AFFDL-TR-68-39
AD0708516

AN APPLICATION OF THE FINITE ELEMENT METHOD TO ELASTIC-PLASTIC PROBLEMS OF PLANE STRESS

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TECHNICAL REPORT AFFDL-TR-68-39

MAY 1970

AIR FORCE FLIGHT DYNAMICS LABORATORY
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

NOTICE

400 - June 1970 - CO455 - 122-2685

AFFDL-TR-68-39

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FOREWORD

This report was prepared by the Illinois Institute of Technology Research Institute and the Air Force Flight Dynamics Laboratory as a joint in-house effort under Project No. 1467, "Structural Analysis Methods."

The computer program presented here was developed through a number of modifications during the period January 1967 through December 1968.

This report has been reviewed and is approved.

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ABSTRACT

A computer program is presented for the small strain analysis of plane structures in the strain hardening elastic-plastic range. The finite element displacement method is used to perform the linear analyses in the iterative scheme. Bar and constant strain isotropic plane stress triangles are available for use in constructing idealizations. The use of ten different sets of material properties, three different material laws, and incremental proportional loading are available as options. Good correlation is shown with available test data and theoretical solutions.

The distribution of this abstract is unlimited.

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SECTION I

INTRODUCTION

This report describes a computer program for the stress analysis of plane structures in the elastic-plastic range by the finite element method. The program can handle bar and triangular plate elements so that it is applicable to trusses and to the analysis of in-plane stresses in reinforced plates. The material behavior is assumed to be isotropic and the user has a choice of three types of stress-strain laws and ten different materials.

A numerical step by step procedure for obtaining solutions which satisfy the requirements of the incremental theory of plasticity for materials which obey the Mises yield condition and the associated flow rule is used in the program. At each step in the solution, an iterative procedure is used to find the correct values of the strain increments. Changes in plastic strain are accounted for by the addition of fictitious plastic forces to the actual loading on the structure in such a way that the deflections of the structure under the modified loading with assumed elastic behavior are equal to the actual deflections. A modified form of the computer program given in Reference 1 is used to obtain elastic solutions.

The finite element method originally developed for elastic stress analysis was extended to apply to inelastic material behavior by Padlog (Reference 2) et al. Further studies of the use of the method in elastic-plastic problems have been made, for example, at MIT (Reference 3) and the California Institute of Technology (Reference 4).

The method has been extended to apply to anisotropic materials by Jensen (Reference 5). Marcal and King (Reference 7) have applied the method to problems of plane stress and strain and to axisymmetric problems. However, the computer programs necessary for the application of the method have not been published. The purpose of this report is to make such a program available.

The following section of the report presents a detailed explanation of the method of analysis. The application of the program is demonstrated by examples in Section III. Directions for the use of the computer program are given in Section IV. Listing of the program and sample data sets are given in the Appendix.

SECTION II

METHOD OF ANALYSIS

The analysis procedure used in the computer program is described here. The method, first used by Padlog for the solution of problems involving plastic flow and creep, is given here in a slightly modified form. The well known formulas for the stiffness of bars and triangular plate elements are first presented. Then the step by step iterative procedure used for the solution of problems in which plastic flow occurs is described.

1. ELEMENT PROPERTIES

The two types of elements to be considered in this analysis (the bar and the triangular plate) are shown in Figure 1. The coordinates of the end points of the bar and the vertices of the triangle are referred to a fixed coordinate system in the plane. The cartesian components of the nodal displacements for each of these elements comprise the element displacement vector X. The ordering of the displacement components is shown in Figure 1. The total element strains designated by the vector ϵ can be expressed in terms of the nodal displacements by an equation of the form

$$\epsilon = BX \tag{1}$$

The stresses are related to the elastic strains $\epsilon^e = \epsilon - \epsilon^p$ by Hooke's law

$$\sigma = C\epsilon^e \tag{2}$$

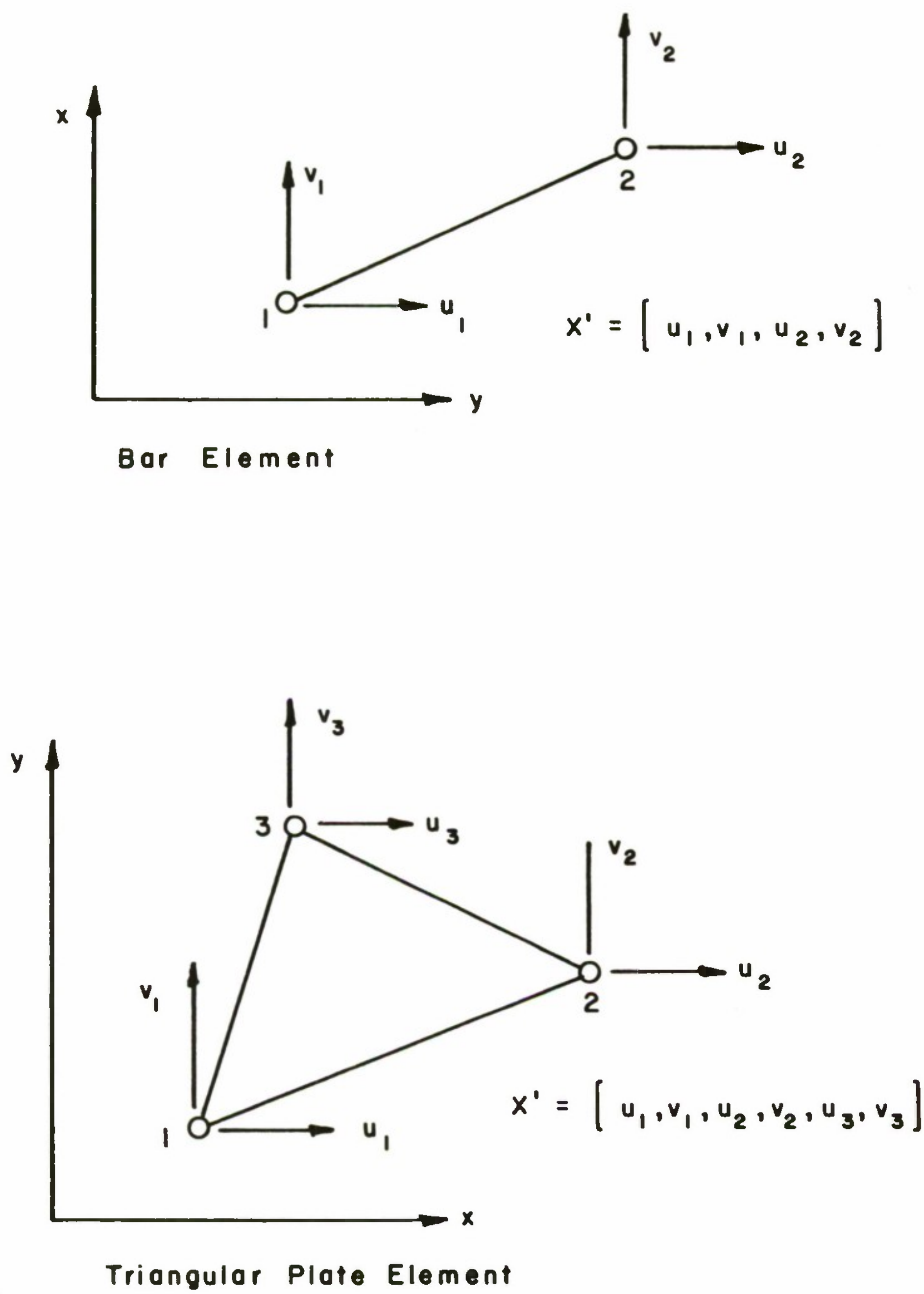


Figure 1. Bar and Plate Elements

The nodal forces F corresponding to given displacements X are found by the principle of virtual work. That is

$$\overline{X}'F = \int_V \overline{\epsilon}'\sigma \, dV \tag{3}$$

where $\overline{X}$ and $\overline{\epsilon}$ are the virtual displacement vector and the corresponding strain vector, respectively, and the integration is carried out over the volume of the element. Prime superscripts denote transposed matrices. Using Equations 1 and 2 in Equation 3 gives the results

$$\overline{X}'\left[F - \int_V B'CB\,dV\,X + \int_V B'C\,dV\,\epsilon^p\right] = 0 \tag{4}$$

Or, since the elements of $\overline{X}$ are arbitrary, then

$$F + F^p = kX \tag{5}$$

where

$$k = \int_V B'CB\,dV \tag{6}$$

is the element stiffness matrix and

$$F^p = D\epsilon^p \tag{7}$$

is the vector of plastic forces in which

$$D = \int_V B'C\,dV \tag{8}$$

The definitions of the nodal force and displacement vectors and of the matrices C, B, D, and k for bars and triangular plate elements are:

<u>Bar Element</u>

$$X' = \left[u_1, v_1, u_2, v_2\right] \tag{9}$$

$$F' = \left[F_{x1}, F_{y1}, F_{x2}, F_{y2}\right] \tag{10}$$

$$B = \frac{1}{L^2}\left[-x_{21}, -y_{21}, x_{21}, y_{21}\right] \tag{11}$$

in which L is the length of the bar and

$$x_{21} = x_2 - x_1 , \quad y_{21} = y_2 - y_1 \quad \text{etc} \tag{12}$$

$$C = E \text{ -- Young's modulus} \tag{13}$$

$$D' = \frac{AE}{L} \left[-x_{21} , -y_{21} , x_{21} , y_{21} \right] \tag{14}$$

where A is the cross-sectional area of the bar

$$k = \frac{AE}{L^3} \begin{bmatrix} k_{11} & -k_{11} \\ -k_{11} & k_{11} \end{bmatrix} \tag{15}$$

in which

$$k_{11} = \begin{bmatrix} x_{21}^2 & x_{21} y_{21} \\ x_{21} y_{21} & y_{21}^2 \end{bmatrix} \tag{16}$$

__Triangular Plate Element__

$$X' = \left[u_1 , v_1 , u_2 , v_2 , u_3 , v_3 \right] \tag{17}$$

$$F' = \left[F_{x1} , F_{y1} , F_{x2} , F_{y2} , F_{x3} , F_{y3} \right] \tag{18}$$

$$\epsilon' = \left[\epsilon_x , \epsilon_y , \gamma_{xy} \right] \tag{19}$$

$$\sigma' = \left[\sigma_x , \sigma_y , \tau_{xy} \right] \tag{20}$$

$$B = \frac{1}{h} \begin{bmatrix} -y_{32} & 0 & y_{31} & 0 & -y_{21} & 0 \\ 0 & x_{32} & 0 & -x_{31} & 0 & x_{21} \\ -x_{32} & -y_{32} & -x_{31} & y_{31} & x_{21} & -y_{21} \end{bmatrix} \tag{21}$$

where

$$h = x_{21} y_{31} - x_{31} y_{21} \tag{22}$$

The absolute value of h equals twice the area of the triangular element.

$$C = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \tag{23}$$

in which ν = Poisson's ratio

$$D = \frac{E\,t\,|h|}{2h\,(1-\nu^2)}\begin{bmatrix} -y_{32} & -\nu y_{32} & C_1 x_{32} \\ \nu x_{32} & x_{32} & -C_1 y_{32} \\ y_{31} & \nu y_{31} & -C_1 x_{31} \\ \nu x_{31} & -x_{31} & C_1 y_{31} \\ -y_{21} & -\nu y_{21} & C_1 x_{21} \\ \nu x_{21} & x_{21} & -C_1 y_{21} \end{bmatrix} \tag{24}$$

where $C_1 = (1-\nu)/2$ and t is the element thickness.

$$k = \frac{E\,t}{2\,|h|\,(1-\nu^2)}\,\overline{k} \tag{25}$$

where

$$\overline{k}_{ij} = \overline{k}_{ji} \tag{26}$$

$$\left.\begin{aligned}
\bar{k}_{11} &= y_{32}^2 , \ \bar{k}_{12} = -\nu x_{32} y_{32} - C_1 x_{32} y_{32} \\
\bar{k}_{13} &= -y_{31} y_{32} - C_1 x_{31} x_{32} , \ \bar{k}_{14} = \nu x_{31} y_{32} + C_1 y_{31} x_{32} \\
\bar{k}_{15} &= y_{21} y_{32} + C_1 x_{21} x_{32} , \ \bar{k}_{16} = -\nu x_{21} y_{32} - C_1 y_{21} x_{32} \\
\bar{k}_{22} &= x_{32}^2 + C_1 y_{32}^2 , \ \bar{k}_{23} = \nu y_{31} x_{32} + C_1 x_{31} y_{32} \\
\bar{k}_{24} &= -x_{31} x_{32} - C_1 y_{31} y_{32} , \ \bar{k}_{25} = -\nu y_{21} x_{32} - C_1 x_{21} y_{32} \\
\bar{k}_{26} &= x_{21} x_{32} + C_1 y_{21} y_{32} , \ \bar{k}_{33} = y_{31}^2 + C_1 x_{31}^2 , \\
\bar{k}_{34} &= -\nu x_{31} y_{31} - C_1 y_{31} x_{31} , \ \bar{k}_{35} = -y_{21} x_{31} - C_1 x_{21} x_{31} \\
\bar{k}_{36} &= \nu x_{21} y_{31} + C_1 y_{21} x_{31} , \ \bar{k}_{44} = x_{31}^2 + C_1 y_{31}^2 , \\
\bar{k}_{45} &= \nu y_{21} x_{31} + C_1 x_{21} y_{31} , \ \bar{k}_{46} = -x_{21} x_{31} - C_1 y_{21} y_{31} \\
\bar{k}_{55} &= y_{21}^2 + C_1 x_{21}^2 , \ \bar{k}_{56} = -\nu x_{21} y_{21} - C_1 x_{21} y_{21} \\
\bar{k}_{66} &= x_{21}^2 + C_1 y_{21}^2
\end{aligned}\right\} \qquad (27)$$

