^2 = -\frac{ag_2 + bg_1}{m_1 g_2}$ $\lambda^2 = -\frac{c + bg_2}{g_1 m_2}$ $\lambda^2 = \frac{ec - bf}{bm_4 - em_2}$ $\lambda^2 = -\frac{eg_2 - g_1 f}{g_1 m_4}$	The trivial solution is the only one possible if all of the stability inequalities are satisfied.

Table 1. Conditions for Pervasive Damping (Cont)

Case	Homogeneous Solution	Particular Solution
IV X_1, C_2, C_3, C_4	$\lambda^2 = -\frac{a}{m_1}$	if $b = 0$ and $-af^2 + acg - e^2c = 0$
	$g_1\lambda^2 = 0$	if $e = 0$ and $af^2 - acg + b^2g = 0$
	$b\lambda^2 = 0$	if $b, e = 0$, and $cg - f^2 = 0$
	$e\lambda^2 = 0$	
V C_1, X_2, X_3, C_4	$\lambda^2 = -\frac{g_1c + bg_2}{g_1m_2}$	if $b = 0$ and $-ce^2 + acg - f^2 = 0$
	$\lambda^2 = -\frac{bd}{bm_3 + g_1g_2}$	if $c = 0$ and $b^2g - 2bfe + f^2a = 0$
	$\lambda^2 = -\frac{g_1f - bg_3}{g_1m_4}$	
	$m_2m_3\lambda^4 + \lambda^2(m_3c + m_2d + g_2^2) + cd = 0$	if $f = 0$ and $-b^2g + acg - e^2c = 0$

Table 1. Conditions for Pervasive Damping (Cont)

Case	Homogeneous Solution	Particular Solution
III X_1, C_2, X_3, C_4	$\lambda^2 = -\frac{g_2 a + b g_1}{m_1 g_2}$	if $e = 0$ and $(b^2 - ac) g + af^2 = 0$
	$m_1 m_3 \lambda^4 + \lambda^2 (am_3 + dm_1 + g_1^2) + ad = 0$	if $b = 0$ and $- af^2 + cag - e^2 c = 0$
	$\lambda^2 = -\frac{g_1 e + a g_3}{m_1 g_3}$	if $e = 0$, $b = 0$ and $f^2 - cg = 0$
	$\lambda^2 = -\frac{bd}{bm_3 + g_1 g_2}$	
	$(bg_3 + eg_2) \lambda = 0$	
	$\lambda^2 = \frac{ed}{g_1 g_3 - em_3}$	

Table 1. Conditions for Pervasive Damping (Cont)

Case	Homogeneous Solution	Particular Solution
V (Cont)	$\lambda^2 = -\frac{(cg_3 + g_2 f)}{m_2 g_3 + g_2 m_4}$	if $b, c, f = 0$
	$m_3 m_4 \lambda^4 + \lambda^2 (m_3 f + m_4 d - g_2 g_3) + df = 0$	if $b, f = 0$ and $ag - e^2 = 0$
		if $c, f = 0$ and $bg = 0$
VI C_1, X_2, C_3, C_4	$\lambda^2 = -\frac{c}{m_2}$	if $b = 0$ and $acg - af^2 - e^2 c = 0$
	$\lambda^{g_2} = 0$	if $f = 0$ and $-gb^2 + acg - e^2 c = 0$
	$\lambda^2 = -\frac{f}{m_4}$	
	$b\lambda = 0$	if $c = 0$ and $-2bef + f^2 a + b^2 g = 0$
		if $b, c, f = 0$
		if $b, f = 0$ and $e^2 - ag = 0$

Table 1. Conditions for Pervasive Damping (Cont)

Case	Homogeneous Solution	Particular Solution
VI (Cont)		if c, f = 0 and bg = 0
VII C_1, C_2, X_3, C_4	$g_1\lambda = 0$ $g_2\lambda = 0$ $\lambda^2 = -\frac{d}{m_3}$ $\lambda g_3 = 0$	Satisfied if the stability inequalities are satisfied.
VIII C_1, C_2, C_3, C_4	—	Satisfied if the stability inequalities are satisfied.

other than the trivial one, and hence is not pervasive, if simultaneously e, f and g are zero or if simultaneously $bg - ef = 0$, $bf - ce = 0$ and $cg - f^2 = 0$.

