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MEMORANDUM

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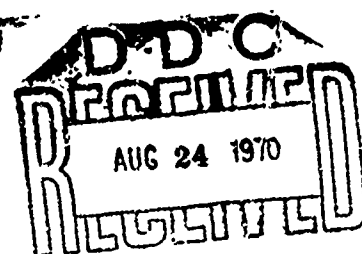
MAY 1970

A SIMPLIFIED WEAPONS EVALUATION MODEL

Roger Snow and Margaret Ryan

PREPARED FOR:

UNITED STATES AIR FORCE PROJECT RAND



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PREFACE

This Memorandum, part of Rand's research on the use of airpower in support of ground operations, is one of a series of studies concerned with the prediction of damage to a single or multiple targets. Aspects of the target-weapon relationship, target-weapon errors, and the coverage problem were considered in RM-4566-PR, *FAST-VAL: A Theoretical Approach to Some General Target Coverage Problems*, March 1966 (For Official Use Only); the computer program, originally designed to compute inputs to the Rand FAST-VAL (Forward Air Strike Evaluation) simulation model, was described in a companion study, RM-4567-PR, *FAST-VAL: Target Coverage Model*, March 1966 (For Official Use Only).

From the point of view of production computations, however, the target coverage program had two serious limitations: the length of computer time required to make the computations, and dependence of precision on the size of the integration cell, which in turn depends on the machine capacity available. To alleviate these problems, a model was designed that replaced the empirical damage function used in the general model with a simpler and far less time-consuming analytic expression. The results of this work were described in an interim reference report, RM-5152-PR, *A Simplified Target Coverage Model*, November 1967 (For Official Use Only). As these results have been incorporated in the present study, RM-5152-PR has been withdrawn and replaced by this Memorandum.

The work of simplification continues with the present study, which describes a model that is sufficiently broad to cover almost all problems that arise in the field of nonnuclear weapons evaluation and a machine program that is designed to make coverage computations for some of the problems considered in RM-4567-PR in a fraction of the time previously needed. Without materially reducing the accuracy of the answers, the computer time required for some problems has been reduced by as much as a factor of 100.

After continued use of the model at Rand and at the Air Force Armament Laboratory, further refinements to the program to make it more efficient both as to

time and accuracy were deemed desirable. An improved method for determining the integration step size was implemented with a considerable increase in performance. The method of fitting an empirical bomblet distribution to an analytical distribution used in the program has been changed so that any smoothing necessary in irregular data is done by the machine rather than by the user, thus avoiding possible input errors. The process of using a multiple wing station configuration was also simplified. A direct input is now provided for the numbers of wing stations and the numbers of weapons per impulse per station.

SUMMARY

Two restrictions permit a simplification of the target coverage model and computer program described in *FAST-VAL: A Theoretical Approach to Some General Target Coverage Problems* [1] and in *FAST-VAL: Target Coverage Model* [2]. The problems are restricted to the case of a gaussian aiming error distribution and a rectangular target area (uniform distribution of the target elements in the area). As a result it is possible to reduce the coverage computations to two stages, each involving a double integration, in contrast to the three stages required in the original model. The second restriction concerns both the assumed form of the damage function and the ballistic error distribution. It is necessary that (1) the damage function be an analytic function, rather than an empirical function; (2) the ballistic error distribution be one of three types: gaussian, uniform, or stick* type; and (3) the damage function be integrable in a closed form with respect to the ballistic error distribution. Under these restrictions, the coverage computations are reduced to a single stage, involving only one double integration. For some cases, it is possible to reduce the problem to a summation of functions in closed form with no integration necessary.

Two types of damage functions are considered, corresponding to two different types of weapon-target effects: a fragment-sensitive target, or one in which the major damage mechanism is due to fragments rather than to a direct impact by the weapon; and an impact-sensitive target, or one for which there is a definite geometric figure that must be impacted by the weapon. For the fragment-sensitive target, the empirical function that is usually obtained from a computer program using fragmentation data is replaced by an analytic function, the "gaussian damage function," fitted through the choice of three parameters. For an impact-sensitive target, it is assumed that the target element is a rectangle, and that there is a fixed probability of damage, given a hit on the element. Under these assumptions, the damage function is exact.

* Stick distribution is defined in App. B.

The two types of damage function, the aiming and ballistic error distributions, and the basic coverage relation are considered in turn, and the expected coverage $K(X,Y)$ is expressed as one double integration in terms of the damage function and the various forms of the aiming and ballistic error distribution. For various coverage problems that occur in practice, an explicit expression for the coverage is derived in terms of the pertinent parameters. The set of formulas developed for the coverage function provides an answer to the weapon-target effectiveness problem that corresponds to most of the current weapon delivery systems.

The FORTRAN program computes numerical answers for each of the target coverage problems. The output of the program is the particular value of K depending on the parameters considered, such as aiming errors ballistic errors, spacing, and weapon-target effectiveness indices.

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1. INTRODUCTION

A number of general target coverage problems were considered in *FAST-VAL: A Theoretical Approach* [1]. The companion report, *FAST-VAL: Target Coverage Model* [2] presents a machine program designed to make coverage computations for some of the problems formulated in Ref. 1. However, from the point of view of production computations, the target coverage program described in Ref. 1 and Ref. 2 suffers from two serious drawbacks: lengthy computation time and dependence of precision on integration step cell size. In some cases a single problem may require 20 minutes of computer time, and the cell size used is usually limited by the memory capacity of the machine as well as the time element in computation. To alleviate these problems, a model was designed that replaced the empirical damage function used in the general model with a simpler and far less time-consuming analytic expression. The results of this work are described in Ref. 3, *A Simplified Target Coverage Model*, and have been incorporated in the present study.*

Continuing the work of simplification, the present study describes a machine program designed to make coverage computations for some of the problems considered in Ref. 2 in a fraction of the time previously needed. The program in Ref. 2 accomplishes the coverage computations in three stages, each involving a double integration. In contrast, this study restricts the problems to the case of a gaussian aiming error distribution and a rectangular target area (uniform distribution of the target elements in the area). As a result, it is possible to reduce the coverage computations to two stages, each involving a double integration. For most problems, these restrictions are acceptable, and assumptions of a gaussian aiming error distribution and of a rectangular target area are usually valid. The second simplification in this study concerns both the assumed form of the damage function and the ballistic error distribution. It is necessary that (1) the damage function be an analytic function, rather than an empirical function; (2) the ballistic error

*Reference 3 will be withdrawn and replaced by this Memorandum.

distribution be one of three types: gaussian, uniform, or stick type; and (3) the damage function be integrable in closed form with respect to the ballistic error distribution. Under these restrictions, the coverage computations are reduced to a single stage, involving only one double integration. For some cases it is possible to reduce the problem to a summation of functions in closed form with no integration necessary.

Two types of damage functions are considered, corresponding to two different types of weapon-target effects: a fragment-sensitive target and an impact-sensitive target. For the fragment-sensitive target, the damage function used in the program in Ref. 2 is an empirical function that is usually obtained from a computer program using the fragmentation data (size, velocity, and distribution of fragments) from arena tests. The simplification used here is to replace this matrix function with an analytic function, the "gaussian damage function" [1], fitted through the choice of three parameters. The form of this function fits most of the empirical nonnuclear weapon-effects tables very well. The use of this function is not new. It has been used in work at Sandia Corporation, where it is called the "casualty function." A Ballistic Research Laboratories report [4] considered the function for the special case of an elliptical target. Current work both at the Strategic Air Command and the Naval Weapons Laboratory has involved use of this function. It also occurs naturally as an approximation to the circular coverage function, where it has been referred to as the "Carleton" approximation. In other uses, it has been called the "Carleton" function.

For an impact-sensitive target, i.e., one for which there is a definite geometric figure that must be impacted by the weapon, it is assumed that the target element is a rectangle, and that there is a fixed probability of damage, given a hit on the element. Under these assumptions, the damage function is exact, i.e., the same as would be used in the program of Ref. 2.

Section 2, "Basic Functions," considers in turn the two types of damage functions, the aiming and ballistic error distributions to be used and the basic coverage relation, which expresses the expected coverage $K(X,Y)$ as one double integration

in terms of the damage function and the various forms of the aiming and ballistic error distributions.

Sections 3 and 4 consider various coverage problems that occur in practice. For each of these, there is derived an explicit expression for the coverage $K(X,Y)$ in terms of the pertinent parameters. Section 3 examines the ripple delivery of weapons with gaussian distribution of both the ballistic and aiming errors (air delivered bombs). Included is one case of an elliptical target area for a fragment-sensitive target, since the methods used here are applicable to this case. Various types of dispenser delivery of weapons are considered in Sec. 4. Section 4.1 discusses deliveries for fixed dispensers such as the SUU-7 and SUU-14 (CBU-1, 2, 3, 7, etc.). For these cases, the ballistic distribution in range is usually available as a table obtained from test data and is fed into the model of Ref. 2 in this form. For use in the model in this Memorandum, this empirical distribution is fitted by a stick distribution through the use of two parameters. Section 4.2 considers the case of a ripple of dispensers, released from the carrier, each of which spreads its subweapons uniformly over a rectangular pattern (Hayes dispenser). A ripple of dispensers, released from the carrier, each of which spreads its subweapons uniformly over an ellipse (Rock-eye), is described in Sec. 4.3. Section 4.4 examines the same case for dispensers that spread their subweapons uniformly over an elliptical annulus (self-dispersing subweapons, such as the CBU-24).

Sections 3 and 4 thus derive a set of formulas for the coverage function $K(X,Y)$ that provide the answer to the weapon-target effectiveness problem corresponding to most of the current weapon delivery systems. Section 5 describes a machine program that computes numerical answers for each of these problems. The output of the machine program is the particular value of K depending on the parameters considered (aiming errors, ballistic errors, spacing, weapon-target effectiveness indices, etc.) and is designated by $X(j)$, where j is an index that designates the particular type of problem: $X(1)$, $X(11)$, $X(5)$, and $X(15)$ are answers to the problems considered in Sec. 3, air-delivered bombs; $X(2)$, $X(12)$, $X(3)$,

X(13), X(4), and X(14) are answers to the problems in Sec. 4.1, deliveries from fixed dispensers; X(103), X(104), X(105), and X(106) are answers to problems in Sec. 4.2, dispensers with a uniform pattern over a rectangle; X(107), X(108), and X(109) are answers to problems in Sec. 4.3, dispensers with a uniform pattern over an ellipse; X(100), X(101), X(102), X(110), X(111), and X(112) are answers to problems considered in Sec. 4.4, dispensers with a uniform pattern over an annulus. Each pertinent formula in Secs. 3 and 4 is designated by the proper X designation to correlate it with the corresponding computer output.

Section 5, a description of the machine program, was planned for the person interested only in running the program and can be used without necessarily referring to the previous sections. Thus, some description of the various problems considered in Secs. 3 and 4 is repeated.

2. BASIC FUNCTIONS

2.1. DAMAGE FUNCTIONS

We will be concerned with two types of damage functions, depending on the weapon-target damage mechanism: (1) a fragment-sensitive target and (2) an impact-sensitive target with a defined area in which a hit is necessary for damage, i.e., the probability of damage if hit, p_{dk} , may be less than 1.

2.1.1. Fragment-sensitive Gaussian Damage Function

As the name implies, a fragment-sensitive weapon-target case is one in which the major damage mechanism is due to fragments rather than to a direct impact by the weapon. In general, the damage function will be computed in matrix form. We will assume the form of the damage function to be

$$(2.1) \quad D(x,y) = D_0 \exp\left(-D_0 \left[\frac{x^2}{R^2(1)} + \frac{y^2}{R^2(2)} \right]\right),$$

where $D(x,y)$ is the probability that a target element at (x,y) is "killed" (damaged at least to a specified degree), and $R(1)$, $R(2)$, and D_0 are parameters to be obtained from empirical data.

In Ref. 1, Sec. 1.6.3, certain parameters are discussed that may be used to characterize a damage function. The weapon radius R is defined as

$$R^2 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{D(x,y) dx dy}{\pi}.$$

Using (2.1), we find that

$$(2.2) \quad R^2 = R(1)R(2).$$

Along any ray θ , the mean square radial damage distance $\overline{r^2}(\theta)$ is, if (2.1) is used,

$$(2.3) \quad \overline{r^2}(\theta) = 2 \int_0^{\infty} r D(r \cos \theta, r \sin \theta) dr$$

$$= \frac{1}{\chi^2(\theta)},$$

where

$$\chi^2(\theta) = \frac{\cos^2 \theta}{R^2(1)} + \frac{\sin^2 \theta}{R^2(2)}.$$

Letting $\theta=0^\circ$ and $\theta=90^\circ$, we find that

$$R^2(1) = \int_0^\infty 2xD(x,0)dx = \overline{r^2}(0^\circ),$$

(2.4)

$$R^2(2) = \int_0^\infty 2yD(0,y)dy = \overline{r^2}(90^\circ).$$

Thus, $R(1)$ and $R(2)$ in (2.1) are simply the mean square radial damage distances along the x and y axes, respectively.

Considering the mean radial damage distance $\overline{r}(\theta)$ and using (2.1), we obtain

$$\overline{r}(\theta) = \int_0^\infty D(r \cos \theta, r \sin \theta) dr$$

(2.5)

$$= \frac{\sqrt{\pi D_0}}{2\chi(\theta)}.$$

The relative damage variance $\Sigma^2(\theta)$ is

$$\Sigma^2(\theta) = 1 - \frac{[\overline{r}(\theta)]^2}{R^2(\theta)} = 1 - D_0 \frac{\pi}{4},$$

(2.6)

$$D_0 = \frac{4[1 - \Sigma^2(\theta)]}{\pi}.$$

Thus, we can view the three parameters in (2.1) as the triplet $[R(1), R(2), \Sigma]$, where $R(1)$ and $R(2)$ are the mean square radial distances along the x and y axes, and Σ^2 is the relative damage variance. Instead of $R(1)$ and $R(2)$, we can use R and ρ , considering the triplet (R, ρ, Σ) , where

$$(2.7) \quad R^2 = R(1)R(2),$$

$$\rho = \frac{R(1)}{R(2)}.$$

The parameter R is the weapon radius as in (2.2). The function D is an elliptic damage function in that constant probability contours are all ellipses with the same eccentricity, $\sqrt{1-R^2(1)/R^2(2)}$. The parameter ρ thus characterizes the eccentricity of the ellipses, i.e., the eccentricity is equal to $\sqrt{1-\rho^2}$. The parameter Σ characterizes the spread or dispersion of the damage function.

2.1.2. Impact-sensitive Damage Function

We define an impact-sensitive target to be one for which there is a definite geometric figure that must be impacted by the weapon or subweapon; i.e., there is a p_{dk} , the probability of damage if hit, that may be less than 1. In general, we will use a rectangle of half dimensions $B(1)$ and $B(2)$. The damage function D is thus

$$(2.8) \quad D(x,y) = p_{dk} \alpha(x, B(1)) \alpha(y, B(2)),$$

where $\alpha(x, B)$ is the cookie-cutter function

$$(2.9) \quad \alpha(x, B) = 1 \quad \text{if} \quad -B \leq x \leq B \\ = 0 \quad \text{otherwise.}$$

We will consider x to be the range direction and y to be the deflection direction. In this study, odd subscripts will refer to dimensions in the x direction and even subscripts to dimensions in the y direction.

2.2. DELIVERY ERRORS

In general, we shall consider the case of multiple delivery of weapons. Each weapon is assumed to be subject to a ballistic error in range and deflection, expressed by a distribution function $T(x,y)$, independent of the other weapons. The whole delivery is considered to be subject to aiming errors in range and deflection, expressed by another distribution function.

2.2.1. Aiming Error Distributions

We shall confine our attention to the case of the aiming error distribution having the form of a noncorrelated bivariate gaussian, i.e.,

$$(2.10) \quad \text{Prob}(x \leq X, y \leq Y) = \int_{-\infty}^X \int_{-\infty}^Y \exp\left(-\frac{1}{2} \left[\frac{x^2}{t^2(1)} + \frac{y^2}{t^2(2)} \right]\right) \frac{dx dy}{2\pi t(1)t(2)},$$

where $t(1)$ and $t(2)$ are the standard deviations in the x and y directions, respectively. For convenience, we define the linear gaussian distribution $G(X)$ and its density function $g(X)=G'(X)$ as follows:

$$(2.11) \quad G(X) = \int_{-\infty}^X g(x) dx = \int_{-\infty}^X \exp\left(-\frac{x^2}{2}\right) \frac{dx}{\sqrt{2\pi}},$$

$$g(x) = \frac{\exp(-x^2/2)}{\sqrt{2\pi}}.$$

Thus, Eq. (2.10) can be written as

$$(2.12) \quad \text{Prob}(x \leq X, y \leq Y) = G\left(\frac{X}{t(1)}\right) G\left(\frac{Y}{t(2)}\right).$$

If we wish to use REP (range probable error) and DEP (deflection probable error), the relations between them and the standard deviations are

$$(2.13) \quad \text{REP} = .6744 t(1),$$

$$\text{DEP} = .6744 t(2).$$

Further, if $t(1)=t(2)=t$, CEP (circular probable error) is given by

$$(2.14) \quad \text{CEP} = 1.1774 t = 1.7459 \text{REP}.$$

2.2.2. Ballistic Error Distributions

We shall assume that the ballistic errors are independent in the x and y directions. Therefore, the ballistic error distribution $T(X,Y)$ is of the form $T(X,Y)=T_1(X)T_2(Y)$. We shall consider three types of linear ballistic error distributions: gaussian, uniform, and stick.

a. Gaussian Ballistic Error. As for the aiming error, we assume for the first ballistic error a linear gaussian error distribution, i.e.,

$$T_1(X) = G\left(\frac{X}{s(1)}\right) = \int_{-\infty}^X \exp\left(-\frac{1}{2} \frac{x^2}{s^2(1)}\right) \frac{dx}{\sqrt{2\pi}s(1)}, \quad (2.15)$$

$$T_2(Y) = G\left(\frac{Y}{s(2)}\right).$$

If the errors are gaussian in both directions, we have

$$T(X, Y) = G\left(\frac{X}{s(1)}\right) G\left(\frac{Y}{s(2)}\right). \quad (2.16)$$

b. Uniform Ballistic Error. For the second ballistic error, we assume a uniform distribution, i.e.,

$$T_1(X) = \int_{-L}^X \alpha(x, L) \frac{dx}{2L}, \quad (2.17)$$

where $\alpha(x, L)$ is defined in (2.9). A similar expression holds in the y direction.

c. Stick Ballistic Error. A third type of ballistic error is the so-called stick type, which is a combination of a uniform and a gaussian distribution, discussed in App. B. As given in (B4), we have

$$T_1(X) = H\left(\frac{X}{s}, \frac{L}{s}\right) = \int_{-\infty}^X h\left(\frac{u}{s}, \frac{L}{s}\right) \frac{du}{s}, \quad (2.18)$$

where

$$H(X, L) = \int_{-\infty}^X h(x, L) dx, \quad (2.19)$$

$$h(x, L) = \frac{1}{2L} \int_{-L}^L g(x+y) dy.$$

If Lemma 3 in App. D is used, the function $H(X, L)$ can be evaluated, for $L \neq 0$, as

$$H(X, L) = (X+L)G(X+L) - (X-L)G(X-L) + g(X+L) - g(X-L), \quad (2.20)$$

where $G(X)$ and $g(x)$ are defined in (2.11).

We note the following limits for h and H :

$$\begin{aligned}
 (2.21) \quad & \lim_{L \rightarrow 0} \frac{h(x/s, L/s)}{s} = \frac{g(x/s)}{s}, \\
 & \lim_{s \rightarrow 0} \frac{h(x/s, L/s)}{s} = \frac{\alpha(x, L)}{2L}, \\
 & \lim_{L \rightarrow 0} H\left(\frac{X}{s}, \frac{L}{s}\right) = G\left(\frac{X}{s}\right), \\
 & \lim_{s \rightarrow 0} H\left(\frac{X}{s}, \frac{L}{s}\right) = \int_{-L}^X \frac{\alpha(x, L)}{2L} dx.
 \end{aligned}$$

2.3. THE BASIC COVERAGE RELATIONS

Define $K(X, Y)$ as the fractional coverage to a target area with center at (X, Y) inflicted by a pattern of weapons subject to a gaussian aiming error with standard deviations $t(1)$ and $t(2)$. The pattern damage function $D_p(u, v)$ is the kill probability to a target element at (u, v) if the center of the pattern is at $(0, 0)$. The function $D_p(u, v)$ depends on the specific problem and will be determined for each case.

Define $K_p(u, v)$ as the point target kill probability to a target element at (u, v) if the aiming point is at $(0, 0)$. We obtain

$$(2.22) \quad K_p(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D_p(u-x, v-y) g\left(\frac{x}{t(1)}\right) g\left(\frac{y}{t(2)}\right) \frac{dx dy}{t(1)t(2)}.$$

The fractional coverage $K(X, Y)$ for a target area A is thus

$$\begin{aligned}
 (2.23) \quad & K(X, Y) = \iint_A K_p(u, v) \frac{dudv}{A} \\
 & = \iint_A \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D_p(u-x, v-y) g\left(\frac{x}{t(1)}\right) g\left(\frac{y}{t(2)}\right) \frac{dx dy dudv}{t(1)t(2)A}.
 \end{aligned}$$

Consider a rectangular area of dimensions $[2A(1), 2A(2)]$. Using (2.23), we obtain

$$(2.24) \quad K(X,Y) = \int_{X-A(1)}^{X+A(1)} \int_{Y-A(2)}^{Y+A(2)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D_p(u-x, v-y) g\left(\frac{x}{t(1)}\right) g\left(\frac{y}{t(2)}\right) \frac{dx dy du dv}{4t(1)t(2)A(1)A(2)}.$$

Through appropriate changes in variable and reversing the order of integration, we obtain

$$(2.25) \quad K(X,Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D_p(x+X, y+Y) \left[\frac{1}{2A(1)} \int_{[x-A(1)]/t(1)}^{[x+A(1)]/t(1)} g(u) du \right] \\ \left[\frac{1}{2A(2)} \int_{[y-A(2)]/t(2)}^{[y+A(2)]/t(2)} g(v) dv \right] dx dy.$$

Using the definition of $h(x,L)$ from (2.19) in (2.25), we get

$$(2.26) \quad K(X,Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D_p(x+X, y+Y) h\left(\frac{x}{t(1)}, \frac{A(1)}{t(1)}\right) h\left(\frac{y}{t(2)}, \frac{A(2)}{t(2)}\right) \frac{dx dy}{t(1)t(2)}.$$

If $D_p(x,y)$ can be expressed without integrals, we have thus reduced the problem of obtaining K to a double integration. An alternate form for K used in the numerical procedure is

$$(2.27) \quad K(X,Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D_p[xt(1)+X, yt(2)+Y] h\left(x, \frac{A(1)}{t(1)}\right) h\left(y, \frac{A(2)}{t(2)}\right) dx dy.$$

For a gaussian target, i.e., one whose location (u,v) is subject to a gaussian distribution with variances $t^2(3)$ and $t^2(4)$ and center at (X,Y) , the probability of kill $K(X,Y)$ is

$$(2.28) \quad K(X,Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} K_p(u,v) g\left(\frac{u-X}{t(3)}\right) g\left(\frac{v-Y}{t(4)}\right) \frac{du dv}{t(3)t(4)},$$

where $K_p(u,v)$ is given in (2.22). Substituting Eq. (2.22) in (2.28) and using Lemma 1 in App. D, we obtain

$$(2.29) \quad K(X, Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D_p(x+X, y+Y) g\left(\frac{x}{t(5)}\right) g\left(\frac{y}{t(6)}\right) \frac{dx dy}{t(5)t(6)},$$

where

$$(2.30) \quad \begin{aligned} t^2(5) &= t^2(1) + t^2(3), \\ t^2(6) &= t^2(2) + t^2(4). \end{aligned}$$

Now from (2.21), we get

$$(2.31) \quad \begin{aligned} &\frac{h(x/t(5), 0)}{t(5)} - \frac{g(x/t(5))}{t(5)}, \\ &\frac{h(y/t(6), 0)}{t(6)} - \frac{g(y/t(6))}{t(6)}. \end{aligned}$$

Thus we can put K in (2.29) in the form

$$(2.32) \quad K(X, Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D_p(x+X, y+Y) h\left(\frac{x}{t(5)}, 0\right) h\left(\frac{y}{t(6)}, 0\right) \frac{dx dy}{t(5)t(6)}.$$

When we compare this equation with Eq. (2.26), we see that the problem is equivalent to that of a point target with the aiming error variances equal to the sum of the actual aiming error variances and the target location variances.

3. RIPPLE DELIVERY WITH GAUSSIAN DELIVERY ERRORS

3.1. RECTANGULAR TARGET AREA: FRAGMENT-SENSITIVE TARGET

Let us consider first a single weapon against a target element at (u, v) subject to ballistic errors according to a gaussian distribution with standard deviations $s(1)$ and $s(2)$. If the ballistic center of impact is at $(0, 0)$, the kill probability $D_b(u, v)$ is given by

$$(3.1) \quad D_b(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D(u-x, v-y) g\left(\frac{x}{s(1)}\right) g\left(\frac{y}{s(2)}\right) \frac{dx dy}{s(1)s(2)},$$

where $D(x, y)$ is the damage function in (2.1). Substituting for D from (2.1) and integrating, using Lemma 1 of App. D, we obtain

$$(3.2) \quad D_b(u, v) = \frac{R(1)R(2)}{2q(1)q(2)} \exp\left(-\frac{1}{2}\left\{\left[\frac{u}{q(1)}\right]^2 + \left[\frac{v}{q(2)}\right]^2\right\}\right),$$

where

$$q^2(1) = \frac{R^2(1)}{2D_0} + s^2(1),$$

$$q^2(2) = \frac{R^2(2)}{2D_0} + s^2(2).$$

Consider the delivery of $N(1)$ weapons against a target element at (u, v) . Each weapon is subject to ballistic errors as above. The $N(1)$ weapons will be delivered in a ripple mode. Examine first a zero aiming error. Then the i th weapon will have a center of impact (CI) at (ξ_i, η_i) . The array of CI's form the ripple pattern. We will use the center of gravity of the pattern as the pattern reference point; i.e., if the reference point is at (x, y) , then

$$(3.3) \quad x = \frac{1}{N(1)} \sum_{i=1}^{N(1)} \xi_i, y = \frac{1}{N(1)} \sum_{i=1}^{N(1)} \eta_i.$$

We will measure a_i and b_i from the reference point, i.e.,

$$a_i = \xi_i - x,$$

$$b_i = \eta_i - y.$$

Thus, we obtain

$$(3.4) \quad \sum_{i=1}^{N(1)} a_i = \sum_{i=1}^{N(1)} b_i = 0.$$

Consider the pattern reference point at (x, y) and a single weapon subject only to ballistic error. The center of impact of the i th weapon is thus at $(x+a_i, y+b_i)$. Let p be the weapon reliability factor, i.e., the probability of the weapon working properly. If the center of the pattern is at $(0, 0)$, the kill probability $D_p(u, v)$ due to the pattern is given by

$$(3.5) \quad D_p(u, v) = 1 - \prod_{i=1}^{N(1)} [1 - p D_b(u - a_i, v - b_i)],$$

where D_b is given by (4.2). Explicitly, we have

$$(3.6) \quad D_p(u, v) = 1 - \prod_{i=1}^{N(1)} \left[1 - \frac{pR(1)R(2)}{2q(1)q(2)} \exp \left(-\frac{1}{2} \left\{ \left[\frac{u-a_i}{q(1)} \right]^2 + \left[\frac{v-b_i}{q(2)} \right]^2 \right\} \right) \right].$$

Using this expression for D_p in (2.27), we obtain the coverage $K(X, Y)$:

$$(3.7) \quad K(X, Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ 1 - \prod_{i=1}^{N(1)} \left[1 - \frac{pR(1)R(2)}{2q(1)q(2)} \exp \left(-\frac{1}{2} \left\{ \left[\frac{xt(1)+X-a_i}{q(1)} \right]^2 + \left[\frac{yt(2)+Y-b_i}{q(2)} \right]^2 \right\} \right) \right] \right\} h \left(x, \frac{A(1)}{t(1)} \right) h \left(y, \frac{A(2)}{t(2)} \right) dx dy.$$

Equation (3.7) is the basis for obtaining $K(11)$ in the machine computation program discussed in Sec. 5.

Expanding the product in (3.6), we obtain D_p as a sum, which, when substituted in (2.24), gives an alternate expression for $K(X, Y)$ that can be integrated term by term. Thus, after expansion, we have

$$(3.8) \quad K(X, Y) = \sum_{k=1}^{N(1)} (-1)^{k-1} \left[\frac{pR(1)R(2)}{2q(1)q(2)} \right]^k \sum_{i_1=1}^{N(1)-k+1} \sum_{i_2=i_1+1}^{N(1)-k+2} \cdots \sum_{i_k=i_{k-1}+1}^{N(1)} \\ \times \int_{X-A(1)}^{X+A(1)} \int_{-\infty}^{\infty} \exp \left(- \sum_{j=1}^k \frac{(u-a_{i_j}-x)^2}{2q^2(1)} \right) g \left(\frac{x}{t(1)} \right) \frac{dx dv}{2t(1)A(1)} \int_{Y-B(1)}^{Y+B(1)} \int_{-\infty}^{\infty} \\ \times \exp \left(- \sum_{j=1}^k \frac{(v-b_{i_j}-y)^2}{2q^2(2)} \right) g \left(\frac{y}{t(2)} \right) \frac{dy dv}{2t(2)A(2)}.$$

Using Lemma 5 in App. D to evaluate the inner integrals in (3.8), we obtain

$$(3.9) \quad K(X, Y) = \sum_{k=1}^{N(1)} (-1)^{k-1} \left[\frac{pR(1)R(2)}{2q(1)q(2)} \right]^k \left[\frac{q(1)q(2)}{kQ(1)Q(2)} \right] \\ \times \sum_{i_1=1}^{N(1)} \int_{X-A(1)}^{X+A(1)} \exp \left(- \frac{[u-\bar{a}(i_1, i_2, \dots, i_k)]^2}{2Q^2(1)} \right) \frac{du}{2A(1)} \\ \times \int_{Y-A(2)}^{Y+A(2)} \exp \left(- \frac{[v-\bar{b}(i_1, i_2, \dots, i_k)]^2}{2Q^2(2)} \right) \frac{dv}{2A(1)} \\ \times \exp \left(- \frac{k[\bar{a}^2(i_1, i_2, \dots, i_k) - \bar{a}^2(i_1, i_2, \dots, i_k)]}{2q^2(1)} \right) \\ \times \exp \left(- \frac{k[\bar{b}^2(i_1, i_2, \dots, i_k) - \bar{b}^2(i_1, i_2, \dots, i_k)]}{2q^2(2)} \right).$$

where

$$\bar{a}^2(i_1, i_2, \dots, i_k) = \frac{1}{k} \sum_{j=1}^k a_{i_j}^2, \quad \bar{a}(i_1, i_2, \dots, i_k) = \frac{1}{k} \sum_{j=1}^k a_{i_j},$$

$$\bar{b}^2(i_1, i_2, \dots, i_k) = \frac{1}{k} \sum_{j=1}^k b_{i_j}^2, \quad \bar{b}(i_1, i_2, \dots, i_k) = \frac{1}{k} \sum_{j=1}^k b_{i_j},$$

$$(3.10) \quad q^2(1) = \frac{R^2(1)}{2D_0} + s^2(1),$$

$$q^2(2) = \frac{R^2(2)}{2D_0} + s^2(2),$$

$$Q^2(1) = t^2(1) + \frac{q^2(1)}{k} = t^2(1) + \frac{R^2(1)/2D_0 + s^2(1)}{k},$$

$$Q^2(2) = t^2(2) + \frac{q^2(2)}{k} = t^2(2) + \frac{R^2(2)/2D_0 + s^2(2)}{k},$$

$$\binom{N(1)}{k} = \frac{N(1)!}{k! [N(1)-k]!},$$

$$\sum \binom{N(1)}{k} = \sum_{i_1=1}^{N(1)-k+1} \sum_{i_2=i_1+1}^{N(1)-k+2} \dots \sum_{i_k=i_{k-1}+1}^{N(1)}.$$

The sum $\sum \binom{N(1)}{k}$ is the sum over the $\binom{N(1)}{k}$ combinations of the $N(1)$ integers $1, 2, 3, \dots, N(1)$ taken k at a time.

If we have a point target, i.e., $A(1)=A(2)=0$, Eq. (3.9) for K becomes

$$(3.11) \quad K(X, Y) = \sum_{k=1}^{N(1)} (-1)^{k-1} \left[\frac{pR(1)R(2)}{2q(1)q(2)} \right]^k \left[\frac{q(1)q(2)}{kQ(1)Q(2)} \right]$$

$$\times \sum \binom{N(1)}{k} \exp\left(-\frac{(X-\bar{a})^2}{2Q^2(1)}\right) \exp\left(-\frac{(Y-\bar{b})^2}{2Q^2(2)}\right) \exp\left(-k\frac{(\bar{a}^2-\bar{a}^2)}{2Q^2(1)}\right) \exp\left(-k\frac{\bar{b}^2-\bar{b}^2}{2Q^2(2)}\right).$$

In the special case of $a_i=b_i=0$, i.e., salvo fire against a point target, the sum $\sum \binom{N(1)}{k}$ in (3.11) is simply $\binom{N(1)}{k}$ since $\bar{a}^2=\bar{a}^2=\bar{b}^2=\bar{b}^2=0$. Thus, we obtain

$$(3.12) \quad K(X, Y) = \sum_{k=1}^{N(1)} (-1)^{k-1} \binom{N(1)}{k} \left[\frac{pR(1)R(2)}{2q(1)q(2)} \right]^k \left[\frac{q(1)q(2)}{kQ(1)Q(2)} \right] \exp\left(-\frac{X^2}{2Q^2(1)}\right) \exp\left(-\frac{Y^2}{2Q^2(2)}\right).$$

For an area target, if the definition of $h(x, L)$ given in (2.19) is used, Eq. (3.9) for K becomes

$$\begin{aligned}
 (3.13) \quad K(X, Y) = & \sum_{k=1}^{N(1)} (-1)^{k-1} \left[\frac{pR(1)R(2)}{2q(1)q(2)} \right]^k \frac{2\pi}{k} \left[\frac{q(1)q(2)}{Q(1)Q(2)} \right] \\
 & \times \sum_k^{N(1)} h\left(\frac{X-\bar{a}}{Q(1)}, \frac{A(1)}{Q(1)}\right) h\left(\frac{Y-\bar{b}}{Q(2)}, \frac{A(2)}{Q(2)}\right) \\
 & \times \exp\left(-\frac{k[\bar{a}^2(i_1, i_2, \dots, i_k) - \bar{a}^2(i_1, i_2, \dots, i_k)]}{2q^2(1)}\right) \\
 & \times \exp\left(-\frac{k[\bar{b}^2(i_1, i_2, \dots, i_k) - \bar{b}^2(i_1, i_2, \dots, i_k)]}{2q^2(2)}\right),
 \end{aligned}$$

where the symbols are defined in (3.10).

In the special case of salvo fire, i.e., $a_i = b_i = 0$, the expression for $K(X, Y)$ in (3.13) simplifies to

$$(3.14) \quad K(X, Y) = \sum_{k=1}^{N(1)} (-1)^{k-1} \binom{N(1)}{k} \left[\frac{pR(1)R(2)}{2q(1)q(2)} \right]^k \frac{2\pi}{k} \left[\frac{q(1)q(2)}{Q(1)Q(2)} \right] h\left(\frac{X}{Q(1)}, \frac{A(1)}{Q(1)}\right) h\left(\frac{Y}{Q(2)}, \frac{A(2)}{Q(2)}\right).$$

Equations (3.13) and (3.14) form the basis for obtaining $X(1)$ in the machine program. Thus, $X(1)$ and $X(11)$ are equivalent expressions in which different computation techniques are used.

3.2. RECTANGULAR TARGET AREA: IMPACT-SENSITIVE TARGET

Consider a single weapon subject only to ballistic errors. Substituting the impact damage function D from (2.8) in the expression for D_b in (3.1), we obtain for $D_b(u, v)$ the equation

$$(3.15) \quad D_b(u, v) = p \int_{u-B(1)}^{u+B(1)} \int_{v-B(2)}^{v+B(2)} g\left(\frac{x}{s(1)}\right) g\left(\frac{y}{s(2)}\right) \frac{dx dy}{s(1)s(2)}.$$

If we define $f(x,y)$ as

$$(3.16) \quad f(x,y) = G(x+y) - G(x-y),$$

D_b from (3.15) becomes

$$(3.17) \quad D_b(u,v) = p_{dk} f\left(\frac{u}{s(1)}, \frac{B(1)}{s(1)}\right) f\left(\frac{v}{s(2)}, \frac{B(2)}{s(2)}\right).$$

Proceeding exactly as in the preceding section on a fragment-sensitive target for a ripple of $N(1)$ weapons, we obtain an expression for $D_p(u,v)$ corresponding to (3.5):

$$(3.18) \quad D_p(u,v) = 1 - \prod_{i=1}^{N(1)} [1 - p' D_b(u-a_i, v-b_i)],$$

where D_b is given in (3.17) and

$$(3.19) \quad p' = pp_{dk},$$

i.e., p' is the product of the reliability factor for the weapon and the probability of damage if hit. Using (3.17) in (3.18) and substituting in (2.27), we obtain for $K(X,Y)$

$$(3.20) \quad K(X,Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ 1 - \prod_{i=1}^{N(1)} \left[1 - p' f\left(\frac{xt(1)+X-a_i}{s(1)}, \frac{B(1)}{s(1)}\right) f\left(\frac{yt(2)+Y-b_i}{s(2)}, \frac{B(2)}{s(2)}\right) \right] \right\} \\ \times h\left(x, \frac{A(1)}{t(1)}\right) h\left(y, \frac{A(2)}{t(2)}\right) dx dy.$$

Equation (3.20) is the basis for obtaining $K(15)$ in the machine program.

Expanding the product in (3.20), we obtain an alternate expression for K , i.e.,

$$(3.21) \quad K(X,Y) = \sum_{k=1}^{N(1)} (-1)^{k-1} \binom{N(1)}{k} \int_{-\infty}^{\infty} \prod_{j=1}^k f\left(\frac{xt(1)+X-a_{ij}}{s(1)}, \frac{B(1)}{s(1)}\right) h\left(x, \frac{A(1)}{t(1)}\right) dx$$

$$\times \int_{-\infty}^{\infty} \prod_{j=1}^k f\left(\frac{yt(2)+Y-b_{ij}}{s(2)}, \frac{B(2)}{s(2)}\right) h\left(y, \frac{A(2)}{t(2)}\right) dy.$$

We note that we have a sum of products involving only a single integration rather than a double integration as in (3.20). For the salvo case, we obtain from (3.21)

$$(3.22) \quad K(X,Y) = \sum_{k=1}^{N(1)} (-1)^{k-1} \binom{N(1)}{k}$$

$$\times \int_{-\infty}^{\infty} f^k\left(\frac{xt(1)+X}{s(1)}, \frac{B(1)}{s(1)}\right) h\left(x, \frac{A(1)}{t(1)}\right) dx \int_{-\infty}^{\infty} f^k\left(\frac{yt(2)+Y}{s(2)}, \frac{B(2)}{s(2)}\right) h\left(y, \frac{A(2)}{t(2)}\right) dy.$$

Equations (3.21) and (3.22) form the basis for obtaining $X(5)$ in the machine program. Again $X(5)$ and $X(15)$ are equivalent expressions for K , but differ in the method of computation.

