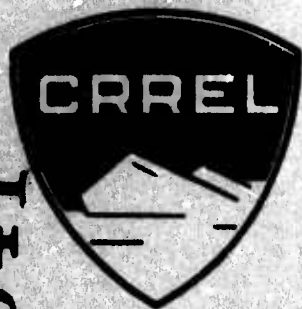


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EXPERIMENTAL AND THEORETICAL STUDIES OF THE MECHANISM OF FROST HEAVING

Bruce Chalmers
and
Kenneth A. Jackson

October 1970

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HANOVER, NEW HAMPSHIRE

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PREFACE

Authority for the investigation reported herein is contained in FY 1958 Instructions and Outline, Military Construction Investigations, Engineering Criteria and Investigations and Studies, Studies of Construction in Areas of Seasonal Frost: Cold Room Studies. The manuscript was originally prepared in 1963.

The study was conducted for the Engineering Division, Directorate of Military Construction, Office, Chief of Engineers. The Military Construction Investigations Program is presently conducted for the Office of Plans, Research and Systems (OPRS), Directorate of Military Construction.

Principal investigators were Dr. Bruce Chalmers and Dr. Kenneth A. Jackson, Division of Engineering and Applied Physics, Harvard University, under Contract DA19-016 Engineering Corps 5745, with the Arctic Construction and Frost Effects Laboratory. ACFEL was merged with the former Snow, Ice and Permafrost Research Establishment (SIPRE) in 1961 to form the Cold Regions Research and Engineering Laboratory (CRREL), Hanover, New Hampshire.

This report was prepared for the Applied Research Branch (Mr. A.F. Wuori, Chief) as a project of the Experimental Engineering Division (Mr. K.A. Linell, Chief). Mr. C.W. Kaplar, Applied Research Branch, was project leader and also technical reviewer of this report.

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NOMENCLATURE

A_i	Surface area of the ice structure
A_v	Surface area of the voids
C	Changes in pressure due to change in water content
D	kC
k	Coefficient of permeability of the soil
K_f, K_u	Thermal conductivities of frozen and unfrozen soil respectively
L	Latent heat per unit volume
h	Depth of ice lens below surface
h_0	Depth of ground water table
h_1	$h_0 - h$
P	Pressure, or energy per unit volume
P_L	Load pressure
P_S	Suction pressure
r	Radius of curvature of the interface
\bar{r}	Average void size
R	Rate of heave
T_E	$273^\circ K$
v	Rate of advance of freezing front
V_i	Volume of ice transformed
V_v	Volume of voids
ΔF_v	Free energy per unit volume of ice
ΔT	Freezing point depression
ρ_w, ρ_i	Densities of water and ice respectively
σ	Solid-liquid surface free energy per unit area

EXPERIMENTAL AND THEORETICAL STUDIES OF THE MECHANISM OF FROST HEAVING

by

Bruce Chalmers and Kenneth A. Jackson

INTRODUCTION

This work is in part an extension of the theory of frost heave developed by the authors and in part an attempt to verify that theory experimentally. The earlier work (Jackson and Chalmers, 1956, 1958)* has precipitated other explanations of frost heave (Penner, 1959)† which do not differ in principle from the original. A brief outline of the theory of frost heave is presented below.

THEORY OF FROST HEAVE IN SOILS

Most soils do not act as nucleating agents for ice. Soils containing water can be undercooled several degrees below zero before the water in them transforms to ice, indicating that a soil particle/ice interface has a relatively large surface free energy. At temperatures in the vicinity of 0°C there is probably a layer of water separating ice from the soil particles (Fig. 1). This layer of water plays an important role in frost heave.

When water is cooled below 0°C, a nucleus of ice must form before the water can transform to ice. Such nuclei do not form readily in ordinary (tap) water, which must be cooled to about -6°C before nucleation takes place. This also happens when a soil containing water is cooled; nucleation first occurs a few degrees below zero, the exact temperature depending on the soil. Once a nucleus of ice has formed, the rest of the water can freeze by the growth of the nucleus. In the case of water, this happens readily; the ice grows rapidly, heating the ice-water mixture to 0°C. The rest of the water transforms to ice as heat is extracted from the system.

In a soil, however, the ice must grow around the soil particles and through the voids between the soil particles. The ice does not come into contact with the soil particles but, as pointed out above, is separated from them by a layer of liquid. The ice which forms is a three-dimensional network with many holes in it, each hole containing a soil particle surrounded by water. The important feature is the large area of ice/water interface that is built into this structure.

Such a network of ice will not form at 0°C, but at some temperature below 0°C, specified by the equation:

$$V_i \Delta F_v - A_i \sigma = 0, \quad (1)$$

* Jackson, K.A. and Chalmers, B. (1956) Study of ice formation in soils. U.S. Army Corps of Engineers, Arctic Construction and Frost Effects Laboratory Technical Report 65.

(1958) Freezing in porous liquids with special reference to frost heave in soils. *Journal of Applied Physics*, vol. 29, p. 1178.

† Penner, E. (1959) The mechanism of frost heaving in soils. Highway Research Board, Bulletin No. 25, p. 1-13.

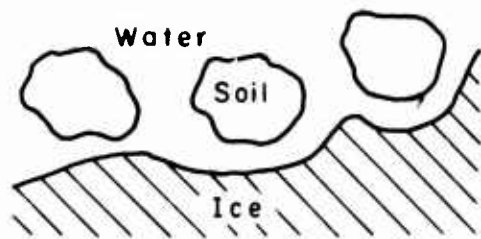


Figure 1. Diagram showing water, soil, ice relationship (freezing upward).

where V_i is the volume of ice transformed, ΔF_v is the free energy per unit volume of the ice transformed, A_i is the surface area of the ice structure, and σ is the solid-liquid surface free energy per unit area. Using

$$\Delta F_v = \frac{L\Delta T}{T_E} \quad (2)$$

(L is the latent heat per unit volume, T_E is 273°K, ΔT is the freezing point depression), the depression

of the freezing temperature of the ice because of its structure is (from eq 1):

$$\Delta T = \frac{A_i \sigma T_E}{V_i L} \quad (3)$$

By enlarging a void in the soil and forming an ice lens, bulk ice can develop in the soil. The formation of ice lenses in a soil is a manifestation of the fact that free ice is a thermodynamically more stable form of ice than a network of ice surrounding soil particles.

The factor A_i/V_i in eq 3, which is the only part that depends on the soil, is the ratio of the surface area built into the ice network to the volume of water that transforms to ice. This factor must be large if ΔT is to be appreciable. Soils for which this factor is small will not exhibit frost heave. These are the coarse sands and gravel, where the voids are large enough for the water in them to behave essentially like bulk water.

For an ice lens to form, water must move through the soil and this requires a pressure gradient in the soil. The faster the flow, the greater must be the pressure gradient. A pressure gradient can be developed in a soil by the freezing front. The details of how this occurs will be outlined in the next section. For our present purposes, it is sufficient to point out that a pressure can be developed from a free energy difference, the magnitude of the pressure being given by

$$P = \frac{L\Delta T}{T_E} \quad (4)$$

where the units are pressure, or energy per unit volume.