A limiting example of the hyperbolic case is where the constant speed rotor is oriented along the pitch axis. The hinge axis is also oriented along the pitch axis. Librations in roll or yaw would not be accompanied by energy dissipation, and so this particular configuration would not be pervasively damped. This will be confirmed presently in terms of the preceding equations. The equations of motion for this special case reduce to

$$\begin{vmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & m_4 \\ 0 & 0 & m_3 & 0 \\ 0 & m_4 & 0 & m_4 \end{vmatrix} \{\ddot{x}\} + \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d \end{vmatrix} \{\dot{x}\}$$

$$+ \begin{vmatrix} 0 & 0 & -g_1 & 0 \\ 0 & 0 & 0 & 0 \\ g_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} \{\dot{x}\} + \begin{vmatrix} a & 0 & 0 & 0 \\ 0 & c & 0 & f \\ 0 & 0 & d & 0 \\ 0 & f & 0 & g \end{vmatrix} \{x\} = \{0\}$$

Consider the possible system motion

$$x^* = [x_1 \; c_2 \; x_3 \; c_4]^T$$

Consider the six possible determinants which have been equated to zero, Eqs. (14) through (19). If the solution to only one of these six possible equations is not satisfied by any of the remaining solutions, then the rank of the W matrix is maximum, and a non-trivial solution would exist. For the special case examined, all but one equation remains, since the parameters g_2, g_3, b, and e are identically zero. Thus, the rank of W is not maximum, and we can conclude that the configuration for the special case of pitch augmentation with a hinge axis along the pitch direction would not be pervasively damped.

To this juncture, we have considered two different methods for determining whether a system is pervasively damped. Definitive judgments about specific configurations, i.e., configurations for which the parameter values are known explicitly, can be made. More general statements about the pervasiveness of damping for classes of configuration parameters is more difficult, although the relationships that must be satisfied in order for there to be no pervasive damping are enumerated.

3. STABILITY INEQUALITIES

In developing the stability inequalities, we find that the existence of a positive definite stiffness matrix is sufficient to insure that the system is asymptotically stable, if the damping is pervasive. The stiffness matrix for the hyperbolic case is represented generically as

$$K = \begin{bmatrix} a & b & 0 & e \\ b & c & 0 & f \\ 0 & 0 & d & 0 \\ e & f & 0 & g \end{bmatrix}$$

The K matrix can be guaranteed to be positive definite if the following inequalities are satisfied:

$$a > 0 \qquad (20)$$

$$ac - b^2 > 0 \qquad (21)$$

$$d > 0 \qquad (22)$$

$$acg + 2bfe - e^2c - b^2g - f^2a > 0 \qquad (23)$$

For the conical case, the stiffness matrix is generically represented by

$$K = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & e & f \\ 0 & e & c & g \\ 0 & f & g & d \end{bmatrix}$$

The stability inequalities for the conical case are

$$a > 0 \tag{24}$$

$$b > 0 \tag{25}$$

$$bc - e^2 > 0 \tag{26}$$

$$bcd + 2efg - f^2c - e^2d - bg^2 > 0 \tag{27}$$

The stability inequalities associated with the conical case will not be developed further, since the performance of this particular configuration is markedly inferior to the performance experienced with the hyperbolic case. This comparison is made more precise in Ref. 8.

For the hyperbolic case, the relationship between the symbols employed in the stiffness matrix and the system parameters is as follows:

$$a = C\varphi C\theta \frac{J_0}{I_1} + \left[3 + C^2\theta\right] \frac{\left(I_2^T - I_3^T\right)}{I_1^1}$$

$$b = \left[\frac{I_3^T - I_2^T}{I_1}\right] S\theta C\theta - \frac{J_0}{I_1} C\varphi S\theta$$

$$c = -\frac{J_0}{I_1} S\varphi S\theta - \frac{\left(I_3^T - I_1\right)}{I_1}\left(3 + S^2\theta\right)$$

$$d = \frac{\left(I_2^T - I_1\right)}{I_1}\left(C^2\theta - S^2\theta\right) + \frac{J_0}{I_1}\left(C\theta C\varphi - S\varphi S\theta\right)$$