3.3. ELLIPTICAL TARGET AREA: FRAGMENT-SENSITIVE TARGET

Consider an elliptical target area with axes $A(3)$ and $A(4)$ in the x and y directions. The fraction coverage $K(X,Y)$ for this target area is given by (2.23). Specifically, we have

$$(3.23) \quad K(X,Y) = \int_A \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D_p(u-X, v-Y) g\left(\frac{x}{t(1)}\right) g\left(\frac{y}{t(2)}\right) \frac{dx dy du dv}{\pi t(1) t(2) A(3) A(4)},$$

where A is defined by

$$\left[\frac{u-X}{A(3)}\right]^2 + \left[\frac{v-Y}{A(4)}\right]^2 \leq 1.$$

For a fragment-sensitive target, the pattern damage function $D_p(u,v)$ is given by Eq. (3.6), the same as was used in Sec. 3.1. Expanding the product in (3.6),

substituting in (3.3), and integrating as in Eq. (3.9), we obtain an expression for K that is similar to (3.9):

$$(3.24) \quad K(X, Y) = \sum_{k=1}^{N(1)} (-1)^{k-1} \left[\frac{pR(1)R(2)}{2q(1)q(2)} \right]^k \left[\frac{q(1)q(2)}{kQ(1)Q(2)} \right]^{\binom{N(1)}{k}} \exp\left(-\frac{k(\bar{a}^2 - \bar{a}^2)}{2q^2(1)}\right) \exp\left(-\frac{k(\bar{b}^2 - \bar{b}^2)}{2q^2(2)}\right) \cdot \int_A \int \exp\left(-\frac{1}{2} \left\{ \left[\frac{u-\bar{a}}{Q(1)} \right]^2 + \left[\frac{v-\bar{b}}{Q(2)} \right]^2 \right\} \right) \frac{dudv}{\pi A(3)A(4)},$$

where A is given in (3.23) and the other symbols are as defined in (3.10).

Consider the integral expression I_2 in (3.24); i.e.,

$$(3.25) \quad I_2 = \int_A \int \exp\left(-\frac{1}{2} \left\{ \left[\frac{u-\bar{a}}{Q(1)} \right]^2 + \left[\frac{v-\bar{b}}{Q(2)} \right]^2 \right\} \right) \frac{dudv}{\pi A(3)A(4)} \\ = \frac{2Q(1)Q(2)}{A(3)A(4)} \left[\int_{C_1} \exp\left(-\frac{1}{2}(u^2+v^2)\right) \frac{dudv}{2\pi} \right],$$

where

$$C_1: \frac{\{[u-(X-\bar{a})]/Q(1)\}^2 + \{[v-(Y-\bar{b})]/Q(2)\}^2}{[A(3)/Q(1)]^2 + [A(4)/Q(2)]^2} \leq 1.$$

Define the offset ellipse function $P(A, B; x, y)$ as the integral of a gaussian distribution of unit standard deviation over an offset ellipse, center at (x, y) , with axes A and B in the x and y directions, respectively; i.e.,

$$(3.26) \quad P(A, B; x, y) \equiv \iint_{C_2} \exp\left(-\frac{1}{2}(\zeta^2 + \eta^2)\right) \frac{d\zeta d\eta}{2\pi},$$

where

$$C_2: \frac{(\zeta-x)^2}{A^2} + \frac{(\eta-y)^2}{B^2} \leq 1.$$

Thus, the integral I_2 in (3.25) becomes

$$I_2 = \frac{2Q(1)Q(2)}{A(3)A(4)} P\left(\frac{A(3)}{Q(1)}, \frac{A(4)}{Q(2)}, \frac{X-\bar{a}}{Q(1)}, \frac{Y-\bar{b}}{Q(2)}\right).$$

Substituting in (3.24), we obtain the expression for $K(X,Y)$:

$$(3.27) \quad K(X,Y) = \sum_{k=1}^{N(1)} (-1)^{k-1} \left[\frac{pR(1)R(2)}{2q(1)q(2)} \right]^k \left[\frac{q(1)q(2)}{kA(3)A(4)} \right] \\ \times \sum_{k=1}^{N(1)} \exp\left(-\frac{k}{2} \left[\frac{\bar{a}^2 - \bar{a}^2}{q^2(1)} + \frac{\bar{b}^2 - \bar{b}^2}{q^2(2)} \right]\right) P\left(\frac{A(3)}{Q(1)}, \frac{A(4)}{Q(2)}, \frac{X-\bar{a}}{Q(1)}, \frac{Y-\bar{b}}{Q(2)}\right),$$

where $P(A,B;X,Y)$ is defined in (3.26). Tables of the offset ellipse function are available in the open literature.

In (3.26), if $A=B=R$, i.e., if the ellipse becomes a circle, the offset ellipse function becomes the offset circle function, i.e.,

$$P(R,R;x,y) = P(R, \sqrt{x^2+y^2}),$$

where $P(R,r)$ is defined as

$$(3.28) \quad P(R,r) = \int_{x^2+y^2 \leq R^2} g(x-r)g(y) dx dy, \\ = \int_0^R x \exp\left(-\frac{1}{2}(x^2+r^2)\right) I_0(rx) dx,$$

where $I_0(x)$ is the zero-order Bessel function of the first kind with imaginary argument.

Consider the special case

$$Q(1)=Q(2) \quad \text{and} \quad A(3)=A(4),$$

i.e., the target is a circle of radius $A(3)$.

$$(3.29) \quad K(X,Y) = \sum_{k=1}^{N(1)} (-1)^{k-1} \left[\frac{pR(1)R(2)}{2q(1)q(2)} \right]^k \left[\frac{q(1)q(2)}{kA(3)^2} \right] \\ \times \sum_{k=1}^{N(1)} \exp \left(-\frac{k}{2} \left[\frac{a^2 - \bar{a}^2}{q^2(1)} + \frac{b^2 - \bar{b}^2}{q^2(2)} \right] \right) P \left(\frac{A(3)}{Q(1)}, \frac{\sqrt{(X-\bar{a})^2 + (Y-\bar{b})^2}}{Q(1)} \right),$$

where $P(R,r)$ is defined in (3.28). For the salvo case, this expression reduces to

$$(3.30) \quad K(X,Y) = \sum_{k=1}^{N(1)} (-1)^{k-1} \binom{N(1)}{k} \left[\frac{pR(1)R(2)}{2q(1)q(2)} \right]^k \left[\frac{q(1)q(2)}{kA(3)^2} \right] P \left(\frac{A(3)}{Q(1)}, \frac{\sqrt{X^2 + Y^2}}{Q(1)} \right).$$

4. DISPENSER-TYPE DELIVERIES AGAINST RECTANGULAR TARGET AREAS

Let us consider a dispenser-type delivery. This type includes both the mass delivery of a number of weapons and the delivery of a sequence of weapons at timed intervals. The latter type is similar to the ripple delivery of the previous section except that the ballistic dispersion of the individual weapon is not necessarily gaussian, and the number delivered may be much larger than in the case of Sec. 3.

The dispensers themselves may stay with the delivery vehicle (no ballistic errors for the dispenser) or may be released in the ripple mode and thus be subject to dispenser ballistic errors. In either case, the subweapons are distributed according to some type of pattern distribution. In addition to the ballistic distributions of Sec. 3, we will also consider cases in which the subweapons are uniformly distributed over a pattern (rectangle, ellipse, annulus). We will view these pattern distributions as ballistic error distributions for the individual subweapons. Thus, we have cases where there are two ballistic error distributions, one for the dispenser and one for the individual subweapons.

We will consider, in turn, the following types of subweapon distribution functions:

1. Stick distribution in range; gaussian distribution in deflection.
2. Uniform distribution in range and deflection (over a rectangle).
3. Uniform distribution over an ellipse.
4. Uniform distribution over an elliptic annulus.

In this section we restrict ourselves to the consideration of either point targets or rectangular target areas of dimensions $[2A(1), 2A(2)]$.

4.1. RIPPLE-TYPE DISPENSER DELIVERIES

We consider first a sequence of $N(2)$ weapons delivered at timed intervals, i.e., with centers of impact forming a pattern $(a_i, b_i), i=1, 2, 3, \dots, N(2)$. This type of delivery is similar to the type of delivery of Sec. 3 except that several

patterns may be delivered at once, either with the same aiming points or with offset aiming points, which involve the delivery of several tubes in a container or from several containers. Further, the ballistic data for the subweapon are usually not obtained for each weapon individually. The ballistic errors in deflection are assumed to be gaussian, but the range errors are governed by an empirical distribution.

We assume each of the $N(2)$ weapons to be aimed individually at the center point of the pattern. We fit the empirical data for range dispersion by assuming that its distribution $K_1(X)$ is the stick distribution:

$$(4.1) \quad K_1(X) = H\left(\frac{X}{q(0)}, \frac{L(1)}{q(0)}\right) = \int_{-\infty}^X h\left(\frac{x}{q(0)}, \frac{L(1)}{q(0)}\right) \frac{dx}{q(0)},$$

where $L(1)$ is the "length" of the stick, and $q(0)$ is a parameter of the distribution. Appendix B contains a discussion of this pattern.

The ballistic distribution in deflection is gaussian with standard deviation $s(2)$. Consider a single weapon against a target element at (u, v) . The kill probability $D_b(u, v)$ is given by

$$(4.2) \quad D_b(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D(u-x, v-y) h\left(\frac{x}{q(0)}, \frac{L(1)}{q(0)}\right) g\left(\frac{y}{s(2)}\right) \frac{dx dy}{s(2)q(0)},$$

where $D(u-x, v-y)$ is the damage function. If $N(2)$ weapons are released from the dispenser with pattern center at $(0, 0)$, the kill probability $D_D(u, v)$ to a target element at (u, v) due to a single dispenser is

$$(4.3) \quad D_D(u, v) = 1 - [1 - p D_b(u, v)]^{N(2)},$$

where $D_b(u, v)$ is as given in (4.2), and p is the subweapon reliability factor. We define r as the probability of a dispenser operating properly, i.e., the dispenser reliability factor. If there are $N(1)$ dispensers with pattern centers at (a_i, b_i) , a pattern of dispensers is formed. Thus, the kill probability $D_p(u, v)$ due to the whole pattern is

$$(4.4) \quad D_p(u,v) = 1 - \prod_{i=1}^{N(1)} [1 - r D_b(u-a_i, v-b_i)]$$

$$= 1 - \prod_{i=1}^{N(1)} [1 - r \{1 - [1 - p D_b(u-a_i, v-b_i)]^{N(2)}\}].$$

4.1.1. Fragment-sensitive Targets

For a fragment-sensitive target, substituting the damage function (2.1) in (4.2), we obtain for $D_b(u,v)$ the expression

$$(4.5) \quad D_b(u,v) = D_0 \int_{-\infty}^{\infty} \exp\left(-D_0 \left[\frac{u-x}{R(1)}\right]^2\right) h\left(\frac{x}{q(0)}, \frac{L(1)}{q(0)}\right) \frac{dx}{q(0)}$$

$$\times \int_{-\infty}^{\infty} \exp\left(-D_0 \left[\frac{v-y}{R(2)}\right]^2\right) g\left(\frac{y}{s(2)}, \frac{s(2)}{s(2)}\right) \frac{dy}{s(2)}.$$

The integrals are evaluated using App. D; Lemma 6 is used for the first integral and Lemma 1 for the second integral. We obtain

$$(4.6) \quad D_b(u,v) = \left[\frac{\sqrt{\pi} R(1)}{q(3)} h\left(\frac{u}{q(3)}, \frac{L(1)}{q(3)}\right) \right] \left[\frac{R(2)}{\sqrt{2} q(2)} \exp\left(-\frac{v^2}{2q^2(2)}\right) \right],$$

where

$$(4.7) \quad q^2(2) = \frac{R^2(2)}{2D_0} + s^2(2),$$

$$q^2(3) = \frac{R^2(1)}{2D_0} + q^2(0).$$

Using (4.6) in (4.4) to get $D_p(u,v)$ and substituting in (3.27) to obtain the coverage $K(X,Y)$, we have

$$(4.8) \quad K(X, Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(1 - \prod_{i=1}^{N(1)} \left\{ 1 - r \left[1 - \frac{p\sqrt{\pi}R(1)}{q(3)} \right] \right. \right. \\ \left. \left. \times h\left(\frac{xt(1)+X-a_i}{q(3)}, \frac{L(1)}{q(3)}\right) \frac{R(2)}{\sqrt{2}q(2)} \exp\left(-\frac{1}{2}\left[\frac{yt(2)+Y-b_i}{q(2)}\right]^2\right) \right\}^{N(2)} \right) \\ \times h\left(x, \frac{A(1)}{t(1)}\right) h\left(y, \frac{A(2)}{t(2)}\right) dx dy.$$

If $r=1$, we obtain

$$(4.9) \quad K(X, Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[1 - \prod_{i=1}^{N(1)} \left\{ 1 - \frac{p\sqrt{\pi}R(1)}{q(3)} h\left(\frac{xt(1)+X-a_i}{q(3)}, \frac{L(1)}{q(3)}\right) \right. \right. \\ \left. \left. \times \frac{R(2)}{\sqrt{2}q(2)} \exp\left(-\frac{1}{2}\left[\frac{yt(2)+Y-b_i}{q(2)}\right]^2\right) \right\}^{N(2)} \right] h\left(x, \frac{A(1)}{t(1)}\right) h\left(y, \frac{A(2)}{t(2)}\right) dx dy.$$

The expression for $K(X, Y)$ in (4.9) forms the basis for X(12) in the machine program.

For a single dispenser, $N(1)=1$, we expand the expression in (4.9) to obtain

$$(4.10) \quad K(X, Y) = \sum_{k=1}^{N(2)} (-1)^{k-1} \binom{N(2)}{k} \left[\frac{p\sqrt{\pi}R(1)}{q(3)} \right]^k \int_{-\infty}^{\infty} h^k\left(\frac{xt(1)+X}{q(3)}, \frac{L(1)}{q(3)}\right) h\left(x, \frac{A(1)}{t(1)}\right) dx \\ \times \left[\frac{R(2)}{\sqrt{2}q(2)} \right]^k \int_{-\infty}^{\infty} \exp\left(-\frac{k}{2}\left[\frac{yt(2)+Y}{q(2)}\right]^2\right) h\left(y, \frac{A(2)}{t(2)}\right) dy.$$

The second integral may be evaluated using Lemma 6 of App. D, so that

$$(4.11) \quad K(X, Y) = \sum_{k=1}^{N(2)} (-1)^{k-1} \binom{N(2)}{k} \left[\frac{R(2)}{\sqrt{2}q(2)} \right]^k \sqrt{\frac{\pi}{k}} h\left(\frac{X}{q(2)}, \frac{A(2)}{Q(2)}\right) \int_{-\infty}^{\infty} h^k\left(\frac{xt(1)+X}{q(3)}, \frac{L(1)}{q(3)}\right) h\left(x, \frac{A(1)}{t(1)}\right) dx,$$

where $q(2)$ and $q(3)$ are given in (4.7) and

$$Q^2(2) = \frac{q^2(2)}{k} + t^2(2).$$

Equation (4.11) forms the basis for $X(2)$ in the machine program, and $X(2)$ and $X(12)$ are thus also equivalent except for the difference in computational methods.

4.1.2. Impact-sensitive Targets

For a hard target-type subweapon, substituting the damage function (3.1) in (4.2), we obtain for $D_b(u, v)$

$$D_b(u, v) = p \int_{u-B(1)}^{u+B(1)} h\left(\frac{x}{q(0)}, \frac{L(1)}{q(0)}\right) \frac{dx}{q(0)} \int_{v-B(2)}^{v+B(2)} g\left(\frac{y}{s(2)}, \frac{L(2)}{s(2)}\right) \frac{dy}{s(2)}.$$

Thus, we can express $D_b(u, v)$ as

$$(4.12) \quad D_b(u, v) = F\left(\frac{u}{q(0)}, \frac{B(1)}{q(0)}, \frac{L(1)}{q(0)}\right) f\left(\frac{v}{s(2)}, \frac{B(2)}{s(2)}\right) p \, dk,$$

where the function $F(x, y, L)$ is defined as

$$F(x, y, L) = H(x+y, L) - H(x-y, L),$$

and $H(x, L)$ is defined in (2.19), and $f(x, y)$ is defined in (3.16). Using (4.12) in (4.4) to obtain $D_p(u, v)$ and substituting in (3.20) to get the coverage $K(X, Y)$, we obtain

$$(4.13) \quad K(X, Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ 1 - \prod_{i=1}^{N(1)} \left[1 - p' F\left(\frac{xt(1) + X - a_i}{q(0)}, \frac{B(1)}{q(0)}, \frac{L(1)}{q(0)}\right) \right. \right. \\ \left. \left. \times f\left(\frac{yt(2) + Y - b_i}{s(2)}, \frac{B(2)}{s(2)}\right) \right]^{N(2)} \right\} h\left(x, \frac{A(1)}{t(1)}\right) h\left(y, \frac{A(2)}{t(2)}\right) dx dy,$$

where p' is as defined in (3.19). The expression for $K(X, Y)$ in (4.13) forms the basis for $X(14)$ in the machine program.

For a single dispenser, $N(1)=1$, we expand the expression in (4.13) to obtain

$$(4.14) \quad K(X, Y) = \sum_{k=1}^{N(2)} (-1)^{k-1} \binom{N(2)}{k} (p')^k \int_{-\infty}^{\infty} F^k \left(\frac{xt(1)+X}{q(0)}, \frac{B(1)}{q(0)}, \frac{L(1)}{q(0)} \right) \\ \times h \left(x, \frac{A(1)}{t(1)} \right) dx \int_{-\infty}^{\infty} f^k \left(\frac{yt(2)+Y}{s(2)}, \frac{B(2)}{s(2)} \right) h \left(y, \frac{A(2)}{t(2)} \right) dy.$$

The expression for $K(X, Y)$ in (4.14) forms the basis for $X(4)$ in the machine program. Again $X(4)$ and $X(14)$ are equivalent but are computed differently.

4.1.3. Approximate Method: Fragment-sensitive Target

In the problem considered in Sec. 4.1.1 for a fragmentation-type weapon, we can simplify the expression for $K(X, Y)$ if the range offsets $a_i = 0$ by an approximation for the range ballistic distribution. Referring to the expression for $D_b(u, v)$ in (4.6), we have

$$\frac{h \left(\frac{u}{q(3)}, \frac{L(1)}{q(3)} \right)}{q(3)},$$

which is a density function for a distribution function. If we replace this function by a uniform distribution $\alpha(u, L(5))/2L(5)$, where α is defined in (2.9), we obtain the approximate expression for $D_b(u, v)$:

$$(4.15) \quad D_b(u, v) = \frac{\sqrt{\pi} R(1)}{2L(5)} \alpha(u, L(5)) \left[\frac{R(2)}{\sqrt{2} q(2)} \exp \left(-\frac{v^2}{2q^2(2)} \right) \right].$$

We will demand that the functions $h(u/q(3), L(1)/q(3))/q(3)$ and $\alpha(u, L(5))/2L(5)$ have the same variance. Then $L(5)$ is determined by

$$(4.16) \quad L^2(5) = L^2(1) + 3q^2(3).$$

Using (4.15) in (4.4) to determine $D_D(u, v)$, we obtain

$$D_D(u, v) = \alpha(u, L(5)) \left[1 - \prod_{i=1}^{N(1)} \left\{ 1 - p \frac{\sqrt{\pi} R(1)}{2L(5)} \frac{R(2)}{\sqrt{2} q(2)} \exp \left(-\frac{1}{2} \left[\frac{v-b_i}{q(2)} \right]^2 \right) \right\}^{N(2)} \right].$$

Substituting for $D_p(u, v)$ in (2.27), we obtain the coverage $K(X, Y)$:

$$(4.17) \quad K(X, Y) = \int_{-\infty}^{\infty} \alpha(xt(1)+X, L(5)) h\left(x, \frac{A(1)}{t(1)}\right) dx \int_{-\infty}^{\infty} \left[1 - \prod_{i=1}^{N(1)} \left\{1 - \frac{p\sqrt{\pi}R(1)R(2)}{2L(5)\sqrt{2}q(2)}\right\}\right. \\ \left. \times \exp\left(-\frac{1}{2}\left[\frac{yt(2)+Y-b_i}{q(2)}\right]^2\right)\right]^{N(2)} h\left(y, \frac{A(2)}{t(2)}\right) dy.$$

Using the definition of $F(x, y, L)$ in (4.12), we find that the first integral in (4.17) is

$$F\left(-\frac{X}{t(1)}, \frac{L(5)}{t(1)}, \frac{A(1)}{t(1)}\right).$$

Since $F(-x, y, L) = F(x, y, L)$, we obtain for $K(X, Y)$ from (4.17)

$$(4.18) \quad K(X, Y) = F\left(\frac{X}{t(1)}, \frac{L(5)}{t(1)}, \frac{A(1)}{t(1)}\right) \int_{-\infty}^{\infty} \left[1 - \prod_{i=1}^{N(1)} \left\{1 - \frac{p\sqrt{\pi}R(1)R(2)}{2L(5)\sqrt{2}q(2)}\right\}\right. \\ \left. \times \exp\left(-\frac{1}{2}\left[\frac{yt(2)+Y-b_i}{q(2)}\right]^2\right)\right]^{N(2)} h\left(y, \frac{A(2)}{t(2)}\right) dy.$$

Equation (4.18) forms the basis for X(13) in the machine program.

If $N(1)=1$, expanding the expression in (4.18), we obtain

$$K(X, Y) = F\left(\frac{X}{t(1)}, \frac{L(5)}{t(1)}, \frac{A(1)}{t(1)}\right) \sum_{k=1}^{N(2)} (-1)^{k-1} \binom{N(2)}{k} \left[\frac{p\sqrt{\pi}R(1)R(2)}{2L(5)\sqrt{2}q(2)}\right]^k \\ \times \int_{-\infty}^{\infty} \exp\left(-\frac{k}{2}\left[\frac{yt(2)+Y}{q(2)}\right]^2\right) h\left(y, \frac{A(2)}{t(2)}\right) dy.$$

Evaluating the integral using Lemma 6 of App. D, we obtain

$$(4.19) \quad K(X, Y) = F\left(\frac{X}{t(1)}, \frac{L(5)}{t(1)}, \frac{A(1)}{t(1)}\right) \sum_{k=1}^{N(2)} (-1)^{k-1} \binom{N(2)}{k} \left[\frac{p\sqrt{\pi}R(1)R(2)}{2L(5)\sqrt{2}q(2)}\right]^k \\ \times \sqrt{\frac{2\pi}{k}} \frac{q(2)}{Q(2)} h\left(\frac{Y}{Q(2)}, \frac{A(2)}{Q(2)}\right).$$

Equation (4.19) for $K(X,Y)$ forms the basis for $X(3)$ in the machine program. The expressions $X(3)$ and $X(13)$ are equivalent here for K when different computational methods are used.

4.2. UNIFORM DISTRIBUTION OVER A RECTANGLE

We consider a ripple of dispensers whose subweapon distribution is uniform in range and deflection over a rectangle. Each dispenser is assumed subject to ballistic errors according to a gaussian distribution with standard deviations $s(3)$ and $s(4)$. The centers of impact of (a_i, b_i) refer to the center point of the dispenser pattern. Each dispenser is assumed to release the subweapons so that the distribution of the subweapon is uniform over a rectangle $[2L(3), 2L(4)]$. If the pattern center is at $(0,0)$, the kill probability $D_b(u,v)$ to a target element at (u,v) due to a specific weapon is

$$(4.20) \quad D_b(u,v) = \int_{-L(3)}^{L(3)} \int_{-L(4)}^{L(4)} D(u-x, v-y) \frac{dx dy}{4L(3)L(4)},$$

where $D(u-x, v-y)$ is the damage function.

If $N(2)$ weapons are released and if the center point of the pattern is at $(0,0)$, the kill probability from a dispenser, $D_D(u,v)$, is

$$(4.21) \quad D_D'(u,v) = 1 - [1 - p D_b(u,v)]^{N(2)},$$

where $D_b(u,v)$ is given by (4.20), and p is the subweapon reliability factor. The dispenser is subject to a gaussian ballistic error, so that $D_D(u,v)$, the dispenser kill probability, is given by

$$(4.22) \quad D_D(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D_D'(u-x, v-y) g\left(\frac{x}{s(3)}\right) g\left(\frac{y}{s(4)}\right) \frac{dx dy}{s(3)s(4)} \\ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ 1 - [1 - p D_b(u-x, v-y)]^{N(2)} \right\} g\left(\frac{x}{s(3)}\right) g\left(\frac{y}{s(4)}\right) \frac{dx dy}{s(3)s(4)}.$$

4.2.1. Fragment-sensitive Target

For a fragment-sensitive target, substituting the damage function (2.1) in (4.20), we obtain

$$D_D(u, v) = \int_{-L(3)}^{L(3)} \int_{-L(4)}^{L(4)} D_0 \exp \left(-D_0 \left\{ \left[\frac{u-x}{R(1)} \right]^2 + \left[\frac{v-y}{R(2)} \right]^2 \right\} \right) \frac{dx dy}{4L(3)L(4)}$$

$$= \frac{\pi R(1)R(2)}{\gamma(1)\gamma(2)} h\left(\frac{u}{\gamma(1)}, \frac{L(3)}{\gamma(1)}\right) h\left(\frac{v}{\gamma(2)}, \frac{L(4)}{\gamma(2)}\right),$$

where

$$\gamma^2(1) = \frac{R^2(1)}{2D_0},$$

$$\gamma^2(2) = \frac{R^2(2)}{2D_0}.$$

The expression for $D_D(u, v)$ from (4.22) is thus

$$(4.23) \quad D_D(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ 1 - \left[1 - \frac{\pi R(1)R(2)}{\gamma(1)\gamma(2)} h\left(\frac{u-x}{\gamma(1)}, \frac{L(3)}{\gamma(1)}\right) h\left(\frac{v-y}{\gamma(2)}, \frac{L(4)}{\gamma(2)}\right) \right]^{N(2)} \right\}$$

$$\times g\left(\frac{x}{s(3)}\right) g\left(\frac{y}{s(4)}\right) \frac{dx dy}{s(3)s(4)}.$$

a. Approximation 1: Ballistic Error, No Edge Effects. The computational methods based on (2.27) will not work if the expression for $D_D(u, v)$ contains an integral. Thus, expression (4.23) must be simplified. One method is to replace the density functions in (4.23):

$$\frac{h\left(\frac{u}{\gamma(1)}, \frac{L(3)}{\gamma(1)}\right)}{\gamma(1)} \quad \text{and} \quad \frac{h\left(\frac{v}{\gamma(2)}, \frac{L(4)}{\gamma(2)}\right)}{\gamma(2)}$$

by the uniform density functions

$$\frac{\alpha(u, L(5))}{2L(5)} \quad \text{and} \quad \frac{\alpha(v, L(6))}{2L(6)},$$

where $\alpha(x, L)$ is as defined in (2.9) and variances are equal. Thus, $L(5)$ and $L(6)$ are given by

$$(4.24) \quad L^2(5) = L^2(3) + 3\gamma^2(1) = L^2(3) + \frac{3R^2(1)}{2D_0},$$

$$L^2(6) = L^2(4) + 3\gamma^2(2) = L^2(4) + \frac{3R^2(2)}{2D_0}.$$

Essentially, we are assuming the ratios $R(1)/L(3)$ and $R(2)/L(4)$ are small. We enlarge the pattern $\{2L(3), 2L(4)\}$ to be $\{2L(5), 2L(6)\}$ and assume no edge effects in the enlarged pattern. Thus, we have

$$(4.25) \quad D_D(u, v) = \left\{ 1 - \left[1 - \frac{p\pi R(1)R(2)}{4L(5)L(6)} \right]^{N(2)} \right\} \frac{1}{4L(5)L(6)} \\ \times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \alpha(u-x, L(5)) \alpha(v-y, L(6)) g\left(\frac{x}{s(3)}\right) g\left(\frac{y}{s(4)}\right) \frac{dx dy}{s(3)s(4)}.$$

If $s(3)$ and $s(4) > 0$ and we use the definition of $f(x, y)$ in (3.16), the expression for $D_D(u, v)$ becomes

$$(4.26) \quad D_D(u, v) = \left\{ 1 - \left[1 - \frac{p\pi R(1)R(2)}{4L(5)L(6)} \right]^{N(2)} \right\} f\left(\frac{u}{s(3)}, \frac{L(5)}{s(3)}\right) f\left(\frac{v}{s(4)}, \frac{L(6)}{s(4)}\right).$$

When we use (4.26) for a ripple of $N(1)$ dispensers with CI's at (a_i, b_i) , the kill probability $D_p(u, v)$ due to the whole ripple is

$$(4.27) \quad D_p(u, v) = 1 - \prod_{i=1}^{N(1)} [1 - r D_D(u - a_i, v - b_i)] \\ = 1 - \prod_{i=1}^{N(1)} \left[1 - r \left\{ 1 - \left[1 - \frac{p\pi R(1)R(2)}{4L(5)L(6)} \right]^{N(2)} \right\} f\left(\frac{u - a_i}{s(3)}, \frac{L(5)}{s(3)}\right) f\left(\frac{v - b_i}{s(4)}, \frac{L(6)}{s(4)}\right) \right],$$

where r is the dispenser reliability factor.

Substituting the expression for $D_p(u,v)$ as given by (4.27) in (2.27), we obtain

$$(4.28) \quad K(X,Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ 1 - \prod_{i=1}^{N(1)} \left[1 - r \left\{ 1 - \left[\frac{p\pi R(1)R(2)}{4L(5)L(6)} \right]^{N(2)} \right\} f\left(\frac{xt(1)+X-a_i}{s(3)}, \frac{L(5)}{s(3)}\right) \right. \right. \\ \left. \left. \times f\left(\frac{yt(2)+Y-b_i}{s(4)}, \frac{L(6)}{s(4)}\right) \right\} h\left(x, \frac{A(1)}{t(1)}\right) h\left(y, \frac{A(2)}{t(2)}\right) dx dy \right\}$$

Equation (4.28) forms the basis for X(103) in the machine program. For only one dispenser, we can integrate the expression in (4.28), using Lemma 6 of App. D:

$$(4.29) \quad K(X,Y) = r \left\{ 1 - \left[\frac{p\pi R(1)R(2)}{4L(5)L(6)} \right]^{N(2)} \right\} F\left(\frac{X}{t(3)}, \frac{L(5)}{t(3)}, \frac{A(1)}{t(3)}\right) F\left(\frac{Y}{t(4)}, \frac{L(6)}{t(4)}, \frac{A(2)}{t(4)}\right),$$

where $F(x,y,L)$ is defined in (4.12) and $t(3)$ and $t(4)$ are defined by

$$(4.30) \quad \begin{aligned} t^2(3) &= t^2(1) + s^2(3), \\ t^2(4) &= t^2(2) + s^2(4). \end{aligned}$$

The expression for the coverage $K(X,Y)$ in (4.29) is for the case of a single dispenser rectangular pattern where the ratio of the subweapon MAE (mean effective area) to the pattern area $[4L(3)L(4)]$ is small. It can also be considered as an approximation to the case discussed in Sec. 4.1.3, "Approximate Method: Fragment-sensitive Target." For the range dimension $L(5)$, we use the value obtained in (4.16), i.e.,

$$L^2(5) \approx L^2(1) + 3q^2(3) = L^2(1) + 3q^2(0) + \frac{3R^2(1)}{2D_0}$$

and equivalently

$$(4.31) \quad L^2(3) \approx L^2(1) + 3q^2(0).$$

We make a similar approximation in deflection, again equating variances, and obtain

$$L^2(6) = 3q^2(2) = 3s^2(2) + \frac{3R^2(2)}{2D_0},$$

and equivalently

$$(4.32) \quad L^2(4) = 3s^2(2).$$

Equation (3.29) with $L(3)$ and $L(4)$ given by (4.31) and (4.32) form the basis for $X(10)$ in the machine program.

b. Approximation 2: Edge Effects, No Ballistic Error. A second approximation for $D(u,v)$ in (4.23) may be made if $s(3)$ and $s(4)$ are small with respect to $L(1)$ and $L(2)$. We then assume $s(3)=s(4)=0$, so that $D_D(u,v)$ is given simply by

$$(4.33) \quad D_D(u,v) = 1 - \left[1 - \frac{p\pi R(1)R(2)}{\gamma(1)\gamma(2)} h\left(\frac{u}{\gamma(1)}, \frac{L(3)}{\gamma(1)}\right) h\left(\frac{v}{\gamma(2)}, \frac{L(4)}{\gamma(2)}\right) \right]^{N(2)}.$$

For a ripple of $N(1)$ dispensers, the kill probability $D_p(u,v)$ due to the whole ripple is

$$(4.34) \quad D_D(u,v) = 1 - \prod_{i=1}^{N(1)} \left[1 - r \left\{ 1 - \left[1 - \frac{p\pi R(1)R(2)}{\gamma(1)\gamma(2)} h\left(\frac{u-a_i}{\gamma(1)}, \frac{L(3)}{\gamma(1)}\right) h\left(\frac{v-b_i}{\gamma(2)}, \frac{L(4)}{\gamma(2)}\right) \right]^{N(2)} \right\} \right].$$

Substituting (4.34) in (2.27), we obtain

$$(4.35) \quad K(X,Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ 1 - \prod_{i=1}^{N(1)} \left[1 - r \left\{ 1 - \left[1 - \frac{p\pi R(1)R(2)}{\gamma(1)\gamma(2)} h\left(\frac{x t(1) + X - a_i}{\gamma(1)}, \frac{L(3)}{\gamma(1)}\right) \right. \right. \right. \right. \\ \left. \left. \left. \times h\left(\frac{y t(2) + Y - b_i}{\gamma(2)}, \frac{L(4)}{\gamma(2)}\right) \right]^{N(2)} \right\} \right] \right\} h\left(x, \frac{A(1)}{t(1)}\right) h\left(y, \frac{A(2)}{t(2)}\right) dx dy.$$

Equation (4.35) forms the basis for $X(104)$ in the machine program. The expressions for $K(X,Y)$ in (4.28) and (4.35) are answers to the same problem but different approximations are used; i.e., $X(103)$ and $X(104)$ are equivalent.

4.2.2. Impact-sensitive Target

For an impact-sensitive target, by substituting the damage function (2.8) in (4.20), we obtain for $D_b(u,v)$

$$(4.36) \quad D_b(u, v) = p \int_{-L(3)}^{L(3)} \int_{-L(4)}^{L(4)} \alpha(u-x, B(1)) \alpha(v-y, B(2)) \frac{dx dy}{4L(3)L(4)}.$$

We will assume that $L(3)/B(1) > 1$ and $L(4)/B(2) > 1$. Define $\beta(x, B, L)$ as the integral of two α functions, i.e.,

$$(4.37) \quad \beta(x, B, L) \equiv \int_{-\infty}^{\infty} \alpha(x-y, B) \alpha(y, L) dy.$$

Thus, we have

$$(4.38) \quad \beta(x, B, L) = \begin{cases} 1 & |x| < L-B, \\ L+B-x & L-B \leq |x| \leq L+B, \\ 0 & |x| > L+B. \end{cases}$$

The function $\beta(x, B, L)/4BL$ is a density function with variance L^2+B^2 . When we use (4.37) and (4.38), $D_b(u, v)$ in (4.36) becomes

$$(4.39) \quad D_b(u, v) = \frac{p \int_{-L(3)}^{L(3)} \int_{-L(4)}^{L(4)} \beta(u-x, B(1), L(3)) \beta(v-y, B(2), L(4))}{4L(3)L(4)}.$$

For a single dispenser, $D_D(u, v)$ is given by

$$(4.40) \quad D_D(u, v) = 1 - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[1 - p' \beta(u-x, B(1), L(3)) \frac{\beta(v-y, B(2), L(4))}{4L(3)L(4)} \right]^{N(2)} \\ \times g\left(\frac{x}{s(3)}\right) g\left(\frac{y}{s(4)}\right) \frac{dx dy}{s(3)s(4)},$$

where $p' = pp_{dk}$ as in (3.19).

a. Approximation 1: Ballistic Error, No Edge Effects. As for the fragmentation case in Sec. 4.2.1, we must make an approximation for D_D in (4.40). As one approximation, we will replace the density function $\beta(u, B(1), L(3))/4B(1)L(3)$ in (4.40) by the density function $\alpha(u, L(5))/2L(5)$, again requiring equal variances. Thus, we let

$$(4.41) \quad \frac{\beta(u, B(1), L(3)) \cdot \alpha(u, L(5))}{4B(1)L(3)} + \frac{\alpha(u, L(5))}{2L(5)},$$

where

$$(4.42) \quad L^2(5) = L^2(3) + B^2(1).$$

Similarly, we let

$$(4.43) \quad \frac{\beta(v, B(2), L(4)) \cdot \alpha(v, L(6))}{4B(2)L(4)} + \frac{\alpha(v, L(6))}{2L(6)},$$

where

$$(4.44) \quad L^2(6) = L^2(4) + B^2(2).$$

Then, in place of (4.40), we have

$$(4.45) \quad D_D(u, v) = 1 - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[1 - p' \alpha(u-x, L(5)) \alpha(v-y, L(6)) \frac{B(1)B(2)}{L(5)L(6)} \right]^{N(2)} \\ \times g\left(\frac{x}{s(3)}\right) g\left(\frac{y}{s(4)}\right) \frac{dx dy}{s(3)s(4)} \\ = \left\{ 1 - \left[1 - \frac{p' B(1)B(2)}{L(5)L(6)} \right]^{N(2)} \right\} f\left(\frac{u}{s(3)}, \frac{L(5)}{s(3)}\right) f\left(\frac{v}{s(4)}, \frac{L(6)}{s(4)}\right).$$

We note that this expression for $D_D(u, v)$ is the same as for a fragmentation weapon in (4.26) except that $4B(1)B(2)$, the area of the target, replaces the $MAE = \pi R(1)R(2)$, and the definitions of $L(5)$ and $L(6)$ in (4.42) and (4.44) are slightly different from those in (4.24). Thus, the expression for $K(X, Y)$ is similar to (4.28):

$$(4.46) \quad K(X, Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ 1 - \prod_{i=1}^{N(1)} \left[1 - p' \left[1 - \frac{p' B(1)B(2)}{L(5)L(6)} \right]^{N(2)} \right] \right\} \\ \times f\left(\frac{xt(1) + X - a_i}{s(3)}, \frac{L(5)}{s(3)}\right) f\left(\frac{yt(2) + Y - b_i}{s(4)}, \frac{L(6)}{s(4)}\right) \left\{ h\left(x, \frac{A(1)}{t(1)}\right) h\left(y, \frac{A(2)}{t(2)}\right) dx dy \right\}.$$

Equation (4.46) is the basis for $X(105)$ in the machine program. For a single dispenser, we obtain

$$(4.47) \quad K(X,Y) = r \left\{ 1 - \left[1 - \frac{p' B(1) B(2)}{L(5) L(6)} \right]^{N(2)} \right\} F\left(\frac{X}{t(3)}, \frac{L(5)}{t(3)}, \frac{A(1)}{t(3)}\right) F\left(\frac{Y}{t(4)}, \frac{L(6)}{t(4)}, \frac{A(2)}{t(4)}\right),$$

where $t(3)$ and $t(4)$ are as in (4.30) and $L(3)$ and $L(4)$ as in (4.42) and (4.44).

b. Approximation 2: Edge Effects, No Ballistic Error. A second approximation is to assume $s(3)=s(4)=0$. Then from (4.40) $D_D(u,v)$ is

$$(4.48) \quad D_D(u,v) = 1 - \left[1 - \frac{p' \beta(u, B(1), L(3)) \beta(v, B(2), L(4))}{4L(3)L(4)} \right]^{N(2)}.$$

For a ripple of $N(1)$ dispensers, we obtain, as in (4.35),

$$(4.49) \quad K(X,Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ 1 - \prod_{i=1}^{N(1)} \left[1 - r \left\{ 1 - \left[1 - p' \frac{\beta(xt(1)+X-a_i, B(1), L(3)) \beta(yt(2)+Y-b_i, B(2), L(4))}{4L(3)L(4)} \right]^{N(2)} \right\} \right] \right\} \\ \times h\left(x, \frac{A(1)}{t(1)}\right) h\left(y, \frac{A(2)}{t(2)}\right) dx dy.$$

Equation (4.49) forms the basis for $X(106)$ in the machine program. Thus, expressions (4.46) and (4.49), i.e., $X(105)$ and $X(106)$, are solutions to the same problem using different approximations.