The soil water can transform to ice by freezing where it is, around the soil particles, or it can be drawn to the ice lenses. The first of these processes occurs only below an undercooling given by eq 3. The second occurs at an undercooling given by eq 4. The rate of flow to the ice lens depends on the pressure developed at the ice lens, which in turn depends on undercooling there. For an ice lens to grow, the competing process, freezing of the soil water *in situ*, must not occur; the undercooling at the ice lens must be less than the value given by eq 3. The maximum pressure which can be developed in the soil is obtained by combining eq 3 and 4:

$$P_{\max} = \frac{A_i \sigma}{V_i} \quad (5)$$

Pressures up to this value can be achieved at the ice lens before the soil water ahead of the ice lens will freeze. The maximum pressure which can be achieved is greater in fine soils than in coarse soils. For finer soils, however, a greater pressure gradient is necessary to obtain a given flow of water to the ice lens. For very impermeable soils, the pressure which can be developed at the ice lens is not sufficient to cause an appreciable flow of water to the ice lens and thus the heave in impermeable soils is small.

The above discussion outlines from a thermodynamic viewpoint the reasons for frost heave and one of the most striking features of frost heave: its dependence on particle size in the soil. A complete description of the frost heave process must include a discussion of heat flow, as well as the factors already mentioned. This will be done in a later section. In the next section, the detailed mechanism for frost heave as proposed by Jackson and Chalmers will be presented.

MECHANISM OF FROST HEAVE

In the last section, it was shown that porous ice having holes filled with soil particles separated from the ice by a thin layer of water was thermodynamically less stable than bulk ice. This bulk property of the ice in a soil can be understood on a microscopic scale.

Consider an interface between ice and water in a soil (Fig. 1). The ice cannot grow closer to the soil particles. If the soil water is to freeze *in situ*, then the ice must grow between the soil particles. This can happen only if the ice interface becomes convex. If the surface of the ice is convex, then the melting temperature of the ice decreases according to the Gibbs-Thompson relationship:

$$\Delta T = \frac{2\sigma T_E}{Lr} \quad (6)$$

where r is the radius of curvature of the interface and the other symbols have the same meaning as above. The ice in the soil cannot freeze unless r is less than the radius of the channels in the soil, so that the soil water will freeze *in situ* only below the temperature given by substituting the appropriate value of r in eq 6.

This equation is remarkably similar to eq 3, except that A_i/V_i has been replaced by $2/r$. The derivation of these two equations was quite different, but they do, in fact, describe the same thing. Consider, for example, a special porous material made up of a series of cylindrical channels of radius r (Fig. 2). Both eq 3 and eq 6 should apply to this material as well as to a soil. Equation 3 states that the freezing temperature of the water in the voids depends on a factor A_v/V_v , where A_v is the surface area of the voids, and V_v is the void volume. If there are n cylinders, h cm long, then

$$A_v = n2\pi rh$$

and

$$V_v = n\pi r^2 h$$

so that

$$\frac{A_v}{V_v} = \frac{2}{r}.$$

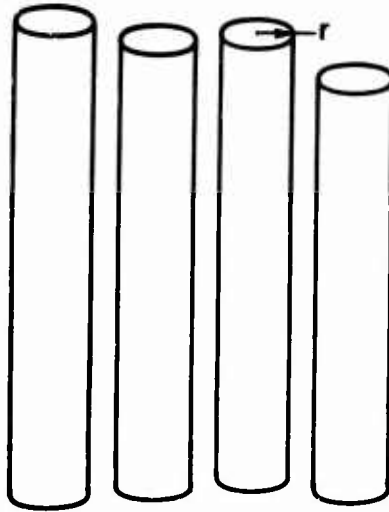


Figure 2. Idealized cylindrical porosity.

In this case, eq 3 and eq 6 are identical. In the general case, eq 6 represents the local melting point depression of ice at some specific region of the interface and eq 3 represents the average of these melting point depressions taken over all the voids and channels in the soil. In the case of Figure 2, since the channels are of uniform diameter, the average depression of the melting temperature is the same as the local depression for an interface in the channel. Since the ice interface cannot grow between the soil particles without assuming a very small curvature in the connecting channels, there will be some conditions where the soil water flows between the soil particles to the ice interface.

There is an equilibrium thickness for the layer of water between the soil particles and the ice. The thickness of this layer of water will vary with temperature, and at a sufficiently low temperature the layer will not exist. It is not important for our purposes how the thickness of the layer changes with temperature; it is sufficient that an equilibrium thickness exists. If the soil water is undercooled, then a little of the water layer separating the ice and soil will freeze, reducing the thickness of this layer below the equilibrium thickness. The equilibrium thickness can be restored by an influx of water from the unfrozen soil (Fig. 3). A steady state thickness of the water layer will be established, less than the equilibrium thickness, so that water is constantly being drawn into the layer and frozen on the ice lens. The layer of water is a region of negative pressure. The pressure could be relieved by the influx of water if more freezing were not constantly occurring. The free energy available in the supercooled water is thus, through the medium of the water layer, converted into a negative pressure in the soil. The maximum pressure which can be achieved depends on the free energy available in the supercooled water, as given by eq 5.

The microscopic viewpoint is in complete accord with the macroscopic viewpoint presented in the last section. Each gives insight into the process of frost heave. Both approaches to the problem demonstrate that frost heave occurs because the presence of soil particles prohibits the freezing of soil water *in situ*, unless the soil water is supercooled. If the soil water is supercooled, it will flow through the soil to a bulk ice interface (an ice lens), resulting in frost heave.

In the following sections we shall try to incorporate some of the other important factors in an attempt to derive a more comprehensive theory of frost heave.

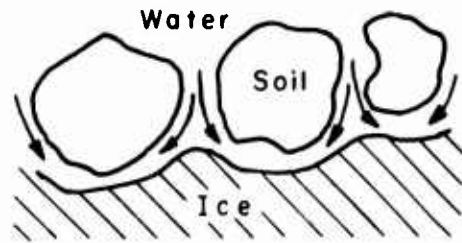


Figure 3. Liquid flow to water layers.

PRESSURE IN THE SOIL

The maximum suction pressure which can be generated in a soil is given by eq 4

$$P_{\max} = \frac{L\Delta T}{T_E},$$

and can be written as

$$P_{\max} = \frac{\sigma A_v}{V_v}, \quad P_{\max} = \frac{2\sigma}{\bar{r}}, \quad (7)$$

where \bar{r} is some average void size in the soil. This pressure must do two things: it must lift the soil plus any surcharge which is above the ice lens, and it must provide a pressure gradient to draw soil water to the ice lens. This is represented by the equation

$$P_{\max} = P_L + P_S, \quad (8)$$

where P_L is the load pressure and P_S is the suction pressure in the soil water at the ice lens.

The suction pressure in the soil is

$$P_S = P_{\max} - P_L.$$

If the pressure gradient in the soil is linear between the ice lens (distance h below the surface, Fig. 4) and the ground water table (distance h_0 below the surface) where the suction pressure is zero (neglecting hydrostatic effects), then the pressure gradient in the soil is

$$\frac{P_S}{h_0 - h} = \frac{P_{\max} - P_L}{h_0 - h}.$$

MECHANISM OF FROST HEAVING

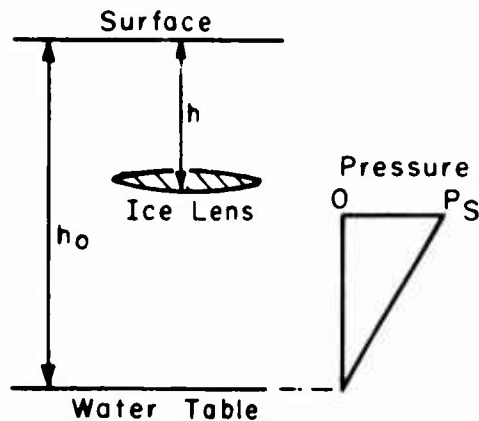


Figure 4. Linear suction pressure distribution.

