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THE ORBIT OF ARIEL 3 (1967-42A)

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MINISTRY OF TECHNOLOGY
FARNBOROUGH HANTS

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SUMMARY

The definitive orbit for Ariel 3 has been computed, from Minitrack observations, for a period of 27½ months from the launch of the satellite. The orbit was represented by a model with seven independent orbital parameters and the values of these parameters were determined, and are listed, at three-day intervals. Typical accuracies are 10^{-5} in eccentricity and 4" in angular parameters, that is, about ½ km in position.

A curious feature of the secular variation of orbital inclination, viz. that the expected decrease of about 0.02° appeared to occur over a three-month period instead of the full 27½-month period, is discussed but has not been explained.

Departmental Reference: Space 327

CONTENTS

1 INTRODUCTION

Ariel 3, the third of the series of satellites being launched in the scientific programme of Anglo-American co-operation, was the first spacecraft to be built entirely in Britain. The five experiments in the payload were concerned[1] with electron density and temperature (University of Birmingham), VLF radio waves (University of Sheffield), cosmic radio noise (Nuffield Radio Astronomy Laboratory, Jodrell Bank), molecular oxygen (Meteorological Office) and terrestrial noise sources (Radio and Space Research Station).

Known as S53 or UK3 before launch, and as Ariel 3 or 1967-42A afterwards, the satellite was placed in a near-polar, near-circular orbit at 18^h UT on 5 May 1967 by a Scout rocket launched from the Western Test Range, California.

The definitive orbit of Ariel 3, as for Ariel 2 [2], has been derived at R.A.E. from Minitrack (interferometer) data provided by NASA. Orbital parameters have been obtained at three-day intervals by the use of the new computer program PROP[3]. They are tabulated, for the first 27½ months of the satellite's lifetime, in this Report. For epochs up to 1968 JAN 16 the work was done on London University's Atlas computer and for subsequent epochs it was done on the ICL 1907 computer at R.A.E.

2 OBSERVATIONS

The STADAN Minitrack network now consists of ten stations - five of the original twelve stations[2] have disappeared and there are three new ones. These are listed in Table 1, with their assumed positions in standard geocentric co-ordinates (x-axis towards the Greenwich meridian).

Observations consist of pairs of direction cosines. Their a priori accuracy (s.d.) has been taken, as usual[3,4], at 0.00029, equivalent to 1' in angular measure, though their true accuracy is believed to be worse than this.

Times are given in the UTC system, i.e. the system defined by WWV time transmissions from America, and have not been corrected during the orbital determination, except that the times of observations made on 1 February 1968 and used at epoch 1968 JAN 31 had to be reduced by 0.1^s to allow for the step advance of UTC at Feb 1.0. Times should be accurate to about 1 ms; no allowance for timing error was made by PROP.

In total, about 10000 Minitrack observations were used, covering the period from launch until about a week before the satellite's transmitter was switched off (1 September 1969); i.e. these were about 12 per day. They were received from NASA on punched cards, suitable for direct input to PROP. The epochs for orbit determination were taken at three-day intervals, and always at midnights (unlike the Ariel 2 epochs which were at ascending nodes). The orbit determination at each epoch used observations over a four-day period, allowing one-day overlaps in the periods of validity of the resulting orbital parameters, but observations were not (in general) used twice; observations on an 'overlap day' were divided into two sets, by alternate allocation, for use with the epochs before and after the overlap day. About 650 of the observations were rejected during analysis, but this includes (a) a rather high rejection rate during the first 3½ months - all the observations from Orroral were being rejected at one stage - and (b) nearly 100 observations, over a period of a month (Jan-Feb, 1968), which all had a one wavelength error in the north-south direction cosine due to a temporary error in the NASA program[5] for processing the raw data; the normal rejection rate was about 4½%.

The number of Minitrack observations per day varied, of course, but there was at least one on every day of the period covered, apart from the week 28 November-4 December 1967, for which there was a complete absence of data.

A few observations of Ariel 3 were made by the Hewitt camera[6] at Malvern. Among these, 8 observations came from a pass on the evening of 10 April 1969 and 12 observations came from two passes close to midnight on 19 June 1969 and 21 June 1969. It was decided to incorporate these 20 observations into re-runs of the orbit determinations at the appropriate two epochs, to see how fit and accuracy were affected. The remaining Hewitt camera observations, and the many visual observations of Ariel 3, have not been used.

3 ORBITAL MODEL

The orbital model of the program PROP is not the same as that used in the analysis of the orbit of Ariel 2. Eccentricity, inclinations etc. are defined slightly differently in the two programs, and the connecting relations are given in Appendix C of Ref.3.

The model allows some choice as to the set of orbital parameters which represent the orbit and which are determined from fitting to observations. The set chosen for the Ariel 3 orbit contained seven parameters, viz, e_o (eccentricity), i_o (inclination), Ω_o (right ascension of the node), ω_o (argument of perigee), M_o (mean anomaly), M_1 (mean motion) and M_2 (half the mean acceleration). The first four parameters are epoch values of mean elements, as defined in Refs.7 and 8 and the last three are the coefficients in the polynomial representation of (mean) mean anomaly:

$$M = M_o + M_1 t + M_2 t^2 \quad ,$$

where t is measured from epoch.

Secular rates of change of e, i, Ω and ω (i.e. the polynomial coefficients e_1, i_1, Ω_1 and ω_1) were computed, inside PROP at the beginning of each iteration of the differential-correction process, as functions of the seven independent parameters[3,8]. These quantities, together with the long-periodic and short-periodic terms computed at each observation time, represented orbit perturbations due to drag and to the earth's zonal harmonics up to J_9. The along-track effect of the tesseral harmonic $J_{2,2}$ was represented as usual[3,8] using the value 1.8×10^{-6}, but, apart from this, tesseral harmonics were neglected. Luni-solar perturbations were ignored; their effect on Ariel 3 over a period of two or three days from any epoch is very small.

The decision not to have an eighth parameter M_3, which would have made the M polynomial a cubic, was justified by some test runs, early in the lifetime, which showed that no significant improvement in fit would result and that the value of M_3 itself would not be significant. For the two epochs during the week of missing data, however, M_3 was included in the model and a reasonable fit thereby obtained to data before and after the gap, covering a period of 10½ days. In retrospect, the decision to omit M_3 is open to question because drag increased fairly steadily through the 27½ months considered (as the values of M_2 in Table 2 show), the maximum effects being at the end of March 1969; repetition of two of the runs, with M_3 included, showed that at this stage significant improvement in fit would be obtained (see section 4).

Apart from the omission of M_3 during periods of high drag, the main limitation of the orbital model is in the neglect of important tesseral-harmonic perturbations and, in particular, of the perturbation in inclination due to $J_{2,2}$. This perturbation has a period of just under 12 hours and an amplitude of about $0^{\circ}.002$, equivalent to a maximum position error of about ¼ km.

4 RESULTS

4.1 Main results

The orbital parameters obtained from the orbit determinations, together with certain additional information, are listed in Table 2. Successive columns of the table provide the following quantities, zero suffixes being omitted from a_o etc:-

Epoch date (0^h UTC understood).
Semi-major axis, a (km).
Eccentricity, e.
Perigee height, h_p (km).
Inclination, i (degrees).
Right ascension of the node, Ω (degrees).
Argument of perigee, ω (degrees).
Mean argument of latitutde, $M_o + \omega$ (degrees).
Mean motion, M_1 (degrees/day).
Half acceleration, M_2 (degrees/day^2).
Number of observations used, N.
Number of observations rejected, K.
Extent of the observations, D (days).
Standard deviation of an observation of unit weight, ε.
Modified Julian Day number of epoch date, MJD.

The orbital parameters are the seven quantities e, i, Ω, ω, M_o, M_1 and M_2, but $M_o + \omega$ is given instead of M_o because of the high correlation between M_o and ω. This correlation arises directly from the fact that the orbit is so nearly circular, and Ref.3 may be consulted for further explanation. (The appropriate value of the control parameter JELTYP[3] was used to give the variance of $M_o + \omega$ directly.)

The semi-major axis, a, is the mean element, as used by Merson[7], defined from M_1 by

$$a = (\mu/M_1)^{\frac{1}{3}} - \tfrac{1}{2} J_2 R^2 (\mu/M_1^2)^{-\frac{1}{3}} (2 - 3 \sin^2 i) (1 - e^2)^{-1\frac{1}{2}} ,$$

where μ is the earth's gravitational constant, J_2 is its second zonal harmonic coefficient and R is its mean equatorial radius, the values from Ref.3 being used.

The perigee height is given by

$$h_p = r_p - R_p ,$$

where*

$$r_p = a(1 - e) + \frac{J_2 R^2}{4a(1 - e^2)} \left\{ \sin^2 i \cos 2\omega - (2 - 3 \sin^2 i) \left(1 + \frac{e}{1 + \sqrt{1-e^2}} - \frac{\sqrt{1-e^2}}{1 + e} \right) \right\}$$

and

$$R_p = R - 21.379 \sin^2 i \sin^2 \omega .$$

The right ascension of the node is nominally referred to the standard PROP equinox[3], but contains a small error due to the fact that the times are given in UTC and no correction to UT1 has been made. To correct Ω to the time PROP equinox (epoch date still understood to be 0^h UTC) add $0^{\circ}.004 \times$ (UT1 - UTC), where the time difference is in seconds.

The 'number of observations used' includes the number rejected; i.e. the parameters have been determined, in the end, from N - K observations.

After nine of the tabulated quanitites - the seven orbital parameters plus semi-major axis and perigee height - are given their computed standard

*The difference between r_p and a (1 - e) is important. Thus, though r_p (or rather h_p) is the right parameter to use when correlating drag behaviour with perigee height, a (1 - e) is the right parameter to work with when studying the effects of the earth's odd harmonics. For Ariel 3 the difference is approximately 1.54 cos 2ω km, and it was the use of r_p instead of a (1 - e) which led to the apparent discrepancy mentioned in section 3.20 of Ref.13.

deviations, to one or two significant figures, the unit in each case being that of the final figure quoted for the main quantity. Every standard deviation includes ε as a factor, where ε is given by

$$\varepsilon = \{\Sigma\ (\mathrm{Res}/0.00029)^2/(2N - 2K - 7)\}^{\frac{1}{2}}\ ;$$

here the summation is over all residuals, Res, in the $N - K$ accepted east-west and north-south direction cosines, and 0.00029 is the a priori accuracy referred to in section 2. Since the actual accuracy is worse than this a priori figure[4], the values of ε in Table 2 are expected to be - and are - larger than unity.

Table 2 was obtained as direct computer output from a program known as TOP (Tabulation of Orbital Parameters). This program takes, as part of its input data, the punched card output from PROP runs, so there should not be any errors in the table.

The secular rates of change e_1, i_1, Ω_1 and ω_1 are not given in Table 2, since they are computed internally by PROP as part of the model. It is remarked, however, that the computation of the J_2^2 component of ω_1 contained an error until the PROP3 version of the program was introduced at the end of January, 1969. PROP2, which had this error in ω_1, was able to compensate for it almost exactly, by fitting a slightly wrong value of M_1, and this was one reason why the error was for a long time undiscovered. To correct the results from the PROP2 runs it was only necessary to correct M_1 by an amount equal to the error in ω_1, and a special program was written to do this. The only reason for mentioning this point is that the values of M_1 in Table 2 are the corrected values, and so are different from the values provided in the first four provisional lists of Ariel 3 parameters to be issued. (To avoid having to ask AWRE Foulness to make a correction to the Ariel 3 telemetry data analysis program after PROP3 had been introduced, it was decided to continue sending incorrect values to AWRE, by adding the appropriate deliberate error to M_1.)

Fig.1 gives a plot of orbital inclination, each value being represented by a vertical line, two standard deviations in length, centred on the fitted value. Fig.2 gives a plot of eccentricity, but most of the time the scale is too small for standard deviations to be shown. Fig.3 shows a short section of the eccentricity curve (covering just over half a period of the perigee) with the scale expanded sufficiently for the standard deviations to be indicated as on the inclination plot. Figs.1 and 2 give, in addition to the definitive inclinations and eccentricities obtained at R.A.E., the SDC (NORAD) values published in Spacetrack bulletins.

4.2 Results involving the parameter M_3

For two of the runs covered by Table 2 the parameter M_3 was included in the orbital model, namely, for those of epochs 1967 NOV 29 and 1967 DEC 2. These were the epochs which occurred during the week when no Minitrack data were supplied. Without M_3 the fit was twice as bad (ε 5.7 for the first epoch instead of 2.8), due to the number of days spanned by the observations. The value of M_3, omitted from Table 2 to retain a regular format, was -0.00069, with standard deviation 0.00003, for both epochs; the same observations were used in both runs, so the second set of elements is really just the first set advanced three days, with small variations in the residuals (and hence a small change in ε) due to limitations of the orbital model.