$$e = S\theta C\theta \frac{I_3^2}{I_1}$$

$$f = -\frac{I_3^2}{I_1}\left(3 + S^2\theta\right)$$

$$g = -\frac{I_3^2}{I_1}\left(3 + S^2\theta\right) + \frac{S}{I_1}$$

Consider the following definitions:*

$$k_1 \triangleq \frac{I_3^1 - I_2^1}{I_1^1}$$

$$k_2 \triangleq \frac{I_2^1 - I_1^1}{I_1^1}$$

$$k_3 \triangleq \frac{I_3^2}{I_1^1}$$

$$K \triangleq \frac{I_1^1 - I_2^1 - I_2^2}{2J_0}$$

$$k_4 = \frac{J_0}{I_1} \qquad (28)$$

*These definitions are not identical to the definitions for inertia parameters that are commonly used in dynamic analyses.

Since the main body of the spacecraft is symmetric, and the control body is a rod with a hinge axis along the body pitch axis, the following relationships hold:

$$I_2^1 = I_1^1$$

$$I_2^2 = I_3^2$$

$$I_1^2 = 0$$

Also

$$k_1 = \frac{I_3^T - I_2^T}{I_1}$$

$$K = -\frac{I_2^2}{2J_0} = -\frac{k_3}{2k_4}$$

$$k_2 = 0 \tag{29}$$

The symbols employed in the stiffness matrix reduce to

$$a = k_4 C\varphi C\theta - k_1\left(C^2\theta + 3\right)$$

$$b = k_1 S\theta C\theta - k_4 C\varphi S\theta$$

$$c = -k_4 S\varphi S\theta - \left(k_1 + k_3\right)\left(S^2\theta + 3\right)$$

$$d = k_3\left(C^2\theta - S^2\theta\right) + k_4(C\varphi C\theta - S\varphi S\theta)$$

$$e = k_3 S\theta C\theta$$

$$f = -k_3\left(S^2\theta + 3\right)$$

$$g = -k_3\left(S^2\theta + 3\right) + \frac{S}{I_1}$$

As defined in Ref. 1, the equation that must be satisfied for equilibrium is

$$\sin(\theta + \varphi) = K \sin 2\theta \qquad 30)$$

So, if K and φ are given, there exist several θ's that satisfy the equation above, but, in general, only one value of θ gives results that are stable. This will be considered in more detail later.

Considering the multidimensionality of the parameter space, it is difficult to give a complete representation of the stability in terms of all the configuration parameters. The size of the parameter space could be reduced in part by limiting the values that the parameters are allowed to take based upon the performance analysis results.

If the parameters φ and K were given, there would exist at least one value of θ from Eq. (30). The proper selection of θ depends upon satisfaction of the inequality in Eq. (22). Since K is specified in advance, there exists a relationship between the parameters k_3 and k_4, where k_3 is a measure of the inertia properties of the control body relative to the main body roll axis and k_4 is a measure of the rotor "strength" relative to the inertia of the main body about the roll body axis. The inequality in Eq. (22) becomes

$$\left(C^2\theta - S^2\theta\right) - \frac{(C\varphi C\theta - S\varphi S\theta)}{2K} > 0$$

or

$$\cos 2\theta - \frac{\cos(\varphi + \theta)}{2K} > 0 \qquad (31)$$

So, given a value of K and φ, there exists a value of θ that satisfies Eq. (30) and the inequality in Eq. (31). Since the parameters k_3 and k_4 are linearly related as in Eq. (29), there exist three independent quantities, k_1, k_3, and S/I, which serve to define the configuration uniquely. By selecting the parameter K, we are constraining rotor "strength" to be linearly related to the parameter k_3. By changing the value of the parameter k_3 we can change the rotor strength relative to a fixed value of the parameter k_3, but we also have a different equilibrium orientation as implied by Eq. (30). The values of K and φ are based upon the performance results in Ref. 8. In Fig. 2, a value of k equal to -0. 175 and φ equal to 2. 34 radians was selected. The value of θ was 3. 775 radians. The orientation of the body is illustrated in Fig. 3. For a given point in k_3, k_1 space, values of the dimensionless parameter S/I and values that are greater identify the stable region.