The flow of water under the influence of such a gradient depends on the permeability of the soil, and is usually measured in inches per second, or some other convenient units of length and time. All the water flowing because of this gradient freezes onto the ice lens, producing heave. The rate of heave, then, is the rate at which water arrives at the ice lens corrected for the density difference on freezing.

The rate of heave R is thus

$$R = \frac{k\rho_w}{\rho_i} \frac{P_{\max} - P_L}{h_0 - h} \quad (9)$$

where k is the coefficient of permeability of the soil and ρ_w and ρ_i are the densities of water and ice, respectively. This equation gives a relationship between parameters of the soil and the test conditions, predicts a rate of heave, and predicts a qualitative dependence of frost heave on the soil. For a coarse soil, $P_{\max} = 2\sigma/\bar{r}$ will be small since \bar{r} is large. For a fine soil, k will be small. Some intermediate soil particle size will produce the maximum frost susceptibility.

Equation 9 predicts the right functional relationship between the various parameters, but it assumes a linear pressure gradient in the soil. It will be shown in a later section that this is a valid assumption for some cases.

HEAT FLOW

The amount of undercooling at the interface is determined by the flux of heat through the soil and by the freezing of water in the soil. The requirement of heat balance at the freezing front can be stated as follows. The heat conducted away through the frozen soil comes from three sources: 1) the heat conducted to the ice lens from the soil below, 2) latent heat from soil water *in situ*, and 3) latent heat from frost heave. In some cases, the specific heat from cooling of the soil must be considered as well, but in the usual case this will be small compared to the other factors. Mathematically, this heat balance can be expressed:

$$K_f \left(\frac{dT}{dx} \right)_{x=h-} = K_u \left(\frac{dT}{dx} \right)_{x=h+} + v n \rho_i L + R \rho_i L \quad (10)$$

where K_f and K_u are the thermal conductivities of the frozen and unfrozen soils, $(dT/dx)_{x=h-}$ and $(dT/dx)_{x=h+}$ are the temperature gradients below and above the freezing front ($x = h$) respectively, v is the rate of advance of the freezing front, and n is the volume fraction of the soil that is water. As before, R , ρ_i , and L are the rate of heave, density of ice, and latent heat of fusion of ice, respectively.

For brevity, let us write $H_f = K_f (dT/dx)_{x=h-}$ and $H_u = K_u (dT/dx)_{x=h+}$. These two quantities will depend on a variety of thermal factors and must be measured or calculated in any specific case.

From eq 10, the rate of advance of the freezing front is:

$$v = \frac{H_f - H_u - \rho_i L R}{n \rho_i L} \quad (11)$$

Substituting for R from eq 9,

$$v = \left[H_f - H_u - \rho_w L k \frac{P_{\max} - P_L}{h_0 - h} \right] \frac{1}{n \rho_i L} \quad (12)$$

From eq 9 and 12, it is evident that the rate of heave will be independent of the rate of advance of the frost front, provided sufficient heat is being carried from the soil. Under these conditions, the rate of heave is given by eq 9. The rate of advance of the freezing front is given by eq 12, and depends on the heat flow conditions. If the heat is extracted too slowly from the soil, however, we can have

$$H_f < H_u + \rho_w L k \frac{P_{\max} - P_L}{h_0 - h} \quad (13)$$

Equation 12 predicts a negative advance of the frost line for $H_f < H_u$; v will actually be negative (melting occurring). In the intermediate region v will be zero, and R will be less than the value given by eq 9. The rate of heave R will depend on heat flow:

$$R = \frac{H_f - H_u}{\rho_i L}$$

for

$$H_u < H_f < H_u + \rho_w L k \frac{P_{\max} - P_L}{h_0 - h} \quad (14)$$

This corresponds to heave occurring with a stationary frost front ($v = 0$).

Depending on how rapidly heat is extracted from the soil, we can have two cases. If heat is extracted slowly, the frost line will be stationary, and heave will occur at a rate which depends on heat flow (eq 14). Above a critical rate of heat extraction given by eq 13, the rate of heave is constant (independent of heat flow), and is given by eq 9. Above this critical rate of heat extraction the rate of advance of the frost line increases as the rate of extraction of heat increases (eq 11).

The rate of extraction of heat from the system is one of the most important factors in determining the total frost heave that will occur in a soil. The flow of heat in a soil depends on the temperature of the soil and of the air over the soil. A detailed analysis of this heat flow has not been attempted here. This analysis is, in general, quite complex in detail, but quite straightforward in principle. The thermal conditions are usually well known, and the problem consists of solving the heat flow equations with various boundary conditions. The heat flow conditions are complicated by the freezing front which acts as a heat source. The strength of this heat source depends on whether or not frost heave is occurring. If the freezing front is stationary and heave is occurring, one set of thermal conditions prevails. If the freezing front is moving, this affects the heat flow another way. The problem is further complicated because the thermal conductivity of the frozen soils depends on how much ice is contained in the soil, i.e. on how much heave has already taken place. The heat flow conditions depend in general, then, on how much heave is occurring, which in turn depends on the heat flow. The two problems must be considered together to be solved completely, and the interaction between the heat flow and the frost heave taken into consideration.

The analysis which has been presented in this section is in terms of the instantaneous heat flow at the freezing front. The rate of heave will thus change during the freezing of the soil as the heat flow conditions change. The analysis can be checked experimentally by measuring the thermal gradients in the vicinity of the freezing front, calculating the heat fluxes from these measurements, and checking the observed heave rates against the heave rates predicted by these equations.

The theory presented above accounts for the principal phenomena of frost heave. It predicts the dependence of the rate of heave on various factors such as the thermal conditions, the soil permeability, the effective soil void size, the surcharge, and the depth to ground water table.

MOTION OF WATER THROUGH AN UNSATURATED SOIL

The tension in an unsaturated soil depends on the water content. In the case of frost heave, a negative pressure (suction or tension) is put on the soil by the freezing process as outlined above. This can lead to a local depletion of water. It is important to determine how easily the water can move through the soil. The frost heave may occur by drawing water from a ground water table, or by depleting the unfrozen soil of soil water. Which of these occurs will have a profound effect on the extent of frost heave.

We will develop the general equations for the motion of water through a soil, and determine what will happen during frost heave in a few cases.

Consider the flux of water across two planes a distance dx apart in a soil (Fig. 5). The flux J_1 across the plane at x_0 is

$$J_1 = k \left(\frac{dP}{dx} \right)_{x = x_0} \quad (15a)$$

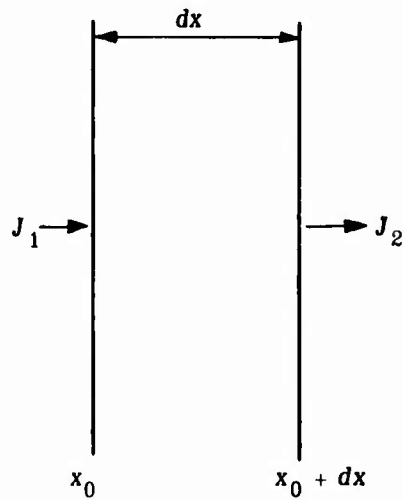


Figure 5. Flux of water across two planes.

where k is the permeability of the soil. At $x_0 + dx$ the flux of water J_2 is

$$J_2 = k \left(\frac{dP}{dx} \right)_{x = x_0 + dx} \quad (15b)$$

If $J_1 \neq J_2$, then the flux of water into the region between x_0 and $x_0 + dx$ is not equal to the flux out, so that there is a net change in the amount of water in this volume. There will be a corresponding change in pressure given by

$$J_1 - J_2 = \frac{1}{C} \frac{dP}{dt} dx, \quad (16)$$

where C is the change in pressure resulting from unit change in water content per unit volume of the soil and dP/dt is the rate of change in pressure between x_0 and $x_0 + dx$.

