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DAMPING OF CALIBERS 0.30 AND 0.50 BULLETS AND 37MM H.E. SHELL

by

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DAMPING OF CALIBERS 0.30 and 0.50 BULLETS AND 37MM H.E. SHELL

Abstract

The damping factors of the caliber 0.30 bullets, ball M1 and M2 and tracer M1; caliber 0.50 bullets, ball M1 and A.P. M2; and 37mm H.E. Shell M54 with P.D. Fuze M56 have been determined experimentally. The corresponding aerodynamic coefficients are tabulated.

ILLUSTRATIONS

TABLES

PREFACE TO FIRST REVISION

In this revision, the values of the cross wind force coefficient were recalculated, using the last term in equation (18) on page 55 of BRL Report No. 276, "Aerodynamics of Small Arms Bullets", revised October 1944. Including this term and the spin factor, equation (22) of the same report becomes:

$$\frac{dz'}{K} = \left(\frac{\omega}{\omega_o}\,\frac{1}{v^2} - \frac{B\rho}{C}\,\frac{z'}{K}\right)dx, \qquad (22)$$

if

$$K = K_L/K_M C_L, \qquad (21)$$

K_L is the cross wind force coefficient, K_M the moment coefficient, C_L the drift coefficient, z' the time derivative of the linear drift, x the horizontal range, v the velocity of the projectile, ω the spin, ω_o the initial spin, B the drag coefficient, C the ballistic coefficient, and ρ the air density ratio.

The values of the cross wind force damping factor, the yawing moment damping factor, the yawing moment coefficient, and the Magnus moment coefficient have been changed to agree with the revised values of the cross wind force coefficient. The sum of the cross wind force and yawing moment damping factors has not been changed.

I. INTRODUCTION

1. Since bullets and shells fired from an airplane attain large yaws, it is necessary to determine their rate of damping in order to calculate their trajectories accurately. It is therefore proposed to collect the best available data on this subject, and to summarize the results. The principal sources of the experimental data are two Ballistic Research Laboratory reports: No. 276, "Aerodynamics of Small Arms Bullets", and No. 354, "Aerodynamics of 37mm H.E. Shell M54." Much of the explanation and most of the tables are taken immediately from these reports.

2. In Report No. 276, the Magnus moment damping factor was disregarded. After comparing the theoretical trajectories, based on the results contained therein, with observations of actual firings from airplanes, Major T. E. Sterne discovered that it was necessary to take account of this factor in order to secure agreement. The evidence indicates that this factor is negative: since this had been believed to be unlikely, in view of Fowler's observations*, its effect had previously been considered negligible.

II. MOULTON- GUION THEORY

3. The yawing moment damping factor of some of the bullets was computed by a method derived from Guion's extension of Moulton's formulas.[1] These are applicable to larger yaws than Fowler's[2], but the results were found to be less accurate because they involved several factors that were not well determined.

4. All the observations indicated that the minimum yaw was zero at all ranges. More accurate measurements may show that it is really different from zero, but it is certainly quite small. This condition allows us to simplify the formulas. Furthermore, we assume that

$$T = L/v_o, \ \Omega/2 = \varphi' v_o,$$

where

T is the period of yaw (in seconds),

L the linear period (in feet),

v_o the muzzle velocity (in ft/sec),

$\Omega/2$ the rate of precession (in radians/sec),

φ' the linear precession (in radians/ft).

* Ref. 2, p. 356.

It should be explained here that this is not the true rate of precession; but the rate of change of orientation at the time of a small maximum yaw:

$$\Omega/2 = AN/2B,$$

where

A is the axial moment of inertia,

B the transverse moment of inertia,

N the spin.

5. This method then reduces to finding the yawing moment damping factor f by the approximate formula:

$$\left\{ \frac{B(1 + w_1)}{2\mu(w_3 - w_1)} \left[\frac{8\varphi' v_o}{3 + w_1} \varphi' L(1 - w_1) + \frac{\pi^2 \alpha^2 v_o}{L} \right] - \frac{2\varphi' v_o B}{\mu(w_3 - w_1)} \varphi' L(1 - w_1) \right\} f = -\Delta\alpha \sin\alpha$$

$$- \left\{ \frac{2\varphi' v_o B}{\mu(w_3 - w_1)} \varphi' L(1 - w_1) - \frac{B(1 + w_1)}{2\mu(w_3 - w_1)} \left[\frac{8\varphi' v_o}{3 + w_1} \varphi' L(1 - w_1) - \frac{\pi^2 \alpha^2 v_o}{L} \right] \right\} \varkappa \qquad (1)$$

where

$$w_1 = \cos\alpha, \qquad (2)$$

$$w_3 = 2s - 1, \qquad (3),$$

$$\mu = \rho d^3 v^2 K_M, \qquad (4)$$

$$\varkappa = \rho d^2 v K_L / m, \qquad (5)$$

α is the average maximum yaw (radians),

$\Delta\alpha$ the variation in maximum yaw during one complete period,

$\varkappa$ the cross wind force damping factor,

s the stability factor,

μ the moment factor,

ρ the air density,

d the caliber,

v the velocity,

m the mass of the projectile,

K_M the moment coefficient,

K_L the cross wind force coefficient.

6. The maximum and minimum yaws, the linear period, and the linear precession are determined by firing the projectile through a series of screens and measuring the major axis of each hole and its orientation with respect to the upwards vertical. The yaw is a function of the major axis, which depends on the shape of the projectile. The screens used in the present tests were made of photographic paper tacked to wooden frames. The holes were exceptionally clear and easy to measure, because the shell left a sharp impression on the colored emulsified face of the paper and the ragged edges were white. In order to induce yaw the muzzle was deformed: a 180° notch was cut in the machine gun barrels; an adapter with a similar notch was screwed to the 37mm tube.

7. The stability factor is defined by the formula

$$s = A^2N^2/4B\mu \qquad (6)$$

but is determined as a function of the maximum yaw and the product of the linear period by the linear precession. The moment coefficient and moment factor may then be calculated from it.

8. The cross wind force coefficient is determined from the observed drift. In order to eliminate the effects of wind and aiming errors, the drift is measured by firing two barrels, one with right hand rifling and one with left hand rifling, in the same mount, at vertical targets. Only the variation in angular drift from 100 to 600 or 1000 yards is considered in calculating the cross wind force coefficient, to which the drift is proportional.

III. FOWLER THEORY

9. If the initial minimum yaw is zero, and if the damping factors are proportional to the velocity, Fowler's formulas (4.041) and (4.042) for the analytical maximum and minimum yaws may be expressed:

$$\alpha = \alpha_o (p_o/p)^{1/2} \exp(-\frac{f + \varkappa}{2v} x) \cosh (\frac{r}{v} x), \qquad (7)$$

$$\beta = -\alpha_o (p_o/p)^{1/2} \exp(-\frac{f + \varkappa}{2v} x) \sinh (\frac{r}{v} x), \qquad (8)$$

where

$$p = (1 - 1/s)^{1/2} \qquad (9)$$

$$r = \frac{f - \varkappa + 2\gamma}{2p}, \qquad (10)$$

α is the maximum yaw,

$|\beta|$ the minimum yaw,

s the stability factor,

f the yawing moment damping factor,*

$\varkappa$ the cross wind force damping factor*

γ the Magnus moment damping factor,*

v the velocity,

x the range (along the trajectory),

and the subscript $_o$ pertains to $x = 0$.

* Major T. E. Sterne[3] has called attention to the fact that these so-called damping factors are misnamed: the real damping factors are

$$\exp(-\frac{f+\varkappa}{2v} x + \frac{r}{v} x), \quad \exp(-\frac{f+\varkappa}{2v} x - \frac{r}{v} x),$$

which are multipliers of the precessional and nutational amplitudes respectively.

10. From (7) and (8), we obtain

$$\alpha^2 - \beta^2 = \alpha_0^2 \, (p_0/p) \, \exp \left(- \frac{f + \varkappa}{v} x\right), \tag{11}$$

$$\beta / \alpha = - \tanh \left(\frac{r}{v} x\right). \tag{12}$$

Therefore, if we know the maximum yaw α_1 at a range x_1 and the maximum and minimum yaws α_2 and $|\beta_2|$ at a range x_2, we can calculate the damping factors by means of the formulas:

$$\left|\frac{r}{v}\right| = \frac{1}{x_2} \tanh^{-1} \left|\beta_2/\alpha_2\right| , \tag{13}$$

$$|\beta_1| = \alpha_1 \tanh\left|\frac{r}{v} x_1\right| , \tag{14}$$

$$f - \varkappa + 2\gamma = 2pr, \tag{15}$$

$$f + \varkappa = \frac{v}{x_2 - x_1} \log_e \frac{p_1(\alpha_1^2 - \beta_1^2)}{p_2(\alpha_2^2 - \beta_2^2)}. \tag{16}$$

The sign of r is positive or negative, according to whether the type of motion is stepped-down or stepped-up, providing the initial minimum yaw is zero. In the stepped-down motion, the orientation decreases in the vicinity of the minimum yaw, or increases at a reduced rate; in the stepped-up motion, it increases at a higher rate than the average precession. These combinations of the damping factors are sufficient to determine the damping. However, the individual damping factors may be found if the cross wind force coefficient has been determined from the drift, so that $\varkappa$ may be computed by formula (5).

