AD

SY-R6-70

ANALYSIS OF RISK FOR THE MATERIEL ACQUISITION PROCESS

PART I: FUNDAMENTALS

BY

JOHN D. HWANG

NOVEMBER 1970

SYSTEMS ANALYSIS DIRECTORATE
U. S. ARMY WEAPONS COMMAND
ROCK ISLAND, ILLINOIS

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED.

SY-R6-70

ANALYSIS OF RISK
FOR THE MATERIEL ACQUISITION PROCESS
PART I: FUNDAMENTALS

By

John D. Hwang

Systems Analysis Directorate
Headquarters, U. S. Army Weapons Command
Rock Island, Illinois

November 1970

TABLE OF CONTENTS

ABSTRACT

This paper is the first in a series devoted to the subject of analysis of risk for the materiel acquisition process. The objective of this introductory paper is three-fold. First, risk analysis is structured to show that it has close affinity to systems analysis and adds a new dimension, in terms of a probability measure, to integrate the three dimensions of cost, time to complete, and performance of a program in the materiel acquisition process. Secondly, numerous applicable techniques of statistical decision theory are presented, plus decision tree analysis and subjective judgment collection. Thirdly, methods for risk analysis of the concept formulation and contract definition phases of the acquisition cycle are exhibited. Research problems are also mentioned for future investigative efforts. Significant payoffs from a risk analysis include the identification of high risk areas, recommendations of additional studies to fill data gaps for better management decision making, a better basis for budget allocation, as well as the discovery of additional program alternatives.

ACKNOWLEDGEMENT

The author is extremely grateful to his colleagues in the Systems Analysis Directorate and the Research and Engineering Directorate, particularly Robert C. Banash, for the many stimulating discussions in the preparation of this paper.

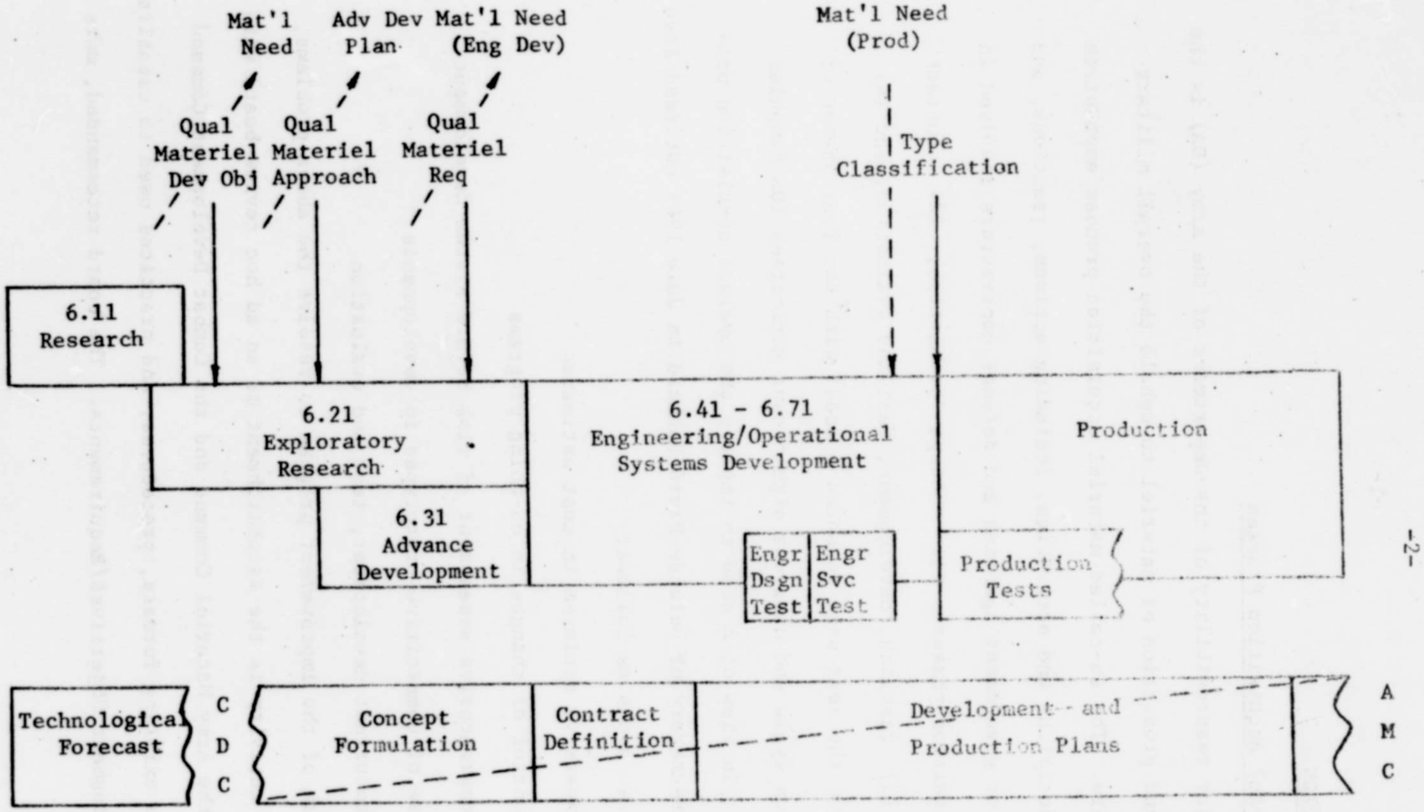
I. INTRODUCTION

A. Materiel Acquisition Process

A major responsibility of the Department of the Army (DA) is the development and production of materiel to uphold the overall military defense posture. The so-called materiel acquisition process emphasizes the flow of decisions and activities, including actions, reactions, and interactions of government agencies and defense contractors involved in the four acquisition phases: (1) concept formulation, (2) contract definition, (3) research, development, test and evaluation, and (4) production. On the next page is shown a model with the four phases of the acquisition cycle and numerous significant activities (DA Pamphlet 11-25, 1968). In view of a need to improve the overall acquisition process, Deputy Secretary of Defense David Packard in July 1969 outlined five specific problem areas as follows:

1. Excessive optimism in cost estimates
2. Control of changes in on-going programs
3. Comprehensive assessment of risk before system development
4. Use of competitive prototypes in developments
5. Concurrent development, test and evaluation.

As an outgrowth of the improvement program to resolve the above problems, one important activity is the establishment of an ad hoc review board with members from the Army Materiel Command and the Combat Development Command to analyze the existing formats, procedures, and practices used to establish Materiel Development Objectives/Requirements. The board recommended, more



ACQUISITION PROCESS AND ACTIVITIES

significantly, a proposed Materiel Need Concept (MN)(Ad Hoc Board, 1970). A Materiel Need is defined as "a DA approved statement of a need for new or improved materiel to provide an initial operational capability by a specific time frame, without regard to a particular technical approach or solution". One main philosophical attitude permeating this concept is the joint face-to-face combat/materiel developer actions. The proposed changes are shown also on the last page. Different formats for the Qualitative Materiel Development Objective (QMDO), Qualitative Materiel Approach (QMA) and Qualitative Materiel Requirements (QMR) are replaced by Materiel Need, Advanced Development Plan (ADP), Materiel Need (Engineering Development)-MN(ED), and Materiel Need (Production)-MN(P).

Deputy Secretary of Defense Packard (1970) further outlined acquisition policy for weapon systems. Throughout development from the conceptual stage to deployment, cost trade-offs between stated operating requirements and engineering design are key considerations. Program schedule should be structured to allow slack time for resolution of problems which inevitably arise in any program. More specifically, he provided the following guidelines:

1. Concept formulation - minimize technical risk by:

    a. Risk assessment - careful assessment of technical problems, consequence of failure, judgment as to effort needed for a practical solution.

    b. System and hardware proofing - actual engineering design and component testing to demonstrate elimination or reduction of technical risks through component or complete system prototyping, or back-up development.

c. Trade-offs (risk avoidance) - risk/cost trade-offs between stated operating requirements and engineering design throughout developmental stage.

2. Full-scale development - authorization to proceed provided:

a. Development Concept Paper (DCP) is prepared to reflect a management plan including appropriate attention to risk assessment, system and hardware proofing, and trade-offs.

b. Defense Systems Acquisition Review Council (DSARC) recommends that adequate risk reduction has been accomplished.

c. Procedures are established to address continually technical and engineering problems in view of possible trade-offs with stated operating requirements, cost and operational readiness date.

d. Problems encountered in concept formulation must be solved. Milestones are established to demonstrate achievement of objectives including appropriate stages of system design and testing of critical items.

e. Consideration is given to all matters such as maintenance, logistic support, training, etc., necessary in a full operating system.

f. Request For Proposals (RFP) for development stage is carefully reviewed for efficient accomplishment of the actual work.

3. Production - DSARC review

a. All milestones demonstrating achievement of a practical engineering design are met.

b. All important engineering problems encountered have been resolved with trade-offs between operating requirements and costs.

c. Start-up production is scheduled to minimize financial commitments until all major problems are resolved.

4. Contracts - contract type tailored to risks involved:

a. Cost-plus-incentive contracts are preferred for advance development and full-scale development; provisions for competative fixed-price subcontracts are included when technical risk can be assessed.

b. Fixed-price contracts are for projects when risks are low; contractor's ability to absorb losses must be a factor considered.

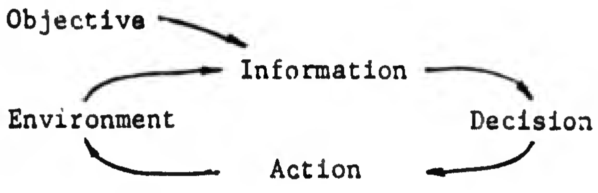
c. A negotiated fixed-price contract after production design can be realistically specified should be considered with encouragement to competition for subsystems.

d. Letter contracts must be minimized; change orders should not be authorized until they are contractually priced, or ceilings are established.

B. Systems Analysis

Decision analyses are involved throughout all echelons of the government agencies and defense contractors as to how the activities and relationships affect the three dimensions of cost, time and effectiveness of materiel programs. To understand decision analysis and acquisition management better, we define several terms as follows (Peck and Scherer, 1962). A subset of materiel developed and produced consists of the weapon systems which are composites of equipments employed as entities to accomplish a mission. An example of a weapon system is a helicopter, a radar unit, plus a set of guns, employed to offset a threat of potential advanced of hostile troops and ground vehicles to our defense territories. An alternative definition of a weapon system is a set of potential military capabilities. The ability of a weapon system to perform a mission is described in terms of three quantities or dimensions: cost, time and effectiveness. Cost reflects the resource commitments required to counter a specific level of potential enemy capability. Time denotes the period during which the weapon system is available for military operations determining the system's effectiveness relative to the military environment within which it operates. Finally, effectiveness reflects a weapon system's technical performance in terms of system characteristics including, say, mobility, fire power, communication, and reliability (the probability that the system can sustain its technical performance potential). Furthermore, a weapon system program decision is defined as the decision to undertake and commit resources to the development of a specific weapon system. A multiplicity of technical, military, financial, and scheduling decisions determine the course of a typical weapons program. In actual program decisions, uncertainties are very real and important complications. More carefully stated, we should say that a weapon system program decision is a "decision under uncertainties".

In the acquisition management, we observe that management is the process of converting information into action (Forrester, 1961). The acquisition management constitutes an information feedback loop illustrates below:



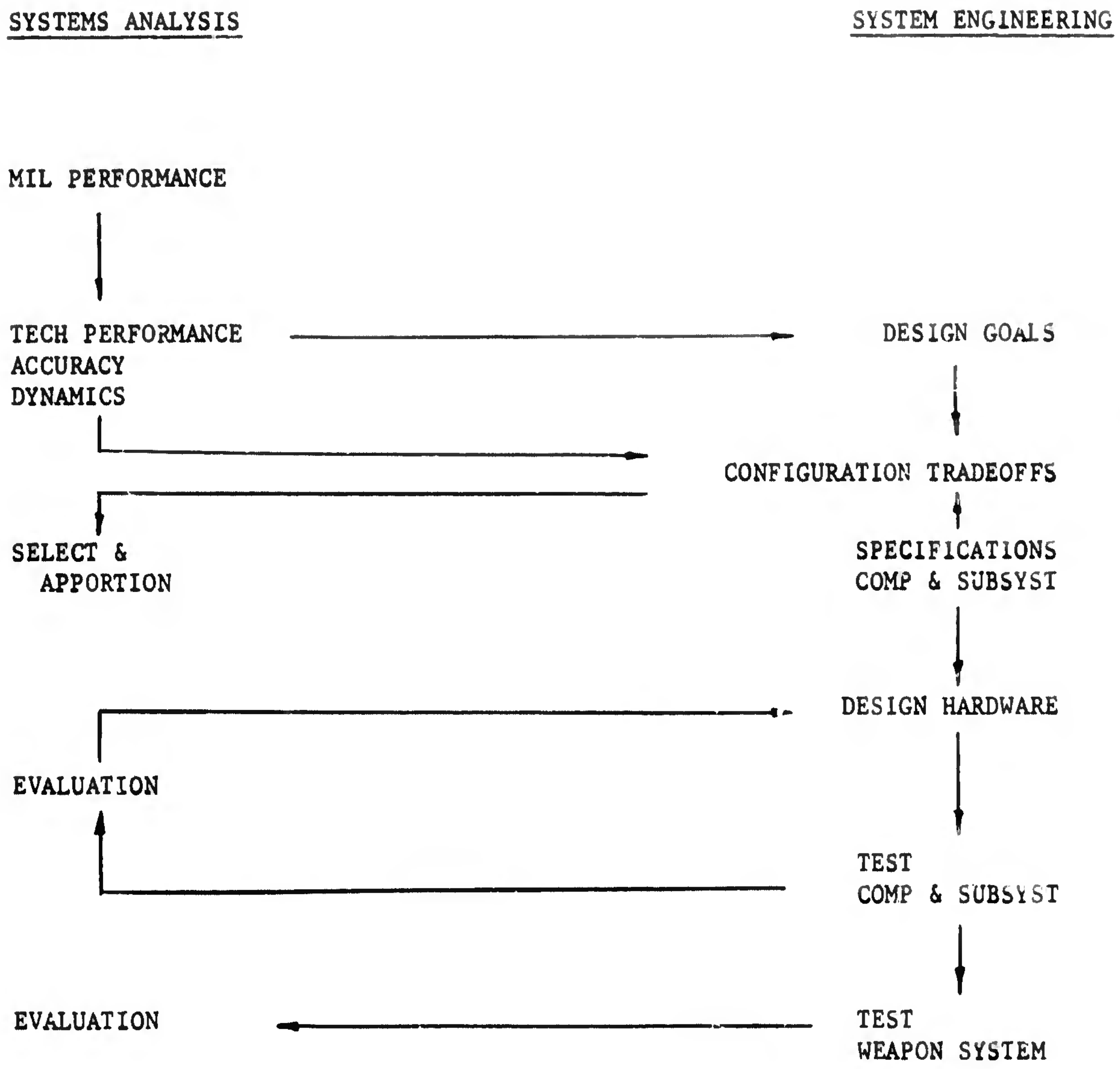
Acquisition management is a complex, multi-loop, and interconnected field. Decisions are made at multiple points throughout the development program; each resulting action generates information that may be used at several but not all decision points. This structure of cascaded and interconnected information-feedback loops, when taken together, describes the management process. The interlocking network of information channels emerges at various points to control physical processes. Every action point in the management network is backed up by a local decision point whose information sources reach into other parts of the organization and the surrounding environment.

