Special Program in Group Theory AFOSR 70-0993 TR

Final Status Report

Grant No. AF-AFOSR-1468-68 1.

Project Task: 9749-02

The activity at the Institute for Advanced Study under the Special Program on Finite Groups and Algebraic Groups took place at the Institute during the academic year 1968-69. The participants who received support from the Air Force Office of Scientific Research have already reported on their research in the Technical Status Report for the academic year 1968-69. Here we shall mainly concentrate on the general activities, in particular on the seminars.

There were two main seminars which took place during the whole academic year:

1. Seminar on Representations of Algebraic Groups and Related Finite Groups, with contributions by A. Borel, R. Carter, J. Humphreys, N. Iwahori, T. Springer, and R. Steinberg. The main theme was the linear representations, both modular and over the complex numbers, of the groups of rational points of algebraic groups defined over finite fields, and of certain related finite groups. Also presented were a number of topics in algebraic groups, which were both relevant to this topic and of independent interest. Notes of this seminar were made, and will appear shortly in the Springer Mathematics Lecture Notes Series.

2. Seminar on Finite Simple Groups, with contributions by R. Brauer, G. Glauberman, D. Gorenstein, K. Harada, G. Higman, Z. Janko, and E. Shult. There was first a series of lectures by Gorenstein addressed to all participants. The seminar then discussed a number of recent advances in the study and construction of finite simple groups.

In addition, there were three series of lectures of shorter duration (from one to two months):

3. Block Theory for the Layman, by J. Alperin, designed to give an introduction for the non-expert to R. Brauer's theory of blocks in representations of finite groups.

4. Simple Groups over Local Fields, by F. Bruhat and J. Tits.

5. Abstract Homomorphisms of Algebraic Groups, by A. Borel and J. Tits.

These last two seminars covered some rather extensive research in progress.

In addition, several participants spoke at the Members' Seminar which meets once a week throughout each academic year.

As was stated in our initial proposal, the goal of the program was twofold: to stimulate progress in each of two areas of group theory (finite groups and linear algebraic groups), and to foster and develop contacts between them. Accordingly, the seminars fulfilled various purposes. As can be seen from the above list, and the individual reports, the program appears to have been successful in both counts. The possibility of prolonged contacts helped a number of members to broaden, and some also found new directions for their work, especially in the framework of the first seminar above which was perhaps the main area of contact between the two groups of specialists. It also allowed much greater progress than would have been otherwise possible for some difficult joint work (in particular, that of Alperin, Brauer, and Gorenstein, and that discussed in seminars 4. and 5. above).

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The individual reports enclosed herewith cover only participants who received some support from the Air Force grant. But the program appears to have been equally stimulating for those supported by other sources. Furthermore, a number of Institute members with peripheral interests also participated in seminars and discussions, and the program, on the whole, fitted very well into, and contributed in considerable measure to, the mathematical life at the Institute.

The following manuscripts were completed and 25 copies of reprints will be forwarded to AFOSR as soon as they are received:

R. Brauer and D. Gorenstein (with J. L. Alperin), FINITE GROUPS WITH QUASI-DIHEDRAL AND WREATHED SYLOW 2-SUBGROUPS

D. Gorenstein, ON FINITE SIMPLE GROUPS OF CHARACTERISTIC 2 TYPE

———————, THE FLATNESS OF SIGNALIZER FUNCTORS ON FINITE GROUPS

D. Gorenstein and K. Harada, A CHARACTERIZATION OF JANKO'S TWO NEW SIMPLE GROUPS

Z. Janko, THE NONEXISTENCE OF A CERTAIN TYPE OF FINITE SIMPLE GROUP

M. Suzuki, CHARACTERIZATIONS OF LINEAR GROUPS

A. Borel, et al., SEMINAR ON ALGEBRAIC GROUPS AND RELATED FINITE GROUPS

Armand Borel
Principal Investigator
March 31, 1970

Technical Status Report for academic year 1968-69:

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Author: Richard Brauer

During the second term of this year I have been working on a paper: Finite groups with quasi-dihedral Sylow-2-subgroups which I am writing in collaboration with Jonathan Alperin and Daniel Gorenstein. On one hand, I have completed a chapter in its final form. I have also made some progress with the work on the final chapter (or a continuation) where the results are not yet complete. In addition, I have worked on: The orders of the finite linear groups of given degree. I have a first draft of a paper in which I shall deal with the subject.