It was stated in section 3 that, for epochs early in the satellite's lifetime, general introduction of M_3 would not have helped. To illustrate, the complete set of parameters, when M_3 is included, for 1967 DEC 20 is as follows (with last-figure standard deviations in brackets):-
$e = 0.007329$ (15), $i = 80.1802$ (19), $\Omega = 239.0276$ (19), $\omega = 155.22$ (10), $M_0 + \omega = 91.7171$ (15), $M_1 = 5433.0021$ (22), $M_2 = 0.0701$ (9) and $M_3 = 0.0007$ (9); the value of ε, viz. 3.6, was actually <u>larger</u> (unrounded value, 3.553 as against 3.546) than for the run without M_3, due to the loss of a degree of freedom. For certain epochs later in the lifetime, however, introduction of M_3 <u>would</u> have led to better fits. This may be illustrated by considering the two worst fits obtained, namely, for epochs 1969 MAR 20 (ε of 5.0 in Table 2) and 1969 MAR 23 (ε of 5.1); on repeating these runs, with M_3 included, the following results were obtained:- for 1969 MAR 20, $e = 0.006902$ (45), $i = 80.1665$ (20), $\Omega = 10.3404$ (19), $\omega = 124.14$ (17), $M_0 + \omega = 189.4465$ (46), $M_1 = 5475.7830$ (24), $M_2 = 0.1282$ (9) and $M_3 = 0.0077$ (9), with $\varepsilon = 3.1$; for 1969 MAR 23, $e = 0.006936$ (49), $i = 80.1677$ (30), $\Omega = 6.4259$ (22), $\omega = 116.38$ (16), $M_0 + \omega = 48.2682$ (47), $M_1 = 5476.6144$ (32), $M_2 = 0.1444$ (12) and $M_3 = 0.0085$ (13), with $\varepsilon = 3.6$.

4.3 Results involving Hewitt camera observations

The runs at epochs 1969 APR 10 and 1969 JUN 21 were repeated, with (respectively 8 and 12) Hewitt camera observations included. The following results were obtained:- for 1969 APR 10, $e = 0.006778$ (7), $i = 80.1671$ (12), $\Omega = 342.9333$ (13), $\omega = 67.28$ (12), $M_0 + \omega = 326.4188$ (10), $M_1 = 5480.4510$ (8) and $M_2 = 0.0850$ (7), with $\varepsilon = 2.0$ and only the same Minitrack observation rejected as was originally rejected; for 1969 JUN 21, $e = 0.005403$ (8), $i = 80.1610$ (10), $\Omega = 248.6570$ (13), $\omega = 175.80$ (6), $M_0 + \omega = 120.8871$ (10), $M_1 = 5490.2793$ (7) and $M_2 = 0.0584$ (3), with $\varepsilon = 2.4$ and two of the Minitrack observations rejected that had previously been accepted. On comparison with corresponding entries in Table 2 it may be seen that for the first run there is little change - the maximum change in a parameter is for $M_0 + \omega$, the change being about twice the original standard deviation, and no standard deviation has decreased by a factor of more than $1\frac{1}{2}$; for the other run, however, there is a large change in eccentricity, nearly five times the original standard deviation, and the standard deviations for e, ω and $M_0 + \omega$ have all been reduced by factors of more than 2. (It is worth remarking that the change in e was caused entirely by the introduction of the Hewitt camera observations, and not at all by the subsequent rejection of two Minitrack observations.)

A reasonable conclusion is that Hewitt camera observations, of high accuracy, are compatible with Minitrack observations of poorer accuracy. For a high-inclination satellite like Ariel 3 the effect is not very significant if the Hewitt camera observations all come from a single pass, but there is a great improvement in accuracy when observations from two or more passes are available.

5 ACCURACY OF POSITION COMPUTATION

As with Ariel 2 it was required, for correlation with on-board experiments, that the definitive orbital parameters should be good enough for position to be computable from them to better than 1 km. In the paper on the Ariel 2 orbit[2] the accuracy of position computation was considered by reference to plots of $\{\sigma^2(x) + \sigma^2(y) + \sigma^2(z)\}^{\frac{1}{2}}$, where the variances $\sigma^2(x)$, $\sigma^2(y)$ and $\sigma^2(z)$ are functions of time and the covariance matrix of the orbital parameters, and by comparison of such plots with plots, during overlap periods, of $\{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2\}^{\frac{1}{2}}$, where x_1, y_1, z_1 denote satellite co-ordinates computed from orbital parameters at the epoch before the given overlap period and x_2, y_2, z_2 denote co-ordinates computed from parameters at the following epoch. This approach would have been equally possible for Ariel 3, using the program PREP[8], but it was decided that it would be adequate to consider the question by looking directly at standard deviations of orbital parameters and interpreting these as maximum position errors after 1½ days.

The justification for this approach is that, with $M_o + \omega$ rather than M_o taken as a parameter, large correlations between parameters did not occur. (Occasional correlations as large as ±0.4 occurred, usually involving e and one other parameter.) For a satellite, like Ariel 3, in an orbit which is nearly circular and not too far from polar, the maximum effects, on position after 1½ days, of one-sigma errors in the parameters are approximately as follows: $2\,a\,\sigma(e)$, $a\,\sigma(i)$, $a\,\sigma(\Omega)$, $2\,a\,e\,\sigma(\omega)$, $a\,\sigma(M_o + \omega)$, $1\frac{1}{2}\,a\,\sigma(M_1)$ and $2\,a\,\sigma(M_2)$, where angle sigmas are now taken to be in radians. The main effects here of e, ω and $M_o + \omega$ are the along-track errors which arise from the expression of argument of latitude in the form

$$u = (M + \omega) + 2\,e\,\sin\{(M + \omega) - \omega\} \quad .$$

Let us consider 'maximum position effects' for two different sets of sigmas: first, the maximum value of each sigma that occurs anywhere in Table 2, and, second, maximum values during, roughly speaking, the best ninety per cent of the time. Denoting sigmas from the two sets by σ_1 and σ_2 respectively (with angles in degrees again), and the corresponding maximum position effects, in km, by MPE_1 and MPE_2, we have the following table:-

	σ_1	MPE_1	σ_2	MPE_2
e	0.000072	1.1	0.000020	0.3
i	0.0043	0.5	0.0030	0.4
Ω	0.0048	0.6	0.0025	0.3
ω	0.51	0.8	0.17	0.3
$M_o + \omega$	0.0074	0.9	0.0031	0.4
M_1	0.0043	0.8	0.0023	0.4
M_2	0.0019	0.5	0.0019	0.5

Since $(\Sigma\ MPE_2^2)^{\frac{1}{2}} = 1.0$ km it is reasonable to claim that the accuracy requirements are met most of the time. If we consider the accuracy of height only, i.e. of $r = a\ \{1 - e \cos\ (M + \omega - \omega)\}$, then only the maximum one-sigma effects $a\ \sigma(e)$ and $a\ e\ \sigma(\omega)$ are significant; this gives a $\Sigma^{\frac{1}{2}}$ of 0.4 km corresponding to the MPE_2 column in the table.

Some comments may be useful on the reason for some of the larger sigmas in Table 2. The large $\sigma(M_1)$ (and hence $\sigma(a)$) and $\sigma(M_2)$ at the first epoch arose partly because this epoch was only 8 hours after launch and partly because of the complete absence of observations between $2^h\ 43^m$ on 6 May 1967 and $20^h\ 18^m$ on 7 May; there was a correlation of -0.988 between the computed values of M_1 and M_2. Similarly, a large $\sigma(M_1)$ arose for epoch 1967 DEC 5 because of the missing data for 3-4 December, which has already been mentioned. High sigmas for epochs from 1968 DEC 20 to 1969 JAN 16, inclusive, arose because of the paucity of observations during this period; with only 14 observations accepted, the run of 1969 JAN 1 gave the largest sigmas, for the parameters Ω and ω, of all the runs in Table 2. High sigmas for epochs from 1969 FEB 27 to 1969 APR 4 arose partly from paucity of observations and partly from the large values of ε during that period; the epochs 1969 MAR 20 and 1969 MAR 23, for which the largest values of ε (and of sigmas for e, i and $M_0 + \omega$) of all the runs in Table 2 were obtained, have already been discussed in section 4.

6 DISCUSSION

Although the perigee height of Ariel 3, during the period considered, was around 500 km, as opposed to about 300 km for Ariel 2, air drag was still important enough to be the chief limitation in the computation of orbital parameters by PROP. The value of M_2, equal to half the mean angular acceleration of the satellite, was only about $0.02^\circ/d^2$ immediately after launch; when this parameter approached or exceeded $0.1^\circ/d^2$, as it did for a few days in October-November 1968 and for longer periods in 1969, or when it changed by more than about $0.1^\circ/d^2$ from epoch to epoch, for example in late December 1967, the orbit does not fit the data so well, as indicated by higher values of ε. The period of validity of a set of orbital parameters is the same as the period spanned by the observations used in determining the parameters, i.e. between 3 and 4 days. If a set of orbital parameters is used to predict beyond the period of validity, then, when M_2 is changing rapidly (and ε is large), error increases rapidly. Now since each set of orbital parameters (after the first) was obtained by iteration from an initial set equivalent to the parameters at the preceding epoch, an immediate guide to the accuracy of three-day prediction - i.e. up to five days from a given epoch - is provided by the largest absolute value for the residuals in the first iteration of the orbit determination at the next epoch. This largest absolute value can change violently. As an extreme example, the figures for a series of successive epochs, starting at 1969 OCT 12, are:- 29, 45, 13, 7, 12, 67, 154, 189, 628, 52, 30, 59, 46, 17, 12, 74, 10; the value 628, equivalent to an angular error of about 10°, occurred for epoch 1968 NOV 5, and was obviously due to the unusually high value (at 1968 NOV 2) of 0.1144 for M_2, which immediately afterwards fell to 0.0530. (A very large magnetic storm occurred on 1 November and had a devastating effect on the upper atmosphere[9].)

The behaviour of the orbital inclination, as evidenced by Fig.1, is worth discussing in detail. There are two distinct features. First, and very striking, is the secular behaviour: i remained essentially constant at 80.18° until the middle of December 1967, then dropped to about 80.163 in a period of about three months, and thereafter again remained essentially constant. Second, there are the superimposed oscillations, in which certain frequencies and amplitudes can fairly readily be seen. It is not entirely easy to explain either of these features.

Apart from resonances - and there should be no relevant resonance associated with the orbital parameters of Ariel 3 - the only known cause of secular variation in the orbital inclination of an earth satellite is the rotation of the atmosphere. Applying the formula of King-Hele and Scott[10], if the atmosphere at a height of 500 km is taken to rotate at twice the angular velocity of the earth ($\Lambda = 2.0$ in Ref.10), then i in Fig.1 should show a secular drop of about 0.02°, i.e. just about what it does show. The rate of drop should be proportional to M_1, however, whereas in Fig.1, as already remarked, the total drop is concentrated into a period of about three months, starting in December 1967.

The phenomenon is sufficiently remarkable for a sceptical reader to wonder whether the inclinations in Fig.1 really are right. Here the SDC values, though less accurate than the R.A.E. values, are useful; they are completely independent of the R.A.E. values, and confirm - not that there was a serious doubt - the secular behaviour indicated by the R.A.E. values.

Slightly more credible, though still unlikely, is the possibility that the sharp drop in i is not a purely secular effect but an oscillation superimposed on the change due to atmospheric rotation. Such an oscillation would require a period of 500 days or more, however, and even then the next cycle might have been expected to appear before the end of the graph. The amplitude of the oscillation would have to be nearly 0.01°. The author is unable to see whence such a term could arise.

There remain two possibilities: a single, complete discontinuity, due for example to meteoric impact, and a genuine (secular) perturbation over the (roughly) three-month period. The former seems very unlikely, though a discontinuity near the beginning of January 1968 cannot be completely ruled out; so we are left with the possibility of a real perturbation. Bearing in mind that a perturbing force, to produce an effect on i without affecting i, has to act in a direction perpendicular to the orbital plane, and that, to avoid cancellation, it has to act in opposite directions at the ascending and descending nodes of the orbit, it is difficult to see what the force can be, other than atmospheric rotation.

Attempts to produce an explanation must therefore degenerate into mere speculation. The three-month period of interest corresponded to a period of maximum solar activity (mean 107 mm solar radiation in excess of 150×10^{-22} W m^{-2} Hz^{-1}), and during this period M_2 was greater than before and after, though not enough greater to explain Fig.1 at once. Could it be that, at heights above 400 km, where no accurate measurements have been made, atmospheric rotation is significantly faster during periods of high solar activity, i.e. that King-Hele and Scott's Λ parameter is considerably less than 2.0 for most of Fig.1, but very much larger during the short period of maximum activity?

