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Aberdeen Proving Ground, Maryland

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by

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U.S. ARMY ABERDEEN RESEARCH AND DEVELOPMENT CENTER
BALLISTIC RESEARCH LABORATORIES
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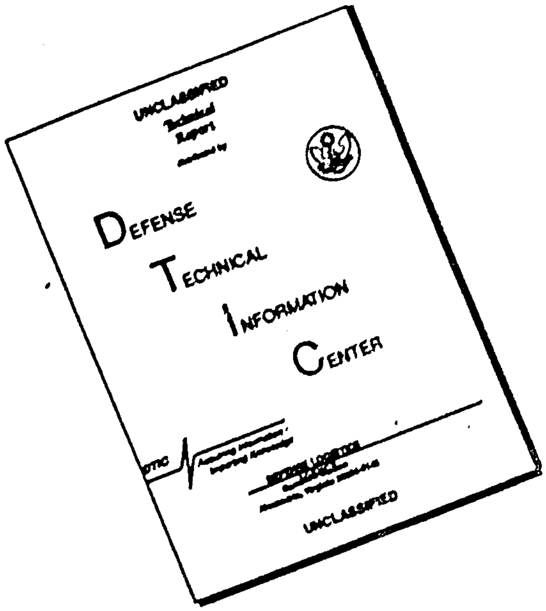

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Laboratory Report No. 242

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THEORY OF A PROCESS FOR SECURING A LARGE INITIAL AMPLITUDE OF YAW FOR BOMB STABILITY TESTS

Abstract

Tests of bomb stability require that a large initial amplitude of yaw be obtained. The initial amplitude of yaw should be slightly greater than 45° but less than 90°, and accordingly 65° has been considered desirable. The initial amplitude of yaw is dependent upon both the initial angle of yaw and the initial angular velocity of yaw. Col. L. E. Simon has suggested an apparatus consisting of a taut cord with one end fixed to a pawl in the bomb bay and with the other end attached to a ring engaging a cut nose plug. The nose plug is cut in the form of an arc of a circle, the arc beginning at the junction of the longitudinal axis of the bomb and the nose plug ring. The initial motion of the bomb in, and immediately below, the bomb bay takes place in a disturbed airflow. The initial motion may be regarded, roughly, as taking place in three regimes. In the first regime the air in the bomb bay may be considered motionless, in the second the air in the lowest portion of the bomb bay and just below it as turbulent but partaking to some extent of the motion of the air far below the airplane, and in the third regime the air may be considered as moving horizontally with uniform speed with respect to the airplane. Without data which are at present unavailable the character of the action of the air is incapable of a precise description. As a consequence the motion of the bomb will be considered as taking place in air which does not move with respect to the airplane until the retaining ring slips from the nose plug, and thereafter in air having uniform horizontal velocity with respect to the airplane. The effects of a gradual entry into the free air stream are neglected as unnecessarily complicated for the present problem. The equations of motion for this constrained system are worked out and a method found for deduction of the point at which the ring should be cut so that the assigned initial amplitude of yaw will be obtained. The method is applied to the Bomb, Demolition, 1000 lb., M44.

I. Purpose of a Stability Test for Bombs

1. Aerodynamic Forces and Torques

The forces tending to change the velocity of the center of mass of the bomb are currently taken to be the weight of the bomb, W, the drag, D, and the lift or cross-wind force, L[1]. These are

$$\left.\begin{aligned} W &= mg \\ D &= K_D \rho d^2 u^2 \\ L &= K_L \rho d^2 u^2 \sin \varepsilon \end{aligned}\right\} \qquad (1)$$

where m denotes the mass of the bomb, g, the acceleration of gravity, ρ, the density of the air, d, the master diameter of the bomb, u, the velocity of the center of mass, ε, the angle of yaw, K_D, the drag coefficient and K_L, the lift coefficient.

It will be assumed that the bomb does not spin, that is, that the bomb does not rotate about its longitudinal axis. The angular motion of the bomb will be supposed to consist only of motion of the longitudinal axis of the bomb in a vertical plane parallel to the axis of the airplane at the instant of release. Let the angle between the axis of the bomb and the horizontal plane through the center of mass be denoted by η and the angle of inclination, or angle between the horizontal plane and the tangent to the trajectory be denoted by θ. It appears from Diagram 1 that

$$\eta = \varepsilon + \theta \qquad (2)$$

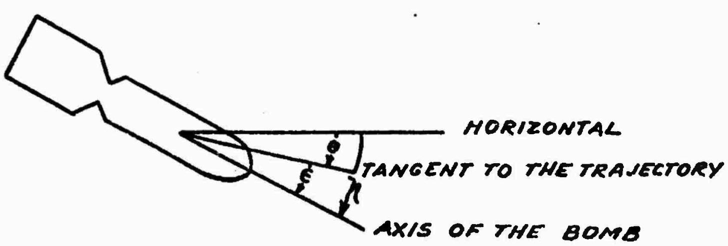


DIAGRAM 1

Let τ denote the torque acting to turn the bomb about its center of mass and I_B the moment of inertia of the bomb about a transverse axis through the center of mass. By the Eulerian physical equation[2]:

[1] The formulae for forces and torques given in this report are, with two exceptions, those employed in Ballistic Research Laboratory Report No. 111 "Aerodynamic Nomenclature and Formulas, Conversion Factors and Drag Functions". The angle of plane yaw is positive when the nose of the bomb is below the tangent to the trajectory, and the expression for lift has the sign of the angle of yaw. The damping torque denoted by Hitchcock as $H\omega$ is here denoted by H in conformity with the well-known work of R. H. Fowler. It is now considered that the force system here given is somewhat incomplete and a thorough study of this system is now being made by E. J. McShane.

[2] J. H. Jeans in "Theoretical Mechanics" derives Euler's physical equation on pages 306-308 and pages 346-348.

$$I_B \frac{d\Omega}{dt} = \tau \quad , \tag{3}$$

The angular velocity of the axis of the bomb with respect to directions fixed with respect to the air is denoted by Ω. Evidently

$$\Omega = \dot{\eta} = \dot{\varepsilon} + \dot{\theta} \quad . \tag{4}$$

The external torque, τ , is the sum of two torques, the restoring torque, denoted by M and the damping torque denoted by H. These are

$$\left.\begin{aligned} M &= -K_M \rho d^3 u^2 \sin \varepsilon \\ H &= -K_H \rho d^4 u \Omega \quad . \end{aligned}\right\} \tag{5}$$

Substitution of the value of τ in the Eulerian Physical Equation (3) results in

$$I_B \frac{d\Omega}{dt} = -K_M \rho d^3 u^2 \sin \quad - K_H \rho d^4 u \Omega \quad . \tag{6}$$

Substitution of the value of Ω from equation (4) in equation (6), transposition of terms and division of both sides of the resulting equation by I_B yields

$$\ddot{\varepsilon} = -\frac{K_M \rho d^3 u^2}{I_B} \sin \varepsilon - \frac{K_H \rho d^4 u(\dot{\varepsilon}+\dot{\theta})}{I_B} - \ddot{\theta} \quad . \tag{7}$$

Let x' denote the air range coordinate of the center of mass counted forward from the release point and y' the corresponding depth. Under standard bombing table conditions the equations of motion of the center of mass are given by[1]:

$$\left.\begin{aligned} \ddot{x}' &= -\frac{D \cos \theta}{m} - \frac{L \sin \theta}{m} \\ \ddot{y}' &= -\frac{D \sin \theta}{m} + \frac{L \cos \theta}{m} + g \quad . \end{aligned}\right\} \tag{8}$$

The angle of inclination of the tangent to the trajectory has been denoted by θ. Then:

$$\theta = \arc \tan \left(\frac{\dot{y}}{\dot{x}}\right) \quad . \tag{9}$$

The equation (7), which governs the motion of the axis of the bomb in the vertical plane, may be simplified considerably under the conditions prevailing during the short time interval which is of present concern. The time interval during which the motion of the axis of the bomb is of interest covers the time between release of the bomb nose plug from the retaining ring and the time when the bomb attains its first maximum yaw, or initial amplitude. This interval is less than a quarter

[1] These equations are given, for example, by F. V. Reno in Ballistic Research Laboratory Report No. 145 "On the Theory of Motion of the Bomb".

period of the yaw, and within this interval the center of mass makes a vertical drop of less than six feet. The shortness of this interval makes it possible to neglect the variation of the density of the air, ρ. Accordingly ρ will be replaced by ρ_o in equation (7).

