EXPERIMENTAL AND THEORETICAL INVESTIGATION OF PAVEMENT DEFLECTIONS

FINAL TECHNICAL REPORT

by

R W KIRWAN principal investigator

T E GLYNN research assistant

Contractor UNIVERSITY OF DUBLIN
ENGINEERING SCHOOL
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EUROPEAN RESEARCH OFFICE

United States Army

CONTRACT NO DAJA37-68-C-1490

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PREFACE

The research described in this report was performed under Contract No. DAJA37-68-C-1490 entitled Experimental and Theoretical Investigation of Pavement Deflections, between the U.S. Army European Research Office and the University of Dublin, Engineering School, Trinity College. The work was conducted for the U.S. Army under Research and Development Project R&D 1537.

The contract represents a continuation of research into behaviour of boulder clay subgrades in flexible pavement design. The work of previous contracts has been extended to include consideration the entire layered system including subgrade.

The sponsorship of the U.S. Army is gratefully acknowledged. We are especially indebted to Messrs. J.F. Appleby and R.J. Ahlvin of U.S. Army Staff for guidance in planning the research. The assistance of Dr. J.C. Byrne and Mr. P.M. Clarke of the T.C.D.Computing Centre is gratefully acknowledged.

ABSTRACT

This report presents a coordinated study involving a laboratory investigation, theoretical analysis and field measurements of stresses induced in flexible pavements by moving wheel loads. The experimental investigation concentrated on improving the simulation of field environmental conditions in laboratory tests. Theoretical work entailed extension of the finite element techniques to computation of permanent displacement in non-linear multilayer pavements. The field work consisted of stress measurements in a two-layer pavement subjected to commercial traffic loading.

The laboratory tests indicated that the modulus of resilience is virtually independent of stress intensity and number of load repetitions provided the boulder clay samples are preconditioned. Conditioning consisted of electro-osmositic saturation and curing at constant moisture content in the current tests.

The finite element program was developed to the stage where either linear or non-linear material properties in any combination can be handled in pavement structures of one to four layers. Results obtained to date in trial problems indicate that the finite element program is working correctly and yields realistic answers to pavement deflections.

A method for eliminating tension is unbound granular layers is presented. Results of the trial problem are given in the report. An investigation of Poisson's ratio for clay soils is included in the section devoted to theoretical analysis.

The results of the set of stress measurements in the subgrade for an unbound gravel layer on a boulder clay subgrace are given. The data have been analysed on a statistical basis. The data are of limited usefulness because displacement measurements were not accomplished.

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CHAPTER I

INTRODUCTION

The research described in this report forms a continuation of the work undertaken in our final report (December 1968) under Contract DAJ A37 - 68 - C - 0206. As in the previous report three aspects related to pavement design are covered herein; laboratory investigations into the behaviour of boulder clay subjected to repeated loading, theoretical analysis of multilayer systems and field measurements of stress and displacement in flexible pavements. The report is presented in three parts corresponding to the current status of the findings in each branch of the project.

Part I.

The experimental work of Part I was performed in the period September 1968 to August 1969. The object of the testing program were as follows:

(i) To seek refinement in techniques of sample preparation, with improved simulation of environmental field conditions in laboratory repeated load tests.

(ii) To investigate the effects of ageing of specimens on the resilient modulus of compacted boulder clays.

(iii) To determine the range of the resilient modulus for single stress levels per specimen as opposed to the stage tests of several increments on the same specimen.

Part. II

This section is a continuation of the theoretical study of multi-layer systems begun under previous contracts.

The development is advanced to the stage where numerical results are available for pavement structures consisting of two to four layers. The analysis provides facility for handling either linear or non-linear material properties in any desired combination. Numerical data for both recoverable and permanent displacements are available: The analysis is based on the finite element method mentioned in our previous report. A readout of the digital computer program with some explanatory annotations is given in Appendix I. The introduction to Part II appears on page 14.

Part III

The work in this section deals with measurement of stress and displacement in a two-layer experimental pavement.

Most of the measurements pertain to the boulder clay subgrade. A brief description of this project appears in our previous report, details of progress to date can be found in Chapters 6 and 7. The introduction to Part III appears on page 48.

Chapter 7, as well as containing conclusions from the investigations, also includes some suggestions for additional research which would appear necessary for future advancement of theoretical pavement design.

CHAPTER 2

EXPERIMENTAL INVESTIGATION OF SOME FACTORS OF INFLUENCE ON RESILIENT MODULUS DETERMINATION.

2. I. General.

The laboratory testing programme was planned with two factors pertaining to repeated load testing in mind.

a) the influence of ageing on the resilient modulus of compacted boulder clay.

b) the influence of stress history on the resilient modulus.

An evaluation of ageing effects is required for setting up routine test procedures, and the influence of stress history has a bearing on the technique to be adopted for conducting repeated load tests. In our previous report only cursory attempts had been made to account for ageing of specimens prior to testing, and most determinations of moduli pertained to stage tests where several increments of load were successively applied to the specimen. The effects on the modulus of applying one deviator stress had not been investigated. That both ageing and stress history should be evaluated for clay samples representative of actual subgrade materials was deemed imperative. The criteria for simulation field compacted boulder clay in service was judged to be the following:

(a) Uniform density throughout specimen.

(b) Degree of saturation to exceed 95%.

(c) Homogenous soil with controlled grading.

Accordingly, the first phase of the programme was to achieve the desired criteria in laboratory prepared samples. The brown boulder clay typical of the upper zone of Dublin area was selected. Thirty samples referred to as the A and B-Series herein were tested.

2. 2. Sample Preparation For Repeated Load Tests.

Grading.

The test specimens were prepared from one bulk sample. The soil was air dried and passed through a 3/8-inch sieve. The natural material is well graded as shown in Particle Size Distribution Chart Fig. 1.

Compaction

Because in practice the moisture content and dry density specified for field compaction are close to the optimum values of the Standard Proctor, the repeated load specimen were moulded dry of Proctor Optimum. Dry densities between Proctor Optimum (117.2 P.C.F.) and Modified A.A.S.H.O. Optimum (127.0 P.C.F.) were employed with an average of 122.0 P.C.F.

The specimens were compacted in six layers with a 5.5. lb. rammer falling 12 inches per blow. To obtain a uniform density, the number of blows varied from layer to layer starting with 30 blows on bottom layer and 21, 22, 23, 24, and 25 blows on remaining five layers.

In the case of the B-Series the compactive effort was increased by using a rammer drop of 18 inches with the same set of blow counts; the tests on the A-Series samples indicated that more consistent trends in moduli determinations are obtained at high densities.

Saturation

The degree of saturation was raised to values exceeding 95% using an improved version of the electro-osmosis cell described in our previous reports. The new cell is shown in Fig 2. Electrophoresis and heating of the specimen were minimised by keeping the current below 150 mA. In general, a potential of 70 volts was found to be sufficient. The pH of the sample was checked before and after a 2 hour treatment - no detectable change in pH was induced in glass electrode PYE pH Meter.

Curing

The ingress of moisture vis electro-osmosis requires some period to permeate the entire clay structure and attain an equilibrium distribution. This is one reason why curing is necessary. Another, is the gain in strength due to thixotropy that is a characteristic of some clays. The overall effects of curing was investigated by testing samples at different ages ranging from 1 to 56 days after compaction.

The specimens were wrapped in cellophane and placed inside a plastic tube 4- inch i.d., the ends of the tube were sealed with plasticene and layers of cellophane. In order to establish the long term efficiency of the ageing cell, a specimen was allowed to stay in one for 9 months. At the end of that period the loss in moisture was 0.1% which is considered negligible.

2.3. Repeated Load Testing.

For the purpose of reporting the results of the repeated load tests, the samples are grouped into A-Series and B-Series tests. The A-Series samples were subjected to a single constant deviator stress per specimen. The deviator stresses employed were 3, 6, 9 and 12 p.s.i. The B-Series were stage loaded in increments of 3 p.s.i. in the range 3 to 12 p.s.i.

The repeated load test equipment is described in our report dated September, 1967. No modifications to the equipment were necessary, but following a thorough checking of the response of rubber and plasticene specimens a defect was found in one triaxial press. Due to fatigue in the springs clamping the proving ring assembly to the hydraulic load capsule, a permanent deviator stress of the order of 2.5 p.s.i. inadvertently acted on the specimen. Unfortunately, the results of the twelve tests were negated before the defect was tracked down. However, the erroneous tests did show that the modulus reaches exaggerately high values if a permanent load accompanies the pulsed deviator stress; it appears, that pre stress raises the modulus of resilience. None of these test results are included herein.

Plots of the Modulus of Resilience-vs-Number of Stress Applications are shown in Figs. 3 through 6.

The A-Series of tests were conducted at six different ageing periods (1, 3, 7, 14, 28, and 56 days). The B-Series were tested after a curing time of 7 days.

2. 4. Special Tests.

A limited number of special static loading tests were run to check the initial magnitude of the deformation modulus. This course was taken because it is difficult to get reliable readings of modulus at less than ten stress applications in the repeated load machine. The results of the static tests are shown in Figs. 7 and 8.

Although intended primarily for initial modulus determinations these tests also provide data on creep under static loading*.

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*The instantaneous modulus was determined in the set of special 'static tests' using a deadweight loading device similar to that described by Spence and Glynn (1962). Electronic recording of deformation is feature of the new machine used in these constant stress tests.

CHAPTER 3.

TEST RESULTS AND DISCUSSIONS.

3. 1. Influence of Ageing of Specimens on Modulus of Resilience.

The modulus of resilience-vs-number of stress applications for different ageing periods is plotted in Fig. 6 . Because, this group of tests were conducted at the same deviator stress and furthermore the moisture content and density is approximately the same for all specimens, the variable "time of ageing" is really evaluated. Referring to Fig. 6 . it is seen that at 1000 stress applications the modulus lies between 2000 and 3000 p.s.i.

The sample aged for 7 days has a modulus of 2500 p.s.i. which represents the mean value of this set. The sample aged for 1 day yields an irregular dependence of modulus on number of stress applications - a trend that is not evident at greater ageing times. In general, higher moduli values pertain to the longer curing periods, but the strength gain is less than 25% in two months. Consequently, the ageing time of 7 days seems entirely adequate for the Dublin boulder clay, and even a 3 day curing period after electro-osmotic saturation will yield a realistic, but somewhat conservative, value for the modulus of resilience. The Dublin boulder clay has a low sensitivity (of the order of 2.5) which undoubtedly accounts for the low dependence of strength on age after moulding. Many boulder clays have low sensitivities, therefore if sensitivity is adopted as an index to curing time, the 7 day ageing period would seem acceptable for glacial tills. It must be emphasised however, that for clays susceptible to thixotropic strength regain, a separate investigation along the lines of the current tests may be needed to access the influence of ageing. In any case, the thixotropic strength regain tends to disappear with repeated stress; applications; the modulus becomes independent of ageing between 1,000 and 10,000 repetitions, Seed et al., (1955).

3.2. Influence of Stress History on Modulus of Resilience.

Some impressions of the influence of stress history on modulus can be gained from the plots of Resilient Modulus-vs- Number of Stress Applications shown in Figs. 3 through 5, and the plots of Instantaneous Modulus-vs-Deviator Stress Fig. 7.

The specimens of the A-Series had no prior stress history whereas stress history enters each deviator stress except the first in the B-Series. The moduli determined in the A and B-test series are in the range 2000 - 3000 p.s.i. The instantaneous module determined in the static tests varies over the same range of values.

The repeated load test results indicate that stress history has only a minor influence on modulus. The trend is one of slight increase in modulus if the specimen had been subjected previously to a lower deviator stress. The static creep tests results shown in Figs. 7 and 8, substantiate this finding. The creep tests exhibit almost the same magnitudes of axial deformation at a given stress intensity irrespective of previous loading sequence.

3. 3. Effect of Dry Density on Modulus of Resilience.

Variations in density can mask the effects of stress history on repeated load test results). If densification accompanies permanent deformation an increase in modulus may be expected. On the other hand, if the permanent deformation is attributable entirely to creep, the modulus will be more or less independent of stress history. To pursue the effect of density on modulus determinations, the plot shown in Fig.9 was prepared. Due to emphasis on control of density and moisture content in this investigation, the data for the modulus-vs-density plot relies on the small range of random departures from the intended density. Nevertheless, these minor variations in density do give a clue to the transition from densification cum creep to purely creep deformation.

On the basis of Fig. 9 it is concluded that the modulus of this soil is independent of dry densities in excess of 122 P.C.F.

3. 4. Effect of Deviator Stress On Modulus of Resilience.

Referring to our previous report and the work of other investigations, Wiffin and Lister (1962) Brown and Pell (1967) Seed et al (1967) it will be noted that earlier determinations of moduli were strongly influenced by magnitude of deviator stress. In general, the greater the magnitude of deviator stress the lower the modulus reported. That this trend has been revoked in the current tests is a significant finding. The current tests indicate that the modulus is virtually independent of deviator stress and number of load repetitions.

The notable exceptions are found in the tests conducted at a deviator stress of 3 p.s.i. but even in the case of low deviator stresses the dependence on stress level is not as pronounced as hitherto reported. The enhanced value of these data are apparently the outcome of sample conditioning prior to repeated load testing. A linear modulus greatly facilitates theoretical analysis of pavement deflections. There are reasons to believe that clays deform resiliently in direct proportion to the applied stress in most field environments.

3. 5. Evaluation of Sample Conditioning Techniques.

The techniques of sample preparation developed in this investigation was motivated by the need to simulate the long term behaviour of clay subgrades in service. It was known beforehand that a flocculated soil structure is obtained as a result of compaction at moisture contents dry of Standard Proctor optimum in most compaction jobs. In temperate moist climates, such as the British Isles or the North West coast region of the United States, there is a tendency for ingress of moisture into compacted fills. It is conservative to assume that the natural moisture content reaches equilibrium at degrees of saturation exceeding 90%

In the current series of tests, the structure, density and final moisture of the repeated load test specimens are presumably representative of many subgrade conditions. However, supplementary tests on existing fill materials are required to confirm this deduction. Unfortunately, the random distribution of coarse aggregate in any fill material makes precise checking difficult. Perhaps X-ray or sonic methods could be used to select samples of existing fill materials.

There is no doubt that the proper conditioning of the freshly moulded soil plays an important role in determination of material parameters. The marked improvement in data trends and consistency of results in this report is thought to be almost entirely due to sample conditioning. Perhaps the most significant single factor is the eradication of stress dependent moduli. This situation is in sharp contrast with our earlier test results, where no concerted effort as made to condition the test specimens.

PART II

CHAPTER 4

DETERMINATION OF STRESS AND DISPLACEMENTS IN FLEXIBLE PAVEMENTS BY NUMERICAL PROCEDURES.

4. 1. Introduction.

The second part of this report deals mainly with an application of the finite element method of numerical analysis to multilayer systems. The main object of the work of this section is to elucidate the role of traffic stress in the distribution of permanent displacement of the pavement surface and individual layers. The analysis has been advanced to the stage where both recoverable and non-recoverable displacements and the complete stress-field for pavement structures consisting of non-linear materials can be determined. Much of the new development is accomplished with the aid of the theoretical relationships derived in Chapters 8 and 9 of our previous report (1968).

Originally the finite element method was devised for elastic analysis of structures with complex boundary conditions, but recent experience with the method has shown its potential in a wider range of problems. While the present endeavour to adapt the finite element technique to non-linear permanent displacement problems was in progress, two papers on topics as widely different as plastic flow and non-linear elasticity appeared.

Meek and Carey (1969) Duncan, Monismith and Wilson (1968). The latter paper included one aspect of the pavement problem namely non-linear behaviour of gravel base courses which had in fact been incorporated by a somewhat different scheme into our analysis.

The diverse range of these investigations and many others attest to superior attributes of the finite element method. However, the homogenous strain condition inherent in the direct stiffness formulation of finite element displacements imposes the restriction that deformations must be infinitesimal in the sense understood in linear elasticity theory. For this reason, the proposed analysis considers the total displacements as being composed of a small recoverable component superimposed on cumulative displacements, each contribution to the permanent displacement being infinitesimal in itself. Another concept basic to our analysis may be stated thus: insofar as mathematical manipulation of non-linear stress-strain relationships is concerned, the cumulative displacement together with instantaneous recoverable displacement after any time interval (or number of load repetitions) may be represented by single valued parametres suitably chosen to reflect the previous loading history of the material. The application of this concept minimises divergence in numerical solution of the large numbers of simultaneous equations common to all finite element formulations. In practice, it means replacing the Incremental

Load and Iteration procedures described by Clough (1968) by a modified iteration procedure.

The digital computer program for the modified iterative process arrives at the solution by cyclic modification of element properties. The element stiffness matrix is adjusted by changing the modulus values assigned to any element in accordance with the previously computered values of stress in the element and the prescribed stress-strain relationship of the material, i.e. the common factor, γ, of the stiffness matrix takes on different values depending upon the stress intensity in an element of the continuum. The magnitude of the secant modulus is constant in linear layers but it is a function of the stress in the case of non-linear layers.

4.2. Manipulation of Materials Properties in Finite Element Analysis.

The properties of the conventional pavement materials may be summarised as follows:-

Bituminous Paving.

Deformation of bituminous materials is highly temperature dependent as shown in Fig. 10 . These data reported by Secor and Monismith (1965) indicates that stress transfer to underlying layers is greatest and more localised in summer. The response of bituminous concrete to constant stress agrees well with linear viscoelastic theory, Pagen (1965). Both effects can be readily assimilated in the finite element analysis. For instance, it appears that the response of bituminous mixes could be expressed in the form:

$$\epsilon_p = m_p \left[\frac{1 + \log_{10} N_J}{1 + \log_{10} N_i} \right] \sigma \qquad (4.1.)$$

where ϵ_p = creep strain [in/in]

m_p = creep compliance $\left[\frac{\text{strain}}{\text{p.s.i.}}\right]$

N_J = Total number of stress applications.

N_i = Number of stress applications to determine m_p, $(1 \leq N_i \leq 10)$.

σ = applied stress (deviator) [p. s.i.]

However, the present computer program is intended for estimating displacements in underlying layers of the pavement, hence only the question of stress transfer is of direct interest at this stage; displacements contributed by creep of bituminous layer is ignored. Therefore, in the work that follows, a single value of the stiffness modulus is assumed for convenience; the stiffness of bituminous mixes is assumed independent of stress over the small variation of stress within a layer for a specified wheel contract pressure.

Granular Base Courses.

The response of granular materials to repeated stress has been

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The factor $\psi = \frac{E_s(1-\nu)}{(1+\nu)(1-2\nu)}$

where E_S = Secant modulus of deformation

ν = Poisson's ratio.

investigated by Biarez (1962), Seed et al (1967). The experimental work indicates that the resilient modulus is strongly dependent on the confining pressure, but the mean stress defined as $\sigma_m = 1/3(\sigma_x + \sigma_y + \sigma_z)$ has a similar unique influence. The mean (or spherical) stress is more satisfactory from a theoretical standpoint; it is the stress component used herein. In terms of the spherical stress the resilient modulus may be written in form:

$$M_r = K\,\sigma_m^{\,n} \qquad (4.2.)$$

where, $0 < n < 1$ and k = constant for any one granular material. The effect of the spherical stress on the modulus of granular soils is shown in Fig. 11.

The inability of granular materials to sustain tensile stresses led to considerable difficulties in the computer program because the effective modulus in the direction of a tensile stress is essentially zero. The tendency to exhibit tensile stresses must be corrected in the case of granular materials.

<u>Cohesive Subgrade Soils.</u>

It appears that displacements seated in clay subgrades are the most significant single cause of pavement distress. Indeed deformation of clays subjected to repeated loading is the main topic of this research. It was shown in our previous report that the behaviour of clays in repeated loading is merely a specialized case of the response to sustained load; the study resulted in a stress-strain relationship which included both the recoverable and creep components of deformation. The expressions derived in Chapter 8 of that report can be organised for ready assimilation into the finite element program as follows:

The resilient modulus, M_r , is assumed independent of the number of stress applications; this conclusion is reached on the basis of the work in Part I of this report. Therefore the recoverable deformation attributable to the clay layer can be expressed in the form:

$$\epsilon_r = \frac{\sigma}{M_r} \qquad (4.3.)$$

where ϵ_r denotes recoverable strain

σ denotes applied stress.

M_r is the resilient modulus

Similarly the deformation can be written as:

$$\epsilon_p = m_p \left[\frac{(1 + \log N_T)^{\sinh\left(\frac{\sigma}{\sigma_o} - 1\right)}}{(1 + \log N_i)^{\sinh\left(\frac{\sigma}{\sigma_o} - 1\right)}} \right] (\sigma - \sigma_o) \qquad (4.4.)$$

where ϵ_p denotes permanent strain

m_p denotes creep compliance $\left[\frac{\text{Strain}}{\text{p.s.i.}}\right]$

N_T is total number of stress applications

N_i is number of stress applications used to determine, m_p.

σ_o denotes threshold stress [p.s.i.]

σ denotes applied stress/induced stress in finite element. For stresses lower than the plastic yield stress the total strain is the absolute sum of the resilient and creep strain viz.

$$\epsilon_t = \epsilon_r + \epsilon_p \qquad (4.\ 5.)$$

The applied stress, σ, can be assigned either the deviator component $(\sigma_{max} - \sigma_{min})$ or the devaitoric tensor $(\sigma_{max} - \sigma_m)$ in deriving the creep compliance, m_p, from test data.

The deviatoric stress possesses more theoretical significance for cohesive materials because shear distortion is a unique function of the deviatoric stress. Hence an option to chance the element stiffness matrix in terms of either deviatoric or deviator components is provided in the program.

4. 3 Unified Approach to Manipulation of Materials Parameters.

The analysis is programmed in such a manner that the first cycle yields only the resilient displacement, δ_r, and 'elastic' stress field, while subsequent cycles produce values of the total displacement, δ_t, and any stress relaxation in overworked elements. The permanent displacement is given by the algebraic difference of displacements in the first and final cycle of the computations. This noteworthy simplification in programming has been effected by lumping the recoverable and non-recoverable deformation into a single parameter for any one material. Cross plots from the strain -

vs-time curves for different stress levels yield the relationship of total deformation to stress intensity. The relationship is assumed to hold identically for test specimen and finite element. The single parameter, secant modulus Es, describes the total deformation up to and including a specified time, t.

In the case of bituminous and clay layers this is achieved by drawing section lines on plots of strain-vs-time for various stress levels. Stress intercepts on the derived curve give an equivalent secant modulus with appropriate limiting stress. Linear viscoelastic materials, such as bituminous mixes, yield a linear relationship while clays give a non-linear stress-vs-strain relationship in any specified loading interval. The method of curve development is shown in Fig. 12.

On the other hand, granular meterials do not exhibit large time dependent deformations, hence the experimental stress-strain relationship or its mathematically equivalent expression can be used directly to give secant moduli-vs-stress level, see Fig. 12. Although mathematical models of materials behaviour facilitates the programming of input date, experimental data present no difficulties.

4. 4. Abridged Description of Finite Element Computer Program.

It is required that the program should be capable of dealing with several layers of material, each with different properties, and any or all of which might be non-linear. The elements which would be used to describe the system would be of the type shown in Fig. 13. This limits the analysis to that of an axi-symmetric solid loaded axi-symmetrically. Solution of the stiffness matrix was to be by successive over relaxation.

The following features were considered desirable.

1. The input should be a minimum consistent with describing the structure and loading adequately, and without imposing restrictions on flexibility. This should help reduce the errors d. to incorrect data.
2. The program should be capable of automatically modifying the stiffness matrix during the analysis of a non-linear system in accordance with the change in stress in each element.
3. It should be possible to limit the number of iterations performed as a safeguard against setting a tolerance for convergence to too small a value.
4. It should be possible to specify a different tolerance for each cycle of the solution of non-linear systems. This is to make it possible to obtain a rough solution in relatively few cycles, make changes to element properties and in the next cycle to have a slightly more accurate solution as fewer element properties are going to be changed. In this way a considerable amount of computer time may be saved.

5. It should be possible to control output at each stage of the solution.

Initially it was hoped that the programs could be incorporated into the I.B.M. System called ICES (Integrated Civil Engineering System) as a subsystem after they had been developed and tested under the RAX time-sharing system. The ICES system is made up of FORTRAN sub-routines but gives dynamic storage facilities together with a very high-level language for each subsystem in ICES. This makes it possible for data to be put in along with descriptions of the data (which may be omitted in certain instances) instead of the usual form of putting figures in certain columns on certain cards. Unfortunately this objective was not implemented.

Limitations

The amount of core available placed a restriction on the size of problem that might be attempted, even using over-relaxation. In its present form the successive over-relaxation program is capable of solving up to 500 equations: that is, the structure may have up to 250 nodes. This appears to be about the maximum that the system will allow.

It was necessary to limit the number of layers in the structure to a fairly small number, and 4 layers was chosen as reasonable. As it was decided to describe the non-linearity of layers by giving explicit stress ranges over which specific values of the resilient modulus or secant modulus modulus were in effect, it was required to limit these in number. It was decided that 5 ranges should give sufficient flexibility.

4.5. Description of Successive Over-Relaxation Program, Dynastco.

A block diagram showing the main sections of the program is given in Fig. 14. The program is designed to solve axisymmetric problems in layered systems with up to four layers. Each layer may be linear or non-linear. The program is capable of modifying material properties in accordance with the stress within each element for non-linear elements using prescribed stress-vs-strain relationships.

The criterion for such changes are prescribed maximum stresses within the element. Solution of the modified structure is then initiated.

A tolerance for the convergence of the iterative over-relaxation process is specified, in the case of non-linear systems a different tolerance may be specified for each cycle. To avoid the possibility of too small a tolerance being set an upper limit for the number of iterations to be performed on each cycle must be specified.

Boundaries and externally applied nodal loads are easily specified in simple forms.

The mainline program consists of two main sections: An input section which also prints input data for checking purposes, and a solution control section which monitors the solution. These two sections are discussed below.

The sub routines have been named sequentially in the fourth position of the name and are of the general form FIN-EL where the gap is occupied by a hexadecimal digit (O through G). It is convenient that there are 16 sub routines. The numbering does not follow any logical sequence through the program so they will be described in numerically ascending order.

FINOEL

Sub-routine to set up matrix, $[\underline{A}^{-1}]$ (See Appendix I). A one dimensional array is used. The upper left quadrant of the matrix is set explicitly and is copied into the lower right quadrant.

FINIEL

Sub-routine to change element stiffness matrix from local to global co-ordinates:

$$[k] = [\underline{A}^{-1}]^T [\bar{k}] [A^{-1}]$$

Two matrix multiplications are involved and a transposition: the former are carried out in FINGEL/MATMN, the latter in the routine itself. Re-organization of the matrix into a partitioned form is also carried out.

FIN2EL

Sub-routine to add element stiffness matrix to the structural stiffness matrix. The element stiffness matrix is multiplied by the constant factor, ψ.

$$\psi = \frac{E_s(1-\nu)}{(1+\nu)(1-2\nu)}$$

which is omitted when setting up the matrix together with the factor of 2π. The structural stiffness matrix is multiplied by this term when all element stiffnesses have been added.

FIN3EL

Sub-routine to perform Successive Over-relaxation. Considerable subscripting is required to calculate the position of entries. Where possible subscripting is reduced to simple incrementing inside loops with full calculation only once before the loop. This is not possible in all cases.

