AN ANALYTIC INVESTIGATION OF AIR TRAFFIC IN THE VICINITY OF TERMINAL AREAS

by

Amedeo R. Odoni

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AMEDEO R. ODONI

Technical Report No. 46

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Adapted from a thesis, supervised by Professor Robert W. Simpson, presented to the Department of Electrical Engineering in partial fulfillment of the requirements for the degree of Doctor of Philosophy, September, 1969. The computation described in this report was performed at the M. I. T. Information Processing Center.

FOREWORD

The Operations Research Center at the Massachusetts Institute of Technology is an interdepartmental activity devoted to graduate education and research in the field of operations research. The work of the Center is supported, in part, by government contracts and industrial grants-in-aid. Supervision and other expenditures associated with the work reported herein were supported (in part) by the Office of Naval Research under Contract Nonr-3963 (06), NR 276-004 and (in part) by the National Science Foundation under Grant GK-1685.

John D. C. Little
Director

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The computations described in Chapter 5 were performed at the M.I.T. Information Processing Center.

Finally, I would like to take this opportunity, in order to express my deep gratitude to Professor Alvin W. Drake of the Electrical Engineering Department, who also acted as a thesis reader. Professor Drake has been my faculty advisor during my years of graduate study at M.I.T. Throughout these - sometimes difficult - years, he has always been a source of inspiration and encouragement both as an excellent teacher and as a friend.

Amedeo R. Odoni

AN ANALYTICAL INVESTIGATION OF AIR-TRAFFIC

IN THE VICINITY OF TERMINAL AREAS

by

AMEDEO R. ODONI

Submitted to the Department of Electrical Engineering on August 18, 1969, in partial fulfillment of the requirements for the Degree of Doctor of Philosophy.

ABSTRACT

Several analytical models for air-traffic in the general air terminal area are constructed, with the purpose of exploring some questions about air-traffic congestion.

Attention is first focused on a single runway which is used exclusively for landings. The distribution for interarrival times of aircraft is obtained under capacity conditions. The inputs to the model are the velocity distribution of incoming aircraft, the distribution for the errors in spacing aircraft and the minimum separation requirements, both in the air and on the ground, as specified by the FAA.

A similar analysis is performed for a runway which is used only for take-offs. The conceivable situation in which landings alternate with take-offs is also investigated. In all cases, the sequence of procedures is analyzed by using an applied probability theory approach.

The results from the basic models are used in order to develop expressions about queuing and delay characteristics associated with various operations. In particular, an original method for modelling the arrivals queue is developed. Comparisons of the expressions derived for this model with those commonly used for estimating aircraft delays, suggest that the latter may be over-estimating runway utilization and average delays. In addition the severe effect of spacing errors on delays is clearly demonstrated.

The possibility of inserting departures between successive arrivals, thus increasing airport capacity, is also examined in detail. Several parameters of interest associated with this situation are computed. Some numerical results are presented along with a qualitative discussion of various issues.

THESIS SUPERVISOR: Robert W. Simpson
TITLE: Associate Professor of Aeronautics and Astronautics

TABLE OF CONTENTS

Chapter I

General Air Traffic Control Considerations

1. Introduction

The problems of our major airports have suddenly gained a central position in the public interest during the last three years. It is a measure of this new interest, that both major candidates in the 1968 presidential campaign felt compelled to make very detailed statements about their plans for alleviating this latest crisis in air transportation. Rather grotesque stories about delays in the air or on the ground inside an airplane, can also be heard around the cocktail party circuit, nowadays.

Briefly, the problems of the air transportation industry can be separated into two very broad classes:

The first class includes the problems of ground transportation to and from the airports, general accessibility of the terminal areas, interconnections among different terminal buildings and problems in the terminal buildings themselves (baggage handling, ticket counters, associated services, even provision of adequate sitting room for all passengers).

In the second class fall all problems associated with regulating the movement of the aircraft themselves, in the air and on the ground -- in other words problems in air-traffic control (ATC). In this report we concern ourselves exclusively with that aspect of this second area which deals with the control of air-traffic in the vicinity of the terminal areas, i.e., with the landing and take-off stages of flying.

More specifically, we will concentrate on operations conducted under poor visibility conditions, i.e., in "instrument-flight-rules" (IFR) weather, as opposed to "visual-flight-rules" (VFR) weather.

A few words should be said about the importance of research in air-traffic control, although the reader may, by now, be very familiar with this wort of argument.

The annual contribution of civil aviation (this includes commercial aviation and general aviation) to the gross national product of the United States is estimated to be of the order of $10 billion [15]. The airlines investment in new equipment for the decade 1965-1975 should be inthe vicinity of $18 billion [34]. These are just two measures of the staggering stakes involved in the air transportation field.

Yet, a 1966 survey of 21 airline presidents by the New York Times, revealed that 18 of the 21 believed that the main problem facing the industry were the delays in getting planes in and out of airports because of congestion in air-traffic.

At the base of the traffic congestion problems, lies the rather peculiar distribution of traffic activity which is heavily concentrated around the major cities of the United States. For example, in 1967, the 50 largest airports accounted for about 40% of the 30 million itinerant operations (departures or arrivals) conducted in some 320 airports with FAA control towers. Primarily responsible for this is the distribution of commercial aviation flights. Twenty percent of all air carrier operations took place in New York, Chicago, or Los Angeles, the three major centers of airline activity [11], for instance.

The effects of this peculiar situation on the movement of air-traffic are catastrophic. The presence of many large airliners in the vicinity of an airport, each of them requiring a lot of airspace for maneuvering and each carrying a large number of passengers, increases the concern for safe operations and for appropriate control of the airways. Because of the increased stakes involved, pilots and controllers cannot rely any more on their visual perception only, and even under VFR conditions, airline pilots request that landings and departures be performed under instrument flight rules. The statistics in this respect are quite impressive: although the total share for air-traffic operations of New York, Chicago and Los Angeles came to 5%, their share of all IFR operations came to 19%, in 1967. Moreover, while only 23% of all aircraft operations were conducted under IFR rules, the corresponding collective percentage for O'Hare (Chicago), JFK International, La Guardia, Newark (New York), Los Angeles, San Francisco, and Washington International was 81% [11]. Four terminal areas (New York, Los Angeles, Chicago, and Washington, D.C.) accounted for about 40% of all instrument operations.

Now, delays are primarily associated with IFR flying. The strict separation standards which go in effect when these rules are used, and the need to use only those runways which are equipped with the appropriate electronic instruments, substantially reduce the rate at which operations are conducted at airports. Delays due to congestion occur, almost exclusively, during the arrival and departure segments of a flight: only 8% of the delays occur because of interference among aircraft during the enroute portion of the flight while the

the corresponding percentages for the arrival and departure parts are 59% and 33% respectively [31].

The cost of these delays to the air transportation industry is quite difficult to estimate. The numbers given usually differ by several million dollars, depending on who is providing the statistics. For 1966, the direct operating costs to the airlines (mainly extra fuel consumption and additional compensation for flight and maintenance crews) were given by the New York Times as $63.6 million. For 1962, Fromm [15] places the cost for time-loss to passengers at $59 million. Although it is not very clear how these numbers were computed, they appear to present a fair picture of the order of magnitude of the losses involved.

Even more important than these direct costs, it is felt here, are the indirect losses incurred: loss of goodwill, deterioration of confidence on the part of the public, and shift of interest to other modes of transportation.

Initially (this is already taking place) the hardest-hit airlines are those which provide primarily short distance flights. Mohawk Airlines, for example, which specializes in short flights in the Northeastern states reported that in 1966, the cost of delays to its flights amounted to 25% of its total operating costs.

The delay problems are, of course, less painful for those airlines which provide primarily medium and long range flights. However, it is rather disconcerting to learn that the expected door-to-door travel time from downtown Chicago to downtown Los Angeles in 1975 is expected to be about the same as it was in 1950, although flight time will have been cut to almost 1/3 of what it used to be [34]. This is certainly a far cry from the visions of those who have always spoken about the glowing future of air travel in this country and throughout the world.

2. Directions in Research on Air-Traffic Control

In this section we have tried to put in perspective the scope of an analytical investigation such as the present one and to relate it to the various efforts for improvements in the current air-traffic control system.

Of all the problems plaguing air transportation today, it appears that the ones which are more likely to stay with us for a long time to come, are those associated with congestion in the air and on the ground at the airport areas. These same problems have been the most vexing and pressing ones for some time now.

Putting it in a simple way, it is very likely that in the very near future: the utilization of airspace during the enroute portion of flights will be much more efficient; there will be adequate terminal buildings for both passengers and aircraft; the reservation and baggage handling systems will become increasingly more effective and foolproof. However, the type of aircraft using the runways will possess substantially the same landing and take-off characteristics as today, and, at the same time, no substantial increase in the number of available runways is foreseen while a 40% increase in the total number of flights is predicted by 1974 [34]. The latter rise is not as spectacular as the projected increase in the number of passengers, or the number of miles flown, nevertheless it is much more than the presently overburdened major airports can bear.

That the characteristics of aircraft (at least as far as landings and take-offs are concerned) are not going to change radically within the next decade or so, appears quite certain, at this point. The use of VTOL and STOL aircraft will increase, of course, but aircraft of this type are presently fit for rather short trips and, more important, can carry only small numbers of passengers. Development of a STOL airplane capable of carrying a large number of passengers for even medium range distances requires initiation of a project which is likely to be a long, expensive, and controversial (witness the SST case) one. Such a project is not presently forthcoming. Hence, VTOL and STOL aircraft will most likely continue to serve as a means of transportation from various points in and around the cities to the airports and back.

It was also stated above that the number of available runways will not increase by much, at least at those places where they are really needed. At this stage, the construction of new airports is more a political than an engineering problem. The planners and engineers have long ago made their case for extension of the old runways and construction of new ones.

However, any program for a major increase in the number of existing runways has become too big for local or even state governments to handle because of: (i) the immense costs of acquiring new land in the vicinity of major air-hubs; (ii) the complex legal and legislative issues involved in such acquisitions; (iii) the strong reaction of communities to the possibility of more noise, air pollution and disruption of life; and (iv) the costs of construction of new highways, transit systems, etc., required for servicing the new airports. Any major action should, therefore, be taken at the federal government level with appropriations of considerable funds, if such a program is to be initiated. This is not to suggest that the author thinks that such a program should rank very high among national priorities competing for the allocation of funds: in fact, he thinks it should not, despite the undeniable fact that air-transportation faces serious difficulties today. But, anyway, such a program seems to be contemplated now. The trouble is that, as past experience has shown, airport construction always takes much longer than anticipated. Therefore, for the next several years one will have to make do with whatever is presently available.

Thus, the problem is, in effect, that of improving the present system so that it can accommodate the increased traffic which is anticipated for the future, to the best degree possible.

In the last three years, ever since the problems of delays to airplanes became particularly acute, articles on air-traffic control have been appearing at an almost bewildering rate in numerous publications. Many of these recent writings discuss in very qualitative terms and in rather vague form "possible solutions" to the problems of our airports. The most often made suggestion, stated generally, is that the air-traffic control system should be "computerized" to a greater extent than now and that more accurate navigation and guidance instruments should be built at the same time.

However, the need for better equipment and more aid from computers has been recognized by the FAA more than a decade ago. What is sorely needed at the present stage is intensive work in two general directions:

(a) Toward the completion of computer hardware and software as well as of guidance and navigational instruments that would alleviate the burden on ground controllers and airline pilots, and would lessen the airspace requirements of individual aircraft.

(b) Toward a better understanding of the present ATC system, including the diagnosis of primary and secondary causes of delays and the exploration of ways to reduce them.

Examples of work of type (a) are:

(i) The radar equipment presently being installed at control centers throughout the country; on the radar screen the controller can read, next to the usual beacon image indicating the position of an aircraft, information about the identity and type of aircraft and about its altitude.

(ii) The new "cooperative" collision avoidance system (CAS) which is in the last stage of testing; this device will detect the presence of aircraft that could constitute a potential collision threat, analyze the threat and command the pilot to make an appropriate avoidance maneuver. Unfortunately, the device works only when both planes in a pair of aircraft on a collision course are equipped with compatible CAS equipment.

Contributions toward an understanding of the causes of delays in the present system have been numerous and can be classified as either analytical investigations or investigations by simulation (real time or fast). A rather extensive review of past analytical work is presented in the next chapter. Some simulation studies are reviewed in the reports of Blumstein [4] and of Simpson [35].

The two approaches are, of course, complementary. A detailed examination of a specific given situation at an airport would seem to require the use of simulation because of the multitude of parameters involved in such a study. On the other hand, an analytical investigation, when it is successful, provides a better understanding of the dependence of various statistics of interest on different parameters. Thus, in a study of rather generalized, abstract models of reality with the intent of investigating various, widely different combinations of system configuration, the analytical approach seems to be indicated.

In any case, one should never lose sight of the limitations and pitfalls of either approach. A simulation is liable to produce results which are statistically "freakish", because of the sampling methods used. This is true, in particular, in cases when systems are examined for configurations that may result in wide variations of system characteristics with time (an example would be the investigation of a queuing situation at an airport operating near the saturation point). In such cases it may be necessary to obtain a huge number of results in order to produce meaningful statistics.

On the other hand, whenever the analytical approach is used, one should remember that models can become so oversimplified as to lose any resemblance to reality.

In addition to providing better insight, analytical and simulation studies of air-traffic can be very valuable in that they may point out the most promising areas for conducting future research. One may, for example, consider the potential value of designing new radar equipment or the potential benefits that may result from altering the minimum separation standards for aircraft. One can also point out to planners, the futility of building, say, a new airport at a location where airspace is so congested that facilities on the ground would be of little help (witness the fourth airport for New York City).

Returning now to our previous discussion, the availability of new equipment coupled with a better understanding of air-traffic control processes may lead to radical improvements and changes in the air transportation system, in the near future. Proposals for such changes appear with increasing frequency.

For example, the need for a more tightly regulated system is becoming increasingly clear. The banning of certain types of general aviation aircraft from the three major New York airports for certain hours of the day (and increased users' fees for the remaining hours) have already produced some welcome relief for commercial aviation. Granted, that this measure may impose some hardships on general aviation fliers (although even that is questionable, because of the availability of several general aviation facilities in the New York area). But for better or worse, the days of laissez faire seem to be over for aviation, at least in certain segments of the country.

This general trend toward a more tightly controlled system will undoubtedly continue. The next step in this direction, will very likely be some effort toward forcing the airlines to spread more evenly their scheduled arrivals and departures throughout the day.

Also as a result of better insight and equipment, we may in the future be more willing to examine the whole air-traffic system from a systems engineering point of view, recognizing the inter-reaction among the various components of the system and the importance of a stable, regulated flow of users through the network. Two different proposals for studies of this type have come to the attention of the author recently.

3. ATC Rules in IFR Weather

An attempt to describe here the present ATC system in all of its aspects would be pointless, since we are only interested in instrument flight rules (IFR) in the vicinity of terminal areas. We will therefore refer the reader to the excellent review of ATC presented by Blumstein [4] and will concentrate here, on the particular topics which are pertinent to this report.

During the enroute portion of a flight, a plane flying under IFR is under the guidance of Air Route Traffic Control Centers (ARTCC). There are 41 such centers today, effectively covering the whole area of the United States. The ARTCC are, then, responsible for surveillance of the network of sky routes, better known as "airways". Airways are defined by radio transmitter pairs called "fixes". When an enroute plane passes above a fix, its time of arrival to the next few fixes is projected. If the controller in an ARTCC detects any conflicts with other projected arrivals at a fix, he takes appropriate steps to avert any violation of the FAA minimum separation standards for the enroute portion of the flights.

When a flight approaches its destination area, control over it is transferred from the nearest Air Route Traffic Control Center to the Approach Control Center of that area. The airspace supervised by Approach Control extends to a radius of about 20-30 miles away from an airport and is called the terminal area. As an aircraft enters a terminal area for a landing, the ARTCC formerly supervising its flight, "hands off" the aircraft to Approach Control. Entry to the terminal area does not take place at just any point along its periphery but at various specific points, marked by radio transmitters. These points are called entry fixes.

After reaching an entry fix, a pilot is instructed on whether to proceed immediately to a landing along a given route, or join a waiting line of aircraft. This waiting line is called a "holding stack" because of the fact that aircraft are "stacked" at different altitudes above a radio transmitter called a "holding fix". The planes in the holding stack fly the so-called "holding pattern", a path having the shape of a racetrack(hence the alternatively used name "racetrack pattern") which usually takes about 6 minutes to cover (hence the name "6-minute pattern"). The vertical separation between successive levels of the stack is 1,000 feet. As the holding stack can extend up to an altitude of 10,000-12,000 feet, it can accommodate up to 7 or 8 different aircraft if needed.

The plane at the lowest level of a holding stack is always the first one to be ordered to leave the stack and proceed to a landing, once the controller decides that such an operation is appropriate. When the bottom plane leaves the stack, the rest of the planes in the stack descend to the immediately lower level, one plane at a time, starting with the plane at the lowest level.

A plane that leaves the stack, proceeds through an area which is called "regulator space" (for reasons to be explained immediately) or simply "funnel" (because of its shape -- see Figure 1). There are several alternative paths that an aircraft may follow through the "funnel". These paths, usually called fan patterns,are determined by the controller on the ground. His objective is to space appropriately aircraft travelling through the "funnel" space, so that they reach the approach gate of the glide path with the distance separation desired by the

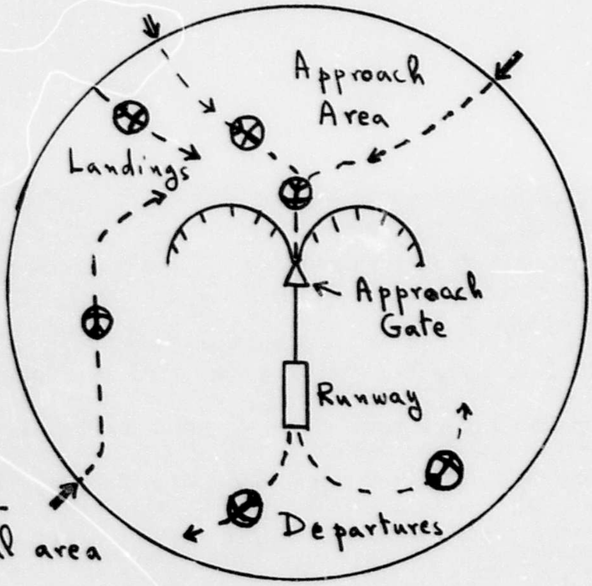
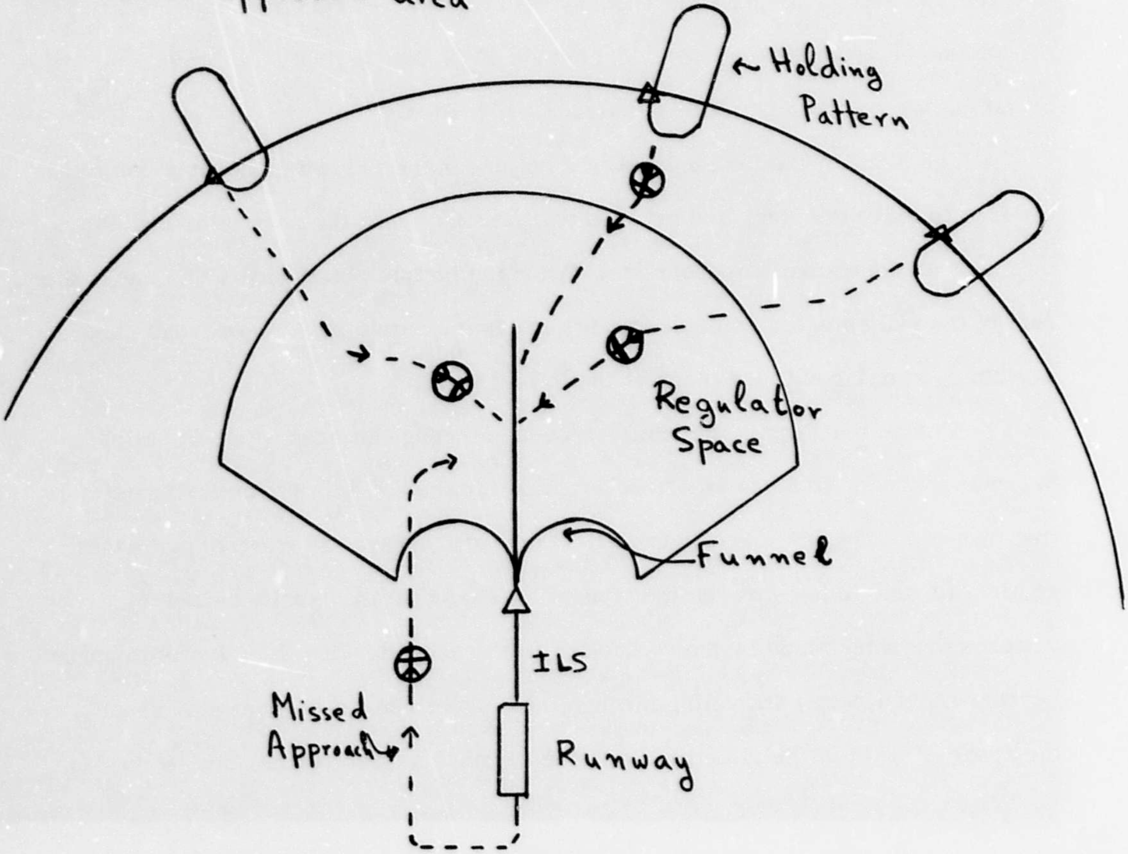


Figure 1.1(a)
The terminal area

Figure 1.1b
The IFR approach area



controller. This explains the term "regulator space".

The last leg of the landing route is called, as already mentioned above, the "common glide path" or "common approach path." This is the final path of descent and the term "common" derives from the fact that all aircraft flying under IFR must fly exactly the same route during this last segment of the flight. That point at the beginning of the glide path where all aircraft must have joined the path is called the "approach gate". Two radio transmitters, the "outer marker" and the "middle marker" indicate to the pilot his distance from the runway.

As soon as the aircraft enters the glide path, control over it is transferred from the Approach Control Center to the Airport Traffic Center which supervises air traffic on the runways, the taxiways, and the airspace in the immediate vicinity of an airport. The Airport Traffic Center is commonly known as "the tower"; it is responsible for giving the final clearance to land, as it exercises visual control over its area of responsibility.

The descent along the glide path is facilitated by the instrument landing system (ILS). Electronic signals (VHF and UHF) create a precise approach path so that the pilot can line up on the runway and descend to the point where he can see the approach lights; from that point on he can proceed visually. The landing aircraft guided by the ILS usually touches down at a point about 1,000 feet from the beginning, the "threshold" of the runway.

Once the plane touches down, it decelerates and leaves the runway at an appropriate taxiway. An FAA rule which is strictly adhered to, is that only one aircraft at a time may occupy a runway which is used for arrivals.

Actually, the controller can exercise very little control over an aircraft once it starts descent along the glide path. Therefore, in order to avoid violation of the single occupancy requirement for runways, the controller must space aircraft appropriately before they reach the approach gate. The way this is done is by estimating the expected arrival times of aircraft and spacing aircraft so that the difference between their arrival times is greater than some minimum time, say t_0. This practice results in what is known as "time separation requirement" on the runway. In other words, arrivals are not permitted to land at time intervals less than t_0 seconds. The commonly used value for t_0 in IFR weather is 90 seconds.

In addition, there are strict minimum distance separation requirements for aircraft in the airspace of the terminal area (the separation requirements are different for aircraft in the enroute portion of the flight). A minimum horizontal separation s_0 (usually 3 miles) must be maintained between aircraft. When aircraft are closer by, they must maintain a vertical separation of 1,000 feet. Thus, when successive aircraft reach the approach gate and throughout their descent through the glide path, they must always maintain a horizontal separation of s_0 miles. The actual length of the common approach path is usually between 6 and 10 miles depending on the airport.

The IFR weather procedures for departures are considerably simpler. After leaving the loading area, a departing aircraft proceeds through the taxiway network to take up its position in the waiting line of departing aircraft. Once permission is granted by the tower for a take-off, the first aircraft in the line of departures enters the runway and lines up to start the take-off roll.

After the departure leaves the runway and clears the "outer threshold"

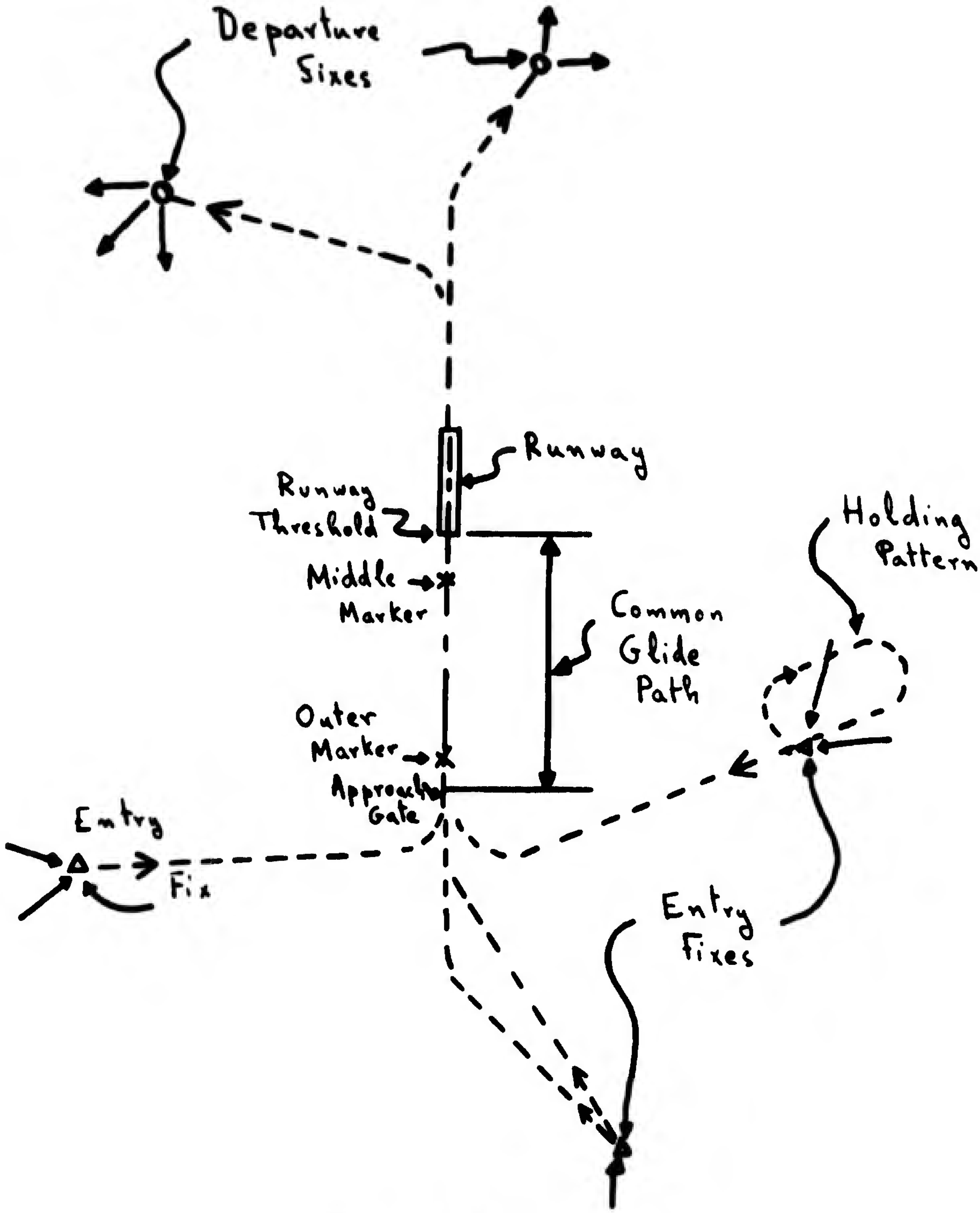


Figure 1.2

IFR Landing and Departure Routes
(Source: Pestalozzi [31])

of the runway, the tower turns it over to the Approach Control Center. There, the controller responsible for departures guides the plane through its climb and to the appropriate departure fix, depending on its destination. After this point, the departing plane leaves the terminal area and control over it is transferred to one of the enroute ARTCC.

The minimum separation standards for take-offs are set as minimum time separations between successive departures. If two successive departures are scheduled to fly the same course, the time separation requirement is 3 minutes; if they will fly the same course initially and will follow diverging courses five minutes after departure time, the separation is 2 minutes; finally, the separation is 1 minute for successive departures following entirely divergent paths. Since the least interdeparture intervals that can be achieved without the existence of minimum time separation standards are of the order of about 60 seconds, this last requirement is usually ignored and successive departures following divergent courses are dispatched as soon as possible; i.e., the second of the two departures receives clearance to take-off as soon as the first of the pair has cleared the outer threshold of the runway.

We have outlined above the essential procedures of air traffic control in IFR weather. In practice, as congestions became worse with the years, several extensions of these procedures have been added. For one thing, there is significant interaction among a local Approach Control Center on the one hand and the enroute ARTCC on the other. Thus, whenever the congestion at a particular airport becomes critical, the enroute controllers initiate delaying action on those aircraft directed toward the location of trouble. Eventually, in addition to the primary holding stacks within the terminal area, secondary stacks are created in neighboring control areas. This, of course, alleviates the pressures on the controllers at the primary congestion area, but has a propagating effect on air traffic in other locations. For example, when many New York area controllers reported "sick" recently (June 18-20, 1969) the resulting tie-up of New York-bound traffic, quickly disrupted air-traffic operations not only throughout the United States, but in Europe as well. An additional reason for the propagating effects of a major tie-up is that many flights are diverted to airports other than their original destination, thus upsetting the schedules of these airports.

Another modification in the procedures, concerns the manner in which airplanes leave the holding stacks (there are usually two or three separate primary holding stacks within the terminal area of a major airport). Instead of clearing one plane at a time from a stack, as mentioned above, a currently used experimental procedure calls for the controller to remove from the stack the bottom three planes each time. The reasons for this change are, on the one hand, the improved ability of the controller to check and control the flight profiles of aircraft (primarily due to better radar equipment) and, on the other, the desire to neutralize to any extent possible the secondary delays which may arise from the geometrical shape of the flying stack. This last point is extensively discussed by Simpson [35].

So far, we have not discussed the question of priorities for the use of airport facilities in IFR weather.

One fundamental rule which still remains a "sacred cow" of air-traffic control is the first-come, first-served principle. Thus, arriving aircraft land according to their order of arrival at the terminal area, and departures take-off in the order they reach the runway used for take-offs. As the reader will readily guess, this priority rule can be adhered to only loosely in the case of arrivals, since it is not always clear who arrived first at a terminal area. It often happens that, because of the different routes available to landing aircraft, a fast plane may bypass a slow one enroute to a holding stack or to the approach gate of the glide path. Of course, once the common glide path has been reached one aircraft can not pass another. With the aforementioned reservations, anyway, we can say that the first-come, first-served principle applies quite well in the case of landings.

Application of first-come, first-served rules is even stricter in the case of departures. The only case in which the tower sometimes intervenes is in order to avoid the formation of a waiting line in which two successive aircraft are bound for the same destination. As mentioned above, there is a three-minute minimum separation requirement for successive departures in this case. The tower controller may therefore attempt to interpose other airplanes between such a pair of departures. This, of course, has to happen prior to the formation of the waiting line. Once aircraft have reached the taxiway, next to the runway which is used for take-offs, there is very little that the controller may do, in terms of rearranging the order of departures.

It remains to discuss the relative priorities of landings and departures. In general, when a common runway is used for both operations, landings have priority over departures. Primarily, the reasons are: safety, since there are dangers of collision between aircraft awaiting to land; reduction of the pressure on the controller since aircraft on the ground are much less of a concern to him, than aircraft circling in the holding pattern; and economy, since the fuel consumption of aircraft in the air is about twice that of aircraft on the ground. Thus it is customary to attempt to land all arriving aircraft in a waiting line before servicing the queue of departing aircraft, whenever a common runway is used.

However, when the line of departing aircraft grows too long or whenever a departure has waited for an unusually long period of time, the controller may decide to delay the landing aircraft and insert some departures between successive strings of arrivals. The one case when landing aircraft may not be delayed in this manner, is when an arrival is already in the glide path. In this case, a controller can not provide clearance for a departure unless the landing aircraft is more than 4 miles away from the runway threshold. In summary, landings have priority over departures, but on occasion the rule is not adhered to strictly.

Chapter II - An Overview of this Study

1. Previous Analytical Reports on ATC

The existing literature on air traffic control is quite voluminous. In this section we will discuss only that particular class of reports which is of immediate interest to this analytical investigation. We are specifically interested in works that examine air traffic operations from the probabilistic analysis point of view. The reader who is interested in other approaches to the air-traffic control problem (simulation studies, systems control studies, etc.) should consult the extensive list of references in [4], [34], and [35]. As for a well-written and comprehensive discussion of air transportation problems and of possible directions for future research, the book by Seifert and Schriever [34] is particularly recommended.

Returning to papers of interest here, we note that the first analytical studies of delays to airport traffic were made in the late 1940s in Great Britain. These studies by Bowen and Pearcey [5], Pearcey [30] and Bell [3] discuss the queuing situations arising under VFR conditions. Bell was the first to discuss the validity of the assumption that arrivals at an airport constitute a Poisson process, an assumption which had already been used by Bowen and Pearcey. The assumption of Poisson arrivals is also used in the present report.

An interesting paper by Galliher and Wheeler [16] appeared in 1958. To the knowledge of the author, this is still the only analytical effort that takes into account the non-stationary character of the arrivals rate. The paper considers a day as divided into discrete intervals. The arrivals rate may change from one interval to another but it remains constant for the duration of each interval. The state of the queue of arrivals at the end of an interval is taken as the input state for the following interval.

The present report does not mention the variation in the arrivals rate during a day. Although this is a serious flaw, it appears that, in a strange way, the assumption of a constant traffic flow may, in the not too distant future, be a fairly accurate one, at least for the daylight and early evening hours. This is due to the fact that the terrible congestions occurring during the peak traffic hours are already forcing a more even scheduling of flights throughout the day, thus smoothing out the peaks and valleys of "the traffic rate vs hour of the day" charts.

Among other interesting papers we might also mention, although slightly outside the scope of this report, the papers of Horonjeff [17] and Rosenshine [33]. Horonjeff deals with the optimum location of high-speed exit taxiways. Rosenshine considers the delays that may arise due to the slowness of communications between the ground and aircraft in the air. As these delays can be expected to increase with the number of aircraft present in the holding stack, the length of service times (time required for communication with the controller) becomes state dependent.

Perhaps the most important analytical report on the subject of air traffic is due to Blumstein [4]. In his investigation of landing operations in bad weather (IFR conditions), Blumstein was the first to point out the importance of the common glide path in determining the capacity of a runway. Blumstein also estimated the landing capacity of a runway under saturation conditions (which he defines as the situation in which aircraft are always awaiting at the beginning of the glide path for clearance to land). His model is extended in this report to include the spacing errors due to the controller.

In another part of his paper, Blumstein derives the probability that a controller may be able to insert a departure between two successive arrivals on a runway. This probability is also derived in the present report under a different set of assumptions.

With reference to Blumstein's work, it is interesting to note that his definition of the capacity of a runway is by no means the generally accepted one. Under contract from the FAA, the Airborne Instruments Laboratory (AIL) conducted a series of ATC studies from 1958 to 1963. One of the results of these studies is the definition of landing capacity as the maximum arrival rate for which the expected delay per arriving aircraft does not exceed a maximum acceptable value (usually 4 minutes). This definition seems to be lacking in the sense that the estimation of expected delays depends strongly on the type of procedures used, the type of aircraft present and the queuing model itself [2].

One of the most interesting studies performed by AIL, is an investigation of possible sites for the fourth New York Airport [24]. In this report the airspace limitations in the New York area are vividly depicted. To avoid conflicts with the airspace requirements of the three existing airports, sites as far away as 87 miles southwest of Manhattan (Lebanon Forest) were considered.

Simpson's voluminous report [35] is the next major contribution on the subject of ATC. The analytical part of his work deals with various aspects of air-traffic such as the "trombone pattern" (in VFR weather), the holding stacks, the laddering process, and the flying patterns in regulator space. Of special interest to this author is Simpson's conclusion that the whole sequence of holding procedures during high-traffic-density periods in IFR weather, may introduce secondary delays in addition to those due exclusively to the limited capacity of the runways. For example, Simpson has pointed out that an aircraft may not be able to leave the holding stack at the instant desired by the controller. The second part of Simpson's report describes an extensive fast-time simulation of air traffic in the vicinity of the terminal area.