It is found readily that

$$\left.\begin{aligned}
\dot{u} &= g \sin \theta - \frac{K_D \rho_o d^2 u^2}{m} \\
\dot{\theta} &= \frac{g \cos \theta}{u} + \frac{K_L \rho_o d^2 u \sin \varepsilon}{m} \\
\ddot{\theta} &= \left[- K_L g \sin \theta + u\dot{u} \frac{\partial K_L}{\partial u} + u\dot{\varepsilon} \frac{\partial K_L}{\partial \varepsilon} + K_L \dot{u} \right] \frac{\rho_o d^2 \sin \varepsilon}{m} \\
&\quad + \frac{\dot{\varepsilon} \rho_o d^2 u K_L \cos \varepsilon}{m} - \frac{g^2 \sin \theta \cos \theta}{u^2} - \frac{g \cos \theta}{u^2} \dot{u} .
\end{aligned}\right\} \qquad (10)$$

The result of substitution of (10) into (7) is:

$$\left.\begin{aligned}
\ddot{\varepsilon} = &- \left[\frac{K_M d}{I_B} + \frac{K_H K_L \rho_o d^4}{m I_B} + \frac{\dot{\varepsilon}}{mu} \frac{\partial K_L}{\partial \varepsilon} + \frac{g \sin \theta}{mu} \frac{\partial K_L}{\partial u} \right. \\
&\left. - \frac{K_D \rho_o d^2 u}{m^2} \frac{\partial K_L}{\partial u} - \frac{K_L K_D}{m^2} \rho_o d^2 \right] \rho_o d^2 u^2 \sin \varepsilon \\
&- \left[\frac{K_L}{m} \cos \varepsilon + \frac{K_H d^2}{I_B} \right] \rho_o d^2 u \dot{\varepsilon} \\
&- \left[\frac{K_D \rho_o d^2}{m} + \frac{K_H \rho_o d^4}{I_B} - 2 \frac{g \sin \theta}{u^2} \right] g \cos \theta .
\end{aligned}\right\} \qquad (11)$$

Mean aerodynamic coefficients, considered independent of the angle of yaw and the air speed, will be employed in the present report. Consequently $\frac{dK_L}{d\varepsilon}$ and $\frac{dK_L}{du}$ will be considered zero, $\sin \varepsilon$ will be taken equal to ε and $\cos \varepsilon = 1$. The result is:

$$\left.\begin{aligned}
\ddot{\varepsilon} = &- \left[\frac{K_M d}{I_B} + \frac{K_H K_L \rho_o d^4}{m I_B} - \frac{K_L K_D \rho_o d^2}{m^2} \right] \rho_o d^2 u \varepsilon \\
&- \left[\frac{K_L}{m} + \frac{K_H d^2}{I_B} \right] \rho_o d^2 u \dot{\varepsilon} \\
&- \left[\frac{K_H \rho_o d^4}{I_B} - \frac{2g \sin \theta}{u^2} + \frac{K_D \rho_o d^2}{m} \right] g \cos \theta .
\end{aligned}\right\} \qquad (12)$$

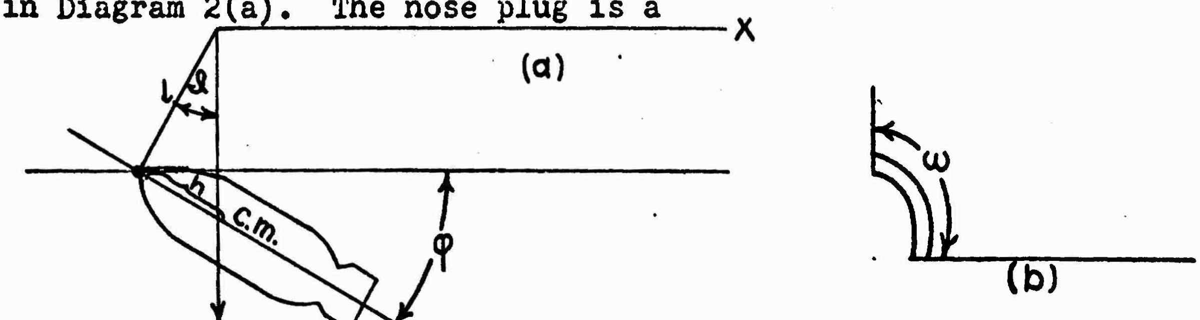
For the short period in time with which the present problem is concerned, $\frac{2g^2 \sin\theta \cos\theta}{u^2}$ is negligible, $\cos\theta$ may be replaced by unity and the airspeed, u, replaced by u_o without appreciable error[1]. The result is:

$$\ddot{\varepsilon} = -\left[\frac{K_M d}{I_B} + \frac{K_H K_L \rho_o d^4}{m I_B} - \frac{K_L K_D \rho_o d^2}{m^2}\right]\rho_o d^2 u_o^2 \varepsilon - \left[\frac{K_L}{m} + \frac{K_H d^2}{I_B}\right]\rho_o d^2 u_o \dot{\varepsilon} - \left[\frac{K_H \rho_o d^4}{I_B} + \frac{K_D \rho_o d^2}{m}\right] g \,. \qquad (13)$$

The moment of inertia, I_B will be replaced by its equivalent in terms of mass and radius of gyration, mk_B^2. Brief numerical computation indicates that $\frac{K_H K_L \rho_o d^3}{m k_B^2}$ and $\frac{K_D K_L \rho_o d}{m}$ are negligible in comparison to $\frac{K_M}{k_B^2}$ and these terms may be dropped. The result is:

$$\ddot{\varepsilon} = -\frac{\rho_o d^3 u_o^2}{m k_B^2} K_M \varepsilon - \left[K_L + \frac{K_H d^2}{k_B^2}\right]\frac{\rho_o d^2 u_o}{m}\dot{\varepsilon} - \left[K_D + \frac{K_H d^2}{k_B^2}\right]\frac{g \rho_o d^2}{m} \,. \qquad (14)$$

A determination of the degree of stability of flight can be made for any given set of conditions if the aerodynamic coefficients are known as functions of the yaw and Mach number. The purpose of a stability test is not to test directly the stability of the bomb, but to measure the aerodynamic coefficients. In performing a stability test, it is desirable to have an initial amplitude of yaw, designated hereafter as A, of at least 30 times the probable error of a measurement of the yaw. On the other hand, the yaw should not exceed 90° as an overturning bomb requires an inconvenient procedure in reduction of angular data. The most suitable initial amplitude of yaw is between 45° and 90°, preferably about 65°. It is proposed, in order to produce an initial amplitude of yaw of any desired magnitude, that the bomb be suspended from the bomb bay by a cord attached to a cut nose plug on the bomb. The arrangement is shown in Diagram 2(a). The nose plug is a



[1] No considerable approximation in the equation for plane yawing has been made previous to (13), at least for small angles of yaw. The solution of (13) is by no means sufficiently accurate for problems other than that examined in this report. A solution of a more accurate character has been made by E. J. McShane.

ring attached to the nose of the bomb. The nose plug ring will be cut at an angle ω from the axis of the bomb, as shown in Diagram (2)(b). The angle ω will be chosen such that the initial angular velocity of yaw and initial angle of yaw will combine to produce the desired initial amplitude of yaw.

2. <u>Causes of the Yaw of Bombs Under Standard Bombing Table Conditions</u>

Demolition bombs are suspended horizontally in the bomb bay. As the axis of the bomb is in the same direction as the line of flight, the initial angle of yaw of the bomb is zero. At the instant of release θ is also zero, and the time rate of change of ε is equal to that of θ, but opposite in sense. The normal initial conditions for the horizontal suspension of demolition bombs are:

$$\left.\begin{aligned} \varepsilon_0 &= 0 \\ \dot{\varepsilon}_0 &= -\dot{\theta}_0 \,. \end{aligned}\right\} \quad (15)$$

With regard to stability tests, the conditions are more general. The origin of time will be taken as the instant the bomb is released from the retaining arc. The angle of yaw, ε, the angle of inclination of the tangent to the trajectory, θ, and the angle of inclination of the axis of the bomb are related by

$$\eta = \varepsilon + \theta \,.$$

Between the instant of release from the shackle and the instant of release from the retaining arc, the angle η is the negative of the angle φ. Then

$$\left.\begin{aligned} \varepsilon &= \eta - \theta = -\varphi - \theta \\ \dot{\varepsilon} &= \dot{\eta} - \dot{\theta} = \dot{\varphi} - \dot{\theta} \,. \end{aligned}\right\} \quad (16)$$

Under the assumption that the air forces are negligible during this regime, it follows that

$$\left.\begin{aligned} y &= l(\cos\psi - \cos\psi_{-\tau}) + h \sin\varphi \\ x &= u_0 t - l(\sin\psi_{-\tau} - \sin\psi) + h(1 - \cos\varphi \end{aligned}\right\} \quad (17)$$

Evaluation of $\dot{y}_0$, $\dot{x}_0$, $\ddot{y}_0$, $\ddot{x}_0$, θ_0 and $\dot{\theta}_0$ can be made easily from (17).