11. If the minimum yaw is zero at all ranges, as the observations have indicated, formula (14) indicates that r is zero. Then, by (15),

$$\gamma = - \frac{f - \varkappa}{2} . \tag{17}$$

12. The yawing moment coefficient K_H is related to f by the formula

$$K_H = fB/\rho d^4 v, \tag{18}$$

and the Magnus moment coefficient K_J is related to γ by the formula

$$K_J = \gamma A/\rho d^4 v. \quad (19)$$

Formula (5) may be written

$$K_L = \varkappa m/\rho d^2 v. \quad (20)$$

Therefore, according to (17), when the minimum yaw is zero,

$$K_J = -\frac{A}{2}\left(\frac{K_H}{B} - \frac{K_L}{md^2}\right). \quad (21)$$

IV. BALL M1 BULLETS

13. The inclosed drawings show the outlines of the projectiles considered in this report. Table I gives the masses prescribed by the official drawings of the bullets and the mean mass, distance from base to center of gravity, and principal moments of inertia of a few sample bullets.

14. The yawing moment damping factor of the ball M1 bullets, calibers 0.30 and 0.50, were determined by the method derived from the Moulton-Guion theory. Table II gives the data that were observed in the stability firings of the caliber 0.30 bullets; Table III contains similar data for the caliber 0.50 bullets. Table IVa gives average values of the observed rate of precession, period of yaw, maximum yaw, and variation of maximum yaw per period, obtained from the data given in Tables II and III, and the mean cross wind force coefficients that were determined from drift firings. Table IVb gives the ratio of the moment factor μ to the transverse moment of inertia B, the two damping factors f and $\varkappa$, and the yawing moment coefficient. The moment factors were computed from the moment coefficients, which were determined from the stability factors: the moment coefficient of the caliber 0.30 bullet decreased considerably as the velocity increased; but that of the caliber 0.50 bullet did not vary much and the average value, 1.24, was used. The air density given in these and other tables is the ratio of the density to the Ordnance Department's standard of 525.9 gr/ft^3 (0.07513 lb/ft^3).

15. The results are not at all accurate on account of the dispersion in the numerous factors involved in the formula for f. Greater accuracy would have been obtained if the damping had been observed over a longer distance, with fewer yaw screens between the observed maximum yaws. Such a procedure was followed with other bullets, but the additional work of determining more accurate damping factors for the ball M1 bullets was not warranted because they were being replaced by the ball M2 bullets.

16. Although the yawing moment coefficient seems to vary with velocity, this variation is not significant. The weighted mean values are 3.6 + 1.5 for the caliber 0.30 ball M1 and 6.0 + 5.5 for the caliber 0.50 ball M1. Both of these bullets have a 9° boat-tail and both have an ogival head, one with a radius of 7 calibers and the other with a radius of 9 calibers.

17. The average cross wind force coefficients of the calibers 0.30 and 0.50 ball M1 bullets are 0.77 and 0.63 respectively. The Magnus moment coefficients required to maintain a zero minimum yaw are - 0.15 and -0.23 respectively.

V. CALIBER 0.50 A.P. M2 BULLET

18. In connection with the stability firing of the caliber 0.50 A.P. M2 bullet, for the sparse distribution, the yaw screens were placed at 2.5-foot intervals from 7.5 to 22.5 feet and from 80 to 95 feet from the muzzle; to determine the damping, some additional screens were placed at 5-foot intervals from 280 to 300 feet from the muzzle. The maximum yaw that occurred between 80 and 95 feet and the one between 280 and 300 feet were used in computing the damping factors by Fowler's formulas.

19. Table V gives the experimental data. Observations indicated that the minimum yaw was zero at all ranges. Table VI gives the average values of the maximum yaw and their approximate distances from the muzzle. It also gives the muzzle velocity and air density ratio obtained in the stability firings, the cross wind force coefficient determined from the drift firings, the damping factors f and x corresponding to the given velocity and air density, and the resulting yawing moment and Magnus moment coefficients. The mean cross wind force coefficient is 0.83; the yawing moment coefficient, 3.2; and the Magnus moment coefficient, -0.10. The effect of the yaw screens on the damping of this bullet was calculated, but the decrease in damping did not appear to be significant.

VI CALIBER 0.30 BALL M2 BULLET

20. The yaws of the caliber 0.30 ball M2 bullet fired for stability were too small to determine the damping accurately. Therefore, ten bullets were fired from a barrel that was cut away half way around for 3/8 inch instead of 1/4 inch. Yaw screens were placed at 2-foot intervals from 8 to 16 feet and from 192 to 200 feet from the muzzle. The maximum yaws and their distances from the muzzle are given in Table VII. The minimum yaw appeared to be zero.

21. The maximum yaw decreased from about 10° at 12 feet to about 5° at 195 feet. The averages and results are given in Table VIII. The cross wind force coefficient determined from drift firings is 0.98. The yawing moment coefficient is 2.6, and the Magnus moment coefficient is -0.09.

VII. CALIBER 0.30 TRACER M1 BULLET

22. In the stability firings of the caliber 0.30 tracer M1 bullet, for the sparse distribution and damping observation, yaw screens were placed at 2.5-foot intervals from 5 to 15 feet, from 37.5 to 50 feet, and from 285 to 300 feet from the muzzle. However, the flight was so erratic, on account of the large yaws attained by the bullets when fired from the 1/4-inch notched barrel, that only one bullet went through the last group of screens. Therefore, five additional rounds were fired, with the third group of screens from 85 to 100 feet from the muzzle: these bullets went through all the screens. The maximum yaws and their distances from the muzzle are given in Table IX. The mimimum yaw was apparently zero.

23. The results are given in Table X. The damping factors were calculated for muzzle velocity of 2741 ft/sec, which corresponds to the standard instrumental velocity of 2700 ft/sec at 78 feet, although the instrumental velocities obtained on May 3rd and 9th respectively were 2528 and 2518 ft/sec. At the time of the drift firings with Mann barrels, instrumental velocities of 2693 and 2666 ft/sec were obtained. The cross wind force coefficient is 1.07; the yawing moment coefficient is 5.4; and the Magnus moment coefficient is -0.22.

VIII. 37MM H.E. SHELL M54

24. The standard mass of the 37mm H.E. Shell M54 with the P.D. Fuze M56 is 1.34 lb. Table XI gives the average physical data for two of these projectiles without the detonator, which weighs 10 grains (0.0014 lb.)

25. This high explosive shell was fired from an Automatic Gun M1A2 (antiaircraft) fitted with a muzzle adapter to increase the yaw. A powder charge was established to give a muzzle velocity of 2000 ft/sec, which is the standard for the Automatic Gun M4 (aircraft). The twist of rifling of the antiaircraft gun is one turn in 30 calibers; that of the aircraft gun is one turn in 25 calibers.

26. Yaw screens were placed at 5-foot intervals from 35 to 75 feet and from 475 to 515 f et from the muzzle (some changes were made for the last five rounds). On some rounds, double screens were used: one screen was fastened to the front of the frame, and the other to the back of it, so that they were about 3-1/2 inches apart. To determine the squares of the maximum and minimum yaws, the square of the

yaw was plotted against the distance: some of these curves are inclosed. The circles indicate the observed values. It is quite evident that the damping was positive and that the minimum yaw was zero at all ranges. Table XII gives the squares of the maximum yaw and their distances from the muzzle.

27. The values of $f + \varkappa$, determined from these data, are given in Table XIII. The average for eight rounds, fired through single screens, reduced to normal air density, is 6.90 per second. For three rounds fired through double screens, the mean is greater; but this increase does not appear to be significant. The cross wind force coefficient, determined from drift firings, is 0.98. Hence, the cross wind force damping factor, $\varkappa$, at normal air density and 2000 ft/sec, is 1.62 per second; and the yawing moment damping factor, f, under the same conditions, is 5.28 per second. Consequently, the yawing moment coefficient is 3.16 and the Magnus moment coefficient is -0.19.

IX. CONCLUSION

28. By observing the yaws of certain small arms bullets and the 37mm H.E. Shell M54, it was found that their maximum yaw decreased and their minimum yaw was very close to zero. Damping factors were calculated from the experimental data. The aerodynamic coefficients determined from these results, together with those of stability and drift firings, are tabulated in Table XIV.

H. P. Hitchcock

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REFERENCES

1 Hitchcock, H.P. Damping of the yaw of a projectile over a complete period. APG BRL file A-IV-33 (1931).

2 Fowler, R. H., E. G. Gallop, C. N. H. Lock and H. W. Richmond. The aerodynamics of a spinning shell. Phil. Trans. Royal Soc. London, A <u>221</u>: 295-387 (1920).

