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INFLUENCE OF THE POSITION OF AN ELLIPTICAL HOLE ON STRESS CONCENTRATION IN A STRETCHED ORTHOTROPIC PLATE

by

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## EDITED TRANSLATION

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INFLUENCE OF THE POSITION OF AN ELLIPTICAL HOLE ON STRESS  
CONCENTRATION IN A STRETCHED ORTHOTROPIC PLATE

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Problems in the concentration of stresses close to holes in anisotropic plates are of major practical interest, especially in connection with the broad application of synthetic anisotropic materials in technology. They are also of interest for the mining industry (study of stresses close to underground shafts, determination of pressure on a shaft lining, etc.). At present approximate solutions of the plane problem for anisotropic media with cutouts having widely varied configurations are known [1], but apparently as yet there has been no detailed investigation of the problem of how stress concentration is influenced by the position of the axis of the hole with respect to the principal directions of elasticity and the direction of action of the forces. Exceptions to this statement are the works of A. S. Dorogobed [2] and V. V. Lipkin [3], where a study was made of the distribution of stresses in a plate with a round hole deformed by tangential and bending forces which are oriented differently with respect to the principal directions.

Given below are some results of an investigation of stresses in an orthotropic plate with an elliptical hole cut at different angles to the principal directions of elasticity, with one-directional stretching.

1. Formulas for stress at the edge of the hole. We shall examine an elastic orthotropic plate weakened by an elliptical hole and stretched in one direction by forces acting in the middle plane. We shall make the following assumptions: 1) the directions  $x$  and  $y$  of the major axes of the hole in general do not coincide with the principal directions of elasticity  $\xi$  and  $\eta$ ; 2) the forces act at an arbitrary angle to the principal axes of the hole, while the edges of the latter are not fastened and are not loaded; 3) the material obeys the generalized Hooke's law and experiences small deformations.

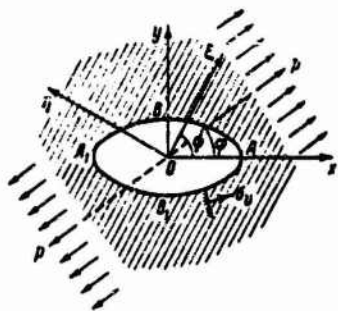


Fig. 1.

We shall introduce the following designations:  $a, b$  - lengths of the semi-axes of the ellipse;  $c = a/b$  ( $a > b$ );  $p$  - intensity of forces;  $\phi$  - their slope to the major axis;  $\psi$  - angle between directions  $\xi$  and  $x$  (Fig. 1);  $E_1$  and  $E_2$  - Young's moduli for tension-compression in directions  $\xi, \eta$ ;  $\nu_1$  and  $\nu_2$  - principal Poisson coefficients;  $G$  - shear modulus characterizing the change in the angle between the principal directions

$$k = \sqrt{\frac{E_1}{E_2}}, \quad n = \sqrt{2\sqrt{\frac{E_1}{E_2} + \frac{E_1}{G}} - 2\nu_1}, \quad m = \frac{E_1}{G} - 2\nu_1 \quad (1.1)$$

$$\begin{aligned} K &= 0.5n(1-k)\sin 2\psi, & M &= 0.5[1-k^2 + (1-m+k^2)\cos 2\psi]\sin 2\psi \\ N &= 0.5n[1+k + (1-k)\cos 2\psi], & N &= 0.25(1-m+k^2)\sin^2 2\psi - k \\ P &= 0.125(3+m+3k^2) + 0.5(1-k^2)\cos 2\psi + 0.125(1-m+k^2)\cos 4\psi \end{aligned} \quad (1.2)$$

The equations of the generalized Hooke's law in the plane stressed state are written as follows:

1) for the major system of coordinates  $\xi, \eta$

$$\epsilon_{\xi} = \frac{1}{E_1} \sigma_{\xi} - \frac{\nu_1}{E_1} \sigma_{\eta}, \quad \epsilon_{\eta} = -\frac{\nu_1}{E_1} \sigma_{\xi} + \frac{1}{E_2} \sigma_{\eta}, \quad \gamma_{\xi\eta} = \frac{1}{G} \tau_{\xi\eta} \quad (1.3)$$

2) for the system  $x, y$

$$\begin{aligned} \epsilon_x &= a_{11}\sigma_x + a_{12}\sigma_y + a_{16}\tau_{xy} \\ \epsilon_y &= a_{12}\sigma_x + a_{22}\sigma_y + a_{26}\tau_{xy} \\ \gamma_{xy} &= a_{16}\sigma_x + a_{26}\sigma_y + a_{66}\tau_{xy} \end{aligned} \quad (1.4)$$

