

*Esti file Copy*

ESD-TR-70-34

# ESD ACCESSION LIST

ESTI Call No. 66214

Copy No. 1 of 7 cys.

## ESD RECORD COPY

RETURN TO  
SCIENTIFIC & TECHNICAL INFORMATION DIVISION  
(ESTI), BUILDING 1211

### Technical Report

478

### Very Long Baseline Interferometry as a Means of Worldwide Time Synchronization

A. E. E. Rogers

J. M. Moran, Jr.

17 February 1970

Prepared under Electronic Systems Division Contract AF 19(628)-5167 by

## Lincoln Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Lexington, Massachusetts



AD704134

This document has been approved for public release and sale;  
its distribution is unlimited.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
LINCOLN LABORATORY

VERY LONG BASELINE INTERFEROMETRY  
AS A MEANS OF WORLDWIDE TIME SYNCHRONIZATION

*A. E. E. ROGERS*

*J. M. MORAN, JR.*

*Group 21*

TECHNICAL REPORT 478

17 FEBRUARY 1970

This document has been approved for public release and sale;  
its distribution is unlimited.

LEXINGTON

MASSACHUSETTS

The work reported in this document was performed at Lincoln Laboratory, a center for research operated by Massachusetts Institute of Technology, with the support of the Department of the Air Force under Contract AF 19(628)-5167.

This report may be reproduced to satisfy needs of U.S. Government agencies.

Non-Lincoln Recipients

**PLEASE DO NOT RETURN**

Permission is given to destroy this document  
when it is no longer needed.

#### ABSTRACT

Extraterrestrial radio waves from sources of small angular size can be used to synchronize clocks at two receiving sites to a high degree of precision. Radio star signals, together with time identification, are recorded on magnetic tape. Tapes are subsequently processed on a digital computer to obtain the synchronization error between the two clocks at the time of observation. The ultimate limit to the precision of the method is the knowledge of the relative signal delay in the atmosphere and ionosphere.

Accepted for the Air Force  
Franklin C. Hudson  
Chief, Lincoln Laboratory Office



## CONTENTS

|                             |     |
|-----------------------------|-----|
| Abstract                    | iii |
| I. Introduction             | 1   |
| II. Principles of Operation | 1   |
| III. Delay Resolution       | 7   |
| IV. Systematic Errors       | 7   |
| V. Signal-to-Noise Ratios   | 7   |
| VI. Conclusion              | 8   |
| References                  | 9   |

# VERY LONG BASELINE INTERFEROMETRY AS A MEANS OF WORLDWIDE TIME SYNCHRONIZATION

## I. INTRODUCTION

Interferometers<sup>1-3</sup> with baselines up to 7,000 km have been used to measure the angular sizes of extraterrestrial radio sources. These interferometers use independent frequency standards of a high spectral purity to convert radio star signals to video. The video signals from each element of the interferometer are encoded, together with time identification, and recorded on magnetic tape. The tapes are subsequently processed on a digital computer to obtain information about the angular size of the radio source.

## II. PRINCIPLES OF OPERATION

If the angular size of a radio source at wavelength  $\lambda$  is smaller than  $\lambda/D$  radians, the electric field time functions sampled at a distance  $D$  apart along the wavefront are identical. Thus, the voltage time functions received by two antennas from a small-diameter source are just