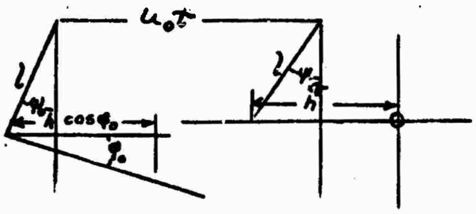


DIAGRAM 3

The results are:

$$
\left.
\begin{aligned}
\dot{y}_o &= h\dot{\varphi}_o \cos\varphi_o - l\dot{\psi}_o \sin\psi_o \\
\dot{x}_o &= u_o + h\dot{\varphi}_o \sin\varphi_o + l\dot{\psi}_o \cos\psi_o \\
\ddot{y}_o &= h\ddot{\varphi}_o \cos\varphi_o - h\dot{\varphi}_o^2 \sin\varphi_o - l\ddot{\psi}_o \sin\psi_o - l\dot{\psi}_o^2 \cos\psi_o \\
\ddot{x}_o &= h\ddot{\varphi}_o \sin\varphi_o + h\dot{\varphi}_o^2 \cos\varphi_o + l\ddot{\psi}_o \cos\psi_o - l\dot{\psi}_o^2 \sin\psi_o \\
\theta_o &= \arctan \frac{h\dot{\varphi}_o \cos\varphi_o - l\dot{\psi}_o \sin\psi_o}{u_o + h\dot{\varphi}_o \sin\varphi_o + l\dot{\psi}_o \cos\psi_o} \\
\dot{\theta}_o &= \frac{\begin{array}{c} u_o h(\ddot{\varphi}_o \cos\varphi_o - \dot{\varphi}_o^2 \sin\varphi_o) - u_o l(\ddot{\psi}_o \sin\psi_o + \dot{\psi}_o^2 \cos\psi_o) - h^2\dot{\varphi}_o^3 \\ - l^2\dot{\psi}_o^3 - hl\cos(\varphi_o - \psi_o)\left[\dot{\varphi}_o\ddot{\psi}_o - \ddot{\varphi}_o\dot{\psi}_o\right] - hl\sin(\varphi_o - \psi_o)\left[\dot{\varphi}_o\dot{\psi}_o^2 + \dot{\varphi}_o^2\dot{\psi}_o\right] \end{array}}{h^2\dot{\varphi}_o^2 + l^2\dot{\psi}_o^2 + 2hl\dot{\varphi}_o\dot{\psi}_o \sin(\varphi_o - \psi_o) + u_o^2 + 2u_o(h\dot{\varphi}_o \sin\varphi_o + l\dot{\psi}_o \cos\psi_o)}
\end{aligned}
\right\} \qquad (18)
$$

The angle θ_o is, for present purposes, small. Brief numerical calculation, employing values given later in this report, shows that all in θ_o or $\dot{\theta}_o$ which do not contain u_o are negligible in comparison with terms that contain u_o. Then

$$
\left.
\begin{aligned}
\varepsilon_o &= -\varphi_o - \frac{h\dot{\varphi}_o \cos\varphi_o - l\dot{\psi}_o \sin\psi_o}{u_o} \\
\dot{\varepsilon}_o &= -\dot{\varphi}_o - \frac{h(\ddot{\varphi}_o \cos\varphi_o - \dot{\varphi}_o^2 \sin\varphi_o) - l(\ddot{\psi}_o \sin\psi_o + \dot{\psi}_o^2 \cos\psi_o)}{u_o}
\end{aligned}
\right\} \qquad (19)
$$

3. <u>An Approximate Determination of the Motion of the Axis of the Bomb in the Vertical Plane</u>

Let

$$
\left.
\begin{aligned}
2s &= \left[K_L + \frac{K_H d^2}{k_B^2}\right] \frac{\rho_o d^2}{m}; \quad p = \left[K_D + \frac{K_H d^2}{k_B^2}\right] \frac{g\rho_o d^2}{m} \\
n^2 &= \left[K_M\right] \frac{\rho_o d^3}{m k_B^2},
\end{aligned}
\right\} \qquad (20)
$$

Substitution of (20) in (14) yields

$$
\ddot{\varepsilon} + 2su_o\dot{\varepsilon} + n^2u_o^2\left(\varepsilon + \frac{p}{n^2u_o^2}\right) = 0 \qquad (21)
$$

Write z for $\varepsilon + \frac{P}{n^2u_o^2}$ and (21) becomes

$$\ddot{z} + 2su_o\dot{z} + n^2u_o^2 z = 0 \quad . \tag{22}$$

This is the well-known equation for free vibrations of a damped oscillator. The solution is:[1]

$$z = Ae^{-su_o t} \cos\left[\sqrt{n^2 - s^2}\, u_o t - \sigma\right] \tag{23}$$

where

$$A = \frac{1}{u_o}\sqrt{\frac{\dot{z}_o^2 + 2u_o s z_o \dot{z}_o + z_o^2 u_o^2 n^2}{n^2 - s^2}} \quad . \tag{24}$$

An easy numerical calculation shows that $\frac{P}{n^2u_o^2}$ is of the order of 0.1 and therefore negligible, and $\dot{z}_o$ is of course equal to $\dot{\varepsilon}_o$. The solution for (22) is:

$$\varepsilon = A\varepsilon^{-su_o t} \cos\left[\sqrt{n^2 - s^2} \quad u_o t - \sigma\right] \tag{25}$$

where

$$A = \frac{1}{u_o}\sqrt{\frac{\dot{\varepsilon}_o^2 + 2u_o s \varepsilon_o \dot{\varepsilon}_o + \varepsilon_o^2 u_o^2 n^2}{n^2 - s^2}} \tag{26}$$

and

$$\sigma = \text{arc} \sin\left(\frac{\dot{\varepsilon}_o + u_o s \varepsilon_o}{\sqrt{\dot{\varepsilon}_o^2 + 2u_o s \varepsilon_o \dot{\varepsilon}_o + \varepsilon_o^2 u_o^2 n^2}}\right) \quad . \tag{27}$$

The natural frequency is:

$$f = u_o\sqrt{n^2 - s^2} \quad . \tag{28}$$

The period is:

$$P = \frac{2\pi}{f} = \frac{2\pi}{u_o\sqrt{n^2 - s^2}} \quad . \tag{29}$$

It is obvious from Diagram 4 that:

$$A_1 = Ae^{-su_o t_1}$$

and

$$A_2 = Ae^{-su_o t_2} \quad . \tag{30}$$

[1] A discussion of this well-known equation and its solution may be found in "Introduction to Theoretical Physics" by Leigh Page, D. Van Nostrand Co. Inc., 1928 on page 63 et seq.

Solving equations (30) for s yields:

$$s = \frac{1}{(t_1-t_2)u_o} \log_e \frac{A_2}{A_1} = -\frac{1}{Pu_o} \log_e \frac{A_2}{A_1} . \qquad (31)$$

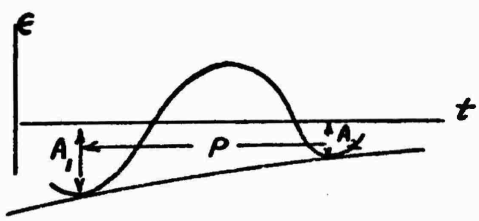
Now:

$$\sqrt{n^2 - s^2} = \frac{2\pi}{Pu_o} ,$$

and therefore

$$n^2 - s^2 = \left(\frac{2\pi}{Pu_o}\right)^2 \qquad (32)$$

$$n^2 = \left(\frac{2\pi}{Pu_o}\right)^2 + s^2 . \qquad (33)$$



The true curve of yaw against time will be approximately represented by the solution (25) for a very short time, about one second. As the vibration is damped, the second amplitude of yaw, A_2, is less than the first ,A_1.