The work of Oliver and of Pestalozzi at Berkeley is also especially important. Pestalozzi's doctoral dissertation [31], in particular, provided the motivation for a good part of the work presented in this report. Pestalozzi noted that, since the service time for a particular incoming aircraft also depends on the type of aircraft that lands just ahead of it, it follows that service times of landing aircraft can not be considered independent random variables. Accordingly, Pestalozzi has attempted to solve a very complicated problem based on a model picturing this situation. He was only partially successful in this effort. The same problem has been tackled with a different point of view in this report, with very good results.

Other papers by Oliver [29], [28], and Pestalozzi [32] touch on such subjects as the delays to arriving aircraft, delays to departing aircraft, and rules for deciding on priorities for prospective users of a runway -- provided, of course, that the usual first-come, first-served rule, now in force, is abandoned. Finally, in this respect, we would like to mention Oliver's lucid review of air traffic research [26] as mandatory reading for anyone interested in this field.

In the last two years, several papers dealing with various topics related to airport capacities and delays have appeared. In particular, we mention in the bibliography some reports from the 17th IATA technical conference on "Major Airport and Terminal Area Problems" [19]. In these papers, a very encouraging trend away from the simple presentation of statistics and the attempt to develop some handbook type formulae, and toward a more substantive discussion of the issues and problems is evident. A report from the Aviation Operational Research Branch of the United Kingdom's Board of Trade [20] is especially interesting. Among other things this report contains the first extensive discussion of what would be a desirable strategy for arranging the order of take-offs that has come to the attention of this writer.

Finally, in addition to the references on queuing theory given in the bibliography, we would like to mention that some of the literature on road traffic has been found to be extremely useful and pertinent to some problems or air traffic control. We have listed some papers of this type by Weiss and others [39], [10], Oliver [27], Oliver and Bisbee [28], Newell [25], and Tanner [36], [37].

In some respects, problems on ATC seem to be easier from the analytical point of view than the corresponding problems on road traffic. For example, in dealing with air-traffic one does not have to consider such complicated issues as driver impatience or multi-lane traffic. On the other hand, in other respects ATC problems are more difficult. For instance, arrivals of take-offs at the edge of a runway can hardly be assumed to constitute a Poisson (random) process, while such an assumption would seem to be valid for arrivals at an intersection of cars on a secondary road. These points will be further discussed in the course of this study.

2. Preview of the remainder of this report

We would now like to present a short "preview" of the material contained in the following chapters. It is felt that in this way the reader will be better able to piece together the significance of the results that will be developed. It will be easier to see where each derivation and analysis fits in the general scheme of this thesis.

The runway is the most important element of the air traffic control system. All flights originate there and are concluded there. It is a facility that serves all aircraft; its careful and judicious use is essential to the smooth flow of air traffic. The remainder of this report concentrates on the important characteristics of this service facility and of the operations that take place on it and in its vicinity.

Roughly, the course to be followed is this: First, we will, more or less, concentrate on the individual user of a runway, in other words, we will focus on a particular aircraft. We will try to follow the sequence of operations it has to go through, when using the runway. We will try to somehow measure the service time required by an aircraft, depending on the type of operation it performs. This is no mean task. There is a huge number of parameters that determine these service times. In fact, the only method one can use in order to summarize what happens to an aircraft is a probabilistic description of service characteristics for each operation. Development of the probabilistic descriptions for individual service times is the subject of Chapter 3.

Equipped with this knowledge one can proceed to examine the collective characteristics associated with the service facility itself. For example, if we have a probabilistic description of the service time for a landing aircraft, what can we say about the capability of a runway to serve, say 25 aircraft per hour? What delays will result from such a situation? What other information is required in order to be able to answer this question, in the first place? And what is a meaningful analytical model for this service system? It is questions like this that we will tackle in chapter 4.

We will now be more specific: In chapter 3, we look closely at three specific models for a runway which is used: (1) for landings only; (2) for take-offs only; (3) by alternating landings with take-offs.

From the model of a runway which is used only for landings, the probability distribution for the time between successive arrivals is obtained under capacity conditions. We use the same definition of "capacity conditions" that Blumstein [4] does: it is the situation when arriving aircraft are always present, awaiting their turn to land. Note, that for the time being, we are not interested in the effects that such saturation conditions may have on aircraft delays (although, we would venture to guess that delays in this case would be "very long"). What we seek is to explore the performance characteristics of the runway when a waiting line of aircraft is present. Then, in chapter 4, we will consider the more realistic situation when the runway operates below the saturation point.

The inputs to this model of the runway which is used for landings are: the probability distribution for the approach speed of incoming aircraft; the probability distribution for the errors in appropriately spacing aircraft; the minimum separation requirements, both in the air and on the ground, as specified by the FAA; and the length of the common glide path that all aircraft have to fly before landing.

We are not saying that these are the only important parameters affecting the landing process. One can readily name other important factors: the wind speed, the local weather conditions, and the local traffic pattern regulations (primarily due to an effort to minimize noise pollution of heavily populated areas). Note, however, that some of these factors can be incorporated in the parameters which are used as inputs (wind speed, for example, would affect approach speeds, and poor visibility would primarily increase spacing errors), while others (like the local traffic patterns) are so topical restrictions that they can hardly be included in generalized, abstract models as the present one. Hence, it is the feeling here that the inputs mentioned in the previous paragraph are by far the most essential ones and are about all that one would like to consider in a study of this type.

We have discussed the above example in more detail than we will others, in order to illustrate the point of this study: We have tried to construct analytical models which are general enough to be applicable to most local conditions, yet precise enough to bring out the importance of the basic parameters and produce results of the correct order of magnitude.

Continuing with the discussion of the basic models, in chapter 3 (part II) we investigate the sequence of procedures involved in the take-off process. Thus, we obtain the probability distribution for the length of interdeparture times from this runway which is used only for take-offs. Again the inputs are basic characteristics of aircraft: the time required for a departing aircraft in order to line up on the runway, the "response" time until the take-off roll begins, and the duration of the take-off run itself.

Similarly, in part III, the landing-followed by take-off-followed by landing procedure, provides an opportunity to study the effects of "switching" from one type of service to another. This also motivates a discussion of controller strategy and of a possible measure of a runway's capacity under the most general conditions conceivable. This latter material is the subject of part IV, the last part of chapter 3.

By that point, two things have been accomplished: first, we have, hopefully, improved our understanding of the basic operations and of the effects of parameter changes on these operations and, second, we have developed basic tools for examination of the runway as a service system.

At this point, we start viewing the runway as a service facility and turn our attention to the delay problems associted with it. Assuming that the arrivals of planes at the terminal area are random (Poisson) and also assuming a first-come, first-served discipline, what can we say about runway utilization and average delays for a given arrival rate per hour? In this report, a new method for analyzing this problem has been developed and several expressions have been derived. In essence, the basic observation is the following: an arriving aircraft will either proceed directly to a landing, or will be delayed in a holding stack until it is cleared to land. In the first case, the service time of an incoming aircraft depends only on its own characteristics and the regulations associated with the runway. In the second case, the landing aircraft's service time is strongly affected by the characteristics of the aircraft landing immediately before it (this point is discussed in detail in chapter 3). Now, in this second case, the landing is performed under saturation conditions as far as this particular delayed aircraft is concerned. The results of chapter 3, therefore, become very pertinent since the probability distribution for the service time of an operation conducted under saturation conditions has already been obtained there.

Proceeding from this observation, a detailed analysis of the problem is presented. The expressions obtained are compared, then, with the commonly used expressions for estimating airport delays.

The second part of chapter 4 is devoted to consideration of delays to aircraft on the ground and to consideration of problems associated with the use of multiple runways. Again the basic effort is to describe complicated procedures (such as the interposition of a departure between two successive landings) by using the basic tools developed in chapter 3. Some of the results point out very clearly the interactions among various operations.

Chapter 5 presents several computational results of interest. It also includes a summary and a list of conclusions.

The operations research oriented reader should also find appendix II interesting reading.

Chapter III

Basic Analytical Models for Runway Operations

1. Introduction

In this chapter we focus attention on the runway, the basic air traffic facility. We consider the cases in which the runway is used for landings only, for take-offs only, and for mixed operations.

We take the broadest possible view, in an effort to develop models which are generally applicable. Consequently the approach is very abstract; we have concentrated on the methods of analysis rather than on specific results.

While some of the derived expressions, especially in part I, may appear to be uncomfortably complicated, it is hoped that the methods demonstrated are explicit enough to make application to any specific airport situation a rather routine matter. An effort has been made to discuss the parameters and variables of interest and to point out what statistics and measurements would be the most useful in analyzing a given case.

Part I

2. Landings only on a single runway

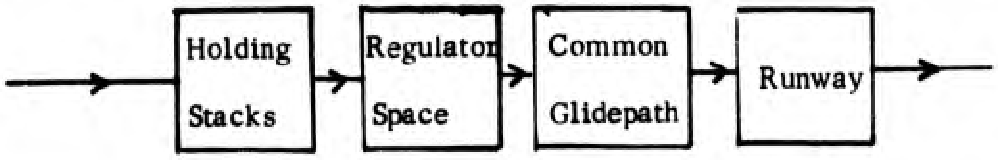
The first model that we will discuss is that of a single runway used exclusively for landings.

Attention will be focused on landings under instrument-flight-rules (IFR) weather. For a treatment of landings under visual-flight-rules (VFR) weather the interested reader may consult the work of Simpson [35].

The present model is an extension of the work of Blumstein [4], whose results can be obtained by examining a special case of the model to be described below.

3. Discussion of the model

An incoming pilot wishing to perform a landing under IFR conditions must follow a certain predetermined route, passing through the various stages of the air-space in the vicinity of the terminal area as indicated below:



Incoming aircraft first join a holding stack and remain there until cleared to land by the controller. Then, following the controller's instructions, the pilot travels through regulator space until the outer marker of the common glide path is reached. From that point on, all aircraft follow the same path descending at an appropriate rate (usually 500 ft/min) and at a speed which remains substantially the same for most of the descent. Finally, the runway is reached, the plane decelerates and leaves the runway at one of the taxiways. These procedures are discussed in considerably more detail in section 1.3.

Let us assume that there is only one runway used exclusively for landings. There are two fundamental questions of interest in relation to this runway:

(a) What is the capacity of this runway?

(b) What are the queuing statistics associated with this service facility, i.e., the runway?

We shall take up the second question in chapter 4, and will presently deal only with the first one.

In order to find the capacity of the runway we must assume, as it has already been pointed out in chapter 2, that at all times, there are aircraft available in the holding stack, awaiting to land.

Blumstein [4] was the first to point out that the bottleneck (and therefore the critical point as far as delays are concerned) of the landing process is the common glide path. Since all aircraft must fly the same route, differences in landing speed become significant. Because of the FAA requirement for a minimum distance separation between two successive aircraft, any plane starting descent along the glide path must be at least that minimum distance behind the preceding airplane. Thus, if the second plane is slower than the first, the distance between them by the time they reach the runway will have increased and the interarrival time gap will be longer than the minimum possible. On the other hand, if the second plane is preceded by a slower one, the distance separation at the beginning of the glide path should be greater than the minimum allowed, so that the "fast" plane will not "catch up" with the "slow" one. This, of course results into longer delays for the other aircraft awaiting clearance to land. In conclusion, we see that the minimum distance separation requirement in the air, and the minimum time separation requirement at the runway, in conjunction with the varying approach speeds of landing aircraft result in lost time and delays. This situation is depicted in Figure 3.1.

However, the delays due to speed differences are not the only ones. Pestalozzi [31] notes that "delays grow much more rapidly than traffic volume" and thus, that "secondary" delays are introduced by procedures associated with traffic congestion. These procedures actually involve the stacking of airplanes, flying of the usual holding pattern and travelling through regulator airspace according to instructions from the controller. As Simpson [35], has shown, the geometry of the holding pattern, for one thing, makes it difficult for aircraft to be in position to leave the stack at the exact instant desired. Secondly, due to inaccuracies of navigational aids; the aircraft characteristics; the wind shifts; and the slow communications procedures, aircraft do not fly through regulator space precisely according to the pattern desired by the controller. Perhaps even more important, it is felt here, is the human element involved, the tendency by the controllers to allow for extra margins of safety under heavy congestion conditions, especially when, in addition, visibility is poor.

The net effect of all the factors outlined above is that the arrival of an aircraft at the outer fix of the glide path under capacity conditions can not be assumed to occur at the instant desired by the controller. Thus the usual assumption made when the capacity of runways is estimated, namely that aircraft are always available at the beginning of the glide path and, therefore, can be optimally spaced by the controller, is not valid to a great extent.

F = "fast" plane
S = "slow" plane

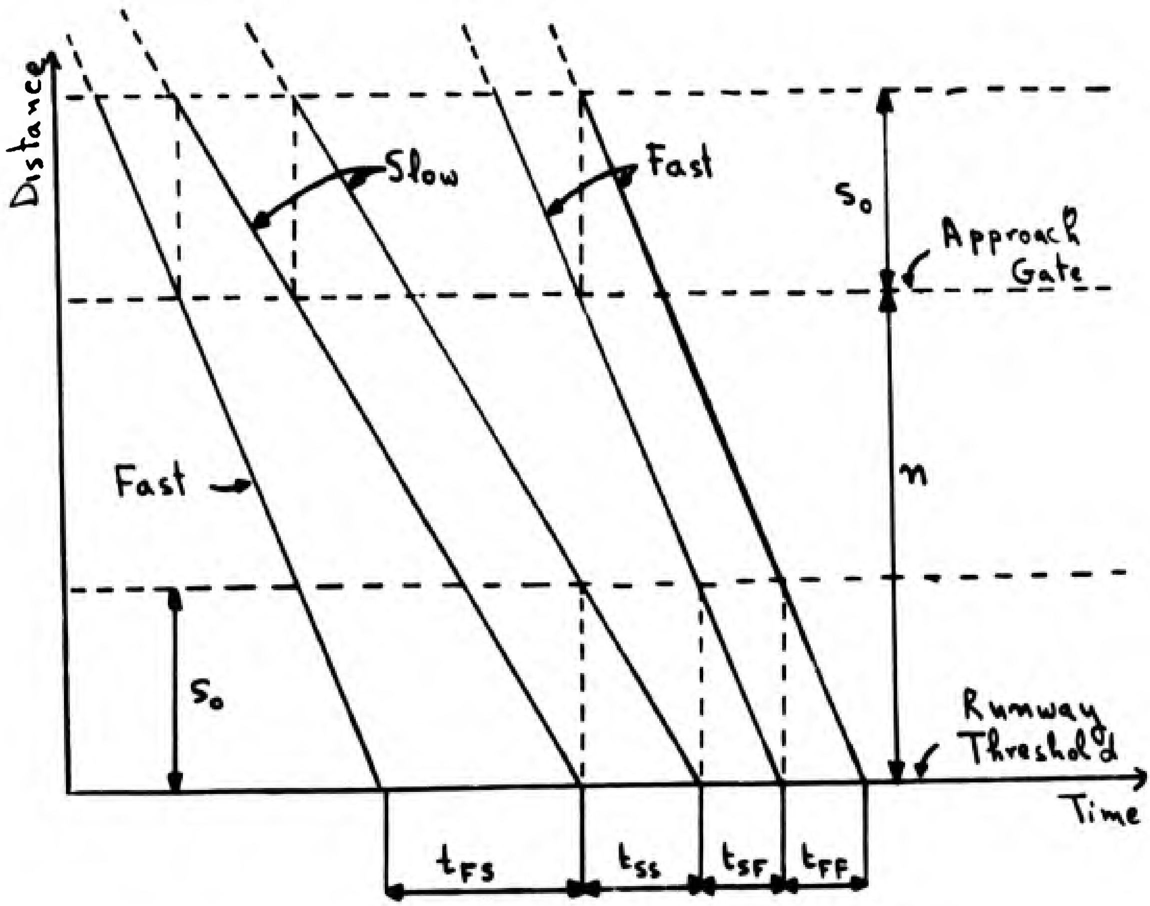


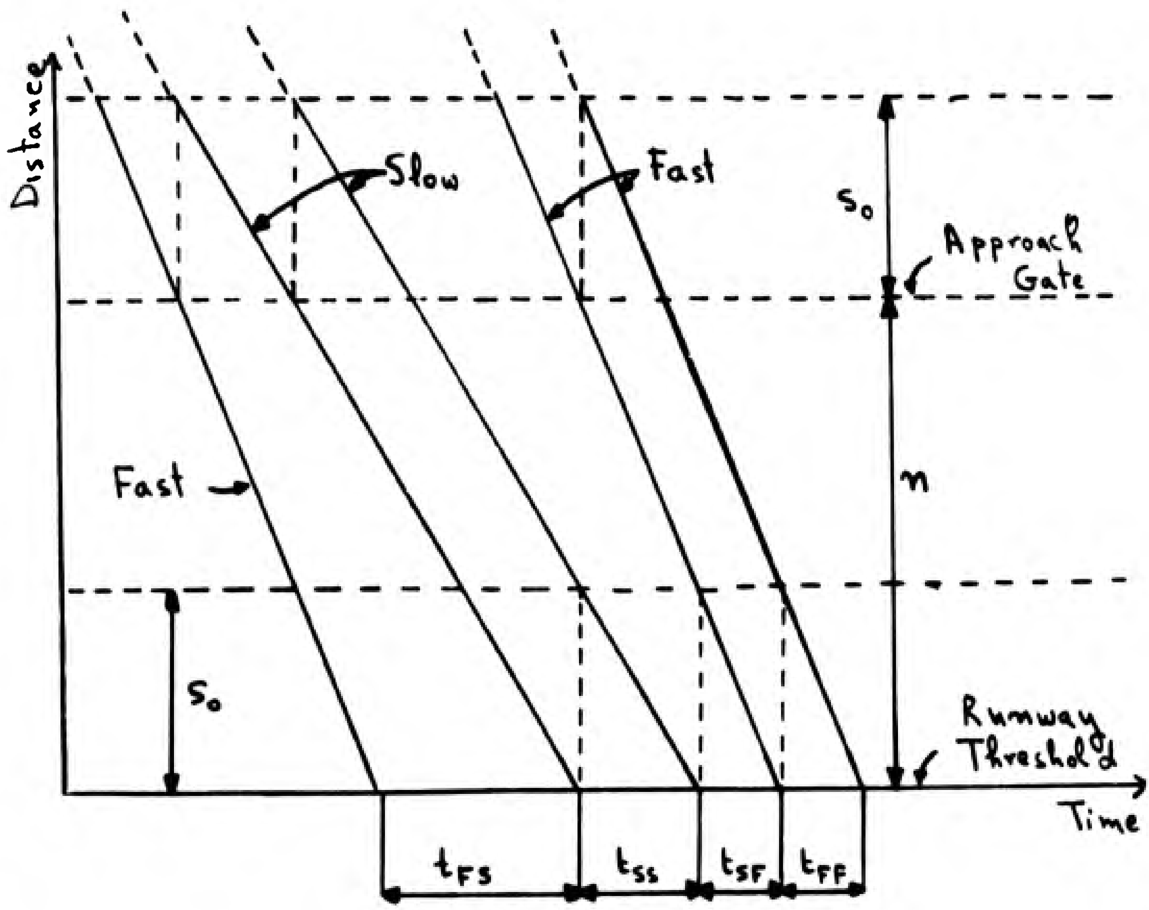
Figure 3.1

Effect of approach speeds on interarrival time gaps for two speed classes (F and S) (Source: Pestalozzi [31])

We see, then, that a more realistic view of the landing process under capacity conditions should lead to the introduction of an additional random quantity to the model of a runway which is used only for landings. This new random quantity is the positioning error at the beginning of the glide path. Thus, the only additional feature of the model to be described below, as compared to Blumstein's model [4], is the introduction of a random variable that represents the distance between two successive aircraft at the outer marker of the glide path. This contrasts with Blumstein's assumption of a deterministic distance separation between the two airplanes equal to the minimum possible separation which is consistent with the safety requirements both in the air and on the runway.

It is believed, here, that this additional feature of the model is a very significant one and makes for a closer correspondence to the real air traffic situation. Through this feature, it is now possible to draw a model that takes into consideration (if an appropriate probability density function for the distance separation between two successive aircraft is used) the secondary delays due to the holding stacks and to all the air-traffic control procedures, in general.

F = "fast" plane
S = "slow" plane



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4. Description of the model

We assume a single runway used exclusively for landings. Aircraft are always available at the holding stacks, thus providing capacity conditions. Aircraft land in accordance with a first-come, first-served discipline without any regard for other characteristics such as speed or origination of flight.

With reference to figure 3.2, the parameters of the model are:*

$n \triangleq$ length of the common glide path

$s_0 \triangleq$ minimum distance separation between two successive landing aircraft at the beginning of the glide path, as required by air traffic regulations.

$t_0 \triangleq$ minimum time separation at the runway between two successive landings as required by air-traffic regulations.

The random variables in the model are:

$v \triangleq$ approach speed of landing aircraft. A probability density function (pdf) $f_v(v_0)$ is associated with v.

$x \triangleq$ distance between two successive landing aircraft at the instant when the first of the pair reaches the outer marker of the common glide path. A pdf $f_x(x_0)$ is associated with x.

$t_{AA} \triangleq$ time between two successive landings. The pdf $f_{t_{AA}}(t)$ for

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* We have tried to use the same notation as Blumstein [4], to the extent possible, in this section.

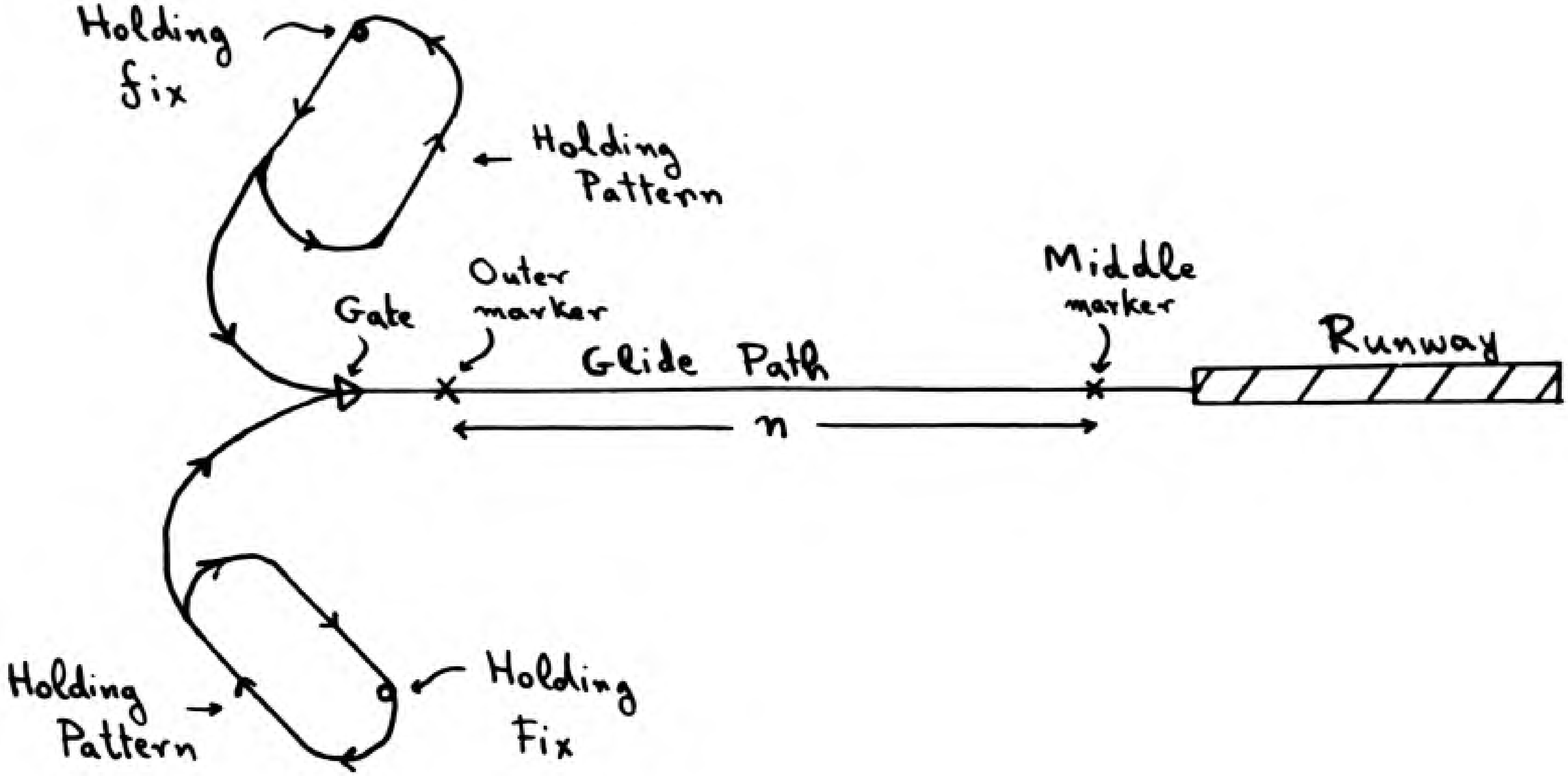


Figure 3.2
Idealized model for a runway used only for landings

the interarrival gaps is unknown and must be derived.

Additional assumptions are that: (i) The speed of an airplane remains constant throughout its descent through the glide path. (ii) The approach speeds of two successive aircraft are statistically independent random variables. (iii) The random variables x and v are statistically independent.

The air traffic control regulations impose two constraint relations on the random variables:

$$x \geqslant s_0 \tag{3.1}$$

and

$$\frac{n+x}{v_2} - \frac{n}{v_1} \geqslant t_0 \tag{3.2}$$

where,

$v_1 \triangleq$ approach speed of first landing aircraft

$v_2 \triangleq$ approach speed of second landing aircraft.

Finally, we have:

$$t_{AA} = \begin{cases} \frac{n+x}{v_2} - \frac{n}{v_1}, & \text{for } \frac{n+x}{v_2} - \frac{n}{v_1} \geqslant t_0 \\ t_0, & \text{otherwise} \end{cases} \tag{3.3}$$

The items of interest are the probability density function for random variable t_{AA}, as well as its first and second moments.

5. Two general results

The random variable t_{AA} is a function of x, v_1 and v_2. Therefore, the probability density function for t_{AA}, as well as the moments of t_{AA}, depend directly on the form of the pdf's $f_x(x_0)$ and $f_v(v_0)$.

In general, we can quote only two results about random variable t_{AA}, without knowledge of $f_x(x_0)$ and $f_v(v_0)$. These results apply to the particular case when the parameters n, s_0 and t_0, as well as the ranges of the positioning distance x and the approach speed v, are such that we always have:

$$t_{AA} = \frac{n+x}{v_2} - \frac{n}{v_1}$$

This is equivalent to saying that the minimum time-separation requirement at the runway, is small enough in comparison with the minimum distance-separation in the air, so as not to affect the landing process.

In this case we have:

$$\text{(i)}\quad E[t_{AA}] = E\left[\frac{n+x}{v_2} - \frac{n}{v_1}\right] = nE\left[\frac{1}{v_2}\right] + E\left[\frac{x}{v_2}\right] - nE\left[\frac{1}{v_1}\right]$$

$$= nE\left[\frac{1}{v}\right] + E[x]\,E\left[\frac{1}{v}\right] - nE\left[\frac{1}{v}\right]$$

$$= E[x]\,E\left[\frac{1}{v}\right], \qquad (3.4)$$

where we have used the independence of x, v_1, and v_2.

$$\text{(ii)}\quad E[t^2_{AA}] = E\left[\left(\frac{n+x}{v_2} - \frac{n}{v_1}\right)^2\right] =$$

$$= E\left[\left(\frac{n+x}{v_2}\right)^2 - 2\left(\frac{n+x}{v_2}\right)\left(\frac{n}{v_1}\right) + \left(\frac{n}{v_1}\right)^2\right]$$

$$= E\left[\frac{n^2+2nx+x^2}{v_2^2}\right] - 2E\left[\frac{n^2+nx}{v_2 v_1}\right] + E\left[\frac{n^2}{v_1^2}\right]$$

$$= \left\{ n^2 + 2n\, E[x] + E[x^2] \right\} E[\frac{1}{v_2{}^2}]$$
$$- 2 \left\{ n^2 + nE[x] \right\} E[\frac{1}{v_2}]\ E[\frac{1}{v_1}] + n^2 E[\frac{1}{v_1^2}]$$
$$= \left\{ n^2 + 2nE[x] + E[x^2] \right\} E[(\frac{1}{v})^2] - 2 \left\{ n^2 + nE[x] \right\} E^2[\frac{1}{v}]$$
$$+ n^2 E[(\frac{1}{v})^2] = (2n^2 + 2nE[x] + E[x^2])\ E[(\frac{1}{v})^2]$$
$$- 2(n^2 + nE[x])E^2[\frac{1}{v}] \qquad (3.5)$$

Thus, if we define a random variable $q \overset{\Delta}{=} \frac{1}{v}$, we have:

$$E[t_{AA}] = E[x]\ E[q] \qquad (3.4a)$$

and,

$$E[t_{AA}{}^2] = (2n^2 + 2nE[x])\ (E[q^2] - E^2[q]) + E[x^2]\ E[q^2]$$
$$= (2n^2 + 2nE[x])\ \sigma_q^2 + E[x^2]\ E[q^2] \qquad (3.5a)$$

Therefore, we see that if the postulated conditions are satisfied, the expected interarrival time is directly proportional to the expected positioning distance, and that the second moment of t_{AA} is directly proportional to both the first and second moments of random variable x.

Although the conditions postulated in this section do not usually hold, the usefulness of expressions (3.4a) and (3.5a) lies in the fact that they clearly point out the importance of positioning errors at the beginning of the glide path. $E[t_{AA}]$ is, of course, just the inverse of the runway capacity, while $E[t_{AA}^2]$ is directly associated with the expected delay of aircraft using the runway (this will be more extensively discussed in the next chapter). When the minimum time separation requirement at the runway does affect landing operations, the effects of the positioning error can not be so clearly demonstrated, as we shall show shortly, but expressions (3.4 a) and (3. 5a) can always be used as an upper limit of the effects of random variable x on the quantities $E[t_{AA}]$ and $E[t_{AA}^2]$.

6. The pdf's $f_x(x_0)$ and $f_v(v_0)$

In order to proceed any further with our present model, we must now specify the probability density functions $f_v(v_0)$ and $f_x(x_0)$.

The pdf for the approach speeds will be taken to be a uniform density function ranging from b to a, i.e.,

$$f_v(v_0) = \begin{cases} \frac{1}{a-b}, & \text{for } b \leqslant v_0 \leqslant a \\ o, & \text{otherwise} \end{cases} \quad (3.6)$$

This uniform pdf for v, was first used by Blumstein [4] in his analytical investigation. He justified the assumption on the ground that aircraft do not fly at discrete speeds, anyway, "particularly in the presence of varying wind conditions"; since the uniform pdf is particularly convenient for analytical purposes, it was, therefore, used in calculating the landing capacity. Furthermore, Blumstein compared the results obtained with the uniform pdf assumption with those obtained by assuming a discrete speed distribution. The close agreement of the results seemed to further justify the original assumption.

More recently, the author has seen some measurements taken at Heathrow airport of London [20]. The measurements make clear the very wide discrepancy between advertised approach speeds for various types of aircraft and actual approach speeds as measured on the radar. Wind conditions and plane loads seem to be especially significant factors in this respect. Some kind of continuous pdf for the velocity v seems to be definitely indicated, therefore. In addition, the more or less even spread of speeds which is reported indicates that the uniform pdf assumption is fairly accurate, after all.

On the other hand, the report cited above, does not contain any information on the mix of planes served by Heathrow airport. Where major United States airports are concerned, however, the last few years have presented a very clear trend toward an increase in the number of fast jet planes using these airports. It is entirely possible, therefore, that the velocity distribution should be assumed to be heavily weighted toward higher speeds instead of being uniform. This was, of course not true at the time of Blumstein's report (1960) when the mix of planes at major US airports was still about 50% propeller and 50% jet planes.

Nevertheless, in spite of the last paragraph, we shall persist with the uniform pdf assumption. A different pdf for v might make the computations more involved, but the analysis would still be substantially the same.

For the pdf of random variable x, we shall again assume a uniform pdf.

$$f_x(x_0) = \begin{cases} \frac{1}{c-d}, & \text{for } d \leq x_0 \leq c \\ 0, & \text{otherwise} \end{cases} \tag{3.7}$$

Normally, we must have that $d=s_0$, i.e., the minimum value of x is just the minimum distance separation allowed.

The assumption for $f_x(x_0)$, here, may seem much more arbitrary than the assumption for $f_v(v_0)$. The fact is that the author has been unable to find any detailed information on x. A report by Koch [22] verifies the existence of the positioning errors but does not attempt any measurements of them. The lack of any statistics on x is actually very understandable, in view of the difficulties involved in measuring the distance between successive aircraft at the instant when the first of them reaches the outer fix of the glide path. This measurement can only be made by observing the controller's radar and taking snapshots of the radar screen at the appropriate instants. Certainly this is an interesting project which may yield very useful results.

Later in this chapter we will indicate the analytical steps required, if the assumption of a "triangular" pdf for x is made (this type of pdf is normally considered much more appropriate for an "error" function). However, the algebra for such a pdf soon becomes too heavy, although it is quite simple to solve for any specific cases by use of a digital computer.

For the time being we shall proceed with the uniform pdf assumption. As already indicated in chapter I, we are more interested in indicating analytical models and orders of magnitude involved, rather than in numerical results which can be easily obtained by simulation.

7. The pdf $f_{t_{AA}}(t)$

We now compute the probability density function for the duration of the interarrival gap t_{AA} under capacity conditions.

We have

$$t_{AA} = \begin{cases} \frac{n+x}{v_2} - \frac{n}{v_1}, & \text{for } \frac{n+x}{v_2} - \frac{n}{v_1} \geqslant t_0 \\ t_0, & \text{otherwise} \end{cases} \tag{3.3}$$

We define a random variable $q \triangleq \frac{1}{v}$. Then, since

$$f_v(v_0) = \begin{cases} \frac{1}{a-b}, & \text{for } b \leqslant v_0 \leqslant a \\ 0, & \text{otherwise} \end{cases} \tag{3.6}$$

we obtain by elementary methods (see appendix B):

$$f_q(q_0) = \begin{cases} \frac{1}{q_0^2} \frac{1}{(a-b)}, & \text{for } \frac{1}{a} \leqslant q_0 \leqslant \frac{1}{b} \\ 0, & \text{otherwise} \end{cases} \tag{3.8}$$

Let us now define a random variable $w \triangleq \frac{n+x}{v} = (n+x)q$. Using again, probability theory techniques (see appendix C) we obtain:

$$f_w(w_0) = \begin{cases} \frac{a^2}{2K} - \frac{L^2}{2Kw_0^2}, & \text{for } \frac{L}{a} \leqslant w_0 < \frac{M}{a} \\ \frac{M^2-L^2}{2Kw_0^2}, & \text{for } \frac{M}{a} \leqslant w_0 < \frac{L}{b} \\ \frac{M^2}{2Kw_0^2} - \frac{b^2}{2K}, & \text{for } \frac{L}{b} \leqslant w_0 \leqslant \frac{M}{b} \\ 0, & \text{otherwise} \end{cases} \tag{3.9}$$

where we have defined,

$$K \triangleq (a-b)(c-d) \tag{3.10}$$

$$M \triangleq n + c \tag{3.11}$$

$$L \triangleq n + d \tag{3.12}$$

Expression (3.9) holds for the assumption that:

$$\frac{M}{a} \leqslant \frac{L}{b} \tag{3.13}$$

If (3.13) does not hold, but instead $\frac{M}{a} > \frac{L}{b}$, $f_w(w_0)$ is slightly different, although of the same form. The derivation of this new $f_w(w_0)$ would again follow the lines outlined in appendix C.

Finally, we define a random variable $y \triangleq \frac{n}{v} = nq$. Since n is a constant, we know that:

$$f_y(y_0) = \frac{1}{n} f_q\left(\frac{y_0}{n}\right) = \begin{cases} \frac{n}{y_0^2} \frac{1}{(a-b)}, & \text{for } \frac{n}{a} \leqslant y_0 \leqslant \frac{n}{b} \\ 0, & \text{otherwise} \end{cases} \tag{3.14}$$

Now (3.3) can be rewritten as:

$$t_{AA} = \begin{cases} w-y, & \text{for } \frac{n+x}{v_2} - \frac{n}{v_1} \geqslant t_0 \\ t_0, & \text{otherwise} \end{cases} \tag{3.15}$$

It must be underlined that w and y are statistically independent random variables in this case, since the substitution from (3.3) was $w = \frac{n+x}{v_2}$ and $y = \frac{n}{v_1}$ and we have already assumed that x, v_1, and v_2 are statistically independent random variables.

Instead of proceeding directly to evaluate the pdf $f_{t_{AA}}(t)$, we shall define an intermediate random variable $r \triangleq w - y$ and will evaluate $f_r(r_0)$, first.