With the constant need to make management decisions on some quantized basis, systems analysis erupted with the McNamara's era in 1961. The theme of McNamara's system analysis is that defense is an economic problem in the efficient allocation and use of resources (Hitch and McKean, 1967). He introduced the so-called three-phase operation: planning - programming - budgeting, in which planning involves cost-effectiveness analysis. Reliance on systems analysis has been very heavy regarding such questions as how much is enough, how should resources be allocated, and what trade-offs among doctrine, weapons, equipment, etc., are feasible for an effective defense posture. All in all, systems analysis has been denoted as "quantized common sense" and used to provide "synthetic experience". Systems analysis/systems engineering can be defined as an explicit logical examination of alternatives by estimating and comparing the impact of each alternative on the costs and/or effectiveness of a given system without violating exogenous constraints imposed on the system under study. One school of thought juxtaposes systems analysis and system engineering by establishing system performance objectives versus design criteria for system elements (DA Technical Manual 38-760, 1969). The next page depicts the interface between systems analysis and systems engineering.

A weapon system program decision involves four fundamental elements: the external threat, cost, the state of the art, and time. The existence of external threat is the reason for developing weapons. Cost enters into the program decision as a constraint. The state of the art can also be viewed as a constraint upon a nation's ability to deal with the external threat. As time passes, the stock of knowledge increases to offset the nation's inability to deal with external threat. Consequently, there is a significant interaction between time, the growth of technical knowledge, and the external threat. Under the circumstances it is important to exploit, quickly, significant advances in the state of the art to maintain the qualitative superiority or at least parity of its weapons inventory. This quickness implies the decision to begin development after technical feasibility is predicted or at any time thereafter. Development lead time, the time interval between decision to begin development and ensuing operational availability of that weapon, is controlled by varying the amount of resources allocated to the development effort.

C. Risk Analysis Overview

Luce and Raiffa (1957) categorize decision making into certainty-risk-uncertainty classifications as follows:


SYSTEMS ANALYSIS
SYSTEM ENGINEERING
MIL PERFORMANCE
TECH PERFORMANCE
ACCURACY
DYNAMICS
DESIGN GOALS
CONFIGURATION TRADEOFFS
SELECT &
APPORTION
SPECIFICATIONS
COMP & SUBSYST
DESIGN HARDWARE
EVALUATION
TEST
COMP & SUBSYST
EVALUATION
TEST
WEAPON SYSTEM

1. Certainty - Each action is known to lead invariably to a specific outcome.

2. Risk - Each action leads to one of a set of possible specific outcomes, each outcome occurring with a known probability.

3. Uncertainty - Each action has as its consequence, a set of possible specific outcomes, but the outcome probabilities are completely unknown or not even meaningful.

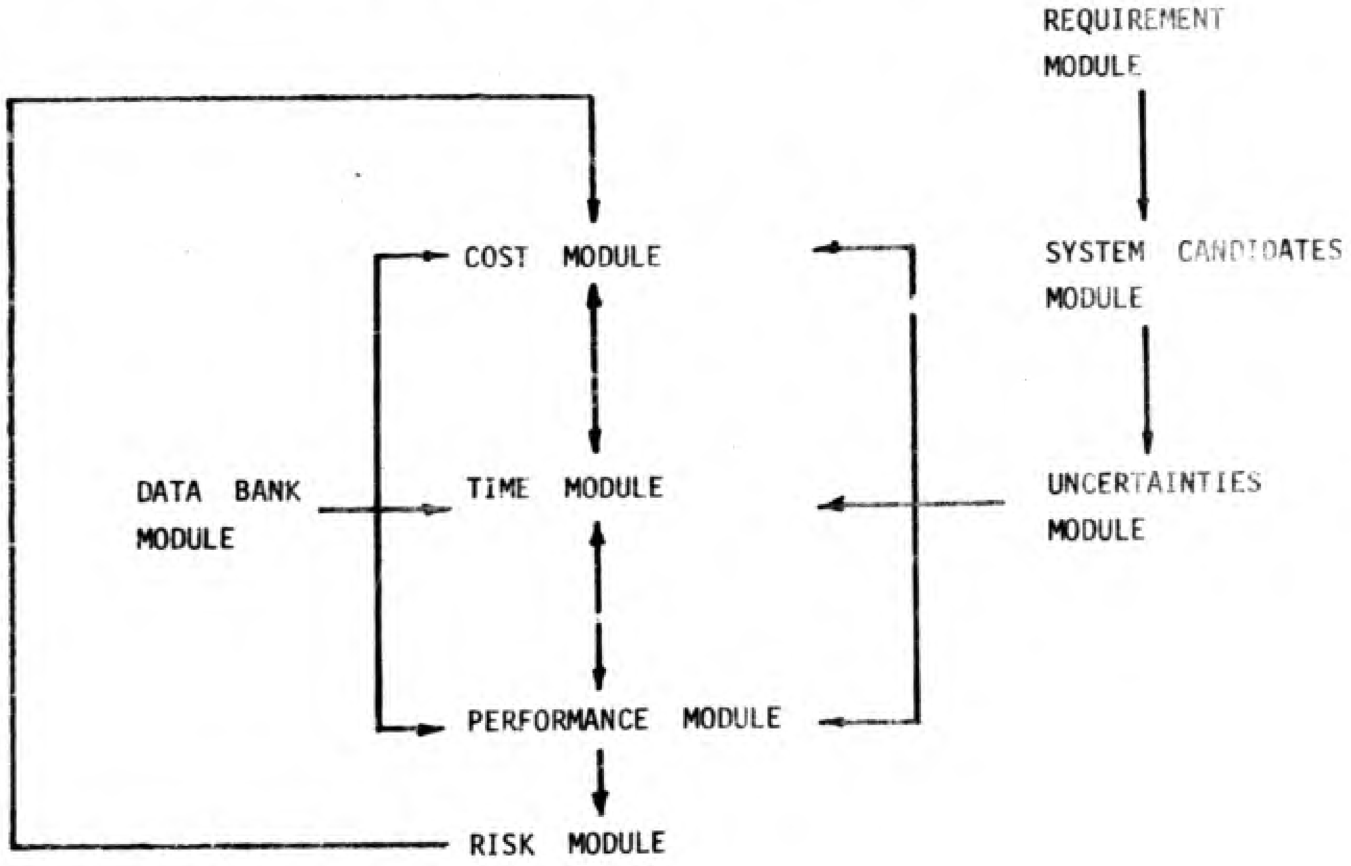
Under these notions, we now examine the decision-making process for the acquisition management. Let us consider again those "global" definitions of cost, time and effectiveness discussed earlier. We can adapt them to the level of materiel development, production, and management. Here, cost is concerned with the amount of money to be allocated to complete the acquisition process. Time denotes the total time needed to acquire the materiel. We define performance, instead of effectiveness, to be the technical capability to attain and sustain its technical potential. In view of cost growth, schedule slippage, and degradation of performance in the course of the materiel acquisition process, we ask ourselves whether or not there is a way to assess program success and to control program problems. Uncertainties exist and affect the three dimensions of cost, time and performance. The assessment of program success constitutes the basis for so-called risk analysis of a program. Risk analysis is defined as the disciplined process, essential to program decision making, involving the application of a broad class of qualitative and quantitative techniques for analyzing, quantifying, and reducing the uncertainties associated with the realization of cost, time, and performance goals of large scale, military projects. Hence, a fourth dimension of risk has been introduced which is used as a common measure to integrate the three dimensions and to effect trade-offs. Risk analysis can also be envisaged as "systems analysis of risk". As materiel acquisition decision making is decision making involving all three categories, risk analysis calls for imagination and ingenuitv, particularly to transform decision making under uncertainty to decision making under risk.

As a basic objective of risk analysis is to create a quantitative and experimental laboratory to study program success, the general methodology for a risk analysis is quite similiar to the steps involved in systems analysis, systems engineering, or industrial dynamics. The steps include the following:

1. identify objective
2. state alternatives
3. collect data
4. construct maodel
5. simulate/apply model
6. validate model
7. obtain criteria and trends

Under the above general scheme, we design a basic modular approach for risk analysis as shown on the next page. The System Candidate Module includes all system candidates under consideration. The Uncertainties Module represents the interface between the environment and the program. Uncertainties affecting the three dimensions are noted. The Data Bank

SCHEME FOR RISK ANALYSIS - A MODULAR APPROACH



Module has two functions: provides a repository of information and data for the model, and serves as a mechanism for updating and maintaining current information. The data are normally in two forms: objective, available data from testing, data bank, or previous studies; and subjective, judgmental values obtained from "experts". The Cost Module represents the three dimensions; moreover, probability distributions of the three dimensions are included in these modules. These distributions are combined to calculate system risk in the Risk Module. Trade-offs among the three dimensions can be conducted to obtain criteria and trends as recommendations for decision analysis. Furthermore, sensitive elements or parameters and high risk areas are identified so that those critical areas are carefully monitored. Unfortunately, the validation phase may not be possible, until such time when actual test data are generated or at normal intervals to up-date the risk analysis studies. It is hoped that the above modular approach can lead to a generalized risk analysis model.

A full-scale, comprehensive risk analysis then involves five distinct phases as shown below:

1. form a risk analysis team
2. collect data
3. develop decision tree
4. simulate by computer
5. construct module for management information system

A team is chosen to include the following disciplines:

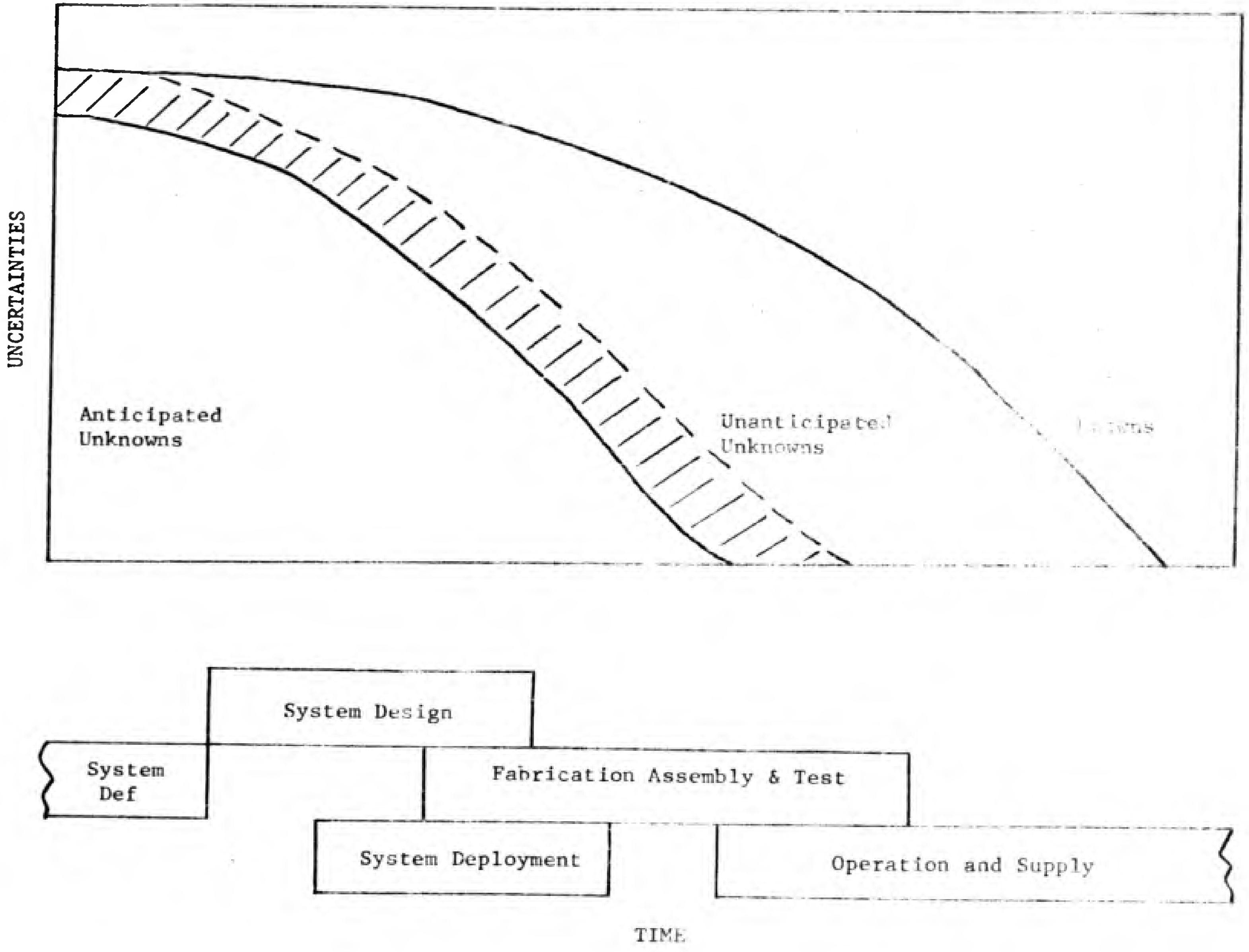
1. operations research/systems analysis
2. program-cost analysis

3. physical sciences/engineering
4. reliability-maintainability
5. military doctrine/combat experience

The Aerospace Industries Association (1969) launched a massive effort to uncover the problem of relating uncertainties in weapon system development. In identifying the essential technical steps, AIA found that there is much similarity in the evolution of programs through uncertainties. A fundamental curve, as shown on page 15, portrays the conversion of unknowns to knowns against time. Uncertainties fall into two main categories: "the things you know you don't know at the start of the program" and "things you don't know you don't know". Thus, we have known-unknowns for which allowances can be made and unknown-unknowns (unk-unks) for which we are unable to plan. A list of uncertainties is attached in Appendix A for reference. An example of some uncertainties which contribute to the three-dimensions of a program, in a qualitative fashion, is shown on page 16. Ideally, of course, we should quantify each uncertainty.