3 Sterne, T. E. The effect of yaw upon aircraft gunfire trajectories. APG BRL Report 345 (1943).

BULLET, CAL. 0.30 BALL M1

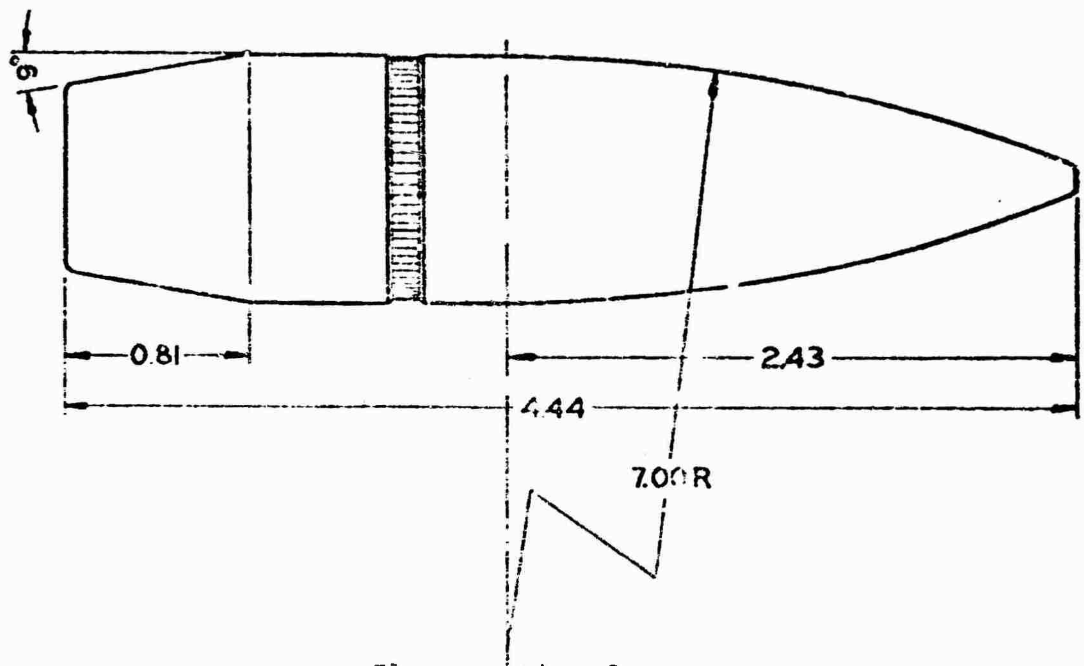
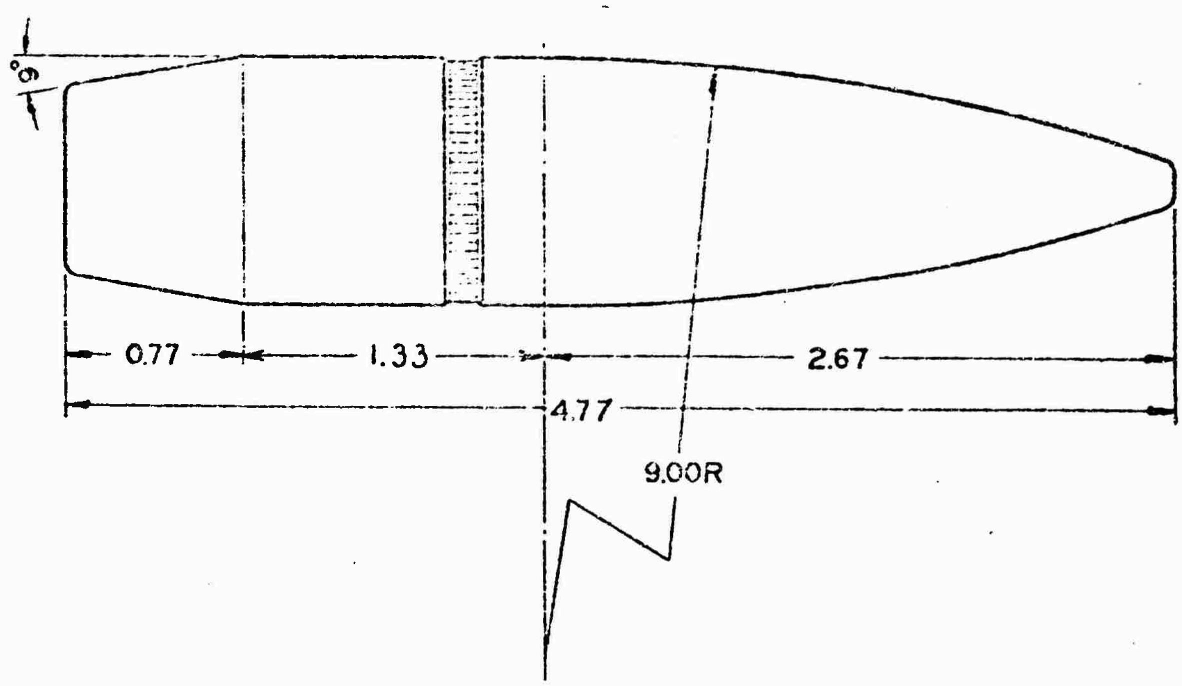