The new values for the deflection vector may be calculated in pairs due to the small coupling between horizontal and vertical deflection at a node. This makes for a smaller program with half the number of loops and half the number of explicit subscript calculation. The saving of time in this section is important as up to 80% of the computing time may be spent in this routine.

Boundary points with fixed (zero) deflections are recognized by the 'diagonal' term being larger than 10^{30}. The calculations are carried out normally until the change in deflection ($\Delta\delta_i$) has been calculated and then the diagonal term is scrutinised. The required chance is made if the test is successful, otherwise this step is skipped. The process is rather wasteful but unless the node is fixed both horizontally and vertically the loop must be completed. The row may not be set to zero with the exception of the diagonal term as in the compressed form symmetry is required. Deleting a row (or column) destroys the symmetry. The diagonal term is not required by other equations so it is used as an indication.

The over-relaxation factor is specified by the user as internal calculation is not possible for equations of the form used.

Convergence is tested for in the following way. The maximum change in value of the deflection vector is computed for each iteration. If this maximum change is less than the specified tolerance iteration is terminated and the number of iterations performed is printed. Control is returned to the mainline program. The maximum number of iterations to be carried out is also specified. Let this be N. The actual number of iterations performed is $N+\frac{N}{10}$ Every $\frac{N}{10}$ iterations the maximum increment in the last iteration is printed together with the number of iterations performed. In this way it is possible to observe the convergence.

FIN4EL

This sub-routine is really a subsidary mainline program. A flow diagram is shown in Fig. 15. The following functions are performed by the program:

1. Set up the stiffness matrix both initially and on subsequent cycles.
2. Determine and print element stresses if required.
3. Alter element properties as necessary for non-linear layers, and give an indication when maximum stresses are exceeded.

4. Indicate the total number of elements that have exceeded the maximum prescribed stresses.

5. Indicate the number of element properties that have been changed.

6. Call the sub-routine FIN5EL to 'fix' the boundaries of the structure.

Three pointers are used to control the program - INIT, PRINT and the linearity indicator for each layer, LINEAR (Layer). The effect of the pointers is to enable certain sections of the program to be skipped when the information generated in that section is not required. Printing of element stresses is also controlled. Alternative paths shown in Fig.15 are determined by these pointers.

The matrix $\left[A^{-1} \right]$ although required in several placed in the program for each element is only generated once. The ordering is such that the matrix is not destroyed until it is required for the last time.

Element stresses are computed using values of the modulus stored in the array ELSTR. For non-linear layers this array is modified on each cycle to give the new values for the secant modulus. The criterion for change is the mean or the deviatoric stresses. As the system is designed for non-tensile materials an indication is given if tension arises in any element that is non-linear. If the maximum stress specified in a non-linear layer is exceeded the modulus for the prescribed stress is assumed and a warning is printed.

FIN5EL

Sub-routine to set boundary points in the stiffness matrix. A value of 10^{40} is placed in each row of the matrix that corresponds to a fixed boundary point. The information is packed in the array BNDRY which is set up from input instructions by the sub-routine FIN9EL. The node number and fixing condition - horizontal, vertical, or both - are placed in one word.

FIN6EL

Sub-routine to form the product prior to stress determination of matrices $[\underline{D}][\underline{B}][\underline{A}^{-1}]$

The product $[\underline{D}][\underline{B}]$ is set explicitly and is post multiplied by $[\underline{A}^{-1}]$. The multiplication is carried out in the sub-routine MATMN.

FIN7EL

Sub-routine to calculate element stresses divided by the constant factor.

$$\psi = \frac{E_s(1-\nu)}{(1+\nu)(1-2\nu)}$$

FIN8EL

Sub-routine to set up the local element stiffness matrix, $\left[\bar{k}\right]$.

FIN9EL

Sub-routine to decode boundary specifications as read in and put them into a form acceptable to FIN5EL.

FINAEL

Sub-routine to decode loading specified and set up the load vector appropriately. Uniform loads are treated as if they were simply supported at the nodes. The routine fails when the maximum limit of a unifrom load exceeds the size of the structure.

FINEEL

Sub-routine to calculate the residual loads at each node in order to obtain an exact solution to the equations as they stand: that is, without altering the deflections. This is a useful guide to the accuracy of the solution as large residual loads imply considerable lack of equilibrium. It is possibly a better guide than the convergence of deflections as small deflections may induce considerable stress changes. It might be possible to specify that iteration is to proceed until the maximum residual load at any node in the structure is less than, say, 1 per cent of the maximum applied load.

FINFEL

Sub-routine to print nodal characteristics at the end of each cycle if required. These include node number, co-ordinates, loads, and

and deflections. An option exists to call FINEEL and print residual loads as well.

FINGEL/MATML

Sub-routine to perform matrix multiplication. Written in 360 Assembly Language this routine is suitable for handling matrices of dimension 8 x 8 only.

MODVAL

Sub-routine to calculate the influence of number of stress applications on the permanent displacement in accordance with Egn(4.4). The number of stress applications and threshold stress are specified within the sub-routine data. The program must be run for each value of stress repetitions required. The inclusion of this sub-routine induces apparent tension in granular materials as the creep displacement of underlying layers produces a local loss of support.

4.6. Tension Corrector for Granular Layers.

Unbound pavement layers present special difficulties in a finite element program for the same reason that closed mathematical solution prove difficult for 'no tension' materials.

One approach is to assume the material to be highly anisotropic with zero (or very small) elastic modulus in the direction indicated by tension and to retain the full value of the modulus in the compressive stress direction, Duncan, Monismith, and Wilson (1968).

Another scheme consists of applying restraining forces at those nodes where tension is indicated in an element, perform an elastic analysis and then re-analyse for a set of equal but opposite 'de-restraining forces'; this is known as the 'Stress Transfer' method, Zienkiewicz et al., (1968).

To implement the zero modulus scheme would require major changes in the program, and the stress transfer method is more complex than necessary in a successive approximation analysis.

By taking advantage of the symmetry of the problem, a simplified system of tension elimination has been devised. The method is solely intended to remove the troublesome tensile stresses in elements close to the axis of the load. In these elements the vertical stress is always compressive but tension appears in hoop and radial directions. Further out from the axis, tension may develop in the vertical stress direction but correction is scarcely worthwhile and has not been implemented here. Starting with virtual work expression for generalised forces β, given by Clough (1968) the tension corrector takes the form:

$$\{\beta\} \quad \left[\int_{vol} [B]^T [D] [B] \right] \{\alpha\}$$

where $\{\beta\}$ denotes generalised nodal force vector.

Substituting the expressions for element stress

$$\{\sigma\} = [D] \{\epsilon\} = [D] [B] \{\alpha\}$$

we may write:

$$\{\beta\} = \left[\int_{Vol} [B]^T \{\sigma_t\} \, dv \right] \qquad (4.\ 6.)$$

where $\{\sigma_t\}$ is the tensile stress of the elastic solution at the geometrical centre of an element.

Equilibrium is automatically satisfied if forces given by above integral are applied at each node of the element. Nodes on the vertical axis are constrained while nodes in (+ve and -ve) radial directions are pulled apart to the positions they should occupy if the material did not sustain tension.

The matrix multiplication and integrations of expression (4. 6.) are given in Appendix II. The computations indicate that vertical as well as horizontal nodal forces are required. However, it appears reasonable to ignore the vertical forces at each node, i.e. to apply only those forces required for horizontal straining of the elements. Thus four unidirectional forces are applied to the nodes of each element where tension registers at its centre. These forces are worked into the analysis in sub-routine FIN4EL. The program must be cycled at a few times for satisfactory relief of tension as the forces applied in one cycle induce tensile stresses in elements previously free of tension. Some changes in numerical factors are made also to minimise 'out of step marching' of tension and creep computation. In general, the tensile stresses are transferred from the lower interface of the granular layer to the underside of the bituminour layers; intuitively this is the correct transposition. In fact, it seems that a more realistic assumption would entail stretching the element until the tension corrector, $\{\beta\}$, induced compressive stress in the element equivalent to the active earth pressure; outwards spreading of particles from the load axis to preserve an active pressure state is more in keeping with the behavious of granular materials subjected to vertical stress. However, reduction of tensile stresses to zero magnitude is tentatively adequate, because it simplifies programming without loss of generality in the procedure.

4.7. Input Section.

The following data are required:

1. Title.
2. The number of layers that are used.
3. For each layer:
   a) The number of rows of elements in the layer and an indication to say whether the layer is linear or not.
   b) Value (s) of secant (or resilient) modulus. Only one value need be specified if the layer is linear. Otherwise 5 values must be specified.

c) If the layer is non-linear, the maximum stress to be allowed for each value of the modulus must be specified, and also the maximum shear stress that is desirable.

d) The value of Poisson's Ratio for the layer. This is a constant even for non-linear layers.

4. The number of elements in each row.

5. The radius of each column of nodes (one more than the number of elements).

6. The vertical ordinate of each row of nodes.

7. The boundary conditions. These consist of:

a) The number of cards specifying boundaries.

b) Five numbers to specify each boundary:

(i) Type declaration: 1. indicates a row.
2. indicates a column.

(ii) Row in which the first node to be set is located.

(iii) Column in which the first node to be set is located.

(iv) The number of nodes to be set.

(v) The boundary condition required:

1. Restrained horizontally.
2. Restrained vertically.
3. Restrained horizontally and vertically.

Note: The input must in the above order and each number separated by at least one space. Input is by means of the "free" format system routine INPUT available on RAX.

8. External loading. This consists of:

   a) The number of separate loadings that are specified.

   b) Five numbers are required to specify each load.

      (i) Type indicator -

         1. Uniform vertical load on a row of nodes.

         2. Uniform horizontal load on a column of nodes.

         3. Point load at a node (horizontal and vertical)

      (ii) Row number of first node to be loaded.

      (iii) Column number of first node to be loaded.

      (iv) Maximum radius (or depth) of loading.
      (types 1 and 2,) or horizontal point load (type 3).

      (v) Loading as a uniform pressure (types 1 and 2), or vertical point load (type 3).

9. Number of cycles to be carried out.

10. For each cycle the maximum number of iterations and a specified tolerance of convergence.

11. Value of Over-relaxation factor.

12. For each cycle specify output requirements:

   1. Output required.

   0. No output.

Six options are available:

(i) Print stresses and Young's Modulus for each element on completion of cycle.

(ii), (iii), (iv), (v) Print graphically radial, tangential, vertical and shear stresses respectively.

(vi) Print nodal characteristics - numbers, co-ordinates, loads and deflections. A two (2) in this position causes residual loads to be calculated and printed also.

Solution Control Section

This section uses information supplied in the input instructions type 10 and 12 to give control to the appropriate sub-routines as the solutions progresses.

Data Structure in Program

The main storage requirement of the program is the stiffness matrix which is stored in a compressed form. The array is of size SF (10,500). The array was made 'horizontal' rather than 'vertical' to allow easy extension to a larger number of equations. Only one statement in the mainline program need be changed if storage is in the 'horizontal' form All sub-routine dimension statements would need to be changed for 'vertical'. Two arrays of size (500) are required for the load and deflection vectors. The current value of the modulus for each element is stored in an array ELSTR (255).

Two 8 x 8 matrices are required for formation of element stiffness matrices and determining element stresses. These serve different purposes in other parts of the program.

The ordinates of the rows and columns of nodes are in two one dimensional arrays of size 20. This limits the number of elements that may be specified in either direction to 19.

Linearity of each layer is stored in the array LINEAR. Value (s) of Young's and Secant Modulus for each layer is stored in the array YMODE. The maximum stress corresponding to these values (for non-linear layers) is stored in LIMITS, and the maximum shear stress for each layer is stored in SHEAR.

The boundary specifications are stored in a packed form in the array BNDRY of size 120. This limits the number of nodes that may be specified as on a rigid boundary to 120.

The maximum number of iterations to be performed on each cycle is stored in REPEAT and the required tolerance in TOLS. Output requirements are stored in OUTPUT. MODVAL has its own input section.

Admittedly the need to insert input date in MODVAL is inconvenient but future work may rectify this defect.

4. 8. Program Checking and Results.

Several checks on the program were made using, for the most part, elastic solutions. For all cases investigated satisfactory agreement between the closed and finite element solution was obtained. In retrospect, it appears that excessive time was spent on this phase, as ancillary programs for these tests are time consuming. Amongst the solutions examined mention may be made of the following.

1. Thick cylinder-vs-Lame Solution.
2. Point load on single layer-vs-Boussinesq Solution.
3. Uniform load on circular area: Ahlvin and Ulery tabulated results.
4. Uniform load on four layer pavement with non-linear elastic properties (Gonzales Bypass) - Duncan, Monismith and Wilson (1968).

The agreement between the published date and test program results can be gauged from inspection of Figs.16 through 18 . It is considered satisfactory from the practical standpoint.

Elastoplastic Solutions.

No published numerical procedures on permanent displacement are available to date. The elastoplastic solution methods proposed by Meek and Carey (1969), Girijavallabhan and Reese (1968), are quite dissimilar to our approach to permanent deformation analysis. Hence it appears that reliance on field measurements is the only option at present.

To ensure that the program was changing the element stiffness matrix in accordance with the prescribed limiting stress criteria, a single homogenous clay layer with a uniform load on a circular area was processed. Some results of this test are shown in Fig. 19.

It can be seen that element properties were changed in a rational pattern and that the displacements are of the correct order. This test and several others for more than one layer indicate that the program yields realistic results. Of course, work on refinements in input data is still in progress. The program supercedes the method of the correspondence principle discussed in our previous report, which is in comparison a crude approximation. Unfortunately, the program makes heavy demands on computer time, up to 20 minutes per run. An effort is underway to reduce machine time by using vertical scanning of data cards and incorporate more Assembly Language.

Typical Results of Permanent Displacement Analysis of Three Layer Pavement.

The trial pavement consists of a bituminous, gravel and clay layer with thicknesses shown in Fig. 20 . Certain simplifying assumptions are included in the program relating to this sample as follows:

1. Full continuity of strains at interfaces, i.e. perfectly rough contact between layers.
2. Gravity stresses are ignored.
3. Tensile stresses in vertical direction may exist.

4. The secant moduli are the same for tension as compression, except where tension is explicitly removed from solution.

5. The flexural rigidity of the bituminous layers is constant.

It appears that items 2, 4, and 5 can be easily revoked for more realistic assumptions. The result for a five cycle run (first run is 'resilient' solution) are shown in Fig. 21 . through . Only the salient quantities are plotted from the complete stress-displacement set of results. Until more experience with the program is gained and further evidence from field measurements is available, the accuracy of the results is not known. However, there is little doubt that the program yields results of the correct order of magnitude. Every effort has been made to make the program flexible to the extent that modifications based on new findings can be introduced without radical changes in the mainline routine; changes where necessary would appear in sub-routines. Although the program is set up to handle four layers, the example deals with a three layer pavement because of output data restrictions in the RAX system.

4.9. Influence of Poisson's Ratio on Stiffness Matrix.

As a result of the earlier theoretical study of the influence of Poisson's Ratio, ν , on the element stiffness matrix, we proposed to investigate experimentally the magnitude of this parameter in the current contract. The experimental work was beset by several difficulties because it was realised that a highly sophisticated technique and instrumentation must be developed if worthwhile experimental data were to emerge. Whilst developing a suitable experimental technique, additional study of earlier measurements on a range of materials was reviewed. In the light our criticisms of analysis based on the assumption $\nu = 0.5$ in the previous report, it is interesting to discover that all values of Poisson's ratio reported in the literature were within the range $0.15 \leq \nu \leq 0.47$ irrespective of the material. For instance india rubber which is sometimes considered incompressible is quoted as having $\nu = 0.45$ Jaeger (1960).

Noteworthy experimental data for tests on ductile metals conducted by Gerard and are reported by Nadai (1963).

These tests results are very interesting and are reproduced in Fig. 22 . Note that the maximum value of Poisson's ratio is 0.46 in these tests.

Furthermore, the so called denominator function, $f(\nu) = 1 - 2\nu^3 - 3\nu^2$ mentioned in our previous report, can yield geometrical interpretations (slopes of tangents and axis intercepts) which seem to indicate a breakdown of conventional analysis when a Poisson's ratio of 0.46 is exceeded. It appears that some materials exhibit mysterious phenomenon akin to microcracking or dislocations before the upper bound on compressibility $\nu = 0{\cdot}5$ is reached.

A particulate substance, such as clay, could conceivably possess non-ideal properties with respect to compressibility, i.e. never possesses $\nu = 0{\cdot}5$ value. Indeed there are reasons to suppose that Poisson's ratio is not a constant when creep accompanies elastic strain. The operative Poisson's ratio, $\hat{\nu}$, consists of the sum of two components, ν' and ν'', on the basis of tests on ductile metals. According to Nadai, the expression relating Poisson's ratio to strain has the form:

$$\hat{\nu} = \frac{\nu' \epsilon_x' + \nu'' \epsilon_x''}{\epsilon_x' + \epsilon_x''} \qquad (4.\ 7.)$$

where ν' and ν'' denote Poisson's ratio corresponding to the elastic ϵ_x', and the permanent, ϵ_x'' components of strain in uniaxial stress field, Nadai, ibid, Vol II pp 31-35.

Presumably a relationship of similar form to expression (4. 7.) holds for cohesive soils.

It appears that no serious difficulty in programming the stiffness matrix would be encountered if Poisson's ratio were introduced in a prescribed manner analogous to expression (4. 7.). Indeed experimental data for stiffness modulus, E, or Poisson's ratio, ν, could be substituted for the functional relationships with few changes in the program. However, functional relationships are preferable because smooth input improves convergence of solutions in a successive over relaxation program.

4.10. Summary

The analysis presented in this chapter replaces the actual multi-layer system by a mathematical model. The corresponding model is the finite element idealisation of the continuum. Moreover, there are other idealisations involved in the analysis. The most notable of these is the idea of combining the instantaneous with the permanent strain accumulated in stipulated number of wheel load applications, N. Mathematically this is equivalent to stating that the total strain materialises immediately the 'N' th wheel load has passed over the structure. The total strain is the algebraic sum of past creep and transient strains, and is governed solely by secant moduli of the materials. Indeed this scheme for including materials properties of widely different characteristics is the distinctive feature of the analysis.

CHAPTER 5

PRACTICAL ASPECTS OF THE FINITE ELEMENT PROGRAM.

5. 1. General Considerations.

Rut formation is a valuable index of pavement performance. A pavement can be considered structurally adequate for a specified wheel load if the depth of rut caused by repeated application of that wheel load reaches a tolerable value which does not increase with additional load applications. The other important index of performance is the degree of surface crazing caused by cyclic flexing of the bituminous wearing layer caused by resilient (and permanent) displacements. Evidently, any method of analysis that provides a quantitative assessment of these phenomena is likely to prove an important contribution to our understanding of the problem of pavement design. Indeed, the average pavement is a critical structure insofar as rather minor departures from an over conservative design can adversely affect the performance.

Therefore, selection of the correct layer thicknesses and a realistic evaluation of the materials properties become an essential part of successful design. The finite element program makes it relatively easy to compare the effects of varying layer thicknesses and provides excellent scope for incorporating materials properties; we think that it deserves the prominence accorded in this report. For the first time, it now appears feasible to obtain a complete picture of the stresses and time-dependent displacements, including rut depths in a multilayer structure on a clay subgrade.

Notwithstanding the limitations of numerical approximation procedures, the finite element approach offers unique practical attributes.

5.2. Limitations.

The most serious limitation is that the analysis is restricted to two dimensional problems with the axially symmetric problem merely a special case. No truly three-dimensional stiffness matrix suitable for inclusion in the finite element analysis of multilayer systems has emerged. A three dimensional matrix proposed by Melosh (1963) is inordinately complex and no substitutes have come to the author's attention. This situation means that the important practical case of a wheel load close to the edge of an unpaved shoulder is not solvable.

However, there is a definite possibility of finding an upper bound to the displacement by placing the wheel load on a truncated cone of sandwich layers as shown in Fig. 23.

A second limitation is involved in the assumption of infinitesimal strains and its corollary i.e. that only displacement functions which are linear combinations of the generalised displacements are admissable. This limitation does not seriously affect the analysis in the case of traffic lanes adjacent to the centre line, but it has detrimental implications in analysis of lanes close to the edge of the pavement. It means that spreading of granular layers and flow of clay subgrades are not amenable to the analysis. Recently, we have attempted to set up a finite strain theory to overcome the problem of horizontally limited layered systems. Unfortunately the effort has not led to useful conclusions, and it is not included in this report.

Consolidation and densification are two additional factors which may contribute to permanent displacement of the pavement surface. Neither phenomenon are included in the current analysis despite the fact that in certain cases they may cause pavement distress.

CHAPTER 6

FIELD MEASUREMENTS - BLESSINGTON TEST PAVEMENT.

6. 1. Introduction.

This Chapter deals with an attempt to measure stresses and displacements in an experimental pavement under commercial traffic loading. Some notes on the experiment were included in our previous report. Although this project has proved difficult to control with University resources of staff and mobile facilities, we have made some progress in long-term field measurements. The original scheme was to construct a three-layer pavement using a bituminous wearing course on a gravel layer with a boulder clay subgrade. The subgrade and base course were placed last year, but due to a number of difficulties with instrumentation no bituminous layer has been laid. For reasons yet to be determined we have failed to obtain satisfactory measurements of displacement. On the other hand, we have had considerable success with stress measurements.

6. 2. Layout and Operation of the Experimental Pavement.

The layout of the Blessington test loop is shown in Fig. 24. All measurements to date pertain to this two-layer system consisting of graded gravel on a compacted boulder clay subgrade. The sensors are placed in the subgrade six inches below the gravel layer. Two load cells and two displacement transducers are placed at each of two locations denoted by 'L' and 'R' in Fig. 24. Thus horizontal and vertical components of the stresses for two thicknesses of gravel cover are provided.

The track has been subjected to quarry traffic over the past year. It is not known how many passes of the various types of vehicle using the route have been accumulated. During construction the upper 15 inches of the subgrade experienced heavy rain which made the track incapable of supporting heavy vehicles. In the interval between construction and the present set of measurements the subgrade gained stability and can now support fully laden trucks without distress. It seems that re-distribution of the pore water, rather than consolidation or densification, is responsible for the improved strength.

The original transducers were removed and replaced by improved versions of the designs described in our previous report. The measurement pertain to the wheel configurations shown in Fig. 25. Merely for identification, the trucks are denoted by R-ST. and C.I.E., the former is a long wheel base and the latter is a short wheel base truck.

6.3. Two-Layer System. Stress Measurements.

During the Summer of 1969 a total of three hundred readings of the stresses imposed by the R-ST. and C.I.E. trucks were taken at different times. This series of tests proved the long-term capabilities of the stress measuring system. On the other hand, the new displacement transducers did not function properly. The defect is though to be caused by a pollysulphide sealant that appears to have penetrated the cable, or the telescopic plate assembly, and immobilised the core drive. Although the trouble arose at an early stage, it was decided to proceed with the stress measurements and later return to the displacement question.

Typical results of the stress measurements are given in Tables I and II. The set of data were analysed by statistical methods mainly because the position of the wheel tracks relative to the sensor locations was not accurately known. This tracking problem led to the invention of the location finder described later in this chapter. However, the statistical analysis indicated good correlation between vertical and horizontal stress measurements. The correlation is demonstrated in the plot regression analysis shown in Fig. 26. where a coefficient of correlation better than 0.9 emerged from the whole set of data. However, the horizontal stresses are less than anticipated. Further checking of cell calibrations are needed to clarify this finding. Until more data are obtained, it is premature to use the field measurements for checking the finite element analysis.

6. 4. Description of Location Finder.

This device simply consists of a length of resistance wire stretched over a gauge sleeper as shown in Fig. 27. The resistance wire is connected into one arm of the bridge of the amplifier demodulator cum U.V. oscillograph. When the wire is depressed onto the sleeper by a passing vehicle the drop in electrical resistance produces an event mark on the trace which serves as a precise indication of the position of the actuator. The assembly could be mounted inside the conventional traffic signal pressure pad.

The device is still in laboratory trials stage where experiments on water proofing are in progress. The location finder will minimise dependence on statistics because in principle every stress reading can be related to the location of the wheel.

CHAPTER 7.

SUMMARY AND CONCLUSIONS.

The presentation is a coordinated study involving theoretical considerations, laboratory experiments, and field measurements. This policy is considered to be a balanced approach which offers many advantages in building up an understanding of pavement behaviour: it is hoped to pursue it in future work.

The following conclusions can be drawn from this research:-

1. With suitable control, the resilient modulus of compacted boulder clays determined in laboratory tests is virtually independent of stress intensity, and number of stress applications.

2. The control involves simulation of field conditions. It appears that electro-osmotic saturation and ageing of specimens are suitable simulation techniques.

3. An ageing period of one week after saturation is adequate for boulder clays of low sensitivity (sensitivities less than 3).

4. Stress history has only a slight effect on modulus determinations provided that the density and degree of saturation are high enough to eleminate deformation attributable

to densification.

5. The finite element method of analysis has been adapted to estimate both the recoverable and non-recoverable components of displacement in multilayer systems. Some computations for axisymmetrical loading on a three layer pavement are presented.

6. Theoretical investigations indicate that for saturated clay Poisson's ratio does not exceed 0.47.

7. A unified approach to handling materials properties in the finite element analysis is given. It entails prescribing the secant modulus of deformation of bituminous, granular, and cohesive layers. The secant moduli are easily derived from laboratory tests.

8. Stress measurements in an experimental two-layer pavement are presented. The data is analysed on a statistical basis. Good correlation between vertical and horizontal stresses in the subgrade is obtained.

The intrinsic value of the finite element analysis suggests that laboratory test procedures should be dictated by the requirement of ready assimilation of the data into the computer program. As an example of this idea the deformation characteristics of granular materials whould be related to the mean stress in reporting test results.

Similarly for clays the testing should be organised such that deformation in terms of the deviatoric stress component is the end result.

Checking of theoretical analysis against field data is of paramount importance; it is hoped to advance this aspect of the research in future work.

TABLE I

RECORD OF STRESS MEASUREMENTS ON BLESSINGTON TEST TRACK.

The following data were derived from histograms of almost 300 stress measurements. The modes of the histograms yielded the following combinations of vertical and horizontal stresses for the wheel loads as indicated:

Stress Measurements for R-ST and C.I.E. trucks.

Vert. Stress V. p.s.i.	Horiz. Stress H. p.s.i.	Contact Pressure p.s.i.	Contact Area Sq. ins.	Speed M.P.H.
3.20	0.45	90	67	10
7.25	0.95	"	152	10
1.50	0.30	"	31	6
3.20	0.35	"	52	6
6.65	0.90	"	127	6
6.00	0.85	"	118	6
1.65	0.25	"	67	10
4.50	0.65	"	152	10
0.70	0.15	"	22	6
1.95	0.20	"	54	6
3.00	0.55	"	129	6
2.80	0.50	"	117	6

Legend

Contact areas determined on basis of wheel load distributions and gross weight.

Contact pressure equal to inflation pressure.

Speed is average value of 12 checks.

TABLE II

STATISTICAL ANALYSIS OF STRESS MEASUREMENTS.