It is important to ask ourselves whether or not risk analysis contributes to acquisition management. A risk analysis should identify the following areas to the acquisition personnel:

1. potential problem areas
2. consequences of failure
3. low risk program areas
4. requirements versus state-of-the-art trade-offs
5. adequacy of acquisition time



UNCERTAINTIES RESOLUTION

UNCERTAINTIES CONTRIBUTING TO
TIME, COST AND PERFORMANCE

UNCERTAINTIES	TIME	COST	PERF.
POOR COST ESTIMATES	L	H	L
REQUIREMENTS CHANGES	M	M	H
DESIGN ERRORS	H	H	H
ANALYTIC ERRORS	H	H	H
ANALYTIC OVERSIGHTS	H	H	H
FABRICATION ERRORS	H	M	M
DEFECTIVE PARTS	H	L	M
DEFECTIVE RAW MATERIAL	H	L	H
CONTRACT INITIATION DELAYS	H	L	0
FUNDING DELAYS	H	M	0
BUDGET CUTS	H	H	L
PRIORITY CONFLICTS	H	0	0
PERSONNEL ACTIONS	H	L	M
FABRICATION DELAYS	H	L	0
DECISION DELAYS	H	L	0

L - LOW PROBABILITY OF BEING AFFECTED IF PROBLEM OCCURS.
M - MODERATE PROBABILITY OF BEING AFFECTED IF PROBLEM OCCURS.
H - HIGH PROBABILITY OF BEING AFFECTED IF PROBLEM OCCURS.
0 - NO INTERACTION.

6. sufficiency of appropriations
7. optimum allocation of funds
8. data gaps/recommend studies and concepts
9. sensitive/critical parameters.

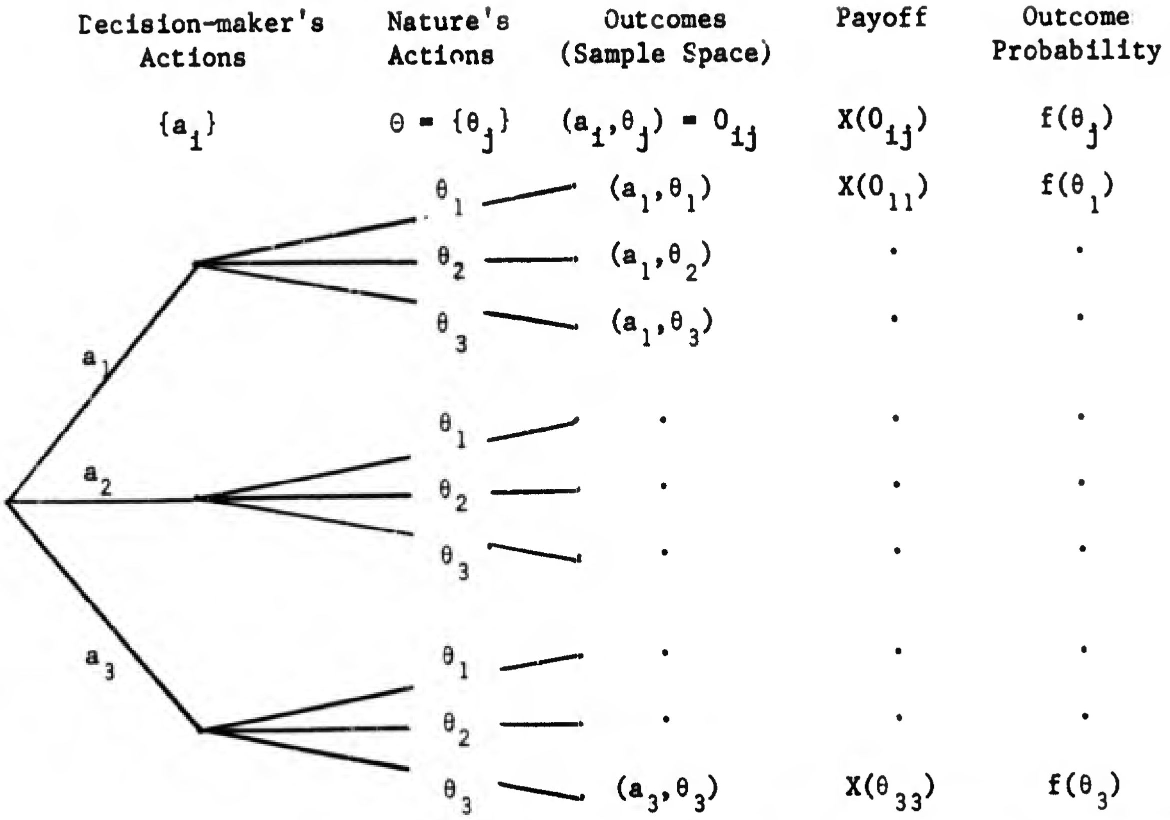
One important aspect of risk analysis is to train all acquisition management personnel to become more conscious of system risk. The realization of risk by all levels of acquisition personnel would increase the probability of project success and control cost growth, schedule slippages, and degradation of performance. Finally, it is clear that risk analysts could place more confidence on the results of the analysis when people come to better grips with subjective risk assessment.

II. Statistical Decision Theory

A. Some Decision Criteria

The materiel acquisition process is a complex management problem consisting of a multitude of tasks with interactions and hierarchy of objectives to achieve goals. Acquisition managers live in a world of continuous change with uncertainties, make decisions, and act on them, often based on very limited information. In this chapter, we present some aspects of statistical decision theory which are relevant to the development of risk analysis methodologies.

We begin with some definitions found in Dyckman et al, (1969). If a process of change leads to more than one possible outcome, these outcomes are called uncertain, and the process is called a stochastic process or a random process. States of nature are considered as any possible state resulting from intransigence of outside force other than the actions by rational opponents and not controllable by the decision maker. A decision criterion for selecting among action is a strategy. A payoff is a real value of a random variable whose domain is the set of outcomes in a decision problem. Suppose $A = \{a_i\}$ is a set of actions, $i \varepsilon I$, where I is some index set. Then a decision is defined as the selection by the decision maker of an element a from the set A. A diagram on the next page shows a simplified decision tree tracing all possible outcomes.



To resolve the decision problem under uncertainty, we offer some criteria below. A criterion is well defined provided a precise algorithm is prescribed which unambiguously selects the acts which are tautologically termed "optimal according to the criterion".

The first criterion involves the Bayesian measure for an action a_i which is the expected payoff X for that action:

$$\underset{\Theta}{E}[X(a_i,\theta)] = \sum_{j} X(O_{ij})\, f_{\Theta}(\theta_j) ,$$

for discrete states, where Θ presents the set of states O_{ij}, and $f_{\Theta}(\theta_j)$ is the state probability, with j in some index set J. A Bayesian decision criterion is the criterion for which the action selected would yield the maximum Bayesian measure.

Risk prescribes some measure of program success in view of all uncertainties. Later, we shall exhibit alternative definitions which are directly applicable to the particular phases of the acquisition cycle. In the following, we present a general definition of risk as found in most statistics texts. Dyckman et al, (1969) begins with a sample which is an observable event. A statistic is a random variable whose values can be computed from a sample. A decision function is a function whose domain is a sample space of observable outcomes and range a set of actions. By an opportunity loss, for outcome 0_{ij}, we mean a function ℓ such that

$$\ell(0_{ij}) = |X(0_{ij}) - X(0_{kj})|,$$

where $X(0_{kj})$ is the most desirable payoff for state j under some defined optimality condition. The risk for a decision function d and a given state of nature θ is the conditional expected loss for the decision function, given the state of nature, i.e.,

$$\underset{Z}{R}(d,\theta) = \underset{Z}{E}[\ell(d,\theta)],$$

where Z is a statistic, and ℓ is the loss function.

We present in the following, which can be skipped if a reader is not familiar with basic measure theory (Royden, 1963), a mathematical definition of risk:

> Consider a state space Ω indexing a family $\mathcal{P} = \{P_\omega : \omega\varepsilon\Omega\}$ over a σ-field β of a sample space χ; a measurable action space $(A,\mathcal{A})$; and a real-valued loss function $L \geq 0$ defined on $\Omega x A x \chi$ which is $\mathcal{A}$-measurable for each $\omega\varepsilon\Omega$ and $x\varepsilon\chi$. A

(randomized) decision function ψ with domain $\chi \times \mathcal{A}$ is such that $\psi(x)(\cdot)$ is a probability measure on $\mathcal{A}$ for each fixed x. If the state is ω, the conditional risk of ψ given x is

$$L(\omega,\psi(x),x) = \int_A L(\omega,a,x)\psi(x)(da).$$

If $L(\omega,\psi(x),x)$ is β-measurable, the unconditional risk is

$$R(\psi,\omega) = \int L(\omega,\psi(x),x)\,dP_\omega(x).$$

Secondly, a maximum criterion is the choice of the act which maximizes the minimum payoff. Thus, each act is appraised by considering the worst state for that act, and the "optimal" choice is the one with the best worst state. This criterion is analogous to the minimax principle: If the payoff is in the context of losses, then the decision maker attempts to minimize the maximum loss for each act.

Thirdly, Savage (1951) suggested an improvement over the maximum criterion with the minimax "risk" criterion. The procedure is as follows. First, to each payoff entry u_{ij}, associate a new table with "risk" payoffs r_{ij}, where r_{ij} is the amount to be added to u_{ij} to equal the maximum payoff in the jth column. An example is shown below:

Payoffs

States / Acts	S_1	S_2
A_1	0	100
A_2	1	1

"Risk" Payoffs

States / Acts	S_1	S_2
A_1	1	0
A_2	0	99

Next, we choose the act that minimizes the maximum risk payoff for each act. This criterion is also called minimax regret criterion.

The fourth criterion of Hurwicz (1951) is the pessimism-optimism index criterion. As the maximum and minimax risk criteria, concentrate on the state with the worst consequences. Hurwicz proposed to look at the best state or at a weighted combination of the best and the worst. For each act a_i, let m_i and M_i be the respective minimum and maximum payoffs over the possible states. Suppose a fixed number α, $0 \leq \alpha \leq 1$, called the pessimism-optimism index, be given. Then we associate with a_i,

$$\alpha m_i + (1 - \alpha)M_i ,$$

and find the act with the highest corresponding value.

We note that for $\alpha = 1$, the criterion is simply the maximum criterion; for $\alpha = 0$, the maximax criterion. To find α, we resort to resolve a simple decision problem by searching for an indifference level among the acts.

Finally, if one is "completely ignorant" as to which state among all states prevails, then he should treat all states equally likely, take an average of all payoffs for that state as the expected payoff, and choose the act with the largest payoff.

Luce and Raiffa (1951), as well as other texts such as Savage (1954), digress on the advantages and disadvantages of each criterion. The interested reader is encouraged to pursue further in these references.

B. Decision-Tree Network Analysis

Classical graph theory and topology in mathematics date as far back as the beginning of mathematics. Engineers, particularly in electrical circuit design analysis and synthesis, have applied the concepts for decades (Kim and Chien, 1962). The subject of activity networks has attracted great attention since the 1950's to decision analysis primarily on account of the following attributes (Prisker and Happ, 1966):

1. Ability to model complex systems by compounding single systems.
2. Mechanistic procedure for obtaining system figure-of-merits from networks.
3. Need for a communication vehicle to discuss the system in terms of its significant features.
4. Means for specifying data requirements for analysis of the system.
5. Starting point for analysis and scheduling of the operational system.

Literature on activity networks and flow graphs can be found in hundreds of references (Lorens, 1964; Busacker and Saaty, 1965). In the area of decision processes, work extends from generalized activity networks (Elmaghraby, 1966), to techniques such as Program Evaluation and Review Techniques (PERT), Critical Path Method (CPM) (Kaufmann and Desbazeille, 1969), Industrial Dynamics (Forrester, 1961), Decision Mapping via Optimum Networks (DEMON) (Charnes, Cooper et al, 1966) to Graphical Evaluation and Review Technique (GERT) (Prisker and Happ, 1966), to name a few.

Basically, a program is charted out by a graph called a program graph or sequence graph whose arcs represent the activities. Assuming the duration of operations or job times are known, we can attribute a value to each arc. The graph nodes are called events and are interpreted as realizations of significant milestones. It is now possible to trace through all possible routes to arrive at the final milestones to obtain total job times. A critical path is defined as a route between two milestones such that the total time is minimal. It is important to note the realization time limit, the date beyond which the whole program becomes delayed, as well as the expected times. A time margin of a job can be defined as the difference between the realization time limit. These times can be calculated to envisage the rigidity of program time.

Thompson (1968) extended the CPM to "CPM under risk" problem to mean the following:

1. A project defined by means of a project graph that indicates the technological precedence conditions among the jobs.
2. In each job in the project, a distribution of job times, i.e., a probability distribution function F(t) such that the job time is less than or equal to t.

This extension involves the application of statistical decision theory to determine subjective probabilities.

