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August 1970
Report 1341-26F

APPLICATIONS OF CUMULATIVE DAMAGE IN THE PREPARATION
OF PARAMETRIC GRAIN DESIGN CURVES AND THE
PREDICTION OF GRAIN FAILURES ON PRESSURIZATION

FINAL REPORT

1 April 1969 through 28 February 1970

VOLUME II - APPENDICES A THROUGH M

By

K. W. Bills, Jr., D. M. Campbell, R. D. Steele and
J. D. McConnell

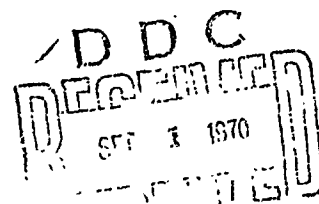
Aerojet Solid Propulsion Company
Sacramento, California

and

Consultants: L. R. Herrmann, University of California and
R. J. Farris, University of Utah

Prepared For

Department of the Navy
Naval Ordnance Systems Command (ORD-0331)
Contract No. N00017-69-C-4423



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APPLICATIONS OF CUMULATIVE DAMAGE IN THE PREPARATION
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VOLUME II - APPENDICES A THRU M

PREPARED FOR

DEPARTMENT OF THE NAVY
NAVAL ORDNANCE SYSTEMS COMMAND (ORD-0331)
CONTRACT NO. N00017-69-C-4423

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Aerojet Solid Propulsion Company

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VOLUME II

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APPENDICES

APPENDIX A

MODULUS DATA INPUT FOR THE COMPUTER

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Appendix A

MODULUS DATA INPUT FOR THE COMPUTER

The relaxation modulus was represented by a sixteen-term Prony series. This relation for the tensile modulus, $E(t)$, is

$$E(t) = A_0 + \sum_{i=1}^m A_i e^{-\beta_i t} \quad (A-1)$$

where A_0 , A_i , and β_i are constants.

For an incompressible material the relaxation modulus in shear, $\mu(t)$, is given by

$$\mu(t) = \frac{E(t)}{3} = \frac{A_c}{3} + \sum_{i=1}^m \frac{A_i}{3} e^{-\beta_i t} \quad (A-2)$$

The viscoelastic stress analyses requires the shear modulus representation.

The Prony series constants employed in the present parameter study are given in Table A-1, for the CTPB propellant, and in Table A-2, for the HTPB propellant.

To complete the viscoelastic description the time-temperature shift function, a_T , is required. Hence, also listed in Tables A-1 and A-2 are the logarithms of a_T at 12 different temperatures from -100° to $+200^\circ\text{F}$, for the two referenced propellants.

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TABLE A-2

MODULUS INPUT FOR THE HTPB PROPELLANT

Prony Series Parameters					
$\log (t/a_T)$ (min)	E psi	β_i (hr ⁻¹)	A_i	$A_i/3$	i
			80	26.67	0
-8	10000	3×10^9	4376.65	1458.9	1
-7	7000	3×10^8	3697.78	1232.6	2
-6	4000	3×10^7	2056.49	685.50	3
-5	2400	3×10^6	896.865	298.96	4
-4	1600	3×10^5	665.701	221.90	5
-3	1050	3×10^4	384.337	128.11	6
-2	720	3×10^3	250.028	83.343	7
-1	500	3×10^2	180.589	60.196	8
0	350	3×10^1	106.550	35.517	9
1	260	3×10^0	67.0877	22.363	10
2	205	3×10^{-1}	35.8845	11.962	11
3	170	3×10^{-2}	39.3243	11.441	12
4	140	3×10^{-3}	24.4644	8.155	13
5	120	3×10^{-4}	15.1300	5.043	14
6	110	3×10^{-5}	-2.4636	-0.8212	15
7	100	3×10^{-6}	33.0018	11.001	16

Time-Temperature Shift Function

Temp., °F	$\log_{10} a_T$
-75	6.90
-50	4.93
-25	3.41
0	2.35
25	1.48
50	0.72
77	0
100	-0.46
125	-0.90
150	-1.24
170	-1.47

APPENDIX B
PARAMETER STUDY FOR HISTORY 1

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APPENDIX B

PARAMETER STUDY FOR HISTORY 1

The results of a series of one-dimensional, thermoviscoelastic analyses are presented graphically in this appendix. All of the analyses were based upon the environmental temperature history described in the text and shown separately in each figure. The data for the CTPB propellant are presented in Figures B-1 to B-6, while those for the HTPB propellant are given in Figures B-7 to B-12.

No analyses of the results are made here.

A. CTPB PROPELLANT

The following graphs give the thermoviscoelastic solutions for these grain designs.

PARAMETER STUDY OF INNER-BORE HOOP STRAINS
CTPB PROPELLANT - HISTORY 1
(B = 4 in.)

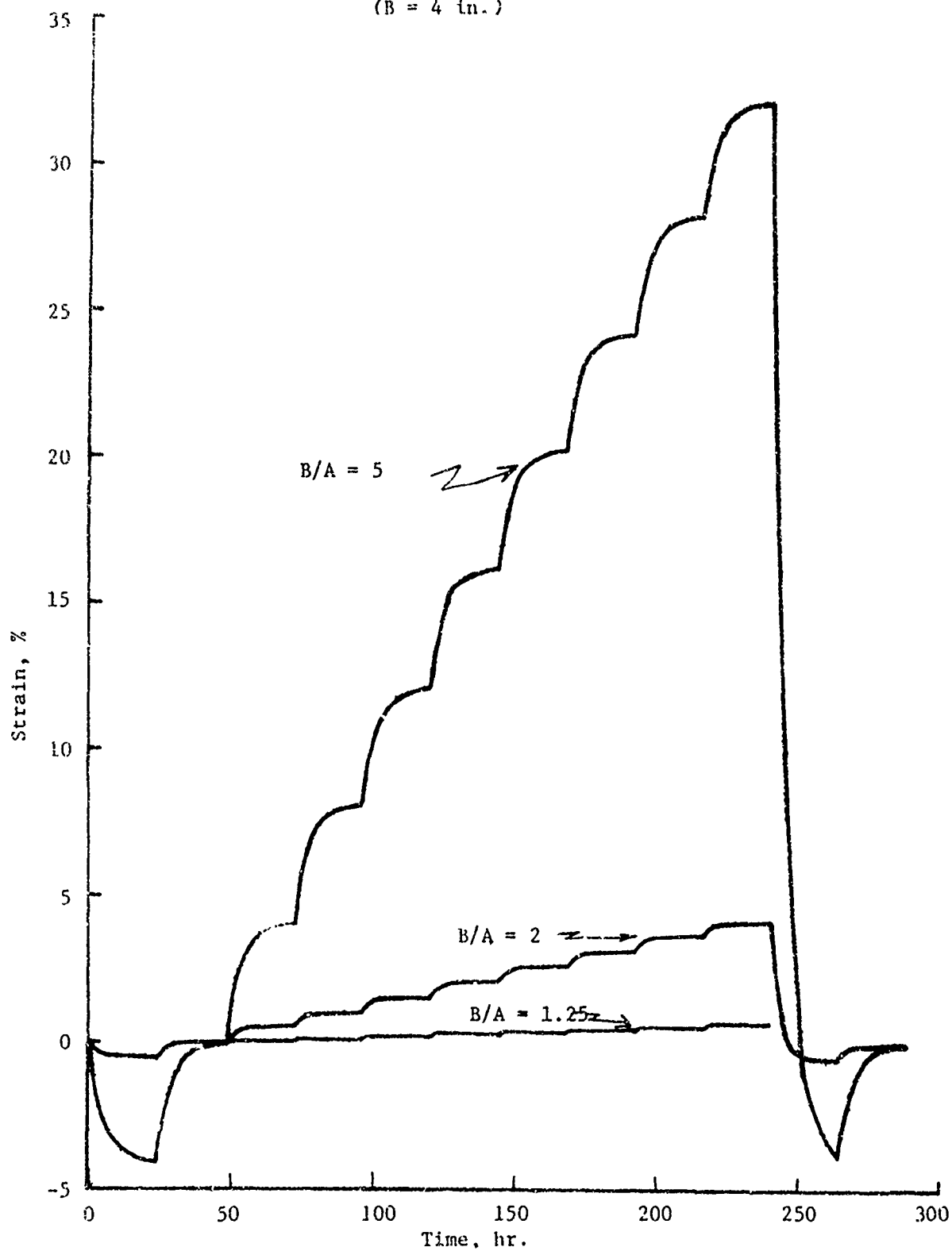


Figure B-1

PARAMETER STUDY OF INNER-BORE HOOP STRAINS
CTPB PROPELLANT - HISTORY 1
(B = 8 in.)

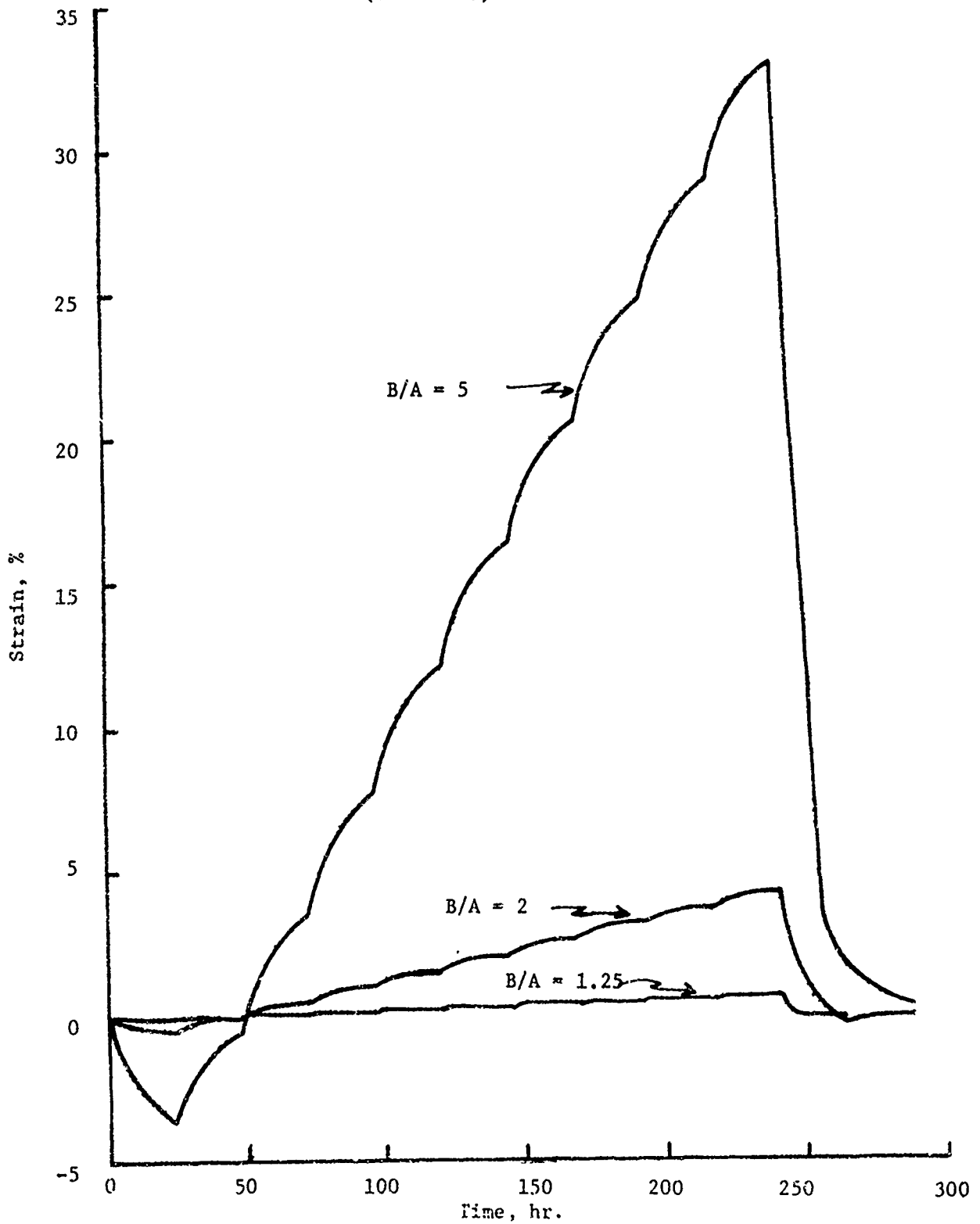
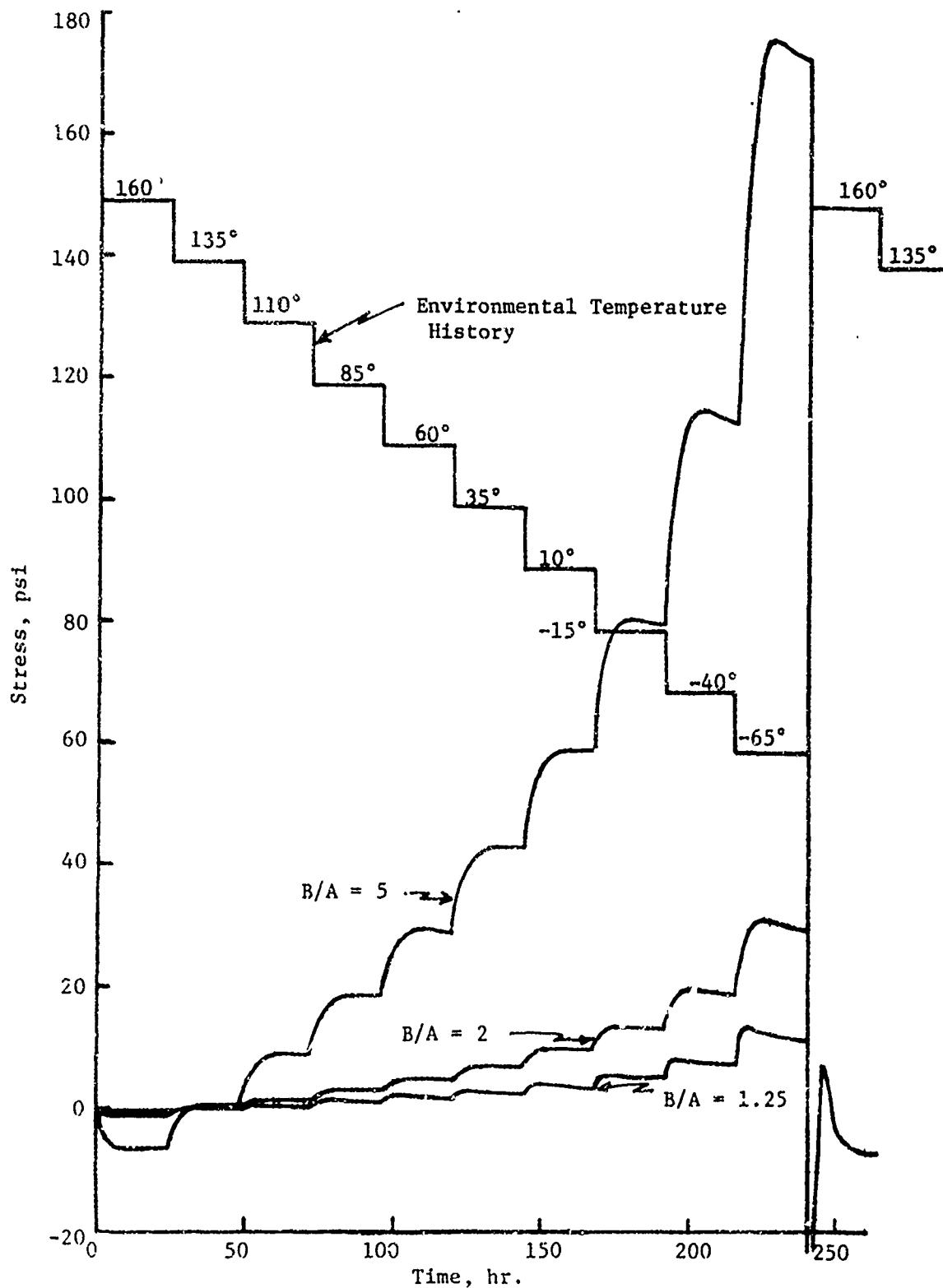


Figure B-2

PARAMETER STUDY OF INNER-BORE HOOP STRESS SOLUTIONS
CTPB PROPELLANT - HISTORY 1
(B = 4 in.)



PARAMETER STUDY OF INNER-BORE HOOP STRESS SOLUTIONS
CTPB PROPELLANT - HISTORY 1
(B = 8 in.)

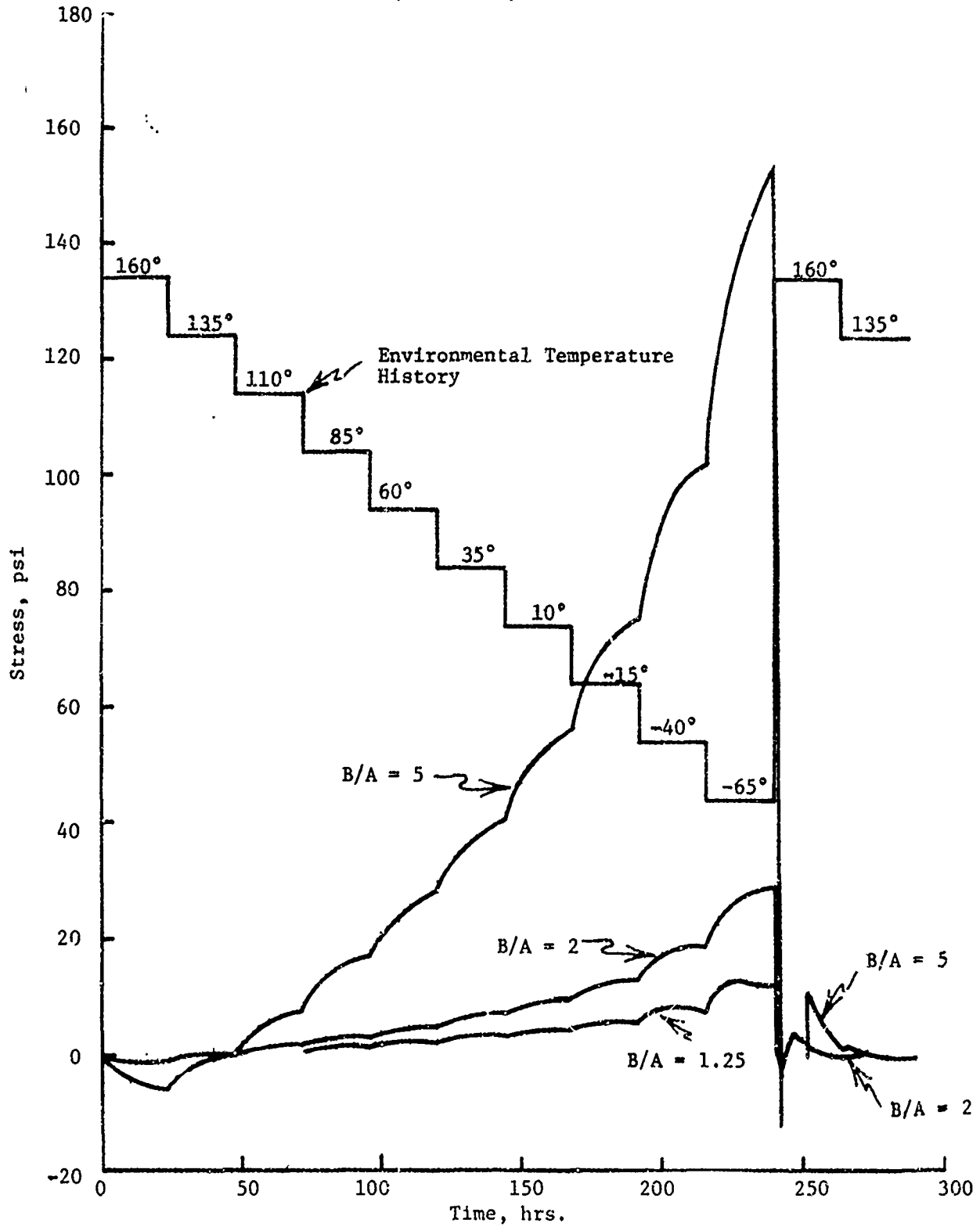


Figure B-4

PARAMETER STUDY OF RADIAL BOND STRESS SOLUTIONS
CTPB PROPELLANT - HISTORY 1

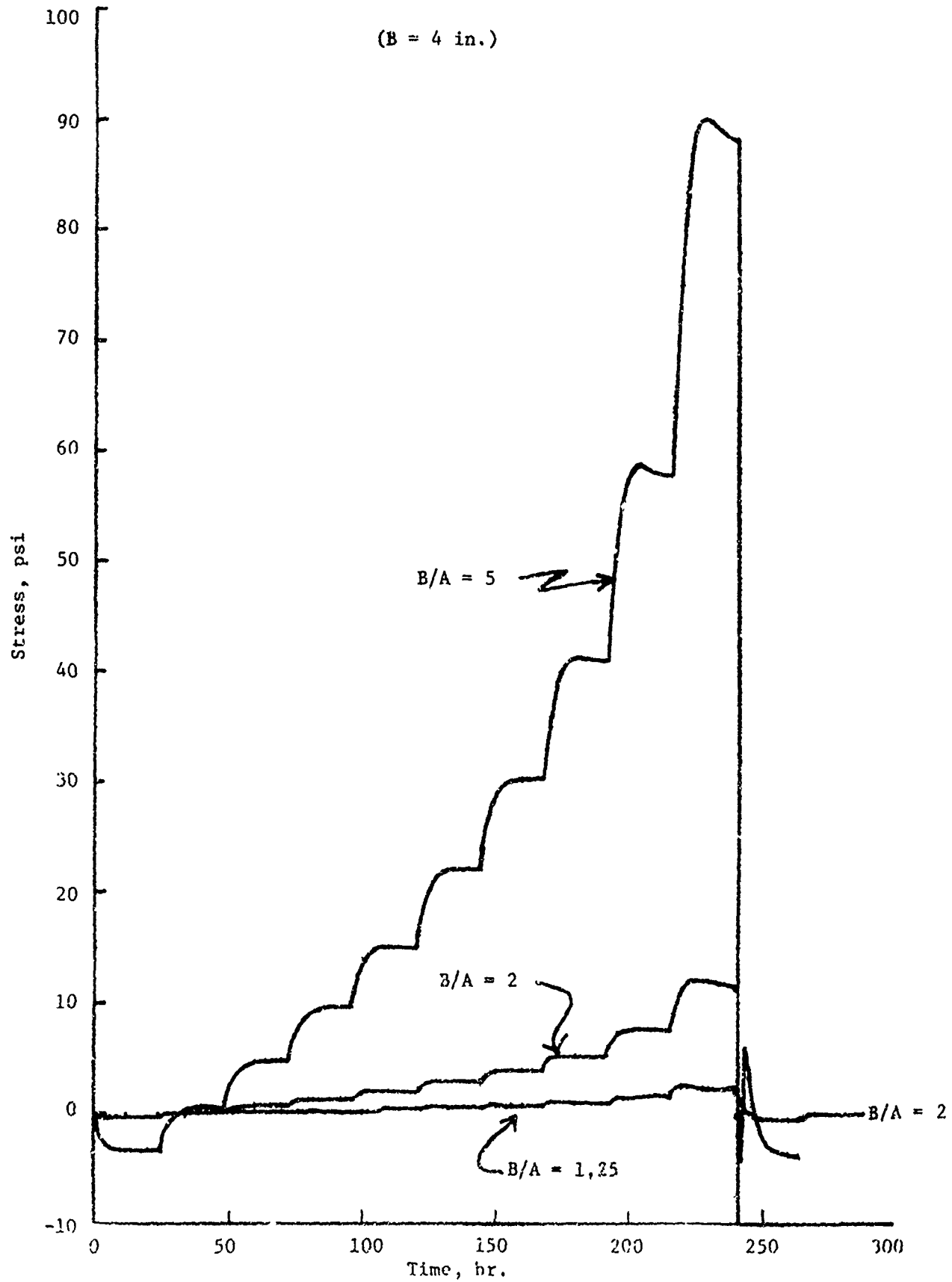


Figure B-5

PARAMETER STUDY OF RADIAL BOND STRESS SOLUTIONS
CTPB PROPELLANT - HISTORY 1

(B = 8 in.)

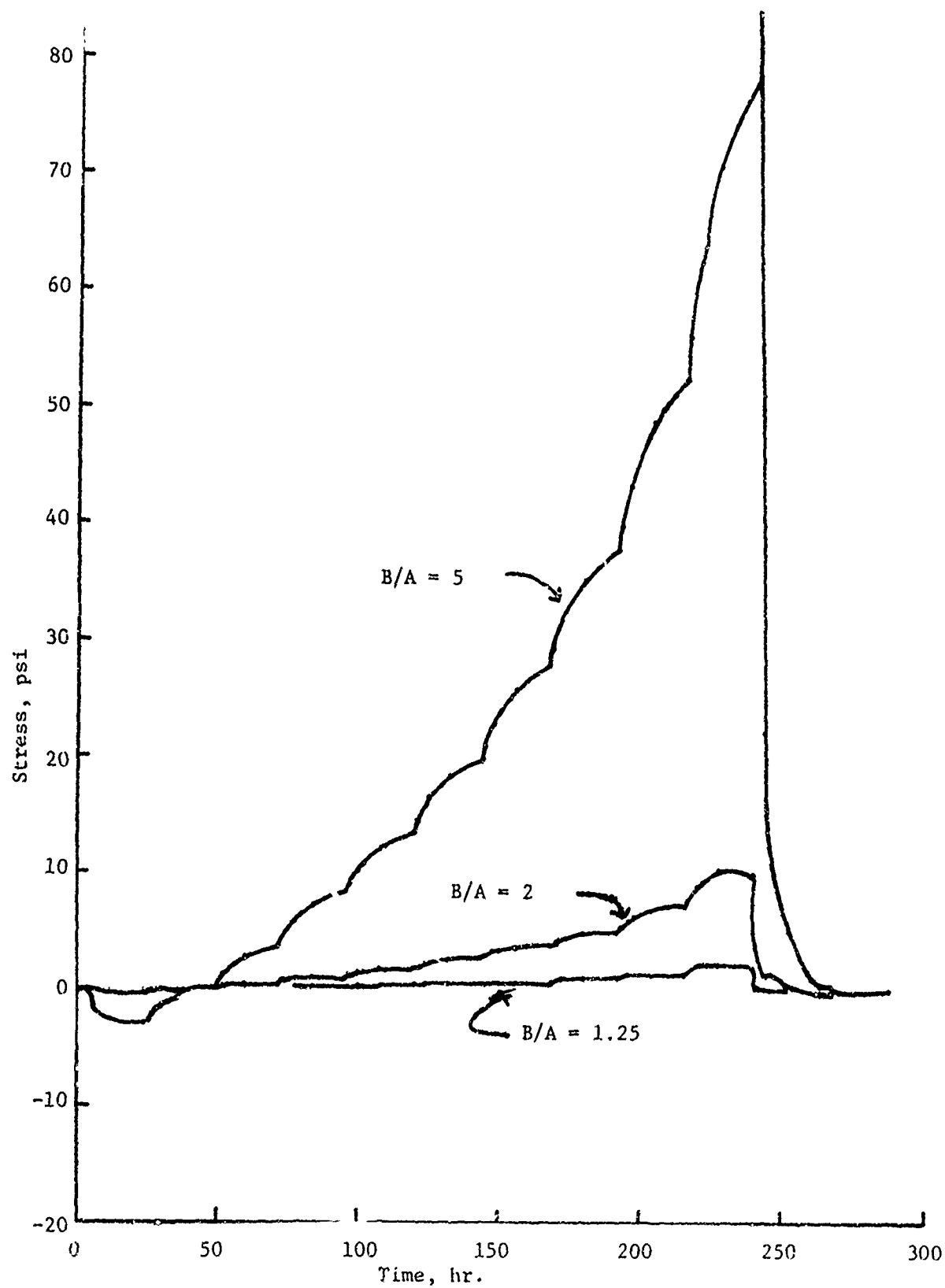


Figure B-6

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Appendix B

B. HTPB PROPELLANT

The following graphs give the thermoviscoelastic solutions for these grain designs.

PARAMETER STUDY OF INNER-BORE HOOP STRAINS
HTPB PROPELLANT - HISTORY 1

(B = 4 in.)

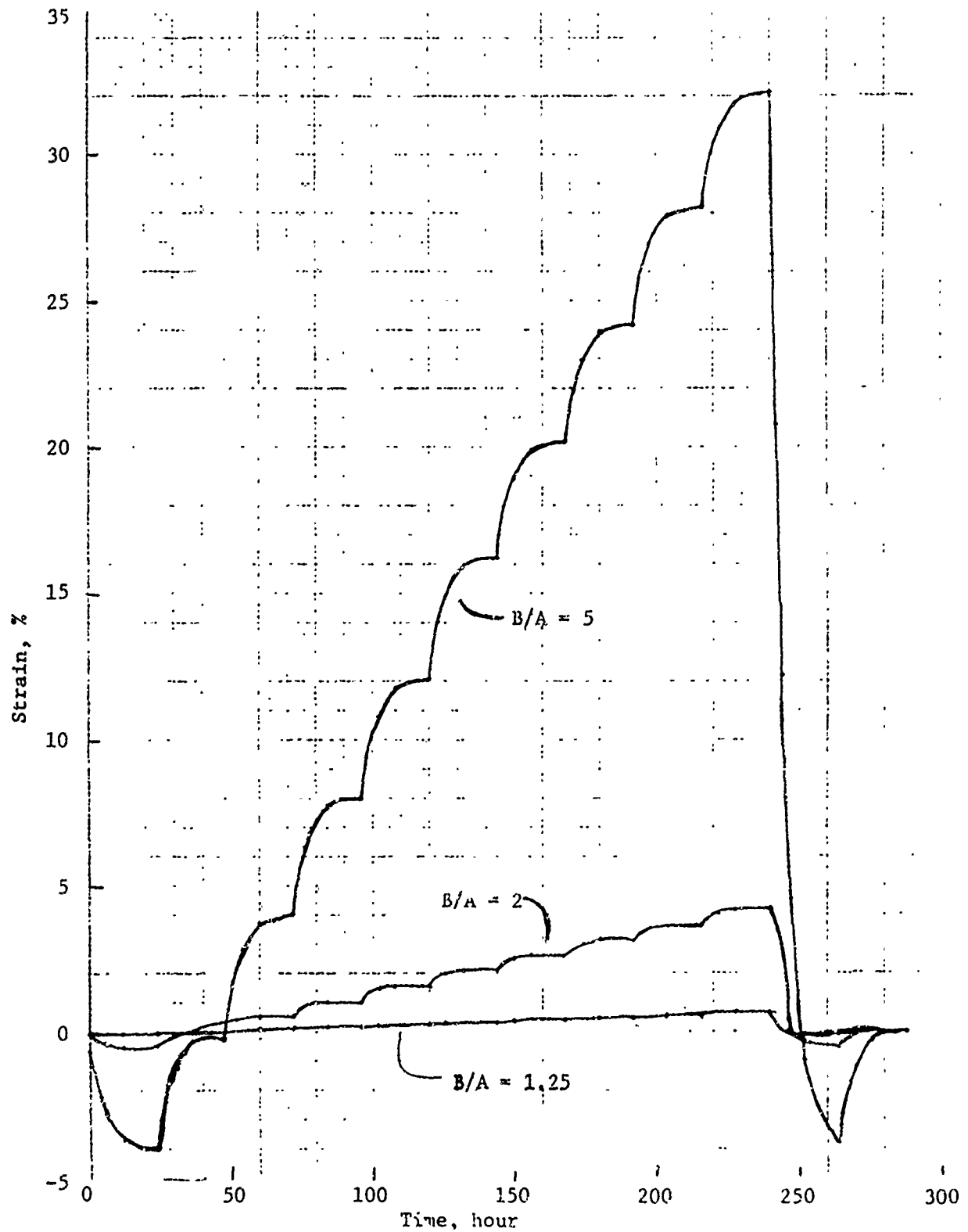


Figure B-7

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PARAMETER STUDY OF INNER-BORE HOOP STRAINS
HTPB PROPELLANT - HISTORY 1

Appendix B

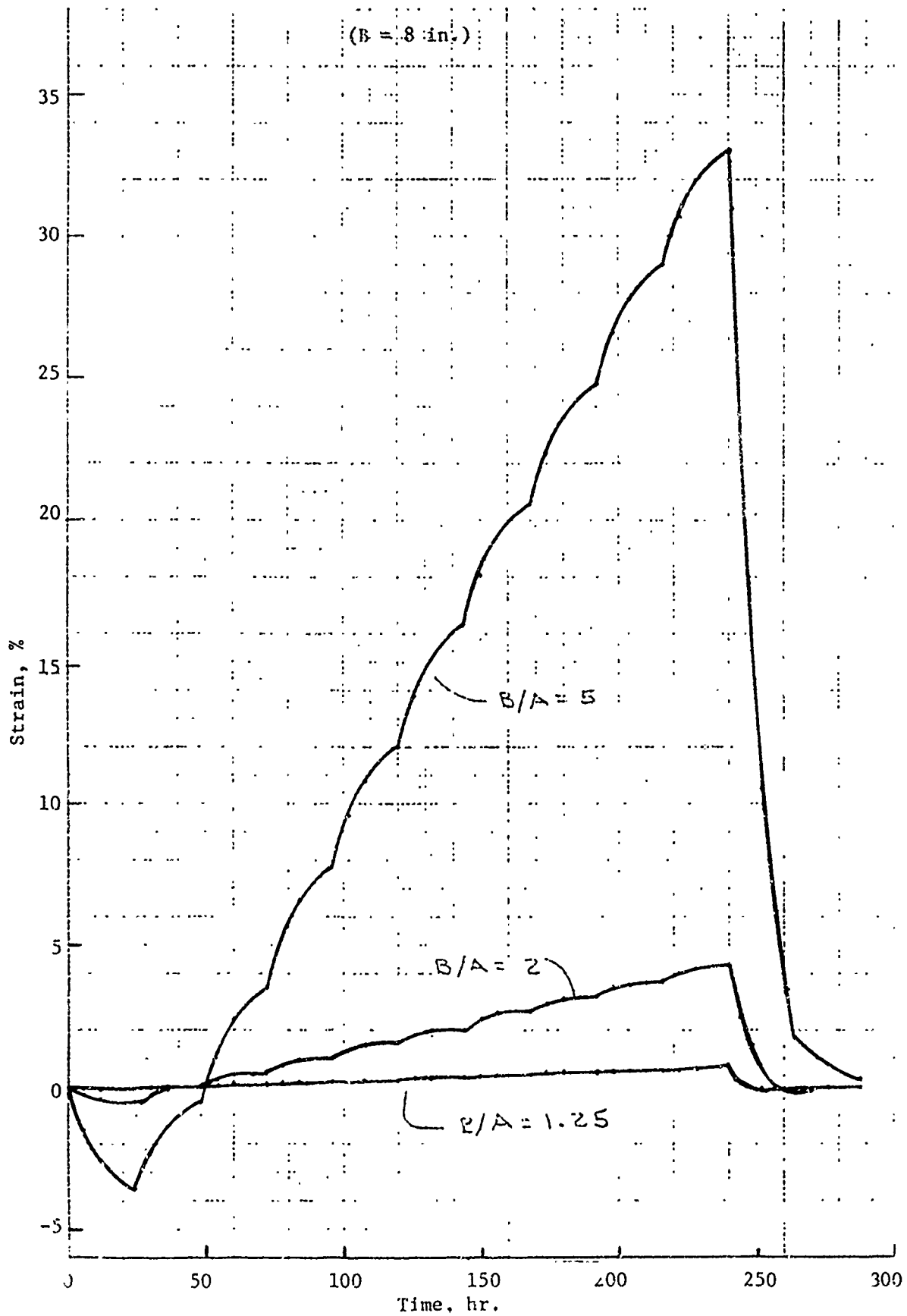


Figure B-8

PARAMETER STUDY OF INNER-BORE HOOP STRESS SOLUTIONS
HTPB PROPELLANT - HISTORY 1

(B = 4 in.)