Applying statistical methods to data in Table I, the correlation coefficient is given by the following expressions:

Standard Deviation $$S_x = \sqrt{\frac{\sum_1^n x_i^2 - n\bar{x}^2}{n-1}}$$

$$\Sigma V^2 = 199.61 \quad , \quad \Sigma H^2 = 3.93$$
$$n\bar{V}^2 = 149.52 \qquad n\bar{H}^2 = 3.12$$
$$n = 12$$

$$S_V = \sqrt{\frac{199.61 - 149.52}{11}} \quad , \quad S_H = \sqrt{\frac{3.93 - 3.12}{11}}$$

$$S_V = 2.133 \qquad S_H = 0.268$$

D_x is deviation of , x_s from mean $\bar{x}$

$$\Sigma D_V = 0 \qquad \Sigma D_H = 0 \qquad \Sigma D_V D_H = 6.231$$

$$S_{VH} = \frac{\Sigma D_V D_H}{n-1} = 0.566$$

$$R = \frac{S_{VH}}{S_V S_H} = 0.99 \longleftarrow$$

The correlation coefficient, R, is very close to unity which indicates that a direct correlation exists between vertical and horizontal stresses in the boulder clay subgrade.

APPENDIX I

SUMMARY OF EXPRESSIONS AND READOUT OF FINITE ELEMENT PROGRAM.

The finite element method is now well documented, therefore a brief outline only of the mainline program and sub-routines is presented herein. The program produces solutions for pavement structure consisting of two to four layers. In its present state of development it is restricted to solution of axially symmetric problems of stress distribution; the important case of horizontally limited layers encountered in unpaved road shoulders is not covered in the present program. Three procedures for solving the set of simultaneous equations were investigated - namely, Gaussian elimination, Choleski triangulation and Successive Over Relaxation. The latter showed definite computational advantages and is adopted for the multilayer program in this report. For the sake of completeness, the expressions given in our previous report (1968) are listed in this Appendix as follows:

1, Express internal displacements, δ, in terms of an assumed displacement function M, where the amplitudes of the displacement function are represented by generalised coordinates, α:

thus:

$$\{\delta(r,z)\} = [M(r,z)]\{\alpha\} \qquad (A.1)$$

2. Evaluate nodal displacement components δ_i - merely a substitution of the coordinates of the nodal points into the displacement function matrix $[M]$ - to obtain:

$$\{\delta_i\} = [A]\{\alpha\} \qquad (A.2)$$

3, Express the generalised coordinates in terms of nodal displacements by matrix inversion;-

$$\{\alpha\} = [A^{-1}]\{\delta_i\} \qquad (A.3)$$

4. Evaluate the element strains, ϵ ,

$$\{\epsilon(r,z)\} = [B(r,z)]\{\alpha\} \qquad (A.4)$$

5. Evaluate the element stress, σ,

$$\{\sigma(r,z)\} = [D]\{\epsilon(r,z)\} = [D][B]\{\alpha\}$$

(A.5)

The specific elastic characteristics of the element material are represented by the stress-strain matrix $[D]$.

6. Compute the generalised coordinate stiffness of the element $[\bar{k}]$ from the expression:

$$[\bar{k}] = \int_{Vol} [B^T][D][B]\,dv$$

(A.6)

7. Transform to the nodal point stiffness, $[k]$ by the congruent transformation:

$$[k] = [A^{-1}]^T [\bar{k}][A^{-1}]$$

(A.7)

8. Superpose the individual element stiffnesses contributing to each nodal point to obtain the total nodal stiffness matrix $[K]$

where
$$[K] - \sum_{1}^{n} [k]$$

(A.8)

9. Assemble and solve the equilibrium equations expressing the relationship between the applied nodal forces $\{R\}$ and the resulting nodal displacements in the form:

$$\{R\} = [K]\{\delta_i\}$$

(A.9)

10 Evaluate the stresses components, σ, from the expression

$$\{\sigma\} = [D][B][A^{-1}]\{\delta_i\}$$

(A.10)

It is convenient to split δ_i into two components of displacement u_i, v_i.

Referring to Figure 13 b, a compatible displacement function for the rectangular element shown is:

$$\{\delta\} = \begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} 1 & r & z & zr & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & r & z & zr \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \vdots \\ \alpha_8 \end{Bmatrix}$$

(A.11)

Expression (A.11) satisfies the homogenous strain condition because the element strains will be linear function of the coordinates for either r or z equal to constants.

The $[A]$ matrix is obtained by introducing the nodal coordinates into the displacement functions, with the following result for the element illustrated:

$$\begin{Bmatrix} u_i \\ u_j \\ u_k \\ u_l \\ v_i \\ v_j \\ v_k \\ v_l \end{Bmatrix} = \begin{bmatrix} 1 & a_i & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & a_j & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & a_i & b & a_i b & 0 & 0 & 0 & 0 \\ 1 & a_j & b & a_j b & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & a_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & a_j & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & a_i & b & a_i b \\ 0 & 0 & 0 & 0 & 1 & a_j & b & a_j b \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_7 \\ \alpha_8 \end{Bmatrix} \qquad (A.12)$$

Element Strains $[B]$

$$\begin{Bmatrix} \epsilon_r \\ \epsilon_\theta \\ \epsilon_z \\ \gamma_{rz} \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & z & 0 & 0 & 0 & 0 \\ \frac{1}{r} & 1 & \frac{z}{r} & z & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & r \\ 0 & 0 & 1 & r & 0 & 1 & 0 & z \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \vdots \\ \alpha_8 \end{Bmatrix} \qquad (A.13)$$

The element properties are defined by the stress-strain matrix $[D]$. For the case of isotropic material with elastic modulus, E, and Poisson's ratio, ν, it takes the form:

Stress - Strain Relationship $[D]$

$$\begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \\ \tau_{rz} \end{Bmatrix} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & 1 & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 1 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \begin{Bmatrix} \epsilon_r \\ \epsilon_\theta \\ \epsilon_z \\ \gamma_{rz} \end{Bmatrix} \qquad (A.14)$$

Finally, the element stiffness, $[\bar{k}]$ in the generalised coordinate system, is obtained by the integration of:

$$[\bar{k}] = \left[\int_{vol} [B^T][D][B]\,dv\right]$$

(A. 6.) (A. 15.)

The author has performed the matrix multiplications and explicit integration of expression (A. 15) the result is given in Fig. 28. The expression for the transformed stiffness $[k]$ is given by:

$$[k] = 2\pi\psi\,[A^{-1}]^T[\bar{k}][A^{-1}]$$

(A. 16.)

LISTING OF

SUCCESSIVE OVER RELAXATION PROGRAM

DYNASTCO

```
/JOB GO,TIME=10
/FTC LIST
      REAL SF(10,500),A(8,8),S(8,8),Z(20),R(20),LOAD(500),DEF(500)
      REAL NU(4),YMODE(4,5),LIMITS(4,5),SHEAR(4),TOLS(5),ELSTR(225)
      INTEGER LINEAR(4),BNDRY(120),ROWS(5),IN(5),OUTPUT(5,7)
      INTEGER REPEAT(5),PRINT,PICTUR(600)
      EQUIVALENCE (SF(2000,1),PICTUR(1))
C
C       MAINLINE PROGRAM
C
C       READ AND WRITE TITLE
    1 READ (5,27) (A(I,1),I=1,20)
   27 FORMAT (20A4)
      WRITE (6,28) (A(I,1),I=1,20)
   28 FORMAT ('1',T20,20A4//)
C       READ NUMBER OF LAYERS IN STRUCTURE
      READ (5,2) NLAYER
      WRITE (6,31) NLAYER
   31 FORMAT (////'   THERE ARE ',I2,' LAYERS IN THE STUCTURE.'/)
    2 FORMAT (7I2)
C       READ NUMBER OF ROWS IN AND CHARACTERISTICS OF EACH LAYER
      DO 5 I=1,NLAYER
      READ (5,2) ROWS(I),LINEAR(I)
      WRITE (6,32) I,ROWS(I)
   32 FORMAT (//'LAYER',I2,' HAS ',I3,'ROWS.')
      READ (5,3) (YMODE(I,J),J=1,5)
    3 FORMAT (10F6.0)
      IF (LINEAR(I))39,4,39
   39 WRITE (6,33) YMODE(I,1)
   33 FORMAT('OYOUNGS MODULUS:',T20,5F10.2)
      GO TO 41
    4 READ (5,3) (LIMITS(I,J),J=1,5),SHEAR(I)
```

```
      WRITE (6,33) (YMODE(I,J),J=1,5)
      WRITE (6,34) (LIMITS(I,J),J=1,5),SHEAR(I)
   34 FORMAT (' LIMITING STRESSES:',T20,5F10.2/'MAX. SHEAR STRESS:',T20,
     1F10.2)
   41 READ (5,3) NU(I)
    5 WRITE (6,35) NU(I)
   35 FORMAT ('OPOISSON''S RATIO:',F6.3)
      I=2
C      SUM THE ROWS TO OBTAIN CUMULATIVE TOTALS
    6 IF(ROWS(I))8,8,7
    7 ROWS(I)=ROWS(I)+ROWS(I-1)
      I=I+1
      GO TO 6
    8 MM=ROWS(NLAYER)
      M=MM+1
C      READ NO. OF ELEMENTS IN EACH ROW
      READ(5,2) NN
      WRITE (6,36) MM,NN
   36 FORMAT ('OTOTAL NUMBER OF ROWS:',T27,I3/' TOTAL NUMBER OF COLUMNS:
     1',T27,I3)
      N=NN+1
C      DETERMINE NO. OF EQUILIBRIUM EQUATIONS
      NO=2*M*N
C      INITIALISE LOAD AND DEFLECTION VECTORS
      DO 9 I=1,NO
      DEF(I)=0.
    9 LOAD(I)=0.
C      READ RADIUS OF EACH COLUMN OF NODES
      READ (5,3) (R(I),I=1,N)
      WRITE (6,37) (R(I),I=1,N)
   37 FORMAT (///' RADIUS OF NODES IN EACH COLUMN:'/2(T10,10F6.2/))
C      READ VERTICAL ORDINATE OF EACH ROW OF NODES
```

```
      READ (5,3) (Z(I),I=1,M)
      WRITE(6,38) (Z(I),I=1,M)
   38 FORMAT ('OVERTICAL ORDINATE OF NODES IN EACH ROW:'/2(T10,10F6.2/))
C       READ BOUNDARY CONDITIONS AND SET 'BNDRY' TO DESCRIBE THESE
      WRITE (6,42)
   42 FORMAT (///' BOUNDARY CONDITIONS:'/)
      READ (5,2) NOFIX
      K=1
      DO 10 I=1,NOFIX
      CALL INPUT(5,IN(1),IN(2),IN(3),IN(4),IN(5))
      WRITE (6,43) (IN(J),J=1,5)
   43 FORMAT ('0',T20,5I4)
   10 CALL FIN9EL(IN,BNDRY,N,K)
C       READ EXTERNAL LOADS AND SET UP LOAD VECTOR
      WRITE (6,44)
   44 FORMAT (///' APPLIED LOADING:'/)
      READ (5,2) NLOADS
      DO 11 I=1,NLOADS
      CALL INPUT(5,IN(1),IN(2),IN(3),S(1,1),S(2,1))
      WRITE (6,45) (IN(J),J=1,3),S(1,1),S(2,1)
   45 FORMAT (' ',T20,3I4,2F8.3)
   11 CALL FINAEL(IN,S,N,LOAD,R,Z)
C       READ NUMBER OF CYCLES TO BE CARRIED OUT
      READ (5,2) NCYCLE
      WRITE (6,46) NCYCLE
C       READ MAXIMUM NUMBER OF ITERATIONS AND SPECIFIED LIMIT OF CONVERGENCE FOR
C       EACH CYCLE
   46 FORMAT (///'NUMBER OF CYCLES:',I5//' NO. OF ITERATIONS',5X,
     1'TOLERANCE'/)
      READ (5,12) (REPEAT(I),TOLS(I),I=1,NCYCLE)
   12 FORMAT (I3,F10.0)
      WRITE (6,47) (REPEAT(I),TOLS(I),I=1,NCYCLE)
```

```
   47 FORMAT (' ',T6,I4,T23,F10.6)
C       READ OVER-RELAXATION FACTOR
      READ (5,3) ORF
   48 FORMAT ('OOVER-RELAXATION FACTOR:',F6.3////)
      WRITE (6,48) ORF
C       READ OUTPUT REQUIREMENTS FOR EACH CYCLE
      DO 13 I=1,NCYCLE
      READ (5,2) (OUTPUT(I,J),J=1,6)
      OUTPUT(I,7)=0
      DO 13 J=2,5
   13 OUTPUT(I,7)=OUTPUT(I,7)+OUTPUT(I,J)
      LAYER=1
      IEL=1
C       DETERMINE INITIAL VALUES OF YOUNG'S MODULUS FOR EACH ELEMENT
C       PLACE RESULT IN EMODE
      DO 20 I=1,MM
      K=1
   14 IF(LINEAR(LAYER))17,15,17
   15 IF(LIMITS(LAYER,K))16,17,17
   16 K=K+1
      GO TO 15
   17 DO 18 J=1,NN
      ELSTR(IEL)=YMODE(LAYER,K)
   18 IEL=IEL+1
      IF(ROWS(LAYER)-I)19,19,20
   19 LAYER=LAYER+1
   20 CONTINUE
C       INITIALISE POINTERS
      NTIMES=NCYCLE+1
      INIT=1
      PRINT=0
C
```

```
C     SOLUTION CONTROL SECTION
C      COMPLETE THE SOLUTION FOR NCYCLES ONLY
C      AN EXTRA CYCLE IS STARTED TO ALLOW FINAL EVALUATION OF STRESS
      DO 26 I=1,NTIMES
C      SET UP THE STIFFNESS MATRIX
      CALL FIN4EL(SF,A,S,NU,YMODE,LIMITS,LINEAR,DEF,ROWS,R,Z,
     1ELSTR,BNDRY,SHEAR,MM,NN,NO,PRINT,INIT)
      PRINT=OUTPUT(I,1)
C      IS THIS THE LAST TIME?
      IF(NCYCLE-I)26,21,21
C      SET TOLERANCE AND MAXIMUM NUMBER OF ITERATIONS FOR THIS CYCLE
   21 TOL=TOLS(I)
      MAXNO=REPEAT(I)
      DO 30 K=1,NO
   30 DEF(K)=DEF(K)*.75
C      CALL RELAXATION ROUTINE
      CALL FIN3EL(SF,NN,NO,LOAD,DEF,TOL,ORF,MAXNO)
C      ARE DEFLECTIONS ETC. REQUIRED ?
      IF(OUTPUT(I,6))23,23,22
C      CALL ROUTINE FOR DEFLECTION AND RESIDUAL LOAD CALCULATION WITH PRINT OUT
   22 K=OUTPUT(I,6)
      CALL FINFEL(SF,LOAD,DEF,M,N,R,Z,K)
C      IS GRAPHIC OUTPUT REQUIRED ?
   23 IF(OUTPUT(I,7))26,26,24
   24 DO 25 J=1,4
   25 IN(J)=OUTPUT(I,J+1)
C      CALL GRAPHIC OUTPUT ROUTINE FOR REQUIRED OUTPUT
      CALL FINBEL(SF,PICTUR,A,S,DEF,IN,MM,NN,R,Z,NU,YMODE,ELSTR,ROWS)
   26 CONTINUE
C      GO TO BEGINNING AND READ INPUT DATA FOR NEXT PROBLEM
      GO TO 1
      END
```

```
/FTC LIST
      SUBROUTINE FIN3EL(SF,NN,NO,LOAD,DEF,TOL,ORF,MAXNO)
      REAL SF(10,1),LOAD(1),DEF(1),INCR
C
C      RELAXATION SUBROUTINE
C
C      SET POINTERS
C
      NOITS=MAXNO/10
      NCYCLE=0
C
C      START ITERATION
C
   15 INCR=0.
      DO 31 I=1,NO,2
C
C      CALCULATE PAIR OF CHANGES TO DEFLECTION VECTOR
C
      DEL=-LOAD(I)
      ALT=-LOAD(I+1)
      K=I-1
      DO 17 J=1,4
      K=K+1
      IF(K-NO)16,16,20
   16 DEL=DEL+SF(J,I)*DEF(K)
   17 ALT=ALT+SF(J,I+1)*DEF(K)
      K=I+2*NN-1
      DO 19 J=5,10
      K=K+1
      IF(K-NO)18,18,20
   18 DEL=DEL+SF(J,I)*DEF(K)
   19 ALT=ALT+SF(J,I+1)*DEF(K)
```

```
   20 K=I
      DO 22 J=3,4
      K=K-1
      IF(K)25,25,21
   21 DEL=DEL+SF(3,K)*DEF(K)
   22 ALT=ALT+SF(4,K)*DEF(K)
      K=I-2*NN+2
      DO 24 J=5,10
      K=K-1
      L=(J-1)/2*2+1
      IF(K)25,25,23
   23 DEL=DEL+SF(L,K)*DEF(K)
   24 ALT=ALT+SF(L+1,K)*DEF(K)
C
C       IS THE POINT RESTRIANED HORIZONTALLY ?
C
   25 IF(SF(1,I)-1.E30)26,28,28
C
C       SET DEFLECTION VECTOR TO NEW VALUE
C
   26 DEF(I)=DEF(I)-ORF*DEL/SF(1,I)
C
C       CHECK FOR MAXIMUM INCREMENT ON CURRENT CYCLE
C
      IF(ABS(DEL/SF(1,I))-INCR)28,28,27
   27 INCR=ABS(DEL/SF(1,I))
C
C       IS THE POINT RESTRIANED VERTICALLY ?
C
   28 IF(SF(2,I+1)-1.E30)29,31,31
C
C       SET DEFLECTION VECTOR TO NEW VALUE
```

```
C
   29 DEF(I+1)=DEF(I+1)-ORF*ALT/SF(2,I+1)
C
C       CHECK FOR MAXIMUM INCREMENT ON CURRENT CYCLE
C
      IF(ABS(ALT/SF(2,I+1))-INCR)31,31,30
   30 INCR=ABS(ALT/SF(2,I+1))
   31 CONTINUE
      NCYCLE=NCYCLE+1
      IF(NCYCLE/NOITS*NOITS-NCYCLE)33,32,33
   32 WRITE (6,36) NCYCLE,INCR
   36 FORMAT ('0   NUMBER OF CYCLES =',I5/'LARGEST INCREMENT ON LAST CYC
     1LE =',E15.6)
C
C       TEST FOR TOO MANY ITERATIONS
C
      IF(NCYCLE-MAXNO)33,33,34
C
C       TEST FOR CONVERGENCE
C
   33 IF(INCR-TOL)34,34,15
   34 WRITE(6,35) NCYCLE
   35 FORMAT(1HOT20,'NUMBER OF CYCLES =',I6)
      RETURN
      END
```

```
N D COMPILER                                                        11/25/69
      SUBROUTINE FIN4EL(A,S,NU,YMODE,LIMITS,LINEAR,ROWS,AI,BI,      11/25/69
     1ELSTR,BNDRY,SHEAR,MM,NN,NO,PRINT,INIT,LOAD)
      REAL A(8,1),S(8,1),YMODE(4,1),LIMITS(4,1),AI(1),BI(1),SHEAR(1)
       REAL NU(1),ELSTR(1),SS(20,20),LOAD(1)
      INTEGER LINEAR(1),BNDRY(1),ROWS(1),ROWNO,SFAIL,PRINT,IN(3)
      COMMON DEF(500),SF(10,500)
C
C   SUBROUTINE TO DETERMINE ELEMENT STRESSES AND (RE)CONSTITUTE STIFFNESS
C     MATRIX 'SF'.
C
C       INITIALISE STIFFNESS MATRIX AND SET POINTERS
      N=NN+1
      ICOL=2*N+3
      DO 35 J=1,10
      DO 35 I=1,NO
   35 SF(J,I)=0.
      IN(1)=2
      DO 1 I=1,NO,2
    1 LOAD(I)=0.
      LAYER=1
      ROWNO=1
      IEL=1
      ICHANG=2
      NCHANG=0
      NFAIL=0
      SFAIL=0
C      TEST FOR PRINTING
      IF(INIT)30,27,30
```

```
27 IF(PRINT)28,30,28
28 WRITE (6,29)
29 FORMAT('1    ELEMENT STRESSES'/'     ================'//T25,'RADIAL'
  1,T40,'HOOP',T55,'VERTICAL',T70,'SHEAR',T85,'MODULUS',T100,'SMEAN',
  2T114,'SDEV'/'LAYER NO. 1'/)
C      DETERMINE STRESSES IN EACH ELEMENT AND SET UP STIFFNESS MATRIX
30 DO 55 I=1,MM
   Z=ABS(BI(I+1)-BI(I))
   Z1=Z/2.
   POISON=NU(LAYER)
   DO 51 J=1,NN
   RI=AI(J)
   RJ=AI(J+1)
   R=(RI+RJ)/2.
   CALL FINOEL(A,RI,RJ,Z)
   IF(INIT)50,36,50
36 IF(PRINT)34,26,34
26 IF(LINEAR(LAYER))50,34,50
C      DETERMINE STRESSES IN ELEMENT
34 CALL FINGEL(A,R,Z1,S,POISON)
   CALL FINTEL(ICOL,ROWNO,S)
   DO 39 K=1,4
39 S(K,1)=S(K,1)*ELSTR(IEL)*(1.-POISON)/((1.+POISON)*(1.-2.*POISON))
   IF(PRINT)31,33,31
31 SMEAN=(S(1,1)+S(2,1)+S(3,1))/3.
   WRITE (6,32) IEL,(S(K,1),K=1,4),ELSTR(IEL),SMEAN
32 FORMAT ('      ELEMENT NO ',I3,6F15.4)
33 IF(LAYER-2)100,110,100
```

```
110 IF(S(1,1)) 50,50,111
111 IN(2)=I
    IN(3)=J
    SS(I,J)=S(2,1)*(RJ-RI)*Z*6.284
    SS(I,J+1)=(S(2,1)+S(1,1))*(RJ-RI)*(RJ+RI)*Z*3.142
    SS(I+1,J)=S(2,1)*(RJ-RI)*Z*6.284
    SS(I+1,J+1)=(S(2,1)+S(1,1))*(RJ-RI)*(RJ+RI)*Z*3.142
    IF(J-1) 113,113,112
112 CALL FINAEL(IN,SS,N,LOAD,AI,BI)
113 IN(3)=J+1
    SS(I,J)=SS(I,J+1)
    CALL FINAEL(IN,SS,N,LOAD,AI,BI)
    GO TO 669
100 IF(LINEAR(LAYER)) 70,70,50
C     CHANGE ELEMENT PRCPERTIES IF REQUIRED
 70 SMIN=-AMAX1(S(1,1),S(2,1),S(3,1))
    SMAX=-AMIN1(S(1,1),S(2,1),S(3,1))
    SMEAN=(S(1,1)+S(2,1)+S(3,1))/3.
    SDEV=-(SMAX+SMEAN)
    WRITE (6,101) SMEAN,SDEV
101 FORMAT ('+',T98,2F12.4)
    IF(SMIN) 37,67,67
 37 WRITE (6,38) IEL
 38 FORMAT('    TENSICN IN ELEMENT ',I3)
    IF(LAYER-2) 68,68,69
 68 GO TO 669
 69 SMAX=-SDEV
C   DETERMINE EFFECTS OF LOAD REPETITIONS
```

INPUT FOR TENSION CORRECTION IN GRANULAR LAYERS

```
      CALL MODVAL(E,SMAX,NCHANG)
      ELSTR(IEL)=E
      GC TC 50
  669 SMAX=-SMEAN
      E=ELSTR(IEL)
      DO 65 JSTR=1,5
      IF(ELSTR(IEL)-YMODE(LAYER,JSTR))65,40,65
   65 CONTINUE
   40 IF(SMAX-LIMITS(LAYER,JSTR))44,45,41
   41 ICHANG=2
      NCHANG=NCHANG+1
      JSTR=JSTR+1
      IF(JSTR-5)40,40,42
   42 WRITE(6,43) IEL
   43 FORMAT(1H 'COMPRESSION FAILURE IN ELEMENT ',I3)
      JSTR=5
      NFAIL=NFAIL+1
   44 IF(JSTR-1)47,47,45
   45 IF(SMAX-LIMITS(LAYER,JSTR-1))46,46,47
   46 ICHANG=2
      NCHANG=NCHANG+1
      JSTR=JSTR-1
      GC TO 44
   47 IF(ABS(S(4,1))-SHEAR(LAYER))66,48,48
   48 WRITE (6,49) IEL
   49 FORMAT(1H 'SHEAR FAILURE IN ELEMENT ',I3)
      SFAIL=SFAIL+1
   66 ELSTR(IEL)=(YMODE(LAYER,JSTR)*2.+ELSTR(IEL))/3.
   50 E=ELSTR(IEL)
C      SET UP STIFFNESS MATRIX
```

```
      CALL FINBEL(S,RI,RJ,Z,POISON)
      CALL FIN1EL(A,S)
      CALL FIN2EL(A,S,ROWNO,N,POISON,E)
      IEL=IEL+1
   51 ROWNO=ROWNO+2
      IF(I-ROWS(LAYER))55,52,52
   52 LAYER=LAYER+1
      IF(PRINT)55,55,53
   53 WRITE (6,54) LAYER
   54 FORMAT('OLAYER NO.',I2/)
   55 ROWNO=ROWNO+2
      GO TO (56,58),ICHANG
   56 IF(INIT)62,63,62
C       PRINT DETAILS OF ELEMENT CHANGES
   63 WRITE (6,57)
   57 FORMAT(1HO'NO ELEMENT PROPERTIES CHANGED')
      GO TO 60
   58 WRITE(6,59) NCHANG
   59 FORMAT(1HOI4,' ELEMENT PROPERTIES CHANGED')
   60 WRITE (6,61) NFAIL,SFAIL
   61 FORMAT(1HOI4, ' ELEMENTS FAILED IN COMPRESSION'/I5,' ELEMENTS FAIL
     1ED IN SHEAR')
C       CALL BOUNDARY FIXING ROUTINE
   62 CALL FIN5EL(BNDRY)
      INIT=0
      RETURN
      END
```

```
ST                                                                              11/25/69
TRAN D COMPILER                                                                11/25/69
         SUBROUTINE MODVAL(SLOPE,SIGMA,NCHANG)
         REAL NI,NJ
         SINH(X)=.5*(EXP(X)-EXP(-X))
         NI=3.
         NJ=7.
         SIGMAO=3.
         IF(SIGMA-SIGMAO)2,2,1
       1 EP=.0015*(SIGMA-SIGMAO)*(NJ/NI)**SINH(SIGMA/SIGMAO-1.)+SIGMA/2000.
         SLOPE=(2*SIGMA/(1000.*EP)+SLOPE)/3.
         NCHANG=NCHANG+1
         RETURN
       2 SLOPE=2.
         RETURN
         END
            SIZE OF COMMON   00000    PROGRAM   C0738
COMPILATION   MODVAL
```

$NI = (1 + \log_{10} N_i)$, $NJ = (1 + \log_{10} N_J)$
$N_i = 100$ $N_J = 10^6$ repetitions here