Combining eq 15 and 16,

$$\frac{k \left(\frac{dP}{dx} \right)_{x = x_0} - k \left(\frac{dP}{dx} \right)_{x = x_0 + dx}}{dx} = \frac{1}{C} \frac{dP}{dt} \quad (17)$$

or

$$\frac{d^2P}{dx^2} = \frac{1}{D} \frac{dP}{dt},$$

where $D = kC$ in cm^2/sec .

A similar analysis in three dimensions gives:

$$\nabla^2 P = \frac{1}{D} \frac{\partial P}{\partial t} \quad (18)$$

This is the diffusion equation. It is mathematically identical to the equation for heat flow or composition changes in a diffusing system. Solutions for it with D constant are well known. For a soil, D is not a constant, since C , the change in water content of the soil per unit pressure change, is very sensitive to the water content of the soil. For the present, we shall use the well-known solutions for the case of constant D to approximate the conditions during frost heave.

SOLUTIONS TO THE DIFFUSION EQUATION

The solution of the diffusion equation is facilitated by using a coordinate system fixed in the freezing front. After sufficient time has elapsed, steady state conditions will be established. (In some cases steady state will not be achieved, and a solution to the complete equation must be obtained.) Under steady state conditions the water content and pressure at a fixed distance from the interface do not change in time. For the case of unidirectional freezing, the diffusion equation becomes

$$D \left(\frac{d^2 P}{dx^2} \right) + v \left(\frac{dP}{dx} \right) = 0. \quad (19)$$

The rate of motion of the freezing front is v , which has the solution

$$P = A + B e^{-(v/D) x} \quad (20)$$

where x is measured from the interface.

Subject to the boundary conditions

$$P = -P_0 \text{ at } x = 0$$

$$P = 0 \text{ at } x = h_0 - h = h_1 \quad (21)$$

where h_1 is the distance from the freezing point to the ground water table,

eq 20 becomes:

$$P = -P_0 \left\{ \frac{e^{(v/D)(h_1 - x)} - 1}{e^{(v/D)h_1} - 1} \right\}. \quad (22)$$

Case I (Fig. 6a): If the ground water table is very far away

$$h_1 \gg \frac{D}{v}$$

$$P = -P_0 e^{-(v/D)x}$$

The ground water table in this case has no effect on the flow of water to the freezing front. All the water is drawn from the soil.

Case II (Fig. 6b): If the ground water table is close to the freezing front

$$h_1 \ll \frac{D}{v} \text{ and } e^{-(v/D)h_1} \approx 1 - \frac{v}{D} h_1; \quad e^{-(v/D)x} \approx 1 - \frac{v}{D} x$$

$$P = -P_0 \left(\frac{h_1 - x}{h_1} \right)$$

Here the pressure gradient is linear, and all the water is drawn from the ground water table to the freezing front.

General case (Fig. 6c):

$$P = -P_0 \left(\frac{e^{(v/D)(h_1 - x)} - 1}{e^{(v/D)h_1} - 1} \right)$$

Here the water reaching the frost line comes partly from the ground water table and partly from the soil.

In each of these cases, the rate of heave depends on the flux of water to the interface, which depends on the pressure gradient at the interface. The flux of water to the interface is given by:

$$J = k \left(\frac{dP}{dx} \right)_{x=0}$$

General case:

$$J = \frac{P_0 v}{C} \frac{1}{1 - e^{-(v/D)h_1}} \quad (23)$$

Case I:

$$J = \frac{P_0 v}{C} \quad h_1 \gg \frac{D}{v}$$

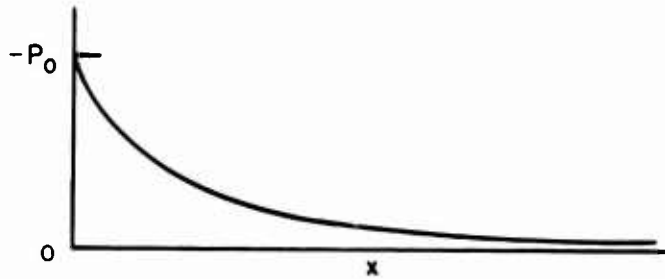


Figure 6a. Theoretical suction pressure distribution with deep water table.

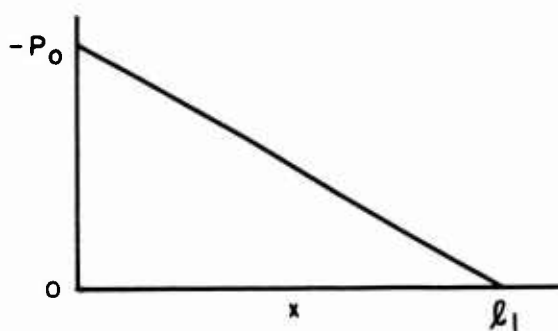


Figure 6b. Theoretical suction pressure with water table close by.

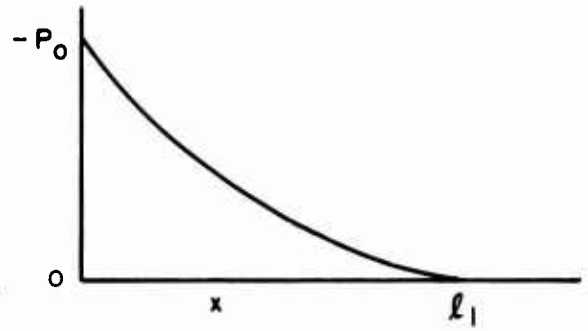


Figure 6c. Theoretical suction pressure distribution with intermediate water table depth.

Case II.

$$J = \frac{kP_0}{h_1}$$

$$h_1 \ll \frac{D}{v}$$

In Case I, the flow of water in the soil depends on the capacity of the soil to hold water; in Case II, it depends on the permeability of the soil and the distance to the water table. The result of Case II was used to develop eq 8.

For a typical frost heaving soil, D is about 10^{-3} sq cm/sec. It will be much larger for a gravel and much smaller for a clay. If the advance of the freezing front is 10^{-5} cm/sec (about 1 cm/day), then $D/v \approx 100$ cm. Case II above is usually appropriate. For faster growth or for less permeable soils, the more general equations should be used. In most of the tests such as conducted at the Arctic Construction and Frost Effects Laboratory, there probably was a linear pressure gradient between the freezing front and the ground water table. The equations developed in the previous section will be valid for these cases. The analysis of this section, however, indicates that most laboratory experiments are done on the borderline of where all the water is drawn from the ground water table. In many instances, and frequently in the field, this will not be so. In the general case, the frost heave depends critically on the availability of water in the soil.