For jobs of uncertain duration, experience shows that the distribution of job times may be approximated by the beta laws (Husic). Estimates of most probable time to complete and of lower and upper bounds are needed to fit to the beta distribution. By a beta density function, we mean a functional form as follows:

$$f(x) = \frac{\Gamma(p + q)}{\Gamma(p)\Gamma(q)} x^{p-1} (1 - x)^{q-1}$$

where $0 \leq x \leq 1$, $p,q > 0$, and the gamma function is defined as follows:

$$\Gamma(p) = \int_0^\infty x^{p-1} e^{-x} dx$$

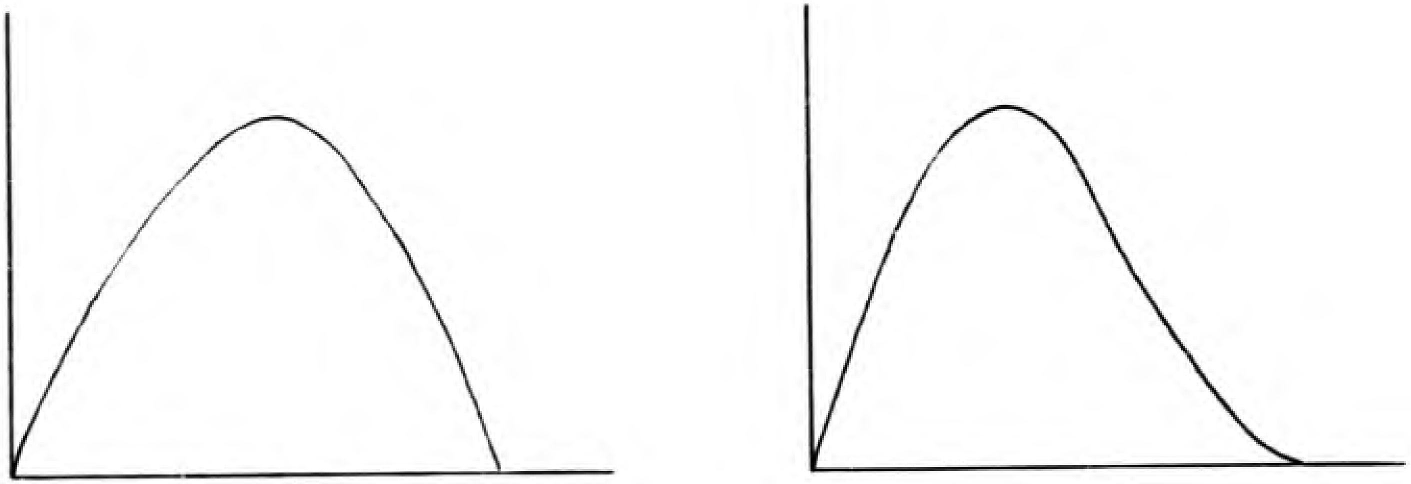
for $p > 0$. The beta probability distribution function is shown below:

$$F(x) = \int_0^x \frac{\Gamma(p + q)}{\Gamma(p)\Gamma(q)} t^{p-1} (1 - t)^{q-1} dt.$$

First, we observe that if $p = q = 2$, then the beta density function is a symmetric curve. For $p \neq q$, the beta density function is skewed. The density functions used to characterize cost depend upon whether or not the estimates made are closer to the median or to a percentile of the range of the bounds. In the former case, the symmetric density function is used; otherwise, a skewed curve is used. These estimates are made consistant by assuming the lower and upper bounds fixed; the mode or the most probable value then becomes the median or that percentile depending whether the symmetric or skewed curve is used. Typical beta probability distribution functions are shown on the next page.

Stochastic networks have the following characteristics (Prisker and Happ, 1966):

1. Each network consists of nodes denoting logical operations and directed branches.



Symmetric

Skewed

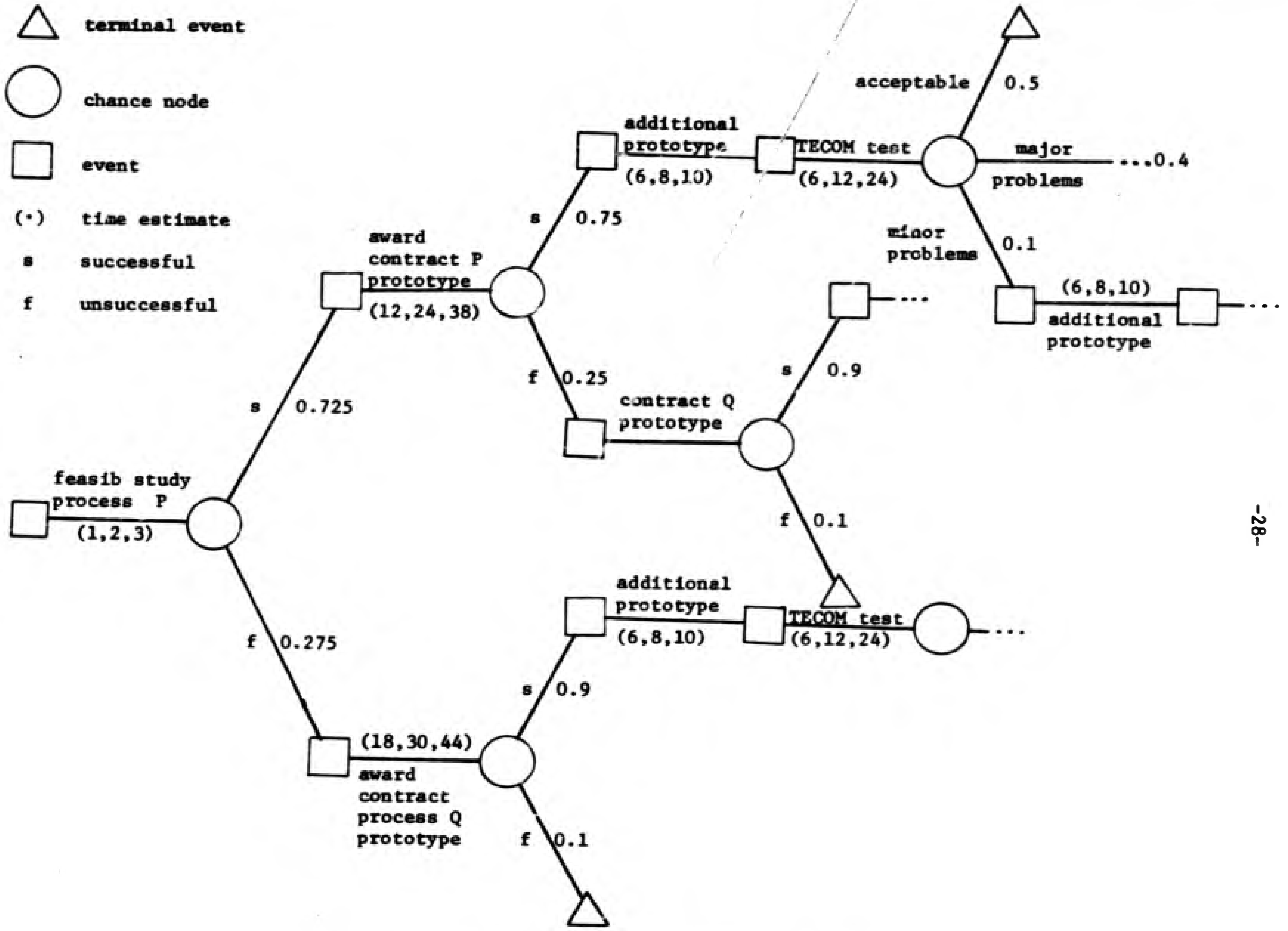
$$f(x) = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)} x^{p-1} (1-x)^{q-1}$$

BETA DENSITY FUNCTIONS

2. A branch carries a probability denoting the activity represented by the network will be performed.
3. Other parameters which may be additive for cost or multiplicative for reliability describe the activities which the branches represent.
4. A realization of a network is a particular set of branches and nodes which describes the network for one experiment.
5. If the time associated with a branch is a random variable, then a realization also implies that a fixed time has been selected for each branch.

Suppose we consider a simple network as shown on the next page with time distributions only. A Monte-Carlo simulation is applied to introduce randomness into the network at each node, thus determining which branch is selected from the event fork. The resulting end position from a Monte-Carlo trial yields a trial value. These trial values generate an "implied" distribution of outcomes and corresponding times required.

O'Flaherty (1970) attempted to outline a historical data base providing inputs to weapon system cost analysis in a form that will permit required outputs to be directly related to historical costs of similar systems. It was concluded there is no uniformity in structure of cost data available from contractors. In recording research, development, and investment costs, a uniform expandable structure compatible with DOD Cost Information Reports (1966) and AR 37-18 (1968) should be adopted with cost elements as follows:



SAMPLE NETWORK FOR ONE PHASE OF ACQUISITION PROCESS

Engineering
Tooling
Quality control
Manufacturing
Purchased equipment
Materiel overhead
Subcontract
General and administrative
Other costs
Profit, loss, or fee

These costs should be further divided between recurring and nonrecurring costs. The operating cost structure of AR 37-18 should be simplified.

From cost studies, it is necessary to obtain a cost breakdown of a system by major subassemblies into research and development, nonrecurring investment, recurring investment, and operating costs (Feyereisen, 1970). Within each cost category, we need the most probable cost, lower and upper bounds, skewed or symmetric, and estimates of cumulative probabilities for the most probable values. A table to chart the breakdown is exhibited on the next page.

To calculate subtotals or total costs for each phase of the acquisition cycle, we must add the costs in the respective categories. This is accomplished by the application of the following theorem from basic probability theory (Parzen, 1960):

Theorem: The probability density function $f_{X_1+X_2}(\cdot)$ of the sum of two independent continuous random variables is the convolution of the two probability density functions $f_{X_1}(\cdot)$ and $f_{X_2}(\cdot)$ of the random variables X_1 and X_2, i.e.,

$$f_{X_1+X_2}(y) = \int_{-\infty}^{\infty} dx\, f_{X_1}(x) f_{X_2}(y-x).$$

COST BREAKDOWN

MAJOR ASSEMBLIES	R & D					NON-RECURRING					RECURRING					OPERATING				
	L	M	H	m	%	L	M	H	m	%	L	M	H	m	%	L	M	H	m	%

L - LOWER BOUND

M - PROBABLE COST

H - UPPER BOUND

m - SKEWED OR SYMMETRIC

% - CUMULATION PROBABILITY FOR M

In our case, for beta cost probability density functions, we have, in general,

$$f_{X_1}(x) = K_1 x^{p_1-1}(1-x)^{q_1-1}, \quad 0 \leq x \leq 1, \quad a \leq X_1 \leq b,$$

and

$$f_{X_2}(x) = K_2 x^{p_2-1}(1-x)^{q_2-1}, \quad 0 \leq x \leq 1, \quad c \leq X_2 \leq d,$$

where K_1 and K_2 are some normalizing constants, and $p_1, p_2, q_1, q_2, > 0$. Hence,

$$f_{X_1+X_2}(y) = \int_{\max(c,y-b)}^{\min(d,y-a)} dx\, f_{X_1}(x) f_{X_2}(y-x) \;,$$

$$\max(c,y-b) \leq X_1+X_2 \leq \min(d,y-a) \;,$$

and the probability distribution function is

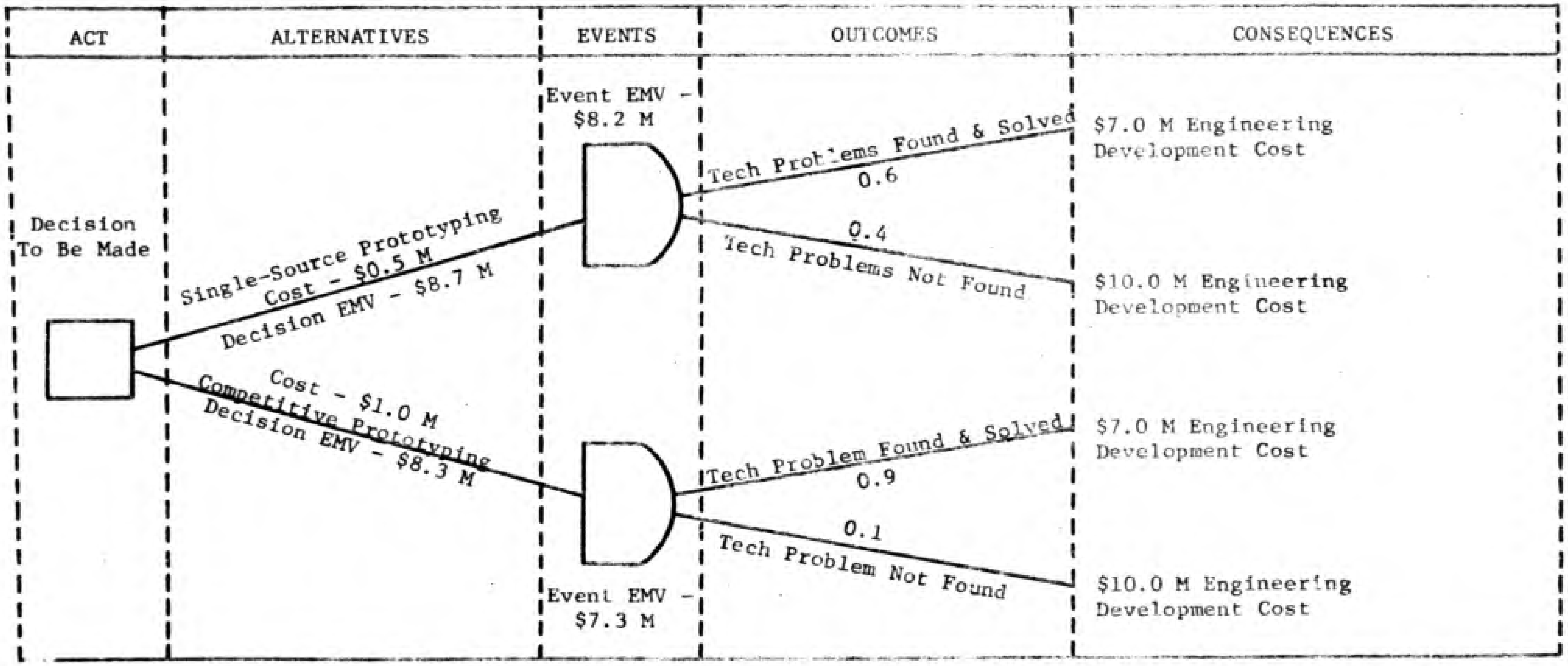
$$F_Y(y) = P[X_1 + X_2 \leq y] = \int_0^y f_{X_1+X_2}(z)dz.$$

These integrals are difficult to calculate analytically, if not impossible; however, digital computer programs are available for evaluation numerically.

We can use a hypothetical example to illustrate how we can quantitatively obtain benefits from a risk analysis.

Suppose we consider a simple decision tree, using a validation prototyping decision as an example as shown on the next page. The decision alternatives are to conduct single-source validation prototyping or to conduct multi-source competitive prototyping. The cost of the various alternatives and the probability of occurrence of each outcome are included on the alternative decision branches. In this case, competitive prototyping

SAMPLE DECISION TREE



with two contractors is twice the cost of validation prototyping with a single contractor. The engineering development cost is lower if the technical problems are found and solved prior to initiating engineering development; thus $7 million is estimated to be engineering development cost, if the technical problems are found and solved, while $10 million will be necessary if the problems are not found.

A technique called "averaging out and folding back" procedure can be applied to reach the most beneficial decision with the most advantageous expected monetary value, denoted by EMV. Calculations indicate the event EMV's as posted on the tree as $8.2 million and $7.3 million for single versus multi-sources prototyping, respectively. Adding the prototyping costs, we have $8.7 million and $8.3 million for the respective decision EMV's.