DIAGRAM 4

The stability test yields observations of the period and the amplitude. Measures of the period and the amplitude can be used to derive the auxiliaries n^2 and s. The aerodynamic coefficients K_M and $K_H + \frac{K_L k_B^2}{d^2}$ can then be found from the formulae:

$$\left.\begin{aligned} K_M &= \frac{n^2 I_B}{\rho_o d^3} \\ K_H + \frac{K_L k_B^2}{d^2} &= \frac{2sI_B}{\rho_o d^4} . \end{aligned}\right\} \qquad (34)$$

A graphical procedure is convenient for the derivation of A_1, A_2 and P. The quantities directly measurable from the photographs are the depth, y, the trail, λ , and the angle η, as functions of count number, c. It will be supposed that the time bears a linear relation to the count number. The constants of the linear relation may be found by a determination of the origin of time and a determination of the rate of the camera, the latter being found by photographing the face of a stop-watch. Then a plot gives:

$$\left.\begin{aligned} y &= y(c) \\ \lambda &= \lambda(c) \\ \eta &= \eta(c) \\ t &= (c-c_o)r . \end{aligned}\right\} \qquad (35)$$

A plot of θ is required. Then:

$$\theta = \text{arc tan}\,\frac{dy}{dx} \doteq \text{arc tan}\,\frac{\Delta y}{\Delta x}$$

$$\doteq \text{arc tan}\,\frac{\Delta y}{u_o\Delta t-\Delta\lambda}\,. \qquad (36)$$

The increments Δy, Δt and Δλ having been read off the plot, θ can be calculated as a function of count number. Reference to Diagram 5 shows that:

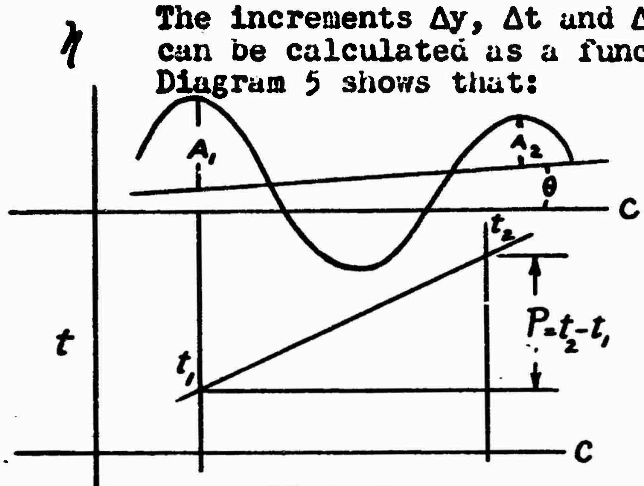


DIAGRAM 5

$$\left.\begin{aligned} A_1 &= \eta_1 - \theta_1 \\ A_2 &= \eta_2 - \theta_2\,. \end{aligned}\right\} \qquad (37)$$

The density of the air at the altitude of release may be deduced from:

$$\rho = \rho_\omega e^{-aY} = 0.075126e^{-0.00003158Y}$$

where ρ is given in pounds per cubic foot and Y is the altitude of release in feet.

In the present study the amplitude of yaw for the stability test will be assigned. This is done by substitution in equation (26):

$$A = \frac{\sqrt{\dot{\varepsilon}_o^{\,2} + 2u_o s\varepsilon_o\dot{\varepsilon}_o + \varepsilon_o^2u_o^2n^2}}{u_o\sqrt{n^2 - s^2}}\,.$$

II. The Bomb Constrained by a Cord From a Point Fixed in The Airplane to a Cut Nose Plug

1. Geometry of Constraint Imposed on the Bomb

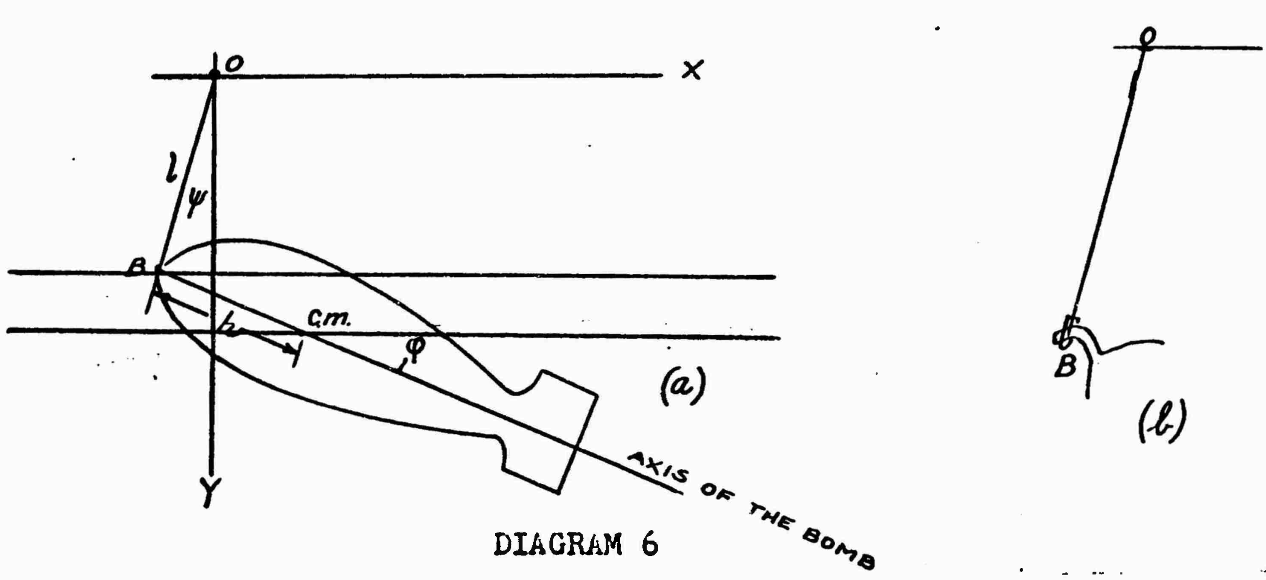


DIAGRAM 6

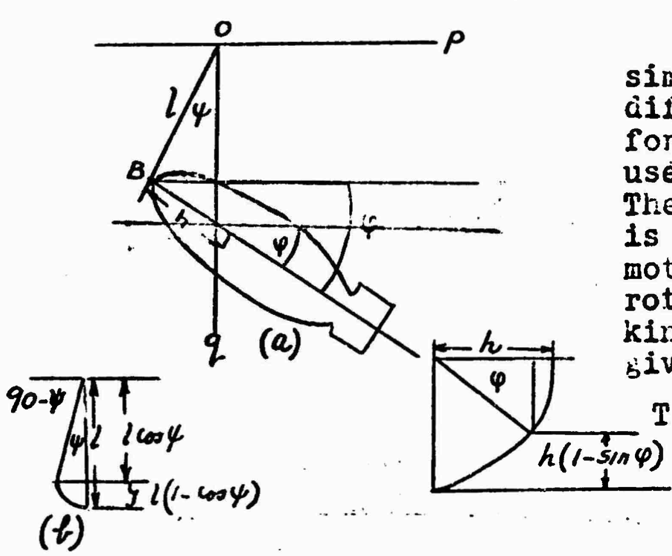
Diagram 6(a) is a schematic representation of the constraint to be imposed on the bomb. At B, there is a cut ring attached to the nose of the bomb. On this ring, will be placed another ring with a cord attached thereto. This cord, OB, is attached to a fixed point in the bomb bay of the airplane as shown in Diagram 6(b).

For purposes of the subsequent development, it is assumed that the cord is inextensible and acts as a rigid body similar to a very thin bar.

There is a rotation of OB about O and also a rotation of the bomb about B. This latter rotation might also be considered as the rotation of the bomb about its center of mass. The problem to be solved is thus one of a system of constrained motion having two degrees of freedom, both of which are rotational.

Let the angle between the cord and the vertical be denoted by ψ and the distance of the center of mass from the nose of the bomb by h . Let φ be the angle between the axis of the bomb and the horizontal.

2. The Equations of Motion and Their Solution



(c)

A most direct and comparatively simple method of developing the differential equations of motion for this system can be found by use of Lagranges' equations.[1] The kinetic energy of the system is partly due to translational motion and partly due to rotational motion. The total kinetic energy of the bomb is given by:[2]

$$T = \frac{1}{2} m \left[\dot{p}^2 + \dot{q}^2 \right] + \frac{1}{2} I_B \dot{\varphi}^2 . \qquad (38)$$

The potential energy, V, consists of two terms, the sum of these terms being due to the height of the center of mass of the bomb above its lowest possible position. The two terms are due to the two distances $l(1 - \cos\psi)$ and $h(1 - \sin\varphi)$ which make up the combined height of the center of mass above its lowest possible position. The two distances are shown by Diagram 7(b) and Diagram 7(c). Evidently:

$$V = mg\left[l(1 - \cos\psi) + h(1 - \sin\varphi) \right] . \qquad (39)$$

[1] An extended discussion of Lagrange's Equations is found in E. J. Routh, "Dynamics of a System of Rigid Bodies", Part I, Chapter VIII.

