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SYNTHESIS OF AERODYNAMIC FORCE CHARACTERISTICS
OF BISYMMETRIC LIFTING VEHICLES

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FOREWORD

The work reported here was originally done under the T-16 Subtask, Comparison of Lifting and Ballistic Reentry Spacecraft for Military Purposes. Results of the initial application of the derived expressions to evaluate glide and landing characteristics of low L/D vehicles are described in IDA Study S-112 (Aug 1963). The derived linearized expressions have been incorporated in the RANGE program and are used in all aircraft landing, lifting-reentry, and horizontal-takeoff launch vehicle trajectory calculations; the linear approximations are fundamental to all other RANGE trajectories involving lift. Subsequent IDA documents whose results make use of these routines include P-237 (Jan 1966), R-114 (Feb 1966), P-427 (May 1968), and P-425 (Jun 1969). This present document is assembled to provide background detail for these aerodynamic subroutines which are in continuing use in RANGE.

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ABSTRACT

The dependences of lift and drag coefficients on angle of attack for bisymmetric lifting vehicles are synthesized using two simplifying assumptions:

1. the axial force coefficient is independent of the angle of attack, and
2. the normal force coefficient increases linearly with the angle of attack.

Good agreement with experimental data is found for the value of the angle of attack at which the maximum L/D occurs and the angle of attack for maximum C_L in the hypersonic regime; applicability for subsonic flight is limited to those angles of attack below the inception of flow separation.

Conditions for terminal equilibrium glide and landing for aircraft are derived from the simplified aerodynamic characteristics.

The following useful rules of thumb are obtained for super and hypersonic vehicles with maximum L/D greater than 1.0:

1. the drag at maximum L/D is very closely twice the zero-lift drag,
2. the L/D at maximum C_L is about 0.8,
3. the angle of attack for maximum C_L is about 48 deg,
4. the ratio of the maximum C_L to the C_L at $L/D_{\max}$ is approximated by $0.5 + L/D_{\max}$, and
5. the ratio of the normal force slope to the maximum lift is closely 2.0 per radian.

NOMENCLATURE

A	Reference area
C_A	Axial force coefficient
C_D	Drag coefficient
C_L	Lift coefficient
C_N	Normal force coefficient
$C_{N\alpha}$	Factor relating C_N and its α dependence
C_V	Vertical force coefficient = vertical force/ $\frac{1}{2}\rho V^2 A$
D	Drag force
f	Ratio of vertical force to effective weight
F_A	Axial force
g_{flare}	Flare acceleration in g's
L	Lift force
V	Velocity
W_{eff}	Effective weight = mass times local acceleration of gravity less the centrifugal force due to velocity
α	Angle of attack
α_0	Initial angle of attack
α_1	Angle of attack at which maximum L/D occurs
α_m	Angle of attack at which maximum vertical force (or C_L) occurs
α_n	Angle of attack at which maximum or minimum C_N occurs
γ	Flight path angle with horizontal, positive upwards
γ_0	Initial or desired flight path angle
ρ	Atmospheric density

CONTENTS

I. Introduction	1
II. Basic Assumptions	3
III. Derivation of Relations	5
IV. Equilibrium Glide	11
V. Recapitulation for Linear Dependence of C_N on α	13
VI. Comparison of Linearized Aerodynamic Characteristics with Experimental Data	19
VII. Conclusions	21
References	23
Appendix A. Program to Calculate Aerodynamic Force Characteristics	24

I. INTRODUCTION

Calculation of the flight path of a point mass vehicle acting under the influence of drag and lift forces requires, in addition to the more commonly available dependences of these forces on Mach and Reynolds numbers, the dependences of the lift-drag ratio and lift or drag on the angle of attack. (With a point mass vehicle the drag and lift are assumed to act through the center of mass; the otherwise interesting moments are assumed to have been cancelled out by hypothetical control surfaces.) A description is given here of the synthesis of these aerodynamic force characteristics for a vehicle configuration simplified but sufficiently general to generate flight behavior information indicative of the performance of most real vehicles in the angle of attack range 0-20 deg for subsonic flight and 0-50 deg for hypersonic flight.

The subject vehicle is required to have two basic limitations. The first is that it should have mirror symmetry in the two perpendicular planes, vertical and horizontal, whose intersection forms the longitudinal axis of the vehicle. Its second limitation, really an extension of the symmetry limitation, is that stability and attitude control are to be supplied by control surfaces or forces giving negligible drag perturbation or mass expenditure. In other words it should be a bisymmetrical body-wing configuration with symmetry not

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only left to right but also with symmetry in the top and bottom surfaces (airfoil) and having a method of attitude control that does not disturb the symmetry. Bodies of revolution, e.g., cylindrical or conical missile bodies, are included in the family of vehicles covered by these limitations. The symmetry limitation is to assure that the force normal to the vehicle is zero for zero angle of incidence between the longitudinal axis and the flight path.

II. BASIC ASSUMPTIONS

Two assumptions about the aerodynamic force characteristics of the subject vehicle are required, from which the further necessary relations are easily derivable. The two basic assumptions for this paper are determined empirically from inspection of a wide range of experimental results. The first is that the axial force coefficient, the relative aerodynamic force in the direction of the vehicle's longitudinal axis, is a constant independent of the angle of attack. The second is that the normal force coefficient, the relative aerodynamic force perpendicular to the longitudinal axis, has a quadratic dependence on the angle of attack. In equation form these assumptions are

$$C_A = \text{constant} \quad (1)$$

$$C_N = C_{N_\alpha} \alpha(1 - \alpha/2\alpha_m) \quad (2)$$

where α_m is the angle at which a maximum or minimum value of C_N occurs.

The first assumption is good, i.e., within 20% in the first 20° of α , for thick delta wings (Ref. 1, Mach 3-6), excellent, i.e., within 10% for $\alpha < 20^\circ$, for cylindrical bodies (Ref. 2, Mach 6.86), but of varying applicability for sharp cones which show either a 20% drop in C_A (Ref. 2, Mach 2) or a 50% increase in C_A (Ref. 5, Mach 6.77) in the first 12° of α . The accuracy of the assumption becomes poorer as the Mach number decreases below 2, but remains within 50% at $\alpha < 12^\circ$ even at low subsonic speeds (Ref. 4).