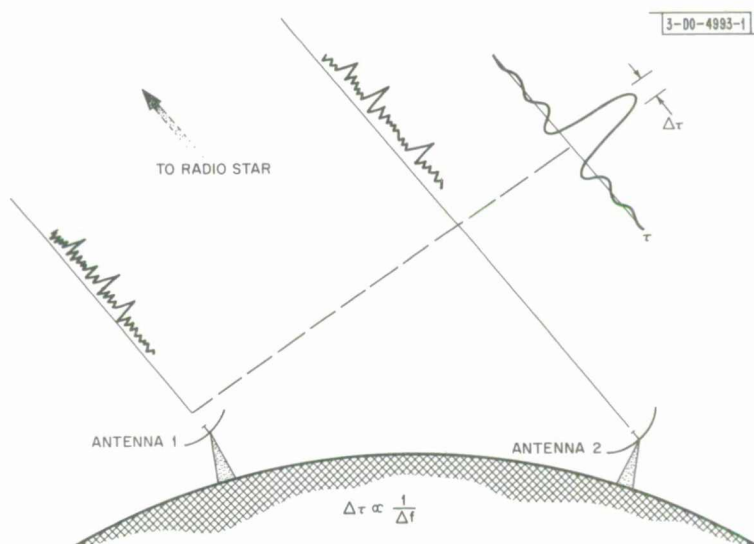


Fig. 1. Long baseline interferometer.

delayed replicas of each other. This is illustrated in Fig. 1. The delay  $\tau_{\text{int}}$  is determined by the geometry shown in Fig. 2. Neglecting delays in the atmosphere and ionosphere,  $\tau_{\text{int}}$  is simply

$$D/C \cos \Theta \quad ,$$

where  $D$  is the length of the baseline,  $C$  is the velocity of propagation, and  $\Theta$  is the angle between the line to the source (assumed at infinity) and the line between sites. In celestial coordinates,

$$\tau_{\text{int}} = \frac{D}{C} [\sin \delta_B \sin \delta_S + \cos \delta_B \cos \delta_S \cos (L_S - L_B)] \quad ,$$

where  $L_S$  and  $\delta_S$  are the hour angle and declination of the source, and  $L_B$  and  $\delta_B$  are the hour angle and declination of the baseline.

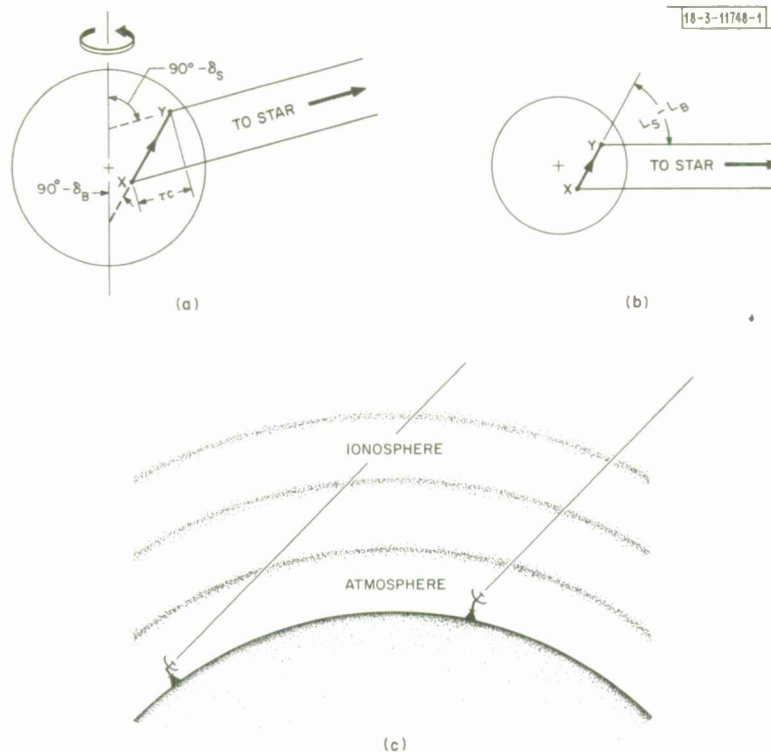


Fig. 2. Geometry of long baseline interferometer.

The geometric delay could be measured by cross correlating the time functions known to occur at specified times. Alternatively, the difference in time between clocks used to identify time function samples can be obtained if the geometric delay is already known. In practice, the time functions cannot be sampled directly but must be converted to video with a local oscillator and mixer chain.