The coefficients of deformation  $a_{1j}$  are expressed through the principal elastic constants (see [1], pp. 45-47). In our designations the formulas will have the form

$$\begin{aligned} a_{11} &= \frac{P}{E_1}, & a_{22} &= \frac{1}{E_1} [P + (k^2 - 1) \cos 2\psi] \\ a_{12} &= \frac{1}{E_1} [-\nu_1 + 0.25(1 - m + k^2) \sin^2 2\psi] \\ a_{66} &= \frac{1}{E_1} \left[ \frac{E_1}{G} + (1 - m + k^2) \sin^2 2\psi \right] \\ a_{16} &= \frac{1}{2E_1} [1 - k^2 + (1 - m + k^2) \cos 2\psi] \sin 2\psi \\ a_{26} &= \frac{1}{2E_1} [1 - k^2 - (1 - m + k^2) \cos 2\psi] \sin 2\psi \end{aligned} \quad (1.5)$$

The equation of the contour of the hole will be taken in parametric form:

$$x = a \cos \phi, \quad y = b \sin \phi \quad (1.6)$$

and we will also introduce the designations

$$\begin{aligned} I^2 &= c^2 \sin^2 \phi + \cos^2 \phi \\ S &= E_1 [a_{11}c^4 \sin^4 \phi - 2a_{16}c^2 \sin^2 \phi \cos^2 \phi + (2a_{12} + a_{66})c^2 \sin^2 \phi \cos^2 \phi - \\ &\quad - 2a_{26}c \sin \phi \cos^2 \phi + a_{22} \cos^4 \phi]. \end{aligned} \quad (1.7)$$

Assuming that the hole is small and is located far from the edge, we will consider the plate to be infinite.

The general solution for the problem of stress distribution in an infinite anisotropic plate with an elliptical hole can be found in the book [1], in which a number of particular cases are also examined, including stretching at arbitrary angle  $\phi$  ([1], pp. 148

and 152, formulas (37.8), (38.11), and (38.12)). The expression for the stress  $\sigma_\theta$  at the edge of the hole with arbitrary anisotropy is quite complicated and is obtained only in complex form, but in the case of an orthotropic plate it is possible to simplify it significantly and to separate the real part. We will present the final formula for the stress:

$$\begin{aligned} \sigma_\theta = \frac{p^2}{S} & \left[ (cN \sin^2 \varphi + (K + cM) \sin \varphi \cos \varphi + (L + cP) \cos^2 \varphi) c \sin^2 \theta - \right. \\ & - \{ [MN + K(L + cP)] c \sin^2 \varphi + [M(K + cM) + cL(L + cP) - LN] \times \\ & \times \sin \varphi \cos \varphi + (K + cM) P \cos^2 \varphi \} \frac{\sin \theta \cos \theta}{P} + \{ [K(K + cM) + \\ & \left. + N(N - cL)] \sin^2 \varphi + [MN + K(L + cP)] \sin \varphi \cos \varphi + PN \cos^2 \varphi \} \frac{\cos^2 \theta}{P} \right]. \end{aligned} \quad (1.8)$$

From this (with  $\phi$  and  $\psi$  equal to 0 or  $(1/2)\pi$ ) we will obtain an expression for stresses for four basic cases, when the hole is cut in such a way that the direction of the major and minor axes of the ellipse coincide with the principal axes of elasticity, and tension is carried out in the direction of the major or minor axis of the ellipse. In particular, during stretching in the direction of the minor axis ( $\phi = \pi/2$ ) we will have

$$\sigma_\theta = \frac{p^2}{S} k [-c^2 \sin^2 \theta + (k + cn) \cos^2 \theta] \quad (\varphi = 0) \quad (1.9)$$

or

$$\sigma_\theta = \frac{p^2}{S} [-kc^2 \sin^2 \theta + (1 + cn) \cos^2 \theta] \quad \left( \varphi = \frac{\pi}{2} \right). \quad (1.10)$$

In this case, when  $c > 1$  it is natural to expect that the greatest stress will be obtained at the ends of the major axis; it is determined according to one of the two formulas:

$$(\sigma_\theta)_{\theta=0} = p \left( 1 + \frac{cn}{k} \right) \quad (\psi = 0) \quad (1.11)$$

or

$$(\sigma_\theta)_{\theta=0} = p(1 + cn) \quad \left( \psi = \frac{\pi}{2} \right). \quad (1.12)$$

The stress on the ends of the major axis is determined as compressive [stress]:

$$(\sigma_\theta)_{\theta=\pi/2} = -pk \quad \text{or} \quad (\sigma_\theta)_{\theta=\pi/2} = -\frac{p}{k}. \quad (1.13)$$

For an isotropic plate ( $k = 1$ ,  $n = 2$ ) the above assertions are indisputable; however, in the case of an anisotropic plate it is possible (at least in theory) to find other relationships between the constants at which the greatest compressive stress exceeds the magnitude of the greatest tensile stress. We will note that stress concentration is significantly influenced by the principal shear modulus  $G$ , entering into the expression for  $n$ , which in an orthotropic plate is not connected with other elastic constants; at small  $G$  the concentration can be very significant.