Apart from the correlation with solar activity, two other interesting (and unexplained) correlations should be mentioned. First, the direction of the spin axis of Ariel 3 has been monitored by RSRS, Slough[1]. At injection the spin axis pointed 69° south, i.e. to a point on the celestial sphere with declination -69°. During the first three months the axis looped towards the south, reaching declination -86° on 19 June 1967, but after this it moved north and the declination remained positive after August 1967, for as long as the satellite was still spinning. The axis looped towards the north, reaching a declination of almost 90° on 25 or 26 February 1968; from early

January to early April 1968, i.e. roughly the period of sharp fall in orbital inclination, was the period during which the declination of the spin axis exceeded 45°. (There was another period, starting about the middle of September 1968, when the declination again exceeded 45°, but this only lasted for about one month instead of three.) Second, the satellite's tape recorder, which contained two litres of air at a pressure of one atmosphere, was operating only intermittently during the period of interest. Until 28 November 1967 the tape recorder worked successfully. It then failed, but recovered and worked, apparently perfectly again, for three periods (of two, four and three weeks), until it failed for the last time on 14 April 1968. It is tempting to speculate that air was leaking, but, even if this was so, it is difficult to see how the right inclination-reducing force could result.

On turning to the oscillations, it is clear from Fig.1 that a number of components, of differing frequency, amplitude and phase, are present. Since $\cos i$ is small and e is very small, the oscillatory perturbations due to the earth's odd harmonics, the amplitude of which is proportional to $e \cos i$, is completely negligible. The effect of the earth's tesseral harmonics, as remarked in section 3, is not negligible, and the amplitude of the $J_{2,2}$ perturbation is more than twice as big as some of the values of $\sigma(i)$ in Table 2; however, such effects should not appear in Fig.1, since they are averaged out during orbit determination. Hence the oscillatory components in Fig.1 may be thought of as being due solely to luni-solar perturbations, for which the various terms in di/dt are given in Ref.11 (equation (31)). The main term in the integral of the equation for di/dt is, for Ariel 3, $0^{\circ}.0015 \cos 2(u_s - \Omega)$, where u_s is the argument of latitutde of the sun; the period of this term is 80 days and a complete cycle may be seen, in Fig.1, for example between MJD 39673 and 39753 and between MJD 39993 and 40073. The next largest terms are combined terms for the sun and moon which, if we ignore the small non-zero value (Ω_m) for the right ascension of the node of the moon's orbit, are given by $0^{\circ}.0012 \cos \Omega$ and $0^{\circ}.0007 \cos 2\Omega$, of period 280 days and 140 days respectively. The fourth largest term is also the principal one in which u_m, the argument of latitude of the moon, appears; ignoring Ω_m again, it is $0^{\circ}.0005 \cos 2(u_m - \Omega)$, of period 12½ days approximately. Other terms are of smaller argument, but a combination of such terms could produce a detectable contribution to the graph of i.

In the absence of a spectral analysis or a complete analysis of all terms from Ref.11 it is difficult to be sure whether the oscillatory component of Fig.1 can be fully explained by luni-solar perturbations. It does appear, however, that Fig.1 contains a sinusoidal term with period about 30 days. A term in $\cos(k\Omega \pm u_m)$, for small integral k, would be appropriate here, but the only terms which arise have the eccentricity of the moon, i.e. 0.055, as a factor, and their amplitudes are too small.

One other known source of sinusoidal contributions to inclination variation should be mentioned. This is the precession and nutation of the earth's axis, which provides the reference with respect to which (the complement of) an orbital inclination is measured. The main contribution is from precession and may be taken from Ref.12 (in which the nutation terms have the wrong sign but the precession term is correct). For Ariel 3 this gives $-0^\circ.0007 \cos \Omega$, and so reduces (to about half) the amplitude of the direct luni-solar perturbation term of argument Ω.

7 CONCLUSIONS

Orbital parameters for the satellite Ariel 3, as for Ariel 2, have been determined at R.A.E., at three-day intervals, from Minitrack observations supplied by NASA. The accuracy of the computed parameters is about the same, in general, as was obtained for Ariel 2, i.e. better than 1 km except on very rare occasions, and should be adequate for experimenters' requirements. (Values of eccentricity were accurate enough to be used in determining the odd harmonics in the geopotential[13].) During periods of high drag, better accuracy could have been obtained by inclusion of an eighth parameter, M_3, in the orbital model, as was done with Ariel 2.

Oribital inclination was determined rather less accurately than for Ariel 2, no doubt due to the fact that there are no Minitrack stations, in either hemisphere, at latitutdes as high as 80°. The accuracy was good enough, however, for an anomalous secular behaviour in inclination to be clearly observable. This behaviour has been discussed but not explained.

Acknowledgement

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Table 1

MINITRACK STATIONS OBSERVING ARIEL 3

Station name	Location	x (km)	y (km)	z (km)
Fort Myers	Fort Myers, Florida, U.S.A.	807.885	-5652.020	2833.549
Johannesburg	Hartebeshoek, South Africa	5084.798	2670.474	-2768.164
Lima	Lima, Peru	1388.818	-6088.429	-1293.207
Newfoundland	St. Johns, Newfoundland	2602.801	-3419.184	4697.694
Quito	Quito, Ecuador	1263.617	-6255.010	-68.856
Santiago	Santiago, Chile	1769.707	-5044.642	-3468.192
Winkfield	Winkfield, England	3983.130	-48.404	4964.711
Ulaska	Fairbanks, Alaska	-2282.332	-1452.667	5756.942
Madagascar	Tananarive, Malagasy	4091.903	4434.373	-2064.537
Orroral	Canberra, Australia	-4447.361	2677.215	-3695.209

DATE	a		e		h_p		i		Ω		ω		$M_0 + \omega$		M_1		M_2		N	K	D	ε	MJD
1967 MAY 6	6930.0593	36	0.007756	11	501.18	8	80.1793	14	169.4546	9	163.21	7	134.1727	8	5419.2235	43	0.0220	19	40	6	2.7	1.8	39616
MAY 9	6929.9546	6	0.007933	5	501.70	4	80.1782	9	165.6410	5	154.27	3	182.4750	5	5419.3466	7	0.0168	5	66	8	2.9	1.5	39619
MAY 12	6929.8707	3	0.008072	5	503.00	4	80.1790	11	161.8309	6	145.50	4	231.0999	5	5419.4449	3	0.0171	3	61	0	3.9	1.7	39622
MAY 15	6929.7787	4	0.008208	6	504.58	4	80.1805	11	158.0188	6	136.79	5	280.0288	6	5419.5528	4	0.0183	4	57	1	3.7	1.8	39625
MAY 18	6929.6831	4	0.008350	8	506.08	5	80.1782	11	154.2069	9	128.36	4	329.2894	8	5419.6651	5	0.0188	5	44	2	3.8	1.7	39628
MAY 21	6929.5814	5	0.008496	11	507.36	8	80.1751	14	150.3958	11	119.99	6	18.8983	11	5419.7843	6	0.0196	5	33	3	4.0	2.2	39631
MAY 24	6929.4573	5	0.008578	7	508.73	5	80.1761	12	146.5827	12	111.41	5	68.8946	8	5419.9299	6	0.0269	6	41	9	3.8	1.7	39634
MAY 27	6929.2057	8	0.008708	13	509.07	9	80.1737	30	142.7737	23	102.76	10	119.4956	17	5420.2252	10	0.0557	9	44	8	3.8	3.5	39637
MAY 30	6928.9673	4	0.008659	5	509.92	3	80.1778	11	138.9548	7	94.70	5	171.0505	7	5420.5052	5	0.0442	4	50	2	3.8	1.7	39640
JUN 2	6928.7651	5	0.008624	5	510.01	4	80.1801	16	135.1617	7	86.40	6	223.3796	7	5420.7427	6	0.0321	5	37	3	3.6	1.7	39643
JUN 5	6928.6101	4	0.008579	6	509.50	4	80.1796	17	131.3280	8	78.21	6	276.3207	8	5420.9246	5	0.0296	5	44	0	3.7	2.0	39646
JUN 8	6928.4620	7	0.008521	8	508.42	5	80.1795	17	127.5145	12	69.94	7	329.8027	10	5421.0985	8	0.0231	8	49	2	3.8	2.5	39649
JUN 11	6928.3730	5	0.008446	6	506.93	4	80.1795	9	123.6980	8	61.58	5	23.7085	7	5421.2030	6	0.0126	5	40	7	3.8	1.5	39652
JUN 14	6928.3165	6	0.008368	8	505.17	6	80.1774	16	119.8865	11	53.55	7	77.8500	10	5421.2693	7	0.0108	6	31	6	3.9	2.0	39655
JUN 17	6928.2475	6	0.008258	6	503.26	4	80.1784	12	116.0692	9	44.98	5	132.2045	7	5421.3503	7	0.0147	6	49	7	3.6	1.8	39658
JUN 20	6928.1621	5	0.008115	6	501.50	4	80.1779	11	112.2529	7	36.20	4	186.8281	6	5421.4506	6	0.0187	5	42	4	3.6	1.5	39661
JUN 23	6928.0563	5	0.007956	8	500.02	5	80.1809	16	108.4613	8	27.17	4	241.7847	7	5421.5749	6	0.0219	5	36	4	3.8	1.5	39664
JUN 26	6927.9258	5	0.007753	10	499.30	7	80.1823	16	104.6257	8	18.01	4	297.1494	7	5421.7282	6	0.0313	5	43	6	3.7	1.8	39667
JUN 29	6927.7744	5	0.007557	11	499.23	7	80.1824	14	100.8101	9	8.78	5	353.0386	8	5421.9060	6	0.0262	5	44	7	3.7	1.9	39670
JUL 2	6927.6453	5	0.007325	11	500.30	7	80.1812	13	96.9950	11	359.41	5	49.4181	8	5422.0575	6	0.0229	6	38	5	3.9	1.9	39673
JUL 5	6927.5565	5	0.007125	8	502.15	6	80.1810	10	93.1785	10	349.77	4	106.1924	8	5422.1618	5	0.0153	5	29	5	3.7	1.6	39676
JUL 8	6927.4913	4	0.006923	8	505.00	5	80.1803	9	89.3650	10	339.99	6	163.2328	9	5422.2384	5	0.0096	5	24	7	3.7	1.1	39679
JUL 11	6927.4530	7	0.006751	12	508.48	8	80.1759	19	85.5491	17	330.04	7	220.4671	10	5422.2832	8	0.0066	8	24	4	3.6	1.8	39682
JUL 14	6927.4053	4	0.006596	7	512.54	5	80.1784	11	81.7288	7	319.59	5	277.8073	6	5422.3394	5	0.0096	4	37	5	3.9	1.3	39685
JUL 17	6927.3508	5	0.006466	5	516.69	4	80.1783	11	77.9113	6	308.77	5	335.3459	5	5422.4036	6	0.0114	5	41	6	3.7	1.3	39688
JUL 20	6927.2797	6	0.006354	8	520.54	5	80.1798	19	74.0948	8	297.59	9	33.0997	8	5422.4869	7	0.0152	7	27	7	3.2	1.5	39691
JUL 23	6927.1967	6	0.006304	6	523.18	4	80.1812	16	70.2774	7	286.40	9	91.1277	7	5422.5844	7	0.0180	6	37	7	3.7	1.7	39694
JUL 26	6927.1007	5	0.006265	6	524.63	4	80.1794	13	66.4588	10	274.96	9	149.4745	8	5422.6972	6	0.0189	5	35	2	3.8	1.8	39697
JUL 29	6927.0095	6	0.006245	8	524.59	5	80.1792	15	62.6421	13	263.55	12	208.1495	11	5422.8043	7	0.0191	7	39	6	3.2	2.3	39700
AUG 1	6926.9181	3	0.006265	4	522.96	3	80.1780	8	58.8224	7	252.33	6	267.1631	6	5422.9116	4	0.0138	4	31	7	3.9	1.1	39703
AUG 4	6926.8528	7	0.006297	7	520.11	5	80.1743	17	55.0065	14	240.87	11	326.4393	10	5422.9882	8	0.0127	7	38	14	3.8	1.9	39706
AUG 7	6926.7767	4	0.006391	5	516.28	4	80.1758	10	51.1846	7	230.03	5	25.9579	6	5423.0777	5	0.0159	4	56	9	3.6	1.6	39709
AUG 10	6926.6964	3	0.006518	5	512.10	3	80.1775	9	47.3641	6	219.51	4	85.7589	5	5423.1721	3	0.0158	3	50	8	3.6	1.3	39712
AUG 13	6926.6030	3	0.006679	6	507.92	4	80.1784	12	43.5456	5	209.12	4	145.8548	5	5423.2819	4	0.0196	3	54	13	3.8	1.4	39715
AUG 16	6926.4962	4	0.006838	9	504.40	6	80.1808	15	39.7269	6	198.97	4	206.3060	7	5423.4075	4	0.0223	4	36	6	3.7	1.4	39718
AUG 19	6926.3602	4	0.007022	10	501.56	7	80.1822	12	35.9091	8	189.06	4	267.1707	7	5423.5673	5	0.0309	5	39	6	3.8	1.8	39721
AUG 22	6926.1988	7	0.007216	12	499.62	8	80.1824	14	32.0900	13	179.27	7	328.5754	10	5423.7570	9	0.0309	7	32	4	3.5	1.9	39724
AUG 25	6926.0251	7	0.007426	16	498.57	11	80.1796	15	28.2697	13	169.59	7	30.5544	12	5423.9610	8	0.0397	8	30	4	3.5	2.3	39727
AUG 28	6925.8251	4	0.007580	7	498.74	5	80.1809	10	24.4674	8	160.26	3	93.2135	6	5424.1961	5	0.0388	5	23	1	3.7	1.1	39730
AUG 31	6925.6073	7	0.007755	14	499.47	9	80.1799	19	20.6299	18	150.91	8	156.5231	13	5424.4521	8	0.0484	8	25	3	3.6	2.2	39733

Table 2. Orbital parameters of Ariel 3

Table 2(cont'd.)