EXPERIMENTAL INVESTIGATION**Choice of experiment**

This experimental investigation was undertaken to elucidate, and if possible verify, the mechanism of frost heave proposed by the authors. It was decided that a soil system was too complex to work with and that a simpler system should be chosen. The essence of the frost heave theory is that supercooled water can exist in a soil because of the small channels between the soil particles. In a soil there are many voids of various sizes interconnected by channels of various sizes, and, from a microscopic point of view, it is difficult to determine what is happening at the freezing front. The 0°C isotherm can be located approximately with thermocouples, but the freezing front itself is difficult to detect visually. It is, moreover, difficult to bring a microscope up to a freezing soil without seriously disturbing the thermal conditions. These observational difficulties, combined with the difficulty in characterizing the void structure of the soil, led to the conclusion that a simpler system should be used for this investigation.

The system which would serve to best advantage would be one in which the "void size" was well known, and the freezing of water in the voids could be observed directly. The porous material would have to be separated from ice by a layer of water near 0°C, as in the case of soil particles.

Several systems were considered. It was decided that fine pore filters created the same difficulties as the soil itself, the great disadvantage being the unknown pore size. A few experiments performed with a fine pore filter demonstrated that "frost heave" did occur in such a system. These experiments are reported below.

Another possibility was to drill small holes in a thin plate by a sparking technique. The smallest holes which can be drilled by this method are about 10 μ in diameter, much too big for these experiments.

The freezing of water between two optically flat pieces of glass was considered. Optically flat glass can be made flat to $\pm 0.1 \mu$ (about $\frac{1}{4}$ wavelength of visible light). The water in this case would be confined on two sides, rather than in a three-dimensional network, as in a soil. The surface-to-volume ratio of water between two optical flats should be sufficient to depress the freezing temperature a measurable amount. A wedge can be made of two optical flats, so that they are 1 μ apart at the wide end, and together at the narrow end. Although this does not seem to be directly related to frost heave, a measurement of the position of the ice interface between two optical flats would give a direct verification of eq 3, which is fundamental to understanding frost heave. Some experiments performed on a system such as this were unsuccessful as reported below. Spherical glass beads, available commercially in a range of sizes, were also investigated as a possibility. Spheres of glass of the same diameter should produce a reasonably well-defined void size. However, the smaller beads available departed significantly from the spheres, so that regular packing could not be achieved, and were also too large for our purposes. The channels connecting the voids would be so large that the depression of the melting point in the voids would be very small.

It was decided that fine bore glass capillaries would serve as a suitable system. Each capillary would represent one void. The void size would thus be well defined and measurable. At first, it was questionable whether sufficiently fine capillaries could be made. The inner diameter should be in the range 0.1 μ to 2.0 μ . Capillaries with bores in this range were made by a method which is described later.

The basic experiments were to involve the measurement of frost heave with a glass capillary. The freezing of water in the capillary would have to be done in a carefully controlled thermal environment. In a soil, the freezing of the soil water, and the latent heat released thereby, control the temperature in the region of the freezing front. Rather large changes in the external thermal conditions can be compensated by motion of the freezing front. Whenever a freezing front occurs in a soil, the thermal conditions around it are similar. In our case, however, no built-in thermostat was available. The thermal mass of our system would be small compared to the thermal mass of its environment. It was, therefore, necessary to construct an environment having a carefully controlled temperature.

The melting point of water in a glass capillary having a $1\text{-}\mu$ radius should be depressed (from eq 6) about 0.1°C . In order to make sensible measurements, the temperature should be controlled to $\pm 0.001^\circ\text{C}$. The temperature of the capillary would then be sufficiently constant to check this depression of the melting temperature, and to observe the effect of changing the temperature a few degrees from this value of -0.1°C .

Temperature-controlled chamber

A constant temperature chamber was constructed. The temperature of this chamber could be fixed in the range 0 to -5°C , and held constant to $\pm 0.001^\circ\text{C}$ for several days. The temperature could also be moved slowly up or down at a rate of about 1/20 of a degree a day.

A schematic diagram of the chamber is shown in Figure 7. The basic principle used to obtain the fine temperature control was thermal damping. A rapidly changing thermal signal was applied to a fairly large thermal mass, resulting in a rapidly changing, but small, amount of heat being added to the mass. The temperature of the thermal mass could not respond to the rapid fluctuations in thermal input, but only to the average thermal input, which was controlled by varying the on-to-off time of the fluctuating thermal input.

The basic cooling unit, a $\frac{1}{2}$ -hp commercial refrigeration unit, was used to cool about 5 gallons of an ethylene glycol - water mixture. Since it is difficult to obtain accurate thermal control by turning a refrigeration unit on and off the unit was run continuously. The temperature of the antifreeze mixture was controlled to $\pm 0.1^\circ\text{C}$ using auxiliary heaters switched on and off by a mercury relay operated by a bimetal temperature-sensitive element. The antifreeze mixture was then pumped by an auxiliary pump through cooling coils in the main cooling chamber, a double-walled box 2 ft x 2 ft x 3 ft. The two aluminum walls were separated by a 2-in. layer of cork insulation. Air in this chamber was circulated by a fan past the cooling coils containing the antifreeze. The temperature of the air was further controlled by low-wattage auxiliary heaters with an on-off switching time of about 1 second. The voltage across the heaters was adjustable to obtain optimum control. The temperature of the air was sensed with a bimetal element which would respond to changes of $\pm 0.005^\circ\text{C}$. The temperature of the air in the main chamber fluctuated fairly rapidly about a very precise mean, constant at a value fixed by the setting on the bimetal sensing element.

The experiments were conducted in a 6 x 6 x 6-in. box constructed of $\frac{1}{4}$ -in. aluminum within the main chamber. The box had a large thermal mass and was supported on non-conducting supports to thermally isolate it from the outer box. The temperature of the box and its interior did not respond to the rapid fluctuations in the temperature of the air around it. The temperature inside the box remained constant to $\pm 0.001^\circ\text{C}$ for extended periods.

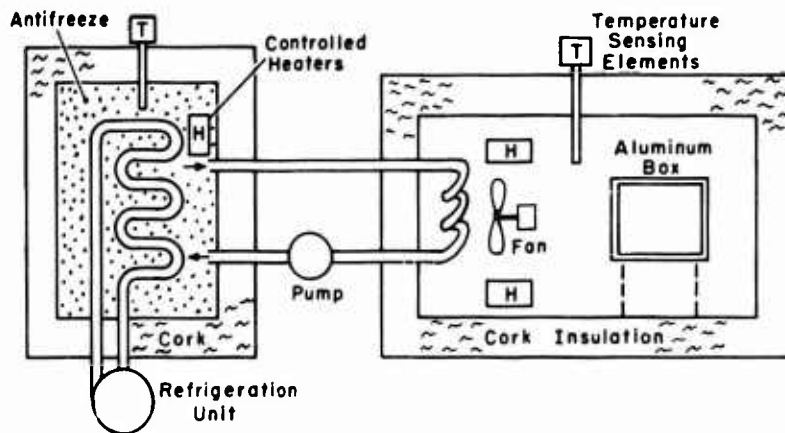


Figure 7. Equipment set-up used in experiment.

Preparation of the capillaries

Capillaries of glass with an internal diameter of about 1μ can be made by drawing down larger glass tubing.

Pyrex tubing, $\frac{1}{4}$ in. OD and $\frac{1}{16}$ in. ID, was first drawn by hand down to about $\frac{1}{16}$ in. OD. Then a weight was suspended from a hook made at the end of the tube and heat (oxygen-gas flame) was applied locally for a few seconds, until the tube started to extend. Heat was then removed and the tube continued to extend for a short time. Frequently a section of the tube reduced to the order of 1μ ID and a few tenths of a millimeter OD.

