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A PRACTICAL OPTIMIZATION DESIGN PROCEDURE FOR STABILITY AUGMENTATION SYSTEMS

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SYSTEMS TECHNOLOGY, INC.

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FOREWORD

The research reported here was accomplished for the United States Air Force by Systems Technology, Inc., Hawthorne, California, under Contract No. F33615-69-C-1359. The program was sponsored by the Air Force Flight Dynamics Laboratory, Aeronautical Systems Division, Wright-Patterson Air Force Base, Ohio, under Project 8219.

The Air Force project engineer was Paul E. Pietrzak, FDCC. The contractor's technical director was Duane T. McRuer. The contractor's principal investigator and project engineer was Robert L. Stapleford.

The research was performed during the period from January 1969 through November 1969. The manuscript was released by the authors for publication in December 1969.

A significant portion of this project was spent on evaluating the relative merits of free-controller and parameter optimization for application to the design of stability augmentation systems. During the course of this investigation several noteworthy developments to linear optimal control theory were made. Because the final decision was to use parameter optimization, those developments are not pertinent to this report, which deals only with the proposed design procedure. They are published separately in AFFDL-TR-70-52.

The opinions expressed here on the relative merits of free-controller and parameter optimization are those of the authors and not necessarily those of the USAF.

This technical report has been reviewed and is approved.

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ABSTRACT

A systematic procedure for the design of aircraft stability augmentation systems is presented. The key features of this procedure are the selection of essential feedbacks from an examination of several handling quality metrics and the use of parameter optimization techniques to determine the numerical values of the SAS parameters. The optimization problem is structured to include both manual and SAS feedbacks. The cost function includes pilot tracking errors and SAS control deflections. A method of selecting the relative weighting is presented.

The feasibility of this procedure is demonstrated by applying it to the longitudinal axis of the F-4 aircraft. Three widely different flight conditions are selected. For all three, the same SAS form (pitch rate and normal acceleration feedbacks to the elevator), the identical problem formulation, and the same method of selecting the cost function weights are used. The resulting systems are judged quite satisfactory and well within the short-period requirements of the current military handling qualities specification.

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FIGURES

TABLES

LIST OF SYMBOLS

a_y Lateral acceleration as measured by an accelerometer

a_z Normal acceleration as measured by an accelerometer

$C_{\ell_{\delta_a}}$ Partial derivative of roll coefficient with respect to aileron deflection

C_{L_α} Partial derivative of lift coefficient with respect to angle of attack

$C_{L_{\delta_f}}$ Partial derivative of lift coefficient with respect to DLC deflection

C_{m_α} Partial derivative of pitch coefficient with respect to angle of attack

$C_{m_{\delta_e}}$ Partial derivative of pitch coefficient with respect to elevator deflection

$C_{m_{\delta_f}}$ Partial derivative of pitch coefficient with respect to DLC deflection

$C_{n_{\delta_r}}$ Partial derivative of yaw coefficient with respect to rudder deflection

C_{y_β} Partial derivative of side force coefficient with respect to sideslip

g Acceleration due to gravity

h Altitude

I_x Roll moment of inertia

I_{xz} Product of inertia

I_y Pitch moment of inertia

I_z Yaw moment of inertia

J Cost function

K Gain, particularized by subscript

ℓ Aerodynamic rolling moment

L Aerodynamic rolling moment divided by aircraft roll moment of inertia, ℓ/I_x

L_η — $\partial L/\partial \eta$, where η = p, r, v, β, δ_a, or δ_r

L'_η — Partial derivative of roll acceleration with respect to η, where η = p, r, v, β, δ_a, or δ_r;

$$L'_\eta = \left[L_\eta + \frac{I_{xz}}{I_x} N_\eta\right]\left[1 - \frac{I_{xz}^2}{I_x I_z}\right]^{-1}$$

m — Aircraft mass

m — Number of linearly independent sensors

m — Aerodynamic pitching moment

M — Mach number

M — Aerodynamic pitching moment divided by aircraft pitch moment of inertia, m/I_y

M_η — Partial derivative of pitch acceleration with respect to η, where η = u, w, $\dot{w}$, α, $\dot{\alpha}$, δ_e, δ_T, or δ_f

n — Order of plant dynamics

n — Aerodynamic yawing moment

n/α — Steady-state normal acceleration change per unit change in angle of attack for an elevator input at constant airspeed and Mach number

n — Normal acceleration load factor in g units

N — Aerodynamic yawing moment divided by aircraft yaw moment of inertia, n/I_z

N_δ^η — Numerator of η/δ transfer function

N_η — $\partial N/\partial \eta$, where η = p, r, v, β, δ_a, or δ_r

N'_η — Partial derivative of yaw acceleration with respect to η, where η = p, r, v, β, δ_a, or δ_r;

$$N'_\eta = \left[N_\eta + \frac{I_{xz}}{I_z} L_\eta\right]\left[1 - \frac{I_{xz}^2}{I_x I_z}\right]^{-1}$$

p — Roll rate

p — Transfer function pole, particularized by subscript

q — Pitch rate, $\dot{\theta}$

q_η	Cost function weight on η
r	Yaw rate
r_g	Yaw rate sensed by yaw rate gyro, $r_g = r \cos \alpha_g + p \sin \alpha_g$
s	Laplace operator
t	Time
T	Time constant, particularized by subscript
u	Velocity perturbation along X-axis
U_o	Aircraft steady-state velocity
v	Velocity perturbation along Y-axis
w	Velocity perturbation along Z-axis
w_g	Vertical gust velocity
x_a	Accelerometer distance forward of c.g.
X,Y,Z	Body-fixed stability axis system, X positive along steady-state velocity vector, Y positive out the right wing, Z positive down
X_η	Partial derivative of linear acceleration along X-axis with respect to η, where η = u, w, α, δ_e, δ_T, or δ_f
Y_c	Controlled-element transfer function
Y_p	Pilot describing function
Y_η	Partial derivative of linear acceleration along Y-axis with respect to η, where η = v, β, δ_a, or δ_r
Z_η	Partial derivative of linear acceleration along Z-axis with respect to η, where η = u, w, α, δ_e, δ_T, or δ_f
α	Angle of attack perturbation
α_g	Inclination of yaw rate gyro relative to stability axes
β	Sideslip angle
γ	Vertical flight path angle
δ_a	Aileron deflection
δ_e	Elevator deflection
δ_f	DLC (direct lift control) deflection

δ_r	Rudder deflection
δ_{SAS}	Control deflection produced by SAS feedbacks
δ_T	Throttle deflection
ζ	Damping ratio of second-order mode, particularized by subscript
θ	Pitch angle perturbation
λ	Lateral flight path perturbation, $\lambda = \psi + \beta$
ν	Observability index
ρ	Atmospheric density
τ_h	Time delay for the altitude perturbation caused by a step elevator input to return to zero
φ	Bank angle
$\Phi_{\eta\eta}$	Power spectral density of η
ψ	Aircraft heading perturbation
ω	Undamped natural frequency of second-order mode, particularized by subscript
ω	Frequency

SECTION I

INTRODUCTION

A. BACKGROUND

There has been a long-standing general need for better communication between the handling qualities specialist and the flight control system designer. In part this derives from the usual statement of handling quality problems in terms of vehicle dynamic characteristics. The vehicle dynamic details are familiar to the handling qualities stability and control specialist, but often more foreign to the flight control system designer, whose view of dynamics emphasizes the closed-loop flight controller-vehicle system. For example, when the handling quality specialist speaks of an "ω_φ/ω_d effect" or the "N'_p effect on heading control" the designer cannot readily translate such "effects" into desirable feedbacks or, for that matter, into desirable response characteristics. Thus the adequacy of a flight control system design from a handling qualities standpoint does not automatically follow from recognition of the handling quality problem areas. In fact, existing handling quality specifications have been criticized on the basis that they do not provide sufficient guidance for design. In essence, the flight control designer could use new criteria which subsume handling quality requirements but are expressed in forms more directly useful for design purposes.

Another, more basic, communication block exists. Handling qualities problems and criteria are characteristically phrased in terms of vehicle-alone dynamic properties. Stability augmentation is recognized, but is considered, just as its name implies, as a means of augmenting vehicle stability derivatives. Consequently there is a tendency to narrowly restrict the conceivable dynamic properties of the vehicle-plus-controller to those intrinsic to an airframe-alone with augmented derivatives. This viewpoint has had great merit in the past, for it tends to minimize the use of automatic control and thus improves reliability and reduces complexity. The narrow view is, however, becoming outmoded for two key reasons — the reliability, weight, and volume penalties associated with automatic control are being overcome with technological advances; and the extension of performance regimes makes the vehicle more unmanageable and thus creates more demands for automatic control solutions. The net effect is to make automatic control so attractive as to be the essential ingredient in primary flight control systems of several next-generation aircraft. In any event, a systems approach in which the closed-loop properties of the airframe-plus-automatic-controller are treated as an entity is needed. Only in this way can we take advantage of the great opportunities for pilot/aircraft integration and compatibility afforded by the broad range of dynamics available in the complete airframe-plus-controller system.

In the new view, the traditional vehicle-centered dynamics may no longer provide a convenient basis for handling quality description because they do not comprehensively define the total system. Indeed, new modes may be present while previously dominant characteristics may be changed drastically or suppressed. Where then, do we turn for a starting point to develop desirable properties for the effective vehicle? The obvious answer is the pilot. Fortunately, advances in pilot dynamic description and pilot/vehicle system analysis, coupled with an improved appreciation for the connections between pilot ratings, pilot dynamics, and system performance, permit us to specify sufficient conditions for good ratings and/or to make reasonable assessments of any given configuration. Thus we have a basis for specifying desirable vehicle characteristics; "desirable" being from the pilot's viewpoint.

The objective of the research program reported here was to utilize the recent advances in pilot/vehicle analysis and modern control theory to develop a systematic procedure for designing stability augmentation systems (SAS). The first phase of the study was a detailed investigation of the relative merits of linear optimal control and parameter optimization techniques as SAS design tools. The second phase was working out the details of the design procedure and applying it to an example problem.

B. OPTIMIZATION TECHNIQUE

Our conclusion was that with the current state of the art, parameter optimization methods are superior to linear optimal control theory for use in a SAS design procedure. Since this conclusion is a key point in the proposed design procedure, and subject to dispute, a fairly detailed comparison of the two optimization techniques is given in Appendix A. Some of the major points of that discussion are summarized below.

A major problem in applying linear optimal control theory to flight control is the conversion of the optimal control into a practical optimal (or near optimal) controller. The optimal controller can be realized in a variety of ways ranging from pure gain feedback of all the state variables to all control points to a few feedbacks with rather high order equalizations. These results must be converted to practical controller mechanizations in which a relatively few easily sensed quantities are fed back through fairly simple equalizations. Because the optimal controls are usually computed for several combinations of flight condition and vehicle configurations, another practicalization difficulty is to provide a means of adjusting the practical controller parameters so as to approximate the optimal solutions over the entire vehicle flight envelope. Both of these problems are major obstacles to the utilization of linear optimal control theory in a SAS design procedure.

On the other hand both problems can be avoided with parameter optimization techniques. In parameter optimization the controller form is specified, and then the computer determines the optimal values for these parameters. Since the controller form is prespecified only practical system mechanizations are considered from the outset. Only easily sensed parameters are used as feedbacks, only reasonable equalization schemes are used, and limitations on the parameters can also be imposed. These limitations might be set to avoid such practical mechanization problems as excessive structural feedback or oversensitivity to sensor noise. The problem of scheduling the controller parameters can also be avoided since the parameter optimization can be carried out simultaneously for a variety of flight conditions and vehicle configurations. In fact, a potential scheduling form can be specified and then the optimization program determines the optimal values of the scheduling parameters.

Parameter optimization also offers several other advantages relative to linear optimal control theory. These include:

1. The final controller configuration is truly optimized in the sense that the optimal values of all available parameters are determined.
2. The inclusion of actuator and sensor dynamics is straightforward and presents no problems.
3. Among the parameters which can be optimized are accelerometer location and rate gyro orientation.
4. Plant and controller nonlinearities can be included.
5. Nonquadratic cost functions can easily be utilized.

C. PROPOSED DESIGN PROCEDURE

The design procedure which was developed is based on pilot model tracking error and SAS control deflection in the performance criterion and parameter optimization techniques. This procedure can be divided into three steps: (1) selection of essential feedbacks, (2) single case optimization, (3) multicase optimization. Each of these steps is outlined below.

To select the essential feedbacks, the basic handling qualities deficiencies of the aircraft must first be identified. This is done by examining several key handling quality metrics for a wide variety of flight conditions. Once the deficiencies have been identified, the selection of practical feedback alternatives to improve the characteristics is rather straightforward. At this point the designer should have one or more practical and economically efficient controller configurations which are inherently capable of providing good vehicle handling qualities.