For a continuum source (a source whose spectrum is nearly flat or nonspectral), the Fourier transform  $S_{xy}(\omega)$  of the cross-correlation function  $R_{xy}(\tau)$  of the video signals (in an upper sideband system) is

$$S_{xy}(\omega) = A \exp \{ i [ \varphi + \Delta\omega t + (\omega + \omega_o) (\tau_{int} + \tau_e) ] \} ,$$

where

$\omega_o$  = local oscillator frequency (radians/sec),

$\omega$  = local frequency,

$\Delta\omega$  = difference in local oscillator frequencies,

$\tau_e$  = time error,

$\varphi$  = instrumental phase,

$\omega_o d\tau_{int}/dt$  = fringe rotation rate.

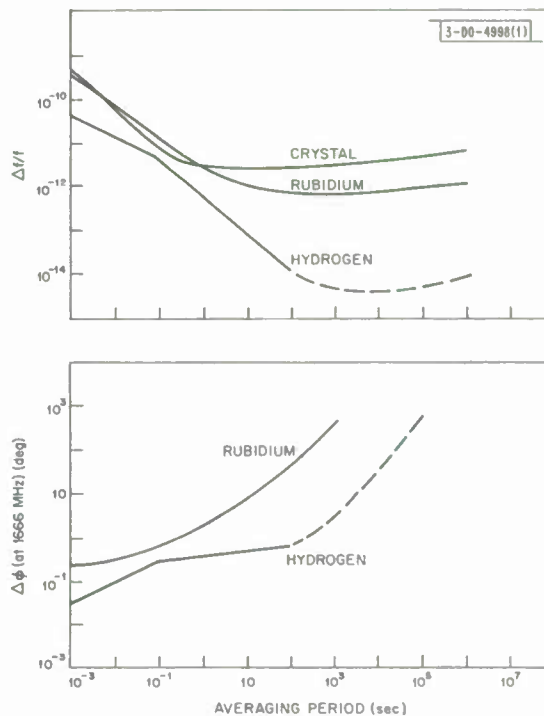
This function has a time-varying phase given by the sum of the local oscillator frequency difference and the "fringe rate." This can be compensated by suitable local oscillator offsets (to cancel most of the fringe rate) and by sine-cosine multiplication in the data reduction program. However, a fundamental "coherent" integration time  $T$  is imposed by requiring

$$|\langle e^{i\varphi(t)} \rangle_T| \approx 1,$$

or the "phase noise" between standards must be small enough to allow coherent integration.

Figure 3 shows how long a coherent integration can be performed at 1666 MHz by requiring  $\Delta\phi$

Fig. 3. Fractional frequency stability and phase noise of various frequency standards. Data were obtained by R. Vessat, Hewlett-Packard Frequency and Time Division, November 1967.



to be less than 100 degrees. The cross-correlation function  $R_{xy}(\tau)$ , after removal of fringe rotation rate, is

$$R_{xy}(\tau) = 2A \cos \varphi \frac{\sin \omega_{\max}(\tau + \tau_{\text{int}} + \tau_e)}{\pi(\tau + \tau_{\text{int}} + \tau_e)} - 2A \sin \varphi \frac{[1 - \cos \omega_{\max}(\tau + \tau_{\text{int}} + \tau_e)]}{\pi(\tau + \tau_{\text{int}} + \tau_e)},$$

where  $\omega_{\max}$  is the bandwidth (radians/sec) of the video recorded. The above shows that the cross correlation is zero except around  $\tau = -(\tau_{\text{int}} + \tau_e)$ . Figure 4 shows sine and cosine components of the cross-correlation function.

To reduce the information rate, and for ease of computation, the video signals are first bandlimited and then infinitely clipped and sampled at a rate equal to twice the bandwidth. Thus, the video data are in binary form (video  $\geq 0$  corresponding to logical 1, video  $< 0$  corresponding to logical 0) and are easily digitally recorded\* in the form of bit strings. The effect of infinite clipping is to preserve zero-crossing information, and a theorem by Van Vleck,<sup>4</sup>

\*At present, time identification is achieved by recording the first bit of each data block at a precisely known time relative to the station clock.