This simple example is designed to show that under a risk analysis of a particular system, a decision maker is in a good position to choose the competitive prototyping route. The extra cost incurred for competitive prototyping actually affords a 30 percent better chance for program success, a possible savings of some $3 million, and avoids time delays. If we assume engineering development cost is around ten percent of the total life-cycle cost, then the possible savings represent a two percent reduction in life-cycle cost.

C. Subjective Judgment

In the case when the state probabilities are not derived through sampling, the important area involving subjective judgment is now examined.

For example, to use the Bayesian criterion, we can rely on the following scheme (Dyckman et al, 1969):

1. Enumerate the relevant states of nature.
2. Assign weights incorporating the decision maker's judgment on the likelihood of occurrence of each state, i.e., so-called a priori probabilities.
3. Determine the outcome values from selecting a particular action.
4. Derive a new a posteriori probability law if new knowledge is available.

This is actually the big step involved in transforming a decision problem under uncertainty to a decision problem under risk.

Savage (1954) proposed a gamble or lottery technique to develop the a priori probability law over the states. The procedure calls for a series of binary yes-no answers to questions structured in terms of simple betting odds. An example was provided as follows:

Suppose an item is requisitioned from a stock with a daily demand from zero to twenty, and the stock is replenished at the end of the day. In order to establish a probability density function, we begin by considering a demand of exactly, say, nine units, and a box containing red and white chips totaling 100. Now, the decision maker is offered the following choices:

1. If tomorrow's demand is equal to nine, he wins $10.
2. Draw a chip from the box, and win $10 if it is white, given the exact composition of numbers of red and white chips before the drawing.

It is clear that if the box contains only red chips, he selects the first option, and he would choose the second option if the box contains all white chips. However, as the red chips are replaced by white chips, an indifference level is reached as to the choosing of either the first option or the second option. This betting experiment is carried out for demand from 5 to 11, and a table is constructed as follows:

Demand	No. of White at indiff. point	Probability assigned
5	2	.02
6	10	.10
7	20	.20
8	25	.25
9	25	.25
10	15	.15
11	3	.03
	100	1.00

We observe that a probability density function can be established. It is important to check for consistency with the decision maker's expectations; otherwise, he should modify his initial assessment or repeat the process until consistency is achieved. When consistency is obtained, and the requirements of a probability law are satisfied, a useable subjective probability law is determined. This law can then be used to calculate the expected payoffs required by the Bayesian decision criterion. We note that the above procedure is useful provided there are small number of states, adequate amount of time, and the decision maker is patient.

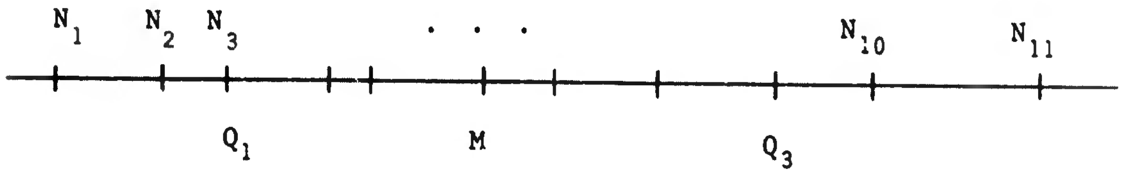
Raiffa and Schlaifer (1961) suggested the following questionnaire to obtain subjective probability distributions for time:

1. What is the time which you think is just as likely to be greater than or less than the actual completion time? (This gives the mean time.)
2. Assuming that the completion time will be greater than the mean, what new time is just as likely to be greater than or less than the actual completion time? (This gives the 75% fractile.)
3. Assuming that the completion time is greater than the answer you gave in (2), what is the new time that is just as likely to be greater than or less than the actual completion time? (This gives 87.5% fractile.)
4. What is the longest time you could imagine the project to take? (This gives 100% fractile.)

Besides the lottery technique, Quade (1968), suggests the possibility of the "delphi" technique primarily used for forecasting and commonly applied throughout the world (Business Week, 1970). A panel of experts is drawn together to make forecasts or evaluations on some particular subjects at hand. Each one is asked to submit his answers anonymously. Next, a composite feedback of all answers is communicated to each panelist, and a second round begins. This process may be repeated a number of times, and hopefully, convergence takes place. By keeping the identities anonymous rather than a committee session, a panelist can more easily change his mind at each iteration and come up with good predictions rather than defending his very original idea. Two examples (Quade, 1968) are presented on the next page to clarify the delphi technique.

First, suppose we wish to arrive at how large a particular number N should be. The following steps are involved:

1. Ask each expert independently to give an estimate of N, arrange responses in order, and group them in quartiles Q_1, M, Q_3 as shown:



2. Communicate Q_1, M, Q_3 to each expert, ask him to reconsider his previous estimate, and if his estimate (old or revised) lies outside the interquartile range $(Q\ ,Q_3)$, state his reason.
3. Communicate the results of this second round, plus the reasons, to the respondents in summary form, and ask for new estimates and arguments.
4. Take the median to represent the group decision as to N, say, in the fourth round, provided the dispersion is "small enough". The "smallness" in dispersion, of course, depends on the criticality of N for the said purposes.

A second example of delphi technique is applied to solicit policy advice. The inquiry itself could be broken down into four to six successive rounds, each based on a suitably formulated questionnaire. Only the first round would necessarily involve all respondents. The first questionnaire may contain the following:

1. Brief background and purpose of study; anonymity, except for approval to ask individuals for further actions.
2. Statement encouraging suggestions and subsidiary questions.
3. Some relevant considerations to spark thoughts.

As a result of sorting, collating, clarifying, eliminating, and editing the responses, a second questionnaire may carry the following format:

Proposals	Desirability					Feasibility					Importance			
	Desirable	Mildly	Doubtful	Mildly Undes.	Undesirable	Definitely Feas	Possibly Feas	Doubtful	Possibly Infeas	Def Infeas	Very	Important	Slightly	Unimportant
1														
2														
.														
.														
.														
M														

The third questionnaire takes the following forms:

1. Eliminated items:

Item	Description	Reason for Elimination		
		Undesirable	Infeasible	Unimportant

2. Acceptable items:

Item	Description

3. Controversial items:

Item	Description	Controversial as to			Your Reasons for Previous/Revised Rating
		Desir	Feas	Impor	

The principle drawback is that it is cumbersome, and the time elasped in processing may present some difficulty to respondents as to their reasons for the ratings. Little is known about its validity. Improvements in this technique offer some possibilities (Helmer, 1967). Some requirements include the possibility of inviting the respondents, as part of the answers, to attribute differential weights to their opinions as self-appraisals of their own competence. It is also possible to automate the experiment with the use of on-line computer consoles for automatic processing and immediate feedbacks. Of course, it would be relatively easy to develop data banks and even banks of mathematical models. One modification suggested by Helmer is an open debate of pros and cons of each estimate without identifying which was made by whom. Another modification is the use of a hierarchical panel structure, whereby a delphi response is collected, not from individuals, but from sub-panels of experts.

III. Risk Analysis

A. Risk analysis for concept formulation

Based on the policy guidance of defense Secretary Packard, we find there are actually three distinct phases of risk analysis; they are the concept formulation phase, contract definition and development phase, and production phase, respectively. We will examine two different phases in turn and show the methodologies involved in each.

First, for concept formulation phase, we have the following prerequisites for obtaining a decision to proceed into Engineering Development:

1. Primarily engineering rather than experimental effort is required, and the technology needed is sufficiently in hand.
2. The mission and performance envelopes are defined.
3. The best technical approaches have been selected.
4. A thorough trade-off analysis has been made.
5. The cost effectiveness of the proposed item has been determined to be favorable in relationship to the cost effectiveness of competing items on a Department-wide basis.
6. Cost and schedule estimates are credible and acceptable.

A risk analysis conducted during this phase is primarily concerned with technical risk; i.e., assessment of technical problems, consequence of failure, judgment as to efforts needed for a practical solution, and cost/risk trade-offs between stated operating requirements and engineering design throughout development stage.

Suppose we consider a weapon system which is composed of N major subassemblies or subsystems S_n, $n = 1,2,\ldots, N$ and each subsystem has M_n alternative candidates S_{nm_n}, $m_n = 1,2,\ldots, M_n$. Then, by permuting the candidate subsystems, we have L candidates systems S^ℓ, $\ell = 1,2,\ldots, L$,

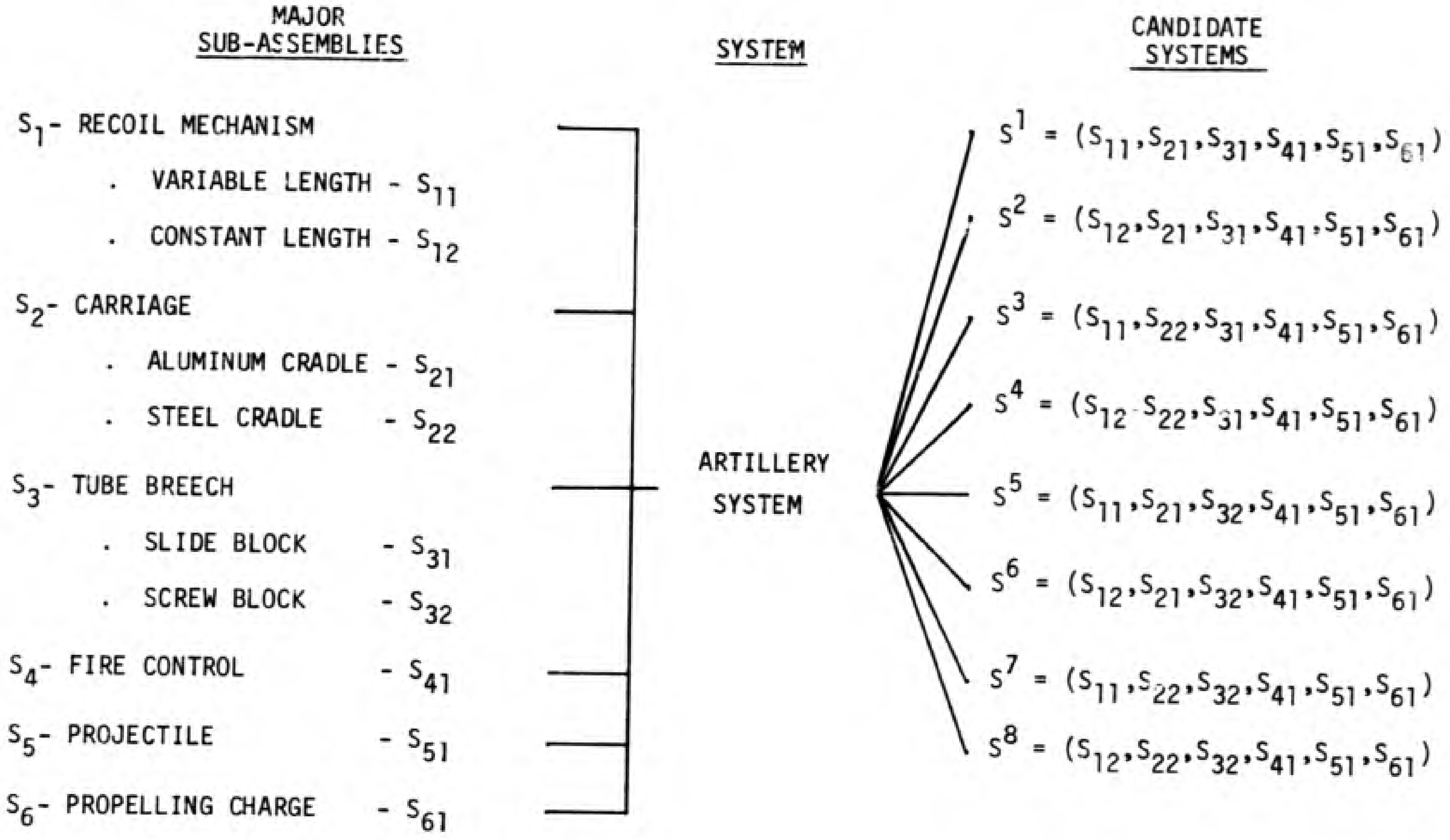
$$L \leq \prod_{n=1}^{N} M_n$$

Each candidate system $S^\ell = (S_{1m_1}, S_{2m_2}, \ldots, S_{Nm_N})$.

An example of candidate systems for an artillery system is shown on the next page.

To determine technical performance relative to the technology, we begin with a detailed analysis of some reference requirements document, such as the QMR or MN. From this document, we group all performance goals and components into categories of performance characteristics and major subassemblies or subsystems, noting the dependency and independency among the performance characteristics.

A candidate system is rated on the basis of certain performance characteristics, e.g., reliability, mobility, fire power, communication, etc. Performance estimates are obtained for a candidate configuration as it is compared with some reference system. One approach is to weight each performance characteristic on a ten-point performance level scale based on the document of the proposed system. This scoring model could be highly sensitive to outcomes and decisions. Considerations, such as


MAJOR SUB-ASSEMBLIES
S_1- RECOIL MECHANISM
. VARIABLE LENGTH - S_{11}
. CONSTANT LENGTH - S_{12}
S_2- CARRIAGE
. ALUMINUM CRADLE - S_{21}
. STEEL CRADLE - S_{22}
S_3- TUBE BREECH
. SLIDE BLOCK - S_{31}
. SCREW BLOCK - S_{32}
S_4- FIRE CONTROL - S_{41}
S_5- PROJECTILE - S_{51}
S_6- PROPELLING CHARGE - S_{61}
SYSTEM
ARTILLERY SYSTEM
CANDIDATE SYSTEMS
$S^1 = (S_{11},S_{21},S_{31},S_{41},S_{51},S_{61})$
$S^2 = (S_{12},S_{21},S_{31},S_{41},S_{51},S_{61})$
$S^3 = (S_{11},S_{22},S_{31},S_{41},S_{51},S_{61})$
$S^4 = (S_{12},S_{22},S_{31},S_{41},S_{51},S_{61})$
$S^5 = (S_{11},S_{21},S_{32},S_{41},S_{51},S_{61})$
$S^6 = (S_{12},S_{21},S_{32},S_{41},S_{51},S_{61})$
$S^7 = (S_{11},S_{22},S_{32},S_{41},S_{51},S_{61})$
$S^8 = (S_{12},S_{22},S_{32},S_{41},S_{51},S_{61})$

the underlying distributions of data, the number of ranking intervals or categories, and the width of the intervals, all contribute to the implications of the final outcomes, Moore and Baker (1969) analyzed the computational analysis of scoring models by validating some twenty projects against economic models and constrained optimization model. It was concluded that it is possible to construct a scoring model which is rank-order consistent with the other models. Secondly, the level of rank-order consistency is a function of the range over which each criterion measurement is defined and of the ability of the model to distinguish between levels across the range. When the estimates are likely to be in error, the model should utilize a large number of scoring intervals. It was under these basic guidance that we designed the ten-point performance level scale.