INPUT THRESHOLD STRESS, σ_o = 3 here

INPUT MODULUS OF RESILIENCE = 2000 here

INPUT CREEP COMPLIANCE m_p = 0.0015 here

```
/FTC LIST
      SUBROUTINE FINOEL(A,RI,RJ,B)
      REAL A(1)
C  SUBROUTINE TO SET UP INVERSE OF 'A' MATRIX
C  RI=INNER RADIUS,   RJ=OUTER RADIUS,   B='HEIGHT' OF ELEMENT
C       MATRIX 'A' IS ONE DIMENSIONAL FOR CONVENIENCE
C       INITIALISE MATRIX TO ZERO
      DO 1 I=1,64
    1 A(I)=0.
C       SET  VALUES IN UPPER LEFT QUADRANT
      A(2)=1./(RI-RJ)
      A(1)=-RJ*A(2)
      A(4)=-A(2)/B
      A(3)=-RJ*A(4)
      A(9)=A(2)*RI
      A(10)=-A(2)
      A(11)=RI*A(4)
      A(12)=-A(4)
      A(19)=-A(3)
      A(20)=A(12)
      A(27)=-A(11)
      A(28)=A(4)
C       COPY UPPER LEFT QUADRANT INTO LOWER RIGHT QUADRANT
      DO 2 I=1,28
    2 A(I+36)=A(I)
      RETURN
      END
```

```
/FTC LIST
      SUBROUTINE FINIEL(A,S)
      REAL A(8,1),S(8,1)
C   SUBROUTINE TO CALCULATE (A-INVERSE)TRANSPOSE*K*(A-INVERSE)
C   MATRIX A CONTAINS A-INVERSE,   MATRIX 'S' CONTAINS LOCAL STIFFNESSES
C       FORM MATRIX PRODUCT |S||A|
      CALL MATMN(S,8,8,A,8)
C       TRANSPOSE MATRIX 'A'
      DO 3 I=1,8
      DO 3 J=I,8
      X=A(I,J)
      A(I,J)=A(J,I)
    3 A(J,I)=X
C       FORM MATRIX PRODUCT  |A||S|
      CALL MATMN(A,8,8,S,8)
C       REORGANISE ROWS AND COLUMNS OF 'A' FOR DIRECT INSERTION INTO
C       STIFFNESS MATRIX
      DO 4 I=1,4
      DO 4 J=1,8
      S(2*I-1,J)=A(I,J)
    4 S(2*I,J)=A(I+4,J)
      DO 5 J=1,4
      DO 5 I=1,8
      A(I,2*J-1)=S(I,J)
    5 A(I,2*J)=S(I,J+4)
      RETURN
      END
```

```
/FTC LIST
      SUBROUTINE FIN2EL(SF,A,S,ROWNO,N,NU,YMODE)
       REAL SF(10,1),A(8,1),NU
      INTEGER S(1),ROWNO,COLNO
C
C  SUBROUTINE TO ADD INDIVIDUAL STIFFNESSES
C  SF IS GENERAL STIFFNESS MATRIX,   A CONTAINS NODAL STIFFNESS TO ADD
C  S IS USED TO CALCULATE COL. NUMBERS IN SF,   ROWNO INDICATES ROW AT
C      WHICH TO START ADDING,   N IS THE NO. OF  NODES PER ROW
C  NU IS THE VALUE OF POISSONS RATIO,   YMODE IS THE VALUE OF YOUNGS MOD
C
C       INITIALISE POINTERS
C
      DO 6 I=1,4
    6 S(I)=I
      DO 7 I=5,8
    7 S(I)=I+2
C
C       COMPUTE FACTOR TO MULTIPLY MATRIX 'A' WITH
C
      TEMP=YMODE*(1.-NU)/((1.+NU)*(1.-2.*NU))
C
C       MULTIPLY MATRIX 'A' BY FACTOR TEMP
C
      DO 8 I=1,64
    8 A(I,1)=A(I,1)*TEMP
      DO 11 I=1,4,2
      M=ROWNO
C
C       ADD THE MATRIX 'A' TO THE GENERAL STIFFNESS MATRIX 'SF'
C
      DO 10 J=1,8
```

```
      COLNO=S(J)
      IF(COLNO) 10,10,9
    9 SF(COLNO,M)=SF(COLNO,M)+A(I,J)
      SF(COLNO,M+1)=SF(COLNO,M+1)+A(I+1,J)
   10 S(J)=S(J)-2
   11 M=M+2
      M=M+2*N-4
      DO 14 I=5,8,2
      DO 13 J=1,8
      S(J)=S(J)-2
      COLNO=S(J)
      IF(COLNO) 13,13,12
   12 SF(COLNO,M)=SF(COLNO,M)+A(I,J)
      SF(COLNO,M+1)=SF(COLNO,M+1)+A(I+1,J)
   13 CONTINUE
   14 M=M+2
      RETURN
      END
```

```
/FTC LIST
      SUBROUTINE FINSEL(SF,BNDRY)
      REAL SF(10,1)
      INTEGER BNDRY(120),ROWNO
C  SUBROUTINE TO SET BOUNDARY CONDITIONS.  EXECUTED BY MAKING STIFFNESS
C     AT NODE  V.LARGE TO GIVE RIGIDITY IN A PARTICULAR DIRECTION.
C              FIXITY :-  1 - HORIZ. FIXED  2 - VERTICAL FIXED
C                         3 - BOTH HORIZ. AND VERTICAL FIXED
C      NODE NUMBER IN FIRST 30 BITS, FIXING CONDITION IN LAST 2 BITS
      DO 66 I=1,120
      ROWNO=BNDRY(I)/4*2-1
      K=BNDRY(I)-(ROWNO+1)*2
      GO TO (62,64,63),K
   62 SF(1,ROWNO)=1.E40
      GO TO 65
   63 SF(1,ROWNO)=1.E40
   64 SF(2,ROWNO+1)=1.E40
   65 IF(BNDRY(I+1))67,67,66
   66 CONTINUE
   67 RETURN
      END
```

```
/FTC LIST
      SUBROUTINE FIN6EL(AF,R,Z,D,V)
      REAL AF(8,8),D(8,8)
C      SUBROUTINE TO SET UP MATRIX PRODUCT |D||AF| FROM WHICH ELEMENT STRESSES
C      MAY BE CALCULATED
C      SET MATRIX 'D'
      DO 10 I=1,64
   10  D(I,1)=0.
      D(4,6)=(1-2*V)/2./(1.-V)
      D(1,7)=V/(1.-V)
      D(2,7)=D(1,7)
      D(1,8)=D(1,7)*R
      D(2,8)=D(1,8)
      D(3,7)=1.
      D(3,8)=R
      D(4,8)=D(4,6)*Z
      D(1,1)=D(1,7)/R
      D(1,2)=1./(1.-V)
      D(1,3)=D(1,1)*Z
      D(1,4)=D(1,2)*Z
      D(2,1)=1./R
      D(2,2)=D(1,2)
      D(2,3)=D(2,1)*Z
      D(2,4)=D(1,4)
      D(3,1)=D(1,1)
      D(3,2)=2.*D(1,7)
      D(3,3)=D(1,3)
      D(3,4)=Z*D(3,2)
      D(4,3)=D(4,6)
      D(4,4)=D(4,3)*R
C      PERFORM MULTIPLICATION
      CALL MATMN(D,4,8,AF,8)
      RETURN
      END
```

```
/FTC LIST
      SUBROUTINE FIN7EL(REACT,ICOL,IROW,STK)
      REAL REACT(1),STK(8,8)
      INTEGER ROW(8)
C
C      SUBROUTINE TO CALCULATE ELEMENT STRESSES
C
C      SET POINTERS
C
      ROW(1)=IROW
      ROW(2)=IROW+2
      ROW(3)=IROW+ICOL-3
      ROW(4)=ROW(3)+2
      DO 10 I=1,4
   10 ROW(I+4)=ROW(I)+1
C
C      CALCULATE STRESSES
C
      DO 30 I=1,4
      STRS=0.
      DO 20 J=1,8
      ICL=ROW(J)
   20 STRS=STRS+STK(I,J)*REACT(ICL)
   30 STK(I,1)=STRS
      RETURN
      END
```

```
/FTC LIST
      SUBROUTINE FINBEL(S,R1,R2,Z,D)
      REAL S(8,8)
C       SUBROUTINE TO SET STIFFNESS MATRIX OF ELEMENT IN LOCAL CO-ORDINATES
C       SET CONSTANTS
      B=1./(1-D)
      A=B*D
      C=(1.-2.*D)/2./(1.-D)
      R=R2-R1
       RP2=R2*R2-R1*R1
      RP4=R2**4-R1**4
      RP3=R2**3-R1**3
      ZP2=Z*Z
      ZP3=Z*ZP2
       DO 10 I=1,8
      DO 10 J=1,8
   10 S(I,J)=0.
C       SET LOWER TRIANGLE
C       CHECK FOR ZERO RADIUS
       IF(R1)20,30,20
   20 S(1,1)=Z*ALOG(R2/R1)
      S(3,1)=S(1,1)*Z/2.
      S(3,3)=ZP3*ALOG(R2/R1)/3.
   30 S(2,1)=B*R*Z
      S(4,1)=B*R*ZP2/2.
      S(7,1)=A*R*Z
      S(4,4)=B*RP2*ZP3/3.+C*RP4*Z/4.
      S(8,1)=A*RP2*Z/2.
      S(2,2)=B*RP2*Z
      S(3,2)=B*R*ZP2/2.
      S(4,2)=B*RP2*ZP2/2.
      S(7,2)=A*RP2*Z
```

```
      S(8,2)=A*RP3*Z*2./3.
      S(3,3)=S(3,3)+C*RP2*Z/2.
       S(7,7)=RP2*Z/2.
       S(7,3)=S(4,1)*D
       S(3,3)=(A+C)*RP2*ZP2/4.
      S(7,4)=S(4,2)*D
      S(8,4)=(2.*A+C)*RP3*ZP2/6.
       S(6,3)=C*S(7,7)
      S(6,6)=S(6,3)
      S(8,6)=C*RP2*ZP2/4.
      S(8,7)=RP3*Z/3.
      S(6,4)=S(8,7)*C
      S(4,3)=B*R*ZP3/3.+S(6,4)
      S(8,8)=C*RP2*ZP3/6.+RP4*Z/4.
C      COPY LOWER TRIANGLE INTO UPPER TRIANGLE
      DO 35 J=1,8
      DO 35 I=J,8
   35 S(J,I)=S(I,J)
      RETURN
      END
```

```
/FTC LIST
      SUBROUTINE FIN9EL(SF,BNDRY,N,I)
      INTEGER SF(1),BNDRY(1)
C       SF(1)= TYPE.    ROW=1       COLUMN=2
C       SF(2)=ROWNO.
C       SF(3)=COLUMN NO.
C       SF(4)=NO. OF POINTS TO BE SET
C       SF(5)=FIXING CONDITION.    HORIZ.=1    VERT.=2      BOTH=3
C       SET BNDRY SO THAT FIRST 30 BITS CONTAIN NODE NUMBER, LAST
C       TWO BITS CONTAIN FIXING CONDITION.
      J=SF(1)
      K=((SF(2)-1)*N+SF(3))*4+SF(5)
      M=SF(4)
      GO TO (80,82),J
   80 DO 81 L=1,M
      BNDRY(I)=K
      K=K+4
   81 I=I+1
      RETURN
   82 DO 83 L=1,M
      BNDRY(I)=K
      K=K+4*N
   83 I=I+1
      RETURN
      END
```

```
/FTC LIST
      SUBROUTINE FINAEL(IN,S,N,LOAD,A,B)
      REAL S(1),A(1),B(1),LOAD(1),LOAD1
      INTEGER IN(1)
C
C     IN(1) IS TYPE INDICATOR.     1=ROW   2=COLUMN   3=POINT LOAD
C     IN(2) IS ROW NUMBER OF FIRST POINT.
C     IN(3) IS COLUMN NUMBER OF FIRST POINT.
C     S(2) IS MAXIMUM RADIUS(DEPTH) OF LOADING, OR HORIZONTAL POINT LOAD
C     S(3) IS UNIFORM LOAD AS PRESSURE , OR VERTICAL POINT LOAD
C
C      DETERMINE ROW NUMBER OF FIRST NODE
C
      K=((IN(2)-1)*N+IN(3))*2-1
      J=IN(1)
      GO TO (90,95,100),J
C
C     VERTICAL LOADING IN A PLANE
C
   90 I=IN(3)
   91 R1=A(I)
      R2=A(I+1)
      IF(S(1)-R2)92,93,93
   92 R2=S(1)
   93 WT=S(2)*(R2*R2-R1*R1)*3.141593
      LOAD1=WT*(R1+R2+R2)*(R2-R1)/(3*(R1+R2)*(A(I+1)-R1))
      LOAD(K+1)=LOAD(K+1)+WT-LOAD1
      LOAD(K+3)=LOAD(K+3)+LOAD1
      I=I+1
      K=K+2
      IF(A(I)-S(1))91,94,94
   94 RETURN
```

```
C
C     HORIZONTAL LOAD ON CYLINDER
C
   95 I=IN(2)
   96 J=K+2*N
      R2=B(I+1)
      IF(R2-S(1))98,98,97
   97 R2=S(1)
   98 WT=S(2)*(R2-B(I))*6.283185
      LOAD1=WT*(R2-B(I))*.5/(B(I+1)-B(I))
      LOAD(J)=LOAD(J)+LOAD1
      LOAD(K)=LOAD(K)+WT-LOAD1
      I=I+1
      K=K+2*N
      IF(B(I)-S(1))96,99,99
   99 RETURN
C
C     POINT LOAD ON A NODE
C
C      HORIZONTAL LOAD
C
  100 LOAD(K)=LOAD(K)+S(1)
C
C      VERTICAL LOAD
C
      LOAD(K+1)=LOAD(K+1)+S(2)
      RETURN
      END
```

```
/FTC LIST
      SUBROUTINE FINEEL(SF,NN,NO,LOAD,DEF)
       REAL SF(10,1),LOAD(1),DEF(1)
C      SUBROUTINE TO CALCULATE RESIDUAL LOADS
C      AT EACH NODE TO GIVE COMPLETE EQUILIBRIUM.
C      SF - STIFFNESS MATRIX.
C      LOAD - LOAD VECTOR
C      DEF - DEFLECTION VECTOR.
C      NN - NUMBER OF FINITE ELEMENTS IN EACH ROW.
C      NO - NUMBER OF ROWS IN STIFFNESS MATRIX
C      RESIDUAL LOADS ARE RETURNED IN SF(1,I).
C      BASICALLY THE SAME ROUTINE AS FIN3EL.
      DO 31 I=1,NO,2
C      FOR ODD NUMBERED ROWS, CALCULATE HORIZONTAL RESIDUAL LOADS
      IF(SF(1,I)-1.E30)151,231,231
  231 SF(1,I)=0.
      GO TO 22
  151 DELTA=LOAD(I)
      DO 18 J=1,4
      K=I+J-1
      IF(K-NO)16,16,18
   16 DELTA=DELTA-SF(J,I)*DEF(K)
      DO 17 J=5,10
      K=I+2*NN+J-5
      IF(K-NO)17,17,18
   17 DELTA=DELTA-SF(J,I)*DEF(K)
   18 DO 19 J=3,4
      K=I-J+2
      IF(K)21,21,19
   19 DELTA=DELTA-SF(3,K)*DEF(K)
      DO 20 J=5,10
      K=I-J-2*(NN-3)
```

```
      L=(J-1)/2*2+1
      IF(K)21,21,20
   20 DELTA=DELTA-SF(L,K)*DEF(K)
   21 SF(1,I)=DELTA
C      FOR EVEN NUMBERED ROWS, CALCULATE VERTICAL RESIDUAL LOADS
   22 IF(SF(2,I+1)-1.E30)232,23,23
   23 SF(1,I+1)=0.
      GO TO 31
  232 DELTA=LOAD(I+1)
      DO 24 J=1,4
      K=I+J-1
      IF(K-NO)24,24,26
   24 DELTA=DELTA-SF(J,I+1)*DEF(K)
      DO 25 J=5,10
      K=I+2*NN+J-5
      IF(K-NO)25,25,26
   25 DELTA=DELTA-SF(J,I+1)*DEF(K)
   26 DO 27 J=3,4
      K=I-J+2
      IF(K)  29,29,27
   27 DELTA=DELTA-SF(4,K)*DEF(K)
      DO 28 J=5,10
      K=I-J-2*(NN-3)
      L=(J+1)/2*2
      IF(K)29,29,28
   28 DELTA=DELTA-SF(L,K)*DEF(K)
   29 SF(1,I+1)=DELTA
   31 CONTINUE
      RETURN
      END
```

```
/FTC LIST
      SUBROUTINE FINFEL(SF,LOAD,DEF,M,N,R,Z,K)
      REAL SF(10,1),LOAD(1),DEF(1),R(1),Z(1)
      INTEGER EQUALS
C
C      SUBROUTINE TO PRINT NODAL CHARACTERISTICS.
C      SF - STIFFNESS MATRIX.
C      LOAD - LOAD VECTOR
C      DEF - DEFLECTION VECTOR.
C      M - NUMBER OF ROWS OF NODES.
C      N - NUMBER OF COLUMNS OF NODES.
C      R - ARRAY CONTAINING RADIUS OF EACH COLUMN OF NODES
C      Z - ARRAY CONTAINING Z-ORDINATE OF EACH NODE.
C      K - INDICATOR :-    K=1, RESIDUAL LOADS NOT REQUIRED,
C                          K=2, RESIDUAL LOADS REQUIRED.
C
      EQUALS=2118139968
      K=K-1
C
C      PRINT HEADING
C
      WRITE (6,170) (EQUALS,I=1,21)
  170 FORMAT ('1',T20,'NODAL CHARACTERISTICS.'/T20,21A1/)
      IF(K) 173,173,171
C
C      PRINT HEADING FOR FULL CHARACTERISTICS, K=2.
C
  171 WRITE (6,172) (EQUALS,I=1,53)
  172 FORMAT('0',T20,'CO-ORDINATES',T46,'INITIAL LOADS',T72,'DEFLECTIONS
     1',T98,'RESIDUAL LOADS'/T5,'NODE NO.',T20,12A1,T46,13A1,T72,11A1,
     2T98,14A1/T5,8A1,T16,'R-ORDINATE  Z-ORDINATE     HORIZONTAL    VERTI
     3CAL      HORIZONTAL    VERTICAL      HORIZONTAL    VERTICAL'/)
```

```
      NN=N-1
      NO=2*M*N
      CALL FINEEL(SF,NN,NO,LOAD,DEF)
      GO TO 175
C
C       PRINT HEADING FOR PARTIAL CHARACTERISTICS, K=1.
C
  173 WRITE (6,174)  (EQUALS,I=1,44)
  174 FORMAT('0',T20,'CO-ORDINATES',T46,'INITIAL LOADS',T72,'DEFLECTIONS
     1'/T5,'NODE NO.',T20,12A1,T46,13A1,T72,11A1/T5,8A1,T16,'R-ORDINATE
     2 Z-ORDINATE    HORIZONTAL    VERTICAL     HORIZONTAL    VERTICAL'/)
  175 IROW=1
      NODE=1
C
C       PRINT REQUIRED DATA.
C
      DO 180 I=1,M
      DO 180 J=1,N
      IF(K) 177,177,176
  176 WRITE (6,178)  NODE,R(J),Z(I),LOAD(IROW),LOAD(IROW+1),DEF(IROW),DEF
     1(IROW+1),SF(1,IROW),SF(1,IROW+1)
      GO TO 179
  177 WRITE (6,178)  NODE,R(J),Z(I),LOAD(IROW),LOAD(IROW+1),DEF(IROW),DEF
     1(IROW+1)
  178 FORMAT (' 'T5,I4,T12,2F12.3,T38,2F12.3,T66,2F12.4,T92,2F12.7)
  179 IROW=IROW+2
  180 NODE=NODE+1
      RETURN
      END
```

```
V2L0   SOURCE

         FINELG START 0
                ENTRY FINGEL
                DC    CL6'FINGEL'
                DC    X'0'

                CNOP  0,4
         *
         *                                  FORTRAN CALL
         *                                  ============
         *
         *                                  CALL FINGEL(NN,NO,LOAD,ORF,MAXNO)
         *
         *                                  COMMON DEF(500),SF(10,500)
         *
000008          USING *,15
         FINGEL STM   14,12,12(13)
                ST    13,SAVE13
         *                                  STORE OVER-RELAXATION FACTOR
                L     2,12(1)
                L     2,0(2)
                ST    2,ORF
                L     2,0(1)                LOCATE 'NN'
                L     2,0(2)
                SLA   2,3                   FORM 'NN5'=2*NN *4(BYTES)
                ST    2,NN5
                L     2,4(1)                LOCATE NO
                L     3,0(2)
                SLA   3,2                   FORM NO*4 IN GPR 3
         *                                  LOAD 'MAXNO' INTO GPR 0 FOR COUNTING
```

```
         L      2,16(1)
         L      0,0(2)
         L      7,8(1)                     ADDRESS OF LOAD(1) IN GPR 7
*                                          FORM 'BAKSTP'=(2*NN)*10 *4(BYTES)
*                                          .            = NN5*10
         LA     9,10
         SR     8,8
         M      8,NN5
         ST     9,BAKSTP
*                                          INITIALISE GPR'S 8,9,12 CONSTANTS
         LA     10,8                       GPR 10= 8
         LR     2,10                       GPR 2 = 8
         LCR    12,10                      GPR 12=-8
         AR     3,12                       GPR 3 = NC-1 FOR 'BXLE' INSTRUCTION
*                                          BEGIN NEW ITERATION
*                                          INITIALISE POINTERS
AGAIN    SR     9,9                        GPR 9 POINTS TO LOAD
         LA     6,2000(4)                  GPR 6 POINTS TO SF(1,'I')
*                                          BEGIN LOOP FOR NEW PAIR OF DEFLECTIONS
LOOP1    LR     5,6
         SDR    0,0
         SDR    2,2
         LE     0,0(7,9)
         LE     2,4(7,9)
         LCER   0,0                        DEL = -LOAD(I)
         LCER   2,2                        ALT = -LOAD(I+1)
```

V2L0 SOURCE

```
         LR    8,9              FOR DEF(I) TO BE INCREMENTED
         LA    11,8(9)          SET LOWER LIMIT FOR LOOP
*                               DO LOOP NUMBER ONE
TESTK1   CR    9,3              IS INCREMENT > 'NO' ?
         BC    2,END1           IF YES THEN END LOOP
         LE    4,0(5)
         ME    4,0(4,8)
         ADR   0,4              DEL = DEL + ...
         LE    4,40(5)
         ME    4,0(4,8)
         ADR   2,4              ALT = ALT + ...
         LE    4,4(5)
         ME    4,4(4,8)
         ADR   0,4              DEL = DEL + ...
         LE    4,44(5)
         ME    4,4(4,8)
         ADR   2,4              ALT = ALT + ...
         AR    5,10             SKIP 2 ROWS IN SF
         BXLE  8,10,TESTK1      INCREMENT DEF('J')
*                               SET POINTERS FOR LOOP 2
         L     8,NN5
         AR    8,9
         LA    11,16(8)
*                               START LOOP 2
```

```
TESTK2 CR     8,3                    IS INCREMENT > 'NO' ?
       BC     2,END1                 IF YES THEN END LOOP
       LE     4,0(5)
       ME     4,0(4,8)
       ADR    0,4                    DEL = DEL + ...
       LE     4,40(5)
       ME     4,0(4,8)
       ADR    2,4                    ALT = ALT + ...
       LE     4,4(5)
       ME     4,4(4,8)
       ADR    0,4                    DEL = DEL + ...
       LE     4,44(5)
       ME     4,4(4,8)
       ADR    2,4                    ALT = ALT +
       AR     5,10                   INCREMENT SF BY 2 ROWS
       BXLE   6,10,TESTK2
END1   LR     8,9
       AR     8,12                   ROW = I-2
       BC     4,END2                 IF I-2 < 0  THEN END 'LOOP'
       LR     5,8
       S      5,SEVEN2
       LE     4,40(5)                2000-40+8 = 1968 SF(3,I-1)
       ME     4,4(4,8)               DEF(I-1)
       ADR    0,4                    DEL = DEL + ...
       LE     4,44(5)                2000-40+12 = 1972 SF(4,I-1)
       ME     4,4(4,8)               DEF(I-1)
       ADR    2,4                    ALT = ALT + ...
       LE     4,0(5)                 SF(3,I-2)
       ME     4,0(4,8)               DEF(I-2)
       ADR    0,4                    DEL = DEL + ...
       LE     4,4(5)                 SF(4,I-2)
       ME     4,0(4,8)               DEF(I-2)
```

```
*                                      TEST FOR FIXITY OF NODE
         CE     4,0(6)                 SF(1,1)
         BC     4,HORFIX               BRANCH IF FIXED HORIZONTALLY
         DE     0,0(6)
         ME     0,ORF
         LCER   0,0
         AE     0,0(4,9)
         STE    0,0(4,9)               DEF(I) = DEF(I) - DEL/SF(1,I)*ORF
HORFIX   CE     4,44(6)                SF(2,I+
         BC     4,VRTFIX               BRANCH IF FIXED VERTICALLY
         DE     2,44(6)
         ME     2,ORF
         LCER   2,2
         AE     2,4(4,9)
         STE    2,4(4,9)               DEF(I+1) = DEF(I+1) - ALT SF(2,I+1)*ORF
VRTFIX   LA     6,80(6)
         BXLE   9,2,LOOP1              INCREMENT LOAD POINTER BY 2 WORDS AND BRA CH
         BCT    0,AGAIN                BRANCH TO START NEW ITERATION
*                                      RETURN
         L      13,SAVE13
         MVI    12(13),X'FF'
         LM     2,12,28(13)
         BCR    15,14
*
*                                      SYMBOL TABLE
*                                      ============
*
SAVE13   DS     F
ORF      DS     F
NNS      DS     F
```

V2LC SOURCE

```
         ADR    2,4             ALT = ALT + ...
*                               SET POINTERS FOR LOOP 4
         LR     8,9
         S      8,NN6           GPR  * = I-2*NN+2
         LR     13,8
         S      13,TWNTY4       UPPER LIMIT
         LA     5,16(6)         SF(5,I)
         S      5,BAKSTP        SF(5,I-NN)
*                               BEGIN LOOP 4
TESTK4   LTR    8,8             IS 'I-NN' NEGATIVE ?
         BC     4,END2          IF YES THEN END LOOP
         LE     4,40(5)         SF(_,_-NN+1)
         ME     4,4(4,8)        DEF(_-NN+1)
         ADR    0,4             DEL = DEL + ...
         LE     4,44(5)         SF(_+1,_-NN+1)
         ME     4,4(4,8)        DEF(_-NN+1)
         ADR    2,4             ALT = ALT + ...
         LE     4,0(5)
         ME     4,0(4,8)
         ADR    0,4             DEL = DEL + ...
         LE     4,4(5)
         ME     4,0(4,8)
         ADR    2,4             ALT = ALT + ...
         S      5,SEVEN2        SF POINTER PLUS TWO ROWS, LESS TWO COLS
         BXH    8,12,TESTK4
END2     LE     4,INFIN
```

```
V2EC   SOURCE

        BAKSTP DS     F
        TWNTY4 DC     F'24'
        SEVEN2 DC     F'72'
        INFIN  DC     E'1.E30'
               END    FINGEL
```

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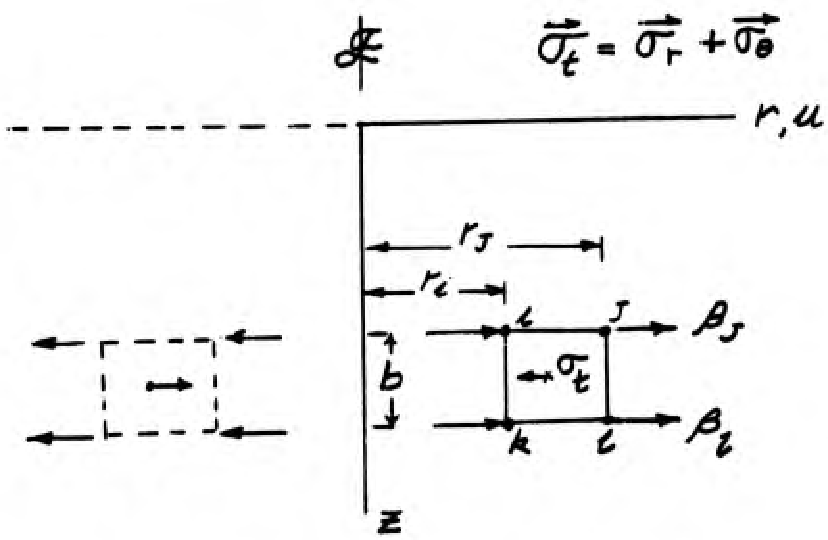
APPENDIX II

MATRIX FOR ELIMINATION OF TENSION IN UNBOUND GRANULAR LAYERS.