4.3. UNIFORM DISTRIBUTION OVER AN ELLIPSE

We consider the same problem as in the previous section except that the sub-weapons are assumed to be uniformly distributed over an ellipse with semimajor axis $T(1)$ and semiminor axis $T(2)$. The kill probability $D_b(u,v)$ is

$$(4.50) \quad D_b(u,v) = \iint_{C_1} D(u-x, v-y) \frac{dx dy}{T(1)T(2)\pi},$$

where C_1 is the area inside the ellipse $[x/T(1)]^2 + [y/T(2)]^2 = 1$. The kill probability $D_D(u, v)$ for a single dispenser against a target element at (u, v) is thus given by (4.22) and (4.21) with $D_b(u, v)$ from (4.50).

4.3.1. Fragment-sensitive Target

For a fragment-sensitive target, substituting the damage function (2.1) in (4.50), we obtain

$$(4.51) \quad D_b(u, v) = \frac{R(1)R(2)}{T(1)T(2)} \iint_{C_1} g\left(\frac{u-x}{Y(1)}\right) g\left(\frac{v-y}{Y(2)}\right) \frac{dx dy}{Y(1)Y(2)},$$

where

$$C_1: \left[\frac{x}{T(1)}\right]^2 + \left[\frac{y}{T(2)}\right]^2 \leq 1,$$

$$Y^2(1) = \frac{R^2(1)}{2D_0},$$

$$Y^2(2) = \frac{R^2(2)}{2D_0}.$$

In Sec. 3.3, the offset ellipse function $P(A, B; x, y)$ was defined in (3.26) by

$$(4.52) \quad P(A, B; x, y) = \iint_{C_2} g(\xi) g(\eta) d\xi d\eta,$$

where $C_2: [(\xi-x)/A]^2 + [(\eta-y)/B]^2 \leq 1$. We can thus express $D_b(u, v)$ in (4.51) as

$$(4.53) \quad D_b(u, v) = \frac{R(1)R(2)}{T(1)T(2)} P\left(\frac{T(1)}{Y(1)}, \frac{T(2)}{Y(2)}; \frac{u}{Y(1)}, \frac{v}{Y(2)}\right),$$

with $P(A, B; x, y)$ as in (4.52). Thus, $D_D(u, v)$, which is similar to (4.23), is

$$(4.54) \quad D_D(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ 1 - \left[1 - \frac{P R(1) R(2)}{T(1) T(2)} P\left(\frac{T(1)}{Y(1)}, \frac{T(2)}{Y(2)}; \frac{u-x}{Y(1)}, \frac{v-y}{Y(2)}\right) \right]^{N(2)} \right\} \\ \times g\left(\frac{x}{S(1)}\right) g\left(\frac{y}{S(2)}\right) \frac{dx dy}{S(1)S(2)}.$$

a. Approximation 1: Ballistic Error, No Edge Effects. As before, we must simplify (4.54). If $T(1)/\gamma(1)$ and $T(2)/\gamma(2)$ are large, we make the following approximation in (4.53):

$$(4.55) \quad D_b(u,v) = \begin{cases} \frac{R(1)R(2)}{T(1)T(2)} & \text{if } \left[\frac{u}{T(1)}\right]^2 + \left[\frac{v}{T(2)}\right]^2 \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Again, this is the approximation often used when we ignore edge effects: The kill probability is the ratio of the MAE to the pattern area inside the pattern and zero outside. Under this approximation, we obtain from (4.22)

$$(4.56) \quad D_D(u,v) = \iint_{C_3} \left\{ 1 - \left[1 - \frac{PR(1)R(2)}{T(1)T(2)} \right]^{N(2)} \right\} g\left(\frac{x}{s(3)}\right) g\left(\frac{y}{s(4)}\right) \frac{dx dy}{s(3)s(4)},$$

where

$$C_3: \left[\frac{x-u}{T(1)}\right]^2 + \left[\frac{y-v}{T(2)}\right]^2 \leq 1.$$

The integral in (4.56) is also an offset ellipse function. Thus, we have

$$(4.57) \quad D_D(u,v) = \left\{ 1 - \left[1 - \frac{PR(1)R(2)}{T(1)T(2)} \right]^{N(2)} \right\} P\left(\frac{T(1)}{s(3)}, \frac{T(2)}{s(4)}; \frac{u}{s(3)}, \frac{v}{s(4)}\right),$$

where $P(A,B;x,y)$ is defined in (4.52).

For a ripple of $N(1)$ dispensers with CI's at (a_i, b_i) , the kill probability $D_p(u,v)$, which is similar to (4.27), is

$$(4.58) \quad D_p(u,v) = 1 - \prod_{i=1}^{N(1)} \left[1 - \left\{ 1 - \left[1 - \frac{PR(1)R(2)}{T(1)T(2)} \right]^{N(2)} \right\} P\left(\frac{T(1)}{s(3)}, \frac{T(2)}{s(4)}; \frac{u-a_i}{s(4)}, \frac{v-b_i}{s(4)}\right) \right].$$

Substituting the expression for $D_p(u,v)$ given in (4.58) in (2.27), we obtain

$$(4.59) \quad K(X, Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ 1 - \prod_{i=1}^{N(1)} \left[1 - r \left\{ 1 - \left[\frac{pR(1)R(2)}{T(1)T(2)} \right] \right\} \right]^{N(2)} \right\} \\ \times P\left(\frac{T(1)}{s(3)}, \frac{T(2)}{s(4)}, \frac{xt(1)+X-a_i}{s(3)}, \frac{yt(2)+Y-b_i}{s(4)}\right) \left[h\left(x, \frac{A(1)}{T(1)}\right) h\left(y, \frac{A(2)}{T(2)}\right) \right] dx dy.$$

Equation (4.59) is the basis for K(107) in the machine program.

If the pattern is circular, $T(1)=T(2)=T$, and the dispenser ballistic errors are equal, $s(3)=s(4)=s$, the expression for $K(X, Y)$ in (4.59) becomes

$$(4.60) \quad K(X, Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ 1 - \prod_{i=1}^{N(1)} \left[1 - r \left\{ 1 - \left[\frac{pR(1)R(2)}{T^2} \right] \right\} \right]^{N(2)} \right\} \\ \times P\left(\frac{T}{s}, \frac{\sqrt{[xt(1)+X-a_i]^2 + [yt(2)+Y-b_i]^2}}{s}\right) \left[h\left(x, \frac{A(1)}{T(1)}\right) h\left(y, \frac{A(2)}{T(2)}\right) \right] dx dy,$$

where $P(R, r)$ is the offset circle function discussed in Sec. 3.3 and defined in (3.28) as

$$(4.61) \quad P(R, r) = \iint_{x^2+y^2 \leq R^2} g(x-r)g(y) dx dy.$$

As discussed in Ref. 1, the circular coverage function $P(R, r)$ may be approximated for R large:

$$(4.62) \quad P(R, r) \approx \int_{r-\sqrt{R^2-1}}^{\infty} g(u) du = 1 - G(r-\sqrt{R^2-1}).$$

This approximation is very close for $R \geq 5$ but should not be used for $R < 3$. Using this approximation in (4.60), we obtain a second approximation:

$$(4.63) \quad K(X, Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ 1 - \prod_{i=1}^{N(1)} \left[1 - r \left\{ 1 - \left[1 - \frac{pR(1)R(2)}{T^2} \right]^{N(2)} \right\} \right. \right. \\ \left. \left. \times \left\{ 1 - G \left(\frac{\sqrt{[xt(1)+X-a_i]^2 + [yt(2)+Y-b_i]^2 - \sqrt{T^2 - s^2}}}{s} \right) \right\} \right] \right\} \\ \times h\left(x, \frac{A(1)}{t(1)}\right) h\left(y, \frac{A(2)}{t(2)}\right) dx dy.$$

b. Approximation 2: Edge Effects, No Ballistic Error. A second approximation in (4.54) is to set $s(3)=s(4)=0$. Then $D_D(u, v)$ from (4.54) is

$$(4.64) \quad D_D(u, v) = 1 - \left[1 - \frac{pR(1)R(2)}{T(1)T(2)} P\left(\frac{T(1)}{\gamma(1)}, \frac{T(2)}{\gamma(2)}, \frac{u}{\gamma(1)}, \frac{v}{\gamma(2)}\right) \right]^{N(2)},$$

where $\gamma(1)$ and $\gamma(2)$ are as in (4.51). For a ripple of $N(1)$ dispensers, $D_p(u, v)$ is thus

$$D_p(u, v) = \prod_{i=1}^{N(1)} \left\{ 1 - r \left\{ 1 - \left[1 - \frac{pR(1)R(2)}{T(1)T(2)} P\left(\frac{T(1)}{\gamma(1)}, \frac{T(2)}{\gamma(2)}, \frac{u-a_i}{\gamma(1)}, \frac{v-b_i}{\gamma(2)}\right) \right]^{N(2)} \right\} \right\}.$$

Substituting in (2.27), we obtain

$$(4.65) \quad K(X, Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ 1 - \prod_{i=1}^{N(1)} \left[1 - r \left\{ 1 - \left[1 - \frac{pR(1)R(2)}{T(1)T(2)} \right. \right. \right. \\ \left. \left. \times P\left(\frac{T(1)}{\gamma(1)}, \frac{T(2)}{\gamma(2)}, \frac{xt(1)+X-a_i}{\gamma(1)}, \frac{yt(2)+Y-b_i}{\gamma(2)}\right) \right]^{N(2)} \right\} \right\} \\ \times h\left(x, \frac{A(1)}{t(1)}\right) h\left(y, \frac{A(2)}{t(2)}\right) dx dy.$$

Equation (4.65) is the basis for X(108) in the machine program. Equations (4.59) and (4.65) are two approximations to the same thing by different methods.

If the pattern is circular, $T(1)=T(2)=T$ and if $R(1)=R(2)=R$, so that $\gamma(1) \cdot \gamma(2) = \gamma$, Eq. (4.65) for $K(X, Y)$ becomes

$$(4.66) \quad K(X, Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(1 - \prod_{i=1}^{N(1)} \left\{ 1 - r \left[1 - \frac{pR^2}{T^2} \right. \right. \right. \\ \left. \left. \left. \times P\left(\frac{T}{Y}, \frac{\sqrt{[xt(1)+X-a_i]^2 + [yt(2)+Y-b_i]^2}}{Y}\right) \right\}^{N(2)} \right] \right) \\ \times h\left(x, \frac{A(1)}{t(1)}\right) h\left(y, \frac{A(2)}{t(2)}\right) dx dy,$$

where $P(R, r)$ is the offset circle function as in (4.61). Equation (4.66) corresponds to Eq. (4.60).

Finally, if we use the approximation for $P(R, r)$ in (4.62), we obtain the second approximation, corresponding to (4.63) for $K(X, Y)$:

$$(4.67) \quad K(X, Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[1 - \prod_{i=1}^{N(1)} \left(1 - r \left\{ 1 - \left[1 - \frac{pR^2}{T^2} \right. \right. \right. \right. \\ \left. \left. \left. \times \left\{ 1 - G\left(\frac{\sqrt{[xt(1)+X-a_i]^2 + [yt(2)+Y-b_i]^2 - \sqrt{T^2 - Y^2}}}{Y}\right) \right\}^{N(2)} \right\} \right) \right] \\ \times h\left(x, \frac{A(1)}{t(1)}\right) h\left(y, \frac{A(2)}{t(2)}\right) dx dy.$$

4.3.2. Impact-sensitive Target

For an impact-sensitive target, substituting (2.8) in (4.50), we obtain

$$(4.68) \quad D_b(u, v) = \frac{p dk}{\pi T(1)T(2)} \iint_{C_4} \alpha(u-x, B(1)) \alpha(v-y, B(2)) dx dy,$$

where $C_4: [x/T(1)]^2 + [y/T(2)]^2 \leq 1$. If $T(1)/B(1)$ and $T(2)/B(2)$ are large, we can make an approximation similar to that used in (4.55), i.e., let

$$(4.69) \quad D_b(u, v) = \begin{cases} p dk \frac{4B(1)B(2)}{\pi T(1)T(2)} & \text{if } \left[\frac{u}{T(1)}\right]^2 + \left[\frac{v}{T(2)}\right]^2 \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Thus, if (4.22) is used, $D_D(u, v)$ becomes

$$D_D(u, v) = \left\{ 1 - \left[1 - \frac{p' 4B(1)B(2)}{\pi T(1)T(2)} \right]^{N(2)} \right\} P\left(\frac{T(1)}{s(3)}, \frac{T(2)}{s(4)}, \frac{u}{s(3)}, \frac{v}{s(4)}\right).$$

This is exactly the same as $D_D(u, v)$ in (4.57) for the fragment-sensitive case, except that we have replaced $R(1)R(2)/T(1)T(2)$ by the ratio $4B(1)B(2)/\pi T(1)T(2)$ and p by p' . Thus, we obtain an expression that corresponds to (4.59):

$$(4.70) \quad K(X, Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ 1 - \prod_{i=1}^{N(1)} \left[1 - r \left\{ 1 - \left[1 - \frac{p' 4B(1)B(2)}{\pi T(1)T(2)} \right]^{N(2)} \right. \right. \right. \\ \left. \left. \left. \times P\left(\frac{T(1)}{s(3)}, \frac{T(2)}{s(4)}, \frac{xt(1)+X-a_i}{s(3)}, \frac{yt(2)+Y-b_i}{s(4)}\right) \right\} \right] \right\} h\left(x, \frac{A(1)}{t(1)}\right) h\left(y, \frac{A(2)}{t(2)}\right) dx dy.$$

Equation (4.70) forms the basis for X(109) in the machine program. For the circular case, Eq. (4.60) applies with the above replacement, i.e.,

$$(4.71) \quad K(X, Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ 1 - \prod_{i=1}^{N(1)} \left[1 - r \left\{ 1 - \left[1 - \frac{p' 4B(1)B(2)}{\pi T^2} \right]^{N(2)} \right. \right. \right. \\ \left. \left. \left. \times P\left(\frac{T}{s}, \frac{\sqrt{[xt(1)+X-a_i]^2 + [yt(2)+Y-b_i]^2}}{s}\right) \right\} \right] \right\} h\left(x, \frac{A(1)}{t(1)}\right) h\left(y, \frac{A(2)}{t(2)}\right) dx dy.$$

Finally, for T/s sufficiently large, we obtain an expression similar to (4.63):

$$(4.72) \quad K(X, Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\prod_{i=1}^{N(1)} \left\{ 1 - r \left[1 - \left[1 - \frac{p' 4B(1)B(2)}{\pi T(1)T(2)} \right]^{N(2)} \right. \right. \right. \\ \left. \left. \left. \times \left\{ 1 - G\left(\frac{\sqrt{[xt(1)+X-a_i]^2 + [yt(2)+Y-b_i]^2 - \sqrt{T^2 - s^2}}}{s}\right) \right\} \right] \right\} \right) \\ \times h\left(x, \frac{A(1)}{t(1)}\right) h\left(y, \frac{A(2)}{t(2)}\right) dx dy.$$

We note that the approximation ignores the edge effects. If the edge effects are important, i.e., $T(1)/B(1)$ and $T(2)/B(2)$ small, we may transform the elliptical pattern into an equal area rectangular pattern and use the results of Sec. 4.2.2. In this case, we would let

$$(4.73) \quad \begin{aligned} L(3) &= \frac{\sqrt{\pi}T'(1)}{2}, \\ L(4) &= \frac{\sqrt{\pi}T(2)}{2}, \end{aligned}$$

and use Eq. (4.49).

4.4. UNIFORM DISTRIBUTION OVER AN ELLIPTIC ANNULUS

We consider the same problem as that of Sec. 4.3, "Uniform Distribution over an Ellipse," except that the subweapons are assumed to be uniformly distributed over an elliptic annulus, the area included between the inner ellipse with semi-axes $W(1)$ and $W(2)$ and the outer ellipse of semi-axes $T(1)$ and $T(2)$. The kill probability $D_b(u,v)$ is now

$$(4.74) \quad D_b(u,v) = \iint_{C_5} \frac{D(u-x, v-y) dx dy}{\pi[T(1)T(2) - W(1)W(2)]},$$

where C_5 is the area between the ellipses $[x/T(1)]^2 + [y/T(2)]^2 = 1$ and $[x/W(1)]^2 + [y/W(2)]^2 = 1$. The kill probability $D_D(u,v)$ for a single dispenser against a target element at (u,v) is thus given by (4.22) and (4.21) with $D_b(u,v)$ now given by (4.74).

4.4.1. Fragment-sensitive Target

For a fragment-sensitive target, substituting the damage function (2.1) in (4.74), we obtain for $D_b(u,v)$ an expression similar to that of Eq. (4.53):

$$D_b(u,v) = \frac{R(1)R(2)}{T(1)T(2) - W(1)W(2)} \left[P\left(\frac{T(1)}{\gamma(1)}, \frac{T(2)}{\gamma(2)}, \frac{u}{\gamma(1)}, \frac{v}{\gamma(2)}\right) - P\left(\frac{W(1)}{\gamma(1)}, \frac{W(2)}{\gamma(2)}, \frac{u}{\gamma(1)}, \frac{v}{\gamma(2)}\right) \right],$$

where $P(A,B;u,v)$ is the offset ellipse function in (3.26) and $\gamma(1)$ and $\gamma(2)$ are as in (4.51), i.e.,

$$\gamma^2(1) = \frac{R^2(1)}{2D_0},$$

$$\gamma^2(2) = \frac{R^2(2)}{2D_0}.$$

Thus, as with Eq. (4.54), $D_D(u,v)$ is now

$$(4.75) \quad D_D(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[1 - \left\{ 1 - \frac{pR(1)R(2)}{T(1)T(2) - W(1)W(2)} \left[P\left(\frac{T(1)}{\gamma(1)}, \frac{T(2)}{\gamma(2)}; \frac{u}{\gamma(1)}, \frac{v}{\gamma(2)}\right) - P\left(\frac{W(1)}{\gamma(1)}, \frac{W(2)}{\gamma(2)}; \frac{u}{\gamma(1)}, \frac{v}{\gamma(2)}\right) \right] \right\}^{N(2)} \right] g\left(\frac{x}{s(3)}\right) g\left(\frac{y}{s(4)}\right) \frac{dx dy}{s(3)s(4)}.$$

a. Approximation i: Ballistic Error, No Edge Effects. If $T(1)/\gamma(1)$, $T(2)/\gamma(2)$, $W(1)/\gamma(1)$, $W(2)/\gamma(2)$ are large, we make the same approximation, as in (4.55), that

$$(4.76) \quad D_b(u,v) = \frac{R(1)R(2)}{T(1)T(2) - W(1)W(2)}$$

if (u,v) is inside the elliptic annulus C_5 and otherwise zero. Under this approximation, $D_D(u,v)$ becomes

$$(4.77) \quad D_D(u,v) = \iint_{C_5} \left\{ 1 - \left[1 - \frac{pR(1)R(2)}{T(1)T(2) - W(1)W(2)} \right]^{N(2)} \right\} g\left(\frac{x}{s(3)}\right) g\left(\frac{y}{s(4)}\right) \frac{dx dy}{s(3)s(4)}.$$

The integral in (4.77) is the difference of two offset ellipse functions; thus

$$(4.78) \quad D_D(u,v) = \left\{ 1 - \left[1 - \frac{pR(1)R(2)}{T(1)T(2) - W(1)W(2)} \right]^{N(2)} \right. \\ \left. \times \left[P\left(\frac{T(1)}{s(3)}, \frac{T(2)}{s(4)}; \frac{u}{s(3)}, \frac{v}{s(4)}\right) - P\left(\frac{W(1)}{s(3)}, \frac{W(2)}{s(4)}; \frac{u}{s(3)}, \frac{v}{s(4)}\right) \right] \right\}.$$

In this equation, $F(k, p; u, v)$ is the offset ellipse function given in Eq. (3.26).

If we proceed as in (4.59), for a ripple of $N(1)$ dispensers with CI's at (a_i, b_i) and a dispenser reliability factor p , the fractional coverage $K(X, Y)$ is

$$(4.79) \quad K(X, Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ 1 - \prod_{i=1}^{N(1)} \left[1 - p \left\{ 1 - \left[1 - \frac{F(k(1), p; u_i(1), v_i(1))}{T(1)W(2) - a_i(1)W(2)} \right]^{R(1)/T(1)} \right\} \right]^{W(2)/T(1)} \right\} \\ \cdot \left[F\left(k(1), p; \frac{xt(1)+X-a_i}{s(1)}, \frac{yt(1)+Y-b_i}{s(1)}\right) - F\left(k(1), p; \frac{xt(1)+X-a_i}{s(1)}, \frac{yt(2)+Y-b_i}{s(2)}\right) \right] \left\{ \right. \\ \left. \cdot F\left(k(2), p; \frac{xt(2)+X-a_i}{s(2)}, \frac{yt(2)+Y-b_i}{s(2)}\right) - F\left(k(2), p; \frac{xt(2)+X-a_i}{s(2)}, \frac{yt(2)+Y-b_i}{s(2)}\right) \right\} \Bigg] \Bigg\} \\ \cdot h\left(x, \frac{A(1)}{t(1)}\right) h\left(y, \frac{A(2)}{t(2)}\right) dx dy.$$

Equation (4.79) is the basis for $K(110)$ in the machine program. It allows a ballistic error but considers no edge effects. Thus, to be valid, the ratios $T(1)/R(1)$ and $T(2)/R(2)$ should be reasonably large, i.e., >5 .

If the pattern is a circular annulus, $T(1)=T(2)=T$ and $W(1)=W(2)=W$, and the ballistic errors are equal, $s(3)=s(4)=s$, the expression $K(X, Y)$, which is similar to (4.60), becomes

$$(4.80) \quad K(X, Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ 1 - \prod_{i=1}^{N(1)} \left[1 - p \left\{ 1 - \left[1 - \frac{F(k(1), p; u_i(1), v_i(1))}{T} \right]^{R(1)/T} \right\} \right]^{W(2)/T} \right\} \\ \cdot \left[F\left(k(1), p; \frac{\sqrt{[xt(1)+X-a_i]^2 + [yt(1)+Y-b_i]^2}}{s}, \frac{\sqrt{[xt(1)+X-a_i]^2 + [yt(1)+Y-b_i]^2}}{s}\right) - F\left(k(1), p; \frac{\sqrt{[xt(1)+X-a_i]^2 + [yt(2)+Y-b_i]^2}}{s}, \frac{\sqrt{[xt(2)+X-a_i]^2 + [yt(2)+Y-b_i]^2}}{s}\right) \right] \left\{ \right. \\ \left. \cdot F\left(k(2), p; \frac{\sqrt{[xt(2)+X-a_i]^2 + [yt(2)+Y-b_i]^2}}{s}, \frac{\sqrt{[xt(2)+X-a_i]^2 + [yt(2)+Y-b_i]^2}}{s}\right) - F\left(k(2), p; \frac{\sqrt{[xt(2)+X-a_i]^2 + [yt(2)+Y-b_i]^2}}{s}, \frac{\sqrt{[xt(2)+X-a_i]^2 + [yt(2)+Y-b_i]^2}}{s}\right) \right\} \Bigg] \Bigg\} \\ \cdot h\left(x, \frac{A(1)}{t(1)}\right) h\left(y, \frac{A(2)}{t(2)}\right) dx dy,$$

where $F(k, p)$ is the offset circle function as in (4.61).

As a second approximation for T/s and W/s large, using the approximation for $F(k, p)$ in (4.62), we obtain an expression that is similar to (4.63):

$$\begin{aligned}
 (4.81) \quad K(X, Y) = & \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ 1 - \prod_{i=1}^{N(1)} \left[1 - r \left\{ 1 - \left[\frac{pR(1)R(2)}{T^2 - W^2} \right] \right\}^{N(2)} \right] \right\} \\
 & \times \left\{ G \left(\frac{\sqrt{[xt(1) + X - a_i]^2 + [yt(2) + Y - b_i]^2 - \sqrt{W^2 - s^2}}}{s} \right) \right. \\
 & \left. - G \left(\frac{\sqrt{[xt(1) + X - a_i]^2 + [yt(2) + Y - b_i]^2 - \sqrt{T^2 - s^2}}}{s} \right) \right\} \\
 & \times h\left(x, \frac{A(1)}{t(1)}\right) h\left(y, \frac{A(2)}{t(2)}\right) dx dy.
 \end{aligned}$$

Equation (4.81) is the basis for X(100) in the machine program. Again, no edge effects are considered and $\min [T/R(1), T/R(2)]$ should be greater than 5. Further, in order that approximation (4.62) be valid, T/s and W/s should be greater than 3.

b. Approximation 2: Edge Effects, No Ballistic Error. The second approximation is to set $s(3)=s(4)=0$. Then, proceeding as in Sec. 4.3.1a for Eqs. (4.64) and (4.65), we obtain for $K(X, Y)$ an equation that corresponds to (4.65):

$$\begin{aligned}
 (4.82) \quad K(X, Y) = & \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(1 - \prod_{i=1}^{N(1)} \left\{ 1 - r \left[1 - \frac{pR(1)R(2)}{T(1)T(2) - W(1)W(2)} \right] \right\} \right. \\
 & \times \left[P \left(\frac{T(1)}{\gamma(1)}, \frac{T(2)}{\gamma(2)}; \frac{xt(1) + X - a_i}{\gamma(1)}, \frac{yt(2) + Y - b_i}{\gamma(2)} \right) \right. \\
 & \left. \left. - P \left(\frac{W(1)}{\gamma(1)}, \frac{W(2)}{\gamma(2)}; \frac{xt(1) + X - a_i}{\gamma(1)}, \frac{yt(2) + Y - b_i}{\gamma(2)} \right) \right] \right\}^{N(2)} \Bigg) \Bigg\} h\left(x, \frac{A(1)}{t(1)}\right) h\left(y, \frac{A(2)}{t(2)}\right) dx dy,
 \end{aligned}$$

where $\gamma(1)$ and $\gamma(2)$ are defined in Eq. (4.51). Equation (4.82) forms the basis for X(111) in the machine program. Equation (4.82) is used when the edge effects are important, while Eq. (4.79) is used when the ballistic errors are important.

If the pattern is a circular annulus $T(1)=T(2)=T$ and $W(1)=W(2)=W$, we obtain an expression that corresponds to (4.66):

$$(4.83) \quad K(X, Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[1 - \prod_{i=1}^{N(1)} \left(1 - r_i \left\{ 1 - \frac{pR^2}{T^2 - W^2} \right. \right. \right. \\ \left. \left. \left. \times \left\{ P \left(\frac{T}{Y}, \frac{\sqrt{[xt(1)+X-a_i]^2 + [yt(2)+Y-b_i]^2}}{Y} \right) \right. \right. \right. \right. \\ \left. \left. \left. - P \left(\frac{W}{Y}, \frac{\sqrt{[xt(1)+X-a_i]^2 + [yt(2)+Y-b_i]^2}}{Y} \right) \right\} \right]^{N(2)} \right\} \right] h \left(x, \frac{A(1)}{t(1)} \right) h \left(y, \frac{A(2)}{t(2)} \right) dx dy.$$

Finally, a second approximation for T/γ and W/γ large that is similar to (4.67) is

$$(4.84) \quad K(X, Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[1 - \prod_{i=1}^{N(1)} \left(1 - r_i \left\{ 1 - \frac{pR^2}{T^2 - W^2} \right. \right. \right. \\ \left. \left. \left. \times \left\{ G \left(\frac{\sqrt{[xt(1)+X-a_i]^2 + [yt(2)+Y-b_i]^2 - \sqrt{W^2 - \gamma^2}}}{Y} \right) \right. \right. \right. \right. \\ \left. \left. \left. - G \left(\frac{\sqrt{[xt(1)+X-a_i]^2 + [yt(2)+Y-b_i]^2 - \sqrt{T^2 - \gamma^2}}}{Y} \right) \right\} \right]^{N(2)} \right\} \right] \\ \times h \left(x, \frac{A(1)}{t(1)} \right) h \left(y, \frac{A(2)}{t(2)} \right) dx dy.$$

Equation (4.84) forms the basis for $X(101)$ in the machine program. In this case, T/R and W/R should be larger than 5.

4.4.2. Impact-sensitive Target

For an impact-sensitive target, $D_D(u, v)$ is the same as in (4.68) for the ellipse, except that the elliptic area C_4 is replaced by the elliptic annulus C_5 of (4.74), if $T(1)/B(1)$, $T(2)/B(2)$, $W(1)/B(1)$, and $W(2)/B(2)$ are sufficiently large, we make the same approximation as in (4.69) over C_5 , i.e.,

$$D_b(u,v) = \begin{cases} \frac{p' 4B(1)B(2)}{\pi[T(1)T(2)-W(1)W(2)]} & \text{if } (u,v) \text{ is inside } C_5, \\ 0 & \text{otherwise.} \end{cases}$$

Under this approximation, $K(X,Y)$, which is similar to (4.70), is

$$(4.85) \quad K(X,Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(1 - \prod_{i=1}^{N(1)} \left\{ 1 - r \left[1 - \frac{p' 4B(1)B(2)}{\pi[T(1)T(2)-W(1)W(2)]} \right] \right\}^{N(2)} \right) \\ \times \left[P\left(\frac{T(1)}{s(3)}, \frac{T(2)}{s(4)}, \frac{xt(1)+X-a_i}{s(3)}, \frac{yt(2)+Y-b_i}{s(4)}\right) \right. \\ \left. - P\left(\frac{W(1)}{s(3)}, \frac{W(2)}{s(4)}, \frac{xt(1)+X-a_i}{s(3)}, \frac{yt(2)+Y-b_i}{s(4)}\right) \right] \Bigg\} h\left(x, \frac{A(1)}{t(1)}\right) h\left(y, \frac{A(2)}{t(2)}\right) dx dy.$$

We note that (4.85) is identical with (4.79) for the fragment-sensitive target except that the ratio $R(1)R(2)/[T(1)T(2)-W(1)W(2)]$ is replaced by $4B(1)B(2)/\pi[T(1)T(2)-W(1)W(2)]$. Equation (4.85) is the basis for $X(112)$ in the machine program.

For the circular case, $T(1)=T(2)=T$ and $W(1)=W(2)=W$, and the ballistic errors $s(3)=s(4)=s$, the expression $K(X,Y)$, which corresponds to (4.81), becomes

$$(4.86) \quad K(X,Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ 1 - \prod_{i=1}^{N(1)} \left[1 - r \left\{ 1 - \left[\frac{p' 4B(1)B(2)}{\pi(T^2-W^2)} \right] \right\}^{N(2)} \right] \right\} \\ \times \left\{ G\left(\frac{\sqrt{[xt(1)+X-a_i]^2 + [yt(2)+Y-b_i]^2 - \sqrt{W^2-s^2}}}{s}\right) \right. \\ \left. - G\left(\frac{\sqrt{[xt(1)+X-a_i]^2 + [yt(2)+Y-b_i]^2 - \sqrt{T^2-s^2}}}{s}\right) \right\} \Bigg\} h\left(x, \frac{A(1)}{t(1)}\right) h\left(y, \frac{A(2)}{t(2)}\right) dx dy.$$

Again, we note that (4.81) and (4.86) are identical except for the factors

$$\frac{4B(1)B(2)}{\pi(T^2-W^2)} \quad \text{and} \quad \frac{R(1)R(2)}{T^2-W^2}.$$

Equation (4.86) forms the basis for $X(102)$ in the machine program. Since no edge effects are considered, $T/B(1)$ and $T/B(2)$ should be greater than 5; further, in order that approximation (4.62) be valid, T/s and W/s should be greater than 3. If a case occurs in which edge effects are important, i.e., $T/B(1)$ or $T/B(2)$, some other method should be used, e.g., $X(109)$.

5. THE COMPUTER PROGRAM

The simplified weapons evaluation model described here is sufficiently broad to cover almost all problems that arise in the field of nonnuclear weapons evaluation. The FORTRAN program is designed to compute the solutions to most of the effectiveness problems discussed in Secs. 3 and 4. After describing the inputs and computational methods that are generally used for all problem types, we consider each problem type that involves different inputs or different methods.

This section has been planned to permit the user to run the program without referring to previous sections. In the event that further investigation is desired, however, the appropriate references are given for each case discussed.

5.1. GENERAL CONSIDERATIONS

The following list provides the inputs required for the program, with references to the sections in which these inputs, their uses and restrictions, are described. Odd integers in the inputs denote a parameter in the x (range) direction; even integers denote a parameter in the y (deflection) direction.

<u>FORTTRAN</u> <u>Inputs</u>	<u>Description</u>	<u>Reference</u>
A3,A4	Target area dimensions	5.1.1
B3,B4	Impact-type target dimensions	5.1.5
R1,R2	Fragment-type target parameters	5.1.5
D1,D2	Fragment-type target parameters	5.1.5
CL1,Q0	Dispenser range ballistic parameters	5.3
GCL3,GCL4	Rectangular pattern half dimensions	5.4
ET1,ET2	Elliptic pattern semiaxes	5.4
W1,W2	Elliptic annulus inner semiaxes	5.4
U1,U2	Number of integration steps	5.1.4
SU1,SU2	Target offsets	5.1.1
S1,S2	Ballistic error standard deviations	5.2
S3,S4	Ballistic error standard deviations for dispenser	5.4
T1,T2	Aiming error standard deviations	5.1.2
A(J),B(J)	Individual weapon aiming point	5.1.3
NXAJ,NXBJ	Flags for aiming point pattern	5.1.3
D,DF	Spacing between successive weapons	5.4
N,NB	Number of weapons, number of subweapons	5.4
SP	Probability of kill if hit, weapon reliability	5.1.5
R	Dispenser reliability	5.4

5.1.1. Target Area: Inputs $A3$, $A4$, $SU1$, $SU2$

The target area in all cases considered is a rectangle with dimensions $A3$ in the x (range) direction and $A4$ in the y (deflection) direction. The inputs $A3$ and/or $A4$ may be zero. The center of the target area may be offset from the aim point at $SU1$, $SU2$, where $SU1$ is the range offset and $SU2$ is the deflection offset.

5.1.2. Aiming Error: Inputs $T1$, $T2$

The aiming error distribution is assumed gaussian (see Sec. 3.2.1) with standard deviations $T1$ and $T2$ in x and y , respectively. In terms of REP and DEP, $AREP=.6744T1$, $ADEP=.6744T2$. If $T1=T2=T$, then $CEP=1.1774T$.

5.1.3. Aiming Point Array: Inputs $A(J)$, $B(J)$, D , DF , N

The aiming pattern is the array of the N desired center of impacts $[A(J), B(J)]$, $J=1, 2, \dots, N$. (See Sec. 3.1.) The points are so defined that $\sum A(J)=0$, $\sum B(J)=0$. The pattern may be a direct input or may be calculated. If the flag $NXAJ=0$, a direct input for the $A(J)$'s is called for and D must be 1; if the flag $NXBJ=0$, the $B(J)$'s are called for and DF must be 1. If $NXAJ=1$, the $A(J)$'s are calculated, using the uniform spacing D ; if $NXBJ=1$, the $B(J)$'s are calculated, using the spacing DF .

5.1.4. Integration Routine: Inputs $U1$, $U2$

The basic integration problem (see Sec. 2.3) is the evaluation of the double integral

$$(5.1) \quad I = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} K(x, y) h\left(x, \frac{A(1)}{T(1)}\right) h\left(y, \frac{A(2)}{T(2)}\right) dx dy,$$

where $K(x, y)$ is a nonnegative integrable function never larger than 1; $A(1)$ and $A(2)$ are the target half dimensions; and $h(x, A)$ is the function defined in Eq.

(2.19) as

$$(5.2) \quad h(x, A) = \frac{1}{2A} \int_{-A}^A g(x+z) dz,$$

where $g(x) = \exp(-x^2/2) / \sqrt{2\pi}$.

We note the limits

$$(5.3) \quad \lim_{T \rightarrow 0} \frac{h(x/T, A/T)}{T} = \frac{a(x, A)}{2A} = \begin{cases} \frac{1}{2A} & \text{if } |x| \leq A, \\ 0 & \text{if } |x| > A, \end{cases}$$

$$\lim_{A \rightarrow 0} h(x, A) = g(x).$$

In order to use a numerical integration routine, we must truncate the integrals in (5.1) through an appropriate choice of the truncation limits $E(1)$ and $E(2)$, obtaining the expression

$$(5.4) \quad I = \int_{-E(1)}^{E(1)} \int_{-E(2)}^{E(2)} K(x, y) h\left(x, \frac{A(1)}{T(1)}\right) h\left(y, \frac{A(2)}{T(2)}\right) dx dy + \varepsilon_1,$$

where ε_1 is the truncation error. To obtain $E(1)$ and $E(2)$ we consider in turn two possible choices for truncation limits. The first set, $E(3)$ and $E(4)$, are based on the fact that $K(x, y) \leq 1$. For truncation limits $E(3)$ and $E(4)$, the corresponding truncation error ε_2 is

$$(5.5) \quad \varepsilon_2 \leq 4 \int_{E(3)}^{\infty} h\left(x, \frac{A(1)}{T(1)}\right) dx \int_{E(4)}^{\infty} h\left(y, \frac{A(2)}{T(2)}\right) dy \\ \leq 4 \left[1 - H\left(E(3), \frac{A(1)}{T(1)}\right) \right] \left[1 - H\left(E(4), \frac{A(2)}{T(2)}\right) \right],$$

where $H(x, A)$ is defined as

$$(5.6) \quad H(x, A) = \int_{-\infty}^x h(y, A) dy.$$

and is evaluated by Eq. (2.20). We determine $E(3)$ and $E(4)$ satisfying the inequalities

$$1-h\left(E(3), \frac{A(1)}{T(1)}\right) \leq 10^{-4},$$

$$1-h\left(E(4), \frac{A(2)}{T(2)}\right) \leq 10^{-4}.$$

Thus the truncation error ϵ_2 is

$$(5.7) \quad \epsilon_2 \leq 4(10)^{-8}.$$

The second set $E(5)$ and $E(6)$ is based on the fact that

$$\int_{-\infty}^{\infty} h(x, A) dx = 1.$$

Thus for truncation limits $E(5)$ and $E(6)$, the truncation error ϵ_3 is

$$(5.8) \quad \epsilon_3 \leq 2 \left[\max_{|x| > E(5)} K(x, y) + \max_{|y| > E(6)} K(x, y) \right].$$

We determine $E(5)$ and $E(6)$ for each function $K(x, y)$ satisfying the inequalities

$$(5.9) \quad \begin{aligned} \max_{|x| > E(5)} K(x, y) &\leq 10^{-4}, \\ \max_{|y| > E(6)} K(x, y) &\leq 10^{-4}, \end{aligned}$$

so that the truncation error ϵ_3 is

$$\epsilon_3 \leq 4(10)^{-4}.$$

The truncation limits $E(1)$ and $E(2)$ are chosen as

$$(5.10) \quad \begin{aligned} E(1) &= \min(E(3), E(5)), \\ E(2) &= \min(E(4), E(6)). \end{aligned}$$

The truncation error ϵ_1 is thus

$$\varepsilon_1 \leq \max(\varepsilon_2, \varepsilon_3).$$

In the numerical integration routine, we choose the number of integration steps $2U(1)$ in the x direction and $2U(2)$ in the y direction. The respective step sizes $e(1)$ and $e(2)$ are thus

$$(5.11) \quad e(1) = \frac{E(1)}{U(1)}, \quad e(2) = \frac{E(2)}{U(2)},$$

where $E(1)$ and $E(2)$ are given in (5.10). We designate the centers of the integration rectangles by $[x(j), y(j)]$ and let

$$(5.12) \quad x(j) = \begin{cases} (2 \cdot j - 1) \frac{e(1)}{2} & \text{if } j \leq U(1), \\ [2 \cdot j - 1 - 2U(1)] \frac{e(1)}{2} & \text{if } j > U(1), \end{cases}$$

$$y(j) = \begin{cases} (2 \cdot j - 1) \frac{e(2)}{2} & \text{if } j \leq U(2), \\ [2 \cdot j - 1 - 2U(2)] \frac{e(2)}{2} & \text{if } j > U(2). \end{cases}$$

The integral I in (5.4) is thus given approximately by

$$(5.13) \quad I \approx \sum_{j=1}^{2U(1)} \sum_{k=1}^{2U(2)} \int_{x(j)-e(1)/2}^{x(j)+e(1)/2} \int_{y(k)-e(2)/2}^{y(k)+e(2)/2} K(\xi, \eta) h\left(\xi, \frac{A(1)}{T(1)}\right) h\left(\eta, \frac{A(2)}{T(2)}\right) d\xi d\eta.$$

Replacing $K(\xi, \eta)$ by its value at the midpoint of the integration rectangle, we find that

$$(5.14) \quad I = \sum_{j=1}^{2U(1)} \sum_{k=1}^{2U(2)} K(x(j), y(k)) \int_{x(j)-e(1)/2}^{x(j)+e(1)/2} h\left(\xi, \frac{A(1)}{T(1)}\right) d\xi$$

$$\times \int_{y(k)-e(2)/2}^{y(k)+e(2)/2} h\left(\eta, \frac{A(2)}{T(2)}\right) d\eta.$$

When we use the expression (5.6) for $H(x, y)$, we obtain for I

$$(5.15) \quad I = \sum_{j=1}^{2U(1)} \sum_{k=1}^{2U(2)} K(x(j), y(k)) \left[H\left(x(j) + \frac{e(1)}{2}, \frac{A(1)}{T(1)}\right) - H\left(x(j) - \frac{e(1)}{2}, \frac{A(1)}{T(1)}\right) \right] \\ \times \left[H\left(y(k) + \frac{e(2)}{2}, \frac{A(2)}{T(2)}\right) - H\left(y(k) - \frac{e(2)}{2}, \frac{A(2)}{T(2)}\right) \right].$$

We define the function $F(x, y, L)$ as

$$(5.16) \quad F(x, y, L) = H(x+y, L) - H(x-y, L).$$

Then, I from Eq. (5.15) is

$$(5.17) \quad I = \sum_{j=1}^{2U(1)} \sum_{k=1}^{2U(2)} K(x(j), y(k)) F\left(x(j), \frac{e(1)}{2}, \frac{A(1)}{T(1)}\right) F\left(y(k), \frac{e(2)}{2}, \frac{A(2)}{T(2)}\right).$$

In terms of the FORTRAN symbols, the integral I is

$$(5.18) \quad I = \sum_{j=1}^{2U1} \sum_{k=1}^{2U2} K6(x(j), y(k)) FF\left(x(j), \frac{SE1}{2}, \frac{A1}{T1}\right) FF\left(y(k), \frac{SE2}{2}, \frac{A2}{T2}\right).$$

If $K(x, y)$ is symmetric in either x or y , the sum in (5.17) need only be taken to $U(1)$ or $U(2)$. If symmetric in both x and y , we obtain

$$(5.19) \quad I = 4 \sum_{j=1}^{U(1)} \sum_{k=1}^{U(2)} K(x(j), y(k)) F\left(x(j), \frac{e(1)}{2}, \frac{A(1)}{T(1)}\right) F\left(y(k), \frac{e(2)}{2}, \frac{A(2)}{T(2)}\right).$$

Equations (5.17) and (5.19) are the basis for the integration routine, subroutine A50. For one dimension, the expressions are single sums and are the basis for the single integration routines.