2. ELASTIC-PLASTIC ANALYSIS

In the elastic range of material behavior the equilibrium equations for a structure composed of plate and bar elements of the type considered here can be written

$$F = KX \qquad (28)$$

where the force and displacement vectors now have as their components the cartesian components of force and displacement at all the nodes and K is the assembled stiffness matrix for the whole structure. The solution of Equation 28 for the unknown displacement is given symbolically by

$$X = K^{-1} F \qquad (29)$$

The displacements known, the element strains can be obtained from Equation 1 and the stresses from Equation 2. However, when the stresses reach the intensity required to cause plastic flow, it becomes necessary to determine the increments of plastic strain caused by the load increment. The material is assumed to obey the Mises yield condition and the associated flow rule. For plane stress the following equations apply

$$\bar{\sigma} = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\,\tau_{xy}^2 \right)^{1/2} = H(\bar{\epsilon}^p) \tag{30}$$

$$\Delta\bar{\epsilon}_x^p = \frac{2}{\sqrt{3}}\left(\Delta\epsilon_x^{p^2} + \Delta\epsilon_x^p\Delta\epsilon_y^p + \Delta\epsilon_y^{p^2} + \frac{1}{4}\gamma_{xy}^{p^2} \right)^{1/2} \tag{31}$$

$$\left.\begin{aligned} \Delta\epsilon_x^p &= \frac{\Delta\bar{\epsilon}^p}{2\bar{\sigma}}(2\sigma_x - \sigma_y) \\ \Delta\epsilon_y^p &= \frac{\Delta\bar{\epsilon}^p}{2\bar{\sigma}}(2\sigma_y - \sigma_x) \\ \Delta\gamma_{xy}^p &= 3\,\frac{\Delta\bar{\epsilon}^p}{\bar{\sigma}}\,\tau_{xy} \end{aligned}\right\} \tag{32}$$

where $\bar{\sigma}$ and $\bar{\epsilon}^p$ are the effective stress and the effective plastic strain, respectively, and where $H(\bar{\epsilon}^p)$ is the stress-plastic strain relation for uniaxial stress.

If it is assumed that the response of the structure to the removal of a load increment will be completely elastic then Equation 28 can be modified to account for plastic flow as follows

$$K\,X = F + F^p \tag{33}$$

where X and F are the displacement and load after the application of the increment and F^p is the vector of plastic forces corresponding to the plastic strains. The plastic strain increments caused by the increment of load must satisfy Equations 32 and for an element undergoing plastic flow the stresses must satisfy the yield condition (Equation 30).

The following step by step iterative method is used to obtain solutions:

1. An increment is given to the applied loads.

2. New values of displacement are found from Equation 33 using the current values of the plastic forces (these will be zero for the first step).

3. The displacements are used to compute total strains, elastic strains, stresses, and the effective stress.

4. If the new value of the effective stress is greater than the largest previous value, the element is plastic and the effective stress is used to determine a new value of the effective strain.

5. Plastic strain increments computed from Equations 32 are added to the current values of the plastic strain and new values of the plastic forces are calculated.

6. If the increment in effective plastic strain is sufficiently small the iteration is complete and a return to step 1 is made, if not a return is made to step 2 and a new cycle begun.

This procedure is applied to each of the elements and the decision to start a new step (apply a load increment) is based on the largest plastic strain increment found among all the elements.

An important feature of the method is the way in which the effective plastic strain is computed from the new value of the effective stress at each iteration. If the inverse of Equation 30 is used to give $\bar{\epsilon}^p$ as a function of $\bar{\sigma}$ the solution may become unstable. This becomes obvious when one considers the case of the elastic, perfectly plastic material for which the inverse of the function $H(\bar{\epsilon}^p)$ does not exist. To avoid this difficulty the "constant strain" method of Reference 2 is used. In this method the total strain ϵ_t is taken equal to the sum of the value of $\bar{\epsilon}^p$ computed in the previous iteration and $\bar{\sigma}/E$.

The stress-strain law can be written in the form

$$\epsilon_t = \frac{\bar{\sigma}}{E} + \bar{\epsilon}^p$$

or

$$\epsilon_t = \frac{H(\bar{\epsilon}^p)}{E} + \bar{\epsilon}^p \tag{34}$$

The new value of $\bar{\epsilon}^p$ can be found from Equation 34 without difficulty.

The criterion used in step 6 of the iterative procedure given above, to decide whether the plastic strains have been determined with sufficient accuracy, is the size of the ratio of the increment in effective plastic strain to $\bar{\sigma}/E$. This ratio is a measure of the difference between the ordinates to the theoretical stress strain curve and the curve that is actually being used at that step in the calculation.

3. STRESS-STRAIN LAWS

The following three types of stress-strain laws are available for use in the computer program. Each of them is a three parameter law.

Type 1 - Ramberg-Osgood Law

$$\epsilon_t = \frac{\sigma}{E} + \frac{3\sigma_1}{7E}\left(\frac{\sigma}{\sigma_1}\right)^n$$

in which

- E — Young's modulus
- σ_1 — secant yield stress (stress at which the secant modulus = 0.7E)
- n — shape factor

Type 2 - Goldberg-Richard Law

$$\sigma = E\epsilon_t\left[1 + \left|\frac{E\epsilon_t}{\sigma_u}\right|^n\right]^{-1/n}$$

in which

- E — Young's modulus
- σ_u — maximum stress
- n — shape factor

Type 3 - Bilinear Law

$$\sigma = E\epsilon_t\,, \qquad \text{for} \quad \sigma < \sigma_y$$

$$\sigma = \sigma_y + E_1\left(\epsilon_t - \frac{\sigma_y}{E}\right), \qquad \text{for} \quad \sigma \geq \sigma_y$$

in which

E — Young's modulus

σ_y — yield stress

E_1 — Slope of the plastic portion of the stress-strain curve

To reduce computing time a linearized form of the Ramberg-Osgood law is used in the program. This law is fitted by a series of straight line segments which match the actual curve at 100 points in the interval $0 \leq \epsilon_t \leq 20\,\sigma_1 / E$. If values of ϵ_t outside this range are encountered the exact formula is used.

SECTION III

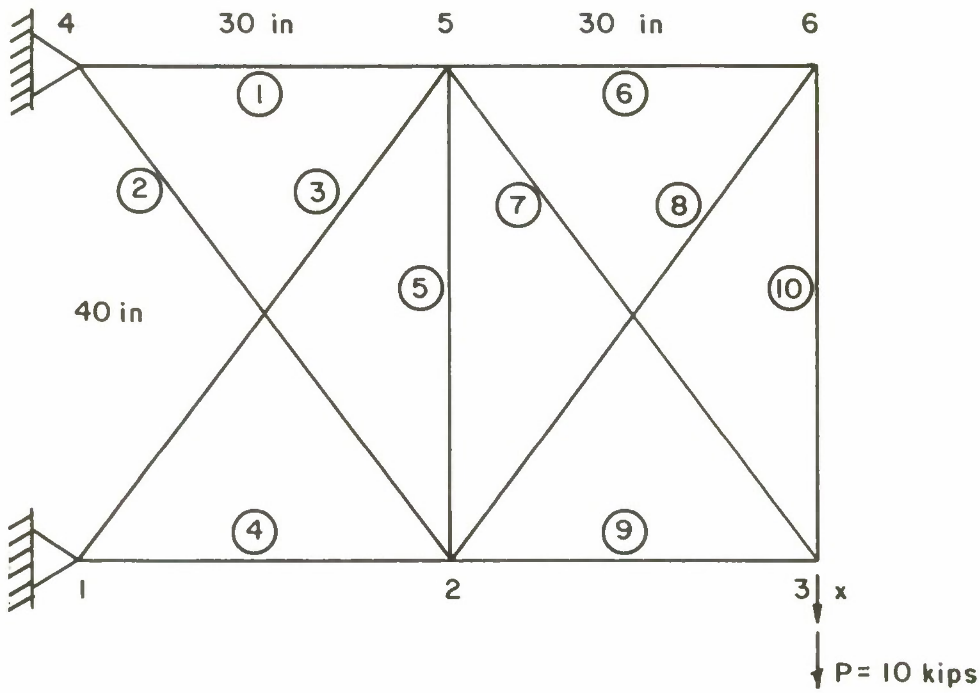
EXAMPLES

1. NONLINEAR TRUSS

To illustrate the use of the program a solution for the member stresses and the tip deflection of the cantilever truss shown in Figure 2 is obtained. A stress-strain relation of the Ramberg-Osgood type is assumed with values of $E = 10.3 \times 10^6$, $\sigma_1 = 40.5 \times 10^3$, and $n = 7$. A solution by another method is given in Reference 6. A comparison of the stresses obtained by the two methods is shown in the table in Figure 2. The tip displacement is shown as a function of the load in Figure 3. The displacements obtained by the two methods agree so closely that both solutions are represented by a single curve.

2. SHEAR LAG SPECIMEN

As a second example, solutions are obtained for the shear lag specimen tested in Reference 3 shown in Figure 4. Two solutions are obtained. In the first solution the same idealization of the structure is used as that used in Reference 3 (see Figure 5). The second solution is found using the idealization shown in Figure 6. Values of the Ramberg-Osgood constants of $E = 10.2 \times 10^6$, $\sigma_1 = 46.6 \times 10^3$, and $n = 10$ were used. These correspond to the values for the RO2 stress-strain curve of Reference 3.



ELEMENT	FORCE, in kips	
	Ref. 6	Present Analysis
1	11.26	11.18
2	6.23	6.19
3	- 6.27	- 6.31
4	-11.24	-11.11
5	- 0.51	- 0.52
6	3.35	3.33
7	6.91	6.95
8	- 5.59	- 5.55
9	- 4.15	- 4.17
10	4.47	4.44