Having selected potential controller forms, the next step is to perform a variety of single case optimizations. By "single case" we mean one combination of flight condition (Mach number, altitude, and normal acceleration) and vehicle configuration (weight and c.g. location).

This step has two objectives. The first is to determine whether additional system complexity is required or whether the system can be simplified. The second is to make a preliminary estimate of the requirements for controller parameter scheduling. For those parameters which appear to require scheduling a preliminary estimate of the form of the scheduling algorithm should be made.

The final step is the multicase optimization. In this step the controller is optimized for a combination of flight conditions and vehicle configurations. The cost function would be the weighted sum of the cost functions for each individual case. The parameters to be optimized would include controller parameters which are held constant over the flight envelope, parameters in the scheduling algorithm, and sensor location and/or orientation. These optimizations can best be done on a large scale hybrid computer with the vehicle dynamics simulated on the analog portion, in fast time, and the optimization logic controlled by the digital portion of the computer.

The multicase optimization would be an iterative process. For each iteration, the acceptability of the system optimized for the range of conditions would be assessed with regard to the individual flight conditions. Whether the system is satisfactory or not, the optimization would be repeated. If the system is satisfactory, attempts to develop a simpler system would be made. If it were unsatisfactory, additional complexity, such as additional feedbacks or more complicated scheduling, would be introduced. This iterative procedure would terminate when the designer became fairly confident of having achieved a minimum complexity, satisfactory system.

Details of the design procedure, such as the performance criterion, were evolved during trial applications for a longitudinal SAS for the F-4 aircraft. Approximately 75 single-case optimizations were computed in our attempts to find criterion and input combinations which would consistently produce good SAS designs. Unfortunately, time and money did not permit a trial application of the final design step, the multicase optimization. However, we were successful in developing a criterion and input combination which worked well for three widely different flight conditions. This success makes us very optimistic about the general applicability of the proposed design procedure.

D. OUTLINE OF THE REPORT

Section II describes the process for the selection of the essential feedbacks. Longitudinal and lateral/directional augmentation are discussed separately. For both axes the discussion starts with a consideration of operational functions, such as attitude control, path control, and gust regulation. For these functions the key stability derivatives and handling quality metrics are given and practical means of modifying these are indicated. Then a variety of competing sets of essential feedbacks are evolved and compared on the basis of equalization and gain scheduling requirements and other practical design problems.

Section III discusses design criteria and how to arrange the optimization problem to provide satisfactory solutions. The objectives of this section are to describe and justify the proposed problem formulation and to discuss the effects of variations in the various elements, such as the cost function or the input characteristics. These discussions are based on the approximately 75 optimization trial results mentioned above.

The final example applications using the selected cost function and input are given in Section IV. These examples are single flight condition optimizations for the longitudinal axis of an F-4 aircraft. Comparisons of the optimized SAS results with current handling qualities specifications are presented.

Section V is a summary.

As noted earlier the more detailed discussion of the relative merits of linear optimal control theory and parameter optimization techniques is given in Appendix A. Appendix B describes the digital computer optimization program which was used in the application examples.

SECTION II

ESSENTIAL FEEDBACKS

A. GENERAL

From an overall systems viewpoint, a flight control system development can be considered in terms of stimulus and response. The stimuli are flight control desires or troubles; these evoke responses in the form of system configurations to satisfy the desires or to remedy the basic problems which underlie the troubles. In the course of these challenge-response confrontations, competitions occur between system concepts, each capable in principle of satisfying the flight control desires or of correcting the fundamental flight control problems. Although all of the system configurations conceived may be possible, some are far more feasible and desirable than others. The competitive factors are many and varied, often subtle, and change with time. But in any given design era, the best systems exhibit their superiority in such competitive factors as:

- Simplicity of mechanization
- Economy of equalization
- Commonality of elements and settings for different operational modes
- Insusceptibility of sensors to unwanted inputs
- Insensitivity to controller tolerances and airframe configuration changes
- Insensitivity to controlled element uncertainties and parasitic nonlinearities
- Small responses to unwanted inputs
- Versatility across vehicles
- Inherent reliability and maintainability

In actuality, of course, few flight control systems have been formally competitive, one with another. Rather the competition has been akin to historical evolution. But the important things to recognize are the encapsulated results of this history: the <u>root problems</u> of flight control and successful system structures which solve these problems. From the systems viewpoint, the systems structures are defined by particular combinations of feedbacks. These are what we call here "essential feedbacks." They constitute those quantities needed for feedback control purposes to solve, in a practical fashion, the several basic flight control problems. Since almost every conceivable airframe motion quantity has been used as a feedback at some time or other, the essential feedbacks constitute in themselves a record of historical successes. This record is extended as new flight control problems appear, or by revolutionary advances in sensor technology or computational capacity in the flight controller.

This philosophical introduction to essential feedbacks is intended to emphasize a continuity between flight control design using conventional techniques and that using optimal control design procedures. For the newer, and hopefully more systematic, approach to design offered by optimal control to be accepted ultimately, the answers obtained must be consonant with those provided by decades of successful conventional practice.

The essential feedbacks which we shall summarize below derive from one or both of two basic flight control system purposes:

- To establish and maintain certain specified equilibrium states of vehicle motion
- To remedy aircraft handling quality deficiencies

The establishment and maintenance of an equilibrium state of motion requires an outer control loop pertinent to the vehicle motion quantity defining that state. But more often than not such outer loops closed about aircraft-alone dynamics do not result in stable, well-damped, rapidly responding systems. Instead, equalization provided by inner loops is used to assist. The inner loops needed to make possible stable, well-damped, rapidly responding outer loop system responses are similar or identical to those needed to alleviate flying quality deficiencies. Consequently, many of the feedbacks needed to accomplish the first purpose are also useful for the second. This will become more evident as we consider the specifics below.

In the following, we shall summarize sets of essential feedbacks needed to solve the more fundamental longitudinal and lateral flight control problems.

The development will be separated for longitudinal and lateral situations. The first step will be a listing of the basic operational modes and/or states of equilibrium desired. These are, in almost all cases, common operational modes for manual and automatic control. The notable exceptions involve highly maneuverable flight, which is conducted primarily under manual control. Since we are considering here that the equilibrium states are accomplished by the pilot, the operational functions listed are, <u>ipso facto</u>, handling quality factors. Then for each of the operational functions the vehicle dynamic parameters, key metrics, and stability derivatives, etc., which together define the key features of the effective controlled element for the particular operational mode are listed in a table. When the key metrics, or certain combinations thereof, have values outside of specifiable ranges, a control system using only the outer-loop feedback will suffer from possible deficiencies. These can be corrected using the control crossfeeds or feedbacks from other variables which are listed in the table.

From the list of potential feedback possibilities, a set of systems are constructed, each amounting to a compatible set of essential feedbacks. To some extent these systems, even though all are practical in that they satisfy operational requirements and, in almost all cases, have been reduced to practice, can still be considered competitive. Thus another table is presented in which each system is appraised with respect to equalization and compensation requirements and with other practical design problems. These tables amount to verbal tradeoff summaries for the basic essential feedbacks.*

B. LONGITUDINAL

The key longitudinal operational modes which also lead to handling quality factors are:

*The background for most of the story presented here in summary form is presented in Chapters 7 and 8 of Ref. 1.

- Pitch attitude control
- Vertical gust regulation
- Path (deviation from prescribed datum) control, front side
- Path control, back side
- Speed control
- Horizontal gust regulation

For each of the operational functions, Table I lists the effective dynamic parameters, key metrics and potential feedbacks. Each entry is briefly discussed below.

Pitch attitude control is a most important longitudinal factor, both in its own right and as a possible inner loop for path and/or speed control. Pitch attitude, θ, is the essential feedback directly specified by the operational function. The aircraft dynamic parameters on which the establishment of an adequate pitch loop depend are usually the short-period frequency, ω_{sp}, and damping ratio, ζ_{sp}, and the time constant of the pitch/elevator numerator, T_{θ_2}. In some circumstances, such as operation on the back side of the thrust required curve, the phugoid characteristics can become important as quantities which limit the speed, accuracy, or damping of the closed-loop response. Pertinent feedbacks which might be used to modify the short period (and phugoid) characteristics are listed beside their appropriate key metric. Notice that if a direct lift control (DLC) is available, either an elevator to DLC cross-feed or one of the indicated feedbacks can be used to modify the pitch zero.

The second function, vertical gust regulation, is closely associated with pitch attitude control. The effective dynamic parameters are the same as for pitch attitude control, with the addition of the stability derivatives Z_w and M_w, in their roles as gain factors acting on gust inputs. With only elevator control, the beneficial potential feedbacks can do little more than increase the short-period damping. Altitude disturbances due to gusts can be reduced by washing out any θ feedback, or by using an a_z or α feedback as the primary means to increase the short-period undamped natural frequency. Similarly, pitch motions can be reduced at the expense of increased

TABLE I

LONGITUDINAL OPERATIONAL FUNCTIONS GOVERNING AUGMENTATION REQUIREMENTS

OPERATIONAL FUNCTIONS	EFFECTIVE DYNAMIC PARAMETERS	KEY METRICS	METHODS OF MODIFYING
1. Pitch attitude control ($\theta_c \to \delta_e$)	$\zeta_{sp}, \omega_{sp}, T_{\theta_2}$	$2\zeta_{sp}\omega_{sp} \doteq -(M_q + M_{\dot{\alpha}} + Z_w)$ $\omega_{sp}^2 \doteq M_q Z_w - M_\alpha$ $\frac{1}{T_{\theta_2}} \doteq -Z_w + M_w \frac{Z_{\delta_e}}{M_{\delta_e}}$	$q \to \delta_e$, $\dot{\alpha} \to \delta_e$, $\dot{a}_z \to \delta_e$ $\theta \to \delta_e$, $\alpha \to \delta_e$, $a_z \to \delta_e$ $\delta_e \to \delta_f$, $\alpha \to \delta_f$, $a_z \to \delta_f$
2. Vertical gust regulation	ζ_{sp} , ω_{sp} , T_{θ_2} , Z_w , M_w	Open-loop $\frac{\theta}{w_g} \doteq \frac{-M_w}{s^2 + 2\zeta_{sp}\omega_{sp}s + \omega_{sp}^2}$ $\frac{\dot{h}}{w_g} \doteq \frac{Z_w(s - M_q - M_{\dot{\alpha}})}{s^2 + 2\zeta_{sp}\omega_{sp}s + \omega_{sp}^2}$ $\theta \to \delta_e$ loop closed $\left(\frac{\dot{h}}{w_g}\right)_{\theta \to \delta_e} \doteq \frac{-1}{T_{\theta_2}s + 1}$	$q \to \delta_e$, $\dot{\alpha} \to \delta_e$, $\dot{a}_z \to \delta_e$ $\alpha \to \delta_e$, $a_z \to \delta_e$ $\alpha \to \delta_f$, $a_z \to \delta_f$
3. Speed control ($u_c \to \delta_e$)	ζ_p , ω_p (or $1/T_{p_1}$, $1/T_{p_2}$)	$2\zeta_p\omega_p$ or $\left(\frac{1}{T_{p_1}} + \frac{1}{T_{p_2}}\right) \doteq -X_u - \frac{M_u(X_\alpha - g)}{Z_w M_q - M_\alpha}$ ω_p^2 or $\frac{1}{T_{p_1}T_{p_2}} \doteq \frac{g(M_w Z_u - M_u Z_w)}{Z_w M_q - M_\alpha}$	$\theta \to \delta_e$, $u \to \delta_e$ $\dot{h} \to \delta_e$
4. Path control, front side ($h_c \to \delta_e$)	Above plus T_{h_2} , T_{h_3}	Above plus $\frac{1}{T_{h_2}} \doteq -\frac{1}{T_{h_3}} \doteq \left[M_\alpha - Z_\alpha \frac{M_{\delta_e}}{Z_{\delta_e}}\right]^{1/2}$ $\tau_h \doteq \left[\frac{12\, Z_{\delta_e}}{-Z_\alpha M_{\delta_e}}\right]^{1/2}$ $\frac{\gamma}{\theta} \doteq \frac{T_{h_1}s + 1}{(T_{\theta_1}s + 1)(T_{\theta_2}s + 1)}$; $\frac{1}{T_{\theta_1}} \doteq -X_u$	$\delta_e \to \delta_f$ See below
5. Path control on the backside of the drag curve ($h_c \to \delta_e$)	T_{h_1}	$\frac{1}{T_{h_1}} \doteq -X_u + \frac{(X_\alpha - g)(M_u Z_{\delta_e} - Z_u M_{\delta_e}) + X_{\delta_e}(M_\alpha Z_u - Z_\alpha M_u)}{M_\alpha Z_{\delta_e} - M_{\delta_e} Z_\alpha}$	$u \to \delta_T$, $\alpha \to \delta_T$ $\delta_e \to \delta_T$
6. Horizontal gust regulation	T_{h_1}		Same as above

altitude disturbances. If DLC is available, both the pitch and altitude gust responses can, in principle, be attenuated.