With reference to figure 3.2, we shall start by computing the probabilities of events A, B, and C, indicated on the joint sample space for random variables w and y. These probabilities will be useful for our subsequent developments. We have:

$$P[A] = \int_{\frac{L}{a}}^{\frac{M}{a}} \int_{\frac{n}{a}}^{\frac{n}{b}} f_{w,y}(w_0, y_0)\, dy_0 dw_0 = \int_{\frac{L}{a}}^{\frac{M}{a}} \int_{\frac{n}{a}}^{\frac{n}{b}} \left(\frac{a^2}{2K} - \frac{L^2}{2Kw_0^2}\right) \frac{n}{y_0^2} \left(\frac{1}{a-b}\right) dy_0 dw_0$$

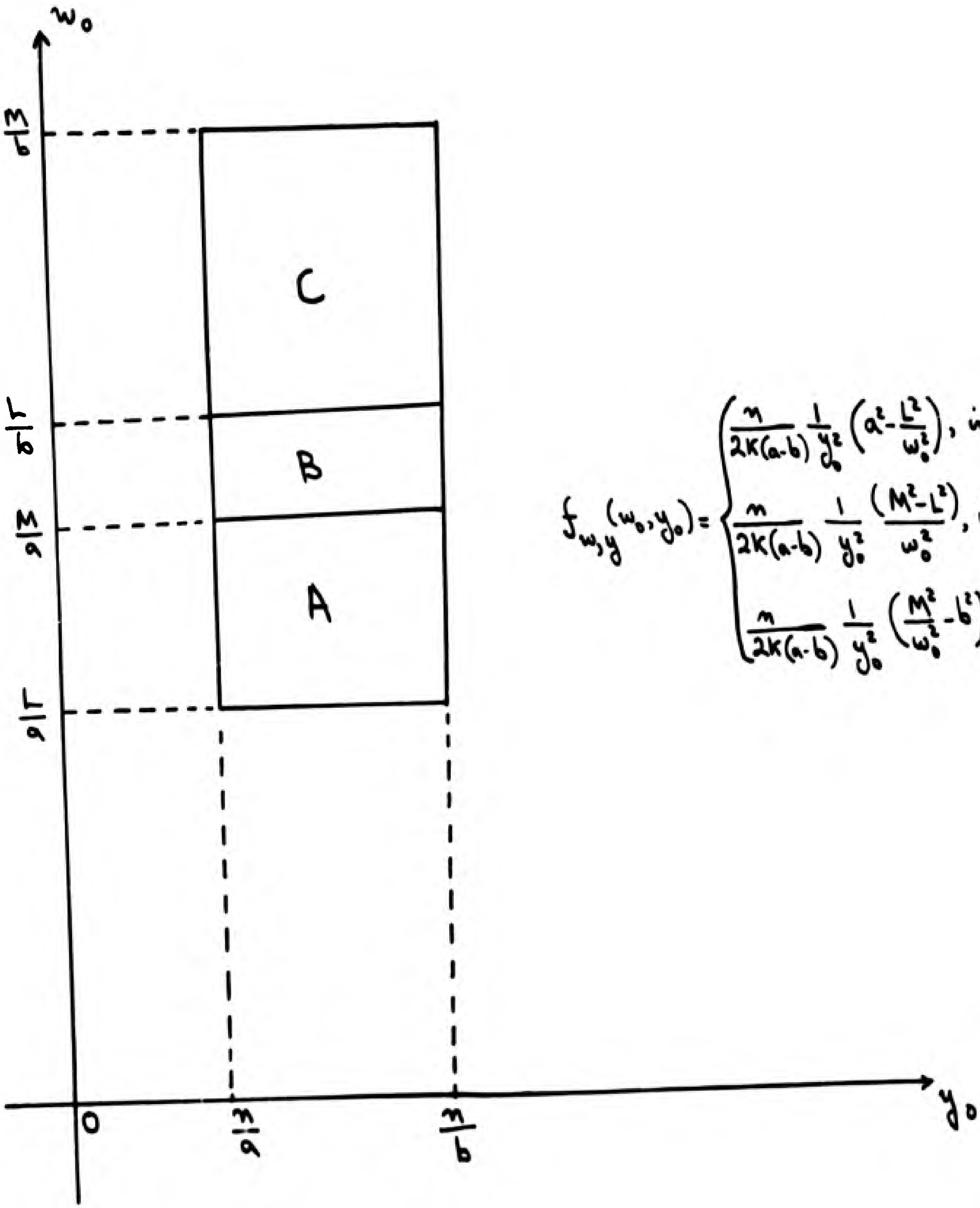
$$= \frac{a}{K} \left[\frac{L^2}{2M} + \frac{M}{2} - L \right] \tag{3.16}$$

$$P[B] = \int_{\frac{M}{a}}^{\frac{L}{b}} \int_{\frac{n}{a}}^{\frac{n}{b}} \left(\frac{M^2 - L^2}{2Kw_0^2}\right) \frac{n}{y_0^2} \left(\frac{1}{a-b}\right) dy_0 dw_0 = \frac{(M^2 - L^2)}{2K} \left[\frac{a}{M} - \frac{b}{L}\right] \tag{3.17}$$

$$P[C] = \int_{\frac{L}{b}}^{\frac{M}{b}} \int_{\frac{n}{a}}^{\frac{n}{b}} \left(\frac{M^2}{2Kw_0^2} - \frac{b^2}{2K}\right) \frac{n}{y_0^2} \left(\frac{1}{a-b}\right) dy_0 dw_0 = \frac{b}{K} \left[\frac{M^2}{2L} + \frac{L}{2} - M\right] \tag{3.18}$$

The cumulative probability function for r will be computed next; we will use the notation $F_{r\leqslant}(r_0) \triangleq Pr[r \leqslant r_0]$.

As can be seen from figure 3.3, the function $F_{r\leqslant}(r_0)$ takes on different values depending on the particular value of r_0. The regions of integration also vary with the relative magnitude of the parameters L, M, n, a, and b. Here, we have performed a complete analysis for the case when the following conditions hold:



$$f_{w,y}(w_0,y_0)=\begin{cases}\dfrac{m}{2K(a-b)}\dfrac{1}{y_0^2}\left(a^2-\dfrac{L^2}{w_0^2}\right), & \text{in } A\\[2ex] \dfrac{m}{2K(a-b)}\dfrac{1}{y_0^2}\dfrac{(M^2-L^2)}{w_0^2}, & \text{in } B\\[2ex] \dfrac{m}{2K(a-b)}\dfrac{1}{y_0^2}\left(\dfrac{M^2}{w_0^2}-b^2\right), & \text{in } C\end{cases}$$

<u>Figure 3.3</u>

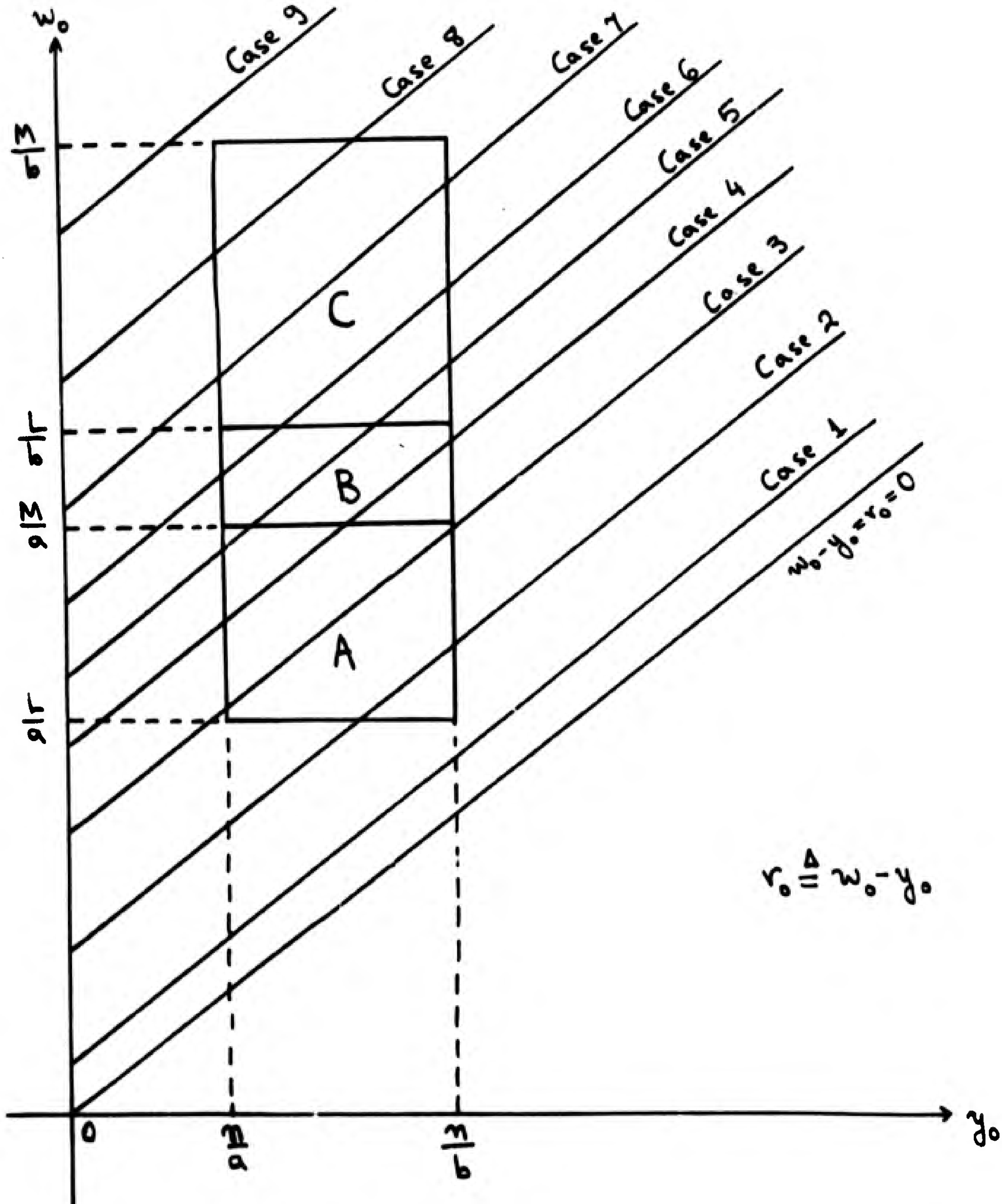


Figure 3.4

As r_o increases, $F_{r_\xi}(r_o)$ takes different values

$$\frac{L}{a} - \frac{n}{a} \leq \frac{M}{a} - \frac{n}{b} \tag{3.19}$$

$$\frac{L}{b} - \frac{n}{b} \leq \frac{M}{a} - \frac{n}{a} \tag{3.20}$$

$$\frac{L}{b} - \frac{n}{a} \leq \frac{M}{b} - \frac{n}{b} \tag{3.21}$$

Any change in the direction of the above inequalities would result in a change of the expressions for $F_{r\leq}(r_0)$ and $f_r(r_0)$. However, the expressions would be of the same form and their derivation would be identical, anyway.

The functions $F_{r\leq}(r_0)$ and $f_r(r_0)$ are derived in appendix D. Since the expressions look quite cumbersome, we will not reproduce them here.

It is now very simple to find expressions for the cumulative probability function $F_{t_{AA}\leq}(t)$ and for the probability density function $f_{t_{AA}}(t)$. We have:

$$F_{t_{AA}\leq}(t) = \begin{cases} 0, & \text{for } t < t_0 \\ F_{r\leq}(t), & \text{for } t_0 \geq t \end{cases} \tag{3.22}$$

and

$$f_{t_{AA}}(t) = \begin{cases} F_{r\leq}(t), & \text{for } t = t_0 \\ f_r(t), & \text{for } t_0 < t \\ 0, & \text{otherwise} \end{cases} \tag{3.23}$$

It should be noted that $f_{t_{AA}}(t)$ has an impulse at $t=t_0$ and is otherwise a continuous pdf.

There are two extreme cases: (i) When:

$$t_0 \leqslant \frac{L}{a} - \frac{n}{b}$$

then,

$$f_{t_{AA}}(t) = f_r(r_0)$$

(This is the case that has been discussed already in section 3.5; in this case the t_0 requirement does not affect the operations, at all.)

(ii) When

$$\frac{M}{b} - \frac{n}{a} \leqslant t_0$$

then,

$f_{t_{AA}}(t)$ is a unit impulse at $t = t_0$.

In this case the t_0 requirement completely determines the time gaps between successive arrivals of aircraft.

Before proceeding with the next topic, it might be expedient to discuss briefly the physical meaning of the cases to which we have referred so often in the above. Take for example case (i) for which $t_0 \leqslant \frac{L}{a} - \frac{n}{b}$. We have defined $L = n + d$ in (3.12), where d is the minimum separation possible, which is consistent with distance-separation requirements at the beginning of the glide path. Thus, the inequality $\frac{L}{a} - \frac{n}{b} \geqslant t_0$ says that even if: (i) the first of a pair of incoming aircraft is as slow as possible (velocity b);

(ii) the second of the pair is as fast as possible (velocity a); and (iii) the separation of the aircraft is the shortest possible at the beginning of the glide path (distance d), still, by the time the aircraft will reach the runway, their time separation $t_{AA} = \frac{L}{a} - \frac{n}{b}$ (in this case) will be greater than the minimum time-separation requirement, t_0, at the beginning of the runway. Since this is the most unfavorable situation imaginable, it is evident that in this case the t_0 requirement does not affect operations, as stated above.

Similar physical interpretations can be given for all the possible cases. As t_0 increases it affects the landing rate to a greater and greater extent. The most extreme case, of course, is when $\frac{M}{b} - \frac{n}{a} \leqslant t_0$ (case (ii) above).

8. The first and second moments of t_{AA}

Of primary interest are, as noted above, the first and second moments of t_{AA}. The first moment, $E[t_{AA}]$ is the inverse of the capacity of the runway, while the second moment will be useful for the queuing considerations of the next chapter.

For the expected value of t_{AA}, we have:

$$E[t_{AA}] = \int_0^{+\infty} t f_{t_{AA}}(t)\, dt$$

$$= t_0 F_{r\leqslant}(t_0) + [1 - F_{r\leqslant}(t_0)] \left(\int_{t_0+}^{\infty} t f_r(t) dt \right) \qquad (3.24)$$

We have used above, the result of (3.23) and we have indicated the lower limit of the last integral as "t_0+" in order to underline the fact that the value $t=t_0$ is not included in the integration.

The relation (3.24) used in conjunction with the expressions for $F_{r\leqslant}(r_0)$ and $f_r(r_0)$, obtained in appendix D, provides the value of $E[t_{AA}]$ for all values of the landing separation requirement t_0.

We present a few cases in order to illustrate the type of results obtained:

(i) For $t_0 \leqslant \frac{L}{a} - \frac{n}{b}$: We can use the result of equation (3.4), since, in this case, the t_0 requirement does not interfere with operations. With the pdf's $f_x(x_0)$ and $f_v(v_0)$, postulated in (3.7) and (3.6), respectively, we have:

$$E[t_{AA}] = E[x]\, E[\frac{1}{v}] = (\frac{c+d}{2}) \left(\frac{\ell n(\frac{a}{b})}{(a-b)} \right)$$

$$= \frac{(c+d)}{2(a-b)} \ell n\, (\frac{a}{b}) \qquad (3.25)$$

It is interesting to rewrite (3.25) by substituting $d = s_0$ (i.e., the minimum allowable separation) and $c=d+(c-d)=s_0+R$, where $R=c-d$ is the range of the positioning error x. Then, we have:

$$E[t_{AA}] = \frac{2S_0+R}{2(a-b)} \ \ell n \left(\frac{a}{b}\right) \tag{3.25a}$$

Thus the expected interarrival gap length is directly proportional to the range R. It is also interesting to note that for R=0, i.e., using the model postulated by Blumstein [4] with no positioning errors, the result above reduces to the one presented in the aforementioned report.

(ii) For $\frac{L}{a} - \frac{n}{b} \leqslant t_0 \leqslant \frac{L}{a} - \frac{n}{a}$:

We first have (see the results of Appendix D):

$$P[t_{AA}=t_0]=F_{r\leqslant}(t_0)=\int_{\frac{L}{a}-t_0}^{\frac{n}{b}} \int_{\frac{L}{a}}^{t_0+y_0} \frac{1}{(a-b)} \frac{n}{y_0^2} \left(\frac{a^2}{2K} - \frac{L^2}{2K w_0^2}\right) dw_0 \, dy_0$$

$$= \frac{n}{(a-b)2K} \left\{ a^2 \ln \frac{na}{b(L-at_0)} - \frac{L^2}{t_0}\left(\frac{b}{n+t_0 b} - \frac{a}{L}\right) \right.$$

$$\left. - \frac{L^2}{t_0^2} \ln \frac{nL}{(L-at_0)(n+bt_0)} - \frac{a^2 b}{n}\left(t_0 + \frac{n}{b} - \frac{L}{a}\right) + \frac{bL^2}{n}\left[\frac{a}{L} - \frac{b}{n+t_0 b}\right] \right\} \tag{3.26}$$

Also:

$$\int_{t_0+}^{\infty} t f_{t_{AA}}(t)dt = \int_{t_0+}^{\infty} t f_r(t)dt =$$

$$= \frac{c+d}{2(a-b)} \ln \frac{a}{b} - \frac{n}{2K(a-b)} \left\{ \frac{a^2 t_0^2 - L^2}{2} \left[\frac{a}{L-at_0} - \frac{b}{n}\right] \right.$$

$$- \frac{a^2}{2}\left[\frac{n}{b} - \frac{L}{a} + t_0\right] + 2aL \ln\left(\frac{na}{b(L-at_0)}\right) + \frac{L^2 b}{n} \ln\left(\frac{L(n+bt_0)}{ab}\right)$$

$$+ \frac{2L^2}{t_0} \ln\left(\frac{(n+bt_0)(L-at_0)}{nL}\right)\Bigg\}$$

(3.27)

Then combining (3.26) and (3.27), we obtain:

$$E[t_{AA}] = t_0 P[t_{AA} = t_0] + \left\{1 - P[t_{AA} = t_0]\right\} \int_{t_0+}^{\infty} t\, f_{t_{AA}}(t)\, dt \qquad (3.28)$$

Obviously then, the expression for $E[t_{AA}]$ is a very complicated one. This is also the case for all other values of the minimum ground separation requirement t_0 in the range between $\frac{L}{a} - \frac{n}{b}$ and $\frac{M}{b} - \frac{n}{a}$. Thus the expressions for $E[t_{AA}]$ for $\frac{L}{a} - \frac{n}{b} \leq t_0 \leq \frac{M}{b} - \frac{n}{a}$ should more profitably be computed by using numerical integration. Programming for such computations should not be too cumbersome, since the expression for $f_{t_{AA}}(t)$ as given from Appendix D, presents a considerable degree of symmetry.

(iii) In the case when $\frac{M}{b} - \frac{n}{a} \leq t_0$, i.e., when the minimum separation requirement on the ground becomes the limiting factor in the operation, we have:

$$E[t_{AA}] = t_0 \qquad (3.29)$$

and thus the operational capacity of the runway is simply given by $\frac{1}{t_0}$.

We are also interested here in the second moment and the variance of the random variable t_{AA}.

For the second moment, we have:

$$E[t_{AA}^2] = \int_{-\infty}^{+\infty} t^2 f_{t_{AA}}(t)dt = t_0^2 \, P[t_{AA}=t_0]$$

$$+ [1 - P[t_{AA}=t_0]] \int_{t_0^+}^{\infty} t^2 f_{t_{AA}}(t)\, dt \qquad (3.30)$$

and for the variance:

$$\sigma^2_{t_{AA}} = E[t^2_{AA}] - E^2[t_{AA}] \qquad (3.31)$$

Then:

(i) For the case when $t_0 \leqslant \frac{L}{a} - \frac{n}{\bar{b}}$, i.e., whenever the t_0 requirement does not interfere with the landing procedures we have

$$E[t^2_{AA}] = E[(w-y)^2] = E[w^2] + E[y^2] - 2E[w]E[y]$$

$$= \frac{1}{3}\frac{M^3-L^3}{a\bar{b}(c-d)} + \frac{n^2}{a\bar{b}} - 2\frac{(M+L)}{2}\frac{n}{(a-b)^2}\ \ell n^2 (\frac{a}{\bar{b}})$$

$$= \frac{1}{a\bar{b}}[\frac{1}{3}\frac{M^3-L^3}{c-d} + n^2] - \frac{(M+L)n}{(a-b)^2}\ \ell n^2 (\frac{a}{\bar{b}})$$

$$= \frac{1}{a\bar{b}}[\frac{c^2+cd+d^2}{3} + (c+d)n + 2n^2] - \frac{(c+d+2n)n}{(a-\bar{b})^2}\ \ell n^2(\frac{a}{\bar{b}}) \qquad (3.32)$$

and using (3.25)

$$\sigma^2_{t_{AA}} = \frac{1}{a\bar{b}}[\frac{c^2+cd+d^2}{3} + (c+d)n + 2n^2]$$

$$- \frac{\ell n^2(\frac{a}{\bar{b}})}{(a-b)^2}[\frac{(c+d)^2}{4} + (c+d)n + 2n^2] \qquad (3.33)$$

It is interesting to note that since $c = s_0 + R$, where R is the range of error, the variance is proportional to R^2, in this case.

(ii) In the special case when $\frac{M}{b} - \frac{n}{a} \leqslant t_0$, i.e., when the duration of the interarrival gap is always t_0, we have

$$E[t_{AA}^2] = t_0^2 \text{ and } \sigma^2_{t_{AA}} = 0. \tag{3.34}$$

(iii) It would be fruitless to try to provide an explicit expression for $E[t_{AA}^2]$ and for $\sigma^2_{t_{AA}}$ for the intermediate cases, because the expressions are extremely cumbersome while providing very little insight as to the effect of the various parameters.

It is possible however to use numerical integration in order to compute these quantities, using the expression for $f_{t_{AA}}(t)$ provided by (3.23). The possibility of such numerical computations has already been discussed in the present section.

9. Analysis with a different pdf for $f_x(x_0)$

In order to illustrate the practical difficulties arising from assuming a more complicated pdf for the positioning error x, we present, in appendices E and F, an analysis of the landing problem under capacity conditions which uses a "triangular" pdf $f_x(x_0)$, instead of the uniform pdf postulated by (3.7).

As the reader can see, the development soon becomes vastly more complicated due to the great increase in the number of cases on hand. Thus, although the method of analysis remains the same, one soon has to resort to some degree of computer simulation.

Part II

10. Departures only from a single runway

Our second fundamental model is that of a single runway used only for departures. Following the pattern set in the previous section, we will first discuss the sequence of procedures involved in arranging departures of aircraft, we will then postulate a probabilistic description of the process, and, finally we will derive a few basic results pertaining to the present model.

11. Sequence of procedures

Once a departing airplane has been instructed to attempt a take-off from a given runway, it proceeds through the taxiways of the airport and takes up a position in the waiting line of aircraft awaiting their turn to depart.

Let us assume here that the runway in question is used only for departures. Then, the following well-known sequence of events takes place when the particular aircraft, on which we are now focusing attention, becomes the first in the waiting line outside the runway:

As soon as the airplane currently on the runway starts the take-off roll, the aircraft of interest, on the apron next to the runway, enters the runway and takes up the appropriate position, set to start its own roll.

In the meanwhile, the take-off currently in progress traverses the runway, becomes airborne and finally clears the so-called "upwind threshold" of the runway, at which instant the runway is considered free for the following plane.

The tower then, gives permission to the following take-off to start its roll. A response period (delay) follows and the next take-off finally begins.

The situation, as described here, calls for the definition of the following random variables:

$x \stackrel{\Delta}{=}$ time interval from the instant when a take-off starts to roll to the instant when the following departure, in the waiting line, is set up on the runway, ready to start rolling.

$y \triangleq$ time interval from the instant when a departure starts the take-off roll to the instant when the departure crosses the upwind threshold

$z \triangleq$ "response period" for a departing aircraft, which is set on the runway, i.e., time interval from the instant when a departure receives permission for a take-off to the instant when it starts to roll.

$t_{DD} \triangleq$ time interval between successive departures from a single runway, used only for take-offs, under capacity conditions.

Then, we have:

$$t_{DD} = \max(x, y) + z \tag{3.35}$$

A few remarks are in order: First, we have assumed that there is no minimum value imposed (for safety reasons) on t_{DD}, the time between successive departures. In other words, t_{DD} is only limited by the duration of the operations described by the random variables x, y, and z. This assumption is only partly true. Whenever two successive aircraft following identical departure courses for more than five minutes arrive for take-off, a minimum separation must indeed be imposed by the controller, ranging in time up to as long as 3 minutes (see also section 1.3). However the controller can anticipate this possibility and can interpose (especially during periods with a high number of movements) a departure following a different course between two successive departures following identical courses (this interposition, of course, must happen before the aircraft line up on the taxiway). Therefore, we will assume, as stated above, that the minimum separation requirement does not interfere with operations in this case.

Secondly we should point out that the random variables with which we shall work here, involve time intervals and not specific characteristics of aircraft like speed (which was used in the previous part of this chapter). The reason is that for any mix of aircraft using a particular airport, the variables x, y, and z used above can be rather easily measured statistically (in contrast, for example, to measuring the time required by a landing plane in order to descend through the glide-path) and, in fact, we have available some quite detailed data [12], [20]. We will then assume in the sequel, that the probability density functions $f_x(x_0)$, $f_y(y_0)$ and $f_z(z_0)$ are known, and we will derive some useful statistics of the interdeparture times random variable, t_{DD}.

12. The probability density function $f_{t_{DD}}(t)$

As stated above we assume that the pdf's $f_x(x_0)$, $f_y(y_0)$ and $f_z(z_0)$ are known.

In addition, x, y, and z are taken to be statistically independent random variables, an assumption which is certainly justified in view of the definitions of x, y, and z given above.

Then, if we define:

$$w = \max(x, y) \tag{3.36}$$

we have

$$\begin{aligned} F_{w\leq}(w_0) &\overset{\Delta}{=} \Pr[w \leq w_0] = \Pr[\max(x,y) \leq w_0] \\ &= \int_{-\infty}^{w_0}\int_{-\infty}^{w_0} f_{x,y}(x_0, y_0)\, dx_0 dy_0 \\ &= \int_{-\infty}^{w_0}\int_{-\infty}^{w_0} f_x(x_0)\, f_y(y_0)\, dx_0 dy_0 \\ &= F_{x\leq}(w_0)\, F_{y\leq}(w_0) \end{aligned} \tag{3.37}$$

where $F_{x\leq}(x_0)$ and $F_{y\leq}(y_0)$ are the cumulative probability functions for random variables x and y, respectively.

Then,

$$f_w(w_0) = \frac{dF_{w\leq}(w_0)}{dw_0} = f_x(w_0)\, F_{y\leq}(w_0) + f_y(w_0)\, F_{x\leq}(w_0) \tag{3.38}$$

Since,

$$t_{DD} = w + z$$

it follows immediately that

$$f_{t_{DD}}(t) = f_w \circledast f_z = \int_{-\infty}^{+\infty} f_w(T)\, f_z(t-T)\, dT$$

$$= \int_{-\infty}^{+\infty} f_x(T)\, F_{y\lessgtr}(T)\, f_z(t-T) dT$$

$$+ \int_{-\infty}^{+\infty} f_y(T) F_{x\lessgtr}(T)\, f_z(t-T) dT \qquad (3.39)$$

where we have used "$\circledast$" to indicate convolution. Expression (3.39) is the one we sought. We can now obtain all the statistics of t_{DD} that may be of interest.

13. <u>An example for a special case</u>

We now provide as an example the case for which we assume the pdf's $f_x(x_0)$, $f_y(y_0)$, and $f_z(z_0)$ to be uniform. In this particular case, the example is very relevant to the actual situation in a typical runway. Measurements taken recently at Heathrow airport [20] seem to indicate that the uniform probability density functions can provide excellent first-order approximations for the distribution of x, y, and z.

Thus we assume:

$$f_x(x_0) = \begin{cases} \dfrac{1}{x_2 - x_1}, & \text{for } x_1 \leqslant x_0 \leqslant x_2 \\ 0, & \text{otherwise} \end{cases} \tag{3.40}$$

$$f_y(y_0) = \begin{cases} \dfrac{1}{y_2 - y_1}, & \text{for } y_1 \leqslant y_0 \leqslant y_2 \\ 0, & \text{otherwise} \end{cases} \tag{3.41}$$

and,

$$f_z(z_0) = \begin{cases} \dfrac{1}{z_2 - z_1}, & \text{for } z_1 \leqslant z_0 \leqslant z_2 \\ 0, & \text{otherwise} \end{cases} \tag{3.42}$$

Then, assuming that $x_1 \leqslant y_1 \leqslant y_2 \leqslant x_2$ and that $x_2 - y_2 \leqslant z_2 - z_1 \leqslant y_2 - y_1$ (the inequalities have been taken so that they reflect the most typical situation in airports), we obtain:

$$
f_{t_{DD}}(t) = \begin{cases}
\frac{1}{C}\left[t^2 - t(2z_1 + y_1 + x_1) + z_1(y_1 + x_1 + z_1) + y_1 x_1\right], & \text{for } y_1 + z_1 \leq t \leq y_1 + z_2 \\
\frac{z_2 - z_1}{C}\left[2t - (z_2 + z_1) - (y_1 + x_1)\right], & \text{for } y_1 + z_2 \leq t \leq y_2 + z_1 \\
\frac{t - z_1 - y_2}{(x_2 - x_1)(z_2 - z_1)} + \frac{1}{C}\left[y_2^2 - (t - z_2)^2 - (y_1 + x_1)(y_2 + z_2 - t)\right], & \text{for } y_2 + z_1 \leq t \leq x_2 + z_1 \\
\frac{x_2 - y_2}{(x_2 - x_1)(z_2 - z_1)} + \frac{1}{C}\left[y_2^2 - (t - z_2)^2 - (y_1 + x_1)(y_2 + z_2 - t)\right], & \text{for } x_2 + z_1 \leq t \leq y_2 + z_2 \\
\frac{x_2 - t + z_2}{(x_2 - x_1)(z_2 - z_1)}, & \text{for } y_2 + z_2 \leq t \leq x_2 + z_2 \\
0, & \text{otherwise}
\end{cases}
\tag{3.43}
$$

For the derivation of the above expression, the reader should consult Appendix G.

Finally, we compute the quantities $E[t_{DD}]$ and $\sigma^2_{t_{DD}}$, the expectation and variance of the interdepartures time. Instead of using (3.43), we will, of course, use

$$E\left[t_{DD}\right] = E\left[w\right] + E\left[z\right] \quad , \text{ and}$$

$$\sigma^2_{t_{DD}} = \sigma^2_w + \sigma^2_z$$

in order to simplify the computations.

From (3.38), providing the form for $f_w(w_0)$, we have:

$$E\left[w\right] = \int_{y_1}^{y_2} w_0 \frac{2w_0 - (y_1+x_1)}{(x_2-x_1)(y_2-y_1)} dw_0 + \int_{y_2}^{x_2} \frac{w_0}{(x_2-x_1)} dw_0$$

$$= \frac{2}{3} \frac{y_2^3 - y_1^3}{(x_2-x_1)(y_2-y_1)} - \frac{1}{2}\left[\frac{(y_1+x_1)(y_2+y_1) - (x_2^2 - y_2^2)}{(x_2-x_1)}\right]$$

$$= \frac{1}{2(x_2-x_1)}\left[\frac{1}{3}\left(y_2^2 + y_1y_2 + y_1^2\right) + x_2^2 - x_1(y_2+y_1)\right] \qquad (3.44)$$

and

$$E\left[w_0^2\right] = \int_{y_1}^{y_2} w_0^2 \frac{2w_0 - (y_1+x_1)}{(x_2-x_1)(y_2-y_1)} dw_0 + \int_{y_2}^{x_2} w_0^2 \frac{1}{x_2-x_1} dw_0$$

$$= \frac{1}{2} \frac{y_2^4 - y_1^4}{(x_2-x_1)(y_2-y_1)} - \frac{1}{3}\left[\frac{(y_1+x_1)(y_2^2 + y_1y_2 + y_1^2) - (x_2^3 - y_2^3)}{(x_2-x_1)}\right]$$

$$= \frac{1}{3(x_2-x_1)}\left[\frac{1}{2}\left(y_2^3 + y_2^2y_1 + y_1^2y_2 + y_1^3\right) + x_2^3 - x_1\left(y_2^2 + y_1y_2 + y_1^2\right)\right] \qquad (3.45)$$

It follows that:

$$E\left[t_{DD}\right] = E\left[w\right] + \frac{1}{2}\left(z_2 + z_1\right) \qquad (3.46)$$

and

$$\sigma^2_{t_{DD}} = E\left[w^2\right] - E^2\left[w\right] + \frac{1}{12}\left(z_2 - z_1\right)^2 \tag{3.47}$$

Part III

14. Alternating arrivals and departures

The final fundamental model that we consider is that of a single runway for which each arrival is followed by a departure. Although a discipline of this type does not seem very realistic, the model provides the opportunity to consider some very important issues in connection with the time required in order to "switch" from one type of service (servicing arrivals) to another (servicing departures).

Let us first describe the stages involved in an arrival-departure-arrival cycle: Following Blumstein's example [4], we will assume that there is a point r_0 miles away from the threshold of the runway at which a landing aircraft becomes "committed to land". If at the instant when the landing aircraft reaches this point the runway is not free, due to the fact that another operation is still going on, the arrival is waved off and a "missed approach" occurs.

If, on the other hand, the runway is free, the arrival on hand proceeds to land. While this goes on, a departing aircraft takes up its position at the holding area next to the beginning of the runway. As soon as the arrival crosses the threshold of the runway, the prospective departure enters the runway and prepares to start a take-off. In the meanwhile, the arrival touches down on the runway, decelerates and eventually enters a taxiway, thus freeing the runway. If by that time the departing aircraft is set to start its take-off roll, it is given permission to do so.

After a certain "response delay" the take-off roll begins. The departure becomes airborne and eventually crosses the so-called "upwind threshold". At that instant the runway becomes available for the next operation. Ideally, at that same instant the next arrival will be at the "committed to land" point (a distance r_0 from the threshold of the runway). However, it is more realistic to assume that, in practice, the landing aircraft will be only in the vicinity of the desired point, due to the usual inaccuracies associated with the navigation and control of landing operations. Thus, there is another time-lapse until the landing aircraft reaches the "committed-to-land" point at which instant the arrival-departure-arrival cycle is completed.

We now describe the aforementioned procedures in terms of the following analytical model:

We define the following random variables:

$t_{ADA} \triangleq$ time required for a full arrival-departure-arrival cycle at a single runway for which each arrival is followed by a departure.

$v \triangleq$ approach speed of a landing aircraft as it descends through the glide path. This speed is assumed to remain constant throughout this portion of the flight.

$g \triangleq$ time interval from the instant when an arrival crosses the runway threshold to the instant when the departure in the holding area next to the runway, lines up on the runway, ready to start its take-off roll.

$h \triangleq$ time interval from the instant when an arrival crosses the runway threshold to the instant when this arrival clears the runway by entering a taxiway.

$z \triangleq$ "response period" for a departing aircraft which is set on the runway, i.e., time interval from the instant when a departure receives permission for a take-off to the instant when it starts to roll.

$y \triangleq$ time interval from the instant when a departure starts the take-off roll to the instant when the departure crosses the upwind threshold.

$e \triangleq$ distance of an arriving aircraft from a point r_0 miles away from the runway threshold at the instant when the preceding departure crosses the upwind threshold. It is assumed that a prospective arrival can never be closer than r_0 miles away at that instant and, hence, that $e \geq 0$.

Finally the only parameter associated with the model is:

$r_0 \triangleq$ the minimum distance from the runway an arriving aircraft can be, before it becomes committed to land.

Combining the definitions above, we obtain:

$$t_{ADA} = \frac{r_0}{v_1} + \max(h, g) + z + y + \frac{e}{v_2} \qquad (3.48)$$

It would be fruitless, of course, to attempt an evaluation of a detailed expression for the probability density function of t_{ADA}. Since it is the sum of five random variables or functions of random variables, the resulting expression (four convolution operations are needed) would be too cumbersome to be of any practical use. We will therefore, concentrate only on deriving the expectation and variance of t_{ADA}. In any case, these are the only quantities that will be of use in our subsequent discussions.

We have already derived the expression for the expectation of a random variable of the form max (. , .) in section 3.13. Using the same expression, (3.44), here (assuming that g and h are uniformly distributed from g_1 to g_2, and from h_1 to h_2, respectively), we obtain:

$$E\left[\max(g,h)\right] = \frac{1}{2(g_2-g_1)}\left[\frac{1}{3}\left(h_2^2+h_1h_2+h_1^2\right)+g_2^2-g_1(h_2+h_1)\right] \qquad (3.49)$$

The above expression holds for $g_1 \leq h_1 \leq h_2 \leq g_2$ which is the most typical situation in the present case.