DATE		a		e		h_p		i		Ω		ω		$M_o+\omega$		M_1		M_2		N	K	D	ε	MJD
1967	SEP 3	6925.3506	5	0.007877	6	500.89	4	80.1831	9	16.8056	6	142.00	3	220.8086	5	5424.7539	6	0.0473	7	37	2	3.0	1.3	39736
	SEP 6	6925.1570	7	0.007968	6	502.76	4	80.1817	13	12.9885	7	133.20	3	285.8482	12	5424.9814	9	0.0337	3	26	0	4.9	1.4	39739
	SEP 9	6924.9797	4	0.008069	5	504.43	4	80.1808	14	9.1667	7	124.74	5	351.5117	6	5425.1899	5	0.0371	5	35	2	3.1	1.5	39742
	SEP 12	6924.8036	5	0.008150	6	505.94	4	80.1819	16	5.3460	8	116.41	7	57.8177	8	5425.3969	5	0.0318	5	32	2	3.7	1.8	39745
	SEP 15	6924.6180	5	0.008207	6	507.17	4	80.1831	12	1.5257	8	108.00	7	124.7375	9	5425.6152	6	0.0357	6	37	1	3.7	2.1	39748
	SEP 18	6924.4613	5	0.008248	6	507.92	4	80.1849	11	357.7073	11	99.67	7	192.2816	9	5425.7995	6	0.0296	6	34	0	3.7	2.0	39751
	SEP 21	6924.2779	7	0.008281	10	507.99	7	80.1839	12	353.8841	15	91.43	10	260.3797	14	5426.0151	8	0.0437	7	37	0	3.7	2.8	39754
	SEP 24	6924.1083	6	0.008284	9	507.57	6	80.1819	12	350.0528	12	83.31	9	329.1769	12	5426.2144	7	0.0266	6	32	0	3.7	2.4	39757
	SEP 27	6923.9703	7	0.008283	11	506.55	7	80.1823	14	346.2408	14	75.32	16	38.6612	20	5426.3768	8	0.0283	8	25	1	3.7	2.5	39760
	SEP 30	6923.7676	8	0.008232	10	505.12	7	80.1824	16	342.4170	15	66.89	13	108.3183	15	5426.6152	9	0.0449	8	28	2	3.4	2.3	39763
	OCT 3	6923.6107	5	0.008151	7	503.40	5	80.1817	10	338.5932	11	58.43	7	178.8867	9	5426.7997	6	0.0244	5	32	0	3.7	2.0	39766
	OCT 6	6923.4813	4	0.008037	6	501.53	4	80.1793	8	334.7685	9	49.76	5	249.9155	7	5426.9518	5	0.0256	5	39	3	3.7	1.7	39769
	OCT 9	6923.3457	5	0.007903	8	499.63	5	80.1786	14	330.9434	11	40.97	6	321.4037	8	5427.1112	6	0.0303	5	40	1	3.8	2.1	39772
	OCT 12	6923.1719	4	0.007719	7	498.10	5	80.1777	11	327.1172	8	32.01	4	33.4439	7	5427.3157	5	0.0320	5	41	3	3.5	1.6	39775
	OCT 15	6923.0109	4	0.007550	7	496.82	5	80.1782	13	323.2937	6	22.96	4	106.0775	6	5427.5051	4	0.0331	4	46	2	3.8	1.6	39778
	OCT 18	6922.8430	4	0.007354	8	496.33	5	80.1783	9	319.4679	6	13.77	3	179.2902	6	5427.7027	5	0.0345	5	42	2	3.4	1.6	39781
	OCT 21	6922.6723	5	0.007144	10	496.72	7	80.1781	12	315.6426	9	4.42	5	253.1044	8	5427.9035	6	0.0336	5	35	1	3.8	2.0	39784
	OCT 24	6922.4796	5	0.006909	10	498.19	7	80.1791	11	311.8163	10	354.89	5	327.5352	9	5428.1303	6	0.0428	5	41	3	3.8	2.1	39787
	OCT 27	6922.2590	9	0.006679	18	500.57	12	80.1774	19	307.9862	19	345.25	10	42.7060	15	5428.3898	10	0.0484	9	35	2	4.0	3.5	39790
	OCT 30	6921.9522	9	0.006472	18	503.64	12	80.1765	18	304.1576	17	335.27	10	118.7789	16	5428.7508	11	0.0624	10	32	3	3.6	3.0	39793
	NOV 2	6921.6745	8	0.006247	15	507.64	10	80.1753	16	300.3293	15	325.02	8	195.9471	12	5429.0776	9	0.0509	7	29	2	3.8	2.6	39796
	NOV 5	6921.4201	5	0.006072	9	511.80	6	80.1763	11	296.4997	10	314.44	6	274.0446	9	5429.3770	6	0.0449	5	33	0	3.7	2.1	39799
	NOV 8	6921.2304	5	0.005914	9	515.94	6	80.1766	12	292.6709	11	303.62	7	352.9327	9	5429.6004	6	0.0341	5	37	2	3.8	2.2	39802
	NOV 11	6921.0603	5	0.005802	7	519.37	5	80.1772	13	288.8397	10	292.52	8	72.4408	8	5429.8007	6	0.0303	5	43	3	3.8	2.1	39805
	NOV 14	6920.8900	4	0.005731	6	521.62	4	80.1769	9	285.0093	9	281.26	6	152.5321	6	5430.0011	5	0.0338	5	44	3	3.9	1.8	39808
	NOV 17	6920.7058	4	0.005709	5	522.26	4	80.1775	11	281.1777	8	269.70	8	233.2411	7	5430.2180	5	0.0398	5	50	3	3.7	2.0	39811
	NOV 20	6920.4900	6	0.005719	7	521.23	5	80.1781	14	277.3662	11	258.21	9	314.6614	8	5430.4722	7	0.0450	6	47	1	3.9	2.4	39814
	NOV 23	6920.2475	7	0.005793	11	518.51	8	80.1769	19	273.5144	14	246.94	11	36.8888	10	5430.7577	8	0.0486	4	43	2	3.7	3.0	39817
	NOV 26	6919.9821	5	0.005880	8	514.73	5	80.1761	15	269.6824	10	235.65	8	120.0260	8	5431.0702	6	0.0492	6	48	0	3.7	2.3	39820
*	NOV 29	6919.7398	7	0.006021	9	510.16	6	80.1789	16	265.8547	10	224.52	7	204.0586	29	5431.3556	9	0.0460	4	58	0	10.5	2.8	39823
*	DEC 2	6919.5208	7	0.006155	10	505.79	7	80.1789	17	262.0189	10	213.80	7	288.9116	31	5431.6136	8	0.0398	2	58	0	10.5	2.9	39826
	DEC 5	6919.3423	19	0.006364	8	501.49	6	80.1781	13	258.1892	8	203.40	6	14.4857	15	5431.8240	23	0.0373	8	37	0	2.8	1.8	39829
	DEC 8	6919.1521	5	0.006557	9	498.10	6	80.1790	13	254.3560	7	193.21	5	100.6832	8	5432.0481	5	0.0355	5	49	1	3.9	2.2	39832
	DEC 11	6918.9895	5	0.006725	9	495.92	6	80.1804	12	250.5240	7	183.47	5	187.5169	7	5432.2396	6	0.0280	5	45	3	3.7	1.9	39835
	DEC 14	6918.8397	6	0.006913	11	494.62	7	80.1814	14	246.6908	10	173.70	6	274.8756	9	5432.4161	7	0.0310	6	41	2	3.7	2.4	39838
	DEC 17	6918.6310	6	0.007125	11	494.01	8	80.1813	14	242.8597	13	164.46	7	2.8371	10	5432.6620	7	0.0503	6	49	0	3.8	2.8	39841
	DEC 20	6918.3194	8	0.007327	15	494.13	10	80.1802	19	239.0276	19	155.22	10	91.7171	15	5433.0292	9	0.0699	9	44	0	3.0	3.5	39844
	DEC 23	6918.0284	7	0.007546	14	494.63	10	80.1789	15	235.1930	18	146.42	9	181.7599	13	5433.3722	8	0.0471	7	46	2	3.8	3.3	39847
	DEC 26	6917.8217	5	0.007693	9	496.03	6	80.1791	10	231.3581	11	137.61	5	272.6506	8	5433.6138	6	0.0350	5	53	5	3.8	2.2	39850
	DEC 29	6917.6294	7	0.007823	13	497.58	9	80.1788	14	227.5230	15	128.99	8	4.2180	12	5433.8424	9	0.0354	8	43	6	3.8	2.8	39853

* FOR THESE TWO EPOCHS THE ADDITIONAL PARAMETER M_3 WAS INCLUDED IN THE ORBITAL MODEL. THE VALUE OBTAINED, IN BOTH CASES, WAS −0·00069 3.