The quadratic C_N dependence on α could be made to follow the data within a few percent up to $\alpha \approx 45^\circ$ throughout the whole range of Mach numbers if α_n were allowed to vary with M . Generally α_n would have a large negative value leading to a C_N vs α curve that is slightly concave upward (Refs. 1, 2, and 5). The value of α_n would become closer to zero as the Mach number increases, leading to a more nonlinear C_N vs α curve.

A linear approximation to assumption 2 ($\alpha_n = \pm \infty$) is generally acceptable (deviation less than 10%) up to $\alpha = 20^\circ$ and sometimes higher. Newtonian impact theory predicts an increase in C_p with α (Ref. 10),

$$\frac{\partial C_p}{\partial \alpha} = 3 C_L$$

that is as much as 30 to 40% greater than supported by experiment (Refs. 11 and 12) and about 50% greater than that given by applying the assumption $C_A = \text{constant}$ to the differential of equation 4 below.

III. DERIVATION OF RELATIONS

The basic relationship is the transformation from the body reference system of forces, i.e., the axial and normal forces on the vehicle, to the flight-path reference system defined by the lift force perpendicular to the flight path and the drag force parallel with the flight path. From the force diagram in Figure 1

$$C_L = C_N \cos \alpha - C_A \sin \alpha \quad (3)$$

$$C_D = C_N \sin \alpha + C_A \cos \alpha \quad (4)$$

$$\text{or } C_L/C_D = \tan (\tan^{-1} C_N/C_A - \alpha) \quad (5)$$

The reverse transformation is

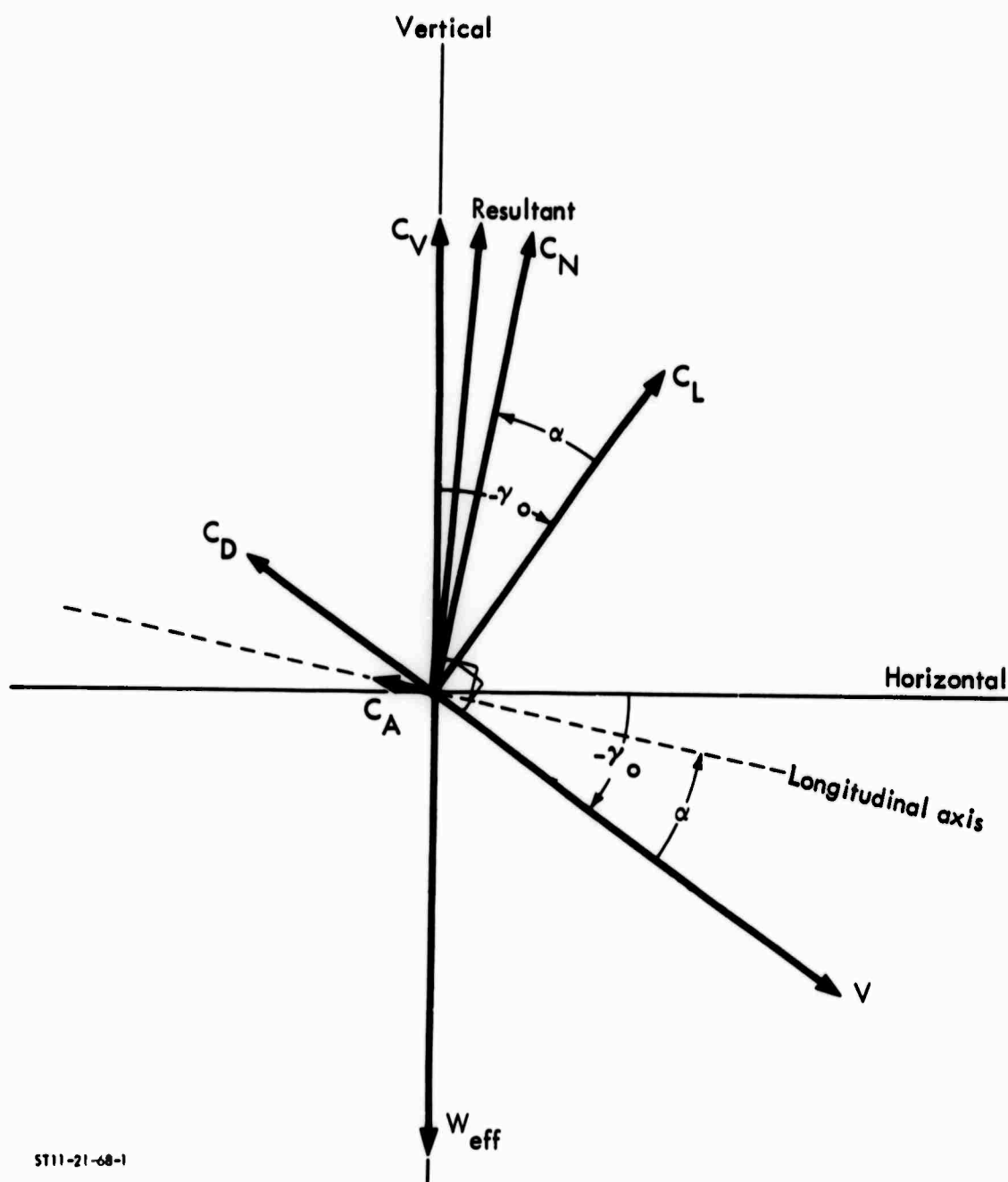
$$C_N = C_L \cos \alpha + C_D \sin \alpha \quad (6)$$

$$C_A = C_D \cos \alpha - C_L \sin \alpha \quad (7)$$

$$\text{or } C_N/C_A = \tan (\tan^{-1} C_L/C_D + \alpha) \quad (8)$$

Differentiation of equation 5 with respect to α (with substitution of equation 2) and setting the result equal to zero gives a preliminary relation for α_1 at which maximum L/D occurs

$$\alpha_1 = \frac{\sqrt{(C_{N\alpha}/C_A) (1 - \alpha_1/\alpha_m) - 1}}{(C_{N\alpha}/C_A) (1 - \alpha_1/2\alpha_m)} \quad (9)$$



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FIGURE 1. Force Diagram for a Gliding Aircraft Showing Relationship of Flight-Path-Oriented Force Coefficients, C_L and C_D , with Body-Oriented Force Coefficients, C_N and C_A , and with an Earth-Oriented Force Coefficient, C_V

or

$$(C_{N\alpha}/C_A) (1 - \alpha_1/\alpha_2) - 1 = (C_{N\alpha}/C_A)^2 \Big|_{\alpha = \alpha_1} \quad (10)$$