Consider the following ten levels V_i where i designates the ith performance characteristic:

$$V_i = \begin{cases} 0\,, & \text{would cause cancellation} \\ 1 & \\ 2\,, & \text{meets minimum acceptability} \\ 3 & \\ 4\,, & \text{meets "essential" criteria} \\ 5 & \\ 6\,, & \text{meets "desirable" criteria} \\ 7 & \\ 8\,, & \text{exceeds "desirable" criteria} \end{cases}$$

An example of the outcome of performance requirement categorization is as follows for a soft recoil artillery system (Seamands and Hwang, 1970):

1. Reliability
2. Mobility, transportability
3. Stability

4. Range, precision
5. Reaction time, rate of fire, traverse capability
6. Growth potential for product improvement
7. Human engineering

Under each performance measurement, technical barriers and potential problem areas can be identified and matched with the five levels of state-of-the-art (Rosenzweig, 1968):

1. Existing technology
2. Scaled version based on existing technology
3. Limited component tests available
4. No laboratory or component work available
5. No laboratory or component work and limited theoretical basis.

The concept of variability in performance can be approached by assigning probability values to each performance level. Thus, we assign, for each performance characteristic i,

$$(P_{i0}(j) \; , \; P_{i1}(j) \; , \; P_{i2}(j) \; , \; \ldots \; , \; P_{i9}(j)) \quad ,$$

where $P_{ik}(j)$ means $P[V_i$ meets performance level $k]$ for the jth subassembly. An example is shown on the next page.

Le us define the following:

$$P_i^{k*}(j) = P[V_i \text{ meets or exceeds performance level } k]$$

for the jth subassembly. Then, if all the performance characteristics are independent,

PERFORMANCE CHARACTERISTICS

	RELIABILITY-MAINT.	MOBILITY	STABILITY	RANGE	RATE OF FIRE	HUMAN ENGINEERING
PERFORMANCE LEVEL	0 1 2 ·· 9	0 1 2 ·· 9	0 1 2 ·· 9	0 1 2 ·· 9	0 1 2 ·· 9	0 1 2 ·· 9
RECOIL MECHANISM						
CARRIAGE						
TUBE-BREECH						
FIRE CONTROL						
PROJECTILE						
CHARGE						

$$P_i^{k^*}(j) = \sum_{k'=k}^{9} P_{ik'}(j) \quad .$$

Therefore, for instance,

$$P_i^{2^*}(j) = \sum_{k'=2}^{9} P_{ik'}(j)$$

would denote the probability that the jth subassembly meets or exceeds minimum acceptability for the ith performance characteristic.

A weapon system will be released for fielding provided it meets or exceeds the minimum acceptability level in all performance characteristics and for all subassemblies. Therefore, the probability $P^2(S^\ell)$ for a candidate system S is, for independent performance characteristics,

$$P^2(S^\ell) = \prod_{j=0}^{N_\ell} \prod_{i=0}^{I} P_i^{2^*}(j) \quad ,$$

where I is the number of performance characteristics.

A definition of risk, based on Williams and Banash (1970), considers the following probabilities:

$$P[\text{performance} \geq \text{minimum acceptability}] = P_f \quad ,$$

$$P[\text{completion time} \leq T] = P_t \quad ,$$

$$P[\text{total cost} \leq C, \text{ given time} \leq T] = P_{c/t} \quad .$$

Risk R can be defined as the probability of failing in at least one of the above categories:

$$R = 1 - P_f P_{c/t} P_t \quad ,$$

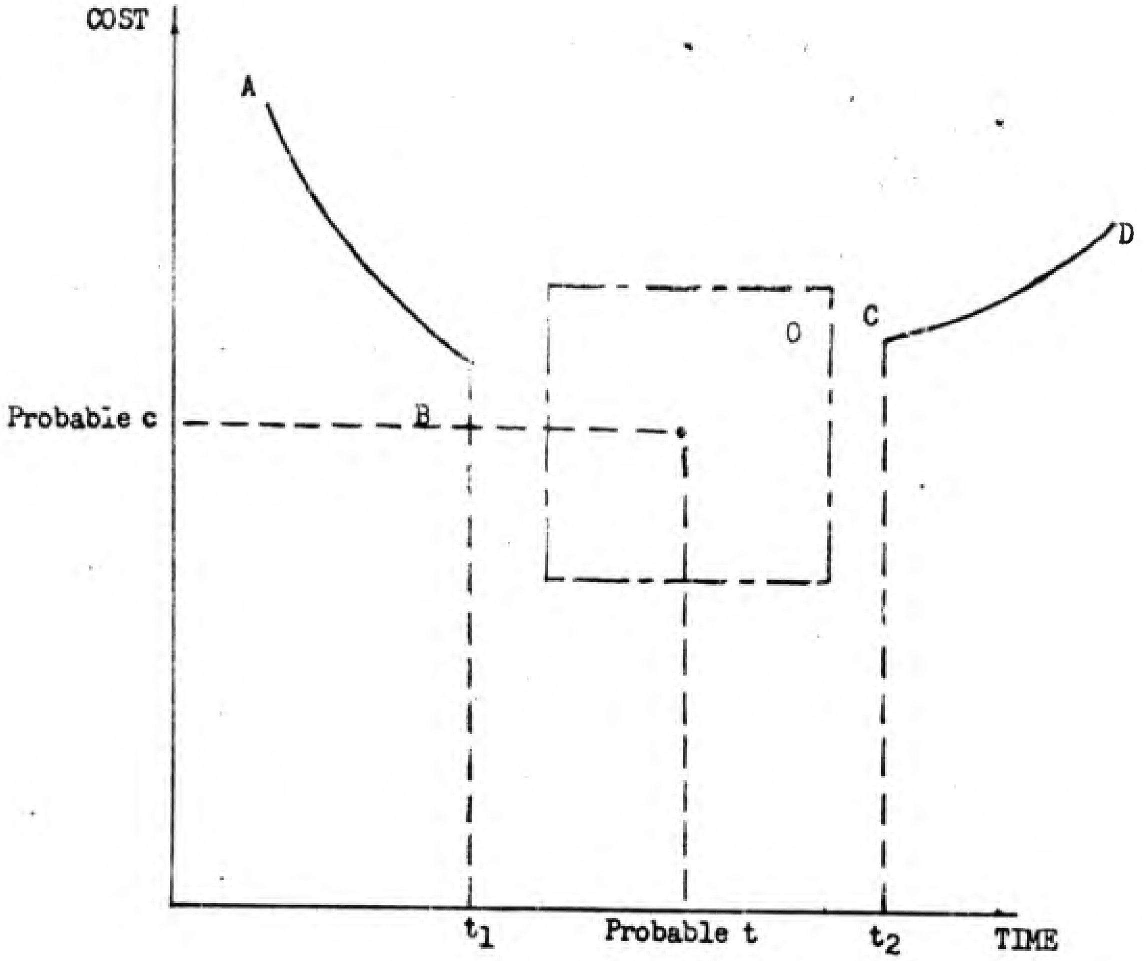
which we denote as the cost-time dependent case.

For the moment, let us ponder on the last expression. Suppose a performance level is fixed. We can graphically portray the relationship between cost and time as shown on the next page. First, it is clear that if we are allowed less time to achieve the same pre-established performance level, we must devote more resources. This accounts for the curve segment AB. Next, it is also clear that if we encounter time delays in the form of decision delays on account of, say, unacceptable performance of the system, again the cost would rise with respect to time. This is portrayed by curve segment CD. For the time period between $t = t_1$ and $t = t_2$, we find that there is a region 0 where the slippage in time schedule of the order of, say around 10% about the mode would not affect the cost growth significantly, and, similarly, a cost change of around 10% about the mode would not affect the time schedule very much. This is a reasonable assumption, as a skillful project manager usually anticipates such small fluctuations in time and cost and should be able to absorb the difference. Of course, this implies that in region 0, the cost and time are independent, and the expression for risk can be simplified to the following:

$$R = 1 - P_f P_c P_t \quad ,$$

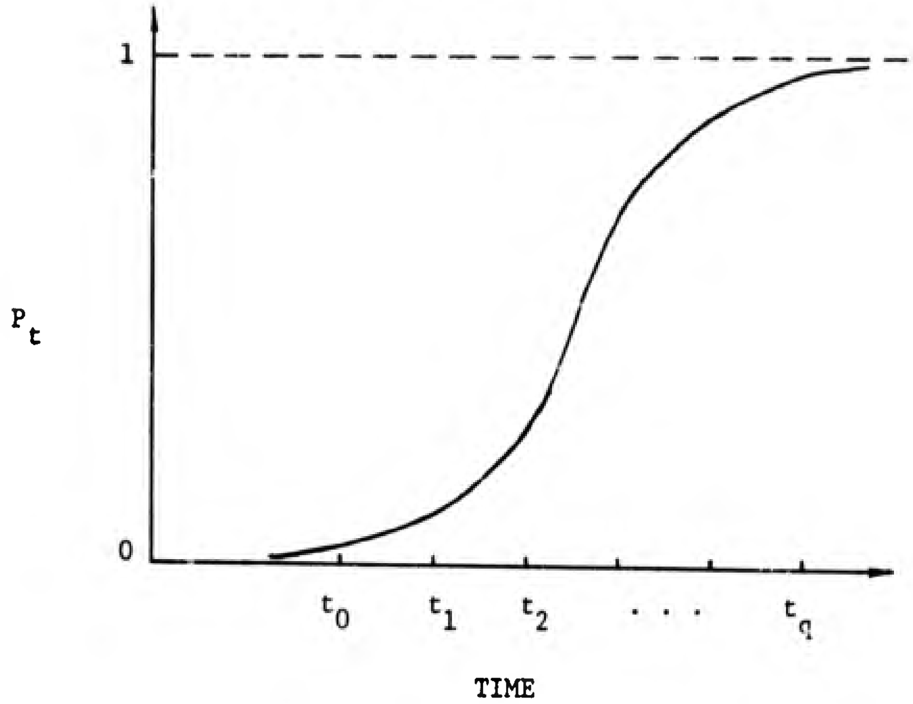
where $P_c = P[\text{total cost} \le C]$. We deonte this as the cost-time independent case.

With this definition of risk, we can graphically present iso-risk contours as described below. To graph risk isograms for the cost-time dependent case, we begin with a probability distribution curve for time.

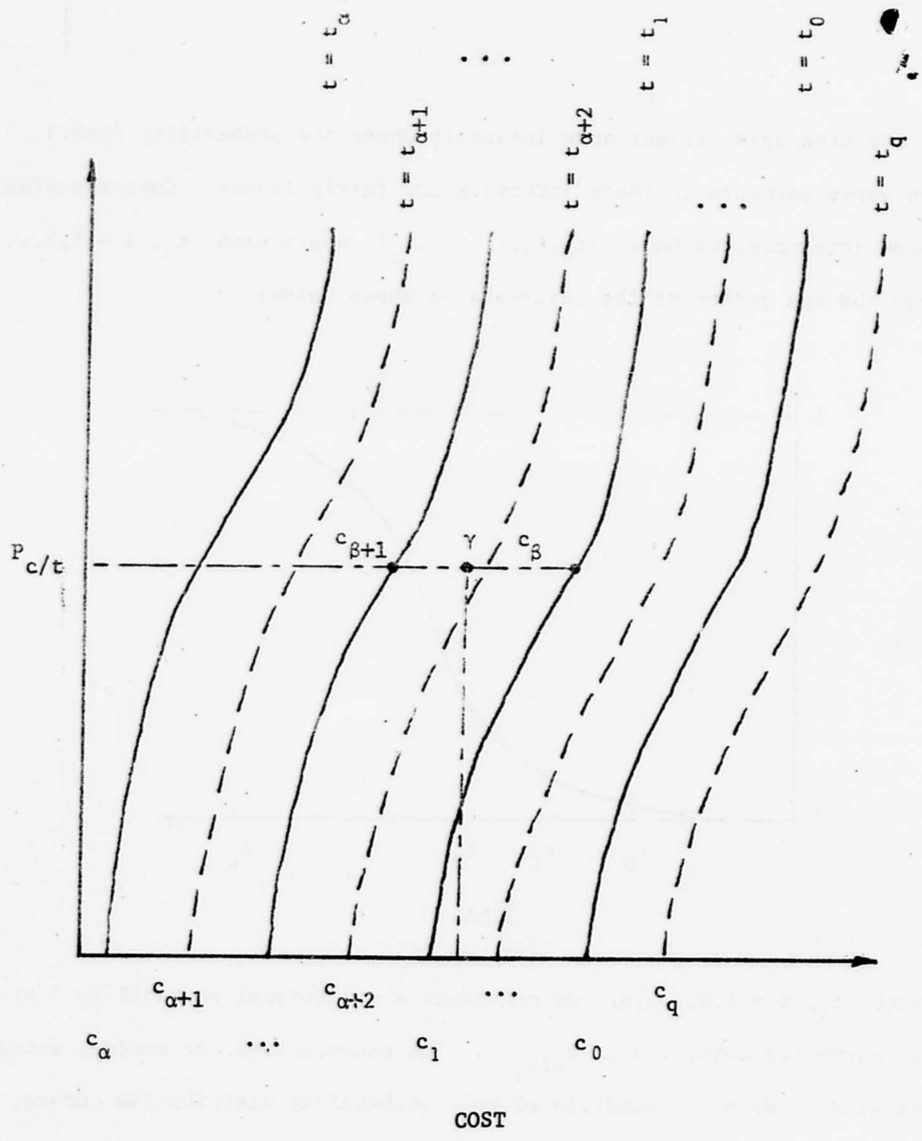


COST - TIME RELATION FOR FIXED PERFORMANCE LEVEL

Along the time axis, select some intervals where the probability distribution curve segments in these intervals are fairly linear. Corresponding to these intervals, we have $(t_0, t_1, t_2, \ldots, t_q)$ where each t_i, $i = 1,2,\ldots,q$, denotes the end points of the intervals as shown below:



For each t_i, $i = 1,2,\ldots,q$, we construct a conditional probability distribution curve for cost, i.e., P_{c/t_i} . The construction, of course, entails a cost study. With q conditional cost probability distribution curves, we can superimpose these cost probability distributions on each other and obtain a composite picture such as one shown on next page.