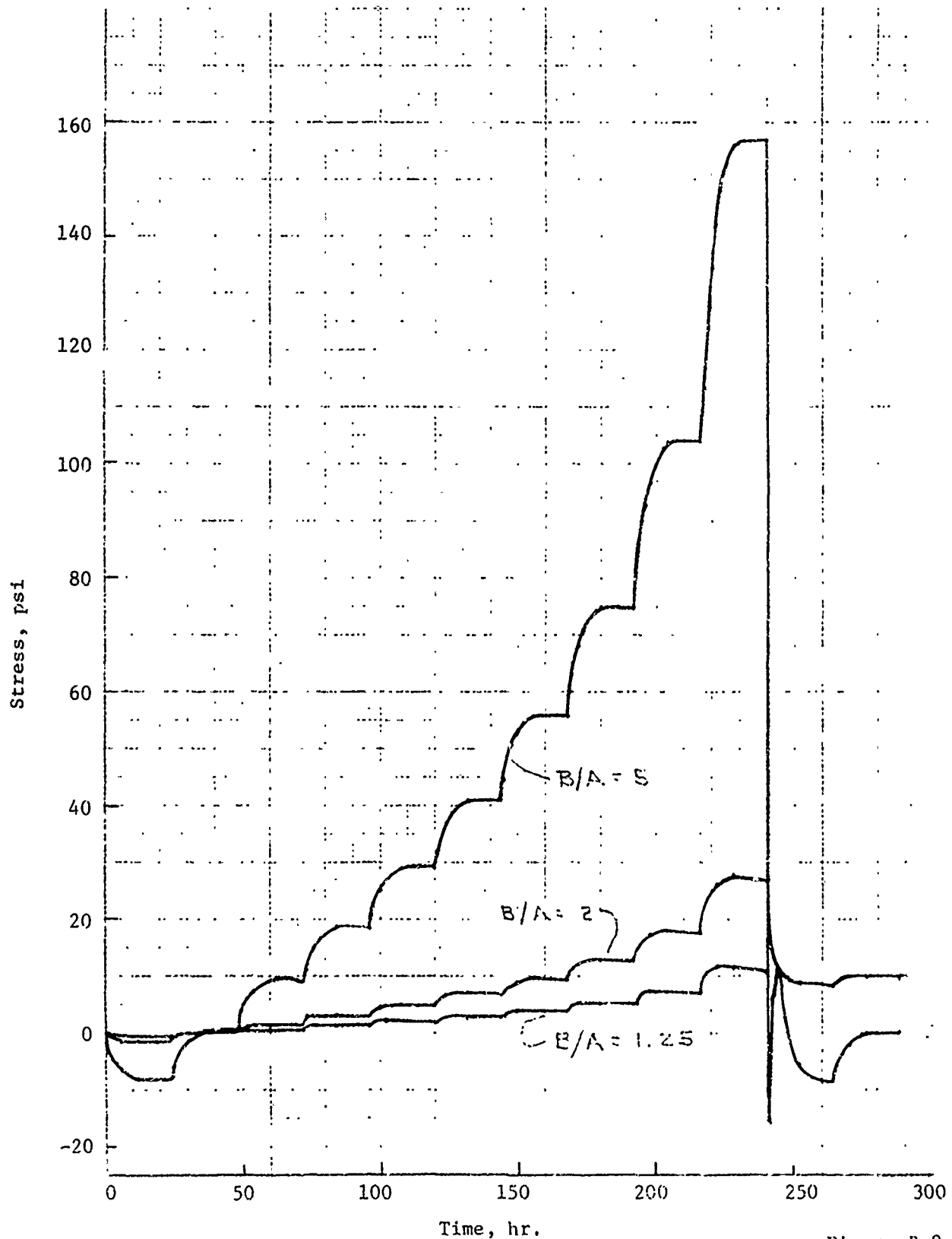
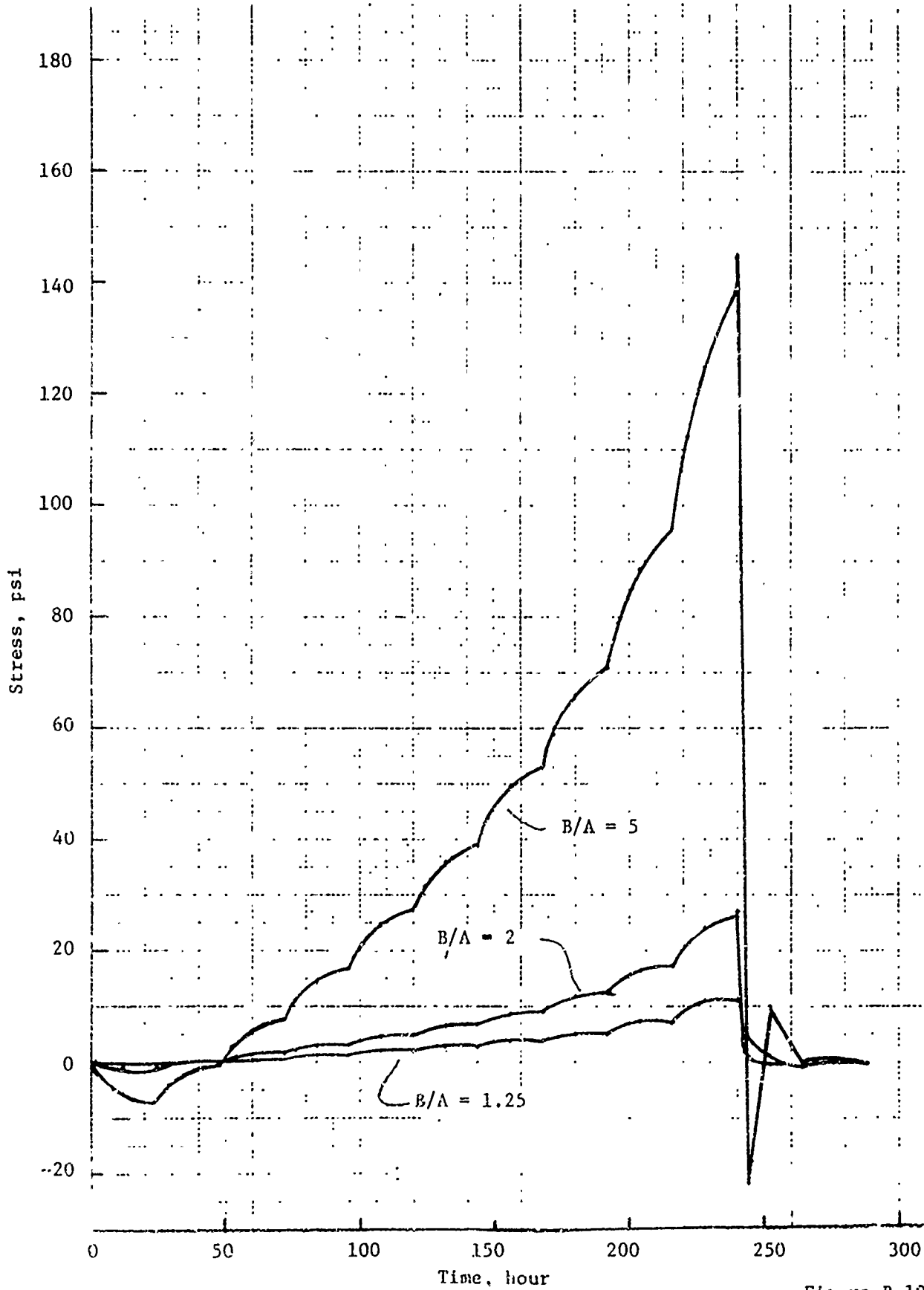


Figure B-9

PARAMETER STUDY OF INNER-BORE HOOP STRESS SOLUTIONS
HTPB PROPELLANT - HISTORY 1
(B = 8 in.)



PARAMETER STUDY OF RADIAL BOND STRESS SOLUTIONS
HTPB PROPELLANT - HISTORY 1
(B = 4 in.)

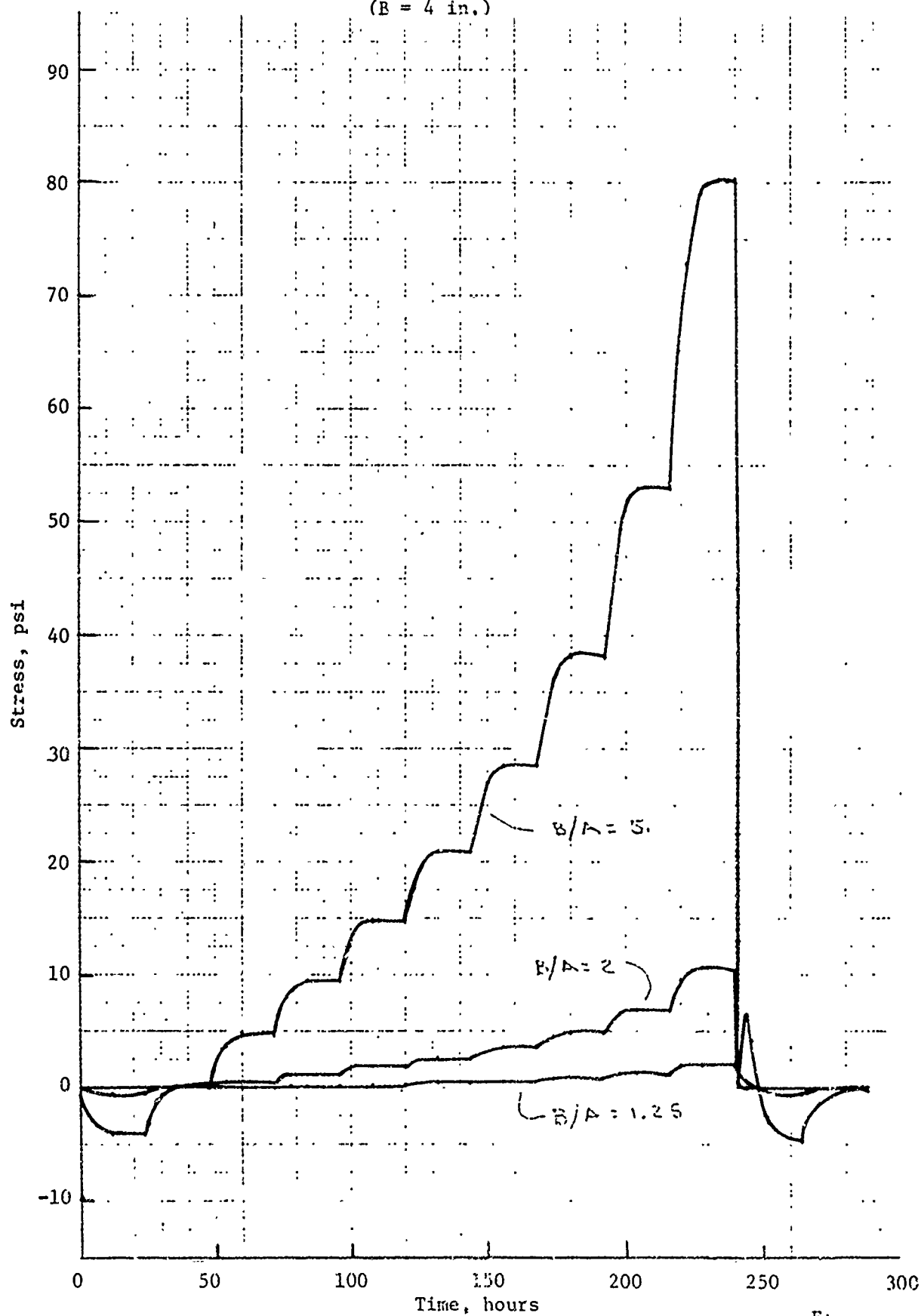
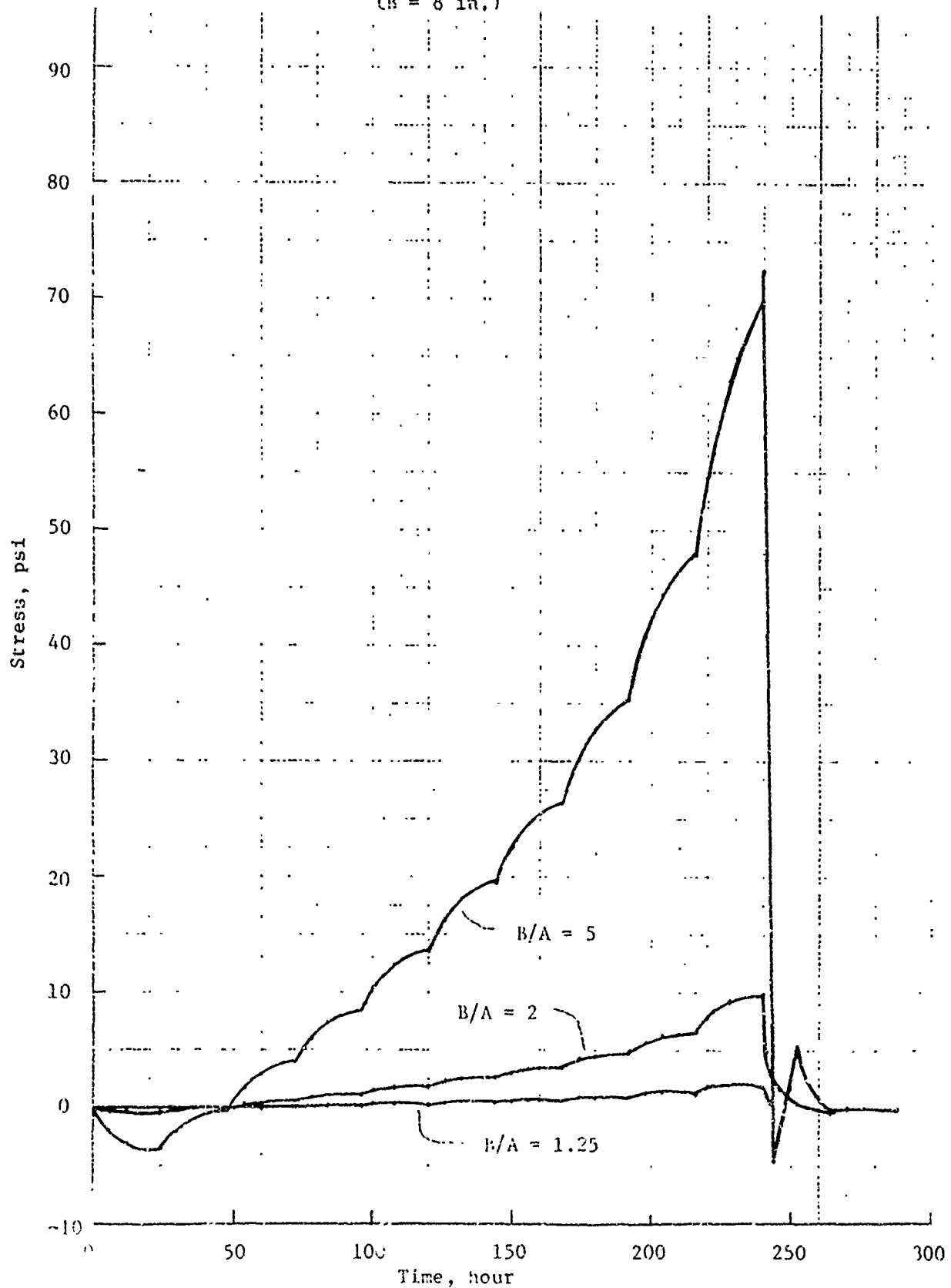


Figure B-11

PARAMETER STUDY OF RADIAL BOND STRESS SOLUTIONS
HTPB PROPELLANT - .. STORY 1
($R = 8$ in.)



APPENDIX C

PARAMETER STUDY FOR HISTORY 2

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APPENDIX C

PARAMETER STUDY FOR HISTORY 2

The results of a series of one-dimensional, thermoviscoelastic analyses are presented graphically in this appendix. All of the results were based upon the environmental temperature history described in the e text and shown separately in each figure. The data for the CTPB and HTPB propellants are presented separately.

No analyses of the results are made here.

A. CTPB PROPELLANT

The following graphs give the thermoviscoelastic solutions for these grain designs.

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Appendix C

PARAMETER STUDY OF INNER-BORE HOOP STRAINS
CTPB PROPELLANT - HISTORY 2

(B = 4 in.)

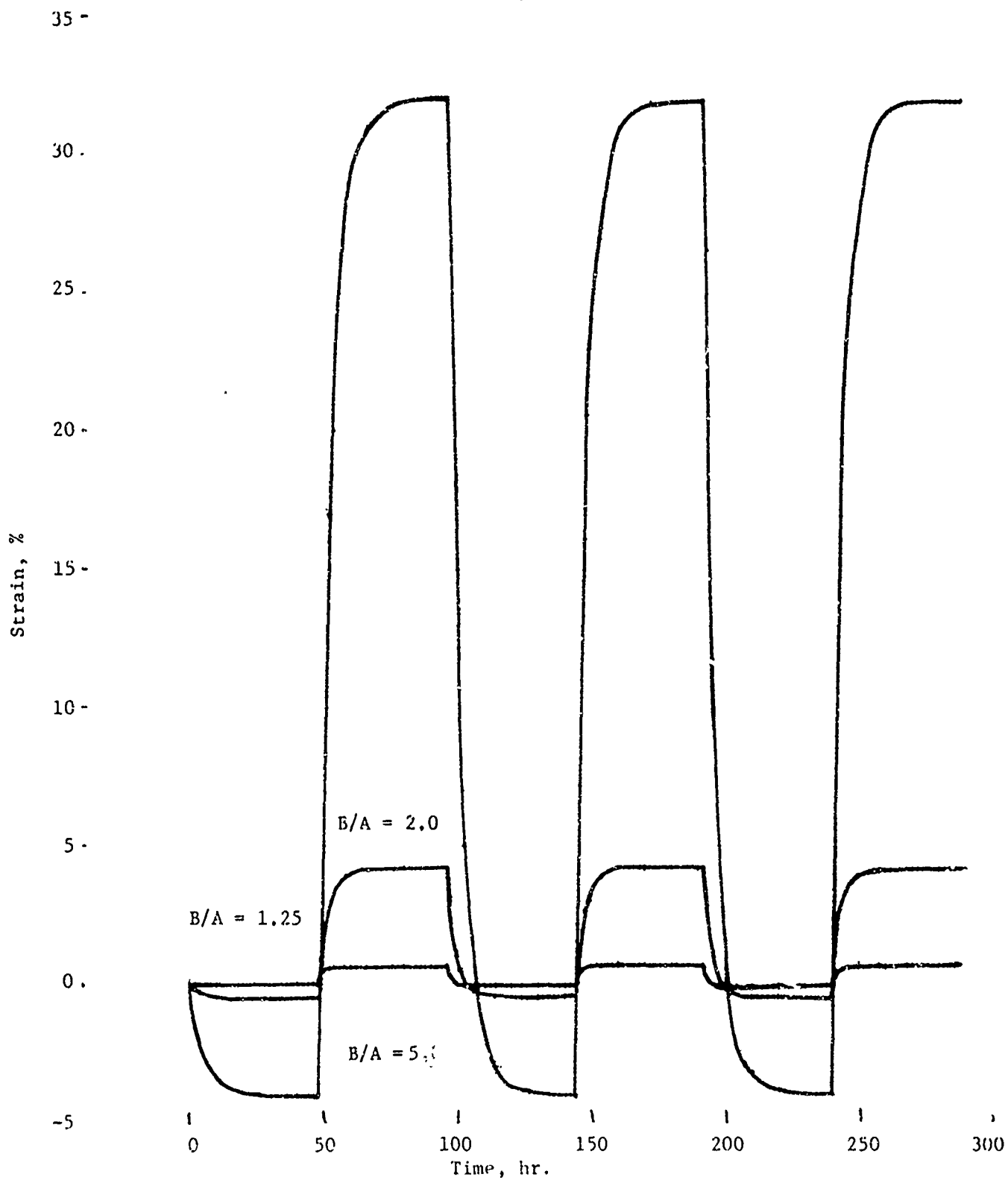


Figure C-1

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Appendix C

PARAMETER STUDY OF INNER-BORE HOOP STRAINS
CTPB PROPELLANT - HISTORY 2

(B = 8 in.)

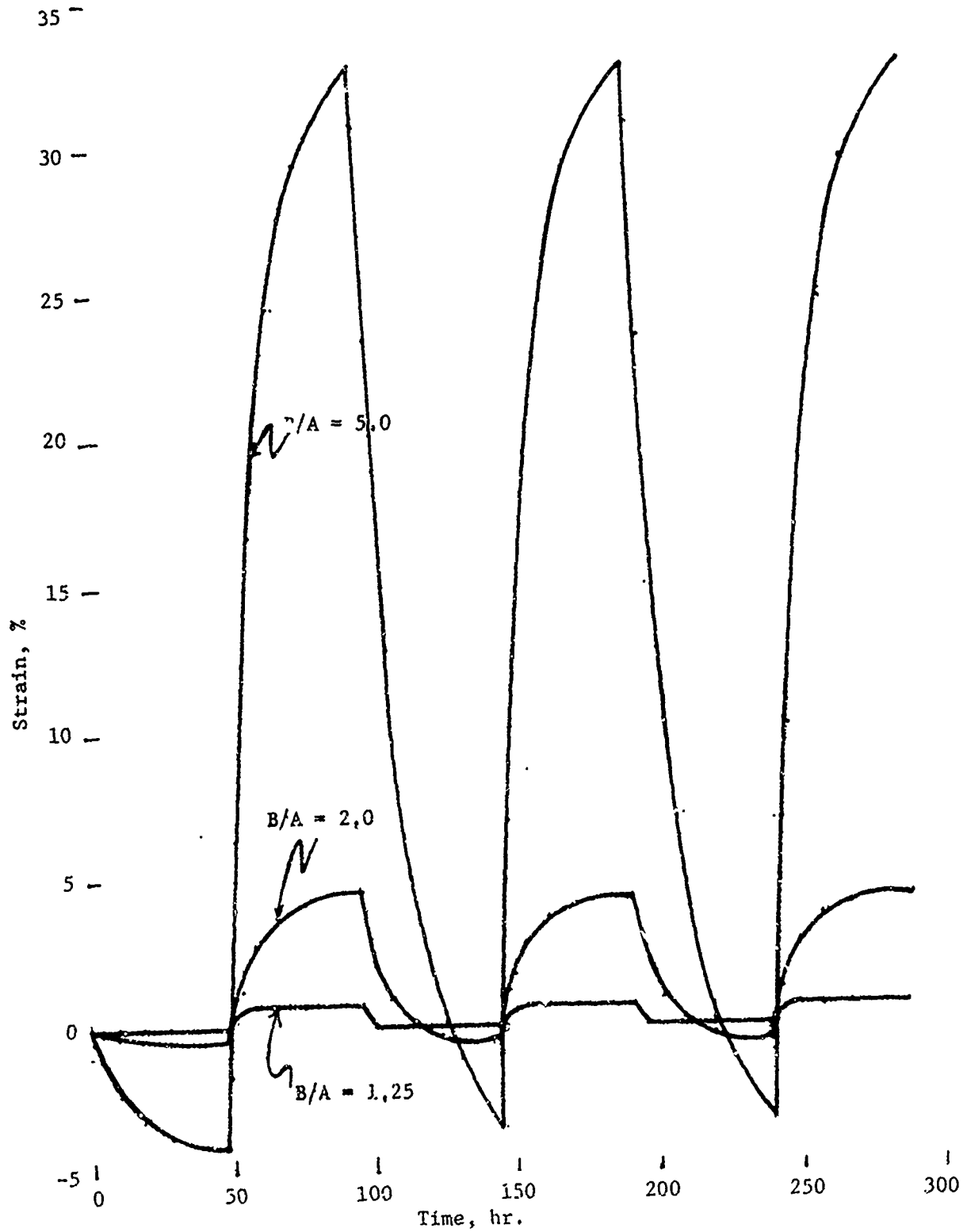


Figure C-2

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PARAMETER STUDY OF INNER-BORE HOOP STRESS
CTPB PROPELLANT - HISTORY 2

(B = 4 in.)

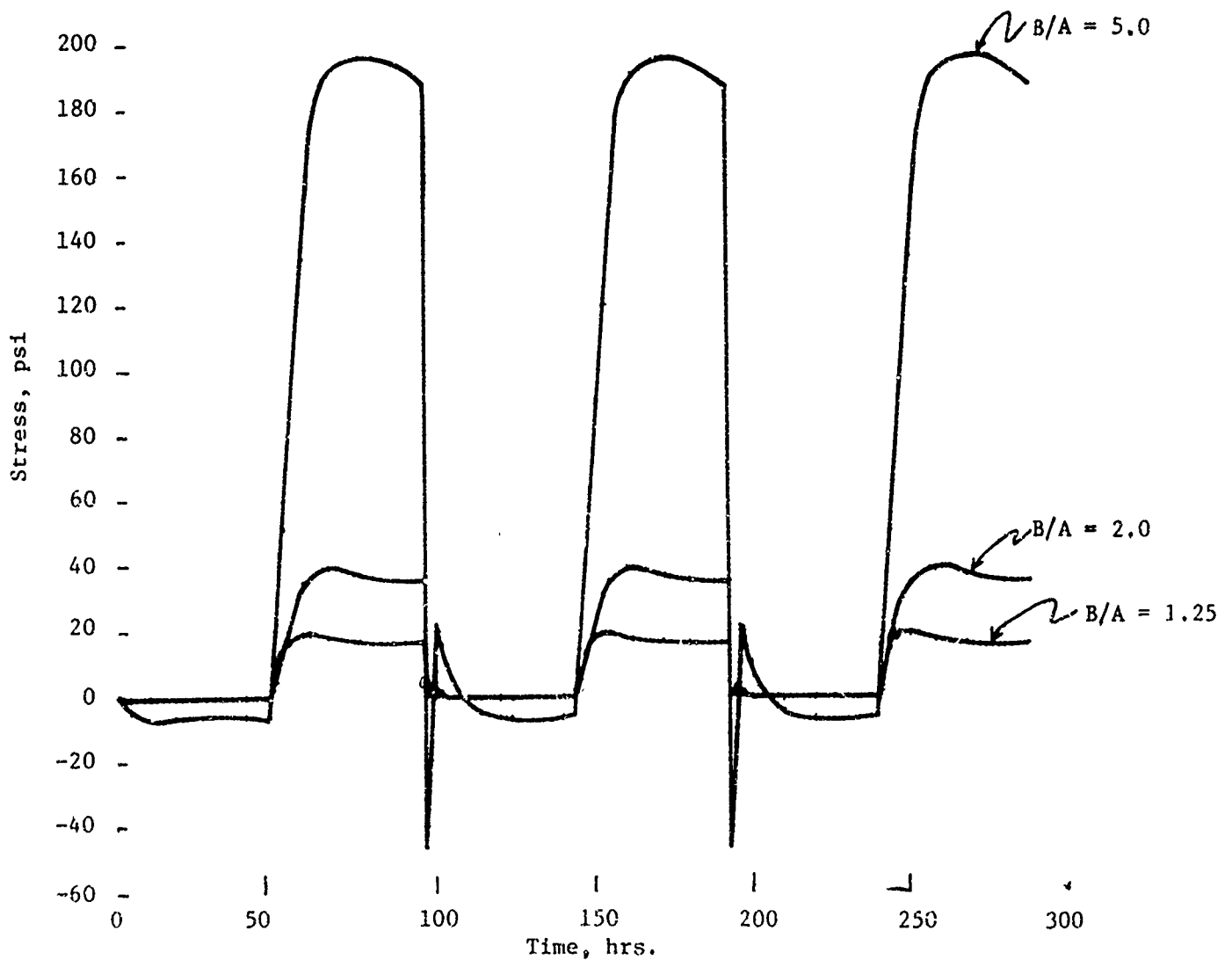


Figure C-3

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Appendix C

PARAMETER STUDY OF INNER-BORE HOOP STRESS
CTPB PROPELLANT - HISTORY 2

(B = 8 in.)

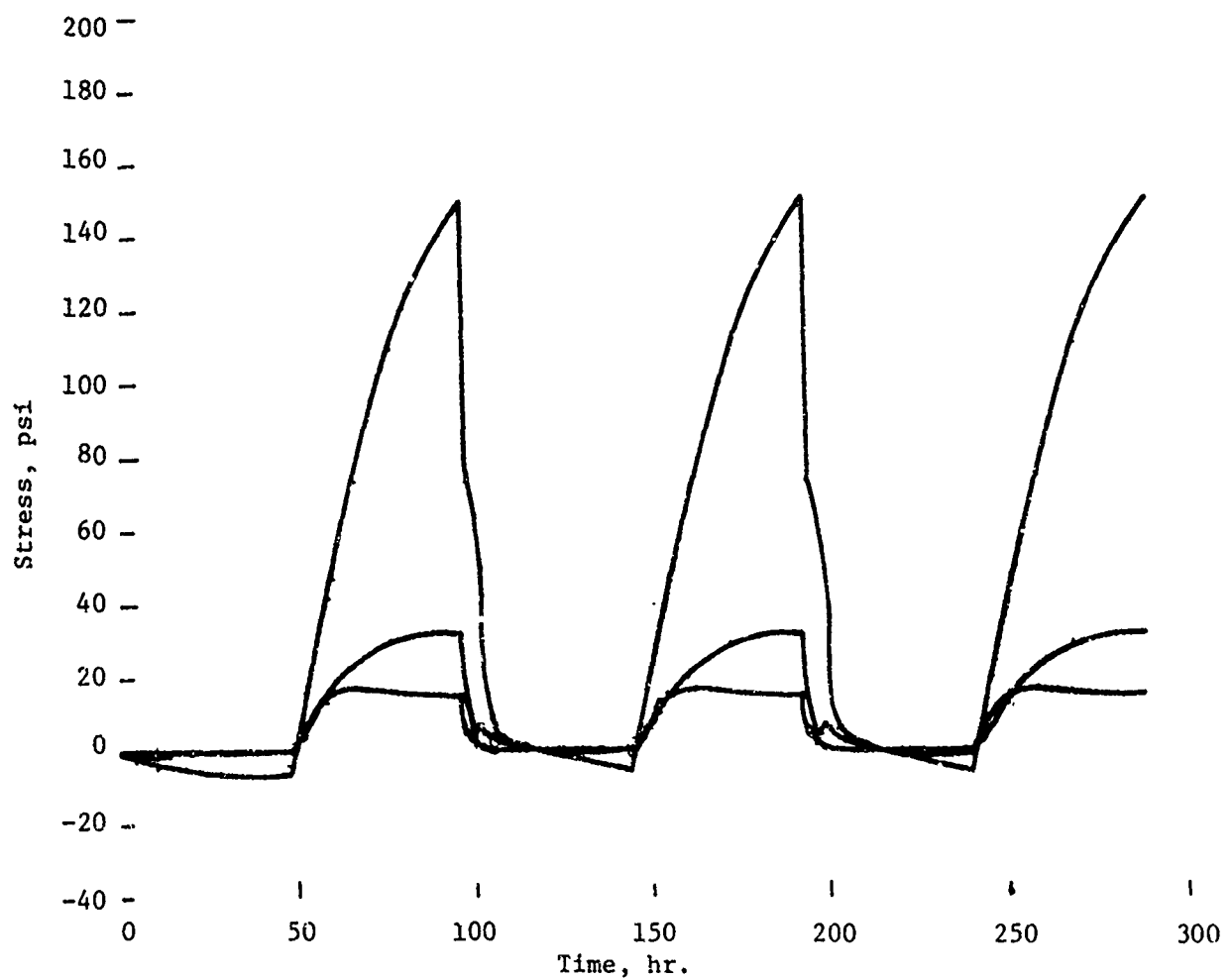


Figure C-4

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Appendix C

PARAMETER STUDY OF RADIAL BOND STRESS SOLUTIONS
CTPB PROPELLANT - HISTORY 2

(B = 4 in.)

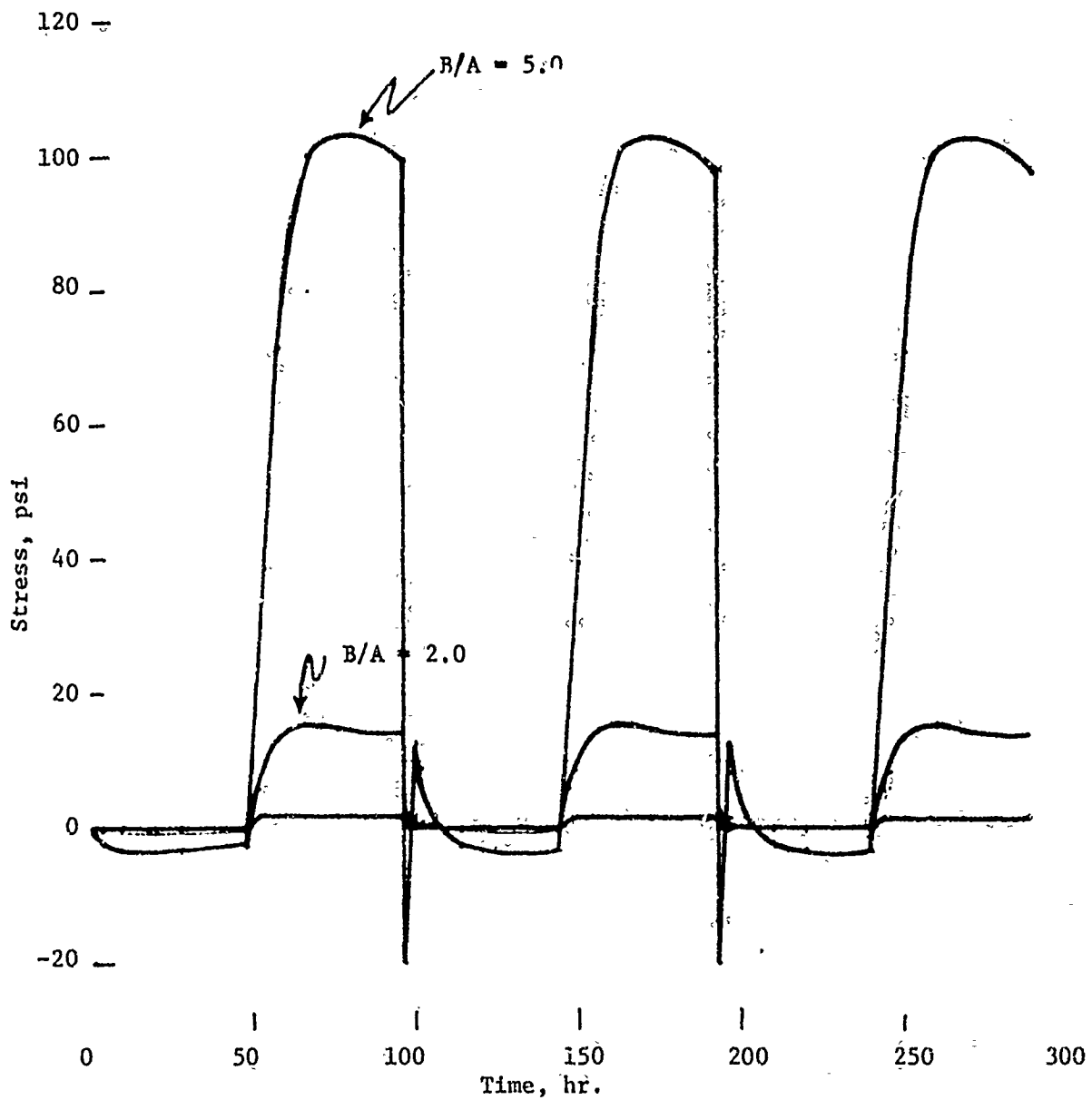


Figure C-5

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Appendix C

PARAMETER STUDY OF RADIAL BOND STRESS SOLUTIONS
CTPB PROPELLANT - HISTORY 2

(B = 8 in.)

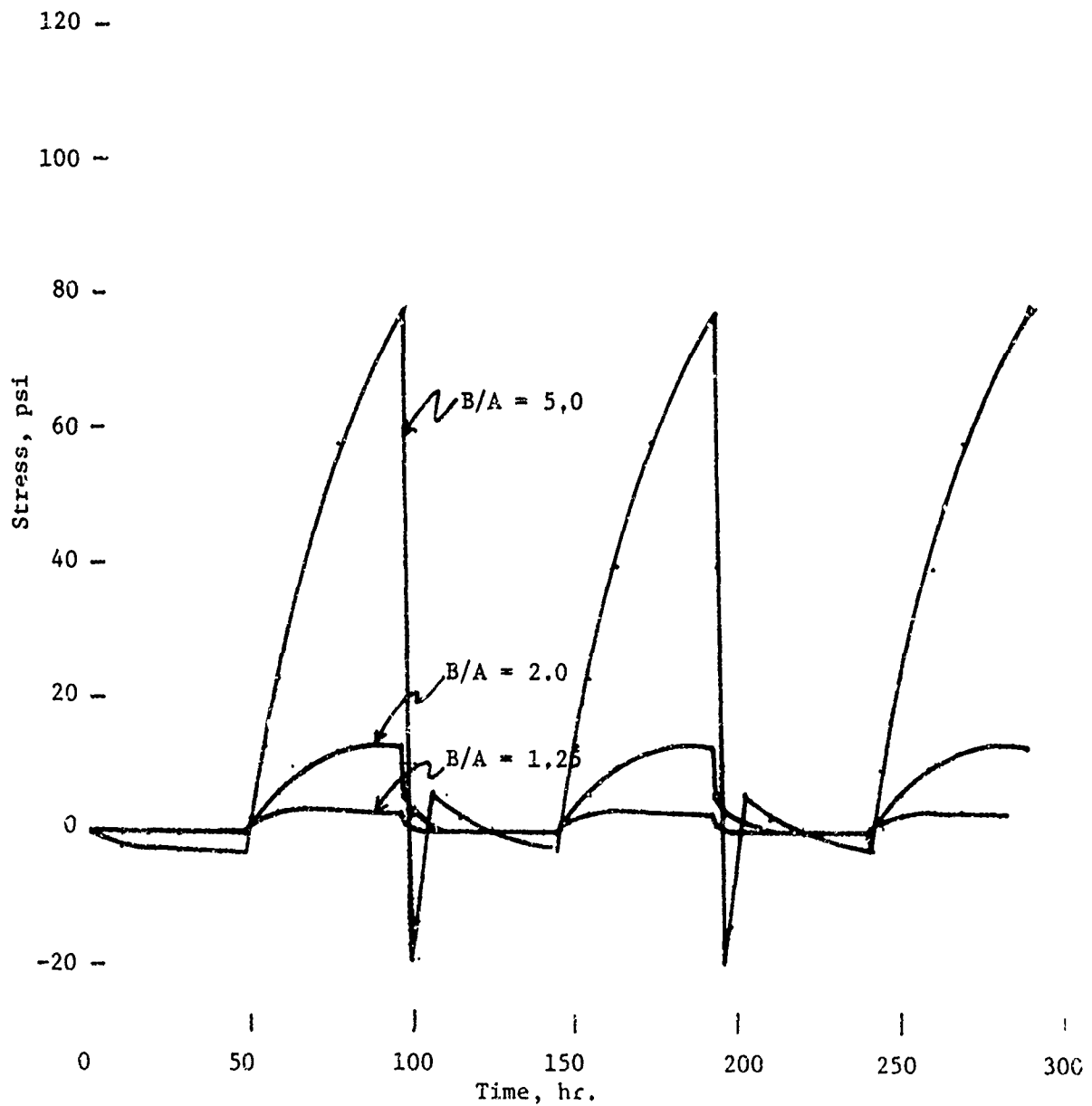


Figure C-6

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Appendix C

B. HTPB PROPELLANT

The following graphs give the thermoviscoelastic solutions for these grain designs.

PARAMETER STUDY OF INNER-BORE HOOP STRAINS
HTPB PROPELLANT - HISTORY 2
(B = 4 in.)