Path control is a key operational mode for normal flying and can become critical for such precision tasks as landing approach, terrain following, and in-flight refueling. The essential feedback is deviation, h, from the datum line defined by the task. [h is used here as the symbol for deviation because "altitude" from a datum line, as defined by $\ddot{h} = -(\dot{w} - U_0 q)$ is the most common deviation of interest.] This by itself is not sufficient to achieve a well damped response of the basic path mode. To provide path damping, either an $\dot{h}$ feedback or an inner pitch attitude control loop is required. For the latter, the key metrics and feedbacks are as described previously for pitch attitude control. For large, short coupled aircraft, the altitude-to-elevator numerator time constants, T_{h_2} and T_{h_3}, can become important as can the effective dead-time for altitude response, τ_h (Ref. 2). The methods of modifying any of these parameters rely primarily on DLC as a device to change Z_{δ_e}.

Horizontal gust regulation is particularly important during critical path control situations, landing in particular. T_{h_1}, the altitude to elevator very-low-frequency numerator time constant, is the key parameter because it is the limiting airspeed time constant when the altitude loop is tightly closed. To improve the airspeed time response $1/T_{h_1}$ can be changed with either of the throttle feedbacks or crossfeed noted. Note further that for altitude errors due to trim changes or steady horizontal speed shifts to return to zero requires the pilot to provide an altitude integration.

For path control when the aircraft is operating on the backside of the drag curve, the key parameter is again T_{h_1}. Because of the backside operation T_{h_1} is negative, and thus an altitude-to-elevator feedback produces a divergence which will be reflected in both altitude and airspeed. The cure is to make T_{h_1} positive. Because this is an elevator transfer function numerator, another control is required, the most effective being the throttle.

Speed control as an operational function is useful for ATC procedures and cruise, and for precise control during landing approach. It is also a potential problem when a divergent phugoid or tuck mode is present. These bring the phugoid characteristics into consideration as key metrics, although they are ordinarily important in this regard only at transonic speeds where large values of M_u are possible. The conventional fixes are an attitude or airspeed feedback to the elevator.

Examination of Table I shows a number of potentially useful feedbacks and crossfeeds. For each, the equalization requirements, practical design problems, and primary functions performed are listed in Table II. For clarity, further elaboration on some of the Table II entries is given below.

The gain variations indicated among the equalization requirements are based on maintaining a constant crossover frequency for each individual feedback. These are overly severe, and smaller gain variations are usually adequate. The listed gain variations should be considered merely as indications of the relative amount of gain changing between competing possibilities.

Feedback of structural modes can be an important practical design problem for any of the feedbacks. However, the relative magnitude of the problem varies greatly depending on the aircraft motion variable being fed back. The problem is the most severe when this is normal acceleration because:

- The a_z/δ rigid-body transfer function is flat at high frequency, i.e., there is no attenuation of higher frequency modes except that inherent in the modes themselves.
- The sensor location is restricted by rigid-body considerations.

In contrast a pitch to elevator feedback is highly attenuated at higher frequencies, even if lead/lag equalization is used, and the gyro location can be selected to minimize structural mode feedback. Consequently, Table II lists structural mode feedback as a practical design problem only for the normal acceleration feedbacks.

TABLE II

LONGITUDINAL COMPETING SYSTEMS

FEEDBACK OR CROSSFEED	PRIMARY FUNCTIONS PERFORMED	EQUALIZATION REQUIREMENTS	PRACTICAL DESIGN PROBLEMS
$q \to \delta_e$	1. Increase short-period damping. 2. Reduce pitch response to gusts.	Gain, K_q, should vary as $\frac{1}{M_{\delta_e}}$, i.e., $K_q \propto \frac{I_y}{\rho U_0^2 C_{m_{\delta_e}}}$	Gain adjustment with flight condition.
$\theta \to \delta_e$	1. Increase short-period damping and frequency. 2. Reduce pitch response to gusts. 3. Increase phugoid damping. 4. Stabilize tuck mode.	1. Gain should vary as $\frac{1}{M_{\delta_e}}$ 2. Lead/lag element desirable	1. Gain adjustment with flight condition. 2. Increased h and a_z response to vertical gusts.
$a_z \to \delta_e$	1. Increase short-period damping and frequency. 2. Reduce h and a_z response to gusts.	1. Gain, K_{a_z}, should vary as $\frac{1}{Z_w M_{\delta_e}}$, i.e., $K_{a_z} \propto \frac{m I_y}{\rho^2 U_0^4 C_{L_\alpha} C_{m_{\delta_e}}}$ 2. Lead/lag element desirable	1. Severe gain adjustment with flight condition. 2. Sensor location adequate for all flight conditions. 3. Structural mode feedback. 4. Increased θ response to vertical gusts.
$\alpha \to \delta_e$	1. Increase short-period damping and frequency. 2. Reduce h and a_z response to vertical gusts.	1. Gain should vary as $\frac{1}{M_{\delta_e}}$ 2. Lead/lag element desirable	1. Gain adjustment with flight condition. 2. Sensor instrumentation. a. Determination of operating point. b. Errors due to aerodynamic interference. c. Elaborate sensor complex required to suppress gust inputs.
$u \to \delta_e$	Stabilize tuck mode.		Sensor instrumentation (see above).
$\alpha \to \delta_T$	Prevent altitude instability.		Sensor instrumentation (see above).
$u \to \delta_T$	Prevent altitude instability.		Sensor instrumentation (see above).
$a_z \to \delta_f$	Reduce h and a_z response to gusts.	1. Gain, K_{a_z}, should vary as $\frac{1}{Z_w Z_{\delta_f}}$, i.e., $K_{a_z} \propto \frac{m^2}{\rho^2 U_0^3 C_{L_\alpha} C_{L_{\delta_f}}}$ 2. Crossfeed, $\delta_f \to \delta_e$, probably desirable to adjust effective $\frac{M_{\delta_f}}{Z_{\delta_f}}$	1. Severe gain adjustment with flight condition. 2. Sensor location adequate for all flight conditions. 3. Structural mode feedback. 4. Probable drag penalty due to direct lift control surface.
$\delta_e \to \delta_f$	Reduce effective T_{θ_2}.	1. Gain, K_{δ_e}, should vary as $\frac{M_{\delta_e}}{M_\alpha Z_{\delta_f}}$, i.e., $K_{\delta_e} \propto \frac{m C_{m_{\delta_e}}}{\rho U_0 C_{m_\alpha} C_{L_{\delta_f}}}$ 2. Washout may be desirable	1. Gain adjustment with flight condition. 2. Probable drag penalty due to direct lift control surface.
$\delta_e \to \delta_T$	Prevent altitude instability.	Low-pass filter desirable	Will not work if stick force/knot gets too low.

The use of airspeed or angle-of-attack feedbacks presents several problems in sensor instrumentation. One is setting the operating point or desired steady-state condition, e.g., setting the reference or command speed in an airspeed feedback. Another problem with any aerodynamic sensor is the errors introduced by variations in the local flow conditions. The flow in the vicinity of the sensor can change significantly with Mach number and angle of attack. The third problem relates to the basic sensitivity of the sensor to gust disturbances. In modern systems the gust inputs are suppressed by using an elaborate sensor complex instead of a single aerodynamic sensor. For example, the sensitivity of an airspeed sensor to horizontal gusts can be reduced by adding, with appropriate gains and equalization, fore-and-aft acceleration and pitch feedbacks.

At this point combinations of feedbacks and crossfeeds can be selected from Table II to meet the requirements implied by Table I. For an aircraft without DLC, the resultant multiloop augmenter system might include:

$$q \rightarrow \delta_e \qquad a_z \text{ or } \alpha \rightarrow \delta_e \qquad u \rightarrow \delta_e$$

A block diagram of this type of system, using a_z instead of α, is shown in Fig. 1. For an aircraft with DLC, the system might also include an $a_z \rightarrow \delta_f$ feedback for gust alleviation or an $\delta_e \rightarrow \delta_f$ crossfeed to augment $1/T_{\theta_2}$. In either case, the following would be added for operation on the backside of the drag curve:

$$u,\ \alpha,\ \text{or } \delta_e \rightarrow \delta_T$$

C. LATERAL DIRECTIONAL

The major operational functions for lateral-directional control are:

- Roll Attitude Control
- Roll Attitude Regulation
- Rudder-Induced Rolling
- Turn Coordination
- Path Control
- Inertial Cross-Coupling

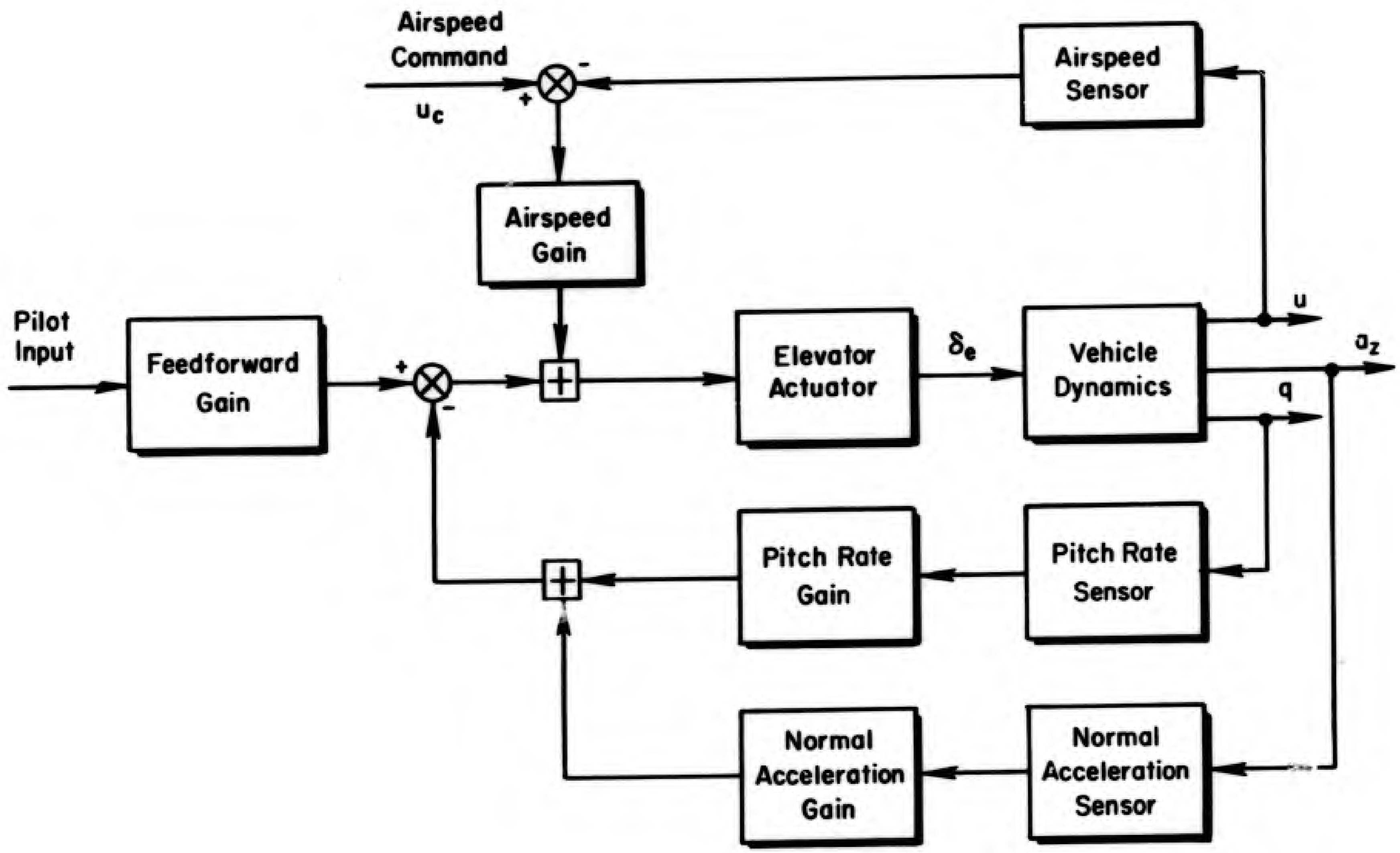

Figure 1. Longitudinal Augmenter Mechanization without DLC

For each of these operational functions, Table III lists the effective dynamic parameters, key metrics, and potential modifying feedbacks and crossfeeds. The progression from one operational function to the next is somewhat more complex than that for longitudinal in that there is more interaction between the several functions. However, a partially sequential buildup is possible.