COMPOSITE DIAGRAM OF CONDITIONAL COST PROBABILITY DISTRIBUTIONS

For $$t_0 < t_1 < t_2 < \dots < t_{\alpha+1} < \dots < t_q ,$$

we have correspondingly

$$c_0 > c_1 > c_2 > \dots > c_\alpha$$

and

$$c_\alpha < c_{\alpha+1} < \dots < c_q .$$

A non-monotonic behavior is entirely possible if the cost analyses reveal erratic cost behavior. The construction of risk isograms is, however, no problem.

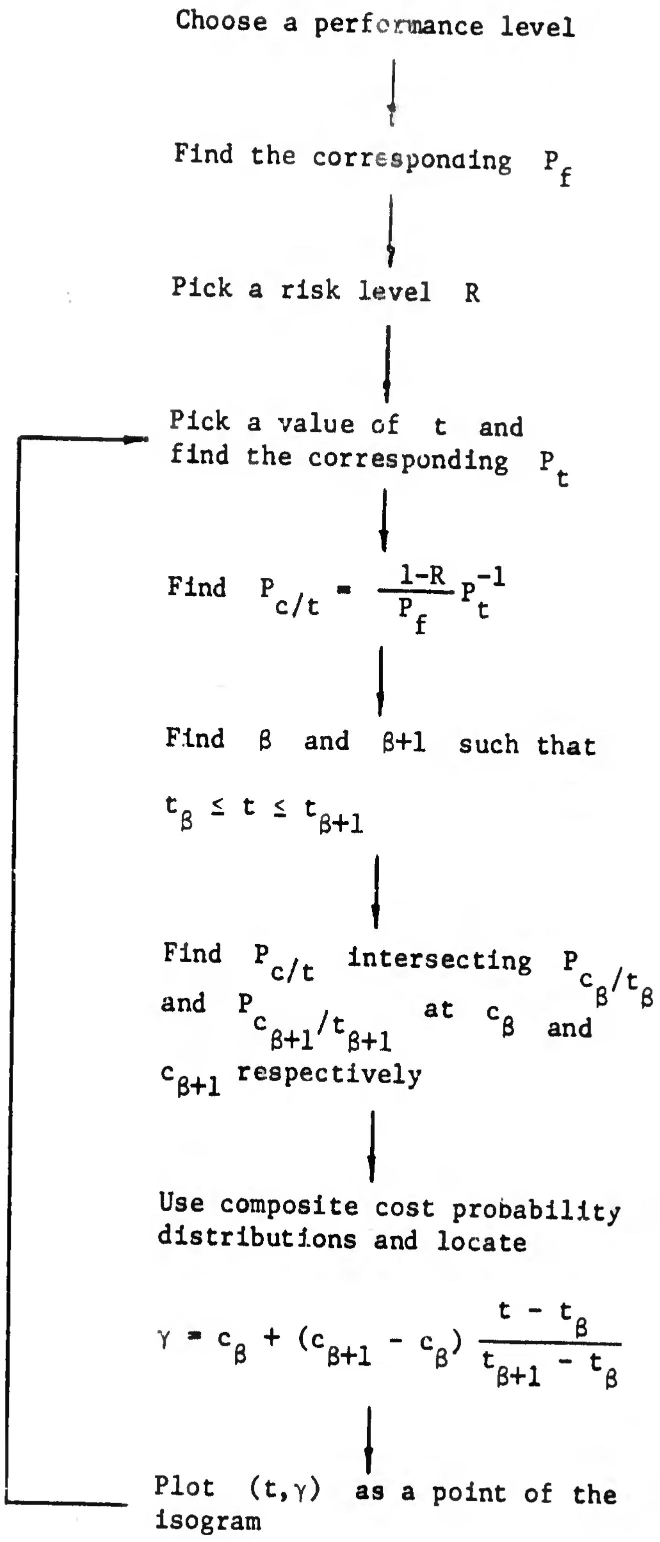
With $R = 1 - P_f P_{c/t} P_t$, we can derive an alternate relation

$$P_{c/t} = \frac{1-R}{P_f} P_t^{-1}.$$

This relation is useful in the construction of risk isograms with respect to time and cost for fixed performance levels. A basic diagram to accomplish this process is shown on page 52. By specifying time t_1 we can find $P_{c/t}$ by the above equation. If $0 \le P_{c/t} \le 1$, then the value of c corresponding to $P_{c/t}$ can be determined. For each t , there exists some β such that the interval $[t_\beta, t_{\beta+1}]$ contains t. Referring to the composite cost probability distribution picture, we see that for the calculated $P_{c/t}$, we wish to locate γ . But this simply calls for a linear interpolation:

$$\frac{t_{\beta+1} - t}{t - t_\beta} = \frac{c_{\beta+1} - \gamma}{\gamma - c_\beta} ,$$

where $c_{\beta+1}$ and c_β are the intersections of $P_{c/t}$ with $P_{c_{\beta+1}/t_{\beta+1}}$ and



CONSTRUCTION OF RISK ISOGRAMS

(Cost-Time Dependent Case)

P_{c_β/t_β} respectively. Solving for γ, we have

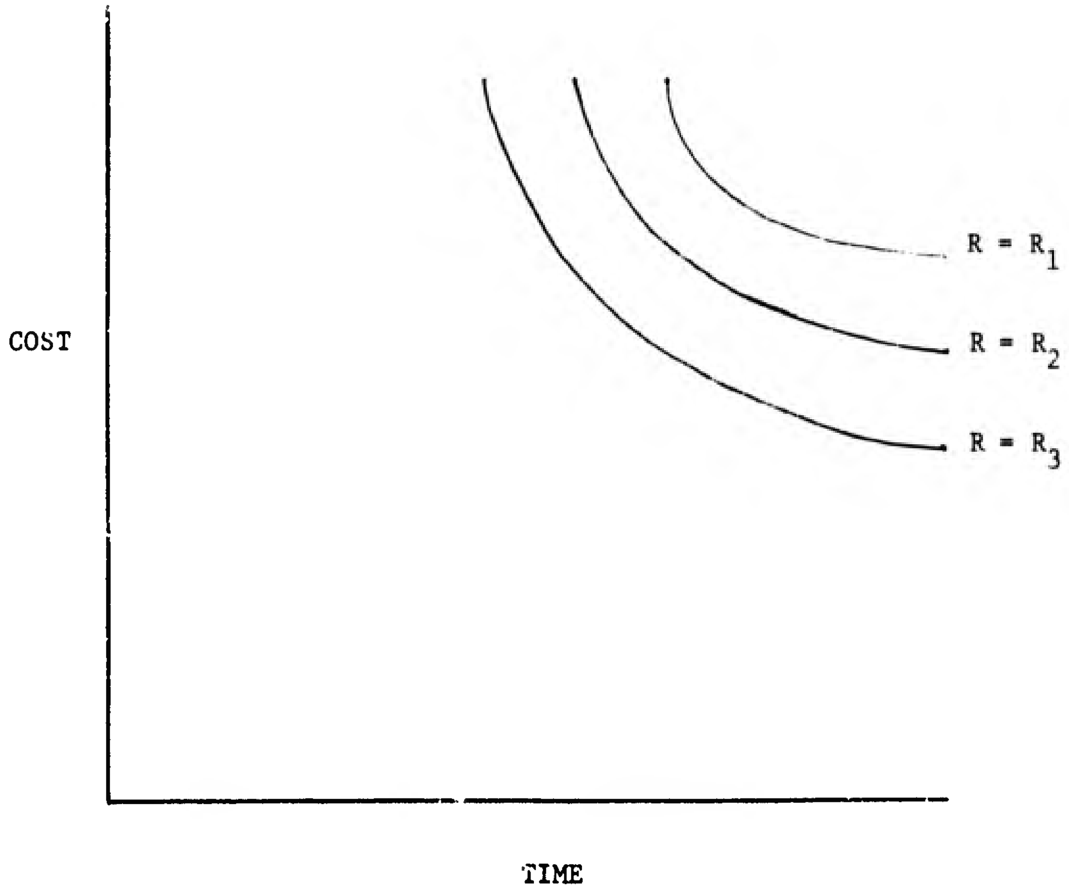
$$\gamma = c_\beta + (c_{\beta+1} - c_\beta) \frac{t - t_\beta}{t_{\beta+1} - t_\beta} .$$

With t and γ, we can plot a point on the isogram; an example of the total resulting curves is shown on the next page. If $P_{c/t}$ is not in the interval [0, 1], no cost will yield a value of $P_{c/t}$; and, performance and time constraints are such that even though cost is not constrained, program risk exceeds the risk specified.

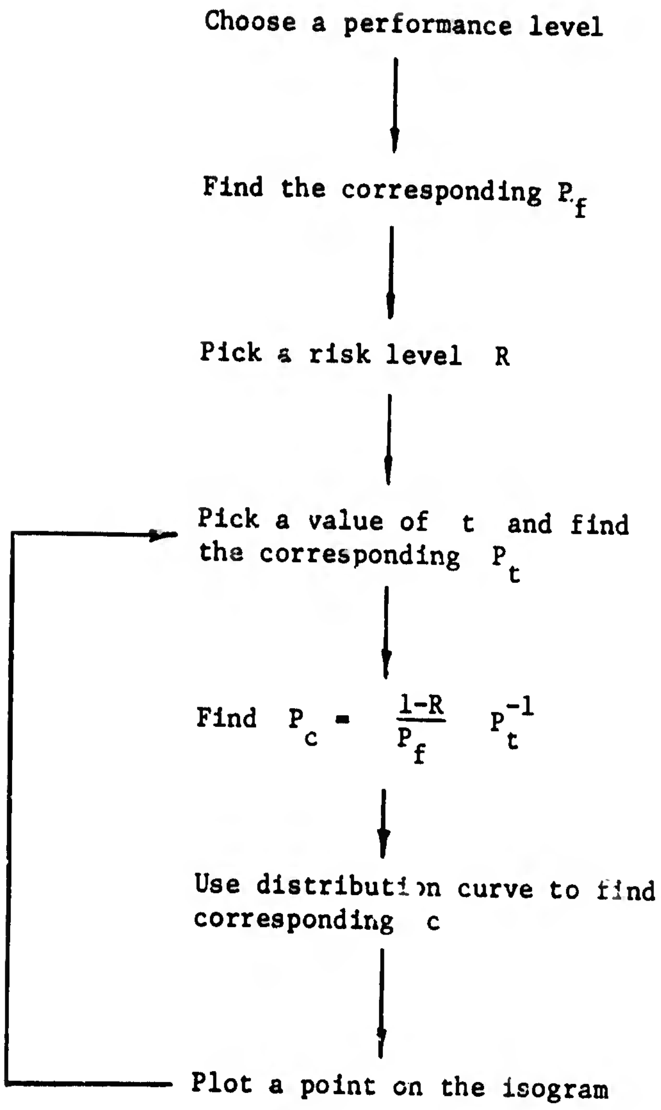
For the cost-time independent case, the risk relation is simply

$$R = 1 - P_f P_c P_t ,$$

The construction of risk isograms is much simplier as it involves no interpolation. The diagram on page 55 shows the scheme and is self-explanatory. In this austere environment, we are often faced with the situation when cost is the ultimate constraint. We are asked to trade between time and performance with constant funding. Under the above definition, we can easily exhibit risk isograms relative to time and performance with a slight change in the computational algorithm.



RISK ISOGRAMS



CONSTRUCTION OF RISK ISOGRAMS

(Cost-Time Independent Case)

B. Risk analysis for contract definition phase

For the contract definition phase, the following objectives are important:

1. Providing a basis for a firm fixed-price or fully structured incentive contract for development.
2. Identification of high risk elements.
3. Detailed specifications for all end items.
4. Verification of technical approaches.
5. Establishment of firm schedules and cost estimates including production engineering, facilities, construction and production hardware to be funded during the development.
6. Establishment of schedules and cost estimates for the total project including production, operation and maintenance.

In this phase, we are concerned with the type of contract which must be tailored to the risks involved. An appropriate definition for risk analysis follows that of Marshall (1969). In the quantification of contractor risk, Marshall suggested three major factors should be included in cost variations. The variation in cost due to the real world uncertainties is one. Additionally, contract structure, including contract type, is involved mainly on risk assumption. Finally, contractors with extensive resources or special goals may treat money in a different way than other contractors, and this treatment presumes contractor utility for money. The quantification of contractor risk R is expressed in terms of the above by the following formulation:

$$R = \mathrm{Var}[u(F(C))],$$

where C denotes a random variable whose value represents the final cost to the customer for a particular project, $F(\cdot)$ is a mathematical expression of contract structure, and $u(\cdot)$ represents a contractors utility function for money. If $G_C(c)$ is the distribution function for the random cost C, then R, with the variance Var of a random quantity, may be expressed as the Stieltje's integral:

$$R = \int_c [u(F(c)) - \bar{u}]^2 \, d\, G_C(c) ,$$

where

$$\bar{u} = \int_c u(F(c)) \, d\, G_C(c) .$$

The variance is a convenient measure of variability, has the requisite that both positive and negative variation about a central value weighs larger variations more heavily than smaller variations, and has a relatively simple mathematical formulation.

On the next page is a table summarizing risk assumption for the various contract types:

1. Firm Fixed Price Contract - contractor provides a product at a fixed price to the customer. Contractor fee is the difference between the price P and the cost C.

2. Fixed Price Incentive Firm Contract - customer pays no more than price ceiling $P_0 = (m+1)c_0 + b$, where c_0 is the cost value at the point of total assumption (PTA); m is the slope of the share time; and b is the fee value for zero cost. (b can be derived from the following:

$$b = \Gamma(C_T) - mC_T ,$$

where C_T is target cost value.) Hence, the contract is specified by the values of target cost, target fee, slope of share line, and ceiling price P_0. An incentive fee is provided to motive the contractor to keep cost down which is commonly expressed in terms of two straight lines.

3. Cost Plus Incentive Fee Contract - similar to Fixed Price Incentive Firm Contract but with no price ceiling, instead a minimum fee C_1 is specified.

4. Cost Plus Fixed Fee Contract - cost is paid by customer and an additional fee F_0, independent of cost, is paid to a contractor.