A most useful defining parameter of the vehicle is the maximum L/D value. Substituting equation 8 in equation 10, we obtain as an intermediate result in terms of $L/D_{\max}$

$$C_{N\alpha}/C_A = \frac{\sec^2 (\tan^{-1} L/D_{\max} + \alpha_1)}{(1 - \alpha_1/\alpha_2)} \quad (11)$$

which, when substituted in equation 9, gives

$$\alpha_1 = \frac{\tan (\tan^{-1} L/D_{\max} + \alpha_1)}{\sec^2 (\tan^{-1} L/D_{\max} + \alpha_1)} \frac{(1 - \alpha_1/\alpha_2)}{(1 - \alpha_1/2\alpha_2)} \quad (12)$$

which can be finally simplified to

$$\alpha_1 = \frac{1}{2} \sin 2 (\tan^{-1} L/D_{\max} + \alpha_1) \frac{(1 - \alpha_1/\alpha_2)}{(1 - \alpha_1/2\alpha_2)} \quad (13)$$

Using this value of α_1 and a combination of equations 2 and 8, a slightly simpler relation than equation 11 is obtained

$$C_{N\alpha}/C_A = \frac{\tan (\tan^{-1} L/D_{\max} + \alpha_1)}{\alpha_1 (1 - \alpha_1/2\alpha_2)} \quad (14)$$

Differentiating equation 3 with respect to α (with substitution of eqs. 1 & 2) and setting equal to zero gives a relation for α_2 at which maximum C_L occurs

$$\alpha_2 \tan \alpha_2 = \frac{C_{N\alpha}/C_A (1 - \alpha_2/\alpha_2) - 1}{C_{N\alpha}/C_A (1 - \alpha_2/2\alpha_2)} \quad (15)$$

For glide at an angle γ_0 negative below the horizontal, the vertical force coefficient (in an earth-oriented system rather than body-oriented or flight-path-oriented, Fig. 1) is

$$C_V = C_N \cos (\gamma_0 + \alpha) - C_A \sin (\gamma_0 + \alpha) \quad (16)$$

Maximizing this with respect to α gives an alternative equation for α_s , of which 15 is a special case for $\gamma_0 = 0$,

$$\alpha_s \tan (\gamma_0 + \alpha_s) = \frac{C_{N\alpha}/C_A (1 - \alpha_s/\alpha_n) - 1}{C_{N\alpha}/C_A (1 - \alpha_s/2\alpha_n)} \quad (17)$$

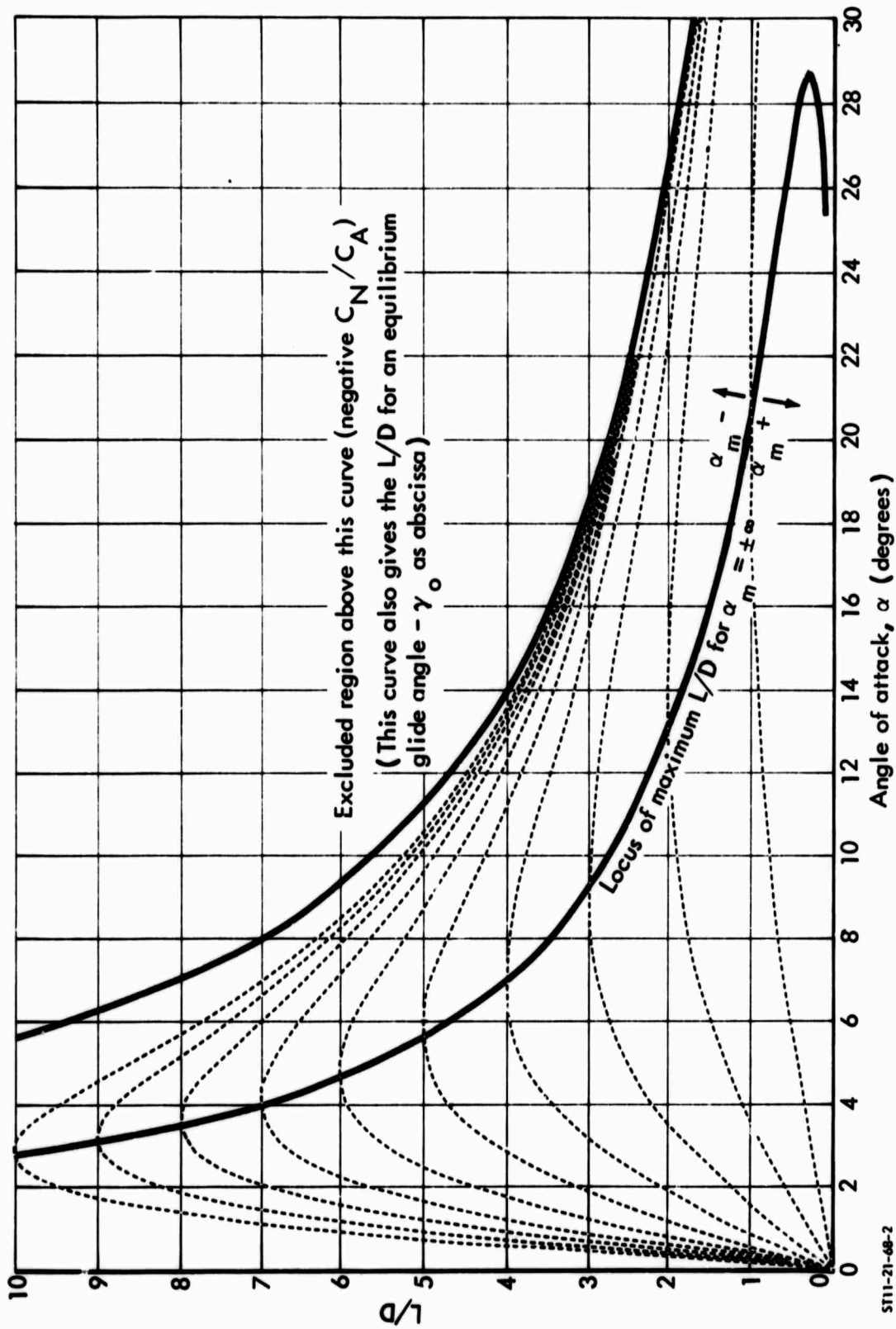
This gives the angle of attack at which the maximum vertical force is available to act against the vehicle weight.