Illustration 1

BULLET, CAL. 0.50 BALL M1



ALL DIMENSIONS IN CALIBERS
Illustration 2

BULLET, CAL. 0.50 A.P. M2

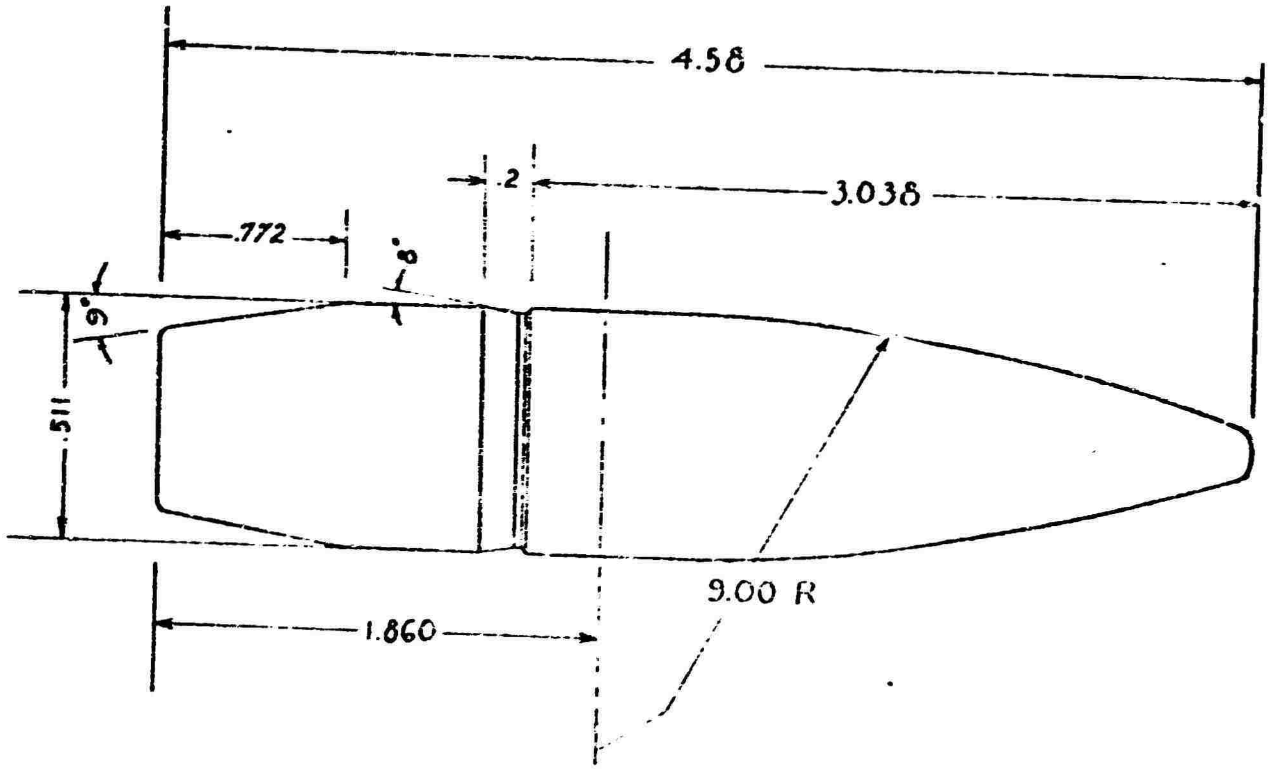


Illustration 3

ALL DIMENSIONS IN CALIBERS

BULLET, CAL. 0.30 BALL M2

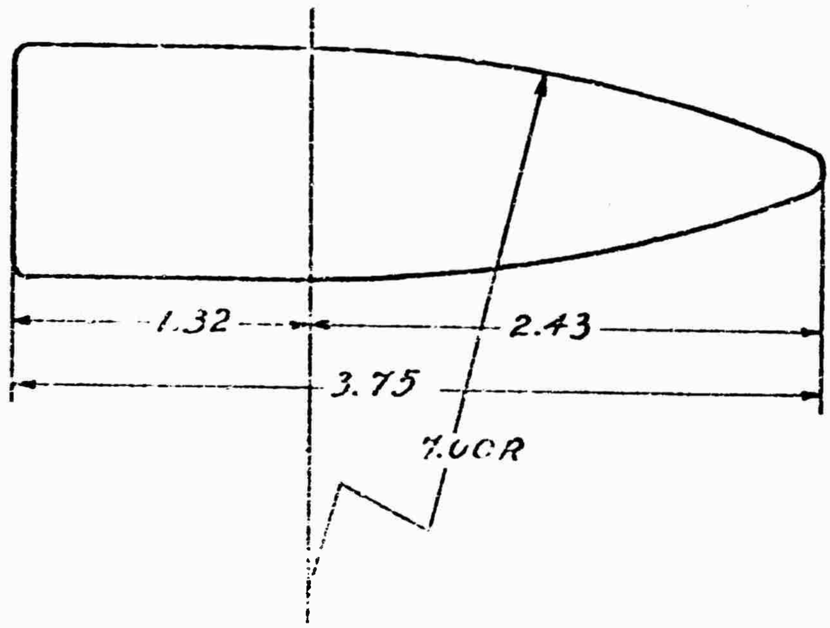


Illustration 4

BULLET, CAL. 0.30 TRACER M1

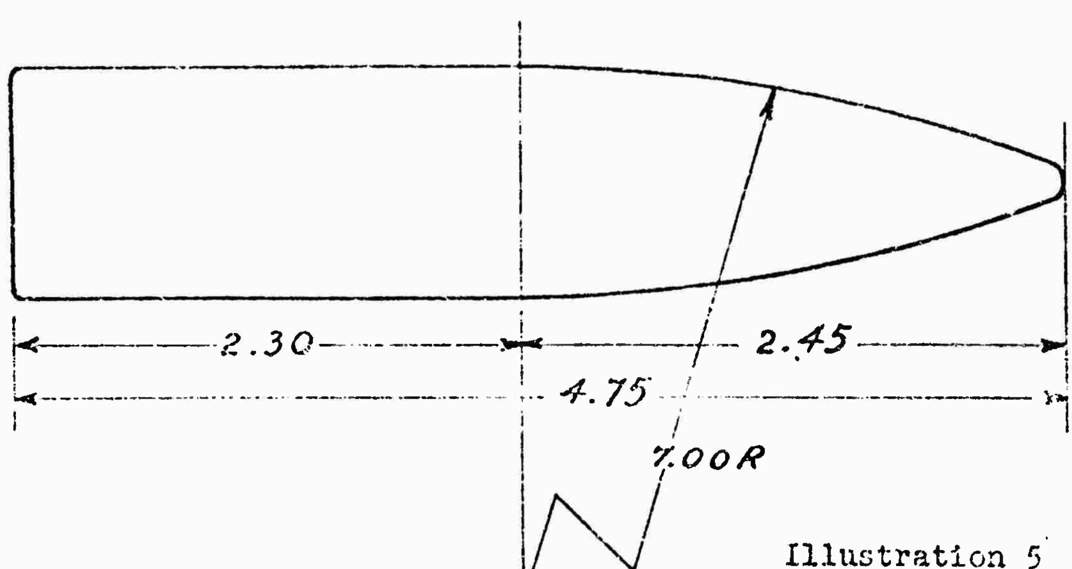
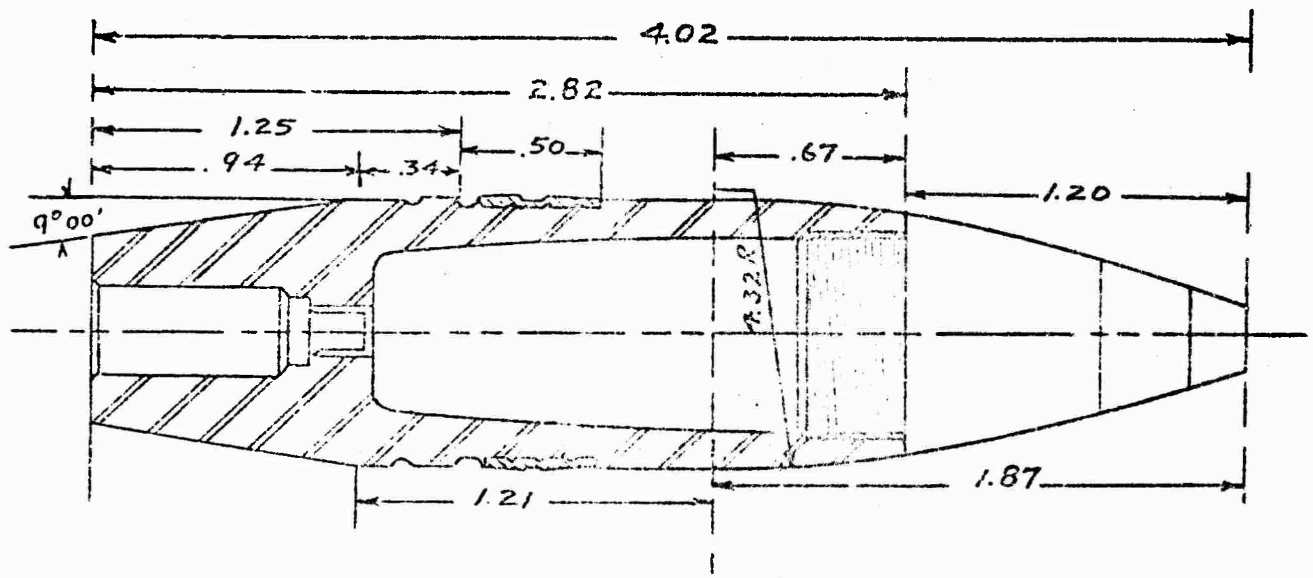


Illustration 5

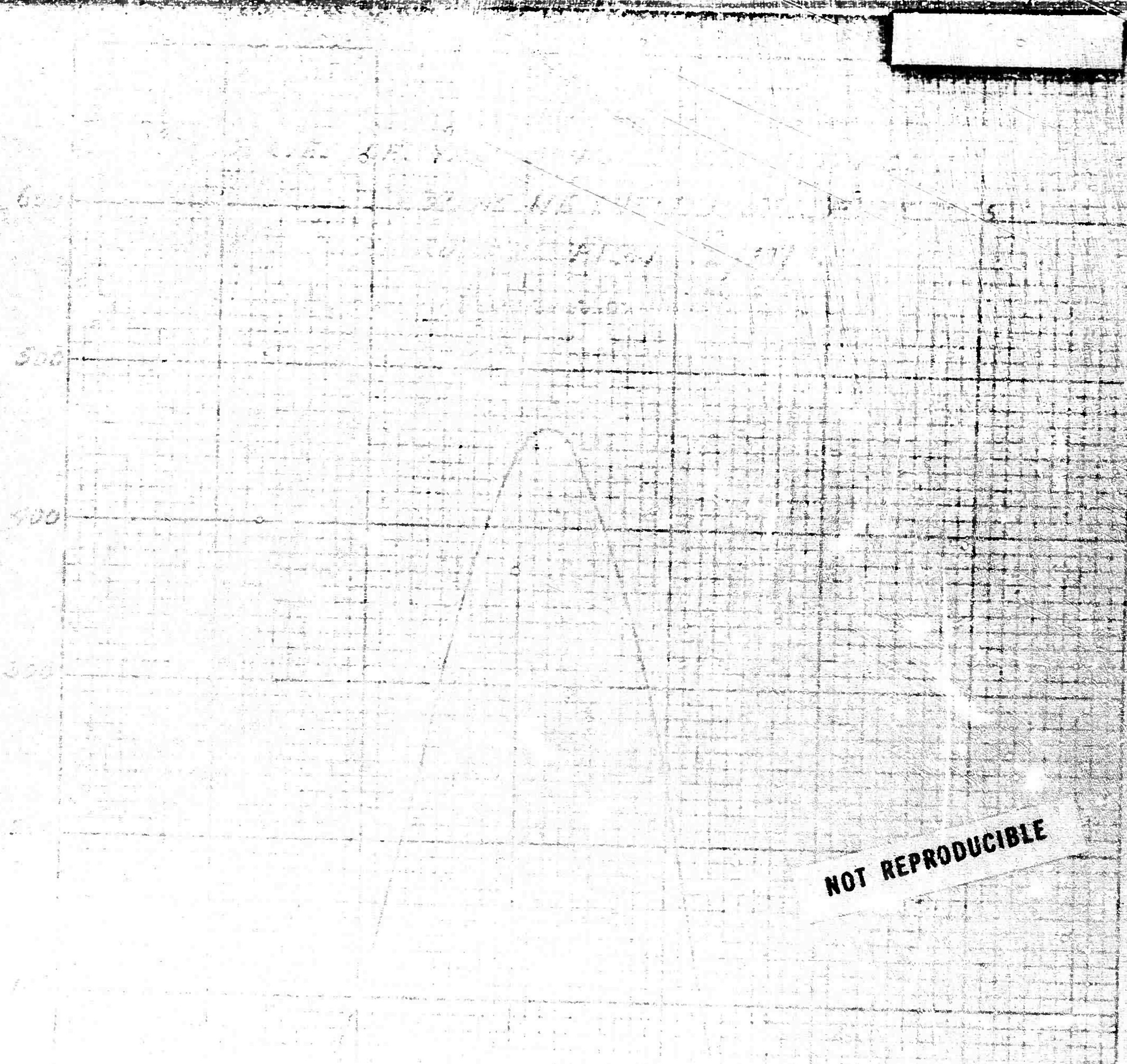
Shell, H.E., 37mm M54, with Fuze, P.D., M56