CONTRACT TYPE	FEE FORMULA	MEASURE OF RISK	LEVEL OF CONTRACTOR RISK
Firm Fixed Price	$F(c) = P-c$	$R = Var(C)$	Full Risk Assumption
Fixed Price Incentive Firm	$F(c) = \begin{cases} mc+b , & c \leq c_0 \\ F(c_0) - c+c_0 , & c > c_0 \end{cases}$ where $F_0 = mc_0 + b + c_0$ $PTA = c_0$	$R = m^2 \int_0^{c_0} (c-C_E)^2 g_C(c) dc + \int_{c_0}^{\infty} (c-C_E)^2 g_C(c) dc$ $C_E = E(C)$	High Risk Assumption
Cost Plus Incentive Fee	$F(c) = \begin{cases} mC_0 + b , & 0 \leq c < C_0 \\ mc + b , & C_0 \leq c \leq C_1 \\ mC_1 + b , & C_1 \leq c \end{cases}$	$R = m^2 \int_{C_0}^{C_1} (c-C_E)^2 g_C(c) dc$	Low Risk Assumption
Cost Plus Fixed Fee	$F(c) = F_0$	$R = 0$	No Assumption of Risk

RISK ASSUMPTION FOR CONTRACT TYPES

IV. CONCLUSION

Risk analysis is by nature an interative process and must be up-dated and validated at regular intervals. It has been proposed that risk analysis be carried out at least during concept formulation, during contract definitions, and prior to a production decision. These analyses should be coordinated with key decision points of the acquisition cycle. Also, a timely risk analysis can be used as a basis for budget appropriation purposes. It is suggested that the variability of system risks be calculated upon each update, for this variability constitutes the quantification of success of the analysis.

So far, we note that there are many wide-open questions which need further research. Some of the questions will be addressed in later papers as a part of this series of technical reports.

The very useful management information systems (MIS) being developed today are designed to help modern managers in their planning controls and to provide program visibility (Hartmann *et al.*, 1968). A risk analysis should be a module of the MIS. The design of a suitable module is one problem at hand which must be handled carefully to insure compatibility with the rest of the MIS modules.

A confidence rating of the analysis by the risk analysis team should be included so as to demonstrate the extent of sophistication of the methodology and the validity of data used in the study. A table on the next page is provided as a sample for such a rating. This area needs refinement.

CONFIDENCE RATINGS (2 DIGIT CODE)

Rating	Methods	Rating	Data
1	The basic method used to perform this analysis is exceptionally well documented and time tested; one or more other techniques have been used to verify the estimate provided.	1	Very complete, well authenticated, highly relevant data, such as recent contractor actuals, official catalog prices, etc. have been used.
2	The basic method used to perform this analysis is well documented, but no doublecheck or authentication has been possible.	2	The data used are generally relevant and from a reputable source, however, they are incomplete, preliminary, or not completely current.
3	The basic method used to perform this analysis has been documented, but has not been widely used or approved.	3	The data used have been obtained from official or standard sources; however, notable inconsistencies, lack of currency, or gaps in data reduce the confidence in the estimate.
4	A highly arbitrary method of analysis has been used.	4	The data used to make the estimate are highly suspect, doubtful relevancy, very sparse in quantity, and characterized by major inconsistencies.
5	The analysis is almost pure guesswork, and little or no confidence can be placed in it.	5	An almost total lack of current, reliable relevant data makes the cost estimate completely uncertain.

Validation of risk analysis methodology is one missing link. Besides waiting for time to pass on the present systems where risk analyses are conducted, we can look into some historical weapon system cases and carry out risk analyses to check what would have happened if the analyses were carried out. This is a non-trivial problem.

Definitions of risk analysis for the acquisition phases should be refined; a good definition for the production phase is lacking.

Sensitivity analyses should be conducted to check which activity network program is most suitable for risk analysis. Dependency among the performance characteristics present some serious problems in risk calculations.

The activity network is closely associated with game theory and programming problems. This area should be investigated to exhibit the equivalence relationship.

Finally, techniques to collect subjective judgment must be developed further.

APPENDIX A

UNCERTAINTIES

National Objectives and Strategies
Present Defense Systems Capabilities
Defined Threat or Proposed Change/Innovation
Current/Future State of Technology
Fiscal Information/Available Resources
Desired Date for Operational Capability
Expected Operational Environment
Mission Responsibility Assignment/Harmonization
Mission Objectives and Priorities
System Operational/Functional Requirements
Performance Envelopes/Design Constraints
Necessary Technology Advance and Risk Assessment
Estimated Program Costs Schedules/Concurrency
Program Approval and Budget Authorization
Rudimentary Development Plans and Objectives
System Performance/Design Requirements
Initial Spec Tree Subsystem Interface Definition
End Item Performance/Design Requirements
Maintenance and Logistics Plans
Test and Evaluation Concepts
Training and Personnel Requirements
Realistic Program Costs and Schedules
Program Management/Development/High Risk Areas
Long Lead Parts, Tooling and Facilities
Applicable Specifications/Waivers
Feasible Design Approach for End Items
Preliminary Drawings for Modules/Units
Reliability/Maintainability Budgets for End Items
Critical Components/Design Areas Identified
Subsystem Specifications
End Item Interfaces Defined
Preliminary Operational Facilities Criteria
Test Facility/Range or Support Agency Requirements
Identified/Approved Engineering Design Changes
End Item Configuration and Acceptance Requirements
DetailedDesign and Assembly Drawings
Circuit Diagrams, Mechanical/Packaging Layouts
Quality Assurance and Test Requirements
Estimated Production Rates/Quantities/Deliveries
Process Specs and Standards
Make or Buy Decisions
Configuration Control Plans

Long Lead Parts/Materials/Tooling Quantities
Parts Lists, Components Specs
Needed On-Dock Delivery Dates
Purchase Authorizations
Material Sources and Market Prices
Permissable Substitution Parts Lists
Receiving and Inspection Instruction
Preliminary Design and Assembly Drawings
Shop Fabrication Instructions
Required Materials and Parts
Test Objectives, Environment, Expected Results
Detailed Test Plans and Procedures
Test Facility, Support Equipment, Instrumentation
Known Configuration of Test Hardware
Test Measurements, Data, Variables, Parameters
Report Documentation Required
Production Line/Material Handling Layouts
Tooling Design Jigs and Fixtures
Production Facilities and Factory Test Equipment
Materials and Parts Inventory On-Hand
Routing, Scheduling and Dispatch Orders
Production Procedures, Plans and Processes
Realistic Cost and Delivery Schedules
Subcontractor Conformance Specs
Inspection Tolerances
End Item Acceptance Test Requirements
Test Objectives, Extreme Environment Conditions
Acceptable Quantity/Time Duration Sample Sized
Test Measurements, Data, Variables, Parameters
Data Reduction and Analysis Procedures
Report Documentation Requirements
Test Objectives, Environment Defined
Acceptable Demonstration Criteria Per System Spec
Detailed Test Plans and Procedures
Test Measurements, Data, Variables and Parameters
Test Site, Support Equipment, Instrumentation
Support from Range/Other Contractors/Agencies
Production Hardware Including Necessary Spares
Other Required System Segments/Elements
Data Reduction and Analysis Procedures
Report Documentation Requirements
Training Course Materials
Required Training Equipment and Facilities
Qualified Instructors
Field Requirements for Trained Personnel
Scheduled Number of Students

Examination for Minimum Acceptable Skill Level
Percentage Expected to Attain Achievement Level
Shipping and Transportation Plans
Receival Inspection Procedures
Operation Facilities Constructed
Support Facilities/Equipment on Hand
Installation, Assembly, Check Out Procedures
Equipment Scheduled Delivery Dates
Realistic Costs and Schedules to Completion
System Performance Demonstration Plans
Operation Plans Instructions and Manuals
Maintenance and Logistics Plans
Personnel Subsystem Evaluation Plans
Reliability, Maintainability, Evaluation Criteria
User Performance Capability Evaluation Criteria
Required Data and Reports on System Performance
Data Reduction/Analysis Techniques/Responsibility
System Acceptance and Turnover Agreement
Transition of Logistic Support Responsibility
Preliminary Follow On Plans
Recommended Changes to System Design
Inputs for Next Generation System Concept
Program Completion Objectives Accomplished
Human Engineering

V. REFERENCES

1. Aerospace Industries Association, "Essential Technical Steps and Related Uncertainties in DOD Weapon System Development", 1969.
2. Anonymous, "Forecasters Turn to Group Guesswork", Business Week, March 14, 1970.
3. Blue Ribbon Defense Panel, "Report to the President and the Secretary Defense on the Department of Defense", July 1970.
4. Blumenthal, Sherman C., Management Information Systems, Prentice-Hall, 1969.
5. Busacker, R. G. and T. L. Saaty, Finite Graphs and Networks, McGraw-Hill, 1965.
6. Byrne, R. *et al.*, "Some New Approaches to Risk", The Accounting Review, January, 1968.
7. Charnes, A. *et al.*, "DEMON: Decision Mapping via Optimum Go-No Networks - A Model for Marketing New Products", Management Science, July 1966.
8. Department of the Army, "A Guide to System Engineering", Technical Manual 38-760, April 1969 (Draft).
9. Department of the Army, "Life Cycle Management Model for Army Systems", Pamphlet No. 11-25, October 1968.
10. Department of the Army, "Weapon/Support Systems Cost Categories and Elements", AR 37-18, 2 July 1968.
11. Elmaghraby, S.E., "On Generalized Activity Network", Journal of Industrial Engineering, November 1966.
12. MG Feyereisen, P. A., "Product Improvement versus New Development", National Capital Chapter, Armed Forces Management Association, Ft. McNair, Washington, D.C., 1970.
13. Hartmann, W., H. Matthes and A. Proeme, Management Information Systems Handbook, McGraw-Hill, 1968.
14. Hitch, C. J. and R. N. McKean, The Economics of Defense in the Nuclear Age, Atheneum, New York, 1967.
15. Hurwicz, L. "Optimality Criteria for Decision Making Under Ignorance", Cowles Commission Discussion Paper, Statistics, No. 370, 1951.

16. Husic, F. J., "Cost Uncertainty Analysis", Research Analysis Corporation, Paper RAC-P-29.

17. Joint CDC/AMC Ad Hoc Board, "Review of Existing Formats/Procedures/Practices Used to Establish Materiel Development Objectives/Requirement", 15 May 1970.

18. Kaufmann, A. and G. Desbazeille, The Critical Path Method, Gorden and Breach, 1969.

19. Kim, W. H. and R. T. Chien, Topological Analysis and Synthesis of Communication Networks, Columbia University Press, New York, 1962.

20. Lorens, C. S., Flowgraphs, McGraw-Hill, 1964.

21. Luce, R. D. and H. Raiffa, Games and Decision, John Wiley, 1957.

22. Marshall, C. W., "Quantification of Contractor Risk", Naval Research Logistics Quarterly, December 1969.

23. Moore, J. R. and N. R. Baker, "Computational Analysis of Scoring Models for R and D Project Selection", Management Science, December 1969.

24. O'Flaherty, J. "Data System for Weapon System Cost Analysis in the Army Materiel Command", Research Analysis Corporation, RAC-TP-398, June 1970.

25. Packard, David, "Weapon Systems Acquisition Policy", Army Research and Development News Magazine, July-August 1970.

26. Parzen, Emanuel, Modern Probability Theory and its Applications, John Wiley, 1960.

27. Prisker, A. A. B. and W. W. Happ, "GERT: Graphical Evaluation and Review Technique - Part I. Fundamentals", Journal of Industrial Engineering, May 1966.

28. Peck, M. J. and F. M. Scherer, The Weapon Acquisition Process: An Economic Analysis, Harvard University, Boston, 1962.

29. Quade, E. S. and W. I. Boucher, Systems Analysis and Policy Planning, American Elsevier, New York, 1968.

30. Raiffa, H., Decision Analysis, Addison-Wesley, 1968.

31. Raiffa, H. and R. Schlaifer, Applied Statistical Decision Theory, Harvard University Press, Cambridge, Mass., 1961.

32. Ringer, L. J., "Numerical Operators for Statistical PERT Critical Path Analysis", Management Science, October 1969.

33. Rosenzweig, H., "Technological Considerations", Systems Analysis and Policy Planning, ed. E. S. Quade and W. I. Boucher, Elsevier, 1968.

34. Royden, H. L., Real Analysis, Macmillan, 1963.

35. Savage, L. J., The Foundations of Statistics, John Wiley, 1954.

36. Seamands, R. and J. D. Hwang, "Risk Analysis of the Soft-Recoil Artillery System XM204", Technical Report, Research and Engineering Directorate, HQ, US Army Weapons Command, Rock Island, Illinois, December 1970. (Confidential)

37. Schlaifer, R., Analysis of Decisions under Uncertainty, McGraw-Hill, 1969.

38. Thompson, G. L., "CPM and DCPM under Risk", Naval Research Logistics Quarterly, June 1968.

39. Williams, J. H. and R. C. Banash, "Pilot Risk Analysis of the 155mm Howitzer, XM198", Project Manager, 155mm Close Support Artillery Weapon System, US Army Weapons Command, Rock Island, Illinois, May 1970. (Confidential)

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* Systems Analysis Directorate Headquarters, U. S. Army Weapons Command	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

ANALYSIS OF RISK FOR THE MATERIEL ACQUISITION PROCESS
PART I: FUNDAMENTALS

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*
Final Technical Report

5. AUTHOR(S) *(First name, middle initial, last name)*

John D. Hwang

6. REPORT DATE November 1970	7a. TOTAL NO. OF PAGES 74	7b. NO. OF REFS 39

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S) SY-R6-70
b. PROJECT NO.	
c.	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
d.	

10. DISTRIBUTION STATEMENT

Distribution of this document is unlimited.

11. SUPPLEMENTARY NOTES	12 SPONSORING MILITARY ACTIVITY Headquarters, U. S. Army Weapons Command Rock Island, Illinois 61201

13. ABSTRACT

This paper is the first in a series devoted to the subject of analysis of risk for the materiel acquisition process. The objective of this introductory paper is three-fold. First, risk analysis is structured to show that it has close affinity to systems analysis and adds a new dimension, in terms of a probability measure, to integrate the three dimensions of cost, time to complete, and performance of a program in the materiel acquisition process. Secondly, numerous applicable techniques of statistical decision theory are presented, plus decision tree analysis and subjective judgment collection. Thirdly, methods for risk analysis of the concept formulation and contract definition phases of the acquisition cycle are exhibited. Research problems are also mentioned for future investigative efforts. Significant payoffs from a risk analysis include the identification of high risk areas, recommendations of additional studies to fill data gaps for better management decision making, a better basis for budget allocation, as well as the discovery of additional program alternatives.

DD FORM 1 NOV 65 1473 REPLACES DD FORM 1473, 1 JAN 64, WHICH IS OBSOLETE FOR ARMY USE.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Analysis of risk						
Materiel acquisition process						
Systems analysis						
Statistical decision analysis						