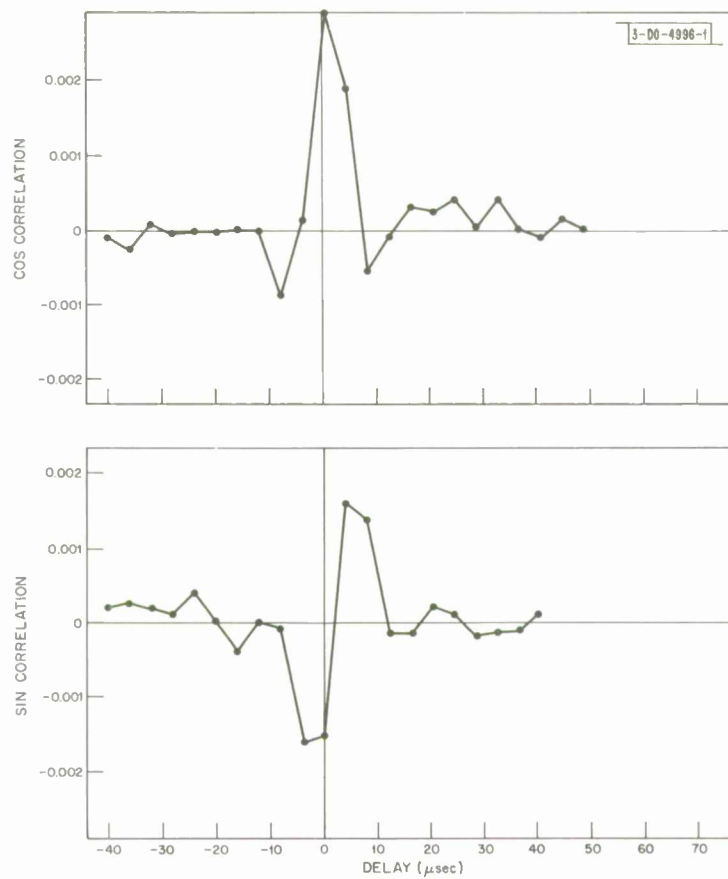


Fig. 4. Cross-correlation function components observed on radio source 3C273 with receiver bandwidth of 120 Hz.

$$\rho_{xy}(\tau) = \sin[\pi/2 \rho_{c_{xy}}(\tau)] \quad ,$$

allows the normalized cross-correlation function of the video signals  $\rho_{xy}(\tau)$  to be estimated from the cross correlation of the bit strings  $\rho_{c_{xy}}(\tau)$ . Figure 5 shows a block diagram of a long baseline interferometer. Figure 6 shows the amplitude of the  $S_{xy}(\omega)$  for different trial fringe rates.

Fig. 5. Long baseline interferometer terminal.

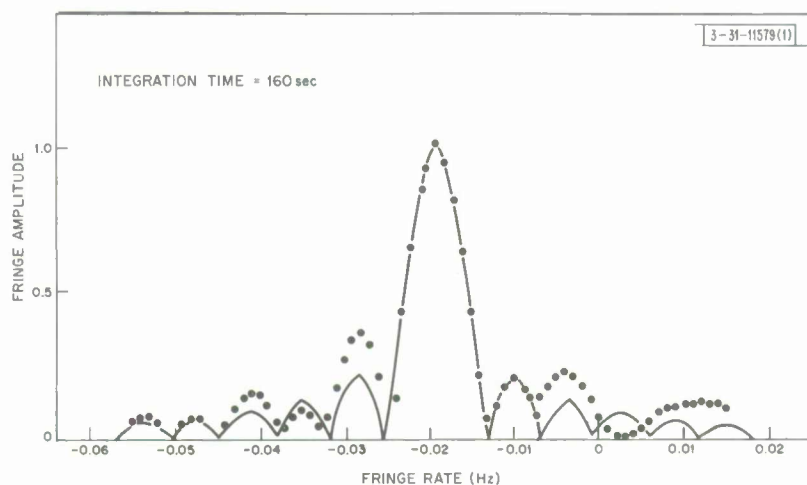
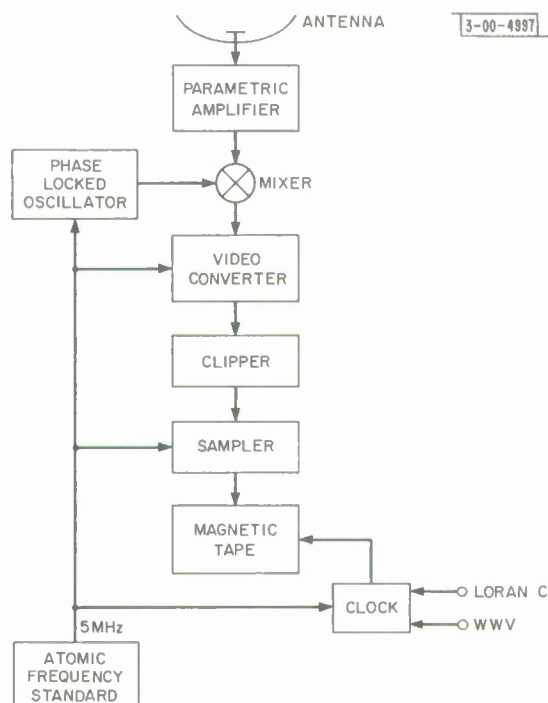