Figure 2. Nonlinear Truss

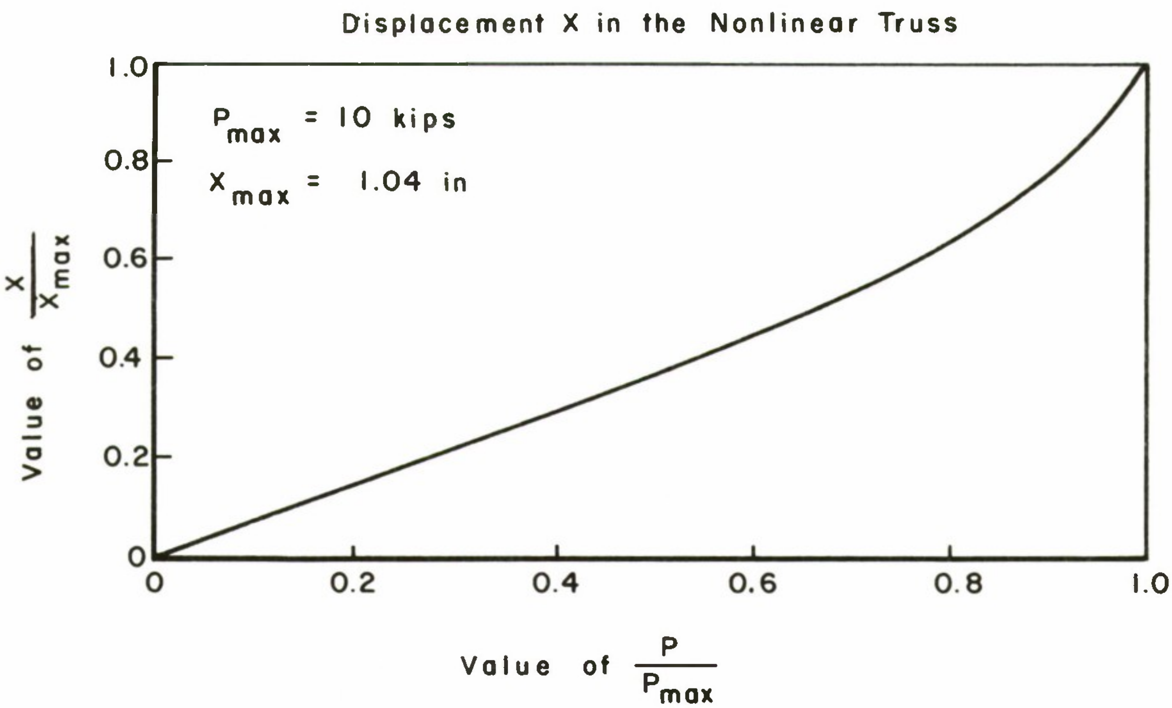


Figure 3. Load Versus Tip Displacement - Nonlinear Truss

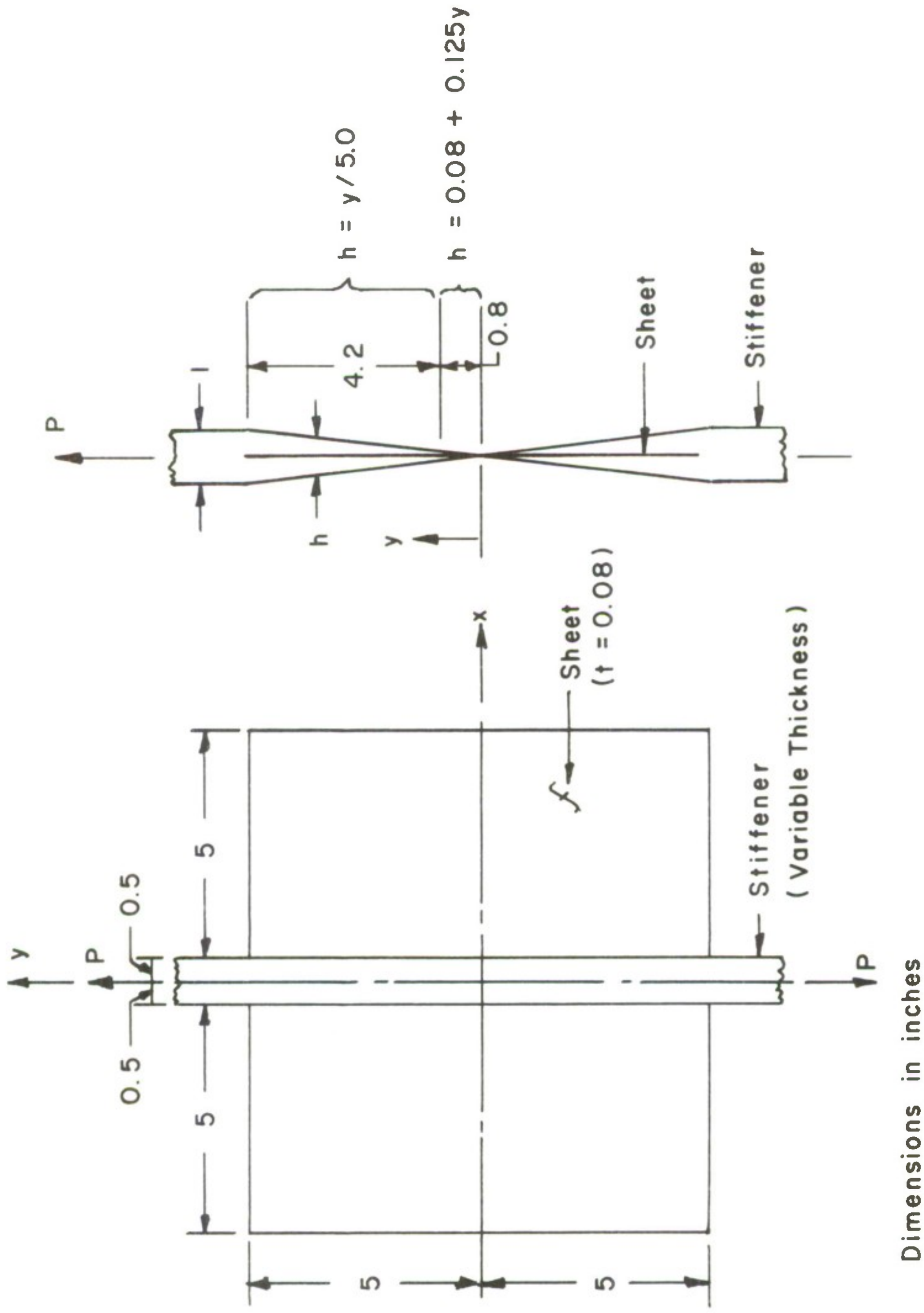


Figure 4. MIT Shear Lag Specimen

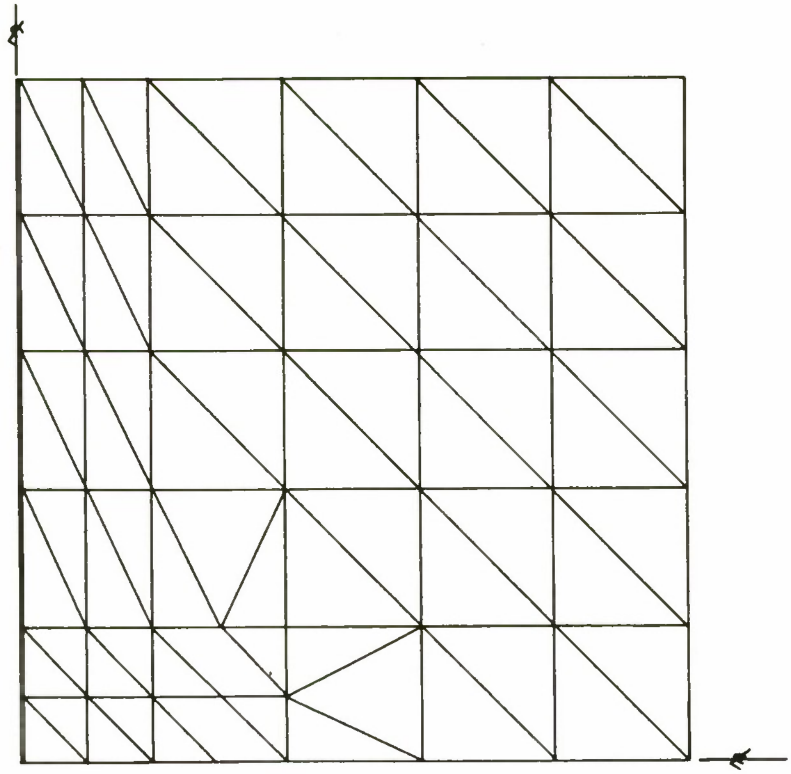


Figure 5. MIT Shear Lag Specimen - Configuration I

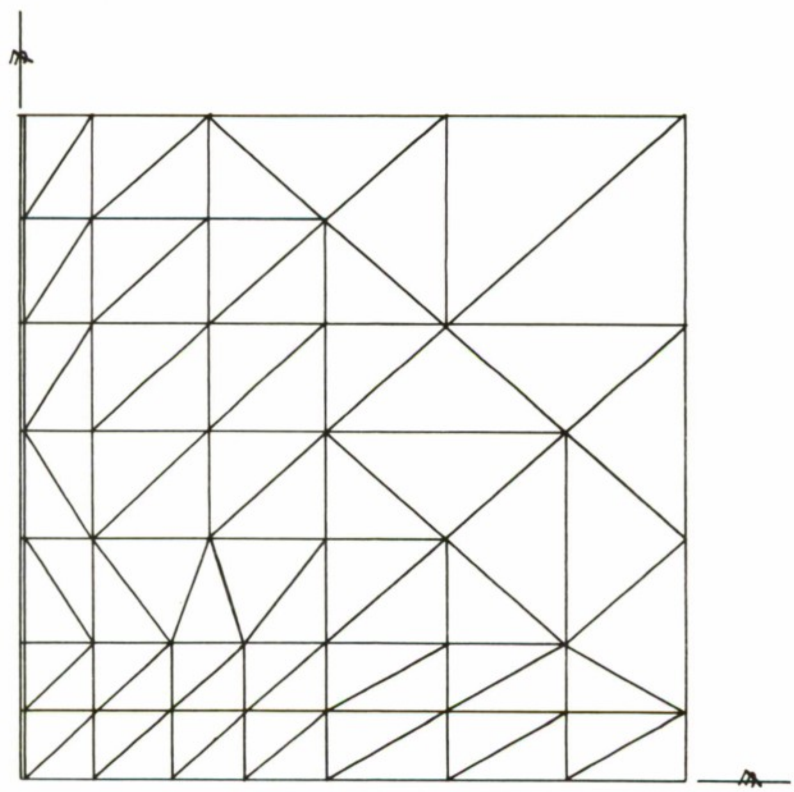


Figure 6. MIT Shear Lag Specimen - Configuration II

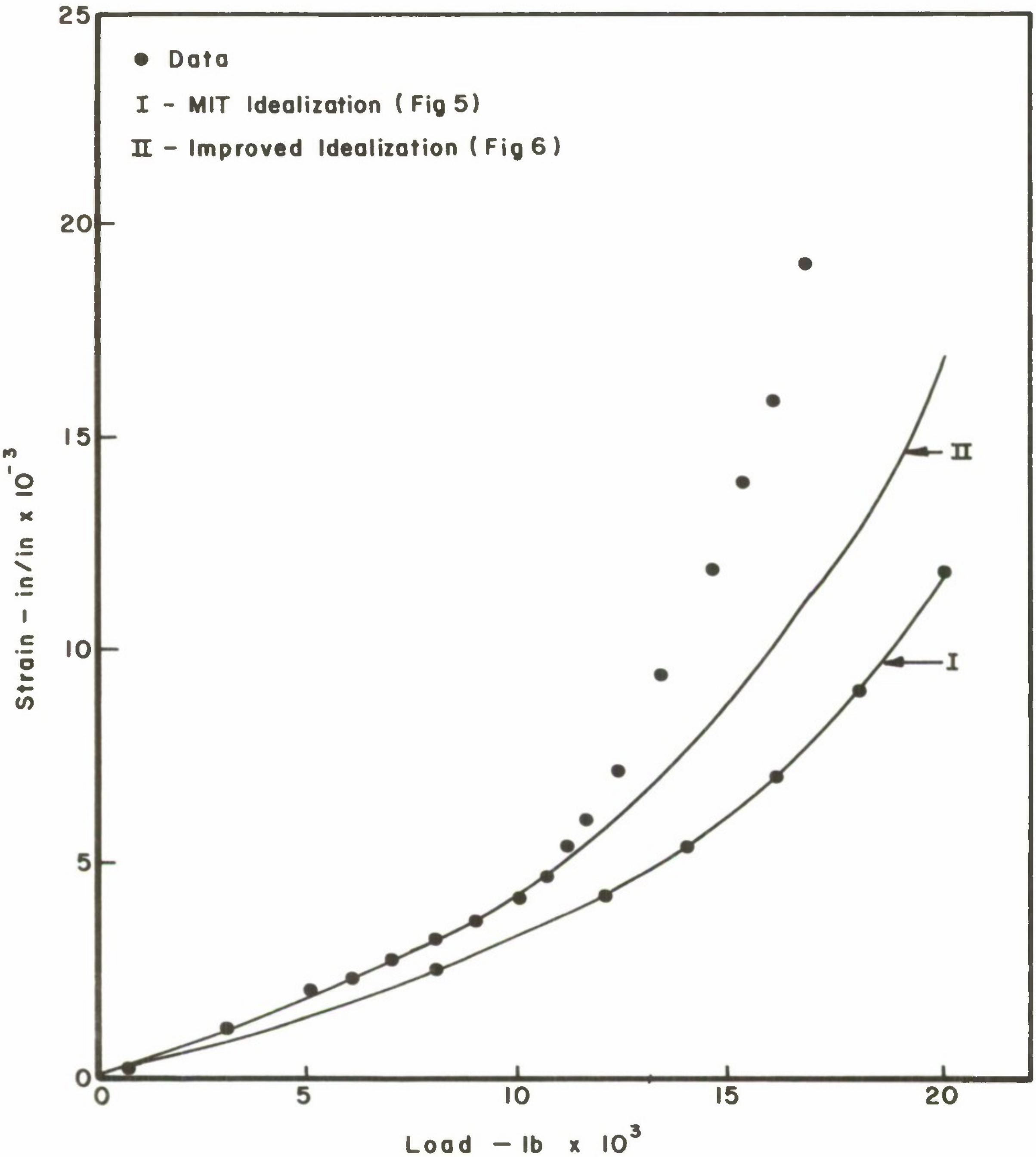


Figure 7. Comparison of Test Results of Shear Lag Specimen with Finite Element Analyses, Axial Strain at Center of Stiffener

A comparison of the results of these solutions with the test data for axial strain at the center of the stiffener is given in Figure 7. While the second solution is in better agreement with the test results than the first, the agreement at large values of the load is poor. A still more refined idealization would probably improve the solution but, as shown in Reference 5, much of the discrepancy is due to the inadequacy of the tensile stress-strain data in the range of large strains.

3. PERFORATED STRIP

Theocaris and Marketos (Reference 8) obtained results for a linear strain-hardening aluminum strip with a ratio of hole diameter to strip width of 1:2. The material properties were

yield stress = 24.3 kg/mm^2
plastic modulus = 225.0 kg/mm^2
Young's modulus = 7000.0 kg/mm^2

The finite element idealization of the test specimen is shown in Figure 8; 116 nodes and 172 triangular elements were used. A comparison of measured and computed values of the maximum strain in the y direction at the edge of the hole is given in Figure 9. The agreement between theory and experiment is fairly good. The same test is used by Marcal and King (Reference 7) for comparison with the results of their analysis and about the same degree of agreement is obtained.