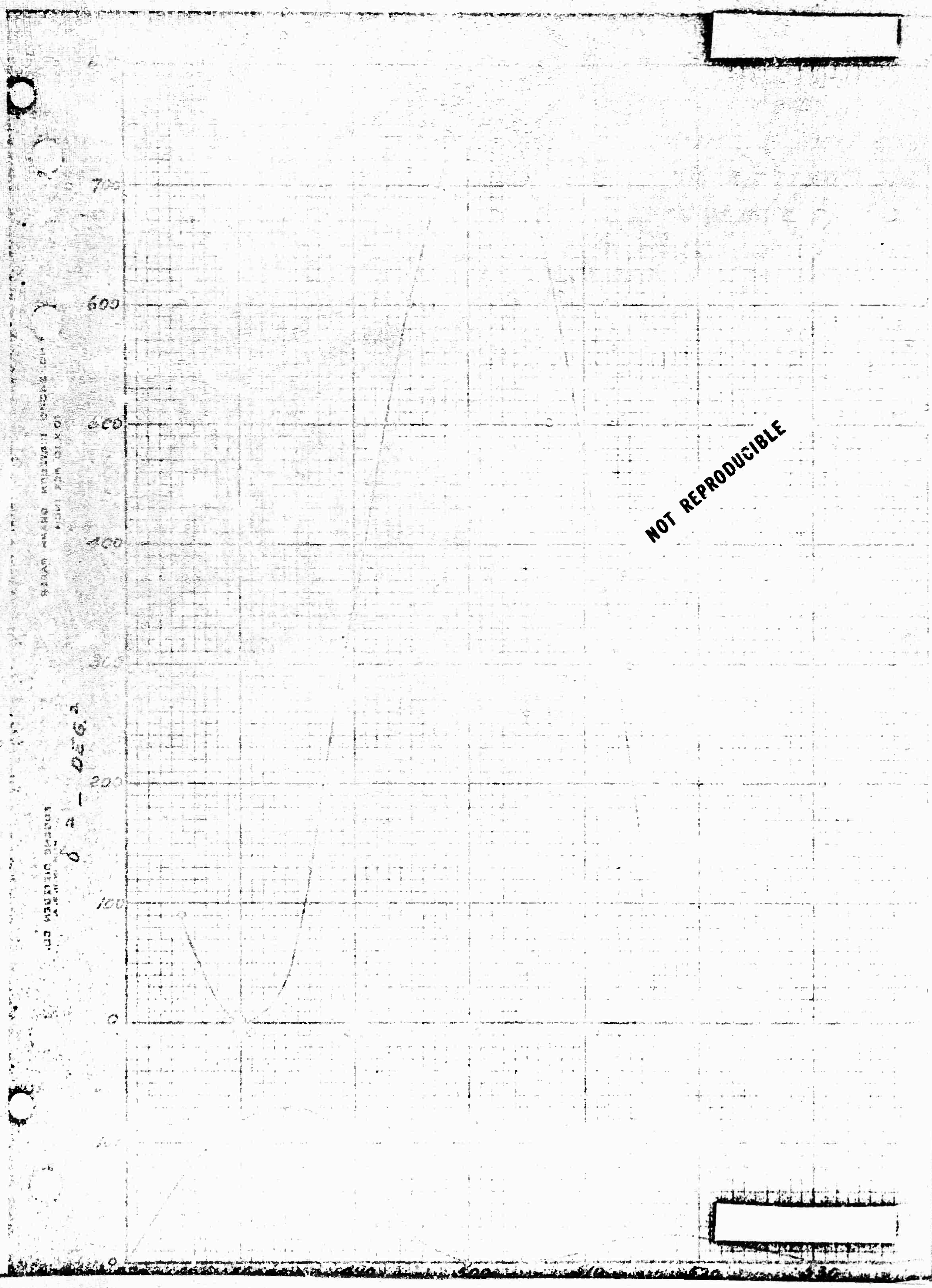


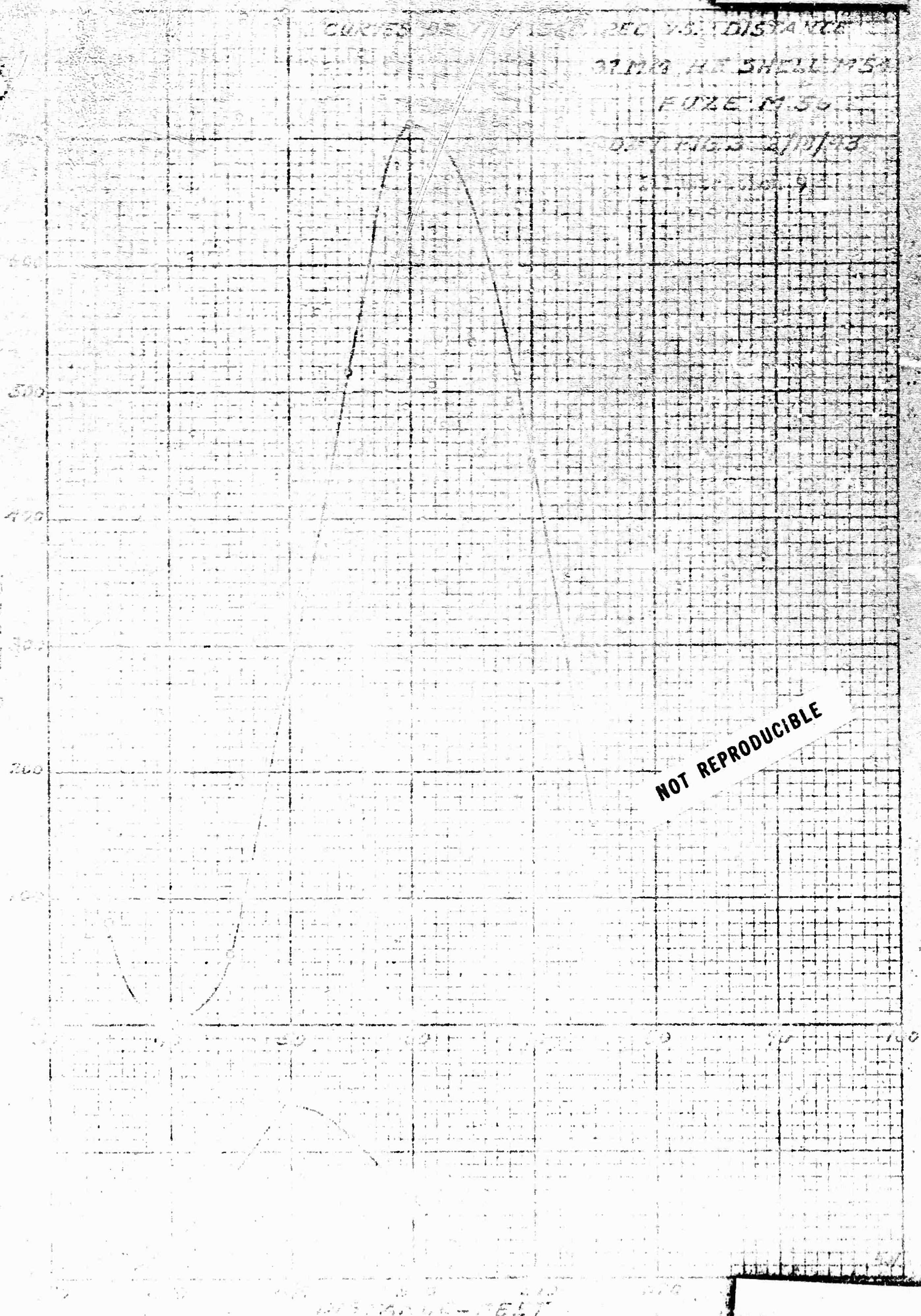
ALL DIMENSIONS IN CALIBERS
1 CAL. = 1.457 IN.

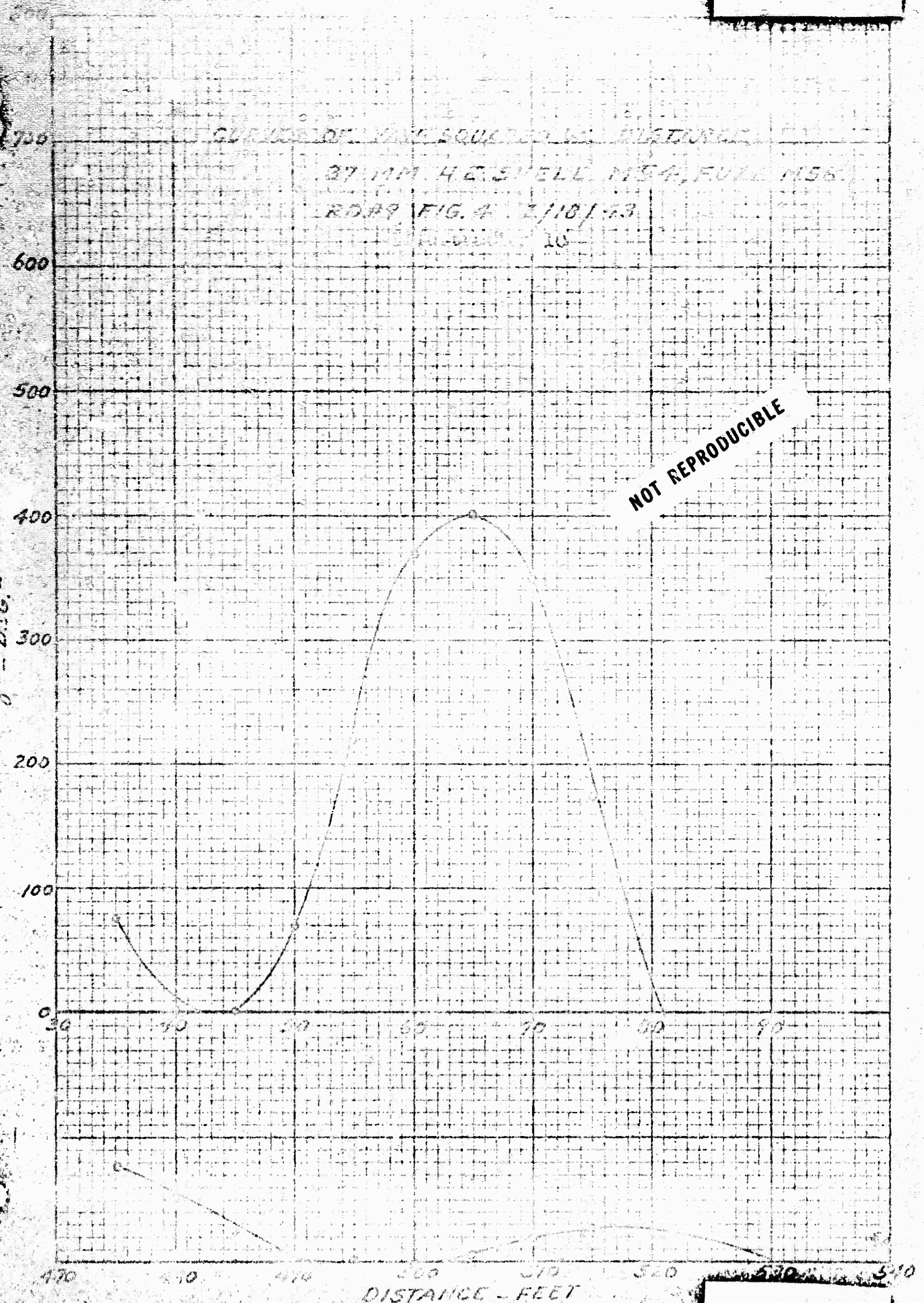
10-5-42 E.W.H.