With an arbitrary orientation of the hole  $\psi$  and stretching at an arbitrary angle of  $\phi$  the theoretical solution of the problem about which is the greatest stress and where it is obtained is made difficult by the complexity of formula (1.8). We will limit ourselves below to investigation of this problem for the particular case of a plate with given elastic constants.

All the formulas (1.8)-(1.13) are also valid for an orthotropic medium weakened by a cavity in the form of an elliptical cylinder located in a state of plane deformation under the action of stretching forces  $p$ ; however in this case the moduli  $E_1$  and  $E_2$  and the coefficient  $\nu_1$  must everywhere be replaced by the corresponding quantities:

$$E_1' = \frac{E_1}{1 - \nu_{13}\nu_{31}}, \quad E_2' = \frac{E_2}{1 - \nu_{23}\nu_{32}}, \quad \nu_1' = \nu_1 + \nu_{13}\nu_{31} \quad (1.14)$$

Here  $\nu_{13}$  and  $\nu_{23}$  are Poisson coefficients which characterize the contraction in the direction of the axis of the cavity during stretching along the principal directions  $\xi$  and  $\eta$ ;  $\nu_{31}$  and  $\nu_{32}$  are coefficients which characterize contraction along directions  $\xi$  and  $\eta$  during stretching along the axis of the cavity (see [1], p. 18).

2. Results of calculations. At present all of the elastic constants which are necessary for the calculations have been determined only for a comparatively small number of anisotropic materials. These include certain minerals (quartz, topaz, beryl, etc.), natural wood (oak, pine, Canadian fir, etc.), delta-wood

[veneer and plywood], aviation plywood, and also SVAM (glass fiber reinforced anisotropic material). The Young moduli and Poisson coefficients were found experimentally for the latter for the case of stretching along the fibers and at an angle of  $45^\circ$  [4], which makes it possible to calculate the principal shear modulus; the following is obtained:

$$E_1 = E_{\max} = 3.46 \cdot 10^8, \quad E_2 = E_{\min} = 3.43 \cdot 10^8, \quad G = 0.84 \cdot 10^8 \quad (\text{in kg/cm}^2) \quad (2.1)$$

$$\nu_1 = \nu_2 = 0.13, \quad k = 1.00, \quad n = 2.42$$

The following are the greatest tensile and compressive stresses in a plate of SVAM with an elliptical hole in which  $c = 2$  and in which the directions of the axes coincide with the principal [directions] under tension along the minor axis (see (1.11)-(1.13)):

$$\sigma_{\max} = 5.84p, \quad \sigma'_{\max} = p$$

For such an isotropic plate  $k = 1$ ,  $n = 2$ ,  $\sigma_{\max} = 5p$ , and  $\sigma'_{\max} = p$ . Comparison of all these numbers shows that SVAM should be related to the category of weakly anisotropic materials, and stresses in a plate of SVAM will differ insignificantly from stresses in an isotropic plate.

Therefore we took a material with stronger anisotropy, a material which has repeatedly been used to illustrate calculations of stresses in anisotropic plates - birch aviation plywood, in which

$$E_1 = 1.2 \cdot 10^8, \quad E_2 = 0.6 \cdot 10^8, \quad G = 0.07 \cdot 10^8 \quad (\text{in kg/cm}^2) \quad (2.3)$$

$$\nu_1 = 0.071, \quad \nu_2 = 0.036, \quad k = \sqrt{2} = 1.414, \quad n = 4.453$$

For the sake of definiteness we took  $a = 2b$  and  $c = 2$ . Tables 1 and 2 give the values of the greatest tensile stresses  $\sigma_{\max}$  and the greatest compressive stresses  $\sigma'_{\max}$  for five cases of tension:  $\phi = 0, 30, 45, 60, \text{ and } 90^\circ$ , and for eight cases of orientation of the hole with respect to the principal directions of elasticity:  $\psi = 0, 30, 45, 60, 90, 120, 135, \text{ and } 150^\circ$  (accurate to 2 decimal places; calculations with higher accuracy are hardly advisable, keeping in mind the approximate character of the initial data (2.3)).