Author: François Bruhat

During my stay at the Institute in the second term of 1968-69 I have continued my research on Tits systems of affine type and on algebraic groups over a local field, in collaboration with Professor J. Tits. The theory of valuations of root data and the theorems of descent of such a valuation have been completely settled. A first draft of a joint paper with Professor Tits has been written (approximately 250 typed pages) and a seminar has been held, by Professor Tits and myself from March 5 to April 23, in which a large part of our results has been exposed.

Author: Charles W. Curtis

In my research during this academic year there are two projects. The first is concerned with modular representations of finite groups with split (B,N)-pairs. In this work, a new and more conceptual formulation was discovered for the earlier results of Curtis and Richen on the classification of irreducible modular representations. New results were obtained on the connections between the irreducible representations and induced representations from parabolic subgroups.

The second project consists of joint work with N. Iwahori. Let G be a finite Chevalley group (or a twisted group), and let B be a Borel subgroup. The irreducible complex characters χ of G such that $\chi \in 1_B^G$ are investigated, using results of Iwahori on the structure of the centralizer rings $\mathrm{End}(1_P^G)$, with P a parabolic subgroup, and results of Curtis and Fossum on degrees of characters and primitive idempotents. Explicit formulas for degrees of characters and primitive idempotents affording them in the complex group algebra CG are found for a large class of characters, namely those irreducible characters χ such that for some parabolic group P, χ appears with multiplicity one in 1_P^G.

Author: Daniel Gorenstein

During my year at the Institute for Advanced Study, I have been working on the following problems:

(1) Completion of the classification of finite groups with quasi-dihedral Sylow 2-subgroups and of the non-character-theoretic portions of the corresponding classification of groups with wreathed Sylow 2-subgroups. This has been a joint effort with J. L. Alperin and R. Brauer.

(2) A preliminary investigation of finite groups of "characteristic 2 type", including various sufficient conditions for all the p-local subgroups of a group G to be p-constrained, p an arbitrary prime.

(3) An improvement of an earlier result on flat A-signalizer functors on finite groups, in which the assumption of flatness was shown to be superfluous.

(4) An improvement of the same earlier result in a second direction - namely, in reducing the minimal rank $m(A)$ for A for which the theorem is applicable from $m(A) \geq 6$ to $m(A) \geq 4$.

(5) A theorem on groups of 2-rank at least 5 in which the centralizer of every involution is 2-constrained. In particular, a classification of simple groups of 2-rank at least 5 in which the centralizer of every involution is solvable of 2-length 1. Jointly with J. H. Walter.

(6) Some general concepts and results concerning 2-components and the 2-layer of a finite group that are needed for the analysis of the centralizers of involutions in finite groups. Jointly with J. H. Walter.

(7) A general result concerning the existence of A-signalizer functors in finite groups which is a part of the same analysis of the centralizers of involutions. Jointly with J. H. Walter.

(8) A classification of simple groups whose Sylow 2-subgroups are of order 2^7 and isomorphic to a Sylow 2-subgroup of one of Janko's two recently discovered new simple groups. This has been primarily an effort of Harada, but the results of (4) above and one of the results of (5) above have been designed in part to apply to this problem.

Author: Graham Higman

The following matters have engaged my attention principally:

(i) A study of certain Steiner triple systems, suggested by work of M. Hall. Some progress was made, but the work has not yet reached a definitive stage.

(ii) The sporadic simple groups, particularly the group forecast and characterized by D. Held. For this group, I obtained an existence proof, showing that if it exists it must be given by certain generators and relations. Handling these generators and relations could only be done by cooperation with an expert on the use of electronic computers in group theory, and, in fact, the necessary computations were done by J. Mackay at the Atlas Laboratory in Chilton, England. A report on this will be published in due course.

(iii) A number of topics in infinite discrete group theory, particularly one-relator groups and torsion free nilpotent groups. This was exclusively in discussions with G. Baumslag; and, while some of what I had to say was decisive, particularly in the construction of counterexamples, what we achieved will show in his published work rather than in mine.