Tension Elimination by Virtual Work Principle.

The tensile stress, σ_t , is eliminated by imposing the forces indicated at the nodes of the element shown in sketch. These forces are derived from considerations of the principle of Virtual Work.

To form $\{\beta\} = \left[\int_v [B]^T \{\sigma_t\}\, dv\right]$, the appropriate matrices for the annular element, rectangular in elevation are as follows:



$$[B^T] \qquad \{\sigma\} \qquad [B^T]\{\sigma\}$$

$$\begin{bmatrix} 0 & 1/r & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & z/r & 0 & 1 \\ z & z & 0 & r \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & r & z \end{bmatrix} \begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \\ \tau_{rz} \end{Bmatrix} = \begin{Bmatrix} 1/r\, \sigma_\theta \\ \sigma_r + \sigma_\theta \\ z/r\, \sigma_\theta + \tau_{rz} \\ z(\sigma_r + \sigma_\theta) + r\,\tau_{rz} \\ 0 \\ \tau_{rz} \\ \sigma_z \\ r\sigma_z + z\,\tau_{rz} \end{Bmatrix}$$

The integral $\left[\int_v [B]^T \{\sigma_t\} \, dv \right]$ with limits $r \Rightarrow r_i$ to r_J and $z \Rightarrow 0$ to b reduces to the expression :

Node	
i	$(r_J - r_i) b \, \sigma_\theta$
J	$\frac{1}{2}(r_J^2 - r_i^2) b \, (\sigma_r + \sigma_\theta)$
k	$\frac{1}{2}(r_J - r_i) b^2 \sigma_\theta + \frac{1}{2}(r_J^2 - r_i^2) b \, \tau_{rz}$
l	$\frac{1}{4}(r_J^2 - r_i^2 \; b^2 (\sigma_r + \sigma_\theta)$
i	0
J	$\frac{1}{2}(r_J^2 - r_i^2) b \, \tau_{rz}$
k	$\frac{1}{2}(r_J^2 - r_i^2) b \, \sigma_z$
l	$\frac{1}{3}(r_J^3 - r_i^3) b \, \sigma_z + \frac{1}{2}(r_J - r_i) b^2 \tau_{rz}$

The first four forces of the set above represent horizontal forces and are included in FIN4EL in a simplified form together with multiplication factor 2π.

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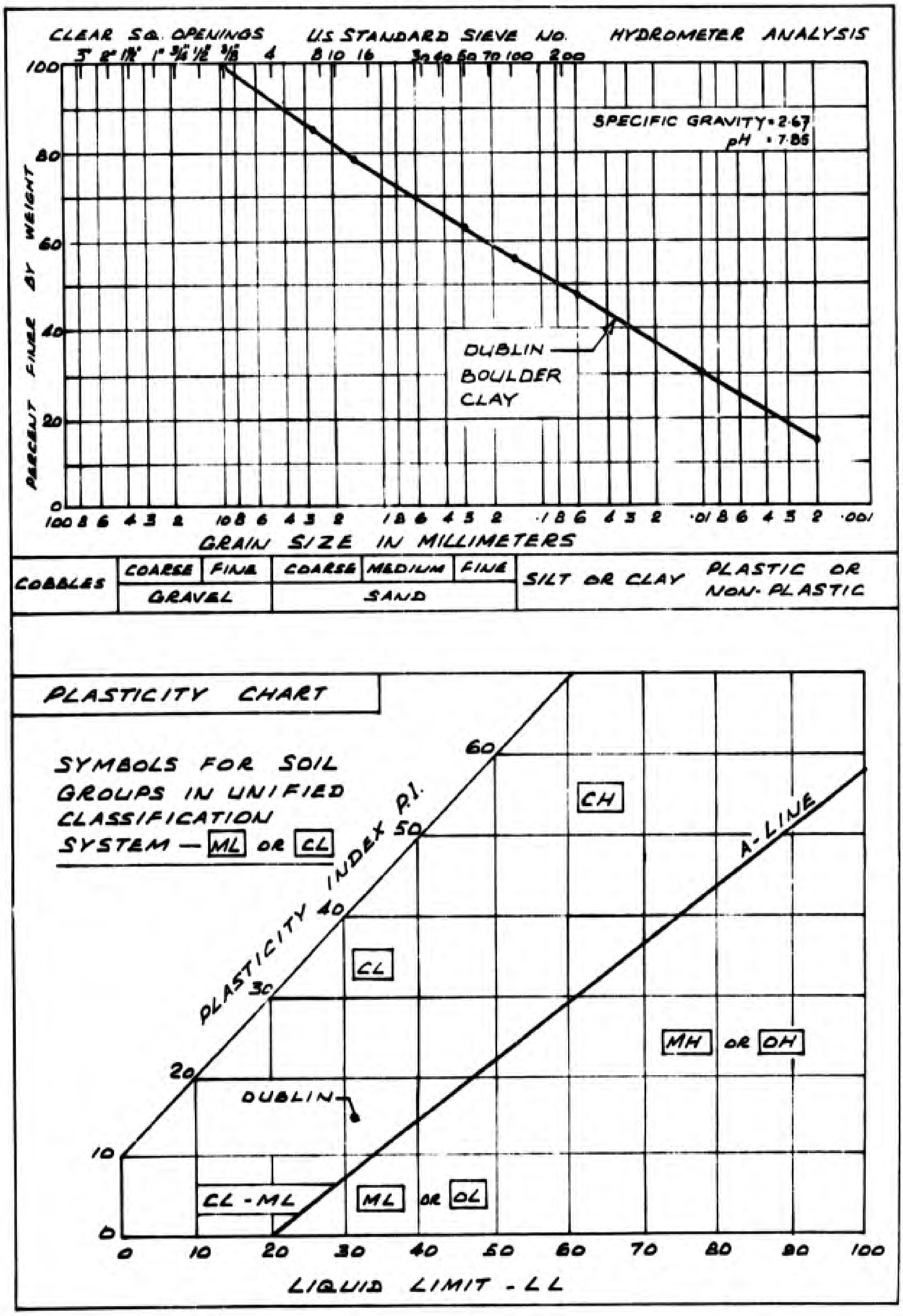
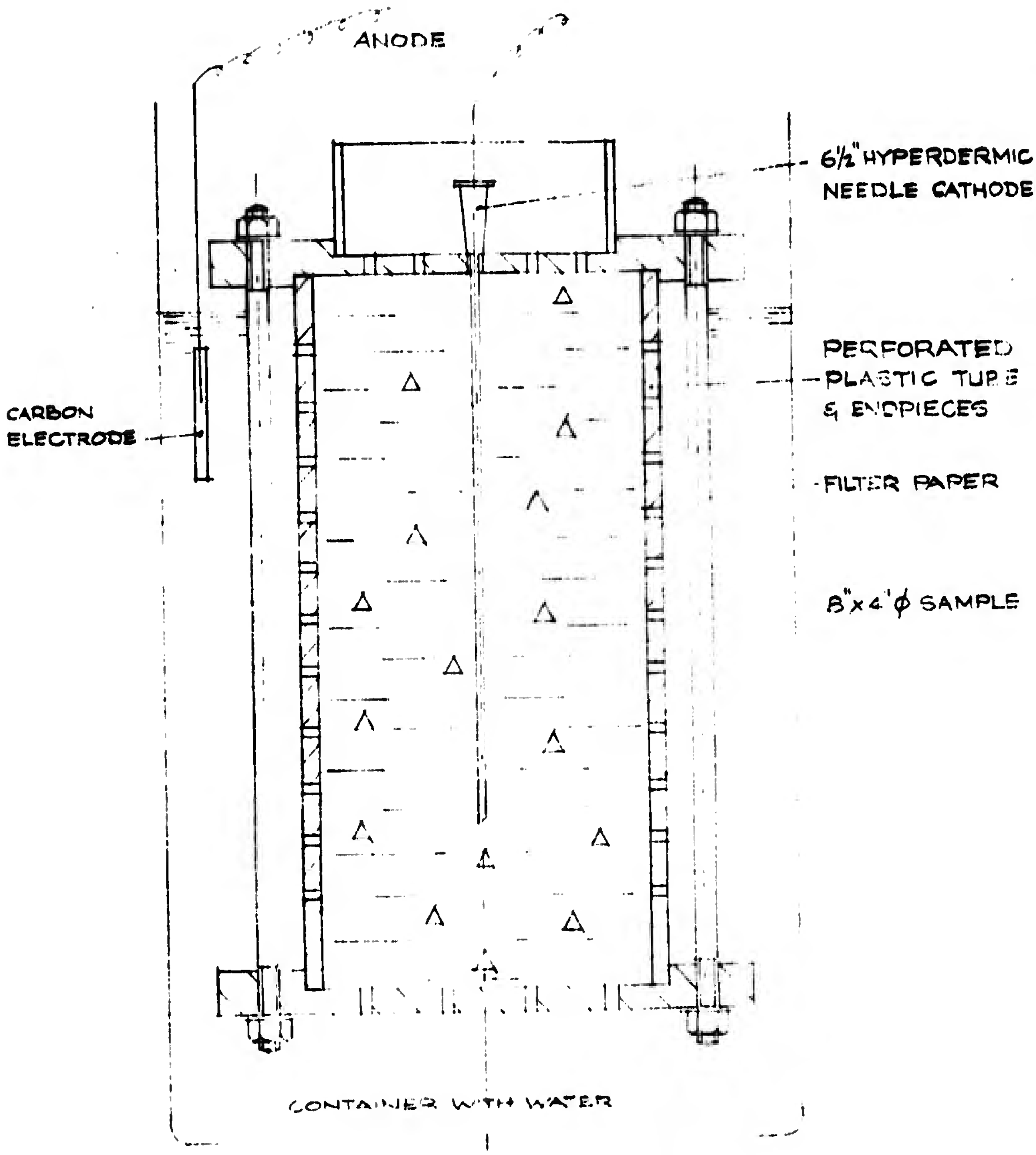
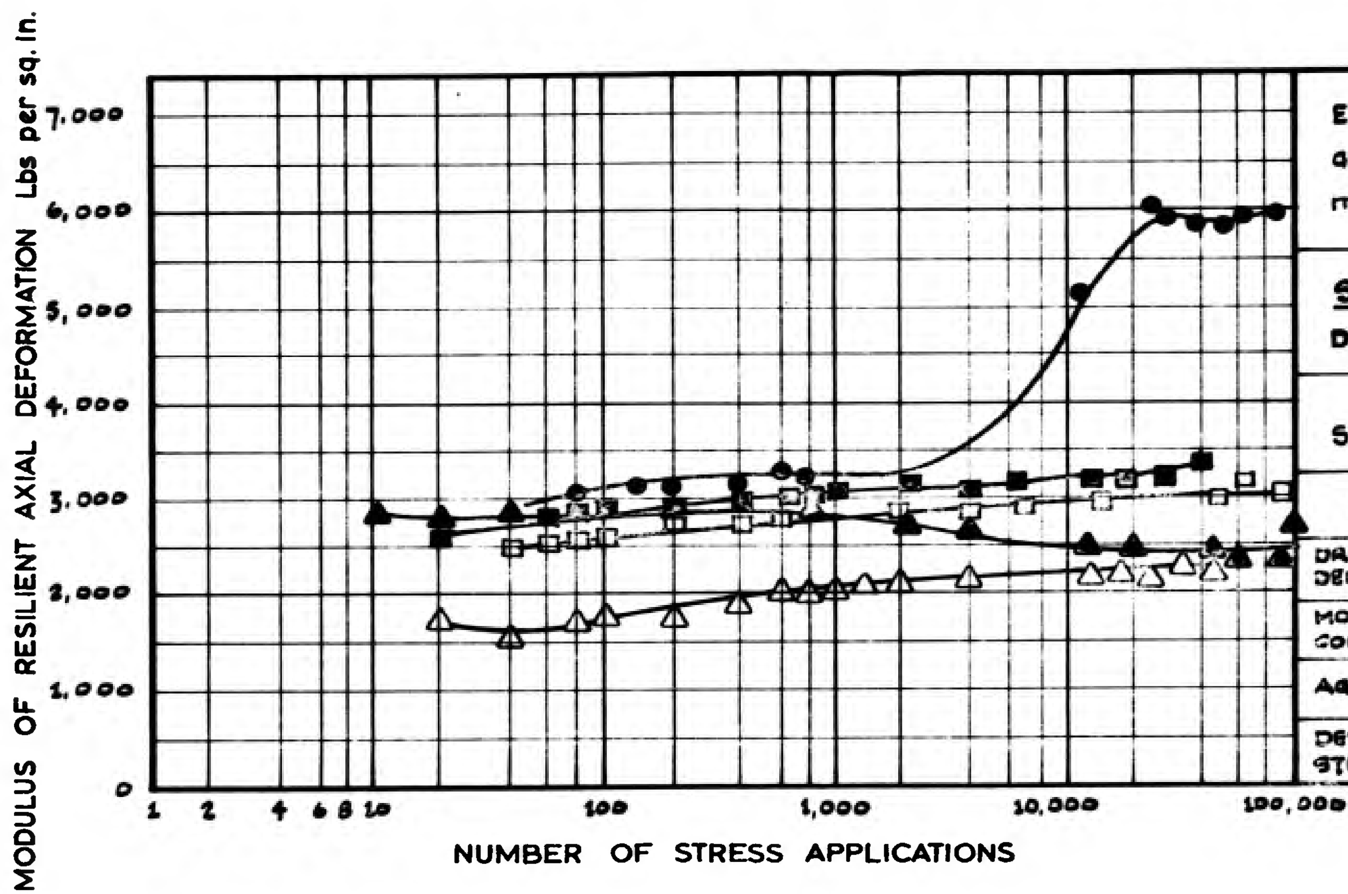


RESULTS OF SOIL CLASSIFICATION TESTS

FIG. 2



ELECTRO-OSMOSIS CELL FOR 8"x4"φ TRIAXIAL SAMPLE



Effects of deviator stress and age of specimen on modulus of resilience.

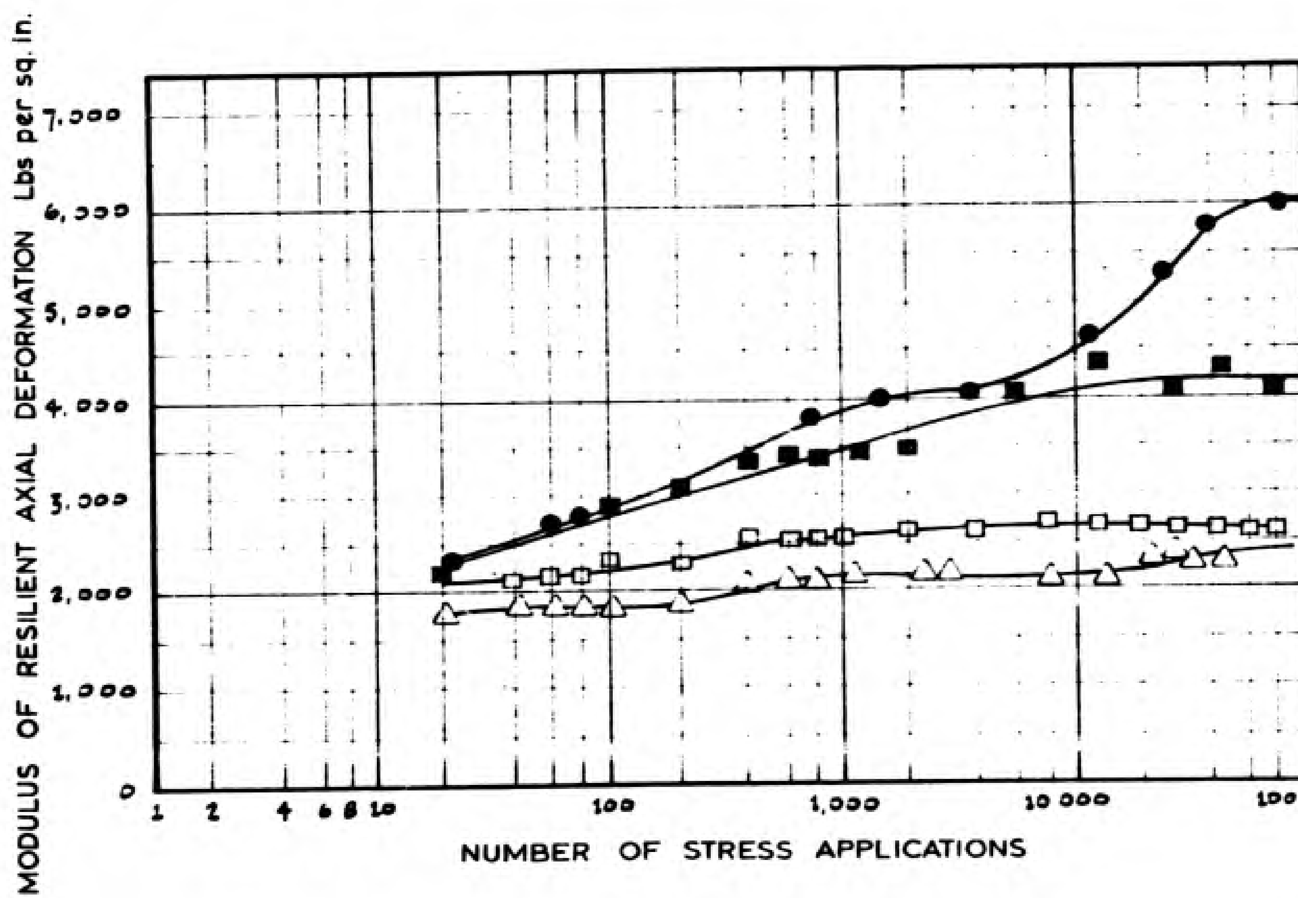
SOIL TYPE :-
DUBLIN BOULDER CLAY

Specimen Size	Diam.	Height
	4"	8"

TEST NO.	A9 ●	A13 ■	A.24 □	A.7 △	A.33 ▲
DRY DENSITY (p.c.f.)	122.5	122.7	121.8	121.3	119.8
MOISTURE CONTENT %	12.7	12.9	13.0	13.05	
AGE (WEEKS)	4	4	2	8	8 DAYS
DEVIATOR STRESS (p.s.i.)	3	9	9	9	6

DYNAMIC MODULUS - vs - NUMBER OF STRESS APPLICATIONS IN REPEATED LOADING TRIAXIAL COMPRESSION TESTS

FIG. 3



Effects of deviator stress and age of specimen on modulus of resilience.

SOIL TYPE :-

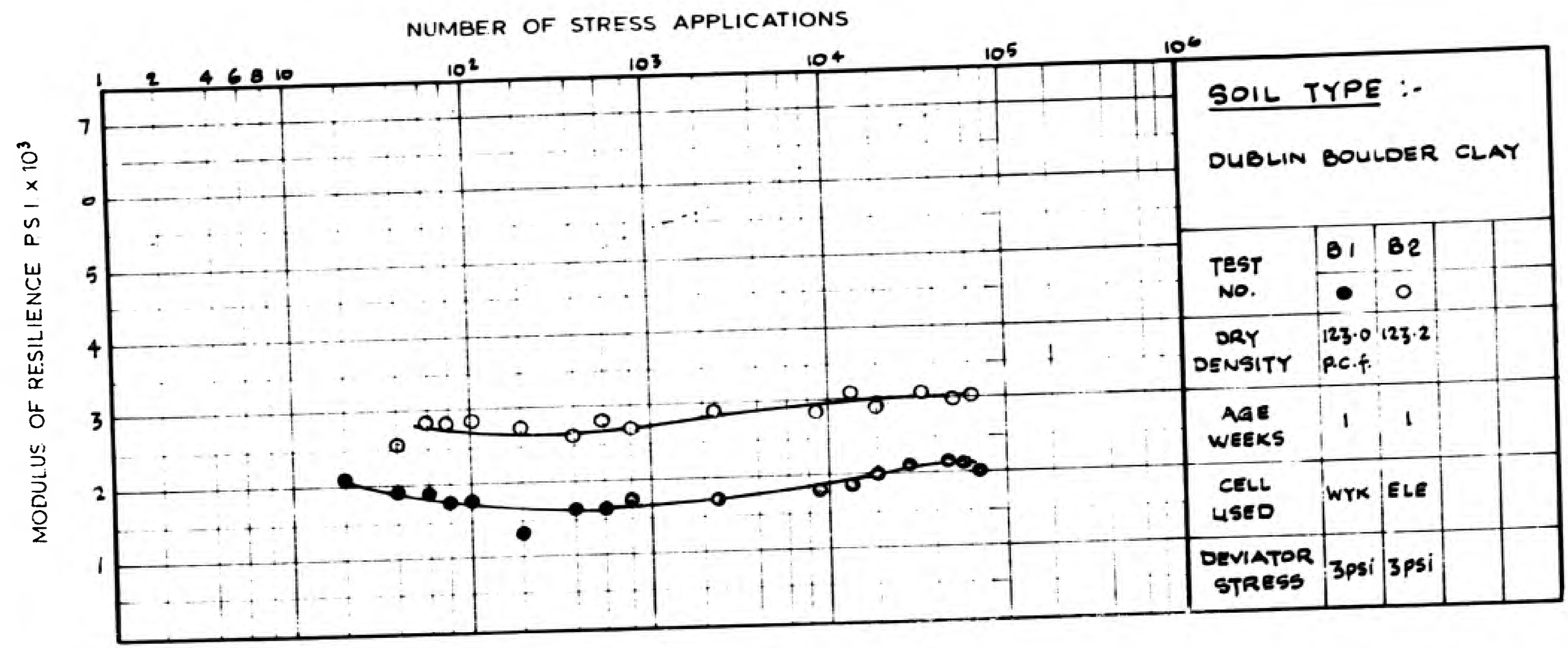
DUBLIN BOULDER CLAY

Specimen Size	Diam.	Height
	4"	8"

TEST NO.	A 25	A 27	A 11	A 15
	△	□	●	■
DRY DENSITY (p.c.f.)	121.0	121.0	124.0	122.0
MOISTURE CONTENT %	13.3	13.2	12.7	12.95
AGE (DAYS)	3	7	3	7
DEVIATOR STRESS (p.s.i.)	9	9	3	3

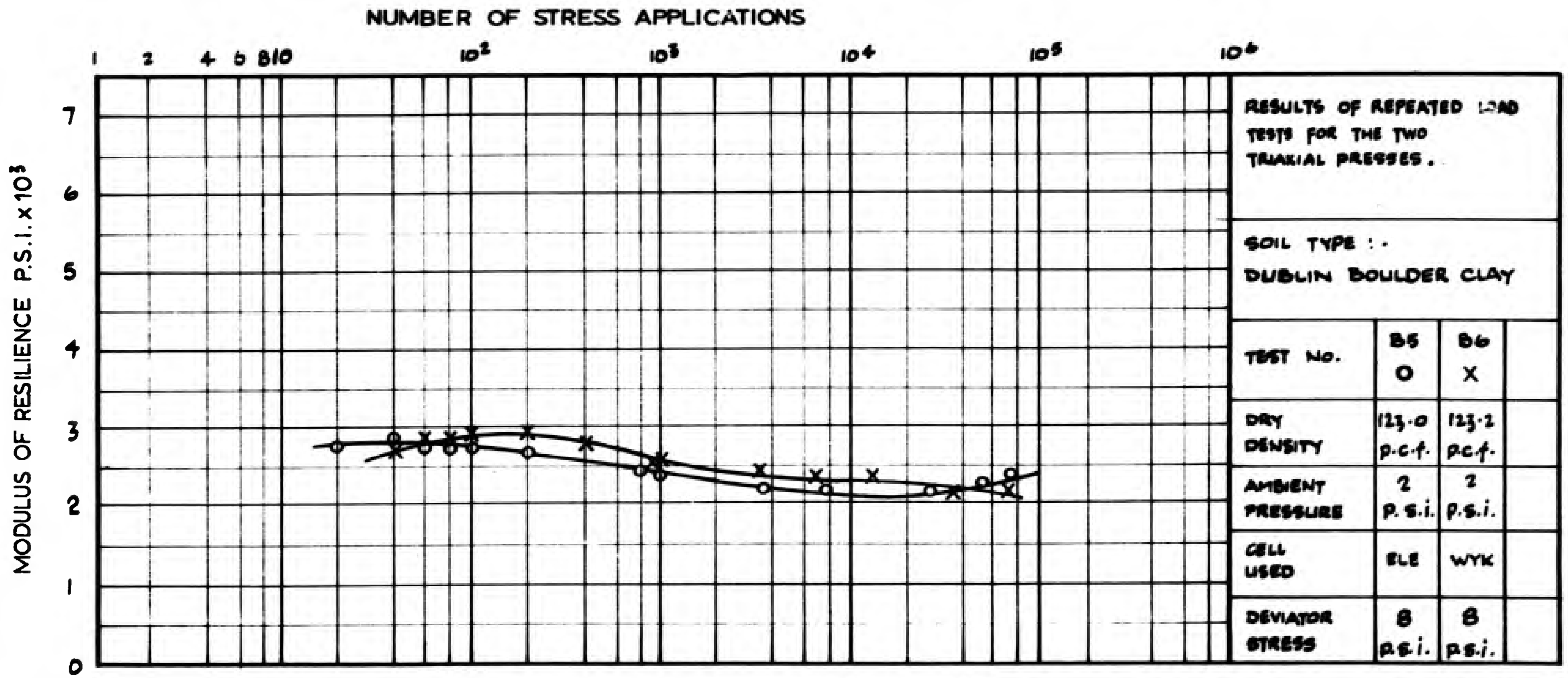
DYNAMIC MODULUS – vs – NUMBER OF STRESS APPLICATIONS IN REPEATED LOADING TRIAXIAL COMPRESSION TESTS

FIG. 4.