Table 1. Greatest tensile stresses  $\sigma_{\max}/p$  (for plywood).

$\phi^\circ$	$\psi=0$	30	45	60	90	120	135	150	180
0	3.23	2.78	2.67	2.21	2.57	2.21	2.67	2.78	3.23
30	3.10	4.92	4.64	4.96	2.55	3.80	4.00	3.78	3.10
45	3.85	5.35	5.84	6.83	6.19	4.28	4.98	5.18	3.85
60	5.58	5.47	6.52	8.20	7.55	4.24	5.33	6.14	5.58
90	7.20	5.77	5.52	7.72	9.91	7.72	5.52	5.77	7.20

Table 2. Greatest compressive stresses  $\sigma'_{\max}/p$  (for plywood):

$\phi^\circ$	$\psi=0$	30	45	60	90	120	135	150	180
0	0.71	0.94	1.18	1.38	1.41	1.38	1.18	0.94	0.71
30	0.98	0.77	0.78	1.24	1.24	1.54	1.80	1.83	0.98
45	1.41	0.76	0.66	1.09	1.16	1.38	1.57	1.82	1.41
60	1.80	0.89	0.63	1.11	1.13	1.24	1.37	1.53	1.60
90	1.41	1.29	1.13	0.92	0.71	0.92	1.13	1.29	1.41

We shall introduce the maximum tensile stresses for an isotropic plate (for such a plate in all cases  $\sigma'_{\max} = p$ ):

$\psi=0$	30	45	60	90
$\sigma_{\max}/p = 2.00$	2.86	2.54	4.20	5.00

Figures 2-5 depict the graphs of distribution of stresses  $\sigma_\theta$  along the contour of a hole for 4 cases:

- 1)  $\phi = \psi = 0$ ; 2)  $\phi = 90^\circ, \psi = 0$ ; 3)  $\phi = 90^\circ, \psi = 45^\circ$ ; 4)  $\phi = 30^\circ, \psi = 60^\circ$ .

The magnitudes of stresses were plotted from the contour of the hole on continuations of the rays drawn from the center through the given points; positive stresses are depicted by arrows directed to the center.

The diagram of loading is shown on each graph. For comparison, a broken line is used to show the graph for an isotropic plate. We will note that at one and the same  $\phi$  the general character of the graphs are retained if  $90^\circ$  is added to the angle  $\psi$ , i.e.,

if the axes  $\xi$  and  $\eta$  change places, only the magnitude of the stresses is changed.

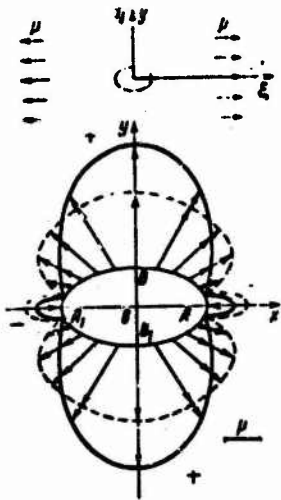


Fig. 2.

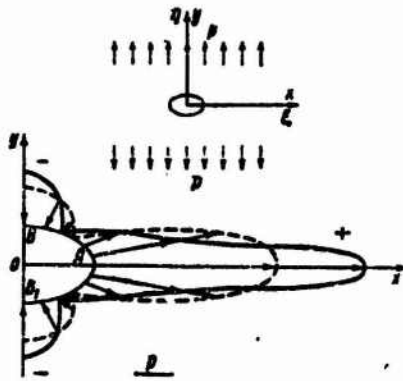


Fig. 3.

Figure 6 shows the change in  $\sigma_{\max}/p$  (curve 1) and  $\sigma_{\max}^i/p$  (curve 2) as a function of  $\psi$  during stretching along the minor axis of the hole; Figure 7 gives the change in  $\sigma_{\max}/p$  as a function of the slope  $\phi$  of forces to the major axis for  $\psi = 0$  (curve 1) and  $\psi = 90^\circ$  (curve 2).

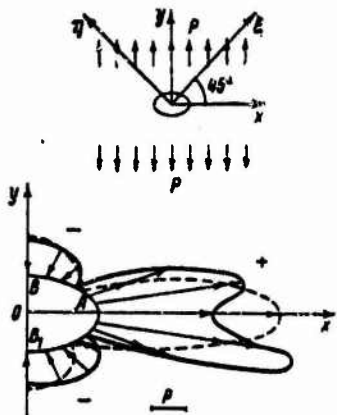


Fig. 4.