One additional relationship is obtained from inspection of equation 8. If $\tan^{-1} (L/D + \alpha)$ is greater than $\pi/2$, then C_N/C_A is negative, which is illogical. Consequently, we have a constraint on the logical values of α for a given value of L/D ,

$$\alpha \leq \pi/2 - \tan^{-1} L/D \quad (18)$$

The region of α greater than this quantity is an excluded region.

Figure 2 shows a set of plots of relations 5, 13, and 18 describing the dependences of L/D on α for vehicles with $\alpha_n = \infty$ and different values of $L/D_{\max}$, the locus of the maxima of L/D vs α curves for vehicles with $\alpha_n = \infty$, and finally the boundary of the excluded region. Figure 3 gives the equivalent dependences to L/D on C_L from equations 3 and 15, with C_L defined in terms of the ratio to $C_{L_{\max}}$.



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FIGURE 2. Basic L/D Dependences: L/D Versus Angle of Attack for Vehicles with Different Values of $L/D_{\max}$ and $\alpha_m = \infty$; Angle of Attack for Maximum L/D Versus $L/D_{\max}$; and L/D Required for an Equilibrium Glide Angle γ_0

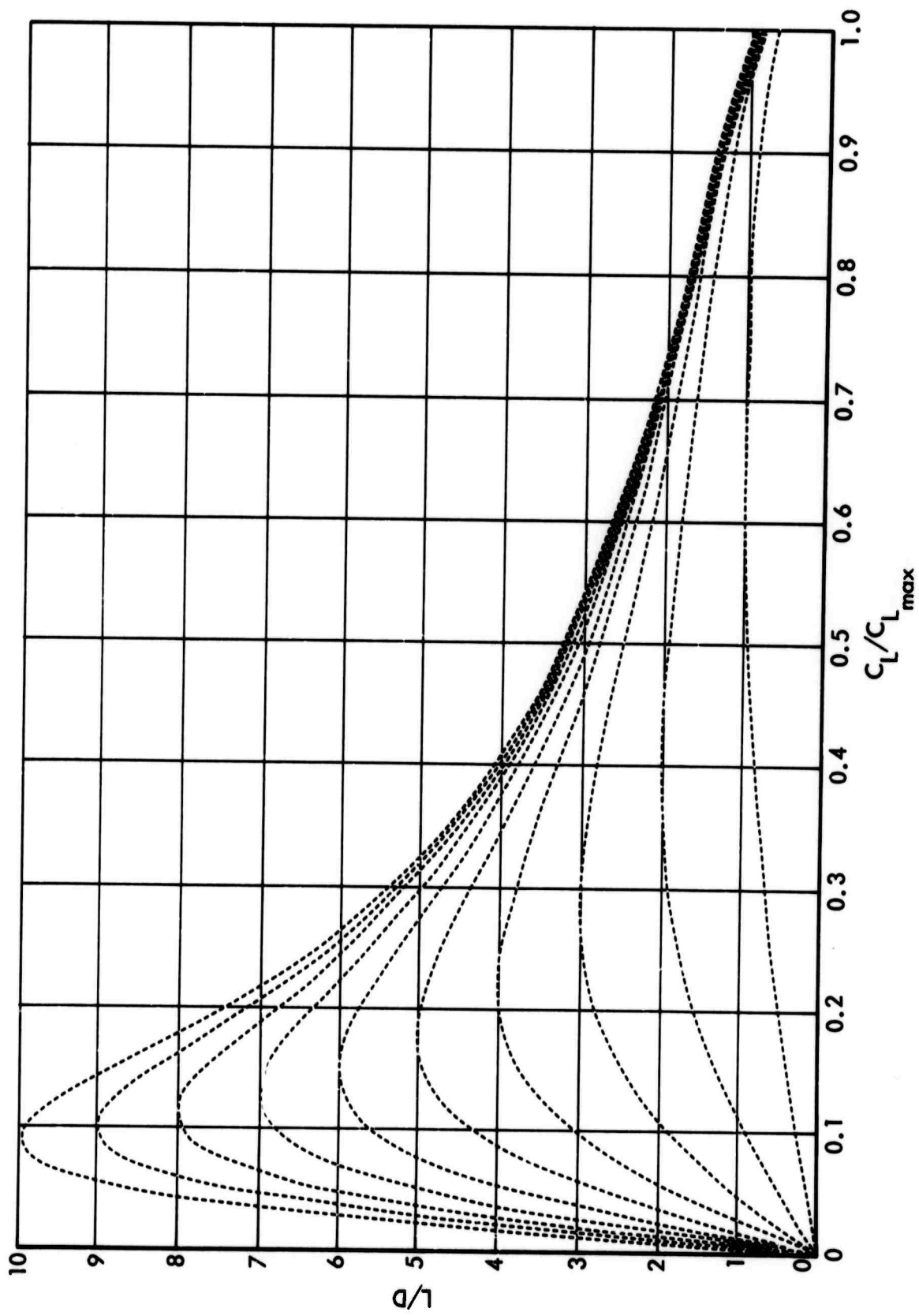


FIGURE 3. L/D Versus C_L (in Terms of $C_{L_{max}}$) for Vehicles with Different Values of L/D_{max} and $\alpha_m = \infty$

IV. EQUILIBRIUM GLIDE

A particular vehicle requires the value of one more parameter in addition to $L/D_{\max}$ to define completely its aerodynamic force characteristics for a point-mass trajectory calculation. That parameter will most likely be its axial force coefficient, i.e., its zero-lift drag coefficient, for subsonic applications or its maximum lift coefficient for hypersonic applications. For a vehicle in subsonic glide, the value of C_A can be obtained, for example, from the vehicle's terminal velocity at a constant glide angle (equilibrium glide condition), in the following manner:

For an equilibrium glide at an angle γ_0 which is measured from the horizontal and negative downward, the resultant of the aerodynamic forces in Fig. 1 is equal and directly opposite to the effective weight W_{eff} , i.e.,

$$L = W_{eff} \cos \gamma_0 \quad (19)$$

$$D = -W_{eff} \sin \gamma_0 \quad (20)$$

or
$$L/D = -\cot \gamma_0 \quad (21)$$

This relationship is the same as the equality in 18 with α replaced by $-\gamma_0$. With $L/D_{\max}$ in place of L/D , this expression also gives the minimum equilibrium glide angle. The boundary of the excluded region in Figure 1 therefore gives the L/D required for equilibrium glide at an angle γ_0 reading $-\gamma_0$ as the abscissa instead of α , or the minimum glide angle, $\gamma_{\min}$, for vehicles with different $L/D_{\max}$.