5.1.5. Damage Functions: Inputs $SP, B3, B4, R1, R2, D1, D2$

Two types of damage functions are used (see Sec. 2.1). We define an *impact-sensitive target* as one for which there is a definite geometric figure (in our case a rectangle of dimensions $B3, B4$), which must be impacted by the weapon or subweapon. A target for which a vulnerable area is given thus belongs in this category. The input parameter SP is the probability of damage if hit. When we define $B1 = B3/2, B2 = B4/2$, D is given by

$$(5.20) \quad D(x,y) = \begin{cases} SP & \text{if } -B1 \leq x \leq B1, -B2 \leq y \leq B2, \\ 0 & \text{otherwise.} \end{cases}$$

We note that $B3$ and $B4$ are the dimensions in the ground plane.

We define a *fragment-sensitive target* as one for which a significant portion of the damage effect is due to the fragmentation of the weapon. Usually, the damage function is determined as a damage matrix using a "lethal area" program. This damage function is approximated by an exponential function (see Sec. 2.1) of the form

$$(5.21) \quad D(x,y) = SP \cdot \sqrt{D1 \cdot D2} \exp\left(-\frac{D1x^2}{R1^2} - \frac{D2y^2}{R2^2}\right).$$

The parameters $D1, D2, R1$, and $R2$ are inputs to be obtained from empirical data. First, we require that $\pi R1 \cdot R2 = MAE$, where the MAE has been determined by some other method. The ratio of $R1$ to $R2$, or the ellipticity of the damage function, may be calculated or estimated. The initial values $D1$ and $D2$ will usually be set equal to one. In this case the input parameter SP is the reliability factor for the individual weapon.

5.2. RIPPLE OF BOMBS: INPUTS $S1, S2, N$

We consider a ripple of N weapons delivered with a common aiming error according to an aiming point pattern. Each weapon is subject individually to a ballistic error according to a gaussian distribution with standard deviations

$S1$ and $S2$. The inputs $S1$ and $S2$ may be zero. Again, in terms of REP and DEP ,
 $BREP = .6744S1$, $BDEP = .6744S2$.

Figure 1 presents a flow diagram of the computer program for the ripple of bombs. The fractional coverages $X(1)$, $X(11)$, $X(5)$, and $X(15)$ are answers to the problems considered in Sec. 3. The program is described as follows:

- The fractional coverage for a ripple of bombs against a *fragment-sensitive target* is $X(11)$. Pertinent inputs are $D1$, $D2$, $R1$, $R2$, $S1$, and $S2$, which are obtained from empirical data. The program uses Eq. (3.7).

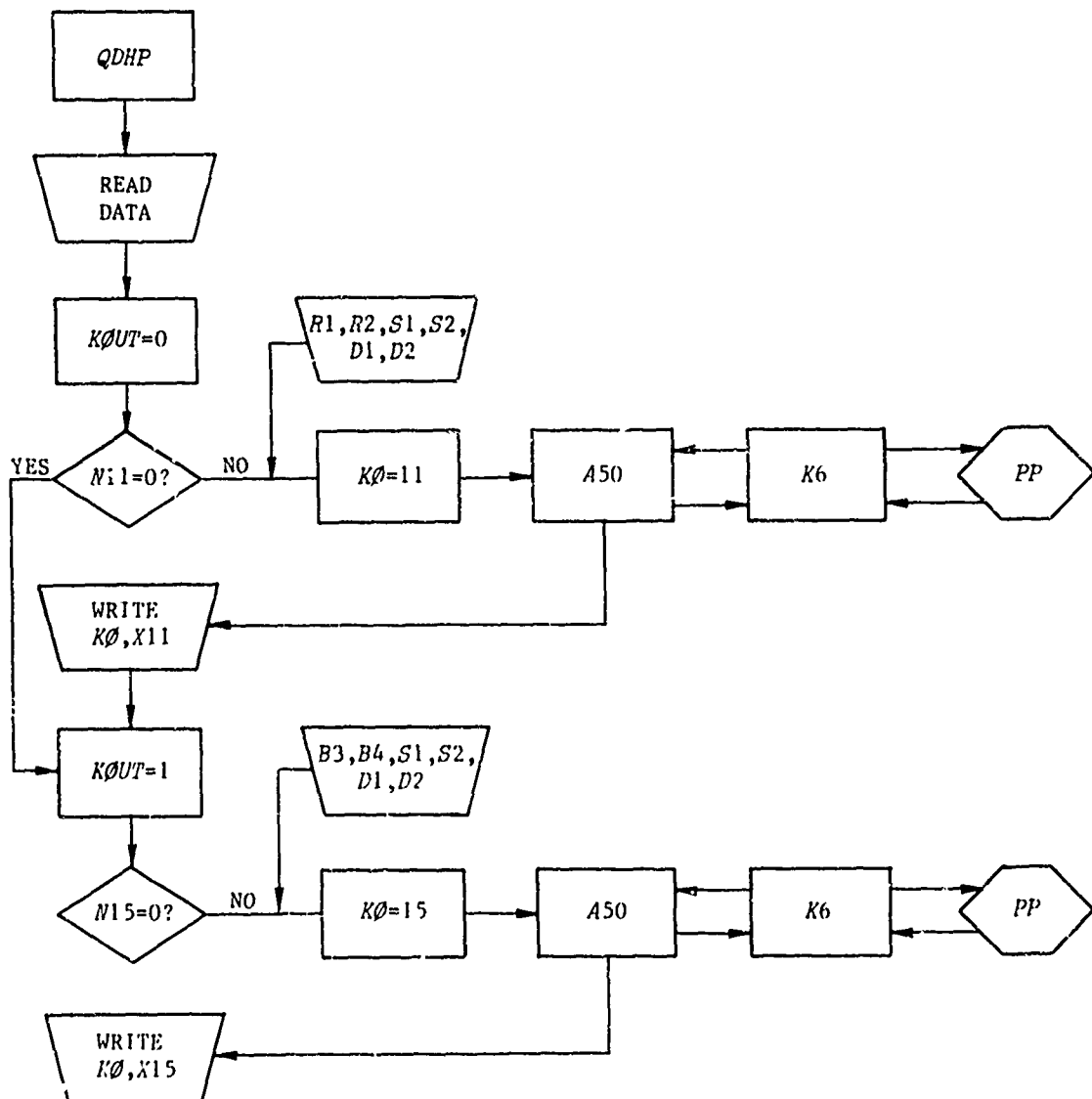


Fig. 1--Ripple of Bombs Delivered with a Common Aiming Error

- The fractional coverage against an *impact-sensitive target* is $X(15)$. This problem uses $B3$ and $B4$ in the computations rather than $R1$, $R2$, $D1$, and $D2$. The program uses Eq. (3.20).
- The fractional coverages $X(1)$ and $X(11)$ are answers to the same problem when different but equivalent expressions are used; $X(5)$ and $X(15)$ are also equivalent. However, $X(1)$ and $X(5)$ are restricted to cases in which the number of weapons, N , is less than or equal to 12. The computation for N very small is faster and more accurate for $X(1)$ and $X(5)$ than it is for $X(11)$ and $X(15)$.
- The flag NO is used to skip parts of the program to compute $X(1)$ or $X(5)$. The code is restricted to the computation of $X(1)$, $X(2)$, $X(3)$, $X(4)$, or $X(5)$ when $NO=0$; when $NO=1$, $X(1)$ through $X(5)$ are omitted; there are no restrictions when $NO=2$. The subroutine $XRØ$ computes $X(1)$ through $X(5)$. Equation (3.13) is used to compute $X(1)$ and Eq. (3.21) to compute $X(5)$.
- The basic flow of the computer model is the same for $X(11)$ and $X(15)$. In subroutine P^* , the $P(x,j)$ function has been redefined for $X(15)$. The flag $KØUT$, which is automatically set and is not a data input, determines which $P(x,j)$ function to use.

5.3. RIPPLE OF FIXED DISPENSERS: INPUTS NB , $S2$, $TABLE$, OR $CL1$, $QØ$

We consider a ripple of N fixed dispensers actuated with a common aiming error according to an aiming point pattern. From each dispenser, NB subweapons are released. In range, the ballistic dispersion for the subweapons within each dispenser is given by an empirical table. In deflection, each subweapon is assumed subject to a gaussian ballistic error with standard deviation $S2$. The range pattern is fitted by an approximating function, the stick distribution $H(x/QØ, CL1/QØ)$, discussed in Sec. 2.2.2c and App. B, through the use of the two parameters $CL1$ and $QØ$.

Figure 2 presents a flow diagram of the computer program for a ripple of fixed dispensers. The fractional coverages $X(2)$, $X(12)$, $X(3)$, $X(13)$, $X(4)$, and $X(14)$ are answers to the problems in Sec. 4.1.

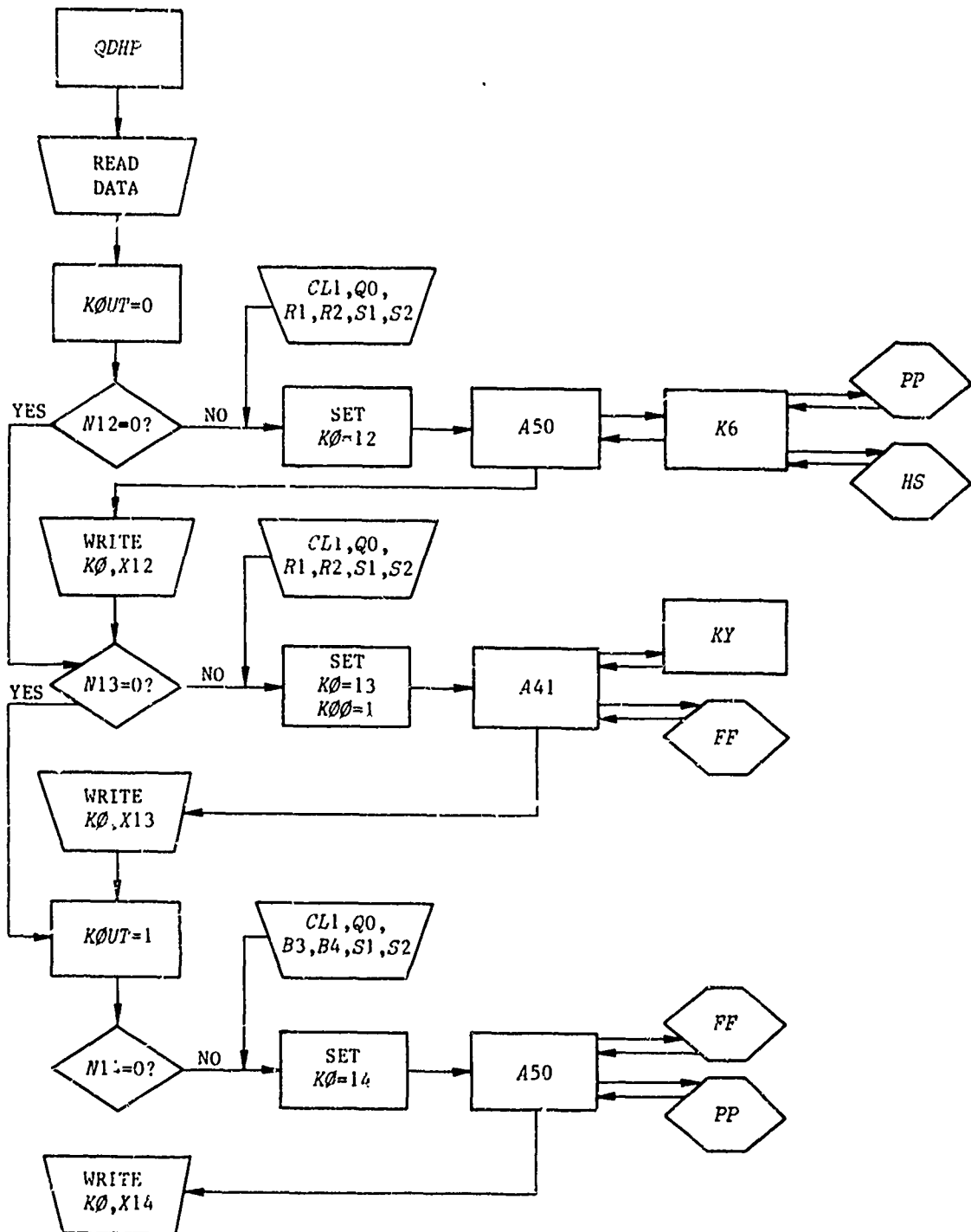


Fig. 2--Ripple of Fixed Dispensers

The program is described as follows:

- The fractional coverage for a ripple of fixed dispensers against a *fragment-sensitive target* is $X(12)$. The inputs are $R1$, $R2$, $D1$, and $D2$; the appropriate reference is Eq. (4.9).
- The approximation to $X(12)$ is $X(13)$. The computation involves only a single integration. It is thus faster than the computation for $X(12)$, but a degree of accuracy is lost. The appropriate reference is Eq. (4.18).
- The fractional coverage for an *impact-sensitive target* is $X(14)$. The inputs $B3$ and $B4$ are necessary. The appropriate reference is Eq. (4.13).
- The fractional coverages $X(12)$, $X(13)$, and $X(14)$ use $CL1$ and $Q0$, which are dispenser range ballistic parameters. If the ballistic data are in tabular form, i.e., $CNPT=1$, and the table is read as an input array, set $CL1=1$ and $Q0=0$. The actual values will then be computed.
- The fractional coverages $X(2)$, $X(3)$, and $X(4)$ are the same as $X(12)$, $X(13)$, and $X(14)$; however, they are restricted to $N=1$ and $NB \leq 30$.

5.4. RIPPLE OF PATTERNS: INPUTS $S3$, $S4$, NB , R

We consider a ripple of N patterns, each containing NB subweapons, delivered with a common aiming error according to an aiming point array for the patterns. The $A(J)$, $B(J)$ of Sec. 5.1.3 are now the pattern aiming points. Each pattern is subject to a gaussian ballistic error with standard deviations $S3$ and $S4$ and to a reliability factor R . The NB subweapons are assumed to be uniformly distributed within the pattern. In the computation we had to make one of two assumptions. The first was that the damage level was uniform throughout the pattern. This assumption ignores the edge effects and is valid whenever the pattern area is much larger (say, 16/1) than the target area $4 \cdot B1 \cdot B2$ or the $MAE = \pi \cdot R1 \cdot R2$. Cases for which this assumption is made will be designated as "no edge effects." For some cases, the edge effects may be considered if the ballistic error standard deviations $S3$ and $S4$ are ignored. These cases will be designated as "edge effects."

5.4.1. Rectangular Patterns: $GCL3$, $GCL4$

Consider a ripple of N rectangular patterns of dimensions $2GCL3$, $2GCL4$, delivered with a common aiming error according to an aiming point array.

Basically, the flow diagram for a ripple of patterns is the same as that for a ripple of fixed dispensers. (See Sec. 5.3.) The coverage $X(103)$ for a fragment-sensitive target, ignoring edge effects, is computed according to Eq. (4.28). The inputs, which are derived from empirical data, are $GCL3$ and $GCL4$, which are the rectangular pattern dimensions; $R1$, $R2$, $D1$, $D2$, R ; and $S3, S4 \neq 0$. The coverage $X(103)$ should be restricted to the cases in which $GCL3/R1$ and $GCL4/R2$ are greater than 5. If these ratios are not greater than 5, the coverage $X(104)$ for a fragment-sensitive target, including edge effects (i.e., $S3=S4=0$), should be used. The coverage $X(104)$ is computed according to Eq. (4.35).

The coverage $X(105)$ for an impact-sensitive target, ignoring edge effects, requires inputs $B3$, $B4$, and $S3, S4 \neq 0$. The ratios $2GCL3/B3$ or $2GCL4/B4$ must be greater than 5. If they are less than 5, the coverage $X(106)$ for an impact-sensitive target, including edge effects, should be used. The coverage $X(105)$ is computed according to Eq. (4.46) and $X(106)$ according to Eq. (4.49).

5.4.2. Ripple of Elliptic Patterns

The flow diagram for the following three cases is the same as that for the corresponding rectangular cases. The coverage $X(107)$ for a ripple of N elliptic patterns against an area of fragment-sensitive targets, ignoring edge effects, is computed according to Eq. (4.59). The coverage $X(107)$ should not be used if the ratios $ET1/R1$ or $ET2/R2$ are less than 5. The coverage $X(108)$ for a ripple of N elliptic patterns against an area of fragment-sensitive targets, including edge effects, should be used instead. The coverage $X(108)$ is computed according to Eq. (4.65).

The coverage $X(109)$ against an area of impact-sensitive targets, ignoring edge effects, is computed according to Eq. (4.70). The coverage $X(109)$ should not be used if the ratios $ET1/B1$ or $ET2/B2$ are less than 5; $S3$ and $S4$ should not be zero.

5.4.3. Elliptic Annulus Patterns: $ET1, ET2, W1, W2, R$

We consider a ripple of N elliptic annulus patterns that are delivered with a common aiming error according to an aiming point array. The axes of the outer ellipses are $ET1$ and $ET2$, while those for the inner ellipse are $W1$ and $W2$. The computation for the elliptic patterns involves the elliptic coverage function, which is time consuming. In each of the following cases, a circular approximation is given, an approach that is fast and in many cases gives a very close approximation. Further, if we are dealing with circular patterns, we may use these approximations in place of the elliptic patterns of the previous section by setting the inner radius to zero.

The coverage $X(110)$ for a ripple of N elliptic patterns against an area of fragment-sensitive targets, ignoring edge effects, is computed according to Eq. (4.79) if the flag $N110=1$. If the ratios $ET1/R1$, $ET2/P2$, $W1/R1$, or $W2/R2$ are less than 5, it is preferable to use $X(111)$, which takes edge effects into account.

For the special case in which the ellipses are circles, $R1=R2$ and $S3=S4$, the special approximation may be used if the ratios $ET1/S3$ and $W1/S3$ are larger than 3. In this case, the coverage $X(100)$ for a ripple of N circular annulus patterns, ignoring edge effects, is computed according to Eq. (4.81) if the flag $N100=1$. Again, we may use $X(100)$ at times in the elliptic case by converting the ellipses to circles of the same area and converting the ballistic errors to the circular case as above. Likewise, $X(100)$ may be used in place of $X(107)$ under the same restrictions by setting the inner radius to zero. The coverage $X(111)$ for a ripple of N elliptic patterns against an area of fragment-sensitive targets, considering edge effects, is computed according to Eq. (4.82), if the flag $N111=1$.

For the special case in which the ellipses are circles and $R1=R2$, the coverage $X(101)$ for a ripple of N circular annulus patterns against an area of fragment-sensitive targets is computed from Eq. (4.84) if the flag $N101=1$. We can use $X(101)$ under these conditions in place of $X(108)$ by setting the inner radius equal to zero.

The coverage $X(112)$ for a ripple of N elliptic annulus patterns against an area of impact-sensitive targets, ignoring edge effects, is computed according to

Eq. (4.85) if the flag $N112=1$. In general, $X(112)$ should not be used if the ratios $ET1/B1$, $ET2/B2$ are less than 5. If the ratios $W1/B1$ or $W2/B2$ are less than 5, it is better to set $W1$ and $W2$ to zero and use $X(109)$.

For the special case in which the ellipses are circles, i.e., $ET1=ET2$ and $W1=W2$, and the ballistic error is circular, i.e., $S3=S4$, a special approximation may be used if the ratios $ET1/S3$ and $W2/S3$ are larger than 3. In this case, the coverage $X(102)$ for a ripple of N circular annulus patterns against an area of impact-sensitive targets, ignoring edge effects, is computed according to Eq. (4.87) if the flag $N102=1$. The same restrictions apply as for $X(112)$ for the ratios $ET1/B1$, $ET2/B2$, $W1/B1$, and $W2/B2$. Since the computation time for $X(102)$ is much smaller than for $X(112)$, it is sometimes worthwhile to use $X(102)$ even in the elliptic case by converting the ellipses to circles of the same area, and by converting the ballistic error to a circular case, letting $S=S3=S4$ be the equivalent ballistic standard deviation in both directions. Likewise, we can use $X(102)$ in place of $X(109)$ under the same restrictions by setting the inner radius to zero.

5.5. SUMMARY

The following tabular listings summarize the problems that are coded and provide a reference for further information.

RIPPLE OF BOMBS, SEC. 5.2

<i>Output</i>	<i>Description</i>	<i>Reference</i>
$X(11)$	Fragment-sensitive target	Eq. (3.7)
$X(1)$	Fragment-sensitive target; limited to the case of $N \leq 12$	Eq. (3.13)
$X(15)$	Impact-sensitive target	Eq. (3.20)
$X(5)$	Impact-sensitive target; limited to the case of $N \leq 12$	Eq. (3.21)

NOTE: The computations for $X(1)$ involve no numerical integrations, merely a finite sum. It is very fast and more accurate than $X(11)$ if N is small.

RIPPLE OF FIXED DISPENSERS, SEC. 5.3

<i>Output</i>	<i>Description</i>	<i>Reference</i>
X(12)	Fragment-sensitive target	Eq. (4.9)
X(2)	Fragment-sensitive target; limited to the case of $N=1$ and $NB \leq 30$	Eq. (4.11)
X(13)	Approximation for X(12); faster in computation time, but not as accurate	Eq. (4.18)
X(3)	Approximation for X(12); limited to the case of $N=1$ and $NB \leq 30$	Lq. (4.19)
X(14)	Impact-sensitive target	Eq. (4.13)
X(4)	Impact-sensitive target; limited to the case of $N=1$ and $NB \leq 30$	Eq. (4.14)

RIPPLE OF RECTANGULAR PATTERNS, SEC. 5.4.1

<i>Output</i>	<i>Description</i>	<i>Reference</i>
X(103)	Fragment-sensitive target, no edge effects	Eq. (4.28)
X(104)	Fragment-sensitive target, with edge effects	Eq. (4.35)
X(105)	Impact-sensitive target, no edge effects	Eq. (4.46)
X(106)	Impact-sensitive target, with edge effects	Eq. (4.49)

RIPPLE OF ELLIPTIC PATTERNS, SEC. 5.4.2

<i>Output</i>	<i>Description</i>	<i>Reference</i>
X(107)	Fragment-sensitive target, no edge effects	Eq. (4.59)
X(108)	Fragment-sensitive target, with edge effects	Eq. (4.65)
X(109)	Impact-sensitive target, no edge effects	Eq. (4.70)

RIPPLE OF ELLIPTIC ANNULUS PATTERNS, SEC. 5.4.3

<i>Output</i>	<i>Description</i>	<i>Reference</i>
X(110)	Fragment-sensitive target, no edge effects	Eq. (4.79)
X(111)	Fragment-sensitive target, with edge effects	Eq. (4.82)
X(112)	Impact-sensitive target, no edge effects	Eq. (4.85)

RIPPLE OF CIRCULAR ANNULUS PATTERNS, SEC. 5.4.3

<i>Output</i>	<i>Description</i>	<i>Reference</i>
X(100)	Fragment-sensitive target, no edge effects	Eq. (4.81)
X(101)	Fragment-sensitive target, with edge effects	Eq. (4.84)
X(102)	Impact-sensitive target, no edge effects	Eq. (4.86)

5.6. QDHP PROGRAM INPUTS

The entire data deck for the QDHP program is read as an array. Space has been provided for up to 1200 entries. A brief description of the entries follows.

- Data (1)=PRO=number of problems to be processed.
- Data (2)=CN=number of weapons.
- Data (3)=CNB=number of bomblets per dispenser.
- Data (4)=CNVI=number of intervals in the ballistic pattern density distribution table plus 1.
- Data (5)=CXAJ=1 if the aiming pattern offsets in range are uniform;
=0 if the aiming pattern offsets are not uniform and must be read in.
- Data (6)=CXBJ=1 if the aiming pattern offsets in deflection are uniform;
=0 otherwise. Must be read in.
- Data (7)=CNPT=1 if there is a ballistic distribution table to be read in;
=0 otherwise.
- Data (8)=CNCF=1 if the distribution of the density values in the ballistic table is cumulative;
=0 otherwise.
- Data (9)=FIMP=the number of weapons per DPI (desired point of impact). This allows more than one weapon to be assigned per DPI; i.e., $CN/(FIMP*AJJ)$ must be an integer ≥ 1 . $FIMP*AJJ$ must equal CN if the weapons are being dropped one at a time.
- Data (10)=ANNO=a special flag to skip parts of the program. Set = 0 to compute any X between X(1) and X(5). Set = 1 to compute any other X and set = 2 to compute a combination of X(1) through X(5) and any other X values.
- Data (11)=AN1=1. Compute X(1). Otherwise, set to 0. This definition of 1 that means "compute" and 0 that means "do not compute" is followed for all flags on the X values. The AN number corresponds to the X number to be computed. X(1) is used for fragment-sensitive targets. Restriction $CN \leq 12$. For larger CN, use X(11).
- Data (12)=AN2=1. Compute X(2). Case of a long, narrow, fixed, dispenser-type pattern. Restrictions ($CN=1$, $CNB \leq 30$). Otherwise, use X(12).
- Data (13)=AN3=1. Compute X(3). An expansion case of X(13). ($CN=1$, $CNB \leq 30$).
- Data (14)=AN4=1. Compute X(4). Used for a dispenser pattern against a hard target. ($CN=1$, $CNB \leq 30$). Otherwise, use X(14).
- Data (15)=AN5=1. Compute X(5). Impact-sensitive target. ($CN \leq 4$). Otherwise, use X(15).
- Data (16)=AN10=1. Compute X(10). Hand method. Rarely used.
- Data (17)=AN11=1. Compute X(11). Used for fragment-sensitive target with $CN \geq 13$.
- Data (18)=AN12=1. Compute X(12). Used for a fixed dispenser weapon against a fragment-sensitive target. Needs R1, R2, CL1, Q0, S1, and S2 inputs.
- Data (19)=AN13=1. Compute X(13). An approximation to X(12). One assumes a uniform distribution of the bomblets over the distance $2 \times CL3$ and a gaussian distribution in deflection.
- Data (20)=AN14=1. Compute X(14). Similar to X(12) for a fixed dispenser pattern against a hard target.
- Data (21)=AN15=1. Compute X(15). Used for impact-sensitive target. Needs R3 and B4 inputs, the target dimensions. ($CN > 4$).
- Data (22)=A100=1. Compute X(100). Case of a ripple of dispensers with a whinut or circular pattern. Used for fragment-sensitive target with no edge effects.
- Data (23)=A101=1. Compute X(101). Fragment-sensitive target with edge effects.
- Data (24)=A102=1. Compute X(102). Vulnerable area, no edge effects.
- Data (25)=A103=1. Compute X(103). Series of rectangles. fragment-sensitive target, but no edge effects.

Data (26)=A104=1. Compute X(104). S3 and S4 must equal 0. Series of rectangles, fragment-sensitive target with edge effects.

Data (27)=A105=1. Compute X(105). Series of rectangles, vulnerable area, but no edge effects.

Data (28)=A106=1. Compute X(106). Series of rectangles, vulnerable area with edge effects. S3 and S4 set to 0.

Data (29)=A107=1. Compute X(107). Must have values for S3 and S4. Series of ellipses, fragment-sensitive target, no edge effects.

Data (30)=A108=1. Compute X(108). S3 and S4 equal 0. Series of ellipses, fragment-sensitive target with edge effects.

Data (31)=A109=1. Compute X(109). S3 and S4 must be > 0. Series of ellipses, vulnerable area, no edge effects.

Data (32)=A110=1. Compute X(110). S3 and S4 must be > 0. Elliptical annulus, fragment-sensitive target, no edge effects.

Data (33)=A111=1. Compute X(111). S3 and S4 must be 0. Elliptical annulus, fragment-sensitive target with edge effects.

Data (34)=A112=1. Compute X(112). S3 and S4 must be > 0. Elliptical annulus, vulnerable area, no edge effects.

Data (35)=D=desired spacing in range in feet between bombs.

Data (36)=DF=desired spacing in deflection in feet between bombs.

Data (37)=SP=the product of the reliability factor and the probability of a kill if hit, providing it is known. Otherwise, 1.

Data (38)=AJJ=number of wing stations ≥ 1 .

Data (39)=A3=target area dimension (length). If you are considering a point target, A3=A4=0.

Data (40)=A4=target area dimension (width).

Data (41)=B3=target dimension for a hard target. Length ≥ 1 .

Data (42)=B4=target dimension for a hard target. Width ≥ 1 .

Data (43)=S1=bomb ballistic standard error in range. (A fixed dispenser is considered a bomb.)

Data (44)=S2=bomb ballistic standard error in deflection.

Data (45)=S3=dispenser ballistic standard error in range.

Data (46)=S4=dispenser ballistic standard error in deflection.

Data (47)=T1=aim standard error in range. $\sigma_x = \text{REF1.4828}$.

Data (48)=T2=aim standard error in deflection.

Data (49)=SU1=coordinate of the aim point in the x direction for an offset target.

Data (50)=SU2=same as Data (49) but in the y direction.

Data (51)=U1=the number of steps used in integration over a quarter of the total space. In general, U1=8. If CEL1 is too small, an increase in U1 will solve the problem.

Data (52)=U2=U1.

Data (53)=CEL1=one-half the length of the dispenser ballistic table.

Data (54)=Q0=standard error of the tabular values in range of the dispenser ballistic table.

Data (55)=R1=effectiveness radii in range in the case of an MAE type of effectiveness index, i.e., $\text{MAE} = \sqrt{R1 \cdot R2}$. R1 ≥ 1 .

Data (56)=R2=effectiveness radii in deflection. R2 ≥ 1 .

Data (57)=D1=initial value constant when a Carlton-type damage function is used. In general, D1=1.

Data (58)=D2=D1.

Data (59)=R=reliability factor for a dispenser, if known. Otherwise, 1.

Data (60)=W1=inner semiaxis of a dispenser elliptical annulus in the x direction.

Data (61)=W2=inner semiaxis of a dispenser elliptical annulus in the y direction.

Data (62)=ET1=outer semiaxis of a dispenser elliptical pattern in the x direction.

Data (63)=ET2=outer semiaxis of a dispenser elliptical pattern in the y direction.

Data (64)=GCL1=half the dispenser rectangular pattern dimension in the x direction.

Data (65)=GCL2=half the dispenser rectangular pattern dimension in the y direction.

Data (66)=UU5=number of integration steps for offset ellipse function. Use 40.

Data (67)= VVF starting point for integration for offset ellipse function. Use 0.
Data (68)= $ANUM=1$ if dispenser deliveries against rectangular target areas are to be computed. $X(100)$ through $X(112)$. Otherwise, 0.
Data (101)= $A=[A(J), J=1, N]$ array. Table of aim points if spacing (D) is not uniform in range. $CXAJ=0$.
Data (301)= $B=[B(J), J=1, N]$ array. Table of aim points if spacing (DF) is not uniform in deflection. $CXBJ=0$.
Data (501)= $SVI=SVI(J), J=1, NVI$. Intervals on the ballistic table. First value must be 0. $NPT=1$.
Data (601)= $BPI=BPI(J), J=1, NVI$. Density values on the ballistic table. $CNCP=0$.
Data (701)= $TI=TI(J), J=1, NVI$. Cumulative density values on the ballistic table. $CNCP=1$.

5.6.1. Input Method Used in QDHP Program

The first card is the number of the first problem to be solved and must be an integer. It is read in on $I5$ format. If the user is cut off because of interval timer overflow before he has processed all his cases, the process allows minimum reassemblage of the data deck without interrupting the run sequence. The entire data array is read in on format $(5I1, I7, 5F12.11)$.

The first five entries are flags to read or skip a data field on that card. A blank or zero means read and store, and a one means skip the field. This system of reading data allows the user to read in only the inputs that he wishes changed for subsequent runs. In the second field of each data card is the index of the first data item on that card. The data are read five items per card. For ease of keypunching, left adjust all data items. A minus sign in column 8 of the last data card of the data deck is essential to mark the end of the set. To run additional cases, add the data cards with the changed parameters and the appropriate flags in columns 1 through 5. The last changed card will have a minus sign in column 8.