Table 2(cont'd.) Orbital parameters of Ariel 3

DATE	a		e		h_p		i		Ω		ω		$M_o + \omega$		M_1		M_2		N	K	D	ε	MJD
1968 JAN 1	6917.4121	7	0.007969	12	498.75	8	80.1787	14	223.6878	13	120.67	9	90.4706	16	5434.0986	8	0.0539	8	32	4	3.8	2.5	39856
JAN 4	6917.1810	5	0.008067	11	499.94	8	80.1756	10	219.8505	10	112.14	6	189.5927	10	5434.3710	6	0.0368	6	34	4	3.8	2.1	39859
JAN 7	6916.9643	5	0.008147	9	500.68	6	80.1759	10	216.0118	11	103.74	6	285.4425	10	5434.6266	4	0.0478	6	35	2	3.7	2.2	39862
JAN 10	6916.7298	7	0.008191	12	500.98	8	80.1743	15	212.1707	13	95.49	8	18.1186	14	5434.9029	8	0.0461	9	20	0	3.2	2.1	39865
JAN 13	6916.4463	8	0.008208	10	500.70	7	80.1733	13	208.3333	13	87.04	7	113.6706	12	5435.2372	9	0.0630	9	29	4	3.3	2.3	39868
JAN 16	6916.1630	8	0.008186	15	499.95	10	80.1721	21	204.4946	14	78.90	10	210.2856	16	5435.5713	9	0.0497	8	27	9	3.7	2.2	39871
JAN 19	6915.9404	5	0.008164	8	498.57	5	80.1709	11	200.6511	10	70.47	6	307.7929	9	5435.8338	6	0.0388	5	35	13	3.7	1.6	39874
JAN 22	6915.7573	5	0.008104	8	496.92	6	80.1731	11	196.8099	10	62.13	7	46.0080	10	5436.0498	6	0.0316	5	37	7	3.7	2.0	39877
JAN 25	6915.5907	5	0.008019	9	495.00	6	80.1728	12	192.9679	13	53.65	8	144.8192	11	5436.2463	6	0.0363	6	36	13	3.7	2.0	39880
JAN 28	6915.3837	7	0.007912	9	492.92	6	80.1697	15	189.1269	15	45.05	9	244.2677	11	5436.4906	8	0.0469	6	37	10	3.9	2.3	39883
JAN 31	6915.1114	5	0.007789	7	490.85	5	80.1701	13	185.2848	12	36.35	8	344.5688	10	5436.8117	6	0.0563	5	35	6	4.0	1.9	39886
FEB 3	6914.7760	7	0.007657	9	489.01	7	80.1697	15	181.4404	15	27.54	10	85.9147	12	5437.2073	9	0.0766	8	30	3	3.7	2.4	39889
FEB 6	6914.4047	8	0.007487	12	487.84	8	80.1698	17	177.5952	16	18.59	10	188.5943	14	5437.6454	10	0.0601	9	31	3	3.6	2.7	39892
FEB 9	6914.1237	12	0.007302	20	487.50	14	80.1680	29	173.7501	25	9.32	13	292.3965	21	5437.9771	14	0.0578	15	29	10	3.6	3.2	39895
FEB 12	6913.8176	12	0.007106	18	488.09	13	80.1698	24	169.9029	24	359.97	17	37.2396	21	5438.3384	14	0.0569	11	34	10	3.7	3.7	39898
FEB 15	6913.6033	10	0.006955	18	489.37	13	80.1704	23	166.0632	22	350.85	14	143.0215	21	5438.5914	12	0.0331	12	30	3	3.8	4.2	39901
FEB 18	6913.4091	6	0.006768	11	491.89	8	80.1684	14	162.2163	13	340.99	6	249.4816	10	5438.8205	7	0.0402	6	26	0	3.1	2.1	39904
FEB 21	6913.1948	5	0.006558	11	495.41	8	80.1651	10	158.3698	10	330.99	6	356.6707	10	5439.0734	6	0.0468	6	32	1	3.7	2.0	39907
FEB 24	6912.9728	5	0.006398	10	499.24	7	80.1651	11	154.5200	11	320.67	6	104.6575	10	5439.3356	6	0.0414	6	39	0	3.8	2.3	39910
FEB 27	6912.7332	6	0.006235	12	503.38	8	80.1647	12	150.6704	12	310.07	8	213.4214	12	5439.6185	7	0.0533	7	35	0	3.7	2.5	39913
MAR 1	6912.4088	7	0.006099	12	507.11	8	80.1633	14	146.8197	13	299.21	10	323.1838	12	5440.0016	9	0.0638	8	42	0	3.8	3.0	39916
MAR 4	6912.1074	6	0.005983	10	510.13	7	80.1616	12	142.9574	12	287.95	9	74.0819	11	5440.3574	8	0.0619	7	40	0	3.8	2.5	39919
MAR 7	6911.8210	7	0.005909	11	511.82	8	80.1612	12	139.1139	13	276.41	12	186.0576	13	5440.6957	8	0.0445	7	36	1	3.8	2.6	39922
MAR 10	6911.6263	5	0.005882	7	511.90	5	80.1610	11	135.2602	11	265.01	10	298.8637	9	5440.9256	6	0.0357	5	37	0	3.8	2.2	39925
MAR 13	6911.4428	5	0.005899	7	510.30	5	80.1603	11	131.4055	11	253.48	9	52.3190	9	5441.1424	6	0.0356	5	40	1	3.7	2.1	39928
MAR 16	6911.2472	5	0.005966	7	507.15	5	80.1605	11	127.5526	12	241.88	9	166.4362	8	5441.3735	6	0.0407	6	44	0	3.7	2.2	39931
MAR 19	6911.0661	4	0.006069	7	503.05	5	80.1588	12	123.6955	11	230.55	7	281.2458	8	5441.5876	5	0.0343	5	46	1	3.9	1.8	39934
MAR 22	6910.8834	4	0.006201	7	498.42	5	80.1596	9	119.8392	8	219.67	5	36.6810	7	5441.8036	5	0.0373	5	47	3	3.9	1.7	39937
MAR 25	6910.6620	4	0.006364	7	494.24	5	80.1601	11	115.9846	6	209.02	4	152.8006	6	5442.0650	4	0.0521	4	51	12	3.9	1.6	39940
MAR 28	6910.3900	4	0.006528	7	490.69	5	80.1594	10	112.1245	6	198.69	4	269.8205	6	5442.3864	4	0.0555	4	55	3	3.9	1.8	39943
MAR 31	6910.0864	3	0.006735	7	487.38	5	80.1589	11	108.2677	6	188.70	4	27.8472	6	5442.7453	4	0.0643	4	52	1	4.0	1.8	39946
APR 3	6909.7825	4	0.006935	7	485.26	5	80.1606	10	104.4090	6	179.16	4	166.9998	6	5443.1045	5	0.0522	5	45	0	3.3	1.6	39949
APR 6	6909.5307	6	0.007131	9	484.21	6	80.1615	15	100.5530	10	169.85	6	267.1078	8	5443.4023	7	0.0523	7	40	0	3.6	2.1	39952
APR 9	6909.3229	4	0.007302	6	484.16	6	80.1630	11	96.6947	8	160.92	5	28.0768	6	5443.6480	5	0.0312	6	45	1	3.8	1.7	39955
APR 12	6909.1514	5	0.007459	9	484.87	6	80.1633	15	92.8358	12	152.18	7	149.6516	11	5443.8507	6	0.0381	7	33	0	3.7	2.2	39958
APR 15	6908.9427	4	0.007606	5	485.07	6	80.1661	9	88.9791	8	143.49	4	271.9061	6	5444.0976	4	0.0417	4	51	5	3.8	1.6	39961
APR 18	6908.7402	4	0.007727	7	487.66	5	80.1662	12	85.1282	9	134.77	5	34.9028	8	5444.3371	5	0.0381	5	38	1	3.7	1.8	39964
APR 21	6908.5658	3	0.007824	4	489.41	3	80.1661	7	81.2630	6	126.28	3	158.5776	5	5444.5434	4	0.0315	3	49	5	3.9	1.4	39967
APR 24	6908.3844	4	0.007937	6	490.79	4	80.1665	10	77.4039	9	117.73	5	282.8510	7	5444.7578	5	0.0379	5	44	1	3.7	1.9	39970
APR 27	6908.1849	4	0.008028	6	491.87	4	80.1643	10	73.5462	10	109.23	5	47.8043	7	5444.9938	5	0.0425	4	41	1	3.8	1.8	39973

Table 2(cont'd.) Orbital parameters of Ariel 3

Table 2(cont'd.)

DATE		a		e		h_p		i		Ω		ω		$M_o + \omega$		M_1		M_2		N	K	D	E	MJD
1968	APR 30	6907.9801	3	0.008078	7	492.61	5	80.1649	8	69.6840	8	100.93	6	173.4934	7	5465.2360	4	0.0395	6	26	0	3.9	1.5	39976
	MAY 3	6907.7640	5	0.008131	8	492.62	6	80.1625	11	65.8286	12	92.49	8	299.9036	10	5465.4916	6	0.0456	6	32	1	3.7	2.1	39979
	MAY 6	6907.5427	5	0.008132	6	492.27	4	80.1621	10	61.9656	10	84.48	7	67.1113	9	5465.7336	6	0.0433	6	35	1	4.0	2.0	39982
	MAY 9	6907.2610	5	0.008115	7	491.27	5	80.1628	9	58.1033	11	76.26	7	195.1821	10	5466.0867	5	0.0554	6	37	1	3.8	2.0	39985
	MAY 12	6906.9929	5	0.008034	6	490.10	4	80.1641	10	54.2407	9	68.07	6	326.2605	7	5466.4039	5	0.0500	5	49	0	3.6	2.0	39988
	MAY 15	6906.7581	4	0.007925	6	488.58	4	80.1654	10	50.3793	6	59.68	6	94.2335	5	5466.6818	4	0.0442	4	39	1	3.9	1.5	39991
	MAY 18	6906.5194	3	0.007787	6	486.88	4	80.1661	11	46.5160	5	51.23	4	225.0190	5	5466.9643	4	0.0507	6	49	2	3.8	1.6	39994
	MAY 21	6906.2481	3	0.007613	5	485.15	4	80.1666	10	42.6530	6	42.56	4	356.7073	5	5467.2855	4	0.0576	3	50	2	3.8	1.5	39997
	MAY 24	6905.9706	4	0.007419	6	483.55	4	80.1682	10	38.7906	7	33.63	4	120.3991	6	5467.6140	4	0.0512	4	42	0	3.8	[illegible]	40000
	MAY 27	6905.7282	4	0.007219	7	482.31	5	80.1690	10	34.9287	8	24.48	4	263.0135	7	5467.9010	5	0.0442	4	51	1	3.8	[illegible]	40003
	MAY 30	6905.5046	4	0.007006	7	481.73	5	80.1687	11	31.0642	8	15.14	4	37.4373	7	5468.1657	5	0.0440	4	41	0	3.8	[illegible]	40006
	JUN 2	6905.3086	3	0.006816	5	481.82	3	80.1663	7	27.2011	7	5.72	4	172.6240	5	5468.3976	4	0.0366	4	44	0	3.6	[illegible]	40009
	JUN 5	6905.1391	4	0.006625	7	482.87	5	80.1649	9	23.3359	9	356.17	6	308.4616	7	5468.5983	5	0.0276	4	33	1	3.7	[illegible]	40012
	JUN 8	6905.0039	4	0.006423	7	485.00	5	80.1631	8	19.4705	7	346.60	4	84.8194	6	5468.7583	5	0.0283	4	43	2	3.8	1.6	40015
	JUN 11	6904.8462	4	0.006215	8	488.14	5	80.1626	8	15.6051	9	336.51	5	221.6810	7	5468.9450	5	0.0366	5	41	1	[illegible]	1.7	40018
	JUN 14	6904.6736	3	0.006044	6	491.83	4	80.1628	7	11.7337	7	326.13	5	359.1641	6	5469.1695	4	0.0293	4	47	0	3.8	1.5	40021
	JUN 17	6904.5393	3	0.005869	5	496.20	3	80.1642	9	7.8672	6	315.14	5	137.1768	5	5469.3085	4	0.0271	3	56	0	3.9	1.5	40024
	JUN 20	6904.4037	3	0.005749	4	500.29	3	80.1641	8	3.9988	5	303.82	5	275.6699	5	5469.4691	4	0.0250	3	50	2	3.8	1.4	40027
	JUN 23	6904.2859	4	0.005690	4	503.53	3	80.1668	8	0.1313	6	292.24	5	54.6118	6	5469.6087	4	0.0221	3	55	1	3.8	1.4	40030
	JUN 26	6904.1790	3	0.005659	4	505.37	3	80.1649	9	356.2621	7	280.56	6	193.9522	5	5449.7353	4	0.0203	3	54	1	3.8	1.5	40033
	JUN 29	6904.0683	3	0.005661	4	506.03	3	80.1673	7	357.3941	6	268.99	5	333.6736	4	5449.8665	4	0.0206	3	56	3	3.8	1.4	40036
	JUL 2	6903.9553	4	0.005686	7	504.93	5	80.1669	10	368.5271	10	257.52	8	113.7938	10	5450.0006	5	0.0205	5	23	0	3.7	1.6	40039
	JUL 5	6903.8567	4	0.005715	5	502.54	4	80.1653	9	364.6595	8	245.99	7	254.2951	7	5450.1171	4	0.0178	4	35	0	3.9	1.6	40042
	JUL 8	6903.7691	4	0.005776	5	499.07	4	80.1637	9	360.7906	10	234.77	8	38.1229	7	5450.2208	4	0.0167	4	27	1	4.0	1.4	40045
	JUL 11	6903.6649	6	0.005892	6	494.66	4	80.1637	9	336.9221	9	223.19	8	176.2630	8	5450.3443	7	0.0261	6	36	0	3.3	1.7	40048
	JUL 14	6903.5221	4	0.006022	7	490.46	5	80.1642	7	333.0530	7	212.56	5	317.8509	6	5450.5136	4	0.0328	4	45	3	3.8	1.6	40051
	JUL 17	6903.3568	3	0.006205	7	486.42	5	80.1641	11	329.1836	6	202.09	4	100.0018	6	5450.7093	4	0.0329	4	46	3	3.7	1.5	40054
	JUL 20	6903.1800	3	0.006383	6	483.28	4	80.1638	9	325.3135	6	192.06	3	262.7520	5	5450.9158	4	0.0350	4	52	1	3.7	1.6	40057
	JUL 23	6903.0030	4	0.006575	6	481.05	4	80.1670	10	321.4435	7	182.48	6	26.1329	6	5451.1287	5	0.0371	4	46	1	3.8	1.5	40060
	JUL 26	6902.8309	4	0.006748	7	479.91	5	80.1693	10	317.5742	9	173.11	5	170.1659	7	5451.3327	5	0.0330	5	38	3	3.7	1.6	40063
	JUL 29	[illegible]	4	0.006907	6	479.79	4	80.1678	9	313.7061	9	163.68	5	316.7536	7	5451.5270	5	0.0284	6	37	1	3.8	1.6	40066
	AUG [illegible]	6902.5368	5	0.007084	9	480.36	6	80.1657	9	309.8376	9	154.40	5	99.8892	7	5451.6812	6	0.0216	6	33	4	3.6	1.6	40069
	AUG 4	6902.4189	5	0.007265	8	481.40	5	80.1635	8	305.9672	8	145.29	4	245.4409	7	5451.8208	5	0.0285	5	32	3	3.5	1.5	40072
	AUG 7	6902.2820	5	0.007409	8	482.93	5	80.1643	9	302.0961	10	136.42	5	31.4460	8	5451.9831	6	0.0300	5	36	0	3.8	1.8	40075
	AUG 10	6902.1300	3	0.007534	5	484.56	3	80.1627	8	298.2265	6	127.86	4	177.9709	5	5452.1632	4	0.0302	3	43	1	3.8	1.4	40078
	AUG 13	6901.9771	4	0.007633	4	486.09	3	80.1617	9	294.3529	7	119.43	5	325.0386	6	5452.3444	4	0.0299	4	35	2	3.5	1.4	40081
	AUG 16	6901.7807	3	0.007674	4	487.58	3	80.1631	11	290.4795	7	111.13	4	112.7018	5	5452.5773	4	0.0424	3	41	1	3.8	1.4	40084
	AUG 19	6901.5850	4	0.007686	5	488.71	3	80.1669	8	286.6073	6	102.90	4	261.1070	6	5452.8095	5	0.0320	5	51	0	3.3	1.3	40087
	AUG 22	6901.4502	4	0.007702	5	489.24	3	80.1663	8	282.7352	8	94.37	4	50.0993	6	5452.9693	4	0.0222	4	49	2	3.9	1.5	40090
	AUG 25	6901.3299	4	0.007716	6	489.06	4	80.1630	9	278.8640	10	86.27	4	199.5206	8	5453.1118	5	0.0226	5	37	0	3.7	2.0	40093