From equation 21 and equation 8 we can derive the angle of attack of the vehicle in this equilibrium glide

$$\alpha_o = -\gamma_o - \cot^{-1} \left[(C_{N_\alpha}/C_A) \alpha_o \left(1 - \frac{\alpha_o}{2\alpha_1} \right) \right] \quad (22)$$

where the solution $\alpha_o < \alpha_1$ is chosen. Finally from equation 7 and equations 19 and 20

$$\begin{aligned} C_A &= C_D \cos \alpha_o - C_L \sin \alpha_o \\ &= - \frac{W_{eff} \sin \gamma_o}{1/2 \rho V^2 A} \cos \alpha_o - \frac{W_{eff} \cos \gamma_o}{1/2 \rho V^2 A} \sin \alpha_o \\ &= \frac{W_{eff}}{1/2 \rho V^2 A} \sin(-\gamma_o - \alpha_o) \end{aligned} \quad (23)$$

If L/D_{max} and C_A are given, then the equilibrium glide velocity is found from equation 23,

$$V = \left[\frac{2}{\rho} \left(\frac{W_{eff}}{C_A A} \right) \sin(-\gamma_o - \alpha_o) \right]^{1/2}$$

V. RECAPITULATION FOR LINEAR DEPENDENCE OF C_N ON α

In subsonic glide situations, only angles of attack less than 20° are of general interest. In this region of values of α a linear dependence of C_N on α may be used with little error, giving some simplification to the equations. The simplified equations are listed below in the order of their calculation with a description of their method of solution. The corresponding FORTRAN subroutines are listed in Appendix A.

The angle of attack for $L/D_{\max}$

$$\alpha_1 = \frac{1}{2} \sin 2 (\tan^{-1} L/D_{\max} + \alpha_1) \quad (13')$$

is a transcendental equation. It is solved by iteration from a first guess that is very close

$$\tilde{\alpha}_1 = \frac{1}{2} (\pi/2 - \tan^{-1} L/D_{\max}) = -\frac{1}{2} \gamma_{\min} \quad (24)$$

Each iteration input after the first is the average of the input and output of the previous iteration.

The ratio of the normal-force-coefficient slope to the axial-force coefficient is a simple direct equation

$$C_{N\alpha}/C_A = \frac{\tan (\tan^{-1} L/D_{\max} + \alpha_1)}{\alpha_1} \quad (14')$$

The angle of attack for maximum vertical force (maximum C_L for horizontal flight)

$$\alpha_a = \frac{C_{N\alpha}/C_A - 1}{(C_{N\alpha}/C_A) \tan(\gamma_o + \alpha_a)} \quad (17')$$

is another transcendental equation to be solved by iteration. The first guess is

$$\tilde{\alpha}_a = 0.86 - 0.64 \gamma_o \quad (25)$$

Each iteration input after the first is a value only one-quarter of the way from the input to the output of the previous iteration, to guarantee convergence for an otherwise divergent process.

With the above equations as programmed in the subroutines in Appendix A, values are calculated for α_1 , α_a , $C_{N\alpha}/C_A$, C_{L_1}/C_A , $C_{L_{\max}}/C_A$, $C_{L_{\max}}/C_{D_1}$, and $\gamma_{\min}$, and are listed in Table 1 for the interesting range of values of $L/D_{\max}$. The ratios $C_{L_{\max}}/C_{L_1}$, $C_{N\alpha}/C_{L_{\max}}$, C_{D_1}/C_A , and $C_{L_{\max}}/C_{D_1}$ are plotted in Fig. 4. The values in the Figure are more limited in their range than the values in the table, and indeed the latter three of them are essentially constant for $L/D_{\max}$ values greater than 1.0.

With the linear C_N dependence, the angle of attack for equilibrium glide becomes

$$\alpha_o = -\gamma_o - \pi/2 + \tan^{-1}((C_{N\alpha}/C_A) \alpha_o) \quad (22')$$

The solution to this equation can best be shown graphically with the aid of Figure 2. Reading up from $-\gamma_o$, as the abscissa, to the boundary curve gives the required L/D . Reading across with this L/D there are in general two intersections, or none, with an L/D vs α dashed curve.

TABLE I
Aerodynamic Force Characteristics of Generalized
Vehicles with Different $L/D_{\max}$