Illustration 6

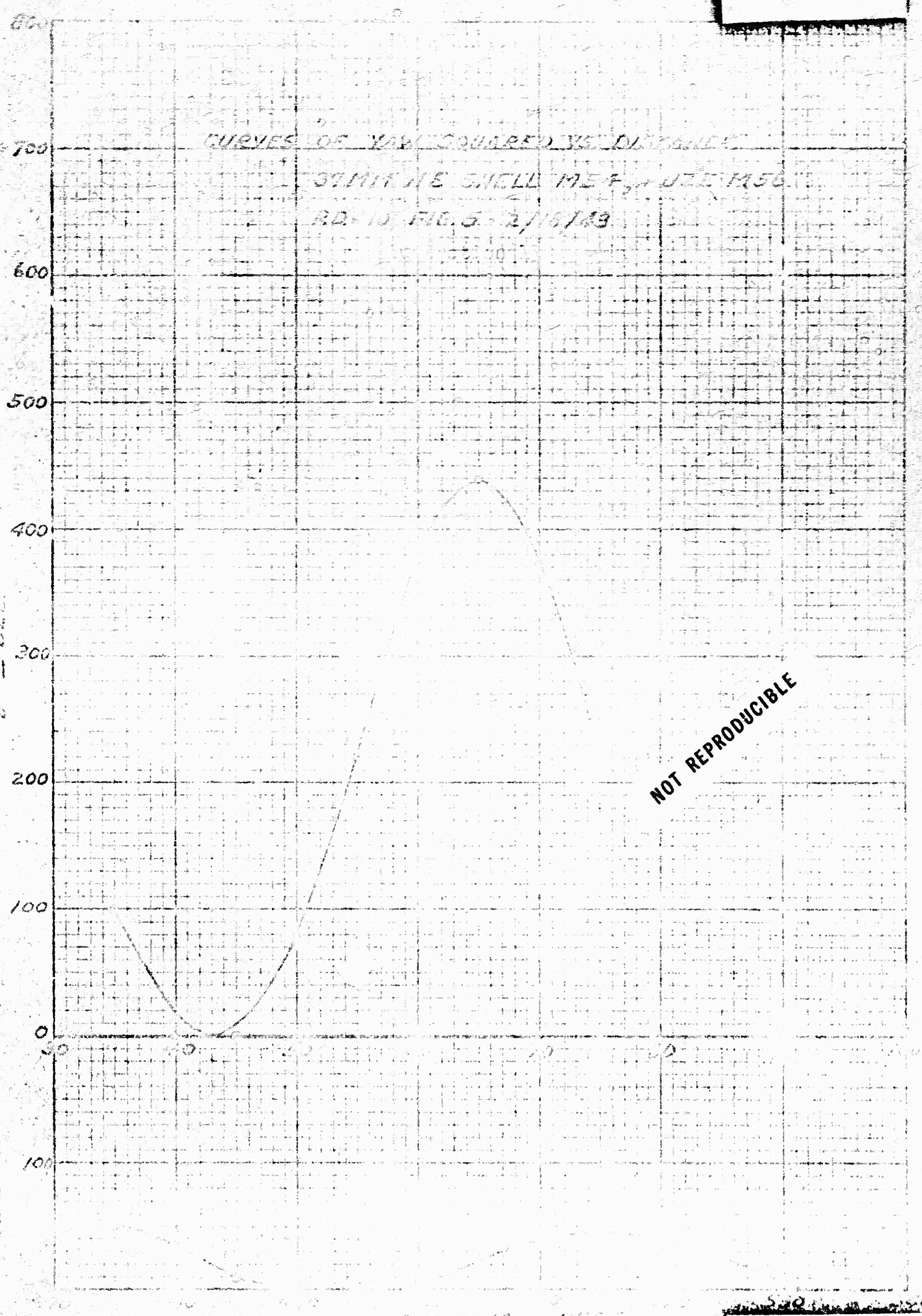

500
400
100

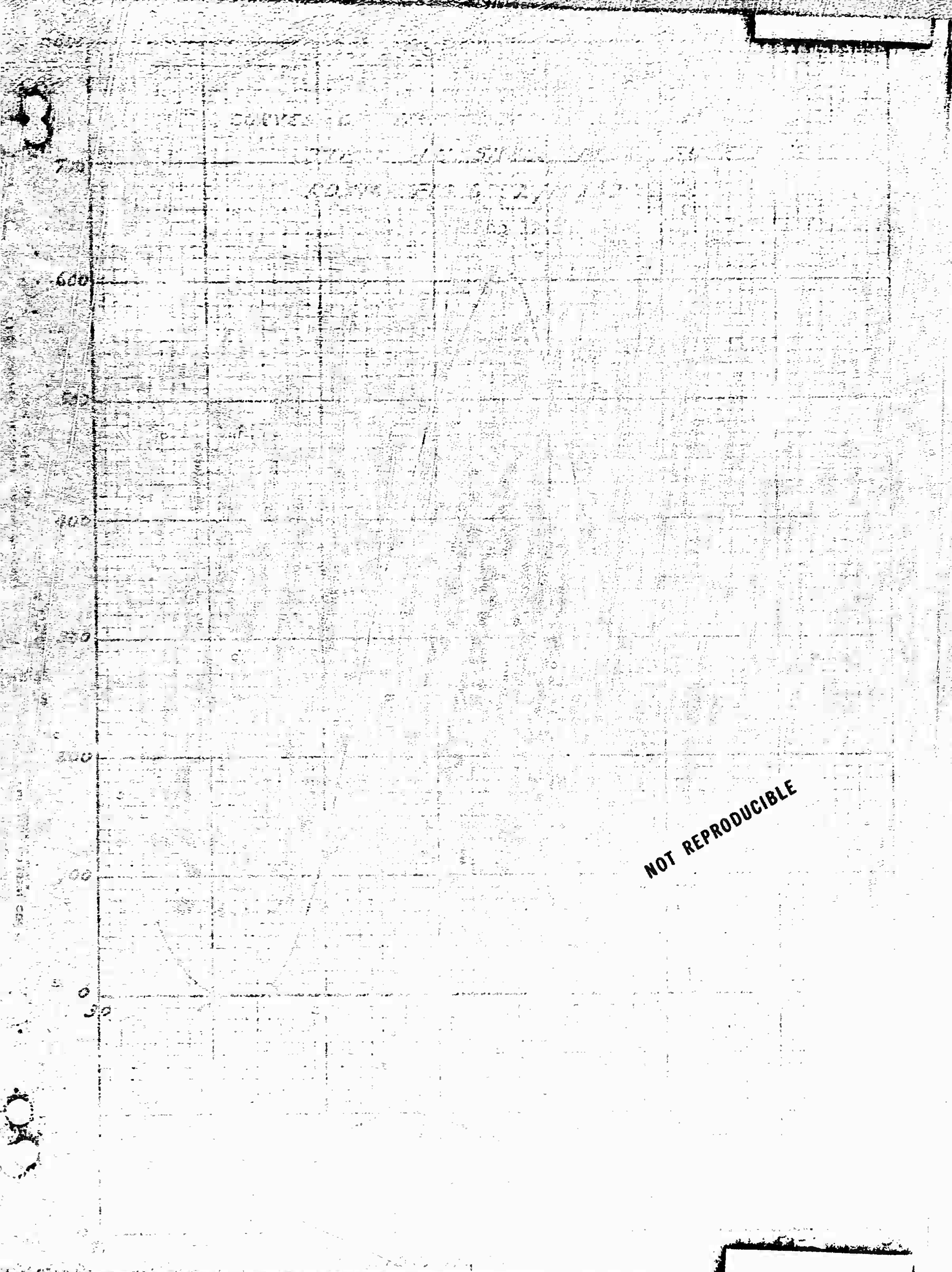

700
600
500
400
300
200
100
0

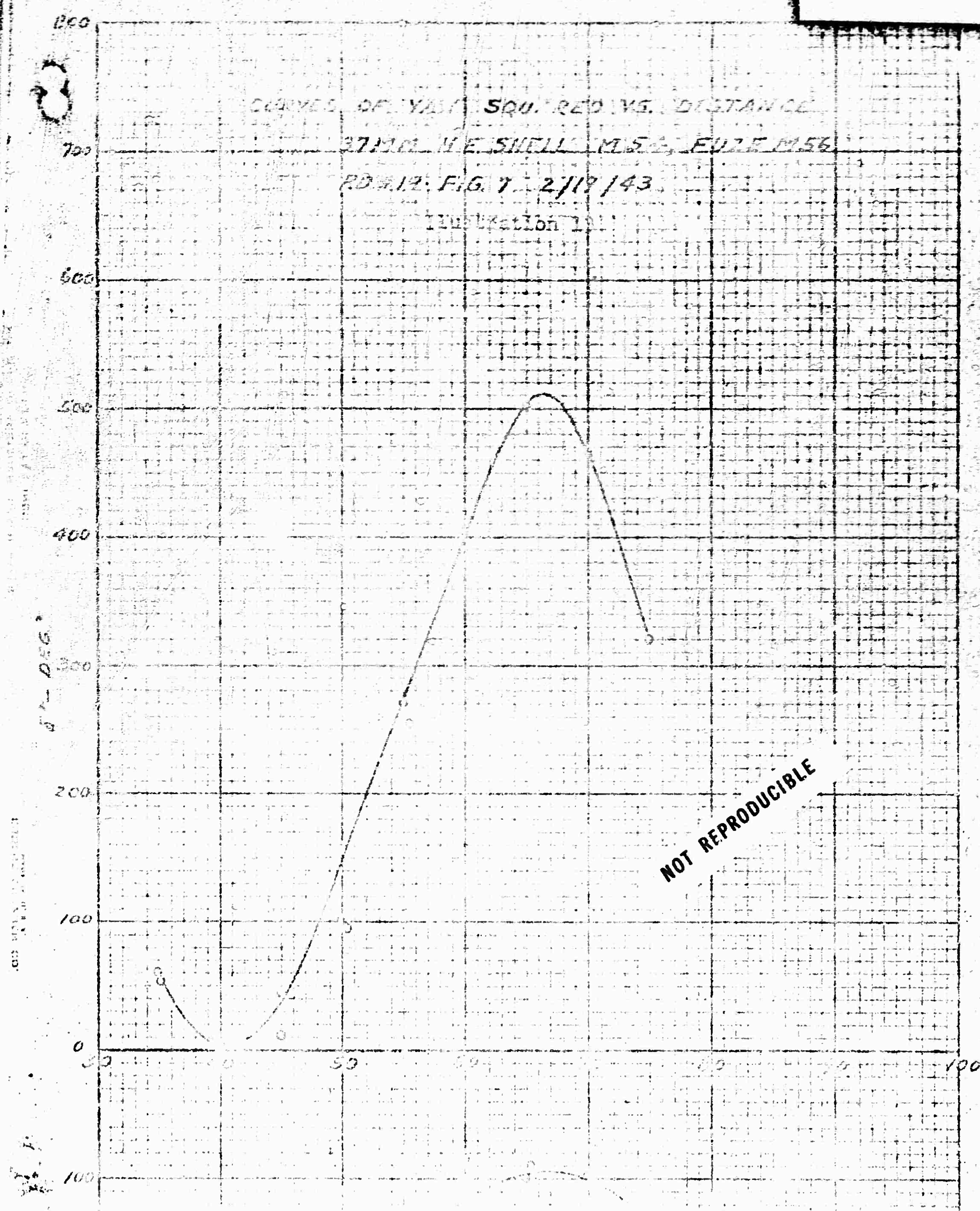

VS. DISTANCE
FUZE



37 MM H.E. SHELL M54, FUZE M56
FIG. 4
10
δ^2 – DEG.^{2}
700
600
500
400
300
200
100
0
30
40
50
60
70
80
90
470
480
490
500
510
520
530
540
DISTANCE – FEET


700
600
500
400
300
200
100
0


600
400
0



OF YAW SQUARED VS. DISTANCE
37MM. H.E. SHELL M54, FUZE M56
PD#12: FIG. 7 2/17/43
Illustration 1
800
700
600
500
400
300
200
100
0
100
50
100
DISTANCE FEET

TABLE I

PHYSICAL CONSTANTS

Caliber	Bullet	Mass grains			Base to C.G.	Moments of Inertia gr.in^2	
		Max.	Tolerance	Mean	cal.	Axial	Transverse
0.30	Ball M1	174.5	-3.0	172	1.827	1.751	16.40
"	Ball M2	152.0	-3.0	151	1.455	1.332	12.13
"	A.P. M2	166.0	-5.0	167	1.980	1.855	20.15
	Tracer M1	152.5	-3.5	149	2.097	1.777	18.57
0.50	Ball M1	753	-18	741	2.043	21.45	244.9
"	Ball M2	711.5	-18	-	-	-	-
"	A.P. M2	718	-17	709	1.922	19.71	217.1
"	Tracer M1	681	-13	674	2.269	20.94	246.5

TABLE II

STABILITY FIRING DATA
BULLET CALIBER 0.30 BALL M1

Round No.	Time 4/12/39	Charge Grains	Muzzle Velocity f/s	Air Temp. °F.	Air Dens. Ratio	Maximum Yaw (deg.) 1st	2nd	3rd	4th	Muzzle to Minimum Yaw (ft) 1st	2nd	3rd	4th	Precession (φ'/s) semi-rev. ft.
1	10:31	32.5		56	1.027	15.4	13.2	12.4		18	38			.119
2	11:15	32.5	2001	54	1.037	9.6	10.5			22.5	44			.119
3	11:54	32.5	2003			3	Approx.							
4	1:05	46	2672			7	Approx.							
5	1:40	32.5	2028	55	1.033	11.8		9.6				51		.122
6	2:03	32.5	1993	55	1.035	10.2		8.0				51		.123
7	2:22	32.5	1926	54	1.036	11.6		8.3				51		.123
8	2:48	46	2684	52	1.039	17.1	15.1	13.1		13.5	26.5	41		.122
9	3:16	46	2688	52	1.039	15.6	14.7	13.9		14	28.5	42		.120
10	3:44	46	2666	51	1.040	17.8	16.5	15.4		14	28	41.5		.120
11	4:01	46	2658	51	1.041	14.2			17.2	13.5		37		.121
12	4:12	46	2681	50	1.041	14.8			9.6	14		37		.121
13	4:22	46	2655	50	1.041	18.4			16.4	13		37		.121
	9/18/39													
14	10:09	Highest		65	1.037	17.5	16.5			11.5	24.5	Hit frame		.120
15	10:34	Practical	2909	65	1.035	12.2	10.7	9.9		14.5		41.5		.120