Similarly the second moment of max (g, h) is given from equation (3.45) as

$$E\left[\left(\max(g,h)\right)^2\right] = \frac{1}{3(g_2-g_1)}\left[\frac{1}{2}\left(h_2^3+h_2^2h_1+h_1h_2^2+h_1^3\right)\right] + \frac{1}{3(g_2-g_1)}\left[g_2^3-g_1\left(h_2^2+h_1h_2+h_1^2\right)\right] \qquad (3.50)$$

and

$$\sigma^2_{max(g,h)} = E\left[(max(g,h))^2\right] - E^2\left[max\,(g,h)\right] \qquad (3.51)$$

We also have that:

$$E\left[\frac{r_o}{v}\right] = r_o\, E\left[\frac{1}{v}\right] \qquad (3.52);$$

$$\sigma^2_{\frac{r_o}{v}} = r_o^2\, \sigma^2_{\frac{1}{v}} \qquad (3.53);$$

$$E\left[\frac{e}{v}\right] = E\left[e\right] E\left[\frac{1}{v}\right] \qquad (3.54);$$

$$\sigma^2_{\frac{e}{v}} = E\left[e^2\right] E\left[\frac{1}{v^2}\right] - E^2\left[e\right] E^2\left[\frac{1}{v}\right] = \sigma^2_e\, E\left[\frac{1}{v^2}\right] + E\left[e^2\right] \sigma^2_{\frac{1}{v}} \qquad (3.55);$$

If v is uniformly distributed from b to a as it has been assumed already in section 3.6 , we have:

$$E\left[\frac{1}{v}\right] = \frac{1}{a-b} \ln\left(\frac{a}{b}\right) \qquad (3.56)$$

and

$$\sigma^2\left[\frac{1}{v}\right] = E\left[\frac{1}{v^2}\right] - E^2\left[\frac{1}{v}\right] = \frac{1}{ab} - \frac{1}{(a-b)^2} \ln^2\left(\frac{a}{b}\right) \qquad (3.57)$$

Thus, we can now write:

$$E\left[t_{ADA}\right] = \left(r_0 + E\left[e\right]\right) E\left[\frac{1}{v}\right] + E\left[\max(g,h)\right] + E\left[z\right] + E\left[y\right] \qquad (3.58)$$

and

$$\sigma^2_{t_{ALA}} = \left(r_0^2 + E\left[e^2\right]\right) \sigma^2_{\frac{1}{v}} + \sigma^2_e \, E\left[\frac{1}{v^2}\right] + \sigma^2_{\max(g,h)} + \sigma^2_z + \sigma^2_y \qquad (3.59)$$

and all the quantities in the two expressions above have been previously derived.

Part IV

15. Capacity of a single runway

Using the results which we have derived so far, we will now develop an expression for estimating the capacity of a runway which is used for both take-offs and landings.

Let us consider, first, the problem of estimating the total time required for a sequence of N operations at a single runway, of which N_A are landings and N_D are departures.

Following Blumstein's [4] method, we will assume, without any loss in generality, that the "zeroth" operation (i.e., the last one before the first of the present group of N operations), as well as the N-th operation, are both arrivals.

Let us now suppose that there are w' "cycles in this sequence of N operations. A cycle is a string of k departures ($k=1, 2, \ldots, N_D$) followed immediately by a string of ℓ landings ($\ell=1, 2, \ldots, N_A$). Therefore, since by assumption the zeroth operation is an arrival, there are w' "switchovers" from strings of landings to strings of take-offs and an equal number of "switchovers" from take-offs to landings.

A switchover from take-offs to landings can be described by the following sequence: After the last departure has lined on the runway and receives permission for a take-off, there is a response delay z, until the take-off roll begins. The departing aircraft must subsequently clear the runway and reach the upwind threshold. This operation consumes a time interval y. At that instant the approaching arrival (first of a string of landings) is r_0+e miles away from the edge of the runway, where r_0 is the distance of the "committed to land" point from the edge of the runway, and e is the spacing error as defined in chapter 3, section 14 (e $\geq$ 0). We have used here the same notation as in chapter 3 above. If the approach speed of the landing aircraft is v, the total switchover time from take-offs to landings is then given by: $z + y + \frac{r+e}{v}$.

Similarly, when a string of landings is followed by a string of departures, the time consumed by the switchover operation is given by max (g, h). From section 3.14, g is the time interval from the instant when an arrival crosses the runway threshold to the instant when the awaiting departure, in the holding area next to the runway, lines up on the runway, ready to start its take-off roll. Also h is the time interval from the instant when an arrival crosses the runway threshold to the instant when this arrival clears the runway by entering a taxiway.

Then the expected total time consumed by the sequence of $N(=N_A+N_D)$ operations is given by:

$$E[t_N] = (N_A - w')\,E[t_{AA}] + w'\,E[\max(g,h)] + (N_D - w')\,E[t_{DD}] + w'\,E\left[z + y + \frac{r_0+e}{v}\right] \qquad (3.60)$$

The first term in (3.60) represents the expected time consumed by landings which are preceded by another landing operation. The third term represents the corresponding time for departures. We can rewrite (3.60) as

$$E[t_N] = (N_A - w')E[t_{AA}] + (N_D - w')E[t_{DD}] + w'\{E[\max(g,l)] + E[y] + E[z] + E[\tfrac{1}{v}](r_0 + E[e])\}$$

and comparing with (3.58), we obtain:

$$E[t_N] = (N_A - w')E[t_{AA}] + (N_D - w')E[t_{DD}] + w'E[t_{ADA}]$$

$$= N_A E[t_{AA}] + N_D E[t_{DD}] + w'(E[t_{ADA}] - E[t_{DD}] - E[t_{AA}]) \quad (3.61)$$

It is interesting to speculate now, on what would be an optimal strategy for performing the N operations so that the quantity $E[t_N]$ is minimized. Roughly, the two strategies available to the controller are: (i) Try to arrange for long strings of landings or departures; (ii) Try to alternate arrivals and departures as often as possible. The former strategy would amount to making w' in (3.61) as small as possible, and the latter to making w' as large as possible.

The decision, of course, hinges on the sign of the sum:

$$E[t_{ADA}] - E[t_{DD}] - E[t_{AA}]$$

We can use our previous results of chapter 3, in order to examine the above sum. We first rewrite t_{AA} as

$$t_{AA} = \max\left\{ \frac{n+s_0+\epsilon}{v_2} - \frac{n}{v_1} , t_0 \right\} \tag{3.62}$$

where s_0 is the minimum distance separation requirement in the air (a constant) and ϵ, a random variable, is the positioning error discussed in section 3.3. In other words, we have rewritten the random variable x of section 3.4 as $x = s_0 + \epsilon$.

Now for the case when $\frac{L}{a} - \frac{n}{b} \geq t_0$ we have from (3.4) that

$$E[t_{AA}] = E[s_0+\epsilon]\, E\left[\frac{1}{v}\right] = (s_0 + E[\epsilon])\, E\left[\frac{1}{v}\right] \tag{3.63}$$

Then, using results already obtained and (3.63), we have:

$$\begin{aligned} E[t_{ADA}] - E[t_{AA}] - E[t_{DD}] &= E[\max(g,h)] + E[y] \\ &+ E[z] + E\left[\frac{1}{v}\right](r_0 + E[e]) - E[\max(x,y)] - E[z] \\ &- (s_0 + E[\epsilon])\, E\left[\frac{1}{v}\right] \end{aligned} \tag{3.64}$$

For the sake of indicating orders of magnitude involved, we will now attempt a rough numerical evaluation of the right hand side of (3.64).

First, for all practical purposes the time elapsing from the instant a departure starts to roll to the instant when the following departure lines up is identical to the time interval between the instant when an arrival crosses the threshold of the runway and the instant when the following departure lines up on the runway (in the case of an arrival which is followed by a departure). Thus we can assume that g and x are identical random variables. The validity of this assumption is borne out by actual measurements at airports [20].

Another reasonable assumption is that $E[\epsilon] \approx E[e]$. Both ϵ and e are errors due to the same human and technical factors and thus their average values must be quite close.

With the above remarks in mind we can rewrite (3.64) as:

$$E[t_{ADA}] - E[t_{AA}] - E[t_{DD}] \approx E[\max(g,h)] + E[y] - E[\max(x,y)] - (s_0 - r_0)E\left[\frac{1}{v}\right] \tag{3.65}$$

Let us attempt to evaluate the right hand side of (3.65) for a set of typical values. We use s_0=3 miles and r_0=2 miles. The approach speeds, v, of landing aircraft are assumed to range uniformly from 100 miles/hr to 150 miles/hr. Random variables y and x are also assumed to be uniformly distributed ranging from 40 to 80 seconds, and from 35 to 90 seconds, respectively. Similarly, h has a uniform density function and ranges from 35 to 80 seconds. Then, using (3.56), (3.49), and (3.44) to estimate

$E[\frac{1}{v}]$, $E[\max(g,h)]$, and $E[\max(x,y)]$, respectively, we obtain from (3.65):

$$E[t_{ADA}] - E[t_{AA}] - E[t_{DD}] \approx 25 \text{ seconds}$$

Thus for this typical situation, it seems that $E[t_{ADA}] > E[t_{AA}] + E[t_{DD}]$. Hence, referring back to equation (3.61), the controller should attempt to minimize w', in other words try to have strings of successive arrivals and of successive departures which are as long as possible.

A couple of remarks must be made with regard to the above discussion. First, we assumed that the rule of assigning priority to landings over departures might be abandoned in favor of having as many operations as possible on the runway. Note, however, that if the controller decides that the best strategy is that of long runs of landings followed by long runs of take-offs, he may still preserve, to some extent, the priority of landings over departures by permitting all awaiting aircraft to land before providing clearance for any take-offs.

Second, adoption of a strategy of alternating arrivals with departures would also mean a significant change in current controller practices. This strategy requires continuous coordination of the activities of the approach control center with those of the "tower". In the present state of affairs inter-action between these two components of ATC is rather loose.

The expected time per operation is now given by dividing $E[t_N]$ in (3.61) by N:

$$\mu \triangleq E[t_N]\cdot\frac{1}{N} = \frac{N_A}{N}E[t_{AA}] + \frac{N_D}{N}\cdot E[t_{DD}] + \frac{\omega'}{N}\left(E[t_{ADA}] - E[t_{AA}]\cdot E[t_{DD}]\right) \tag{3.66}$$

This equation, then, can provide the basis for finding the operational capacity of a runway during high congestion periods for any given ratio of arrivals and departures:

We define $p_A \triangleq \frac{N_A}{N}$ and $p_D \triangleq \frac{N_D}{N}$ as the fraction of operations which is represented by arrivals and departures, respectively. Also, since $\frac{N}{w'}$ is the average length of a cycle, we define a quantity $w \triangleq \frac{2w'}{N}$, which represents the average length of a run of arrivals or of departures.

Then we may rewrite (3.66) as:

$$\mu = p_A E[t_{AA}] + p_D E[t_{DD}] + \frac{w}{2}\left(E[t_{ADA}] - E[t_{DD}] - E[t_{AA}]\right) \tag{3.67}$$

and the operational capacity of the runway under these circumstances is given by $\frac{1}{\mu}$.

Blumstein in his report has used the assumption that $\frac{N_A}{N} \approx \frac{N_D}{N} \approx \frac{1}{2}$, in the long run. It is felt here, however, that because the ratio $\frac{N_A}{N_D}$ varies rather widely depending on the time of the day at a typical major airport, it is better to leave μ in the form of (3.67). It must be understood that p_A and p_D, as well as w, are functions of time. Then (3.67) can be used to provide a good estimate of a runway's capacity for a given time interval, during the day. The expression holds, as well, for runways which are not used for mixed operations. For example, if the runway is used only for landings, we have $p_A=1$, $p_D=0$, and $w=0$ thus giving $\frac{1}{\gamma} = \frac{1}{E[t_{AA}]}$ for the capacity of the runway, as we should.

In this section, we have not considered the situation in which a departure may be interposed between two successive landings (because the time gap between the landings is long enough) without disrupting the landing process. This problem is discussed in chapter 4.

Chapter IV

Queuing Considerations

1. Introduction

In Chapter III, we derived the probability density function for t_{AA}, the duration of the gap between two successive arrivals of aircraft, under capacity conditions, at a runway used only for landings. We used the pdf to compute the capacity of the runway under a certain set of assumptions.

In this chapter, we will use the pdf for the interarrival gap, $f_{t_{AA}}(t)$, in a different manner. Specifically, we shall take a new point of view: The runway is now regarded as a service facility. Aircraft arrive at the vicinity of this facility in a certain fashion. If the facility is free, they proceed to land. If, on the other hand, the facility is servicing other aircraft or if there is already a queue (stack) of arriving aircraft, the last arrival joins the end of the queue and awaits its turn to land. In this case the time between the successive completions of service to aircraft, i.e., the time between successive landings is just the random variable t_{AA} with the probability density function $f_{t_{AA}}(t)$.

We are, therefore, confronted with a classical queuing situation: for the rest of this chapter, we shall draw heavily on results of queuing theory and we shall try to explore such questions as: What is the average waiting time per arriving aircraft? What is the average number of airplanes in the holding stack? What portion of the time is the facility idle?

In the second half of this chapter, we examine the problem of attempting to insert take-offs between landings of aircraft at a runway. It is stipulated that take-offs must not disrupt the landing process. Thus a departing aircraft must wait for an appropriate gap between two successive arrivals. Such a gap should be long enough to allow a take-off to take place without violation of separation standards.

Several quantities of interest are derived, such as the expected waiting time of a particular departing aircraft until a suitable gap appears, or the expected number of take-offs that can be inserted per unit time.

Finally, the case of airports with more than one runway in service is discussed briefly. The purpose is to show how the results that have been obtained previously can be extended to the multiple runway case. This last section is simply a slight extension of the work of Blumstein.

2. Single runway, used for landings

Any investigation of the queuing situation at a service facility requires the specification of the following items: (a) Probability distribution for the time between successive arrivals of prospective users of the facility. (b) Probability distribution for the time required in order to provide service to a user. (c) Information about the facility itself, including such items as the number of servers, the relative priorities assigned to prospective users, the maximum size allowed for the waiting line, etc.

With reference to our present consideration of a runway serving only landing aircraft, we have already specified items (b) and (c). The service time distribution is the pdf for the interarrival gap length $f_{t_{AA}}(t)$. We examine the case where service is assumed to be provided on a strict first-come, first-served basis and the maximum size of the queue is considered to be infinite, for all practical purposes.

As for item (a), it has been generally assumed in past investigations [3], [4], [31] that arrivals of airplanes at the outer fix of the glide path occur in a random fashion in time. In other words, arrivals constitute a Poisson process. For the rest of this chapter, we will use this assumption.

As Pestalozzi [31] points out, it is difficult to verify the assumption of Poisson arrivals. As aircraft approach the terminal area, they interact to an ever-increasing extent with each other because of airspace limitations. In addition, the flow of airplanes into the terminal area is regulated by ground control during periods of congestion. The probabilistic distribution of the arrival instants is, therefore, altered by these effects. We can expect, in general, that as congestion increases, arrival instants become less and less random because of increased interference from the ground. Our real assumption, therefore, is that aircraft arrivals would be Poisson, if each airplane was allowed to proceed free to the beginning of the glide path. This assumption can be well supported by theoretical arguments, as well.

It is interesting to note that the latter is, in fact, the only assumption necessary for our present problem. This is so since our service time distribution $f_{t_{AA}}(t)$ does take into consideration the effects of interference among aircraft prior to their arrival at the glide path. This is achieved through the use of the positioning error, i.e., random variable x (see Chapter III).

To summarize, so far, we have a single server queue, with Poisson arrivals, say at a rate of λ arrivals per hour, and a service time distribution $f_{t_{AA}}(t)$. Service is offerred on a first-come, first-served basis. We can, therefore, use some well known results from the theory of M/G/1 queues.

We will now quote, without proof, some expressions, which will be of use to our work. There are many references which the reader may consult for the derivation of these formulae and for discussion [21], [7], [9].

These expressions apply only under steady-state conditions, i.e., when all transient effects have died out at the system under consideration.

For such conditions to exist (for the model under consideration here) it must be that: $\lambda < [E[t_{AA}]]^{-1}$

We would also like to point out that as λ approaches $[E[t_{AA}]]^{-1}$ from below, the characteristics of the system display deviations of increasing magnitude from the expected steady-state conditions.

The first expression provides the fraction of time the runway is free, with an arrival rate λ and a pdf for service times equal to $f_{t_{AA}}(t)$.

We have

$$\pi_0 = Pr[\text{runway is free}] = 1 - \lambda E[t_{AA}] \tag{4.1}$$

The expected number of airplanes awaiting for service under steady-state conditions (including the one currently being serviced) is:

$$E[X] = \lambda E[t_{AA}] + \frac{\lambda^2 E[t_{AA}^2]}{2(1 - \lambda E[t_{AA}])} \tag{4.2}$$

The average time a plane spends in the queue, i.e., the delay experience due to the existence of other aircraft is given by:

$$E[w] = \frac{\lambda E[t_{AA}^2]}{2(1 - \lambda E[t_{AA}])} \tag{4.3}$$

Finally, we quote the expression for the geometric transform of the probability distribution for the number of planes in the system at any particular time, under steady state conditions.

If we define

$\pi_n \triangleq$ steady state probability that there are n landing aircraft in the system (queue, regulator space, and glide path)

then,

$$\pi^T(z) = \sum_{n=0}^{\infty} z^n \pi_n = (1-\lambda E[t_{AA}])\frac{a^T(z)(1-z)}{a^T(z)-z} \tag{4.4}$$

where

$$a^T(z) = \tilde{f}_{t_{AA}}(\lambda-\lambda z) \tag{4.5}$$

or $a^T(z)$ is the Laplace transform $\tilde{f}_{t_{AA}}(s)$ of $f_{t_{AA}}(t)$, with s substituted by $\lambda-\lambda z$.

Expressions (4.1) through (4.5), can be used in order to estimate some useful statistics concerning the queuing situation on hand. Blumstein [4] provides such an analysis in his report for his analytical model and we can do the same here, by using the expressions for $f_{t_{AA}}(t)$, $E[t_{AA}]$, and $E[t_{AA}^2]$ derived in chapter III.

Before proceeding in this way, however, we will examine, in the next section, the present model more closely and will find that certain correction terms are required for the expressions quoted above.

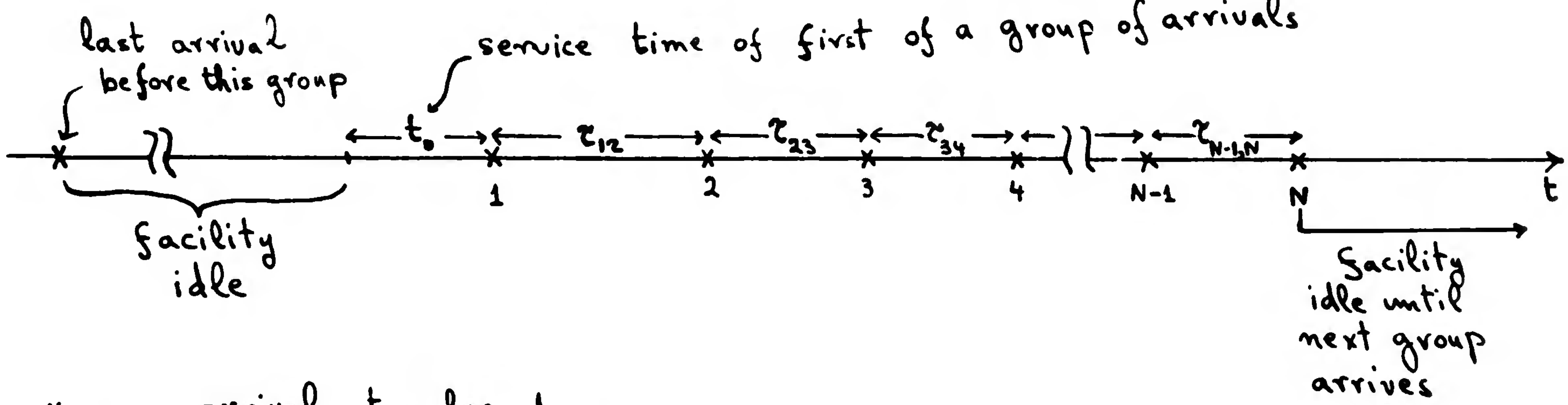
3. A closer look at the queuing model

We have so far discussed an M|G|1 type queue in connection with a runway used only for landings. In this discussion, we have used the pdf $f_{t_{AA}}(t)$ for the time between successive landings as the service time distribution.

A closer look at the situation will, however, reveal a difficulty. The random variable t_{AA} has been defined as the time gap between arrivals, under capacity conditions, i.e., when planes are always available at the holding stack. Thus, t_{AA} depends, as we have seen in Chapter III, on the characteristics (we have focused on speed, specifically) of both the first and the second of a pair of incoming aircraft.

Let us now refer to figure 4.1. A sequence of N successive arrivals is indicated there. Our assumption from the previous section is that the time gap τ_{12} is defined as the service time of the second aircraft. Similarly, τ_{23}, $\tau_{34}, \ldots, \tau_{N-1,N}$ are the service times of the 3rd, 4th,...Nth aircraft, respectively. Random variables τ_{12}, $\tau_{23}, \ldots$ $\tau_{N-1,N}$ are identically distributed and independent with a pdf $f_{t_{AA}}(t)$.

What can we say, however, about the service time of the first airplane, the one which approaches an "idle" facility? The answer depdns on the kind of service the runway is used for. If, for example, the runway is used only for landings, the service time for the first of a group of N successive arrivals should be set equal to t_0, the minimum



x = an arrival touches down on the runway at this instant

$\tau_{i,i+1}$ = time interval between i-th and (i+1)st arrival

Figure 4.1

A sequence of N arrivals at a runway

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time separation requirement as specified by ATC rules. In other words, there must be at least a time interval t_0 between any previous operation on the runway and the arrival of this first of a group of N airplanes. Hence the first arrival, in effect, "occupies" the runway for time t_0.

If, on the other hand, the controller inserts take-offs, whenever there are no landing aircraft, the service time for the first of a group of N arrivals should be set equal to the time interval which starts at the instant when the arrival reaches the "committed to land" point of the glide path (see Chapter I.3) and ends at the instant when the arrival touches down at the runway. (Note, that it is assumed here that arrivals have strict priority over departures and, therefore, the time of an arrival can not be delayed because of a departure.)

The important point here is that the service time for the first of a group of arrivals does not have the same probability distribution as the rest of the arrivals in the group. For one thing, in the case of the first arrival after an idle period, there is no preceding airplane which can interfere with the present arrival.

We are faced, therefore, with a queuing situation in which we have one kind of service time distribution when an arriving customer finds the facility idle, and a different kind of service time distribution when the customer enters the facility immediately after service to a previous customer has been completed. A queue of this type will be discussed.

presently, as a generalized problem and the results will be subsequently applied to the particular situation on hand.

4. An M|G|1 queue with state-dependent service times

Let us consider a single server facility with first-come, first-served discipline and an unlimited queue length allowed. Arrivals of prospective customers at the facility occur in a Poisson manner at a rate of λ arrivals per unit time.

Since we will postulate non-exponential service times distribution, we must use the usual imbedded Markov chain technique. Then, we look at the service facility the instant after a service completion and define:

$X_n \triangleq$ number of customers in the queue the instant after the n-th service completion.

Then, let:

$\beta_0(t) \triangleq$ service time distribution for the next customer to be serviced given that $X_n = 0$

$\beta(t) \triangleq$ service time distribution for the next customer to be serviced given that $X_n > 0$.

We assume that $\beta_0(t)$ has finite moments and we let $b_0, \gamma_0, \delta_0 \ldots$ be the first, second, third, ... moments associated with $\beta_0(t)$. Similarly, we assume that $\beta(t)$ has finite moments and we let $b, \gamma, \delta, \ldots$ be the first, second, third, ... moments associated with $\beta(t)$.

If we define A_{N+1} as the number of arrivals during the service of the (n+1)st customer, we have:

$$X_{n+1} = \begin{cases} X_n + A_{n+1} - 1, & \text{if } X_n > 0 \\ X_n + A_{n+1}, & \text{if } X_n = 0 \end{cases} \tag{4.6}$$

Now, let us define:

$${}_0a_k \triangleq \Pr[A_{n+1}=k \mid X_n=0] = \int_0^\infty \frac{(\lambda t)^k e^{-\lambda t}}{k!} \beta_0(t)dt \tag{4.7}$$

and

$$a_k \triangleq \Pr[A_{n+1}=k \mid X_n>0] = \int_0^\infty \frac{(\lambda t)^k e^{-\lambda t}}{k!} \beta(t)dt \tag{4.8}$$

Then, taking transforms, we have:

$${}_0a^T(z) = \sum_{k=0}^\infty {}_0a_k z^k = \int_0^\infty \sum_{k=0}^\infty \frac{z^k(\lambda t)^k e^{-\lambda t}}{k!} \beta_0(t)dt$$

$$= \int_0^\infty e^{-(\lambda-\lambda z)t} \beta_0(t)dt = \tilde{\beta}_0(\lambda-\lambda z) \tag{4.9}$$

and similarly,

$$a^T(z) = \tilde{\beta}(\lambda-\lambda z) \tag{4.10}$$

where $\tilde{\beta}_0(s)$ and $\tilde{\beta}(s)$ are the Laplace transforms of $\beta_0(t)$ and $\beta(t)$, respectively.

Since the transition probability matrix $[p_{ij}] \triangleq [\Pr[X_{n+1}=j \mid X_n=i]]$ is given by:

$$[p_{ij}] = \begin{bmatrix} {}_0a_0 & {}_0a_1 & {}_0a_2 & \cdots & \\ a_0 & a_1 & a_2 & \cdots & \\ 0 & a_0 & a_1 & \cdots & \\ 0 & 0 & a_0 & a_1 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \end{bmatrix} \tag{4.11}$$

we obtain the following equations for the steady state probabilities $\pi_j \stackrel{\Delta}{=} \lim_{n\to\infty}$ $P[X_n=j \mid X_0=i]$ (the conditions for a steady state to exist will be discussed later):

$$_0a_0\,\pi_0 + a_0\pi_1 = \pi_0$$

$$_0a_1\pi_0 + a_1\pi_1 + a_0\pi_2 = \pi_1$$

$$_0a_n\pi_0 + a_n\pi_1 + a_{n-1}\pi_2 + \dots + a_0\pi_{n+1} = \pi_n$$

and, in general,

$$_0a_n\pi_0 + \sum_{i=0}^{n} a_{n-i}\pi_{i+1} = \pi_n\,, \qquad n \geq 0 \tag{4.12}$$

Then, taking transforms, we obtain:

$$\pi^T(z) = \sum_{n=0}^{\infty} \pi_n z^n = \pi_0 \sum_{n=0}^{\infty} {}_0a_n z^n + \sum_{n=0}^{\infty} z^n \sum_{i=0}^{n} \pi_{i+1} a_{n-i}$$

$$= {}_0a^T(z)\,\pi_0 + \sum_{i=0}^{\infty} z^i\pi_{i+1} \sum_{n=i}^{\infty} a_{n-i} z^{n-i} =$$

$$= {}_0a^T(z)\pi_0 + a^T(z) \sum_{i=0}^{\infty} z^i\pi_{i+1} =$$

$$= {}_0a^T(z)\pi_0 + a^T(z)\,\frac{1}{z}\,[\,\pi^T(z) - \pi_0]$$

From which we obtain:

$$\pi^T(z) = \pi_0\,\frac{a^T(z) - z\,{}_0a^T(z)}{a^T(z) - z}$$

Now, since $\pi^T(1) = 1$, we have:

$$\pi_0 = \lim_{z \to 1} \frac{a^T(z) - z}{a^T(z) - z\,{}_0a^T(z)}$$

Using L'Hopital's rule for the limit, we obtain:

$$\pi_0 = \lim_{z \to 1} \frac{(a^T(z))' - 1}{(a^T(z))' - ({}_0a^T(z)) - z({}_0a^T(z))'}$$

But,

$$(a^T(z))' \Big|_{z=1} = (\tilde{\beta}(\lambda - \lambda z))' \Big|_{z=1} = -\lambda(\tilde{\beta}(s))' \Big|_{s=0} = \lambda b$$

and similarly,

$$({}_0a^T(z))' \Big|_{z=1} = \lambda b_0$$

Then,

$$\pi_0 = \frac{\lambda b - 1}{\lambda b - 1 - \lambda b_0} = \frac{1 - \lambda b}{1 - \lambda(b - b_0)} \qquad (4.13)$$

and

$$\pi^T(z) = \frac{1 - \lambda b}{1 - \lambda(b - b_0)} \; \frac{a^T(z) - z\,{}_0a^T(z)}{a^T(z) - z} \qquad (4.14)$$

The geometric transform of the steady-state probabilities π_i has now been obtained, in terms of known quantities. The probabilities π_i can be obtained now, at least theoretically, from:

$$\pi_i = \frac{1}{i!} \frac{d^i[\pi^T(z)]}{dz^i} \Big|_{z=0} \qquad (4.15)$$

The expected number in queue with the system in steady state can, of course, be obtained from $(\pi^T(z))'\big|_{z=1}$. Instead of differentiating directly, we will use here the method of Taylor series expansion.

We have:

$$a^T(z) = \tilde{\beta}(\lambda-\lambda z) = 1 - (\tilde{\beta}(\lambda-\lambda z))'\big|_{z=1}(1-z)$$

$$+ \frac{1}{2}(\tilde{\beta}(\lambda-\lambda z))''\big|_{z=1}(1-z)^2 - \ldots$$

$$= 1 - \lambda b(1-z) + \frac{\gamma\lambda^2}{2}(1-z)^2 \ldots$$

and similarly,

$$_0a^T(z) - z\,_0a^T(z) = 1-\lambda b(1-z) + \frac{\gamma\lambda^2}{2}(1-z)^2 - \ldots$$

$$-z + \lambda b_0 z(1-z) - \frac{\gamma_0\lambda^2}{2} z(1-z)^2 - \ldots$$

$$= 1-z - (1-z)\lambda(b-b_0z) + (1-z)^2\frac{\lambda^2}{2}(\gamma-\gamma_0 z) - \ldots$$

$$= (1-z)\left[1-\lambda(b-b_0z) + (1-z)\frac{\lambda^2}{2}(\gamma-\gamma_0 z) - \ldots\right]$$

Also,

$$a^T(z) - z = 1-z-\lambda b(1-z) + \frac{\gamma\lambda^2}{2}(1-z)^2 - \ldots$$

$$= (1-z)\left[1-\lambda b + \frac{\gamma\lambda^2}{2}(1-z) - \ldots\right]$$

Substitution in (4.14), the expression for $\pi^T(z)$, then gives:

$$\pi^T(z) = \frac{1-\lambda b}{[1-\lambda(b-b_0)]}\;\frac{[1-\lambda(b-b_0z)+(1-z)\frac{\lambda^2}{2}(\gamma-\gamma_0z)-\ldots]}{[1-\lambda b+\frac{\gamma\lambda^2}{2}(1-z)-\ldots]}$$

$$= \frac{1-\lambda(b-b_0z) + (1-z)\frac{\lambda^2}{2}(\gamma-\gamma_0 z)-\ldots}{[1-\lambda(b-b_0)]\left[1+\frac{\gamma\lambda^2}{2(1-\lambda b)}(1-z)-\ldots\right]}$$

$$= \frac{[1-\lambda(b-b_0z)+(1-z)\frac{\lambda^2}{2}(\gamma-\gamma_0z) - \ldots]}{[1-\lambda(b-b_0)]}[1 - \frac{\gamma\lambda^2(1-z)}{2(1-\lambda b)} + \ldots]$$

$$= \frac{1-\lambda(b-b_0z)+(1-z)[\frac{\lambda^2}{2}(\gamma-\gamma_0z) - \frac{\gamma\lambda^2}{2(1-\lambda b)} + \frac{\lambda(b-b_0z)\gamma\lambda^2}{2(1-\lambda b)}] + R}{1-\lambda(b-b_0)}$$

where R is a sum of terms containing factors $(1-z)^n$, with $n \geq 2$.

Then,

$$E[X] = (\pi^T(x))'\Big|_{z=1} = \frac{1}{1-\lambda(b-b_0)}[\lambda b_0 - \frac{\lambda^2}{2}(\gamma-\gamma_0) + \frac{\gamma\lambda^2}{2(1-\lambda b)} - \frac{\lambda(b-b_0)\gamma\lambda^2}{2(1-\lambda b)}]$$

or

$$E[X] = \frac{1}{1-\lambda(b-b_0)}[\lambda b_0 + \frac{\gamma_0\lambda^2}{2} + \frac{\lambda b_0\gamma\lambda^2}{2(1-\lambda b)}] \qquad (4.16)$$

is the expected number in the queue.

For other moments, appropriate expressions involving derivatives of $\pi^T(z)$ evaluated at z=1, can be used.

The next item of interest is the waiting time of an arbitrary customer with the queue in the steady state: Let w be the time a customer waits for service and S his service time. Thus, the time the customer spends in the system is $t = w + S$. If we take expectations, we have:

$$E[t] = E[w] + E[S] \qquad (4.17)$$

Using (4.13),

$$E[S] = \pi_0 b_0 + (1-\pi_0)b = \frac{1-\lambda b}{1-\lambda(b-b_0)} b_0 + [1 - \frac{1-\lambda b}{1-\lambda(b-b_0)}] b$$

$$= \frac{b_0}{1-\lambda(b-b_0)} \qquad (4.18)$$

Note, also, that $\lambda E[t]$ is the expected number of customers that arrive during the total time the present customer spends in the system. But this is just the expected number in the queue when this customer leaves, or:

$$E[X] = \lambda E[t] = \lambda E[w] + E[S] \tag{4.19}$$

Thus, we finally have:

$$E[w] = \lambda^{-1} E[X] - E[S]$$

and using (4.16) and (4.18), we obtain:

$$E[w] = \frac{1}{1-\lambda(b-b_0)} \left[\frac{\gamma_0 \lambda}{2} + \frac{(\lambda b_0)(\gamma\lambda)}{2(1-\lambda b)} \right] \tag{4.20}$$

Several remarks are in order now:

(i) The only requirement for the realization of a steady state condition for the system described above is that $\lambda b < 1$.

(ii) If we define p_n to be the general-time probability that there are n customers in the system (the fraction of time the system spends in state n), it can be shown that

$$p_n = \pi_n \tag{4.21}$$

The proof of this relation is identical with the proof of the same relation for the common M/G/1 queue. We will not, therefore, reproduce it here, but instead refer the reader to Cox and Miller [7]. It must also be noted, that we have already assumed this relation in arguing that $E[X] = \lambda E[t]$ (equation (4.19)).

(iii) If $\beta_0(t) = \beta(t)$, expressions (4.13), (4.14), (4.16), and (4.20) reduce to the well-known expressions for the M/G/1 queue, as they should. In addition, these expressions exhibit the same general characteristics as the common M|G|1 expressions. For instance, the average delay still depends on the second moments of the service-time distributions $\beta_0(t)$ and $\beta(t)$. On the other hand, we have some "correction terms" in these expressions whose effect will be discussed in detail later.

(iv) The model analyzed above can be applied to several situations, besides the ones related to air-traffic control with which we are concerned here.

For example, we can think of the case of a machine which is turned off whenever there is no "job" awaiting for service by the machine. Then after a "job" arrives, the machine has to be turned on again and some time must be allowed for warm-up. This makes the service-time distribution for those "jobs" which find the machine idle (and, therefore, turned off) different from the usual service-time distribution. Several interesting questions arise in connection with this conceivable situation but they are outside the scope of this thesis.

5. Application to the case of a runway used for landings

We apply, now, the results of the preceding section, to the case of the runway which is used only for landings.

With reference to section 4.4, we have:

$$\beta(t) = f_{t_{AA}}(t) \qquad (4.22)$$

$$b = E[t_{AA}] \qquad (4.23)$$

$$\gamma = E[t_{AA}^2] \qquad (4.24)$$

$$a^T(z) = \tilde{\beta}(\lambda - \lambda z) = \tilde{f}_{t_{AA}}(\lambda - \lambda z) \qquad (4.25)$$

We must now discuss $\beta_0(t)$, the service time distribution for the first of a sequence of n arrivals (n=1, 2, ...) which does not interact with any preceding landing aircraft.

Since we assume here that no other operations are permitted on the runway except landings and since landings can not be spaced closer than t_0 seconds apart, we will take the service time for the first airplane to be equal to this constant t_0. In other words, we assume that for t_0 seconds prior to the touch-down of the first of a group of landings, the runway is, in a sense, "occupied" by this landing plane.