$L/D_{\max}$	α_1	α_2	$C_{N\alpha}/C_A$	C_{L_1}/C_A	$C_{L_{\max}}/C_A$	$C_{L_{\max}}/C_D$	$-Y_{\min}$
.100	25.307	28.274	1.361	.116	.118	.090	
.200	28.101	34.108	1.675	.254	.265	.191	
.300	28.643	37.621	2.024	.409	.442	.276	
.400	28.209	40.035	2.421	.578	.652	.352	
.500	27.289	41.796	2.874	.758	.897	.418	63.43
.600	26.132	43.124	3.389	.947	1.178	.476	
.700	24.875	44.150	3.968	1.142	1.497	.526	
.800	23.602	44.955	4.616	1.342	1.857	.568	
.900	22.359	45.597	5.336	1.545	2.257	.604	
1.000	21.173	46.116	6.128	1.750	2.698	.635	45.00
1.100	20.056	46.538	6.994	1.957	3.182	.661	
1.200	19.014	46.885	7.936	2.164	3.709	.684	
1.300	18.046	47.175	8.955	2.372	4.278	.703	
1.400	17.150	47.417	10.050	2.580	4.892	.719	
1.500	16.323	47.622	11.223	2.787	5.549	.733	33.69
1.600	15.558	47.796	12.475	2.995	6.250	.746	
1.700	14.852	47.945	13.804	3.202	6.995	.756	
1.800	14.199	48.074	15.213	3.409	7.785	.766	
1.900	13.594	48.186	16.700	3.616	8.619	.774	
2.000	13.034	48.283	18.267	3.823	9.497	.781	26.55
2.100	12.513	48.369	19.912	4.029	10.420	.788	
2.200	12.029	48.444	21.637	4.235	11.387	.793	
2.300	11.579	48.511	23.441	4.440	12.399	.798	
2.400	11.158	48.570	25.325	4.645	13.456	.803	
2.500	10.765	48.623	27.289	4.850	14.557	.807	21.82
2.600	10.397	48.671	29.332	5.055	15.704	.811	
2.700	10.052	48.714	31.455	5.259	16.894	.814	
2.800	9.728	48.752	33.657	5.464	18.130	.817	
2.900	9.423	48.787	35.940	5.667	19.410	.820	
3.000	9.136	48.819	38.302	5.871	20.736	.822	18.43
3.100	8.865	48.847	40.744	6.075	22.106	.824	
3.200	8.609	48.874	43.265	6.278	23.520	.827	
3.300	8.367	48.898	45.867	6.481	24.980	.828	
3.400	8.138	48.920	48.548	6.685	26.484	.830	
3.500	7.920	48.940	51.310	6.887	28.034	.832	15.94
3.600	7.714	48.959	54.151	7.090	29.628	.833	
3.700	7.517	48.976	57.072	7.293	31.267	.835	
3.800	7.331	48.992	60.073	7.495	32.951	.836	
3.900	7.152	49.007	63.154	7.698	34.679	.837	
4.000	6.983	49.021	66.315	7.900	36.453	.838	14.04
4.500	6.239	49.077	83.319	8.911	45.993	.843	12.52
5.000	5.636	49.117	102.322	9.919	56.655	.846	11.31
5.500	5.138	49.147	123.324	10.926	68.439	.848	10.30
6.000	4.720	49.170	146.325	11.932	81.345	.850	9.46
6.500	4.365	49.188	171.326	12.937	95.373	.852	8.75
7.000	4.058	49.203	198.327	13.941	110.523	.853	8.13
7.500	3.792	49.215	227.328	14.945	126.795	.854	7.59
8.000	3.558	49.224	258.329	15.948	144.190	.855	7.13
8.500	3.351	49.232	291.329	16.951	162.706	.855	6.71
9.000	3.167	49.239	326.330	17.954	182.345	.856	6.34
9.500	3.002	49.244	363.330	18.956	203.105	.856	6.01
10.000	2.853	49.249	402.331	19.959	224.988	.857	5.70

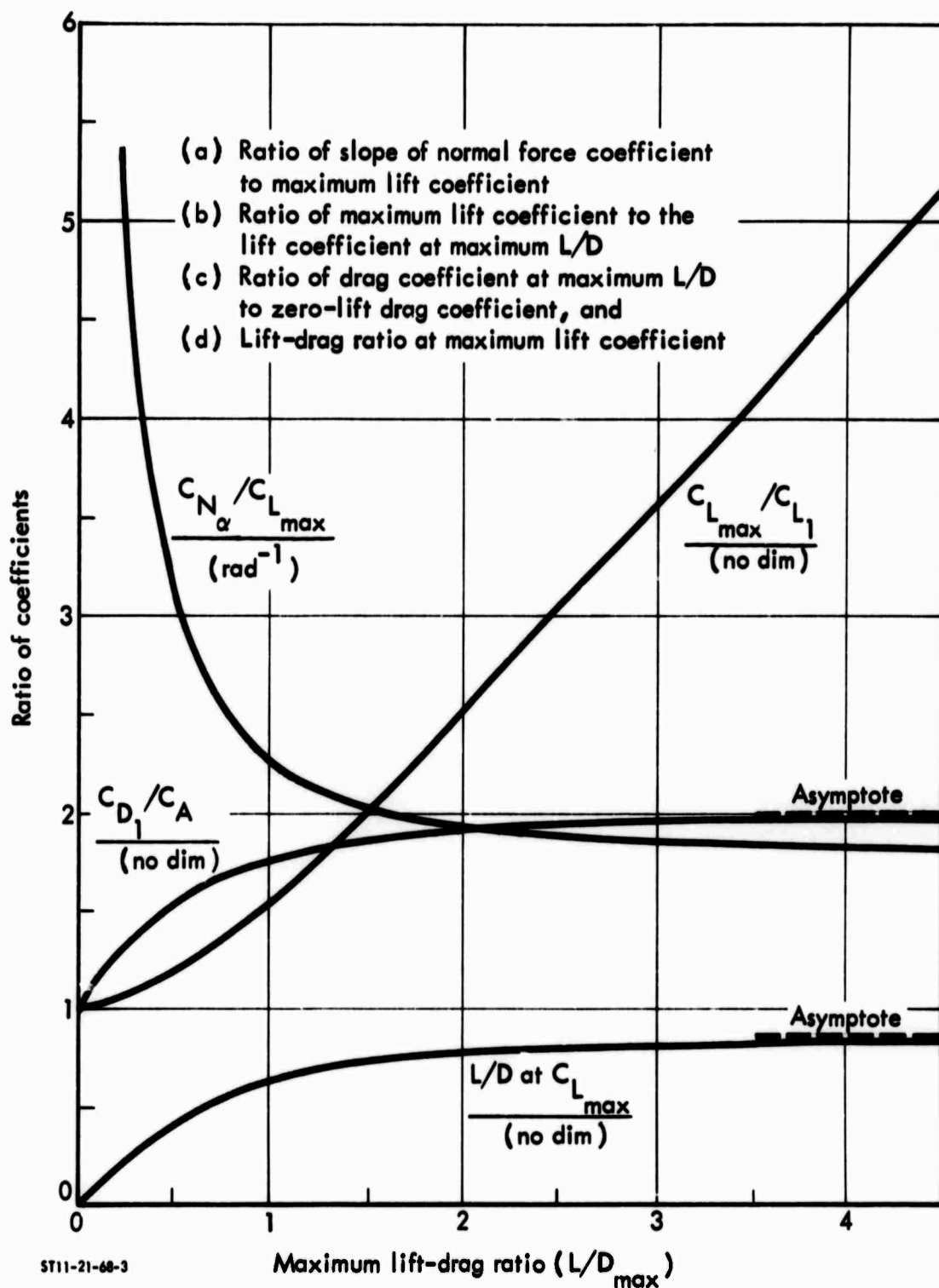


FIGURE 4. Linearized Aerodynamic Characteristics of Low L/D Aircraft

The left, lower α , intersection is of greater interest because it represents an aircraft with a greater velocity excess above stall. Unfortunately, a simple iterative process converges on the other intersection. Consequently, equation 22' is solved by trial and linear interpolation with the first two trial values $1/2 \alpha_1$ and $1/4 \alpha_1$.