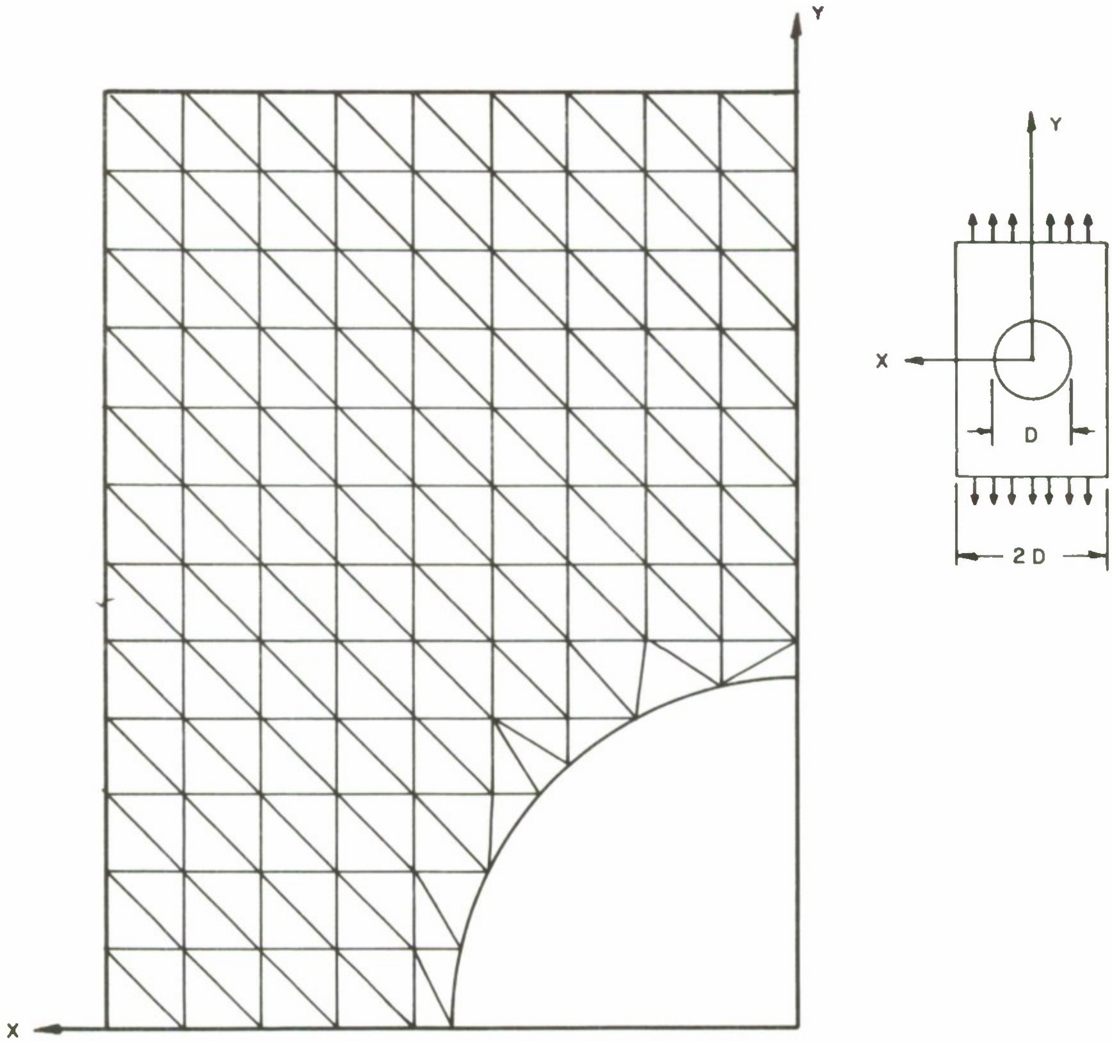


Figure 8. Perforated Strip Finite Element Idealization

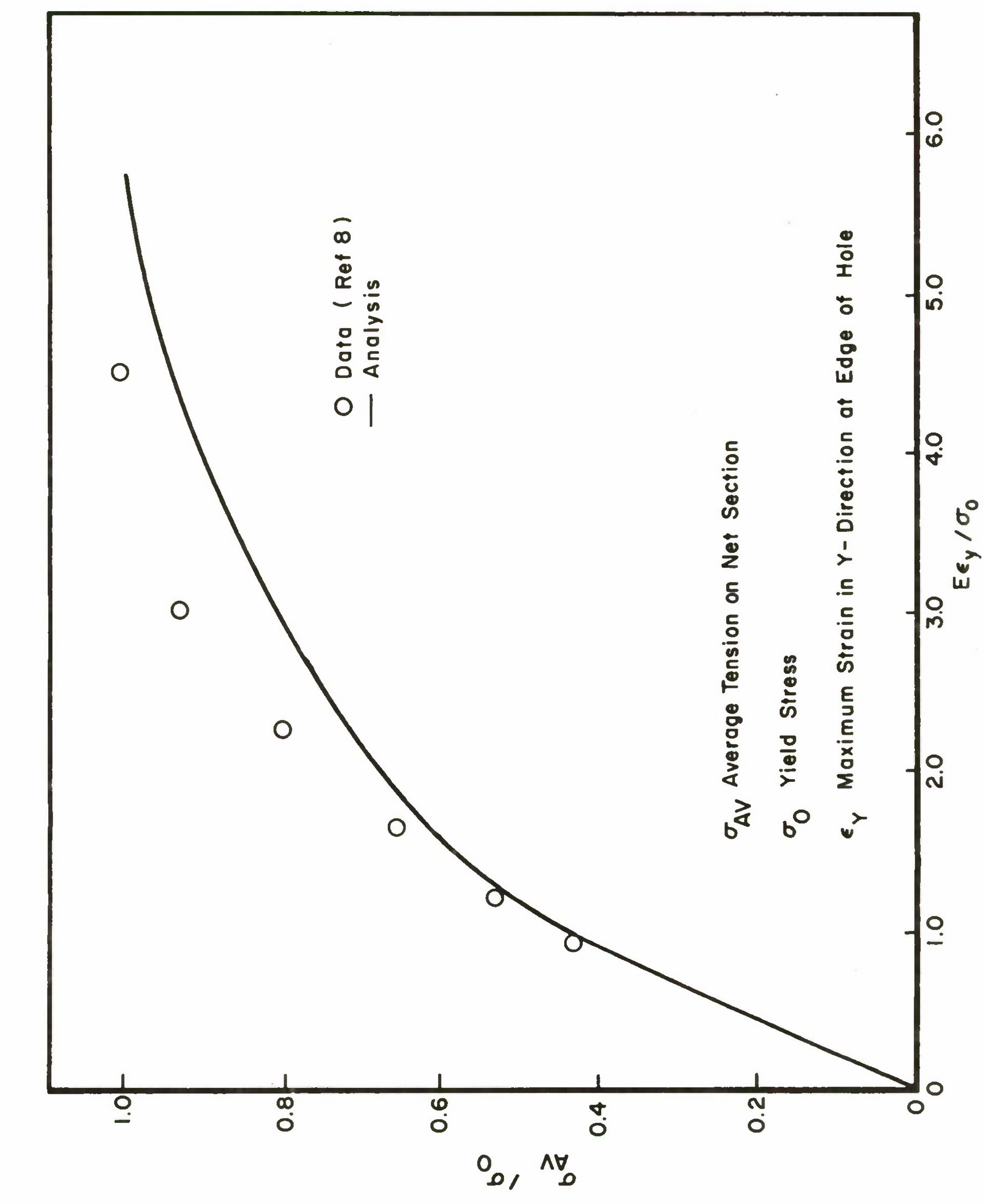


Figure 9. Maximum Values of E ϵ_y/σ_o for Perforated Strip

SECTION IV

DESCRIPTION OF COMPUTER PROGRAM

1. INTRODUCTION

The FORTRAN IV program for the elastic-plastic analysis of plane structures composed of bar and triangular plate elements is described in this section. The correspondence between the program variables and the stress-strain law parameters for each of the three laws available is given in Table I.

TABLE I

CORRESPONDENCE BETWEEN PROGRAM VARIABLES AND STRESS-STRAIN LAW PARAMETERS

Stress-Strain Law	Program Variables				
	ILAW	E	EE1	EE2	PRR
Ramberg-Osgood	1	E	σ_1	n	ν
Goldberg-Richard	2	E	σ_u	n	ν
Bilinear	3	E	σ_y	E_1	ν

2. INPUT DATA AND DESCRIPTION OF OUTPUT

The geometry of the structure is determined by specifying the x and y coordinates of each node with respect to a fixed set of coordinate axes and the thickness (cross-sectional area in the case of bars) of the elements. Up to 225 nodes and 400 elements can be handled. The program uses a subroutine for the solution of simultaneous equations in band form written by Professor E. L. Wilson of the University of

California. Great economies in storage requirements and in time required for solution are achieved in this way; however, the bandwidth of the equations defined by the idealization of the structure is limited in size. To meet this limitation on bandwidth the difference between the node numbers on any element must be 9 or less. Instructions for increasing the bandwidth are given in Table II.

The displacement components in the x and y direction can be specified at any node or a node can be required to move along a line with a specified slope. Boundary conditions can be specified at up to 29 nodes.

The x and y components of load can be specified at any node. Distributed loads must be treated as concentrated at the nodes.

The number of equal increments (steps) into which the applied loads and specified displacements are to be divided is specified as input. It is also necessary to specify the number of the increment at which the solution is to start. For example, if a number of increments NDIV = 20 is specified and a value of the starting increments KSTART = 5 is used, one quarter of the load (displacement) will be applied in the first step, the rest in 15 equal increments. If it is desired to stop the solution at an intermediate step a value of KSTOP may be specified. If the unloading solution is desired the value IUNLD = 1 is used.

An error tolerance must be specified as input. After each cycle of iteration the maximum error among all the elements is compared with the specified tolerance. If the tolerance is met the next load

TABLE II

PROGRAM MODIFICATION

The bandwidth is governed by the difference between the node numbers of a given element. The largest such difference J determines the bandwidth in this program by the formula MBAND = 2 * J + 3. This number cannot be greater than 22 in the present program. This is a rather small bandwidth, but it allows 225 nodes in a 32K core machine. To change the applicable problem size of the program in terms of the basic problem size parameters the following dimensions have to be changed:

	To Change:	Change Dimensions of:
1.	Number of materials	EE, EE1, EE2, PRR, and TAB in common statements (presently 10)
2.	Number of elements	I1, I2, I3, I4, NTYPE, Z, SEF, SET, EEP, EXPL, EYP, EXYP in common statement, JX in main program, and modify equivalence statements containing JX (presently 400)
3.	Number of nodes	B, X, XCORD, Y, ICODE, FP, F in common statements, FE in main program, ICODE in subroutine DCODE, XX in subroutine ELEM (presently 225 or 2 x 225 = 450)
4.	Bandwidth	B in common statements (presently 22)
5.	Number of nodes with boundary conditions	BC in common statements (presently 30)

increment is applied, if not, the iteration is continued. If the tolerance on error is not met when the allowable number of iterations is reached the solution is stopped. A detailed description of the input data format is given in Table III.

TABLE III

INPUT DATA FORMAT*

Card 1	TITLE CARD (72H)			
Col	1-72	Any alphanumeric information		
Card 2	PROPERTIES CARD (14I5)			
Col	1- 5	NNODE	-	number of nodes (maximum 225)
	6-10	NELEM	-	number of elements (maximum 400)
	11-15	ILAW	-	1 Ramberg-Osgood Law
			-	2 Goldberg-Richard Law
			-	3 Bilinear Law
	16-20	IUNLD	-	1 Unloading following loading
			-	0 Loading only
	21-25	MAT	-	number of materials used (maximum 10)
	26-30	MAXBND	-	maximum bandwidth, MAXBND = 22 for this program
	31-35	NBC	-	number of boundary conditions with prescribed displacement. The maximum number is 30 in this program.
Card 3	MATERIAL PROPERTIES CARDS (E15.8, 3F10.5)			
Col	1-15	EE	-	modulus of elasticity
	16-25	EE1	-	secant yield stress, ultimate stress, yield stress
	26-35	PRR	-	Poisson's ratio
	36-45	EE2	-	shape parameter, plastic modulus
Card 4	CONTROL CARD (6I5, F10.0)			
Col	1- 5	NDIV	-	number of load increments
	6-10	NIT	-	maximum number of iterations per step
	11-15	NPRINT	-	print output for each NPRINT increment. (e.g., if NPRINT = 3, for increments 3, 6, 9 etc.)
	16-20	KSTART	-	number of increments at which solution is to start
	21-25	KSTOP	-	number of increments at which solution is to stop
	26-30	NLOAD	-	number of nodes at which loads are specified
	31-40	TOL	-	error tolerance

TABLE III (CONTD)

Card 5	NODE CARDS (4I5, 5F10.0)		
Col	1- 5	Node number	
	6-10	IBCX =	1, if displacement in x-direction is specified
	11-15	IBCY =	1, if displacement in y-direction is specified
	16-20	IBCS =	1, if slope is specified
	21-30	XCORD -	x coordinate of the node
	31-40	YCORD -	y coordinate of this node
	41-50	BCI -	specified displacement in x-direction
	51-60	BC2 -	specified displacement in y-direction
	61-70	BC3 -	specified slope at the node
Card 6	ELEMENT CARDS (5I5, F10.0)		
Col	1- 5	Element number	
	6-10	I1 -	nodes defining the element
	11-15	I2 -	nodes defining the element
	16-20	I3 -	nodes defining the element
	21-25	NTYPE -	material type
	26-35	Z -	element thickness or cross-sectional area
Card 7	LOAD CARDS (I5, 2F10.0)		
	1- 5	Node number	
	6-15	x-component of force	
	16-25	y-component of force	

*NOTE: Input data information in Table III is self explanatory. The use of more than one material, however, may need some clarification. The number of materials "MAT" specified in the field of card 2 defines the number of material properties cards. The sequencing of these cards in turn defines "NTYPE" in the element card, for example, if the element uses material specified in the second material properties card, integer 2 is placed in the field corresponding to "NTYPE".

The nodal forces and displacements, the maximum error and the number of the element in which it occurs are printed out at the end of each step (increment). The cartesian components, principal values, and direction of stress and strain are printed out at the user's option by specifying a value of NPRNT as input. For example, a value of NPRNT = 3 will cause the stresses and strains to be printed out for increment numbers divisible by three. The directions of the principal axes of stress are defined by

$$\phi = -\frac{1}{2}\tan^{-1}\frac{2\tau}{\sigma_x - \sigma_y}, \quad -\frac{\pi}{2} < \phi < \frac{\pi}{2}$$

The value of ϕ in degrees is printed out. In the case of strain the principal directions are defined by

$$\phi = -\frac{1}{2}\tan^{-1}\frac{\gamma}{\epsilon_x - \epsilon_y}, \quad -\frac{\pi}{2} < \phi < \frac{\pi}{2}$$

This value is also printed out since in general the principal axes of stress and total strain do not coincide when plastic flow has taken place.

The effective stress and the effective plastic strain are also given as output for each element.