[2] The coordinate, p, of equations (38) et seq., is unrelated to the temporary parameter, p, introduced in Section I, 3.

By definition:

$$I_B = mk_B^2 \tag{40}$$

where k_B is the radius of gyration of the bomb about a transverse axis through the center of mass. The Lagrangian function, L, is the difference between the kinetic and potential energies.

$$L = T - V = \frac{1}{2} m \left[\dot{p}^2 + \dot{q}^2\right] + \frac{1}{2} I_B \dot{\varphi}^2 - mg\left[l(1 - \cos\psi) + h(1 - \sin\varphi)\right]. \tag{41}$$

It is obvious from Figure 7(a) that:

$$\left.\begin{aligned} p &= - l \sin\psi + h \cos\varphi \\ q &= + l \cos\psi + h \sin\varphi \, . \end{aligned}\right\} \tag{42}$$

Therefore:

$$\left.\begin{aligned} \dot{p} &= - l\dot{\psi} \cos\psi - h\dot{\varphi} \sin\varphi \\ \dot{q} &= - l\dot{\psi} \sin\psi + h\dot{\varphi} \cos\varphi \, . \end{aligned}\right\} \tag{43}$$

The result of squaring and adding equations (43) is:

$$\begin{aligned} \dot{p}^2 + \dot{q}^2 &= l^2\dot{\psi}^2 + h^2\dot{\varphi}^2 - 2hl\dot{\varphi}\dot{\psi} (\sin\psi \cos\varphi - \cos\psi \sin\varphi) \\ &= l^2\dot{\psi}^2 + h^2\dot{\varphi}^2 - 2hl\dot{\varphi}\dot{\psi} \sin(\psi - \varphi) \, . \end{aligned} \tag{44}$$

Substitution of this value in equation (41) yields:

$$\begin{aligned} L &= \frac{1}{2} m\left[l^2\dot{\psi}^2 + h^2\dot{\varphi}^2 - 2hl\dot{\varphi}\dot{\psi} \sin(\psi - \varphi)\right] \\ &\quad + \frac{1}{2} I_B\dot{\varphi}^2 - mg\left[l(1 - \cos\psi) + h(1 - \sin\varphi)\right] \\ &= \frac{1}{2} m \left[(k^2 + h^2)\dot{\varphi}^2 - 2hl\dot{\varphi}\dot{\psi} \sin(\psi - \varphi)\right] \\ &\quad + mgh \sin\varphi + mgl \cos\psi - mgh - mgl \, . \end{aligned} \tag{45}$$

Lagrange's equations for finite forces are: [1]

$$\left.\begin{aligned} &\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) = \frac{\partial L}{\partial \varphi} \\ \text{and} \quad &\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\psi}}\right) = \frac{\partial L}{\partial \psi} \, . \end{aligned}\right\} \tag{46}$$

The required derivatives for Lagrange's equations are found readily. After substitution of these derivatives into equations (46) and slight reduction it is found that:

[1] Footnote, P. 11, supra.

$$m\left[k^2 + h^2\right]\ddot{\varphi} = mgh\cos\varphi + mh l\ddot{\psi}\sin(\psi-\varphi) + mhl\dot{\psi}^2\cos(\psi-\varphi) \quad (47)$$

$$ml^2\ddot{\psi} = -mgl\sin\psi + mhl\ddot{\varphi}\sin(\psi-\varphi) - mhl\dot{\varphi}^2\cos(\psi-\varphi).$$

As a check upon these results it may be noted that if $l = 0$, the first of equations (47) becomes:

$$m\left[k^2 + h^2\right]\ddot{\varphi} = mgh\cos\varphi\ ,$$

which is the equation for a compound pendulum. Also it may be noted that if $h = 0$, the second of equations (47) becomes:

$$ml^2\ddot{\psi} = -mgl\sin\psi,$$

which is the equation for a simple pendulum.

A slight modification of equations (47) will be advantageous. Evidently:

$$\ddot{\varphi} = \frac{gh}{k^2+h^2}\cos\varphi + \frac{hl}{k^2+h^2}\ddot{\psi}\sin(\psi-\varphi) + \frac{hl}{k^2+h^2}\dot{\psi}^2\cos(\psi-\varphi) \quad (48)$$

$$\ddot{\psi} = -\frac{g}{l}\sin\psi + \frac{h}{l}\ddot{\varphi}\sin(\psi-\varphi) - \frac{h}{l}\dot{\varphi}^2\cos(\psi-\varphi).$$

The angles φ and ψ are small in the sense that:

$$\sin\varphi \doteq \varphi \qquad \cos\varphi \doteq 1 - \frac{\varphi^2}{2} \quad (49)$$

$$\sin\psi \doteq \psi \qquad \cos\psi \doteq 1 - \frac{\psi^2}{2}.$$

Equations (48) become:

$$\ddot{\varphi} = \frac{gh}{k^2+h^2}\cos\varphi + \frac{hl\ddot{\psi}}{k^2+h^2}(\psi\cos\varphi - \varphi\cos\psi) + \frac{hl}{k^2+h^2}\dot{\psi}^2(\cos\psi\cos\varphi + \psi\varphi) \quad (50)$$

$$\ddot{\psi} = -\frac{g}{l}\psi + \frac{h}{l}\ddot{\varphi}(\psi\cos\varphi - \varphi\cos\psi) - \frac{h}{l}\dot{\varphi}^2(\cos\psi\cos\varphi + \psi\varphi)$$

The equations (50) are the differential equations of motion of the system under investigation.

The initial conditions are:

$$\varphi(0) = \varphi_o = 0 \qquad \psi(0) = \psi_o$$
$$\dot{\varphi}(0) = \dot{\varphi}_o = 0 \qquad \dot{\psi}(0) = \dot{\psi}_o = 0 \; . \qquad (51)$$

The analytical solution for these equations will now be developed. Assume the following power series form for φ and ψ:

$$\varphi = \varphi_o + \dot{\varphi}_o t + a_2 t^2 + a_3 t^3 + \ldots$$
$$\psi = \psi_o + \dot{\psi}_o t + b_2 t^2 + b_3 t^3 + \ldots \; . \qquad (52)$$

The equations (52) may be simplified slightly by substitution of the initial conditions as given by equations (51). The results are:

$$\varphi =$$
$$\psi = \psi_o + b_2 t^2 + b_3 t^3 + b_4 t^4 + \ldots \qquad (53)$$

For the small interval involved here, approximately one quarter of a second, both series are convergent. However, as the value of t approaches 0.25 second, terms of higher than the fourth order must be taken into account in order to obtain a sufficiently accurate result for present purposes. On the other hand, fourth order terms may be neglected for very small values of t.

Differentiation of equations (53) leads to:

$$\dot{\varphi} = 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + \ldots$$
$$\dot{\psi} = 2b_2 t + 3b_3 t^2 + 4b_4 t^3 + \ldots \qquad (54)$$

and

$$\ddot{\varphi} = 2a_2 + 6a_3 t + 12a_4 t^2 + \ldots$$
$$\ddot{\psi} = 2b_2 + 6b_3 t + 12b_4 t^2 + \ldots \; . \qquad (55)$$

Substituting (53), (54) and (55) in the differential equations (50) and neglecting all terms of or higher orders in the time results in:

$$2a_2 + 6a_3 t + 12a_4 t^2 + \ldots$$

$$= \frac{gh}{k^2+h^2} + \frac{hl}{k^2+h^2}\left\{2b_2\psi_o+6b_3\psi_o t+12b_4\psi_o t^2+2b_2\left[b_2-a_2\left\{1-\frac{\psi_o^2}{2}\right\}\right]t^2\right\}$$

$$+\frac{hl}{k^2+h^2}\left[4b_2^2 t^2\left(1-\frac{\psi_o^2}{2}\right)\right] + \ldots$$

$$2b_2 + 6b_3 t + 12b_4 t^2 + \ldots \qquad (56)$$

$$= \frac{-g}{l}(\psi_o+b_2t^2)+\frac{h}{l}\left[2a_2\psi_o+6a_3\psi_o t+12a_4\psi_o t^2+2a_2\left(b_2-a_2\left\{1-\frac{\psi_o^2}{2}\right\}\right)t^2\right]$$

$$-\frac{h}{l}\left[4a_2^2 t^2\left(1-\frac{\psi_o^2}{2}\right)\right] + \ldots$$