Fig. 6. "Fringe rate" spectrum of 3C273 observed with coherent integration time of 160 seconds compared with theoretical  $|\sin x/x|$  function which would be observed in the absence of phase noise.

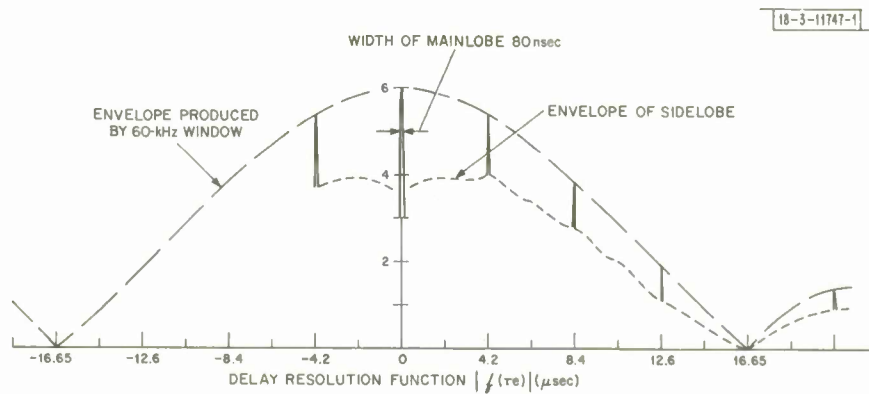


Fig. 7. Theoretical delay resolution function for multiwindow system.

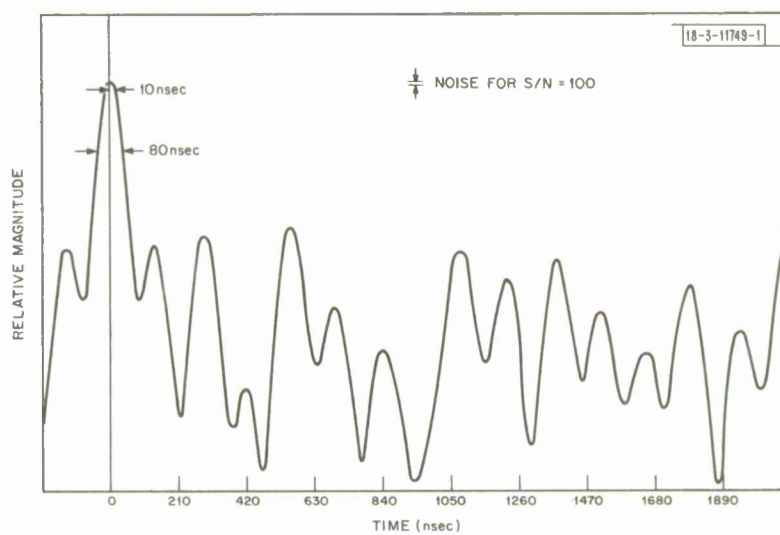


Fig. 8. Sidelobe structure of function shown in Fig. 7.