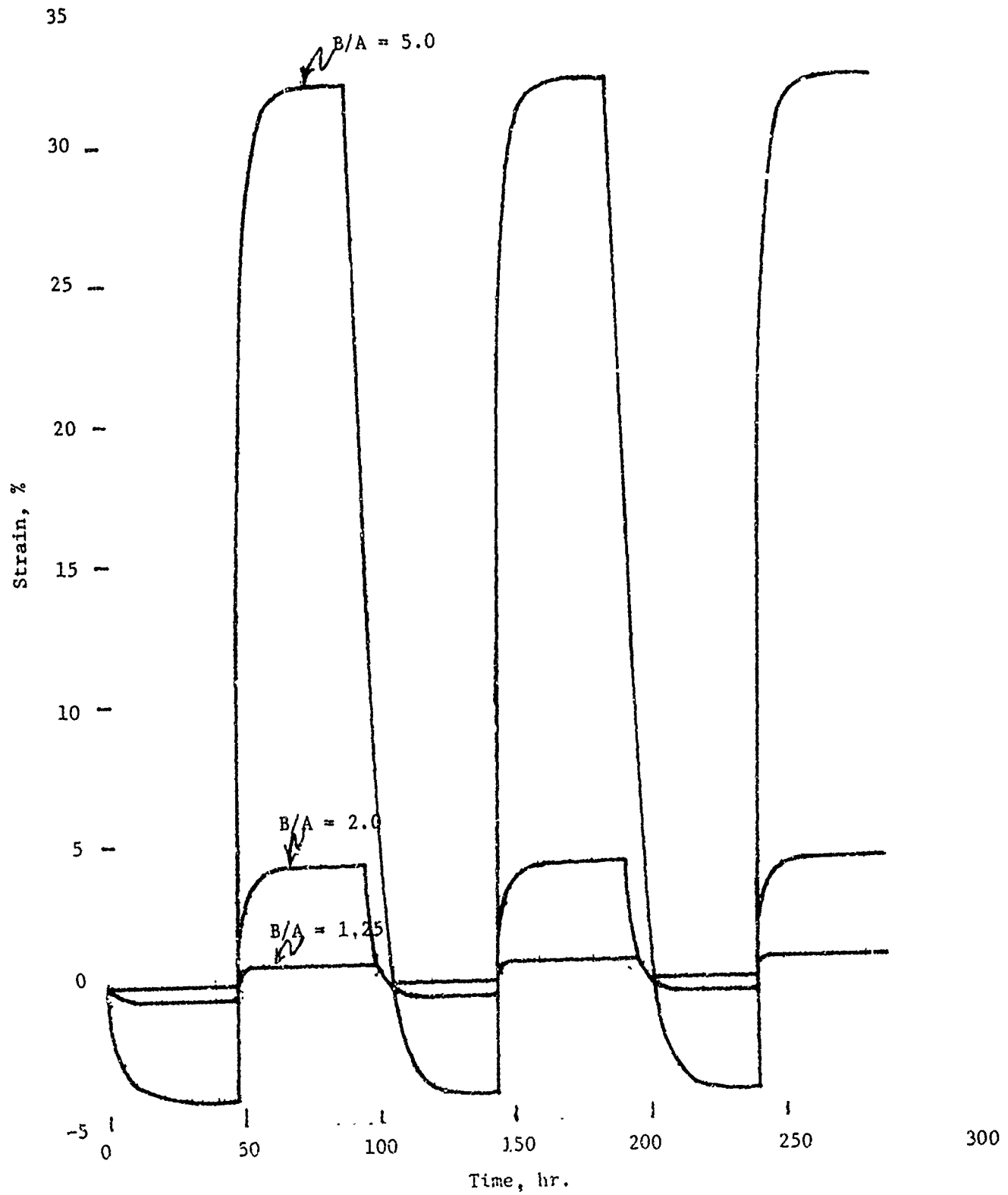


Figure C-7

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Appendix C

PARAMETER STUDY OF INNER-BORE HOOP STRAINS
HTPB PROPELLANT - HISTORY 2

(B = 8 in.)

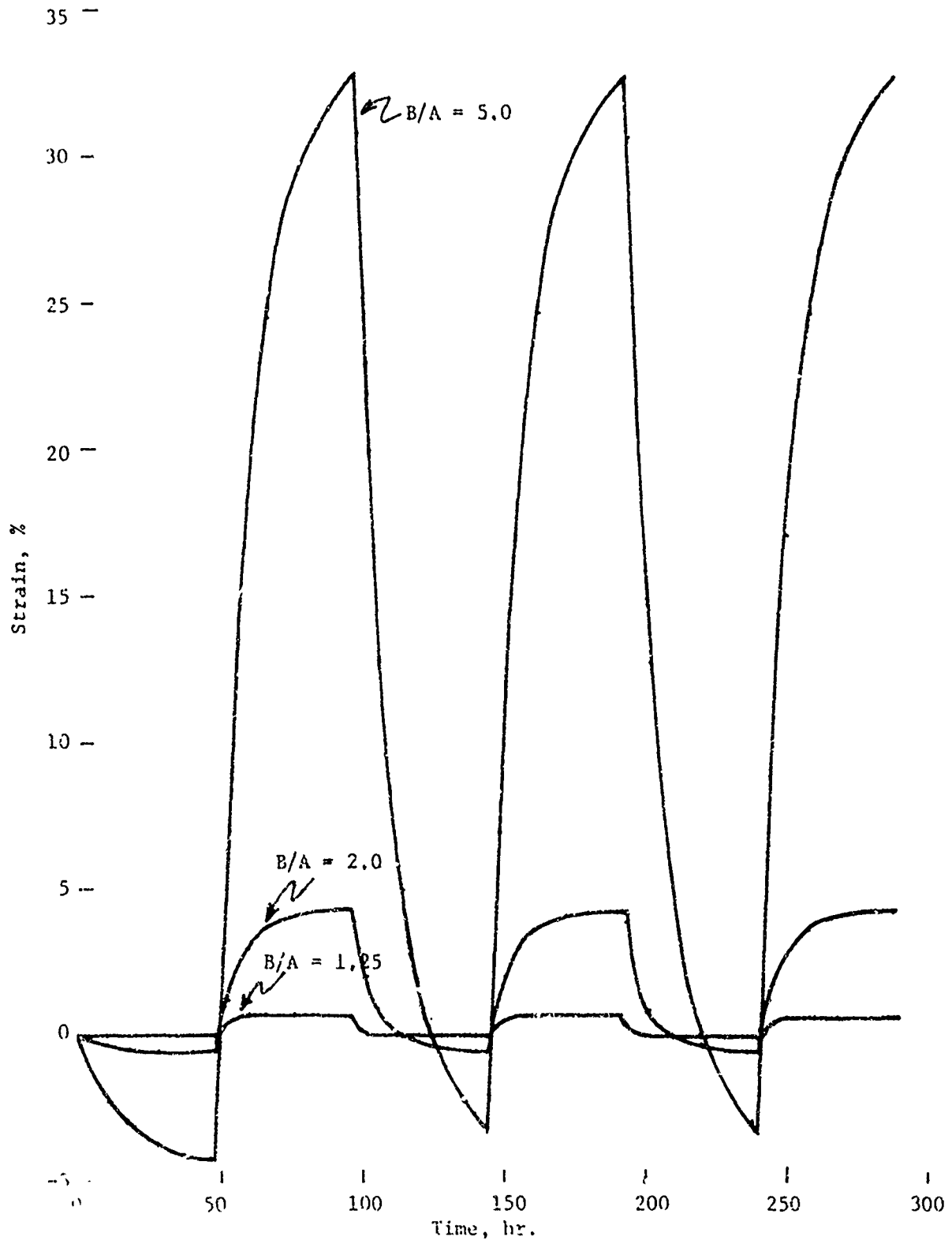


Figure C-8

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Appendix C

PARAMETER STUDY OF INNER-BORE HOOP STRESS SOLUTIONS
HTPB PROPELLANT - HISTORY 2

(B = 4 in.)

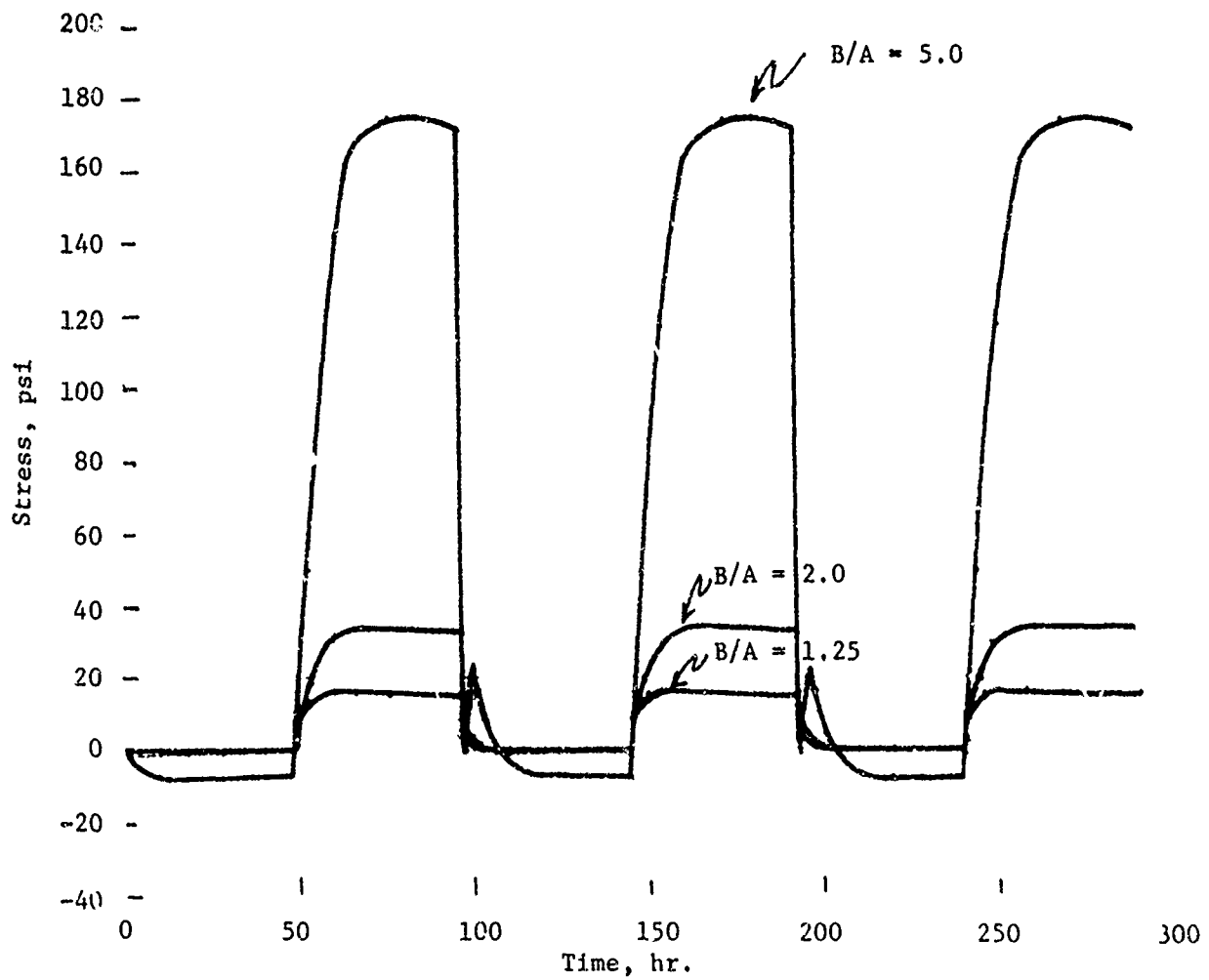


Figure C-9

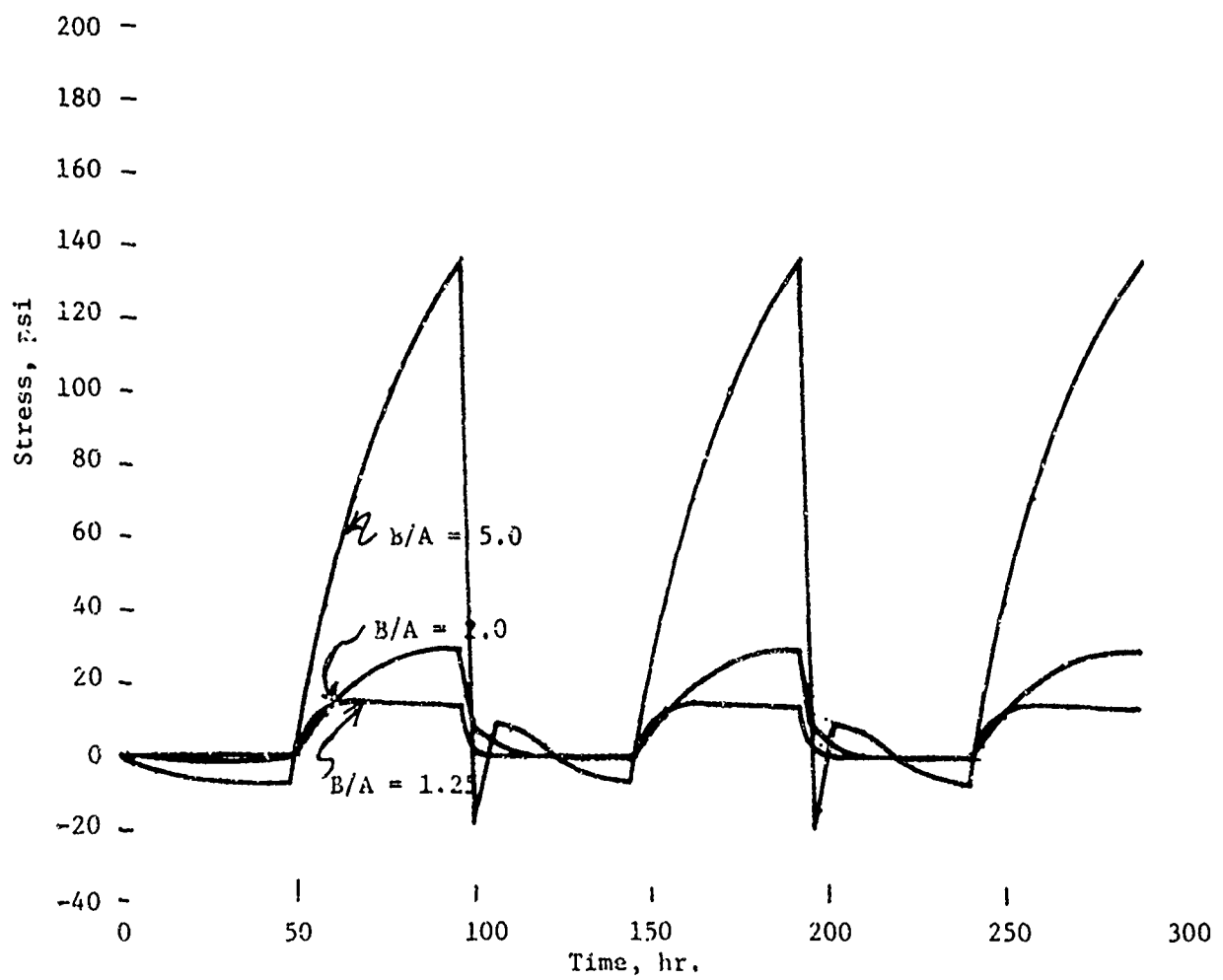
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Appendix C

PARAMETER STUDY OF INNER-BORE HOOP STRESS SOLUTIONS
HTPB PROPELLANT - HISTORY 2

(B = 8 in.)



PARAMETER STUDY OF RADIAL BOND STRESS SOLUTIONS
HTPB PROPELLANT - HISTORY 2

(B = 4 in.)

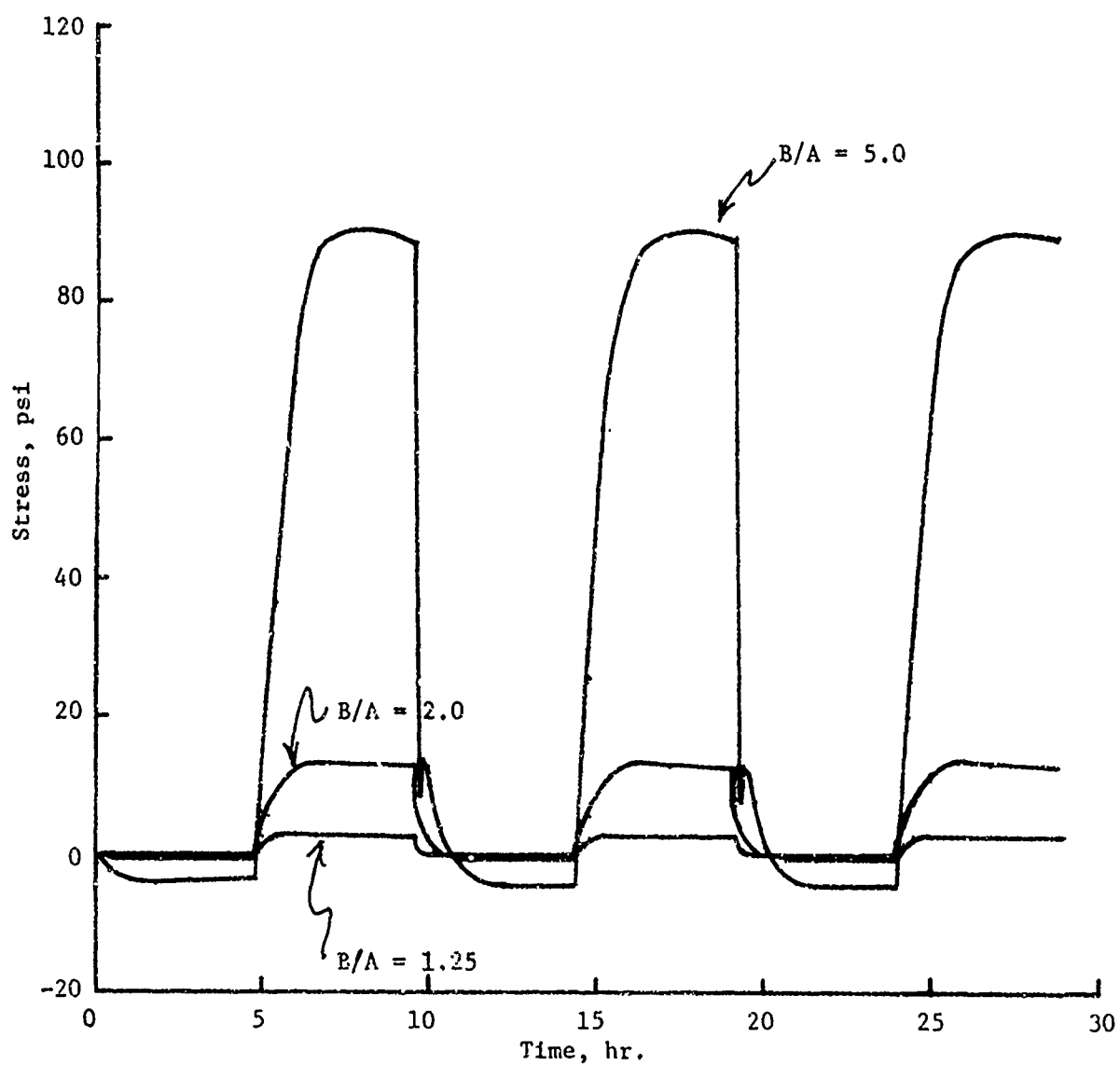


Figure C-11

Appendix C

PARAMETER STUDY OF RADIAL BOND STRESS SOLUTIONS
HTPB PROPELLANT - HISTORY 2
(B = 8 in.)

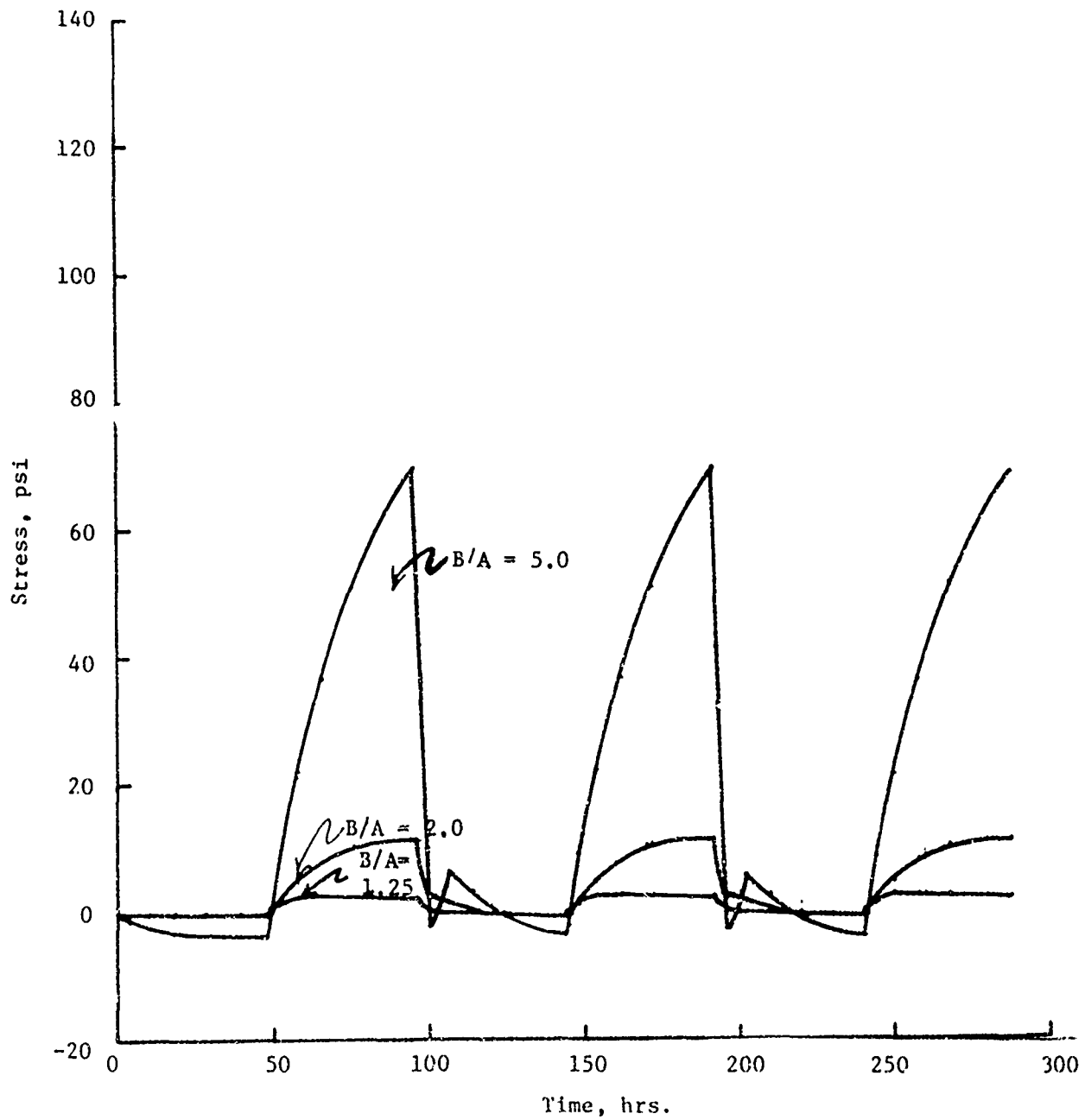


Figure C-12

APPENDIX D

A NEW NORMALIZED RELATION FOR THE RELAXATION MODULUS

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APPENDIX D

A NEW NORMALIZED RELATION FOR THE RELAXATION MODULUS

A. NORMALIZATION OF THE SHEAR MODULUS

Conventional viscoelastic stress analyses involve the relaxation modulus in shear, $\mu(t)$. The basic relations for this modulus require difficult to perform experiments, the results of which are not always satisfactory. It was desirable, therefore, to devise a simpler to use expression for this modulus. This was done starting with the relation now used,

$$\mu(t) = \mu_e + (\mu_g - \mu_e) \int_{-\infty}^{\infty} h e^{-t/\tau} d\ln\tau \quad (D-1)$$

where μ_g is the glassy shear modulus

μ_e is the equilibrium shear modulus

h represents a continuous distribution of relaxation times (normalized)

t is the time of the test

τ is a relaxation time

Since μ_g is difficult to evaluate experimentally an attractive substitute was sought and found.

We solved for $\mu(t)$ at some specific time, like one minute, to obtain $\mu(1)$.

$$\mu(1) = \mu_e + (\mu_g - \mu_e) \int_{-\infty}^{\infty} h e^{-1/\tau} d\ln\tau \quad (D-2)$$

Letting

$$C = \int_{-\infty}^{\infty} h e^{-1/\tau} d\ln\tau \quad (D-3)$$

where C is a constant, we solve for $\mu_g - \mu_e$ in Equation (D-2) using Equation (D-3) and inserting the result into Equation (D-1) gives

$$\mu(t) = \mu_e + (\mu(1) - \mu_e) \int_{-\infty}^{\infty} \frac{h e^{-1/\tau}}{C} d\ln\tau \quad (D-4)$$

For practical experimental purposes the constant C can be combined with h to give h', a quantity which is experimentally identified in the same manner as h.

Thus, the new relation becomes

$$\mu(t) = \mu_e + (\mu(1) - \mu_e) \int_{-\infty}^{\infty} h' e^{-t/\tau} d\ln\tau \quad (D-5)$$

In engineering practice it is unnecessary to evaluate h'. Instead, a graphical plot of $\mu(t)$ versus time is usually sufficient. When normalized results are required a plot of $\frac{\mu(t) - \mu_e}{\mu(1) - \mu_e}$ versus the time is equivalent

to a plot of $\int_{-\infty}^{\infty} h' e^{-t/\tau} d\ln\tau$ versus the time.

Obviously, a broad range of relaxation curves can be obtained for a given distribution of relaxation times, h'.

B. NORMALIZATION OF THE PRONY SERIES

The new relation permits a normalization of the Prony Series as well. This relation in its usual form is given as

$$\mu(t) = \alpha_0 + \sum_{m=1}^m \alpha_m e^{-\beta_m t} \quad (D-6)$$

where the α_m and β_m are constants

α_0 is the equilibrium relaxation modulus

We can normalize Equation (D-6) to give a form similar to that of Equation (D-5). First we replace α_0 by the equivalent term μ_e then normalize the constants α_m , as shown below, to give

$$\mu(t) = \mu_e + (\mu(1) - \mu_e) \sum_{m=1}^m \alpha'_m e^{-\beta_m t} \quad (D-7)$$

where

$$\alpha'_m = \alpha_m / (\mu(1) - \mu_e) \quad (D-8)$$

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Equation (D-7) forms the engineering basis of our normalization procedures. This normalization method defines the modulus in terms of two easily determined parameters, μ_e and $\mu(1) - \mu_e$. These same parameters can be used to normalize the stress and strain data from our engineering analyses.

APPENDIX E
INCREMENTAL ANALYSIS PROCEDURE

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APPENDIX E

INCREMENTAL ANALYSIS PROCEDURE

In an attempt to minimize accumulated numerical errors in our linear viscoelastic analyses, Dr. Herrmann developed this new approach. The "total stress analysis" was replaced by an "incremental analysis procedure"; i.e., instead of solving for the total stress and strain in the propellant for a given point in time, one could solve instead for the incremental changes in stress and strain. The incremental equations are very similar to those previously reported for the "total analysis" with the following exceptions (the equations referred to by number are reported in Reference (E-1)).

Consider first, Equation (13),

$$S_{ijN} = 2\mu_N e_{ijN} + L_{ijN} \quad (13)$$

The quantities S_{ijN} and e_{ijN} in Equation (13) need to be interpreted as the incremental changes in stress and strain during time step N (i.e., $S_{ijN} = 2\mu_N \Delta e_{ijN} + L_{ijN}$ where $S_{ijN} = S_{ijN-1} + \Delta S_{ijN}$, etc.). Equation (18)

$$L_{ijN} = 2 [\chi_{ijN} - (\mu_N - \alpha_0) e_{ijN-1}] \quad (18)$$

is replaced by

$$L_{ijN} = 2 \sum_{m=1}^M \alpha_m [e^{-\alpha_m \tau_N} - 1] C_{ijNm}$$

Equation (19)

$$X_{ijN} = \sum_{m=1}^M \alpha_m C_{ijNm} \quad (19)$$

is eliminated and Equation (24)

$$C_{ijNm} = e^{-\beta_m \Delta \xi_N} [C_{ijN-1,m} + (e_{ijN-1} - e_{ijN-2}) J_{N-1,m}] \quad (24)$$

becomes

$$C_{ijNm} = e^{-\beta_m \Delta \xi_{N-1}} C_{ijN-1,m} + \Delta e_{ijN-1} J_{N-1,m} \quad (24)$$

Finally, Equations (1), (2) and (3)

$$\tau_{ij} = S_{ij} + \delta_{ij} \sigma, \quad \sigma = \frac{1}{33} \tau_{ii} \quad (1)$$

$$\epsilon_{ij} = e_{ij} + \delta_{ij} \frac{\theta}{3}, \quad \theta = \epsilon_{ii} \quad (2)$$

$$\sigma = K (9 - 3\alpha \Delta T) \quad (3)$$

where

K = Bulk modulus
 α = Coefficient of linear thermal expansion
 $\Delta T = T(x, t) - T_0$
 T_0 = Initial stress free reference temperature

are replaced by their corresponding incremental forms (Note: $\Delta T_N = T_N - T_{N-1}$).

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Appendix E

REFERENCES

- E-1 Herrmann, L. R., and Peterson, F. E., "A Numerical Procedure for Viscoelastic Stress Analysis", Bulletin of the 7th Meeting of the ICRPG Mechanical Behavior Working Group, CPIA Publication No. 177, p. 155 (October 1968).

APPENDIX F

PRONY SERIES CURVE FIT ANALYSIS

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APPENDIX F

PRONY SERIES CURVE FIT ANALYSIS

The shear relaxation modulus for most solid propellants has been found to fit the following series function with relatively few terms:

$$\phi(t) = A_0 + \sum_{i=1}^n A_i e^{-\beta_i t} \quad (F-1)$$

Equation (F-1) is a "Prony" series with two unknown coefficients A_i and β_i . The method of collocation is used to find these coefficients:

$$\text{Let } \beta_i = \frac{1}{2t_i} \quad (F-2)$$

Substitution of Equation (F-2) into Equation (F-1) gives

$$\phi = A_0 + \sum_{i=1}^n A_i e^{-\frac{t}{2t_i}} \quad (F-3)$$

Now, choose n points for the evaluation of ϕ

$$\phi_j = A_0 + \sum_{i=1}^n A_i e^{-\frac{t_j}{2t_i}} \quad (j = 1, n) \quad (F-4)$$

Equations (F-4) are sufficient to solve for the A_i . In matrix notation;

$$[\bar{E}] \{A_i\} = \{\phi_j - A_0\} \quad (F-5)$$

where,

$$E_{ji} = e^{-\frac{t_j}{2t_i}}$$

Equation (F-5) is readily solved for A_i ;

$$\{A_i\} = [E]^{-1} \{\phi_j - A_0\} \quad (F-6)$$

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Appendix F

Equation (F-6) has been programmed for computer solution from a times burning terminal in the BASIC language. A listing of the program is given on Page F-3. The order of data input is given below:

1. n
2. A_0
3. t_j ($j = 1, n$)
4. ϕ_j ($j = 1, n$)

Data statements 600 to 9000 may be used for data. Sample data statements are shown below:

600 Data 7,100

610 Data 1E-4, 1E-3, 1E-2, 1E-1, 1, 10, 100

620 Data 3000, 1800, 1000, 620, 410, 320, 280

A sample run with this data is shown on Page F-4.

PROGRAM LISTING

```

100 REM "PRONY SERIES CURVE FIT"
110 DIM T(20,1),F(20,1),A(20,20),C(20,20)
120 DIM F(1,20),C(1,20)
130 READ N,E1
140 MAT REAL I(N,1)
150 MAT F = ZER(1,N)
160 MAT C = ZER(1,N)
170 MAT F = TRN(T)
180 PRINT "INPUT TIMES AND MODULI"
190 MAT PRINT F;
200 MAT A = ZER(N,N)
210 FOR J=2 TO N
220 LET E = -T(J,1)/T(1,1)
230 LET A(J,J) = 0.60653
240 IF E < -20.0 THEN 280
250 LET A(1,J) = EXP(0.5/E)
260 LET A(J,1) = EXP(0.5*E)
270 (9 TO 300
280 LET A(1,J) = 1.0
290 LET A(J,1) = 0.0
300 NEXT J
310 LET A(1,1) = 0.60653
320 LET K1 = N - 1
330 FOR I=2 TO K1
340 LET L = 1
350 LET K2 = K1 - I + 2
360 FOR J=2 TO K2
370 LET L = L + 1
380 LET A(J,L) = A(1,1)
390 LET A(L,J) = A(1,1)
400 NEXT J
410 NEXT I
420 MAT C = ZER(N,N)
430 MAT C = INV(A)
440 MAT READ E(N,1)
450 MAT C = TRN(E)
460 MAT PRINT C;
470 FOR J=1 TO N
480 LET E(J,1) = F(J,1) - F1
490 NEXT J
500 MAT T = C*F
510 PRINT "SOLUTION"
520 MAT F = TRN(T)
530 MAT PRINT F;
540 (1 TO 400
550 END

```

END

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Appendix F

SAMPLE RUN

PRON 9:57 SF FRI 02/13/70

INPUT TIMES AND MODULI

.0001 .001 .01 .1 1 10 100

3000 1800 1000 620 410 320 230

SOLUTION

1370.27 1031.23 464.66 247.646 184.314 -106.615 297.955

OUT OF DATA IN 440

APPENDIX G

INCLUSION OF NON-ZERO THICKNESS STRESSES IN PLANE STRESS ANALYSES

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APPENDIX G

INCLUSION OF NON-ZERO THICKNESS STRESSES
IN PLANE STRESS ANALYSES

The generalized plane stress option was modified so that the stress throughout the thickness of the body, may be specified as a constant rather than zero. The values of the thickness stress constitutes one of the input parameters. The inclusion of a non-zero thickness stress required the modification of the governing variational equation.

It was determined that the appropriate form of the variational function, for a nearly incompressible material ^(G-1) is (for $T = T_N$):

$$\begin{aligned}
 F_N = \iint \{ & \frac{2\mu_N}{3} [(\Delta\epsilon_{x_N})^2 + (\Delta\epsilon_{y_N})^2 - (\Delta\epsilon_{x_N})(\Delta\epsilon_{y_N})] + \frac{1}{2} \mu_N (\Delta\gamma_{x_N})^2 \\
 & + (\mu_N \Delta\bar{H}_N - \Delta\beta_N/3)(\Delta\epsilon_{x_N} + \Delta\epsilon_{y_N}) + L_{xx_N} \Delta\epsilon_{x_N} + L_{yy_N} \Delta\epsilon_{y_N} \\
 & + L_{xy_N} \Delta\gamma_{xy_N} - \frac{(\mu_N)^2}{2\bar{K}_N} (\Delta\bar{H}_N)^2 + \Delta\bar{H}_N \left(\frac{\Delta\beta_N}{2} - 3\mu_N \alpha \Delta T_N \right) \\
 & - \Delta F_{x_N} \Delta u_{x_N} - \Delta F_{y_N} \Delta u_{y_N} \} dx dy - \int \Delta u_N \cdot \Delta(\text{applied boundary load}) ds
 \end{aligned}$$

where

$$\Delta \epsilon_{xx_N} = \frac{2\mu_N}{3} (2\Delta \epsilon_{x_N} - \Delta \epsilon_{y_N}) + \mu_N \Delta \bar{H}_N + L_{xx_N} - \frac{\Delta \beta_N}{3}$$

$$\Delta \epsilon_{zz_N} = \frac{1}{3} (\Delta \epsilon_{x_N} + \Delta \epsilon_{y_N}) - \frac{\Delta \bar{H}_N}{2} + \frac{2}{3\mu_N} (\Delta \sigma_N - L_{zz_N}) + \frac{1}{3} \alpha \Delta T_N$$

$$\Delta \beta_N = 2\mu_N \alpha \Delta T_N + (\Delta \sigma_N - L_{zz_N})$$

$$\bar{K}_N = \frac{8\mu_N(1 - \frac{\mu_N}{6K})}{3(1 + \frac{4\mu_N}{3K})}$$

The symbol $\Delta \sigma_N$ denotes the incremental change in the specified thickness stress, L_{zz_N} denotes the history term associated with ϵ_z , the other symbols have their usual meanings.

REFERENCES

- G-1 Herrmann, L. R., "Elasticity Equations for Incompressible and Nearly Incompressible Materials by a Variational Theorem", AIAA J., Vol. 3, No. 10, pp. 1896-1901, October 1965.

APPENDIX H

A COMPUTER PROGRAM FOR VISCOELASTIC SOLIDS
OF REVOLUTION SUBJECTED TO TIME - VARYING THERMAL
AND MECHANICAL LOAD ENVIRONMENTS

VERSION 2.1

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Appendix H

A COMPUTER PROGRAM FOR VISCOELASTIC SOLIDS OF REVOLUTION
SUBJECTED TO TIME-VARYING THERMAL AND MECHANICAL LOAD ENVIRONMENTS

- VERSION 2.1 -

I. INTRODUCTION

The purpose of the computer code described in this section is to perform viscoelastic stress analyses. The analyses is applicable to arbitrary revolved solids and plane structures subjected to loads of mechanical or thermal origin. The program is segmented into two (2) phases: (1) Transient Heat Transfer Analysis; and (2) Viscoelastic Stress Analysis. The purpose of the heat transfer phase is to generate temperature distributions in the body as a function of time which are used subsequently in performing the stress analysis. The two phases of the program can be used in sequence within a given job or each phase can be used separately.