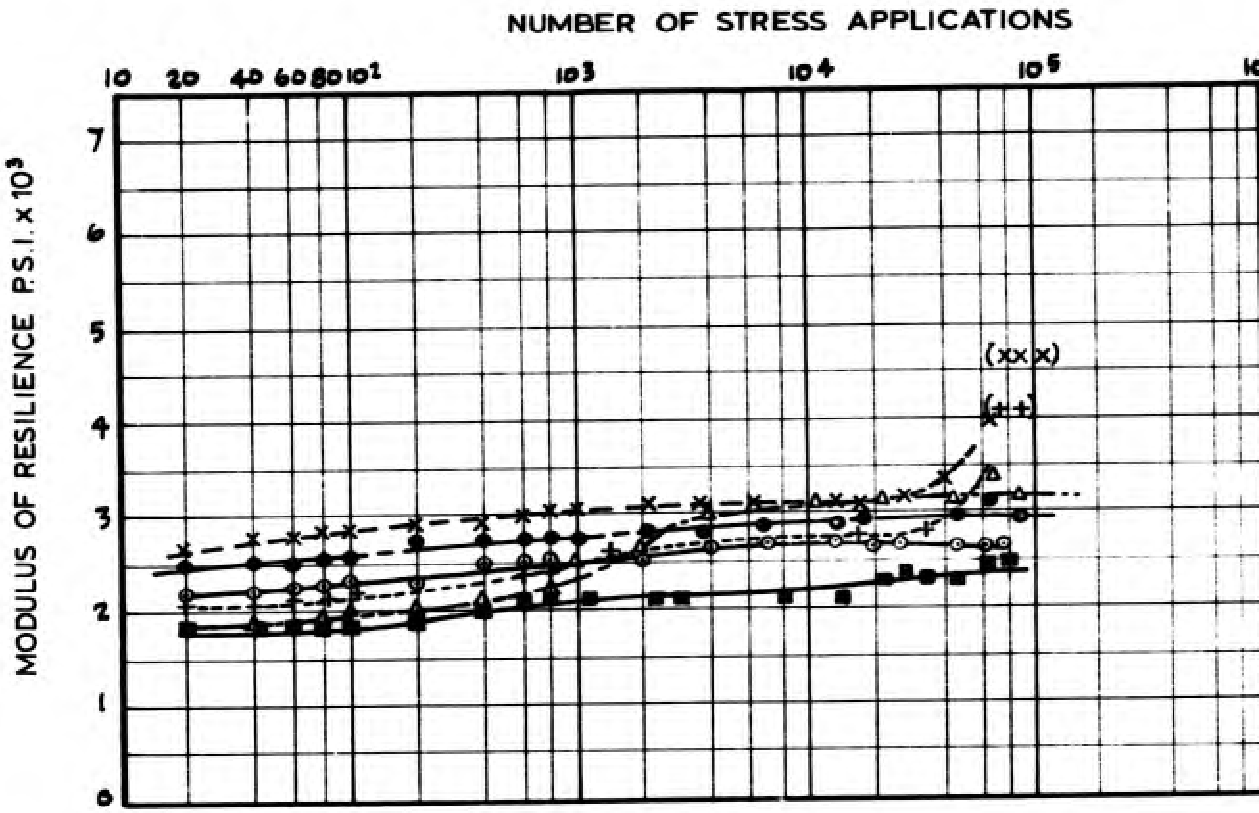
RESULTS OF REPEATED LOADING TRIAXIAL COMPRESSION TEST

FIG. 5 a.



RESULTS OF REPEATED LOADING TRIAXIAL COMPRESSION TEST

FIG. 5 b.



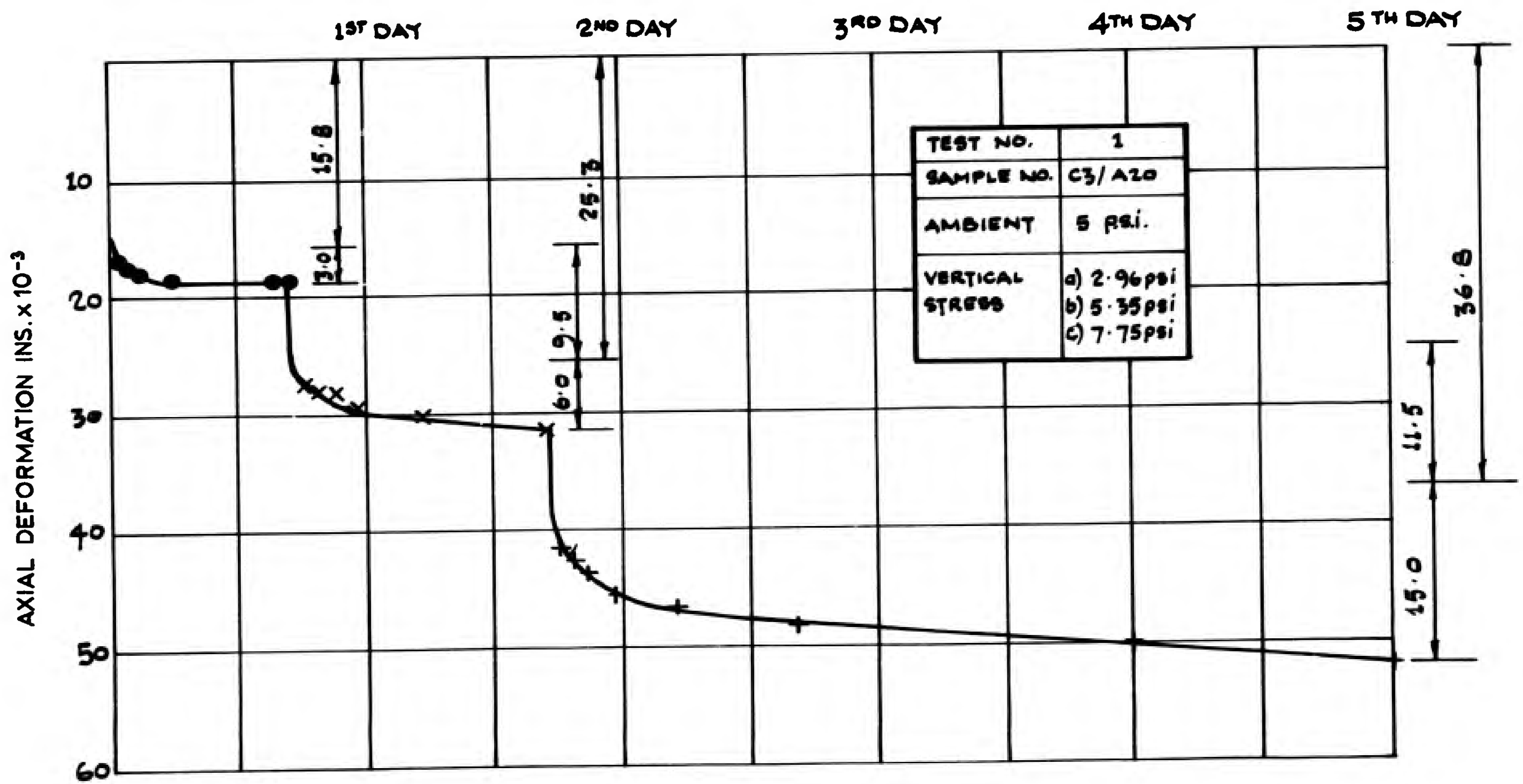
Effects of deviator stress and age of specimens on modulus of resilience.

SOIL TYPE :- DUBLIN BOULDER CLAY

TEST NO.	A5	A25	A27	A29	A13	A7
	Δ	■	⊙	●	×	+
DRY DENSITY	121·0 p.c.f.	121·0	121·0	121·8	122·7	121·3
AGE DAYS	1	3	7	14	28	56
DEVIATOR STRESS	9psi	9psi	9psi	9psi	9psi	9psi

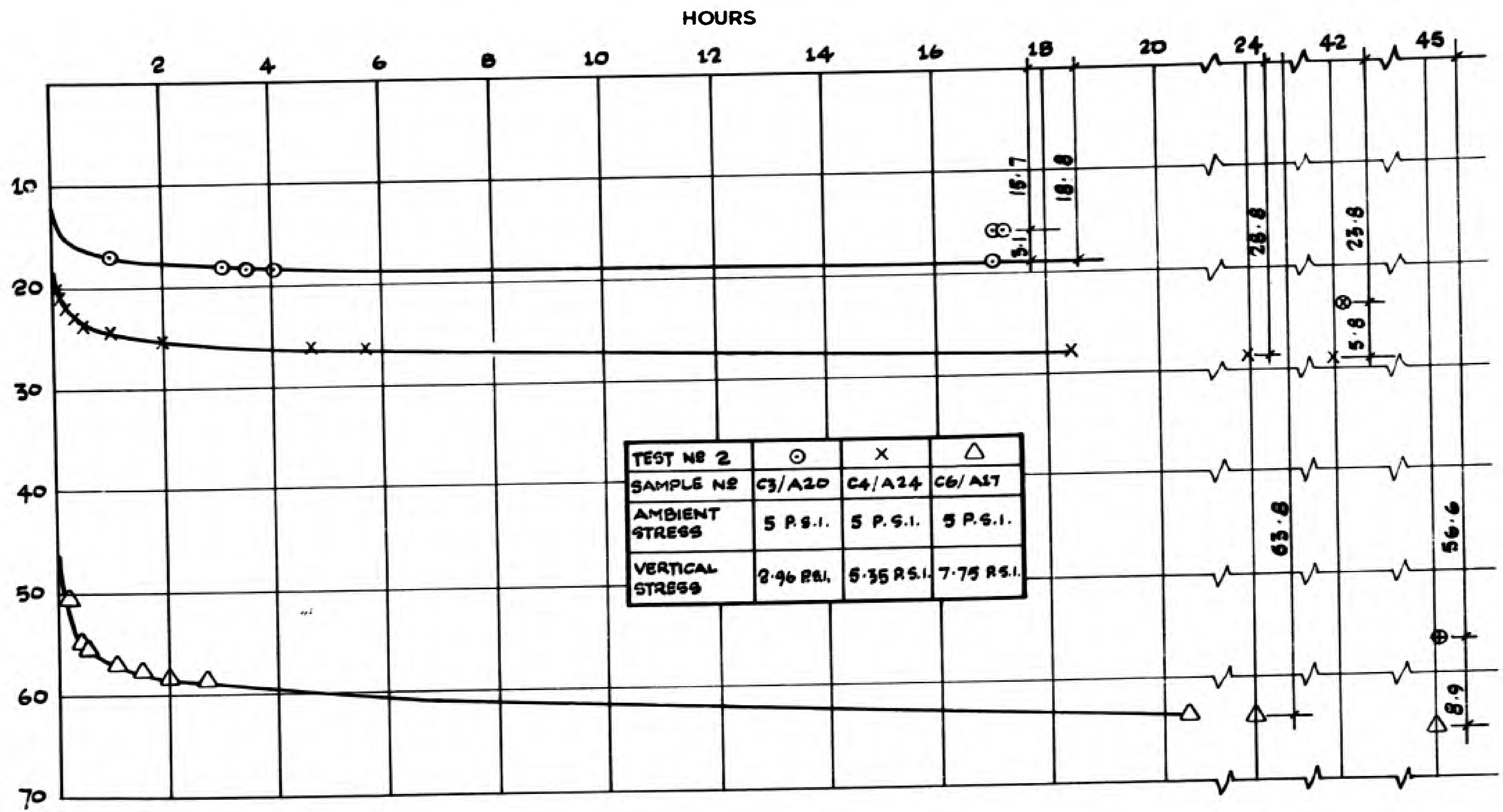
RESULTS OF REPEATED LOADING TRIAXIAL COMPRESSION TESTS

FIG. 6



STATIC CREEP TEST ON AN 8"x4" DIAM. SPECIMEN

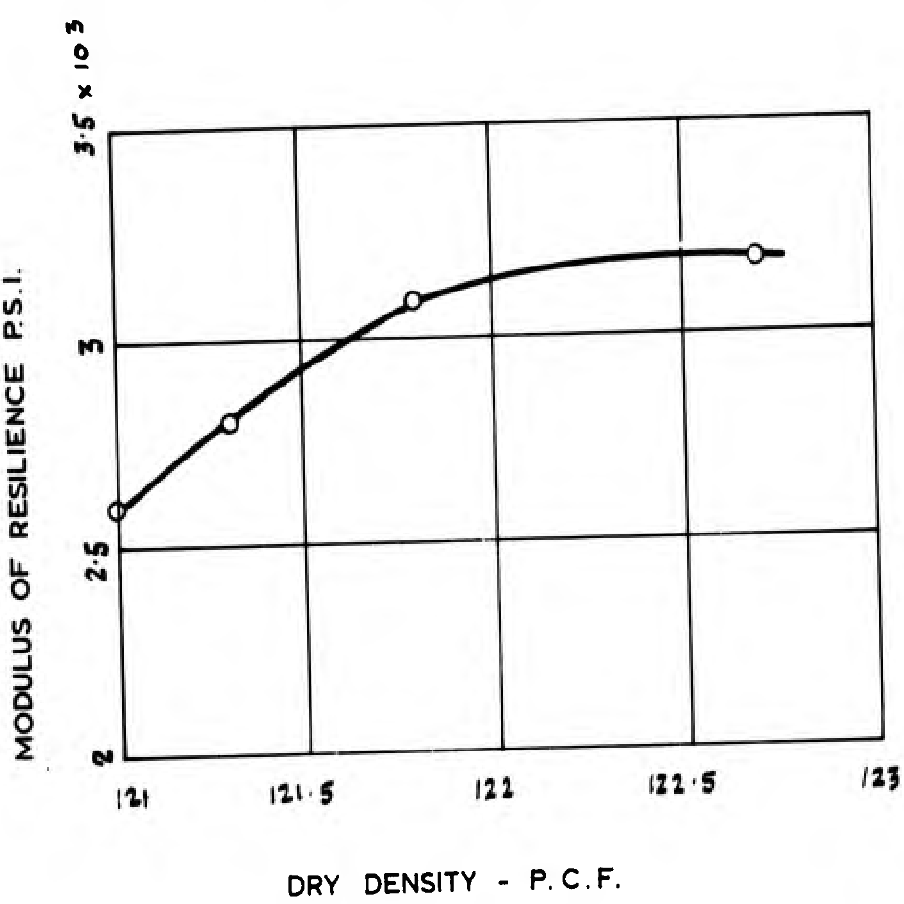
FIG. 7



RESULTS OF STATIC CREEP TESTS ON THREE 8" x 4" DIAMETER SPECIMENS

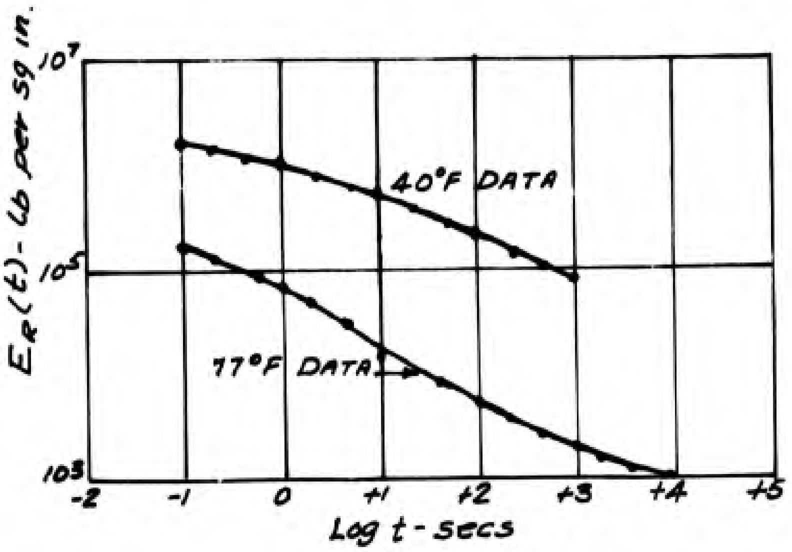
FIG. 8

FIG. 9

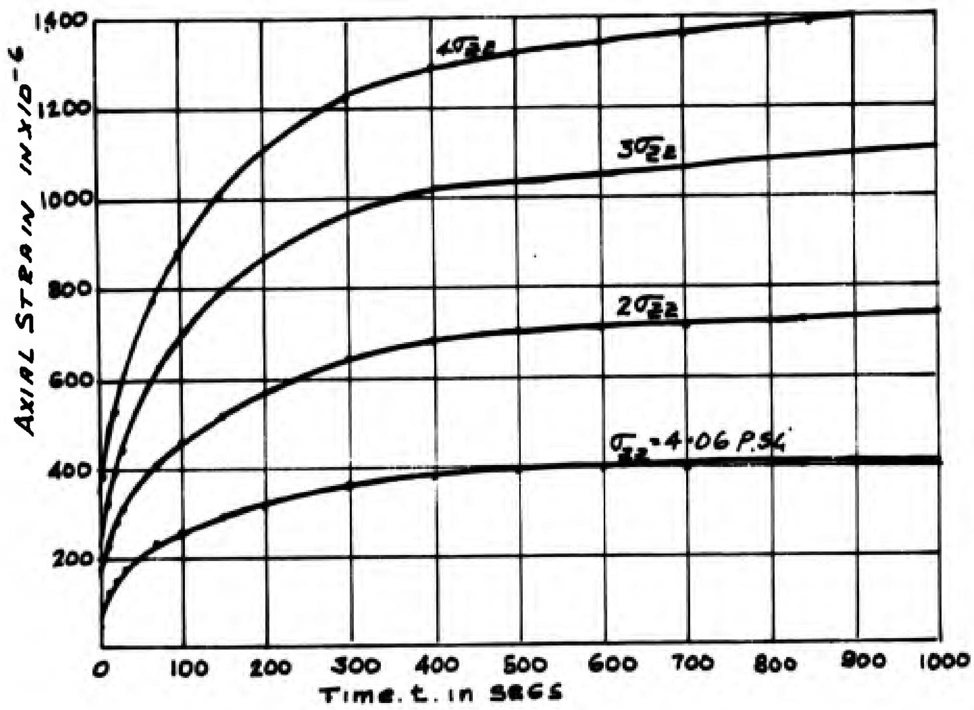


RESILIENT MODULUS - VS - DRY DENSITY :
DUBLIN BOULDER CLAY

FIG. 10



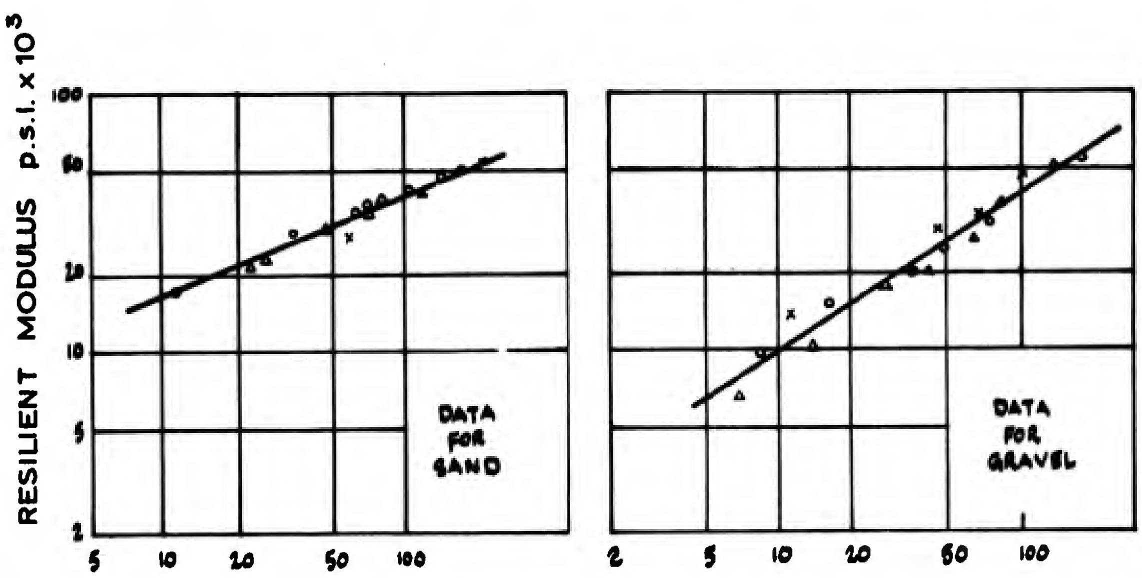
ELASTIC MODULI AT 40°F AND 77°F VS TIME.
(AFTER MONISMITH-SECOR 1965)



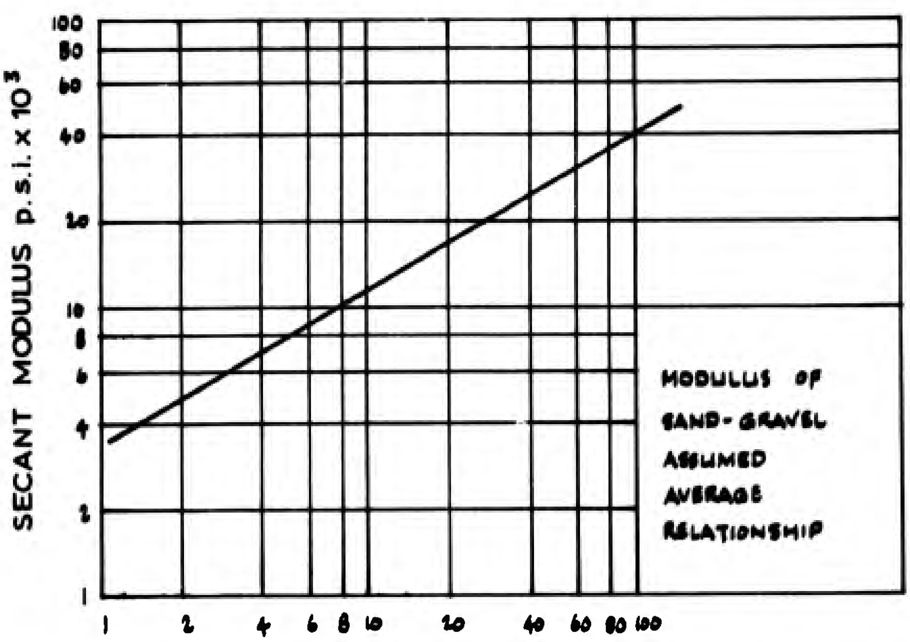
VISCOELASTIC RESPONSE OF BITUMINOUS CONCRETE.
(AFTER PAGEN 1965.)

PROPERTIES OF BITUMINOUS MIXES

FIG. 11



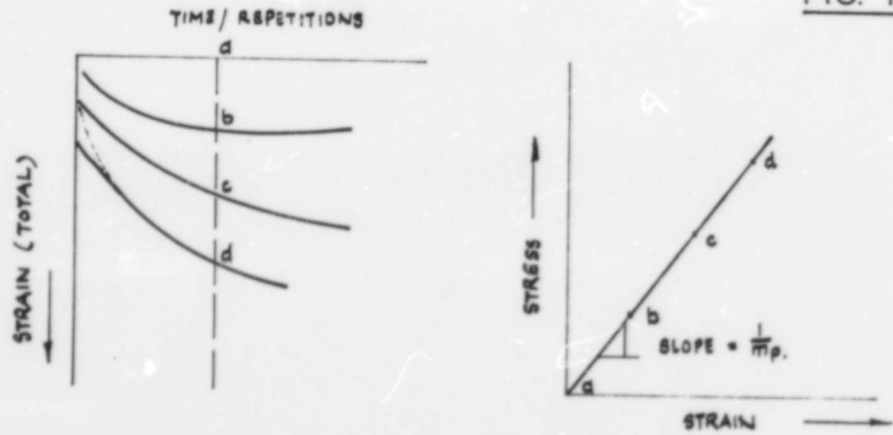
SUM OF PRINCIPAL STRESSES $\theta = (\sigma_1 + \sigma_2 + \sigma_3)$ p.s.i.
(AFTER SEED ET. AL., 1967)



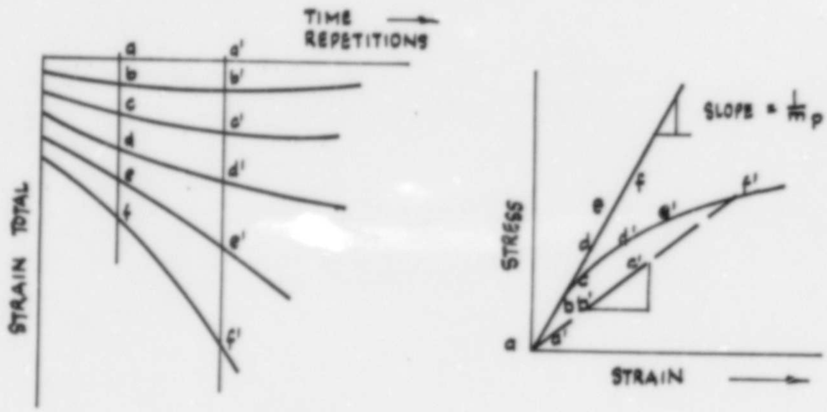
MEAN STRESS ($\theta/3$) p.s.i.