The listing on the opposite page presents typical data entries for three cases; the computer program for the simplified weapons evaluation model follows:

Columns												
1-5	8	10-12	13	14-24	25	26-36	37	38-48	49	50-60	61	62-72
		1	±	PRO	±	CN	±	CNB	±	CNVI	±	CXAJ
		6		CXBJ		CNPT		CNCP		FIMP		ANNO
		11		AN1		AN2		AN3		AN4		AN5
		16		AN10		AN11		AN12		AN13		AN14
		21		AN15		A100		A101		A102		A103
		26		A104		A105		A106		A107		A108
		31		A109		A110		A111		A112		D
		36		DF		SP		AJJ		A3		A4
		41		B3		B4		S1		S2		S3
		46		S4		T1		T2		SU1		SU2
		51		U1		U2		CL1		Q0		R1
		56		R2		D1		D2		R		W1
		61		W2		ET1		ET2		GCL3		GCL4
		66		UU5		VV5		ANUM				
		101		A								
		301		B								
		501		SVI								
		601		BPI								
		701		TI								
10011	-	46				T1		T2				
10010	-	1				CN						
11101	-	6								FIMP		
10011	-	46				T1		T2				

5.7. COMPUTER PROGRAM FOR SIMPLIFIED WEAPONS EVALUATION MODEL (For System 360/65)

```

C   UPDATED MASTER DECK  QDHP PROGRAM RM-5677 AS LOADED ON DISC AT RAND
      DIMENSION A(200),B(200),A22(200),B22(200),DATA(1200)
      DIMENSION SVI(100),BPI(100),TI(100)
      COMMON DATA
      COMMON SE4,SE3,U3,U4,Z,SQ1,SQ2,Q1,Q2,CL2,PI,SP1,SQ3,C,CL3,AT1,AT2
      COMMON BS1,BS2,SE12,SE22,S5,C45,SW3,SW4,ST1,SU3,SU4,AL1,AL2,SW1
      COMMON FCL3,FCL4,ES1,ES2,ER1,ER2,ETB1,ETB2,WR1,WR2,WB1,WB2,WS1,WS2
      COMMON A22,B22,A1,A2,B1,B2
      COMMON SQR2,SP12,PI2,AJMX,BJMX,TU1,TU2,T12,T22
      COMMON R32,RD1,RD2,SPR,SW12,SW22,SRR
      COMMON SN100,SN101,SN102,SN110,SN112,X112
      COMMON X100,X101,X102,X103,X104,X105,X106,X107,X108,X109,X110,X111
      COMMON N100,N101,N102,N110,N112,INUM
      COMMON KK,KOUT,KOO,KO,K11,LU3,LU4,N,NB,LU5
      EQUIVALENCE (DATA(1),PRO),(DATA(2),CN),(DATA(3),CNB),(DATA(4),CNVI
1  ),(DATA(5),CXAJ),(DATA(6),CXBJ),(DATA(7),CNPT),(DATA(8),CNCP),
2  (DATA(9),FIMP),(DATA(10),ANNO),(DATA(11),AN1),(DATA(12),AN2),
3  (DATA(13),AN3),(DATA(14),AN4),(DATA(15),AN5),(DATA(16),AN10),
4  (DATA(17),AN11),(DATA(18),AN12),(DATA(19),AN13),(DATA(20),AN14),
5  (DATA(21),AN15),(DATA(22),A100),(DATA(23),A101),(DATA(24),A102),
6  (DATA(25),A103),(DATA(26),A104),(DATA(27),A105),(DATA(28),A106),
7  (DATA(29),A107),(DATA(30),A108),(DATA(31),A109),(DATA(32),A110)
      EQUIVALENCE (DATA(33),A111),(DATA(34),A112),(DATA(35),D),(DATA(36)
1  ,DF),(DATA(37),SP),(DATA(38),AJJ),(DATA(39),A3),(DATA(40),A4),
2  (DATA(41),B3),(DATA(42),B4),(DATA(43),S1),(DATA(44),S2),(DATA(45)
3  ,S3),(DATA(46),S4),(DATA(47),T1),(DATA(48),T2),(DATA(49),SU1),
4  (DATA(50),SU2),(DATA(51),U1),(DATA(52),U2),(DATA(53),CL1),
5  (DATA(54),QO),(DATA(55),R1),(DATA(56),R2),(DATA(57),D1),(DATA(58)
6  ,D2),(DATA(59),R),(DATA(60),W1),(DATA(61),W2),(DATA(62),ET1),
7  (DATA(63),ET2),(DATA(64),GCL3),(DATA(65),GCL4)
      EQUIVALENCE (DATA(66),UU5),(DATA(67),VV5),(DATA(68),ANUM)
      EQUIVALENCE (DATA(101),A),(DATA(301),B),(DATA(501),SVI),(DATA(601)
1  ,BPI),(DATA(701),TI)
2  FORMAT (8F10.4)
3  FORMAT (1H0,6X,1HD9X,2HDF,8X,2HSP,9X,1HN,9X,2HNB,7X,3HNV1,7X,3HNN4
1/3F10.2,4I10)
4  FORMAT (1H03X,13HALL A(J) = 0.)
5  FORMAT (1H03X,13HALL B(J) = 0.)
6  FORMAT(6E20.8)
7  FORMAT(2A4,1X,11,6I5)
8  FORMAT (1H1,1X,2I10,6I5)
9  FORMAT (1H05X,3HCL1,7X,2HQ0,8X,2HU3,8X,2HU4,8X,3HSE3,7X,3HSE4,9X,1
1HE,9X,1HC/1H 2X,8F10.5)
10 FORMAT (1H0,4X,2HA3,8X,2HA4,8X,2HB3,8X,2HB4,8X,2HS1,8X,2HS2,8X,2HS
13,8X,2HS4/8F10.4)
11 FORMAT (1H0,4X,2HT1,8X,2HT2,7X,3HSU1,7X,3HSU2,8X,2HU1,8X,2HU2,8X,2
1HL1,8X,2HQ0/8F10.4)
12 FORMAT (1H03X,4HA(J))
13 FORMAT (1H03X,4HB(J))
14 FORMAT (16I5)

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15 FORMAT(1H05X,4HX(1),6X,4HX(2),6X,4HX(3),6X,4HX(4),6X,4HX(5)//4X,5F
110.6)
16 FORMAT(1H05X,6F10.6,3E20.8)
17 FORMAT(1H05X,5HX(10),5X,5HX(11),5X,5HX(12),5X,5HX(13),5X,5HX(14),5
1X,5HX(15),5X,5HX(30)//4X,7F10.6//1H 5X,6HX(100),4X,6HX(101),4X,6HX
2(102),4X,6HX(103),4X,6HX(104),4X,6HX(105),4X,6HX(106)//4X,7F10.6//
31H 5X,6HX(107),4X,6HX(108),4X,6HX(109),4X,6HX(110),4X,6HX(111),4X,
46HX(112)//4X,7F10.6)
19 FORMAY(1H0,4X,3HET1,7X,3HET2,6X,4HGCL3,6X,4HGCL4,7X,3HUU5,7X,3HV
15/8F10.4)
22 FORMAT(1H02X,2HNO,3X,2HN1,3X,2HN2,3X,2HN3,3X,2HN4,3X,2HN5.5H N10,
15H N11,5H N12,5H N13,5H N14,5H N15/16I5)
24 FORMAT(1H0,4X,2HR1,8X,2HR2,8X,2HD1,8X,2HD2,9X,1HR,8X,2HW1,8X,2HW2
1/8F10.4)
25 FORMAT(1H02X,3H NO,5H N100,5H N101,5H N102,5H N103,5H N104,5H N10
15,5H N106,5H N107,5H N108,5H N109,5H N110,5H N111,5H N112/16I5)
2006 FORMAT(1H011H EQUATIONS ARE INCONSISTENT. CL1 MUST SATISFY BOTH
1CL1 < OR = TO C*SQRT(3.) AND CL1 > OR = TO 1./2./AJJ. Q0=0./61H IF
2THE TWO ABOVE VALUES FOR CL1 ARE CLOSE, CHOOSE ONE AND GO.)
2007 FORMAT(1H0,61H CHECK INPUTS AJJ AND FIMP. CN/(AJJ*FIMP) MUST BE AN
1 INTEGER.)
732 FORMAT(1H0,76H THERE IS NO SOLUTION. THE BOMBLET IMPACT POINTS
1 ARE BUNCHED MORE THAN IN/1H ,66H A GAUSSIAN DISTRIBUTION. WE HAVE
2 ASSUMED A GAUSSIAN DISTRIBUTION,/1H ,87H I.E. WE HAVE SET CL1=0.
3 AND Q0=C, THE STANDARD DEVIATION OF THE TABULAR DISTRIBUTION.)
742 FORMAT(1H0,70H THERE IS NO SOLUTION. PROBABLY THE TABULAR DISTR
1IBUTION IS BIMODAL,/1H ,77H WE HAVE USED A UNIFORM DISTRIBUTION
2FOR THE RANGE DISPENSER BOMBLET IMPACT/1H ,47H PATTERN, I.E. WE S
3ET Q0=0. AND CL1=C*SQRT(3).)
32 FORMAT(I2)
34 FORMAT(1H0,6X,6HSV(I,J))
35 FORMAT(1H0,6X,5HTI(J))
36 FORMAT(1H0,6X,6HBPI(J))
39 FORMAT(1H03X,4HKO =I5,F10.6)
DO 1003 I = 1,1200
DATA(I) = 0.
1003 CONTINUE
PI = 3.14159265
PI2 = PI * 2.
SPI = SQRT(PI)
SPI2 = SPI * .5
SQR3 = SQRT(3.)
SQR2 = SQRT(2.)
READ (5,14) NUM
1111 CALL DECRD (DATA)
40 NPRO = PRO + .000001
N = CN + .000001
NB = CNB + .000001
NVI = CNVI + .000001
NXAJ = CXAJ + .000001
NXBJ = CXBJ + .000001
NPT = CNPT + .000001
NCPT = CNCP + .000001
NIMP = FIMP + .000001
NN4 = AJJ + .000001
INUM = ANUM + .000001
NO = ANNO + .000001
N1 = AN1 + .000001
N2 = AN2 + .000001

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N3 = AN3 + .000001
N4 = AN4 + .000001
N5 = AN5 + .000001
N10 = AN10 + .000001
N11 = AN11 + .000001
N12 = AN12 + .000001
N13 = AN13 + .000001
N14 = AN14 + .000001
N15 = AN15 + .000001
IF(INUM.EQ.0) GO TO 55
N100 = A100 + .000001
N101 = A101 + .000001
N102 = A102 + .000001
N103 = A103 + .000001
N104 = A104 + .000001
N105 = A105 + .000001
N106 = A106 + .000001
N107 = A107 + .000001
N108 = A108 + .000001
N109 = A109 + .000001
N110 = A110 + .000001
N111 = A111 + .000001
N112 = A112 + .000001
55 LU5 = UU5 + .000001
WRITE (6,9) NUM,NPRO,NXAJ,NXBJ,NPT,NCPT,NIMP,INUM
IF(INUM.GT.0) GO TO 57
WRITE (6,22) NO,N1,N2,N3,N4,N5,N10,N11,N12,N13,N14,N15
GO TO 56
57 WRITE(6,25) NO,N100,N101,N102,N103,N104,N105,N106,N107,N108,N109,
N110,N111,N112
56 WRITE(6,3) D,DF,SP,H,NB,NVI,NN4
ANN4 = AJJ
ANN3 = CN/(ANN4*FIMP)
IZ = IFIX(ANN3)
AIP = IZ
FP3 = ANN3 - AIP
IF(FP3.EQ.0.) GO TO 58
WRITE (6,2007)
GO TO 392
58 WRITE(6,10) A3,A4,B3,B4,S1,S2,S3,S4
WRITE (6,11) T1,T2,SU1,SU2,U1,U2,CL1,Q0
WRITE (6,24) R1,R2,D1,D2,R,W1,W2
WRITE (6,19) ET1,ET2,GCL3,GCL4,UU5,VV5
IF(NPT.EQ.0) GO TO 93
WRITE(6,34)
WRITE (6,2) (SVI(J),J=1,NVI)
IF(NCPT.EQ.0) GO TO 94
WRITE(6,35)
WRITE(6,2) (TI(J),J=1,NVI)
SPI(1) = 0.
DO 92 K = 2,NVI
SPI(K) = TI(K) - TI(K-1)
92 CONTINUE
94 WRITE(6,36)
WRITE (6,2) (BPI(J),J=1,NVI)
TI(1) = 0.
DO 27 K = 2,NVI
TI(K) = TI(K-1)+BPI(K)
27 CONTINUE

```

```

93  SN100 = 0.
    SN101 = 0.
    SN102 = 0.
    SN110 = 0.
    SN112 = 0.
    IF(T1.EQ.0.) T1= .00000001
    IF(T2.EQ.0.) T2= .00000001
    A1 = A3 * .5
    A2 = A4 * .5
    B1 = B3 * .5
    B2 = B4 * .5
    CN1 = CN - 1.
    CN12= (CN+1.)*.5
    S12 = S1*S1
    S22 = S2*S2
    TU1 = T1*U1
    TU2 = T2*U2
    ST1 = SU1/T1
    T12 = T1*T1
    T22 = T2*T2
    AT1 = A1/T1
    AT2 = A2/T2
    ST2 = SU2/T2
    BT1 = B1/T1
    R12 = P1*R1
    R22 = R2*R2
    BS1 = 0.
    BS2 = 0.
    IF(S1.NE.0.) BS1 = B1/S1
    IF(S2.NE.0.) BS2 = B2/S2
10112 D12 = D1*D1
      D22 = D2*D2
      NK1 = N - 1
      RD1 = R12/(2.*D1)
      RD2 = R22/(2.*D2)
      AL1 = SQRT(RD1)
      AL2 = SQRT(RD2)
      X1 = 0.
      X2 = 0.
      X3 = 0.
      X4 = 0.
      X5 = 0.
      X10 = 0.
      X11 = 0.
      X12 = 0.
      X13 = 0.
      X14 = 0.
      X15 = 0.
      X30 = 0.
      X100 = 0.
      X101 = 0.
      X102 = 0.
      X103 = 0.
      X104 = 0.
      X105 = 0.
      X106 = 0.
      X107 = 0.
      X108 = 0.
      X109 = 0.

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```
X11C = 0.  
X111 = 0.  
X112 = 0.  
E = 0.  
C = 0.  
KO = 0  
KPC = 0  
NRST = 0  
CO 300 J = 1,N  
IP = 1 + (J-1)/(NIMP*NN4)  
CIP = IP  
IF(NXAJ.NE.1) GO TO 1798  
A(J) = C*(CIP-ANN3*.5-.5)  
SD3 = D  
1798 A22(J) = A(J)*A(J)  
IF(NXBJ.NE.1) GO TO 1799  
AJ = J  
JF = (AJ-1.)/ANN4  
FPC = JF  
B(J) = ((2.*AJ-1.)-ANN4*(2.*FPC41.))*DF*.5  
SD4 = DF  
1799 B22(J) = B(J)*B(J)  
300 CONTINUE  
IF(NXAJ.EQ.1) GO TO 1795  
IF(N.NE.1) GO TO 301  
SC3 = 0.  
GC TC 1795  
301 AJMX = A(1)  
AJMN = A(1)  
CO 302 J = 2,N  
IF(A(J).GT.AJMX) GC TC 303  
IF(A(J).LT.AJMN) AJMN = A(J)  
GO TO 302  
303 AJMX = A(J)  
302 CONTINUE  
SC3 = (AJMX-AJMN)/CN1  
1795 IF(NXBJ.EQ.1) GC TC 1994  
IF(N.NE.1) GC TC 1796  
SD4 = 0.  
GC TC 1994  
1796 BJMX = B(1)  
BJMN = B(1)  
CO 304 J = 2,N  
IF(B(J).GT.BJMX) GC TC 305  
IF(B(J).LT.BJMN) BJMN = B(J)  
GC TC 304  
305 BJMX = B(J)  
304 CONTINUE  
SC4 = (BJMX-BJMN)/CN1  
1994 IF(SC3.EQ.0.) GC TC 1996  
WRITE(6,12)  
WRITE(6, 2)(A(J),J=1,N)  
1797 IF(SC4.EQ.0.) GO TO 1997  
WRITE(6,13)  
WRITE(6, 2)(B(J),J=1,N)  
GC TO 1995  
1996 WRITE (6,4)  
GO TO 1797
```

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1997 WRITE (6,5)
1995 IF(CL1.NE.1.) GO TO 312
700 SZ = 0.
    IF(NPT.EQ.0) GO TO 312
711 SUSM = 0.
    SMLM = 0.
    KPO = 1
    DO 716 LL = 2,NVI
        V2 = (SVI(LL) + SVI(LL-1))*0.5
        SUSM = SUSM + V2*BPI(LL)
        V3 = SVI(LL)*SVI(LL)+SVI(LL)*SVI(LL-1)+SVI(LL-1)*SVI(LL-1)
        SMLM = SMLM + V3/3.*BPI(LL)
716 CONTINUE
    C = SQRT(SMLM-SUSM*SUSM)
    DO 701 K = 1,NVI
        IF(TI(K).GE..25) GO TO 702
701 CONTINUE
702 J = K
    SX1 = SVI(J-1) + (SVI(J)-SVI(J-1))*((.25-TI(J-1))/(TI(J)-TI(J-1)))
    DO 703 K = 1,NVI
        IF(TI(K).GE..75) GO TO 704
703 CONTINUE
704 I=K
    SX2 = SVI(I-1)+(SVI(I)-SVI(I-1))*((.75-TI(I-1))/(TI(I)-TI(I-1)))
    SX3 = (SX2-SX1)*0.5
    SX3C = SX3/C
    IF(SX3C.GE..6743) GO TO 7028
    WRITE(6,732)
    GO TO 7085
7028 IF(SX3C.LE.(SQ3*.5+.0001)) GO TO 7030
    WRITE(6,742)
    GO TO 708
7030 IF(ABS(SX3C-SQ3*.5).LE..0001) GO TO 708
    IF(ABS(SX3C-.6744).LE..0001) GO TO 7085
    Z1 = 0.
    Z2 = C.
    SZ = C*.5
7040 CSZ = SQRT(3.*(C*C-SZ*SZ))
    CALL FF(0.,SX3,CSZ,SZ,AF)
    CK = AF-.5
    IF(ABS(CK).LE..0001) GO TO 707
    IF(CK.LT.0.) Z1=SZ
    IF(CK.GE.0.) Z2=SZ
    SZ = (Z1+Z2)*0.5
    IF(ABS(Z1-Z2).LE..01) GO TO 707
    GO TO 7040
707 Q0 = SZ
    CL1 = SQRT(3.*(C*C-SZ*SZ))
    GO TO 709
708 Q0 = 0.
    CL1 = SQ3*C
    GO TO 709
7085 CL1 = 0.
    Q0 = C
709 KPO = 1
312 Q02 = Q0 * Q0
    SSR = SP * SPI * R1
    SPR = SP*PI*R1*R2
    SQ1 = SQRT(RD1 + S12)

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SQ3 = SQRT(Q02 + RD1)
Q1 = SQRT(SQ1*SQ1 + T12)
SQ2 = SQRT(R22/(2.*D2)+ S22)
Q2 = SQRT(SQ2*SQ2 + T22)
1800 SSU3 = G.
SSU4 = 0.
AJMX = ABS(SU1 - A(1))
BJMX = ABS(SU2 - B(1))
IF(N.EQ.1) GO TO 3031
DO 3030 J = 2,N
ASU1 = ABS(SU1 - A(J))
BSU2 = ABS(SU2 - B(J))
IF(ASU1.GT.AJMX) AJMX = ASU1
IF(BSU2.GT.BJMX) BJMX = BSU2
3030 CONTINUE
3031 DO 306 J = 1,N
NJ1 = N-J+1
SSU3 = SSU3 + ABS(A(NJ1)+A(J))
SSU4 = SSU4 + ABS(B(NJ1)+B(J))
306 CONTINUE
SU3 = SU1 + SSU3
SU4 = SU2 + SSU4
KK = 1
IF(SU3.EQ.0.) GO TO 3165
U3 = 2.*U1
GO TO 3166
3165 U3 = U1
3166 IF(SU4.EQ.0.) GO TO 3167
U4 = 2.*U2
GO TO 317
3167 U4 = U2
317 LU3 = U3
LU4 = U4
IF(T1.EQ..00000001) GO TO 3192
SE1 = (4.+AT1)/U1
SE12 = SE1 * .5
AK = U3
GO TO 313
314 AK = AK - 1.
313 AX = FXJ(AK)
CALL H(AX,AT1,HXL)
AA = 1.-HXL
IF(AA.GT..0001) GO TO 3175
GO TO 314
3175 SE3 =(SE12 + AX)/U1
318 SE2 = (4.+AT2)/U2
SE22 = SE2 * .5
AK = U4
GO TO 3181
3182 AK = AK - 1.
3181 AY = FYJ(AK)
CALL H(AY,AT2,HXL)
AA = 1.-HXL
IF(AA.GT..0001) GO TO 3185
GO TO 3182
3185 SE4 =(SE22 + AY)/U2
3192 IF(N0.EQ.0) GO TO 340
KOUT = 0
3002 IF(INUM.GT.0) CALL Z100(N103,N104,N105,N106,N107,N108,N109,N111)

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IF(N11.EQ.0) GO TO 324
KO = 11
IF(T1.EQ..00000001) GO TO 633
GSE1=(AJMX+4.*SQ1)/TU1
GSE2 = (BJMX + 4.*SQ2)/TU2
SE12 = AMIN1(SE3,GSE1)*.5
SE22 = AMIN1(SE4,GSE2)*.5
633 CALL A50
630 X11 = Z
WRITE (6,39) KO,X11
324 IF(N12.EQ.0) GO TO 325
KO = 12
IF(T1.EQ..00000001) GO TO 723
GSE1 = (AJMX + CL1 + 4.*SQ3)/TU1
GSE2 = (BJMX + 4.*SQ2)/TU2
SE12 = AMIN1(SE3,GSE1)*.5
SE22 = AMIN1(SE4,GSE2)*.5
723 C = SSR/SQ3
CALL A50
735 X12 = Z
WRITE (6,39) KO,X12
325 IF(N13.EQ.0) GO TO 332
820 CL3 = SQRT(CL1**2 + 3.*SQ3**2)
IF(T1.EQ..00000001) GO TO 825
GSE2 = (BJMX + 4.*SQ2)/TU2
SE22 = AMIN1(SE4,GSE2)*.5
825 C = (SSR * .5)/CL3
KOO = 1
KO = 13
CALL A41
840 X13 = Z
WRITE (6,39) KO,X13
332 KOUT = 1
IF(N15.EQ.0) GO TO 334
KO = 15
IF(T1.EQ..00000001) GO TO 3033
GSE1 = (AJMX + B1 + 4.*S1)/TU1
GSE2 = (BJMX + B2 + 4.*S2)/TU2
SE12 = AMIN1(SE3,GSE1)*.5
SE22 = AMIN1(SE4,GSE2)*.5
3033 CALL A50
X15 = Z
WRITE (6,39) KO,X15
334 IF(N14.EQ.0) GO TO 335
KO = 14
IF(T1.EQ..00000001) GO TO 3341
GSE1 = (AJMX + CL1 + B1 + 4.*Q0)/TU1
GSE2 = (BJMX + B2 + 4.*S2)/TU2
SE12 = AMIN1(SE3,GSE1)*.5
SE22 = AMIN1(SE4,GSE2)*.5
3341 CALL A50
X14 = Z
WRITE (6,39) KO,X14
335 IF(N10.EQ.0) GO TO 336
1515 CL3 = SQRT(CL1**2 + 3.*SQ3**2)
CL4 = SQ2*SQR3
CALL FF(SU1,CL3,A1,T1,AF)
ZO = AF
CALL FF(SU2,CL4,A2,T2,AF)

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Z = ZO * AF
SRC = SP*PI/4.*R1/CL3*R2/CL4
153 X10 = Z * K * (1.-(1.-SRC)**NB)
CALL FF(SU1,B1,CL3,T1,AF)
ZO = AF
CALL FF(SU1,B2,CL4,T2,AF)
Z = ZO * AF
IF(A1.NE.0.) GO TO 1560
X30 = 1. - (1.-SP*Z)**NB
GO TO 336
1560 X30 = 0.
336 IF(NO.EQ.1) GO TO 392
340 IF(SD4.NE.0.) GO TO 392
IF(T1.EQ..00000001) GO TO 3400
GSE1 = (AJMX + B1+4.*S1)/TU1
SE12 = AMIN1(SE3,GSE1)*.5
3400 CALL XRO(X1,X2,X3,X4,X5,SD3,SD4)
WRITE (6,15) X1,X2,X3,X4,X5
392 WRITE (6,17) X10,X11,X12,X13,X14,X15,X30,X100,X101,X102,X103,X104,
1X105,X106,X107,X108,X109,X110,X111,X112
IF(KPD.EQ.0) GO TO 1112
1113 CL1 = 1.
QO = 0.
1112 IF(NUM.EQ.NPRO) GO TO 2222
CALL DECRD (DATA)
NUM = NUM + 1
IF(SN100.NE.0.) N100 = SN100
IF(SN101.NE.0.) N101 = SN101
IF(SN102.NE.0.) N102 = SN102
IF(SN110.NE.0.) N110 = SN110
IF(SN112.NE.0.) N112 = SN112
GO TO 40
2222 CALL EXIT
END

```

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SUBROUTINE Z100(N103,N104,N105,N106,N107,N108,N109,N111)
DIMENSION A(200),B(200),A22(200),B22(200),DATA(1200)
COMMON DATA
COMMON SE4,SE3,U3,U4,Z,SQ1,SQ2,Q1,Q2,CL2,PI,SPI,SQ3,C,CL3,AT1,AT2
COMMON BS1,BS2,SE12,SE22,S5,C45,SW3,SW4,ST1,SU3,SU4,AL1,AL2,SW1
COMMON FCL3,FCL4,ES1,ES2,ER1,ER2,ETB1,ETB2,WR1,WR2,WB1,WB2,WS1,WS2
COMMON A22,B22,A1,A2,B1,B2
COMMON SQR2,SP12,PI2,AJMX,BJMX,TU1,TU2,T12,T22
COMMON RD2,RD1,RD2,SPR,SK12,SW22,SRR
COMMON SN100,SN101,SN102,SN110,SN112,X112
COMMON X100,X101,X102,X103,X104,X105,X106,X107,X108,X109,X110,X111
COMMON N100,N101,N102,N110,N112,INUM
COMMON KK,KOUT,KOO,KO,K11,LU3,LU4,N,NB,LU5
EQUIVALENCE (DATA(33),A111),(DATA(34),A112),(DATA(35),D1),(DATA(36)
1 ,DF),(DATA(37),SP),(DATA(38),AJJ),(DATA(39),A3),(DATA(40),A4),
2 (DATA(41),B3),(DATA(42),B4),(DATA(43),S1),(DATA(44),S2),(DATA(45)
3 ,S3),(DATA(46),S4),(DATA(47),T1),(DATA(48),T2),(DATA(49),SU1),
4 (DATA(50),SU2),(DATA(51),U1),(DATA(52),U2),(DATA(53),CL1),
5 (DATA(54),GO),(DATA(55),R1),(DATA(56),R2),(DATA(57),D1),(DATA(58)
6 ,D2),(DATA(59),R),(DATA(60),W1),(DATA(61),W2),(DATA(62),ET1),
7 (DATA(63),ET2),(DATA(64),GCL3),(DATA(65),GCL4)
EQUIVALENCE (DATA(101),A),(DATA(301),B)
2 FORMAT (8F10.4)

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```

6 FORMAT(6E20.8)
15 FORMAT (1H0,3X,35HGCL3 AND GCL4 MUST BE > 0. FOR KO =14)
30 FORMAT(1H03X,3H$W1,8X,2H$S,8X,2HR3/8F10.4)
39 FORMAT(1H03X,4HKO =15,F10.6)
53 FORMAT(1H02X,3H$S3 OR $4 INPUTS ARE NOT VALID FOR KO =14)
54 FORMAT(1H02X,4HKO =15,2X,65H$3 AND $4 MUST BE 0. FOR THIS CASE TO
  BE VALID. USE KO=1 CASE. )
2001 FORMAT(1H02X,82H N100,N101,N110,N112 HAVE BEEN SET TO ZERO BECAUSE
  THE MIN(ET1/R1,ET2/R2) WAS LE 5.)
2002 FORMAT (1H02X,77H N100,N101,N102,N110 HAVE BEEN SET TO ZERO BECAUS
  E THE MIN(W1/R1,W2/R2) LE 5.)
2003 FORMAT (1H02X,57H N100,N102 HAVE BEEN SET TO ZERO BECAUSE SW2/$5
  IS LE 3.)
2004 FORMAT(1H02X,105H N102,N112 HAVE BEEN SET TO ZERO BECAUSE EITHER
  THE MIN(ET1/B1,ET2/B2) OR THE MIN(W1/B1,W2/B2) WAS LE 5.)
2005 FORMAT(1H02X,78H RATIO OF SW1/$5 LT 3. BUT GT. 0. GIVES TOO GREAT
  AN ERROR FOR X100,X101,X102.)
  R3 = SQRT(R1*R2)
  R12 = R3 * R3
  S5 = SQRT(S3*S4)
  IF(S5.EQ.0.) S5 = .1
  S52 = S5 * S5
  SW2 = SQRT(ET1*ET2)
  SW1 = SQRT(W1*W2)
  SW5 = SW2/S5
  WS14 = SW1/S5
  IF(WS14.EQ.0.) GO TO 1001
  IF(WS14.GE.3.) GO TO 1001
  IF(N100.EQ.1) GO TO 1002
  IF(N101.EQ.1) GO TO 1002
  IF(N102.NE.1) GO TO 1001
1002 SN100 = N100
  SN101 = N101
  SN102 = N102
  N100 = 0
  N101 = 0
  N102 = 0
  WRITE (6,2005)
1001 SW12= SW1*SW1
  SW22= SW2*SW2
  WRITE (6,30)SW1,S5,R3
  SSR = SP*SP1*R1
  SRR = SP*R1*R2
  S4HB = SP*4.*B1*B2
  SWSW = SW22-SW12
3002 ER1 = ET1/R1
  ER2 = ET2/R2
  ETB1= ET1/B1
  ETB2 = ET2/B2
  WR1 = W1/R1
  WR2 = W2/R2
  WB1 = W1/B1
  WB2 = W2/B2
  ER = AMIN1(ER1,ER2)
  WR = AMIN1(WR1,WR2)
  EB = AMIN1(ETB1,ETB2)
  WB = AMIN1(WB1,WB2)
  IF(S3.EQ.0.) GO TO 3000
  IF(S4.EQ.0.) GO TO 3000

```

```

WS1 = W1/S3
WS2 = W2/S4
ES1 = ET1/S3
ES2 = ET2/S4
GO TO 3001
3000 ES1 = 0.
     ES2 = 0.
     WS1 = 0.
     WS2 = 0.
3001 IF(ER.GT.5.) GO TO 3086
     IF(N100.EQ.1) GO TO 3006
     IF(N101.EQ.1) GO TO 3006
     IF(N110.NE.1) GO TO 3086
3006 SN100 = N100
     SN101 = N101
     SN110 = N110
     N100 = 0
     N101 = 0
     N110 = 0
     WRITE (6,2001)
3086 IF(WR.GT.5.) GO TO 3087
     IF(WR.EQ.0.) GO TO 3087
     IF(SW1.EQ.0.) GO TO 3087
     IF(N100.EQ.1) GO TO 3007
     IF(N101.EQ.1) GO TO 3007
     IF(N102.EQ.1) GO TO 3007
     IF(N110.NE.1) GO TO 3087
3007 SN100 = N100
     SN101 = N101
     SN102 = N102
     SN110 = N110
     N100 = 0
     N101 = 0
     N102 = 0
     N110 = 0
     WRITE (6,2002)
3087 IF(SW5.GT.3.) GO TO 3088
     IF(N100.EQ.1) GO TO 3008
     IF(N102.NE.1) GO TO 3088
3008 SN100 = N100
     SN102 = N102
     N100 = 0
     N102 = 0
     WRITE (6,2003)
3088 IF(N102.EQ.1) GO TO 3090
     IF(N112.NE.1) GO TO 3192
3089 IF(EB.LE.5.) GO TO 3089
     IF(WB.GT.5.) GO TO 3192
     IF(SW1.EQ.0.) GO TO 3192
3089 SN102 = N102
     SN112 = N112
     N102 = 0
     N112 = 0
     WRITE (6,2004)
3192 IF(N100.EQ.0) GO TO 3265
     KD = 100
     C45 = (1.-(1.-SP*R32/SWSW)**NB) * R
5210 IF(SW1.EQ.0.) GO TO 4531
     SW3 = SQRT(-1.+SW12/S52)

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```

GO TO 4532
4531 SW3 = 0.
4532 SW4 = SQRT(-1.+SW22/S52)
IF(T1.EQ..00000001) GO TO 4533
GSE1 = (AJMX + ET1 + 4.*S3)/TU1
GSE2 = (BJMX + ET2 + 4.*S4)/TU2
SE12 = AMIN1(SE3,GSE1)*.5
SE22 = AMIN1(SE4,GSE2)*.5
4533 IF(SW1.EQ.0.) KO = 1001
IF(N100.EQ.0) GO TO 5211
CALL A50
X100 = 2
WRITE (6,39) KO,X100
3265 IF(N102.EQ.0) GO TO 3270
KO = 102
C45 = R*(1.-(1.-S4HB/(PI*SWSW))*NB)
IF(N100.EQ.0) GO TO 5210
5211 IF(SW1.EQ.0.) KO = 1021
CALL A50
X102 = 2
WRITE (6,39) KO,X102
3270 IF(N101.EQ.0) GO TO 333
IF(T1.EQ..00000001) GO TO 4534
GSE1 = (AJMX + ET1 + 4.*AL1)/TU1
GSE2 = (BJMX + ET2 + 4.*AL2)/TU2
SE12 = AMIN1(SE3,GSE1)*.5
SE22 = AMIN1(SE4,GSE2)*.5
4534 KO = 101
IF(S3.GT.0.) GO TO 4620
IF(S4.GT.0.) GO TO 4620
C45 = (SP*R32)/SWSW
AL12 = AL1*AL2
IF(SW1.EQ.0.) GO TO 4615
SW3 = SQRT(SW12/AL12-1.)
GO TO 4616
4615 KO = 1011
SW3 = 0.
4616 SW4 = SQRT(SW22/AL12-1.)
CALL A50
X101 = 2
WRITE (6,39) KO,X101
GO TO 333
4620 WRITE (6,53) KI
333 FCL3 = SQRT(GCL3*GCL3 + 3.*R01)
FCL4 = SQRT(GCL4*GCL4 + 3.*R02)
IF(N103.EQ.0) GO TO 337
IF(T1.EQ..00000001) GO TO 4535
GSE1 = (AJMX + FCL3 + 4.*S3)/TU1
GSE2 = (BJMX + FCL4 + 4.*S4)/TU2
SE12 = AMIN1(SE3,GSE1)*.5
SE22 = AMIN1(SE4,GSE2)*.5
4535 KO = 103
C45 = R*(1.-(1.-SPR/(4.*FCL3*FCL4))*NB)
CALL A50
X103 = 2
WRITE (6,39) KO,X103
337 IF(N104.EQ.0) GO TO 338
IF(T1.EQ..00000001) GO TO 4536
GSE1 = (AJMX + GCL3 + 4.*AL1)/TU1

```

```
GSE2 = (BJMX + GCL4 + 4.*AL2)/TU2
SE12 = AMIN1(SE3,GSE1)*.5
SE22 = AMIN1(SE4,GSE2)*.5
4536 KO = 104
IF(S3.GT.0.) GO TO 3371
IF(S4.GT.0.) GO TO 3371
IF(GCL3.EQ.0.) GO TO 3372
C45 = SPR/(4.*GCL3*GCL4)
CALL A50
X104 = Z
WRITE (6,39) KO,X104
GO TO 338
3371 WRITE (6,54) KO
GO TO 338
3372 WRITE(6,15) KO
338 FCL3 = SQRT(GCL3*GCL3 + B1*B1)
FCL4 = SQRT(GCL4*GCL4 + B2*B2)
IF(N105.EQ.0) GO TO 339
IF(T1.EQ..00000001) GO TO 4537
GSE1 = (AJMX + FCL3 + 4.*S3)/TU1
GSE2 = (BJMX + FCL4 + 4.*S4)/TU2
SE12 = AMIN1(SE3,GSE1)*.5
SE22 = AMIN1(SE4,GSE2)*.5
4537 KO = 105
C45 = R*(1.-(1.-(SP *B1*B2)/(FCL3*FCL4))*NB)
CALL A50
X105 = Z
WRITE (6,39) KO,X105
339 IF(N106.EQ.0) GO TO 341
IF(T1.EQ..00000001) GO TO 4538
GSE1 = (AJMX + GCL3 + B1)/TU1
GSE2 = (BJMX + GCL4 + B2)/TU2
SE12 = AMIN1(SE3,GSE1)*.5
SE22 = AMIN1(SE4,GSE2)*.5
4538 KO = 106
IF(S3.GT.0.) GO TO 3391
IF(S4.GT.0.) GO TO 3391
IF(GCL3.EQ.0.) GO TO 3392
CALL A50
X106 = Z
WRITE (6,39) KO,X106
GO TO 341
3391 WRITE (6,54) KO
GO TO 341
3392 WRITE(6,15) KO
341 IF(N107.EQ.0) GO TO 342
IF(T1.EQ..00000001) GO TO 4539
GSE1 = (AJMX + ET1 + 4.*S3)/TU1
GSE2 = (BJMX + ET2 + 4.*S4)/TU2
SE12 = AMIN1(SE3,GSE1)*.5
SE22 = AMIN1(SE4,GSE2)*.5
4539 KO = 107
IF(S3.EQ.0.) GO TO 3421
IF(S4.EQ.0.) GO TO 3421
C45 = R*(1.-(1.-SRR/SW22))*NB)
CALL A50
X107 = Z
WRITE (6,39) KO,X107
GO TO 342
```

```

3421 WRITE (6,53) KU
342 IF(N108.EQ.0) GO TO 343
    IF(T1.EQ..00000001) GO TO 4540
    GSE1 = (AJMX + ET1 + 4.*AL1)/TU1
    GSE2 = (BJMX + ET2 + 4.*AL2)/TU2
    SE12 = AMIN1(SE3,GSE1)*.5
    SE22 = AMIN1(SE4,GSE2)*.5
4540 KO = 108
    IF(S3.GT.0.) GO TO 3451
    IF(S4.GT.0.) GO TO 3451
    C45 = SRR/SW22
    CALL A50
    X108 = Z
    WRITE (6,39) KO,X108
    GO TO 343
3451 WRITE (6,53) KO
343 IF(N109.EQ.0) GO TO 3271
    IF(T1.EQ..00000001) GO TO 4541
    GSE1 = (AJMX + ET1 + 4.*S3)/TU1
    GSE2 = (BJMX + ET2 + 4.*S4)/TU2
    SE12 = AMIN1(SE3,GSE1)*.5
    SE22 = AMIN1(SE4,GSE2)*.5
4541 KO = 109
    IF(S3.EQ.0.) GO TO 3431
    IF(S4.EQ.0.) GO TO 3431
    C45 = R*(1.-(1.-S4BH/(SW22*PI))*NR)
    CALL A50
    X109 = Z
    WRITE (6,39) KO,X109
    GO TO 3271
3431 WRITE (6,53) KO
3271 IF(N110.EQ.0) GO TO 3273
    IF(T1.EQ..00000001) GO TO 4542
    GSE1 = (AJMX + ET1 + 4.*S3)/TU1
    GSE2 = (BJMX + ET2 + 4.*S4)/TU2
    SE12 = AMIN1(SE3,GSE1)*.5
    SE22 = AMIN1(SE4,GSE2)*.5
4542 KO = 110
    IF(S3.EQ.0.) GO TO 3272
    IF(S4.EQ.0.) GO TO 3272
    C45 = R*(1.-(1.-SRR/SWSW))*NR)
    CALL A50
    X110 = Z
    WRITE (6,39) KO,X110
    GO TO 3273
3272 WRITE (6,53) KO
3273 IF(N111.EQ.0) GO TO 3275
    IF(T1.EQ..00000001) GO TO 4543
    GSE1 = (AJMX + ET1 + 4.*AL1)/TU1
    GSE2 = (BJMX + ET2 + 4.*AL2)/TU2
    SE12 = AMIN1(SE3,GSE1)*.5
    SE22 = AMIN1(SE4,GSE2)*.5
4543 KO = 111
    IF(S3.GT.0.) GO TO 3274
    IF(S4.GT.0.) GO TO 3274
    C45 = SRR/SWSW
    CALL A50
    X111 = Z
    WRITE (6,39) KO,X111

```

```

GO TO 3275
3274 WRITE (6,53) KO
3275 IF(N112.EQ.0) GO TO 332
IF(T1.EQ..00000001) GO TO 4544
GSE1 = (AJMX + ET1 + 4.*S3)/TU1
GSE2 = (BJMX + ET2 + 4.*S4)/TU2
SE12 = AMIN1(SE3,GSE1)*.5
SE22 = AMIN1(SE4,GSE2)*.5
4544 KO = 112
IF(S3.EQ.0.) GO TO 3276
IF(S4.EQ.0.) GO TO 3276
C45 = R*(1.-(1.-S4BB/(PI*SWSW))*NB)
CALL A50
X112 = Z
WRITE (6,39) KO,X112
GO TO 332
3276 WRITE (6,53) KO
332 RETURN
END

```

```

SUBROUTINE A50
REAL SS(150)/150*0.0/
DIMENSION DATA(1200)
DIMENSION A22(200),B22(200)
COMMON DATA
COMMON SE4,SE3,U3,U4,Z,SQ1,SQ2,Q1,Q2,CL2,PI,SPI,SQ3,C,CL3,AT1,AT2
COMMON BS1,BS2,SE12,SE22,S5,C45,SW3,SW4,ST1,SU3,SU4,AL1,AL2,SW1
COMMON FCL3,FCL4,ES1,ES2,ER1,EK2,ETB1,ETB2,WR1,WR2,WB1,WB2,WS1,WS2
COMMON A22,B22,A1,A2,B1,B2
COMMON SQR2,SPI2,PI2,AJMX,BJMX,TU1,TU2,T12,T22
COMMON R32,RO1,RO2,SPR,SW12,SW22,SRR
COMMON SN100,SN101,SN102,SN110,SN112,X112
COMMON X100,X101,X102,X103,X104,X105,X106,X107,X108,X109,X110,X111
COMMON N100,N101,N102,N110,N112,INUM
COMMON KK,KOUT,KOO,KO,X11,LU3,LU4,N,NB,LU5
EQUIVALENCE (DATA(33),P111),(DATA(34),A112),(DATA(35),D),(DATA(36),
1 ,DF),(DATA(37),SP),(DATA(38),AJJ),(DATA(39),A3),(DATA(40),A4),
2 (DATA(41),B3),(DATA(42),B4),(DATA(43),S1),(DATA(44),S2),(DATA(45),
3 ,S3),(DATA(46),S4),(DATA(47),T1),(DATA(48),T2),(DATA(49),SU1),
4 (DATA(50),SU2),(DATA(51),U1),(DATA(52),U2),(DATA(53),CL1),
5 (DATA(54),QO),(DATA(55),R1),(DATA(56),R2),(DATA(57),D1),(DATA(58),
6 ,D2),(DATA(59),R),(DATA(60),W1),(DATA(61),W2),(DATA(62),ET1),
7 (DATA(63),ET2),(DATA(64),GCL3),(DATA(65),GCL4)
1 FORMAT (1H02X,3HSE1,7X,3HSE2/2F10.8)
IF(T1.EQ..00000001) GO TO 5057
505 ZSUM = 0.
DO 51 J = 1,LU3
AJ = J
SSUM = 0.
X = FXJ(AJ)
DO 512 K = 1,LU4
AK = K
Y = FYJ(AK)
CALL K6(X,Y,PK6)
CALL FF(Y,SE22,AT2,1.,AF)
SSUM = SSUM + PK6 * AF
512 CONTINUE
53 SS(J) = SSUM