The method of solution employs the finite element procedure for solving the spatial problems (heat conduction and stress analysis) and time marching techniques to evaluate temperatures/stresses at successive points in time. The transient heat transfer problem is solved using the procedure developed by Wilson and Nickell*. Knowing thermal and mechanical loads as a function of time and having available the viscoelastic properties of the material(s), a set of equivalent elastic parameters is defined for a particular point in time; the equivalent elastic problem is posed using the procedure given by Herrmann and Peterson**. An elastic stress analysis must be performed at each point in time for which the viscoelastic response of the body is required. The stress analysis problem is solved using a finite element method given by Herrmann***.

II. APPLICATIONS

The principal application of the program is to stress analysis of solid propellant grains maintained in time varying temperature environments. A typical application might be the grain stress analysis of a motor system subjected to thermal cycling. Mechanical loads may also be applied to the body either isothermally or with simultaneous time varying temperature changes. The kinds of mechanical loading considered in the analysis include surface pressures, body forces (spin and/or axial accelerations), concentrated nodal forces, and specified nodal displacements; all mechanical loads can vary with time in accordance with user-supplied table.

* Wilson, E. L., and Nickell, R. E., "Application of the Finite Element Method to Heat Conduction Analysis", Nuclear Engineering and Design 4 (1966), p. 276-286. North Holland Publishing Company, Amsterdam.

** Herrmann, L. R., and Peterson, F. E., "A Numerical Procedure for Viscoelastic Stress Analysis", CPIA - 7th Meeting, Working Group on Mechanical Behavior (ICRPG), November 1968.

*** Herrmann, L. R., "Elasticity Equations for Incompressible Materials by A Variational Theorem", Journal of the AIAA, p. 1896-1900, October 1965.

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Appendix H

A. PHYSICAL PROPERTIES

The program can be used to perform simple elastic solutions, time-dependent elastic or thermoelastic analyses, or thermoviscoelastic analyses. A viscoelastic analysis requires a complete material property characterization including the master relaxation curve and shift function for each time dependent material, bulk modulus, expansion coefficient, density, etc. Table H-1 illustrates the thermal and mechanical properties data required to perform the thermoviscoelastic stress analysis of a bipropellant grain with a separate viscoelastic liner and an elastic case.

Shift function data is accepted by the program in the form of a table of $\log a_T$ versus T , ($^{\circ}F$); shift factors at temperatures other than those supplied in the input table are determined using linear interpolation between given points.

Relaxation data must be input as the coefficients (A_i, β_i) in a Prony Series fit to the experimental data. The relaxation behavior in shear must be expressed as an exponential series.

$$\phi(t) = A_0 + \sum_{i=1}^n A_i e^{-\beta_i t} \quad (H-1)$$

where A_0 is the shear equilibrium modulus and (A_i, β_i) are found from curve fitting* calculations based on experimental data. Each viscoelastic material must have its relaxation behavior expressed in a separate series expansion. Elastic materials have no terms in the series other than A_0 . If there are no terms in the series expansion ($M = 0$) the program will not read a shift function table.

The special problem of bulk rapid pressurization of a propellant grain where a pressure shift function a_P is required is handled in a different manner. This type of problem^P can only be run isothermally with no superimposed thermal loads. In this case the shift function input values are interpreted as the product $a_P a_T$ as a function pressure where a_T is a constant for the temperature under consideration.

* One recommended curve fitting procedure is a "collocation method" originated by Schapery and summarized in the ICRPG Solid Propellant Mechanical Behavior Manual starting at Section 2.2-4 (June 1963). The method involves assuming values for the constants β_i and solving a set of linear simultaneous equations for the constants A_i . A time-share routine called "PRONY" has been programmed to perform the calculations at Aerojet.

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PHYSICAL PROPERTY INFORMATION REQUIRED
FOR A THERMOVISCOELASTIC STRESS ANALYSIS

PROBLEM IDENTIFICATION:

Propellant No. 1 -
Propellant No. 2 -
Liner -
Case -

Physical Property	Propellant No. 1	Propellant No. 2	Liner	Case
Coefficient of Linear Thermal Expansion ($^{\circ}\text{F}^{-1}$)				
Density (lb-in.^{-3})				
Specific Heat Capacity ($\text{btu-lb}^{-1} \text{ } ^{\circ}\text{F}^{-1}$)				
Thermal Conductivity, ($\text{Btu-in.}^{-1} \text{ hr}^{-1} \text{ } ^{\circ}\text{F}^{-1}$)				
Bulk Modulus (lb-in.^{-2})				
Shear Relaxation Modulus ($\text{lb.-in.}^{-2} \text{ vs. hr}$)	(Table or curve as a function of time)			
Shift Factor ($a_T \text{ vs. } ^{\circ}\text{F}$)	(Table or curve as a function of temperature)			
Elastic Shear Modulus (lb-in.^{-2})				

Table H-1

III. DATA INPUT DESCRIPTION

This section supplies information necessary for the preparation of data input cards. The input sequence is separated into four major groups:

1. Grid Definition
2. Solution Time and Temperature Information
3. Transient Heat Transfer Solution Data
4. Stress Analysis Information

A. NOMENCLATURE

The abbreviation "cc" used below stands for "card columns". The variable names assigned to the various parameters used by the program are given below in upper case letters; for example, "NMF" stands for the total number of nodes in the finite element mesh. All variables starting with any of the letters I, J, K, L, M, N are to be input to the program as integers (i.e., without a decimal point). All integers are to be packed to the right of the field specified by the "cc" numbers. Any variable whose first letter is not an I, J, ..., N is a real number requiring a decimal; "R(N)", for example, is the radius of the 'N-th' nodal point (entered in cc 6-15). If $R(N) = 13.45$, then the number "13.45" can be placed anywhere in the field: cc 6-15. Real numbers can also be input in "E" format; 13.45 could be entered as 1.345E1, .1345E2, 1345.E-2, etc., providing the set of characters is packed to the right of the cc field.

Whenever applicable, units of the variables are stated in symbolic notations. The symbols used below are defined as follows:

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(F) = Force units
(L) = Length units
(T) = Time units
(°F) = Temperature units
(R) = Radian
(Btu) = Thermal Heat Flow Units

Thus, the quantity $(F) (L)^{-2}$ written after the shear modulus means "psi" if pounds and inches are the units chosen by the user. There are no units, conversion factors, etc., built into the program; thus, once a set of units is chosen it must be used consistently throughout the analysis.

B. SEQUENCE OF OPERATIONS

The flow of program execution is controlled by three (3) user-supplied variables:

IPF = Plot Control Flag
NTEM = Temperature Information Flag
JOB = Job Control Flag

Table H-2 contains values which can be assigned to these variables showing what operation results from a particular specification. Certain combinations are not possible. For example, if $IPF < 0$, the program reads mesh data, prepares a plot and returns control to that portion of the program which looks for the next job; thus, the variables "NTEM" and "JOB" cannot be specified. "JOB" has no meaning unless $IPF \geq 0$ and $NTEM = 0$.

The option $NTEM = 1$ is impractical for real problems because of the amount of card data involved; this option is useful when solving "check cases" with temperature distributions generated from an analytical expression or formula. The $NTEM = 2$ option saves re-running the temperature problem if only the mechanical properties or mechanical loads change. $NTEM = 3$ is used for the isothermal problem in which the loading is mechanical in origin. Viscoelastic materials exhibit temperature dependence, so material temperatures must be defined even though there are no driving thermal strains in the body to be analyzed.

The variable "JOB" (which is only defined if $NTEM = 0$) controls what the program does with the results of the heat transfer solution and where the program goes after the heat transfer calculations. If $JOB = 0$, element temperatures are saved temporarily for use in the stress analysis to follow. $JOB = 1$ results in the same operation as $JOB = 0$, but in addition the element temperatures are saved for use in a later analysis (or series of analyses). If $JOB = 2$, the temperatures are saved (and printed at user specified time intervals) on a tape for use at some other time; at this point the program is through with this job and looks for another. $JOB = 3$ allows the user to run the program solely as a heat transfer analysis.

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EXECUTION CONTROL VARIABLES

FLAG	VALUE	OPERATION PERFORMED
IPF	< 0	Read grid data only, plot grid and <u>stop</u> .
	= 0	Read all data, <u>execute</u> the job <u>without</u> a plot.
	> 0	Read all data, <u>execute</u> the job <u>with</u> a plot.
NTEM	= 0	Calculate the element temperatures using the heat transfer analysis.
	= 1	Read the element temperatures from card input.
	= 2	Read the element temperatures from a tape created during a previous run.
	= 3	Read the element temperatures from card input and use the same distribution for all solution time points (isothermal response).
JOB	= 0	Run the heat transfer problem and use the results to perform the stress analysis.
	= 1	Run the heat transfer problem, save the element temperatures on a permanent file (which can be used as input to subsequent jobs) and use the results to perform the stress analysis.
	= 2	Run the heat transfer problem, save the element temperatures on a permanent file, and <u>stop</u> .
	= 3	Run the heat transfer problem and <u>stop</u> .

Table H-2

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C. DATA CARD INPUT

1. Grid Definition

a. Start Cards

(1) First Card (A3)

cc

1-3 Enter the characters "TVA"

(2) Title Card (12A6)

cc

1-72 HED Title information for job and plot (center about cc 36
for plot)

b. Control Card (One card, 3I5)

cc

1-5 NNP Number of nodal points $\leq$ 273

6-10 NEL Number of quadrilateral elements $\leq$ 240

11-15 IPF Plot flag
< 0, plot only
= 0, run only
> 0, plot and run

c. Node Coordinate Cards (I5, 2F10.3, I5, 2F10.3, I5)

cc

1-5 N Node Number

6-15 R(N) Radial Coordinate (L)

16-25 Z(N) Axial Coordinate (L)

26-30 NE Ending Node Point Number

31-40 R(NE) Radial Coordinate (L)

41-50 Z(NE) Axial Coordinate (L)

51-55 NI Node Number Increment

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If the ending node number NE is not zero (or blank), then nodes will be generated in equal distance increments along a line between node N and node NE. The first generated node is assigned the number $N + NI$; the second generated node number is $N + 2NI$, etc. Note that the node number difference $(NE - N)$ must be positive and divisible by NI.

If NI is omitted it will be assigned a value of "1" automatically.

d. Element Numbering Cards (815)

cc

1-5	N	Element Number
6-10	IX (N, 1)	{ Node numbers describing the corner points of the quadrilateral }
11-15	IX (N, 2)	
16-20	IX (N, 3)	
21-25	IX (N, 4)	
26-30	MN(N)	Material Number
31-35	NIG	Number of elements to be generated
36-40	NNI	Node Number Increment

If the number of elements to be generated (NIG) is not zero, then NIG elements will be generated. The first generated element is assigned the number $N + 1$; the second generated element is numbered $N + 2$, etc. The node numbers of the second generated element are found by adding NNI to the node numbers of the first generated element, etc. If NNI is omitted it will be assigned a value of "1".

The material number assigned to generated elements is the same as that for element N. Material numbers are not necessary if the program is to plot the grid only ($IPF < 0$).

If $IPF < 0$; data input ends here.

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Appendix B

2. Solution Time and Temperature Information

a. Control Card (One card 2I5, F10.0, 2I5, F10.0).

cc

- | | | |
|-------|------|--|
| 1-5 | NDT | Total number of time increments to be used in the solution for element temperatures and/or stresses. |
| 6-10 | NTR | Number of regions on the time axis having the same value of time increment ratio. ≤ 50 |
| 11-20 | TMF | Value of time at the end of the NDT th increment. (T) |
| 21-25 | NBCF | Number of functions describing time dependent boundary conditions. ≤ 10 |
| 26-30 | NTEM | Temperature information flag:

= 0 Element temperatures are to be calculated using the heat transfer analysis.

= 1 Element temperatures are to be read from cards.

= 2 Element temperatures are to be read from a tape created on a previous run.

= 3 Element temperatures are to be read from cards, and this distribution is to be used for all solution time points. |
| 31-40 | TZ | Temperature of every element at time zero (°F) (stress free temperature). |

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b. Solution Time Points

At least one card in this section

(1) First Card (3 (I5,2F10.0))

cc

1-5	NTI(1)	Number of increments for which the time increment ratio C(1) remains constant in region 1.
6-15	DT (1)	Value of time after NTI(1) time increments in region 1.
16-25	C (1)	Time increment ratio in region 1.
26-30	NTI (2)	Same as cc 1-5 for region 2.
31-40	DT (2)	Value of time after NTI (1) HNTI (2) time increments (T)
41-50	C (2)	Same as cc 16-25 for region 2
51-55	NTI (3)	
56-65	DT (3)	Region 3
66-75	C (3)	

(2) Second Card (3 (I5, 2 F10.0)) (If required)

cc

1-5	NTI (4)	
6-15	DT (4)	Region 4
16-25	C (4)	

... Etc.

Use as many cards in this section as are required to enter NTR groups of (NTI (I), DT (I), C (I)). Note that

NTR
 $\sum_{i=1} \text{NTI}_i = \text{NDT}$, otherwise an error message will be issued by the program.

The solution time points in region r are found using:

$$t_{i+1} = t_i + C_r * (t_i - t_{i-1})$$

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This method of solution time point input has the effect of automatically producing small time steps at the beginning of the region while continuously widening the time steps toward the end of the region. As an example, if 50 solution time points are specified over a 24 hour period with $C = 1.1$ the first solution time point will occur at .0206 hrs (1.24 minutes) and the last solution step ($t_{50} - t_0$) will be 2.19 hours (132 minutes).

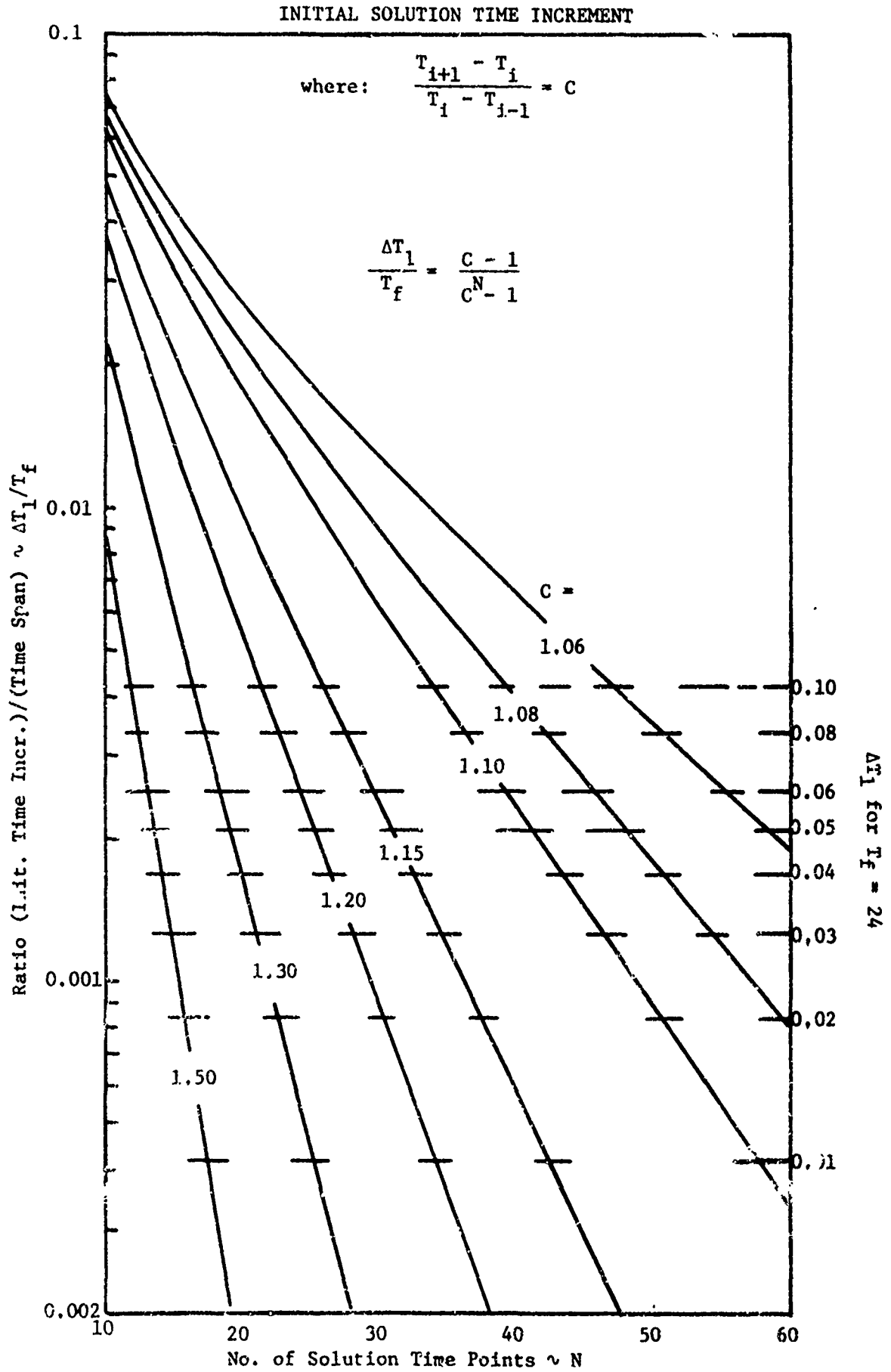
It is important to choose C such that the smallest time step will not be so small as to produce a reduced time falling off the mechanical properties table or the final time step too large. The following equations may be used to find these values:

$$\frac{\Delta T_i}{T_f} = \frac{C-1}{C^N-1} \quad \frac{\Delta T_N}{T_i} = \frac{C^{N-1}(C-1)}{C^N-1} \quad (H-2)$$

where

- C = time increment ratio
- ΔT_i = Initial solution time step
- ΔN = total number of solution time points
- ΔT_N = final time step
- T_f = final value of time ($T_0 = 0$)

These two equations are plotted for various values of C in Figures H-1 and H-2 respectively.



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FINAL SOLUTION TIME INCREMENT

where: $\frac{T_{i+1} - T_i}{T_i - T_{i-1}} = C$

$$\frac{\Delta T_N}{T_f} = \frac{C^{N-1} (C - 1)}{C^N - 1}$$

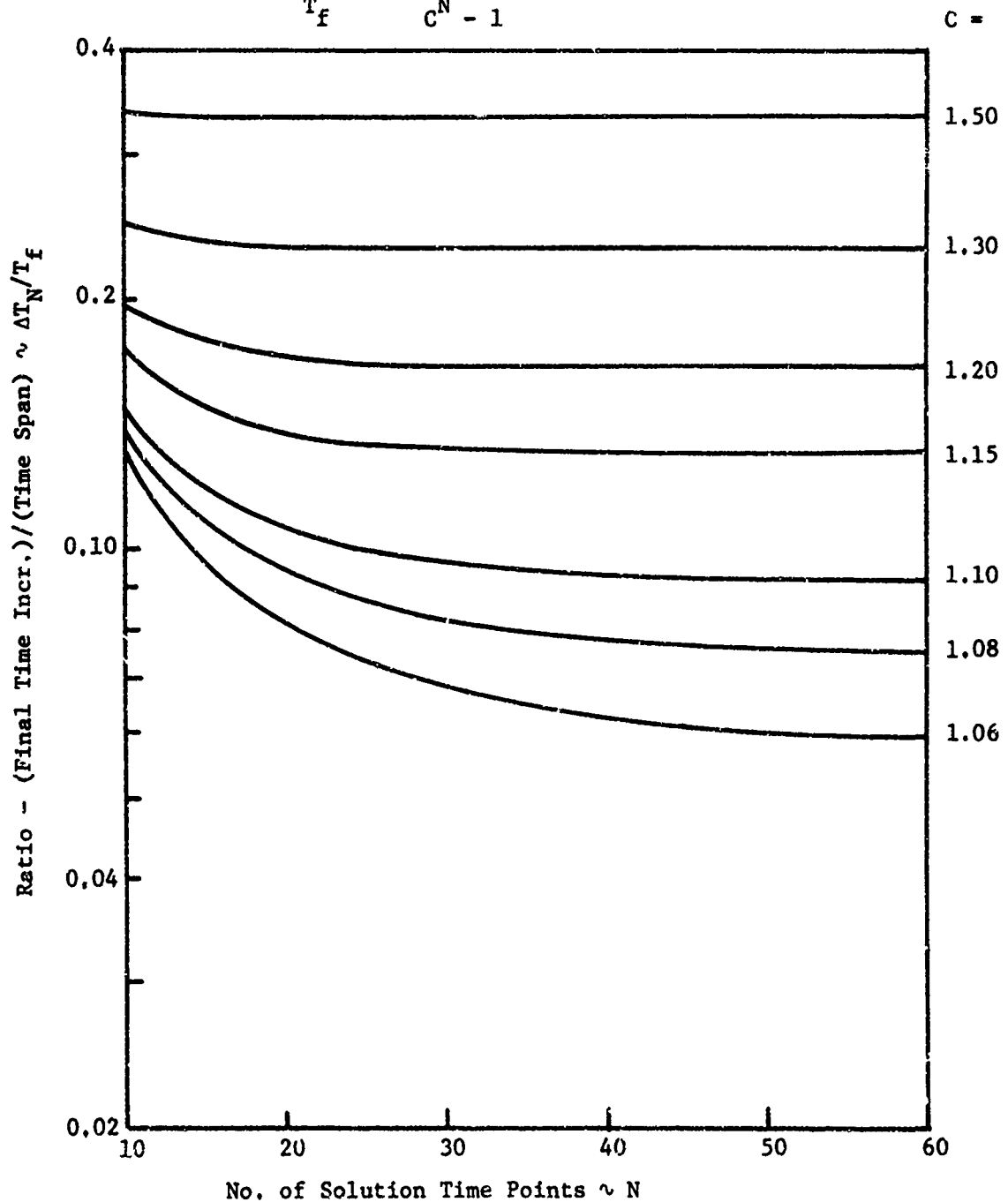


Figure H-2

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c. Boundary Condition Function Cards

Skip this section if NBCF = 0

(1) Card One (2I5)

cc

1-5	N	Number assigned to this function
6-10	NPTS(N)	Number of points in the table describing this function. ≤ 30

(2) Card Two (8F10.0)

cc

1-10	TFN (N, 1)	Time at point 1, t_1 (T)
11-20	FN (N, 1)	Value of the function at time t_1 , f_1
21-30	TFN(N, 2)	Time at point 2, t_2 (T)
31-40	FN (N, 2)	Value of the function at time t_2 , f_2
41-50	TFN (N, 3)	} Point 3
51-60	FN (N, 3)	
61-70	TFN(N, 4)	} Point 4
71-80	FN (N, 4)	

(3) Card Three (8F10.0) (If required)

cc

1-10	TFN (N, 5)	} Point 5
11-20	FN (N, 5)	

... etc.

Use as many cards 2, 3, etc., in this section as are required to enter NPTS(N) pairs of (TFN(N, I), FN(N, I)) which define this function (Number N). There are NBCF sets of cards 1, 2, 3, etc. in this section. Data for a new function begins on a new card 1.

Figure H-3 represents a function which might be used to describe the pressure transient in a motor. The last point in the table must have a value of time which is greater than or equal to the length of solution period (TMF); for this case $16.0 > 14.0 = \text{TMF}$ (Figure 3.1).

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EXAMPLE BOUNDARY CONDITION FUNCTION

I	TFN (3, I)	FN (3, I)
1	0.0	0.0
2	2.0	0.8
3	4.0	1.0
4	5.0	0.7
5	16.0	0.7

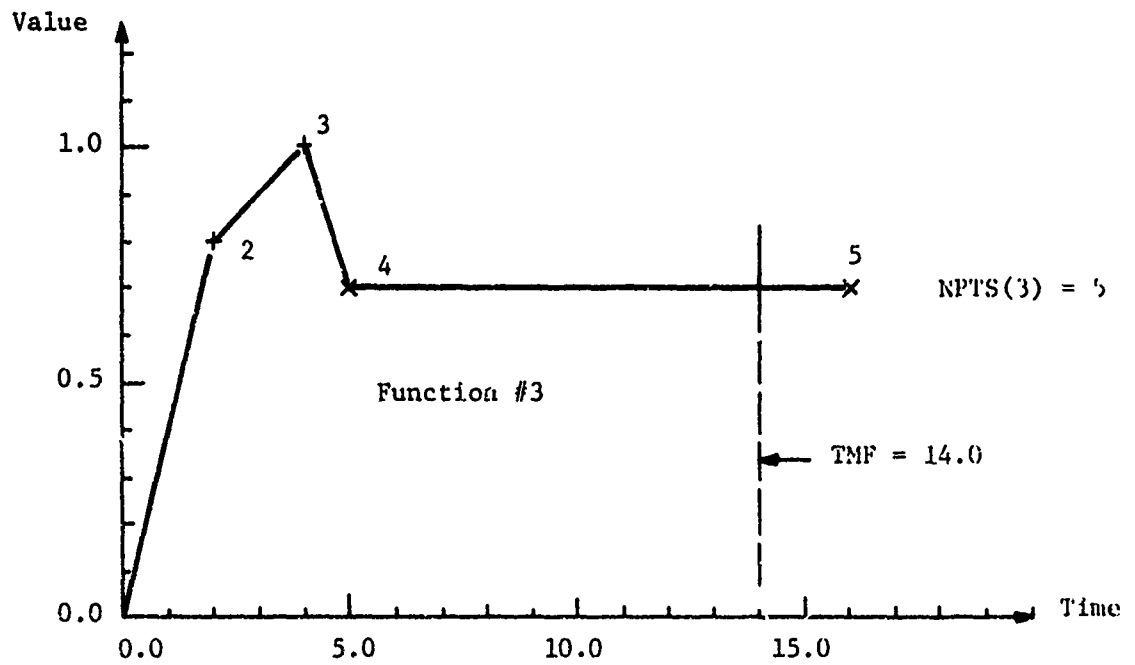


Figure H-3

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If no data are input in this section (NBCF = 0), then all boundary specifications are independent of time and boundary values are assumed to be imposed as step function at $t = 0^+$.

At least one card in this section

• • • •

1-5	NO(9)	Output Interval
6-10	NPR(9)	Number of print operations at this interval
11-15	NO (10)	Output Interval
16-20	NPR (10)	Number of print operations at this interval

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Suppose that temperature output is required at time points 2, 4, 7 and 8 of Figure 2.1, then one card would be input in this section:

Card 1: 2,2,3,1,1,1 ; [2 (2) + 3 (1) + 1 (1) = 8].

If temperatures at all 8 points is requested, then

Card 1: 1, 8 ; [1 (8) = 8].

e. Average Element Temperature Cards (3I5, F10.0)

(Card 2-a is the reference for values of NDT, NTEM and TZ).

Skip this section if NTEM = 0, or NTEM = 2
--

cc

1-5	NELS	Starting element number for this group.
6-10	NELG	Number of elements with the same average temperature as element number NELS.
11-15	NELI	Element number increment.
16-25	TAVG	Average element temperature for this group of elements. (°F)

If NTEM = 1, there are NDT sets of element temperatures that must be defined in this section (one set for each of the NDT time points contained in the solution period). All element temperatures must be prescribed at a given time point before proceeding to the next point. If every element has a different temperature, then NEL cards (with cc 6-15 blank) must be prepared for that time point. It is possible to generate element temperatures at a time point if several elements are at the same temperature. NELG elements are assigned average temperatures TAVG. The number assigned to the first generated element is NELS + NELI; the second is NELS + 2 (NELI), etc.

Suppose that a body described using 50 elements (NEL = 50) is at a uniform 77°F, when $t \leq 0$ (TZ = 77.0), and at the end of the first time increment elements 1-25 drop to 60°F while elements 26-50 reach 45°F. Two cards are required to define the element temperature distribution at the end of time increment one ($t = t_1$):

Card 1: 1, 24, 1, 60.0

Card 2: 26, 24, 1, 45.0

If NTEM = 3, then there is only one set of element temperatures which must be input in this section. This temperature distribution applied to all solution time points; the body starts at $t = 0^-$ at a uniform temperature TZ (at which the body is assumed to be "stress free"), and at $t = 0^+$ the element temperatures assume the values prescribed in this section of the data input and remain constant for all time, $t > 0$.

3. Transient Heat Transfer Solution Data

Skip this section if NTEM = 1, 2, or 3

a. Control Card (One card; 4I5)

cc

1-5 NMAT Number of materials with different thermal properties. ≤ 10

6-10 NNBC Number of nodal point boundary conditions (temperatures or heat fluxes)

11-15 NCBC Number of convection boundary conditions ≤ 65

16-20 JOB Job Control Flag

 = 0 : run heat transfer and use the results to perform the stress analysis.

 = 1 : run heat transfer, save element temperatures on a permanent file, and use the results to perform the stress analysis.

 = 2 : run heat transfer, save the results on a permanent file, and stop.

 = 3 : run heat transfer and stop.

21-25 KAT KAT = 0 Axisymmetric Analysis
 KAT $\neq$ 0 Planar Analysis

b. Material Cards (1I0, 6F10.0)

cc

1-10 N Material number

11-20 XCOND(N) Conductivity: K_{rr} , (Btu)(T) $^{-1}$ (L) $^{-2}$ ($^{\circ}$ F/L)

21-30 YCOND(N) Conductivity: K_{zz}

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31-40 XYCOND(N)	Conductivity: K_{rz}
41-50 SPHT (N)	Specific Heat, $(\text{Btu})(F)^{-1}(\text{°F})^{-1}$
51-60 DENS(N)	Weight density, $(F)(L)^{-3}$
61-70 QX(N)	Heat generated per unit volume, $(\text{Btu})(T)^{-1}(L)$

Use one card for each different material number assigned in the element array (cards 1-d); NMAT cards must be prepared in this section - order is unimportant.

If the thermal conductivity of a material is independent of direction, then

$$K_{rr} = K_{zz} = k$$

$$k_{rz} = 0$$

The heat generated per unit volume is assumed to be constant with time.

c. Nodal Point Boundary Conditions (215, F10.0, 15)

Skip this section if NNBC = 0

cc

1-5	N	Node Number
6-10	KODE(N)	Boundary condition type $\left\{ \begin{array}{l} = 0, \text{ externally supplied heat flux} \\ = 1, \text{ prescribed node temperature} \end{array} \right.$
11-20	T(N)	Boundary value amplitude $\left\{ \begin{array}{l} = \text{Heat flux (KODE(N) = 0), } (\text{Btu})(T)^{-1} (R) \\ = \text{Temperature (KODE(N) = 1), } (\text{°F}) \end{array} \right.$
21-25	NFN(N)	Function number

All nodal points not specified in this section are assumed to have externally supplied heat flux of zero for all values of time.

A function number equal to zero (or blank) means that the prescribed boundary condition is applied at time zero and remains constant for all time, $t > 0$.

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The functions assigned in this section must have been defined previously in Section 2-c. For time varying boundary conditions, the magnitude of the boundary value at some time t is found by selecting the value of the function at t and then multiplying this value times the boundary value amplitude (cc 11-20). A given function can be used to describe any number of boundary conditions.

d. Convection Boundary Conditions

Skip this section if NCBC = 0

(NCBC Cards, 215, 2F10.0, 15)

cc

1-5	I(N)	Node number i
6-10	J(N)	Node number j
11-20	H(N)	Heat transfer coefficient, h : $(\text{BTU})(T)^{-1}(L)^{-2} (^{\circ}\text{F})^{-1}$ $q = h (T - T_o)$
21-30	TE(N)	Environmental temperature amplitude, $\bar{T}_o$
31-35	NFCV(N)	Function number

If the environmental temperature T_o is time dependent, then a non-zero function number must be specified in cc 31-35. $\bar{T}_o$ will be multiplied by the appropriate value of the function of time t in order to establish the value of environmental temperature, T_o .

If the environment does not change temperature with time, then $\text{NFCV}(N) = 0$ and $T_o = T_o$, constant for $t > 0$.

If JOB = 2 or 3 (Card 3-a), then data ends here

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4. Stress Analysis Information

The program will not read data in this section if
NTEM = 0 (card 2a) and JOB = 2 or 3 (card 3a)

a. Title card (20A4)

cc

1-8 HED Any alpha numeric information (printed with
the solution)

b. Control Card (5I5, 2(F10.0, I5), 3I5, F10.0)

cc

1-5	NMAT	Number of materials	≤ 4
6-10	NCMN	Number of elements with material identification numbers which are to be redefined.	
11-15	NBCN	Number of node points for which boundary cards are used	≤ 60
16-20	NPC	Number of pressure cards	≤ 55
21-25	NDMG	Number of elements for damage evaluation	≤ 20
26-35	ANGV	Angular velocity amplitude, (R) (T) ⁻¹	
36-40	NFAV	Function number for ANGV	
41-50	AZZ	Axial acceleration amplitude, (L) (T) ⁻²	
51-55	NFAZ	Function number for AZZ	
56-60	IDSF	Pressure boundary condition function no. for use of pressurization shift function data	
61-65	IPSC	Geometry type flag	
		IPSC = 0 Axisymmetric analysis	
		= 1 Plane strain analysis	
		= 2 Generalized plane strain analysis	
		= 3 Generalized plane stress analysis	
66-70	NSM	Material No. (if IPSC = 2) of case material	
71-80	SZV	Value of normal stress (if IPSC = 3) (F) (L) ⁻²	

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NMAT is the total number of materials (viscoelastic and elastic).

If the material I.D. numbers assigned to the elements in Section 1-d are appropriate for both the heat transfer and stress analyses, then NCMN is set to zero (or blank). NBCN is a count of nodes at which force and/or displacement boundary values are specified. NPC is the total number of element sides subjected to pressure loads.

IDSP is the boundary condition function number which describes the bulk pressure as a function of time when an isothermal pressurization case is being run. For a thermal analysis leave this field blank. This function will be used to find the pressurization shift function values in the properties determination and also will be used in the damage calculations. When IPSC - 2 a special generalized plane strain analysis will be run. The normal strain will be set equal to the average thermal strain ($\alpha_c \Delta T_c$) in the case which is found by the material number NSM specified in cc 66-70.

c. Material Properties

(1) Control Card (3I5, 4F10.0)

cc

1-5 K Material number

6-10 NON(K) Number of terms in the Prony series representation of the shear relaxation function. ≤ 16

11-15 NSFP(K) Number of points in the shift function table. ≤ 16

16-25 APO(K) Equilibrium shear modulus, (F) (L)⁻²

26-35 XK(K) Bulk modulus, (F)(L)⁻²

36-45 AIP(K) Linear coefficient of thermal expansion, (L)(L)⁻¹(°F)⁻¹

46-55 DENS(K) Mass density, (F)(L)⁻⁴(T)²

The shear relaxation function is written in the form:

$$\phi(t) = A_0 + \sum_{i=1}^M A_i e^{-\beta_i t}$$

where

NON(K) = M for the Kth material
APO(K) = A₀ for the Kth material

An elastic material is input by leaving cc 6-15 blank and entering the shear and bulk moduli in cc 16-25 and cc 26-35, respectively. A non-zero value of density is required for the calculation of body forces arising from specified values of spin velocity and/or axial acceleration.

(2) Prony Series Coefficients Card(s) (8F10.0)

Skip this section if NON(K) = 0

cc

1-10	AP(K, 1)	A ₁ for material K	(F)(L) ⁻²
11-20	BP(K, 1)	β_1	(T) ⁻¹
21-30	AP(K, 2)	A ₂	(F)(L) ⁻²
31-40	BP(K, 2)	β_2	(T) ⁻¹
41-50	AP(K, 3)	A ₃	(F)(L) ⁻²

etc.

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Use as many cards in this ſection as are required to ſpecify
NON(K) pairs of (A_1 , B_1); four pairs per card.

(3) Shift Function Table (3F10.0)

Skip this ſection if NSFP(K) = 0

cc

1-10	FST(K,1)	Temperature at 1st point, T_1 (°F)
11-20	FS(K,1)	$\text{Log}_{10} a_{T_1}$
21-30	FST(K,2)	Temperature at 2nd point, T_2 (°F)
31-40	FS(K,2)	$\text{Log}_{10} a_{T_2}$
41-50	FST(K,3)	etc.

Use as many cards in this ſection as are required to ſpecify
NSFP(K) pairs of (T_1 , $\text{log}_{10} a_T$); four pairs per card. If IDSP > 0 (card

4b) FST (K, 1) will be interpreted as pressures and FS (K, 1) are $\text{Log}_{10} a_p$.

d. Element Material Numbers

Skip this ſection if NCMN = 0

cc

1-5	N1 (1)	Element number
6-10	N2 (1)	Material number assigned to element N1 (1)
11-15	N3 (1)	Number of elements with the ſame material number as element N1 (1)
16-20	N4 (1)	Element number increment
21-25	N1 (2)	Element number
26-30	N2 (3)	Material number assigned to element N1 (2)
31-35	N3 (2)	Number of elements with the ſame material number as element N1 (2)

etc.

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Use as many cards in this section as are required to re-define the material numbers of NCMN elements; 16 entries per card are possible.

Suppose that a 50 element problem is to have all its material numbers changed (NCMN = 50), and all even numbered elements are material 1 while all odd elements are material 2; one card describing these changes would read:

[1, 2, 24, 2, 2, 1, 24, 2]

e. Node Point Boundary Specification(s)

At least one card in this section

(I5, 2(I5, F10.0, I5))

cc

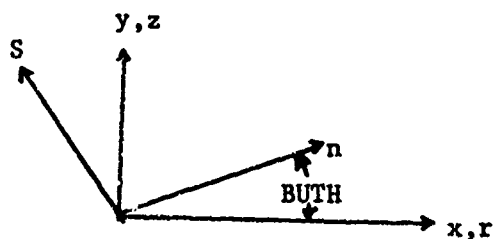
1-5	K	Node point number (NB(N) = K)	
6-10	NFLR(N)	Radial boundary condition type	
		$\begin{cases} = 0; \text{externally applied } \underline{\text{force}} \\ = 1; \text{specified } \underline{\text{displacement}} \end{cases}$	$\begin{matrix} (F) (R)^{-1} \\ (L) \end{matrix}$
11-20	BVR(N)	Radial boundary value amplitude	
21-25	NFNR(N)	Function Number	
26-30	NFLZ(N)	Axial boundary condition type	
		$\begin{cases} = 0; \text{externally applied } \underline{\text{force}} \\ = 1; \text{specified } \underline{\text{displacement}} \end{cases}$	$\begin{matrix} (F) (R)^{-1} \\ (L) \end{matrix}$
31-40	BVZ(N)	Axial boundary value amplitude	
41-45	NFNZ(N)	Function number	
46-55	BUTH(N)	Skew boundary angle (degrees)	

A total of NBCN cards must be prepared in this section. The axial displacement at one node must be specified as a minimum requirement.