Table 2(cont'd.) Orbital parameters of Ariel 3

DATE		a		e		h_p		i		Ω		ω		$M_0+\omega$		M_1		M_2		N	K	D	ε	MJD
1968	AUG 28	6901.2363	4	0.007710	5	488.33	3	80.1598	8	274.9904	8	78.05	5	349.3393	7	5453.2227	5	0.0172	4	41	1	3.9	1.7	40096
	AUG 31	6901.1391	5	0.007675	6	487.16	4	80.1595	11	271.1167	10	69.93	6	139.4698	7	5453.3379	6	0.0241	6	45	4	3.4	1.8	40099
	SEP 3	6900.9737	3	0.007581	5	485.69	3	80.1585	9	267.2414	6	61.48	4	290.0609	5	5453.5340	4	0.0359	3	51	4	3.9	1.5	40102
	SEP 6	6900.7820	3	0.007447	5	484.07	3	80.1596	9	263.3647	6	53.05	4	81.2989	5	5453.7614	4	0.0388	3	56	3	4.0	1.6	40105
	SEP 9	6900.5445	5	0.007264	7	482.46	5	80.1617	13	259.4891	9	44.30	5	233.2764	7	5454.0432	6	0.0509	5	50	5	3.8	2.0	40108
	SEP 12	6900.2949	6	0.007087	8	480.79	5	80.1637	11	255.6141	11	35.62	6	26.1639	8	5454.3392	7	0.0538	6	44	3	3.7	2.2	40111
	SEP 15	6899.9888	5	0.006916	7	479.27	5	80.1611	12	251.7396	12	26.84	7	179.9848	9	5454.7023	6	0.0614	6	35	4	3.6	2.1	40114
	SEP 18	6899.7099	7	0.006712	11	478.52	8	80.1593	13	247.8628	14	18.17	10	334.9123	13	5455.0331	8	0.0492	9	35	5	3.6	2.5	40117
	SEP 21	6899.4432	4	0.006527	8	478.23	5	80.1593	9	243.9842	9	8.91	5	130.7726	8	5455.3696	5	0.0563	5	46	3	3.7	2.0	40120
	SEP 24	6899.1494	4	0.006293	7	479.14	5	80.1621	11	240.1058	7	359.46	4	287.6238	6	5455.6983	5	0.0598	4	49	2	3.8	1.7	40123
	SEP 27	6898.8431	4	0.006070	7	480.99	5	80.1637	12	236.2269	7	349.20	5	85.5477	7	5456.0617	5	0.0649	4	52	5	3.7	1.7	40126
	SEP 30	6898.5021	6	0.005836	9	483.97	6	80.1651	15	232.3490	12	338.68	8	244.6195	9	5456.4665	7	0.0705	6	48	3	3.8	2.6	40129
	OCT 3	6898.1135	6	0.005656	10	487.47	7	80.1627	13	228.4721	12	327.96	8	44.9762	9	5456.9277	7	0.0802	7	42	2	3.7	2.6	40132
	OCT 6	6897.7831	5	0.005460	9	491.69	6	80.1629	11	224.5910	11	317.13	8	206.6828	10	5457.3200	6	0.0576	7	37	1	3.7	2.3	40135
	OCT 9	6897.4690	5	0.005303	9	495.81	6	80.1628	10	220.7095	11	306.12	9	9.4797	9	5457.4029	6	0.0624	6	41	3	3.7	2.2	40138
	OCT 12	6897.1632	7	0.005183	8	499.67	6	80.1634	16	216.8296	14	294.34	13	173.3799	12	5458.0561	8	0.0678	7	52	2	3.9	3.0	40141
	OCT 15	6896.8286	6	0.005114	9	501.82	6	80.1628	18	212.9478	13	282.18	13	338.4629	11	5458.4534	7	0.0539	6	44	3	3.9	2.8	40144
	OCT 18	6896.5598	4	0.005125	5	502.27	4	80.1681	11	209.0681	8	270.07	8	146.5825	7	5458.7729	5	0.0561	5	40	4	3.9	1.9	40147
	OCT 21	6896.2740	6	0.005127	8	501.23	6	80.1670	12	205.1853	13	258.17	10	311.6912	9	5459.1123	7	0.0558	6	40	2	3.8	2.3	40150
	OCT 24	6896.0038	13	0.005139	18	498.74	12	80.1633	27	201.3062	29	246.21	29	119.7787	23	5459.4332	16	0.0620	15	30	0	3.8	4.7	40153
	OCT 27	6895.6671	7	0.005219	10	494.79	7	80.1643	16	197.4232	15	234.52	13	288.9530	12	5459.8[illegible]32	8	0.0651	7	41	1	3.9	2.7	40156
	OCT 30	6895.2881	5	0.005353	10	489.96	7	80.1634	15	193.5377	9	222.93	8	99.3534	9	5460.2835	4	0.0896	5	43	3	3.8	2.3	40159
	NOV 2	6894.7621	7	0.005512	11	485.12	8	80.1677	17	189.6515	12	212.05	9	271.3615	10	5460.9089	[illegible]	0.1144	8	44	4	3.7	2.8	40162
	NOV 5	6894.3693	6	0.005676	10	481.04	7	80.1669	15	185.7665	13	201.63	9	88.1874	11	5461.3756	[illegible]	0.0530	7	41	0	3.7	2.8	40165
	NOV 8	6894.1241	6	0.005829	9	477.99	6	80.1614	11	181.8826	13	191.12	8	260.0250	10	5461.6670	7	.0520	7	31	0	3.7	2.1	40168
	NOV 11	6893.8485	5	0.006028	9	475.70	6	80.1615	11	177.9947	11	180.89	6	75.7875	9	5461.9947	6	.0533	6	42	1	3.7	2.1	40171
	NOV 14	6893.5965	4	0.006217	8	474.54	6	80.1604	12	174.1077	8	171.37	6	252.5015	8	5462.2043	5	0.0483	5	45	0	3.9	2.1	40174
	NOV 17	6893.3393	6	0.006397	10	474.26	7	80.1629	16	170.2159	12	162.42	8	70.0896	10	5462.6003	7	0.0587	7	45	2	3.8	2.7	40177
	NOV 20	6893.0485	5	0.006560	7	474.81	5	80.1633	11	166.3274	11	153.25	7	248.7003	9	5462.9461	6	0.0501	5	33	2	3.9	2.0	40180
	NOV 23	6892.8017	4	0.006700	9	476.09	6	80.1616	11	162.4388	12	144.09	5	68.2564	8	5463.2395	5	0.0452	5	29	0	3.7	1.7	40183
	NOV 26	6892.5514	6	0.006848	9	477.55	6	80.1599	12	158.5515	11	135.12	8	248.6585	9	5463.5372	7	0.0561	5	34	1	4.0	2.2	40186
	NOV 29	6892.2691	6	0.006962	6	479.05	4	80.1579	13	154.6575	8	126.64	7	70.0409	7	5463.8730	7	0.0514	7	42	1	3.6	2.1	40189
	DEC 2	6892.0138	4	0.007028	5	480.65	4	80.1622	10	150.7666	9	118.36	5	252.3680	7	5464.1769	5	0.0541	5	42	2	3.8	1.8	40192
	DEC 5	6891.7244	5	0.007098	7	481.82	5	80.1608	12	146.8746	11	109.80	6	75.6673	8	5464.5211	5	0.0571	5	32	0	3.8	1.9	40195
	DEC 8	6891.4845	6	0.007163	9	482.47	6	80.1587	13	142.9844	12	101.53	7	259.9584	9	5464.8065	7	0.0403	6	36	0	3.8	2.2	40198
	DEC 11	6891.2708	6	0.007182	8	482.77	6	80.1563	17	139.0901	9	93.25	9	85.0047	9	5465.060[illegible]	7	0.0450	5	25	0	3.8	1.7	40201
	DEC 14	6891.0563	8	0.007135	11	482.80	7	80.1623	18	135.1948	12	84.79	8	270.8492	12	5465.3162	10	0.0351	9	32	3	3.5	2.2	40204
	DEC 17	6890.8845	6	0.007096	9	482.09	6	80.1657	14	131.2986	14	76.53	8	97.[illegible]585	9	5465.5208	7	0.0345	6	26	0	3.7	2.1	40207
	DEC 20	6890.6933	11	0.007070	66	480.60	45	80.1524	37	127.4235	46	68.23	18	284.4834	61	5465.7480	13	0.0433	11	18	0	3.2	2.6	40210
	DEC 23	6890.4331	4	0.006964	37	479.06	25	80.1572	16	123.5193	11	59.90	12	112.4083	38	5466.0378	7	0.0585	7	22	1	3.4	1.6	40213