By analyzing the tables and graphs (as well as the results of calculating stresses on different points of the contour, which we did not do), it is possible to draw a number of conclusions relative to a plate made of aviation plywood; they give certain bases for judgment about the behavior under tension of a plate made from a different material, in which  $k > 1$  and  $n > 2$ . We will note the most important of these.

1. The contour of the hole is divided into four segments which are symmetrical

with respect to the center; these segments are subjected alternately to the action of tensile and compressive stresses. Here the greatest compressive stresses are, in all cases, significantly smaller than the greatest tensile stresses.

2. In all cases the maximum tensile stress is found to be greater than that in the same isotropic plate, or in other words, the concentration coefficient for the plywood plate is greater than that for an isotropic plate (with equal  $\phi$ ).

3. The greatest of all possible  $\sigma_{\max} = 9.91 p$  is obtained in the case when the opening is cut in such a way that the direction of its minor axis coincides with a direction of the maximum Young's modulus (fibers of the housing) and stretching is accomplished in the direction of the minor axis.

4. Among the other cases examined, the following turned out to be "unsuitable," in the sense of maximum tensile stresses:

- 1)  $\psi = 60^\circ, \varphi = 60^\circ, \sigma_{\max} = 8.20 p$
- 2)  $\psi = 60^\circ, \varphi = 90^\circ, \sigma_{\max} = 7.72 p$
- 3)  $\psi = 90^\circ, \varphi = 60^\circ, \sigma_{\max} = 7.55 p$ .

The places of greatest stress are located close to the ends of the major axis (angular distance from the x-axis does not exceed  $8^\circ$ ).

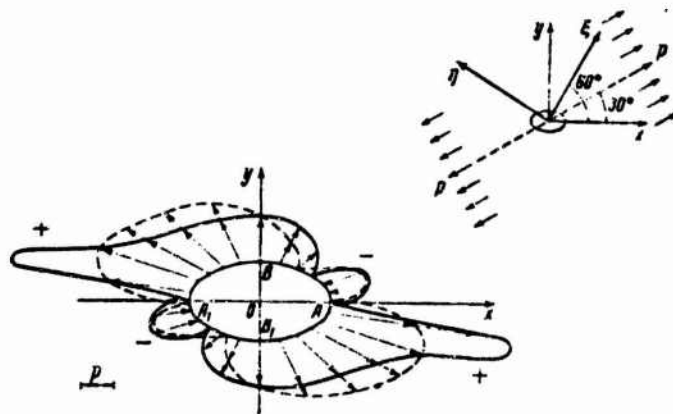


Fig. 5.

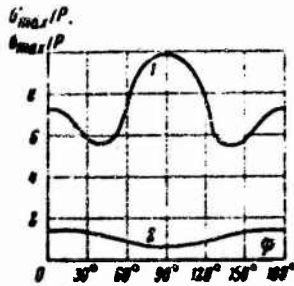


Fig. 6.

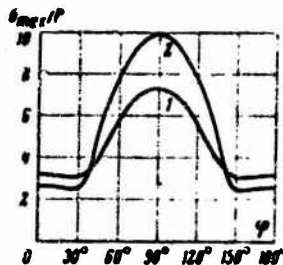


Fig. 7.

5. During stretching along the minor axis the least of all possible maximum tensile stresses,  $\sigma_{max} = 5.52p$ , is obtained in the case when the axis of the hole is directed at an angle of  $45^\circ$  to the principal directions of elasticity.

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25-ABSTRACT (UNCL, 0) ABSTRACT OF REPORT  The problem of stress concentration near holes in anisotropic plates is of considerable practical interest, especially in connection with synthetic anisotropic materials, with the stresses near underground workings, with the determination of the load on pit props, etc. The problem considered is the stress around the boundary of an elliptical hole in an orthotropic plate. The plate possesses two perpendicular principal elasticity directions in its plane; one of these directions makes an angle with the major axis of the elliptical hole, and an angle with the direction of the applied tension. It is assumed that the material obeys the generalized Hooke's law, that the strains are small, and that the plate can be regarded as infinite. The formula for the stress round the boundary of the hole is quoted from "Anisotropic Plates" by S. G. Lekhnitskiy, Gostekhizdat, 1957 (Ref. 1), and is discussed numerically using the elastic constants of glass fibre anisotropic material and of birch aviation plywood. The glass fibre material is weakly anisotropic and the stress distributions do not differ significantly from those in an isotropic material. For the plywood, the values of the maximum tensile and compressive stresses at the edge of the hole are calculated. (The parent document contains seven graphs, two tables showing greatest tensile stresses and greatest compressive stresses for plywood which are included on microfiche.)				