Difficulty was encountered in trying to measure the diameter of the bore by observing the end of the tube. Fortunately, it can be shown that the apparent diameter of the bore of a tube viewed from the side is the actual diameter of the tube times the refractive index of the glass, provided the bore is small compared to the outside diameter. A microscope was used to measure the apparent diameter of the bore looking from the side of the tube. The apparent diameter was divided by the refractive index of the glass to obtain the actual bore diameter. The capillaries were made so that the decrease in internal diameter occurred over as short a distance as possible. The large bore part of the tube could then be used as a water reservoir for the fine bore part. Usually, the tube bore narrowed from $\frac{1}{16}$ in. to 1μ in less than 1 cm, and the $1\text{-}\mu$ capillary was about 1 cm long. If these distances were much larger, feeding problems would occur during the experiment. A photograph of a tube drawn to a capillary is shown in Figure 8.

An unexpected difficulty arose in trying to fill these tubes with water. An air pocket would form in the part of the tube near the capillary, trapped between the water in the capillary and the water in the rest of the tube. The flow rate of air or water through the capillary was so slow that the bubble could not be forced out through the $1\text{-}\mu$ capillary. Moderate success was achieved by using a hypodermic syringe to fill the tubes. The best method of eliminating air pockets was to pour water into the capillary in a vacuum.

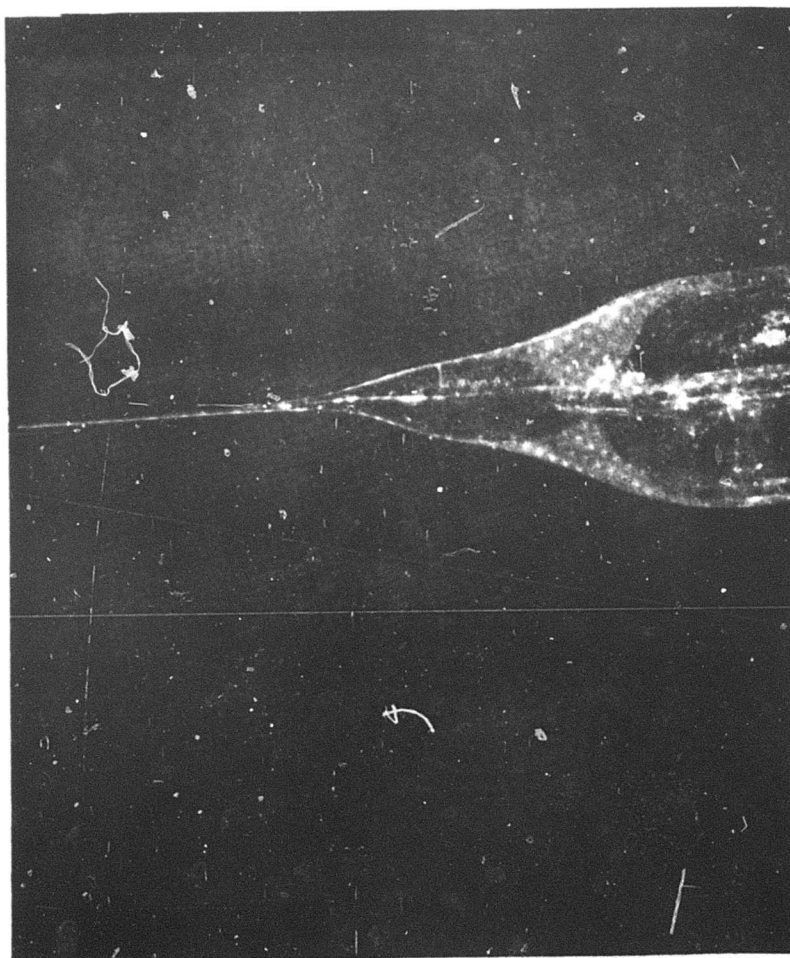


Figure 8. Close-up of typical capillary used in experiments.

RESULTS

Experiments with filter paper

The filter paper experiments were not quantitative in nature, since the effective pore size of the paper, claimed by the manufacturer to be on the order of a few microns, was unknown. The difficulty with the filter paper, as with a soil, is that the pore system is ill defined. Are the larger, or smaller, channels more important in frost heave?

Several experiments were attempted using a double container with ice on one side and water on the other side (Fig. 9). It was expected that the ice would draw all the water through the filter paper, as in ice lens formation, but the flow rate of the water through even this fine-pore filter paper was too large. Gravity feeding resulted in too rapid transfer of the water to the ice side of the container, so that free water could be seen on the ice side. Furthermore, ice always formed on the water side before all the water flowed to the other side. One large pore in the filter paper would suffice as a channel to propagate the ice through the filter paper. These two effects ruined the experiment. This cannot happen in a soil, since a large channel will not lead to a large volume of free water "on the other side."

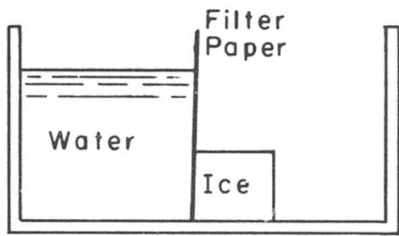


Figure 9. Filter paper experiment set-up.

A different geometry was tried. A jar was filled with water, covered with filter paper, and a small ice chip placed on top. The jar was then cooled to just below 0°C . In a few days, ice resembling hoarfrost grew about $\frac{1}{2}$ in. out of the filter paper (Fig. 10). There could be no gravity feeding in this case. This type of experiment was terminated by the formation of ice within the jar.

These experiments show that the frost heave phenomenon is not limited to soils.