There is a well-known theorem which states that the development of a function in a power series is unique. Therefore the coefficients of the same powers of t on both sides of equations (56) must be the same. Equating coefficients of the same powers of t on both sides of (56) leads to explicit forms of a_2, a_3, a_4 and b_2, b_3 and b_4.

$$2a_2 = \frac{gh}{k^2+h^2} + \frac{hl}{k^2+h^2}\left[2b_2\psi_o\right]$$

$$6a_3 = \frac{hl}{k^2+h^2}\left[6b_3\psi_o\right]$$

$$12a_4 = \frac{hl}{k^2+h^2}\left\{12b_4\psi_o+2b_2\left[b_2-a_2\left(1-\frac{\psi_o^2}{2}\right)\right]\right\}+\frac{hl}{k^2+h^2}\left[4b_2^2\left(1-\frac{\psi_o^2}{2}\right)\right]$$

$$2b_2 = -\frac{g}{l}\psi_o + \frac{h}{l}(2a_2\psi_o)$$

$$6b_3 = \frac{+h}{l}(6a_3\psi_o)$$

$$12b_4 = \frac{-g}{l}b_2 + \frac{h}{l}\left[12a_4\psi_o + 2a_2\left(b_2 - a_2\left\{1-\frac{\psi_o^2}{2}\right\}\right)\right]$$

$$-\frac{h}{l}\left[4a_2^2\left(1-\frac{\psi_o^2}{2}\right)\right].$$

On examination of the above equations it is seen that $a_3 = c_1 b_3$ and $b_3 = c_2 a_3$. This is true only if $c_1 = \frac{1}{c_2}$ or $a_3 = b_3 = 0$. The latter is the case here.

Now

$$a_2 = \frac{gh}{2(k^2+h^2)} + \frac{hl}{k^2+h^2}\left[b_2\psi_o\right]$$

and

$$b_2 = -\frac{g}{2l}\psi_o + \frac{h}{l}\left[a_2\psi_o\right].$$

Simultaneous solution of these two equations results in:

$$\left.\begin{aligned} a_2 &= \frac{gh\left[1-\psi_o^2\right]}{2\left[k^2+h^2-h^2\psi_o^2\right]} \\ b_2 &= \frac{-g\,\psi_o k^2}{2l\left[k^2+h^2-h^2\psi_o^2\right]}. \end{aligned}\right\} \qquad (57)$$

Similarly:

$$\left.\begin{aligned} a_4 &= \frac{hl}{12(k^2+h^2)}\left\{12b_4\psi_o + 2b_2\left[b_2 - a_2\left\{1-\frac{\psi_o^2}{2}\right\}\right]\right\} \\ &\quad + \frac{hl}{12(k^2+h^2)}\left[4b_2^2\left(1-\frac{\psi_o^2}{2}\right)\right] \\ b_4 &= \frac{-g}{12l}b_2 + \frac{h}{12l}\left\{12a_4\psi_o + 2a_2\left[b_2 - a_2\left\{1-\frac{\psi_o^2}{2}\right\}\right]\right\} \\ &\quad - \frac{h}{12l}\left[4a_2^2\left(1-\frac{\psi_o^2}{2}\right)\right]. \end{aligned}\right\} \qquad (58)$$

The solution [1] of these equations is:

$$a_4 = \frac{2g^2hk^4\psi_o^2\left[4-\psi_o^2\right] - 3lg^2h^4\psi_o\left[2-5\psi_o^2+4\psi_o^4-\psi_o^6\right] + lg^2h^2k^2\psi_o\left[2-3\psi_o^2+\psi_o^4\right]}{48l\left[k^2+h^2-h^2\psi_o^2\right]^3}$$

$$b_4 = \frac{2g^2k^4\psi_o(k^2+h^2) + g^2h^2k\psi_o^2\left\{2k^2\psi_o(3-\psi_o^2)\right\} - 3lg^2h^5\left[2-5\psi_o^2+4\psi_o^4-\psi_o^6\right]}{48l^2\left[k^2+h^2-h^2\psi_o^2\right]^3} - \frac{lg^2h^3k^2\left[6-17\psi_o^2-15\psi_o^4-4\psi_o^6\right]}{48l^2\left[k^2+h^2-h^2\psi_o^2\right]^3}. \qquad (59)$$

The leading terms of the initial motion solution are obtained by substitution of these values for a_2, a_4, b_2 and b_4 in the following equations:

$$\left.\begin{aligned} \varphi &= a_2t^2 + a_4t^4 + \ldots \\ \psi &= \psi_o + b_2t^2 + b_4t^4 + \ldots \end{aligned}\right\} \qquad (60)$$

In addition to the foregoing solution by power series development in the independent variable, there are other possible methods of solution. But, inasmuch as only the initial motion is of interest for the present problem, more powerful and general methods of solving differential equations will not be employed.

[1] The values of a_2 and b_2 are first substituted in (58). Then the values of a_4 and b_4 are obtained by simultaneous solution.

The equations of motion may also be solved by numerical integration in any particular case. The method of numerical integration to be employed in an example later in this report is referred to as Moulton's Method.[1] The result of solution by numerical integration and the initial lines of the numerical work will be given later. The example given is that of the BOMB, Demolition, 1000 lb., M44, but the method is general and can be applied to bombs other than the M44.

III. Application to the BOMB, Demolition, 1000 lb., M44

The proposed physical plan with respect to the BOMB, Demolition, 1000 lb., M44 is shown in Diagram 8.

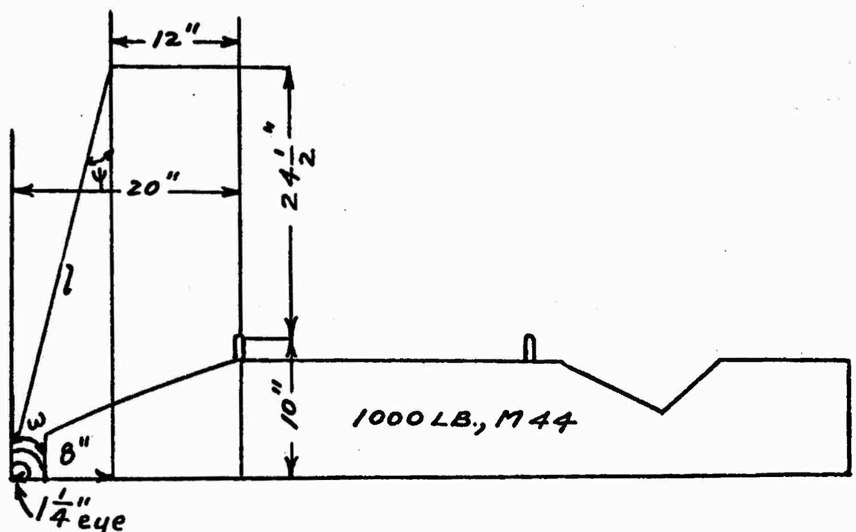


DIAGRAM 8

The equations of motion for this bomb are:

$$\left.\begin{aligned} \ddot{\varphi} &= 10.9375\cos\varphi + 1.0040\,\ddot{\psi}(\psi\cos\varphi-\varphi\cos\psi) + 1.0040\,\dot{\psi}^2(\cos\psi\cos\varphi+\psi \\ \ddot{\psi} &= -10.8942\,\psi + 0.7499\,\ddot{\varphi}(\psi\cos\varphi-\varphi\cos\psi) - 0.7499\,\dot{\varphi}^2(\cos\psi\cos\varphi+\psi\varphi) \end{aligned}\right\} \quad (61)$$

The computing schedule for numerical integration and several lines of computation are shown in Table I. These can be used as models in any subsequent investigations.

The numerical integration was continued to a time equal to about one third of a second to make certain that it covered a range resulting in values of A exceeding 65°.

[1] The proof of this method was first worked out by F. R. Moulton in 1914. The method and proof were published in 1918 in a blue-print pamphlet issued by the Ballistic Branch of the Ordnance Dept., U.S.A. and were reproduced in 1926 in Moulton's "New Method in Exterior Ballistics", Chapter V.