```

```

CALL FF(X,SE12,AT1,1.,AF)
ZSUM = ZSUM + SS(J)*AF
51 CONTINUE
5057 IF(T1.GT..00000001) GO TO 5055
SE1 = A1/U1
SE12 = SE1*.5
SE2 = A2/U2
SE22 = SE2*.5
ZSUM = 0.
DO 5100 J = 1,LU3
AJ = J
SSUM = 0.
X = FXJ(AJ)
DO 5200 K = 1,LU4
AK = K
Y = FYJ(AK)
CALL K6(X,Y,PK6)
SSUM = SSUM + PK6
5200 CONTINUE
SS(J) = SSUM/U4
ZSUM = ZSUM + SS(J)
5100 CONTINUE
Z = ZSUM/U3
GO TO 507
5055 Z = ZSUM
IF(SU3.EQ.0.) Z = 2.*Z
IF(SU4.EQ.0.) GO TO 506
GO TO 507
506 Z = 2.* Z
507 RETURN
END

```

```

SUBROUTINE PP(X,J,PXJ)
DIMENSION DATA(1200)
DIMENSION A22(200),B22(200)
COMMON DATA
COMMON SE4,SE3,U3,U4,Z,SQ1,SQ2,Q1,Q2,CL2,PI,SQ3,C,CL3,AT1,AT2
COMMON BS1,BS2,SE12,SE22,S5,C45,SW3,SW4,ST1,SU3,SU4,AL1,AL2,SW1
COMMON FCL3,FCL4,ES1,ES2,ER1,ER2,ETB1,ETB2,WR1,WR2,WB1,WB2,WS1,WS2
COMMON A22,B22,A1,A2,B1,B2
COMMON SQR2,SP12,PI2,AJMX,BJMX,TU1,TU2,T12,T22
COMMON R32,RD1,RD2,SPR,SW12,SW22,SRR
COMMON SN100,SN101,SN102,SN110,SN112,X112
COMMON X100,X101,X102,X103,X104,X105,X106,X107,X108,X109,X110,X111
COMMON N100,N101,N102,N110,N112,INUM
COMMON KK,KOUT,K00,K0,K11,LU3,LU4,N,NB,LJ5
EQUIVALENCE (DATA(43),S1),(DATA(44),S2),(DATA(55),R1),(DATA(56),R2
1)
PXJ = 0.
IF(KOUT.EQ.1) GO TO 1
IF(J.EQ.2) GO TO 5
SQ7 = 7.*SQ1
IF(X.GT.SQ7) GO TO 100
PXJ = R1/SQ1*EXP(-(X/SQ1)**2*.5)/SQR2
GO TO 100
5 SQ8 = 7.*SQ2
IF(X.GT.SQ8) GO TO 100
2 PXJ = R2/SQ2*EXP(-(X/SQ2)**2*.5)/SQR2

```

```

GO TO 100
1 IF(J.EQ.2) GO TO 3
  CALL SF(X,B1,S1,SMF)
  PXJ = SMF
  GO TO 100
3 CALL SF(X,B2,S2,SMF)
  PXJ = SMF
100 RETURN
END

```

```

SUBROUTINE XRD(X1,X2,X3,X4,X5,SD3,SD4)
DIMENSION DATA(1200)
DIMENSION A(200),B(200),A22(200),B22(200)
DIMENSION I(200),ISTOP(200),LSTOP(200),KSTOP(200)
COMMON DATA
COMMON SE4,SE3,U3,U4,Z,SQ1,SQ2,Q1,Q2,CL2,PI,SPI,SQ3,C,CL3,AT1,AT2
COMMON BS1,BS2,SE12,SE22,S5,C45,SW3,SW4,ST1,SU3,SU4,AL1,AL2,SW1
COMMON FCL3,FCL4,ES1,ES2,ER1,ER2,ETB1,ETB2,WR1,WR2,WB1,WB2,WS1,WS2
COMMON A22,B22,A1,A2,B1,B2
COMMON SQR2,SPI2,PI2,AJMX,BJMX,TU1,TU2,T12,T22
COMMON R32,RD1,RD2,SPR,SW12,SW22,SRR
COMMON SN100,SN101,SN102,SN110,SN112,X112
COMMON X100,X101,X102,X103,X104,X105,X106,X107,X108,X109,X110,X111
COMMON N100,N101,N102,N110,N112,INUM
COMMON KK,KOUT,KOD,KO,K11,LU3,LU4,N,NB,LUS
EQUIVALENCE (DATA(1),PRO),(DATA(2),CN),(DATA(3),CNB),(DATA(4),CNVI
1 ),(DATA(5),CXAJ),(DATA(6),CXBJ),(DATA(7),CNPT),(DATA(8),CNCP),
2 (DATA(9),FIMP),(DATA(10),ANNO),(DATA(11),AN1),(DATA(12),AN2),
3 (DATA(13),AN3),(DATA(14),AN4),(DATA(15),AN5)
EQUIVALENCE (DATA(33),A111),(DATA(34),A112),(DATA(35),D),(DATA(36)
1 ,DF),(DATA(37),SP),(DATA(38),AJJ),(DATA(39),A3),(DATA(40),A4),
2 (DATA(41),B3),(DATA(42),B4),(DATA(43),S1),(DATA(44),S2),(DATA(45)
3 ,S3),(DATA(46),S4),(DATA(47),T1),(DATA(48),T2),(DATA(49),SU1),
4 (DATA(50),SU2),(DATA(51),U1),(DATA(52),U2),(DATA(53),CL1),
5 (DATA(54),Q0),(DATA(55),R1),(DATA(56),R2),(DATA(57),D1),(DATA(58)
6 ,D2),(DATA(59),R)
EQUIVALENCE (DATA(101),A),(DATA(301),B)
N1 = AN1 + .000001
N2 = AN2 + .000001
N3 = AN3 + .000001
N4 = AN4 + .000001
N5 = AN5 + .000001
AU1 = ABS(SU1)
AU2 = ABS(SU2)
CN1 = CN - 1.
NK1 = N - 1
DO 1 J = 1,N
  I(J) = 0
  ISTOP(J) = 0
  LSTOP(J) = 0
  KSTOP(J) = 0
1 CONTINUE
  IF(N1.NE.0) GO TO 2
  IF(N5.EQ.0) GO TO 362
2 DO 390 KK = 1,N
  IF(N.GT.12) GO TO 392
  ONEG = 1.
  LOT = 0

```

```

KA = KK-1
FKK= KK
WX1 = 0.
W5 = 0.
IF(MOD(KA,2))1321,1322,1321
:321 ONEG = -ONEG
1322 K00 = 2
      IF(T1.EQ..00000001) GO TO 1324
      GSE2 = (AU2+B2+4.*S2)/TU2
      SE22 = AMINI(SE4,GSE2)*.5
1324 CALL A41
1430 W7 = Z
      344 Q2 = SQRT(SQ2**2/FKK+T22)
      Q1 = SQRT(SQ1**2/FKK+T12)
      CALL V(2,SU2,VXJ)
      W6 = VXJ
      350 IF(N5.NE.0) GO TO 510
      LOT = 1
      352 IF(N1.EQ.0) GO TO 388
      410 IF(SD3.NE.0.) GO TO 425
      162 CALL V(1,SU1,VXJ)
      WX1= VXJ*YF(KK)
      O = WX1 * W6
      GO TO 385
      510 IF(SD3.NE.0.) GO TO 520
1820 K11 = 3
      CALL A40
      184 W5 = Z
      O = W5*W7
      GO TO 383
      520 CONTINUE
      425 DO 426 J = 1, KK
      KSTOP(J) = N-KK+J
      IF(J.NE.1) GO TO 105
      I(J) = J
      LSTOP(J) = KSTOP(J)
      GO TO 106
      105 I(J) = I(J-1) + 1
      LSTOP(J) = I(1)+NK1-KK+J
      106 ISTOP(J) = MINO(KSTOP(J),LSTOP(J))
      426 CONTINUE
      IF(LOT.EQ.1) GO TO 429
      IF(N5.NE.0)GO TO 192
      429 CALL F(I,ABAR,ASQ,BBAR,BSQ)
      UMA = SU1 - ABAR
      AAM = ASQ - ABAR**2
      SQ11= 2.*SQ1*SQ1
      CALL V(1,UMA,VXJ)
      Z = VXJ * EXP(-FKK * AAM/SQ11)
      175 WX1 = WX1+ Z * SP**KK
      GO TO 1107
      192 ZSUM = 0.
4026 DO 193 IJ = 1, LU3
      AIJ = IJ
      AI = FXJ(AIJ)
      CALL PKX(1,AI,PO)
      APO = PO
      CALL FF(AI,SE12,AT1,1.,AF)
      ZSUM = ZSUM + APO*AF

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```

193 CONTINUE
  Z = ZSUM * SP**KK
  IF(SU1.NE.0.) GO TO 195
  Z = 2.* Z
195 W5 = W5 + Z
1107 IS = KK
  GO TO 101
104 IS = IS + 1
  ISS = IS - 1
  I(ISS) = I(ISS) + 1
  IF(IS-KK) 107,108,107
107 LSTOP(IS+1) = I(1) + NK1 - KK + IS + 1
  ISTOP(IS+1) = MIN0(KSTOP(IS+1),LSTOP(IS+1))
108 GO TO 102
101 I(IS) = I(IS) + 1
  IF(IS - KK) 109,102,109
109 LSTOP(IS+1) = I(1) + NK1 - KK + IS + 1
  ISTOP(IS+1) = MIN0(KSTOP(IS+1),LSTOP(IS+1))
102 IF(I(IS).GT.ISTOP(IS)) GO TO 103
  IF(IS.NE.KK) GO TO 104
  IF(LOT.EQ.1) GO TO 428
  IF(N5.NE.0)GO TO 427
428 CALL F(I,ABAR,ASQ,BBAR,BSQ)
  UMA = SU1 - ABAR
  AAM = ASQ - ABAR**2
  SQ11= 2.*SQ1*SQ1
  CALL V(1,UMA,VXJ)
  Z = VXJ * EXP(-FKK*AAM/SQ11)
  WX1 = WX1 + Z*SP**KK
  GO TO 101
427 ZSUM = 0.
4276 DO 194 IJ = 1,LU3
  AIJ = IJ
  AI = FXJ(AIJ)
  CALL PKX(I,AI,PO)
  APO = PO
  CALL FF(AI,SE12,AT1,1.,AF)
  ZSUM = ZSUM + APO*AF
194 CONTINUE
  Z = ZSUM * SP**KK
  IF(SU3.NE.0.) GO TO 191
  Z = 2. * Z
191 W5 = W5 + Z
  GO TO 101
103 IS = IS-1
  IF(IS.EQ.0) GO TO 382
  IF(I(IS).EQ.ISTOP(IS)) GO TO 103
  GO TO 101
382 IF(LOT-1)383,385,385
383 X5 = X5 + ONEG*W5*W7
  O = W5*W7
  LOT = 1
  GO TO 352
385 X1 = X1 + ONEG*WX1*W6
  O = WX1*W6
388 IF(O.LE..0001) GO TO 362
390 CONTINUE
362 IF(N.GT.1) GO TO 392
  IF(NB.GT.30) GO TO 392

```

```

IF(NB.EQ.0) GO TO 392
KOUT = 1000
N4SV = 0
N3SV = 0
N2SV = 0
DO 400 KK = 1,NB
IF(N4.NE.0) GO TO 404
IF(N3.NE.0) GO TO 404
IF(N2.EQ.0) GO TO 405
404 FKK = KK
Q2 = SQRT(SQ2**2/FKK+T22)
Q1 = SQRT(SQ1**2/FKK+T12)
KA = KK-1
ONEG = 1.
IF(MOD(KA,2)) 401,402,401
401 ONEG = -ONEG
402 IF(B2.EQ.0.) GO TO 403
IF(N4.EQ.0) GO TO 403
KOO = 2
IF(T1.EQ..00000001) GO TO 1325
GSE2 = (AU2+B2+4.*S2)/TU2
SE22 = AMIN1(SE4,GSE2)*.5
1325 CALL A41
W7 = Z
403 CALL V(2,SU2,VXJ)
W6 = VXJ
IF(N4.NE.0) GO TO 1320
347 IF(N3.NE.0) GO TO 1215
348 IF(N2.NE.0) GO TO 1120
GO TO 400
1320 K11 = 2
IF(T1.EQ..00000001) GO TO 1323
GSE1 = (AU1+CL1+B1+4.*Q0)/TU1
SE12 = AMIN1(SE3,GSE1)*.5
1323 CALL A40
W4 = Z
X4 = X4 + ONEG*W4*W7
O = W4 * W7
IF(O.GT..0001) GO TO 347
N4SV = N4
N4 = 0
GO TO 347
1215 CL3 = SQRT(CL1**2 + 3.* SQ3**2)
W31=(SPI2 *R1/CL3)**KK
CALL FF(SU1,CL3,A1,T1,AF)
W3 = W31 * AF * YF(KK)
X3 = X3 + ONEG * W3*W6
O = W3 * W6
IF(O.GT..0001) GO TO 348
N3SV = N3
N3 = 0
GO TO 348
1120 K11 = 1
IF(T1.EQ..00000001) GO TO 1121
GSE1 = (AU1+CL1+4.*SQ3)/TU1
SE12 = AMIN1(SE3,GSE1)*.5
1121 CALL A40
WX2 = Z
X2 = X2 + ONEG * WX2 * W6

```

```

O = WX2 * W6
IF(O.GT..00G1) GO TO 400
N2SV = N2
N2 = 0
400 CONTINUE
405 IF(N4SV.NE.O) N4 = N4SV
   IF(N3SV.NE.O) N3 = N3SV
   IF(N2SV.NE.O) N2 = N2SV
392 RETURN
END

```

```

SUBROUTINE A41
DIMENSION DATA(1200)
DIMENSION A22(200),B22(200)
COMMON DATA
COMMON SE4,SE3,U3,U4,Z,SQ1,SQ2,Q1,Q2,CL2,PI,SP1,SQ3,C,CL3,AT1,AT2
COMMON BS1,BS2,SE12,SE22,S5,C45,SW3,SW4,ST1,SU3,SU4,AL1,AL2,SW1
COMMON FCL3,FCL4,ES1,ES2,ER1,ER2,ETB1,ETB2,WR1,WR2,WB1,WB2,WS1,WS2
COMMON A22,B22,A1,A2,B1,B2
COMMON SQR2,SP12,PI2,AJMX,BJMX,TU1,TU2,T12,T22
COMMON R32,RD1,RD2,SPR,SW12,SW22,SRR
COMMON SN100,SN101,SN102,SN110,SN112,X112
COMMON X100,X101,X102,X103,X104,X105,X106,X107,X108,X109,X110,X111
COMMON N100,N101,N102,N110,N112,INUM
COMMON KK,KOUT,KOO,KO,K11,LU3,LU4,N,NB,LU5
EQUIVALENCE (DATA(52),U2)
ZSUM = 0.
6 DO 4 J = 1,LU4
  AJ = J
  YJ = FYJ(AJ)
  CALL KY(YJ,ZKY)
  ZK1 = ZKY
  CALL FF(YJ,SE22,AT2,1.,AF)
  ZSUM = ZSUM + ZK1*AF
4 CONTINUE
Z = 2.* ZSUM
IF(SU4.EQ.O.) GO TO 100
Z = Z * .5
100 RETURN
END

```

```

SUBROUTINE V(J,X,VXJ)
DIMENSION DATA(1200)
DIMENSION A22(200),B22(200)
COMMON DATA
COMMON SE4,SE3,U3,U4,Z,SQ1,SQ2,Q1,Q2,CL2,PI,SP1,SQ3,C,CL3,AT1,AT2
COMMON BS1,BS2,SE12,SE22,S5,C45,SW3,SW4,ST1,SU3,SU4,AL1,AL2,SW1
COMMON FCL3,FCL4,ES1,ES2,ER1,ER2,ETB1,ETB2,WR1,WR2,WB1,WB2,WS1,WS2
COMMON A22,B22,A1,A2,B1,B2
COMMON SQR2,SP12,PI2,AJMX,BJMX,TU1,TU2,T12,T22
COMMON R32,RD1,RD2,SPR,SW12,SW22,SRR
COMMON SN100,SN101,SN102,SN110,SN112,X112
COMMON X100,X101,X102,X103,X104,X105,X106,X107,X108,X109,X110,X111
COMMON N100,N101,N102,N110,N112,INUM
COMMON KK,KOUT,KOO,KO,K11,LU3,LU4,N,NB,LU5
EQUIVALENCE (DATA(55),R1),(DATA(56),R2)
VXJ = 0.

```

```

AKK = KK
P2OK = SQRT(2.*PI/AKK)
IF(J.EQ.2) GO TO 2
RP =(R1/(SQ1*SQR2))*KK * P2OK * SQ1/Q1
AXQ = X/Q1
AQ1 = A1/Q1
CALL HS(AXQ,AQ1,SH)
VXJ = RP * SH
GO TO 1
2 AXQ = X/Q2
AQ2 = A2/Q2
CALL HS(AXQ,AQ2,SH)
VXJ =(R2/(SQ2*SQR2))*KK * P2OK * SQ2/Q2 * SH
1 RETURN
END

```

```

FUNCTION SMG(XX)
DIMENSION DATA(1200)
DIMENSION A22(200),B22(200)
COMMON DATA
COMMON SE4,SE3,U3,U4,Z,SQ1,SQ2,Q1,Q2,CL2,PI,SPI,SQ3,C,CL3,AT1,AT2
COMMON BS1,BS2,SE12,SE22,S5,C43,SW3,SW4,ST1,SU3,SU4,AL1,AL2,SW1
COMMON FCL3,FCL4,ES1,ES2,ER1,ER2,ETB1,ETB2,WR1,WR2,WB1,WB2,WS1,WS2
COMMON A22,B22,A1,A2,B1,B2
COMMON SQR2,SP12,PI2,AJMX,BJMX,TU1,TU2,T12,T22
COMMON R32,RD1,RD2,SPR,SW12,SW22,SRR
COMMON SN100,SN101,SN102,SN110,SN112,X112
COMMON X100,X101,X102,X103,X104,X105,X106,X107,X108,X109,X110,X111
COMMON N100,N101,N102,N110,N112,INUM
COMMON KK,KOUT,KOO,KO,K11,LU3,LU4,N,NB,LU5
SMG = EXP(-XX*XX*.5)/SQRT(PI2)
1 RETURN
END

```

```

FUNCTION YF(KKK)
DIMENSION DATA(1200)
DIMENSION A22(200),B22(200)
COMMON DATA
COMMON SE4,SE3,U3,U4,Z,SQ1,SQ2,Q1,Q2,CL2,PI,SPI,SQ3,C,CL3,AT1,AT2
COMMON BS1,BS2,SE12,SE22,S5,C45,SW3,SW4,ST1,SU3,SU4,AL1,AL2,SW1
COMMON FCL3,FCL4,ES1,ES2,ER1,ER2,ETB1,ETB2,WR1,WR2,WB1,WB2,WS1,WS2
COMMON A22,B22,A1,A2,B1,B2
COMMON SQR2,SP12,PI2,AJMX,BJMX,TU1,TU2,T12,T22
COMMON R32,RD1,RD2,SPR,SW12,SW22,SRR
COMMON SN100,SN101,SN102,SN110,SN112,X112
COMMON X100,X101,X102,X103,X104,X105,X106,X107,X108,X109,X110,X111
COMMON N100,N101,N102,N110,N112,INUM
COMMON KK,KOUT,KOO,KO,K11,LU3,LU4,N,NB,LU5
EQUIVALENCE (DATA(37),SP)
AND = N
IF(KOUT.EQ.1000) AND = NB
YF = 1.
DO 1 J = 1,KKK
AJ = J
YF=YF * (AND-AJ+1.)/AJ * SP
1 CONTINUE
2 RETURN

```

END

```

FUNCTION FXJ(XX)
  DIMENSION DATA(1200)
  DIMENSION A22(200),B22(200)
  COMMON DATA
  COMMON SE4,SE3,U3,U4,Z,SQ1,SQ2,Q1,Q2,CL2,PI,SPI,SQ3,C,CL3,AT1,AT2
  COMMON BS1,BS2,SE12,SE22,S5,C45,SW3,SW4,ST1,SU3,SU4,AL1,AL2,SW1
  COMMON FCL3,FCL4,ES1,ES2,ER1,ER2,ETB1,ETB2,WR1,WR2,WB1,WB2,WS1,WS2
  COMMON A22,B22,A1,A2,B1,B2
  COMMON SQR2,SPI2,PI2,AJMX,BJMX,TU1,TU2,T12,T22
  COMMON R32,RD1,RD2,SPR,SW12,SW22,SRR
  COMMON SN100,SN101,SN102,SN110,SN112,X112
  COMMON X100,X101,X102,X103,X104,X105,X106,X107,X108,X109,X110,X111
  COMMON N100,N101,N102,N110,N112,INUM
  COMMON KK,KOUT,KOD,KO,K11,LU3,LU4,N,NB,LU5
  EQUIVALENCE (DATA(51),U1)
  IF(XX.NE.0.) GO TO 2
  FXJ = 0.
  GO TO 3
2 IF(SU3.NE.0.) GO TO 4
  FXJ = (2.*XX-1.) * SE12
  GO TO 3
4 FXJ = (2.*XX-1.-2.*U1) * SE12
3 RETURN
END

```

```

FUNCTION FYJ(XX)
  DIMENSION DATA(1200)
  DIMENSION A22(200),B22(200)
  COMMON DATA
  COMMON SE4,SE3,U3,U4,Z,SQ1,SQ2,Q1,Q2,CL2,PI,SPI,SQ3,C,CL3,AT1,AT2
  COMMON BS1,BS2,SE12,SE22,S5,C45,SW3,SW4,ST1,SU3,SU4,AL1,AL2,SW1
  COMMON FCL3,FCL4,ES1,ES2,ER1,ER2,ETB1,ETB2,WR1,WR2,WB1,WB2,WS1,WS2
  COMMON A22,B22,A1,A2,B1,B2
  COMMON SQR2,SPI2,PI2,AJMX,BJMX,TU1,TU2,T12,T22
  COMMON R32,RD1,RD2,SPR,SW12,SW22,SRR
  COMMON SN100,SN101,SN102,SN110,SN112,X112
  COMMON X100,X101,X102,X103,X104,X105,X106,X107,X108,X109,X110,X111
  COMMON N100,N101,N102,N110,N112,INUM
  COMMON KK,KOUT,KOD,KO,K11,LU3,LU4,N,NB,LU5
  EQUIVALENCE (DATA(52),U2)
  IF(XX.NE.0.) GO TO 2
  FYJ = 0.
  GO TO 3
2 IF(SU4.NE.0.) GO TO 4
  FYJ = (2.*XX-1.) * SE22
  GO TO 3
4 FYJ = (2.*XX-1.-2.*U2) * SE22
3 RETURN
END

```

```

SUBROUTINE A40
  DIMENSION DATA(1200)
  DIMENSION A22(200),B22(200)
  COMMON DATA

```

```

COMMON SE4,SE3,U3,U4,Z,SQ1,SQ2,Q1,Q2,CL2,PI,SPI,SQ3,C,CL3,AT1,AT2
COMMON BS1,BS2,SE12,SE22,S5,C45,SW3,SW4,ST1,SU3,SU4,AL1,AL2,SW1
COMMON FCL3,FCL4,ES1,ES2,ER1,ER2,ETB1,ETB2,WR1,WR2,WB1,WB2,WS1,WS2
COMMON A22,B22,A1,A2,B1,B2
COMMON SQR2,SPI2,PI2,AJMX,BJMX,TU1,TU2,T12,T22
COMMON R32,RD1,RD2,SPR,SW12,SW22,SRR
COMMON SN100,SN101,SN102,SN110,SN112,X112
COMMON X100,X101,X102,X103,X104,X105,X106,X107,X108,X109,X110,X111
COMMON N100,N101,N102,N110,N112,INUM
COMMON KK,KOUT,KOO,KO,K11,LU3,LU4,N,NB,LU5
EQUIVALENCE (DATA(51),U1)
ASUM = 0.
6 DO 4 J = 1,LU3
  AJ = J
  XJ = FXJ(AJ)
  CALL KX(XJ,ZKX)
  ZK1 = ZKX
  CALL FF(XJ,SE12,AT1,1.,AF)
  ASUM = ASUM + ZK1*AF
4 CONTINUE
  Z = 2.*ASUM*YF(KK)
  IF(SU3.EQ.0.) GO TO 100
  Z = Z * .5
100 RETURN
END

```

```

SUBROUTINE KX(ZA,ZKX)
DIMENSION DATA(1200)
DIMENSION A22(200),B22(200)
COMMON DATA
COMMON SE4,SE3,U3,U4,Z,SQ1,SQ2,Q1,Q2,CL2,PI,SPI,SQ3,C,CL3,AT1,AT2
COMMON BS1,BS2,SE12,SE22,S5,C45,SW3,SW4,ST1,SU3,SU4,AL1,AL2,SW1
COMMON FCL3,FCL4,ES1,ES2,ER1,ER2,ETB1,ETB2,WR1,WR2,WB1,WB2,WS1,WS2
COMMON A22,B22,A1,A2,B1,B2
COMMON SQR2,SPI2,PI2,AJMX,BJMX,TU1,TU2,T12,T22
COMMON R32,RD1,RD2,SPR,SW12,SW22,SRR
COMMON SN100,SN101,SN102,SN110,SN112,X112
COMMON X100,X101,X102,X103,X104,X105,X106,X107,X108,X109,X110,X111
COMMON N100,N101,N102,N110,N112,INUM
COMMON KK,KOUT,KOO,KO,K11,LU3,LU4,N,NB,LU5
EQUIVALENCE (DATA(43),S1),(DATA(47),T1),(DATA(49),SU1),(DATA(53),C
1L1),(DATA(54),QO),(DATA(55),R1),(DATA(56),R2)
ZKX = 0.
IF(K11.EQ.1) GO TO 1
IF(K11.EQ.2) GO TO 2
IF(K11.EQ.3) GO TO 3
1 XTS =(ZA * T1 + SU1)/SQ3
  CLS = CL1/SQ3
  CALL HS(XTS,CLS,SH)
  ZKX =(SH * SPI * R1/SQ3)**KK
  GO TO 100
2 XT = (ZA*T1+SU1)
  CALL FF(XT,B1,CL1,QO,AF)
  ZKX = AF ** KK
  GO TO 100
3 XT =(ZA*T1+SU1)
  CALL SF(XT,B1,S1,SMF)
  ZKX = SMF**KK

```

100 RETURN
END

```

SUBROUTINE KY(ZZ,ZKY)
  DIMENSION DATA(1200)
  DIMENSION A(200),B(200),A22(200),B22(200)
  COMMON DATA
  COMMON SE4,SE3,U3,U4,Z,SQ1,SQ2,Q1,Q2,CL2,PI,SPI,SQ3,C,CL3,AT1,AT2
  COMMON BS1,BS2,SE12,SE22,S5,C45,SW3,SW4,ST1,SU3,SU4,AL1,AL2,SW1
  COMMON FCL3,FCL4,ES1,ES2,ER1,ER2,ETB1,ETB2,WR1,WR2,WB1,WB2,WS1,WS2
  COMMON A22,B22,A1,A2,B1,B2
  COMMON SQR2,SPI2,PI2,AJMX,BJMX,TU1,TU2,T12,T22
  COMMON R32,RD1,RD2,SPR,SW12,SW22,SRR
  COMMON SN100,SN101,SN102,SN110,SN112,X112
  COMMON X100,X101,X102,X103,X104,X105,X106,X107,X108,X109,X110,X111
  COMMON N100,N101,N102,N110,N112,INUM
  COMMON KK,KOUT,KOO,KO,K11,LU3,LU4,N,NB,LU5
  EQUIVALENCE (DATA(44),S2),(DATA(47),T1),(DATA(48),T2),(DATA(49),S
1U1),(DATA(50),SU2)
  EQUIVALENCE (DATA(101),A),(DATA(301),B)
  ZKY = 0.
  IF(KOO.EQ.1) GO TO 1
  IF(KOO.EQ.2) GO TO 2
1 ZKK = 1.
  DO 3 J = 1,N
    AJS = SU1 - A(J)
    CALL FF(AJS,CL3,A1,T1,AF)
    ZAF = AF
    YTB = ZZ * T2 - B(J) + SU2
    CALL PP(YTB,Z,PXJ)
    ZKK = ZKK * (1. - (1. - (1. - C*PXJ)**NB))
3 CONTINUE
  ZKY = (1. - ZKK) * ZAF
  GO TO 100
2 YTS = ZZ*T2+SU2
  CALL SF(YTS,B2,S2,SMF)
  ZKY = SMF**KK
100 RETURN
END

```

```

SUBROUTINE F(I,ABAR,ASQ,BBAR,RSQ)
  DIMENSION DATA(1200),I(200)
  DIMENSION A(200),B(200),A22(200),B22(200)
  COMMON DATA
  COMMON SE4,SE3,U3,U4,Z,SQ1,SQ2,Q1,Q2,CL2,PI,SPI,SQ3,C,CL3,AT1,AT2
  COMMON BS1,BS2,SE12,SE22,S5,C45,SW3,SW4,ST1,SU3,SU4,AL1,AL2,SW1
  COMMON FCL3,FCL4,ES1,ES2,ER1,ER2,ETB1,ETB2,WR1,WR2,WB1,WB2,WS1,WS2
  COMMON A22,B22,A1,A2,B1,B2
  COMMON SQR2,SPI2,PI2,AJMX,BJMX,TU1,TU2,T12,T22
  COMMON R32,RD1,RD2,SPR,SW12,SW22,SRR
  COMMON SN100,SN101,SN102,SN110,SN112,X112
  COMMON X100,X101,X102,X103,X104,X105,X106,X107,X108,X109,X110,X111
  COMMON N100,N101,N102,N110,N112,INUM
  COMMON KK,KOUT,KOO,KO,K11,LU3,LU4,N,NB,LU5
  EQUIVALENCE (DATA(101),A),(DATA(301),B)
  AK = KK
  ABAR = 0.

```

```

ASQ = 0.
BBAR = 0.
BSQ = 0.
DO 1 J = 1, KK
M1 = I(J)
ABAR = ABAR + A(M1)/AK
ASQ = ASQ + A22(M1)/AK
BBAR = BBAR + B(M1)/AK
BSQ = BSQ + B22(M1)/AK
1 CONTINUE
RETURN
END

```

```

SUBROUTINE PKX(I, AIJ, PO)
DIMENSION DATA(1200), I(200)
DIMENSION A(200), B(200), A22(200), B22(200)
COMMON DATA
COMMON SE4, SE3, U3, U4, Z, SQ1, SQ2, Q1, Q2, CL2, PI, SPI, SQ3, C, CL3, AT1, AT2
COMMON BS1, BS2, SE12, SE22, S5, C45, SW3, SW4, ST1, SU3, SU4, AL1, AL2, SW1
COMMON FCL3, FCL4, ES1, ES2, ER1, ER2, ETB1, ETB2, WR1, WR2, WB1, WB2, WS1, WS2
COMMON A22, B22, A1, A2, B1, B2
COMMON SQR2, SPI2, PI2, AJMX, BJMX, TU1, TU2, T12, T22
COMMON R32, RD1, RD2, SPR, SW12, SW22, SRR
COMMON SN100, SN101, SN102, SN110, SN112, X112
COMMON X100, X101, X102, X103, X104, X105, X106, X107, X108, X109, X110, X111
COMMON N100, N101, N102, N110, N112, INUM
COMMON KK, KOUT, KOO, KO, K11, LU3, LU4, N, NB, LU5
EQUIVALENCE (DATA(43), S1), (DATA(47), T1), (DATA(49), SU1)
EQUIVALENCE (DATA(101), A), (DATA(301), B)
PO = 1.
DO 1 J = 1, KK
M1 = I(J)
SFX = (AIJ*T1+SU1-A(M1))
CALL SF(SFX, B1, S1, SMF)
PO = PO * SMF
1 CONTINUE
RETURN
END

```

```

SUBROUTINE K6(XX, YY, PK6)
REAL XUA(200)/200*0.0/, YUB(200)/200*0.0/
DIMENSION DATA(1200)
DIMENSION A(200), B(200), A22(200), B22(200)
COMMON DATA
COMMON SE4, SE3, U3, U4, Z, SQ1, SQ2, Q1, Q2, CL2, PI, SPI, SQ3, C, CL3, AT1, AT2
COMMON BS1, BS2, SE12, SE22, S5, C45, SW3, SW4, ST1, SU3, SU4, AL1, AL2, SW1
COMMON FCL3, FCL4, ES1, ES2, ER1, ER2, ETB1, ETB2, WR1, WR2, WB1, WB2, WS1, WS2
COMMON A22, B22, A1, A2, B1, B2
COMMON SQR2, SPI2, PI2, AJMX, BJMX, TU1, TU2, T12, T22
COMMON R32, RD1, RD2, SPR, SW12, SW22, SRR
COMMON SN100, SN101, SN102, SN110, SN112, X112
COMMON X100, X101, X102, X103, X104, X105, X106, X107, X108, X109, X110, X111
COMMON N100, N101, N102, N110, N112, INUM
COMMON KK, KOUT, KOO, KO, K11, LU3, LU4, N, NB, LU5
EQUIVALENCE (DATA(33), A11), (DATA(34), A12), (DATA(35), D), (DATA(36),
1 DF), (DATA(37), SP), (DATA(38), AJJ), (DATA(39), A3), (DATA(40), A4),
2 (DATA(41), B3), (DATA(42), B4), (DATA(43), S1), (DATA(44), S2), (DATA(45)

```

```

3 ,S3),(DATA(46),S4),(DATA(47),T1),(DATA(48),T2),(DATA(49),SU1),
4 (DATA(50),SU2),(DATA(51),U1),(DATA(52),U2),(DATA(53),CL1),
5 (DATA(54),Q0),(DATA(55),R1),(DATA(56),R2),(DATA(57),D1),(DATA(58)
6 ,D2),(DATA(59),R),(DATA(60),W1),(DATA(61),W2),(DATA(62),ET1),
7 (DATA(63),ET2),(DATA(64),GCL3),(DATA(65),GCL4)
EQUIVALENCE (DATA(101),A),(DATA(301),B)
PK6 = 0.
PKP = 1.
DO 500 J = 1,N
IF(T1.EQ..000000001) GO TO 501
XUA(J) = XX * T1 + SU1 - A(J)
YUB(J) = YY * T2 + SU2 - B(J)
GO TO 500
501 XUA(J) = XX + SU1 - A(J)
YUB(J) = YY + SU2 - B(J)
500 CONTINUE
IF(K0.EQ.11)GO TO 11
IF(K0.EQ.12)GO TO 12
IF(K0.EQ.14)GO TO 14
IF(K0.EQ.15)GO TO 11
IF(K0.EQ.100) GO TO 4
IF(K0.EQ.1001) GO TO 5
IF(K0.EQ.101) GO TO 101
IF(X0.EQ.1011) GO TO 1011
IF(K0.EQ.102) GO TO 4
IF(K0.EQ.1021) GO TO 5
IF(K0.EQ.103) GO TO 103
IF(K0.EQ.104) GO TO 104
IF(K0.EQ.105) GO TO 105
IF(K0.EQ.106) GO TO 106
IF(K0.EQ.107) GO TO 107
IF(K0.EQ.108) GO TO 108
IF(K0.EQ.109) GO TO 109
IF(K0.EQ.110) GO TO 110
IF(K0.EQ.111) GO TO 111
IF(K0.EQ.112) GO TO 112
11 DO 99 J = 1,N
XTA = XUA(J)
CALL PP(XTA,1,PXJ)
PX1 = PXJ
YTA = YUB(J)
CALL PP(YTA,2,PXJ)
PY1 = PYJ
PKP = PKP * (1.-PX1*PY1*SP)
99 CONTINUE
GO TO 100
12 HLS = CL1/SQ3
DO 98 J = 1,N
YTS = YUB(J)
CALL PP(YTS,2,PXJ)
PP1 = PXJ
XTS = XUA(J)/SQ3
CALL HS(XTS,HLS,SH)
PKP = PKP*(1.-PP1*C*SH)*NB
98 CONTINUE
GO TO 100
14 DO 97 J = 1,N
XTS = XUA(J)
CALL FF(XTS,B1,CL1,Q0,AF)

```

```

TJX = SP * AF
YTS = YUB(J)
CALL PP(YTS,2,PXJ)
PKP = PKP*(1.-TJX*PXJ)**NB
97 CONTINUE
GO TO 100
4 DO 96 J = 1,N
FJXY = SQRT(XUA(J)**2+YUB(J)**2)/S5
X = FJXY - SW3
Y = FJXY - SW4
PKP = PKP * (1.- C45*(BIGG(X)-BIGG(Y)))
96 CONTINUE
GO TO 100
5 DO 95 J = 1,N
Y = SQRT(XUA(J)**2+YUB(J)**2)/S5 - SW4
PKP = PKP * (1.-C45*(1.-BIGG(Y)))
95 CONTINUE
GO TO 100
101 AL = SQRT(AL1*AL2)
DO 86 J = 1,N
FJXY = SQRT(XUA(J)**2+YUB(J)**2)/AL
X = FJXY - SW3
Y = FJXY - SW4
PKP = PKP*(1.-R*(1.- (1.-C45*(BIGG(X)-BIGG(Y)))*NB))
86 CONTINUE
GO TO 100
1011 AL = SQRT(AL1*AL2)
DO 83 J = 1,N
Y = SQRT(XUA(J)**2+YUB(J)**2)/AL -SW4
PKP = PKP*(1.-R*(1.- (1.-C45*(1.-BIGG(Y)))*NB))
83 CONTINUE
GO TO 100
103 DO 94 J = 1,N
X = XUA(J)
Y = YUB(J)
CALL SF(X,FCL3,S3,SMF)
SFA = SMF
CALL SF(Y,FCL4,S4,SMF)
PKP = PKP * (1.-C45*SFA*SMF)
94 CONTINUE
GO TO 100
104 DO 93 J = 1,N
X = XUA(J)
Y = YUB(J)
CALL SF(X,GCL3,AL1,SMF)
SFA = SMF
CALL SF(Y,GCL4,AL2,SMF)
PKP = PKP*(1.-R*(1.- (1.-C45*SFA*SMF)*NB))
93 CONTINUE
GO TO 100
105 DO 92 J = 1,N
X = XUA(J)
Y = YUB(J)
CALL SF(X,FCL3,S3,SMF)
SFA = SMF
CALL SF(Y,FCL4,S4,SMF)
PKP = PKP * (1.- C45*SFA*SMF)
92 CONTINUE
GO TO 100