GRANULAR MATERIALS - MODULUS - VS - STRESS COMPONENT

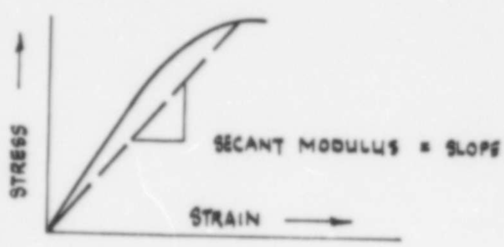
FIG. 12



(a) LINEAR VISCOELASTIC - BITUMINOUS MIXES



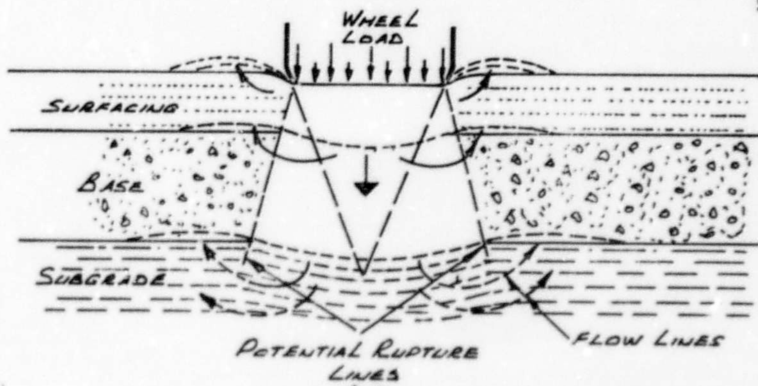
(b) NON-LINEAR CREEP CLAYS



(c) GRANULAR MATERIAL

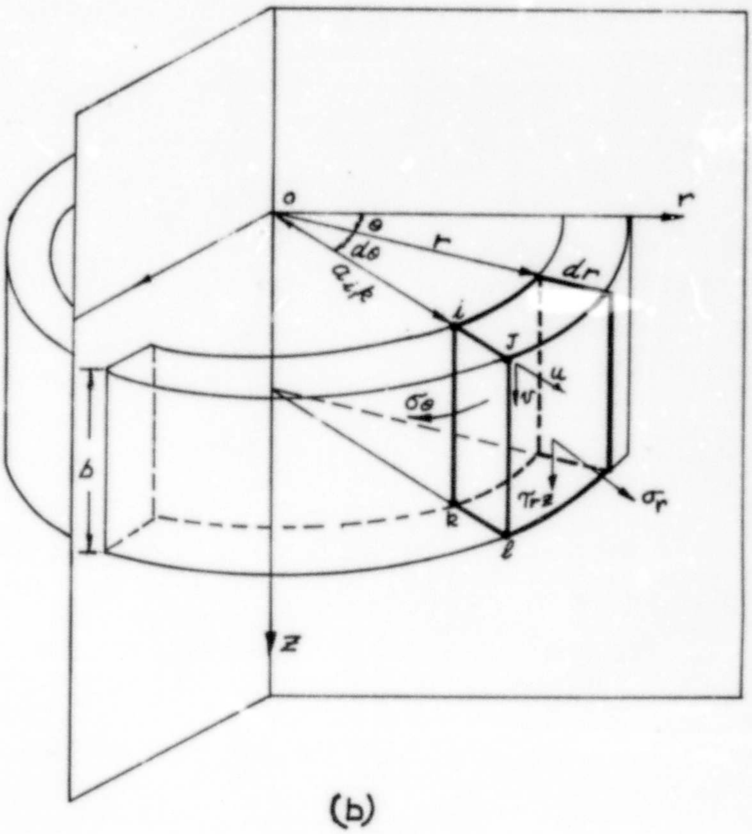
DEVELOPMENT OF STRESS - VS - STRAIN PLOTS FOR PAVEMENT COMPONENTS

FIG.13



(a)

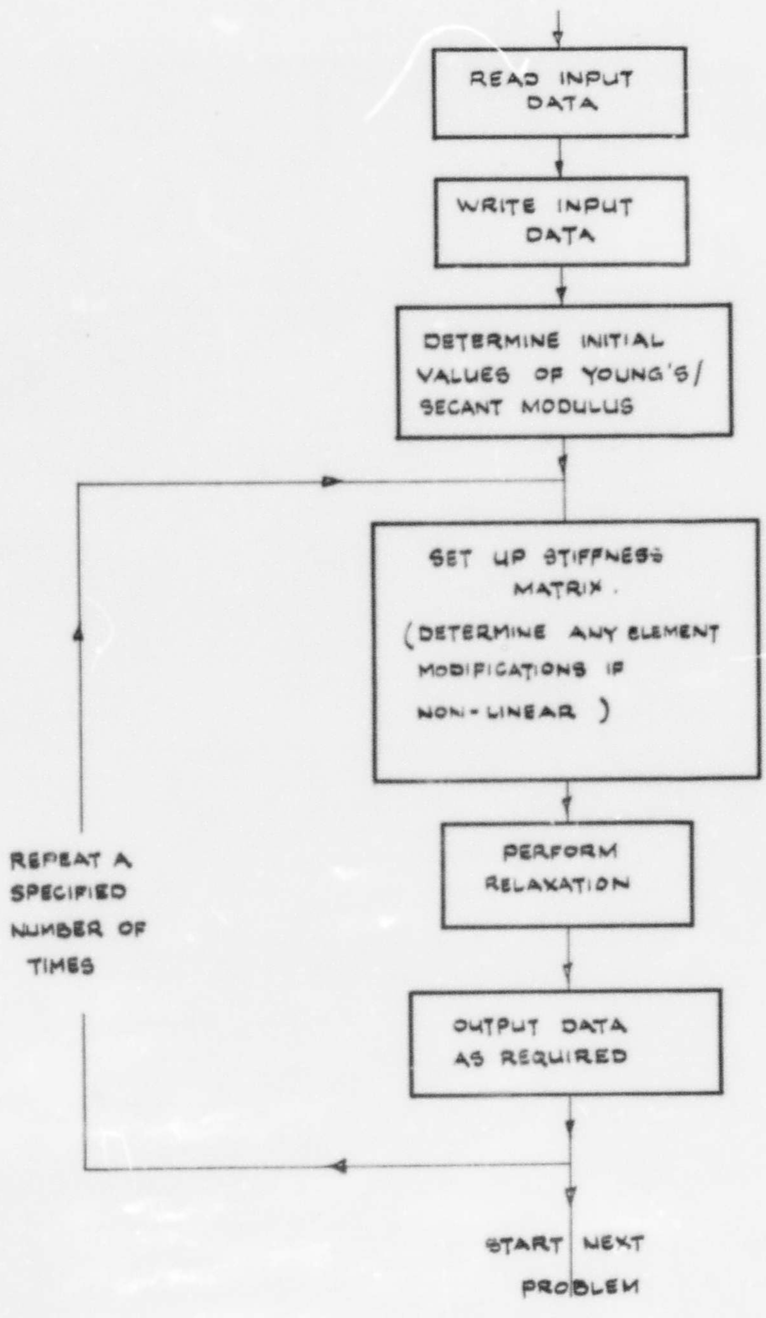
DISPLACEMENT MODES IN MULTI-LAYER PAVEMENT



(b)

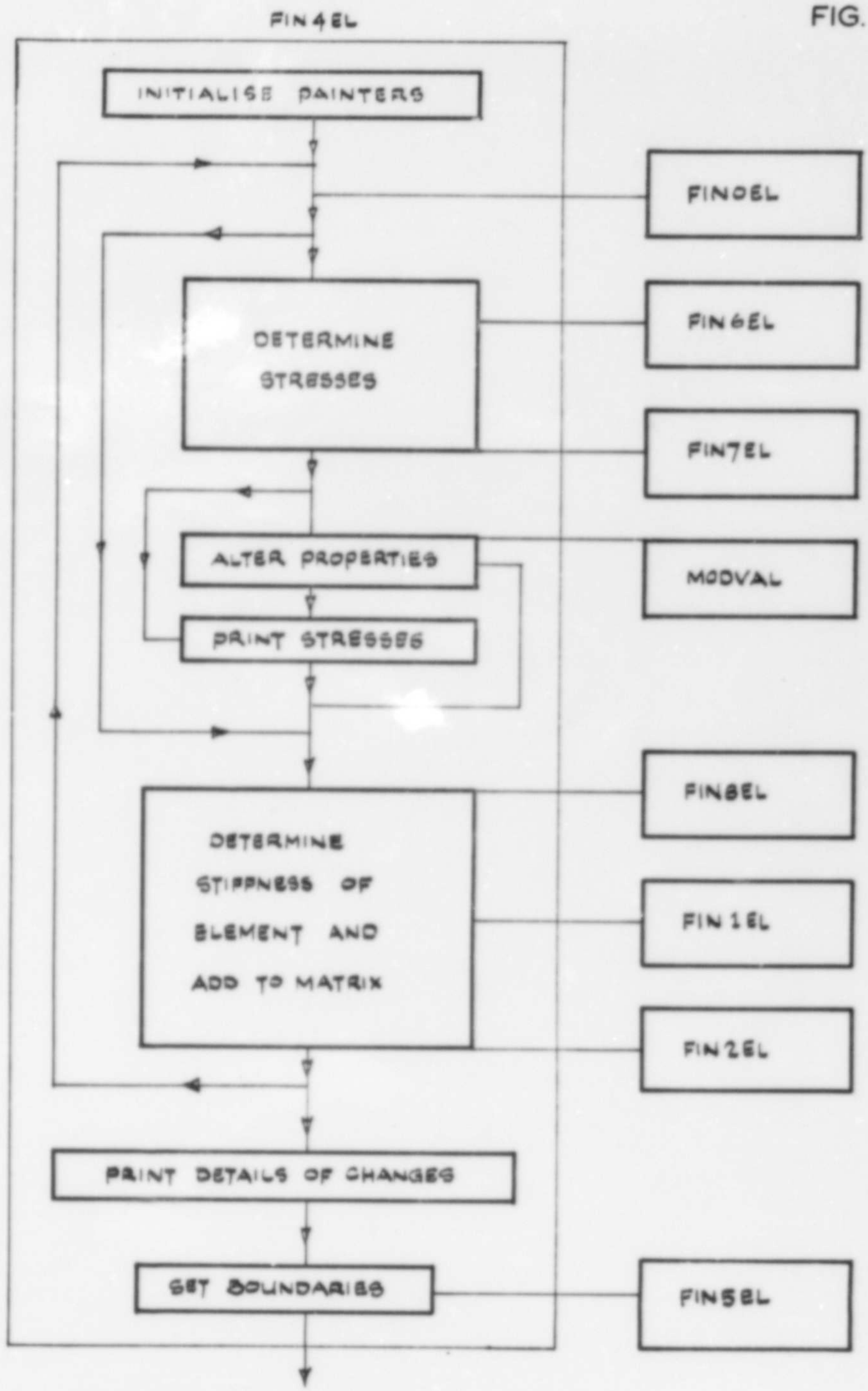
RECTANGULAR ELEMENT : COMPONENTS OF STRESS AND DISPLACEMENT IN CYLINDRICAL COORDINATES