Those operational functions listed above which serve to define an equilibrium state of vehicle motion directly provide some essential feedbacks at the outset. In particular, the need for roll attitude control and regulation and path control implies bank angle and lateral flight path or heading as necessary feedback quantities. These are presumed to be supplied by the pilot. The other essential feedbacks are those needed to make control systems having φ, λ, or ψ as outer loops feasible. These are the functions of the augmenter systems.

The first function is roll attitude control. This is of great importance as banking to rotate the lift vector is the primary means used to change the horizontal flight path of the airplane. In an airplane with idealized rolling characteristics, the roll response dynamic parameters are the roll subsidence time constant, T_R, and the maximum steady-state roll rate, p_{max}. The latter is a basic performance limitation for the aircraft and can be modified only by configuration changes or by aileron-to-rudder crossfeeds. The roll subsidence time constant, on the other hand, is subject to direct change using a feedback of rolling velocity to the ailerons. If this is accomplished with a series link which subtracts from the aileron due to pilot, the maximum rolling velocity will be reduced by the proportion $(\delta_{a_{max}} - \delta_{a_{augmenter}})/\delta_{a_{max}}$. This can be avoided by feedforward command augmentation.

To actually obtain the inherently available roll control properties, the dutch roll characteristics must often be modified. In many aircraft, this is a requirement because ordinary aileron-induced rolling maneuvers excite excessive dutch roll oscillations, sometimes to the point of instability of the closed-loop roll attitude control system. As long as the dutch roll of the aircraft alone is stable, this tendency to instability under control can be eliminated by making $\omega_\varphi/\omega_d \leq 1$ or by increasing the

TABLE III

LATERAL/DIRECTIONAL OPERATIONAL FUNCTIONS GOVERNING AUGMENTATION REQUIREMENTS

OPERATIONAL FUNCTIONS	EFFECTIVE DYNAMIC PARAMETERS	KEY METRICS	FEEDBACK OR CROSSFEED
1. Roll Attitude Control ($\varphi_c \to \delta_a$)	T_R	$\frac{1}{T_R} \doteq -L'_p + \left(N'_p - \frac{g}{U_o}\right)\frac{L'_\beta}{N'_\beta}$	$p \to \delta_a$
	p_{max}	$p_{max} = \left(\frac{\omega_\varphi}{\omega_d}\right)^2 T_R L'_{\delta_a}\delta_{a_{max}}$	$\delta_a \to \delta_r$
	ω_φ/ω_d	$\left(\frac{\omega_\varphi}{\omega_d}\right)^2 \doteq 1 - \frac{N'_{\delta_a}L'_\beta}{L'_{\delta_a}N'_\beta}$	$\delta_a \to \delta_r$
	$\zeta_\varphi\omega_\varphi - \zeta_d\omega_d$	$\zeta_\varphi\omega_\varphi - \zeta_d\omega_d \doteq \frac{L'_\beta}{2N'_\beta}\left(N'_p - \frac{g}{U_o}\right)$	$p \to \delta_r$, $\delta_a \to \delta_r$
	$\zeta_d\omega_d$	$\zeta_d\omega_d \doteq -\frac{1}{2}\left[N'_r + Y_v + \frac{L'_\beta}{N'_\beta}\left(N'_p - \frac{g}{U_o}\right)\right]$	$r \to \delta_r$, $\dot\beta \to \delta_r$, $\dot a_y \to \delta_r$
2. Roll Attitude Regulation (Gust and Yaw Input Disturbances)	$\left\|\frac{\varphi}{\beta}\right\|_d$, $\left\|\frac{\dot p}{\beta}\right\|_d$	$\left\|\frac{\varphi}{\beta}\right\|_d \doteq \left\|\frac{L'_\beta}{N'_\beta}\right\| \frac{1}{\sqrt{1 + \frac{(1 - 2\zeta_d\omega_d T_R)}{(\omega_d T_R)^2}}}$; $\left\|\frac{\dot p}{\beta}\right\|_d \doteq N'_\beta \left\|\frac{\varphi}{\beta}\right\|_d$	$\beta \to \delta_r$, $a_y \to \delta_r$ $\beta \to \delta_a$, $a_y \to \delta_a$ $p \to \delta_a$
	ω_d	$\omega_d \doteq \sqrt{N'_\beta}$	$\beta \to \delta_r$, $a_y \to \delta_r$
	$\zeta_d\omega_d$	As above Roll response to random side gust (Ref. 3) $\frac{\sigma_\varphi}{\sigma_{\beta_g}} \doteq \frac{\omega_g}{2\zeta_d\omega_d}\frac{\|\varphi/\beta\|_d}{[1 + (\omega_g/\omega_d)^2]}$ where ω_g is gust break frequency Roll response to step side gust $\frac{\varphi}{\beta_g} \doteq \left\|\frac{\varphi}{\beta}\right\|_d \left[\frac{\omega_d T_R}{\sqrt{1 + (\omega_d T_R)^2}} e^{-t/T_R} + e^{-\zeta_d\omega_d t}\sin(\omega_d t - \tan^{-1}\omega_d T_R)\right]$	$r \to \delta_r$, $\dot\beta \to \delta_r$, $\dot a_y \to \delta_r$
3. Rudder-Induced Rolling Maneuver	Same as above	Same as above	Same as above
4. Turn Coordination		Rudder deflection to coordinate $\delta_r \doteq \frac{1}{N'_{\delta_r}}\left[-N'_{\delta_a}\delta_a + \left(\frac{g}{U_o} - N'_p\right)p\right]$	$\delta_a \to \delta_r$ $\delta_a \to \delta_r$ $p \to \delta_r$
5. Path Control ($\lambda_c \to \delta_a$) or ($\psi_c \to \delta_a$)		$\frac{N^\lambda_{\delta_a}}{N^\varphi_{\delta_a}} \doteq \frac{g}{U_o s} + \frac{Y_v}{s}\frac{N^\beta_{\delta_a}}{N^\varphi_{\delta_a}}$, $\frac{N^\psi_{\delta_a}}{N^\varphi_{\delta_a}} \doteq \frac{g}{U_o s} - \frac{(s - Y_v)}{s}\frac{N^\beta_{\delta_a}}{N^\varphi_{\delta_a}}$ $\frac{N^\beta_{\delta_a}}{N^\varphi_{\delta_a}} \doteq \frac{-\frac{N'_{\delta_a}}{L'_{\delta_a}}s^2 + \left[\frac{N'_{\delta_a}}{L'_{\delta_a}}L'_p - \left(N'_p - \frac{g}{U_o}\right)\right]s + \frac{g}{U_o}\left(\frac{N'_{\delta_a}}{L'_{\delta_a}}L'_r - N'_r\right)}{s^2 + 2\zeta_\varphi\omega_\varphi s + \omega_\varphi^2}$	$\delta_a \to \delta_r$ $p \to \delta_r$ $\beta \to \delta_r$ $a_y \to \delta_r$
6. Inertial cross coupling Pitch/yaw divergence at critical roll rate Roll rate build-up from pitch input		$p_{crit} = \sqrt{N'_\beta}$ $\frac{p(s)}{p_o} \doteq 1 + \frac{L'_\beta[(I_y - I_x)/I_z]\dot\theta(s)}{(s - L'_p)(s^2 - N'_r s + N'_\beta)}$ (Ref. 4) $\left.\frac{p(t)}{p_o}\right\|_{t\to\infty} \doteq \left\{1 + \frac{L'_\beta}{N'_\beta L'_p}\left(\frac{I_y - I_x}{I_z}\right)\left[\dot\theta(t)\right]_{t\to\infty}\right\}^{-1}$ (Ref. 4)	$\beta \to \delta_r$ $a_y \to \delta_r$ $r \to \delta_r$ $p \to \delta_a$ $\beta \to \delta_r$ $a_y \to \delta_r$

dutch roll damping. ω_φ/ω_d is adjusted by reducing the effective N'_{δ_a}, thereby reducing the degree of dutch roll excitation. Increasing the dut h roll damping has both this effect and the additional merits of reducing dutch roll settling time and improving the roll response to side gusts. This will be described further below.

Several metrics relating to the function of roll attitude regulation and yaw-roll coupling are listed next. With most high effective dihedral aircraft, any yawing moment due to gusts, engine out, asymmetric store releases, or whatever, is converted via dihedral effect into rolling moments. When the listed metrics are out of bounds, the major fixes are to increase the directional stability, the dutch roll damping, and the roll damping. The metrics here are the same as those for the next item — rudder-induced rolling. Because low values are desirable for disturbance regulation and higher values for deliberate rudder-induced rolls, a compromise must often be made between the factors.

When the roll attitude control and dutch roll properties are properly adjusted, the airplane can be made to bank effectively. However, to bank is not necessarily to turn, at least not immediately. So the function of turn coordination enters our considerations. The conventional definition of a coordinated turn is that the sideslip is zero. To the extent that the dutch roll mode is excited in turning maneuvers at either entry or exit, the sideslip cannot be zero. Dutch roll can be excited in aileron-only maneuvers by N'_{δ_a}, which is the primary quantity making ω_φ different than ω_d, or by N'_p, which principally affects the difference in roll numerator and dutch roll denominator dampings. The modification of these stability derivatives by aileron-to-rudder crossfeed or roll rate to rudder feedback, provides an effective means to reduce transient yawing during rolling maneuvers. A supplementary solution is to reduce the effects of these dutch roll exciting terms by increasing the directional stability.

The turn coordination factors are also important for path control using aileron. Turn coordination difficulties make λ/φ or ψ/φ depart from the ideal g/U_0s relationship. Consequently, the bandwidth attainable by the pilot in closing a path control loop is reduced, with an associated degradation in control precision. Fortunately, the correction of turn coordination during turn entry and exit is tantamount to solution of these path control problems.

Finally, large maneuvers require us to consider inertial cross-coupling reduction as an operational function. There are two important inertial cross-coupling problems. One is the classic pitch/yaw divergence, which occurs at a critical roll rate (approximated here for the situation where $\omega_d < \omega_{sp}$ and $\zeta_d \doteq 0$). A function of the control system is then to increase the critical roll rate beyond that which will ever be used in the operation of the aircraft. The second inertial cross-coupling problem is the one of roll rate buildup, which occurs if the aircraft is pitched nose-down during a rolling maneuver. This can be quite severe for swept-wing aircraft with high effective dihedral, low directional stability, and low roll damping. Increased directional stiffness and roll damping are the most effective solutions to this problem.

From the above, we see that the most important functions of the augmentation (inner loop) control systems are to provide any needed improvements in roll damping, directional stability, and dutch roll damping, and to reduce the excitation of the dutch roll in turning maneuvers. There are, as shown by the "Feedback or Crossfeed" column in Table III, a number of sets of possible systems which might be used to perform these functions. These are considered as competing systems in Table IV, where equalization and compensation requirements and practical problems are listed as tradeoff factors.

For the functions of improving roll damping and reducing roll response time, there is no competition, as roll-rate-to-aileron feedback is the only practical solution. The gain adjustment requirement given in Table IV is that necessary to provide a constant increment in roll damping. However, in many cases fixed-gain systems have been satisfactory and the changes in roll damping increment across the flight spectrum have been acceptable. Perhaps the most significant practical problem with the $p \rightarrow \delta_a$ feedback is that failure of this loop could provide an instability in a $r \rightarrow \delta_r$ loop unless ω_r/ω_d is much less than unity with the roll damper off. This can be serious if a yaw-rate-to-rudder loop is used and could require a fail-operational roll damper or possibly a failure detection system which cuts out the directional augmentation if the roll damper fails.