TABLE I: LAYOUT FOR NUMERICAL INTEGRATION

$\frac{gh}{k^2+h^2}=10.9375 \text{sec.}^{-2}$; $\frac{h\ell}{k^2+h^2}=1.0040$; $\frac{g}{\ell}=10.8942 \text{sec.}^{-2}$; $\frac{h}{\ell}=0.7499$; $\ddot{\varphi}=\frac{gh}{k^2+h^2}\cos\varphi+\frac{h\ell}{k^2+h^2}\ddot{\psi}(\psi\cos\varphi-\varphi\cos\psi)+\frac{h\ell}{k^2+h^2}\dot{\psi}^2(\cos\psi\cos\varphi+\psi\varphi)$

t (sec.)	$\dot{\varphi}$ (rad. / sec.)	Δ'	φ (rad.)	Δ'	$\dot{\psi}$ (rad. / sec.)	Δ'	ψ (rad.)	Δ'	$\cos\varphi$	$\frac{gh}{k^2+h^2}\cos\psi$ (rad. / sec.^{2})	$\cos\psi$
0.00	0.000		0.0000		0.000		0.2279		1.00000	10.9375	0.9741
0.01	0.108	108	0.0005	5	0.006	6	0.2279	0	1.00000	10.9375	0.9741
0.02	0.216	108	0.0021	16	0.013	7	0.2278	-1	1.00000	10.9375	0.9742

t (sec.)	$(\psi\cos\varphi-\varphi\cos\psi)$ (rad.)	$\frac{h\ell\ddot{\psi}}{k^2+h^2}(\psi\cos\varphi-\varphi\cos\psi)$ (rad. / sec.^{2})	$\dot{\psi}^2$ (rad.^{2} / sec.^{2})	$\dot{\varphi}^2$ (rad.^{2} / sec.^{2})	$\psi\varphi$ (rad.^{2})	$(\cos\psi\cos\varphi+\psi\varphi)$	$\frac{h\ell}{k^2+h^2}\dot{\psi}^2(\cos\psi\cos\varphi+\psi\varphi)$ (rad.^{2} / sec.^{2})	$\ddot{\varphi}$ (rad. / sec.^{2})	Δ'
0.00	0.2279	-0.146	0.000	0.000	0.000	0.974	0.000	10.79	
0.01	0.2274	-0.148	0.000	0.012	0.000	0.974	0.000	10.79	0
0.02	0.2258	-0.156	0.000	0.047	0.000	0.974	0.000	10.78	-1

$$\ddot{\psi} = \frac{-g}{l}\psi + \frac{h}{l}\ddot{\varphi}(\psi\cos\varphi - \varphi\cos\psi) - \frac{h}{l}\dot{\varphi}^2(\cos\psi\cos\varphi + \psi\varphi)$$

t (sec.)	$-\frac{g}{l}\psi$ (rad. / sec.^{2})	$\frac{h}{l}\ddot{\varphi}(\psi\cos\varphi - \varphi\cos\psi)$ (rad. / sec.^{2})	$-\frac{h}{l}\dot{\varphi}^2(\cos\psi\cos\varphi + \psi\varphi)$ (rad.^{2} / sec.^{2})	$\ddot{\psi}$ (rad. / sec.^{2})	Δ'	Δ''
0.00	-2.483	1.844	-0.000	- 0.64		
0.01	-2.483	1.840	-0.009	- 0.65	-1	
0.02	-2.482	1.825	-0.034	-0.69	-4	-3

By substitution of the numerical coefficients in equations (53), (54) and (55), the following results are obtained for the initial motion solution:

$$\left.\begin{aligned} \varphi &= 5.3957t^2 - 2.1700t^4 + \ldots && \text{(rad.)} \\ \psi &= 0.2279 - 0.3192t^2 - 10.9300t^4 + \ldots && \text{(rad.)} \end{aligned}\right\} \quad (62)$$

$$\left.\begin{aligned} \dot{\varphi} &= 2 \times 5.3957t - 4 \times 2.1700t^3 + \ldots && \text{(rad./sec.)} \\ \dot{\psi} &= -2 \times 0.3192t - 4 \times 10.9300t^3 + \ldots && \text{(rad./sec.)} \end{aligned}\right\} \quad (63)$$

$$\left.\begin{aligned} \ddot{\varphi} &= 2 \times 5.3957 - 12 \times 2.1700t^2 + \ldots && \text{(rad./sec.}^2\text{)} \\ \ddot{\psi} &= -2 \times 0.3192 - 12 \times 10.9300t^2 + \ldots && \text{(rad./sec.}^2\text{)} \end{aligned}\right\} \quad (64)$$

Table II gives a comparison of the results obtained by numerical integration and by substitution in the analytical solution at one tenth and at two tenths of a second.

TABLE II

Numerical Integration:

t (sec.)	φ (rad.)	$\dot{\varphi}$ (rad./sec.)	$\ddot{\varphi}$ (rad./sec.^{2})	ψ (rad.)	$\dot{\psi}$ (rad./sec.)	$\ddot{\psi}$ (rad./sec.^{2})
0.10	0.0537	1.071	10.60	0.2237	-0.107	-1.92
0.20	0.2138	2.137	11.00	0.1981	-0.468	-5.71

Analytical Solution:

t (sec.)	φ (rad.)	$\dot{\varphi}$ (rad./sec.)	$\ddot{\varphi}$ (rad./sec.^{2})	ψ (rad.)	$\dot{\psi}$ (rad./sec.)	$\ddot{\psi}$ (rad./sec.^{2})
0.10	0.0537	1.070+	10.53	0.2236	-0.108	-1.95
0.20	0.2124	2.089	9.75	0.1976	-0.477	-5.88

It will be noted that at a time equal to one tenth of a second, the two methods give results which are in very close agreement but that at a time equal to two tenths of a second there is a small divergence. This small divergence is due to terms of order higher than the fourth. These higher order terms must be taken into account in the analytical solution for values of t greater than about 0.10 second. When t is smaller than 0.10 second the results given by equations (62), (63), and (64) are sufficiently accurate. Computation of the coefficients of the terms higher than the fourth order would be extremely onerous. The best procedure is to carry out the numerical integration first and follow with a check by the

TABLE III: COMPUTATION OF AMPLITUDE RESULTING FROM RELEASE AT t

$$\varepsilon_o = -\varphi_o - \frac{h\dot{\varphi}_o \cos\varphi_o - l\dot{\psi}_o \sin\psi_o}{u_o}; \quad \dot{\varepsilon}_o = -\dot{\varphi}_o - \frac{h(\ddot{\varphi}_o \cos\varphi_o - \dot{\varphi}_o^2 \sin\varphi_o) - l(\ddot{\psi}_o \sin\psi_o + \dot{\psi}_o^2 \cos\psi_o)}{u_o}$$

V = 140 mi. hr.$^{-1}$ [illegible]$_o$ = 151.50 mi. hr.$^{-1}$ u_o = 222.20 ft. sec.$^{-1}$ Y = 5000 ft.

$$A = \frac{\sqrt{\dot{\varepsilon}_o^2 + 2u_o s\varepsilon_o\dot{\varepsilon}_o + \varepsilon_o^2 u_o^2 n^2}}{u_o\sqrt{n^2 - s^2}}$$

t	$\cos\varphi_o$	$\dot{\varphi}_o$	$\dot{\varphi}_o \cos\varphi_o$	$h\dot{\varphi}_o \cos\varphi_o$	$\dot{\psi}_o$	$\sin\psi_o$	$\dot{\psi}_o \sin\psi_o$	$l\dot{\psi}_o \sin\psi_o$
0.00	1.00000	0.000	0.000	0.000	0.000	0.22593	0.000	0.000
0.01	1.00000	0.108	0.108	0.239	-0.006	0.22593	-0.001	-0.003
0.02	0.99999	0.216	0.216	0.478	-0.013	0.22583	-0.003	-0.009

t	$h\dot{\varphi}_o \cos\varphi_o - l\dot{\psi}_o \sin\psi_o$	$\frac{-h\dot{\varphi}_o \cos\varphi_o - l\dot{\psi}_o \sin\psi_o}{u_o}$	$-\varphi_o$	ε_o	$\ddot{\varphi}_o$	$\ddot{\varphi}_o \cos\varphi_o$	$\dot{\varphi}_o^2$	$\sin\varphi_o$	$\dot{\varphi}_o^2 \sin\varphi$
0.00	0.000	0.0000	0.0000	0.0000	10.79	10.79	0.000	0.00000	0.00
0.01	0.242	-0.0011	-0.0005	-0.0016	10.79	10.79	0.012	0.00050	0.00
0.02	0.487	-0.0022	-0.0021	-0.0043	10.78	10.78	0.047	0.00210	0.00