In Fig. 4 the parameter value of k was changed to -0. 165, implying a greater rotor "strength" for a comparable value of the parameter k_3. But, the equilibrium position is changed from a value of θ equal to 3. 775 radians to a value of θ equal to 3. 784 radians as well. For a given value of k_3, and the increased rotor "strength," the instability boundary associated with the dimensionless parameter S/I shows an almost imperceptible change. In Fig. 5, the value of φ was changed to 2. 44 radians, and the value of K was equal to -0. 175. The equilibrium orientation would necessarily change as well. Again, the instability boundary associated with the dimensionless parameter S/I shows a very slight change for a given parameter k_3.

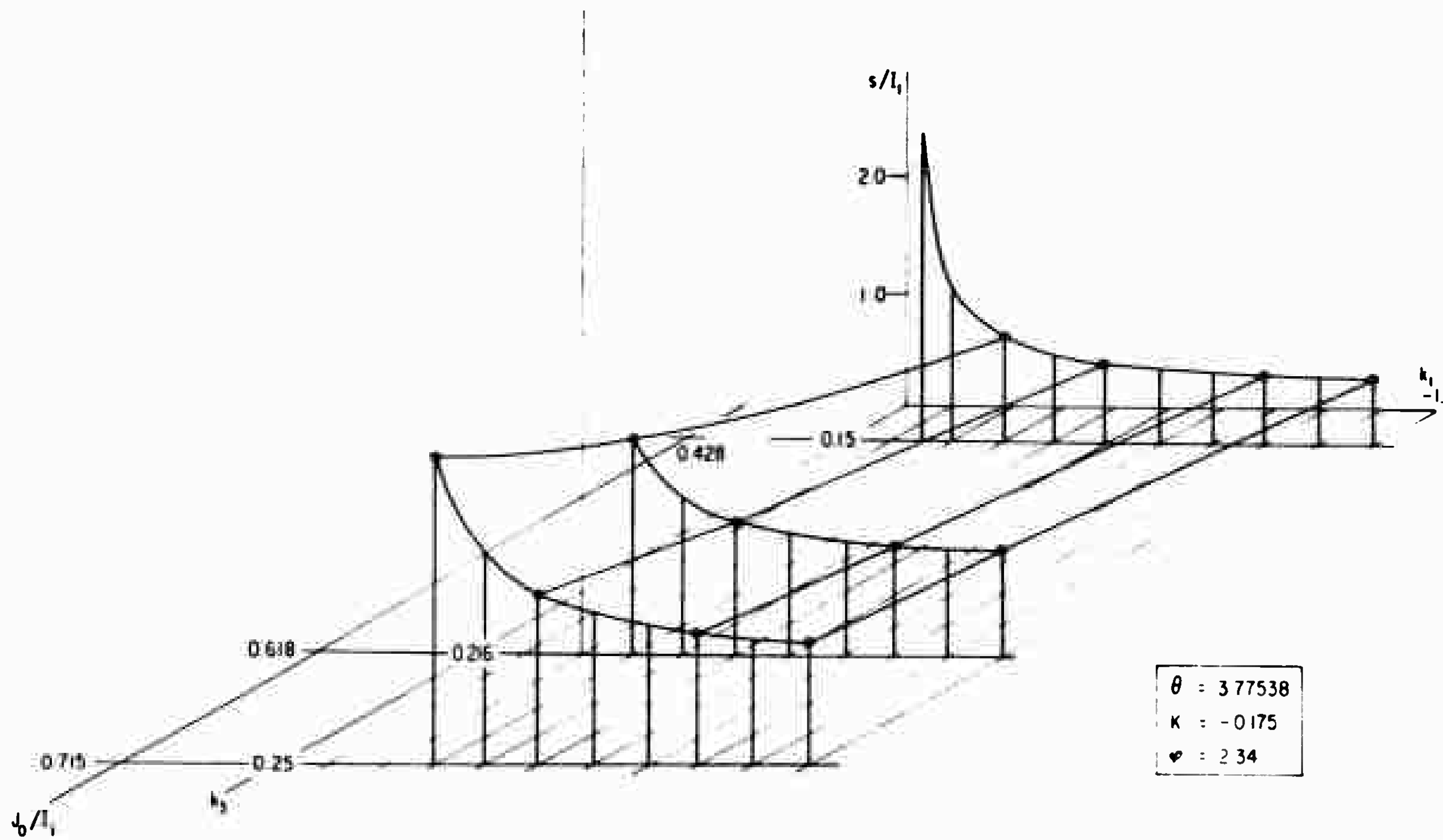

Figure 2. Stability Diagram - Hyperbolic Case (where K = -0.175 and φ = 2.34)

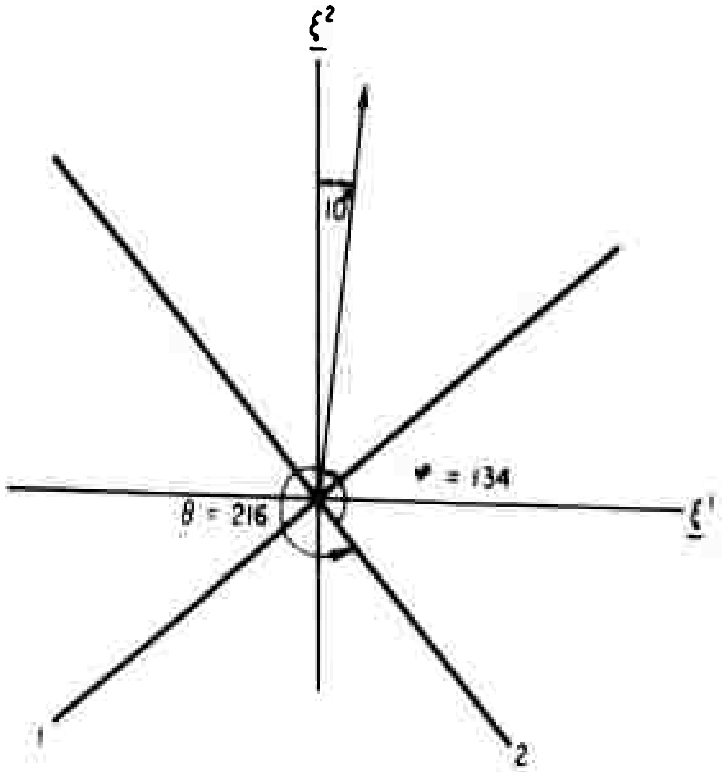

Figure 3. Orientation of the Principal Axes of Inertia of the Composite Satellite System - Hyperbolic Case