Table 2(cont'd.) Orbital parameters of Ariel 3

DATE	a	e	h_p	i	Ω	ω	$M_0 + \omega$	M_1	M_2	N	K	D	Ɛ	MJD
1968 DEC 26	6890.1374 9	0.006742 43	477.68 30	80.1606 25	119.6199 16	50.77 20	301.3706 44	5466.6099 11	0.0563 11	23	0	3.3	2.6	60216
DEC 29	6889.8541 8	0.006608 46	475.62 32	80.1584 15	115.7192 16	41.95 27	131.3575 46	5466.7475 10	0.0561 9	15	0	3.2	1.9	60219
1969 JAN 1	6889.5906 10	0.006590 58	473.01 40	80.1556 37	111.8412 48	33.63 41	322.3101 62	5467.0611 12	0.0474 12	16	1	3.2	2.6	60222
JAN 4	6889.3649 7	0.006404 26	471.89 18	80.1579 22	107.9390 11	25.39 31	156.1453 42	5467.3297 9	0.0458 7	21	0	3.2	1.6	60225
JAN 7	6889.1349 9	0.006202 26	471.07 18	80.1653 19	104.0360 13	15.46 42	346.7988 47	5467.6039 10	0.0462 10	20	1	3.4	2.3	60228
JAN 10	6888.9023 7	0.006004 17	471.11 12	80.1673 15	100.1342 16	5.63 45	18[illegible].2897 42	5467.8810 8	0.0385 9	18	0	3.2	1.8	60231
JAN 13	6888.7267 8	0.005855 19	471.85 13	80.1661 26	96.2368 41	356.86 51	14.4846 49	5468.0901 9	0.0368 8	18	0	3.5	2.1	60234
JAN 16	6888.5214 7	0.005631 11	474.06 7	80.1603 15	92.3459 9	346.76 33	209.3294 34	5468.3344 8	0.0505 8	21	1	3.0	1.4	60237
JAN 19	6888.2698 5	0.005452 9	476.92 6	80.1619 14	88.4418 9	336.51 20	45.0316 19	5468.6342 6	0.0458 6	34	0	3.5	1.8	60240
JAN 22	6888.0800 4	0.005262 7	480.73 5	80.1662 9	84.5396 8	325.62 6	241.5445 7	5468.8605 5	0.0308 5	27	0	3.5	1.4	60243
JAN 25	6887.9095 8	0.005144 17	484.74 12	80.1601 26	80.6454 30	314.89 11	78.6350 14	5469.0634 10	0.0464 9	26	1	3.4	2.5	60246
JAN 28	6887.6762 4	0.004972 7	489.13 5	80.1601 11	76.7421 6	303.42 7	276.4896 7	5469.3414 4	0.0420 4	31	1	3.6	1.3	60249
JAN 31	6887.4917 4	0.004898 6	492.44 4	80.1630 12	72.8382 7	291.51 8	115.1033 6	5469.5614 5	0.0338 5	38	0	3.5	1.5	60252
FEB 3	6887.2796 9	0.004820 14	494.68 10	80.1647 27	68.9338 22	279.26 18	314.3486 14	5469.8142 11	0.0587 10	27	1	3.9	3.1	60255
FEB 6	6887.0068 6	0.004783 9	495.07 6	80.1545 20	65.0393 18	267.28 15	154.3403 9	5470.1413 7	0.0498 7	25	0	3.2	1.8	60258
FEB 9	6886.7448 3	0.004800 4	493.58 3	80.1603 10	61.1309 5	255.14 7	355.4619 5	5470.4515 4	0.0511 3	44	1	3.9	1.4	60261
FEB 12	6886.4480 7	0.004898 9	490.21 6	80.1623 14	57.2259 12	243.30 10	197.7323 9	5470.8053 8	0.0710 9	46	3	3.7	2.6	60264
FEB 15	6886.2096 4	0.004973 5	486.14 3	80.1599 8	53.3220 7	231.36 7	40.8675 6	5471.0895 5	0.0374 4	42	2	3.8	1.6	60267
FEB 18	6886.0037 6	0.005094 11	481.54 8	80.1606 17	49.4164 12	219.73 11	244.7257 11	5471.3350 7	0.0416 7	35	1	3.7	2.6	60270
FEB 21	6885.7394 5	0.005277 9	476.90 6	80.1612 13	45.5080 7	208.79 5	89.3765 6	5471.6501 6	0.0650 5	39	4	3.8	1.6	60273
FEB 24	6885.3901 4	0.005471 8	472.86 6	80.1639 10	41.6038 8	198.27 6	295.1513 7	5472.0668 5	0.0758 5	31	1	3.7	1.7	60276
FEB 27	6885.0175 10	0.005462 16	469.78 11	80.1614 20	37.6972 21	188.05 23	142.2337 26	5472.5111 12	0.0797 11	26	1	3.8	3.5	60279
MAR 2	6884.5708 9	0.005846 16	467.75 11	80.1661 23	33.7915 20	177.72 24	350.7906 28	5473.0441 11	0.0832 10	25	1	3.7	2.5	60282
MAR 5	6884.2249 8	0.006078 33	466.54 22	80.1635 23	29.8816 21	168.13 50	200.8176 54	5473.4567 9	0.0616 10	23	1	3.7	2.7	60285
MAR 8	6883.8985 7	0.006259 24	466.51 16	80.1620 12	25.9763 11	158.86 29	51.9716 34	5473.8461 8	0.0702 7	22	1	3.6	2.1	60288
MAR 11	6883.5562 8	0.006429 41	467.13 28	80.1648 21	22.0688 18	149.96 32	264.3410 53	5474.2547 9	0.0699 9	29	0	3.7	3.1	60291
MAR 14	6883.2018 7	0.006626 47	467.91 32	80.1645 19	18.1565 15	141.26 25	117.9756 48	5474.6777 8	0.0661 8	27	2	3.8	2.4	60294
MAR 17	6882.8018 10	0.006791 50	469.04 34	80.1637 19	14.2503 18	132.56 23	338.8772 49	5475.1551 12	0.0937 10	23	0	3.6	3.1	60297
MAR 20	6882.2606 13	0.006876 72	470.35 49	80.1664 32	10.3406 31	124.43 26	189.4465 74	5475.8013 15	0.1264 14	26	0	3.7	5.0	60300
MAR 23	6881.5632 15	0.006826 66	472.25 45	80.1682 43	6.4266 32	116.00 22	48.2578 64	5476.6342 18	0.1478 16	26	1	3.9	5.1	60303
MAR 26	6880.8331 11	0.007002 43	471.99 29	80.1629 25	2.5158 24	108.[illegible] 15	269.7521 38	5477.5061 13	0.1131 13	23	1	3.6	4.0	60306
MAR 29	6880.3043 9	0.007105 49	471.91 34	80.1670 23	358.6041 17	100.[illegible] 13	133.4476 48	5478.1379 11	0.1083 11	22	0	3.4	2.8	60309
APR 1	6879.7769 8	0.007015 22	472.55 15	80.1668 19	354.6855 15	92.[illegible] 15	359.0223 23	5478.7682 9	0.0994 10	23	0	3.2	2.3	60312
APR 4	6879.2599 9	0.006955 24	472.26 17	80.1622 25	350.7747 21	83.80 21	226.4504 27	5479.3859 11	0.0983 10	24	0	3.6	3.4	60315
APR 7	6878.8026 6	0.006921 13	471.21 9	80.1640 14	346.8569 15	75.96 16	95.6466 20	5479.9326 7	0.0896 7	26	0	3.6	2.4	60318
APR 10	6878.3687 7	0.006795 10	470.10 7	80.1660 17	342.9326 16	67.49 15	326.4219 15	5480.4516 9	0.0845 7	29	1	3.6	2.0	60321
APR 13	6877.9565 10	0.006681 12	468.48 8	80.1627 15	339.0175 16	59.40 24	198.7106 23	5480.9465 12	0.0863 9	26	1	3.6	3.0	60324
APR 16	6877.4873 5	0.006556 7	466.49 5	80.1630 15	335.0973 11	51.05 13	72.5388 15	5481.5056 6	0.0997 6	29	1	3.7	1.9	60327
APR 19	6877.0112 7	0.006389 10	464.46 7	80.1611 19	331.1758 20	42.28 20	308.1365 16	5482.0748 9	0.0805 10	30	2	3.6	3.0	60330
APR 22	6876.6464 5	0.006195 11	462.80 7	80.1607 15	327.2526 14	33.40 18	185.2050 27	5482.5111 6	0.0749 9	21	2	3.7	2.4	60333

Table 2(cont'd.) Orbital parameters of Ariel 3

DATE	a		e		h_p		i		Ω		ω		$M_o + \omega$		M_1		M_2		N	K	D	E	MJD
1969 APR 25	6876.2819	5	0.006014	9	461.36	6	80.1628	12	323.3265	10	24.47	9	63.5871	9	5482.9473	6	0.0685	5	44	8	3.8	2.1	40336
APR 28	6875.9602	5	0.005825	8	460.48	6	80.1614	12	319.4033	11	14.93	9	303.1928	9	5483.3322	6	0.0660	5	35	6	3.9	1.9	40339
MAY 1	6875.6463	5	0.005616	7	460.62	5	80.1590	9	315.4796	7	5.86	5	183.9548	6	5483.7078	6	0.0544	4	44	7	3.8	1.6	40342
MAY 4	6875.3863	4	0.005412	7	461.68	4	80.1623	10	311.5497	7	355.75	5	65.7378	6	5484.0191	4	0.0467	4	45	4	3.6	1.6	40345
MAY 7	6875.1606	7	0.005219	12	463.77	8	80.1603	15	307.6229	14	345.59	10	308.3815	12	5484.2892	9	0.0452	6	35	5	3.8	2.6	40348
MAY 10	6874.9046	4	0.005047	6	466.70	4	80.1612	11	303.6977	7	335.24	7	191.8551	6	5484.5957	5	0.0574	4	43	7	3.6	1.6	40351
MAY 13	6874.5943	10	0.004851	14	470.78	10	80.1636	22	299.7671	23	323.86	24	76.3403	19	5484.9672	12	0.0728	11	26	3	3.7	3.1	40354
MAY 16	6874.1685	11	0.004711	16	474.81	11	80.1597	21	295.8419	21	312.39	32	322.1446	25	5485.4770	13	0.0837	11	32	1	3.7	4.2	40357
MAY 19	6873.8195	7	0.004600	7	478.54	5	80.1605	14	291.9102	11	301.09	16	209.4250	11	5485.8949	8	0.0618	8	31	0	3.7	1.8	40360
MAY 22	6873.5133	10	0.004528	9	481.52	6	80.1615	11	287.9846	15	289.23	18	97.8301	13	5486.2617	12	0.0597	10	26	1	3.9	1.9	40363
MAY 25	6873.2236	4	0.004478	8	483.24	5	80.1597	9	284.0519	8	276.76	13	347.3093	10	5486.6087	5	0.0560	5	32	0	3.6	1.6	40366
MAY 28	6872.9662	6	0.004494	11	482.96	7	80.1627	10	280.1204	10	264.41	11	237.7840	10	5486.9171	7	0.0485	6	27	0	3.6	1.6	40369
MAY 31	6872.7293	5	0.004512	11	481.16	7	80.1617	9	276.1912	10	252.43	11	129.1257	10	5487.2009	5	0.0471	5	28	1	3.7	1.6	40372
JUN 3	6872.5101	5	0.004593	12	477.70	8	80.1615	12	272.2571	9	240.47	8	21.2968	10	5487.4636	6	0.0424	6	33	2	3.6	1.8	40375
JUN 6	6872.2487	6	0.004680	16	473.52	11	80.1618	10	268.3288	13	228.95	15	274.2790	17	5487.7769	7	0.0604	6	24	1	3.6	1.8	40378
JUN 9	6871.8698	6	0.004795	15	468.85	11	80.1593	14	264.3932	9	217.45	9	168.3830	13	5488.2308	7	0.0889	6	28	2	3.6	1.8	40381
JUN 12	6871.4237	6	0.004971	16	464.24	11	80.1635	11	260.4588	14	206.72	11	64.0384	14	5488.7656	8	0.0900	8	27	2	3.3	2.2	40384
JUN 15	6870.9359	5	0.005120	14	460.51	10	80.1613	11	256.5276	9	196.06	9	321.3446	12	5489.3503	7	0.1005	5	33	4	3.9	2.0	40387
JUN 18	6870.4774	13	0.005321	23	457.54	16	80.1632	26	252.5907	24	186.39	24	220.4339	23	5489.9001	15	0.0770	13	21	1	3.5	3.4	40390
JUN 21	6870.1597	6	0.005473	15	456.04	11	80.1597	12	248.6569	15	176.15	15	120.8883	18	5490.2810	7	0.0588	7	22	0	3.7	2.3	40393
JUN 24	6869.9059	6	0.005645	10	455.47	7	80.1595	13	244.7181	11	166.67	14	22.3648	12	5490.5854	8	0.0477	9	33	3	3.3	2.1	40396
JUN 27	6869.6849	6	0.005835	12	455.58	8	80.1623	12	240.7828	14	157.56	14	284.6949	15	5490.8505	7	0.0375	7	24	1	3.6	2.2	40399
JUN 30	6869.5072	6	0.005933	8	456.63	6	80.1584	15	236.8446	10	148.51	15	187.7126	13	5491.0636	8	0.0376	8	33	3	3.6	2.0	40402
JUL 3	6869.2841	6	0.006124	8	457.98	6	80.1593	15	232.9084	20	139.79	16	91.4356	13	5491.3312	7	0.0450	6	25	1	3.6	2.0	40405
JUL 6	6869.0454	5	0.006253	6	459.47	4	80.1572	10	228.9696	8	131.26	11	355.9285	10	5491.6175	6	0.0522	6	36	1	3.5	1.9	40408
JUL 9	6868.7654	7	0.006349	8	461.03	5	80.1595	11	225.0307	14	122.80	13	261.4793	13	5491.9535	8	0.0569	8	25	1	3.3	1.9	40411
JUL 12	6868.4837	5	0.006442	6	462.27	4	80.1572	11	221.0918	8	114.44	8	167.9902	8	5492.2914	6	0.0565	6	33	1	3.7	1.8	40414
JUL 15	6868.2055	5	0.006472	8	463.39	5	80.1594	12	217.1502	11	106.46	7	75.5250	9	5492.6254	7	0.0485	7	35	1	3.8	2.3	40417
JUL 18	6868.0054	4	0.006532	6	463.82	4	80.1574	8	213.2111	8	98.37	6	343.9226	7	5492.8655	5	0.0363	4	40	1	3.7	1.9	40420
JUL 21	6867.8334	5	0.006515	5	464.13	4	80.1588	10	209.2683	8	90.44	6	252.9781	6	5493.0720	5	0.0326	5	45	6	3.2	1.8	40423
JUL 24	6867.6693	4	0.006514	5	463.66	3	80.1578	10	205.3303	8	82.25	6	162.6319	7	5493.2688	5	0.0332	5	38	2	3.6	1.7	40426
JUL 27	6867.4870	5	0.006444	6	462.98	4	80.1560	12	201.3852	8	74.26	7	72.8838	7	5493.4877	6	0.0438	5	42	2	3.6	2.0	40429
JUL 30	6867.2866	7	0.006365	9	461.73	6	80.1578	14	197.4469	13	66.10	12	343.8586	11	5493.7282	8	0.0398	7	30	1	3.7	2.6	40432
AUG 2	6867.0288	6	0.006254	7	460.22	5	80.1590	12	193.5018	10	58.14	12	255.6002	11	5494.0378	7	0.0620	6	31	3	3.7	1.9	40435
AUG 5	6866.6660	6	0.006114	9	458.40	6	80.1591	12	189.5630	13	49.86	17	168.4558	14	5494.4734	7	0.0798	6	21	1	3.7	2.2	40438
AUG 8	6866.2557	5	0.005933	7	456.60	5	80.1606	12	185.6171	10	41.28	14	82.7318	13	5494.9661	6	0.0857	5	31	0	3.7	1.9	40441
AUG 11	6865.8483	6	0.005744	8	454.97	5	80.1594	11	181.6753	11	32.73	15	358.5244	12	5495.4555	8	0.0716	7	31	0	3.7	2.4	40444
AUG 14	6865.4953	7	0.005556	13	453.71	9	80.1603	14	177.7275	16	24.16	20	275.6700	17	5495.8795	8	0.0639	8	27	1	3.7	2.5	40447
AUG 17	6865.2333	5	0.005317	7	453.29	5	80.1625	11	173.7831	9	14.83	18	193.9412	14	5496.1943	6	0.0467	6	27	0	3.6	1.8	40450
AUG 20	6865.0070	10	0.005100	28	453.57	19	80.1600	15	169.8373	16	5.63	26	113.0650	30	5496.4660	12	0.0453	9	15	0	3.7	2.2	40453
AUG 23	6864.7924	7	0.004881	13	454.77	9	80.1580	14	165.8943	9	356.30	46	32.9843	38	5496.7239	9	0.0463	8	21	0	3.2	1.9	40456