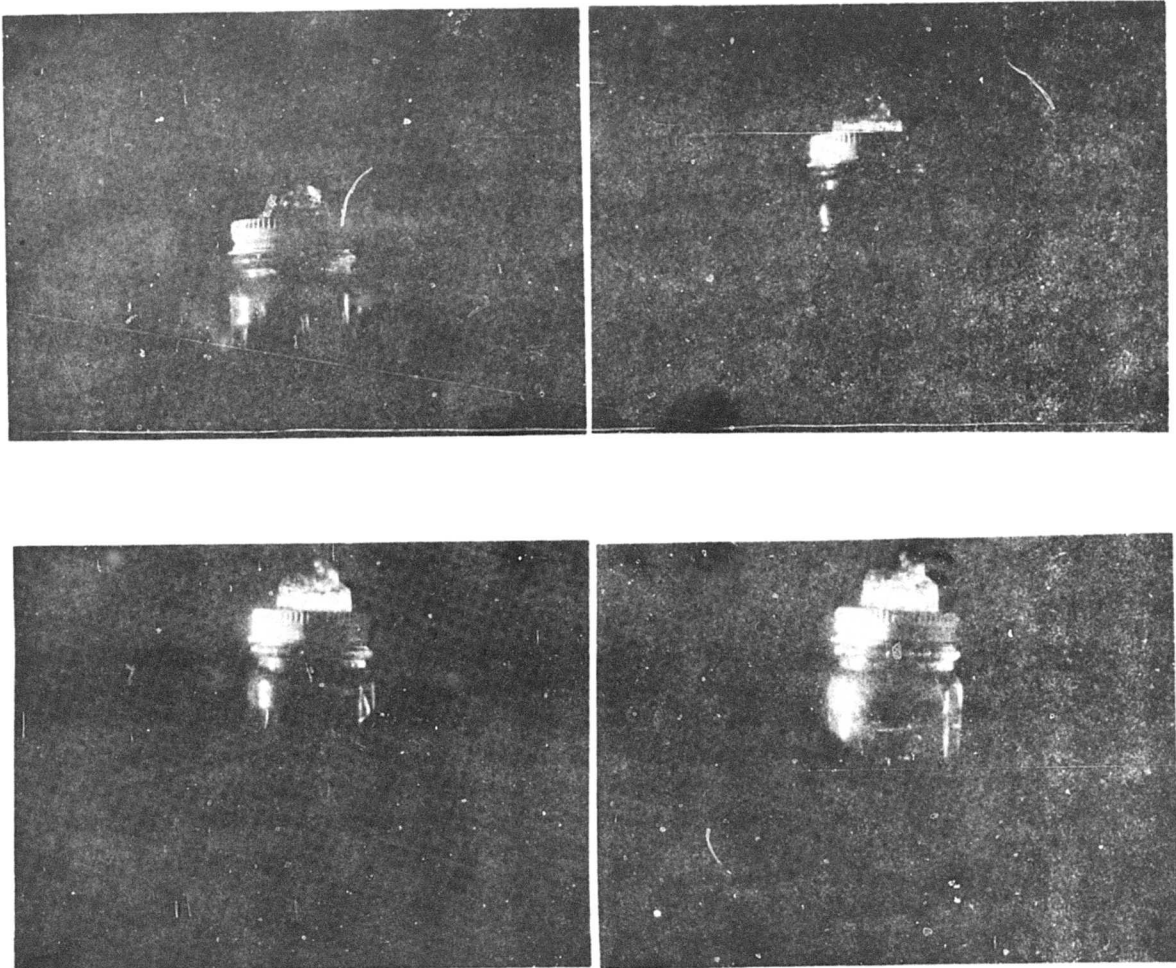


Figure 10. Ice growth from jar using filter paper.

Experiments with optical flats

Two pieces of glass with optically flat surfaces were used to make a wedge of water. The optical flats were first made chemically clean by soaking them in chemical cleaning solution. They were then held $1\text{-}\mu$ apart along one edge and together along another edge and distilled water was introduced between them (Fig. 11). The whole system was cooled to just below 0°C . It was expected that a line separating ice and water would be observed. The position of the interface at an undercooling ΔT should achieve equilibrium when curvature r of the interface is given by

$$r = \frac{\sigma T_E}{\Delta T} \quad (24)$$

The radius r can be determined from the lateral position of the interface

$$r = \frac{t}{2} = \frac{t_1 h}{2h_1} \quad (25)$$

assuming contact angle α between water and glass is 0 as shown in Figure 12. The position of the interface could thus be measured, and r determined for several different ΔT 's. This experiment also did not work as expected. The interface, where it could be seen, was unexpectedly irregular (Fig. 13). In this case no r could be determined and in other cases the interface could not be seen at all, even under polarized light. This type of experiment was reluctantly abandoned.

Experiments with capillaries

The first experiments attempted with capillaries involved the microscopic observation of a freezing front in a capillary. If the freezing front could be established within a capillary, the temperatures at which advance and retreat of the interface were observed would provide direct verification of eq 6. It was also hoped that an ice interface could be established outside the capillary, and frost heave observed directly as water was drawn from the capillary to the ice.

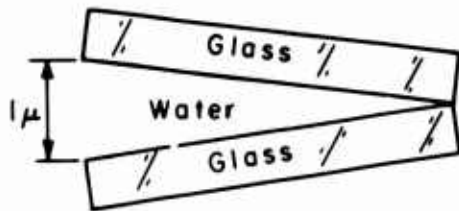


Figure 11. Sketch of set-up using optical flats.

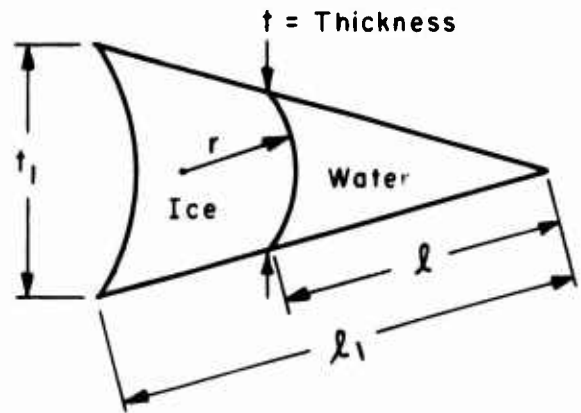


Figure 12. Geometric relationship for computing r in wedge.

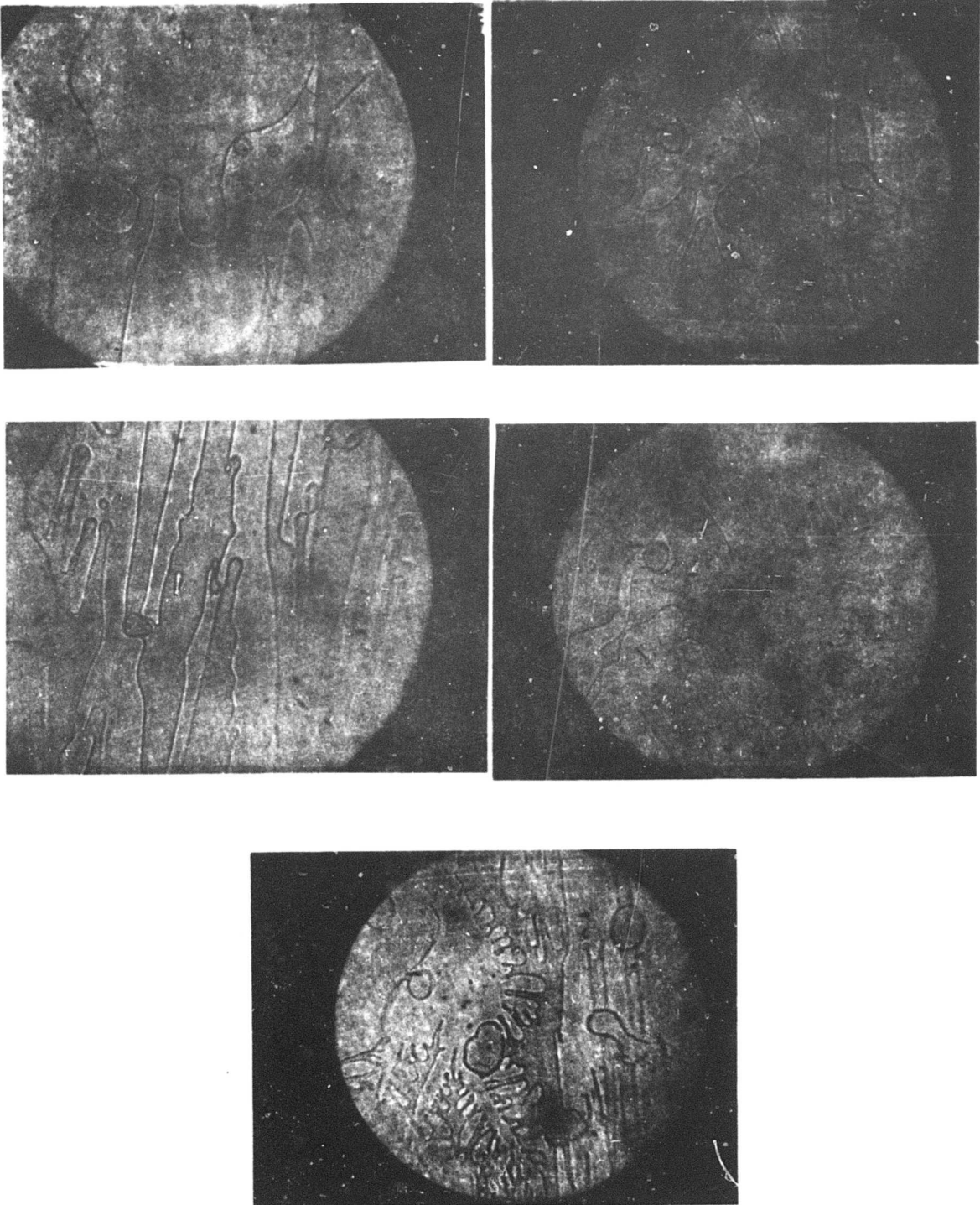


Figure 13. Ice interfaces in wedge experiments.