```

```

106 DO 91 J = 1,N
    XTA = XUA(J)
    YTB = YUB(J)
    CALL FF(XTA,B1,GCL3,0.,AF)
    AF1 = AF
    CALL FF(YTB,B2,GCL4,0.,AF)
    PKP = PKP * (1.-R*(1.-(1.-SP*AF1*AF)**NB))
91 CONTINUE
    GO TO 100
107 DO 90 J = 1,N
    XTS = XUA(J)/S3
    YTS = YUB(J)/S4
    CALL A113(ES1,ES2,XTS,YTS,P113)
    PKP = PKP * (1.- C45*P113)
90 CONTINUE
    GO TO 100
108 TA1 = ET1/AL1
    TA2 = ET2/AL2
    DO 89 J = 1,N
    XTA = XUA(J)/AL1
    YTB = YUB(J)/AL2
    CALL A113(TA1,TA2,XTA,YTB,P113)
    PKP = PKP*(1.-R*(1.-(1.-C45*P113)**NB))
89 CONTINUE
    GO TO 100
109 DO 88 J = 1,N
    XTA = XUA(J)/S3
    YTB = YUB(J)/S4
    CALL A113(ES1,ES2,XTA,YTB,P113)
    PKP = PKP * (1.- C45*P113)
88 CONTINUE
    GO TO 100
110 DO 87 J = 1,N
    XTA = XUA(J)/S3
    YTB = YUB(J)/S4
    CALL A113(ES1,ES2,XTA,YTB,P113)
    PA = P113
    CALL A113(WS1,WS2,XTA,YTB,P113)
    PKP = PKP * (1.-C45*(PA-P113))
87 CONTINUE
    GO TO 100
111 TA1 = ET1/AL1
    TA2 = ET2/AL2
    WA1 = W1/AL1
    WA2 = W2/AL2
    DO 85 J = 1,N
    XTA = XUA(J)/AL1
    YTB = YUB(J)/AL2
    CALL A113(TA1,TA2,XTA,YTB,P113)
    PA = P113
    CALL A113(WA1,WA2,XTA,YTB,P113)
    PKP = PKP * (1.-R*(1.-(1.-C45*(PA-P113)**NB))
85 CONTINUE
    GO TO 100
112 DO 84 J = 1,N
    XTA = XUA(J)/S3
    YTB = YUB(J)/S4
    CALL A113(ES1,ES2,XTA,YTB,P113)
    PA = P113

```

```

CALL A113(WS1,WS2,XTA,YTB,P113)
PKP = PKP * (1.- C45*(PA-P113))
84 CONTINUE
100 PK6 = 1.-PKP
RETURN
END

```

```

SUBROUTINE A113(AZ,BZ,XX,YY,P113)
DIMENSION DATA(1200)
DIMENSION A22(200),B22(200)
COMMON DATA
COMMON SE4,SE3,U3,U4,Z,SQ1,SQ2,Q1,Q2,CL2,P1,SPI,SQ3,C,CL3,A11,AT2
COMMON BS1,BS2,SE12,SE22,S5,C45,SW3,SW4,ST1,SU3,SU4,AL1,AL2,SW1
COMMON FCL3,FCL4,ES1,ES2,ER1,ER2,ETB1,ETB2,WR1,WR2,WB1,WB2,WS1,WS2
COMMON A22,B22,A1,A2,B1,B2
COMMON SQR2,SPI2,PI2,AJMX,BJMX,TU1,TU2,T12,T22
COMMON R32,RD1,RD2,SPR,SH12,SW22,SRR
COMMON SN100,SN101,SN102,SN110,SN112,X112
COMMON X100,X101,X102,X103,X104,X105,X106,X107,X108,X109,X110,X111
COMMON N100,N101,N102,N110,N112,INUM
COMMON KK,KOUT,KOO,KO,K11,LU3,LU4,N,NB,LU5
EQUIVALENCE (DATA(66),UU5),(DATA(67),VV5)
E5 = 2.*PI/UU5
SK = VV5
PSUM = 0.
2 DO 5 J = 1,LU5
  IF(J.EQ.1) GO TO 1
  SK = SK + E5
1 SSK = SIN(SK)
  COSK = COS(SK)
  ZK = BZ * XX * COSK + AZ * YY * SSK + AZ * BZ
  CK = (AZ * COSK + XX)**2 + (BZ * SSK + YY)**2
  XYZJ = Z113(CK)
  PSUM = PSUM + ZK * XYZJ/UU5
5 CONTINUE
10 P113 = PSUM
RETURN
END

```

```

FUNCTION BIGG(T)
S2 = SQRT(2.)
TA = T/S2
BIGG = .5 * (1. + ERF(TA))
1 RETURN
END

```

```

SUBROUTINE DECRD(DATA)
C   DECIMAL CARD READ SUBROUTINE
C   MAIN PROGRAM MUST CONTAIN A DIMENSION STATEMENT AND CORRESPONDING
C   EQUIVALENT STATEMENT.
C   ARRAY DATA MUST BE FIRST ENTRY IN BLANK COMMON IN PROGRAM.
DIMENSION DRBU(5), DATA(1), I1(5)
9 READ (5,10) (I1(I),I=1,5),I1STL,(DRBU(J),J=1,5)
  J = IABS(I1STL)
  DO 30 I=1,5
    IF (I1(I).NE.0) GO TO 30

```

```

20 DATA(J) = DRBU(I)
30 J = J+1
   IF (I1STL) 50,40,9
40 WRITE (6,900) (I1(I),I=1,5),I1STL,(DRBU(J),J=1,5)
   CALL EXIT
50 RETURN
10 FORMAT (5I1,17,5F12.11)
900 FORMAT (1H1,86H***ADDRESS PORTION OF IP CARD = ZERO. RUN TERMINAT
   ED. CARD IN ERROR PRINTED BELOW.***//1H0,5I1,17,5F12.11)
END

```

```

SUBROUTINE FF(OX,OY,OL,S,AF)
AF = 0.
XPY = OX + OY
XMY = OX - OY
OL2 = 2.* OL
CALL EE(XPY,OL,S,EEE)
E1 = EEE
CALL EE(XMY,OL,S,EEE)
E2 = EEE
AF = E1-E2
RETURN
END

```

```

SUBROUTINE H(YX,YL,HXL)
HXL = 0.
IF(YL.EQ.0.) GO TO 1
XPL = YX + YL
XML = YX - YL
HXL = (XPL*BIGG(XPL)-XML*BIGG(XML)+SMG(XPL)-SMG(XML))/(2.*YL)
GO TO 2
1 HXL = BIGG(YX)
2 RETURN
END

```

```

SUBROUTINE EE(X,CL,S,EEE)
IF(S.EQ.0.) GO TO 1
XS = X/S
SL = CL/S
CALL H(XS,SL,HXL)
EEE= HXL
GO TO 5
1 AL = -CL
IF(X.LT.AL) GO TO 2
IF(X.GE.CL) GO TO 3
EEC= (X+CL)/(2.*CL)
GO TO 5
2 EEE= 0.
GO TO 5
3 EEE= 1.
5 RETURN
END

```

```

SUBROUTINE SF(OX,OY,S,SMF)
IF(S.EQ.0.) GO TO 1

```

```
XPY =(OX + OY)/S
XMY =(OX-OY)/S
SMF = BIGG(XPY)-BIGG(XMY)
GO TO 5
1 IF(ABS(OX).LE.OY) GO TO 2
  SMF = 0.
  GO TO 5
2 SMF = 1.
5 RETURN
END
```

```
SUBROUTINE HS(OX,OL,SH)
IF(OL.EQ.0.) GO TO 1
CALL SF(OX,OL,1.,SMF)
SH = SMF/(2.*OL)
GO TO 2
1 SH = SMG(OX)
2 RETURN
END
```

```
FUNCTION Z113(SO)
IF(SO.LE..001) GO TO 1
Z113 = (1.-EXP(-SO/2.))/SO
GO TO 2
1 Z113 = .5 - SO/8. + SO**2/48.
2 RETURN
END
```

5.8. TEST CASES FOR A RIPPLE OF BOMBS AND FOR A RIPPLE OF FIXED DISPENSERS

Test Case 1

	1	23	1	1	0	2	0				
N0	N1	N2	N3	N4	N5	N10	N11	N12	N13	N14	N15
1	0	0	0	0	0	0	1	0	0	0	0
D	DF	SP	N	NS	NVI	NN4					
40.00	20.00	0.95	24	1	0	3					
A3	A4	B3	B4	S1	S2	S3	S4				
400.0000	200.0000	1.0000	1.0000	30.0000	20.0000	0.0	0.0				
T1	T2	SU1	SU2	U1	U2	L1	Q0				
150.0000	100.0000	0.0	0.0	8.0000	8.0000	0.0	0.0				
R1	R2	D1	D2	R	W1	W2					
15.0000	30.0000	1.0000	1.0000	0.0	0.0	0.0					
ET1	ET2	GCL3	GCL4	UU5	VV5						
0.0	0.0	0.0	0.0	40.0000	0.0						
A(J)											
-60.0000	-60.0000	-60.0000	-60.0000	-60.0000	-60.0000	-20.0000	-20.0000				
-20.0000	-20.0000	-20.0000	-20.0000	-20.0000	-20.0000	20.0000	20.0000				
20.0000	20.0000	60.0000	60.0000	60.0000	60.0000	60.0000	60.0000				
B(J)											
-20.0000	-0.0000	20.0000	-20.0000	-0.0000	20.0000	-20.0000	-0.0000				
20.0000	-20.0000	-0.0000	20.0000	-20.0000	-0.0000	20.0000	-20.0000				
-0.0000	20.0000	-20.0000	-0.0000	20.0000	-20.0000	-0.0000	20.0000				
KD =	11	0.122739									
X(10)	X(11)	X(12)	X(13)	X(14)	X(15)	X(30)					
0.0	0.122739	0.0	0.0	0.0	0.0	0.0					
X(100)	X(101)	X(102)	X(103)	X(104)	X(105)	X(106)					
0.0	0.0	0.0	0.0	0.0	0.0	0.0					
X(107)	X(108)	X(109)	X(110)	X(111)	X(112)						
0.0	0.0	0.0	0.0	0.0	0.0						

$X(11)$ is the expected fraction damage to an area target from a ripple of $N(1)$ bombs, each subject to a gaussian ballistic error and the whole ripple subject to a gaussian aiming error. The area is rectangular and oriented along and perpendicular to the line of flight. The major damage is fragmentation. $X(11)$ uses $R1$ and $R2$ entries and $B3=B4=i$.

Test Case 2

	2		23		1		1		0		0		2		0
NO	N1	N2	N3	N4	N5	N10	N11	N12	N13	N14	N15				
1	0	0	0	0	0	0	0	0	0	0	1				
D	DF			SP		N		NB		NVI		NN4			
40.00	20.00			0.95		24		1		0		3			
A3	A4			B3		B4		S1		S2		S3		S4	
400.0000	200.0000			20.0000		20.0000		30.0000		20.0000		0.0		0.0	
T1	T2			SU1		SU2		U1		U2		L1		Q0	
150.0000	100.0000			0.0		0.0		8.0000		8.0000		0.0		0.0	
R1	R2			D1		D2		R		W1		W2			
1.0000	1.0000			1.0000		1.0000		0.0		0.0		0.0			
ET1	ET2			GCL3		GCL4		UUS		VV5					
0.0	0.0			0.0		0.0		40.0000		0.0					
A(J)															
-60.0000	-60.0000			-60.0000		-60.0000		-60.0000		-60.0000		-20.0000		-20.0000	
-20.0000	-20.0000			-20.0000		-20.0000		20.0000		20.0000		20.0000		20.0000	
20.0000	20.0000			60.0000		60.0000		60.0000		60.0000		60.0000		60.0000	
B(J)															
-20.0000	-0.0000			20.0000		-20.0000		-0.0000		20.0000		-20.0000		-0.0000	
20.0000	-20.0000			-0.0000		20.0000		-20.0000		-0.0000		20.0000		-20.0000	
-0.0000	20.0000			-20.0000		-0.0000		20.0000		-20.0000		-0.0000		20.0000	
K0 =	15			0.048656											
X(10)	X(11)			X(12)		X(13)		X(14)		X(15)		X(30)			
0.0	0.0			0.0		0.0		0.0		0.048656		0.0			
X(100)	X(101)			X(102)		X(103)		X(104)		X(105)		X(106)			
0.0	0.0			0.0		0.0		0.0		0.0		0.0			
X(107)	X(108)			X(109)		X(110)		X(111)		X(112)					
0.0	0.0			0.0		0.0		0.0		0.0					

X(15) is the same method of delivery as X(11) but the damage is based on impact (cookie-cutter damage function). X(15) uses B3 and B4 entries and R1=R2=1.

Test Case 3

	3		23		1		1		0		0		1		0
N0	N1	N2	N3	N4	N5	N10	N11	N12	N13	N14	N15				
0	1	0	0	0	0	0	0	0	0	0	0				
D		DF		SP		N		NB		NVI		NN4			
20.00		0.0		0.95		4		1		0		1			
A3		A4		B3		B4		S1		S2		S3		S4	
400.0000		200.0000		1.0000		1.0000		30.0000		20.0000		0.0		0.0	
T1		T2		SU1		SU2		U1		U2		L1		Q0	
150.0000		100.0000		0.0		0.0		8.0000		8.0000		0.0		0.0	
R1		R2		D1		D2		R		W1		W2			
15.0000		30.0000		1.0000		1.0000		0.0		0.0		0.0			
ET1		ET2		GCL3		GCL4		UU5		VV5					
0.0		0.0		0.0		0.0		40.0000		0.0					
A(J)															
-30.0000		-10.0000		10.0000		30.0000									
ALL B(J) = 0.															
X(1)	X(2)	X(3)	X(4)	X(5)											
0.031374	0.0	0.0	0.0	0.0											
X(10)	X(11)	X(12)	X(13)	X(14)	X(15)	X(30)									
0.0	0.0	0.0	0.0	0.0	0.0	0.0									
X(100)	X(101)	X(102)	X(103)	X(104)	X(105)	X(106)									
0.0	0.0	0.0	0.0	0.0	0.0	0.0									
X(107)	X(108)	X(109)	X(110)	X(111)	X(112)										
0.0	0.0	0.0	0.0	0.0	0.0										

X(1) is the same as X(11). The method of computation is different and is faster than X(11) provided $1 \leq i \leq 12$. This restriction has been built in to the program.

$X(5)$ is the same as $X(15)$. It should be restricted to $N(1) \leq 4$.

Test Case 5

```

      5      23      1      1      1      0      1      0
NO  N1  N2  N3  N4  N5  N10  N11  N12  N13  N14  N15
 1   0   0   0   0   0   0   0   1   0   0   0

      D      DF      SP      N      NB      NVI      NN4
      0.0    30.00    0.95      2     100     15      1

      A3      A4      B3      B4      S1      S2      S3      S4
800.0000 200.0000 1.0000 1.0000 1.0000 20.0000 0.0 0.0

      T1      T2      SU1      SU2      U1      U2      L1      Q0
150.0000 100.0000 0.0 0.0 8.0000 8.0000 1.0000 0.0

      R1      R2      D1      D2      R      W1      W2
7.5000 15.0000 1.0000 1.0000 0.0 0.0 0.0

      ET1      ET2      GCL3      GCL4      UU5      VV5
0.0 0.0 0.0 0.0 40.0000 0.0

      SVI(J)
0.0 100.0000 200.0000 300.0000 400.0000 500.0000 600.0000 700.0000
800.0000 900.0000 1000.0000 1100.0000 1200.0000 1300.0000 1400.0000

      BPI(J)
0.0 0.0200 0.0400 0.0600 0.0800 0.1000 0.1000 0.1000
0.1000 0.1000 0.1000 0.0800 0.0600 0.0400 0.0200

ALL A(J) = 0.

B(J)
0.0 0.0

KO = 12 0.141609

      X(10)      X(11)      X(12)      X(13)      X(14)      X(15)      X(30)
0.0 0.0 0.141609 0.0 0.0 0.0 0.0

      X(100)      X(101)      X(102)      X(103)      X(104)      X(105)      X(106)
0.0 0.0 0.0 0.0 0.0 0.0 0.0

      X(107)      X(108)      X(109)      X(110)      X(111)      X(112)
0.0 0.0 0.0 0.0 0.0 0.0

```

X(12) gives the expected fraction damage to an area target from a ripple of fixed dispensers, where the ballistic dispersion of the bomblets in range is given by a table. The deflection dispersion is gaussian, as is the aiming error. The major damage effect is fragmentation. R_1 , R_2 , CL_1 , Q_0 , S_2 , and $B_3=B_4=1$ are required inputs.

Test Case 6

```

        6      23      1      1      1      0      1      0
NO      N1      N2      N3      N4      N5      N10     N11     N12     N13     N14     N15
 1       0       0       0       0       0       0       0       0       1       0       0

      D      DF      SP      N      NB      NVI      NN4
      0.0    30.00    0.95      2     100      15       1

      43      A4      B3      B4      S1      S2      S3      S4
800.0000 200.0000 1.0000 1.0000 1.0000 20.0000 0.0      0.0

      T1      T2      SU1      SU2      U1      U2      L1      Q0
150.0000 100.0000 0.0      0.0      8.0000 8.0000 1.0000 0.0

      R1      R2      D1      D2      R      W1      W2
7.5000 15.0000 1.0000 1.0000 0.0      0.0      0.0

      ET1      ET2      GCL3      GCL4      UU5      VV5
0.0      0.0      0.0      0.0      40.0000 0.0

      SVI(J)
0.0      100.0000 200.0000 300.0000 400.0000 500.0000 600.0000 700.0000
800.0000 900.0000 1000.0000 1100.0000 1200.0000 1300.0000 1400.0000

      BPI(J)
0.0      0.0200 0.0400 0.0600 0.0800 0.1000 0.1000 0.1000
0.1000 0.1000 0.1000 0.0800 0.0600 0.0400 0.0200

      ALL A(J) = 0.

      B(J)
0.0      0.0

      KO = 13 0.139480

      X(10)      X(11)      X(12)      X(13)      X(14)      X(15)      X(30)
0.0      0.0      0.0      0.139480 0.0      0.0      0.0

      X(100)      X(101)      X(102)      X(103)      X(104)      X(105)      X(106)
0.0      0.0      0.0      0.0      0.0      0.0      0.0

      X(107)      X(108)      X(109)      X(110)      X(111)      X(112)
0.0      0.0      0.0      0.0      0.0      0.0

```

X(13) is an approximation to X(12).

Test Case 7

	7		23		1		1		1		0		1		0		
N0	N1	N2	N3	N4	N5	N10	N11	N12	N13	N14	N15						
1	0	0	0	0	0	0	0	0	0	1	0						
D		DF		SP		N		NB		NVI		NN4					
0.0		30.00		0.95		2		100		15		1					
A3		A4		B3		B4		S1		S2		S3		S4			
800.0000		200.0000		15.0000		15.0000		1.0000		20.0000		0.0		0.0			
T1		T2		SU1		SU2		U1		U2		L1		Q0			
150.0000		100.0000		0.0		0.0		8.0000		8.0000		1.0000		0.0			
R1		R2		D1		D2		R		W1		W2					
1.0000		1.0000		1.0000		1.0000		0.0		0.0		0.0					
ET1		ET2		GCL3		GCL4		U05		VV5							
0.0		0.0		0.0		0.0		40.0000		0.0							
SVI(J)																	
0.0	100.0000	200.0000	300.0000	400.0000	500.0000	600.0000	700.0000										
800.0000	900.0000	1000.0000	1100.0000	1200.0000	1300.0000	1400.0000											
BPI(J)																	
0.0	0.0200	0.0400	0.0600	0.0800	0.1000	0.1000	0.1000										
0.1000	0.1000	0.1000	0.0800	0.0600	0.0400	0.0200											
ALL A(J) = 0.																	
B(J)																	
0.0	0.0																
KO = 14 0.099638																	
X(10)	X(11)	X(12)	X(13)	X(14)	X(15)	X(16)											
0.0	0.0	0.0	0.0	0.099638	0.0	0.0											
X(100)	X(101)	X(102)	X(103)	X(104)	X(105)	X(106)											
0.0	0.0	0.0	0.0	0.0	0.0	0.0											
X(107)	X(108)	X(109)	X(110)	X(111)	X(112)												
0.0	0.0	0.0	0.0	0.0	0.0												

X(14) is for the same type of delivery as X(12) but the damage effect is based on impact (cookie-cutter damage function). Required inputs are B3, B4, and R1=R2=1.

Test Case 8

	8		23		1		1		1		0		1		0
N0	N1	N2	N3	N4	N5	N10	N11	N12	N13	N14	N15				
0	0	1	0	0	0	0	0	0	0	0	0				
D		DF		SP		N		NB		NVI		NN4			
C.0		0.0		0.95		1		30		15		1			
A3	A4		B3	B4		S1	S2	S3	S4						
300.0000	200.0000		1.0000	1.0000		1.0000	20.0000	0.0	0.0						
T1	T2		SU1	SU2		U1	U2	L1	Q0						
150.0000	100.0000		0.0	0.0		8.0000	8.0000	1.0000	0.0						
R1	R2		D1	D2		R	W1	W2							
7.5000	15.0000		1.0000	1.0000		0.0	0.0	0.0							
ET1	ET2		GCL3	GCL4		UU5	VV5								
0.0	0.0		0.0	0.0		40.0000	0.0								
SVI(J)															
0.0	100.0000	200.0000	300.0000	400.0000	500.0000	600.0000	700.0000								
800.0000	900.0000	1000.0000	1100.0000	1200.0000	1300.0000	1400.0000									
BPI(J)															
0.0	0.0200	0.0400	0.0600	0.0800	0.1000	0.1000	0.1000								
0.1000	0.1000	0.1000	0.0800	0.0600	0.0400	0.0200	0.1000								
ALL A(J) = 0.															
ALL B(J) = 0.															
X(1)	X(2)	X(3)	X(4)	X(5)											
0.0	0.028578	0.0	0.0	0.0											
X(10)	X(11)	X(12)	X(13)	X(14)	X(15)	X(30)									
0.0	0.0	0.0	0.0	0.0	0.0	0.0									
X(100)	X(101)	X(102)	X(103)	X(104)	X(105)	X(106)									
0.0	0.0	0.0	0.0	0.0	0.0	0.0									
X(107)	X(108)	X(109)	X(110)	X(111)	X(112)										
0.0	0.0	0.0	0.0	0.0	0.0										

X(2) is the same as X(12) except for the method of computation. It applies only to one dispenser and is valid for a small number of bomblets ($NB \leq 30$).

Test Case 9

	9		23		1		1		1		0		1		0	
N0	N1	N2	N3	N4	N5	N10	N11	N12	N13	N14	N15					
0	0	0	1	0	0	0	0	0	0	0	0					
	D		DF		SP		N		NB		NV1		NN4			
	0.0		0.0		0.95		1		30		15		1			
A3		A4		B3		B4		S1		S2		S3		S4		
800.0000		200.0000		1.0000		1.0000		1.0000		20.0000		0.0		0.0		
T1		T2		SU1		SU2		U1		U2		L1		Q0		
150.0000		100.0000		0.0		0.0		8.0000		6.0000		1.0000		0.0		
R1		R2		D1		D2		R		W1		W2				
7.5000		15.0000		1.0000		1.0000		0.0		0.0		0.0				
ET1		ET2		GCL3		GCL4		UUS		VVS						
0.0		0.0		0.0		0.0		40.0000		0.0						
SVI(J)																
0.0	100.0000	200.0000	300.0000	400.0000	500.0000	600.0000	700.0000									
800.0000	900.0000	1000.0000	1100.0000	1200.0000	1300.0000	1400.0000										
BPI(J)																
0.0	0.0200	0.0400	0.0600	0.0800	0.1000	0.1000	0.1000									
0.1000	0.1000	0.1000	0.0800	0.0600	0.0400	0.0200										
ALL A(J) = 0.																
ALL B(J) = 0.																
X(1)	X(2)	X(3)	X(4)	X(5)												
0.0	0.0	0.027904	0.0	0.0												
X(10)	X(11)	X(12)	X(13)	X(14)	X(15)	X(30)										
0.0	0.0	0.0	0.0	0.0	0.0	0.0										
X(100)	X(101)	X(102)	X(103)	X(104)	X(105)	X(106)										
0.0	0.0	0.0	0.0	0.0	0.0	0.0										
X(107)	X(108)	X(109)	X(110)	X(111)	X(112)											
0.0	0.0	0.0	0.0	0.0	0.0											

X(3) is the same as X(13) except for the method of computation. It applies only to one dispenser and is valid for a small number of bomblets ($NB \leq 30$).

Test Case 10

	10		23		1		1		1		0		1		0
NO	N1	N2	N3	N4	N5	N10	N11	N12	N13	N14	N15				
0	0	0	0	1	0	0	0	0	0	0	0				
D		DF		SP		N		NE		NVI		NN4			
0.0		0.0		0.95		1		30		15		1			
A3	A4		B3	B4		S1	S2	S3	S4						
800.0000	200.0000		15.0000	15.0000		1.0000	20.0000	0.0	0.0						
T1	T2		SU1	SU2		U1	U2	L1	Q0						
150.0000	100.0000		0.0	0.0		8.0000	8.0000	1.0000	0.0						
R1	R2		D1	D2		R	W1	W2							
1.0000	1.0000		1.0000	1.0000		0.0	0.0	0.0							
ET1	ET2		GCL3	GCL4		UU5	VV5								
0.0	0.0		0.0	0.0		40.0000	0.0								
SVI(J)															
0.0	100.0000	200.0000	300.0000	400.0000	500.0000	600.0000	700.0000								
800.0000	900.0000	1000.0000	1100.0000	1200.0000	1300.0000	1400.0000									
BPI(J)															
0.0	0.0200	0.0400	0.0600	0.0800	0.1000	0.1000	0.1000								
0.1000	0.1000	0.1000	0.0800	0.0600	0.0400	0.0200									
ALL A(J) = 0.															
ALL B(J) = 0.															
X(1)	X(2)	X(3)	X(4)	X(5)											
0.0	0.0	0.0	0.018550	0.0											
X(10)	X(11)	X(12)	X(13)	X(14)	X(15)	X(30)									
0.0	0.0	0.0	0.0	0.0	0.0	0.0									
X(100)	X(101)	X(102)	X(103)	X(104)	X(105)	X(106)									
0.0	0.0	0.0	0.0	0.0	0.0	0.0									
X(107)	X(108)	X(109)	X(110)	X(111)	X(112)										
0.0	0.0	0.0	0.0	0.0	0.0										

X(4) is the same as X(14) except for the method of computation. It applies only to one dispenser and is valid for a small number of bomblets ($NB \leq 30$).

5.9. TEST CASES FOR A RIPPLE OF DISPENSERS, RECTANGULAR BOMBLET PATTERN

The series X(103) through X(106) applies to ripples of dispensers released from the carrier, which provide a rectangular pattern of bomblet impact points. X(103) applies to the case in which the major damage effect is from fragmentation, while X(105) is for the cookie-cutter damage effect case. The ballistic dispersion is taken into account for these cases. If the MAE of an individual bomblet is appreciable (say, at least a tenth) of the total dispenser pattern area, the edge effects (effect of a bomblet outside the pattern area) become important. In these cases, X(104) is used for the fragmentation case and X(106) for the cookie-cutter case. In both the latter, the dispenser ballistic dispersion is ignored. $S1=S2=S3=S4=0$.

Test Case 11

11	23	1	1	0	0	1	1
----	----	---	---	---	---	---	---

NO	N100	N101	N102	N103	N104	N105	N106	N107	N108	N109	N110	N111	N112
1	0	0	0	1	0	0	0	0	0	0	0	0	0

D	DF	SP	N	NB	NVI	NN4
0.0	0.0	1.00	1	300	0	1

A3	A4	B3	B4	S1	S2	S3	S4
320.0000	320.0000	1.0000	1.0000	0.0	0.0	50.0000	30.0000

T1	T2	SU1	SU2	U1	U2	L1	Q0
200.0000	150.0000	0.0	0.0	8.0000	8.0000	0.0	0.0

R1	R2	D1	D2	R	W1	W2
7.5000	15.0000	1.0000	1.0000	0.9000	0.0	0.0

ET1	ET2	GCL3	GCL4	UUS	VV5
0.0	0.0	180.0000	150.0000	40.0000	0.0

ALL A(J) = 0.

ALL B(J) = 0.

SW1	S5	R3
0.0	38.7298	10.6066

K0 = 103 0.191514

X(10)	X(11)	X(12)	X(13)	X(14)	X(15)	X(30)
0.0	0.0	0.0	0.0	0.0	0.0	0.0

X(100)	X(101)	X(102)	X(103)	X(104)	X(105)	X(106)
0.0	0.0	0.0	0.191514	0.0	0.0	0.0

X(107)	X(108)	X(109)	X(110)	X(111)	X(112)
0.0	0.0	0.0	0.0	0.0	0.0

Test Case 12

12	23	1	1	0	0	1	1
----	----	---	---	---	---	---	---

NG	N100	N101	N102	N103	N104	N105	N106	N107	N108	N109	N110	N111	N112
1	0	0	0	0	0	1	0	0	0	0	0	0	0

D	DF	SP	N	NB	NVI	NN4
0.0	0.0	1.00	1	300	0	1

A3	A4	B3	B4	S1	S2	S3	S4
320.0000	320.0000	10.0000	10.0000	0.0	0.0	50.0000	30.0000

T1	T2	SU1	SU2	U1	U2	L1	Q0
200.0000	150.0000	0.0	0.0	8.0000	8.0000	0.0	0.0

R1	R2	U1	D2	R	H1	W2
1.0000	1.0000	1.0000	1.0000	0.9000	0.0	0.0

ET1	ET2	GCL3	GCL4	UU5	VV5
0.0	0.0	180.0000	150.0000	40.0000	0.0

ALL A(J) = 0.

ALL B(J) = 0.

SW1	S5	R3
0.0	38.7298	1.0000

KD = 105 0.074095

X(10)	X(11)	X(12)	X(13)	X(14)	X(15)	X(30)
0.0	0.0	0.0	0.0	0.0	0.0	0.0

X(100)	X(101)	X(102)	X(103)	X(104)	X(105)	X(106)
0.0	0.0	0.0	0.0	0.0	0.074095	0.0

X(107)	X(108)	X(109)	X(110)	X(111)	X(112)
0.0	0.0	0.0	0.0	0.0	0.0

Test Case 13

13	23	1	1	0	0	1	1	
----	----	---	---	---	---	---	---	--

NO	N100	N101	N102	N103	N104	N105	N106	N107	N108	N109	N110	N111	N112
1	0	0	0	0	1	0	0	0	0	0	0	0	0

D	DF	SP	N	NB	NV1	NN4
0.0	0.0	1.00	1	20	0	1

A3	A4	B3	B4	S1	S2	S3	S4
320.0000	320.0000	1.0000	1.0000	0.0	0.0	0.0	0.0

T1	T2	SU1	SU2	U1	U2	L1	Q0
200.0000	150.0000	0.0	0.0	8.0000	8.0000	0.0	0.0

R1	R2	D1	D2	R	W1	W2
30.0000	40.0000	1.0000	1.0000	0.9000	0.0	0.0

ET1	ET2	GCL3	GCL4	UUS	VV5
0.0	0.0	180.0000	150.0000	40.0000	0.0

ALL A(J) = 0.

ALL B(J) = 0.

SW1	S5	R3
0.0	0.1000	34.6410

K0 = 104 0.165298

X(10)	X(11)	X(12)	X(13)	X(14)	X(15)	X(30)
0.0	0.0	0.0	0.0	0.0	0.0	0.0

X(100)	X(101)	X(102)	X(103)	X(104)	X(105)	X(106)
0.0	0.0	0.0	0.0	0.165298	0.0	0.0

X(107)	X(108)	X(109)	X(110)	X(111)	X(112)
0.0	0.0	0.0	0.0	0.0	0.0

Test Case 14

14	23	1	1	0	0	1	1										
NO	N100	N101	N102	N103	N104	N105	N106	N107	N108	N109	N110	N111	N112				
1	0	0	0	0	0	0	1	0	0	0	0	0	0				
D	DF	SP				N		NB		NV1		MN4					
0.0	0.0	1.00				1		20		0		1					
A3	A4	b3				B4		S1		S2		S3		S4			
320.0000	320.0000	70.0000				70.0000		0.0		0.0		0.0		0.0			
T1	T2	SU1				SU2		U1		U2		L1		Q0			
200.0000	150.0000	0.0				0.0		8.0000		8.0000		0.0		0.0			
R1	R2	D1				D2		R		W1		W2					
1.0000	1.0000	1.0000				1.0000		0.9000		0.0		0.0					
ET1	ET2	GCL3				GCL4		UU5		VV5							
0.0	0.0	180.0000				150.0000		40.0000		0.0							

ALL A(J) = 0.

ALL B(J) = 0.

SW1	S5	R3
0.0	0.1000	1.0000

K0 = 106 0.199237

X(10)	X(11)	X(12)	X(13)	X(14)	X(15)	X(30)
0.0	0.0	0.0	0.0	0.0	0.0	0.0
X(100)	X(101)	X(102)	X(103)	X(104)	X(105)	X(106)
0.0	0.0	0.0	0.0	0.0	0.0	0.199237
X(107)	X(108)	X(109)	X(110)	X(111)	X(112)	
0.0	0.0	0.0	0.0	0.0	0.0	

5.10. TEST CASES FOR A RIPPLE OF DISPENSERS, ELLIPTIC BOMBLET PATTERN

X(107) through X(109) apply to ripples of dispensers released from the carrier, which provide an elliptic pattern of bomblet impact points. X(107) applies to the case in which the major damage effect is fragmentation, while X(109) applies to the cookie-cutter type damage effects. The ballistic dispersion is taken into account. If the MAE of an individual bomblet is appreciable compared with the total dispenser pattern area, the edge effects (effect of a bomblet outside the dispenser pattern area) become important. In this case, X(108) is used for a fragmentation effect. For the cookie-cutter effect, one may use an equivalent rectangular pattern (equal area and ratio) in X(106) above.

Test Case 15

```

      15      23      1      1      0      0      1      1
NO N100 N101 N102 N103 N104 N105 N106 N107 N108 N109 N110 N111 N112
1      0      0      0      0      0      0      0      1      0      0      0      0      0
      D      DF      SP      N      NB      NV1      NN4
    100.00      0.0      1.00      2      400      0      1
      A3      A4      B3      B4      S1      S2      S3      S4
    320.0000    320.0000    1.0000    1.0000    0.0      0.0    50.0000    30.0000
      T1      T2      SU1      SU2      U1      U2      L1      Q0
    200.0000    150.0000      0.0      0.0      8.0000    8.0000      0.0      0.0
      R1      R2      D1      D2      R      W1      W2
      7.0000     14.0000     1.0000     1.0000     0.9000     0.0      0.0
      ET1      ET2      GCL3      GCL4      UU5      VV5
    200.0000    150.0000      0.0      0.0     40.0000     0.0
      A(J)
    -50.0000     50.0000
      ALL B(J) = 0.
      SW1      S5      R3
      0.0      36.7298     9.8995
      KO = 107 0.305239
      X(10)      X(11)      X(12)      X(13)      X(14)      X(15)      X(30)
      0.0      0.0      0.0      0.0      0.0      0.0      0.0
      X(100)      X(101)      X(102)      X(103)      X(104)      X(105)      X(106)
      0.0      0.0      0.0      0.0      0.0      0.0      0.0
      X(107)      X(108)      X(109)      X(110)      X(111)      X(112)
      0.305239      0.0      0.0      0.0      0.0      0.0

```

Test Case 16

```

      16      23      1      1      0      0      1      1
NO N100 N101 N102 N103 N104 N105 N106 N107 N108 N109 N110 N111 N112
1      0      0      0      0      0      0      0      0      1      0      0      0      0

      D      DF      SP      N      NB      NVI      NN4
100.00      0.0      1.00      2      400      0      1

      A3      A4      B3      B4      S1      S2      S3      S4
320.0000      320.0000      1.0000      1.0000      0.0      0.0      0.0      0.0

      T1      T2      SU1      SU2      U1      U2      L1      Q0
200.0000      150.0000      0.0      0.0      8.0000      8.0000      0.0      0.0

      R1      R2      D1      D2      R      W1      W2
7.0000      14.0000      1.0000      1.0000      0.9000      0.0      0.0

      ET1      ET2      GCL3      GCL4      UU5      VV5
200.0000      150.0000      0.0      0.0      40.0000      0.0

      A(J)
-50.0000      50.0000

      ALL B(J) = 0.

      SW1      S5      R3
0.0      0.1000      9.8995

      KO = 108 0.311146

      X(10)      X(11)      X(12)      X(13)      X(14)      X(15)      X(30)
0.0      0.0      0.0      0.0      0.0      0.0      0.0

      X(100)      X(101)      X(102)      X(103)      X(104)      X(105)      X(106)
0.0      0.0      0.0      0.0      0.0      0.0      0.0

      X(107)      X(108)      X(109)      X(110)      X(111)      X(112)
0.0      0.311146      0.0      0.0      0.0      0.0

```

Test Case 17

```

      17      23      1      1      0      0      1      1
NO N100 N101 N102 N103 N104 N105 N106 N107 N108 N109 N110 N111 N112
1    0    0    0    0    0    0    0    C    0    1    0    C    0

      D      DF      SP      N      NB      NV1      NN4
100.00    0.0    1.00    2    400    0    1

      A3      A4      B3      B4      S1      S2      S3      S4
320.0000  320.0000  10.0000  10.0000  0.0    0.0    50.0000  30.0000

      T1      T2      SU1      SU2      U1      U2      L1      Q0
200.0000  150.0000  0.0    0.0    8.0000  8.0000  0.0    0.0

      R1      R2      D1      D2      R      W1      W2
7.0000    14.0000  1.0000  1.0000  0.9000  0.0    0.0

      ET1      ET2      GCL3      GCL4      UU5      VV5
200.0000  150.0000  0.0    0.0    40.0000  0.0

      A(J)
-50.0000  50.0000

      ALL B(J) = 0.

      SW1      S5      R3
0.0    38.7298  9.8995

      KN = 109 0.166712

      X(10)      X(11)      X(12)      X(13)      X(14)      X(15)      X(30)
0.0    0.0    0.0    0.0    0.0    0.0    0.0

      X(100)      X(101)      X(102)      X(103)      X(104)      X(105)      X(106)
0.0    0.0    0.0    0.0    0.0    0.0    0.0

      X(107)      X(108)      X(109)      X(110)      X(111)      X(112)
0.0    0.0    0.166712  0.0    0.0    0.0

```

5.11. TEST CASES FOR A RIPPLE OF DISPENSERS, ELLIPTIC ANNULUS BOMBLET PATTERN

X(110) through X(112) are the same, respectively, as X(107) through X(109) except that the dispenser pattern is an elliptic annulus. X(110) applies to the fragmentation case while X(112) applies to the cookie-cutter case. X(111) is the fragmentation case when the edge effects must be taken into account.