Table 2(concl'd.) Orbital parameters of Ariel 3

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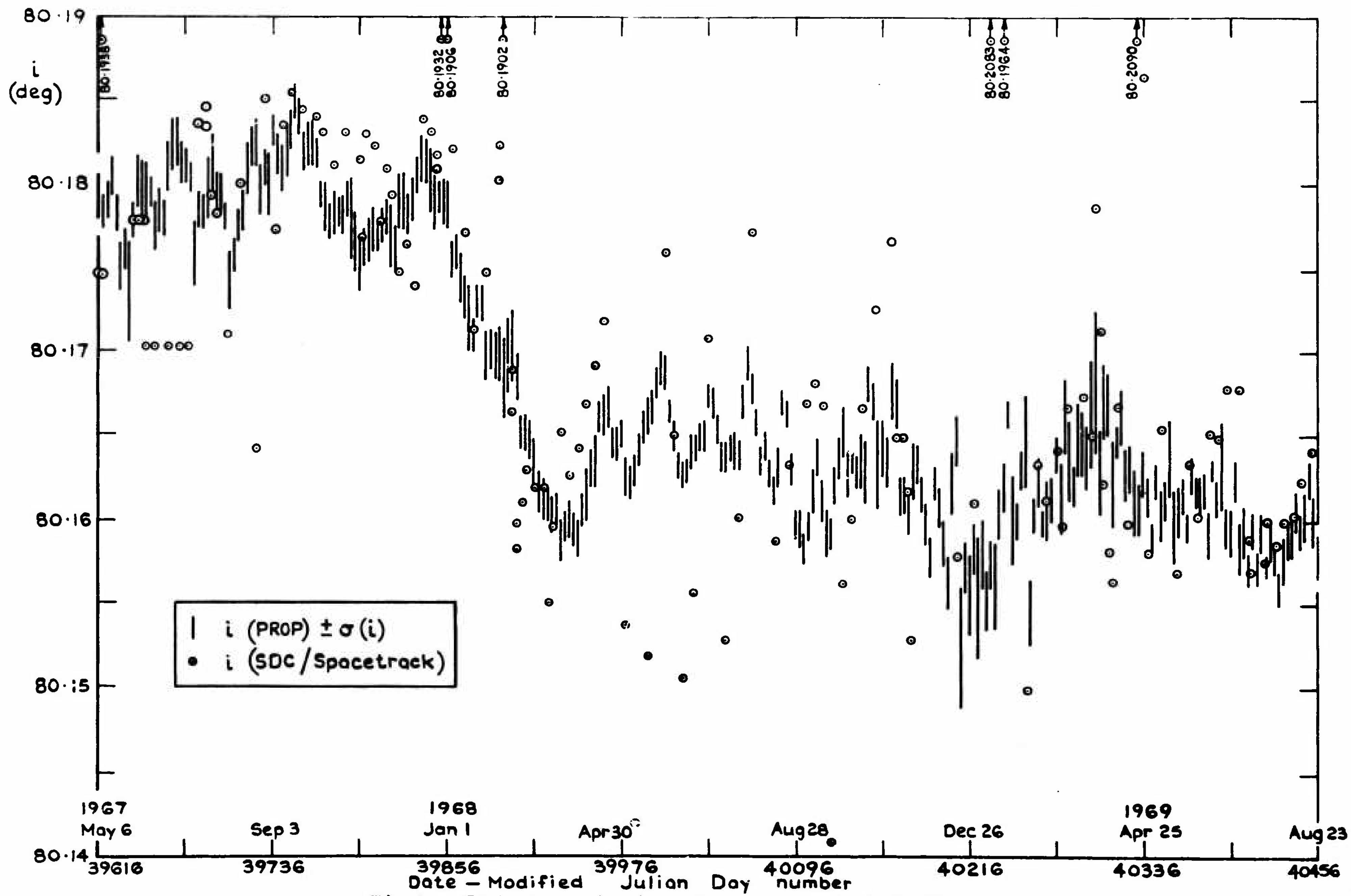

Fig.1 Orbital inclination of Ariel 3

004 902144

Fig.1

Fig. 2

004 902145

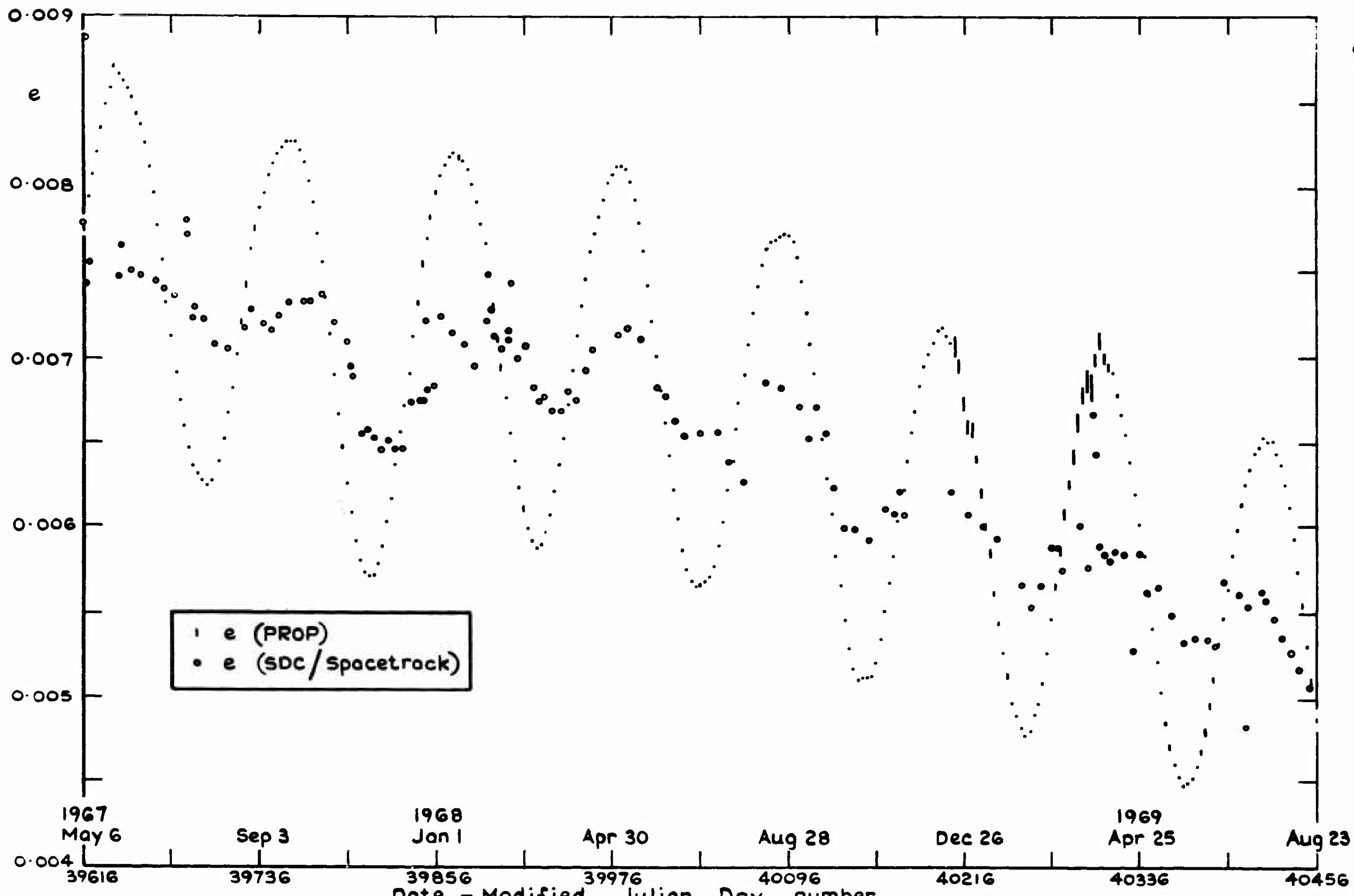

Fig.2 Orbital eccentricity of Ariel 3

Fig. 3

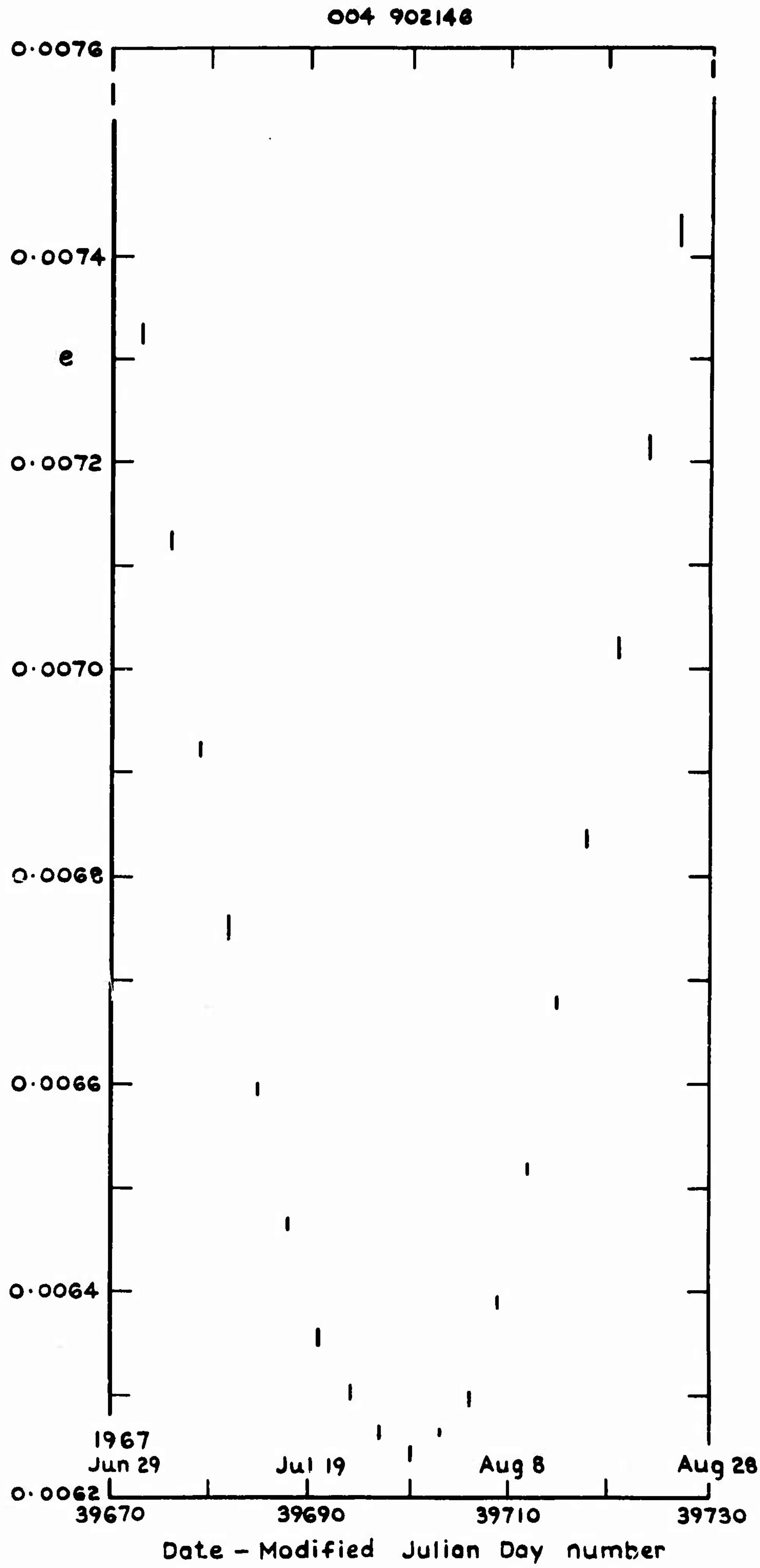

Fig. 3 Eccentricity of Ariel 3 during 60 days, on expanded scale

ROYAL AIRCRAFT ESTABLISHMENT

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by

R. H. Gooding

ADDENDUM

The striking change of i (orbital inclination), from 80.18° in December 1967 to about 80.163° in March 1968, was remarked upon in section 6, but left unexplained. It is believed that the explanation is now known. During this period Ariel 3 was passing through a resonance associated[14] with the earth's tesseral harmonics of order 15. In fact the mean motion of the satellite was exactly commensurate with the earth's rotation rate just before 0 hours on MJD 39889 (3 February 1968), and the variable $M + \omega + 15 (\Omega - n_o t)$ of Ref.14 varied by no more that 120° from its resonance value, during the three-month period. The author, in section 6, dismissed resonance, thinking of luni-solar resonance rather than tesseral-harmonic resonance, and discussed alternative explanations which must now be considered irrelevant.

ERRATA

Page 7, line 3: for the first 'M_1' read 'M_1^2'.
Page 14, line 13: for the second 'i' read 'M_1'.

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