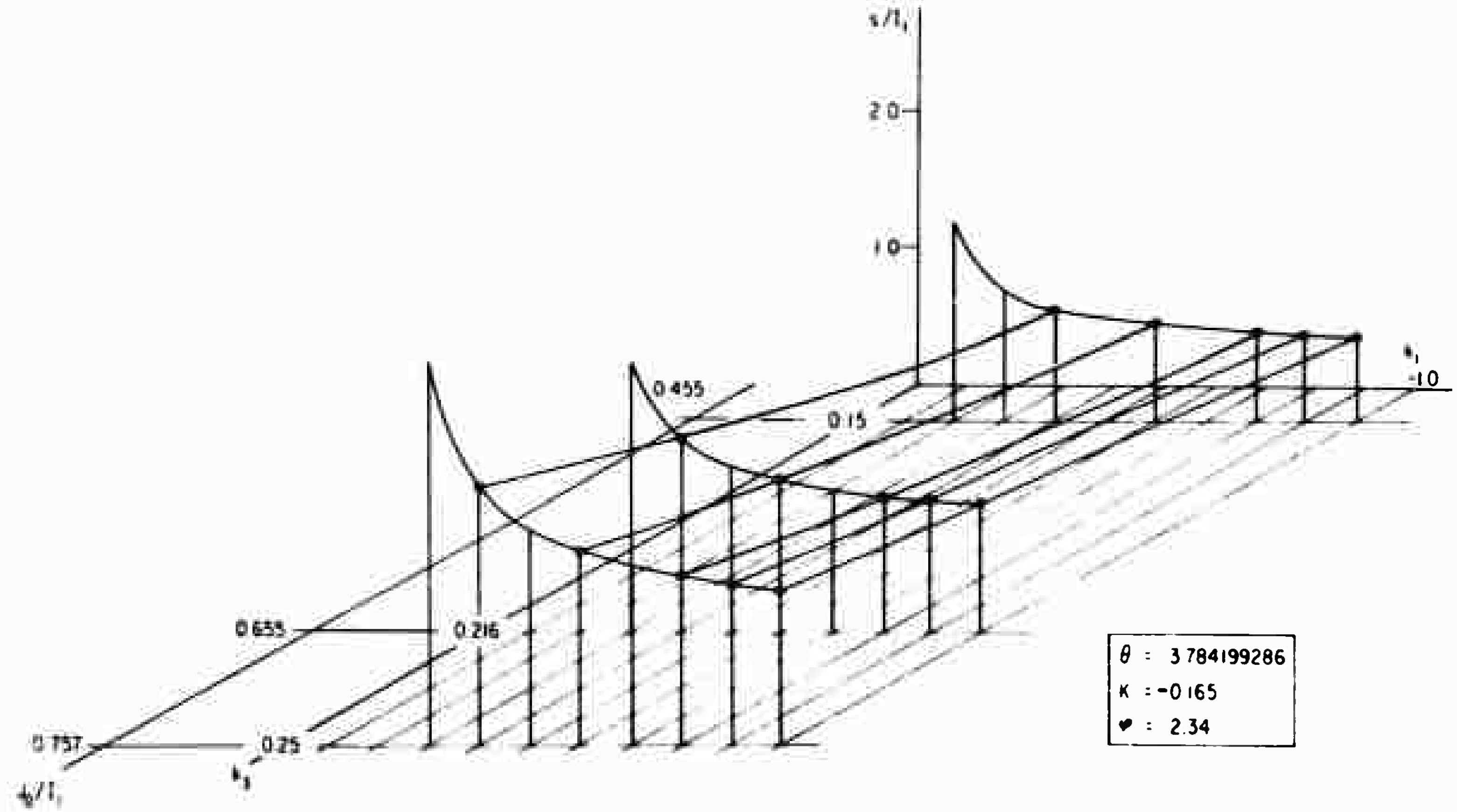

Figure 4. Stability Diagram - Hyperbolic Case (where K = -0.165 and $\varphi = 2.34$)