The input data is printed out at the start of the program to aid in problem identification and checking.

REFERENCES

1. J. S. Prezmieniecki and L. Berke, Digital Computer Program for the Analysis of Aerospace Structures by the Matrix Displacement Method, FDL-TDR-64-18, April 1964.

2. J. Padlog, R. D. Huff, and G. F. Holloway, Unelastic Behavior of Structures Subjected to Cyclic, Thermal, and Mechanical Stressing Conditions, WADD-TR-60-271, December 1960.

3. J. H. Percy, W. A. Loden, and D. R. Navaratna, A Study of Matrix Analysis Methods for Inelastic Structures, RDT-TDR-63-4032, October 1963.

4. J. L. Swedlow and W. H. Yang, Stiffness Analysis of Elasto-Plastic Plates, AFRPL-TR-66-5, January 1966.

5. W. R. Jensen et al, Matrix Analysis Methods for Inelastic Structures.

6. R. M. Richard and J. E. Goldberg, Analysis of Nonlinear Structures: Force Method, Proc. ASCE, Jour. Struct. Div., December 1965.

7. P. V. Marcal and I. P. King, Elastic-Plastic Analysis of Two-Dimensional Stress Systems by the Finite Element Method, Int. J. Mech. Sci., Vol. 9, pp 143-155, 1967.

8. P. S. Theocaris and E. Marketos, Elastic-Plastic Analysis of Perforated Strips of Strain-Hardening Material, J. Mech Phys. Solids, Vol. 12, pp. 377-390, 1964.

APPENDIX

COMPUTER PROGRAM LISTING

The FORTRAN IV Source Program and three sample data cases are listed. The first case is the nonlinear truss with Ramberg-Osgood representation of one material. This data is associated with Figures 2 and 3. The second case is the same truss problem slightly changed to show the introduction of more than one material. The third case is Configuration I for the MIT test specimen.