The axial force coefficient, or zero-lift drag coefficient, is given by equation 23 without change. Since the atmospheric density enters in equation 23, the above set of equations for equilibrium glide will give different values at different altitudes.

In subsequent flight to an altitude with a different atmospheric density, the vehicle will require an angle of attack to maintain zero vertical acceleration (which is not necessarily the same as equilibrium glide as defined here by equations 19 and 20) given by solution of the following equation by trial and linear interpolation

$$(C_{N_Q}/C_A) \alpha F_A \cos (\gamma + \alpha) - F_A \sin (\gamma + \alpha) - W_{eff} = 0 \quad (26)$$

The first guess for α is

$$\tilde{\alpha} = W_{eff} / ((C_{N_Q}/C_A) F_A) \quad (27)$$

If a given glide slope γ_0 is to be maintained instead of zero vertical acceleration, the effective weight, W_{eff} , is replaced in equations 26 and 27 by fW_{eff} , where f has a value near 1 given by

$$f \sim 1 - (\gamma - \gamma_0) \quad (28)$$

For a flare maneuver requiring a pullup acceleration g_{flare} , the expression

$$f = 1 + g_{flare} \quad (29)$$

gives very closely the proper flare acceleration.

For horizontal flight after a flare, equation 28 is used with $\gamma_0 = 0$.

Using the above relations, the performance in the glide, flare, and landing regimes has been calculated with the IDA program RANGE for general bisymmetric lifting vehicles with different wing loading and $L/D_{\max}$. The results have been reported in Ref. 6.

VI. COMPARISON OF LINEARIZED AERODYNAMIC CHARACTERISTICS WITH EXPERIMENTAL DATA

Wind-tunnel tests of some bisymmetric thick delta-wing models at Mach 3, 4.5, and 6, are reported in Ref 1. Equation 13' predicts the angle of attack α_1 giving $L/D_{\max}$ to about 1° accuracy out of 15° . Equations 17' and 14' predict the $C_{L_{\max}}$ angle of attack α_2 about 10 percent low for the most non-linear C_N vs α curves.

The X-20 Dyna-Soar vehicle could hardly be called symmetrical above and below the horizontal plane but it is still interesting to compare the calculations (linearized equations 13', 14', and 17') with its behavior measured in wind-tunnel tests (Refs. 4 and 7). The table below gives the comparison. The angles of attack are measured above the zero-lift attitude.

	Subsonic ($L/D_{\max} = 4$)		Hypersonic ($L/D_{\max} = 2$)	
	<u>α_1</u>	<u>α_2</u>	<u>α_1</u>	<u>α_2</u>
Measured	8°	40°	12°	46°
Calculated	7°	49°	13°	48°

The relatively poor subsonic agreement in α_2 is a result of flow separation.

Lift/drag data are plotted in Fig. 5 for the above vehicles and for three others superimposed on the linearized L/D vs α curves from Fig. 2. The curves reproduce the dependences shown by these data generally within 2 deg of angle of attack.

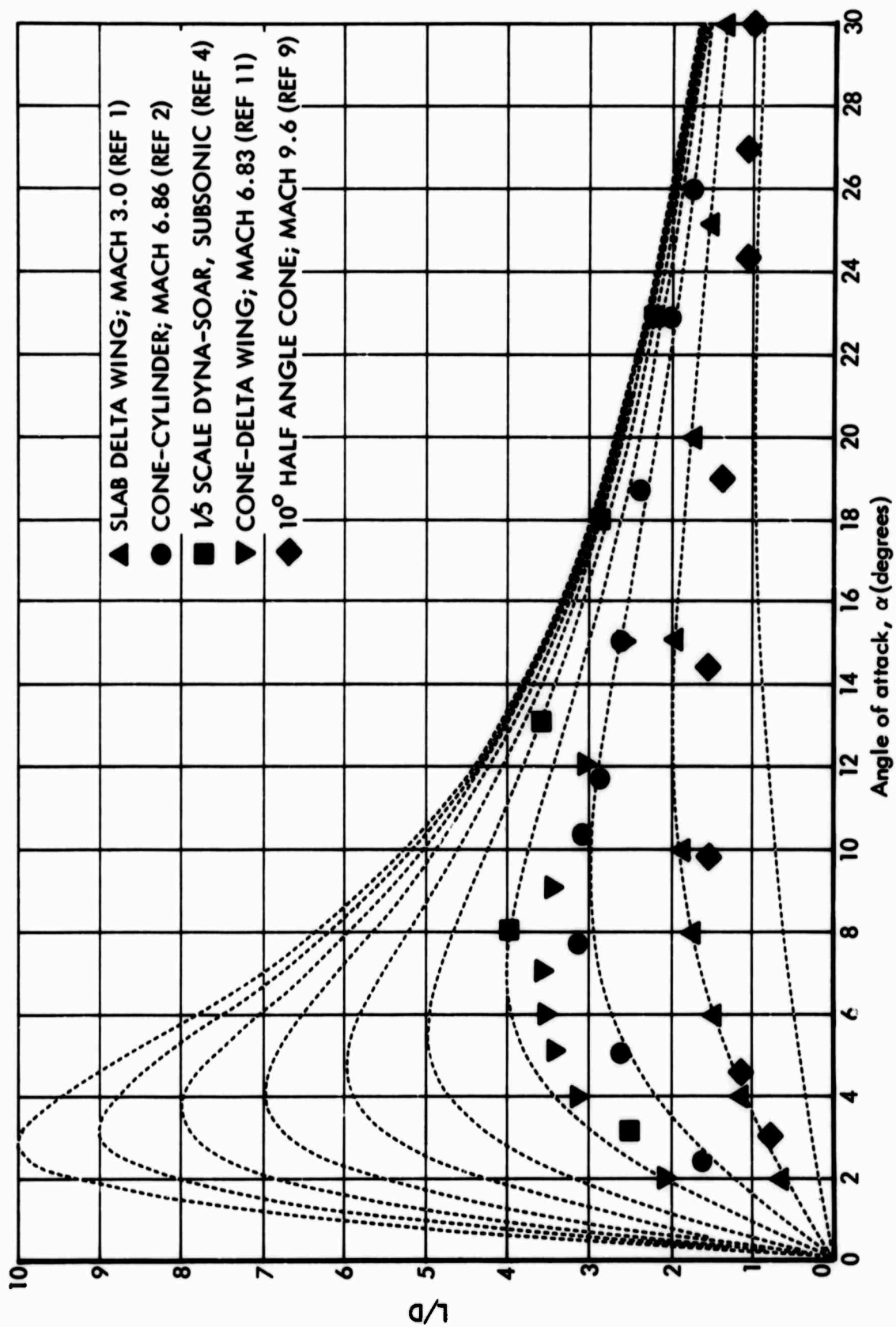


FIGURE 5. Comparison of Linearized Aerodynamic Characteristics with Experimental Data

VII. CONCLUSIONS

Useful relations for estimating the aerodynamic force characteristics of general bisymmetric lifting vehicles in all regimes of flight can be derived from the assumptions of a linear variation of the normal force coefficient with angle of attack and an independence of the axial force coefficient on the angle of attack. Good agreement with wind-tunnel measurements is observed.