Positive boundary values are in the direction of the positive coordinate axes.

Zero (or blank) function numbers assigned to boundary value components implies time independence (constant for $t > 0$). The variation of a boundary value with time is determined by multiplying amplitude times the appropriate value of the corresponding function. All nodes not specified in this section are assumed to have no externally applied loads and are free to displace as the solution dictates.

The shew boundary is shown in the figure below. If $BUTH(N) \neq 0$ the boundary conditions are expressed in the n - s system.



f. Pressure Loads

Skip this section if $NPC = 0$

cc

1-5	IPC(N)	Node number i
6-10	JPC(N)	Node number j
11-20	PR(N)	Pressure amplitude (F) (L) ⁻²
21-25	NFNP(N)	Function

There are NPC cards in this section. $NFNP(N) = 0$ means that pressure is applied as a step function at $t = 0^+$.

Positive pressure acts in the direction shown in Figure 4.1.

g. Damage Parameters

Skip this section if NDMG = 0

cc

1-5	LDMG(N)	Element no. whose damage is to be evaluated
6-15	STZR(N)	σ_{to}
16-25	TZR(N)	t_o See below
26-35	SCR(N)	σ_{cr}

There are NDMG cards in this section. LDMG(N) may be positive or negative. If positive, the hoop stress will be used in damage calculations; if negative the maximum principal r-z stress will be used. The damage is evaluated using

$$PED_t = \frac{1}{a_T} \int_0^t \frac{(\sigma(t') - \sigma_{cr} + P(t'))^B}{(\sigma_{to} - \sigma_{cr})^B a_p (P(t'))} dt$$

If IDSP = 0 (card 4b) then $P(t') = 0$ for all times. If IDSP > 0 the $P(t')$ will be found from the boundary condition function indicated by IDSP.

Data input ends at this point

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IV. PROGRAM OUTPUT DESCRIPTION

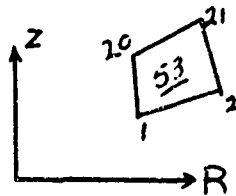
Output from the program includes:

1. Input geometry, material properties and solution time information in a self-explanatory format.
2. Nodal point temperatures at the specified time-point interval.
3. The radial, tangential, axial and shear stresses and strains and the average element temperature at the specified time-print interval.
4. The damage rate and accumulated damage for specific elements at each time-print interval.

The printed program output consists of (1) "echo" reproduction of the data input cards identified in self-explanatory format, and (2) results of the analysis which might include nodal point temperatures, element stresses/strains, node displacements, etc.

A. ELEMENT NODE NUMBERING

It should be noted that element node number data may not be listed in the same order as these data appear on the input card. The program logic permutes the order of the element node numbers so that the largest node number is always last (4th) in the printed list for each element. For example, if element number 53 (shown below)



is input as (1, 2, 21, 20), the program will print the data as (20, 1, 2, 21) so that "21" is last while the original counter clockwise order is preserved. Efficiency is gained if the user specifies the 4th node as the largest for every element.

The reason for having the largest node as last one in the sequence for a given element is that three (3) equations are assigned to this node as opposed to only two (2) equations per node for the others. A node number that is not the largest one in any element has the R and Z displacement components as its unknowns (2 total). All other nodes have R, Z displacement components plus the "mean pressure variable" H as unknowns at that point (3 total). Thus, node "21" of element "53" above has assigned to it U_{R21} , U_{Z21} , and H_{53} as unknowns.

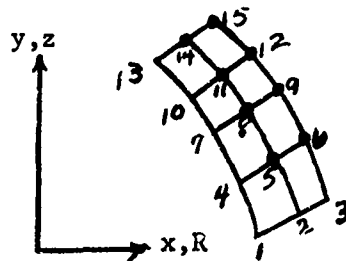
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In order to assign equations uniquely to the unknown values of "H" for every element, the program has the following restriction:

The finite element quadrilateral mesh must be numbered so that any node number is maximum in only one (1) element.

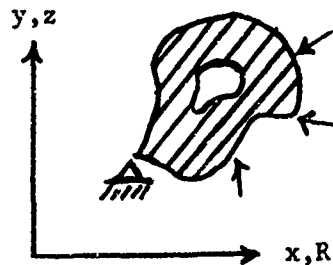
With a "layered" numbering scheme (across the "narrow" direction of the grid) the restriction is complied with automatically:



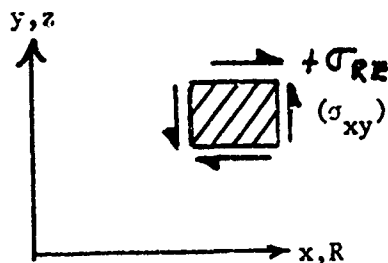
The "heavy" nodes (●) shown above have three (3) unknowns associated with them.

B. SIGN CONVENTION

All positive vector quantities (forces/displacements) are in the directions of the positive coordinate axes providing the grid is positioned in the "positive" R-Z quadrant:



Normal stress (strains) are tensile when positive and the shear stress (strain) sign convention is shown below:



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$$PED_t = \frac{1}{a_t} \int_0^t \frac{(\sigma(t') - \sigma_{cr} + P(t'))^B}{(\sigma_{to} - \sigma_{cr})^B a_p (P(t'))} dt'$$

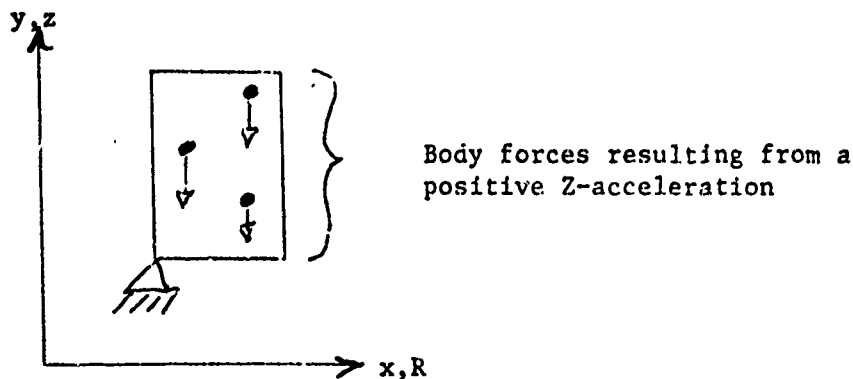
If IDSP = 0 (Card 4b) then $P(t') = 0$ for all times. If IDSP > 0 the $P(t')$ will be found from the boundary condition function indicated by IDSP.

The value of $P(t')$ is that portion of the inner-bore firing pressure, $P_i(t)$, which is transmitted to the point in the grain where the principal stress, $\sigma(t')$, is evaluated. The relation of $P(t')$ to $P_i(t')$ in infinite-length cylinders is well known () (See Section IV of this report). For finite length cylinders the ratio of $P(t')$ to $P_i(t)$ must be obtained from the stress analysis.

For metal cases $P(t')$ seldom differs from $P_i(t')$ by more than 5%. For most firing problems this difference is not significant and $P(t')$ can be approximated by $P_i(t')$.

Data input ends at this point

A positive spin velocity produces hoop tension, and positive axial acceleration causes body forces to be applied on the body acting in the (-Z) direction:



V. EXAMPLE PROBLEMS

A. TRANSIENT HEAT CONDUCTION IN A LONG CYLINDER

The purpose of this example is to illustrate the use of the program in solving heat conduction problems. A long, hollow cylinder constructed from a single material is initially at a uniform temperature of 0°F ; then, at $t = 0$ the outer surface of the cylinder is instantaneously heated to 1°F along its entire length. The ends of the cylinder are insulated against axial heat flow so that at any axial station the heat flow is purely radial.

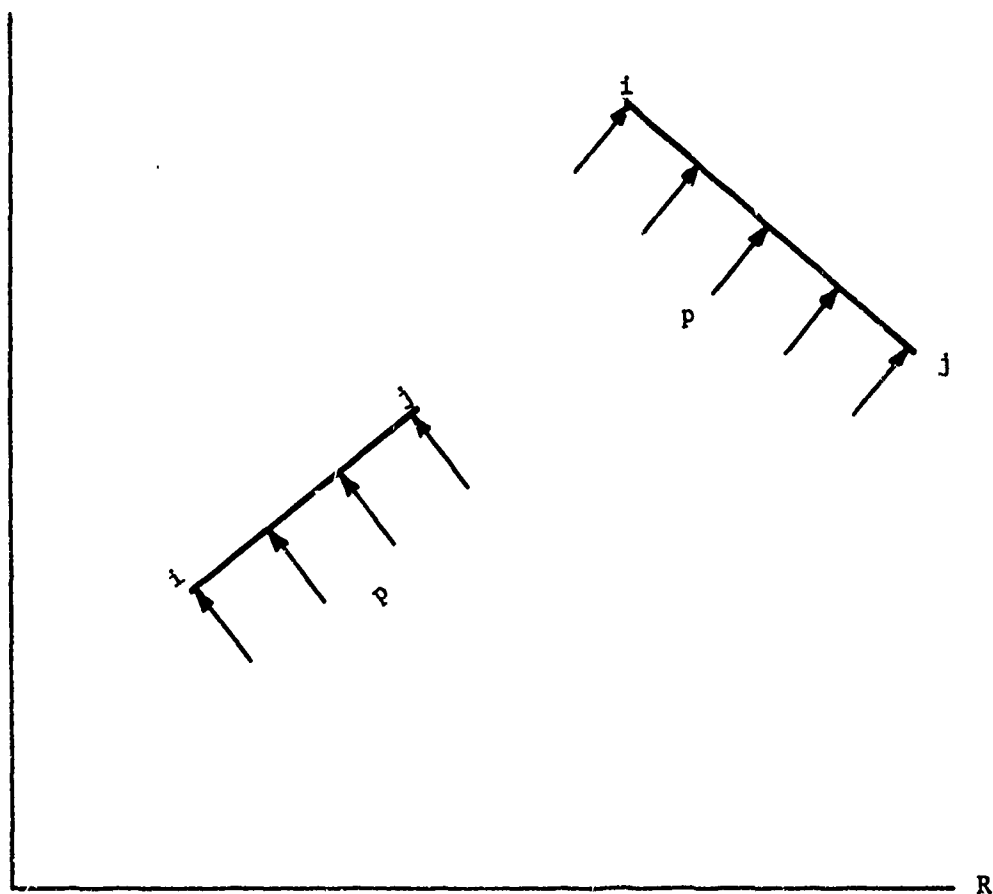
Figure H-4 shows a one (1) inch slice of the cylinder whose inner and outer radii are 1.00" and 2.00", respectively. The slice has been modeled with a mesh composed of ten (10) equal sized elements; the "Z" axis is the center-line of the cylinder. At time zero the temperature of nodes 11 and 22 is changed from 0°F ($TZ = 0.0$) to 1°F , and as time proceeds the interior of the cylinder begins to warm up; thermal equilibrium is reached when the entire cylinder is at a uniform 1°F . Nodes 1-10 and 12-21 are insulated in the sense that no externally supplied heat enters the body at these points. The program assumes that all nodes not specifically included as boundary condition nodes are insulated against externally supplied heat flow; i.e., for all non-boundary nodes, the amount of heat entering a node must balance the amount of heat leaving that node in a unit of time.

The solution span is sub-divided into fifty (50) equal time increments of 0.01 hours each so that the time at the end of solution is 0.050 hours ($TMF = 0.05$), Figure H-5.

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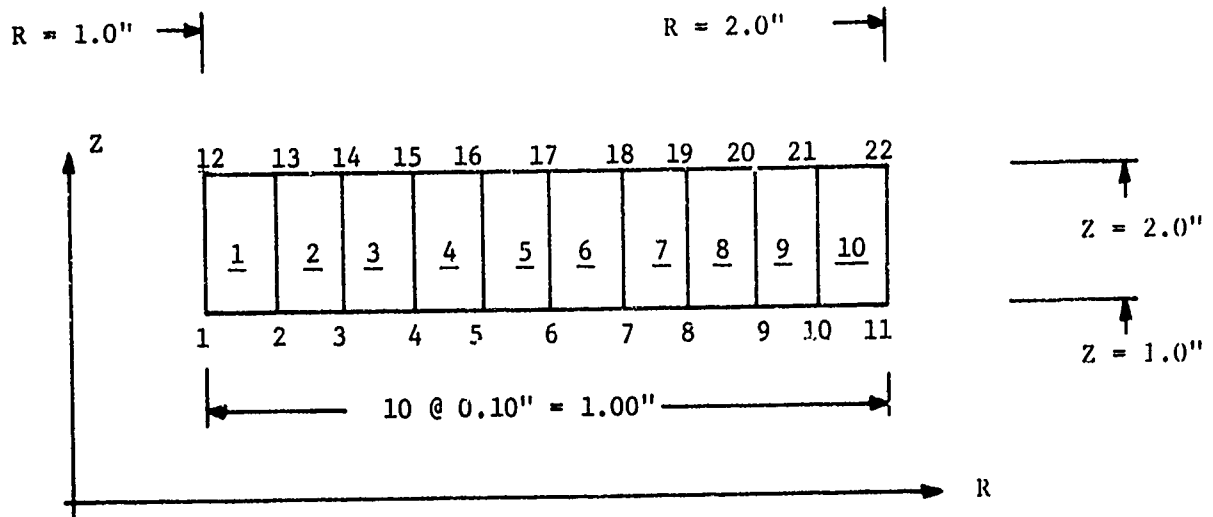
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SIGN CONVENTION FOR PRESSURE LOADS

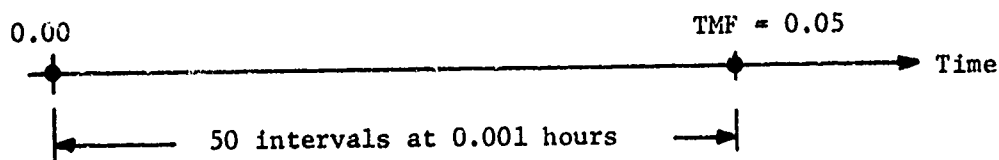


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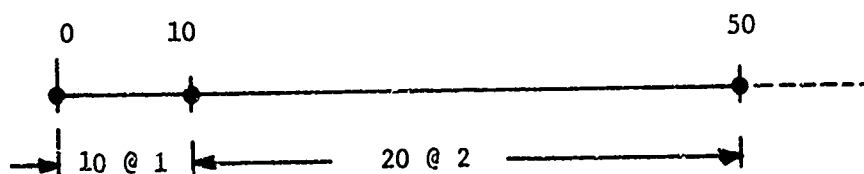
Appendix H



a. Mesh For Example Problem A



a. Solution Time Point Array



b. Solution Output Intervals

Figure H-5: Solution Time Point and Output Schedules

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The output printing schedule is set up so that results are printed at every solution time point for the first ten (10) increments and then at every other solution for the remaining forty (40) increments, Figure H-5.

10 print operations at an interval of 1 = 10
20 print operations at an interval of 2 = 40
50 increments

The thermal properties of the material are summarized as follows:

$$K_{rr} = 0.20 \text{ (Btu)(hr.)}^{-1} \text{ (in.)}^{-2} \text{ (}^{\circ}\text{F/in.)}^{-1}$$

$$K_{zz} = 0.20$$

$$K_{rz} = 0.$$

$$\text{Specific Heat} = 0.20 \text{ (Btu) (lb.)}^{-1} \text{ (}^{\circ}\text{F)}^{-1}$$

$$\text{Weight Density} = 0.20 \text{ (lb.) (in.)}^{-3}$$

$$\text{Heat generated per unit volume per unit time} = 0.*$$

The data cards for this job are shown in Table H-3.

Figure H-6 is a plot of temperature versus time for three (3) points in the cylinder: (1) outer surface (nodes 11 or 22); (2) mid-radius (nodes 6 or 17); and, (3) inner-radius (nodes 1 or 12). For long times, all nodes approach 1°F in the limit.

A value of 3 was assigned to the control variable "JOB" causing a termination of execution after the heat transfer solution. If the results were to be saved**, then the node temperatures would be averaged for each element before saving the results.

B. ELASTIC RING SUBJECTED TO AXIAL TENSION

The purpose of this example is to illustrate the use of the program in solving elastic problems. A hollow, short cylinder is subjected to an axial tension of 6000 psi on one surface and restrained (without radial shear) against axial displacement on the opposite end (Figure H-7). The problem has been modeled with four (4) quadrilateral elements as shown in Figure H-8. The applied stress has been converted to equivalent concentrated loads of 4000, 12000, and 8000 lbs./radian applied at nodes 7, 8 and 9, respectively.

Output from the computer program is shown in Figure H-9 a, b and c.

For purposes of data preparation, unit heat generation is treated as a physical property which can vary from material to material.

**JOB = 1 or 2

360 FIREARM CODING FORM

CODER _____ DATE _____

JOB: _____ DECK _____ SEQ _____
COL 73. 75 .76. 77. 78

[illegible]

Table H-

TRANSIENT TEMPERATURES IN THE CYLINDER OF EXAMPLE A

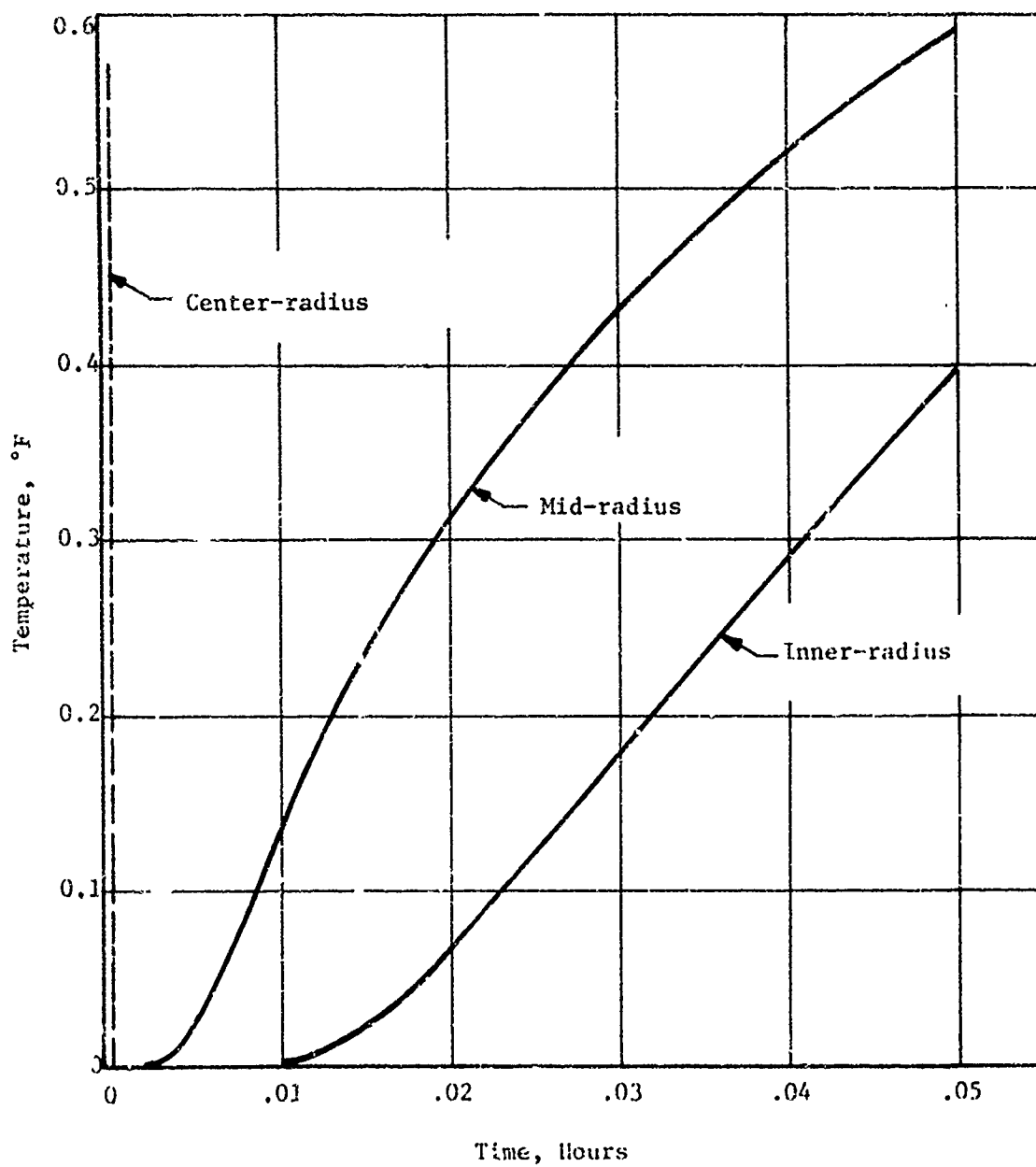
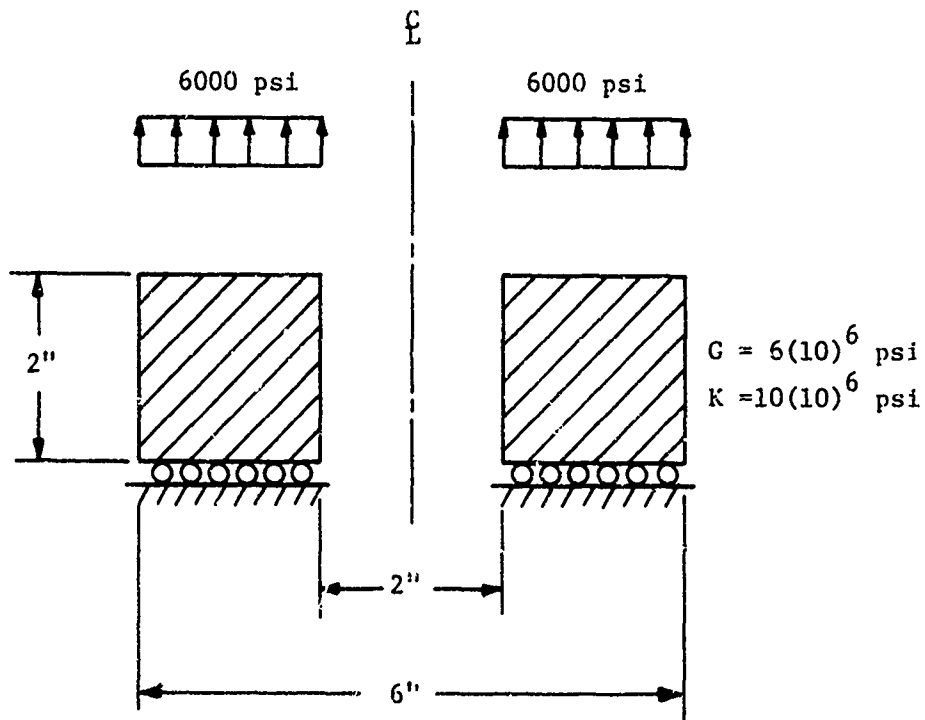


Figure H-6

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EXAMPLE B: AXIAL LOADING OF AN ELASTIC CYLINDER



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GRID FOR EXAMPLE B

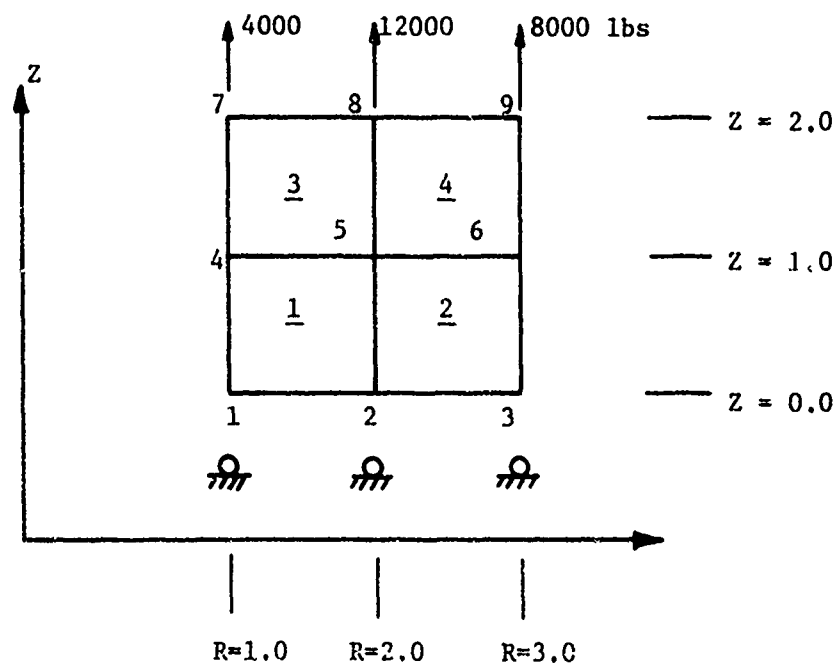


Figure H-8

TWO-DIMENSIONAL THERMOVISCOELASTIC ANALYSIS

P/A. DIRECT-AXIAL-LOADING (LRH SAMPLE) (HOLLOW CYLINDER)

LRH-1.1

CONTROL PARAMETERS

NUMBER OF NODES 5
NUMBER OF ELEMENTS 4

NODE COORDINATE DATA

NODE	R-COORDINATE	Z-COORDINATE
1	1.00000	0.0
2	2.00000	0.0
3	3.00000	0.0
4	1.00000	1.00000
5	2.00000	1.00000
6	3.00000	1.00000
7	1.00000	2.00000
8	2.00000	2.00000
9	3.00000	2.00000

ELEMENT DATA

ELEMENT	NODE 1	NODE 2	NODE 3	NODE 4	MATERIAL
1	4	1	2	5	1
2	5	2	3	6	1
3	7	4	5	8	1
4	8	5	6	9	1

TIME - TEMPERATURE CONTROL INFORMATION

TOTAL NUMBER OF TIME INCREMENTS.....	1
NUMBER OF REGIONS WITH EQUAL TIME INCREMENTS.....	1
TIME AT END OF LAST INCREMENT.....	60.000
NUMBER OF TIME DEPENDENT BOUNDARY CONDITION FUNCTIONS..	0
ELEMENT TEMPERATURE FLAG.....	3
STRESS-FREE TEMPERATURE.....	0.0

SOLUTION TIME POINTS

REGION	NUMBER OF INCREMENTS	INCREMENT VALUE
1	1	60.0000

ELEMENT TEMPERATURE PRINT CONTROL

OUTPUT INTERVAL	PRINT OPERATIONS
1	1

AVERAGE ELEMENT TEMPERATURES

(ELEMENT TEMPERATURE DISTRIBUTION CONSTANT)

SOLUTION TIME VALUE = 0.60000E 02

ELEMENT	AVERAGE TEMPERATURE
1	0.0
2	0.0
3	0.0
4	0.0

Figure H-9a

APPENDIX H

MECHANICAL PROPERTIES INFORMATION

P/A = 6000.0 PSI, AXIAL

NUMBER OF MATERIALS.....	1
NUMBER OF ELEMENTS WITH REDEFINED MATERIALS.....	0
NUMBER OF NODES WITH SPECIFIED BOUNDARY VALUES.....	8
NUMBER OF PRESSURE CARDS.....	0
ANGULAR VELOCITY AMPLITUDE.....	0.0
ANGULAR VELOCITY FUNCTION.....	0
AXIAL ACCELERATION AMPLITUDE.....	0.0
AXIAL ACCELERATION FUNCTION.....	0
OUTPUT INTERVAL FLAG.....	0
DISPLACEMENT PRINT CONTROL FLAG.....	0

MATERIAL DATA

MATERIAL NUMBER	TERMS IN PRONY SERIES	POINTS IN SHIFT FUNCTION	EQUILIBRIUM MODULUS	BULK MODULUS	EXPANSION COEFFICIENT
1	0	0	6000000.000	10000000.000	0.0

SERIES REPRESENTATION OF THE SHEAR RELAXATION FUNCTIONS

MATERIAL NUMBER	TERM IN THE SERIES	COEFFICIENT ALPHA(I)	EXPONENT BETA(I)
1	ELASTIC MATERIAL (NO SERIES EXPANSION)		

TABULATION OF TIME-TEMPERATURE SHIFT FUNCTIONS

MATERIAL NUMBER	POINT NUMBER	TEMPERATURE T(I)	SHIFT FACTOR LOG-10 A(T(I))
1	TEMPERATURE INDEPENDENT MATERIAL		

NOT REPRODUCIBLE

NODAL POINT BOUNDARY CONDITIONS

NODE NUMBER	TYPE CODE	P A D I A L BOUNDARY VALUE	FUNCTION NUMBER	TYPE CODE	A X I A L BOUNDARY VALUE	FUNCTION NUMBER
1	0	C.C	0	1	0.0	0
2	0	C.C	0	1	0.0	0
3	0	C.C	0	1	0.0	0
4	0	C.C	0	0	0.0	0
5	0	C.C	0	0	0.0	0
6	0	C.C	0	0	0.0	0
7	0	C.C	0	0	0.400000E 04	0
8	0	C.C	0	0	0.120000E 05	0
9	0	C.C	0	0	0.900000E 04	0

NODE	DISPLACEMENT	TIME =
1	-1.00000E-04 0.0	6.00000E 01
2	-2.00000E-04 0.0	
3	-3.00001E-04 0.0	
4	-1.00001E-04 4.00000E-04	
5	-2.00001E-04 4.00001E-04	
6	-1.00002E-04 4.00001E-04	
7	-1.00003E-04 3.00001E-04	
8	-2.00003E-04 8.00003E-04	
9	-3.00003E-04 9.00003E-04	

Figure H-9b

APPENDIX H

	SIGW	SIGTH	SIGZ	SIGRZ
1	-1.22070E-03	-7.41250E-03	6.00000E 03	-4.14729E-03
2	-4.39453E-03	1.22070E-03	6.00002E 03	-2.48838E-03
3	-9.76562E-03	-2.73437E-02	6.00001E 03	-4.36557E-05
4	-4.15039E-03	-9.76562E-03	6.00001E 03	-3.97267E-03

	EPR	FPT	FPZ	EPRZ	TEM
	-9.39799E-05	-1.00000E-04	4.00000E-04	-6.91216E-10	0.0
	-1.00001E-04	-1.00000E-04	4.00002E-04	-4.14730E-10	0.0
	-9.99799E-05	-1.00001E-04	4.00002E-04	-7.27596E-12	0.0
	-1.00000E-04	-1.00001E-04	4.00001E-04	-6.62112E-10	0.0

Figure H-9c

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Appendix II

The solution time span which has no meaning when constant loads are applied to elastic media consists of one (1) time increment arbitrarily selected as being 60 seconds long; one print operation is performed at the end of the first (and only) time increment. The average temperature of all elements is read in as 0°F. The pertinent mechanical properties are the shear ($G = 6(10)^6$ psi) and bulk ($K = 10(10)^6$ psi) moduli of the material. There were eight (8) of the nine (9) nodes designated as boundary nodes; nodes 4 and 6 are unloaded and unrestrained and do not have to appear in the statement of boundary conditions. Nodes 4 and 6 were inadvertently retained from a previous data deck.

The results consist of R and Z displacements of each node, average element stresses and strains and the average element temperatures all quoted at $t_1 = 60$ seconds. The applied axial stress is recovered exactly as 6000 psi (SIGZ) constant in each element.

C. PLANE-STRAIN VISCOELASTIC CYLINDER, EXAMPLE PROBLEM

The problem is a long cylindrical bore propellant grain (1.50 I.D., 8.0 O.D.) in a .060 steel case. The element geometry is shown in Figure H-10. The propellant relaxation and shift function curves are shown in Figures H-11 and H-12. Sixteen decade reduced time points were used to perform a prony series curve fit to the relaxation function and the coefficients are shown in Table H-4.

The motor, initially stress free at 135°F, was subjected to the thermal environment, through a heat transfer coefficient at the case, shown in Figure H-13. All nodes in the system were fixed in the axial direction to simulate a plane strain condition and the linear cumulative damage was calculated for two elements in the system, one at bore and one next to the case - grain bond.

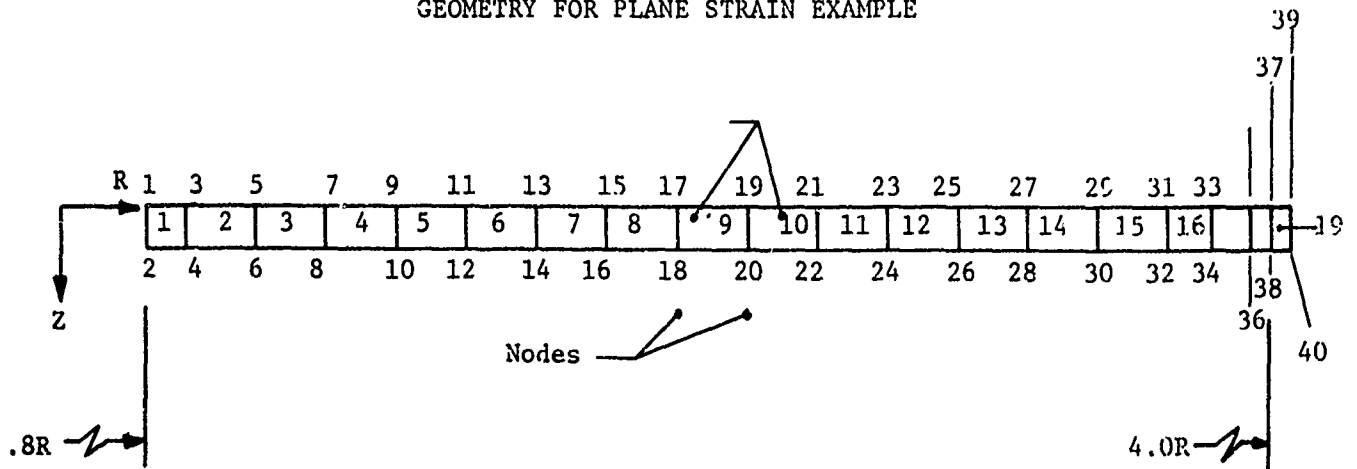
A listing of the data input cards required to describe this problem is shown in Table H-5. Line 2 is the title; line 3 is the control card; lines 4-9 describe the node point coordinates; lines 10, 11 establish the element I.D.; line 12 is the solution time point and reference temperature control and lines 13-29 describe the solution time points. Lines 31-36 describe the free stream temperature function. Line 37 describes the output print interval. Lines 39 and 40 are the thermal properties for the grain and case. Line 41 describes the convection boundary condition. Line 42 is the stress analysis title card and 43 is the stress control card. Lines 45-48 are the relaxation series coefficients. Lines 49-51 are the shift function points. Line 52 describes the case properties. Lines 53-92 are the displacement boundary conditions. The last two lines give the damage parameters.

Selected portions of the output from this analysis are shown in Table H-6.