Author: Nagoyoshi Iwahori

During the second term of this year and my stay at the Institute for Advanced Study I have been working jointly with Charles W. Curtis in trying to determine the complex character table of a finite group G which has a BN-pair (B,N). Due to the difficulty of the problem, we are still on our way to our final goal.

Author: Tonny A. Springer

During my stay at the Institute in the second term of this year I worked in the following subjects:

(1) conjugacy classes in semi-simple groups;

(2) representations of finite algebraic groups;

(3) eigenvalues of elements of Weyl groups..

(1) and (2) are connected with the topics discussed in a weekly seminar, so there was also an expository reason for my interest in these subjects at this time.

Author: Robert Steinberg

The concrete publications resulting from my stay at the Institute for Advanced Study from February to April 1969 will be part of a paper on central extensions of finite groups and a chapter or two in a set of notes coming out under the editorship of A. Borel, the result of a seminar on algebraic groups. But the main benefits of my stay were the renewed contacts that I made, with the attendant person-to-person exchanges of information.

Author: Michio Suzuki

I have been working on charact- izations of simple groups by the structure of centralizers of involutions. I obtained the following results: Let H be the centralizer of an involution of $PSL(m,2^n)$ which is contained in the center of a Sylow 2-group. Let G be a finite simple group which satisfies the conditions (1) G contains H and (2) The centralizer in G of any involution of the center of H coincides with H. If $n > 1$, then G is isomorphic to $PSL(n,2^m)$. If $n = 1$, there are exceptions. The alternating

groups of degree 6 and 9, the Mathieu group of degree 24, the simple group recently discovered by combined efforts of D. Held and G. Higman are the exceptions for $m \leq 5$. If $m \geq 6$, G contains a subgroup isomorphic to PSL(m,2). It remains open to decide if there are any exceptions for $m \geq 6$.

Generalization of the above theorem for non-simple groups has been obtained and similar works for other classical groups are underway.

Author: Jacques Tits

During my stay at the Institute for Advanced Study (February 20 - April 20, 1969) I have mainly been engaged in joint work with Professor A. Borel and with Professor F. Bruhat.

The work with F. Bruhat, on algebraic groups over local fields, was initiated four years ago. Here, we gave a series of expository talks on the results obtained so far, and pushed further our researches on the subject; in particular, we considerably improved our understanding of the newly introduced notion of valuation of a root data.

The work with A. Borel - also a continuation - concerns the study of "abstract" homomorphisms between groups of rational points of algebraic simple groups. Among other things, we established some basic lemmas on the action of a Levi subgroup of a parabolic subgroup P of a simple algebraic group on the unipotent radical of P. Also this work led to a series of seminar talks by A. Borel and myself.

Besides these main activities, I attended other seminars or lectures, and took part in many discussions with specialists of the theory of algebraic groups as well as of the theory of finite groups. This led me for instance to the construction of a counter-example to a conjecture of Professor C. Sims on the orbit structure of permutation groups.

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13. ABSTRACT

The activity at the Institute for Advanced Study under the Special Program on Finite Groups and Algebraic Groups took place at the Institute during the Academic Year 1968-69. There were two main seminars which took place during the whole academic year: 1. Seminar on Representations of Algebraic Groups and Related Finite Groups. The main theme was the linear representations, both modular and over the complex numbers, of groups of rational points of algebraic groups defined over finite fields, and of certain related finite groups. Also presented were a number of topics in algebraic groups, which were both relevant to this topic and of independent interest. 2. Seminar on Finite Simple Groups. There was first a series of lectures addressed to all participants. The seminar then discussed a number of recent advances in the study and construction of finite simple groups. In addition, there were three series of lectures of shorter duration (from one to two months): 3. Block Theory for the Layman, designed to give an introduction for the non-expert to R. Brauer's theory of blocks in representations of finite groups. 4. Simple Groups over Local Fields, and 5. Abstract Homomorphisms of Algebraic Groups. In addition, several participants spoke at the Members' Seminar which meets once a week throughout each academic year.

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