The source deck of the computer program described herein can be obtained by contacting AFFDL (FDTR/BERKE), WPAFB-Ohio, 45433. (513-25-53418).

```
$IBJOB
$IBFTC VER7                                                                      VER70000
C        ELASTIC PLASTIC FINITE ELEMENT PROGAM                                   VER70001
C        WITH THREE STRESS STRAIN LAW OPTIONS                                    VER70002
      COMMON/AOD/  EE(10),EE1(10),EE2(10),PRR(10)                                VER70003
      COMMON  E,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,                    VER70004
     1        N2,NELEM,KEL,ILAW,MAT,NBC,                                         VER70005
     2        B(450,22),BC(30,3),TAB(101,20),IFIX(2),                            VER70006
     3        X(450),XCORO(225),Y(225),ICODE(225),                               VER70007
     4        FP(450),F(450),                                                    VER70008
     5        I1(400),I2(400),I3(400),NTYPE(400),Z(400),I4(400),                 VER70009
     6        SEF(400),SET(400),EEP(400),EXPL(400),EYP(400),EXYP(400),           VER70010
     7    NNOOE,MBAND                                                            VER70011
      DIMENSION JX(400,3),FE(450)                                                VER70012
      EQUIVALENCE (JX,I1),(JX(401),I2),(JX( 801),I3)                             VER70013
      EQUIVALENCE (IFIX(1),IBCX),(IFIX(2),IBCY)                                  VER70014
C                                                                                VER70015
C  **** READ AND PRINT DATA ****                                                 VER70016
C                                                                                VER70017
   10 READ  (5,20)                                                               VER70018
      IONE=1                                                                     VER70019
   20 FORMAT(72H    BCO INFORMATION                                              VER70020
     1              )                                                            VER70021
      WRITE (6,30)                                                               VER70022
   30 FORMAT(1H1)                                                                VER70023
      WRITE (6,20)                                                               VER70024
      READ(5,40) NNOOE,NELEM,ILAW,IUNLO,MAT,MAXBND,NBC                           VER70025
   40 FORMAT(14I5)                                                               VER70026
      READ(5,50) (EE(I),EE1(I)          ,PRR(I),EE2(I),I=1,MAT)                  VER70027
   50 FORMAT(E15.8,3F10.5)                                                       VER70028
      READ(5,60)NDIV,NIT,NPRNT,KSTART,KSTOP,NLOAD,TOL                            VER70029
   60 FORMAT(6I5,F10.0)                                                          VER70030
      IF(KSTOP.EQ.0) KSTOP=NDIV                                                  VER70031
      GO TO(70,90,110),ILAW                                                      VER70032
   70 WRITE(6,80) (I,EE(I),EE1(I),EE2(I),PRR(I),TOL,I=1,MAT)                     VER70033
   80 FORMAT(1H014X18HRAMBERG OSGOOD LAW/                                        VER70034
     115X,30HMATERIAL---------------------  I3/                                  VER70035
     215X30HMOOULUS OF ELASTICITY-------- E12.4/                                 VER70036
     315X30HSECANT YIELO STRESS---------- E12.4/                                 VER70037
     415X30HSHAPE PARAMETER-------------- E12.4/                                 VER70038
     515X30HPOISSON'S RATIO-------------- F8.4/                                  VER70039
     615X30HERROR TOLERANCE-------------- F8.4)                                  VER70040
      GO TO 130                                                                  VER70041
   90 WRITE(6,100) (I,EE(I),EE1(I),EE2(I),PRR(I),TOL,I=1,MAT)                    VER70042
  100 FORMAT(1H014X20HGOLOBERG RICHARO LAW/                                      VER70043
     115X,30HMATERIAL---------------------  I3/                                  VER70044
     215X30HMOOULUS OF ELASTICITY-------- E12.4/                                 VER70045
     315X30HULTIMATE STRESS-------------- E12.4/                                 VER70046
     415X30HSHAPE PARAMETER-------------- E12.4/                                 VER70047
     515X30HPOISSON'S RATIO-------------- F8.4/                                  VER70048
     615X30HERROR TOLERANCE-------------- F8.4)                                  VER70049
      GO TO 130                                                                  VER70050
  110 WRITE(6,120) (I,EE(I),EE1(I),EE2(I),PRR(I),TOL,I=1,MAT)                    VER70051
  120 FORMAT(1H014X12HBILINEAR LAW/                                              VER70052
     115X,30HMATERIAL---------------------  I3/                                  VER70053
     215X30HMODULUS OF ELASTICITY-------- E12.4/                                 VER70054
     315X30HYIELO STRESS----------------- E12.4/                                 VER70055
     415X30HPLASTIC MODULUS-------------- E12.4/                                 VER70056
     515X30HPOISSON'S RATIO-------------- F8.4/                                  VER70057
     615X30HERROR TOLERANCE-------------- F8.4)                                  VER70058
  130 WRITE (6,140)NNOOE,NELEM,NOIV,NIT                                          VER70059
  140 FORMAT(1H014X30HNO. OF NODES            NNOOE =I4/15X30HNO. OF ELEMVER70060
```

```
     1ENTS         NELEM =I4/15X30HNO. OF STEPS             NDIV =I4/15X30
     2HNO. OF ITERATIONS/STEP    NIT =I4)
      DO 150 I=1,NBC
      DO 150 J=1,3
  150 BC(I,J)=0
      IC=1
      WRITE(6,160)
  160 FORMAT(25HOBOUNDARY CONDITION ARRAY/10HO NODAL PT15X1HX23X1HY
     120X7HSLIDING/1H 14X4HCODE7X5HVALUE9X4HCODE7X5HVALUE9X4HCODE
     27X5HVALUE)
C
C  **** NODE COORDINATES AND BOUNDARY CONDITIONS ****
C
      DO 200 J=1,NNODE
      READ(5,170) K,IBCX,IBCY,IBCS,XCORD(K),Y(K),BC1,BC2,BC3
  170 FORMAT(4I5,5F10.0)
      IF(IBCX+IBCY+IBCS.NE.0) WRITE(6,180)K,IBCX,BC1,IBCY,BC2,IBCS,BC3
  180 FORMAT(I7,3X,3(I8,1PE17.7))
      ICODE(K)=IBCS+10*IBCY+100*IBCX
      IF(BC1+BC2+BC3.EQ.0.) GO TO 200
      ICODE(K)=ICODE(K)+IC*1000
      BC(IC,1)=BC1
      BC(IC,2)=BC2
      BC(IC,3)=BC3
      IC=IC+1
      IF(IC.LE.NBC)GO TO 200
      WRITE(6,190)
  190 FORMAT(54HO MORE THAN 29 NODES HAVE NON ZERO BOUNDARY CONDITIONS)
      GO TO 620
  200 CONTINUE
C
C  ****  ELEMENT PROPERTIES  ****
C
      READ(5,210)(K,I1(K),I2(K),I3(K),NTYPE(K),Z(K),J=1,NELEM)
  210 FORMAT(5I5,F10.0)
C
C  ****  LOADS  ****
C
      N2=2*NNODE
      DO 220 K=1,N2
  220 F(K)=0
      IF(NLOAD.EQ.0) GO TO 250
      DO 230 K=1,NLOAD
  230 READ(5,240)J,F(2*J-1),F(2*J)
  240 FORMAT(I5,2F10.0)
  250 CONTINUE
      WRITE(6,260)(K,XCORD(K),F(2*K-1), Y(K),F(2*K),ICODE(K),K=1,NNODE)
  260 FORMAT(10HO NODAL PT8X7HX-COORD8X7HX-FORCE8X7HY-COORD8X7HY-FORCE
     111X4HCODE//(4X,I3,5X4F15.4,I15))
      WRITE(6,270)
  270 FORMAT(1H0///10X,90HELEMENT    NODE 1    NODE 2    NODE 3    ELEMEN
     1T TYPE      AREA OR THICK.      MATERIAL TYPE,//)
      DO 300 K=1,NELEM
      IF(I3(K).EQ.0) WRITE(6,280) K,I1(K),I2(K),I3(K),Z(K),NTYPE(K)
      IF(I3(K).NE.0) WRITE(6,290) K,I1(K),I2(K),I3(K),Z(K),NTYPE(K)
  280 FORMAT(  6X,4I9,11X,3HBAR,E22.5,I14)
  290 FORMAT(  6X,4I9,11X,5HPLATE,E20.5,I14)
  300 CONTINUE
C
C     INITIALIZATION
C
  310 XDIV=NDIV
```

```
      GO TO(320,340,340),ILAW                                           VER70123
  320 CONTINUE                                                          VER70124
      DO 330 I=1,MAT                                                    VER70125
      E=EE(I)                                                           VER70126
      E1=EE1(I)                                                         VER70127
      E2=EE2(I)                                                         VER70128
      CC=E1/E                                                           VER70129
      G1=(7.0*E/3.0)**(1.0/E2)*E1**(1.0-1.0/E2)                         VER70130
      CALL TABLE(I)                                                     VER70131
  330 CONTINUE                                                          VER70132
  340 CONTINUE                                                          VER70133
C                                                                       VER70134
C  **** OETERMINE BANO WIDTH ****                                       VER70135
C                                                                       VER70136
      00 350 K=1,NELEM                                                  VER70137
      I4(K)=I3(K)                                                       VER70138
  350 IF(I3(K).EQ.0) JX(K,3)=JX(K,1)                                    VER70139
      J=0                                                               VER70140
      00 380 N=1,NELEM                                                  VER70141
      DO 380 I=1,3                                                      VER70142
      DO 370 L=1,3                                                      VER70143
      KK=IABS(JX(N,I)-JX(N,L))                                          VER70144
      IF(KK-J)370,370,360                                               VER70145
  360 J=KK                                                              VER70146
  370 CONTINUE                                                          VER70147
  380 CONTINUE                                                          VER70148
      MBAND=2*J+3                                                       VER70149
      IF(MBAND.GT.MAXBND) WRITE(6,390) MBAND                            VER70150
      IF(MBANO.GT.MAXBND) GO TO 10                                      VER70151
  390 FORMAT(1H010X20HBAND WIOTH TOO LARGE5X6HMBAND=I4)                 VER70152
      DO 400 I=1,NELEM                                                  VER70153
  400 I3(I)=I4(I)                                                       VER70154
      00 410 I=1,N2                                                     VER70155
      00 410 J=1,MBANO                                                  VER70156
  410 B(I,J)=0.                                                         VER70157
C                                                                       VER70158
C     CALCULATION OF STIFFNESS MATRIX                                   VER70159
C                                                                       VER70160
      CALL STIFF                                                        VER70161
C                                                                       VER70162
C  **** REDUCE MATRIX ****                                              VER70163
C                                                                       VER70164
      CALL SYMSOL(1)                                                    VER70165
C                                                                       VER70166
C  **** INCREMENT LOAOS,AOD PLASTIC FORCES ANO SOLVE FOR DISPLACEMENTS**VER70167
C                                                                       VER70168
      DO 420 I=1,NELEM                                                  VER70169
      SET(I)=0                                                          VER70170
      SEF(I)=0                                                          VER70171
      EEP(I)=0                                                          VER70172
      EXPL(I)=0                                                         VER70173
      EYP(I)=0                                                          VER70174
  420 EXYP(I)=0                                                         VER70175
      KO=KSTART-1                                                       VER70176
      KU=KO                                                             VER70177
      DO 430 I=1,N2                                                     VER70178
  430 FP(I)=0                                                           VER70179
      GO TO 490                                                         VER70180
  440 WRITE (6,450)KU,(I,FE(2*I-1),FE(2*I),X(2*I-1),X(2*I),I=1,NNODE)   VER70181
  450 FORMAT(1H120X38HFORCES AND DISPLACEMENTS FOR INCREMENT,I4//10X4HNOVER70182
     1DE5X7HX-FORCE8X7HY-FORCE9X8HX-OISPL.7X8HY-DISPL./(9XI3,2F15.3,5X2EVER70183
     215.4 ))                                                           VER70184
```

```
      WRITE (6,460)XERR,KEL,IT                                          VER70185
  460 FORMAT(13HOMAX. ERROR =F8.5,5X14HIN ELEMENT NO.I4,5X17HNO. OF ITERVER70186
     1ATIONSI4)                                                         VER70187
  470 IF(MOD(KO,NPRNT))490,480,490                                      VER70188
  480 CALL OUTPT                                                        VER70189
      IF(KO.EQ.KSTOP)GO TO 620                                          VER70190
      GO TO 500                                                         VER70191
  490 IF(KO.EQ.KSTOP) CALL OUTPT                                        VER70192
      IF(KO.EQ.KSTOP)GO TO 620                                          VER70193
  500 KO=KO+IONE                                                        VER70194
      KU=KU+1                                                           VER70195
      IF(KO-NDIV)510,510,620                                            VER70196
  510 XKO=KO                                                            VER70197
      DO 520 I=1,N2                                                     VER70198
  520 FE(I)=XKO/XDIV*F(I)                                               VER70199
      DO 530 K=1,NELEM                                                  VER70200
  530 SEF(K)=SET(K)                                                     VER70201
      IT=0                                                              VER70202
  540 DO 570 I=1,NNODE                                                  VER70203
      IF(ICODE(I).EQ.0) GO TO 570                                       VER70204
      CALL DCODE(ICODE,I,IBCS,IBCX,IBCY,IC,IX,IY,NBC)                   VER70205
      IF(IBCS.NE.1) GO TO 550                                           VER70206
      ALF=BC(IC,3)                                                      VER70207
      FP(IX)=FP(IX)+ALF*FP(IY)                                          VER70208
      FP(IY)=0.                                                         VER70209
  550 DO 560 N=1,2                                                      VER70210
      IF(IFIX(N).NE.1) GO TO 560                                        VER70211
      IR=IX+N-1                                                         VER70212
      FP(IR)=0.                                                         VER70213
  560 CONTINUE                                                          VER70214
  570 CONTINUE                                                          VER70215
C                                                                       VER70216
C   **** SOLVE FOR DISPLACEMENTS ****                                   VER70217
C                                                                       VER70218
      DO 580 I=1,N2                                                     VER70219
  580 X(I)=FE(I)+FP(I)                                                  VER70220
      CALL SYMSOL(2)                                                    VER70221
C                                                                       VER70222
C     CALCULATE TOTAL STRAINS,STRESSES AND PLASTIC                      VER70223
C        FORCES AND STRAINS FOR EACH ELEMENT                            VER70224
C                                                                       VER70225
      DO 590 I=1,N2                                                     VER70226
  590 FP(I)=0                                                           VER70227
      XERR=0.0                                                          VER70228
      KEL=0                                                             VER70229
      CALL STRAIN                                                       VER70230
C                                                                       VER70231
C   **** PICK LARGEST ERROR AND DETERMINE WHEN TO REITERATE ****        VER70232
C                                                                       VER70233
      IT=IT+1                                                           VER70234
      IF(XERR-TOL)440,440,600                                           VER70235
  600 IF(IT-NIT)540,610,610                                             VER70236
  610 KO=NDIV                                                           VER70237
      IUNLD=0                                                           VER70238
      GO TO 440                                                         VER70239
  620 IF(IUNLD.EQ.0) GO TO 10                                           VER70240
      IUNLD=0                                                           VER70241
      IONE=-1                                                           VER70242
      KSTOP=0                                                           VER70243
      GO TO 440                                                         VER70244
      END                                                               VER70245
$IBFTC ELM                                                              ELM 0000
```

```
      SUBROUTINE ELEM(M)                                                ELM 0001
      COMMON/ADD/  EE(10),EE1(10),EE2(10),PRR(10)                       ELM 0002
      COMMON  E,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,           ELM 0003
     1        N2,NELEM,KEL,ILAW,MAT,NBC,                                ELM 0004
     2        B(450,22),BC(30,3),TAB(101,20),IFIX(2),                   ELM 0005
     3        X(450),XCORD(225),Y(225),ICODE(225),                      ELM 0006
     4        FP(450),F(450),                                           ELM 0007
     5        I1(400),I2(400),I3(400),NTYPE(400),Z(400),I4(400),        ELM 0008
     6        SEF(400),SET(400),EEP(400),EXPL(400),EYP(400),EXYP(400),  ELM 0009
     7       NNODE,MBAND                                                ELM 0010
      DIMENSION XX(225)                                                 ELM 0011
      EQUIVALENCE (XCORD,XX)                                            ELM 0012
      J1=I1(M)                                                          ELM 0013
      J2=I2(M)                                                          ELM 0014
      J3=I3(M)                                                          ELM 0015
      X21=XX(J2)-XX(J1)                                                 ELM 0016
      Y21=Y(J2)-Y(J1)                                                   ELM 0017
      IF(J3.EQ.0) GO TO 10                                              ELM 0018
      Y32=Y(J3)-Y(J2)                                                   ELM 0019
      Y31=Y(J3)-Y(J1)                                                   ELM 0020
      X32=XX(J3)-XX(J2)                                                 ELM 0021
      X31=XX(J3)-XX(J1)                                                 ELM 0022
      RETURN                                                            ELM 0023
   10 Y32=SQRT(X21**2+Y21**2)                                           ELM 0024
      RETURN                                                            ELM 0025
      END                                                               ELM 0026
$IBFTC DCOD                                                             DCOD0000
CDCOD                                                                   DCOD0001
      SUBROUTINE DCODE(ICODE,I,IBCS,IBCX,IBCY,IC,IX,IY,NBC)             DCOD0002
      DIMENSION ICODE(225)                                              DCOD0003
      IBCS=MOD(ICODE(I),10)                                             DCOD0004
      IBCX=MOD(ICODE(I),1000)/100                                       DCOD0005
      IBCY=MOD(ICODE(I),100)/10                                         DCOD0006
      IC=MOD(ICODE(I),100000)/1000                                      DCOD0007
      IX=2*I-1                                                          DCOD0008
      IY=IX+1                                                           DCOD0009
      IF(IC.EQ.0) IC=NBC                                                DCOD0010
      RETURN                                                            DCOD0011
      END                                                               DCOD0012
$IBFTC PNEW                                                             PNEW0000
C2222   PLASTIC STRAIN DETERMINATION                                    PNEW0001
      SUBROUTINE PNEW1 (EEPK,EET,E,E1,E2)                               PNEW0002
      J=1                                                               PNEW0003
      XU=EET                                                            PNEW0004
      XL=0                                                              PNEW0005
   10 EEPK=.5*(XL+XU)                                                   PNEW0006
   20 Y=EEPK+E1/E*EEPK**(1.0/E2)-EET                                    PNEW0007
   30 YP=1.0+E1/E/E2*EEPK**(1.0/E2-1.0)                                 PNEW0008
      J=J+1                                                             PNEW0009
      IF(J-50)40,40,100                                                 PNEW0010
   40 IF(Y)50,100,60                                                    PNEW0011
   50 XL=EEPK                                                           PNEW0012
      GO TO 70                                                          PNEW0013
   60 XU=EEPK                                                           PNEW0014
   70 XT=EEPK-Y/YP                                                      PNEW0015
      IF(XU-XT)10,10,80                                                 PNEW0016
   80 IF(XT-XL)10,10,90                                                 PNEW0017
   90 EEPK=XT                                                           PNEW0018
      DIFF=ABS(Y/YP/EEPK)                                               PNEW0019
      IF(DIFF-.00001)100,100,20                                         PNEW0020
  100 RETURN                                                            PNEW0021
      END                                                               PNEW0022
```

```
$IBFTC PLSTR                                                              PLST0000
CTABL     STRAIN-PLASTIC STRAIN TABLE                                     PLST0001
      SUBROUTINE TABLE(K)                                                 PLST0002
      COMMDN/ADD/  EE(10),EE1(10),EE2(10),PRR(10)                         PLST0003
      CDMMDN   E,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,            PLST0004
     1         N2,NELEM,KEL,ILAW,MAT,NBC,                                 PLST0005
     2         B(450,22),BC(30,3),TAB(101,20),IFIX(2),                    PLST0006
     3         X(450),XCDRD(225),Y(225),ICODE(225),                       PLST0007
     4         FP(450),F(450),                                            PLST0008
     5         I1(400),I2(400),I3(400),NTYPE(400),Z(400),I4(400),         PLST0009
     6         SEF(400),SET(400),EEP(400),EXPL(400),EYP(400),EXYP(400),   PLST0010
     7     NNODE,MBAND                                                    PLST0011
      II2=2*K                                                             PLST0012
      II1=II2-1                                                           PLST0013
      TAB(1,II1)=0.                                                       PLST0014
      TAB(1,II2)=0.                                                       PLST0015
      DO 20 I=1,101                                                       PLST0016
      IF(I-1)20,20,10                                                     PLST0017
   10 TAB(I,II1)=FLDAT(I-1)*CC/5.                                         PLST0018
      EET=TAB(I,II1)                                                      PLST0019
      CALL PNEW1(EEPK,EET,E,G1,E2)                                        PLST0020
      TAB(I,II2)=EEPK                                                     PLST0021