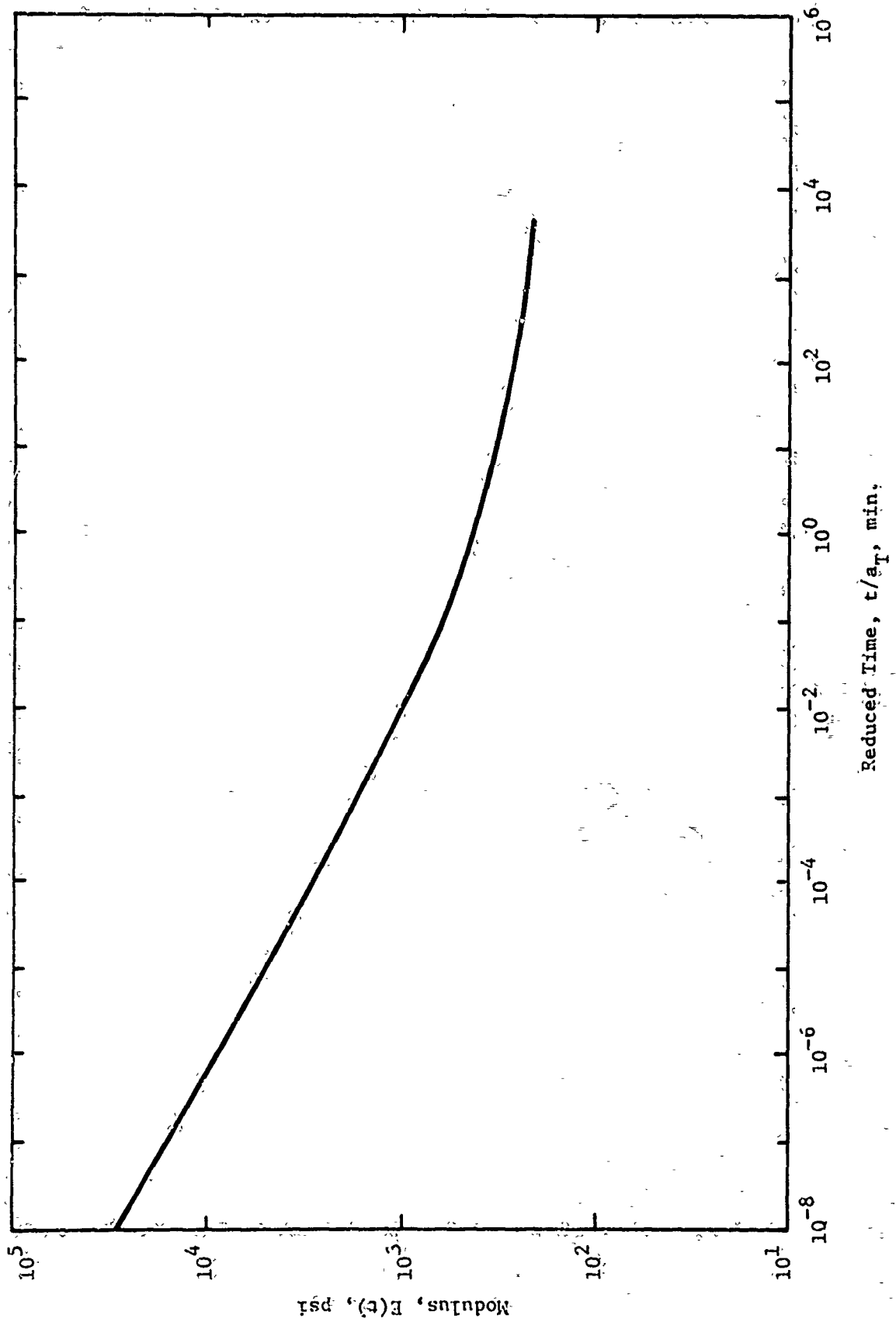
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Appendix II

GEOMETRY FOR PLANE STRAIN EXAMPLE



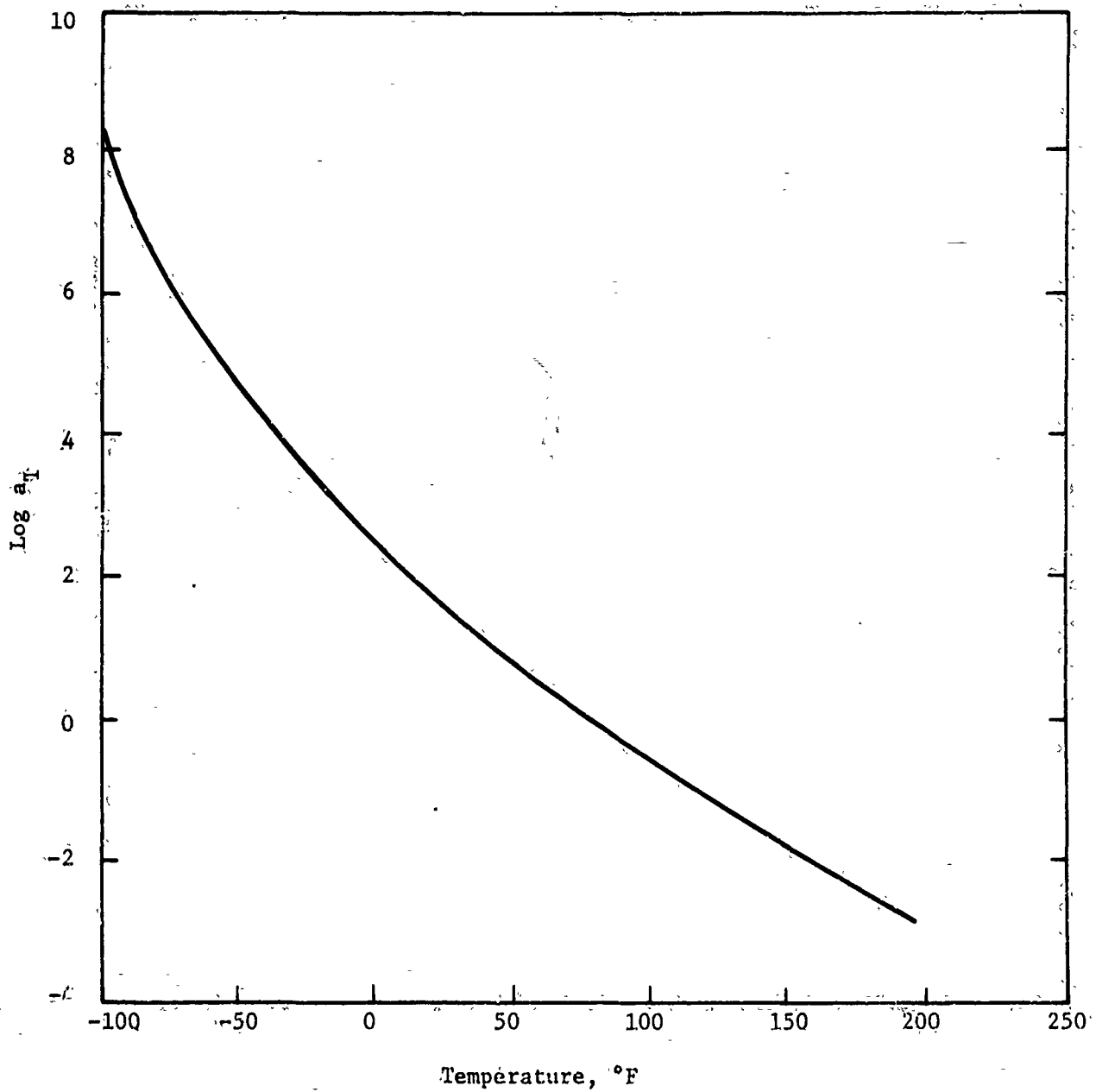
RELAXATION MODULUS OF A CTPB PROPELLANT



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Appendix H

TIME TEMPERATURE SHIFT FACTORS FOR A CTPB PROPELLANT



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Appendix H

Propellant Relaxation Function

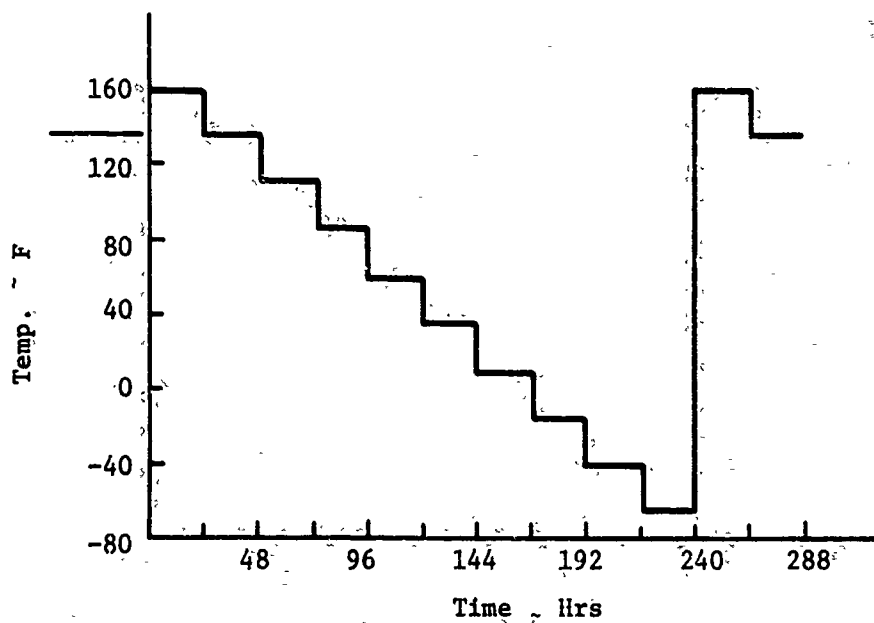
$\log(t/a_T)$ (min)	E_r (psi)	B_i (hr ⁻¹)	A_i	$A_i/3$	i
			100	33.333	0
-9	44000	3×10^{10}	25836	8612	1
-8	29000	3×10^9	12087	4029	2
-7	14000	3×10^8	6919.2	2306.4	3
-6	8300	3×10^7	3915.5	1305.2	4
-5	5000	3×10^6	2435.2	811.40	5
-4	3000	3×10^5	1341.9	447.3	6
-3	1800	3×10^4	1033.9	344.63	7
-2	1000	3×10^3	457.98	152.66	8
-1	620	3×10^2	265.80	88.60	9
0	410	3×10^1	137.74	45.91	10
1	320	3×10^0	.16481	.0546	11
2	280	3×10^{-1}	113.47	37.82	12
3	210	3×10^{-2}	-2.9029	-.9676	13
4	190	3×10^{-3}	60.2506	20.0835	14
5	150	3×10^{-4}	4.26975	1.42325	15
6	130	3×10^{-5}	49.4143	1.64714	16

$$E_r = A_0 + \sum_{i=1}^{16} A_i e^{-B_i t}$$

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Appendix H

FREE-STREAM TEMPERATURE
($H = .0125 \text{ BTU/HR/IN.}^2/\text{F}$)



APPENDIX H

Data Input for Example Problem 3

"VISCOELASTIC CHARACTER STUDY OF A 5.00:1:20 EQUIVALENT PLANE STRAIN MODEL"[illegible]

39 40 .0125 1.0
VISCOELASTIC PARAMETER STUDY $\eta/A=5.00$ $B=4$ EQUIVALENT PLANE STRAIN

17-00
 18-00
 19-00
 20-00
 21-00
 22-00
 23-00
 24-00
 25-00
 26-00
 27-00
 28-00
 29-00
 30-00
 31-00
 32-00
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 85-00
 86-00
 87-00
 88-00
 89-00
 90-00
 91-00
 92-00
 93-00
 94-00
 95-00
 96-00
 97-00
 98-00
 99-00
 100-00

Table H-6

Table 3.4
Computer Output for Example Problem C

TWO-DIMENSIONAL TIME/INVISCOELASTIC ANALYSIS
VISCOELASTIC PARAMETER STUDY B/A=9.00 B=4 EQUIVALENT PLANE STRAIN MIP=1

CONTROL PARAMETERS

NUMBER OF NODES 40
NUMBER OF ELEMENTS 19

NODE COORDINATE DATA

NODE	R-COORDINATE	Z-COORDINATE
1	0.00000	0.00000
2	0.50000	0.00000
3	0.50000	0.00000
4	0.50000	0.00000
5	1.00000	0.00000
6	1.00000	0.00000
7	1.00000	0.00000
8	1.00000	0.00000
9	1.00000	0.00000
10	1.00000	0.00000
11	1.00000	0.00000
12	1.00000	0.00000
13	1.00000	0.00000
14	1.00000	0.00000
15	1.00000	0.00000
16	1.00000	0.00000
17	1.00000	0.00000
18	1.00000	0.00000
19	1.00000	0.00000
20	1.00000	0.00000
21	1.00000	0.00000
22	1.00000	0.00000
23	1.00000	0.00000
24	1.00000	0.00000
25	1.00000	0.00000
26	1.00000	0.00000
27	1.00000	0.00000
28	1.00000	0.00000
29	1.00000	0.00000
30	1.00000	0.00000
31	1.00000	0.00000
32	1.00000	0.00000
33	1.00000	0.00000
34	1.00000	0.00000
35	1.00000	0.00000
36	1.00000	0.00000
37	1.00000	0.00000
38	1.00000	0.00000
39	1.00000	0.00000
40	1.00000	0.00000

APPENDIX H

APPENDIX H

Table 5.6 (cont.)

ELEMENT DATA	ELEMENT	NODE 1	NODE 2	NODE 3	NODE 4	MATERIAL
	1	2	1	3	4	1
	2	3	2	4	1	1
	3	4	3	1	2	1
	4	1	4	2	3	1
	5	2	1	3	4	1
	6	3	2	4	1	1
	7	4	3	1	2	1
	8	1	4	2	3	1
	9	2	1	3	4	1
	10	3	2	4	1	1
	11	4	3	1	2	1
	12	1	4	2	3	1
	13	2	1	3	4	1
	14	3	2	4	1	1
	15	4	3	1	2	1
	16	1	4	2	3	1
	17	2	1	3	4	1
	18	3	2	4	1	1
	19	4	3	1	2	1

TIME-TEMPERATURE CONTROL INFORMATION

TOTAL NUMBER OF TIME INCREMENTS..... 336
 NUMBER OF REGIONS WITH EQUAL TIME INCREMENTS..... 84
 TIME AT END OF LAST INCREMENT..... 288.000
 NUMBER OF TIME DEPENDENT BOUNDARY CONDITION FUNCTIONS..... 1
 ELEMENT TEMPERATURE FLAG..... 0
 STRESS-FREE TEMPERATURE..... 135.000

APPENDIX H

Table 5.4. (cont.)

SOLUTION TIME POINTS	REGION	NUMBER OF INCREMENTS	INCREMENT VALUE
1	1	1	0.0250
2	1	1	0.0550
3	1	1	0.1700
4	1	1	0.4500
5	1	1	0.8000
6	1	1	1.5000
7	1	1	3.0000
8	1	1	0.0250
9	1	1	0.0550
10	1	1	0.1700
11	1	1	0.4500
12	1	1	0.8000
13	1	1	1.5000
14	1	1	3.0000
15	1	1	0.0250
16	1	1	0.0550
17	1	1	0.1700
18	1	1	0.4500
19	1	1	0.8000
20	1	1	1.5000
21	1	1	3.0000
22	1	1	0.0250
23	1	1	0.0550
24	1	1	0.1700
25	1	1	0.4500
26	1	1	0.8000
27	1	1	1.5000
28	1	1	3.0000
29	1	1	0.0250
30	1	1	0.0550
31	1	1	0.1700
32	1	1	0.4500
33	1	1	0.8000
34	1	1	1.5000
35	1	1	3.0000
36	1	1	0.0250
37	1	1	0.0550
38	1	1	0.1700
39	1	1	0.4500
40	1	1	0.8000
41	1	1	1.5000
42	1	1	3.0000
43	1	1	0.0250
44	1	1	0.0550
45	1	1	0.1700
46	1	1	0.4500
47	1	1	0.8000
48	1	1	1.5000
49	1	1	3.0000
50	1	1	0.0250
51	1	1	0.0550
52	1	1	0.1700
53	1	1	0.4500
54	1	1	0.8000
55	1	1	1.5000
56	1	1	3.0000
57	1	1	0.0250
58	1	1	0.0550
59	1	1	0.1700
60	1	1	0.4500
61	1	1	0.8000
62	1	1	1.5000
63	1	1	3.0000
64	1	1	0.0250
65	1	1	0.0550
66	1	1	0.1700
67	1	1	0.4500
68	1	1	0.8000
69	1	1	1.5000
70	1	1	3.0000
71	1	1	0.0250
72	1	1	0.0550
73	1	1	0.1700
74	1	1	0.4500
75	1	1	0.8000
76	1	1	1.5000
77	1	1	3.0000
78	1	1	0.0250
79	1	1	0.0550
80	1	1	0.1700
81	1	1	0.4500
82	1	1	0.8000
83	1	1	1.5000
84	1	1	3.0000
85	1	1	0.0250
86	1	1	0.0550
87	1	1	0.1700
88	1	1	0.4500
89	1	1	0.8000
90	1	1	1.5000
91	1	1	3.0000
92	1	1	0.0250
93	1	1	0.0550
94	1	1	0.1700
95	1	1	0.4500
96	1	1	0.8000
97	1	1	1.5000
98	1	1	3.0000
99	1	1	0.0250
100	1	1	0.0550
101	1	1	0.1700
102	1	1	0.4500
103	1	1	0.8000
104	1	1	1.5000
105	1	1	3.0000
106	1	1	0.0250
107	1	1	0.0550
108	1	1	0.1700
109	1	1	0.4500
110	1	1	0.8000
111	1	1	1.5000
112	1	1	3.0000
113	1	1	0.0250
114	1	1	0.0550
115	1	1	0.1700
116	1	1	0.4500
117	1	1	0.8000
118	1	1	1.5000
119	1	1	3.0000
120	1	1	0.0250
121	1	1	0.0550
122	1	1	0.1700
123	1	1	0.4500
124	1	1	0.8000
125	1	1	1.5000
126	1	1	3.0000
127	1	1	0.0250
128	1	1	0.0550
129	1	1	0.1700
130	1	1	0.4500
131	1	1	0.8000
132	1	1	1.5000
133	1	1	3.0000
134	1	1	0.0250
135	1	1	0.0550
136	1	1	0.1700
137	1	1	0.4500
138	1	1	0.8000
139	1	1	1.5000
140	1	1	3.0000
141	1	1	0.0250
142	1	1	0.0550
143	1	1	0.1700
144	1	1	0.4500
145	1	1	0.8000
146	1	1	1.5000
147	1	1	3.0000
148	1	1	0.0250
149	1	1	0.0550
150	1	1	0.1700
151	1	1	0.4500
152	1	1	0.8000
153	1	1	1.5000
154	1	1	3.0000
155	1	1	0.0250
156	1	1	0.0550
157	1	1	0.1700
158	1	1	0.4500
159	1	1	0.8000
160	1	1	1.5000
161	1	1	3.0000
162	1	1	0.0250
163	1	1	0.0550
164	1	1	0.1700
165	1	1	0.4500
166	1	1	0.8000
167	1	1	1.5000
168	1	1	3.0000
169	1	1	0.0250
170	1	1	0.0550
171	1	1	0.1700
172	1	1	0.4500
173	1	1	0.8000
174	1	1	1.5000
175	1	1	3.0000
176	1	1	0.0250
177	1	1	0.0550
178	1	1	0.1700
179	1	1	0.4500
180	1	1	0.8000
181	1	1	1.5000
182	1	1	3.0000
183	1	1	0.0250
184	1	1	0.0550
185	1	1	0.1700
186	1	1	0.4500
187	1	1	0.8000
188	1	1	1.5000
189	1	1	3.0000
190	1	1	0.0250
191	1	1	0.0550
192	1	1	0.1700
193	1	1	0.4500
194	1	1	0.8000
195	1	1	1.5000
196	1	1	3.0000
197	1	1	0.0250
198	1	1	0.0550
199	1	1	0.1700
200	1	1	0.4500
201	1	1	0.8000
202	1	1	1.5000
203	1	1	3.0000
204	1	1	0.0250
205	1	1	0.0550
206	1	1	0.1700
207	1	1	0.4500
208	1	1	0.8000
209	1	1	1.5000
210	1	1	3.0000
211	1	1	0.0250
212	1	1	0.0550
213	1	1	0.1700
214	1	1	0.4500
215	1	1	0.8000
216	1	1	1.5000
217	1	1	3.0000
218	1	1	0.0250
219	1	1	0.0550
220	1	1	0.1700
221	1	1	0.4500
222	1	1	0.8000
223	1	1	1.5000
224	1	1	3.0000
225	1	1	0.0250
226	1	1	0.0550
227	1	1	0.1700
228	1	1	0.4500
229	1	1	0.8000
230	1	1	1.5000
231	1	1	3.0000
232	1	1	0.0250
233	1	1	0.0550
234	1	1	0.1700
235	1	1	0.4500
236	1	1	0.8000
237	1	1	1.5000
238	1	1	3.0000
239	1	1	0.0250
240	1	1	0.0550
241	1	1	0.1700
242	1	1	0.4500
243	1	1	0.8000
244	1	1	1.5000
245	1	1	3.0000
246	1	1	0.0250
247	1	1	0.0550
248	1	1	0.1700
249	1	1	0.4500
250	1	1	0.8000
251	1	1	1.5000
252	1	1	3.0000
253	1	1	0.0250
254	1	1	0.0550
255	1	1	0.1700
256	1	1	0.4500
257	1	1	0.8000
258	1	1	1.5000
259	1	1	3.0000
260	1	1	0.0250
261	1	1	0.0550
262	1	1	0.1700
263	1	1	0.4500
264	1	1	0.8000
265	1	1	1.5000
266	1	1	3.0000
267	1	1	0.0250
268	1	1	0.0550
269	1	1	0.1700
270	1	1	0.4500
271	1	1	0.8000
272	1	1	1.5000
273	1	1	3.0000
274	1	1	0.0250
275	1	1	0.0550
276	1	1	0.1700
277	1	1	0.4500
278	1	1	0.8000
279	1	1	1.5000
280	1	1	3.0000
281	1	1	0.0250
282	1	1	0.0550
283	1	1	0.1700
284	1	1	0.4500
285	1	1	0.8000
286	1	1	1.5000
287	1	1	3.0000
288	1	1	0.0250
289	1	1	0.0550
290	1	1	0.1700
291	1	1	0.4500
292	1	1	0.8000
293	1	1	1.5000
294	1	1	3.0000
295	1	1	0.0250
296	1	1	0.0550
297	1	1	0.1700
298	1	1	0.4500
299	1	1	0.8000
300	1	1	1.5000
301	1	1	3.0000
302	1	1	0.0250
303	1	1	0.0550
304	1	1	0.1700
305	1	1	0.4500
306	1	1	0.8000
307	1	1	1.5000
308	1	1	3.0000
309	1	1	0.0250
310	1	1	0.0550
311	1	1	0.1700
312	1	1	0.4500
313	1	1	0.8000
314	1	1	1.5000
315	1	1	3.0000
316	1	1	0.0250
317	1	1	0.0550
318	1	1	0.1700
319	1	1	0.4500
320	1	1	0.8000
321	1	1	1.5000
322	1	1	3.0000
323	1	1	0.0250
324	1	1	0.0550
325	1	1	0.1700
326	1	1	0.4500
327	1	1	0.8000
328	1	1	1.5000
329	1	1	3.0000
330	1	1	0.0250
331	1	1	0.0550
332	1	1	0.1700
333	1	1	0.4500
334	1	1	0.8000
335	1	1	1.5000
336	1	1	3.0000
337	1	1	0.0250
338	1	1	0.0550
339	1	1	0.1700
340	1	1	0.4500
341	1	1	0.8000
342	1	1	1.5000
343	1	1	

Table 5.4 (cont.)

SOLUTION TO THE TRANSIENT HEAT CONDUCTION PROBLEM

[illegible]

APPENDIX H.

Table 5.4 (cont.)

MECHANICAL PROPERTIES INFORMATION

VISCOELASTIC PARAMETER STUDY 8/A/3.00 8% EQUIVALENT PLANE STRAIN

NUMBER OF MATERIALS..... 2
 NUMBER OF ELEMENTS WITH PREDEFINED MATERIALS..... 40
 NUMBER OF NODES WITH SPECIFIED BOUNDARY VALUES..... 2
 NUMBER OF PRESSURE CARDS..... 0
 NUMBER OF ELEMENTS FOR DAMAGE EVALUATION..... 0
 ANGULAR VELOCITY AMPLITUDE..... 0.0
 AXIAL ACCELERATION AMPLITUDE..... 0.0
 AXIAL ACCELERATION FUNCTION..... 0
 DISPLACEMENT PRINT CONTROL FLAG..... 0

MATERIALS DATA

MATERIAL NUMBER	TERMS IN PRONY SERIES	POINTS IN SHIFT FUNCTION	EQUILIBRIUM MODULUS	BULK MODULUS	EXPANSION COEFFICIENT	HEIGHT DENSITY
1	16	12	33.300	500000.000	0.56700E-04	0.0
2	16	0	11500000.000	25000000.000	0.59000E-03	0.0

SERIES REPRESENTATION OF THE SHEAR RELAXATION FUNCTIONS

MATERIAL NUMBER	TERM IN THE SERIES	COEFFICIENT ALPHA(I)	EXPONENT BETA(I)
1	1	0.86120E 04	0.30000E 11
1	2	0.40290E 04	0.30000E 10
1	3	0.23064E 04	0.30000E 09
1	4	0.13052E 04	0.30000E 08
1	5	0.81140E 03	0.30000E 07
1	6	0.44730E 03	0.30000E 06
1	7	0.34464E 03	0.30000E 05
1	8	0.13260E 03	0.30000E 04
1	9	0.88600E 02	0.30000E 03
1	10	0.45910E 02	0.30000E 02
1	11	0.56004E 01	0.30000E 01
1	12	0.37820E 02	0.30000E 00
1	13	0.96760E 00	0.30000E -01
1	14	0.20083E 02	0.30000E -02
1	15	0.14232E 01	0.30000E -03
1	16	0.18471E 01	0.30000E -04

2 ELASTIC MATERIAL (NO SERIES EXPANSION)

TABULATION OF TIME-TEMPERATURE SHIFT FUNCTIONS

MATERIAL NUMBER	POINT NUMBER	TEMPERATURE T(I)	SHIFT FACTOR LOG-10 ALT(I)
1	1	-100.000	8.2800
1	2	-75.000	6.0000
1	3	-50.000	4.6000
1	4	-25.000	3.5400
1	5	0.0	2.5200
1	6	25.000	1.6000
1	7	50.000	0.8000
1	8	75.000	0.0
1	9	100.000	-0.5600
1	10	125.000	-1.2200
1	11	150.000	-1.8200
1	12	200.000	-2.9400

2 TEMPERATURE INDEPENDENT MATERIAL

Table H-6 Cont'd.

APPENDIX H

Table 5.4 (cont.)

NODAL POINT BOUNDARY CONDITIONS

NODE NUMBER	TYPE CODE	1 A D I A L BOUNDARY VALUE	FUNCTION NUMBER	TYPE CODE	A X I A L BOUNDARY VALUE	FUNCTION NUMBER
1	0	0.00	0	1	0.00	0
2	0	0.00	0	1	0.00	0
3	0	0.00	0	1	0.00	0
4	0	0.00	0	1	0.00	0
5	0	0.00	0	1	0.00	0
6	0	0.00	0	1	0.00	0
7	0	0.00	0	1	0.00	0
8	0	0.00	0	1	0.00	0
9	0	0.00	0	1	0.00	0
10	0	0.00	0	1	0.00	0
11	0	0.00	0	1	0.00	0
12	0	0.00	0	1	0.00	0
13	0	0.00	0	1	0.00	0
14	0	0.00	0	1	0.00	0
15	0	0.00	0	1	0.00	0
16	0	0.00	0	1	0.00	0
17	0	0.00	0	1	0.00	0
18	0	0.00	0	1	0.00	0
19	0	0.00	0	1	0.00	0
20	0	0.00	0	1	0.00	0
21	0	0.00	0	1	0.00	0
22	0	0.00	0	1	0.00	0
23	0	0.00	0	1	0.00	0
24	0	0.00	0	1	0.00	0
25	0	0.00	0	1	0.00	0
26	0	0.00	0	1	0.00	0
27	0	0.00	0	1	0.00	0
28	0	0.00	0	1	0.00	0
29	0	0.00	0	1	0.00	0
30	0	0.00	0	1	0.00	0
31	0	0.00	0	1	0.00	0
32	0	0.00	0	1	0.00	0
33	0	0.00	0	1	0.00	0
34	0	0.00	0	1	0.00	0
35	0	0.00	0	1	0.00	0
36	0	0.00	0	1	0.00	0
37	0	0.00	0	1	0.00	0
38	0	0.00	0	1	0.00	0
39	0	0.00	0	1	0.00	0
40	0	0.00	0	1	0.00	0

DAMAGE PARAMETERS ... MSG. EL. NO. = DAMAGE BASED ON MAX. PRINC. M-L STRESS
 MSG. EL. NO. = DAMAGE BASED ON HOOP STRESS

ELEMENT SIGNA(T-ZERO)	T-ZERO	SIGMA(T-ZERO)	9
-10	0.00	0.00	0.43
10	0.00	0.00	0.43

Table H-6 Cont'd

050918i

TIME = 1.00000000-01

TIME = 3.200000000D-01

ATTENTION = 1.0000000000

Table H-6 Cont'd

NOT REPRODUCIBLE

CC-0.1000000000000000

[illegible]

$\tau_1 \approx 2.6000000000000000$

[illegible]

T14E = 4.8000000000000000

[illegible]

NOT REPRODUCIBLE

10-206700030 U

Year	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032	2033	2034	2035	2036	2037	2038	2039	2040	2041	2042	2043	2044	2045	2046	2047	2048	2049	2050	2051	2052	2053	2054	2055	2056	2057	2058	2059	2060	2061	2062	2063	2064	2065	2066	2067	2068	2069	2070	2071	2072	2073	2074	2075	2076	2077	2078	2079	2080	2081	2082	2083	2084	2085	2086	2087	2088	2089	2090	2091	2092	2093	2094	2095	2096	2097	2098	2099	2100
1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032	2033	2034	2035	2036	2037	2038	2039	2040	2041	2042	2043	2044	2045	2046	2047	2048	2049	2050	2051	2052	2053	2054	2055	2056	2057	2058	2059	2060	2061	2062	2063	2064	2065	2066	2067	2068	2069	2070	2071	2072	2073	2074	2075	2076	2077	2078	2079	2080	2081	2082	2083	2084	2085	2086	2087	2088	2089	2090	2091	2092	2093	2094	2095	2096	2097	2098	2099	2100	

TIME = 4.0000000000000000

[illegible]

TIW = 1.2630300 32

[illegible]

NOT REPRODUCIBLE

Y141 = 1.4433300 12

Time	Lat	Long	Alt	Temp	Hum	Wind	Dir	Speed	Pressure	Clouds	Visibility	Remarks
0000	34 15 N	122 05 W	1000	55.0	85	10	090	10	1013.5	0	10	Clear
0100	34 15 N	122 05 W	1000	55.0	85	10	090	10	1013.5	0	10	Clear
0200	34 15 N	122 05 W	1000	55.0	85	10	090	10	1013.5	0	10	Clear
0300	34 15 N	122 05 W	1000	55.0	85	10	090	10	1013.5	0	10	Clear
0400	34 15 N	122 05 W	1000	55.0	85	10	090	10	1013.5	0	10	Clear
0500	34 15 N	122 05 W	1000	55.0	85	10	090	10	1013.5	0	10	Clear
0600	34 15 N	122 05 W	1000	55.0	85	10	090	10	1013.5	0	10	Clear
0700	34 15 N	122 05 W	1000	55.0	85	10	090	10	1013.5	0	10	Clear
0800	34 15 N	122 05 W	1000	55.0	85	10	090	10	1013.5	0	10	Clear
0900	34 15 N	122 05 W	1000	55.0	85	10	090	10	1013.5	0	10	Clear
1000	34 15 N	122 05 W	1000	55.0	85	10	090	10	1013.5	0	10	Clear
1100	34 15 N	122 05 W	1000	55.0	85	10	090	10	1013.5	0	10	Clear
1200	34 15 N	122 05 W	1000	55.0	85	10	090	10	1013.5	0	10	Clear
1300	34 15 N	122 05 W	1000	55.0	85	10	090	10	1013.5	0	10	Clear
1400	34 15 N	122 05 W	1000	55.0	85	10	090	10	1013.5	0	10	Clear
1500	34 15 N	122 05 W	1000	55.0	85	10	090	10	1013.5	0	10	Clear
1600	34 15 N	122 05 W	1000	55.0	85	10	090	10	1013.5	0	10	Clear
1700	34 15 N	122 05 W	1000	55.0	85	10	090	10	1013.5	0	10	Clear
1800	34 15 N	122 05 W	1000	55.0	85	10	090	10	1013.5	0	10	Clear
1900	34 15 N	122 05 W	1000	55.0	85	10	090	10	1013.5	0	10	Clear
2000	34 15 N	122 05 W	1000	55.0	85	10	090	10	1013.5	0	10	Clear
2100	34 15 N	122 05 W	1000	55.0	85	10	090	10	1013.5	0	10	Clear
2200	34 15 N	122 05 W	1000	55.0	85	10	090	10	1013.5	0	10	Clear
2300	34 15 N	122 05 W	1000	55.0	85	10	090	10	1013.5	0	10	Clear

TIME 3-1000000000 02

ITEM	QTY	UNIT	PRICE	TOTAL	TAX	NET	GST	GRAND TOTAL
1	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
2	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
3	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
4	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
5	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
6	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
7	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
8	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
9	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
10	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
11	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
12	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
13	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
14	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
15	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
16	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
17	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
18	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
19	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
20	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
21	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
22	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
23	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
24	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
25	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
26	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
27	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
28	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
29	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
30	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
31	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
32	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
33	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
34	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
35	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
36	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
37	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
38	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
39	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
40	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
41	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
42	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
43	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
44	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
45	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
46	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
47	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
48	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
49	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
50	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
51	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
52	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
53	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
54	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
55	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
56	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
57	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
58	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
59	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
60	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
61	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
62	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
63	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
64	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
65	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
66	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
67	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
68	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
69	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
70	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
71	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
72	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
73	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
74	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
75	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
76	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
77	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
78	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
79	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
80	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
81	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
82	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
83	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
84	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
85	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
86	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
87	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
88	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
89	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
90	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
91	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
92	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
93	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
94	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
95	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
96	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
97	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
98	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
99	1	EA	1.00	1.00	0.00	1.00	0.00	1.00
100	1	EA	1.00	1.00	0.00	1.00	0.00	1.00