TABLE IV

LATERAL/DIRECTIONAL COMPETING SYSTEMS

PRIMARY CONTROLLER FUNCTIONS	FEEDBACKS	EQUALIZATION REQUIREMENTS	PRACTICAL PROBLEMS
1. Improve roll response 2. Reduce roll sensitivity to gusts and other inputs 3. Reduce ω_r/ω_d	$p \rightarrow \delta_a$	1. Gain inversely proportional to L'_{δ_a}, i.e., $K_p \propto \frac{I_x}{\rho U^2 C_{l_{\delta_a}}}$ 2. Use p command system so feedback does not oppose pilot inputs and reduce p_{max}	1. Gain adjustment to compensate for changes in dynamic pressure, Mach no. effects on $C_{l_{\delta_a}}$, and loading effects on I_x. 2. Effects of flaps, slats, and other aerodynamic devices on L'_{δ_a}. 3. Failure may result in unstable $r \rightarrow \delta_r$ loop.
1. Increase directional stability 2. Increase dutch roll damping 3. Reduce inertial cross coupling 4. Improve turn coordination	$\beta \rightarrow \delta_r$	1. Gain inversely proportional to N'_{δ_r}, i.e., $K_\beta \propto \frac{I_z}{\rho U^2 C_{n_{\delta_r}}}$ 2. Lead/lag element	1. Instrumenting β sensor (type and location) 2. Gain adjustment to compensate for changes in dynamic pressure and Mach no. effects on $C_{n_{\delta_r}}$. 3. Lead/lag equalization on sensor possibly sensitive to gusty inputs.
	$\beta \rightarrow \delta_r$ $r \rightarrow \delta_r$	1. Both gains inversely proportional to N'_{δ_r} (see above) 2. Washout in $r \rightarrow \delta_r$ feedback	1. Instrumenting β sensor 2. Gain adjustments 3. Unless $\omega_r/\omega_d \ll 1$, $r \rightarrow \delta_R$ does not greatly increase dutch roll damping (may be unstable if $p \rightarrow \delta_a$ fails). 4. Location of washout time constant; washout effect may be deleterious in spin recovery. 5. Rate gyro inclination effects.
	$a_y \rightarrow \delta_r$	1. Gain inversely proportional to $Y_\beta N'_{\delta_r}$, i.e., $K_{a_y} \propto \frac{m I_z}{\rho^2 U^4 C_{y_\beta} C_{n_{\delta_r}}}$ 2. Lead/lag element	1. Severe range requirements on gain adjustments. 2. Finding a sensor location adequate for all flight conditions. 3. Gain limited by high-frequency modes (actuator and structural). 4. Location of lead/lag time constants.
	$a_y \rightarrow \delta_r$ $r \rightarrow \delta_r$	1. a_y gain inversely proportional to $Y_\beta N'_{\delta_r}$ (see above) 2. r gain inversely proportional to N'_{δ_r} 3. Washout in $r \rightarrow \delta_r$ feedback	1. Severe range requirements on gain adjustments. 2. Finding an accelerometer location adequate for all flight conditions. 3. a_y gain limited by high-frequency modes but not as much as equalized $a_y \rightarrow \delta_r$. 4. Unless $\omega_r/\omega_d \ll 1$, $r \rightarrow \delta_r$ does not greatly increase dutch roll damping (may be unstable if $p \rightarrow \delta_a$ fails). 5. Location of washout time constant; washout effect may be deleterious in spin recovery. 6. Rate gyro inclination effects.
1. Provide precise turn coordination 2. Eliminate lateral/ coupling	$p \rightarrow \delta_r$	1. Lead/lag element 2. Gain and lead time constant complex functions of several parameters.	1. Programming gain and lead time constant for various flight configurations and conditions. 2. May be deleterious near stall and in spins.
	$\delta_a \rightarrow \delta_r$	1. Lead/lag or lag/lead element 2. Gain and time constants complex functions of several parameters.	1. Programming gain and time constants for various flight configurations and conditions. 2. May significantly increase pilot rudder input required for deliberate sideslip maneuvers.

Table IV shows four competing directional control systems involving different combinations of rudder feedbacks. The first one is a feedback of sideslip with lead/lag equalization. The gain adjustment indicated in the table is that required to maintain a constant increment in directional stability and dutch roll damping. By far the most significant problem with this system is that of obtaining an adequate sensor. If a good sideslip sensor (or sensor complex) were available, this system would certainly be preferred over the others.

In the second system, yaw rate feedback has been substituted for the lead/lag equalization in the first system. This eliminates the problem of lead equalization on a possibly noisy sensor, but introduces all the problems associated with yaw-rate-to-rudder feedback. These are primarily associated with relatively large ω_r/ω_d (near unity), rate gyro inclination relative to stability axes, and the use of a washout to prevent the rudder from opposing a steady turn. Even with washout, excessive gain on the yaw rate feedback is detrimental to turn coordination as the feedback tries to keep the airplane from turning.

In the other two systems lateral acceleration has been substituted for sideslip. This substitution gets around the problem of the sideslip sensor, but introduces several others. For one thing the gain adjustments required for an acceleration feedback are more severe than for the sideslip systems. Also, it is necessary to locate the accelerometer fairly close to the airplane center of rotation for rudder inputs. Because the center of rotation can shift appreciably with changes in Mach number and vehicle loading, a serious practical problem is finding an accelerometer location which is adequate for all flight and loading conditions.

Another problem related to that of sensor location is the gain limitation imposed by high-frequency modes, such as actuator and structural. As illustrated in Fig. 2, this gain limitation difficulty becomes more severe as the accelerometer is moved further from the center of rotation. The problem is, of course, further accentuated if lead/lag equalization is used on the accelerometer feedback.

While increases in directional stability will improve turn coordination, the resulting coordination may not be sufficiently precise for such mission

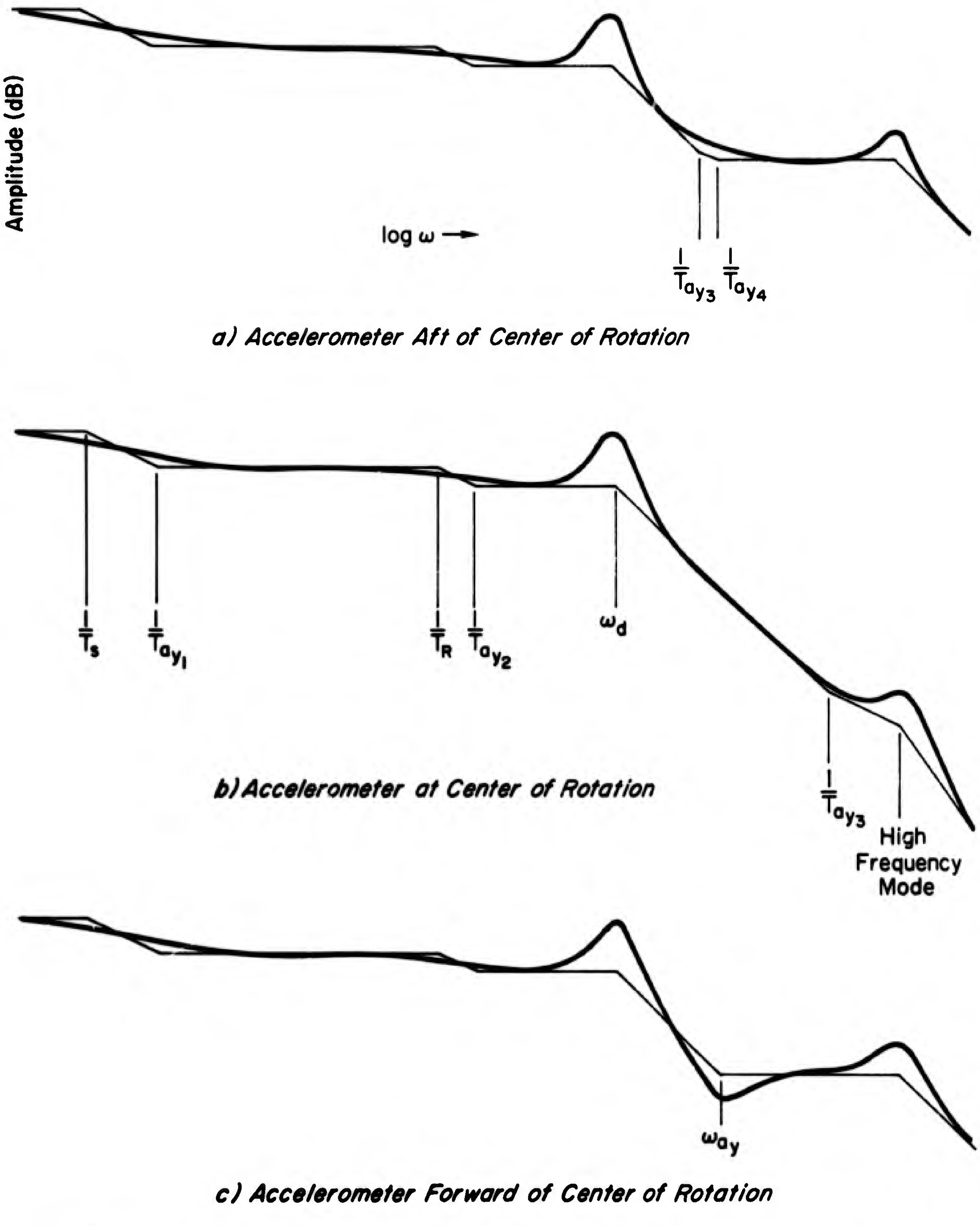

Figure 2. Typical $a_y \rightarrow \delta_r$ Bode Diagram with a High Frequency Mode

phases as intercept, landing, and tactical support. In these cases, the above systems may be supplemented with a special precise turn coordination device. Two possibilities are shown in Table IV, roll-rate-to-rudder feedback and aileron-to-rudder crossfeed. The basic purpose of these devices is to augment both N'_{δ_a} and N'_p, and to offset the adverse effects of the $r \rightarrow \delta_r$ feedback, if one is present. If the yaw rate feedback is

$$\delta_r = \frac{-K_r s}{s + 1/T_{WO}} r_g \doteq \frac{-K_r s}{s + 1/T_{WO}} (r + \alpha_g p) \quad (1)$$

where r_g = yaw rate measured by the gyro

α_g = gyro inclination relative to stability axes

Then for coordination, the turn coordination system should provide an additional rudder input given by:*

$$\Delta\delta_r \doteq \left[\frac{K_r(\alpha_g s + g/U_o)}{s + 1/T_{WO}} + \frac{(g/U_o - N'_p)}{N'_{\delta_r}}\right] p - \frac{N'_{\delta_a}}{N'_{\delta_r}} \delta_a \quad (2)$$

A pure gain $p \rightarrow \delta_r$ feedback will augment N'_p, but to augment N'_{δ_a} we attempt to reproduce δ_a (or $\dot{p}$) by putting lead in the $p \rightarrow \delta_r$ feedback. To offset the adverse effects of a $r \rightarrow \delta_r$ feedback, we need additional lead/lag or lag/lead equalization in the $p \rightarrow \delta_r$ feedback, where the lag breakpoint is at the same frequency as that in the $r \rightarrow \delta_r$ washout.

With the $\delta_a \rightarrow \delta_r$ crossfeed, a pure gain augments N'_{δ_a} and we attempt to reproduce p by putting δ_a through a suitable lag. If there is no $r \rightarrow \delta_r$ feedback, a lag/lead or lead/lag crossfeed should be adequate. With a $r \rightarrow \delta_r$ feedback, more complicated crossfeed equalization may be required for adequate turn coordination, see Eq. 2.

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*Equation 2 is derived by combining the conventional yaw equation, Eq. 1 , and the approximation, $r \doteq (g/U_o)\varphi$, and neglecting N'_r.

Theoretically either the $p \rightarrow \delta_r$ feedback, the $\delta_a \rightarrow \delta_r$ crossfeed, or a combination can greatly improve the turn coordination. With any of these schemes the primary practical problem is in programming the gains and equalization time constants for the various flight conditions and configurations. Consequently the final selection of a turn coordination system will be greatly influenced by the relative magnitudes of the scheduling problem in the particular application.

The block diagrams of Figs. 3 and 4 are typical versions of control systems which incorporate the essential features. The primary differences between these two mechanizations are in the crossfeed and in the dutch roll damping features. Either of these systems can be used as the basis for a parameter optimization procedure.