Some extremely simple rules of thumb are found from the linearized aerodynamic characteristics relations. In the range of maximum L/D above 1.0 the following approximate rules hold:

1. The drag at the angle of attack for maximum L/D is very nearly twice the zero-lift drag, or

$$C_{D_1}/C_A \approx 2$$

and

$$C_{L_1}/C_A \approx 2 L/D_{\max}$$

2. The lift-drag ratio at maximum lift coefficient is about 0.8 (cf. Ref. 8), or $L/D @ C_{L_{\max}} \approx 0.8$
3. The angle of attack for maximum lift coefficient is about 48 deg, or

$$\alpha_s \approx 48 \text{ deg}$$

4. The ratio of the maximum lift coefficient to the lift coefficient at maximum L/D is given by

$$C_{L_{\max}}/C_{L_1} \approx 0.5 + L/D_{\max}$$

and from the first rule above

$$C_{L_{\max}}/C_A \approx L/D_{\max} + 2 (L/D_{\max})^2$$

5. The ratio of the normal force slope to the maximum lift decreases only from 2.25 per radian at $L/D_{\max} = 1.0$ to 1.80 per radian at $L/D_{\max} = 5.0$, so

$$C_{N\alpha}/C_{L_{\max}} \approx 2 \text{ rad}^{-1}$$

For lifting bodies, the hypersonic maximum lift coefficient is about 0.6 (Refs. 1, 7, and 8), referenced to the planform area, so

$$C_{N\alpha} \approx 1.2 \text{ rad}^{-1} \text{ for lifting bodies.}$$

For cones, the hypersonic maximum lift coefficient is about 1.0 (Refs. 2 and 5), referenced to the base area, so

$$C_{N\alpha} \approx 2.0 \text{ rad}^{-1} \text{ for cones.}$$

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APPENDIX A

Calculation of Aerodynamic Force Characteristics Using Equations 13',
14', 24', 17', and 25'; and Program to Generate Table I.

```

100  RADANG = .0174532925
110  DO 7 LDMAX = 1,40
120  ALPH1 = 0
130  CNACA = 0
140  FLDMAX = LDMAX
150  FLDMAX = 0.1 * FLDMAX
160  CALL FLT(FLDMAX, ALPH1, CNACA)
170  ALPH2 = ALP(CNACA,ZERO)
180  CL1CA = CNACA * ALPH1 * COSF(ALPH1) - SINF(ALPH1)
190  CLMCA = CNACA * ALPH2 * COSF(ALPH2) - SINF(ALPH2)
200  CLMCD = TANF(ATANF(CNACA * ALPH2) - ALPH2)
210  ALPH1D = ALPH1/RADANG
220  ALPH2D = ALPH2/RADANG
230  7 PRINT 9, FLDMAX, ALPH1D, ALPH2D, CNACA, CL1CA, CLMCA, CLMCD
240  9 FORMAT (7F10.3)
250  END

260  SUBROUTINE FLT(X,Y,C)
270  PI = 3.1415926536
280  Z = ATANF (X)
290  IF (B) 4,4,5
300  1 Y1 = 0.5 * SINF(2. * (Z + Y)) -----(13')
310  IF (ABSF(Y1 - Y) - 1.E-4) 3,3,2
320  2 Y = (Y1 + Y)/2.
330  GO TO 1
340  3 Y = (Y1 + Y)/2.
350  C = TANF(Z + Y)/Y-----(14')
360  H = Y/(PI/2. - Z)
370  RETURN
380  4 Y = (PI/2. - Z) * 0.48897-----(24')
390  GO TO 1
400  5 Y = (PI/2. - Z) * B
410  GO TO 1
420  END

430  FUNCTION ALP(X,Y)
440  PI = 3.1415926536
450  IF (A) 6, 6, 7
460  1 ZY = Z + Y
470  Z1 = (X - 1.)/(X * TAN(ZY))-----(17')
480  IF (ABSF(Z1 - Z) - 1.E - 4) 3, 3, 2
490  2 Z = Z + (Z1 - Z) * 0.25
500  GOTO 1
510  3 Z = Z + (Z1 - Z) * 0.25
520  IF (Z - PI/2.) 5, 5, 4
530  4 Z = PI/2.
540  5 ALP = Z
550  IF (ABSF(Y) - .01) 9, 9, 8
560  6 Z = 0.86 - Y * 0.64----- (25')
570  GO TO 1
580  7 Z = 0.86 - Y * A
590  GO TO 1
600  8 A = (Z - 0.86)/(-Y)
610  RETURN
620  9 A = 0.64
630  END

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13. ABSTRACT The dependences of lift and drag coefficients on angle of attack for bisymmetric lifting vehicles are synthesized using two simplifying assumptions: (1) the axial force coefficient is independent of the angle of attack, and (2) the normal force coefficient increases linearly with the angle of attack. Good agreement with experimental data is found for the value of the angle of attack at which the maximum L/D occurs and the angle of attack for maximum C_L in the hypersonic regime; applicability for subsonic flight is limited to those angles of attack below the inception of flow separation. Conditions for terminal equilibrium glide and landing for aircraft are derived from the simplified aerodynamic characteristics. With the linearized normal force, the following useful rules of thumb are obtained for super and hypersonic vehicles with maximum L/D greater than 1.0: (1) the drag at maximum L/D is very closely twice the zero-lift drag, (2) the L/D at maximum C_L is about 0.8, (3) the angle of attack for maximum C_L is about 48 deg, (4) the ratio of the maximum C_L to the C_L at L/D max is approximated by $0.5 + L/D_{max}$, and (5) the ratio of the normal force slope to the maximum lift is closely 2.0 per radian.			