Then,

$$\beta_0(t) = \begin{cases} 1, & \text{for } t=t_0 \\ 0, & \text{otherwise} \end{cases} \qquad (4.26)$$

$$b_0 = t_0 \qquad (4.27)$$

$$\gamma_0 = t_0^2 \tag{4.28}$$

and

$$a^T(z) = e^{-st_0} \tag{4.29}$$

Of course, some other pdf $\beta_0(t)$ can be used, but it is felt that for our present analysis the assumption of a constant service time, is not unreasonable.

At the end of this chapter, we present some numerical results derived by using the above assumptions to model some typical situations.

Finally, it should be pointed out that, in physical terms, one should be careful about defining what is a "first of a group of arrivals", with reference to our queuing model. If we want to be consistent we must say that any arriving aircraft that proceeds through the terminal area and up to the gate of the glide path without taking any delaying action due to the presence of other aircraft constitutes such an arrival. Thus, we have excluded from the class of "firsts" not only those aircraft thtat have to join the holding stack (as we obviously should), but also these planes which are instructed by the controller to follow longer routes to the gate in order to avoid conflict with other arrivals.

It should be clear, then, that for high traffic density periods very few aircraft would qualify for the characterization of "first of a platoon".

6. Some take-offs inserted between landings

We will now consider the case of a runway which is used for landing aircraft, while, at the same time, take-offs are inserted whenever the time-interval between two successive arrivals is long enough to permit such an insertion. The primary item of interest is the expected number of take-offs that can be inserted per unit of time, under various conditions. In the process, we will also obtain the expected delay of a departing aircraft awaiting for an opportunity to take-off.

The model is now described in detail: Consider a runway which is used for landings. The probability density function for the time-gap between two successive touch-downs of landing aircraft at the beginning of the runway is given by $f_{t_{AA}}(t)$. Departing aircraft are dispatched to the waiting apron near the beginning of the runway (see fig. 4.2).

A random variable t_D, describes the total time needed by the departing airplane in order to enter the runway (once it receives permission to), start the roll, and leave the runway. The required time t_D is a random variable with a probability density function $f_{t_D}(t)$, describing the time needed for a take-off.

If for a particular departing aircraft we have $t_1 + T \leq t_2$, where t_1 is the value of t_D for this airplane, t_2 is the duration of the present gap and T, a constant, is an arbitrary safety margin, then the waiting departure is ordered to enter the runway and start the take-off. The assumption here

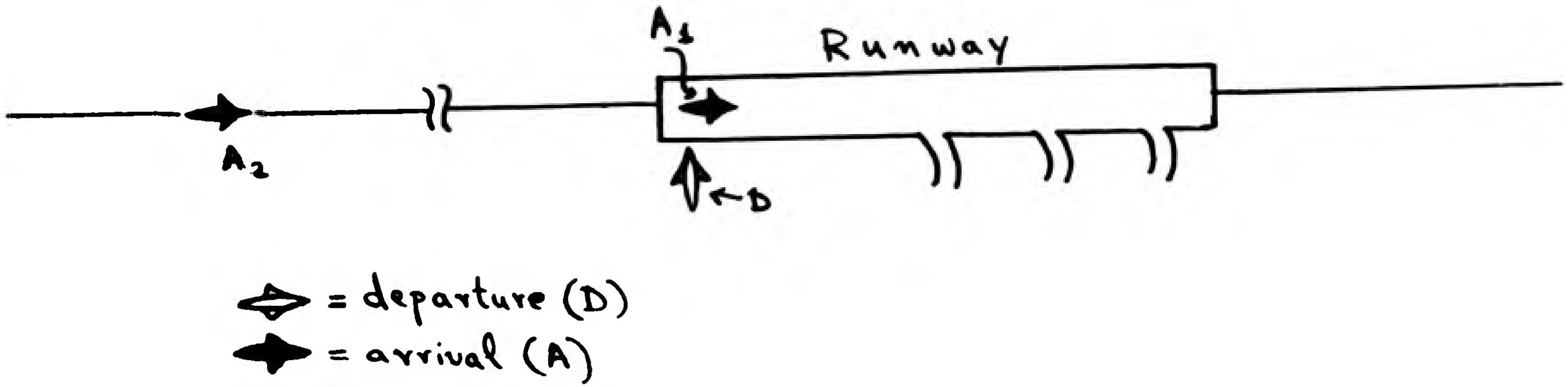


Figure 4.2

A departure awaiting for an appropriate gap between successive arrivals is that the tower can estimate the length of the interarrival gap in advance and, therefore, that the decision of whether or not to insert a take-off can be made as soon as the first of an incoming pair of landings touches down on the runway.

7. Waiting time until a suitable gap is found

We will now estimate the expected delay for a given aircraft awaiting to take-off under capacity conditions (landing aircraft always available). We will assume that the departing aircraft arrives at the apron next to the runway at a random instant and will refer to that instant as the "instant of incidence".

Such a situation is depicted in figure 4.3. The instant of incidence is indicated by t=A. The probability density function for the remaining time y, of the present interval is given by

$$f_y(y_0) = \frac{1 - \int_0^{y_0} f_{t_{AA}}(t)\,dt}{E[t_{AA}]} = \frac{1 - F_{t_{AA}}(y_0)}{E[t_{AA}]} \tag{4.30}$$

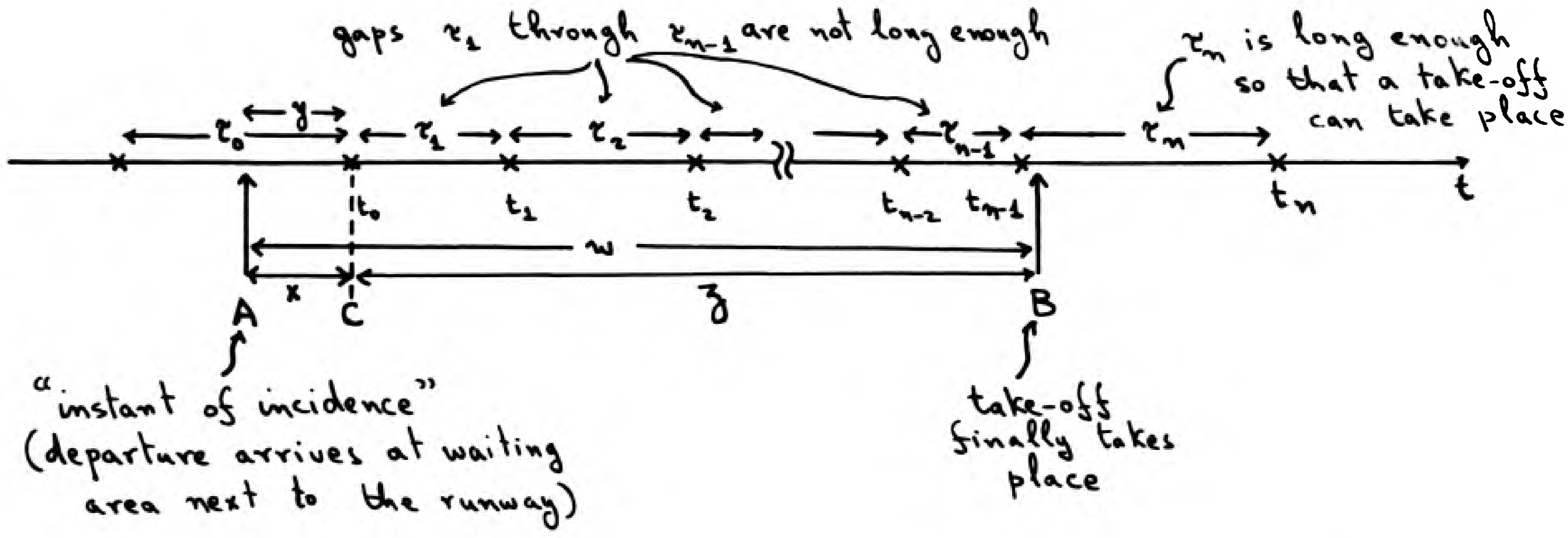
For the derivation of this expression the reader may consult, for example, Drake [8], page 152 .

Then, the probability that the departure time t_D of the departing airplane is small enough so that take-off can take place immediately is given by:

$$P[t_D + T < y] = \int_0^\infty P[t_D + T \le y_0 \mid y_0]\, f_y(y_0)\, dy_0$$

$$= \int_0^\infty P[t_D \le y_0 - T \mid y_0]\, f_y(y_0)\, dy_0 = \int_0^\infty P[t_D \le y_0 - T]\, f_y(y_0)\, dy_0$$

the last relation following from the independence of random variables t_D, the time needed for the departure, and y, the remaining time until the next



$\times \triangleq$ instants at which an arrival crosses the beginning of the runway

$\tau_i \triangleq$ interarrival gaps

Figure 4.3

landing.

Then, since

$$P[t_D \leq y_0 - T] = \int_0^{y_0 - T} f_{t_D}(t)\,dt \triangleq F_{t_D \leq}(y_0 - T)$$

we obtain

$$P_0 \triangleq P[t_D + T \leq y] = \int_0^{\infty} F_{t_D \leq}(y_0 - T)\, f_y(y_0)\, dy_0 \tag{4.31}$$

Therefore, with probability P_0, the waiting time of this departing aircraft will be equal to 0.

Now, with probability $1-P_0$, we have:

$$E[w] = E[x] + E[z], \tag{4.32}$$

where we have defined:

$w \triangleq$ waiting time until take-off starts (time interval from A to B on figure 4.3)

$x \triangleq$ waiting time until the first landing that occurs after the instant of incidence given that a take-off does not take place immediately (time interval from t=A to t=C, on figure 4.3).

$z \triangleq$ waiting time from the first landing after the instant of incidence until the take-off finally occurs (time interval from t=C to t=B on figure 4.3).

We hasten to emphasize that random variable x does not have the same distribution as random variable y, since x is the remaining time of the first interval conditional on no take-off action having taken place.

We find now, the probability density function $f_x(x_0)$. (See also figure 4.3).

$$f_x(x_0) = f_y(x_0 \mid t_D + T > x_0) = \frac{\int_{x_0-T}^{+\infty} f_{y,t_D}(x_0,t)\,dt}{P[t_D + T > x_0]}$$

$$= \frac{\int_{x_0-T}^{\infty} f_y(x_0) f_{t_D}(t)\,dt}{1 - P[t_D + T \leq x_0]}$$

or

$$f_x(x_0) = \frac{f_y(x_0)[1 - F_{t_D \leq}(x_0 - T)]}{1 - P_0} \qquad (4.33)$$

where P_0 is the quantity defined by (4.31). Thus,

$$E[x] = \int_0^{\infty} x_0 f_x(x_0)\,dx_0 = \frac{1}{1-P_0} \int_0^{\infty} t f_y(t)[1 - F_{t_D \leq}(t - T)]\,dt \qquad (4.34)$$

The last item that we must estimate now, is $E[z]$ (see (4.32)). Referring back to figure 4.3, we note that z is given by:

$$z = \tau_1 + \tau_2 + \ldots + \tau_{n-1}$$

where n is a random variable and $\tau_1, \ldots, \tau_{n-1}$ are independent, identically distributed random variables. Thus, z is the sum of a random number of random variables.

In effect, the situation depicted here is a sequence of n independent trials. The first n-1 trials result in failure (no take-off inserted) while the last trial is a gap long enough to permit a take-off.

If we define

$$P \triangleq P[t_D + T \leqslant t_{AA}], \tag{4.35}$$

we obtain,

$$\begin{aligned} P &= \int_0^{\infty} P[t_D + T \leqslant t \mid t]\, f_{t_{AA}}(t)\, dt \\ &= \int_0^{\infty} P[t_D \leqslant t - T \mid t]\, f_{t_{AA}}(t)\, dt \\ &= \int_0^{\infty} P[t_D \leqslant t - T]\, f_{t_{AA}}(t) dt = \\ &= \int_0^{\infty} F_{t_D \leqslant}(t-T)\, f_{t_{AA}}(t)\, dt \end{aligned} \tag{4.36}$$

Then,

$$\begin{aligned} E[z] &= E[n-1]\; E[\tau_i] = \left\{ E[n] - 1 \right\} E[\tau_i] \\ &= (\frac{1}{P} - 1)\; E[\tau_i] = \frac{1-P}{P}\; E[\tau_i] \end{aligned} \tag{4.37}$$

where $i = 1, 2, \ldots, n-1$

Note that τ_i $(i=1,2,\ldots,n-1)$ is the length of an interarrival gap conditional on no merging action having taken place. Then, we find:

$$f_{\tau_i}(t) = f_{t_{AA}}(t \mid t_D + T > t) = \frac{\int_{t-T}^{+\infty} f_{t_{AA},t_D}(t,\tau)\,d\tau}{P[t_D + T > t]}$$

$$= \frac{\int_{t-T}^{+\infty} f_{t_{AA}}(t) f_{t_D}(\tau)\,d\tau}{1 - P[t_D + T \leq t]} = \frac{f_{t_{AA}}(t)[1 - F_{t_{D\leq}}(t-T)]}{1-P}$$

where P is the probability defined by (4.36). Thus,

$$E[\tau_i] = \int_0^{\infty} t f_{\tau_i}(t)\,dt = \frac{1}{1-P} \int_0^{\infty} t f_{t_{AA}}(t)\,[1 - F_{t_{D\leq}}(t-T)]\,dt \qquad (4.38)$$

and

$$E[z] = \frac{1}{P} \int_0^{\infty} t f_{t_{AA}}(t)\,[1 - F_{t_{D\leq}}(t-T)]\,dt \qquad (4.39)$$

Finally combining (4.31), (4.34), and (4.39) we obtain

$$E[w] = P_0 \cdot 0 + (1-P_0)\,[E[x] + E[z]]$$

$$= \int_0^{\infty} t f_y(t)[1 - F_{t_{D\leq}}(t-T)]\,dt$$

$$+ \frac{1-P_0}{P} \int^{\infty} t f_{t_{AA}}(t)\,[1 - F_{t_{D\leq}}(t-T)]\,dt \qquad (4.40)$$

We also estimate the second moment of the expected waiting time for this departing aircraft.

Referring again to figure 4.3, we have:

With probability P_0, we have $E[w^2] = 0$ (the take-off takes place immediately).

With probability $1-P_0$:

$$E[w^2] = E[(x+z)^2] = E[x^2]+E[z^2]+2E[x]E[z] \tag{4.41}$$

Now,

$$E[x^2] = \int_0^{\infty} x_0^2 \, f_x(x_0)dx_0 = \frac{1}{1-P_0}\int_0^{\infty} t^2 f_y(t)[1-F_{t_D}(t-T)]\,dt \tag{4.42}$$

and $E[x]$ and $E[z]$ are given by (4.34) and (4.39), respectively.

Finally we have:

$$E[z^2] = E\left[\left(\sum_{i=1}^{N-1}\tau_i\right)^2\right] = \sum_{n=1}^{\infty} E\left[\left(\sum_{i=1}^{n-1}\tau_i\right)^2 \Big| N=n\right]P[N=n]$$

$$= \sum_{n=1}^{\infty} E\left[\left(\sum_{i=1}^{n-1}\tau_i\right)^2\right]P[N=n] = \sum_{n=1}^{\infty} E\left[\sum_{i=1}^{n-1}\tau_i^2 + \sum_{i=1}^{n-1}\sum_{\substack{j=1\\ i\neq j}}^{n-1}\tau_i\tau_j\right]P[N=n] = \sum_{n=1}^{\infty}\left[\sum_{i=1}^{n-1}E[\tau_i^2] + \sum_{i=1}^{n-1}\sum_{\substack{j=1\\ i\neq j}}^{n-1}E[\tau_i\tau_j]\right]$$

$$\cdot P[N=n] = \sum_{n=1}^{\infty}\left[(n-1)E[\tau_i^2] + (n-1)(n-2)E[\tau_i]E[\tau_j]\right]P[N=n]$$

$$= E[\tau_i^2]E[N-1] + E^2[\tau_i]E[N^2-2N+2] \tag{4.43}$$

In the above we have used repeatedly, the fact that N and τ_i (i=1,2,... N-1) are independent random variables. N, of course, is the geometric random variable representing the number of gaps up to and including the first gap which is appropriate for inserting a take-off.

We have now:

$$E[\tau_i^2] = \int_0^\infty t^2 f_{\tau_i}(t)dt = \frac{1}{1-P}\int_0^\infty t^2 f_{t_{AA}}(t)[1-F_{t_{D\leq}}(t-T)]dt \quad (4.44)$$

$E[\tau_i]$ is given by (4.38), and $E[N^2] = \frac{1-P}{P^2} + \frac{1}{P^2}$.

Then, substituting in (4.37):

$$E[z^2] = \frac{1-P}{P} E[\tau_i^2] + [\frac{2-P}{P^2} - \frac{2}{P} + 2] E^2[\tau_i] \quad (4.45)$$

Finally, combining all the expressions in (4.41), we obtain:

$$E[w^2] = \int_0^\infty t^2 f_y(t)[1-F_{t_{D\leq}}(t-T)]dt + \frac{1-P_0}{P}\int_0^\infty t^2 f_{t_{AA}}(t)[1-F_{t_{D\leq}}(t-T)]dt$$

$$+ \frac{1-P_0}{(1-P)^2}[\frac{2-P}{P^2} - \frac{2}{P} + 2]\left(\left(\int_0^\infty t f_{t_{AA}}(t)[1-F_{t_{D\leq}}(t-T)]dt\right)^2\right.$$

$$+ 2\left(\int_0^\infty t f_y(t)[1-F_{t_{D\leq}}(t-T)dt\right)\left(\frac{1-P}{P}\int_0^\infty t f_{t_{AA}}(t)[1-F_{t_{D\leq}}(t-T)]dt\right)$$

(4.46)

Expressions (4.40) and (4.46) are the ones we were seeking. We have now expressed the first and second moments of the waiting time of a plane which awaits for an opportunity to take off at a runway used for landings, in terms of only: (i) the pdf for the length of the time interval between two successive leandings and (ii) the pdf for the time required for a take-off.

The expressions for $E[w]$ and $E[w^2]$ agree with the expressions derived by Weiss [39], through a different method, for the waiting time of a car that awaits on a secondary road for an opportunity to enter into the stream of cars on a main road. The reader will readily recognize the similarities between the two problems.

8. Number of take-offs inserted

In the previous section, we obtained an expression for the expected waiting time of a departing airplane, which awaits at the beginning of a runway used for landings until a suitable gap appears.

We now proceed to estimate the expected number of departures that can be inserted at this runway, under various traffic conditions.

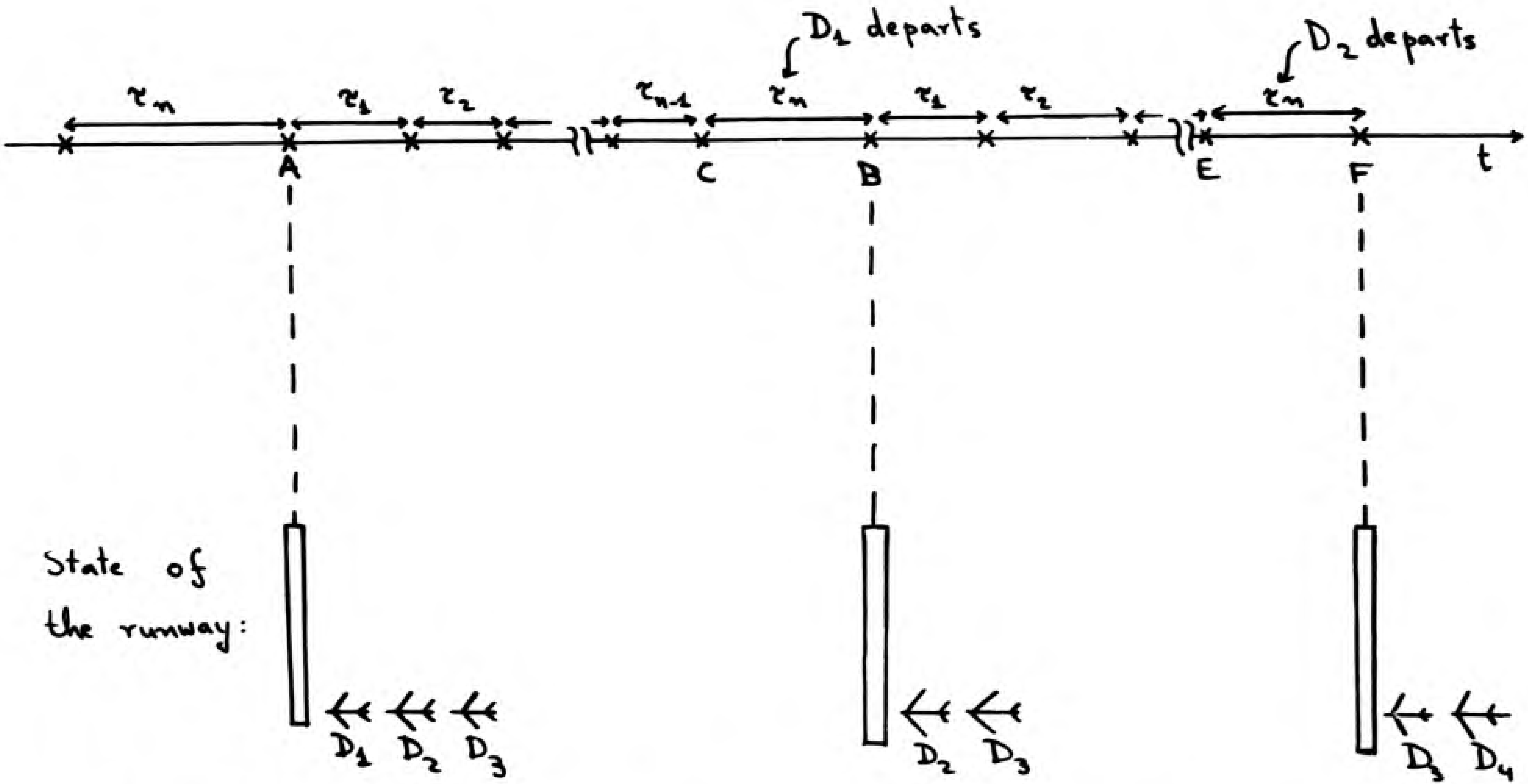
We will postulate the following two conditions:

(1) There are always departing aircraft available at the runway. In other words, we are looking for the capacity of the runway as far as insertions of take-offs are concerned.

(2) At most, only one take-off can be inserted in a gap between successive landings whenever there is a group of arriving aircraft awaiting to land. This assumption seems to be born out by an arithmetic calculation of the typical range of values that the interarrival gap length can take under the assumptions of chapter III.

Let us now refer to figure 4.4. Since there is always a queue of departing aircraft at the beginning of the runway, under assumption (1), and only one departure, at most, is permitted per gap, under assumption (2), we can only have departures at the beginning of gaps, as indicated.

We have already estimated, in the previous section, the expected time the first airplane in the departing queue spends "watching" gaps which are too short (time gap from t=A to t=C on the figure). Now, the expected



τ_n are long enough gaps to permit interposition of a take-off

x – indicates instant of a landing

Figure 4.4

Interposing departures between arrivals

duration of a gap long enough for the insertion of a departure must be found. Using the same notation as in the preceding section, we have for the pdf of τ_n, the length of an interarrival gap conditional on the gap's being long enough to accept a take-off (from t=C to t=B on the figure):

$$f_{\tau_n}(t) = f_{t_{AA}}(t \mid t_D+T \leq t) = \frac{\int_0^{t-T} f_{t_{AA},t_D}(t,\tau)d\tau}{P[t_D+T \leq t]}$$

$$= \frac{\int_0^{t-T} f_{t_{AA}}(t)\, f_{t_D}(\tau)d\tau}{P[t_D+T \leq t]}$$

$$= \frac{f_{t_{AA}}(t)\, F_{t_{D\leq}}(t-T)}{P} \tag{4.47}$$

where P is the probability specified by (4.36). Thus,

$$E[\tau_n] = \int_0^{\infty} tf_{\tau_n}(t) = \frac{1}{P}\int_0^{\infty} tf_{t_{AA}}(t)\, F_{t_{D\leq}}(t-T)dt \tag{4.48}$$

Then, the total time span taken up by the first airplane in the departing queue is given by:

$$D = E[z] + E[\tau_n] = \frac{1}{P}\int_0^{\infty} tf_{t_{AA}}(t)[1-F_{t_{D\leq}}(t-T)]\, dt$$

$$+\frac{1}{P}\int_0^{\infty} tf_{t_{AA}}(t)\, F_{t_{D\leq}}(t-T)dt = \frac{1}{P}\int_0^{\infty} tf_{t_{AA}}(t)\, dt$$

or

$$D = \frac{E[t_{AA}]}{P} \tag{4.49}$$

Of course, this result could have been obtained without using the results of section 4.7.

We now examine two different traffic density situations:

(i) When the runway is used to capacity, i.e., when there are no idle periods from landings, the expected number of departures that can be served by the runway per unit time is:

$$N_1 = \frac{1}{D} = \frac{P}{E[t_{AA}]}, \tag{4.50}$$

provided that assumptions (1) and (2) of this section hold.

(ii) When the rate of arrivals is less than the rate of service $\frac{1}{E[t_{AA}]}$, the runway is idle, as far as the servicing of landings is concerned, for a fraction of time equal to (see (4.13)):

$$\pi_0 = \frac{1 - \lambda E[t_{AA}]}{1 - \lambda(E[t_{AA}] - E[t_f])},$$

where $E[t_f]$ is the expected service time for the first of a group of 1, 2, ..., landings as defined in section

For this fraction of time, it is possible to occupy the runway with departures only. In chapter III, we found an expression for $E[t_{DD}]$, the expected time gap between successive departures (equation (3.46)). Thus, the number of departures that can be served during that period of time when there are no landings is given by:

$$\pi_0 \cdot \frac{1}{E[t_{DD}]}$$

Then, the total expected number of departures per unit time, under these traffic conditions is given by:

$$N_2 = \pi_0 \frac{1}{E[t_{DD}]} + (1-\pi_0)\frac{P}{E[t_{AA}]}$$

$$= \frac{1}{1-\lambda(E[t_{AA}]-E[t_f])}\left[\frac{1-\lambda E[t_{AA}]}{E[t_{DD}]} + \frac{\lambda E[t_f]P}{E[t_{AA}]}\right] \quad (4.51)$$

The only quantities in the above expression which have not yet been specified are $E[t_f]$ and P. An expression for P has already been found ((4.36)), but it is in terms of the probability distribution $F_{t_{D\leq}}(t)$, which we have not yet found. We now proceed to complete these points.

9. Estimation of $f_{t_D}(t)$ and $E[t_f]$

We estimate the pdf for random variable t_D, the total time needed by the departing airplane in order to enter the runway between two arrivals, start the take-off roll, and leave the runway.

A very similar derivation was performed in chapter III, when we estimated the pdf for random variable t_{DD}, the time gap between successive departures for a runway that is uesd only for departures. We will, therefore, make the development below a very brief one.

We assume that the departing aircraft awaits at the beginning of the runway. As soon as a landing aircraft crosses the beginning of the runway, the departing aircraft receives instructions on whether or not to enter the runway depending on the duration of the gap between the present landing and the one following that. If permission for a take-off is given, the procedure starts immediately.

Let us define random variables:

$x \triangleq$ time lapse from the instant a landing aircraft crosses the beginning of the runway to the instant when it leaves the runway through a taxiway.

$y \triangleq$ time interval from the instant when the departing aircraft receives permission to leave the apron next to the beginning of the runway, to the instant when it is lined up on the runway and ready to start to roll.

$z \triangleq$ time interval from the instant when the departure starts to roll until the instant when it leaves the runway.

Then we have

$$t_D = \max(x, y) + z \tag{4.52}$$

Assuming that x, y, and z are independent random variables and defining,

$$w \overset{\Delta}{=} \max(x, y)$$

we have:

$$f_w(w_0) = \frac{d}{dw_0} F_{w\leqslant}(w_0) = \frac{d}{dw_0} \int_{-\infty}^{w_0}\int_{-\infty}^{w_0} f_{x,y}(x_0, y_0)dx_0\, dy_0$$

$$= \frac{d}{dw_0} \int_{-\infty}^{w_0}\int_{-\infty}^{w_0} f_x(x_0)f_y(y_0)dx_0 dy_0$$

$$= \frac{d}{dw_0} (F_{y\leqslant}(w_0)F_{x\leqslant}(w_0))$$

$$= F_{y\leqslant}(w_0)f_x(w_0) + F_{x\leqslant}(w_0)f_y(w_0) \qquad (4.53)$$

Since $t_D = w + z$ (with w and z independent) we may now use the convolution integral and write:

$$f_{t_D}(t) = \int_{-\infty}^{+\infty} f_w(w_0)f_z(t-w_0)dw_0 \qquad (4.54)$$

Thus, given the pdf' s $f_x(x_0)$, $f_y(y_0)$ and $f_z(z_0)$ for the aircraft using the runway in question, we can find the probability density function $f_{t_D}(t)$. This latter pdf is used, as indicated by (4.36), to compute P, the probability of inserting a departure between successive arrivals.

Finally, we conclude by stating our assumptions about t_f, the service time for the first of a group of landings in the case when it is permitted to occupy the runway with departures, whenever no arrivals are available.

As we have done in chapter III, section 15, we shall postulate that when an arrival approaches a runway after the runway has been occupied by a departing aircraft, it is necessary that the departure will have cleared the runway before the landing aircraft reaches a point r_0 miles from the beginning of the runway. If this does not happen the arrival is waved off and a missed approach occurs. Thus, the time for which the runway is, in effect, "occupied" by the first arrival after a set of departures is given by $t_f = \frac{r_0}{V}$.

Then we have:

$$F_{t_{f\leq}}(t) = P[t_f \leq t] = P[\frac{r_0}{V} \leq t] = P[\frac{r_0}{t} \leq V]$$

$$= 1 - P[V < \frac{r_0}{t}] = 1 - F_{V\leq}(\frac{r_0}{t}) , \quad (4.55)$$

for a continuous $F_{V\leq}(v)$. And,

$$f_{t_f}(t) = \frac{d}{dt} F_{t_{f\leq}}(t) = \frac{d}{dt}[1 - F_{V\leq}(\frac{r_0}{t})]$$

$$= \frac{r_0}{t^2} f_V(\frac{r_0}{t}) \quad (4.56)$$

Thus, given the approach speed profile of the aircraft using the runway, we can find all statistics pertaining to t_f, including $E[t_f]$ of equation (4.51).

For example for the pdf $f_V(v_0)$ used in chapter III, i.e.,

$$f_V(v_0) = \begin{cases} \frac{1}{a-b}, & \text{for } b \leq v \leq a \\ 0, & \text{otherwise} \end{cases}$$

we have

$$f_{t_f}(t) = \begin{cases} \frac{r_0}{t^2} \frac{1}{(a-b)}, & \text{for } \frac{r_0}{a} \leq t \leq \frac{r_0}{b} \\ 0, & \text{otherwise} \end{cases}$$

and

$$E[t_f] = \frac{r_0}{(a-b)} \ln(\frac{a}{b}) \quad (4.57)$$

10. Operations capacity with insertions

In section 3.15, we derived an expression for the capacity of a runway which is used both for landings and for take-offs. Relation (3.67) expressed the average service time per operation in terms of the fraction of arrivals, p_A, and departures, p_D, during a given time interval ($p_A+p_D=1$) and in terms of w, the reciprocal of the average length of a run of landings or of take-offs during the time interval:

$$\gamma = p_A E[t_{AA}] + p_D E[t_{DD}] + \frac{w}{2}\left(E[t_{ADA}] - E[t_{DD}] - E[t_{AA}]\right) \tag{4.58}$$

Expressions for $E[t_{AA}]$, $E[t_{DD}]$ and $E[t_{ADA}]$ have, of course, been derived in chapter 3.

At that time, the possibility of inserting a departure between successive arrivals without disrupting the landing process was not considered. However, in this chapter, we have shown in Section 4.7 that there is a probability P given by (4.36) that such an interposition can indeed be achieved. Therefore, we have to revise (4.58) in order to take account of this possibility.

Let us again assume that out of a total of N operations, there are N_A landings and N_D take-offs. Let also w' be the number of cycles in this sequence of operations. However, we now have to change our definition of a cycle somewhat: A cycle is a run of k departures followed by a run of h arrivals; but a run of arrivals may also be interspersed with some departures as long as those departures did not delay the arrival process. For example, the sequence $A_1A_2D_1A_3A_4D_2A_5$ may still be considered a run of 5 arrivals as long as the gaps between arrivals A_2 and A_3 and between A_4 and A_5 were long enough to permit interposition of departures D_1 and D_2.

With this in mind, we still have N_A-w' pairs of successive arrivals as explained in section 3.15. This is, then, the number of opportunities available for inserting a departure between two successive arrivals. Thus, the expected number of interpositions is given by $(N_A - w')P$ and the total time consumed by the N operations is now given by:

$$E[t_N] = \begin{cases} (N_A - w')E[t_{AA}] + \{N_D - w' - (N_A - w')P\} E[t_{DD}] + \\ \quad + w' E[t_{ADA}] \quad , \text{ if } N_A \cdot P < N_D \\ N_A E[t_{AA}] \quad , \text{ if } N_A \cdot P \geq N_D \end{cases} \tag{4.59}$$

What the second part of (4.59) says is that it may be possible for sufficiently large P or N_A, or for a small number of departures N_D, to insert all take-offs between landings without delaying the landing process.

Rearranging terms in (4.59) and dividing through by N, we obtain:

$$
\mu \triangleq \frac{E[t_N]}{N} = \begin{cases} p_A E[t_{AA}] + (p_D - p_A \cdot P) E[t_{DD}] \\ \quad - \frac{w}{2}\left(E[t_{ADA}] - E[t_{AA}] - (1-P) E[t_{DC}]\right), \\ \qquad\qquad \text{for } \mu < \frac{p_D}{p_A} \\ p_A E[t_{AA}], \quad \text{for } \mu \geq \frac{p_D}{p_A} \end{cases} \tag{4.60}
$$

where, we have again defined, as in section 3.15, $p_A \triangleq \frac{N_A}{N}$, $p_D \triangleq \frac{N_D}{N}$, and $w \triangleq \frac{2w'}{N}$.

Expression (4.60) can be used to estimate the operations capcity $\frac{1}{\mu}$ at a runway for a given traffic mix for any time interval, whenever inter-position of take-offs between successive landings is permitted.

11. Interference among operations on different runways

In this section, we attempt to indicate briefly how some of the previously obtained results may be applied to the case of an airport with more than one runway in service.

In general, runway configurations at an airport can be separated into two classes: (1) parallel runways and (2) intersecting runways. The latter class includes pairs of runways which may not actually intersect, but whose extended center axes do, as indicated in figure 4.5. We will concentrate, for the moment, on the case of intersecting runways and will discuss parallel runways very briefly, later on.

In particular, we focus attention on the intersecting pair configuration of figure 4.5(a). The typical strategy for the use of such a pair of runways is to employ one of them for landings only and the other for take-offs. The runway used for landings must, of course, be equipped with an instrument landing system for bad weather conditions.

Assume now that arrivals are alternated with departures. Let us examine the sequence of operations taking place and compare this situation with what happens on a single runway.