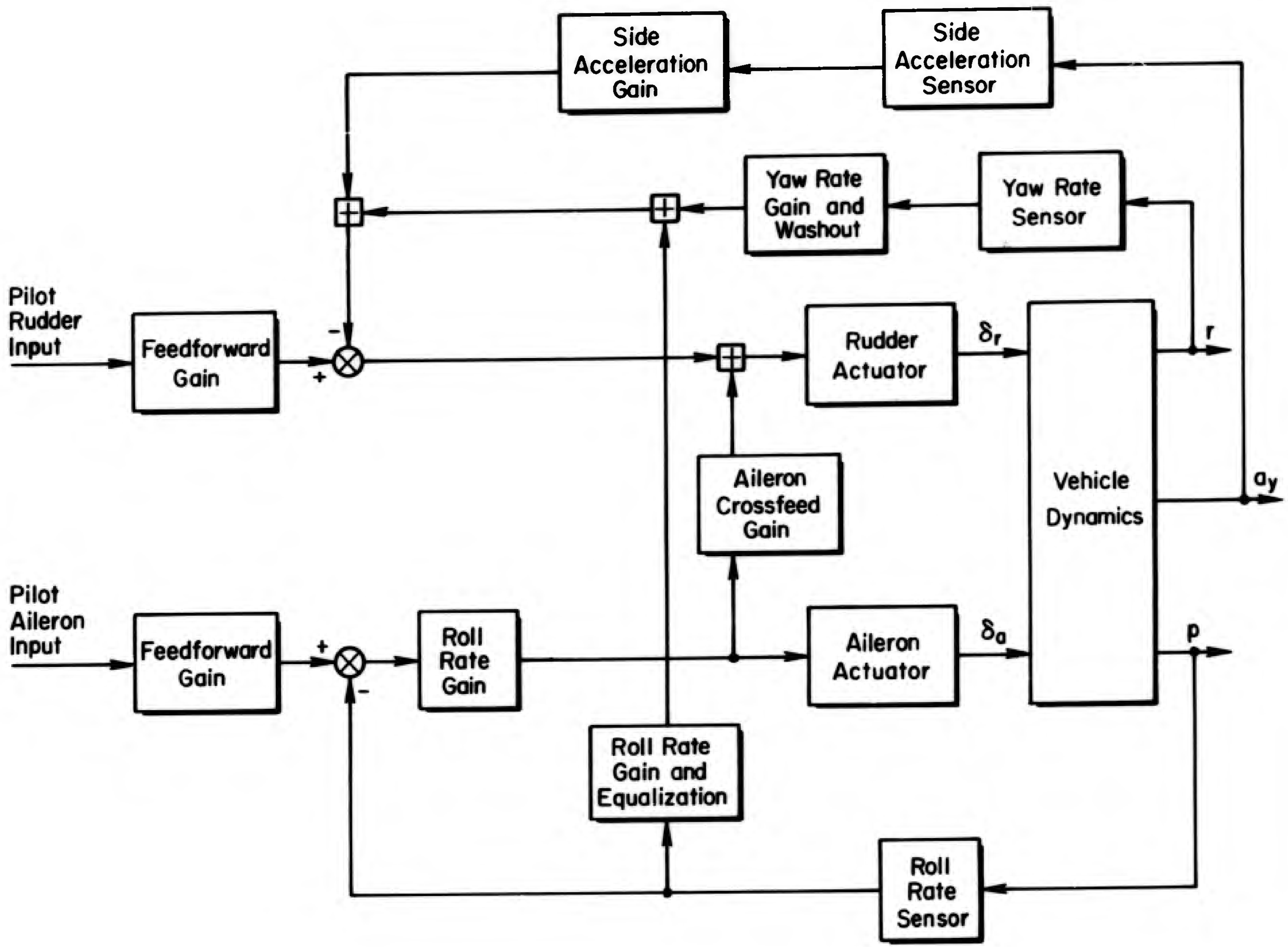

Figure 3. A Basic Mechanization of Essential Feedbacks; Acceleration Plus Yaw Rate

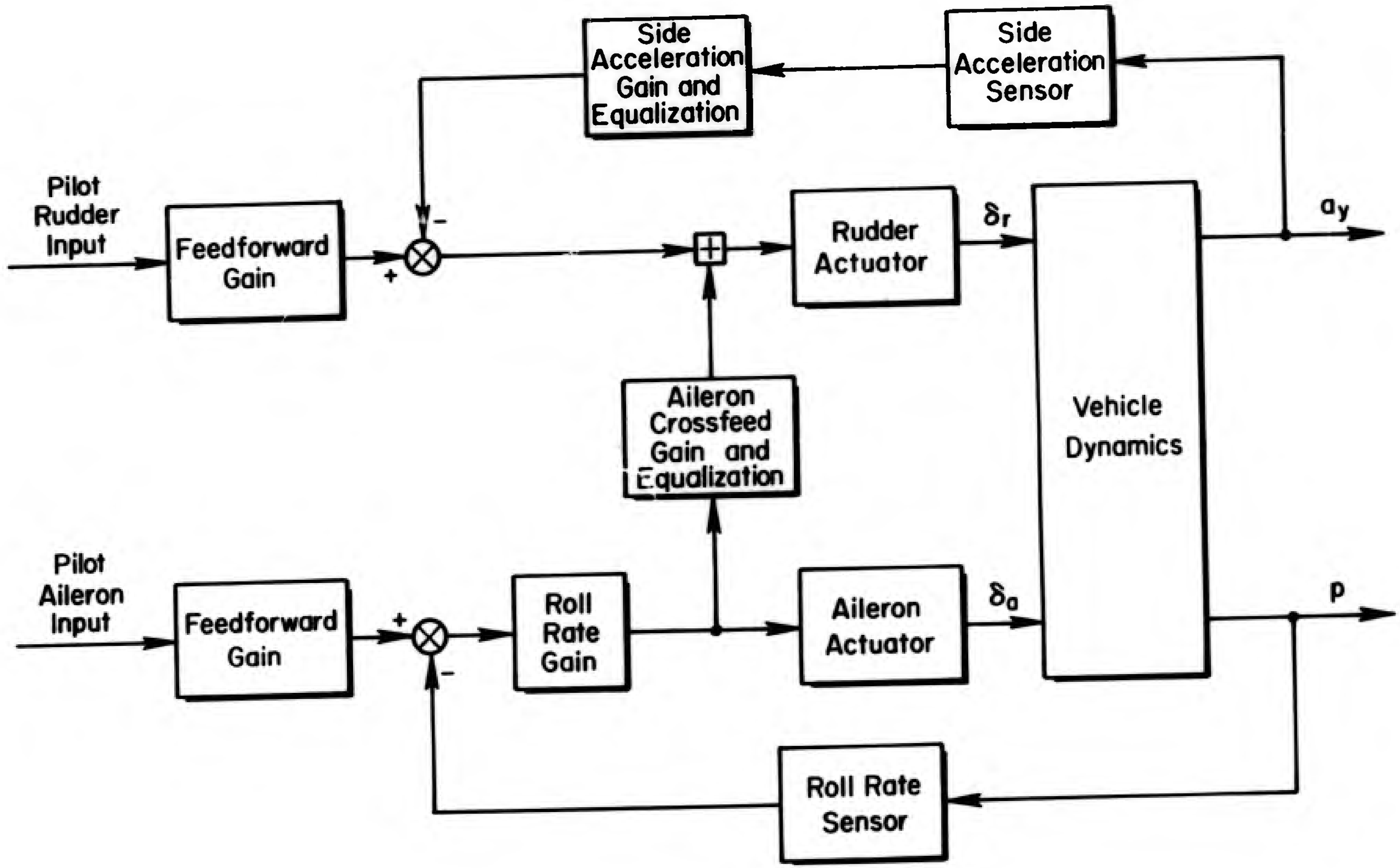

Figure 4. A Basic Mechanization of Essential Feedbacks; Equalized Acceleration

SECTION III

DESIGN CRITERION

A. GENERAL PROBLEM STRUCTURE

In general, a controller optimization problem can have the following elements (see Fig. 5):

- Plant and controller
- Inputs
- Model
- Error filters
- Cost function

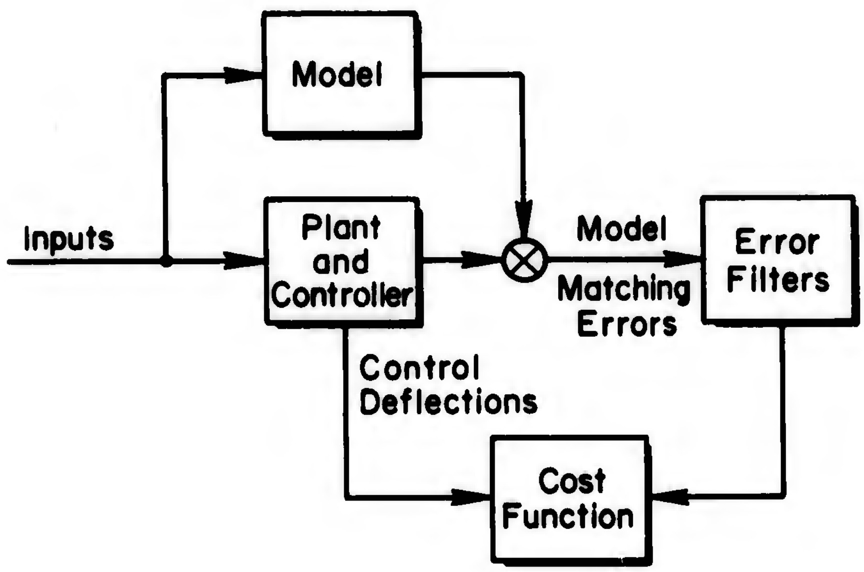

Figure 5 . General Problem Structure

The physical elements in this list are the inputs, which represent the control environment, and the plant and controller, which constitute the control system. The remaining elements, model, error filters, and cost function, together amount to a system specification; usually they are not physically in the final system realization. Our present objective is to specialize this general structure to one appropriate for a SAS design procedure. To gain insight in what this might be, we will start by considering the effects of changes in each of the various elements. These, hopefully, will point the way to a basic specific SAS design structure.

B. PLANT AND CONTROLLER

A key problem in the system description is the definition of the plant and controller. This includes, as a minimum, the aircraft and the SAS but may or may not incorporate the pilot and associated manual control feedbacks. To distinguish between the two possibilities hereafter, the term PIL (pilot in the loop) will be used when manual feedbacks are included and POOL (pilot out of the loop) when manual feedbacks are not present. The relative advantages and disadvantages of these two plant and controller structures are summarized in Table V and discussed below.

TABLE V

RELATIVE MERITS OF PIL AND POOL STRUCTURES

PLANT AND CONTROLLER STRUCTURE	ADVANTAGES	DISADVANTAGES
PIL (Pilot in the loop)	1. Can use simple models of pilot/aircraft dynamics 2. More directly compatible with basic pilot requirements	1. Complicates multi-case optimization program 2. SAS/aircraft combination can be unstable
POOL (Pilot out of the loop)	1. Fewer parameters to optimize 2. More directly compatible with current handling qualities specifications	1. Optimal model-matching system may be unacceptable 2. Can be overly conservative with regard to SAS/aircraft stability

One advantage of PIL is that simple model forms can be used in the specification elements. These can be as simple as unity gain for command inputs and zero gain for disturbance inputs. Because of the direct inclusion of the manual feedbacks in the problem, the model need only reflect the desired pilot/aircraft characteristics, such as good command following and disturbance suppression. On the other hand, with POOL we need a model of the SAS/aircraft dynamics, which if adequately matched, would permit satisfactory manual control. In multiloop tasks (e.g., path outer loop and attitude inner loop) appropriate models for each loop are required.

The other advantage of PIL is that it is more directly compatible with basic pilot requirements. These requirements are adequate command following and disturbance suppression without excessive pilot lead. The restriction on pilot lead can be included in the optimization program by limiting pilot lead to those values which have little effect on pilot ratings (roughly 1 sec).

The major disadvantage of PIL is that it complicates the multicase optimization program. The complication arises because different kinds of restrictions are imposed on SAS and pilot parameter variations between cases. Restrictions on the SAS parameters will be one of the following:

- Constant over all cases
- Varied according to a completely specified algorithm
- Varied according to an algorithm of specified form (numerical values of which are to be optimized)

By contrast, there can be no such restrictions on the pilot parameters. Instead, they should be optimized for each flight condition according to the adjustment rules of pilot/vehicle system analysis. These are basic behavioral characteristics of the pilot which, in a sense, satisfy his internally specified dynamic performance criteria.

There are two solutions to the practical problems of multicase optimization with PIL. One is to include in the list of parameters to be optimized as many sets of pilot variables as there are different cases to be considered. This approach, while relatively straightforward, can significantly increase the number of parameters to be optimized. The second solution is to use a two-step optimization program which operates as follows:

1. Initial estimates of pilot and SAS parameters are inserted as inputs for each case.

2. The program does a multicase optimization of the SAS parameters while holding the pilot parameters fixed.

3. The program optimizes the pilot parameters for each case separately.

4. Steps 2 and 3 are iterated until the overall optimal solution is obtained.

The other PIL disadvantage is only a potential problem and has not arisen in any examples to date. Since the PIL structure imposes no direct restraints on SAS/aircraft dynamics, theoretically the SAS/aircraft combination could be unstable, with the pilot closures providing a stable overall system. This problem can probably be avoided by putting certain restrictions on the SAS parameters.

Let us now consider the advantages of POOL. First, there are fewer parameters to be optimized as the pilot parameters are omitted. Second, this problem structure is more directly compatible with existing handling qualities specifications. All current specifications consider only SAS/aircraft dynamics and not pilot/SAS/aircraft characteristics.

The major problem with POOL is that the controller evolved from an optimal model-matching solution is frequently unacceptable. To illustrate this point, we have used this problem structure for the longitudinal augmentation of the F-4 at two different flight conditions. The results for both, and how these were unacceptable, are described below.

One flight condition investigated was for Mach 1.2 at an altitude of 5,000 ft. The airplane pitch/elevator response is, using the short-period approximation:

$$\frac{q}{\delta_e} = \frac{52.1(s + 1.54)}{s^2 + 2(0.29)(7.3)s + (7.3)^2} \qquad (3)$$

In terms of the current specification, the only handling qualities deficiency at this condition is a low short-period damping ratio. We therefore attempted to use the POOL structure to optimize a pitch damper. The problem formulation used is shown in Fig. 6. The model,

$$\frac{q_m}{\delta_{e_c}} = s \tag{4}$$

or

$$\frac{\theta_m}{\delta_{e_c}} = 1 \tag{5}$$

was selected because:

- Pure gain controlled elements are given very good pilot ratings.*
- The aircraft pitch attitude response is nearly pure gain in the frequency region of the normal pilot/aircraft system attitude control crossover frequency.
- The model magnitude approximates that of the aircraft in the region of pilot crossover frequencies.