### III. DELAY RESOLUTION

From the expression for the cross-correlation function, it is clear that the delay resolution is of the order of  $(2\pi/\omega_{\max})$ , since this is approximately the half-power width of the magnitude of the cross-correlation function. If the signal-to-noise ratio is good enough, resolution should be up to one order of magnitude better. However, if recorder bandwidth is limited, the delay resolution can be improved by sampling and multiplexing several narrowband video windows within a wide band. When this procedure is adopted, the delay resolution function (cross-correlation functions convolved with expected functions and summed) is shown in Figs. 7 and 8 for one proposed multiwindow scheme.

### IV. SYSTEMATIC ERRORS

The long baseline interferometer as a precise method of time synchronization is limited by the precision with which the geometric delay can be computed. The computation of geometric delay involves a knowledge of source positions, baseline vector, and sidereal time. Without first using the interferometer as a survey instrument, the geometric delay can only be computed reliably to within 20 nanoseconds. For baselines within a continent, it might be possible to reduce errors below 20 nanoseconds. With errors this large, it is not important to take careful account of ionospheric and atmospheric delays which should not amount to more than 20 nanoseconds at 1.6 GHz except at low elevations. However, a careful use of a series of observations aimed at determining both time synchronization and baseline vector should result in an order of magnitude improvement if the ionospheric delay can be correctly predicted.

### V. SIGNAL-TO-NOISE RATIOS

Up to this point, we have considered the video signals to be due only to the radio source. With moderate antenna sizes and conventional radiometric receivers, the noise far exceeds the source signal. However, the noise signals are uncorrelated and, provided that enough bits are processed, it is possible to observe very weak signals. The signal-to-noise ratio (S/N) after coherent integration is

$$\frac{S}{N} = \sqrt{\frac{T_{a_x} T_{a_y}}{T_{s_x} T_{s_y}}} \sqrt{BT} \quad ,$$

where the first factor is the geometric mean of the signal-to-noise ratio in the two systems before data processing, and the second factor is  $1/\sqrt{2}$  times the square root of number of bits processed. Table I gives the signal-to-noise ratios for various antenna combinations for 100 seconds integration or  $7.2 \times 10^7$  bits processed. If a signal-to-noise ratio of 4 is regarded as adequate, a combination on the NRAO 140-foot and any 20-foot antenna with 100°K system temperature could yield this desired signal-to-noise ratio with only about  $10^6$  bits processed.  $10^6$  bits could be transmitted over a normal HF radio channel in about 5 minutes.

| TABLE I<br>SIGNAL-TO-NOISE RATIO AT 1.6 GHz ON 3C273<br>(100 Seconds Integration or $7 \times 10^7$ Bits Processed) |                                          |                            |                            |      |
|---------------------------------------------------------------------------------------------------------------------|------------------------------------------|----------------------------|----------------------------|------|
| Antenna 1                                                                                                           | Antenna 2                                | $(T_A/T_S)_{\text{Ant 1}}$ | $(T_A/T_S)_{\text{Ant 2}}$ | S/N* |
| Hoystock 120-foot                                                                                                   | NRAO 140-foot                            | 1/100                      | 1/23                       | 100  |
| Hoystock 120-foot                                                                                                   | Any 60-foot antenna                      | 1/100                      | 1/200                      | 40   |
| NRAO 140-foot                                                                                                       | Any 20-foot antenna<br>with 100°K system | 1/25                       | $\frac{1}{(49 \times 25)}$ | 40   |
| *S/N based on 14 flux-units unresolved from 3C273 at 1.6 GHz.                                                       |                                          |                            |                            |      |