First, as in the case of a single runway, no operation may take place on the runway which is used for landings, say runway 1, after the instant when an arriving aircraft has crossed the "committed to land" point, r_0 miles away from the edge of the runway. Thus the arrival "occupies" runway 1 --

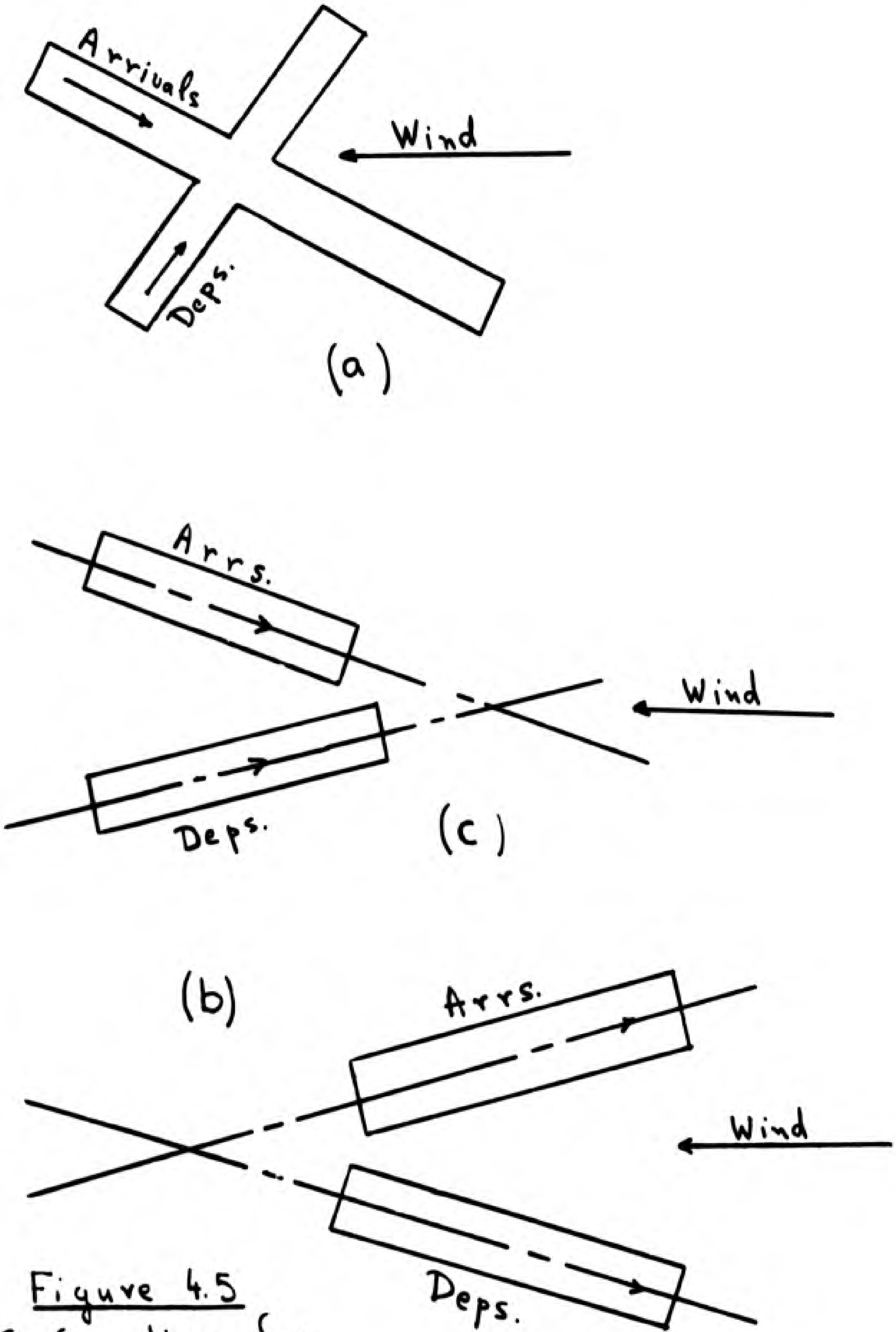


Figure 4.5
Configurations for intersecting runways

while at the same time precluding initiation of any operations on runway 2 -- for a time interval given by $\frac{r_0}{v} + h'$, where v is the approach speed and h' is the time elapsing from the instant when the arrival touches down on the runway to the instant when it crosses the intersection with runway 2. At that instant the departure, which, we assume, has already lined up at the beginning of runway 2, receives clearance for a take-off. After some response time z, the take-off roll starts, and a time y' later the departing aircraft crosses the intersection with runway 1. At that instant, ideally, the next arrival must be r_0 miles away from the edge of runway 1. Assuming the usual positioning error e, the total time consumed by the arrival-departure-arrival cycle is given by

$$t'_{ADA} = \frac{r_0}{v_1} + h' + z + y' + \frac{e}{v_2} \tag{4.61}$$

Comparing with the expression for t_{ADA} given by (3.48):

$$t_{ADA} = \frac{r_0}{v_1} + \max(g,h) + z + y + \frac{e}{v_2}$$

we see that the average time saved per operation cycle is given by:

$$E[t_{ADA}] - E[t'_{ADA}] = \left\{ E[\max(g,h)] - E[h'] \right\} + \left\{ E[y] - E[y'] \right\} \tag{4.62}$$

In other words, the gain is due on the one hand, to the fact that both the arrival and the departure occupy the service facility (meaning the runways) for less time (h' vs h, and y' vs y, respectively) and, at the same time the time g, needed for the lining up of the departure is no longer a consideration when two runways are available.

Note, first, that as the quantities h' and y' decrease, i.e., as the intersection approaches the beginning of the runways, operations on one runway affect the ones conducted on the other runway to a lesser and lesser extent. In the limiting case when the runways intersect on the leeward side, the runways are independent. (See figure 4.5(b).) This configuration, however, is weather-dependent. When the wind direction changes, a departure has to wait until a successful landing is assured on the other runway (figure 4.5(c)).

Note, also, that it is of little advantage in the case of intersecting runways to insert a departure between two successive arrivals on the runway which is used for landings. The reason is that the interposed operation will hinder operations on the other runway.

The most common arrangement of runways at major airports is in parallel. This arrangement has proved the most efficient one from the point of view of increasing the capacity of the airport. If there is sufficient distance between them, two parallel runways can operate independently. The minimum distance required for independent operation for the present generation of aircraft and with the guidance and navigational equipment currently available is 3,000 feet.

Suppose there are two independent parallel runways in service at an airport. As Blumstein has pointed out, in order to minimize delay it is, in general, "a desirable policy to divide the load (in terms of time) equally among the two runways, since delay is a convex function (i.e., non-negative second derivative) of runway utilization in all normal queuing situations. If the load is divided unequally, the additional delay incurred by the additional utilization on one runway would exceed the delay avoided by decreasing the utilization of the other".

Blumstein proceeds to comment that when the two parallel runways are independent, their operations capacity is twice that of a single runway. We would like to add that an even higher operations capacity can be achieved, if appropriate selection is made of those departing aircraft which will be interposed between successive landings. Consider, for example, the case when one runway, say runway A, is used only for landings, while a second runway B is used primarily for departures and for a few arrivals as well. This, incidentally, is the most common situation at airports. Suppose now, that the controller wishes to insert a few take-offs, without disrupting the landings, at runway A. The controller has now the opportunity of selecting those departing aircraft which have the most appropriate characteristics for this purpose.

Note, that the controller would like to maximize P, the probability of interposing a departure. From (4.36) we have:

$$P = \int_0^{\infty} F_{t_{D\leq}}(t-T)\, f_{t_{AA}}(t)\, dt \tag{4.36}$$

The quantity $f_{t_{AA}}(t)$ can not be affected because the landing process can not be tampered with by assumption.

However, from (4.52):

$$t_D = \max(x, y) + z \tag{4.52}$$

where y and z are random variables directly associated with the type of departing aircraft. In particular, z is the "response" time of an airplane before it starts a take-off and y is the time needed by the departing aircraft in order to line up on the runway (see section 4.9). Thus, an effort can be made to pick aircraft for which $F_{t_{D\leq}}(t-T)$ increases (toward its final value of 1) for as small a value of t, as possible. In the most extreme case we can have P=1. This happens if $F_{t_{D\leq}}(t_{min}-T) = 1$ where t_{min} is the minimum value that random variable t_{AA} can take.

There is a large number of unsolved problems associated with the operation of systems of two parallel runways. One type of queuing problem on which considerable time was spent with little success is described in Appendix H. It may be of interest to future investigators.

Chapter V

Numerical Results and Conclusions

Some Computational Results

A numerical investigation of some expressions derived in chapters 3 and 4 was undertaken. Attention was primarily focused on the model of the single runway used only for landings. The effect of some of the parameters on the rate of operations was examined.

In addition, results obtained from the queuing model postulated in chapter 4 were compared in detail with results obtained by using the usual formulae for the M/G/1 queue.

Computations were performed for the following combinations of parameters:

(1) N = glide path length = 4, 5, 6, 7, 8 miles

(2) s_0 = minimum distance separation requirement in the air = 3, 3.5, 4, 4.5, 5 miles.

(3) R = range of spacing error = 0, 0.5, 1, 1.5, 2, 2.5, 3 miles

(4) v = approach speed of landing aircraft = uniformly distributed between 120-140 miles per hour.

(5) t_0 = minimum time separation requirement between successive landings on the runway = 90 seconds.

(6) b_0 = runway occupancy time for aircraft not arriving in platoons = 90 seconds (i.e., same as minimum time separation requirement)

(7) γ_0^2 = second moment of runway occupancy time for aircraft not arriving in platoons = $(90)^2$ seconds^2

(8) λ = arrivals rate = from 5 and up to 30 aircraft per hour.

Arrivals of aircraft are assumed to be Poisson. Item (3) requires some explanation: We assume, as in chapter 3, that the spacing error is a uniformly distributed random variable. Thus, when R, the range of error, is, say, 2 miles, this means that the average spacing error per aircraft is 1 mile.

A sample of the numerical results is presented in the figures of this chapter. Please note that, where applicable, dashed curves (----) were obtained by using the usual queue formulae (specifically expressions (4.1), (4.2), and (4.3)). On the other hand, curves drawn with a continuous line (———) are associated with the corresponding formulae developed in this thesis, namely (4.13), (4.16), and (4.20). The quantities b, b_0, γ, and γ_0 are given by (4.23), (4.27), (4.24), and (4.28), respectively.

Otherwise, the figures should be self-explanatory. We will, therefore, limit ourselves here to a qualitative discussion of the results.

First, numerical computations indicate a very strong dependence of such items as the average delay per landing aircraft and average service time on the distance separations at the beginning of the glide path. In particular, the effects of increasing either the distance separation requirement set by the FAA or the average spacing error are vividly evident.

In this respect we would like to point out that the variance of the average service time, $E[t_{AA}]$ per aircraft, increases faster with the spacing error than with the distance-separation requirement. Consider for example the following case: For a glide path length equal to 6 miles and for approach speeds uniformly distributed between 120-140 miles per hour, the following set of results was obtained:

S_0	R	Av. Error = $\frac{R}{2}$	$E[t_{AA}]$	$\sigma^2_{t_{AA}}$
3 miles	2.5 miles	1.25 miles	117.9 seconds	619 seconds^2
3.5 miles	1.5 miles	0.75 miles	117.9 seconds	362 seconds^2
4 miles	0.5 miles	0.25 miles	117.9 seconds	233 seconds^2

In other words, although the average distance separation remains constant (4.25 miles), and hence the expected service time is constant, the variance decreases with a decrease in the spacing error. As average delays also depend on the variance of service time, it is extremely important that spacing errors be minimized, to the extent possible. It is, then, important to obtain accurate measurements of what errors actually occur.

A second interesting aspect of the investigation was the comparison between the usual queuing expressions and the ones developed here. In general, the former tend to overestimate delays, average number of aircraft present and runway utilization. This is true, of course, under the assumption that average service times (as well as second moments of service times) for aircraft arriving in platoons are at least as large as the corresponding quantities for aircraft which proceed to a landing without going through the stacking procedures. The assumption, then, is that $b \geqslant b_0$ and that $\gamma \geqslant \gamma_0$, in (4.13), (4.16) and (4.20).

The discrepancies between the results obtained from the two methods are most evident for medium utilization rates (for instance for arrival rates of 20-25 aircraft per hour). It seems, that it is certainly worth exploring the queuing model described in chapter 4, by taking actual measurements at airports.

The reader should also note the well-known, by now, phenomenon of rapid increases in average expected delays, when the arrivals rate approaches a critical value. Moreover, one should be aware of the fact that the values exhibited here are only average ones and that in practice one can have wide variations of the system as the saturation point is approached. Delays appear to become critical in the case of arrivals, as the number of aircraft approaches the range of 26 to 32 per hour, for the situation examined here.

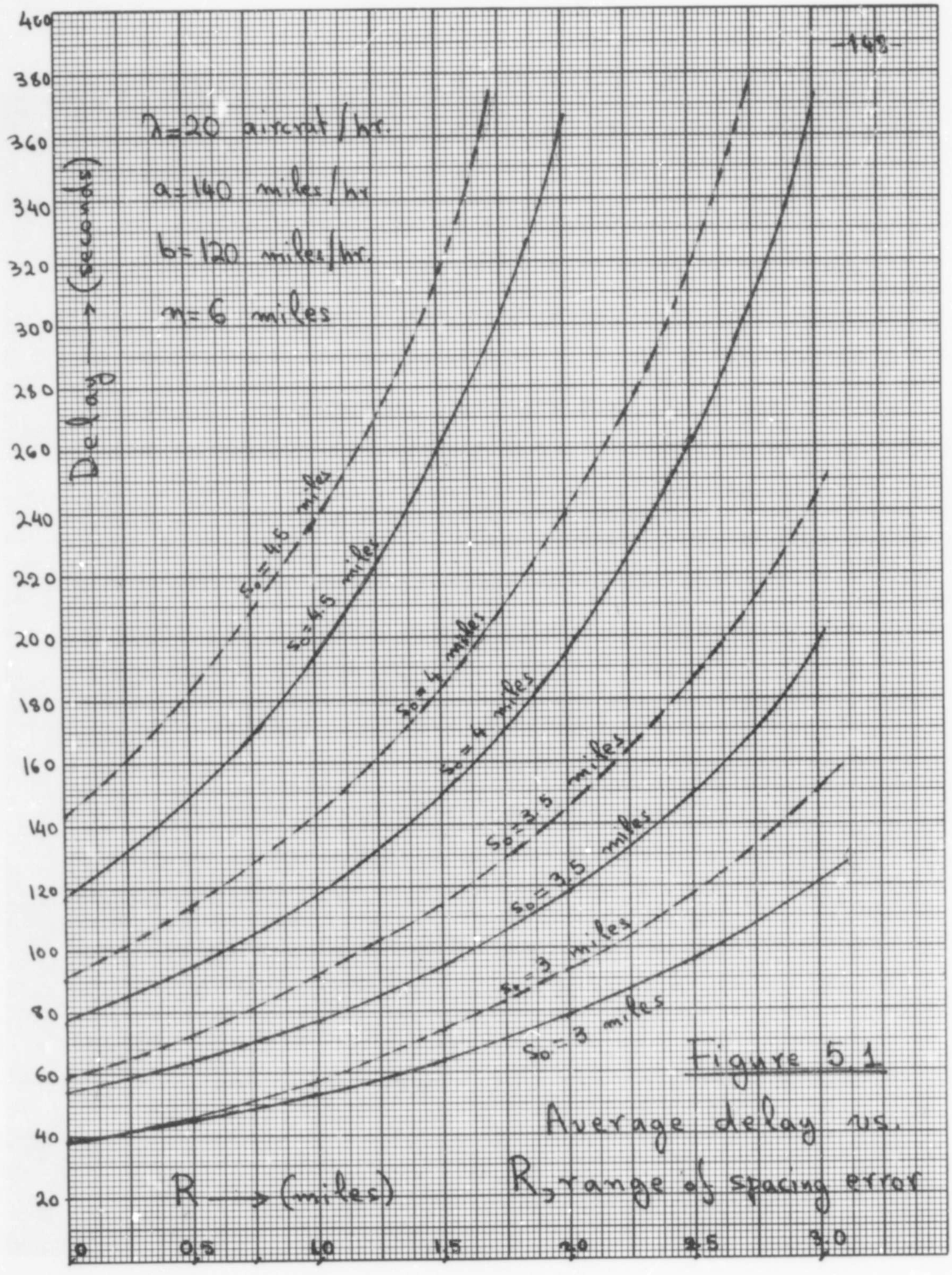


Figure 5.1

Average delay vs. R, range of spacing error

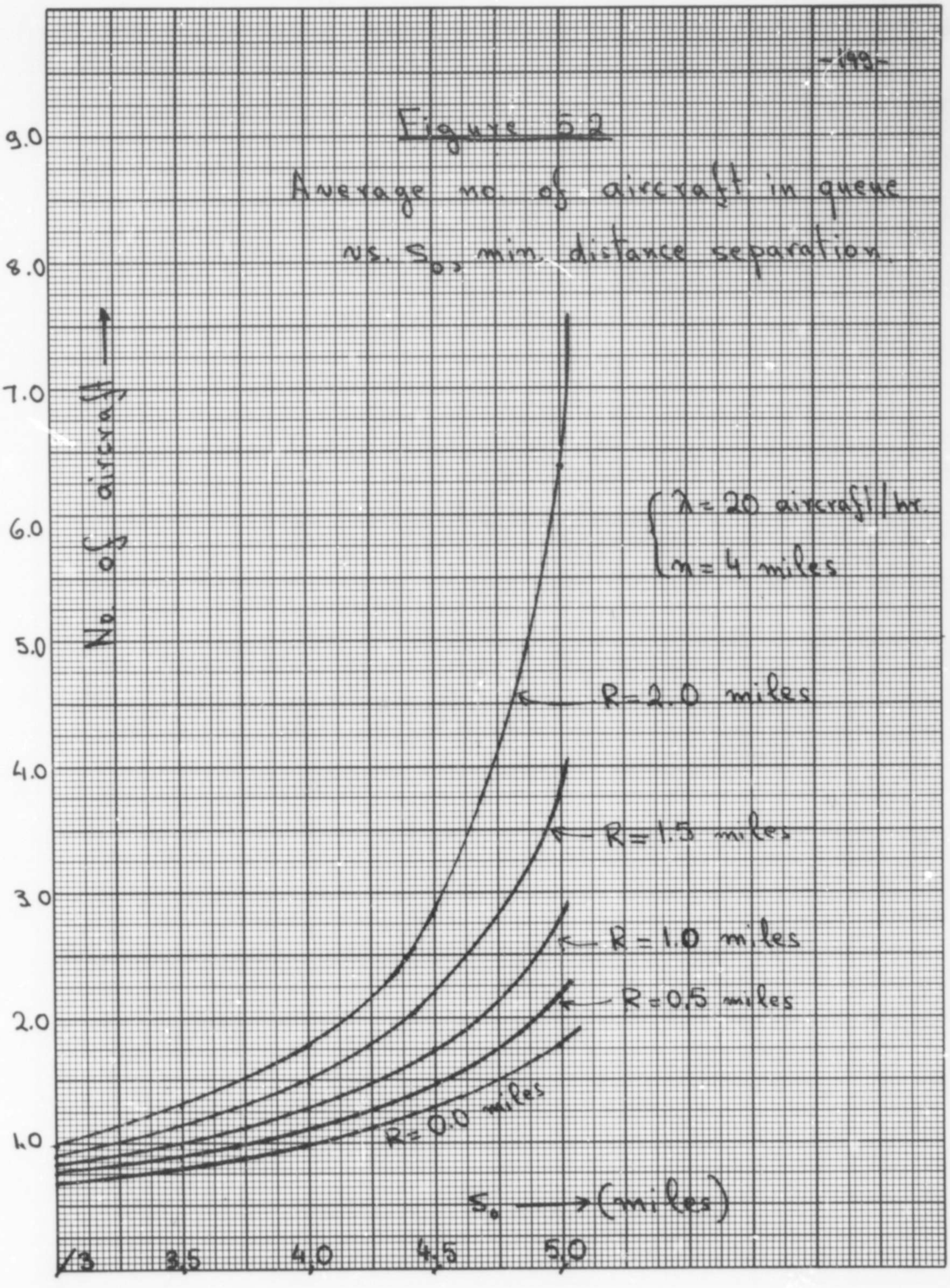


Figure 52
Average no. of aircraft in queue
vs. S₀, min. distance separation.
λ = 20 aircraft/hr.
n = 4 miles
R = 2.0 miles
R = 1.5 miles
R = 1.0 miles
R = 0.5 miles
R = 0.0 miles
No. of aircraft
S₀ (miles)
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
3
3.5
4.0
4.5
5.0

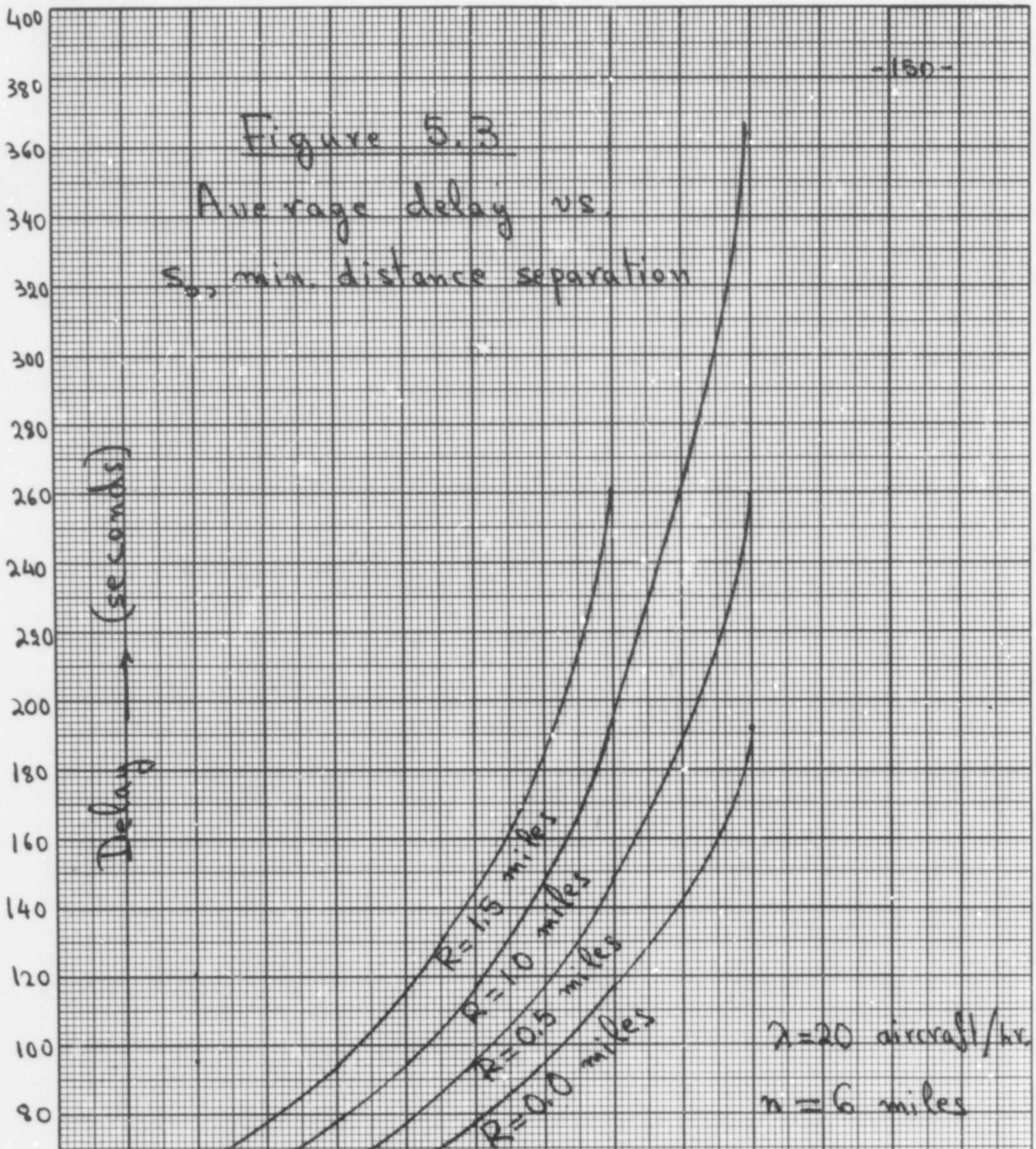


Figure 5.3
Average delay vs.
S_0, min. distance separation
Delay → (seconds)
S_0 → (miles)
R=1.5 miles
R=1.0 miles
R=0.5 miles
R=0.0 miles
λ=20 aircraft/hr.
n=6 miles
400
380
360
340
320
300
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0
3.0
3.5
4.0
4.5
5.0

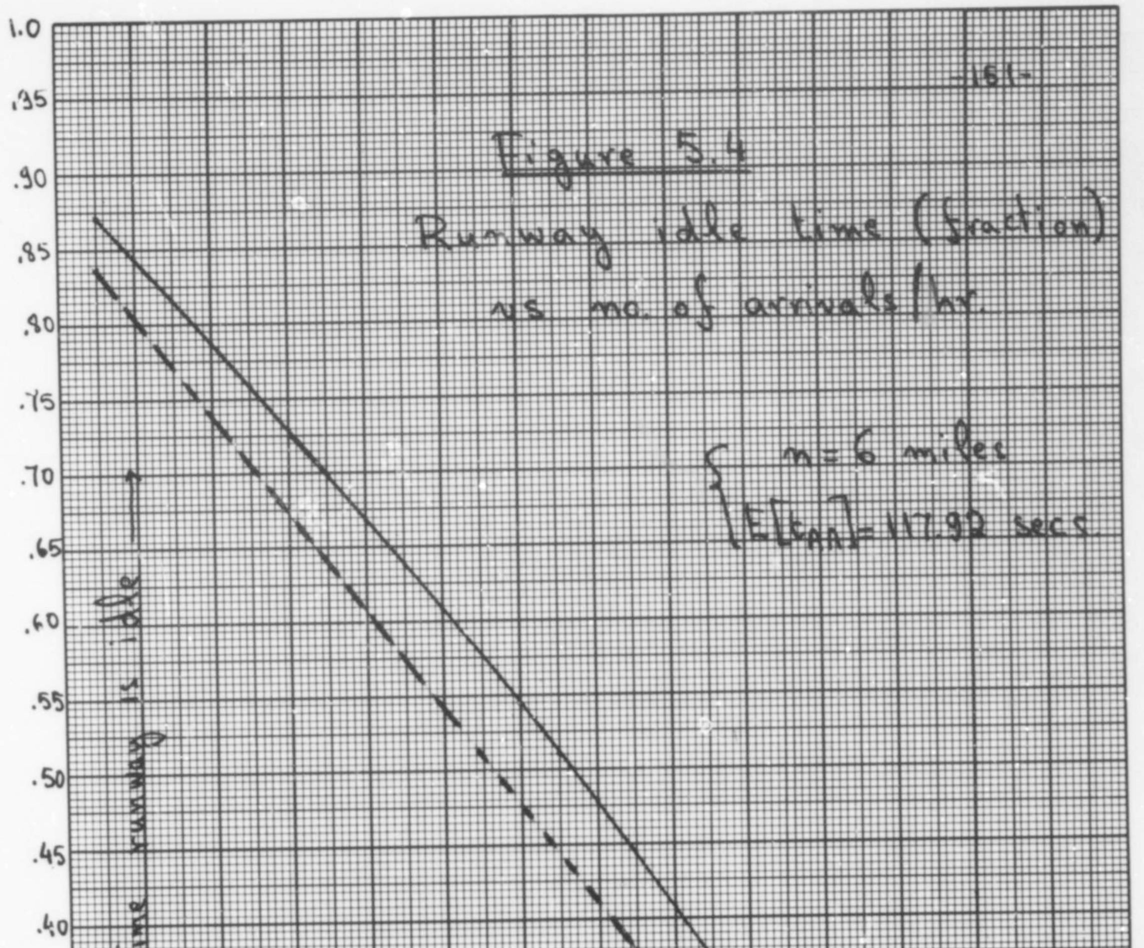


Figure 5.4
Runway idle time (fraction) vs. no. of arrivals/hr.
n = 6 miles
E[t_AR] = 117.92 secs.
Fraction of time runway is idle
No. of arrivals/hr.
1.0
.95
.90
.85
.80
.75
.70
.65
.60
.55
.50
.45
.40
.35
.30
.25
.20
.15
.10
.05
0
5
6
8
10
12
14
16
18
20
22
24
26
28
30
32

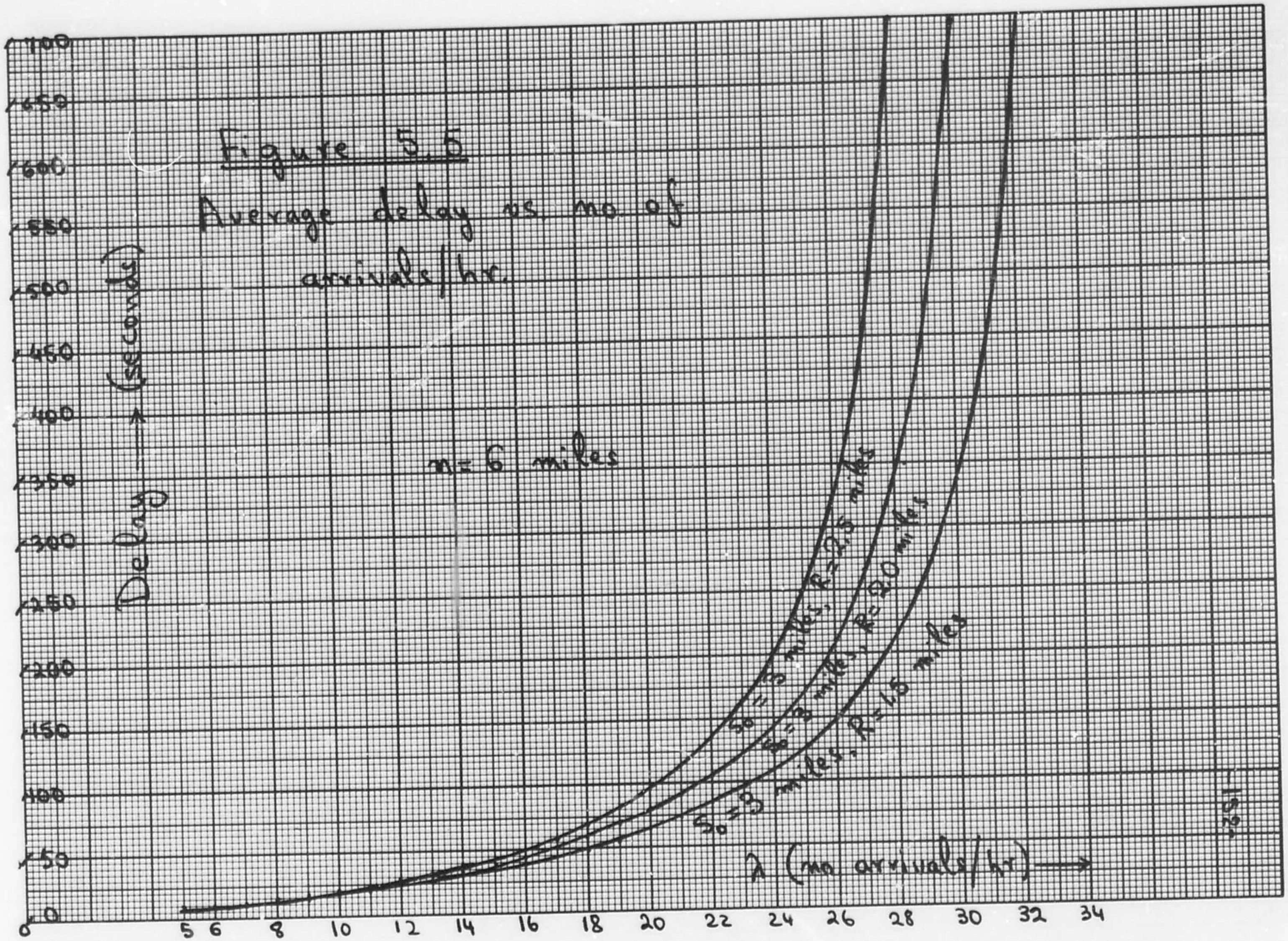


Figure 5.5

Average delay vs. no of arrivals/hr.

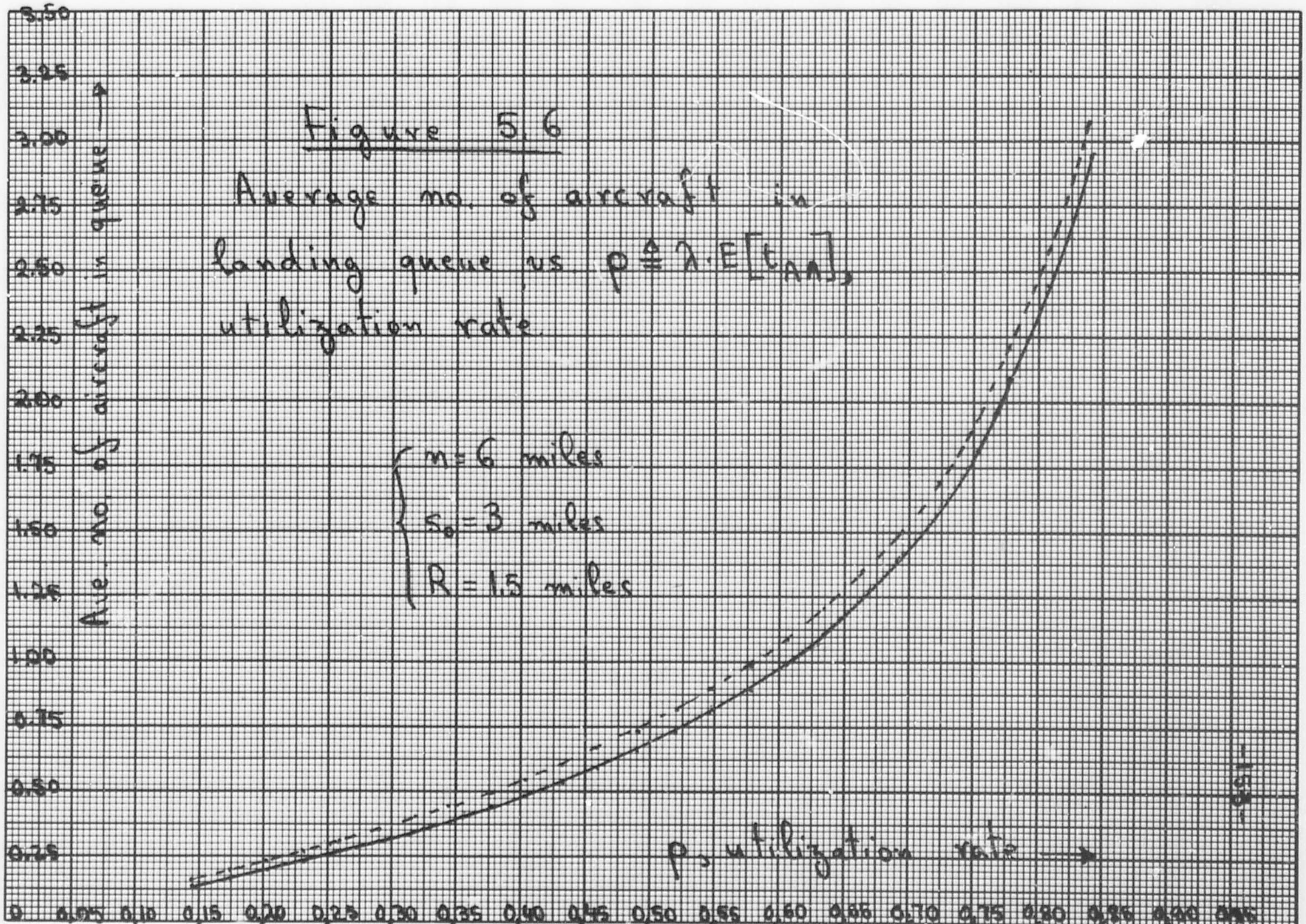


Figure 5.6

Average no. of aircraft in landing queue vs $\rho \triangleq \lambda \cdot E[t_{AA}]$, utilization rate

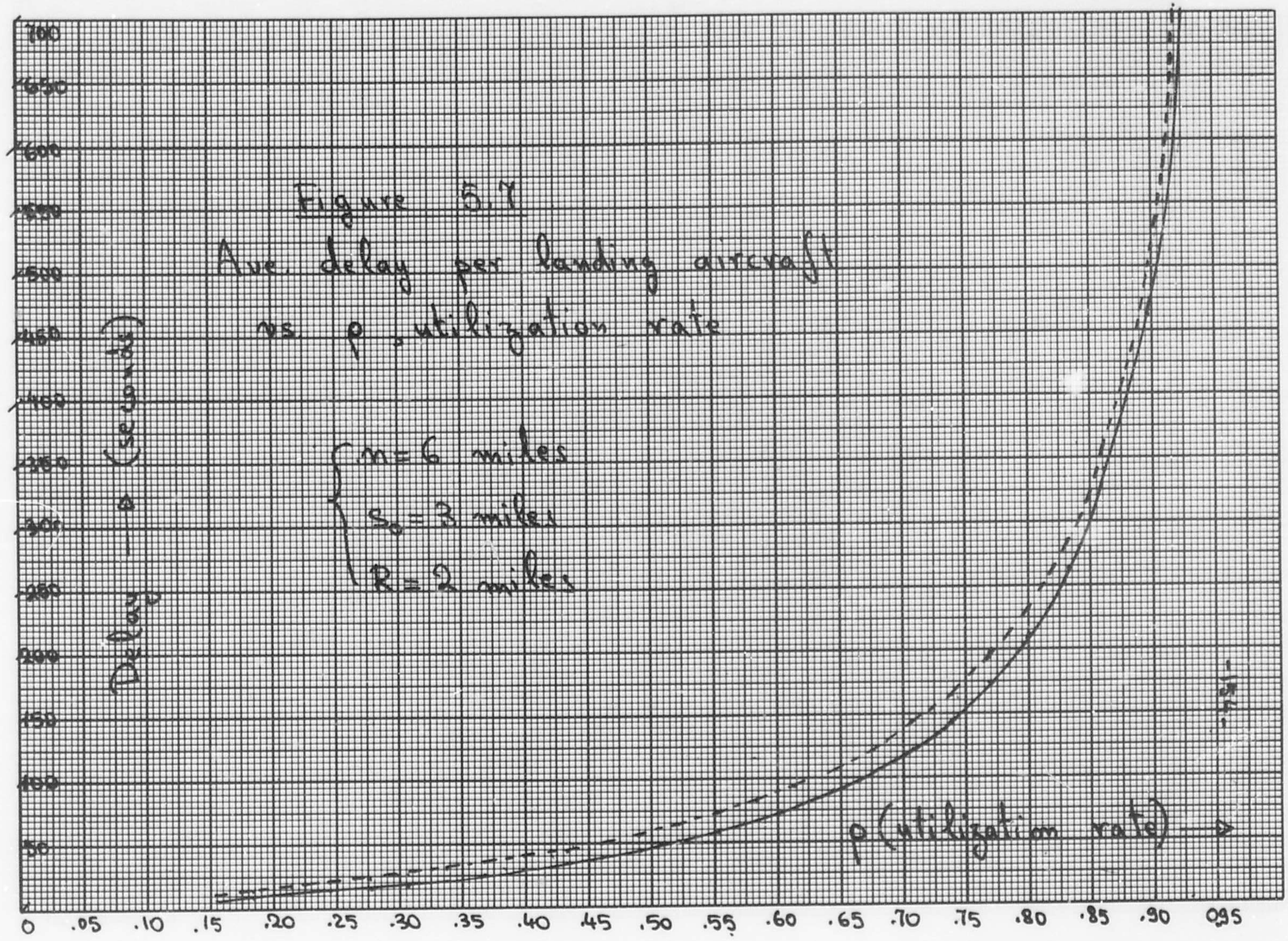


Figure 5.7
Ave. delay per landing aircraft
vs. ρ, utilization rate
n = 6 miles
S₀ = 3 miles
R = 2 miles
Delay → (seconds)
ρ (utilization rate) →
700
650
600
550
500
450
400
350
300
250
200
150
100
50
0
.05
.10
.15
.20
.25
.30
.35
.40
.45
.50
.55
.60
.65
.70
.75
.80
.85
.90
0.95
-154-

Summary and conclusions

In this report, we have attempted to present a meaningful discussion of some issues connected with air traffic control in the vicinity of terminal areas; to develop some analytical models of the operations that take place; to describe and examine the important parameters in each model; and, finally, to indicate analytical methods for attacking some of the problems of interest.