Initially, the error filter was a bandpass type of form

$$Y_f = \frac{s}{(s+p_{f_1})(s+p_{f_2})} \tag{6}$$

This was done to emphasize the model-matching errors over the frequency range about the pilot/vehicle crossover frequency.

We expected the optimal solution would somewhat increase the short-period damping so that the model and aircraft q/δ_e magnitudes would be

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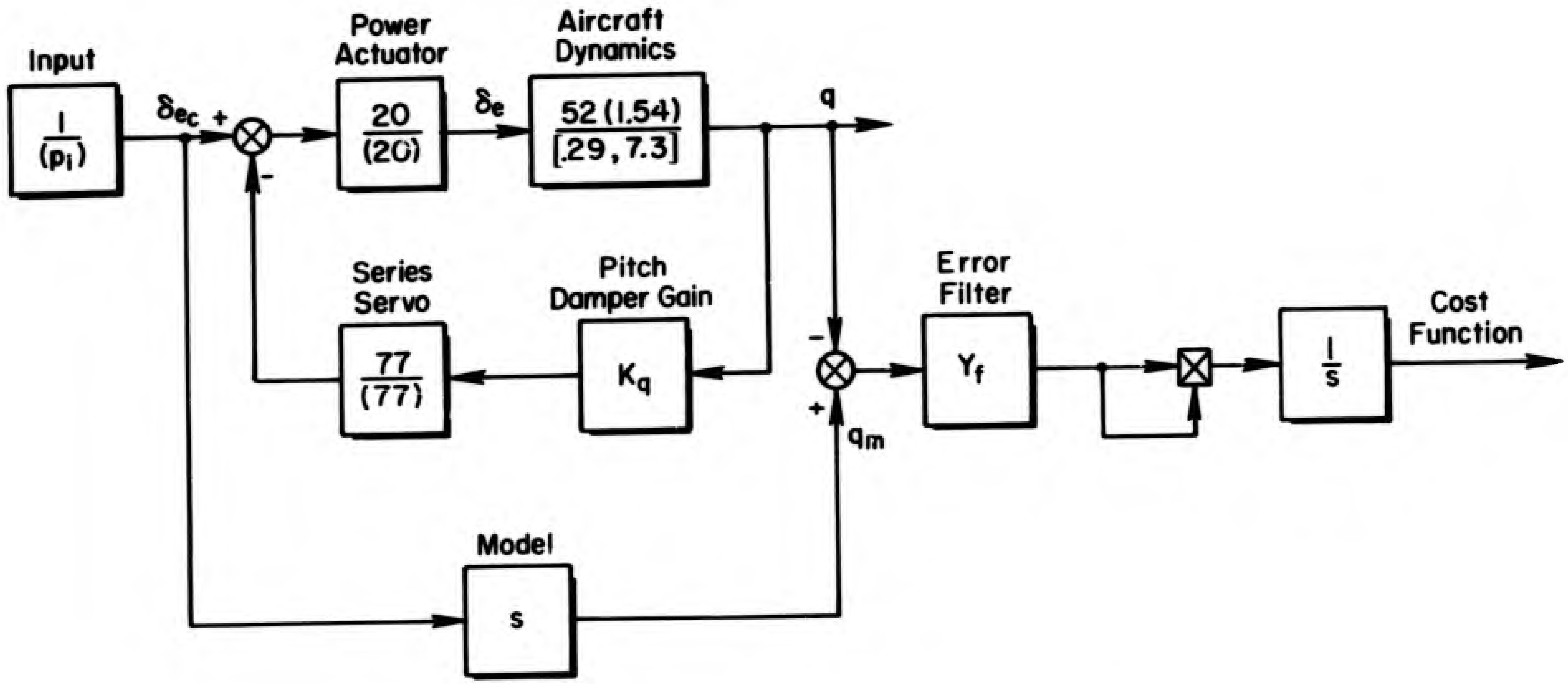
*A pure gain controlled element usually has slightly better pilot tracking performance and slightly worse pilot rating than a K/s controlled element. The pilot uses low frequency lag equalization but this does not significantly affect his rating.



(p) denotes (s+p) ; [ζ, ω] denotes [$s^2 + 2\zeta\omega s + \omega^2$]

Figure 6. POOL Example A

more nearly equal over the frequency range of interest. To our surprise, the optimal solutions for a wide range of inputs and filters gave extremely high pitch damper gains, $K_q \doteq 2$ sec, see Table VI. At this gain, the short-period frequency is greatly increased, to 41 rad/sec, and the damping ratio is extremely low, 0.012.

Later analysis uncovered the source of the problem. The cost function can be written as:

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{\omega^2 + p_i^2} |Y_f(j\omega)|^2 \left| \frac{q_m}{\delta_{e_c}}(j\omega) - \frac{q}{\delta_{e_c}}(j\omega)\right|^2 d\omega$$

At frequencies above 5 rad/sec the model response, $q_m/\delta_{e_c}(j\omega)$, and the aircraft response, $q/\delta_{e_c}(j\omega)$, are more than 90 deg out of phase. In this situation, the way to minimize the magnitude of the vector difference is to reduce the amplitude of the aircraft response as much as possible. The vehicle response can only increase the difference vector. Consequently, the optimal solution is to increase the feedback gain almost to the point of instability. The optimal solution is, thus, very sensitive to the model amplitude and phase characteristics relative to those of the aircraft.

The other flight condition for which we tried the POOL structure was Mach 0.7 at an altitude of 40,000 ft. The airplane pitch/elevator response is

$$\frac{q}{\delta_e} = \frac{4.16(s+0.275)}{s^2+2(0.472)(0.686)s+(0.686)^2} \tag{7}$$

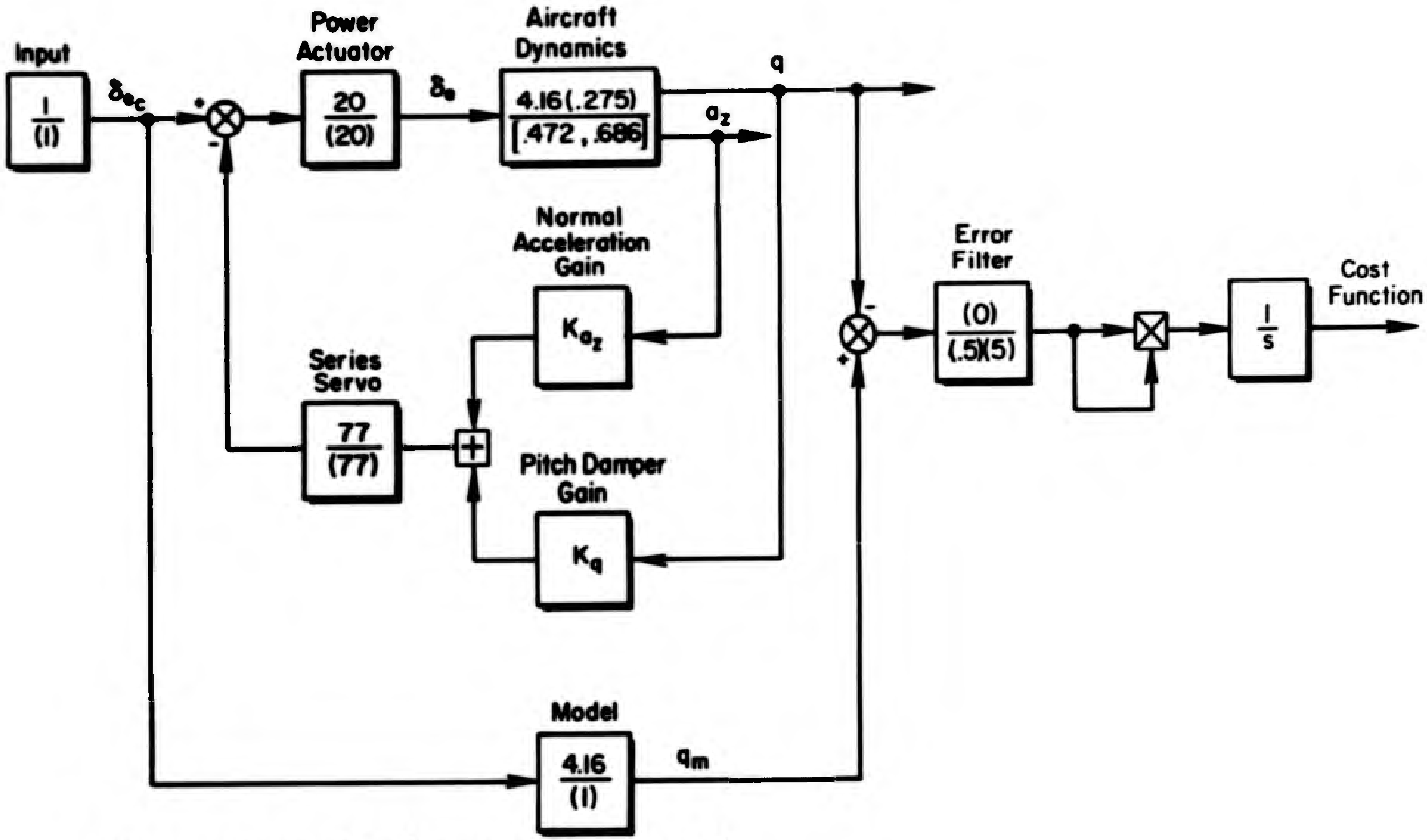
Because of the low short-period frequency, the selected augmentation used feedbacks of pitch rate and normal acceleration (measured at the center of rotation for elevator inputs) to the elevator (see Fig. 7). Because the short-period frequency is so low, the aircraft pitch response, θ/δ_e, looks like that of K/s^2 in the region of normal pilot/vehicle system crossover frequencies. Ideally, the augmentation should make the response more like K/s. However, it is not necessary to go all the way to K/s since controlled elements of $K/s(s+1)$ are considered adequate for good pilot ratings. Therefore, the model selected was

TABLE VI

POOL EXAMPLE A

P_i	Y_f	$(K_q)_{opt}$
10	$\frac{(0)}{(0.1)(10)}$	2.04
10	$\frac{(0)}{(0.2)(10)}$	2.04
10	$\frac{(0)}{(1)(10)}$	2.04
10	$\frac{(0)}{(2)(10)}$	2.04
10	$\frac{(0)}{(5)(10)}$	2.04
10	$\frac{(0)}{(10)(10)}$	2.03
1	$\frac{(0)}{(0.1)(10)}$	2.08
1	$\frac{(0)}{(1)(10)}$	2.06
1	$\frac{(0)}{(0.1)(1)}$	2.18
0.1	$\frac{(0)}{(0.1)(1)}$	2.20
2	$\frac{(0)}{(3)(3)}$	2.05
3	$\frac{(0)}{(3)(3)}$	2.05
0.1	$\frac{1}{(0.1)(10)}$	2.20
0.1	$\frac{1}{(0.1)(5)}$	2.20
0.1	$\frac{1}{(1)(100)}$	2.09
0.1	$\frac{1}{(2)(100)}$	2.07

Note: (p) denotes (s + p)



(p) denotes $(s+p)$; $[\zeta, \omega]$ denotes $[s^2 + 2\zeta\omega s + \omega^2]$

Figure 7. POOL Example B

$$\frac{\theta_m}{\delta_{e_c}} = \frac{4.16}{s(s+1)} \tag{8}$$

or

$$\frac{q_m}{\delta_{e_c}} = \frac{4.16}{(s+1)} \tag{9}$$

As before, the error filter was selected to emphasize the frequency region about pilot crossover.

The optimal solution for this case was quite surprising. The pitch rate gain was reasonable (0.154 sec), but the accelerometer gain had the wrong sign (K_{a_z} = 0.00047 rad/ft/sec^2) and was destabilizing. In fact, this accelerometer feedback reduces the short-period mode to two aperiodic modes, $(s+0.251)$ and $(s+0.421)$. The addition of the pitch rate feedback moves the poles to $(s+0.271)$ and $(s+1.090)$. As a result, the SAS/aircraft response

$$\left(\frac{q}{\delta_e}\right)'' = \frac{4.05\left(\frac{s}{0.275}+1\right)}{\left(\frac{s}{0.271}+1\right)\left(\frac{s}{1.090}+1\right)\left(\frac{s}{19.07}+1\right)} \tag{10}$$

is a very good match with the model; however, the augmentation is unacceptable.