   20 CDNTINUE                                                            PLST0022
      RETURN                                                              PLST0023
      END                                                                 PLST0024
$IBFTC STIF                                                               STIF0000
      SUBRDUTINE STIFF                                                    STIF0001
      CDMMDN/ADD/  EE(10),EE1(10),EE2(10),PRR(10)                         STIF0002
      COMMON   E,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,            STIF0003
     1         N2,NELEM,KEL,ILAW,MAT,NBC,                                 STIF0004
     2         B(450,22),BC(30,3),TAB(101,20),IFIX(2),                    STIF0005
     3         X(450),XCDRD(225),Y(225),ICDDE(225),                       STIF0006
     4         FP(450),F(450),                                            STIF0007
     5         I1(400),I2(400),I3(400),NTYPE(400),Z(400),I4(400),         STIF0008
     6         SEF(400),SET(400),EEP(400),EXPL(400),EYP(400),EXYP(400),   STIF0009
     7     NNDDE,MBAND                                                    STIF0010
      DIMENSION FFP(2),INDDE(3),LNODE(6),DSK(6,6)                         STIF0011
      EQUIVALENCE (IFIX(1),IBCX),(IFIX(2),IBCY)                           STIF0012
      EQUIVALENCE(INODE(1),J1),(INODE(2),J2),(INODE(3),J3)                STIF0013
      DO 80 M=1,NELEM                                                     STIF0014
      J1=I1(M)                                                            STIF0015
      J2=I2(M)                                                            STIF0016
      J3=I3(M)                                                            STIF0017
      CALL ELEM(M)                                                        STIF0018
      NTY=NTYPE(M)                                                        STIF0019
      E=EE(NTY)                                                           STIF0020
      PR=PRR(NTY)                                                         STIF0021
      IF(J3.EQ.0) GO TO 30                                                STIF0022
      IF(NTY.LE.MAT) GO TD10                                              STIF0023
      GD TO 160                                                           STIF0024
C                                                                         STIF0025
C  **** STIFFNESS MATRIX CALCULATIONS FDR TRIANGULAR ELEMENTS ****        STIF0026
C                                                                         STIF0027
   10 JMAT=6                                                              STIF0028
      AE=E*Z(M)                                                           STIF0029
      A123=X21*Y31-X31*Y21                                                STIF0030
      A123=ABS(A123)                                                      STIF0031
      ET1=AE/(2.0*A123*(1.0-PR**2))                                       STIF0032
      ET2=AE/(4.0*A123*(1.0+PR))                                          STIF0033
      DSK(1,1)= ET1*Y32**2        +ET2*X32**2                             STIF0034
      DSK(2,1)=-ET1*PR*Y32*X32    -ET2*Y32*X32                            STIF0035
      DSK(2,2)= ET1*X32**2        +ET2*Y32**2                             STIF0036
```

```
      DSK(3,1)=-ET1*Y31*Y32         -ET2*X32*X31
      DSK(3,2)= ET1*PR*Y31*X32      +ET2*Y32*X31
      DSK(3,3)=ET1*Y31**2          +ET2*X31**2
      DSK(4,1)= ET1*PR*Y32*X31      +ET2*Y31*X32
      DSK(4,2)=-ET1*X31*X32         -ET2*Y31*Y32
      DSK(4,3)=-ET1*PR*Y31*X31      -ET2*Y31*X31
      DSK(4,4)= ET1*X31**2          +ET2*Y31**2
      DSK(5,1)= ET1*Y21*Y32         +ET2*X32*X21
      DSK(5,2)=-ET1*PR*Y21*X32      -ET2*Y32*X21
      DSK(5,3)=-ET1*Y31*Y21         -ET2*X31*X21
      DSK(5,4)= ET1*PR*Y21*X31      +ET2*Y31*X21
      DSK(5,5)= ET1*Y21**2          +ET2*X21**2
      DSK(6,1)=-ET1*PR*Y32*X21      -ET2*Y21*X32
      DSK(6,2)= ET1*X32*X21         +ET2*Y21*Y32
      DSK(6,3)= ET1*PR*Y31*X21      +ET2*Y21*X31
      DSK(6,4)=-ET1*X31*X21         -ET2*Y21*Y31
      DSK(6,5)=-ET1*PR*Y21*X21      -ET2*Y21*X21
      DSK(6,6)= ET1*X21**2          +ET2*Y21**2
      DO 20 I=1,JMAT
      DO 20 J=I,JMAT
   20 DSK(I,J)=DSK(J,I)
      GO TO 50
C
C  **** STIFFNESS MATRIX CALCULATIONS FOR BARS ****
C
   30 ET1=Z(M)*E/Y32**3
      FFP(1)=X21
      FFP(2)=Y21
      DO 40 I=1,2
      DO 40 J=1,2
      DSK(I,J)=ET1*FFP(I)*FFP(J)
      DSK(I+2,J)=-DSK(I,J)
      DSK(I,J+2)=-DSK(I,J)
   40 DSK(I+2,J+2)=DSK(I,J)
      JMAT=4
C
C  **** INCORPORATION OF ELEMENT MATRICES INTO
C                       COMPLETE STIFFNESS MATRIX ****
C
   50 JMAT2=JMAT/2
      K=0
      DO 60 I=1,JMAT2
      DO 60 J=1,2
      K=K+1
   60 LNODE(K)=2*INODE(I)-2+J
      DO 80 I=1,JMAT
      KI=LNODE(I)
      DO 80 J=1,JMAT
      KJ=LNODE(J)
      IF(KJ-KI)80,70,70
   70 K=KJ-KI+1
      B(KI,K)=B(KI,K)+DSK(I,J)
   80 CONTINUE
C
C  **** DISPLACEMENT BOUNDARY CONDITIONS ****
C
      DO 150 I=1,NNODE
      IF(ICODE(I).EQ.0) GO TO 150
      CALL DCODE(ICODE,I,IBCS,IBCX,IBCY,IC,IX,IY,NBC)
      IF(IBCS.NE.1) GO TO 110
      ALF=BC(IC,3)
      B(IX,1)=B(IX,1)+ALF*(ALF*(B(IY,1)+1.)+2.*B(IX,2))
```

```
      B(IX,2)=-ALF
      B(IY,1)=1.
      F(IX)=ALF*F(IY)+F(IX)
      F(IY)=C
      KL=IX-MBAND+2
      KU=IX+MBAND-1
      IF(KL.LT.1) KL=1
      IF(KU.GT.N2)KU=N2
      DO 100 K=KL,KU
      IF(K.EQ.IX.OR.K.EQ.IY) GO TO 100
      IF(K.GT.IY) GO TO 90
      L=IX-K+1
      B(K,L)=B(K,L)+ALF*B(K,L+1)
      B(K,L+1)=0.
      GO TO 100
   90 L=K-IX+1
      B(IX,L)=B(IX,L)+ALF*B(IY,L-1)
      B(IY,L-1)=C.
  100 CONTINUE
  110 DO 140 N=1,2
      IF(IFIX(N).NE.1) GO TO 140
      IR=IX+N-1
      ML=IR-MBAND+1
      MU=IR+MBAND-1
      IF(ML.LT.1) ML=1
      IF(MU.GT.N2)MU=N2
      DO 130 M=ML,MU
      L=IR-M+1
      IF(L.LE.1) GO TO 120
      F(M)=F(M)-B(M,L)*BC(IC,N)
      B(M,L)=0.
      GO TO 130
  120 L=M-IR+1
      F(M)=F(M)-B(IR,L)*BC(IC,N)
      B(IR,L)=0.
  130 CONTINUE
      B(IR,1)=1.
      F(IR)=BC(IC,N)
  140 CONTINUE
  150 CONTINUE
      RETURN
  160 WRITE(6,170) M
  170 FORMAT(1H010X32HINVALID ELEMENT CODE ELEMENT NO.I4)
      STOP
      END
C
$IBFTC SYMSL
      SUBROUTINE SYMSOL(KKK)
      COMMON/ADD/  EE(10),EE1(10),EE2(10),PRR(10)
      COMMON  E,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,
     1        N2,NELEM,KEL,ILAW,MAT,NBC,
     2        B(450,22),BC(30,3),TAB(101,20),IFIX(2),
     3        X(450),XCORD(225),Y(225),ICODE(225),
     4        FP(450),F(450),
     5        I1(400),I2(400),I3(400),NTYPE(400),Z(400),I4(400),
     6        SEF(400),SET(400),EEP(400),EXPL(400),EYP(400),EXYP(400),
     7    NNODE,MBAND
C
      NN=N2
      MM=MBAND
      GO TO (10,60),KKK
```

```
C                                                                        SYMS0015
C     REDUCE MATRIX                                                      SYMS0016
C                                                                        SYMS0017
   10 DO 50 N=1,NN                                                       SYMS0018
      DO 40 L=2,MM                                                       SYMS0019
      C=B(N,L)/B(N,1)                                                    SYMS0020
      I=N+L-1                                                            SYMS0021
      IF(NN-I) 40,20,20                                                  SYMS0022
   20 J=0                                                                SYMS0023
      DO 30 K=L,MM                                                       SYMS0024
      J=J+1                                                              SYMS0025
   30 B(I,J)=B(I,J)-C*B(N,K)                                             SYMS0026
   40 B(N,L)=C                                                           SYMS0027
   50 CONTINUE                                                           SYMS0028
      GO TO 130                                                          SYMS0029
C                                                                        SYMS0030
C     REDUCE VECTOR                                                      SYMS0031
C                                                                        SYMS0032
   60 DO 80 N=1,NN                                                       SYMS0033
      DO 70 L=2,MM                                                       SYMS0034
      I=N+L-1                                                            SYMS0035
      IF(NN-I) 80,70,70                                                  SYMS0036
   70 X (I)=X (I)-B(N,L)*X (N)                                           SYMS0037
   80 X (N)=X (N)/B(N,1)                                                 SYMS0038
C                                                                        SYMS0039
C     BACK SUBSTITUTION                                                  SYMS0040
C                                                                        SYMS0041
      N=NN                                                               SYMS0042
   90 N=N-1                                                              SYMS0043
      IF(N) 100,130,100                                                  SYMS0044
  100 DO 120 K=2,MM                                                      SYMS0045
      L=N+K-1                                                            SYMS0046
      IF(NN-L) 120,110,110                                               SYMS0047
  110 X (N)=X (N)-B(N,K)*X (L)                                           SYMS0048
  120 CONTINUE                                                           SYMS0049
      GO TO 90                                                           SYMS0050
C                                                                        SYMS0051
  130 RETURN                                                             SYMS0052
C                                                                        SYMS0053
      END                                                                SYMS0054
$IBFTC STRN                                                              STRN0000
      SUBROUTINE STRAIN                                                  STRN0001
      COMMON/ADD/  EE(10),EE1(10),EE2(10),PRR(10)                        STRN0002
      COMMON  E,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,            STRN0003
     1        N2,NELEM,KEL,ILAW,MAT,NBC,                                 STRN0004
     2        B(450,22),BC(30,3),TAB(101,20),IFIX(2),                    STRN0005
     3        X(450),XCORD(225),Y(225),ICODE(225),                       STRN0006
     4        FP(450),F(450),                                            STRN0007
     5        I1(400),I2(400),I3(400),NTYPE(400),Z(400),I4(400),         STRN0008
     6        SEF(400),SET(400),EEP(400),EXPL(400),EYP(400),EXYP(400),   STRN0009
     7       NNODE,MBAND                                                 STRN0010
      DO 130 K=1,NELEM                                                   STRN0011
      J1=2*I1(K)-1                                                       STRN0012
      J2=2*I1(K)                                                         STRN0013
      J3=2*I2(K)-1                                                       STRN0014
      J4=2*I2(K)                                                         STRN0015
      J5=2*I3(K)-1                                                       STRN0016
      J6=2*I3(K)                                                         STRN0017
      CALL ELEM(K)                                                       STRN0018
      NTY=NTYPE(K)                                                       STRN0019
      E=EE(NTY)                                                          STRN0020
      E1=EE1(NTY)                                                        STRN0021
```

```
      E2=EE2(   NTY)
      CC=E1/E
      PR=PRR(   NTY)
      IF(ILAW.EQ.1)           G1=(7.*E/3.)**(1./E2)*E1**(1.-1./E2)
      IF(ILAW.GT.1) G1=E1
      EPR=E/(1.0-PR*PR)
      IF(I3(K).EQ.0) GO TO 60
C
C **** TRIANGULAR ELEMENT CALCULATIONS ****
C
   10 A123=X21*Y31-X31*Y21
      SN=A123/ABS(A123)
      EXT=(-Y32*X(J1)+Y31*X(J3)-Y21*X(J5))/A123
      EYT=( X32*X(J2)-X31*X(J4)+X21*X(J6))/A123
      EXYT=(X32*X(J1)-Y32*X(J2)-X31*X(J3)+Y31*X(J4)
     1     +X21*X(J5)-Y21*X(J6))/A123
      EXE=EXT-EXPL(K)
      EYE=EYT-EYP(K)
      EXYE=EXYT-EXYP(K)
      SX=EPR*(EXE+PR*EYE)
      SY=EPR*(EYE+PR*EXE)
      SXY=E/(1.0+PR)*EXYE/2.0
      SE=SQRT(SX**2-SX*SY+SY**2+3.0*SXY**2)
      CRIT=ABS(SE)-SEF(K)
      IF(CRIT)40,40,20
   20 EET=SE/E+EEP(K)
      CALL STRSTN(EET,EEPK,SETK,NTY)
      DEEP=EEPK-EEP(K)
   30 EEP(K)=EEPK
      SET(K)=SETK
      EXPL(K)=DEEP/SE*(SX-SY/2.0)+EXPL(K)
      EYP(K)=DEEP/SE*(SY-SX/2.0)+EYP(K)
      EXYP(K)=3.0*DEEP/SE*SXY+EXYP(K)
      ERR=E*DEEP/SE
      ERR=ABS(ERR)
      GO TO 50
   40 ERR=0.0
   50 CONTINUE
      Q1=E*Z(K)/(1.0-PR**2)/2.0*SN
      Q2=E*Z(K)/(1.0+PR)/4.0*SN
      EXPT=EXPL(K)
      EYPT=EYP(K)
      EXYPT=EXYP(K)
      FP(J1)=-Q1*Y32*EXPT-Q1*Y32*PR*EYPT+Q2*X32*EXYPT+FP(J1)
      FP(J2)= Q1*X32*PR*EXPT+Q1*X32*EYPT-Q2*Y32*EXYPT+FP(J2)
      FP(J3)= Q1*Y31*EXPT+Q1*Y31*PR*EYPT-Q2*X31*EXYPT+FP(J3)
      FP(J4)=-Q1*X31*PR*EXPT-Q1*X31*EYPT+Q2*Y31*EXYPT+FP(J4)
      FP(J5)=-Q1*Y21*EXPT-Q1*Y21*PR*EYPT+Q2*X21*EXYPT+FP(J5)
      FP(J6)= Q1*X21*PR*EXPT+Q1*X21*EYPT-Q2*Y21*EXYPT+FP(J6)
      GO TO 110
C
C **** BAR CALCULATIONS ****
C
   60 EET=(X21*(X(J3)-X(J1))+Y21*(X(J4)-X(J2)))/Y32**2
      STRN=ABS(EET-EXPL(K))
      SIGN=(EET-EXPL(K))/STRN
      SE=E*STRN
      CRIT=SE-SEF(K)
      EET=STRN+EEP(K)
      IF(CRIT)90,90,70
   70 CONTINUE
      CALL STRSTN(EET,EEPK,SETK,NTY)
```

```
      DEEP=EEPK-EEP(K)                                                  STRN0084
   80 EEP(K)=EEPK                                                       STRN0085
      SET(K)=SETK                                                       STRN0086
      EXPL(K)=EXPL(K)+SIGN*DEEP                                         STRN0087
      ERR=E*DEEP/SE                                                     STRN0088
      ERR=ABS(ERR)                                                      STRN0089
      GO TO 100                                                         STRN0090
   90 ERR=0.0                                                           STRN0091
  100 CONTINUE                                                          STRN0092
      EXPT=EXPL(K)                                                      STRN0093
      Q1=E*Z(K)/Y32                                                     STRN0094
      FP(J1)=FP(J1)-Q1*X21*EXPT                                         STRN0095
      FP(J2)=FP(J2)-Q1*Y21*EXPT                                         STRN0096
      FP(J3)=FP(J3)+Q1*X21*EXPT                                         STRN0097
      FP(J4)=FP(J4)+Q1*Y21*EXPT                                         STRN0098
  110 IF(ERR-XERR)130,130,120                                           STRN0099
  120 XERR=ERR                                                          STRN0100
      KEL=K                                                             STRN0101
  130 CONTINUE                                                          STRN0102
      RETURN                                                            STRN0103
      END                                                               STRN0104
$IBFTC STRST                                                            STRS0000
      SUBROUTINE STRSTN(EET,EEPK,SETK,NTY)                              STRS0001
      COMMON/ADD/  EE(10),EE1(10),EE2(10),PRR(10)                       STRS0002
      COMMON   E,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,          STRS0003
     1         N2,NELEM,KEL,ILAW,MAT,NBC,                               STRS0004
     2         B(450,22),BC(30,3),TAB(101,20),IFIX(2),                  STRS0005
     3         X(450),XCORD(225),Y(225),ICODE(225),                     STRS0006
     4         FP(450),F(450),                                          STRS0007
     5         I1(400),I2(400),I3(400),NTYPE(400),Z(400),I4(400),       STRS0008
     6         SEF(400),SET(400),EEP(400),EXPL(400),EYP(400),EXYP(400), STRS0009
     7    NNODE,MBAND                                                   STRS0010
      GO TO(10,50,60),ILAW                                              STRS0011
   10 J=5.0*EET/CC+1.0                                                  STRS0012
      NT2=2*NTY                                                         STRS0013
      NT1=NT2-1                                                         STRS0014
      IF(J-101)20,30,30                                                 STRS0015
   20 EEPK=TAB(J,NT2)+(TAB(J+1,NT2)-TAB(J,NT2))*(EET-TAB(J,NT1))/       STRS0016
     1(TAB(J+1,NT1)-TAB(J,NT1))                                         STRS0017
      GO TO 40                                                          STRS0018
   30 CALL PNEW1(EEPK,EET,E,G1,E2)                                      STRS0019
   40 SETK=G1*EEPK**(1.0/E2)                                            STRS0020
      RETURN                                                            STRS0021
   50 SETK=E*EET/(1.+(ABS(E*EET/G1))**E2)**(1./E2)                      STRS0022
      EEPK=EET-SETK/E                                                   STRS0023
      RETURN                                                            STRS0024
   60 EC=G1/E                                                           STRS0025
      IF(EET-EC)70,70,80                                                STRS0026
   70 EEPK=0.                                                           STRS0027
      SETK=E*EET                                                        STRS0028
      RETURN                                                            STRS0029
   80 EEPK=EET-EC                                                       STRS0030
      SETK=G1+E2*EEPK                                                   STRS0031
      RETURN                                                            STRS0032
      END                                                               STRS0033
$IBFTC OTPUT                                                            OTPU0000
C12345          OUTPUT SUBROUTINE                                       OTPU0001
      SUBROUTINE OUTPT                                                  OTPU0002
      COMMON/ADD/  EE(10),EE1(10),EE2(10),PRR(10)                       OTPU0003
      COMMON   E,CC,G1,E2,PR,EPR,X21,Y21,X31,Y31,X32,Y32,XERR,          OTPU0004
     1        NN2,NELEM,KEL,ILAW,MAT,NBC,                               OTPU0005
     2         B(450,22),BC(30,3),TAB(101,20),IFIX(2),                  OTPU0006
```

```
     3        X(450),XCORD(225),Y(225),ICODE(225),
     4        FP(450),F(450),
     5        I1(400),I2(400),I3(400),NTYPE(400),Z(400),I4(400),
     6        SEF(400),SET(400),EEP(400),EXPL(400),EYP(400),EXYP(400),
     7       NNODE,MBAND
      L=0
      DO 70 K=1,NELEM
C
C   **** TRIANGULAR ELEMENT CALCULATIONS ****
C
      NTY=NTYPE(K)
      E=EE(NTY)
      PR= PRR(NTY)
      EPR=E/(1.-PR*PR)
      IF(I3(K).EQ.0) GO TO 70
   10 CALL ELEM(K)
      A123=X21*Y31-X31*Y21
      J1=2*I1(K)-1
      J2=2*I1(K)
      J3=2*I2(K)-1
      J4=2*I2(K)
      J5=2*I3(K)-1
      J6=2*I3(K)
      EXT=(-Y32*X(J1)+Y31*X(J3)-Y21*X(J5))/A123
      EYT=( X32*X(J2)-X31*X(J4)+X21*X(J6))/A123
      EXYT=(X32*X(J1)-Y32*X(J2)-X31*X(J3)+Y31*X(J4)
     1    +X21*X(J5)-Y21*X(J6))/A123
      EXE=EXT-EXPL(K)
      EYE=EYT-EYP(K)
      EXYE=EXYT-EXYP(K)
      SX=EPR*(EXE+PR*EYE)
      SY=EPR*(EYE+PR*EXE)
      SXY=E/(1.0+PR)*EXYE/2.0
      PE2=SQRT((.5*(SX-SY))**2+SXY**2)
      PHI=.5*ATAN2((-2.0*SXY),(SX-SY))*57.29578
      PH2=.5*ATAN2((-EXYT),(EXT-EYT))*57.29578
      PS1=.5*(SX+SY)
      SIGE1=PS1+PE2
      SIGE2=PS1-PE2
      PST1=.5*(EXT+EYT)
      PET2=SQRT((.5*(EXT-EYT))**2+EXYT**2/4.0)
      STRE1=PST1+PET2
      STRE2=PST1-PET2
      PET2=2.0*PET2
      N1=I1(K)
      N2=I2(K)
      N3=I3(K)
      XC=XCORD(N1)+(X21+X31)/3.0
      YC=Y(N1)+(Y21+Y31)/3.0
      EPE=2.*SQRT((EXPL(K)**2+EXPL(K)*EYP(K)+EYP(K)**2+EXYP(K)**2/4.)/3.
     1)
      SE=SQRT(SX**2-SX*SY+SY**2+3.*SXY**2)
      L=L+1
      IF(MOD(L-1,14))30,20,30
   20 WRITE (6,40)
   30 WRITE(6,50)K,XC,YC,SX,SY,SXY,SIGE1,SIGE2,PE2,PHI,PH2,EXT,EYT,   EX
     1YT,STRE1,STRE2,PET2
   40 FORMAT(9H1EL. NO./5X11HCOORDINATES28X33HS T R E S S E S / S T R A
     1I N S  /8H      PHI7X1HX8X1HY6X9H    TAU-XX6X9H    TAU-YY6X9H    TAU-X
     2Y8X7HMAXIMUM8X7HMINIMUM6X9HMAX SHEAR )
   50 FORMAT(1H0I7,0PF8.3,F9.3,1P6E15.4/1H 0PF7.2,F8.2,9X1P6E15.4)
      WRITE(6,60) SE,EPE
```

```
   60 FORMAT(1H 23H***** EFFECTIVE STRESS=E12.5,23H***** EFFECTIVE STRAI
     1N=E12.5)
   70 CONTINUE
C
C    **** BAR CALCULATIONS ****
C
      J=0
      DO 120 K=1,NELEM
      NTY=NTYPE(K)
      E=EE(   NTY)
      PR=PRR(   NTY)
      J1=2*I1(K)-1
      J2=2*I1(K)
      J3=2*I2(K)-1
      J4=2*I2(K)
      IF(I3(K).NE.0) GO TO 120
   80 CALL ELEM(K)
      EET=(X21*(X(J3)-X(J1))+Y21*(X(J4)-X(J2)))/Y32**2
      SE=E*(EET-EXPL(K))
      SEMZK=SE*Z(K)
      K1=I1(K)
      K2=I2(K)
      IF(J)100,90,100
   90 WRITE (6,130)
      J=1
  100 CONTINUE
  110 WRITE(6,140)K,K1,K2,SE,EET,SEMZK
  120 CONTINUE
  130 FORMAT(9H0BAR NO. 6X10HNODE NOS. 8X7HSTRESS 8X7HSTRAIN,4X,
     113HMEMBER FORCES)
  140 FORMAT(1H0 3I8,2E15.5,2X,E15.5)
      RETURN
      END
$DATA
   NONLINEAR TRUSS PROC. ASCE DEC. 1965 ONE MATERIAL RAMBERG OSGOOD LAW
   6  10     1     1     1       22    30
 0.10000000E+05   40.50000     0.3          7.0
   10    20     1     1   10      1     .01
    1     1     1
    2                    30.
    3                    60.
    4     1     1                      40.
    5                    30.          40.
    6                    60.           40.
    1     4     5     0     1      .25
    2     4     2     0     1     .20
    3     1     5     0     1     .20
    4     1     2     0     1     .25
    5     2     5     0     1     .20
    6     5     6     0     1     .25
    7     5     3     0     1     .20
    8     2     6     0     1     .20
    9     2     3     0     1      .25
   10     3     6     0     1     .20
    3                -10.
   NONLINEAR TRUSS PROC. ASCE DEC. 1965 TWO MATERIALS RAMBERG OSGOOD LAW
   6  10     1     0     2       22    30
 0.10000000E+05   65.00000     0.3          8.0
 0.10000000E+05   45.00000     0.3          8.0
   10    20     1     1   10      1     .01
    1     1     1
    2                    30.
```

```
   3                    60.
   4    1    1                      40.
   5                    30.        40.
   6                    60.         40.
   1    4    5    0    1    .25
   2    4    2    0    2   .20
   3    1    5    0    2   .20
   4    1    2    0    1   .25
   5    2    5    0    1   .20
   6    5    6    0    1   .25
   7    5    3    0    2   .20
   8    2    6    0    2   .20
   9    2    3    0    1    .25
  10    3    6    0    1   .20
   3              -10.
  MIT SHEAR LAG PROBLEM  CONFIGURATION 1
  49   78    2              1   22   30
0.10200000E+08 52000.       0.3          5.
  10   20    1    1         1  0.01
   1    1    1   0.0
   2         1   0.5
   3         1   1.0
   4         1   1.5
   5         1   2.0
   6    1        0.0         0.5
   7             0.5         0.5
   8             1.0         0.5
   9             1.5         0.5
  10             2.0         0.5
  11         1   3.0
  12         1   4.0
  13         1   5.0
  14    1        0.0         1.0
  15             0.5         1.0
  16             1.0         1.0
  17             1.5         1.0
  18             2.0         1.0
  19             3.0         1.0
  20             4.0         1.0
  21             5.0         1.0
  22    1        0.0         2.0
  23             0.5         2.0
  24             1.0         2.0
  25             2.0         2.0
  26             3.0         2.0
  27             4.0         2.0
  28             5.0         2.0
  29    1        0.0         3.0
  30             0.5         3.0
  31             1.0         3.0
  32             2.0         3.0
  33             3.0         3.0
  34             4.0         3.0
  35             5.0         3.0
  36    1        0.0         4.0
  37             0.5         4.0
  38             1.0         4.0
  39             2.0         4.0
  40             3.0         4.0
  41             4.0         4.0
  42             5.0         4.0
  43    1        0.0         5.0
```