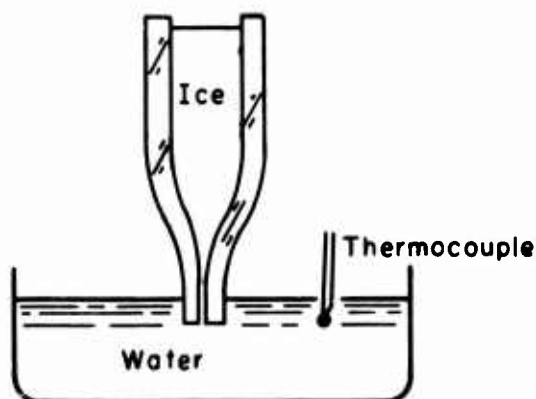


Figure 14. Sketch of experiment using ice in capillary tube.

The capillaries, filled with distilled water, were placed in the inner chamber of the temperature-controlled box. A special low-conductivity, long focal length microscope was built into the side of the box for direct visual observation of the capillary. A seeding device designed to introduce ice into the inner chamber was touched to the end of the capillary without disturbing the thermal conditions of the inner chamber. The seeding operation could be watched at $200\times$ magnification. At all test temperatures (down to -1°C), a layer of water was observed on the surface of the seed crystal of ice. Although many attempts were made, at no time was any ice interface found within a capillary. No ice grew out of the end of the capillary; it would have been observed had it occurred. It is not known whether an ice/water interface was not established within the capillary, or whether it was established and could not be seen.

A simpler experiment was devised (Fig. 14). Ice was introduced into the wide part of a tube which narrowed to a capillary. The open end of the capillary was placed in a small container of water. The ice should propagate through the capillary to freeze the water in the container. The freezing of the water in the container could be detected by the recalescence of the temperature of the water as indicated by a thermocouple. A cooling rate was imposed on the system such that there was sufficient time for ice to grow through the capillary at a temperature which constituted a small departure from the temperature given by eq 6. A satisfactory cooling rate as determined from published growth rate data (Hillig, 1958)* was $0.1^{\circ}\text{C}/\text{day}$ for this size capillary. Cooling from 0°C to -1°C was achieved in 10 days. For the smaller bore capillaries, lower freezing temperatures were expected, so faster cooling rates could be tolerated without impairing the accuracy of the measurement. An accurate value of ΔT could be determined easily by this method. Immediately before freezing starts, the thermocouple reads $-\Delta T^{\circ}\text{C}$. The freezing process raises the temperature of the thermocouple to 0°C . The radius of the tube could be determined microscopically as indicated above.

With this configuration, a rather unfortunate zone-refining effect occurred. Any solutes or impurities rejected by the ice freezing in the large (1/16-in.) bore tube collected in the small (approximately $1\text{-}\mu$) capillary. After several attempts were unsuccessful, the geometry of Figure 15 was adopted. The thermocouple is placed in the water, which is now in the wide tube from which the capillary is drawn, and freezing occurs from the bottom up. With this geometry, the impurities rejected by the ice will not specifically collect in the capillary. The ice in the capillary can be nucleated. The impurities now assume their normal roles as the ice grows along the capillary, without the magnifying effect of the drastically decreasing bore. The rest of the experiment is outlined above.

* Hillig, W.B. (1958) On growth and perfection of crystals. In *The kinetics of freezing in ice* (Doremus, Roberts, and Turnbull, Editors). New York: John Wiley and Sons, p. 350.

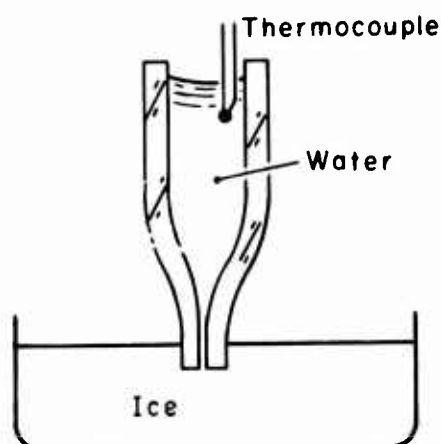


Figure 15. Sketch of experiment using water in capillary tube.

The results obtained from several runs of two weeks duration (Table I) are erratic at best. The ΔT of 0.15°C for a $0.55\text{-}\mu$ radius is more or less in agreement with the expected values. If this is correct, then all the other ΔT 's are too large by about a factor of ten. These erratic results could be caused by impurities lowering the freezing temperature or microscopic particles of dust or dirt partially blocking the capillaries. The tubes were cleaned carefully before drawing down to capillaries, and filled with "clean" water. The drawing was done in air, and the distilled water was in contact with air. Either could introduce dust particles to block the capillaries partially. More careful control of these conditions might have eliminated the difficulties.

TABLE I.

Tube radius	ΔT
0.55μ	0.15°C , $> 1^{\circ}\text{C}$
1.25μ	0.5°C
1.70μ	0.56°C
1.75μ	0.52°C

CONCLUSIONS

This report contains a discussion of the Jackson and Chalmers theory of frost heave. A more complete theory of frost heave is also presented. This theory takes into account the local thermal conditions in the soil and the permeability of the soil. The theory predicts (or explains) stationary ice lens formation, where there is no advance of the frost line, and also predicts a rate of heave which is independent of the rate of advance of the freezing front. The theory assumes that a soil can be represented by a single characteristic void size. For real cases, the soils are not as uniform and homogeneous as assumed.

The flow of water through a soil under tension was considered, and the flow to a frost line studied in detail. It was concluded that in the laboratory most of the water is drawn up to the ice

lenses from the water supply. In the field this will frequently not be the case, and the water will be drawn to the ice lenses by reducing the water content of the rest of the soil.

Several different experiments were attempted to verify directly the Jackson and Chalmers theory of frost heave. These were, by and large, unsuccessful. None of the experiments disproves the theory, nor does much to substantiate it. Direct experimental verification of the Jackson and Chalmers theory proved far more difficult than anticipated. The experiments took more time (e.g. 2-week runs) and were more trouble than originally expected. The equipment constructed for accurate temperature control functioned adequately. Occasional failures during a long run caused delays and were annoying, although the equipment had to work well to make any runs of this sort.

The experiments were reaching a state where some good results might have been achieved had the research continued. The first experimental approach used was simple, though time-consuming, and indications are that the method will work. Minor improvement of technique could well produce satisfactory results.

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