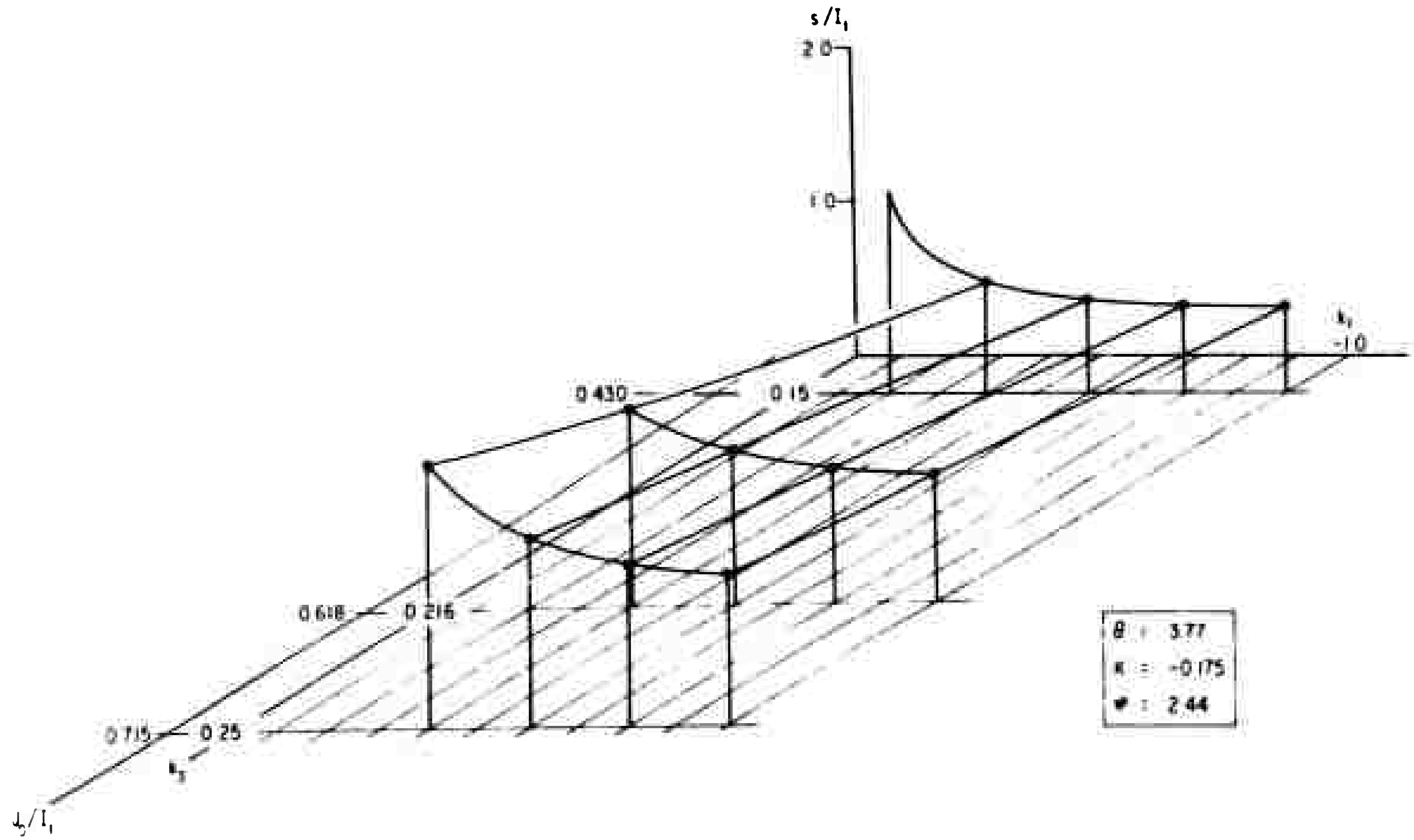

Figure 5. Stability Diagram - Hyperbolic Case (where K = -0.175, and φ = 2.44)

REFERENCES

1. Williamson, R. K., "Rotational Dynamics and Equilibrium Orientations of a Gravitationally Stabilized Satellite Augmented by a Constant Speed Rotor," Report No. TR-0066(5306)-7, Aerospace Corporation, El Segundo, California (69 Jul 15).

2. Zajac, E. E., "Comments on Stability of Damped Mechanical Systems and a Further Extension," AIAA Journal, 3:1749-1750, 1965.

3. Pringle, R., On the Capture, Stability, and Passive Damping of Artificial Satellites, PhD Dissertation, Department of Engineering, Stanford University, April, 1964.

4. Likins, P. W., "Attitude Stability of Space Vehicles," Unpublished course notes, University of California at Los Angeles, Los Angeles, California.

5. Likins, P. W. and D. L. Mingori, "Liapunov Stability Analysis of Freely Spinning Systems," Proceedings of the Eighteenth International Astronautical Congress, Belgrade, Yugoslavia, 1967.

6. Roberson, R. E., "Note on the Thomson-Tate-Chetaev Stability Theorem," to be published.

7. Connell, G. M., "Asymptotic Stability of Second Order Linear Systems with Semi-Definite Damping," to be published.

8. Williamson, R. K., "Results and Conclusions of a Study of a Gravitationally Stabilized Satellite Augmented by a Constant Speed Rotor," Report No. TR-0066(5306)-9, Aerospace Corporation, El Segundo, California (69 Jul 15).

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13 ABSTRACT

The efficacy of a particular satellite design cannot be guaranteed without first examining the problem of pervasiveness of damping. Two recent contributions in the field of motion stability analysis to the general problem of identifying whether a system is pervasively damped are discussed. If the damping is indeed pervasive, then a corollary to the Thomson-Tait-Chetaev stability theorem can be used to establish conditions for which the system is asymptotically stable. These results are illustrated in terms of a gravitationally stabilized vehicle that has been augmented by means of constant speed rotors.

14 KEY WORDS
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