#### Other Strong Sources in Unresolved Flux-Units

|          |    |          |   |
|----------|----|----------|---|
| CTA21    | 6  | 3C287    | 3 |
| 3C84     | 3  | 3C286    | 3 |
| NRAO 140 | 3  | 3C309.1  | 3 |
| NRAO 150 | 4  | 1510-08  | 3 |
| 3C19     | 3  | 3C345    | 5 |
| 3C120    | 3  | NRAO 530 | 4 |
| 3C138    | 3  | 3C380    | 3 |
| 3C147    | 4  | 3C418    | 4 |
| 1127-14  | 6  | 2127+04  | 4 |
| 1148-00  | 3  | 2203-18  | 3 |
| 3C268.1  | 3  | 3C446    | 3 |
| 3C273    | 14 | CTA102   | 6 |
| 3C279    | 8  | 3C454.3  | 9 |

## VI. CONCLUSION

The very long baseline interferometry technique should be able to provide worldwide time synchronization to  $0.05 \mu\text{sec}$  for time standards located at the world's major radio astronomical observatories and surveillance radar establishments. At first, data could be recorded on standard computer tape and processed on general-purpose computers. However, if synchronization were required on a close to real-time basis, HF radio links could provide data links to a computer.

#### REFERENCES

1. N. W. Broten, et al., "Long Baseline Interferometry: A New Technique," Science 156, 1952 (1967).
2. C. Bare, et al., "Interferometer Experiment with Independent Local Oscillators," Science 156, 189 (1967).
3. J. M. Moran, et al., "The 18-cm Flux of the Unresolved Component of 3C273," Astrophys. J. 151, L99 (1968).
4. J. A. Van Vleck and D. Middleton, "The Spectrum of Clipped Noise," Proc. IEEE 54, 2 (1966).

| DOCUMENT CONTROL DATA - R&D                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                             |                      |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------|----------------------|
| (Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                             |                      |
| 1. ORIGINATING ACTIVITY (Corporate author)<br><br>Lincoln Laboratory, M.I.T.                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | 2a. REPORT SECURITY CLASSIFICATION<br>Unclassified                                          |                      |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 2b. GROUP<br>None                                                                           |                      |
| 3. REPORT TITLE<br><br>Very Long Baseline Interferometry as a Means of Worldwide Time Synchronization                                                                                                                                                                                                                                                                                                                                                                                                                                                    |                                                                                             |                      |
| 4. DESCRIPTIVE NOTES (Type of report and inclusive dates)<br>Technical Report                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                                                                                             |                      |
| 5. AUTHOR(S) (Last name, first name, initial)<br><br>Rogers, Alan E. E.                      Moran, James M., Jr.                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                                                                             |                      |
| 6. REPORT DATE<br>17 February 1970                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | 7a. TOTAL NO. OF PAGES<br>16                                                                | 7b. NO. OF REFS<br>4 |
| 8a. CONTRACT OR GRANT NO. AF 19(628)-5167                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 9a. ORIGINATOR'S REPORT NUMBER(S)<br>Technical Report 478                                   |                      |
| b. PROJECT NO. 649L                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)<br>ESD-TR-70-34 |                      |
| c.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                             |                      |
| d.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                             |                      |
| 10. AVAILABILITY/LIMITATION NOTICES<br><br>This document has been approved for public release and sale; its distribution is unlimited.                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                             |                      |
| 11. SUPPLEMENTARY NOTES<br><br>None                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | 12. SPONSORING MILITARY ACTIVITY<br><br>Air Force Systems Command, USAF                     |                      |
| 13. ABSTRACT<br><br><p>Extraterrestrial radio waves from sources of small angular size can be used to synchronize clocks at two receiving sites to a high degree of precision. Radio star signals, together with time identification, are recorded on magnetic tape. Tapes are subsequently processed on a digital computer to obtain the synchronization error between the two clocks at the time of observation. The ultimate limit to the precision of the method is the knowledge of the relative signal delay in the atmosphere and ionosphere.</p> |                                                                                             |                      |
| 14. KEY WORDS<br><br>interferometry                                      frequency standards<br>worldwide time synchronization              radiometry                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                             |                      |