In the following paragraphs we will try to summarize the highlights of this thesis and to indicate possible areas for future research:

1. A model for a single runway which is used only for landings has been described. We have shown how one can find the probability distribution for the time gap between successive landings given the probability distributions for: (i) the approach speeds of aircraft that use the runway; (ii) the errors in spacing aircraft at the beginning of the glide path. Other inputs are the length of the common glide path and the minimum separation requirements set by the FAA.

   In particular, the consideration of spacing errors in the model seems to be original.

2. The complexity of the expression for the probability distribution of the interarrival gap depends heavily on the original assumptions. Unless very simple types of probability density functions are used as inputs, the analysis becomes extremely cumbersome.

3. A model for a single runway which is used only for departures was also described. In this case the analysis is much simpler, and the data that must be collected are much easier to obtain.

4. The case of a runway on which departures are alternated with arrivals was considered. The model provides an opportunity to examine closely the "switchover" procedures involved. It was also shown how it can be used as a basis for a rational search for the best strategy for arranging the sequence of arrivals and departures.

5. The problem of delays to arriving aircraft was investigated by using queuing theory methods. It was observed that because service times for landing aircraft are often functions of the approach speeds of pairs of successive planes, the commonly used queuing formulae may not be quite applicable since they are based on the assumption of independent service times.

A queuing analysis was then performed by distinguishing two classes of aircraft: (i) Those arriving unimpeded by other aircraft and (ii) those arriving in platoons. Different service time distributions were assumed for each of the two classes.

Simple explicit expressions were obtained for quantities such as runway utilization, the average number of planes awaiting and the average delay per aircraft.

6. The possibility of improving runway utilization by inserting departures between successive landings (whenever a gap is sufficiently long) was considered. On the one hand, we are interested in the average waiting time for a departing aircraft until a suitable gap is found. At the same time, the expected number of departures that can be interposed is important.

The probability of interposing a departure was found to be a function of the probability density function for the length of interarrival gaps and of the probability distribution for the duration of a take-off operation.

This points up the possibility of increasing the interposition rate at airports with parallel runways. If arrivals are allocated at one runway and departures at another; and if an effort is simultaneously made to interpose some departures at the runway which is used for landings, those aircraft for which the probability of interposition is maximized should be selected.

7. Numerical computations indicate the strong dependence of such items as average service time and average delay per landing aircraft on the parameters of the system, namely the length of the glide path, the separation requirements and the spacing errors. In view of this fact, a series of measurements was proposed in order to determine the extent of spacing errors in actual operations.

8. Comparison of the commonly used queuing expressions for the delays to landing aircraft with the new expressions developed here shows that

the former tend to overestimate runway utilization, the expected number of aircraft in the waiting line, and the average waiting time. The discrepancies are larger on a percentage basis for medium utilization rates when there is still a significant number of aircraft which proceed unimpeded to a landing. In general, the queuing model developed here has much intuitive appeal and provides most reasonable numerical results. It is worth examining further its validity by actual measurements on the field, preferably at hours when the traffic intensity is not high.

9. The numerical results point out all too clearly, the way in which delays increase drastically when a certain arrival rate range is exceeded. This is even truer for delays as estimated by the model developed here. It appears that for a typical major airport the delays become critical when the arrival rate per runway approaches the range of 28-32 aircraft per hour.

10. Another phenomenon which is painfully clear is the effect of any increase in spacing inaccuracies and errors on the average delay. As these errors tend to occur primarily under poor weather conditions (because of the extra cautiousness of both controller and pilot) this seems to offer an explanation for the acute problems that occur under such conditions, despite the availability of all-weather equipment.

11. The effects of an airport's layout (i.e., of the position of runways relative to each other) on the operations capacity were discussed briefly. A more detailed examination should focus on a specific airport and examine the special conditions existing there.

12. Considerable time and effort was spent in an attempt to find exact analytical solutions to more complicated queuing problems motivated by air traffic control situations. The mathematical difficulties, however, appear to be formidable. For instance, a consistently vexing problem for the analyst is that departing aircraft can hardly be assumed to arrive at the queue of take-offs in a Poisson manner. In a way, it rather seems that we have bulk arrivals in this case, due to the well-known tendency of airlines to bunch their flights on the hour and on the half-hour.

Investigations by simulation then, seem to be indicated for such complex problems. Due to the tremendous number of possibilities for scheduling operations, it is recommended that, if such research efforts are ever undertaken, the investigator concentrate on precisely described situations possibly focusing on traffic patterns at a specific airport.

This, of course, brings up the subject of future research possibilities: The first and most obvious course is to attempt an extensive examination of the models developed here by obtaining the needed data from the literature and from actual measurements on the field. The author hopes to make an effort in this direction in the near future.

For the operations researcher, air traffic control seems to provide an unending source of interesting problems in queuing theory. Indeed, as one acquires a better insight in the area, he discovers more and more intriguing setups that can motivate exciting and pertinent work. A couple of problems of this type have already been described in this report.

Finally, one can proceed in more radical directions by considering situations and procedures which are not permitted in the present state of affairs. For example one may wish to consider changes in the assignment of priorities, altering traffic separation rules, segregating traffic, etc. Some reports on these subjects have already appeared. The field seems wide open in this respect, especially if one wishes to assign costs to particular types of delays and so on.

Bibliography

1] Air Transport Association of America, Facts and Figures, 1967.

2] Airborne Instruments Laboratory, Inc., Airport Capacity - A Handbook for Analyzing Airport Designs and to Determine Practical Movement Rates and Aircraft Operating Costs, Deer Park, N.Y. (1963).

3] Bell, G.E., Operational Research into Air Traffic Control, J. Royal Aeronautical Society, 52, No. 12, pp. 799-812 (1948).

4] Blumstein, A., An Analytical Investigation of Airport Capcity, Ph.D. thesis, Cornell University (1960).

5] Bowen, E.G., and T. Pearcey, Delays in the Flow of Air Traffic, J. Royal Aeronautical Society, 52, No. 4, pp. 251-258 (1948).

6] Cobham, A., Priority Assignment in Waiting Line Problems, J. Operations Research Soc.,2, p. 70-76 (1954).

7] Cox, D.R., and W. L. Smith, Queues, London: Methuen and Co. Ltd. (1961).

8] Drake, Alvin W., Fundamentals of Applied Probability Theory, New York: McGraw-Hill (1967).

9] Eisenberg, Martin, Multiqueues with Changeover Times, Technical Report No. 35, Operations Research Center, MIT, April 1968.

10] Evans, D. H., R. Herman and G. H. Weiss, The Highway Merging and Queuing Problem, Operations Research, 12, pp. 832-857 (1964).

11] Federal Aviation Administration, FAA Air Traffic Activity: Calendar Year 1967, Dept. of Transportation (1968).

12] Federal Aviation Administration, Enroute IFR Air Traffic Survey: 1967, Dept. of Transportation (1968).

13] Federal Aviation Administration, FAA Statistical Handbook of Aviation 1967, Department of Transportation (1968).

14] Federal Aviation Administration, National Airspace Utilization System, June 1962.

15] Fromm, Gary, Civil Aviation Expenditures, from: Measuring Benefits of Government Investment, Robert Dorfman, editor, The Brookings Institution, Washington, D. C. (1965).

16] Galliher, H.P. and R. C. Wheeler, Nonstationary Queuing Probabilities for Landing Congestion of Aircraft, Operations Research, 6, pp. 264-275 (1958).

17] Horonjeff, Robert, R. C. Grassi, R. R. Read, and G. Ahlborn, A Mathematical Model for Locating Exit Taxiways, Univ. of California, Inst. of Transportation (1959).

18] Horonjeff, Robert, Planning and Design of Airports, New York: McGraw-Hill (1962).

19] Proceedings of the 17th IATA Technical Conference on Major Airport and Terminal Area Problems, Vols. I and II, Lucern, Oct. 9-14, 1967. In particular note reports no. 3, 18, 22, 28, 33, 38, 42, 45, 48, 50, 52, 60, 61, 73, 74, 75, 76, 82.

20] ___________, The Aviation Operational Research Branch, Board of Trade, United Kingdom, An Investigation of Factors Affecting the Capacity of Runways, Vol. II, pp. 277-307.

21] Kendall, D.G., Some Problems in the Theory of Queues, J. Royal Statistical Soc., 13, pp. 151-185 (1951).

22] Koch, William H., Computer Aided Approach Spacing for Terminal Air Traffic Control, Dept. of Transportation, FAA Report, Atlantic City, New Jersey (1968).

23] Morse, P.M., Queues, Inventories, and Maintenance, New York: Wiley (1958).

24] The Port of New York Authority, A Report on Airport Requirements and Sites in the Metropolitan New Jersey-New York Area, (1961)

25] Newell, G. F., Queues for a Fixed-Cycle Traffic Light, Ann. Math. Statistics, 31, pp. 589-597 (1960).

26] Oliver, R. M., Delays in Terminal Air Traffic Control, Journal of Aircraft, 1, No. 3,(May-June 1964).

27] Oliver, R. M., Distribution of Gaps and Blocks in a Traffic Stream, Operations Research, 10, pp. 197-217 (1962).

28] Oliver, R.M., and E. F. Bisbee, Queuing for Gaps in High-flow Traffic Streams, Operations Research, 10, pp. 105-114 (1962).

29] Oliver, R.M., Delays to Aircraft Serviced by the Glide Path, Operational Research Quarterly, 13, No. 2, pp. 201-209 (1962).

30] Pearcey, T., Delays in the Landing of Air Traffic, J. Royal Aeronautical Soc., 52, No. 12, pp. 799-812 (1948).

31] Pestalozzi, Gerold, Delays to Air Traffic in a Terminal Area, Ph.D. Thesis, University of California, Berkeley (1966).

32] Pestalozzi, Gerold, Priority Rules for Runway Use, Operations Research, 12, No. 6, pp. 941-950 (1964).

33] Rosenshine, Mathew, Queues with State-Dependent Service Times, Transportation Research, 1, pp. 97-104 (1967).

34] Schriever, B.A., and W. W. Seifert, Air Transportation 1975 and Beyond: A Systems Approach, Cambridge, Mass.: The MIT Press (1968)

35] Simpson, Robert W., An Analytical Investigation of Air Traffic Operations in the Terminal Area, Ph.D. Thesis, MIT, Cambridge, Mass. (1964).

36] Tanner, J.C., A Theoretical Analysis of Delays at an Uncontrolled Intersection, Biometrika, 49, pp. 163-170 (1962).

37] Tanner, J.C., A Problem of Interference Between Two Queues, Biometrika, 40, pp. 58-69 (1953).

38] Webster, F. V., Traffic Signal Settings, Dept. of Scientific and Industrial Research, Road Research Technical Paper no. 39, London H.M.S.O. (1958)

39] Weiss, G. H. and A. A. Maradudin, Some Problems in Traffic Delay, Operations Research, 10, pp. 74-104 (1962).

40] White, H. and L. S. Christie, Queuing with Preemptive Priorities or with Breakdown, Operations Research, 6, pp. 79-95 (1958).

Appendix A

Glossary

1. AIRPORT TRAFFIC CONTROL TOWER - A central operations facility in the terminal air traffic control system, consisting of a tower cab structure, including an associated IFR room if radar equipped, using air/ground communications and/or radar, visual signaling and other devices, to provide safe and expeditious movement of terminal air traffic.

2. AIR ROUTE TRAFFIC CONTROL CENTER - A central operations facility in the air route traffic control systems using air/ground communications and/or radar, primarily providing enroute separation and safe, expeditious movement of aircraft operating under instrument flight rules within the controlled airspace of that center.

3. AIR TRAFFIC - Aircraft operating in the air or on an airport surface, exclusive of loading ramps and parking areas.

4. AIRWAY - A portion of the navigable airspace of the United States designated by the FAA Administrator as a Federal airway.

5. APPROACH CONTROL FACILITY - A terminal area traffic control facility providing service to arriving and/or departing IFR flights and, on occasion, VFR flights.

6. ARTCC - Air Route Traffic Control Center.

7. ATC - Air Traffic Control.

8. CENTER AREA - The specified airspace within which an air route traffic control center provides air traffic control and flight advisory service.

9. ENROUTE AIR TRAFFIC CONTROL SERVICE - Air traffic control service provided to IFR flights, generally by centers, when such flights are operating between departure and destination terminal areas.

10. FAA - Federal Aviation Administration.

11. GENERAL AVIATION AIRCRAFT - All civil aircraft except those classified as air carriers.

12. IFR - Instrument Flight Rules.

13. IFR CONDITIONS - Weather conditions below the minimum prescribed for flight under VFR.

14. INSTRUMENT APPROACH - An approach to an airport with intent to land by an aircraft flying in accordance with an IFR flight plan, when the visibility is less than 3 miles and/or when the ceiling is at or below the minimum initial approach altitude.

15. INSTRUMENT FLIGHT RULES - FAA rules that govern the procedures for conducting instrument flight.

16. INSTRUMENT OPERATION - The arrival at or departure from an airport of an aircraft operating in accordance with an IFR flight plan or the provision of IFR separation from other aircraft by a terminal traffic control facility.

17. ILS - Instrument Landing System.

18. TOWER - See Airport Traffic Control Tower

19. VFR - Visual Flight Rules

20. VFR CONDITIONS - Basic weather conditions prescribed for flight under VFR.

Appendix B

Derivation of $f_q(q_0)$

We define $q = \frac{1}{v}$ and we know that:

$$f_v(v_0) = \begin{cases} \frac{1}{a-b}, & \text{for } b \leqslant v_0 \leqslant a \\ 0, & \text{otherwise} \end{cases}$$

Then, defining $F_{q\leqslant}(q_0) \triangleq Pr[q \leqslant q_0]$

(i) For $q_0 \leqslant \frac{1}{a}$: $F_{q\leqslant}(q_0) = 0$

(ii) For $\frac{1}{a} \leqslant q_0 \leqslant \frac{1}{b}$:

$$F_{q\leqslant}(q_0) = Pr\left[\frac{1}{v} \leqslant q_0\right] = Pr\left[\frac{1}{q_0} \leqslant v\right] = 1 - Pr\left[v < \frac{1}{q_0}\right]$$

$$= \int_b^{\frac{1}{q_0}} f_v(v_0)\, dv_0 = \int_b^{\frac{1}{q_0}} \frac{dv_0}{a-b} = \frac{\frac{1}{q_0} - b}{a-b}$$

(iii) For $\frac{1}{b} \leqslant q_0$: $F_{q\leqslant}(q_0) = 1$

and differntiating, we obtain:

$$f_q(q_0) = \frac{d\, F_{q\leqslant}(q_0)}{d\, q_0} = \begin{cases} \frac{1}{q_0^2} \frac{1}{(a-b)}, & \frac{1}{a} \leqslant q_0 \leqslant \frac{1}{b} \\ 0, & \text{otherwise.} \end{cases}$$

Appendix C

Derivation of $f_w(w_0)$

Referring to figure C.1, and defining $F_{w\leq}(w_0) \triangleq \Pr[w \leq w_0]$,

we obtain:

(i) For $w_0 \leq \frac{L}{a}$: $F_{w\leq}(w_0) = 0$.

(ii) For $\frac{L}{a} \leq w_0 < \frac{M}{a}$:

$$F_{w\leq}(w_0) = \int_{\frac{1}{a}}^{\frac{w_0}{L}} \int_{L}^{\frac{w_0}{q_0}} \frac{1}{Kq_0^2}\, dx_0\, dq_0 = \frac{L^2}{2Kw_0} + \frac{w_0 a^2}{2K} - \frac{La}{K}$$

(iii) For $\frac{M}{a} \leq w_0 \leq \frac{L}{b}$:

$$F_{w\leq}(w_0) = \int_{\frac{1}{a}}^{\frac{w_0}{M}} \int_{L}^{M} \frac{1}{Kq_0^2}\, dx_0\, dq_0 + \int_{\frac{w_0}{M}}^{\frac{w_0}{L}} \int_{L}^{\frac{w_0}{q_0}} \frac{1}{Kq_0^2}\, dx_0\, dq_0$$

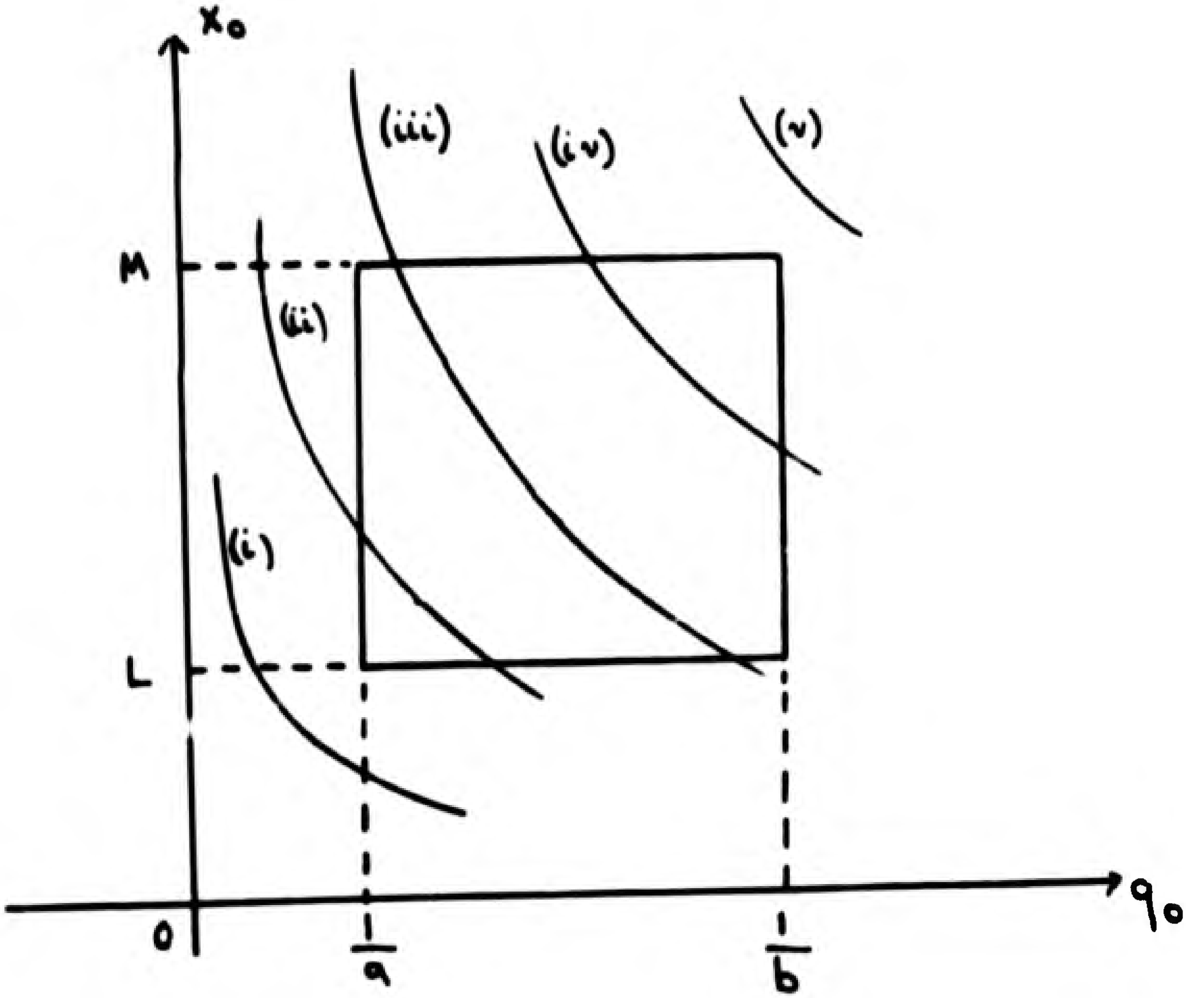
$$= \frac{(M-L)a}{K} - \frac{M^2-L^2}{2Kw_0}$$

(iv) For $\frac{L}{b} \leq w_0 < \frac{M}{b}$:

$$F_{w\leq}(w_0) = 1 - \int_{\frac{w_0}{M}}^{\frac{1}{b}} \int_{\frac{w_0}{q_0}}^{M} \frac{1}{Kq_0^2}\, dx_0\, dq_0 = 1 + \frac{Mb}{K} - \frac{M^2}{2Kw_0} - \frac{w_0 b^2}{2K}$$

(v) For $\frac{M}{b} \leq w_0$: $F_{w\leq}(w_0) = 1$

Differentiating, we obtain:



$$f_{x,q}(x_0,q_0) = \begin{cases} \dfrac{1}{Kq_0^2}, & \text{for } M \geqslant x_0 \geqslant L, \quad \frac{1}{b} \geqslant q_0 \geqslant \frac{1}{a} \\ 0, & \text{otherwise} \end{cases}$$

<u>Figure C.1</u>

$$f_w(w_0) = \frac{dF_{w_f}(w_0)}{dw_0} = \begin{cases} \frac{a^2}{2K} - \frac{L^2}{2Kw_0^2}, \text{ for } \frac{L}{a} \leq w_0 < \frac{M}{a} \\ \frac{M^2 - L^2}{2Kw_0^2}, \text{ for } \frac{M}{a} \leq w_0 < \frac{L}{b} \\ \frac{M^2}{2Kw_0^2} - \frac{b^2}{2K}, \text{ for } \frac{L}{b} \leq w_0 < \frac{M}{b} \\ 0, \text{ otherwise} \end{cases}$$

Appendix D

Computation of $F_{r_{\leq}}(r_0)$ and $f_r(r_0)$

Referring to figure 3.4 , we have first:

$$f_{w,y}(w_0,y_0) = \begin{cases} \dfrac{1}{(a-b)} \dfrac{n}{y_0^2} \left(\dfrac{a^2}{2K} - \dfrac{L^2}{2Kw_0^2} \right), & \text{in } A \\[2ex] \dfrac{1}{(a-b)} \dfrac{n}{y_0^2} \left(\dfrac{M^2-L^2}{2Kw_0^2} \right), & \text{in } B \\[2ex] \dfrac{1}{(a-b)} \dfrac{n}{y_0^2} \left(\dfrac{M^2}{2Kw_0^2} - \dfrac{b^2}{2K} \right), & \text{in } C \\[2ex] 0, & \text{otherwise} \end{cases}$$

After defining $r \triangleq w-y$, we obtain for the cumulative probability distribution $F_{r_{\leq}}(r_0)$:

<u>Case 1</u>: For $r_0 \leq \frac{L}{a} - \frac{n}{b}$, $F_r(r_0) = 0$,

<u>Case 2</u>: For $\frac{L}{a} - \frac{n}{b} \leq r_0 < \frac{L}{a} - \frac{n}{a}$:

$$F_{r_{\leq}}(r_0) = \int_{\frac{L}{a} - r_0}^{\frac{n}{b}} \int_{\frac{L}{a}}^{r_0+y_0} \frac{1}{(a-b)} \frac{n}{y_0^2} \left(\frac{a^2}{2K} - \frac{L^2}{2Kw_0^2} \right) dw_0 \, dy_0$$

<u>Case 3</u>: For $\frac{L}{a} - \frac{n}{a} \leq r_0 < \frac{M}{a} - \frac{n}{b}$:

$$F_{r\leq}(r_0) = \int_{\frac{L}{a}}^{r_0+\frac{n}{a}} \int_{\frac{n}{a}}^{\frac{n}{b}} \frac{1}{(a-b)}\left(\frac{a^2}{2K} - \frac{L^2}{2Kw_0^2}\right)\frac{n}{y_0^2}\, dy_0\, dw_0$$

$$+ \int_{r_0+\frac{n}{a}}^{r_0+\frac{n}{b}} \int_{w_0-r_0}^{\frac{n}{b}} \frac{1}{(a-b)}\, \frac{n}{y_0^2}\left(\frac{a^2}{2K} - \frac{L^2}{2Kw_0^2}\right) dy_0\, dw_0$$

Case 4: For $\frac{M}{a} - \frac{n}{b} \leq r_0 < \frac{L}{b} - \frac{n}{b}$:

$$F_{r\leq}(r_0) = P[A] - \int_{r_0+\frac{n}{a}}^{\frac{M}{a}} \int_{\frac{n}{a}}^{w_0-r_0} \frac{1}{(a-b)}\, \frac{n}{y_0^2}\left(\frac{a^2}{2K} - \frac{L^2}{2Kw_0^2}\right) dy_0\, dw_0$$

$$+ \int_{\frac{M}{a}}^{r_0+\frac{n}{b}} \int_{w_0-r_0}^{\frac{n}{b}} \frac{1}{(a-b)}\, \frac{n}{y_0^2}\, \frac{(M^2-L^2)}{2Kw_0^2}\, dy_0\, dw_0$$

Case 5: For $\frac{L}{b} - \frac{n}{b} \leq r_0 < \frac{M}{a} - \frac{n}{a}$:

$$F_{r\leq}(r_0) = P[A] + P[B] + \int_{\frac{L}{b}}^{r_0+\frac{n}{a}} \int_{\frac{n}{a}}^{\frac{n}{b}} \frac{1}{(a-b)}\, \frac{n}{y_0^2}\left(\frac{M^2}{2Kw_0^2} - \frac{b^2}{2K}\right) dy_0\, dw_0$$

$$+ \int_{r_0+\frac{n}{a}}^{r_0+\frac{n}{b}} \int_{w_0-r_0}^{\frac{n}{b}} \frac{1}{(a-b)}\, \frac{n}{y_0^2}\left(\frac{M^2}{2Kw_0^2} - \frac{b^2}{2k}\right) dy_0\, dw_0$$

$$+\int_{\frac{M}{a}-r_0}^{\frac{L}{b}-r_0}\int_{\frac{M}{a}}^{r_0+y_0}\frac{n(M^2-L^2)}{2k(a-b)}\,\frac{1}{y_0^2 w_0^2}\,dw_0\,dy_0$$

$$+\int_{\frac{L}{b}}^{r_0+\frac{n}{b}}\int_{w_0-r_0}^{\frac{n}{b}}\frac{1}{(a-b)}\,\frac{n}{y_0^2}\left(\frac{M^2}{2Kw_0^2}-\frac{b^2}{2K}\right)dy_0\,dw_0$$

<u>Case 6</u>: For $\frac{M}{a}-\frac{n}{a}\leqslant r_0<\frac{L}{b}-\frac{n}{a}$:

$$F_{r\leq}(r_0)=P[A]+P[B]-\int_{\frac{n}{a}}^{\frac{L}{b}-r_0}\int_{r_0+y_0}^{\frac{L}{b}}\frac{n(M^2-L^2)}{2K(a-b)}\,\frac{dw_0\,dy_0}{w_0^2\,y_0^2}$$

$$+\int_{\frac{L}{b}}^{r_0+\frac{n}{b}}\int_{w_0-r_0}^{\frac{n}{b}}\frac{1}{(a-b)}\,\frac{n}{y_0^2}\left(\frac{M^2}{2Kw_0^2}-\frac{b^2}{2K}\right)dy_0\,dw_0$$

<u>Case 7</u>: For $\frac{L}{b}-\frac{n}{a}\leqslant r_0<\frac{M}{b}-\frac{n}{b}$:

$$F_{r\leq}(r_0)=P[A]+P[B]+\int_{\frac{L}{b}}^{r_0+\frac{n}{a}}\int_{\frac{n}{a}}^{\frac{n}{b}}\frac{n}{(a-b)\,y_0^2}\left(\frac{M^2}{2Kw_0^2}-\frac{b^2}{2K}\right)dy_0\,dw_0$$

$$+\int_{r_0+\frac{n}{a}}^{r_0+\frac{n}{b}}\int_{w_0-r_0}^{\frac{n}{b}}\frac{1}{(a-b)}\,\frac{n}{y_0^2}\left(\frac{M^2}{2Kw_0^2}-\frac{b^2}{2K}\right)dy_0\,dw_0$$

<u>Case 8</u>: For $\frac{M}{b}-\frac{n}{b}\leqslant r_0<\frac{M}{b}-\frac{n}{a}$:

$$F_{r\leq}(r_0)=1-\int_{r_0+\frac{n}{a}}^{\frac{M}{b}}\int_{\frac{n}{a}}^{w_0-r_0}\left(\frac{M^2}{2kw_0^2}-\frac{b^2}{2k}\right)\frac{n}{(a-b)\,y_0^2}\,dy_0\,dw_0$$

Case 9: For $\frac{M}{b} - \frac{n}{a} \leqslant r_0$: $F_{r \leqslant}(r_0) = 1$.

The quantities P[A], P[B], P[C] in the expressions above, are given by (3.16), (3.17), and (3.18) respectively.

Then, differentiating the expression for $F_{r \leqslant}(r_0)$, we obtain:

(i) For $\frac{L}{a} - \frac{n}{b} \leqslant r_0 < \frac{L}{a} - \frac{n}{a}$:

$$f_r(r_0) = \frac{n}{(a-b)2K}\left\{-\frac{ba^2}{n} + \frac{a^3}{L-ar_0} + \frac{L^3}{r_0^2}\left[\frac{(2n+br_0)b}{(r_0b+n)n} - \frac{(2L-ar_0)a}{L(L-ar_0)}\right] + \frac{2L^2}{r_0^3}\ln\left[\frac{(r_0b+n)(L-ar_0)}{nL}\right]\right\}$$

(ii) For $\frac{L}{a} - \frac{n}{a} \leqslant r_0 < \frac{M}{a} - \frac{n}{b}$:

$$f_r(r_0) = \frac{a^2}{2K} + \frac{n}{(a-b)2K}\left\{\frac{-L^2}{r_0^2}\left[\frac{(2n+ar_0)a}{(r_0a+n)n} - \frac{(2n+br_0)b}{(br_0+n)n}\right] - \frac{2L^2}{r_0^3}\ln\left[\frac{(r_0b+n)(r_0a+n)}{n^2}\right]\right\}$$

(iii) For $\frac{M}{a} - \frac{n}{b} \leqslant r_0 < \frac{L}{b} - \frac{n}{b}$:

$$f_r(r_0) = \frac{n}{(a-b)2K}\left\{-\frac{a}{M-ar_0} + \frac{a^3}{n} + \frac{L^2}{r_0^2}\left[\frac{(2M-ar_0)a}{(M-ar_0)M} - \frac{(2n+ar_0)a}{(n+ar_0)n}\right] + \frac{2L^2}{r_0^3}\ln\left[\frac{(M-ar_0)(n+ar_0)}{Mn}\right]\right\}$$

$$+\frac{(M^2-L^2)n}{2K(a-b)}\left\{-\frac{(2n+br_0)b}{r_0^2(r_0b+n)n}-\frac{2}{r_0^3}\ln\left(\frac{n}{r_0b+n}\right)\right.$$

$$\left.+\frac{(2M-ar_0)a}{r_0^2(M-ar_0)M}+\frac{2}{r_0^3}\ln\frac{(M-ar_0)}{M}\right\}$$

(iv) For $\frac{L}{b}-\frac{n}{b}\leqslant r_0<\frac{M}{a}-\frac{n}{a}$:

$$f_r(r_0)=\frac{n}{(a-b)2K}\left\{-\frac{a}{M-ar_0}+\frac{a^3}{n}+\frac{L^2}{r_0^2}\left[\frac{(2M-ar_0)a}{(M-ar_0)M}\right.\right.$$

$$\left.\left.-\frac{(2n+ar_0)a}{(n+ar_0)n}+\frac{2L^2}{r_0^3}\ln\left[\frac{(M-ar_0)(n+ar_0)}{Mn}\right]\right]\right\}$$

$$+\frac{(M^2-L^2)n}{2K(a-b)}\left\{\left[\frac{(br_0-2L)b}{r_0^2L(L-br_0)}+\frac{(2M-ar_0)a}{r_0^2M(M-ar_0)}\right]\right.$$

$$\left.+\frac{2}{t_0^3}\ln\frac{L(M-ar_0)}{(L-ar_0)a}\right\}+\frac{n}{(a-b)2K}\left\{\frac{b^3}{n}-\frac{b}{L-br_0}\right.$$

$$+\frac{M^2}{r_0^2}\left[-\frac{(2n+br_0)b}{n(n+br_0)}+\frac{(2L-br_0)b}{L(L-br_0)}\right]-\frac{2M^2}{r_0^3}$$

$$\left.\ln\left[\frac{nL}{(n+br_0)(L-br_0)}\right]\right\}$$

(v) For $\frac{M}{a} - \frac{n}{a} \leqslant r_0 < \frac{L}{b} - \frac{n}{a}$:

$$f_r(r_0) = \frac{(M^2-L^2)n}{2K(a-b)}\left[-\frac{(2L-br_0)b}{r_0^2(L-br_0)L} + \frac{(2n+ar_0)a}{r_0^2(n+ar_0)n}\right.$$

$$\left. + \frac{2}{r_0^3} \frac{Ln}{(L-br_0)(n-ar_0)}\right] + \frac{n}{(a-b)2K}\left\{\frac{b^3}{n} - \frac{b}{L-br_0}\right.$$

$$+ \frac{M^2}{r_0^2}\left[-\frac{(2n+br_0)b}{n(n+br_0)} + \frac{(2L-br_0)b}{L(L-br_0)}\right] - \frac{2M^2}{r_0^3}$$

$$\left. \ln \frac{nL}{(n+br_0)(L-br_0)}\right\}$$

(vi) For $\frac{L}{b} - \frac{n}{a} \leqslant r_0 < \frac{M}{b} - \frac{n}{b}$:

$$f_r(r_0) = -\frac{b^2}{2K} + \frac{n}{(a-b)2K}\left\{\frac{M^2}{r_0^2}\left[\frac{(2n+ar_0)a}{(ar_0+n)n} - \frac{(2n+br_0)b}{(br_0+n)n}\right]\right.$$

$$\left. - \frac{2M}{r_0^3} \ln \frac{(r_0b+n)(r_0a+n)}{n^2}\right\}$$

(vii) For $\frac{M}{b} - \frac{n}{b} < t_0 < \frac{M}{b} - \frac{n}{a}$:

$$f_r(r_0) = \frac{n}{(a-b)2K}\Bigg\{\frac{b^3}{M-br_0} - \frac{b^2a}{n} + \frac{M^2}{r_0^2}\Bigg[\frac{(2n+ar_0)a}{(ar_0+n)n}$$

$$- \frac{(2M-br_0)b}{(M-br_0)M} - \frac{2M^2}{r_0^3}\ln\left[\frac{(M-br_0)(n+ar_0)}{Mn}\right]\Bigg\}$$

(viii) Otherwise: $f_r(r_0) = 0$.