Test Case 18

```

      18      23      1      1      0      0      1      1
NO N100 N101 N102 N103 N104 N105 N106 N107 N108 N109 N110 N111 N112
1      0      0      0      0      0      0      0      0      0      0      0      0      1
      D      DF      SP      N      NB      NV1      NN4
    100.00      0.0      1.00      2      400      0      1
      A3      A4      B3      B4      S1      S2      S3      S4
    320.0000    320.0000    10.0000    10.0000    0.0      0.0    50.0000    30.0000
      T1      T2      SU1      SU2      U1      U2      L1      Q0
    200.0000    150.0000      0.0      0.0      8.0000    8.0000      0.0      0.0
      R1      R2      D1      D2      R      W1      W2
      7.0000     14.0000     .0000     1.0000     0.9000    100.0000    75.0000
      ET1      ET2      GCL3      GCL4      UUS      VV5
    200.0000    150.0000      0.0      0.0     40.0000      0.0
      A(J)
    -50.0000     50.0000
      ALL B(J) = 0.
      SW1      S5      R3
    86.6025    38.7298    9.8995
      KO = 112 0.153198
      X(10)      X(11)      X(12)      X(13)      X(14)      X(15)      X(30)
      0.0        0.0        0.0        0.0        0.0        0.0        0.0
      X(100)     X(101)     X(102)     X(103)     X(104)     X(105)     X(106)
      0.0        0.0        0.0        0.0        0.0        0.0        0.0
      X(107)     X(108)     X(109)     X(110)     X(111)     X(112)
      0.0        0.0        0.0        0.0        0.0        0.153198

```

Test Case 19

```

19      23      1      1      0      0      1      1
NO N100 N101 N102 N103 N104 N105 N106 N107 N108 N109 N110 N111 N112
1      0      0      0      0      0      0      0      0      0      0      1      0      0

      D      DF      SP      N      NB      NVI      NN4
100.00      0.0      1.00      2      400      0      1

      A3      A4      B3      B4      S1      S2      S3      S4
320.0000 320.0000 1.0000 1.0000 0.0 0.0 50.0000 30.0000

      T1      T2      SU1      SU2      U1      U2      L1      Q0
200.0000 150.0000 0.0 0.0 8.0000 8.0000 0.0 0.0

      R1      R2      D1      D2      R      W1      W2
7.0000 14.0000 1.0000 1.0000 0.9000 100.0000 75.0000

      ET1      ET2      GCL3      GCL4      UU5      VV5
200.0000 150.0000 0.0 0.0 40.0000 0.0

      A(J)
-50.0000 50.0000

      ALL B(J) = 0.

      SW1      S5      R3
86.6025 38.7298 9.8995

      KO = 110 0.268495

      X(10)      X(11)      X(12)      X(13)      X(14)      X(15)      X(16)
0.0 0.0 0.0 0.0 0.0 0.0 0.0

      X(100)      X(101)      X(102)      X(103)      X(104)      X(105)      X(106)
0.0 0.0 0.0 0.0 0.0 0.0 0.0

      X(107)      X(108)      X(109)      X(110)      X(111)      X(112)
0.0 0.0 0.0 0.268495 0.0 0.0

```

Test Case 20

```

      2C      23      1      1      0      0      1      1
NO N100 N101 N102 N103 N104 N105 N106 N107 N108 N109 N110 N111 N112
1      0      0      0      0      0      0      0      0      0      0      0      1      0

      D      DF      SP      N      NB      NV1      NM4
100.00      0.0      1.00      2      40      0      1

      A3      A4      B3      B4      S1      S2      S3      S4
320.0000 320.0000 1.0000 1.0000 0.0 0.0 0.0 0.0

      T1      T2      SU1      SU2      U1      U2      L1      Q0
200.0000 150.0000 0.0 0.0 8.0000 8.0000 0.0 0.0

      R1      R2      D1      D2      R      W1      W2
30.0000 40.0000 1.0000 1.0000 0.9000 100.0000 100.0000

      ET1      ET2      GCL3      GCL4      UU5      VV5
200.0000 200.0000 0.0 0.0 40.0000 0.0

      A(I)
-50.0000 50.0000

      ALL B(J) = 0.

      SW1      S5      R3
100.0000 0.1000 34.6410

      KN = 111 0.383570

      X(10)      X(11)      X(12)      X(13)      X(14)      X(15)      X(30)
0.0 0.0 0.0 0.0 0.0 0.0 0.0

      X(100)      X(101)      X(102)      X(103)      X(104)      X(105)      X(106)
0.0 0.0 0.0 0.0 0.0 0.0 0.0

      X(107)      X(108)      X(109)      X(110)      X(111)      X(112)
0.0 0.0 0.0 0.0 0.383570 0.0

```

5.12. TEST CASES FOR A RIPPLE OF DISPENSERS, CIRCULAR ANNULUS BOMBLET PATTERN

X(100) through X(102) are the same, respectively, as X(110) through X(112) except that the bomblet patterns are circular. Further, an approximation is made that is valid only when the ratio of the pattern radius to the ballistic error is sufficiently large. X(100) applies to the fragmentation case while X(102) applies to the cookie-cutter case. X(101) is the fragmentation case when edge effects must be taken into account.

Test Case 21

21	23	1	1	0	0	1	1
----	----	---	---	---	---	---	---

N0	N100	N101	N102	N103	N104	N105	N106	N107	N108	N109	N110	N111	N112
1	1	0	0	0	0	0	0	0	0	0	0	0	0

D	DF	SP	N	NB	NVI	NN4
30.00	0.0	1.00	6	200	0	1

A3	A4	B3	B4	S1	S2	S3	S4
400.0000	200.0000	1.0000	1.0000	0.0	0.0	30.0000	30.0000

T1	T2	SU1	SU2	U1	U2	L1	Q0
200.0000	150.0000	0.0	0.0	8.0000	8.0000	0.0	0.0

R1	R2	D1	D2	R	W1	W2
7.0000	14.0000	1.0000	1.0000	0.9000	100.0000	100.0000

ET1	ET2	GCL3	GCL4	U05	VV5
200.0000	200.0000	0.0	0.0	40.0000	0.0

A(JP)

-75.0000	-45.0000	-15.0000	15.0000	45.0000	75.0000
----------	----------	----------	---------	---------	---------

ALL B() = 0.

SW1	S5	R3
100.0000	30.0000	9.8995

K0 = 100 0.419609

X(10)	X(11)	X(12)	X(13)	X(14)	X(15)	X(30)
0.0	0.0	0.0	0.0	0.0	0.0	0.0

X(100)	X(101)	X(102)	X(103)	X(104)	X(105)	X(106)
0.419609	0.0	0.0	0.0	0.0	0.0	0.0

X(107)	X(108)	X(109)	X(110)	X(111)	X(112)
0.0	0.0	0.0	0.0	0.0	0.0

Test Case 22

```

      22      23      1      1      0      0      1      1
NO N100 N101 N102 N103 N104 N105 N106 N107 N108 N109 N110 N111 N112
1      0      0      1      0      0      0      0      0      0      0      0      0      0

      D      DF      SP      N      NB      NVI      NN4
      30.00      C.0      1.00      6      200      0      1

      A3      A4      B3      B4      S1      S2      S3      S4
      400.0000      200.0000      10.0000      10.0000      0.0      0.0      30.0000      30.0000

      T1      T2      SU1      SU2      U1      U2      L1      Q0
      200.0000      150.0000      0.0      0.0      8.0000      8.0000      0.0      0.0

      R1      R2      D1      D2      R      W1      W2
      1.0000      1.0000      1.0000      1.0000      0.9000      100.0000      100.0000

      ET1      ET2      GCL3      GCL4      UU5      VV5
      200.0000      200.0000      0.0      0.0      40.0000      0.0

      A(J);
      -75.0000      -45.0000      -15.0000      15.0000      45.0000      75.0000

      ALL B(J) = 0.

      SW1      S5      R3
      100.0000      30.0000      1.0000

      KO = 102 0.226833

      X(10)      X(11)      X(12)      X(13)      X(14)      X(15)      X(30)
      0.0      0.0      0.0      0.0      0.0      0.0      0.0

      X(100)      X(101)      X(102)      X(103)      X(104)      X(105)      X(106)
      0.0      0.0      0.226833      0.0      0.0      0.0      0.0

      X(107)      X(108)      X(109)      X(110)      X(111)      X(112)
      0.0      0.0      0.0      0.0      0.0      0.0

```

Test Case 23

```

      23      23      1      1      0      0      1      1
NO N100 N101 N102 N103 N104 N105 N106 N107 N108 N109 N110 N111 N112
1      0      1      0      0      0      0      0      0      0      0      0      0      0
      D      DF      SP      N      NB      NVI      NN4
      30.00      0.0      1.00      6      20      0      1
      A3      A4      B3      B4      S1      S2      S3      S4
400.0000 200.0000 1.0000 1.0000 0.0 0.0 0.0 0.0
      T1      T2      SU1      SU2      U1      U2      L1      Q0
200.0000 150.0000 0.0 0.0 8.0000 8.0000 0.0 0.0
      R1      R2      D1      D2      R      W1      W2
15.0000 19.0000 1.0000 1.0000 0.9000 100.0000 100.0000
      ET1      ET2      GCL3      GCL4      UU5      VV5
200.0000 200.0000 0.0 0.0 40.0000 0.0
      A(3)
-75.0000 -45.0000 -15.0000 15.0000 45.0000 75.0000
      ALL B(J) = 0.
      SW1      S5      R3
100.0000 0.1000 16.8819
      KO = 101 0.211221
      X(10)      X(11)      X(12)      X(13)      X(14)      X(15)      X(30)
      0.0      0.0      0.0      0.0      0.0      0.0      0.0
      X(100)      X(101)      X(102)      X(103)      X(104)      X(105)      X(106)
      0.0      0.211221 0.0 0.0 0.0 0.0 0.0
      X(107)      X(108)      X(109)      X(110)      X(111)      X(112)
      0.0      0.0      0.0      0.0      0.0      0.0

```

Appendix A

TRAIN DISTRIBUTION

Consider a train of N weapons delivered in a pattern. The ballistic centers of impact forming the pattern are at points $(a_i, b_i), i=1, 2, \dots, N$. The origin is selected such that $\sum a_i = \sum b_i = 0$. Each weapon is subject independently to a ballistic error whose distribution B is assumed to be of the form

$$B(X, Y) = B_1(X)B_2(Y),$$

where $B_1(X)$ is the distribution of ballistic errors in range (x) and $B_2(Y)$ is the distribution in deflection (y). Let $b_1(x)$ and $b_2(y)$ be the corresponding density function, i.e.,

$$B_1(X) = \int_{-\infty}^X b_1(x) dx.$$

We shall assume that the distributions have mean zero and standard deviations s_x and s_y , respectively.

The form of the functions b_1 and b_2 , the values of s_x and s_y , and the pattern values (a_i, b_i) must be determined through some testing procedure. If the data are sufficiently complete (individual data for each weapon in a train), the individual distributions B_1 and B_2 can be used as required in the methods of Ref. [1]. In general, B_1 and B_2 can be arbitrary distributions. For normal train bombing, the values of a_i and b_i can be determined from the delivery conditions and intervalometer settings. For the methodology of Sec. 3, it is necessary that B_1 and B_2 be gaussian in form, so that only s_x and s_y are needed. The spacings a_i and b_i are, of course, necessary as further inputs. However, for some of the cluster-type weapons, the complete data are not available. Although the complete data are available in other cases, the simplification considered here is usually sufficient.

Let us assume that the test data in range are only sufficient to estimate the expected number of weapons landing in a set of range intervals. Specifically, for a

set of K intervals between the points $X_0, X_1, X_2, \dots, X_K$, we obtain the corresponding number of weapons $N_1, N_2, \dots, N_K$, where the total number is $N = \sum_{i=1}^K N_i$. We form the percentage distribution within intervals

$$w_i = \frac{N_i}{N},$$

and the sample density

$$(A.1) \quad p_i = \frac{w_i}{X_i - X_{i-1}}, \quad i=1, 2, \dots, K.$$

We shall view the p_i as an estimate of the probability density function $\rho(x)$ of a single weapon, where $P(X) = \int_{-\infty}^X \rho(x) dx$ is the probability that an individual weapon landed $\leq X$. Basically, we are using as an approximate model one in which each weapon is randomly drawn from the positions in the train. We therefore assume that

$$(A.2) \quad \rho(X) = p_i, \quad X_{i-1} \leq X \leq X_i,$$

$$P(X_j) = \sum_{i=1}^j p_i, \quad j=1, 2, 3, \dots, K.$$

The first moment $\bar{X}$ and variance S^2 of the distribution $P(X)$ are given by

$$(A.3) \quad \bar{X} = \int z \rho(z) dz = \sum_{i=1}^K p_i \left(\frac{X_{i-1} + X_i}{2} \right),$$

$$S^2 = \int z^2 \rho(z) dz = \sum_{i=1}^K w_i \left(\frac{X_{i-1}^2 + X_i X_{i-1} + X_i^2}{3} \right) - \bar{X}^2.$$

The above model, in effect, assumes that each weapon in the pattern is aimed at the center of the pattern. For symmetric patterns, the center is estimated as the mean point $\bar{X}$. However, the data are often quite irregular, so that some smooth-

ing must be done. We have chosen to use the quartiles $X_{.25}$ and $X_{.75}$ as characteristics to be used in the fitting process, since their use introduces a type of smoothing. The center X_c then is defined as the midpoint between the quartiles

$$(A.4) \quad X_c = \frac{X_{.25} + X_{.75}}{2}.$$

For symmetric patterns, $X_c = \bar{X}$. Thus $X_{.25}$ and $X_{.75}$ are defined by the equations

$$P(X_{.25}) = .25,$$

$$P(X_{.75}) = .75.$$

Define I and J by the conditions

$$(A.5) \quad P(X_I) < .25, \quad P(X_{I+1}) > .25,$$

$$P(X_J) < .75, \quad P(X_{J+1}) > .75.$$

Then $X_{.25}$ and $X_{.75}$ are given by the equations

$$(A.6) \quad X_{.25} = X_I + \frac{(.25 - P(X_I))(X_{I+1} - X_I)}{P(X_{I+1}) - P(X_I)},$$

$$X_{.75} = X_J + \frac{(.75 - P(X_J))(X_{J+1} - X_J)}{P(X_{J+1}) - P(X_J)},$$

where $P(X_j)$ is given in (A.2).

Finally, making the change in variable $x = X - X_c$ to center the coordinate system on the pattern center and using (A.4), we obtain the center x_c and the quartiles $x_{.25}$ and $x_{.75}$:

$$x_c = 0,$$

$$(A.7) \quad x_{.75} = X_{.75} - X_c = \frac{X_{.75} - X_{.25}}{2},$$

$$x_{.25} = X_{.25} - X_c = \left(\frac{X_{.25} - X_{.75}}{2} \right) = -x_{.75},$$

where $X_{.25}$ and $X_{.75}$ are given in (A.6). Thus, we have characterized the data by three characteristics: x_c , S , and $x_{.75}$.

In this case, we do not have enough information to determine the form of the ballistic distribution $B_1(X)$ or of the pattern a_i . Let us assume that the form of $B_1(X)$ is gaussian with variance s_x^2 and that the a_i are known. Then, for a delivery of N weapons, the expected number falling short of X is

$$\sum_{i=1}^N B_1\left(\frac{x-a_i}{s_x}\right) = \sum_{i=1}^N G\left(\frac{x-a_i}{s_x}\right),$$

and the percentage of weapons expected to fall short of X is

$$Q(x) = \frac{1}{N} \sum_{i=1}^N G\left(\frac{x-a_i}{s_x}\right).$$

The function $P(x)$ is an approximation to this function, $Q(x)$. We will make the assumption that the N weapons are evenly spaced, i.e.,

$$(A.8) \quad a_i = (2i - N - 1)d,$$

where $2d$ is the spacing. We have characterized F by three parameters: $x_c = 0$, variance S^2 , and upper quartile $x_{.75}$. We will determine values for d and s_x under the restriction that the distributions $Q(x)$ and $P(x)$ have the same variances and quartiles. Our conditions are thus

$$S^2 = s_x^2 + \frac{d^2}{N} \sum_{i=1}^N (2i - N - 1)^2,$$

$$(A.9) \quad \frac{1}{N} \sum_{i=1}^N G\left(\frac{x_{.75} - a_i}{s_x}\right) = .75,$$

$$\frac{1}{N} \sum_{i=1}^N G\left(\frac{x_{.25} - a_i}{s_x}\right) = .25.$$

Since $x_{.25} = -x_{.75}$, the latter two are equivalent. Subtracting and combining, we obtain the equivalent condition

$$(A.10) \quad \frac{1}{N} \sum_{i=1}^N \left[G\left(\frac{x_{.75} - a_i}{s_x}\right) - G\left(\frac{-x_{.75} - a_i}{s_x}\right) \right] = \frac{1}{N} \sum_{i=1}^N f\left(\frac{a_i}{s_x}, \frac{x_{.75}}{s_x}\right) = .5,$$

where $f(x, y)$ is defined as

$$f(x, y) = G(x+y) - G(x-y),$$

and thus $f(-x, y) = f(x, y)$. In summary, our conditions are

$$s^2 = s_x^2 + \frac{d^2}{N} \sum_{i=1}^N (2i - N - 1)^2,$$

$$(A.11) \quad \frac{1}{N} \sum_{i=1}^N f\left(\frac{a_i}{s_x}, \frac{x_{.75}}{s_x}\right) = .5,$$

$$x_c = 0.$$

i.e., d and s_x are determined from these conditions.

Appendix B

STICK DISTRIBUTION

There are occasions when the ballistic dispersion of a cluster of weapons may be approximated by what will be called the "stick distribution." Consider the problem of throwing a stick at a line perpendicular to the stick in an attempt to cover the line. Assume a stick of length $2L$, with the center aimed at the point 0. Assume that the error in throwing the stick is governed by a gaussian distribution with variance s^2 . The probability of covering the offset point x is given by

$$\text{Prob}(\text{point at } x \text{ covered}) = \int_{x-L}^{x+L} g\left(\frac{y}{s}\right) \frac{dy}{s},$$

where $g(x) = \exp(-x^2/2)/\sqrt{2\pi}$. Define the function $h(x, L)$ as

$$(B.1) \quad h(x, L) = \frac{1}{2L} \int_{x-L}^{x+L} g(y) dy.$$

The function $h(x, L)$ can be viewed as a probability density function, since

$$(B.2) \quad \int_{-\infty}^{\infty} h(x, L) dx = 1.$$

We will call the distribution function $H(x, L)$ the stick distribution, i.e.,

$$(B.3) \quad H(X, L) = \int_{-\infty}^X h(y, L) dy = \int_{-\infty}^X \frac{1}{2L} \int_{y-L}^{y+L} g(z) dz dy.$$

More generally, we have

$$(B.4) \quad H\left(\frac{X}{s}, \frac{L}{s}\right) = \int_{-\infty}^{X/s} h\left(y, \frac{L}{s}\right) dy = \int_{-\infty}^X h\left(\frac{y}{s}, \frac{L}{s}\right) \frac{dy}{s}.$$

The first moment of $H(X/s, L/s)$ is

$$(B.5) \quad \int_{-\infty}^{\infty} y h\left(\frac{y}{s}, \frac{L}{s}\right) \frac{dy}{s} = 0,$$

and the variance is

$$(B.6) \quad \int_{-\infty}^{\infty} y^2 h\left(\frac{y}{s}, \frac{L}{s}\right) \frac{y^2}{s} dy = s^2 + \frac{L^2}{3}.$$

Note that we can express H in terms of G and g . Using Lemma 3 of App. D and (B.3), we obtain

$$(B.7) \quad H(X, L) = \int_{-\infty}^X \frac{1}{2L} [G(y+L) - G(y-L)] dy$$

$$= \frac{1}{2L} [(X+L)G(X+L) - (X-L)G(X-L) + g(X+L) - g(X-L)].$$

The stick distribution above is often a good approximation to the train distribution $P(x)$ discussed in the preceding section. As in App. A, assume $P(x)$ to be characterized by the three parameters $x_0 = 0$, variance S^2 , and upper quartile $x_{.75}$, ($x_{.25} = -x_{.75}$). Requiring that P and its approximation H have the same center coordinate, variances, and quartiles, we obtain the conditions

$$S^2 = s^2 + \frac{L^2}{3},$$

$$(B.8) \quad H\left(\frac{x_{.75}}{s}, \frac{L}{s}\right) = .75,$$

$$H\left(\frac{x_{.25}}{s}, \frac{L}{s}\right) = .25,$$

where S and $x_{.75}$ are given in (A.3) and (A.7). Subtracting the last two equations in (B.8) and using the relation $x_{.25} = -x_{.75}$, we obtain an alternate condition

$$H\left(\frac{x_{.75}}{s}, \frac{L}{s}\right) - H\left(\frac{-x_{.75}}{s}, \frac{L}{s}\right) = .5.$$

Thus, we may express the conditions (B.8) as

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$$x_0 = 0,$$

$$S = s^2 + \frac{L^2}{3},$$

$$F\left(0, \frac{x}{s}, \frac{L}{s}\right) = .5,$$

where the function $F(x, y, L)$ is defined as in (4.12) as

$$F(x, y, L) = H(x+y, L) - H(x-y, L).$$

Appendix C

THE ELLIPTIC COVERAGE FUNCTION

The elliptic coverage function $P(R_1, R_2; a, b)$ is defined as the integral of a circular gaussian distribution of unit variance over the area enclosed by an offset ellipse with center at (a, b) and axes R_1, R_2 in the x and y directions. Thus, we have

$$(C.1) \quad P(R_1, R_2; a, b) \equiv \iint_{A_1} g(x)g(y) dx dy,$$

where

$$A_1: \left(\frac{x-a}{R_1}\right)^2 + \left(\frac{y-b}{R_2}\right)^2 \leq 1$$

and

$$g(x) = \frac{\exp(-x^2/2)}{\sqrt{2\pi}}.$$

Making the change of variable $\xi = x\sigma_x$, $\eta = y\sigma_y$, we obtain

$$(C.2) \quad P\left(\frac{R_1}{\sigma_x}, \frac{R_2}{\sigma_y}; \frac{a}{\sigma_x}, \frac{b}{\sigma_y}\right) = \iint_{A_1} g\left(\frac{\xi}{\sigma_x}\right) g\left(\frac{\eta}{\sigma_y}\right) \frac{d\xi d\eta}{\sigma_x \sigma_y},$$

i.e., an equivalent definition is the integral of a bivariate gaussian over an offset ellipse. Similarly, we obtain

$$(C.3) \quad P\left(\frac{R}{\sigma_x}, \frac{R}{\sigma_y}; \frac{a}{\sigma_x}, \frac{b}{\sigma_y}\right) = \iint_{A_2} g\left(\frac{\xi}{\sigma_x}\right) g\left(\frac{\eta}{\sigma_y}\right) \frac{d\xi d\eta}{\sigma_x \sigma_y},$$

where

$$A_2: (x-a)^2 + (y-b)^2 \leq R^2.$$

Another definition is the integral of a bivariate gaussian over an offset circle.

Using (C.1), if $R_1 = R_2 = R$ the elliptic coverage function becomes the circular coverage function $P(R, r)$, i.e.,

$$P(R, R; a, b) = P(R, \sqrt{a^2 + b^2}).$$

From Ref. 1, alternate expressions for $P(R, r)$ are

$$(C.4) \quad \begin{aligned} P(R, r) &= \int_0^R \frac{\exp(-(r^2 + z^2))}{2} I_0(zr) dz \\ &= R \int_r^\infty \frac{\exp(-(R^2 + z^2))}{2} I_1(Rz) dz, \end{aligned}$$

where $I_0(z)$ and $I_1(z)$ are Bessel functions of the first kind of imaginary argument.

If $a=b=0$, the elliptic coverage function is expressible in terms of the circular coverage function (Ref. 7). Thus, from Ref. 2, if $R_1 > R_2$, we have

$$(C.5) \quad P(R_1, R_2; 0, 0) = P\left(\frac{R_1 + R_2}{2}, \frac{R_1 - R_2}{2}\right) - P\left(\frac{R_1 - R_2}{2}, \frac{R_1 + R_2}{2}\right).$$

For $a \neq b$, $R_1 \neq R_2$, the elliptic coverage function has been evaluated by various computational schemes. Included here is a computational scheme well adapted for high-speed computers based on an unpublished paper by Oliver Gross at Rand. Using the definition in (C.1) and converting to polar coordinates, we obtain

$$(C.6) \quad P(R_1, R_2; a, b) = \iint_{A_3} \frac{\exp(-\rho^2)}{2} \frac{d\rho d\theta}{2\pi},$$

where

$$(C.7) \quad A_3: \left(\frac{\rho \cos\theta - a}{R_1}\right)^2 + \left(\frac{\rho \sin\theta - b}{R_2}\right)^2 \leq 1.$$

Consider first the case where the origin is inside the ellipse in (C.7). Then, we obtain

$$(C.8) \quad \begin{aligned} P(R_1, R_2; a, b) &= \int_0^{2\pi} \int_0^{r(\theta)} \frac{\exp(-\rho^2)}{2} \rho d\rho d\theta \\ &= \frac{1}{2\pi} \int_0^{2\pi} \left[1 - \frac{\exp(-r^2(\theta))}{2}\right] d\theta, \end{aligned}$$

where $r(\theta)$ is the radial distance at angle θ to the ellipse E_1 obtained from (C.7) as

$$(C.9) \quad E_1: \left[\frac{r(\theta) \cos \theta - a}{R_1} \right]^2 + \left[\frac{r(\theta) \sin \theta - b}{R_2} \right]^2 = 1,$$

i.e.,

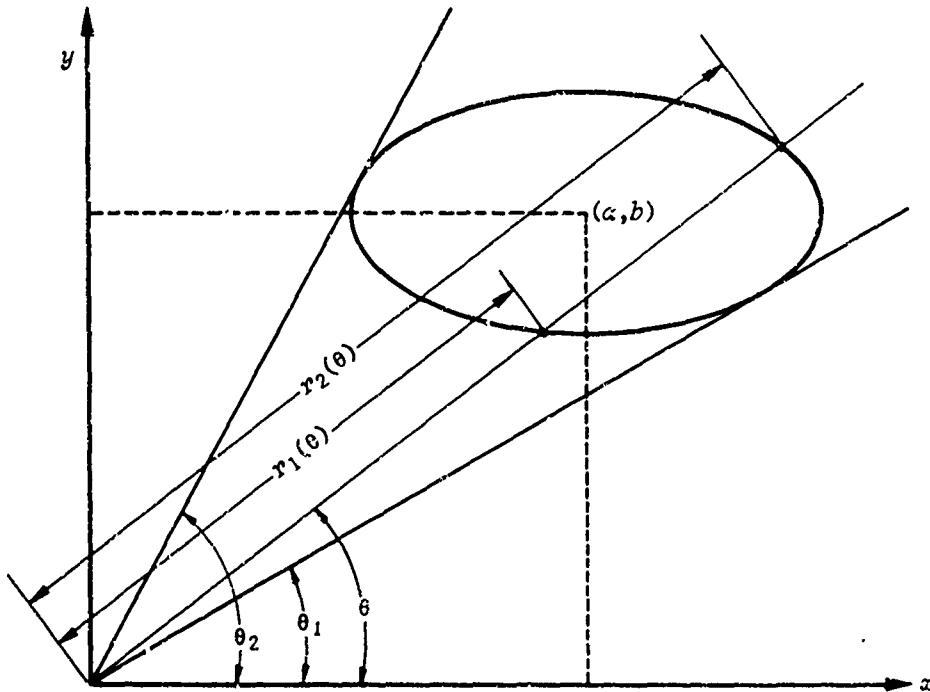
$$(C.10) \quad P(R_1, R_2; a, b) = \frac{1}{2\pi} \int_{E_1} \left[1 - \frac{\exp(-r^2(\theta))}{2} \right] d\theta,$$

where the integration is around the boundary of the ellipse E_1 given by (C.10).

If the origin is not inside the ellipse in (C.7), $P(R_1, R_2; a, b)$ is given by

$$(C.11) \quad P(R_1, R_2; a, b) = \int_{\theta_1}^{\theta_2} \int_0^{r_2(\theta)} \frac{\exp(-\rho^2)}{2} \frac{d\rho d\theta}{2\pi} - \int_{\theta_1}^{\theta_2} \int_0^{r_1(\theta)} \rho \exp\left(-\frac{\rho^2}{2}\right) \frac{d\rho d\theta}{2\pi},$$

where θ_1 and θ_2 are the angular extremities of the offset ellipse, $r_2(\theta)$ is the larger radial distance to the ellipse at angle θ , and $r_1(\theta)$ is the smaller radial distance, as shown in the diagram below.



Thus, when we integrate, we obtain

$$(C.12) \quad P(R_1, R_2; a, b) = \int_{\theta_1}^{\theta_2} \left[1 - \exp\left(-\frac{r_2^2(\theta)}{2}\right) \right] \frac{d\theta}{2\pi} - \int_{\theta_1}^{\theta_2} \left[1 - \exp\left(-\frac{r_1^2(\theta)}{2}\right) \right] \frac{d\theta}{2\pi},$$

where $r_2(\theta)$ is the radial distance to the ellipse E_1 of (C.9) on the far side from θ_1 to θ_2 , and $r_1(\theta)$ is the radial distance to E_1 on the rear side from θ_1 to θ_2 .

Thus, we have

$$(C.13) \quad P(R_1, R_2; a, b) = \int_{\theta_1}^{\theta_2} \left[1 - \exp\left(-\frac{r_2^2(\theta)}{2}\right) \right] \frac{d\theta}{2\pi} + \int_{\theta_2}^{\theta_1} \left[1 - \exp\left(-\frac{r_1^2(\theta)}{2}\right) \right] \frac{d\theta}{2\pi} \\ = \frac{1}{2\pi} \int_{E_1} \left[1 - \exp\left(-\frac{r^2(\theta)}{2}\right) \right] d\theta,$$

where as in (C.10), the integration is around the boundaries of the ellipse E_1 in (C.10). Thus, (C.10) holds for both cases. Let us make the transformation on θ :

$$(C.14) \quad \begin{aligned} r \cos\theta &= R_1 \cos\varphi + a, \\ r \sin\theta &= R_2 \sin\varphi + b, \end{aligned}$$

so that

$$(C.15) \quad \begin{aligned} r^2 &= (R_1 \cos\varphi + a)^2 + (R_2 \sin\varphi + b)^2, \\ \tan\theta &= \frac{R_2 \sin\varphi + b}{R_1 \cos\varphi + a}, \\ d\theta &= \frac{R_1 R_2 + a R_2 \cos\varphi + b R_1 \sin\varphi}{r^2} d\varphi. \end{aligned}$$

Under this transformation, the offset ellipse E_1 changes into the unit circle with center at the origin. Thus, in (C.10) φ goes from 0 to 2π , so

$$(C.16) \quad P(R_1, R_2; a, b) = \frac{1}{2\pi} \int_0^{2\pi} \frac{1 - \exp(-r^2/2)}{r^2} (R_1 R_2 + a R_2 \cos\varphi + b R_1 \sin\varphi) d\varphi,$$

where r is given in (C.15). Thus, the problem of evaluating $P(R_1, R_2; a, b)$ has been reduced to a simple quadrature over the interval $(0, 2\pi)$.

Using a trapezoidal type of numerical integration with N intervals over the range $(0, 2\pi)$, i.e., intervals of length $2\pi/N$ and the midpoint of the first interval at φ_0 , we obtain

$$(C.17) \quad P(R_1, R_2; a, b) = \frac{1}{N} \sum_{j=1}^N Y \left\{ r^2 \left[\varphi_0 + \frac{2\pi(j-1)}{N} \right] \right\} W \left[\varphi_0 + \frac{2\pi(j-1)}{N} \right],$$

where

$$(C.18) \quad \begin{aligned} r^2(\varphi) &= (R_1 \cos \varphi + a)^2 + (R_2 \sin \varphi + b)^2, \\ Y(\rho) &= \begin{cases} \frac{1 - \exp(-\rho/2)}{\rho} & \text{if } \rho \geq .001, \\ \frac{1}{2} \rho + \frac{\rho^2}{48} & \text{if } \rho < .001, \end{cases} \end{aligned}$$

$$W(\varphi) = R_1 R_2 + a R_2 \cos \varphi + b R_1 \sin \varphi.$$

The series approximation for $Y(\rho)$ for $\rho < .001$ avoids roundoff errors for ρ small. The accuracy is, of course, a function of the number of steps N . For the ratio of R_1/R_2 reasonable, i.e., between $\frac{1}{4}$ and 4, a value of $N=80$ seems to be sufficient. For extreme cases, i.e., a ratio of 1000 to 1, N must be taken much larger.

Appendix D

USEFUL EXPRESSIONS

We will be using several relations involving the gaussian or normal distribution $G(X)$ and its density function $g(X) = G'(X)$, i.e.,

$$(D.1) \quad G(X) = \int_{-\infty}^X g(y) dy = \int_{-\infty}^X \frac{\exp(-y^2/2)}{\sqrt{2\pi}} dy,$$

$$(D.2) \quad g(y) = \frac{\exp(-y^2/2)}{\sqrt{2\pi}}.$$

For completeness, we include these relations as lemmas, together with short derivations.

Lemma 1. Let A , s , and t be real, $s > 0, t > 0$. Then,

$$(D.3) \quad I_1 = \int_{-\infty}^{\infty} G\left(\frac{x+A}{s}\right) g\left(\frac{x}{t}\right) \frac{dx}{t} = G\left(\frac{A}{\sqrt{s^2+t^2}}\right),$$

$$(D.4) \quad I_2 = \int_{-\infty}^{\infty} g\left(\frac{x+A}{s}\right) g\left(\frac{x}{t}\right) \frac{dx}{st} = \frac{g(A/\sqrt{s^2+t^2})}{\sqrt{s^2+t^2}}.$$

Proof. Using (D.1), (D.2), and a change in variable, we obtain

$$I_1 = \int_{-\infty}^{\infty} \int_{-\infty}^A \exp\left(-\frac{1}{2} \left[\frac{(x+y)^2}{s^2} + \frac{x^2}{t^2} \right]\right) \frac{dy dx}{2\pi st}.$$

Changing the order of integration, expanding, and integrating, we obtain

$$I_1 = \int_{-\infty}^A \exp\left(-\frac{y^2}{2(s^2+t^2)}\right) \frac{dy}{\sqrt{(2\pi)(s^2+t^2)}} = G\left(\frac{A}{\sqrt{s^2+t^2}}\right).$$

Similarly, we have

$$I_2 = \int_{-\infty}^{\infty} \exp\left(-\frac{1}{2} \left[\frac{(x+A)^2}{s^2} + \frac{x^2}{t^2} \right]\right) \frac{dx}{2\pi st} = \frac{\exp(-A^2/2(s^2+t^2))}{\sqrt{2\pi(s^2+t^2)}} = g\left(\frac{A/\sqrt{s^2+t^2}}{\sqrt{s^2+t^2}}\right).$$

We note that (D.4) can be expressed in a slightly different form:

$$\int_{-\infty}^{\infty} \exp\left(-\frac{(x+A)^2}{2s^2}\right) g(x, t) \frac{dx}{t} = \frac{s}{\sqrt{s^2+t^2}} \exp\left(-\frac{A^2}{2(s^2+t^2)}\right).$$

Lemma 2. Let A, B , and s be real, $A \geq B$ and $s > 0$. Then,

$$(D.5) \quad I_3 = \int_{-\infty}^{\infty} \left[G\left(\frac{x+A}{s}\right) - G\left(\frac{x+B}{s}\right) \right] dx = A - B.$$

Proof. Using (D.1) and (D.2) in the expression for I_1 , and making a change in variable, we obtain

$$I_3 = \int_{-\infty}^{\infty} \int_B^A \exp\left(-\frac{(x+y)^2}{2s^2}\right) \frac{dy dx}{2\pi s}.$$

Changing the order of integration, we have

$$I_3 = \int_B^A dy = A - B.$$

Lemma 3. Let A, B, a, b , and t be real, $A \geq B$, $a \geq b$, $t > 0$. Then,

$$(D.6) \quad \int_{-\infty}^A G(x) dx = AG(A) + g(A),$$

$$(D.7) \quad \begin{aligned} I_4 &= \int_B^A \left[G\left(\frac{x+a}{t}\right) - G\left(\frac{x+b}{t}\right) \right] dx \\ &= (A+a)G\left(\frac{A+a}{t}\right) + (B+b)G\left(\frac{B+b}{t}\right) \\ &\quad - (A+b)G\left(\frac{A+b}{t}\right) - (B+a)G\left(\frac{B+a}{t}\right) \\ &\quad + t \left[g\left(\frac{A+a}{t}\right) + g\left(\frac{B+b}{t}\right) - g\left(\frac{B+a}{t}\right) - g\left(\frac{A+b}{t}\right) \right]. \end{aligned}$$

Proof. Integrating by parts, we obtain (D.6) immediately:

$$\int_{-\infty}^A G(x) dx = AG(A) + g(A).$$

Using (D.6), we obtain

$$(D.8) \quad \int_{-\infty}^A G\left(\frac{x+a}{t}\right) dx = (A+a)G\left(\frac{A+a}{t}\right) + tg(A+a).$$

We may express I_4 in the form

$$(D.9) \quad I_4 = \int_{-\infty}^A \left[G\left(\frac{x+a}{t}\right) dx - \int_{-\infty}^A G\left(\frac{x+b}{t}\right) dx \right] + \int_{-\infty}^B \left[G\left(\frac{x+b}{t}\right) dx - \int_{-\infty}^B G\left(\frac{x+a}{t}\right) dx \right],$$

and using (D.8) in each of the four integrals in (D.9), we obtain (D.7).

Lemma 4. Let A , L , and t be real and positive. Then,

$$(D.10) \quad I_5 = \int_{-A}^A \left[G\left(\frac{x+L}{t}\right) - G\left(\frac{x-L}{t}\right) \right] dx \\ = 2 \left[AH\left(\frac{A}{t}, \frac{L}{t}\right) + LH\left(\frac{L}{t}, \frac{A}{t}\right) \right] + \left[g\left(\frac{A+L}{t}\right) + g\left(\frac{A-L}{t}\right) \right].$$

Proof. In (D.7) of Lemma 3, let $A=A$, $B=-A$, $a=L$, $b=-L$. Then (D.10) follows, since $g(x)=g(-x)$.

Lemma 5. Let u and a_i be real and s_i and t be real and positive for $n=1, 2, \dots, N$. Then,

$$(D.11) \quad I_6 = \int_{-\infty}^{\infty} \exp\left(-\sum_{i=1}^n \frac{(u-a_i-x)^2}{2s_i^2}\right) \exp\left(-\frac{x^2}{2t^2} - \frac{dx}{\sqrt{2\pi}t}\right) \\ = \frac{s}{\sqrt{s^2+t^2}} \exp\left(-\frac{a^2-\bar{a}^2}{2s^2}\right) \exp\left(-\frac{(u-a)^2}{2(s^2+t^2)}\right),$$

where

$$\frac{1}{s^2} = \sum_{i=1}^n \frac{1}{s_i^2},$$

$$\bar{a} = s^2 \sum_{i=1}^n \frac{a_i}{s_i^2},$$

$$\overline{a^2} = s^2 \sum_{i=1}^n \frac{a_i^2}{s_i^2}.$$

Proof. When we use the definitions of s , $\bar{a}$, and $\overline{a^2}$ above, the first exponential under the integral in the definition of I_6 becomes

$$\exp\left(-\frac{x^2 - 2x(u - \bar{a}) + u^2 - 2u\bar{a} + \overline{a^2}}{2s^2}\right) = \exp\left(-\frac{[x - (u - \bar{a})]^2}{2s^2}\right) \exp\left(-\frac{\overline{a^2} - \bar{a}^2}{2s^2}\right).$$

Thus, I_6 becomes

$$(D.12) \quad I_6 = \exp\left(-\frac{\overline{a^2} - \bar{a}^2}{2s^2}\right) \int_{-\infty}^{\infty} \exp\left(-\frac{[x - (u - \bar{a})]^2}{2s^2}\right) \exp\left(-\frac{x^2}{2t^2} - \frac{dx}{\sqrt{2\pi t}}\right).$$

Using Lemma 1, we obtain

$$(D.13) \quad I_6 = \frac{s}{\sqrt{s^2 + t^2}} \exp\left(-\frac{\overline{a^2} - \bar{a}^2}{2s^2}\right) \exp\left(-\frac{(u - \bar{a})^2}{2(s^2 + t^2)}\right).$$

Lemma 6. Let $h(x, L) = [G(x+L) - G(x-1)]/2L$. Then if t and s are real and positive,

$$I_7 = \int_{-\infty}^{\infty} h\left(\frac{x}{t}, \frac{L}{t}\right) g\left(\frac{u-x}{s}\right) \frac{dx}{st} = h\left(\frac{u/t'}{t'}, \frac{L/t'}{t'}\right),$$

where

$$t = \sqrt{s^2 + t^2}.$$

Proof.

$$I_7 = \frac{1}{2L} \int_{-\infty}^{\infty} \int_{(x-L)/t}^{(x+L)/t} g(y) f\left(\frac{u-x}{s}\right) \frac{dy dx}{s}.$$

Making appropriate changes in variables and inverting the order of integration, we obtain

$$I_7 = \frac{1}{2L} \int_{-L}^L \int_{-\infty}^{\infty} g\left(\frac{u+\xi+\eta}{t}\right) g\left(\frac{\xi}{s}\right) \frac{d\xi d\eta}{st}.$$

Using Lemma 1, we have

$$I_7 = \int_{-L}^L g\left(\frac{u+\eta}{t'}\right) \frac{d\eta}{2Lt'} = \frac{h(u/t', L/t')}{t'}.$$

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