16	11:00		2930	66	1.033	10.9	Doubtful							
17	11:23	loading	2851	66	1.031	15.3	14.5	13.6		13.5	26.5	39.0		.120
18	11:40	Ditto		66	1.029	12.35			11.4	13.5			48.8	.120
19	11:55	Ditto	2848	66	1.028	13.4			13.4	13.5			48.5	.120
20	12:49	Ditto	2924	69	1.026	11.15			10.95	13.5		36.5		.120

TABLE III

STABILITY FIRING DATA
BULLET, CALIBER 0.50 BALL M1

Round No.	Time 1939 4/17	Charge Grains	Muzzle Velocity f/s	Air Temp. F.	Air Dens. Ratio	Maximum Yaw (deg.) 1st	Maximum Yaw (deg.) 4th	Muzzle to Minimum Yaw (ft) 1st	2nd	3rd	4th	Precession (p'/π)
1	2:01	170				21.8		Hit frames at 50 and 53 ft.				
2	2:40	170				13.8		Hit frame at 50 ft.				
3	3:19	170				13.5	?					?
4	3:59	170	2166	43	.94[illegible]	1[illegible].7	14.0	26.5	50	75	102.5	.0671
	4/18											
5	9:32	170	2095	45	.922	10.2	9.9a	37.5	63.5	?		.0681
6	9:20	213	2507	45	.92[illegible]	11.6	10.3	27	51	77		.0672
7	9:58	213	2480	45	.919	10.2	8.9	31	50a	80		.0657
8	10:32	213	2497	46	.917	11.5	10.9	27	50	76		.0659
9	10:58	170	2213	46	.918	14.5	14.0	25.5	?	72a	96	.0669
10	11:17	170	2202	46	.919	11.5	9.8	27	?	78		.0672
11	11:37	167	2127	47	.917	12.2	9.0	29a	?	75		.0671
12	11:55	216	2539			4.5	4.3	?	?	?		.0670
13	12:49	217	2636	48	.918	8.1	7.6a	?	?	74		.0670
14	1:12	217	2534	48	.918	8.8	7.4	27	?	75		.0673
	4/19											
15	[illegible]	low	1961	62	1.03[illegible]	14.0	21.8					
16	9:08	low	1996	64	1.02[illegible]	15.2	12.2					
17	9:23	low	1996	64	1.027	16.4	13.4			79.5		.0688
18	9:59	low	1929	65	1.024	13.5	10.0			76.5		.0679
19	10:28	full	2925	66	1.023	[illegible].2	3.0			?		.0686
20	10:49	full		66	1.022	2.[illegible]	2.0			77.0		.0700
21	11:11	full	2937	68	1.021	[illegible] a	3			?		
22	11:30	full	2911	68	1.020	3.4	1.6			?		
23	11:42	full	2945	68	1.020	1.7	2.4				99.	.0736
24	11:57	full	2925	68	1.019	1.7	1.3				92	.0724
25	12:54	low	2004	70	1.011	[illegible] a	?				92	.0724
26	1:06	low		70	1.013	13.6	10.2				?	
27	1:19	low		70	1.012	10.2	[illegible]			57.5a		.0672
28	1:37	low	2004	70	1.010	1[illegible].[illegible]	8.8			?		.0630
										77.5a		.0[illegible]