Appendix E

The landing model with a different $f_x(x_0)$

We discuss the case in which we assume a probability density function $f_x(x_0)$ for the "positioning error" different from the uniform pdf postulated by (3.7). The primary purpose of this discussion is to illustrate how the assumption of a slightly more complicated pdf would make the use of some degree of computer simulation practically mandatory. Theoretically, however the method of analysis is identical to the one we followed above.

Thus we start by assuming:

$$f_v(v_0) = \begin{cases} \frac{1}{a-b}, & \text{for } b \leq v_0 \leq a \\ 0, & \text{otherwise} \end{cases} \tag{3.6}$$

and

$$f_x(x_0) = \begin{cases} \frac{4x_0}{(c-d)^2} - \frac{4d}{(c-d)^2}, & \text{for } d \leq x_0 \leq \frac{c+d}{2} \\ -\frac{4x_0}{(c-d)^2} + \frac{4c}{(c-d)^2}, & \text{for } \frac{c+d}{2} \leq x_0 \leq c \\ 0, & \text{otherwise} \end{cases} \tag{E.1}$$

i.e., $f_x(x_0)$ is a "triangle" pdf centered at $\frac{c+d}{2}$.

We still have:

$$t_{AA} = \begin{cases} \dfrac{n+x}{v_2} - \dfrac{n}{v_1}, & \text{for } \dfrac{n+x}{v_2} - \dfrac{n}{v_1} \geq t_0 \\ t_0, & \text{otherwise} \end{cases} \tag{3.3}$$

As before, we define:

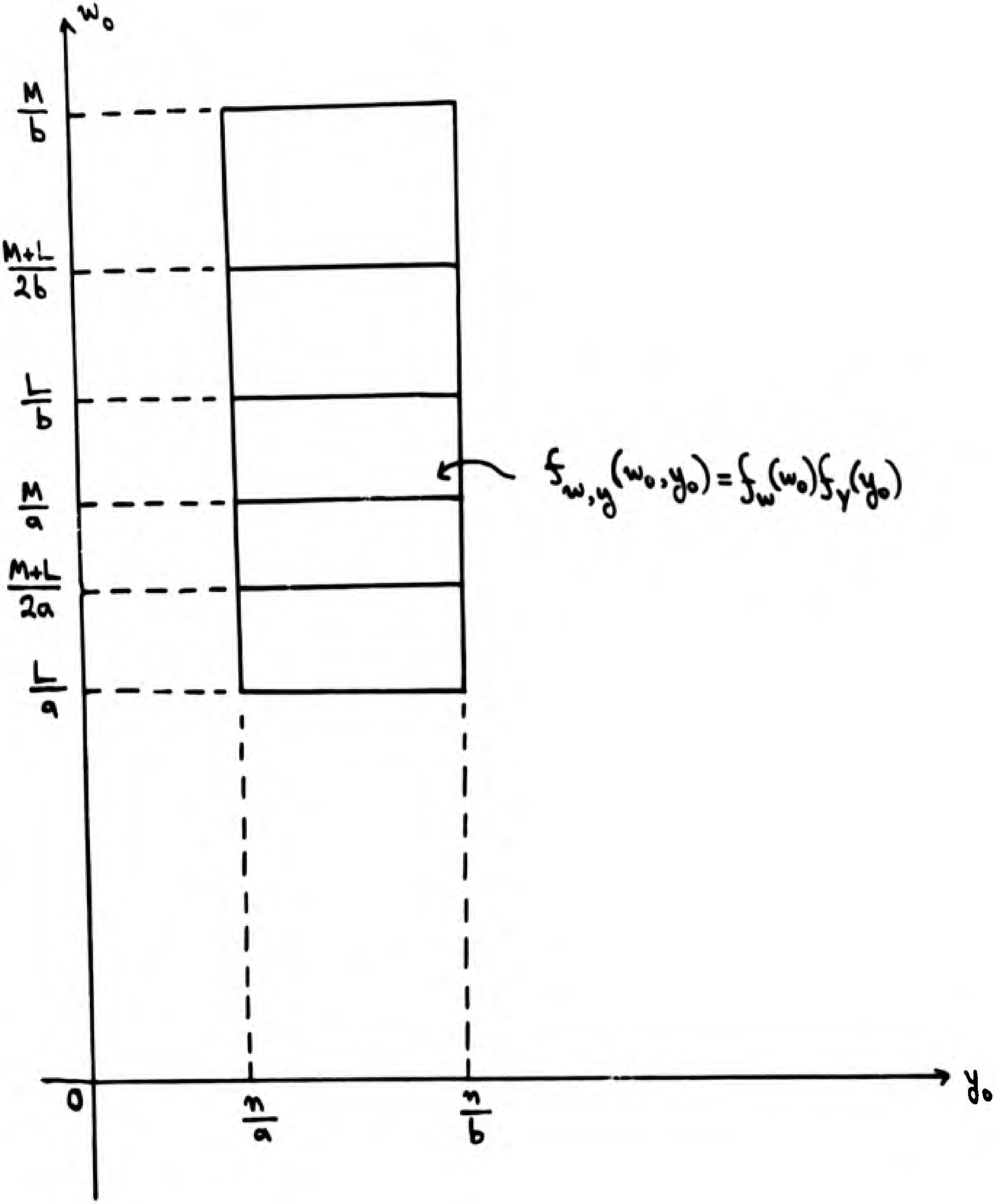
$$w \triangleq \frac{n+x}{v} \quad \text{and} \quad y \triangleq \frac{n}{v} .$$

Then, we have:

$$f_y(y_0) = \begin{cases} \dfrac{n}{y_0^2} \dfrac{1}{(a-b)}, & \text{for } \dfrac{n}{a} \leq y_0 \leq \dfrac{n}{b} \\ 0, & \text{otherwise} \end{cases} \tag{3.14}$$

However the value of $f_w(w_0)$ (see Appendix F) is now given by:

$$f_w(w_0) = \begin{cases} \dfrac{1}{k}\left[\dfrac{L^3}{6w_0^2} - \dfrac{La^2}{2} + \dfrac{w_0 a^3}{3}\right], & \text{for } \dfrac{L}{a} \leq w_0 < \dfrac{M+L}{2a} \\ \dfrac{1}{6kw_0^2}\left[L^3 - \dfrac{(M+L)^3}{4}\right] - \dfrac{w_0 a^3}{3k} + \dfrac{Ma^2}{2k}, & \text{for } \dfrac{M+L}{2a} \leq w_0 < \dfrac{M}{a} \\ \dfrac{1}{6kw_0^2}\left[L^3 + M^3 - \dfrac{(M+L)^3}{4}\right], & \text{for } \dfrac{M}{a} \leq w_0 \leq \dfrac{L}{b} \\ \dfrac{1}{6kw_0^2}\left[M^3 - \dfrac{(M+L)^3}{4}\right] - \dfrac{w_0 b^3}{3k} + \dfrac{Lb^2}{2k}, & \text{for } \dfrac{L}{b} \leq w_0 < \dfrac{M+L}{2b} \\ \dfrac{1}{k}\left[\dfrac{M^3}{6w_0^2} - \dfrac{Mb^2}{2} + \dfrac{w_0 b^3}{3}\right], & \dfrac{M+L}{2b} \leq w_0 < \dfrac{M}{b} \end{cases} \tag{E.2}$$



w_0
$\frac{M}{b}$
$\frac{M+L}{2b}$
$\frac{L}{b}$
$\frac{M}{a}$
$\frac{M+L}{2a}$
$\frac{L}{a}$
$f_{w,y}(w_0,y_0) = f_w(w_0)f_y(y_0)$
0
$\frac{n}{a}$
$\frac{n}{b}$
y_0

Figure E.1

$f_w(w_0)$ is equal to 0 otherwise.

In the above, we have defined

$$K \overset{\Delta}{=} \frac{4}{(c-d)^2(a-b)} = \frac{4}{(M-L)^2(a-b)} \tag{E.3}$$

while M and L are given by (3.11) and (3.12), respectively.

Then, if, as before, we define

$$r \overset{\Delta}{=} w - y$$

we are confronted by the situation pictured in figure E.1. Immediately, it is apparent, by comparison with figure 3.3, that the number of cases at hand has greatly increased and analysis becomes that much more complicated.

It is, therefore, preferable at this stage to obtain the form of the pdf for r by simulation methods.

Appendix F

Computation of $f_w(w_0)$ for Appendix E

First of all, if $q \triangleq \frac{1}{v}$ and since v is a uniformly distributed random variable between a and b, we have (see appendix B):

$$f_q(q_0) = \begin{cases} \frac{1}{q_0^2}\frac{1}{(a-b)} , & \text{for } \frac{1}{a} \leq q_0 \leq \frac{1}{b} \\ 0 , & \text{otherwise} \end{cases}$$

Then the joint density function $f_{x,q}(x_0, q_0)$, for the "triangular" pdf $f_x(x_0)$ given by (E.1), is:

$$f_{x,q}(x_0,q_0) = f_x(x_0)\, f_q(q_0) = \begin{cases} \frac{x_0-d}{Kq_0^2} , & \text{for } d \leq x_0 \leq \frac{c+d}{2} , \frac{1}{a} \leq q_0 \leq \frac{1}{b} \\ \frac{c-x_0}{Kq_0^2} , & \text{for } \frac{c+d}{2} \leq x_0 \leq c , \frac{1}{a} \leq q_0 \leq \frac{1}{b} \\ 0 , & \text{otherwise} \end{cases}$$

where $k \triangleq \frac{4}{(c-d)^2(a-b)}$.

Then if we define $w \triangleq \frac{n+x}{v} = (n+x)q$, we obtain the situation pictured in figure F.1.

Then, for the various cases pictured we obtain:

(i) For $w_0 \leq \frac{L}{a}$: $F_{w\varsigma}(w_0) = P[w \leq w_0] = 0$

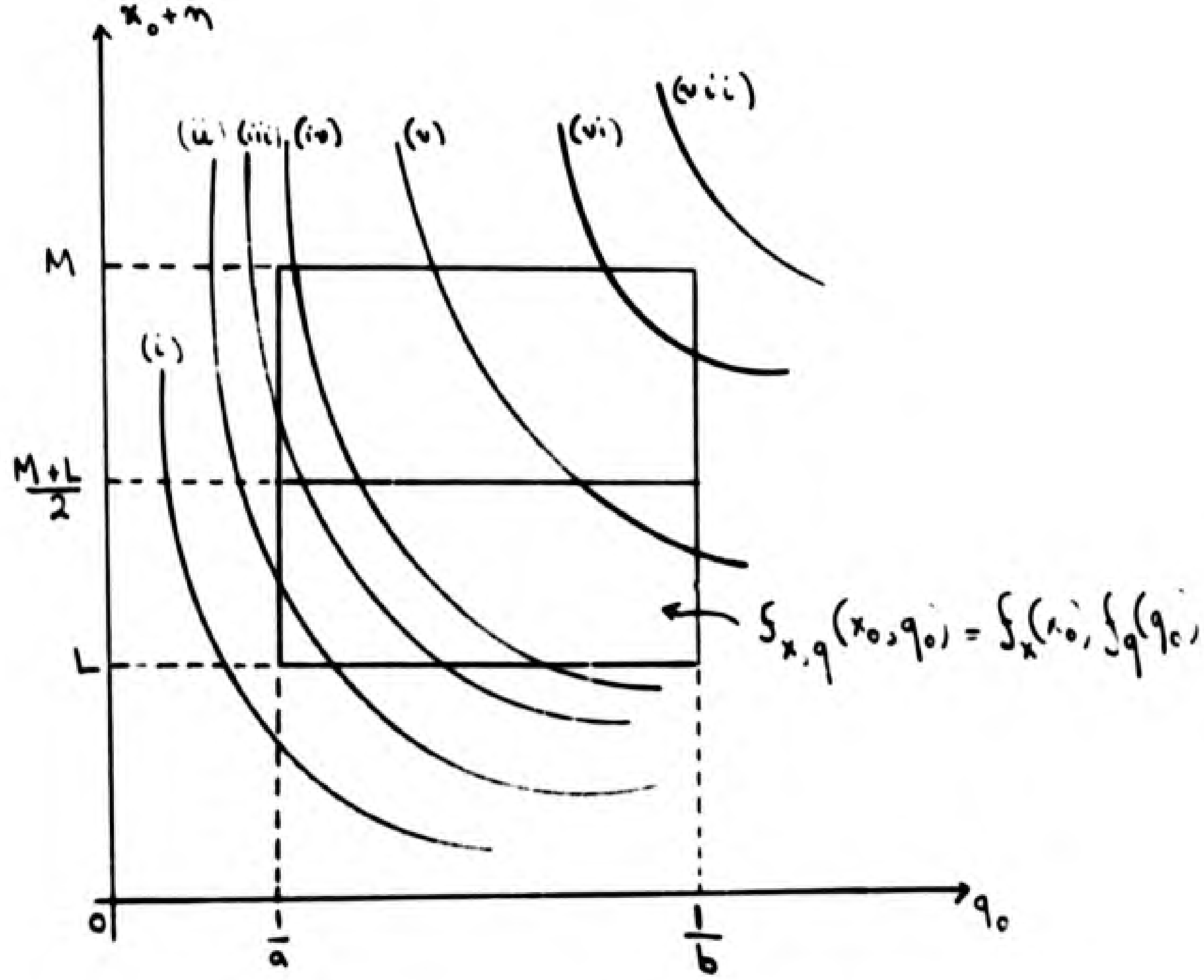


Figure F.1

(ii) For $\frac{L}{a} \leqslant w_0 < \frac{M+L}{2a}$:

$$F_{w\leqslant}(w_0) = \int_{\frac{1}{a}}^{\frac{w_0}{L}} \int_{L}^{\frac{w_0}{q_0}} \frac{x_0 - L}{Kq_0^2}\, dx_0\, dq_0$$

(iii) For $\frac{M+L}{2a} \leqslant w_0 < \frac{M}{a}$:

$$F_{w\leqslant}(w_0) = \int_{\frac{1}{a}}^{\frac{2w_0}{M+L}} \int_{L}^{\frac{M+L}{2}} \frac{x_0 - L}{Kq_0^2}\, dx_0 dq_0 + \int_{\frac{2w_0}{M+L}}^{\frac{w_0}{L}} \int_{L}^{\frac{w_0}{q_0}} \frac{x_0 - L}{Kq_0^2}\, dx_0 dq_0$$

$$+ \int_{\frac{1}{a}}^{\frac{2w_0}{M+L}} \int_{\frac{M+L}{2}}^{\frac{w_0}{q_0}} \frac{M - x_0}{q_0^2}\, dx_0 dq_0$$

(iv) For $\frac{M}{a} \leqslant w_0 \leqslant \frac{L}{b}$:

$$F_{w\leqslant}(w_0) = \int_{\frac{1}{a}}^{\frac{2w_0}{M+L}} \int_{L}^{\frac{M+L}{2}} \frac{(x_0 - L)}{Kq_0^2}\, dx_0 dq_0 + \int_{\frac{2w_0}{M+L}}^{\frac{w_0}{L}} \int_{L}^{\frac{w_0}{q_0}} \frac{x_0 - L}{Kq_0^2}\, dx_0\, dq_0$$

$$+\int_{\frac{1}{a}}^{\frac{w_0}{M}}\int_{\frac{M+L}{2}}^{M}\frac{M-x_0}{Kq_0^2}\,dx_0\,dq_0+\int_{\frac{w_0}{M}}^{\frac{2w_0}{M+L}}\int_{\frac{M+L}{2}}^{\frac{w_0}{q_0}}\frac{M-x_0}{Kq_0^2}\,dx_0\,dq_0$$

(v) For $\frac{L}{b}\leqslant w_0\leqslant\frac{M+L}{2b}$:

$$F_{w_\leqslant}(w_0)=\frac{1}{2}-\int_{\frac{2w_0}{M+L}}^{\frac{1}{b}}\int_{\frac{w_0}{q_0}}^{\frac{M+L}{2}}\frac{x_0-L}{Kq_0^2}\,dx_0\,dq_0$$

$$+\int_{\frac{1}{a}}^{\frac{w_0}{M}}\int_{\frac{M+L}{2}}^{M}\frac{M-x_0}{Kq_0^2}\,dx_0\,dq_0+\int_{\frac{w_0}{M}}^{\frac{2w_0}{M+L}}\int_{\frac{M+L}{2}}^{\frac{w_0}{q_0}}\frac{M-x_0}{Kq_0^2}\,dx_0\,dq_0$$

(vi) For $\frac{M+L}{2b}\leqslant w_0<\frac{M}{b}$:

$$F_{w_\leqslant}(w_0)=1-\int_{\frac{w_0}{M}}^{\frac{1}{b}}\int_{\frac{w_0}{q_0}}^{M}\frac{M-x_0}{Kq_0^2}\,dx_0\,dq_0$$

(vii) For $\frac{M}{b}\leqslant w_0$:

$$F_{w_\leqslant}(w_0)=1$$

Then differentiating the expression for $F_{w_\leqslant}(w_0)$, we obtain:

$$
f_w(w_0) = \begin{cases}
\frac{1}{K}\left[\frac{L^3}{6w_0^2} - \frac{La^2}{2} + \frac{w_0 a^3}{3}\right], \text{ for } \frac{L}{a} \leq w_0 \leq \frac{M+L}{2a} \\[2ex]
\frac{1}{6Kw_0^2}\left[L^3 - \frac{(M+L)^3}{4}\right] - \frac{w_0 a^3}{3K} + \frac{Ma^2}{2K}, \text{ for } \\
\qquad \frac{M+L}{2a} \leq w_0 < \frac{M}{a} \\[2ex]
\frac{1}{6Kw_0^2}\left[L^3 + M^3 - \frac{(M+L)^3}{4}\right], \text{ for } \frac{M}{a} \leq w_0 < \frac{L}{b} \\[2ex]
\frac{1}{6Kw_0^2}\left[M^3 - \frac{(M+L)^3}{4}\right] - \frac{w_0 b^3}{3K} + \frac{Lb^2}{2K}, \text{ for } \\
\qquad \frac{L}{b} \leq w_0 < \frac{M+L}{2b} \\[2ex]
\frac{1}{K}\left[\frac{M^3}{6w_0^2} - \frac{Mb^2}{2} + \frac{w_0 b^3}{3}\right], \text{ for } \frac{M+L}{2b} \leq w_0 \leq \frac{M}{b} \\[2ex]
0, \text{ otherwise}
\end{cases}
$$

This expression is used in (E.2) of appendix E.

Appendix G

Derivation of $f_{t_{DD}}(t)$

We use the uniform pdf's $f_x(x_0)$, $f_y(y_0)$ and $f_z(z_0)$ as given by (3.40), (3.41), and (3.42), respectively, to derive the pdf for random variable t_{DD}.

Assuming that $x_1 \leqslant y_1 \leqslant y_2 \leqslant x_2$, we have using (3.39):

$$f_w(w_0) = \begin{cases} \dfrac{2w_0 - (y_1 + x_1)}{(x_2 - x_1)(y_2 - y_1)}, & \text{for } y_1 \leqslant w_0 \leqslant y_2 \\ \dfrac{1}{x_2 - x_1}, & \text{for } y_2 \leqslant w_0 \leqslant x_2 \\ 0, & \text{otherwise} \end{cases}$$

where $w \triangleq \max(x, y)$. The pdf $f_w(w_0)$ is plotted in figure G.1.

In order to find $f_{t_{DD}}(t)$, we must now convolve $f_w(w_0)$ with $f_z(z_0)$. The operation is pictured in figure G.2, for the case when $x_2 - y_2 \leqslant z_2 - z_1 \leqslant y_2 - y_1$ (these inequalities have been selected to represent the typically prevailing situation in airports). Then we obtain:

(1) For $t \quad y_1 + z_1 : f_{t_{DD}}(t) = 0$

(2) For $y_1 + z_1 \leqslant t < y_1 + z_2$:

$$f_{t_{DD}}(t) = \int_{-\infty}^{+\infty} f_w(T)\, f_z(t-T)\, dT = \int_{y_1}^{t-z_1} \frac{2T-(y_1+x_1)}{C}\, dT$$

$$= \frac{1}{C}\left[(t-z_1)^2 - y_1^2 - (y_1+x_1)(t-z_1) + (y_1+x_1)y_1\right]$$

$$= \frac{1}{C}\left[t^2 - t(2z_1+y_1+x_1) + z_1(y_1+x_1+z_1) + y_1x_1\right]$$

where we have defined $C \triangleq (z_2-z_1)(y_2-y_1)(x_2-x_1)$

(3) For $y_1+z_2 \leqslant t < y_2+z_1$:

$$f_{t_{DD}}(t) = \int_{t-z_2}^{t-z_1} \frac{2T-(y_1+x_1)}{C}\, dT$$

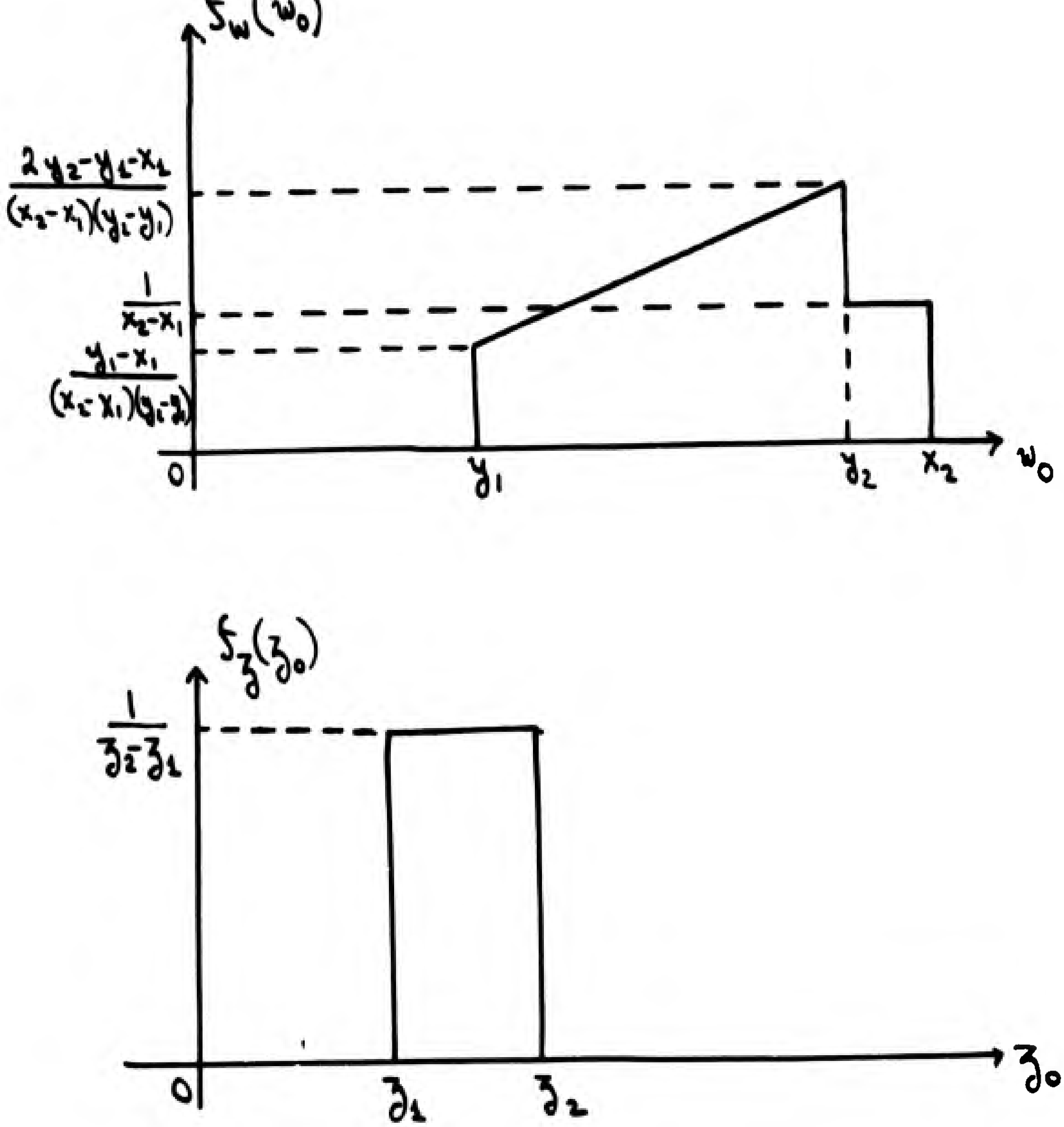
$$= \frac{z_2-z_1}{C}\left[2t-(z_2+z_1)-(y_1+x_1)\right]$$

(4) For $y_2+z_1 \leqslant t < x_2+z_1$:

$$f_{t_{DD}}(t) = \int_{y_2}^{t-z_1} \frac{1}{(x_2-x_1)(z_2-z_1)}\, dT + \int_{t-z_2}^{y_2} \frac{2T-(y_1+x_1)}{C}\, dT$$

$$= \frac{t-z_1-y_2}{(x_2-x_1)(z_2-z_1)} + \frac{1}{C}\left[y_2^2-(t-z_2)^2-(y_1+x_1)(y_2+z_2-t)\right]$$

(5) For $x_2+z_1 \leqslant t < y_2+z_2$:



<u>Figure G.1</u>

The probability density functions $f_w(w_0)$ and $f_z(z_0)$ must be convolved

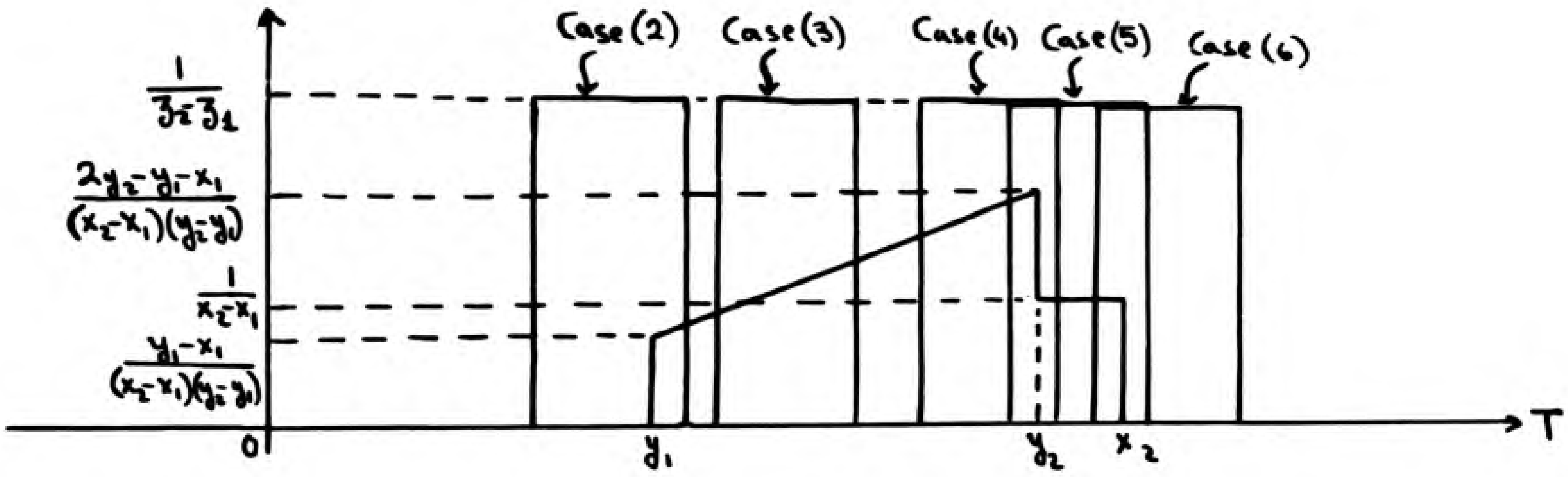


Figure G.2

The convolution operation

$$f_{t_{DD}}(t) = \int_{y_2}^{x_2} \frac{1}{(x_2-x_1)(z_2-z_1)}\, dT + \int_{t-z_2}^{y_2} \frac{2T-(y_1+x_1)}{C}\, dT$$

$$= \frac{x_2-y_2}{(x_2-x_1)(z_2-z_1)} + \frac{1}{C}\left[y_2^2 - (t-z_2)^2 - (y_1+x_1)(y_2+z_2-t)\right]$$

(6) For $y_2+z_2 \leqslant t < x_2+z_2$:

$$f_{t_{DD}}(t) = \int_{t-z_2}^{x_2} \frac{1}{(x_2-x_1)(z_2-z_1)}\, dT = \frac{x_2-t+z_2}{(x_2-x_1)(z_2-z_1)}$$

(7) For $x_2+z_2 \leqslant t$: $f_{t_{DD}}(t) = 0$.

Appendix H

A Multiqueue Multiserver Problem

Several interesting queuing problems can be associated with the two parallel runway system. These problems, however, are usually extremely difficult from the analytical point of view. We describe briefly two queuing situations on the study of which considerable time has been spent.

Consider the case of a short runway (used for landing of small planes) in parallel with a regular runway serving primarily commercial aircraft. There is nothing that precludes use of the regular runway by small aircraft whenever no commercial airplanes are present. But commercial aircraft always retain priority for the use of the major runway. These aircraft can not of course use the short runway.

Another example of a problem of similar type is the system of two parallel runways one of which is used for landings and the other for take-offs. Again when one runway is idle, it is desirable to divert some operations from the other in order to take advantage of the availability of a service facility.

Both of the above situations are excellent examples of systems with two parallel servers and with priority service. Their analysis involves some formidable difficulties. First, where departures are concerned, it is very doubtful that they occur in a Poisson manner. As a matter of fact, it is particularly difficult to provide a probabilistic description of the time instants at which departures are scheduled at major airports. The reason is primarily the well-known habit of airlines to bunch all departures on the hour and the half-hour.

Then, with general service times for both of the two parallel queues, the concept of the renewal epochs (otherwise known as the concept of the imbedded Markov process) can not be used. The reason is that at no instant in time is the system memoryless, any more.

Thus, problems of this type are all but unsolvable unless some simplifying assumptions are made. Even then, the analysis is extremely involved.

In the sequel, we present a simplified abstract version of the first example presented above and we outline a possible approach to it.

We consider the case of two parllel service facilities, say facility A and facility B. There are also two types of customers: type 1 and type 2. Type 1 customers are served by facility A and arrive at facility A in a Poisson manner at a rate of λ_1 per unit of time.

Type 2 customers are primarily served by facility B where they arrive at random instants (Poisson arrivals) at the rate of λ_2 per unit of time. However, if it happens that at a particular instant there are no type 1 customers at facility A, while at the same time there are type 2 customers waiting at facility B, facility A begins service on a type 2 customer. In other words, facility A provides priority service with type 1 customers having top priority. In addition, we assume that the priority is non-preemptive (i.e., if a type 1 customer arrives while a type 2 customer is being served at facility A then the type 1 customer has to wait for the completion of service to the type 2 customer). Service at both facilities is also assumed to be provided on a first-come, first-served basis.

It is postulated that all service time distributions are exponential. Mean service times are: $\frac{1}{\mu_1}$ for type 1 customers at facility A; $\frac{1}{\mu_2}$ for type 2 customers at facility B; and $\frac{1}{\mu_3}$ for type 2 customers at facility A.

We now write the equations for the steady-state probabilities describing this system. A three dimensional state-space must be used. Each state is specified by a set of index numbers (m, n_1, n_2) where:

m = the type of customer presently being serviced by facility A. Thus m = 1 if a type 1 customer is presently in A, m = 2 if a type 2 customer is presently in A, and m = 0 if A is idle at the moment.

n_1 = number of customers of type 1 waiting to be served. Thus n_1 can take any value from 0 to $+\infty$.

n_2 = number of customers of type 2 present in the system including the one, if any, presently being serviced in facility B.

Then if we denote a steady state probability by $P(m, n_1, n_2)$ we obtain the following set of equations:

$$(\lambda_1+\lambda_2)P(0,0,0) = \mu_1 P(1,0,0)+\mu_2 P(0,0,1)+\mu_3 P(2,0,0) \qquad \text{(H.1)}$$

$$(\mu_1+\lambda_1+\lambda_2)P(1,0,0)=\lambda_1P(0,0,0)+\mu_1P(1,1,0)+\mu_2P(1,0,1)+\mu_3P(2,1,0) \tag{H.2}$$

$$(\mu_2+\lambda_1+\lambda_2)P(0,0,1)=\lambda_2P(0,0,0)+\mu_1P(1,0,1)+\mu_2P(0,0,2)+\mu_3P(2,0,1) \tag{H.3}$$

$$(\mu_3+\lambda_1+\lambda_2)P(2,0,0)=\mu_2P(2,0,1) \tag{H.4}$$

$$\begin{aligned}(\mu_1+\lambda_1+\lambda_2)P(1,n_1,0)&=\lambda_1P(1,n_1-1,0)+\mu_1P(1,n_1+1,0)\\ &+\mu_2P(1,n_1,1)+\mu_3P(2,n_1+1,0),\ n_1=1,\ldots,+\infty\end{aligned} \tag{H.5}$$

$$\begin{aligned}(\mu_3+\lambda_1+\lambda_2)P(2,n_1,0)&=\lambda_1P(2,n_1-1,0)+\mu_2P(2,n_1,1)\\ &n_1=1,\ldots,+\infty\end{aligned} \tag{H.6}$$

$$\begin{aligned}(\mu_1+\mu_2+\lambda_1+\lambda_2)P(1,0,n_2)&=\lambda_1P(0,0,n_2)+\lambda_2P(1,0,n_2-1)\\ &+\mu_1P(1,1,n_2)+\mu_2P(1,0,n_2+1)+\mu_3P(2,1,n_2)\\ &n_2=1,\ldots,+\infty\end{aligned} \tag{H.7}$$

$$\begin{aligned}(\mu_3+\mu_2+\lambda_1+\lambda_2)P(2,0,n_2)&=\lambda_2P(2,0,n_2-1)+\mu_1P(1,0,n_2+1)\\ &+\mu_2P(2,0,n_2+1)+\mu_3P(2,0,n_2+1)\\ &n_2=1,\ldots,+\infty\end{aligned} \tag{H.8}$$

$$\begin{aligned}(\mu_1+\mu_2+\lambda_1+\lambda_2)P(1,n_1,n_2)&=\lambda_1P(1,n_1-1,n_2)+\lambda_2P(1,n_1,n_2-1)+\mu_1P(1,n_1+1,n_2)\\ &+\mu_2P(1,n_1,n_2+1)+\mu_3P(2,n\mu+1,n_2)\ \begin{cases}n_1=1,\ldots,+\infty\\ n_2=1,\ldots,+\infty\end{cases}\end{aligned} \tag{H.9}$$

$$(\mu_3+\mu_2+\lambda_1+\lambda_2)P(2,n_1,n_2) = \lambda_1 P(2,n_1-1,n_2) + \lambda_2 P(2,n_1,n_2-1) + \mu_2 P(2,n_1,n_2+1)$$

$$\begin{cases} m = 1,\ldots,+\infty \\ n = 1,\ldots,+\infty \end{cases} \tag{H.10}$$

The above system of equations describes the steady-state condition of the system. As it must be obvious to the reader, however, they are of very little help in their present form. An extensive effort was made to obtain a closed form expression for the geometric transform $\Pi^g(z_1, z_2, z_3)$ of the probabilities $P(m, n_1, n_2)$.

Such an expression was indeed obtained, after some very heavy algebra. However, the right-hand-side of the expression $\Pi^g(z_1, z_2, z_3)$ still contained several unknown terms and, therefore, it was impossible to obtain any moments, etc., of interest in this case.

It was thought, anyway, that presentation of this appendix, besides being possibly of interest for future research, could lend some insight into the complexities that some relatively simple and well-defined problems, as the present one, may involve.

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13 ABSTRACT Several analytical models for air-traffic in the general air terminal area are constructed, to explore some questions about air-traffic congestion. (U)

Attention is first focused on a single runway which is used exclusively for landings. The distribution for interarrival times of aircraft is obtained under capacity conditions. The inputs to the model are the velocity distribution of incoming aircraft, the distribution for the errors in spacing aircraft and the minimum separation requirements, both in the air and on the ground, as specified by the FAA. (U)

A similar analysis is performed for a runway which is used only for take-offs. The conceivable situation in which landings alternate with take-offs is also investigated. In all cases, the sequence of procedures is analyzed by using an applied probability theory approach. (U)

The results from the basic models are used in order to develop expressions about queuing and delay characteristics associated with various operations. In particular, an original method for modelling the arrivals queue is developed. Comparisons of the expressions derived from this model with those commonly used for estimating aircraft delays suggest that the latter may be over-estimating runway utilization and average delays. In addition the severe effect of spacing errors on delays is clearly demonstrated. (U)

The possibility of inserting departures between successive arrivals, thus increasing airport capacity, is also examined in detail. Several parameters of interest associated with this situation are computed. Some numerical results are presented along with a qualitative discussion of various issues. (U)

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