FIG. 14



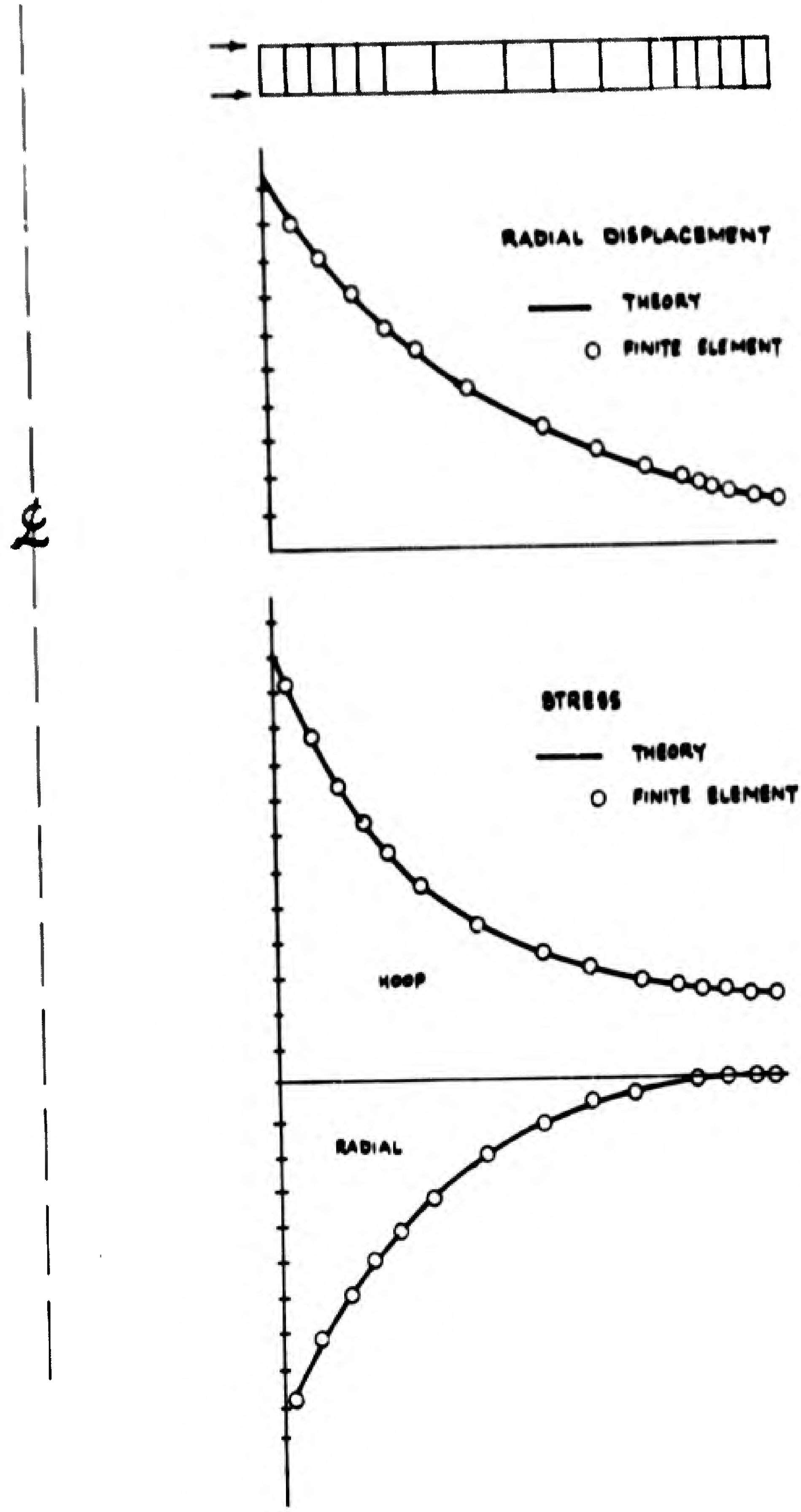
FLOW DIAGRAM - FINITE ELEMENT PROGRAM

FIG. 15

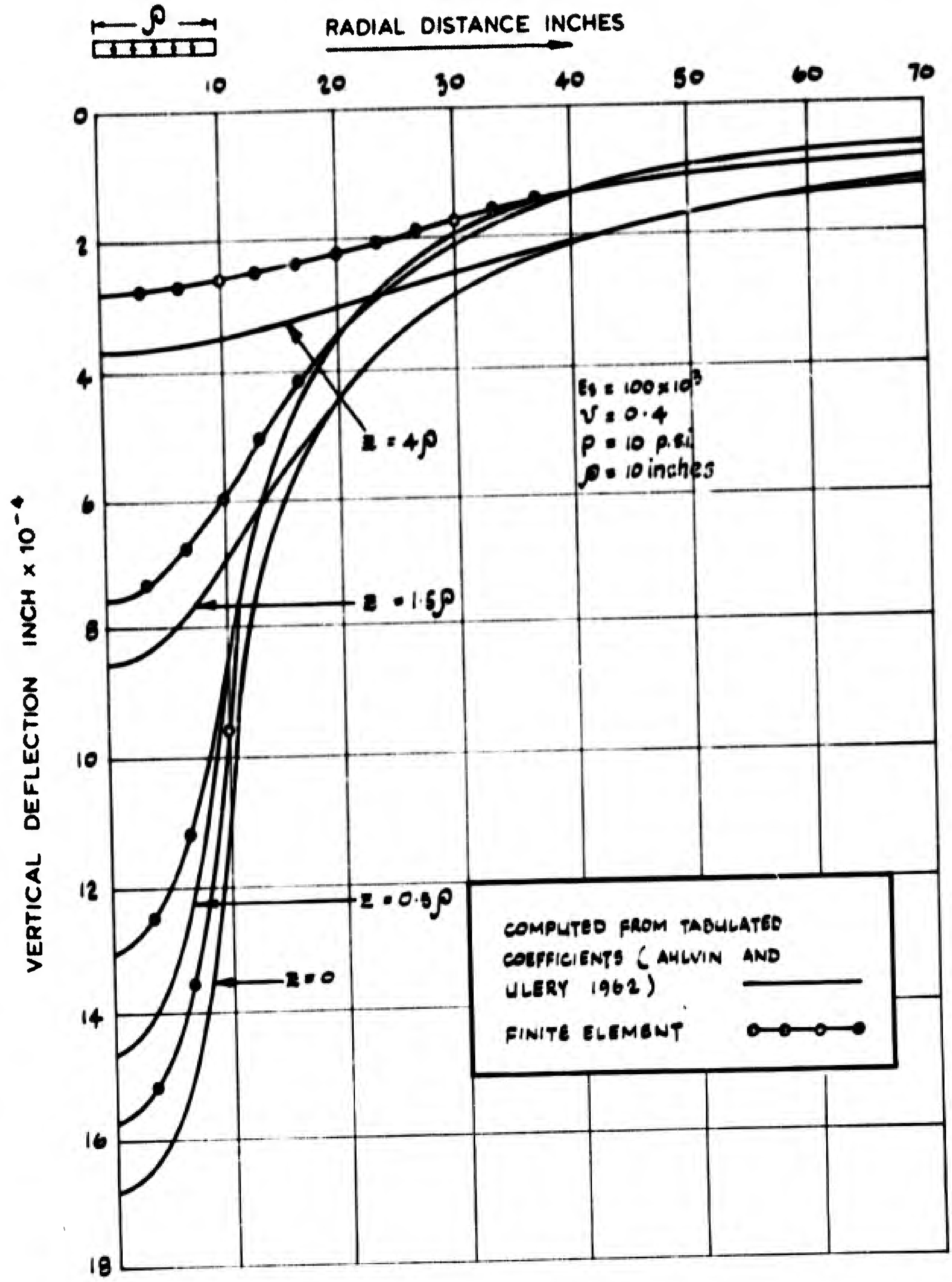


FLOW DIAGRAM - SUBROUTINE FIN4EL

FIG. 16

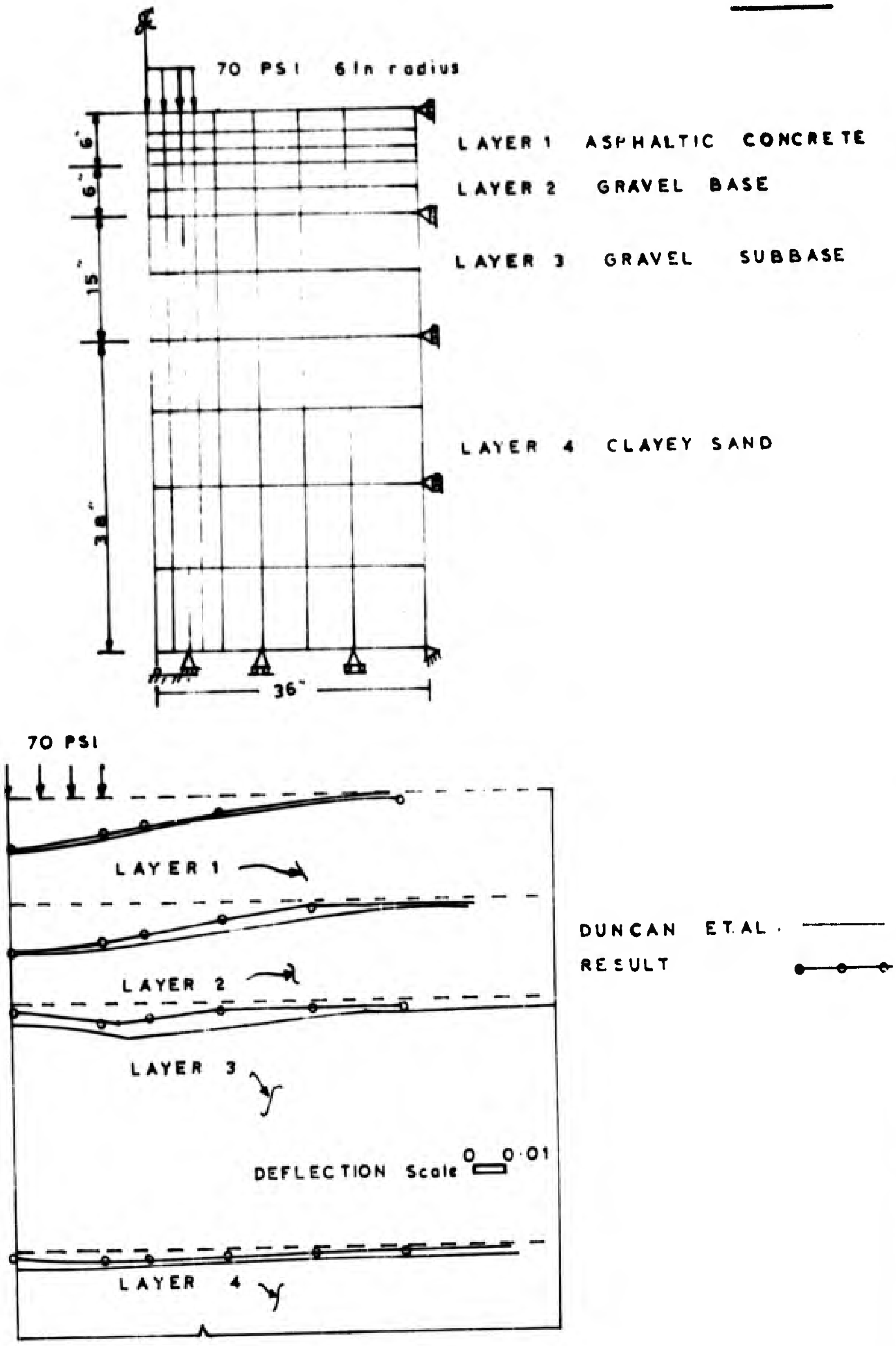


PROGRAM TEST-THICK CYLINDER



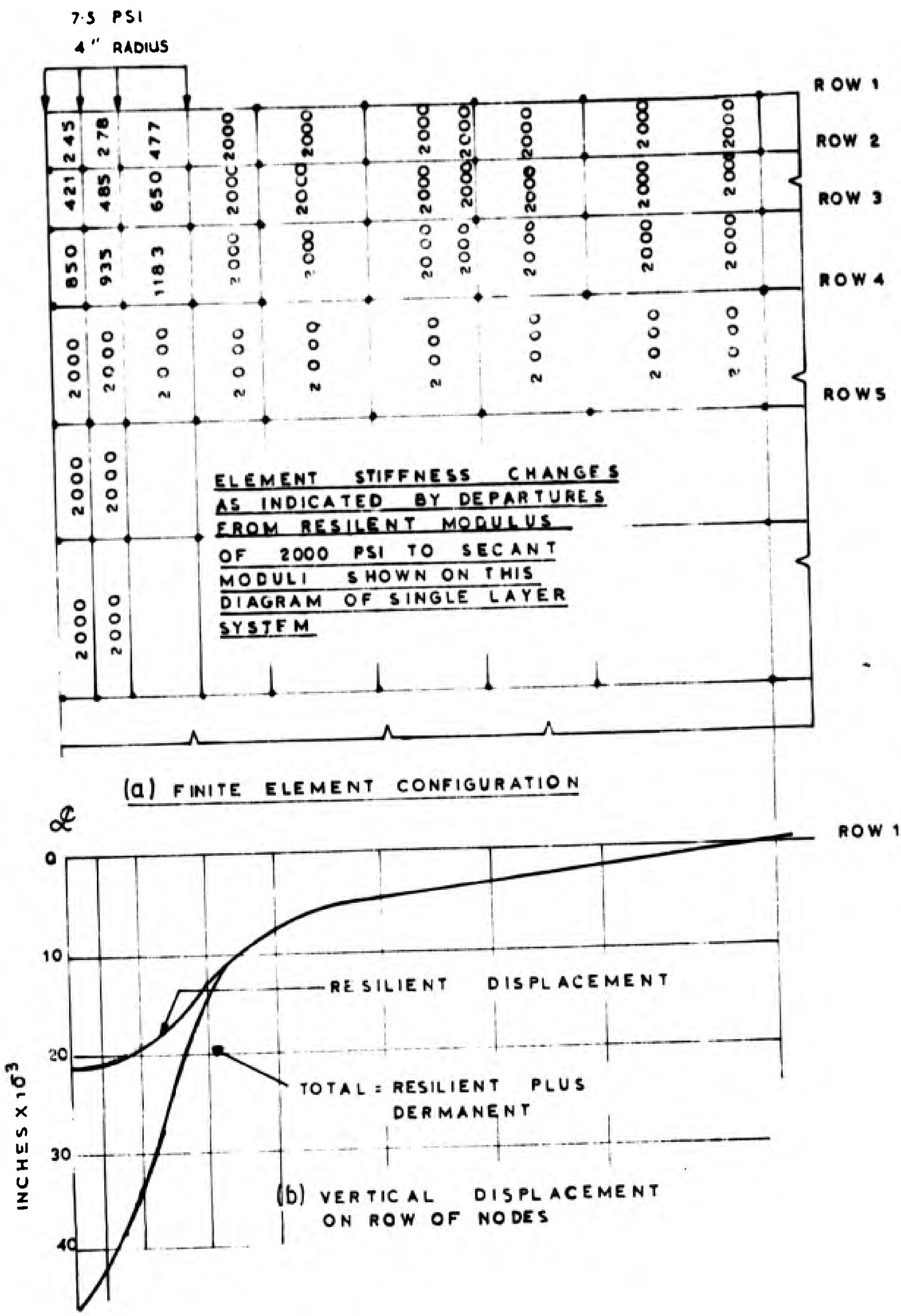
PROGRAM TEST-TABULATED DATA V.S. FINITE ELEMENT ANALYSIS

FIG. 18



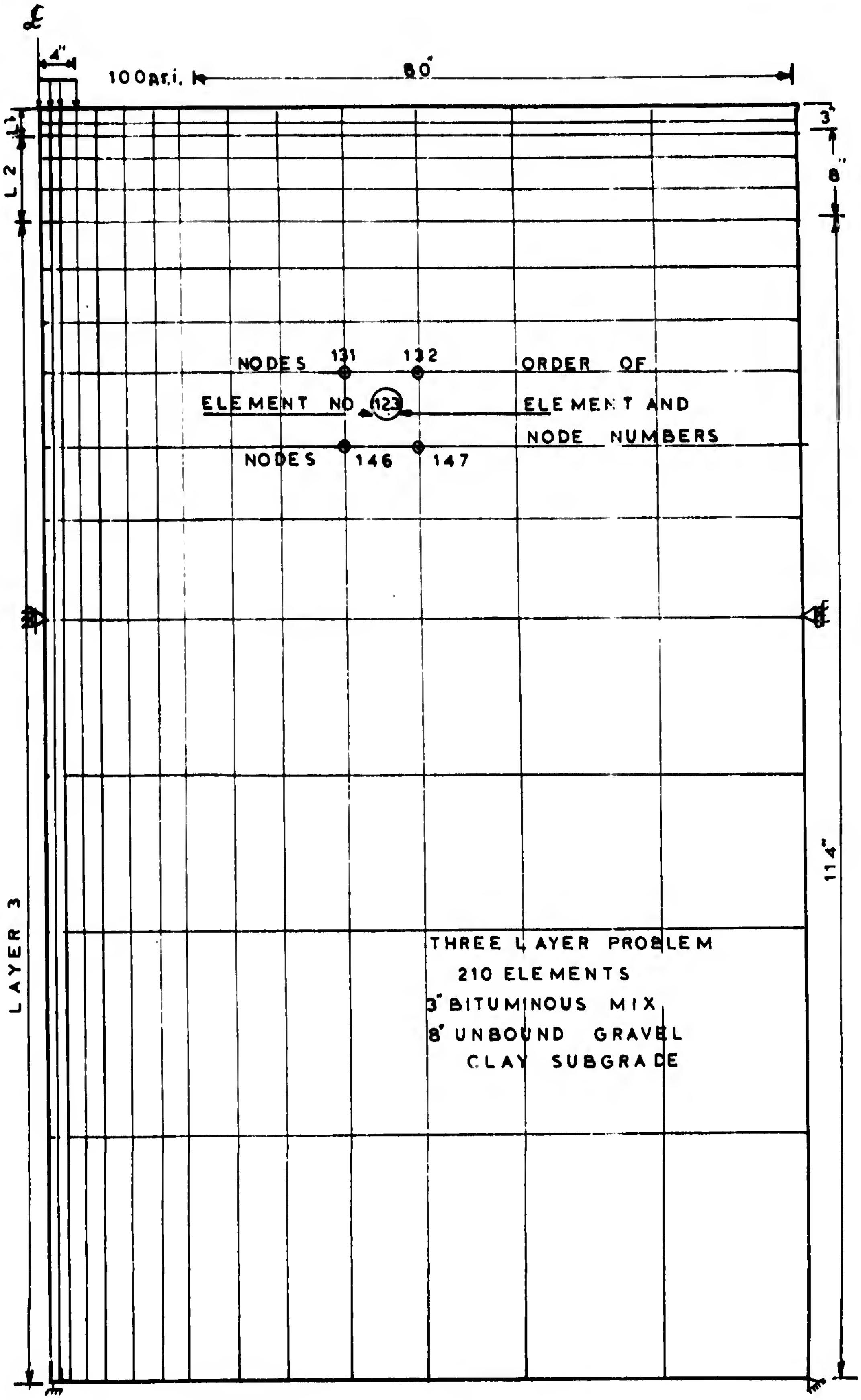
PROGRAM TEST — GONZALES BYPASS

FIG. 19



PROGRAM TEST: PERMANENT DISPLACEMENTS

FIG. 20

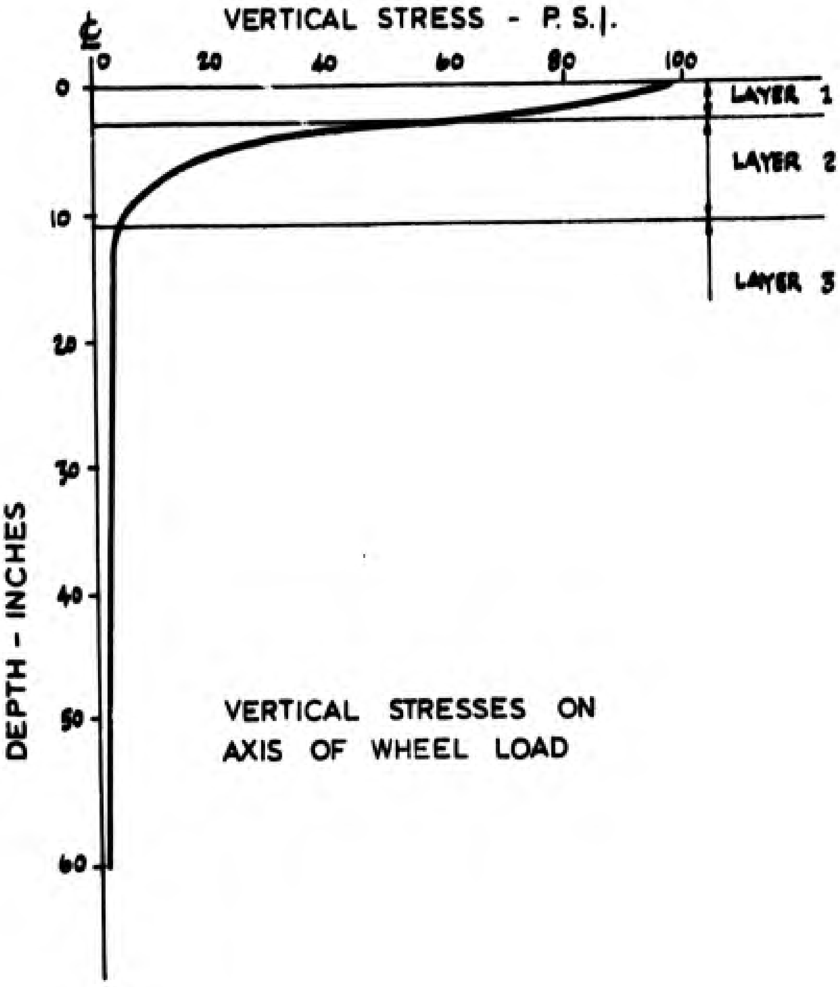


FINITE ELEMENT CONFIGURATION

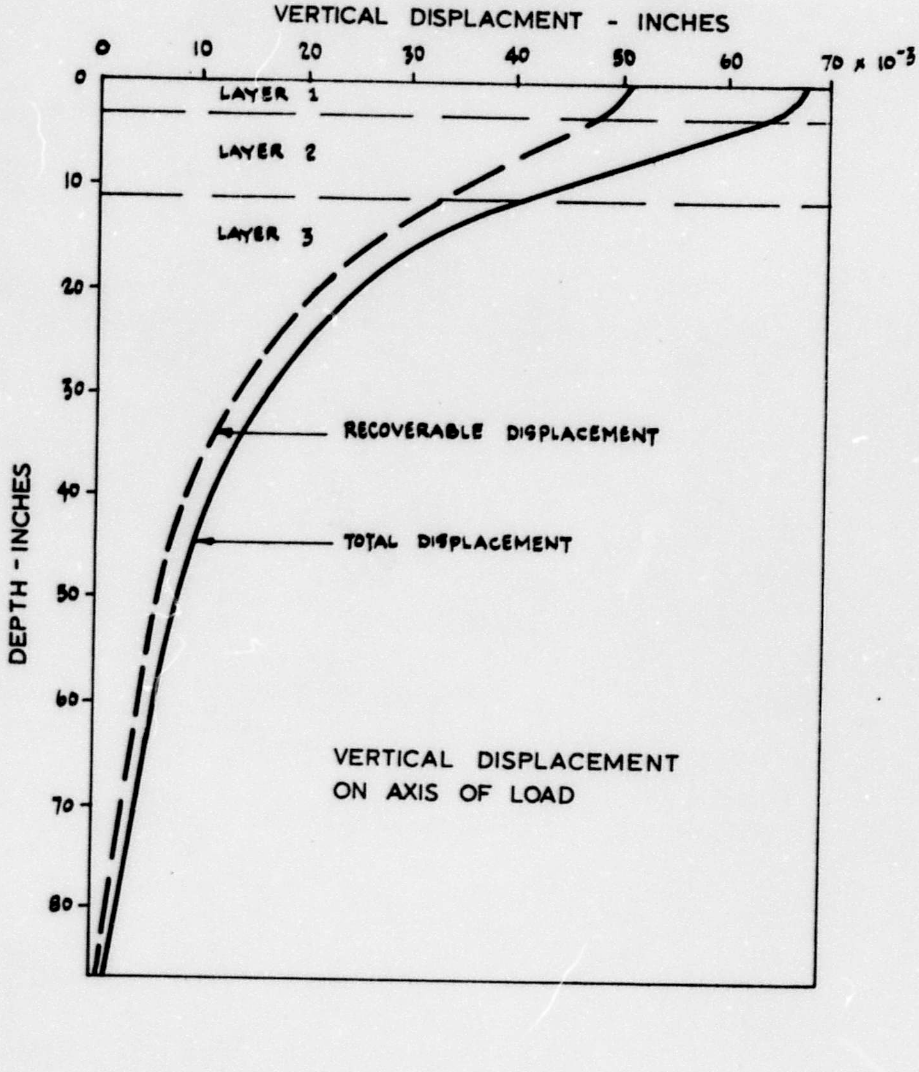
FIG. 21a.

INITIAL PARAMETERS

$E_1 = 50 \times 10^3$ p.s.i, $\nu = 0.35$
$E_2 = 20 \times 10^3$ p.s.i, $\nu = 0.35$
$E_3 = 2 \times 10^3$ p.s.i, $\nu = 0.46$ $\sigma_o = 3$ p.s.i.

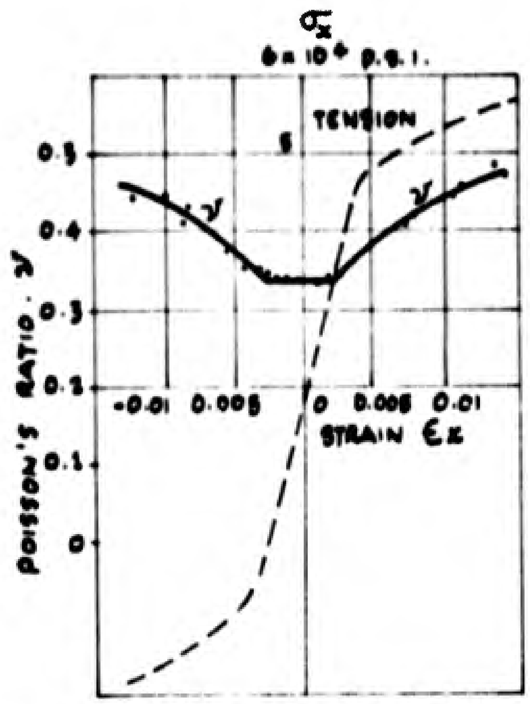


FINITE ELEMENT ANALYSIS OF THREE LAYER PAVEMENT.



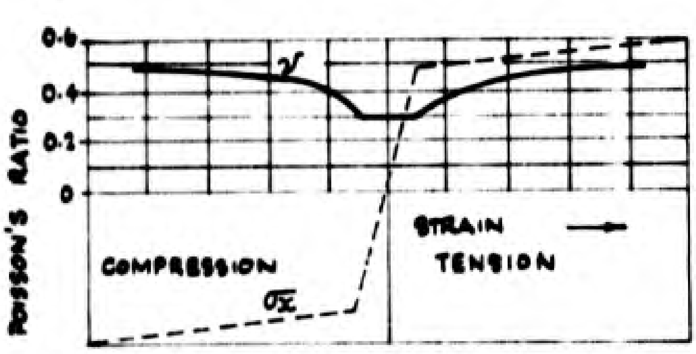
FINITE ELEMENT ANALYSIS OF THREE LAYER PAVEMENT.

FIG. 22



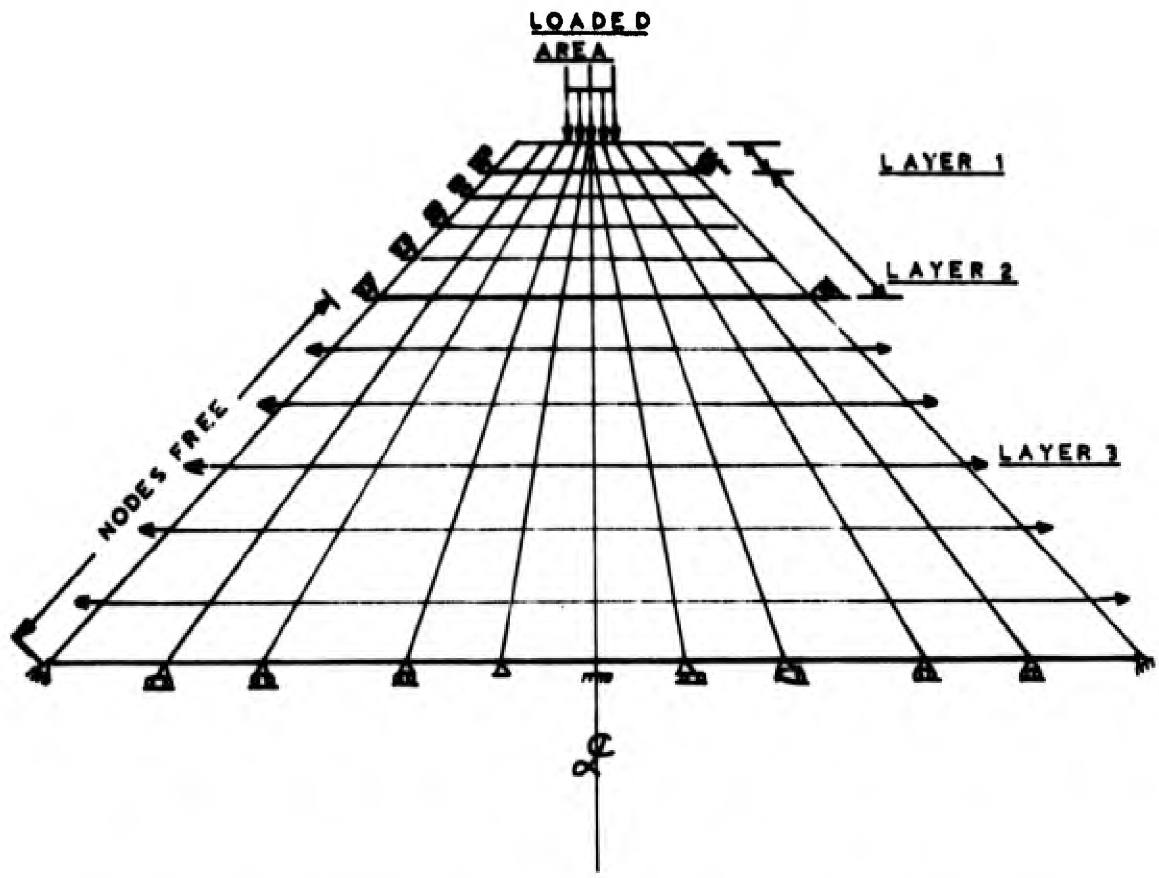
STRESS STRAIN CURVE AND POISSON'S RATIO FOR AN ALUMINIUM ALLOY.

(AFTER GERARD AND WINDHORN 1950) SEE NADAI 1963)



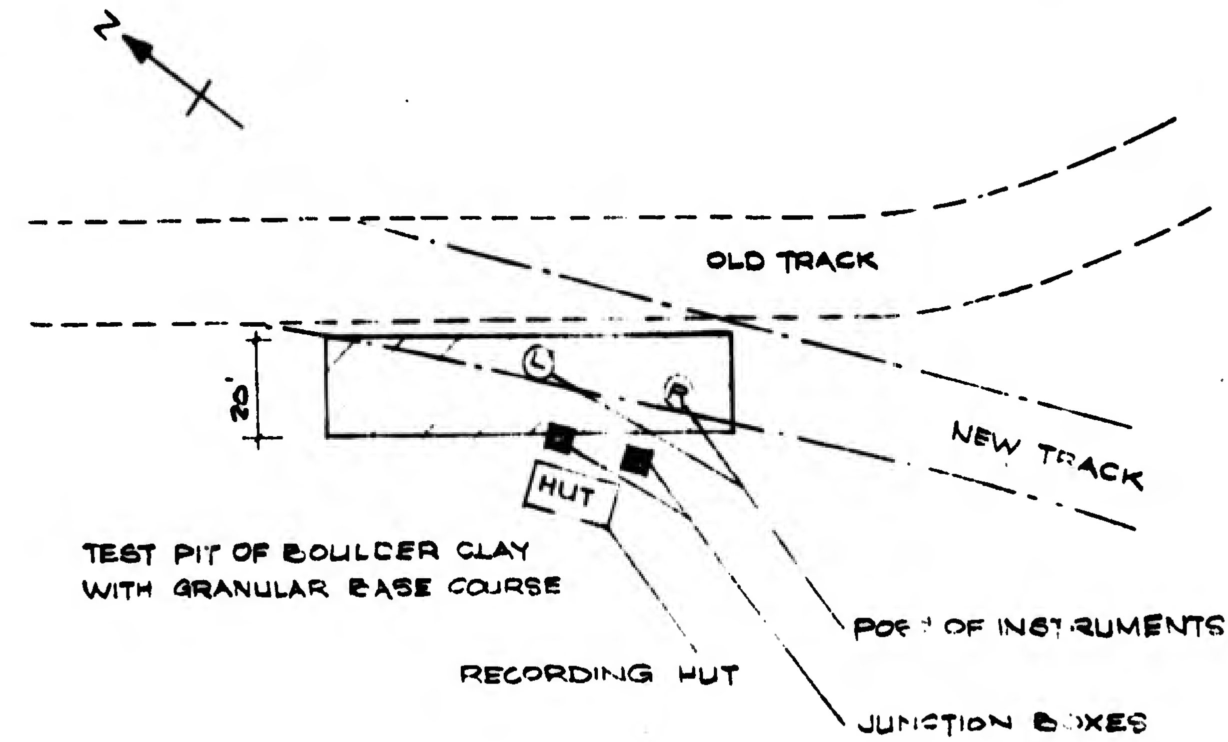
IDEALISED STRESS - STRAIN CURVE AND POISSONS RATIO (AFTER NADAI 1963)

POISSON'S RATIO FOR SOME ELASTOPLASTIC MATERIALS

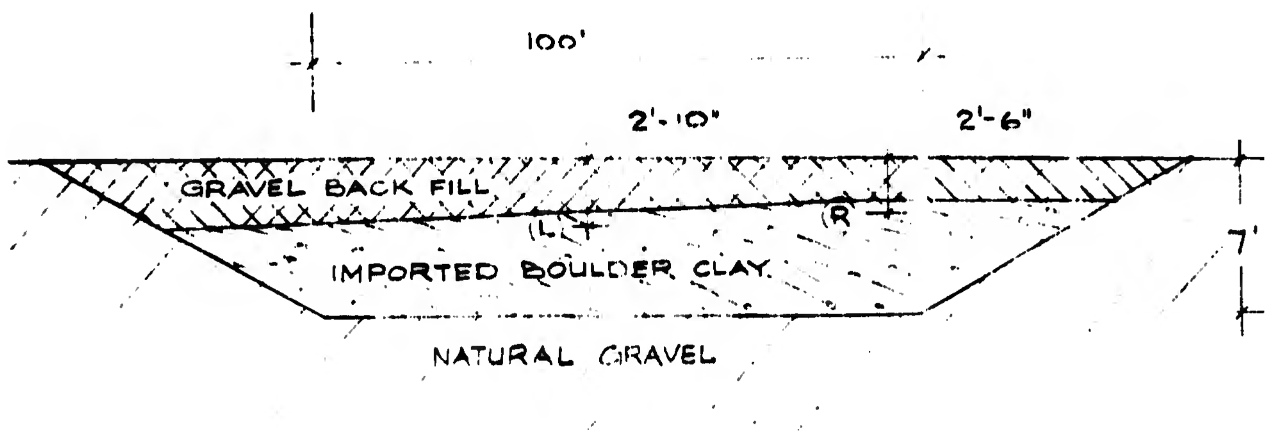


FINITE ELEMENT CONFIGURATION TO REPRESENT LAYERS IN A HORIZONTALLY LIMITED STRUCTURE

FIG. 24



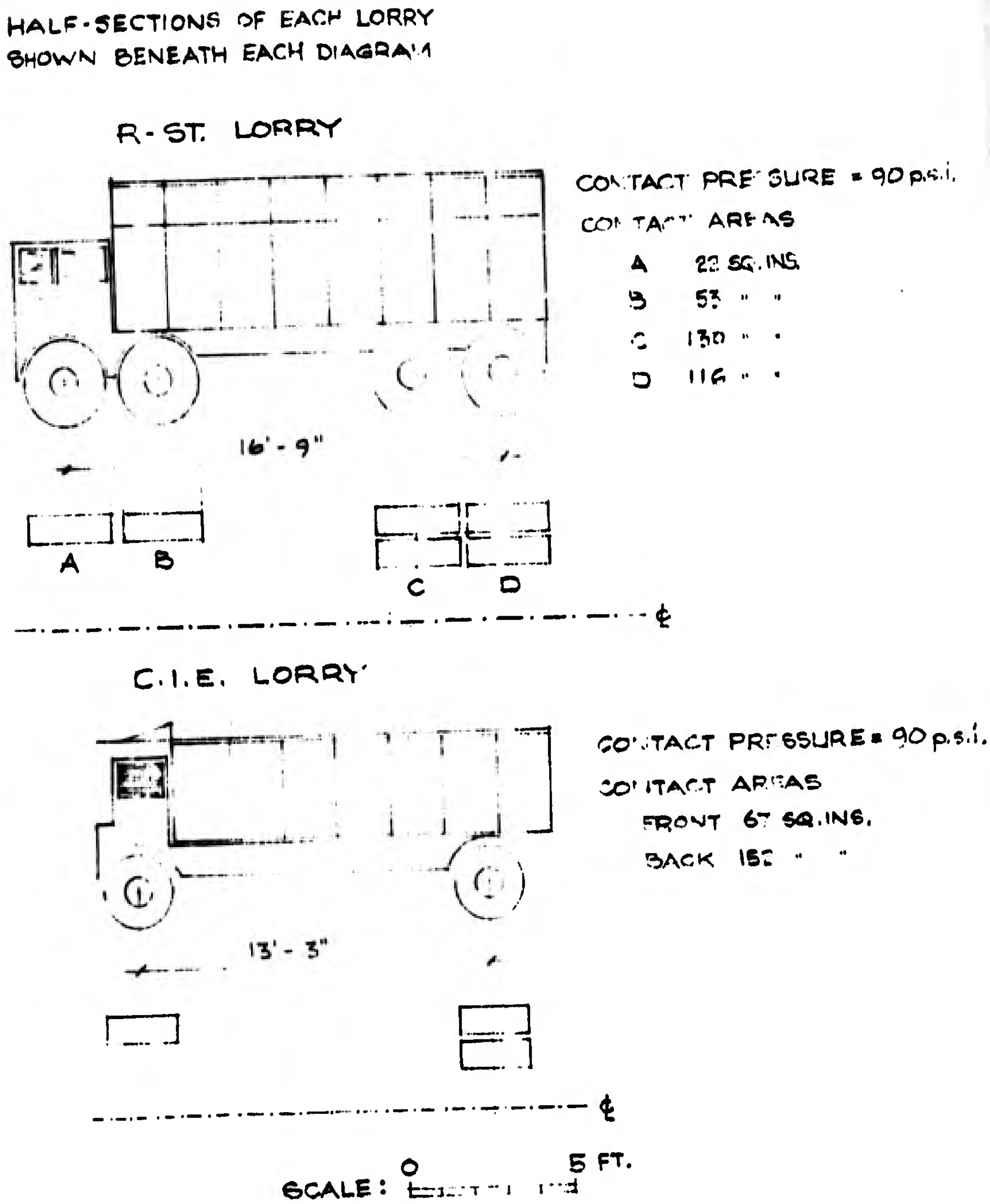
PLAN OF TEST SITE



LONGITUDINAL SECTION OF TEST PIT SHOWING POSITIONING OF THE TWO LOAD AND DISPLACEMENT SENSING DEVICES (N.T.S.)

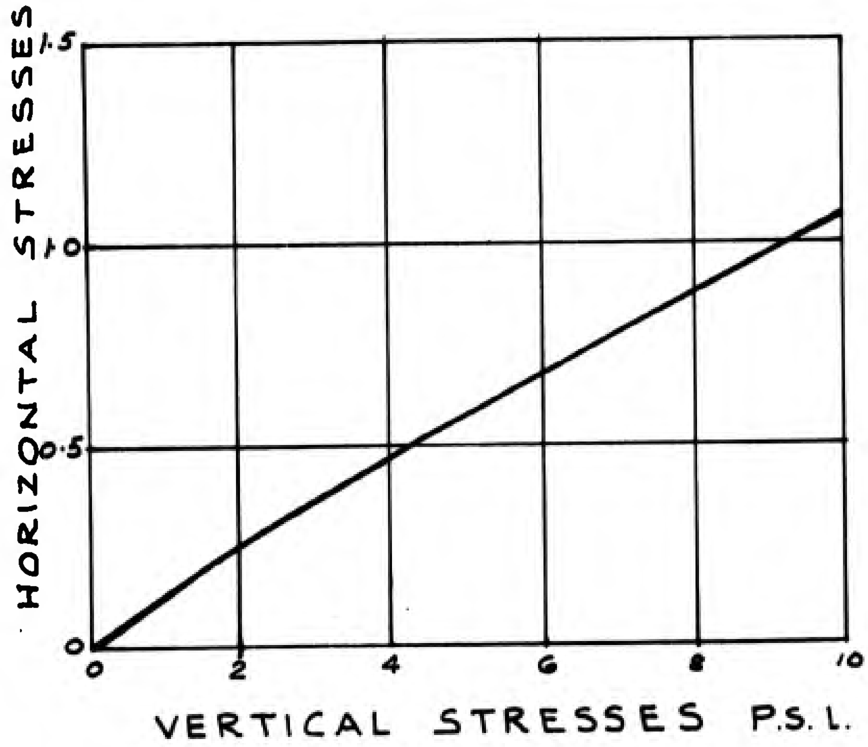
BLESSINGTON TEST TRACK

FIG. 25



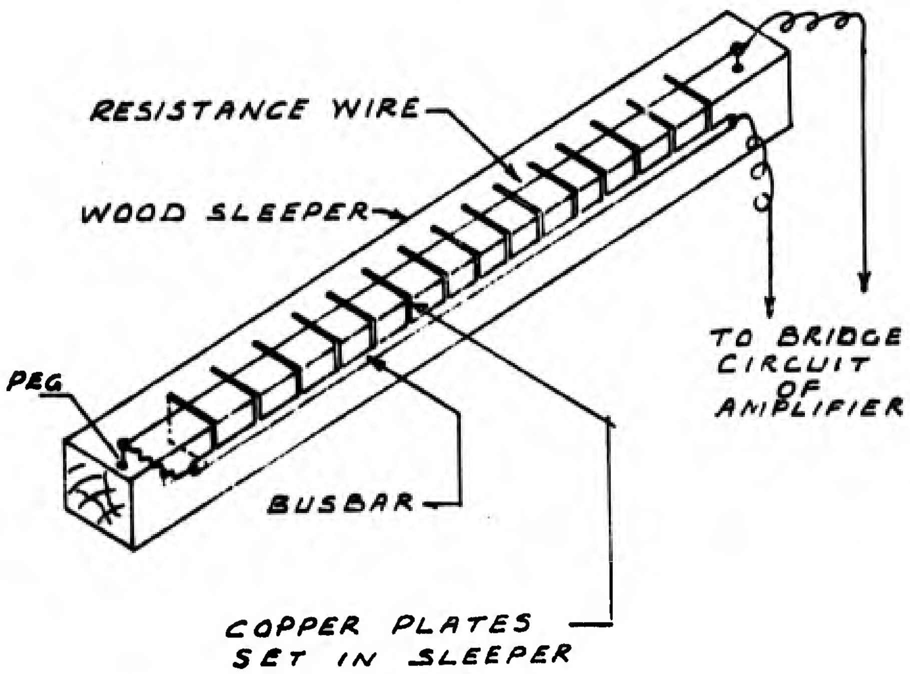
VEHICLE WHEEL CONFIGURATIONS

FIG. 26



REGRESSION ANALYSIS OF FIELD DATA

FIG. 27



WHEEL LOCATION FINDER

$$\left[\begin{array}{c|c|c|c|c|c|c|c}
z \ln r & & & & & & & \\
m r z & m r^2 z & & \text{SYMMETRICAL} & & & & \\
\frac{1}{2} z^2 \ln r & \frac{1}{2} m r z^2 & \frac{1}{3} z^3 \ln r + \frac{1}{2} n r^2 z & & & & & \\
\frac{1}{2} m r z^2 & \frac{1}{2} m r^2 z^2 & \frac{1}{3}(m r z^3 + n r^3 z) & \frac{1}{3} m r^2 z^3 + \frac{1}{4} n r^4 z & & & & \\
0 & 0 & 0 & 0 & 0 & & & \\
0 & 0 & \frac{1}{2} n r^2 z & \frac{1}{3} n r^3 z & 0 & \frac{1}{2} n r^2 z & & \\
L r z & L r^2 z & \frac{1}{2} L r z^2 & \frac{1}{2} L r^2 z^2 & 0 & 0 & \frac{1}{2} r^2 z & \\
\frac{1}{2} L r^2 z & \frac{2}{3} L r^3 z & \frac{1}{4}(L+n) r^2 z^2 & \frac{1}{6}(2L+n) r^3 z^2 & 0 & \frac{1}{4} n r^2 z^2 & \frac{1}{3} r^3 z & \frac{1}{6} n r^2 z^3 + \frac{1}{4} r^4 z
\end{array}\right]$$

$$L = \frac{\nu}{1-\nu}$$

$$m = 1 + \frac{\nu}{1-\nu}$$

$$n = \frac{1-2\nu}{2(1-\nu)}$$

INTEGRATION LIMITS

$$f(r)\Big]_{a_i}^{a_j} \quad , \quad f(z)\Big]_0^b$$

STIFFNESS MATRIX $[\bar{k}]$ FOR RECTANGULAR ELEMENTS

FIG. 28

UNCLASSIFIED
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U. S. Army Research & Development Group (Europe), APO New York 09757

13. ABSTRACT
The project represents a coordinated study involving a laboratory investigation, theoretical analysis and field measurements of stresses induced in flexible pavements by moving wheel loads. The laboratory tests indicated that the modulus of resilience is virtually independent of stress intensity and number of load repetitions provided the boulder clay samples are preconditioned. The theoretical (finite) element program was developed to the stage where either linear or non-linear material properties in any combination can be handled in pavement structures of one to four layers. Results of trial problems indicate that the finite element program is working correctly and yields realistic answers to pavement deflections. Results are presented.

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