44			0.5			5.0
45			1.0			5.0
46			2.0			5.0
47			3.0			5.0
48			4.0			5.0
49			5.0			5.0
1	1	2	6	1	.08	
2	2	6	7	1	.08	
3	2	3	7	1	.08	
4	3	7	8	1	.08	
5	3	4	8	1	.08	
6	4	8	9	1	.08	
7	4	5	9	1	.08	
8	5	9	10	1	.08	
9	5	10	11	1	.08	
10	10	11	19	1	.08	
11	11	12	19	1	.08	
12	12	19	20	1	.08	
13	12	13	20	1	.08	
14	13	20	21	1	.08	
15	6	7	14	1	.08	
16	7	14	15	1	.08	
17	7	8	15	1	.08	
18	8	15	16	1	.08	
19	8	9	16	1	.08	
20	9	16	17	1	.08	
21	9	10	17	1	.08	
22	10	17	18	1	.08	
23	10	18	19	1	.08	
24	14	15	22	1	.08	
25	15	22	23	1	.08	
26	15	16	23	1	.08	
27	16	23	24	1	.08	
28	16	17	24	1	.08	
29	17	24	25	1	.08	
30	17	18	25	1	.08	
31	18	19	25	1	.08	
32	19	25	26	1	.08	
33	19	20	26	1	.08	
34	20	26	27	1	.08	
35	20	21	27	1	.08	
36	21	27	28	1	.08	
37	22	23	29	1	.08	
38	23	29	30	1	.08	
39	23	24	30	1	.08	
40	24	30	31	1	.08	
41	24	25	31	1	.08	
42	25	31	32	1	.08	
43	25	26	32	1	.08	
44	26	32	33	1	.08	
45	26	27	33	1	.08	
46	27	33	34	1	.08	
47	27	28	34	1	.08	
48	28	34	35	1	.08	
49	29	30	36	1	.08	
50	30	36	37	1	.08	
51	30	31	37	1	.08	
52	31	37	38	1	.08	
53	31	32	38	1	.08	
54	32	38	39	1	.08	
55	33	39	32	1	.08	
56	33	39	40	1	.08	

```
      57   33   34   40     1 .08
      58   34   40   41     1 .08
      59   34   35   41     1 .08
      60   35   41   42     1 .08
      61   36   37   43     1 .08
      62   37   43   44     1 .08
      63   37   38   44     1 .08
      64   38   44   45     1 .08
      65   38   39   45     1 .08
      66   39   45   46     1 .08
      67   39   40   46     1 .08
      68   40   46   47     1 .08
      69   40   41   47     1 .08
      70   41   47   48     1 .08
      71   41   42   48     1 .08
      72   42   48   49     1 .08
      73    1    6          10.0452
      74    6   14          10.0754
      75   14   22          10.15
      76   22   29          10.25
      77   29   36          10.35
      78   36   43          10.45
      43             10000.
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UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Air Force Flight Dynamics Laboratory Wright-Patterson Air Force Base, Ohio	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

AN APPLICATION OF THE FINITE ELEMENT METHOD TO ELASTIC-PLASTIC PROBLEMS OF PLANE STRESS

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

5. AUTHOR(S) (First name, middle initial, last name)

Salmon, M.
Berke, L.
Sandhu, R.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
April 1970	57	8

8a. CONTRACT OR GRANT NO. b. PROJECT NO. 1467 c. Task No. 146701 d.	9a. ORIGINATOR'S REPORT NUMBER(S) AFFDL-TR-68-39 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

This document has been approved for public release and sale; its distribution is unlimited. In DDC. Aval frm CFSTI.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Air Force Flight Dynamics Laboratory Wright-Patterson Air Force Base, Ohio 45433

13. ABSTRACT

A computer program is presented for the small strain analysis of plane structures in the strain hardening elastic-plastic range. The finite element displacement method is used to perform the linear analyses in the iterative scheme. Bar and constant strain isotropic plane stress triangles are available for use in constructing structural idealizations. The use of ten different sets of material properties, three different material laws, and incremental proportional loading are available as options. Good correlation is shown with available test data and theoretical solutions.

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14. KEY WORDS	LINK A		LINK B		LINK C	
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Elasticity						
Plasticity						
Structural Analysis						
Displacement Method						
Finite Element Method						

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