t	$\ddot{\varphi}_o \cos\varphi_o - \dot{\varphi}_o^2 \sin\varphi_o$	$h(\ddot{\varphi}_o \cos\varphi_o - \dot{\varphi}_o^2 \sin\varphi_o)$	$\ddot{\psi}_o$	$\ddot{\psi}_o \sin\psi_o$	$\dot{\psi}_o^2$	$\cos\psi_o$	$\dot{\psi}_o^2 \cos\psi_o$	$\ddot{\psi}_o \sin\psi_o + \dot{\psi}_o^2 \cos\psi_o$	$l(\ddot{\psi}_o \sin\psi_o + \dot{\psi}_o^2 \cos\psi_o)$
0.00	10.79	23.88	-0.64	-0.14	0.000	0.9741	0.00	-0.14	-0.41
0.01	10.79	23.88	-0.65	-0.15	0.000	0.9741	0.00	-0.15	-0.44
0.02	10.78	23.86	-0.69	-0.16	0.000	0.9742	0.00	-0.16	-0.47

s = 0.0020928

t	$h(\ddot{\varphi}_o \cos\varphi_o - \dot{\varphi}_o^2 \sin\varphi_o) - l(\ddot{\psi}_o \sin\psi_o + \dot{\psi}_o^2 \cos\psi_o)$	$\left\{\frac{h(\ddot{\varphi}_o \cos\varphi_o - \dot{\varphi}_o^2 \sin\varphi_o) - l(\ddot{\psi}_o \sin\psi_o + \dot{\psi}_o^2 \cos\psi_o)}{u_o}\right\}$	ε_o	$\dot{\varepsilon}_o$	$\varepsilon_o\dot{\varepsilon}_o$	$2u_o\varepsilon_o\dot{\varepsilon}_o$	$2su_o\varepsilon_o\dot{\varepsilon}_o$	ε_o^2
0.00	24.29	0.1093	0.0000	-0.109	0.0000	0.00	0.0000	0.00000
0.01	24.32	0.1095	-0.0016	-0.218	0.0003	0.13	0.0003	0.00000
0.02	24.33	0.1095	-0.0043	-0.326	0.0014	0.62	0.0013	0.00002

t	$\varepsilon_o u_o$	$\varepsilon_o^2 u_o^2$	$\varepsilon_o^2 u_o^2 n^2$	$\dot{\varepsilon}_o^2$	$\dot{\varepsilon}_o^2 + 2su_o\varepsilon_o\dot{\varepsilon}_o + \varepsilon_o^2 u_o^2 n^2$	$\sqrt{\dot{\varepsilon}_o^2 + 2su_o\varepsilon_o\dot{\varepsilon}_o + \varepsilon_o^2 u_o^2 n^2}$	A [RAD.]
[illegible]	-0.00	0.0	0.0000	0.0119	0.0219	0.109	0.0428
0.01	-0.36	0.1	0.0000	0.0475	0.0478	0.219	0.0860
0.02	-0.96	0.9	0.0001	0.1063	0.1077	0.328	0.1288

analytical solution until a value of t is reached such that terms of order higher than the fourth can not be neglected. If the values up to that point as obtained by the two methods agree with each other to within an assigned limit of divergence the numerical integration can be considered as checked.

For the computation of amplitude, the aerodynamic coefficients, K_M and $K_H + K_L \frac{k_B^2}{d^2}$ were necessarily assumed as no data were available from which these coefficients could be calculated for the BOMB, Demolition, 1000-lb., M44. The stability test will subsequently make it possible to calculate accurate values of the aerodynamic coefficients for this bomb. However, for the present study, values of K_M and $K_H + K_L \frac{k_B^2}{d^2}$ for bombs of similar shape were used in order to compute the approximate cutting point of the ring necessary to produce the desired amplitude of yaw. The values of K_M and $K_H + K_L \frac{k_B^2}{d^2}$ for the BOMB, Demolition, 1000-lb., M44 were assumed to be the same as those for the M30. The coefficients for the latter bomb are smaller in absolute value than the corresponding coefficients for other bombs of the current M series.

The values of s and n^2 were computed by:

$$n^2 = \frac{K_M \rho d^3}{I_B}$$

and

$$2s = \left[K_L + \frac{K_H d^2}{k_B^2}\right] \frac{\rho_o d^2}{m} .$$

Table III shows the arrangement for the computation of amplitude. The values of ε_o and $\dot{\varepsilon}_o$ are obtained from:

$$\varepsilon_o = -\varphi_\tau - \frac{h\dot{\varphi}_\tau \cos\varphi_\tau - l\dot{\psi}_\tau \sin\psi_\tau}{u_o}$$

and

$$\dot{\varepsilon}_o = -\dot{\varphi}_\tau - \frac{h(\ddot{\varphi}_\tau \cos\varphi_\tau - \dot{\varphi}_\tau^2 \sin\varphi_\tau) - l(\ddot{\psi}_\tau \sin\psi_\tau + \dot{\psi}_\tau^2 \cos\psi_\tau)}{u_o}$$

A is computed from:

$$A = \frac{\sqrt{\dot{\varepsilon}_o^2 + 2u_o s\varepsilon_o\dot{\varepsilon}_o + \varepsilon_o^2 u_o^2 n^2}}{u_o\sqrt{n^2 - s^2}} . \qquad (26)$$

Table III was continued past the value of t at which A became equal to 65° or 1.1345 radians. The value of t for which A became equal to 1.1345 radians was calculated by inverse interpolation. In this case, $t_{A=65°} = 0.2336$ sec. The values of φ_{t_A} and ψ_{t_A} were then computed by direct interpolation. These values were:

$$\varphi_{t_A} = 0.2920 \text{ rad.}$$

$$\psi_{t_A} = 0.1788 \text{ rad.}$$

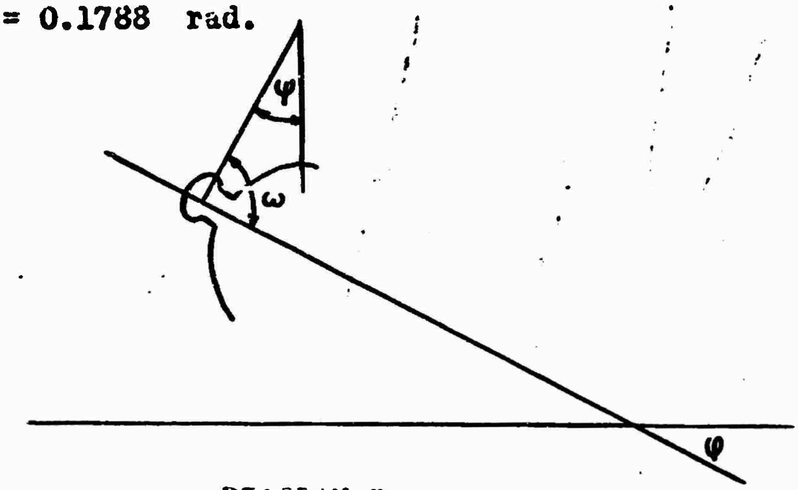


DIAGRAM 9

The angle at which the ring is to be cut is denoted by ω. This angle is shown in Diagram 9. It may be found from Figure 9 that:

$$\omega_o = \varphi_{t_A} + \left(\frac{\pi}{2} - \psi_{t_A}\right). \qquad (65)$$

Substitution of numerical values for φ_{t_A} and ψ_{t_A} leads to:

$$\omega_o = 0.2920 + \left(\frac{\pi}{2} - 0.1788\right)$$

$$= 0.2920 + 1.3920 = 1.6840 \text{ rad.}$$

or $$\omega_o = 96.4861° \qquad (66)$$

The value ω_o, however, does not include the effect of the static friction between the nose plug and the retaining ring on the suspension cord. Dr. A. C. Charters of the Ballistic Research Laboratory has recommended that the cut end of the nose plug be rounded off. The paint on the plug was removed and both the suspension ring and nose plug were filed and then smoothed with No. 1 sandpaper. Finally both the suspension ring and the nose plug were thoroughly lubricated with a graphite base grease. In this manner, the static friction was reduced to a minimum. Dr. Charters also suggested bearing the nose plug on a roller or ball race attached to a thin rod instead of suspension by ring and cord. This would eliminate the static friction and only the effect of the much smaller rolling friction would have to be dealt with. In this instance, however, it is estimated that the static friction will cause approximately 7° delay before the ring begins to slip along the plug. The correct value for the cut angle, ω, is therefore given by

$$\omega_{corr.} = 89.5^{\circ}.$$

~~This~~ report describes a ~~current~~ method for obtaining a large initial amplitude of yaw for tests ~~recently~~ conducted under the direction of Colonel L. E. Simon. The numerical data for the BOMB, Demolition, 1000-lb., M44 have been furnished the writer by Mr. E. S. Martin. The mathematical portions of the work have been verified by Mr. Herbert Roistacher and Mr. Simon Mowshowitz. The numerical integration of the equations of motion has been performed by Miss Elsie Magnuson.

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