$\log = 1.920000000 \times 72$

STB	SLTIN	SLT2	SLT3	SLT4	SLT5	SLT6	SLT7	SLT8	SLT9	SLT10	SLT11	SLT12	SLT13	SLT14	SLT15	SLT16	SLT17	SLT18	SLT19	SLT20	SLT21	SLT22	SLT23	SLT24	SLT25	SLT26	SLT27	SLT28	SLT29	SLT30	SLT31	SLT32	SLT33	SLT34	SLT35	SLT36	SLT37	SLT38	SLT39	SLT40	SLT41	SLT42	SLT43	SLT44	SLT45	SLT46	SLT47	SLT48	SLT49	SLT50	SLT51	SLT52	SLT53	SLT54	SLT55	SLT56	SLT57	SLT58	SLT59	SLT60	SLT61	SLT62	SLT63	SLT64	SLT65	SLT66	SLT67	SLT68	SLT69	SLT70	SLT71	SLT72	SLT73	SLT74	SLT75	SLT76	SLT77	SLT78	SLT79	SLT80	SLT81	SLT82	SLT83	SLT84	SLT85	SLT86	SLT87	SLT88	SLT89	SLT90	SLT91	SLT92	SLT93	SLT94	SLT95	SLT96	SLT97	SLT98	SLT99	SLT100	SLT101	SLT102	SLT103	SLT104	SLT105	SLT106	SLT107	SLT108	SLT109	SLT110	SLT111	SLT112	SLT113	SLT114	SLT115	SLT116	SLT117	SLT118	SLT119	SLT120	SLT121	SLT122	SLT123	SLT124	SLT125	SLT126	SLT127	SLT128	SLT129	SLT130	SLT131	SLT132	SLT133	SLT134	SLT135	SLT136	SLT137	SLT138	SLT139	SLT140	SLT141	SLT142	SLT143	SLT144	SLT145	SLT146	SLT147	SLT148	SLT149	SLT150	SLT151	SLT152	SLT153	SLT154	SLT155	SLT156	SLT157	SLT158	SLT159	SLT160	SLT161	SLT162	SLT163	SLT164	SLT165	SLT166	SLT167	SLT168	SLT169	SLT170	SLT171	SLT172	SLT173	SLT174	SLT175	SLT176	SLT177	SLT178	SLT179	SLT180	SLT181	SLT182	SLT183	SLT184	SLT185	SLT186	SLT187	SLT188	SLT189	SLT190	SLT191	SLT192	SLT193	SLT194	SLT195	SLT196	SLT197	SLT198	SLT199	SLT200	SLT201	SLT202	SLT203	SLT204	SLT205	SLT206	SLT207	SLT208	SLT209	SLT210	SLT211	SLT212	SLT213	SLT214	SLT215	SLT216	SLT217	SLT218	SLT219	SLT220	SLT221	SLT222	SLT223	SLT224	SLT225	SLT226	SLT227	SLT228	SLT229	SLT230	SLT231	SLT232	SLT233	SLT234	SLT235	SLT236	SLT237	SLT238	SLT239	SLT240	SLT241	SLT242	SLT243	SLT244	SLT245	SLT246	SLT247	SLT248	SLT249	SLT250	SLT251	SLT252	SLT253	SLT254	SLT255	SLT256	SLT257	SLT258	SLT259	SLT260	SLT261	SLT262	SLT263	SLT264	SLT265	SLT266	SLT267	SLT268	SLT269	SLT270	SLT271	SLT272	SLT273	SLT274	SLT275	SLT276	SLT277	SLT278	SLT279	SLT280	SLT281	SLT282	SLT283	SLT284	SLT285	SLT286	SLT287	SLT288	SLT289	SLT290	SLT291	SLT292	SLT293	SLT294	SLT295	SLT296	SLT297	SLT298	SLT299	SLT300	SLT301	SLT302	SLT303	SLT304	SLT305	SLT306	SLT307	SLT308	SLT309	SLT310	SLT311	SLT312	SLT313	SLT314	SLT315	SLT316	SLT317	SLT318	SLT319	SLT320	SLT321	SLT322	SLT323	SLT324	SLT325	SLT326	SLT327	SLT328	SLT329	SLT330	SLT331	SLT332	SLT333	SLT334	SLT335	SLT336	SLT337	SLT338	SLT339	SLT340	SLT341	SLT342	SLT343	SLT344	SLT345	SLT346	SLT347	SLT348	SLT349	SLT350	SLT351	SLT352	SLT353	SLT354	SLT355	SLT356	SLT357	SLT358	SLT359	SLT360	SLT361	SLT362	SLT363	SLT364	SLT365	SLT366	SLT367	SLT368	SLT369	SLT370	SLT371	SLT372	SLT373	SLT374	SLT375	SLT376	SLT377	SLT378	SLT379	SLT380	SLT381	SLT382	SLT383	SLT384	SLT385	SLT386	SLT387	SLT388	SLT389	SLT390	SLT391	SLT392	SLT393	SLT394	SLT395	SLT396	SLT397	SLT398	SLT399	SLT400	SLT401	SLT402	SLT403	SLT404	SLT405	SLT406	SLT407	SLT408	SLT409	SLT410	SLT411	SLT412	SLT413	SLT414	SLT415	SLT416	SLT417	SLT418	SLT419	SLT420	SLT421	SLT422	SLT423	SLT424	SLT425	SLT426	SLT427	SLT428	SLT429	SLT430	SLT431	SLT432	SLT433	SLT434	SLT435	SLT436	SLT437	SLT438	SLT439	SLT440	SLT441	SLT442	SLT443	SLT444	SLT445	SLT446	SLT447	SLT448	SLT449	SLT450	SLT451	SLT452	SLT453	SLT454	SLT455	SLT456	SLT457	SLT458	SLT459	SLT460	SLT461	SLT462	SLT463	SLT464	SLT465	SLT466	SLT467	SLT468	SLT469	SLT470	SLT471	SLT472	SLT473	SLT474	SLT475	SLT476	SLT477	SLT478	SLT479	SLT480	SLT481	SLT482	SLT483	SLT484	SLT485	SLT486	SLT487	SLT488	SLT489	SLT490	SLT491	SLT492	SLT493	SLT494	SLT495	SLT496	SLT497	SLT498	SLT499	SLT500	SLT501	SLT502	SLT503	SLT504	SLT505	SLT506	SLT507	SLT508	SLT509	SLT510	SLT511	SLT512	SLT513	SLT514	SLT515	SLT516	SLT517	SLT518	SLT519	SLT520	SLT521	SLT522	SLT523	SLT524	SLT525	SLT526	SLT527	SLT528	SLT529	SLT530	SLT531	SLT532	SLT533	SLT534	SLT535	SLT536	SLT537	SLT538	SLT539	SLT540	SLT541	SLT542	SLT543	SLT544	SLT545	SLT546	SLT547	SLT548	SLT549	SLT550	SLT551	SLT552	SLT553	SLT554	SLT555	SLT556	SLT557	SLT558	SLT559	SLT560	SLT561	SLT562	SLT563	SLT564	SLT565	SLT566	SLT567	SLT568	SLT569	SLT570	SLT571	SLT572	SLT573	SLT574	SLT575	SLT576	SLT577	SLT578	SLT579	SLT580	SLT581	SLT582	SLT583	SLT584	SLT585	SLT586	SLT587	SLT588	SLT589	SLT590	SLT591	SLT592	SLT593	SLT594	SLT595	SLT596	SLT597	SLT598	SLT599	SLT600	SLT601	SLT602	SLT603	SLT604	SLT605	SLT606	SLT607	SLT608	SLT609	SLT610	SLT611	SLT612	SLT613	SLT614	SLT615	SLT616	SLT617	SLT618	SLT619	SLT620	SLT621	SLT622	SLT623	SLT624	SLT625	SLT626	SLT627	SLT628	SLT629	SLT630	SLT631	SLT632	SLT633	SLT634	SLT635	SLT636	SLT637	SLT638	SLT639	SLT640	SLT641	SLT642	SLT643	SLT644	SLT645	SLT646	SLT647	SLT648	SLT649	SLT650	SLT651	SLT652	SLT653	SLT654	SLT655	SLT656	SLT657	SLT658	SLT659	SLT660	SLT661	SLT662	SLT663	SLT664	SLT665	SLT666	SLT667	SLT668	SLT669	SLT670	SLT671	SLT672	SLT673	SLT674	SLT675	SLT676	SLT677	SLT678	SLT679	SLT680	SLT681	SLT682	SLT683	SLT684	SLT685	SLT686	SLT687	SLT688	SLT689	SLT690	SLT691	SLT692	SLT693	SLT694	SLT695	SLT696	SLT697	SLT698	SLT699	SLT700	SLT701	SLT702	SLT703	SLT704	SLT705	SLT706	SLT707	SLT708	SLT709	SLT710	SLT711	SLT712	SLT713	SLT714	SLT715	SLT716	SLT717	SLT718	SLT719	SLT720	SLT721	SLT722	SLT723	SLT724	SLT725	SLT726	SLT727	SLT728	SLT729	SLT730	SLT731	SLT732	SLT733	SLT734	SLT735	SLT736	SLT737	SLT738	SLT739	SLT740	SLT741	SLT742	SLT743	SLT744	SLT745	SLT746	SLT747	SLT748	SLT749	SLT750	SLT751	SLT752	SLT753	SLT754	SLT755	SLT756	SLT757	SLT758	SLT759	SLT760	SLT761	SLT762	SLT763	SLT764	SLT765	SLT766	SLT767	SLT768	SLT769	SLT770	SLT771	SLT772	SLT773	SLT774	SLT775	SLT776	SLT777	SLT778	SLT779	SLT780	SLT781	SLT782	SLT783	SLT784	SLT785	SLT786	SLT787	SLT788	SLT789	SLT790	SLT791	SLT792	SLT793	SLT794	SLT795	SLT796	SLT797	SLT798	SLT799	SLT800	SLT801	SLT802	SLT803	SLT804	SLT805	SLT806	SLT807	SLT808	SLT809	SLT810	SLT811	SLT812	SLT813	SLT814	SLT815	SLT816	SLT817	SLT818	SLT819	SLT820	SLT821	SLT822	SLT823	SLT824	SLT825	SLT826	SLT827	SLT828	SLT829	SLT830	SLT831	SLT832	SLT833	SLT834	SLT835	SLT836	SLT837	SLT838	SLT839	SLT840	SLT841	SLT842	SLT843	SLT844	SLT845	SLT846	SLT847	SLT848	SLT849	SLT850	SLT851	SLT852	SLT853	SLT854	SLT855	SLT856	SLT857	SLT858	SLT859	SLT860	SLT861	SLT862	SLT863	SLT864	SLT865	SLT866	SLT867	SLT868	SLT869	SLT870	SLT871	SLT872	SLT873	SLT874	SLT875	SLT876	SLT877	SLT878	SLT879	SLT880	SLT881	SLT882	SLT883	SLT884	SLT885	SLT886	SLT887	SLT888	SLT889	SLT890	SLT891	SLT892	SLT893	SLT894	SLT895	SLT896	SLT897	SLT898	SLT899	SLT900	SLT901	SLT902	SLT903	SLT904	SLT905	SLT906	SLT907	SLT908	SLT909	SLT910	SLT911	SLT912	SLT913	SLT914	SLT915	SLT916	SLT917	SLT918	SLT919	SLT920	SLT921	SLT922	SLT923	SLT924	SLT925	SLT926	SLT927	SLT928	SLT929	SLT930	SLT931	SLT932	SLT933	SLT934	SLT935	SLT936	SLT937	SLT938	SLT939	SLT940	SLT941	SLT942	SLT943	SLT944	SLT945	SLT946	SLT947	SLT948	SLT949	SLT950	SLT951	SLT952	SLT953	SLT954	SLT955	SLT956	SLT957	SLT958	SLT959	SLT960	SLT961	SLT962	SLT963	SLT964	SLT965	SLT966	SLT967	SLT968	SLT969	SLT970	SLT971	SLT972	SLT973	SLT974	SLT975	SLT976	SLT977	SLT978	SLT979	SLT980	SLT981	SLT982	SLT983	SLT984	SLT985	SLT986	SLT987	SLT988	SLT989	SLT990	SLT991	SLT992	SLT993	SLT994	SLT995	SLT996	SLT997	SLT998	SLT999	SLT1000	SLT1001	SLT1002	SLT1003	SLT1004	SLT1005	SLT1006	SLT1007	SLT1008	SLT1009	SLT1010	SLT1011	SLT1012	SLT1013	SLT1014	SLT1015	SLT1016	SLT1017	SLT1018	SLT1019	SLT1020	SLT1021	SLT1022	SLT1023	SLT1024	SLT1025	SLT1026	SLT1027	SLT1028	SLT1029	SLT1030	SLT1031	SLT1032	SLT1033	SLT1034	SLT1035	SLT1036	SLT1037	SLT1038	SLT1039	SLT1040	SLT1041	SLT1042	SLT1043	SLT1044	SLT1045	SLT1046	SLT1047	SLT1048	SLT1049	SLT1050	SLT1051	SLT1052	SLT1053	SLT1054	SLT1055	SLT1056	SLT1057	SLT1058	SLT1059	SLT1060	SLT1061	SLT1062	SLT1063	SLT1064	SLT1065	SLT1066	SLT1067	SLT1068	SLT1069	SLT1070	SLT1071	SLT1072	SLT1073	SLT1074	SLT1075	SLT1076	SLT1077	SLT1078	SLT1079	SLT1080	SLT1081	SLT1082	SLT1083	SLT1084	SLT1085	SLT1086	SLT1087	SLT1088	SLT1089	SLT1090	SLT1091	SLT1092	SLT1093	SLT1094	SLT1095	SLT1096	SLT1097	SLT1098	SLT1099	SLT1100	SLT1101	SLT1102	SLT1103	SLT1104	SLT1105	SLT1106	SLT1107	SLT1108	SLT1109	SLT1110	SLT1111	SLT1112	SLT1113	SLT1114	SLT1115	SLT1116	SLT1117	SLT1118	SLT1119	SLT1120	SLT1121	SLT1122	SLT1123	SLT1124	SLT1125	SLT1126	SLT1127	SLT1128	SLT1129	SLT1130	SLT1131	SLT1132	SLT1133	SLT1134	SLT1135	SLT1136	SLT1137	SLT1138	SLT1139	SLT1140	SLT1141	SLT1142	SLT1143	SLT1144	SLT1145	SLT1146	SLT1147	SLT1148	SLT1149	SLT1150	SLT1151	SLT1152	SLT1153	SLT1154	SLT1155	SLT1156	SLT1157	SLT1158	SLT1159	SLT1160	SLT1161	SLT1162	SLT1163	SLT1164	SLT1165	SLT1166	SLT1167	SLT1168	SLT1169	SLT1170	SLT1171	SLT1172	SLT1173	SLT1174	SLT1175	SLT1176	SLT1177	SLT1178	SLT1179	SLT1180	SLT1181	SLT1182	SLT1183	SLT1184	SLT1185	SLT1186	SLT1187	SLT1188	SLT1189	SLT1190	SLT1191	SLT1192	SLT1193	SLT1194	SLT1195	SLT1196	SLT1197	SLT1198	SLT1199	SLT1200	SLT1201	SLT1202	SLT1203	SLT1204	SLT1205	SLT1206	SLT1207	SLT1208	SLT1209	SLT1210	SLT1211	SLT1212	SLT1213	SLT1214	SLT1215	SLT1216	SLT1217	SLT1218	SLT1219	SLT1220	SLT1221	SLT1222	SLT1223	SLT1224	SLT1225	SLT1
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H-58

APPENDIX H

NOT REPRODUCIBLE

Table 3.4 (cont.)

TIME = 2.150000000 02

SIGR	SIGZ	SIGYH	SIGZ	SIGRZ	EPH	EPZ	EPZ	TEM
1 0.90E	00	1.15E 02	0.35E 01	0.15E 00	1.12E 01	2.03E 01	1.58E 00	-4.05E 01
2 2.44E	00	1.07E 02	0.23E 01	0.10E 00	0.90E 00	0.00	1.77E 00	0.00
3 4.16E	00	0.73E 01	0.14E 00	0.05E 00	0.22E 00	0.00	1.75E 00	0.00
4 4.91E	00	0.04E 01	0.14E 00	0.00E 00	0.22E 00	0.00	1.75E 00	0.00
5 3.17E	00	0.72E 01	0.14E 00	0.00E 00	0.22E 00	0.00	1.75E 00	0.00
6 5.34E	00	0.04E 01	0.14E 00	0.00E 00	0.22E 00	0.00	1.75E 00	0.00
7 5.55E	00	0.77E 01	0.14E 00	0.00E 00	0.22E 00	0.00	1.75E 00	0.00
8 5.55E	00	0.04E 01	0.14E 00	0.00E 00	0.22E 00	0.00	1.75E 00	0.00
9 5.70E	00	0.04E 01	0.14E 00	0.00E 00	0.22E 00	0.00	1.75E 00	0.00
10 5.70E	00	0.04E 01	0.14E 00	0.00E 00	0.22E 00	0.00	1.75E 00	0.00
11 5.70E	00	0.04E 01	0.14E 00	0.00E 00	0.22E 00	0.00	1.75E 00	0.00
12 5.70E	00	0.04E 01	0.14E 00	0.00E 00	0.22E 00	0.00	1.75E 00	0.00
13 5.70E	00	0.04E 01	0.14E 00	0.00E 00	0.22E 00	0.00	1.75E 00	0.00
14 5.70E	00	0.04E 01	0.14E 00	0.00E 00	0.22E 00	0.00	1.75E 00	0.00
15 5.70E	00	0.04E 01	0.14E 00	0.00E 00	0.22E 00	0.00	1.75E 00	0.00
16 5.70E	00	0.04E 01	0.14E 00	0.00E 00	0.22E 00	0.00	1.75E 00	0.00
17 5.70E	00	0.04E 01	0.14E 00	0.00E 00	0.22E 00	0.00	1.75E 00	0.00
18 5.70E	00	0.04E 01	0.14E 00	0.00E 00	0.22E 00	0.00	1.75E 00	0.00
19 2.99E	01	4.07E 03	2.58E 00	5.68E 01	1.10E 03	1.97E 03	2.47E 00	-4.05E 01

TIME = 2.400000000 02

SIGR	SIGZ	SIGYH	SIGZ	SIGRZ	EPH	EPZ	EPZ	TEM
1 1.06E	01	1.77E 02	0.80E 01	1.32E 00	1.50E 01	3.42E 01	2.07E 00	-6.01E 01
2 3.42E	01	1.50E 02	0.51E 01	1.42E 00	0.91E 01	2.48E 01	2.15E 00	0.00
3 6.74E	01	1.50E 02	0.51E 01	1.42E 00	0.91E 01	2.48E 01	2.15E 00	0.00
4 7.00E	01	1.50E 02	0.51E 01	1.42E 00	0.91E 01	2.48E 01	2.15E 00	0.00
5 7.00E	01	1.50E 02	0.51E 01	1.42E 00	0.91E 01	2.48E 01	2.15E 00	0.00
6 7.00E	01	1.50E 02	0.51E 01	1.42E 00	0.91E 01	2.48E 01	2.15E 00	0.00
7 7.00E	01	1.50E 02	0.51E 01	1.42E 00	0.91E 01	2.48E 01	2.15E 00	0.00
8 7.00E	01	1.50E 02	0.51E 01	1.42E 00	0.91E 01	2.48E 01	2.15E 00	0.00
9 7.00E	01	1.50E 02	0.51E 01	1.42E 00	0.91E 01	2.48E 01	2.15E 00	0.00
10 7.00E	01	1.50E 02	0.51E 01	1.42E 00	0.91E 01	2.48E 01	2.15E 00	0.00
11 7.00E	01	1.50E 02	0.51E 01	1.42E 00	0.91E 01	2.48E 01	2.15E 00	0.00
12 7.00E	01	1.50E 02	0.51E 01	1.42E 00	0.91E 01	2.48E 01	2.15E 00	0.00
13 7.00E	01	1.50E 02	0.51E 01	1.42E 00	0.91E 01	2.48E 01	2.15E 00	0.00
14 7.00E	01	1.50E 02	0.51E 01	1.42E 00	0.91E 01	2.48E 01	2.15E 00	0.00
15 7.00E	01	1.50E 02	0.51E 01	1.42E 00	0.91E 01	2.48E 01	2.15E 00	0.00
16 7.00E	01	1.50E 02	0.51E 01	1.42E 00	0.91E 01	2.48E 01	2.15E 00	0.00
17 7.00E	01	1.50E 02	0.51E 01	1.42E 00	0.91E 01	2.48E 01	2.15E 00	0.00
18 7.00E	01	1.50E 02	0.51E 01	1.42E 00	0.91E 01	2.48E 01	2.15E 00	0.00
19 4.58E	01	0.21E 03	3.30E 00	0.34E 01	1.03E 03	1.77E 03	2.04E 00	-6.01E 01

TIME = 2.600000000 02

SIGR	SIGZ	SIGYH	SIGZ	SIGRZ	EPH	EPZ	EPZ	TEM
1 7.04E	01	7.18E 00	4.14E 00	7.78E 00	4.24E 02	3.65E 02	2.17E 00	1.58E 02
2 2.21E	00	0.82E 00	0.00	0.00E 00	0.00E 00	0.00	1.95E 00	0.00
3 3.04E	00	0.91E 00	0.00	0.00E 00	0.00E 00	0.00	1.95E 00	0.00
4 3.33E	00	0.39E 00	0.00	0.00E 00	0.00E 00	0.00	1.95E 00	0.00
5 3.33E	00	0.07E 00	0.00	0.00E 00	0.00E 00	0.00	1.95E 00	0.00
6 3.33E	00	0.07E 00	0.00	0.00E 00	0.00E 00	0.00	1.95E 00	0.00
7 3.33E	00	0.07E 00	0.00	0.00E 00	0.00E 00	0.00	1.95E 00	0.00
8 3.33E	00	0.07E 00	0.00	0.00E 00	0.00E 00	0.00	1.95E 00	0.00
9 3.33E	00	0.07E 00	0.00	0.00E 00	0.00E 00	0.00	1.95E 00	0.00
10 3.33E	00	0.07E 00	0.00	0.00E 00	0.00E 00	0.00	1.95E 00	0.00
11 3.33E	00	0.07E 00	0.00	0.00E 00	0.00E 00	0.00	1.95E 00	0.00
12 3.33E	00	0.07E 00	0.00	0.00E 00	0.00E 00	0.00	1.95E 00	0.00
13 3.33E	00	0.07E 00	0.00	0.00E 00	0.00E 00	0.00	1.95E 00	0.00
14 3.33E	00	0.07E 00	0.00	0.00E 00	0.00E 00	0.00	1.95E 00	0.00
15 3.33E	00	0.07E 00	0.00	0.00E 00	0.00E 00	0.00	1.95E 00	0.00
16 3.33E	00	0.07E 00	0.00	0.00E 00	0.00E 00	0.00	1.95E 00	0.00
17 3.33E	00	0.07E 00	0.00	0.00E 00	0.00E 00	0.00	1.95E 00	0.00
18 3.33E	00	0.07E 00	0.00	0.00E 00	0.00E 00	0.00	1.95E 00	0.00
19 2.18E	00	0.00E 00	0.00	0.00E 00	0.00E 00	0.00	1.95E 00	0.00

Table H-6 Cont'd

APPENDIX II

Table 5.6 (cont.)

[illegible]**Table H-6-Cont'd**

APPENDIX I

NON-LINEAR ANALYSIS BASED ON PROPELLANT DILATATION

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Appendix I

NON-LINEAR ANALYSIS BASED ON PROPELLANT DILATATION

In general, the response of grains to various loading conditions is mainly determined by the dilatational behavior of the propellant, hence, it would appear that accounting for the non-linear dilatational response of propellant is of primary importance.

Although insufficient experimental data is available to permit a comprehensive characterization of the non-linear dilatational effects, it was felt that an evaluation could be obtained by utilizing a description which qualitatively accounts for the behavior reported for a limited class of stress states in References I-1 and I-2, and which predicts reasonable results for other stress states. Because of the tentative nature of the characterization, it was deemed desirable to approximate the non-linear behavior by an equation which could be incorporated into the existing analysis with a minimum amount of effort.

With the above objectives in mind the following approximations were made: (1) The thermo-rheologically simple linear viscoelastic distortional stress-strain law and the temperature induced volume change relationships would remain unchanged; (2) the stress induced dilatational response would be approximated as elastic. It was felt that the viscoelastic dilatation (compare Figures 11 and 12 of Reference I-2), is of secondary importance as compared to the basic non-linear effects. Additionally, the lack of experimental evidence concerning dilatation during unloading precludes its consideration at this time. (3) The stress-dilatation relationship would be considered to be temperature independent (it was felt that the actual temperature dependence was of secondary importance). (4) The characterization would be limited to relatively small strains. (5) The elastic relationship between the dilatation and the stress state could be expressed as:

$$(\theta - 3\alpha\Delta T) \approx f(g(\tau_{ij})) = f(\tau_{ij}) \quad (I-1)$$

where "f" is a non-linear algebraic function of the quantity "g", $g(\tau_{ij})$ is in turn a non-linear function of the stress invariants. The stress invariants are defined by the equations

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$$\Theta_1 = \tau_{11} + \tau_{22} + \tau_{33} \quad (I-2)$$

$$\Theta_2 = \begin{vmatrix} \tau_{22} & \tau_{23} \\ \tau_{32} & \tau_{33} \end{vmatrix} + \begin{vmatrix} \tau_{11} & \tau_{31} \\ \tau_{31} & \tau_{33} \end{vmatrix} + \begin{vmatrix} \tau_{11} & \tau_{12} \\ \tau_{12} & \tau_{22} \end{vmatrix}$$

$$\Theta_3 = \begin{vmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{12} & \tau_{22} & \tau_{23} \\ \tau_{13} & \tau_{23} & \tau_{33} \end{vmatrix}$$

For linear theory we have $g = \Theta_1$ and $f = \frac{K}{3} g$ i.e.,

$$(\Theta - 3\alpha\Delta T) = K \frac{\Theta_1}{3} \quad (I-3)$$

Appropriate forms of the functions "f" and "g" were determined in the following manner. For a uniaxial state of stress (i.e., $\tau_{ij} = 0$ except $\tau_{11} \neq 0$) noting that $\Theta_2 = \Theta_3 = 0$, it was assumed that

$$g(\Theta_1) \approx \Theta_1 = \tau_{11}, \text{ i.e.,}$$

$$(\Theta - 3\alpha\Delta T) \approx f(\tau_{11}) \quad (I-4)$$

Inspecting results from uniaxial tests (and uniaxial tests with small superimposed hydrostatic pressures) the form of the function "f" was determined. The form of the function "g" was determined by inspecting the results of uniaxial tests with various levels of superimposed hydrostatic pressures. Lastly, the forms of the functions "f" and "g" were appropriately modified so that the predictions for other stress states would intuitively agree with the physical explanation given in Reference (I-2) for the non-linear dilatation phenomenon.

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The experimental results presented in Figures 8, 9, 10, 12 and 14 of Reference (I-2) were plotted in the form $\theta \sim \tau_{11}$ in Figure I-1. Although these data certainly demonstrate a dependence upon temperature and strain rate (and of course a dependence upon "volume of loading"), insufficient data were available to permit a description of this dependence, hence, it was neglected. No data was available for compressive stress states, however, it was anticipated that for negative stresses the relationship would be relatively linear. The data presented in Figure I-1 (i.e., the function "f") was approximated by a hyperbola, i.e., the relationship between θ and the function "g" was written as:

$$\theta = \frac{P_2 + P}{2} + \frac{\beta_2 - \beta}{2} + B \quad (I-5)$$

where

$$\beta_1 = \theta_o + \frac{1}{K_T} (g_1 - g_o)$$

$$B = \frac{\beta_1 (\theta_o - g_o/K_c)}{1 - \frac{\beta_1 K_g}{g_1}}$$

$$\beta_2 = \frac{BK_T}{g_1} - \frac{g}{K_c}$$

$$\beta = \frac{1}{K_T} (g_1 - g)$$

The meaning of the parameters K_T , K_c , g_1 , g_o and θ_o is illustrated in

VOLUME CHANGE
(Data taken from Reference (7))

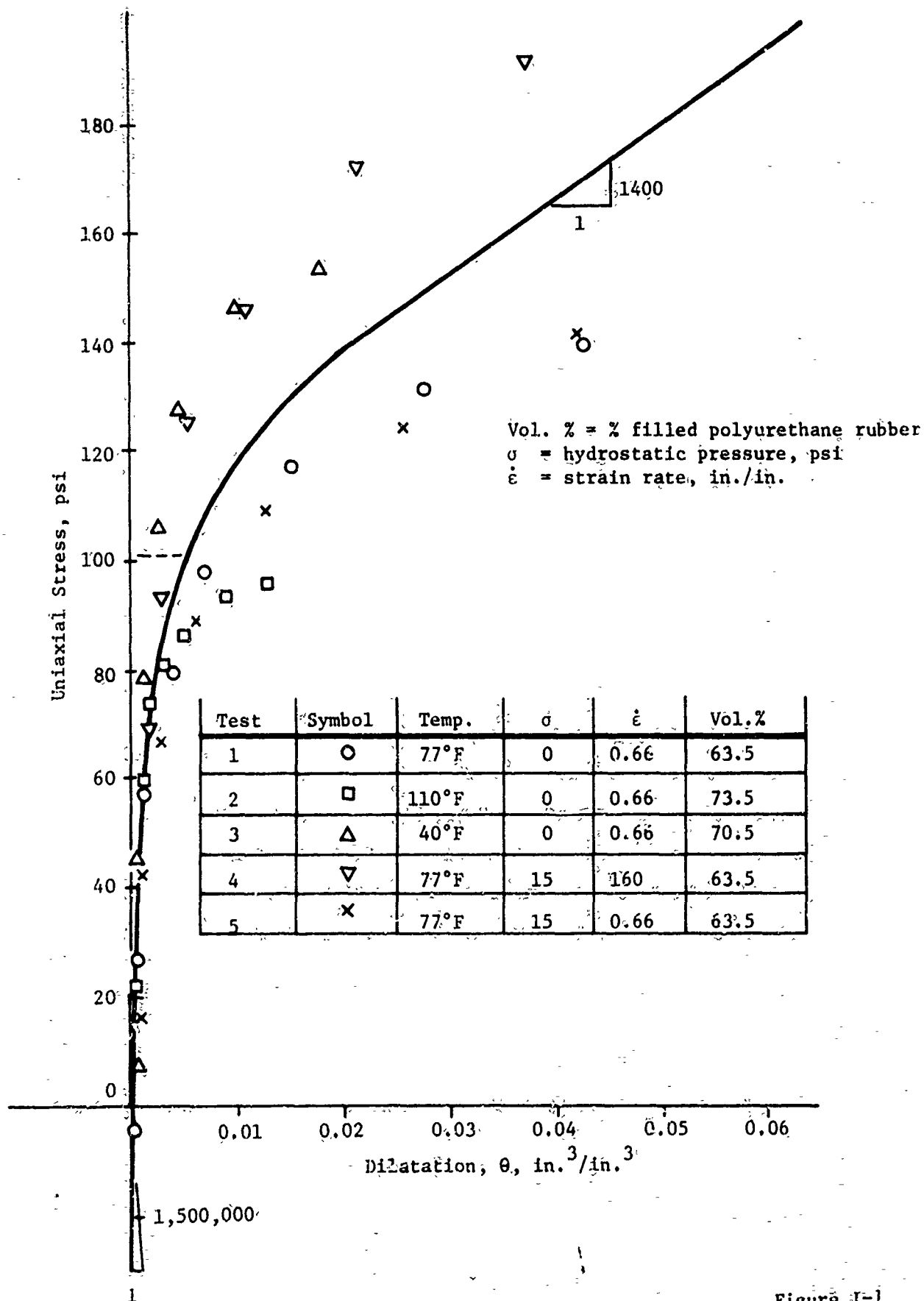


Figure I-1

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Figure I-2. The form of the function $g(\theta_1)$ was determined by considering the data presented in Figure 1 of Reference (I-2). A number of different functional forms were investigated from among which the following form was selected.

$$g(\theta_1) = \theta_1 - K_1 \theta_1^2 + K_2 \theta_1^3 + K_3 \theta_1^3 \quad (I-6)$$

The values of K_1 , K_2 , and K_3 were selected by noting that Equation (I-1) states that for a given amount of dilatation the value of "g" should be a constant. Thus for the different stress states (corresponding to different values of hydrostatic pressure) given in Figure 1 of Reference (I-2) an attempt was made to select the function "g" so that for a given value of θ all states would yield a common value of "g". The following values were found for the parameters

$$K_1 = 4.9 \times 10^{-3}$$

$$K_2 = 2.3 \times 10^{-5}$$

$$K_3 = 5.7 \times 10^{-6}$$

Using these parameters the following table was constructed

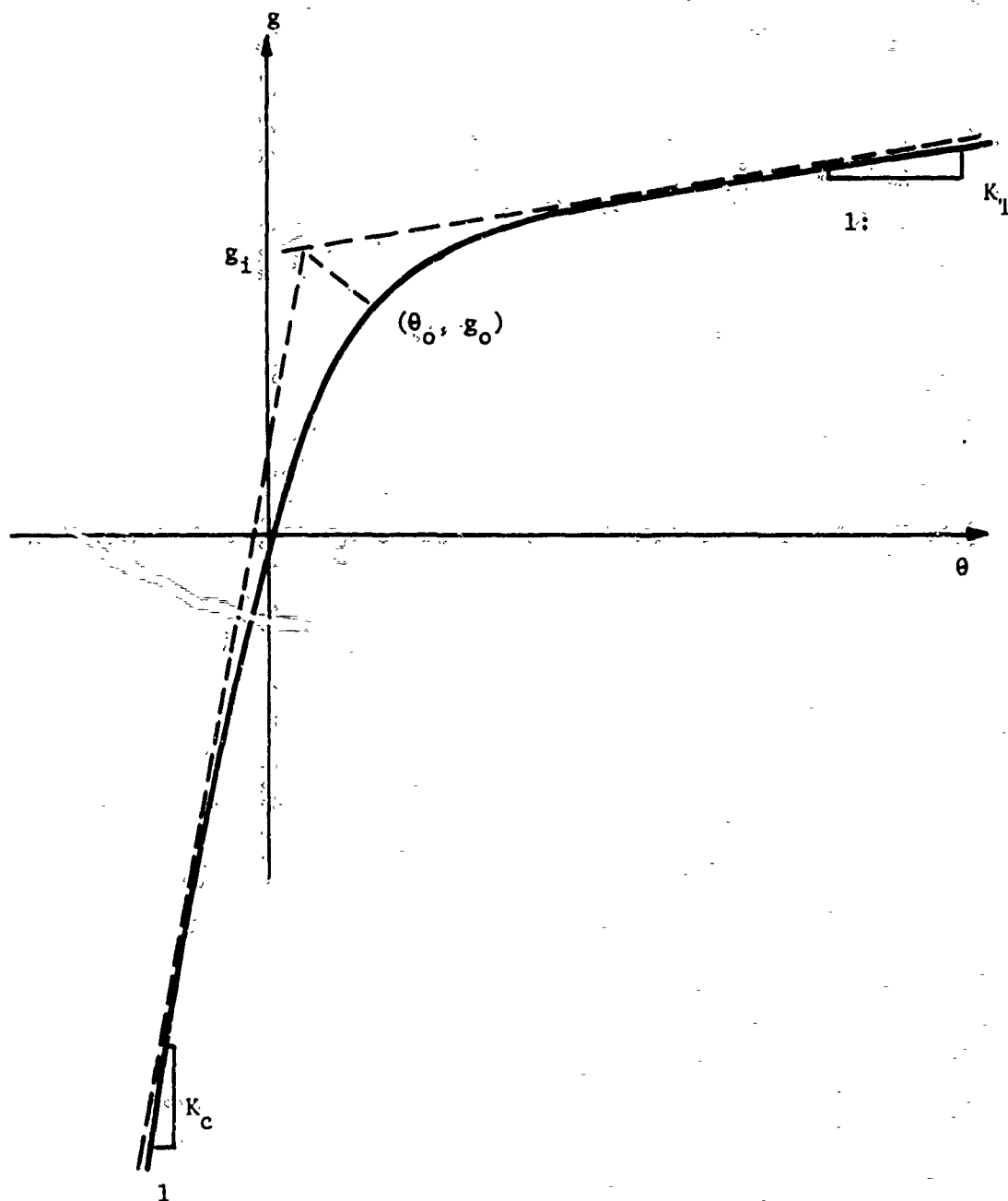
1	2	3	4	5
Hydrostatic Pressure, psi	$\theta = .005$		$\theta = .01$	
	g	$\theta/3$	g	$\theta/3$
15	66	54	84	68
65	66	-5	99	14
165	101	-150	125	-115

If the dilatational response could be described by a linear elastic law all the entries in Column 3 (and in Column 5) would need to be approximately equal, the extreme grossness of this approximation is evident. If the non-linear function "g" is to represent the experimental data all the entries in Column 2 (and in Column 4) would need to be

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PARAMETERS DEFINING HYPERBOLIC
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approximately equal, while this is not true, the very substantial improvement of the non-linear representation as compared to the linear representation is evident.

While no experimental information was available for stress states other than the uniaxial test with superimposed hydrostatic pressure, it is extremely important that the proposed model predicts reasonable results for other stress states as these stress states may occur in a given grain problem. Utilizing the proposed non-linear model dilatation was predicted for the following stress states:

- (a) Uniaxial stress with superimposed hydrostatic pressure (Figure I-3)
- (b) Hydrostatic pressure (Figure I-4)
- (c) Biaxial stress with superimposed hydrostatic pressure (Figure I-5)
- (d) Simple shear with superimposed hydrostatic pressure (Figure I-6)

The straight lines in Figure I-3 indicate the predictions of the linear model, the curved lines the predictions of the non-linear model. The predictions of the non-linear model qualitatively appear to be reasonable.

The incorporation of the non-linear dilatational relationship (Equation (I-1)) into the existing thermoviscoelastic analyses presented some difficulties. The existing thermoviscoelastic analysis employs the following incremental relationship for dilatation (for time increment N):

$$(\Delta\theta_N - 3\alpha\Delta T_N) = K_N \frac{\Delta\theta_N}{3} + \chi_N t_{N-1} \leq t \leq t_N \quad (I-7)$$

In the previous linear analysis K_N was considered to be a constant (the elastic bulk modulus) and $\chi_N = 0$. The utilization of Equation (I-7) as an approximation to Equation (I-1) was accomplished as follows:

Having obtained the solution for time t_{N-1} consider the utilization of Equation (I-7) for $t_{N-1} \leq t \leq t_N$. Symbolically denoting the stress state at time t_{N-1} as $\tau_{ij,N-1}$ and the incremental change in stress state for

$t_{N-2} \leq t_{N-1}$ as $\Delta\tau_{ij,N-1}$. The following two associated stress states are defined:

Appendix 1

DILATATIONAL PREDICTIONS FOR UNIAXIAL STRESS WITH SUPERIMPOSED HYDROSTATIC PRESSURE

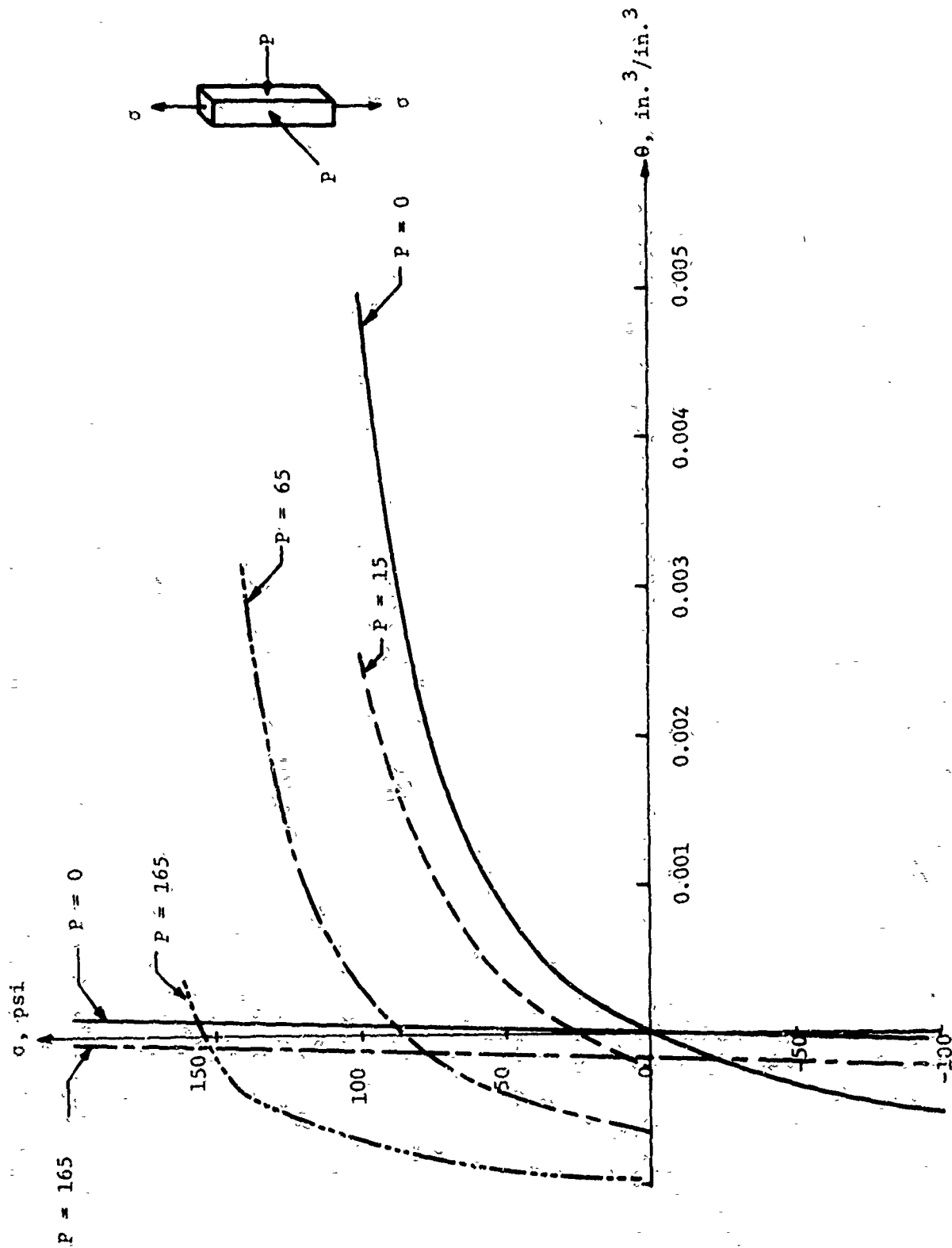
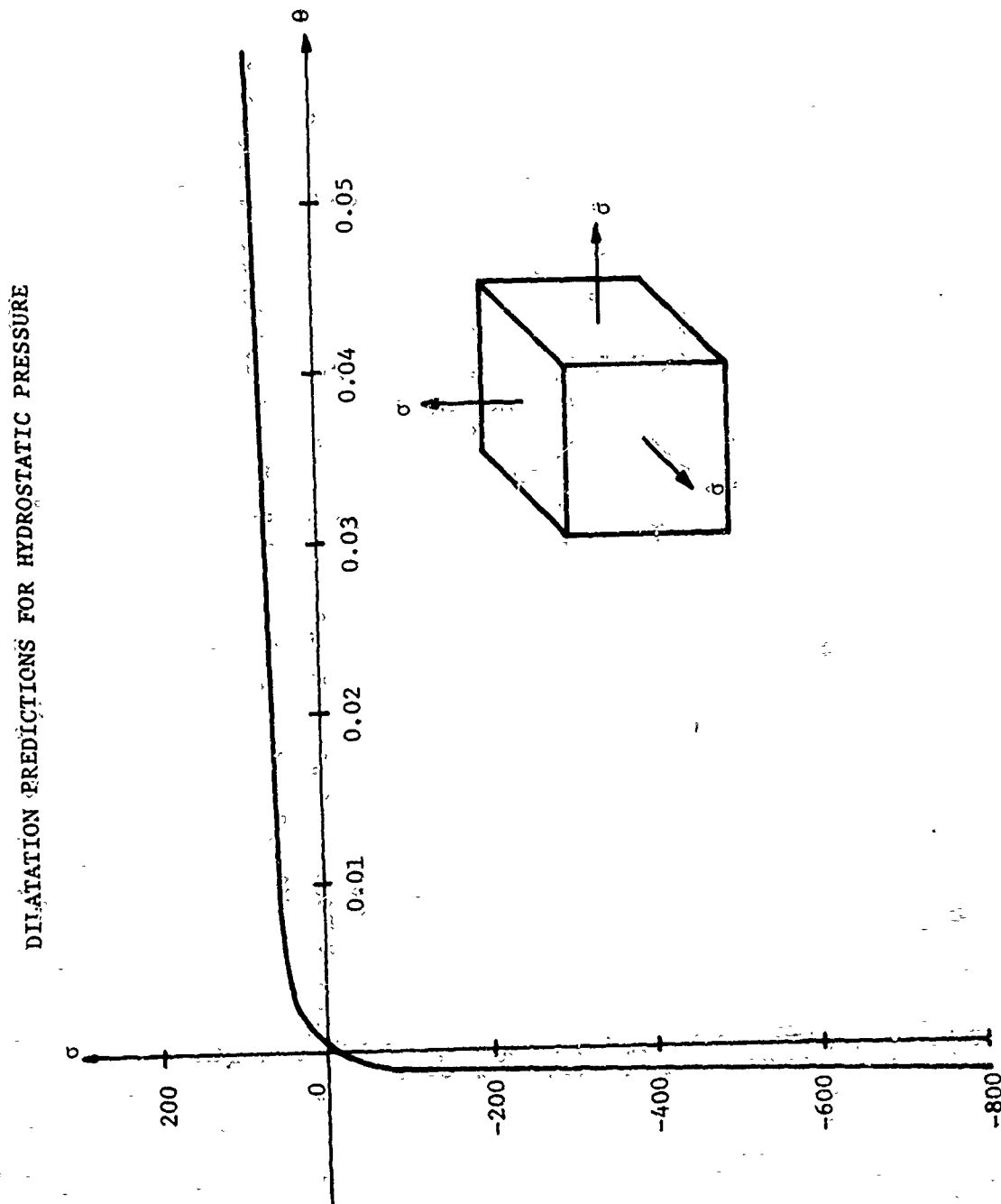
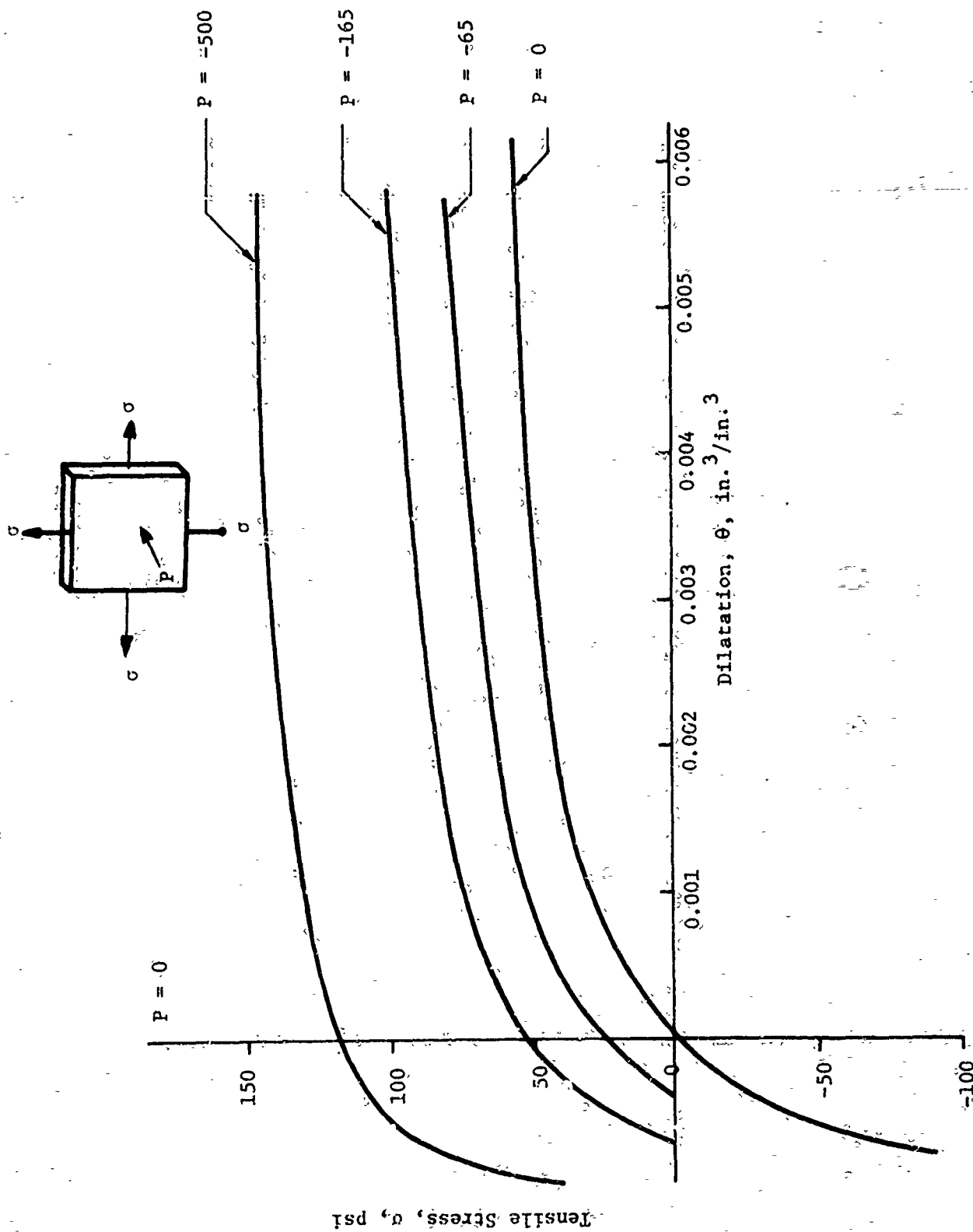


Figure I-3.





DILATATIONAL PREDICTIONS FOR BIAXIAL STRESS WITH SUPERIMPOSED HYDROSTATIC PRESSURE

Figure I-5

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Appendix 1

DILATATIONAL PREDICTIONS FOR SHEAR WITH SUPERIMPOSED HYDROSTATIC PRESSURE

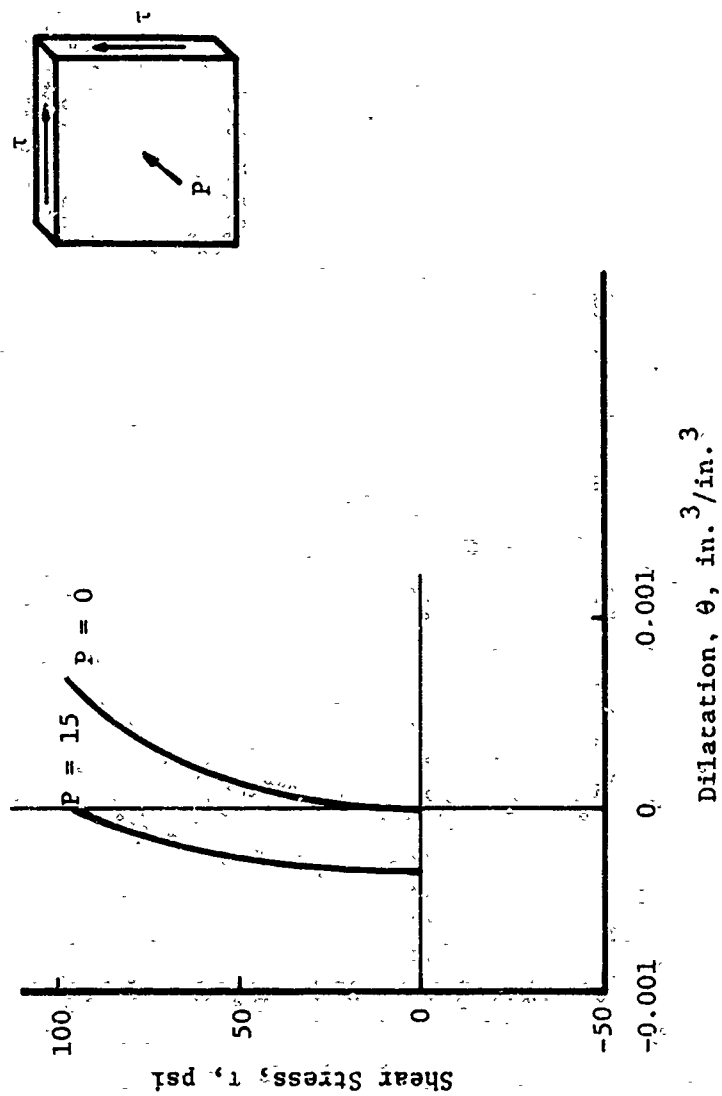


Figure I-6

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$$\tau_{ija} \equiv \tau_{ij_{N-1}} + \Delta \tau_{ij_{N-1}} \quad (I-8)$$

$$\tau_{ijb} \equiv \tau_{ij_{N-1}} + \Delta \tau_{ij_{N-1}} \quad (I-9)$$

Utilizing Equations (I-1), (I-2), (I-8), and (I-9), the following quantities are calculated θ_a , θ_b , θ_a and θ_b . The following approximate relationship is written for values of θ between θ_a and θ_b

$$\theta = \frac{\theta_a \theta_b - \theta_b \theta_a}{\theta_a - \theta_b} + \frac{\theta_a - \theta_b}{\theta_a - \theta_b} \theta \quad (I-10)$$

Assuming that the absolute value of the change in stress during the increment $\Delta \tau_N$ will be of the same order of magnitude as $|\Delta \tau_{ij_{N-1}}|$

(i.e., using Equation (I-10) and writing $\theta_N = \theta_{N-1} + \Delta \theta_N$ and $\theta_N = \theta_{N-1} + \Delta \theta_N$ yields

$$\theta_{N-1} + \Delta \theta_N = \frac{\theta_a \theta_b - \theta_b \theta_a}{\theta_a - \theta_b} + \frac{\theta_a - \theta_b}{\theta_a - \theta_b} (\theta_{N-1} + \Delta \theta_N) \quad (I-11)$$

Comparing the above expression to Equation (I-7) yields

$$K_N = \frac{\theta_a - \theta_b}{\theta_a - \theta_b}$$

$$X_N = \frac{\theta_a \theta_b - \theta_b \theta_a}{\theta_a - \theta_b} + \frac{\theta_a - \theta_b}{\theta_a - \theta_b} \theta_{N-1} - \theta_{N-1}$$

Because of the secant approximation to the non-linear function, in general, $X_N \neq 0$.

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References.

- I-1 Farris, R. J., "The Character of the Stress-Strain Function for Highly Filled Elastomers", Trans. of the Soc. of Rheology 12:2, 303-314 (1968).
- I-2 Farris, R. J., "The Influence of Vacuole Formation on the Response and Failure of Filled Elastomers", Trans. of the Soc. of Rheology 12:2, 315-334 (1968).

APPENDIX J

BASIC CUMULATIVE DAMAGE EQUATIONS

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BASIC CUMULATIVE DAMAGE EQUATIONS

The LCD relation merely states the manner in which damage can be added. To apply the relation in a three-dimensional stress field requires a time-dependent, stress failure criterion. An empirical approach showed that the applicable relation is a time-dependent, maximum principal stress, failure criterion (MPS).

Combining the LCD and MPS criteria leads to integral relations which take full account of the past stress-time-temperature history at a point in the grain.

1. Linear Cumulative Damage Relations

The very simple relation, applicable to solid propellant failures, is defined in terms of tests made under a constantly imposed "true" stress, σ_t . For a number, N, of discrete stress levels the accumulated damage fraction, ΣD , is given by the following linear relation:

$$\Sigma D = \frac{1}{P(n)} \sum_{i=1}^N \frac{\Delta t_i}{\bar{t}_{fi}} \quad (J-1)$$

where ΣD is the cumulative damage

$P(n)$ is a statistical distribution parameter and relates the n^{th} test specimen in the distribution to the mean of the population.

Δt_i is the increment of time the specimen is exposed to the i^{th} "true" stress level

$\bar{t}_{fi}$ is the mean time-to-failure for the population of specimens if the specimens saw only the i^{th} "true" stress level.

The accumulated damage, ΣD , thus gives the fraction of that damage required to fail the specimen. Thus, by definition $\Sigma D = 1$ at failure.

The parameter, $P(n)$, is a highly useful term in that it provides the focal point for all the statistical studies pertinent to the cumulative damage relations. In its simplest form $P(n)$ defines the position of the individual failure with respect to the mean of the distribution of these failures. This is seen on considering specimens held under a single stress, where $P(n)$ equals the ratio of the time-to-failure, $t_f(n)$, for the n^{th} individual divided by the mean time-to-failure for the entire population, $\bar{t}_f$.

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Thus, we have

$$P(n) = t_f(n) / \bar{t}_f \quad (J-2)$$

The cumulative damage relation merely preserves this concept of $P(n)$ for conditions of complicated loading histories.

In Reference J-1, it was shown that $P(n)$ is independent of the stress level. This justifies placing the parameter outside the summation in Equation (J-1).

Statistical evaluations of $P(n)$ obtained from solid propellant failure testing, show the parameter to follow approximately a log normal distribution.

2. Time-Dependent Maximum Principal Stress Failure Criterion

The most general form of the MPS failure criterion for solid propellants appears to be a modified power-law relation. However, a simple power-law approximation to this criterion covers all of the practical motor problems that we have met to date. This power-law relation has the following form:

$$\bar{t}_f = t_o a_T \left(\frac{\sigma_t - \sigma_{cr}}{\sigma_{t_o} - \sigma_{cr}} \right)^{-B} \quad (J-3)$$

- where
- σ_t is the "true" stress applied to the specimen.
 - $\bar{t}_f$ is the mean time-to-failure of specimens held under the constant true stress, σ_t .
 - t_o is the unit value of the time for whatever units are used in measuring t_f .
 - σ_{t_o} is the true stress required to fail the specimen in the time t_o .
 - σ_{cr} is the critical true stress below which no failures are observed.
 - a_T is the time-temperature shift relation.

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It has been found to apply to solid propellant behavior with correlation coefficients usually exceeding 0.98.

The limiting stress, σ_{cr} , is difficult to evaluate because it is usually not large compared with the relatively large variability typical of solid propellants. It appears to be negligibly small (except for the effects of pressure) for all the propellants tested to date.

The cumulative damage relation is generalized as follows. First, Equation (J-3) is combined with Equation (J-1) to obtain the cumulative damage relation in terms of discrete stress levels, loading time intervals, and test temperatures.

$$\Sigma D = \frac{1}{P(n)} \sum \left[\frac{\sigma_{ti} - \sigma_{cr}}{\sigma_{to} - \sigma_{cr}} \right]^B \frac{\Delta t_i}{t_o a_T} \quad (J-4)$$

where σ_{ti} is the i^{th} stress level.

For continuous changes in the stress and in the temperature, Equation (J-4) becomes:

$$\Sigma D \approx \frac{1}{P(n) (\sigma_{to} - \sigma_{cr})^B t_o} \int_0^t \frac{(\sigma_t - \sigma_{cr})^B}{a_T(t)} dt \quad (J-5)$$

where σ_t , the true stress, is now a function of time.

$a_T(t)$ is the time-temperature shift relation with the temperature expressed as a function of time.

Equation (J-5) represents a general form of the linear cumulative damage relation for solid propellants. This equation permits the summation of damage for any type of thermal or mechanical loading history, provided the stresses, times and temperatures are known. Also, Equation (J-5) can be applied without change to three dimensional stress problems.

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REFERENCES

- J-1 Bills, K. W., Jr., Peterson, F. E., Steele, R. D., and Sampson, R. C.,
"Development of Criteria for Solid Propellant Screening and
Preliminary Engineering Design", Aerojet Report 1159-81F
(December 1968).

APPENDIX K

STUDY OF PROPELLANT FAILURE UNDER PRESSURE

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APPENDIX K

STUDY OF PROPELLANT FAILURE UNDER PRESSURE

A. FAILURE MECHANISMS

The basic mechanisms of propellant failure have been extensively studied under current and past research programs (K-1, K-2). Failure is considered to be a three step process beginning with vacuole formation. The vacuoles may initiate in the polymer or at the binder-filler interface. The associated volume change may be so negligible that the process is termed a "Mullin's Effect". The second step is the extension of the vacuoles until they overlap producing a thin membrane of polymer between them. The third and final step is the production of a critical, biaxial, stress field in one or more of the thin membranes. The resulting tears propagate very slowly relative to the speed of sound in the propellant. This failure mechanism is the basis of the considerations made later.

The maximum principal stress failure criterion, Equation (J-3), contains a simple stress difference term, $\sigma_t - \sigma_{cr}$. Some feel that this indicates a maximum shear stress criterion to be operative. In biaxial tension this is expressed by

$$\tau(\text{critical}) = (\sigma_1 - \sigma_2)/2 \quad (K-1)$$

where $\tau(\text{critical})$ is the maximum shear strength of the material
 σ_1 and σ_2 are principal stresses

Considering the stress difference terms only, for both failure criteria, leads to the tabulation in Table K-1. It is assumed that a maximum principal stress, σ , is imposed in tension. The remaining stresses are tabulated.

Table K-1 shows that the two criteria give identical results for simple uniaxial and strip-biaxial tensile data from which the MPS criterion was derived. However, the two criteria differ greatly when applied to the triaxial, tensile, stress field of the poker chip specimen. In this case, the stress difference for the maximum shear stress criterion is dependent upon $1-k$, which is very small, indicating that very large axial stresses, σ , would be required to fail the specimen. On the other hand, the MPS criterion gives a relatively large value for the stress difference term, $\sigma_t - \sigma_{cr}$. This difference, like those for the uniaxial and biaxial tensile data, equals the axial stress on the specimen. Tests on poker chip specimens showed the MPS criterion to hold, (K-2) while the maximum shear stress criterion is grossly in error.

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TABLE K-1

STRESS DIFFERENCES AT ATMOSPHERIC PRESSURE

Test Mode	MPS Criterion			Maximum Shear Stress Criterion		
	σ_t	σ_{cr}^*	$\sigma_t - \sigma_{cr}$	σ_1	σ_2	$\sigma_1 - \sigma_2$
Uniaxial Tensile Test	σ	0	σ	σ	0	σ
Strip-Biaxial Tensile Test	σ	0	σ	σ	0	σ
Poker Chip Tensile Test	σ	0	σ	σ	$k\sigma$	$(1-k)\sigma$

* σ_{cr} is set equal to zero, which appears to be the case for many composite propellants.

** The maximum tensile stress at the center of the specimen.

*** k is a proportionality factor which, in this case, is close to one.

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This result is not surprising in view of the failure mechanisms summarized above. Tensile failures in uniaxial, biaxial, and poker chip specimens follow the same mechanism. Once the vacuoles are produced all three test modes become equivalent, in that failure is produced by extension of a foam-like material until the biaxial stress in the membranes exceeds the critical value and tearing starts.

B. EVALUATING THE TIME-PRESSURE SHIFT FACTOR

The time-pressure shift factor for propellants is best understood on referring to the foam-like failure behavior mentioned above. Before the vacuoles are formed, or when they are still quite small, a_p equals one (up to about 1500 psi pressure). This was shown for tensile moduli by Wiegand, (K-3) Hazelton (K-4) and Lim and Tshoegel (K-5). At this point, the a_p values are those for the solid binder which requires large pressure changes to effect significant changes in a_p . (K-6 to 14). On the other hand, propellant failure data are sensitive to relatively small pressure changes. Dilatation constitutes the only physical difference in the propellant between the points where initial tensile moduli and failure data are obtained. At failure, this dilatation is extensive and can be partially suppressed by superimposed hydrostatic pressures. As shown by Farris (K-15) and Lim and Tshoegel (K-5), the dilatation at failure is positive, even at the higher pressure levels.

Taken together these observations indicate that a_p provides a measure of the suppression of vacuole formation. But, σ_{cr} is also a measure of the same effect. That is, σ_{cr} is the stress level at which dilatation is completely suppressed; or, conversely, the stress level where vacuoles are first formed. Also, σ_{cr} is pressure dependent, defining the point of vacuole formation over the pressure range. Thus, a_p must be a simple function of σ_{cr} . That is,

$$a_p = f(\sigma_{cr}) \quad (K-2)$$

Equation (K-2) tells us how to evaluate a_p deep in the grain. The critical stress acts in the same direction as the maximum principal stress, σ_t , and it includes that portion of the inner-bore pressure that is effective deep within the grain and which acts in the same direction as σ_t . The fraction, f , of the inner-bore pressure that acts within the grain can be obtained directly from existing stress analyses. Using this fraction the critical stress becomes, on using Equation (14) of the text,

$$\sigma_{cr} = (F/A)_{cr} = fP_i \quad (K-3)$$

where P_i is the inner-bore pressure

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The appropriate values of a_p for use in Equation (K-2) are obtained from empirically obtained curve of a_p versus pressure, the desired value of a_p read from the curve at a pressure numerically equal to σ_{cr} , the latter being obtained from Equation (K-3).

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Appendix K

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APPENDIX L
EFFECTS OF PREVIOUS DAMAGE

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APPENDIX L

EFFECTS OF PREVIOUS DAMAGE

The cumulative damage relations can be used to evaluate the effects of previous damage upon propellant failure data. This may be done using the LCD analysis of a single test-to-failure curve which duplicates a given failure condition in the motor. When considering the inner-bore failure of a grain on motor pressurization we use a high rate, uniaxial tensile test performed under a superimposed hydrostatic pressure.

The cumulative damage relation appropriate to this analysis is derived here, starting with Equation (16) from the text. For this analysis, considerations are simplified by assuming the pressure and temperature to be held constant. Thus a_p and a_T become constants and may be taken outside the integral in Equation (16).

For the previously undamaged specimen the time-to-break, t_b , and the maximum true stress become normalizing parameters for the integral term, as originally shown in Reference (L-1). Thus, we let

$$A = \int_0^{t_b/t_b} \frac{(\sigma_t - \sigma_{cr})^B}{(\sigma_{tM} - \sigma_{cr})^B} d(t/t_b) \quad (L-1)$$

Equation (16) after normalization by substituting A, gives

$$\Sigma D = 1 = \frac{(\sigma_{tM} - \sigma_{cr})^B t_b A}{P(n) (\sigma_{to} - \sigma_{cr})^B t_o a_T a_p} \quad (L-2)$$

Prior to failure we can use the same normalization terms, t_b and $(\sigma_{tM} - \sigma_{cr})^B$, but take the quantity A prior to failure with the upper integration limit the time $(t < t_b)$ at which we wish to assess the damage. Thus,

$$A(t) = \int_0^{t/t_b} \frac{(\sigma_t - \sigma_{cr})^B}{(\sigma_{tM} - \sigma_{cr})^B} d(t/t_b) \quad (L-3)$$

and

$$\Sigma D(t) = \frac{(\sigma_{tM} - \sigma_{cr})^B t_b A(t)}{P(n) (\sigma_{to} - \sigma_{cr})^B t_o a_p a_p} \quad (L-4)$$

This relation is greatly simplified on dividing Equation (L-4) by (L-2) to give

$$\Sigma D(t) = A(t)/A \quad (L-5)$$

Considering Equation (L-5), if the specimen which had been previously damaged fails at the time t , then the total damage on the specimen, by definition, must equal one. Since $\Sigma D(t)$ is less than one, the difference must be accounted for by the previous damage, ΣD_p . Thus, $\Sigma D(t)$ becomes

$$\Sigma D(t) = 1 - \Sigma D_p \quad (L-6)$$

The values of A and $A(t)$ can be obtained by integration of the data from a single uniaxial test record, from which the ratio $A(t)/A$ and hence ΣD_p can be plotted versus testing time. From the original test record we can make a plot of the stress versus time and the strain versus the time. Using these three plots then for failure at any given testing time, t , corresponding values for the stress, the strain and ΣD_p can be read off. Plotting the values of σ and ϵ versus ΣD_p gives the desired failure curves and shows the reduction in the undamaged^p values ϵ_b and σ_b caused by the previous damage.

APPENDIX M
INPUT DATA FOR PRESSURIZATION TESTS
ON A PBAN PROPELLANT

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APPENDIX M

INPUT DATA FOR PRESSURIZATION TESTS ON A PBAN PROPELLANT

The data input procedures follow those shown in Appendix A. The Prony Series constants are tabulated in Table M-1. The time-temperature and the time-pressure shift factors are tabulated in Table M-2.

The cumulative damage parameters for this propellant are as follows:

$$B = 8.75$$

$$\sigma_{to} = 71 \text{ psi}$$

$$t_o = 1 \text{ min.}$$

For reference purposes we have provided a master relaxation curve, Figure M-1, and the curves for a_T and a_p , Figures M-2 and M-3, respectively.

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TABLE M-1

MODULUS INPUT FOR PBAN PROPELLANT

<u>log t/a_T, min.</u>	<u>E(T) psi</u>	<u>B_T min.⁻¹</u>	<u>A_T/3</u>	<u>i</u>
			8	0
-11	11,000	5 x 10 ¹⁰	3241.4	1
-10	6,600	5 x 10 ⁸	1187.7	2
-9	3,700	5 x 10 ⁶	287.07	3
-8	1,850	5 x 10 ⁴	108.14	4
-7	1,200	5 x 10 ²	45.09	5
-6	750	5 x 10 ⁰	32.153	6
-5	550	5 x 10 ⁻²	18.209	7
-4	398	5 x 10 ⁻⁴	14.289	8

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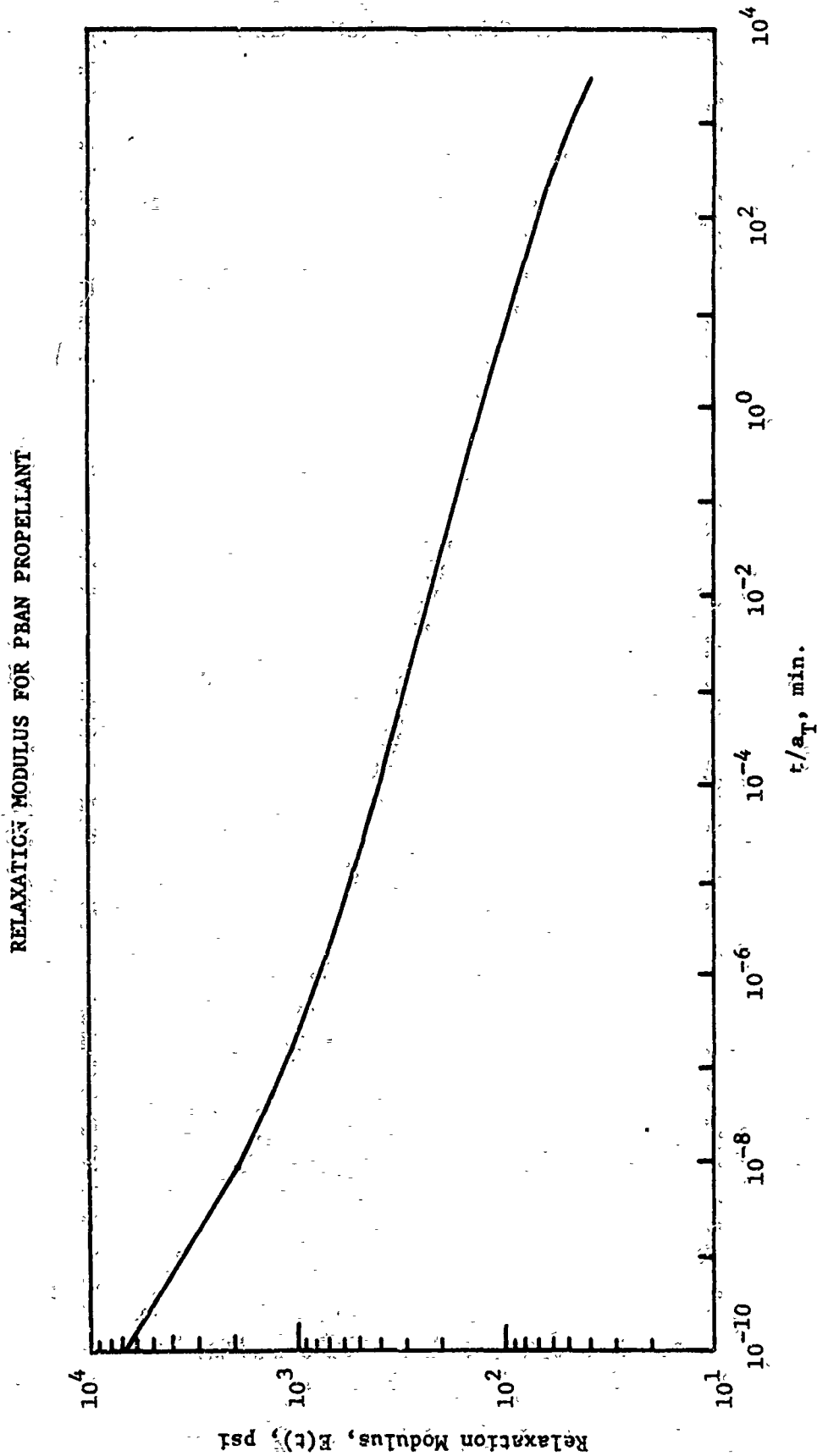
TABLE M-2
SHIFT FACTORS FOR PBAN PROPELLANT

TIME-TEMPERATURE SHIFT FACTORS

<u>Temp., °F</u>	<u>$\log_{10} a_T$</u>
180	-2.508
150	-1.903
110	-0.876
77	0
40	2.097
0	4.057
-40	7.008
-75	10.528

TIME-PRESSURE SHIFT FACTOR

<u>Press., psig.</u>	<u>a_p</u>	<u>$\log_{10} a_p$</u>
0	1	0
200	51	1.71
600	150	2.176
1000	250	2.398



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TIME-TEMPERATURE SHIFT FACTOR FOR PBAN PROPELLANT

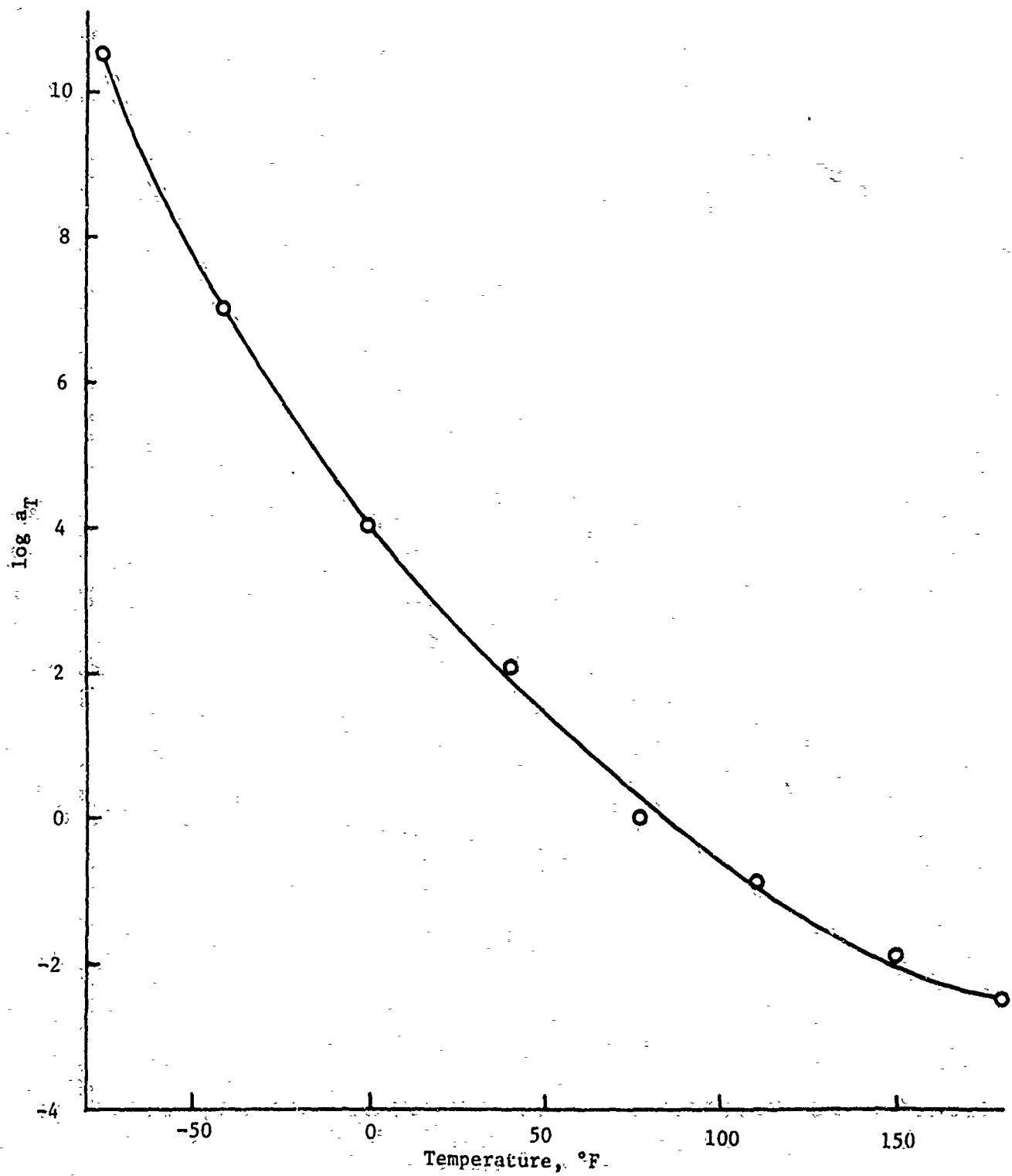
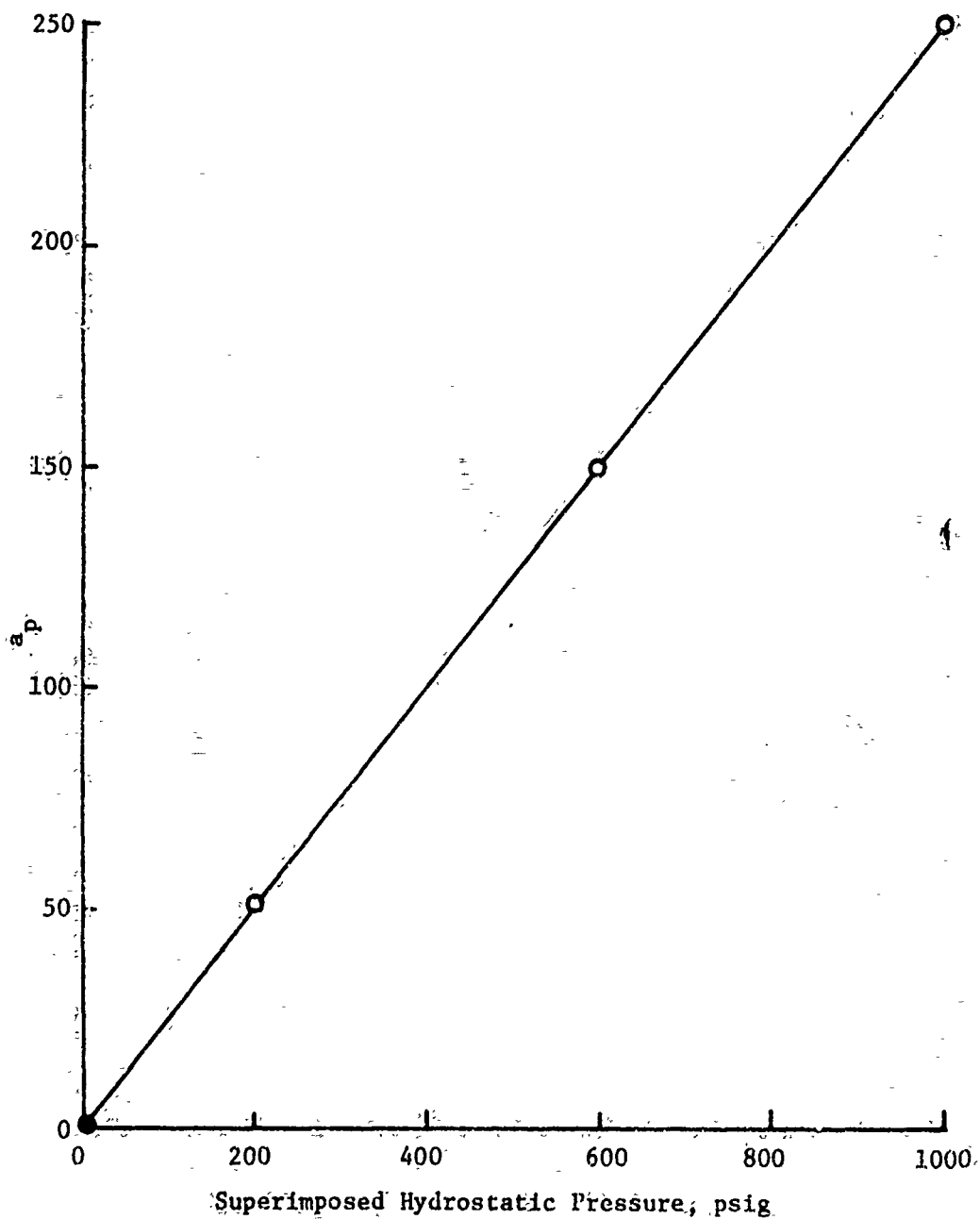


Figure M-2

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Appendix M

TIME-PRESSURE SHIFT FACTOR FOR PBAN PROPELLANT



Superimposed Hydrostatic Pressure, psig

Figure M-3