NOT REPRODUCIBLE

TABLE IVa

DAMPING DATA

Bullets, Cal. 0.30 and 0.50, Ball M1

Caliber	Rate of Pre-cession	Cross Wind Force Coef.	Velo-city	Rounds con-sidered	Period Mean	P.E.	Maximum Yaw Mean	P.E.	Variation per period	P.E.
d in.	φ' /ft.	K_L	v ft/sec		L ft.		α deg.		Δα deg.	
0.30	0.380	0.77	1990	1,5,6,7	17.5	.37	10.9	.83	-1.34	.007
0.30	0.380	0.77	2672	8 - 13	13.1	.19	15.7	.94	-1.32	.199
0.30	0.380	0.77	2892	14 - 20*	12.6	.17	12.9	.65	-0.57	.146
0.50	0.215	0.63	1982	16,17,26,	25.5	.53	13.7	.84	-0.98	.061
0.50	0.215	0.63	2929	28,19,22, 24	24.6	.67	2.37	.38	-0.28	.125

* Except 16

TABLE IVb

DAMPING FACTORS

Bullets, Cal. 0.30 and 0.50, Ball M1

Caliber	Velo-city	Air Density ratio	Moment Ratio Mean	P.E.	Cross Wind Force Damping Factor Mean	P.E.	Yawing Moment Damping Factor Mean	P.E.	Yawing Moment Coef. Mean	P.E.
d in.	v ft/sec	ρ	μ/B 10^5/sec²		ϰ /sec		f /sec		K_H	
0.30	1990	1.033	3.66	.107	3.1	.12	20.3	29.9	5.5	8.1
0.30	2672	1.040	5.63	.071	4.1	.16	28.7	15.0	5.7	3.0
0.30	2892	1.031	5.97	.168	4.4	.17	14.9	9.1	2.8	1.7
0.50	1982	1.022	1.11	.030	1.57	.10	10.8	10.9	5.7	5.8
0.50	2929	1.020	2.43	.052	2.32	.15	26.2	52.3	9.4	18.8

TABLE V

STABILITY FIRING DATA

Bullet, Cal. 0.50, A.P. M2

Round No.	Time 1940 July	Air Density ratio	Max. Yaw deg		Muzzle to Min. Yaw ft.	
			5th	Last	5th	Last
10	101555	.961	8.2	4.8	95.5	291
13	111505	.953	12.4	7.5	94.2	292
15	111531	.954	14.0	9.6	91.2	302
17	111605	.955	11.3	7.5	94.7	290

TABLE VI

DAMPING FACTORS

Bullet, Cal. 0.50, A.P. M2

		Mean	P.E.
Muzzle Velocity	v_o	3112 ft/sec	
Air Density ratio	ρ/ρ_o	0.956	
Cross wind force coef.	K_L	0.83	0.11
Cross wind force damping factor	$\varkappa$	3.2 /sec	0.4
Maximum yaw:			
5th		11.5°	0.8
Last		7.4°	0.7
Muzzle to Max. Yaw:			
5th		85 ft	
Last		285 ft	
Yawing Moment Damping Factor	f	9.4 /sec	3.3
Yawing Moment Coef.	K_H	3.2	1.1
Magnus Moment Coef.	K_J	-0.10	

TABLE VII

MAXIMUM YAWS

Bullet, Cal. 0.30, ball M2

B.M.G. No. 227,735 Barrel No. 218

Estimated Instrumental Velocity 2740 ft/sec

Round No.	Time 1941 9/4	Max. Yaw deg.		Muzzle to Max. Yaw ft.	
1	1:00	4.00	.28	12.00	196.00
2		9.81	2.00	11.60	194.00
3		11.00	7.06	11.75	194.05
4		9.01	3.00	11.90	192.20
5		10.50	4.39	12.00	193.80
6		10.65	5.00	11.70	196.20
7		12.05	6.10	12.00	193.90
8		12.73	6.16	12.00	196.33
9		10.56	6.10	11.60	195.90
10	1:30	11.30	5.50	11.90	196.00
Ave.	P.M.	10.17	4.66	11.84	194.84
P.E.		.51	.44	.04	.30

TABLE VIII

DAMPING FACTORS

Bullet, Cal. 0.30, ball M2

		Mean	P.E.
Muzzle Velocity	v_o	2790 ft/sec	
Density ratio	ρ/ρ_o	0.984	
Cross Wind Force Coef.	K_L	0.98	0.090
Cross Wind Force Damping Factor	$\varkappa$	5.7 /sec	0.5
Maximum Yaw:			
2nd		10.17°	0.51
Last		4.66°	0.44
Distance to Max. Yaw:			
2nd		11.8 ft	0.04
Last		194.8 ft	0.30
Yawing Moment Damping Factor	f	17.3 /ft	3.1
Yawing Moment Coef.	K_M	2.6	0.5
Magnus Moment Coef.	K_J	-0.09	

TABLE IX

MAXIMUM YAWS

Bullet, Cal. 0.30, Tracer M1

Round No.	Time 1941 May	Maximum Yaw deg.			Muzzle to Max. Yaw ft.		
28	040045	11.2	2.0	-	50	291	-
171	092243	11.9	10.9	9.9	6	42	90
172	092256	13.6	11.8	8.5	5	39	96
173	092310	17.2	13.7	10.2	6	38	94
174	092320	12.2	11.3	5.5	6	42	90
175	092331	14.5	12.4	8.2	6	39	95
171 to 175	Mean	13.9	12.0	8.5	5.8	40.0	93.0
	P.E.	0.5	0.4	0.5	0.1	0.1	0.9

TABLE X

DAMPING FACTORS

Bullet, Cal. 0.30, Tracer M1

Maximum yaw	deg.	α_1	11.2	13.9	12.0	
		α_2	2.0	12.0	8.6	
Distance to max. yaw		x_1	50	5.8	40.0	
		x_2	291	40.0	93.0	
Air density	ratio	ρ/ρ_o	1.051	0.996	0.996	
						Ave.
Sum of damping factors (At normal density)	/sec	$f+ \varkappa$	38.8 37.0	23.2 23.3	34.0 34.1	31.5
Muzzle velocity	ft/sec	v_o				2741
Cross wind force coef.		K_L				1.07
Cross wind force damping factor (At normal density)	/sec	$\varkappa$				6.8
Yawing moment damping factor	/sec	f				24.7
Yawing moment coef.		K_H				5.4
Magnus moment coef.		K_J				-0.22

TABLE XI

PHYSICAL DATA

37mm H.E. Shell M54

P.D. Fuze M56

Length	5.828 in.
Diameter	1.502 in.
Mass	1.328 lb.
C.G. to base	2.238 in.
C.G. to base	1.536 cal.
Moments of Inertia	
Axial	0.480 $lb.in^2$
Transverse	2.814 $lb.in^2$

TABLE XII

DAMPING FACTOR FIRING DATA

37mm H.E. Shell M54 with Inert Fuze M56

37mm Tube M1A2, No. 1280, with Muzzle Adapter

Round No.	Time 1943	Distance to Maximum Yaw Ft.		Square of Maximum Yaw Deg.^{2}		Density Ratio
	Feb.	x_1	x_2	α_1^2	α_2^2	ρ/ρ_o
1	181145	Missed screens beyond 490 ft.				1.120
2	181257	62	510	460	100	1.112
3	181340	62	484	780	130	1.107
4	181355	Hit frame at 480 ft.				1.105
5	181405	Hit frame at 475 ft.				1.104
6	181435	65	480	390	60	1.102
7	181450	61	491	710	140	1.101
8	181510	72	511	120	25	1.100
9	181520	64	475	400	80	1.099
10	181525	65	515	440	50	1.098
11	181535	?	480	?	7	1.097
12	181550	?	?	?	?	1.096
13	181600	67	486	360	70	1.095
14	191430	67	514	370	30	1.031
15	191450	72	?	290	?	1.031
16	191510	66	498	610	105	1.030
17	191525	72	?	295	?	1.030
18	191535	70	?	280	?	1.030
19	191600	67	496	510	105	1.030

All minimum yaws were apparently 0.

Rounds 1 to 13 were fired through single yaw screens.

Rounds 14 to 19 were fired through double yaw screens.

TABLE XIII

DAMPING FACTORS

37mm H.E. Shell M54 with Inert Fuze M56

37mm Tube M1A2, No. 1280, with Muzzle Adapter

Round No.	$f + \kappa$ Observed /sec	$f + \kappa$ At Normal Air Density /sec		$r = \frac{f - \kappa + 2 \text{[illegible]}}{2p}$ /sec
2	6.37	5.73		0
3	8.00	7.23		0
6	8.59	7.79		0
7	7.12	6.46		0
8	6.72	6.11		0
9	7.40	6.74		0
10	9.24	8.42		0
13	7.39	6.75		0
14	10.88	10.56		0
16	7.79	7.57		0
19	7.01	6.80		0
		Mean	P.E. of Mean	
2 - 13		6.90	.21	0
14 - 19		8.31	.77	0
Diff.		1.41	.80	0

Estimated muzzle velocity = v = 2000 ft/sec.
Standard stability factor = s = 1.62
Ballistic coefficient = C_5 = 0.69

TABLE XIV

Aerodynamic Coefficients

Projectile			Velocity	Moment Coef.	Cross Wind Force Coef.	Yawing Moment Coef.	Magnus Moment Coef.
Kind	Cal.	Mod.	ft/sec	K_M	K_L	K_H	K_J
Ball	.30	M2	2740	0.51	0.98	2.6	-0.09
Ball	.30	M1	2600	1.09	0.77	3.6	-0.15
Ball	.50	M1	2800	1.24	0.63	6.0	-0.23
A.P.	.50	M2	2900	0.97	0.83	3.2	-0.10
Tracer	.30	M1	2700	0.73	1.07	5.4	-0.22
H.E.	37mm	M54	2000	1.89	0.98	3.2	-0.19