The above examples clearly demonstrate that minimization of model-matching errors using the POOL structure can result in unacceptable SAS systems. To avoid this problem, one must use more complex models which represent fairly accurately the dynamics of the aircraft and a good augmentation system. To derive such a model for each flight condition is roughly equivalent to designing the SAS, so why bother with an optimization procedure?

The second disadvantage of POOL is the requirement that the SAS/aircraft system be stable. This is inherently necessary if the cost function integration is carried out over a long time interval. Thus, acceptably slow divergent spiral modes would not be tolerated. However, this problem can easily be circumvented. A fixed, low-gain feedback could be added to stabilize the spiral mode. The design procedure could then be applied to the augmented aircraft. Finally, the initial feedback could be deleted from the system.

After consideration of the above factors, it was decided that the PIL problem structure had considerably more promise than POOL for use in a practical SAS design procedure. The difficulties involved in specifying an adequate model for the POOL structure make that approach unattractive. Consequently, most of our effort on this project was devoted to using the PIL form and it is the one recommended in our proposed design procedure. The remainder of this report will consider only this form.

C. COST FUNCTION

A quadratic type cost function is:

$$J = \sum_i \int_0^\infty [q_i x_i(t)]^2 \, dt \tag{11}$$

where x_i is any pilot/SAS/aircraft response variable
q_i is the cost function weight on x_i

This is a particularly convenient form. With a linear constant coefficient system the cost function can be evaluated without having to integrate the equations of motion (see Appendix B). Other types of cost function can be used with parameter optimization and, if the optimization is done on a hybrid computer, a wide variety of cost functions could be applied with little difficulty. However, for most problems the quadratic form is adequate and computationally convenient, especially for digital optimization. For these reasons, only the quadratic form of Eq. 11 will be considered further.

Given the cost function form, one still has the problem of picking which response variables to include and their relative weights (the q_i's in Eq. 11). Clearly, the model-matching errors should be included, but are additional terms necessary? Several examples were run with only an error term in the cost function. In some cases this resulted in extremely large feedback gains. With a pitch damper, large gain makes the pitch/elevator response look like K/s out to high frequencies. Consequently, nice pilot closures are achieved and the error response at low frequencies is extremely small. However, the damping of the high frequency modes in these instances was obviously too low, e.g., damping ratio of 0.02 and frequency of 38 rad/sec.

A simple, straightforward method of restricting the feedback gains is to include control deflection in the cost function. When using the PIL problem structure, this approach would restrict both pilot and SAS gains, whereas we are only interested in restricting the SAS. Pilot gains are adequately limited by closed-loop stability because of fixed lag elements in the pilot describing function. Consequently, instead of using total control deflection in the cost function we used only the control deflection due to the SAS feedbacks.

With two terms in the cost function there is the problem of selecting the relative weights. For guidance in solving this problem we borrowed an idea from linear optimal control theory, Ref. 5. The basic idea is as follows:

- Given a model reference optimization where the plant output, x, is to follow the model for frequencies up to ω_1
- The relative cost function weights on control deflection, δ, and model-matching error, should be

$$\frac{q_\delta}{q_\epsilon} \doteq \left|\frac{x}{\delta}(j\omega_1)\right| \qquad (12)$$

This concept was employed to the problem at hand and the frequency, ω_1, was selected as approximately that of pilot/vehicle system crossover. Example applications indicated that a frequency of 4 rad/sec was generally satisfactory.

For longitudinal SAS design with manual response to a pitch command, the relative weights used were*

$$\frac{q_{\delta_{SAS}}}{q} \doteq \left| \frac{\theta}{\delta_e}(j4) \right| \tag{13}$$

For some situations, it will be more appropriate to consider manual closure of path loops, as well as attitude inner loops, and path commands. Then it would probably be better to use a frequency typical of manual crossover for a path loop, roughly 1 rad/sec.

D. INPUT CHARACTERISTICS

For all the examples which were run, the input command was assumed to be of the form

$$\theta_c = \frac{1}{s + p_i} \tag{14}$$

In selecting the value of p_i, Eq. 14 should be interpreted as the transient analog of a random input with power spectral shape (where k is an arbitrary constant)

$$\Phi_{\theta_c\theta_c}(\omega) = \frac{k}{\omega^2 + p_i^2} \tag{15}$$

rather than as a deterministic input. Viewed in this light, p_i should be set at the bandwidth of typical pitch commands. About 1 rad/sec is an appropriate order of magnitude for this. Much lower values were also considered for a number of cases, but the results were unsatisfactory, particularly for the Mach 0.7 at 40,000 ft example. In this case the bare airplane short period frequency was low and an accelerometer feedback was

*For simplicity, the magnitude of the $|\theta/\delta_e(j4)|$ asymptote was used rather than the actual value.

put in to augment it (plus pitch rate feedback for damping). With the low p_i's the optimal solutions always had too low an accelerometer gain, frequently of the wrong sign. In other words, with a low bandwidth input, the optimum solution was a low short period frequency. Therefore a value of 1 rad/sec was chosen for p_i.

For situations containing manual path control, it would probably be desirable to reduce the input frequency. Path command and disturbances are generally considerably lower in bandwidth than their attitude control equivalents. For these cases a value of roughly 0.5 rad/sec is suggested as a starting point.

E. MODEL DYNAMICS

For command inputs the simplest possible model is unity gain. Since this proved to be satisfactory in the example applications, it was selected. However, brief investigations were also conducted with two other models. One was

$$\left(\frac{\theta}{\theta_c}\right)_{model} = \frac{3}{s+3} \tag{16}$$

This is the closed-loop transfer function for

$$Y_pY_c = \frac{3}{s} \tag{17}$$

The other model was

$$\left(\frac{\theta}{\theta_c}\right)_{model} = \frac{3(-s+4.3)}{[s^2+2(0.18)(3.6)s+(3.6)^2]} \tag{18}$$

which resulted from closing the loop around

$$Y_pY_c = \frac{3}{s}\frac{(-s+4.3)}{(s+4.3)} \doteq \frac{3}{s}e^{-0.47s} \tag{19}$$

Neither of these gave better results than the simple unity gain model.

F. ERROR FILTER CHARACTERISTICS

The original reason for providing an error filter was to have a direct method of emphasizing certain frequencies. However, changing the input characteristics has approximately the same effect. In fact, if the cost function consists only of a model matching error, the input and filter dynamics are completely interchangeable, e.g., Fig. 6 or 7. By properly choosing the input dynamics and cost function, good results were obtained without an error filter.

G. PROPOSED PROBLEM STRUCTURE

The above paragraphs have discussed the various elements involved in the formulation of the SAS optimization problem. At this point we will summarize the proposed method of structuring the problem. This presentation will first consider situations in which the primary concern is manual control of attitude alone. Then extensions to manual path control will be discussed.

An example of manual attitude control is pitch control with the elevator. The proposed problem structure for this situation is shown in Fig. 8 for the case of only SAS feedbacks to the elevator. In Section IV this form is applied to the F-4 at three flight conditions. Note that because the model is assumed to be unity gain, the model matching error is the same as the manual tracking error, θ_e.

The form of the pilot describing function, Y_p, is not shown in Fig. 8 . The recommended form is

$$Y_p = \frac{K_p}{0.1s+1}\left(\frac{s-16}{s+16}\right)^2 = \frac{10K_p}{(s+10)}\left(\frac{s-16}{s+16}\right)^2 \qquad (20)$$

The fixed elements represent typical pilot lags. The poles and zeros at 16 are an approximation to a 0.25 sec time delay. The gain, K_p, is a parameter to be optimized along with the SAS parameters.

It should be noted that the pilot model of Eq. 20 has no lead term. Lead was omitted primarily because we found this improved the SAS

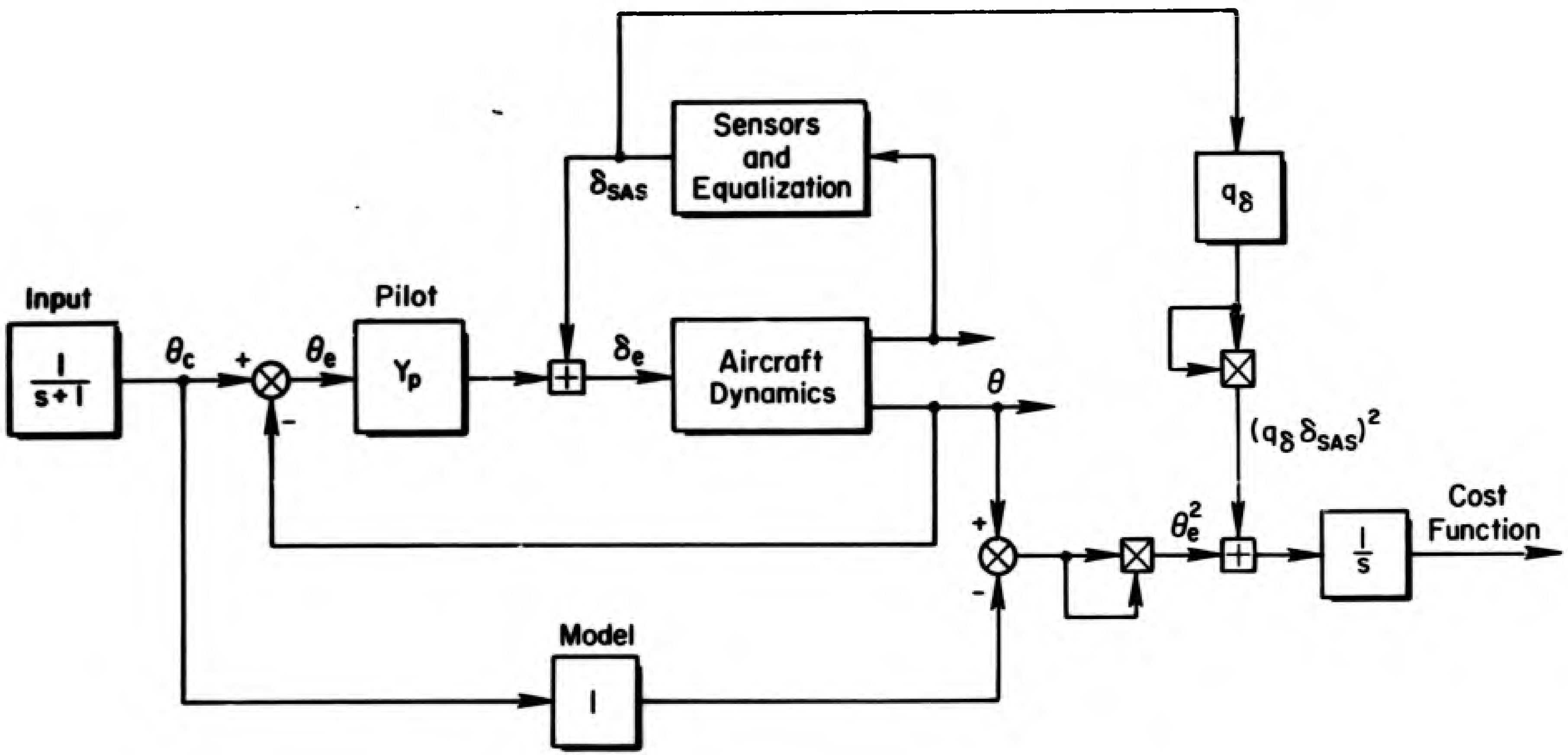


Figure 8. Proposed Problem Structure for Manual Pitch Control

characteristics in our example applications. In particular, including pilot lead led to a problem with the normal acceleration feedback to the elevator. In the low dynamic pressure examples, this feedback is needed to augment the short-period frequency. However, when lead was included in the pilot model the acceleration gain was too low and frequently had the wrong sign. Removing the lead in the pilot model allowed adequate augmentation of the short-period frequency. The pilot model of Eq. 20 is also desirable for another reason. Since lead degrades pilot ratings, we would like to optimize pilot/aircraft performance without the pilot having to use lead.

The method of selecting the relative cost function weight on SAS control, q_{δ}, was described in subsection B. The weight is the magnitude of the pitch/elevator transfer function asymptote at 4 rad/sec.

In some situations, such as landing approach, it may be necessary to consider manual path control which is generally a multiloop task. Although we have not actually tried a problem of this type, we can estimate the modifications which would be required. First, the manual feedback structure would have to be expanded to include the outer path loop. The error term in the cost function should be changed to the outer loop or path error and the relative weight on SAS control changed accordingly. The relative weight on SAS control should be the path/elevator transfer function at roughly 1 rad/sec. Finally, it would probably be desirable to change the input characteristics. Path inputs and disturbances are generally lower bandwidth than attitude ones. Accordingly, an input time constant on the order of 2 sec, rather than 1 sec, is more appropriate.

SECTION IV

APPLICATION EXAMPLES

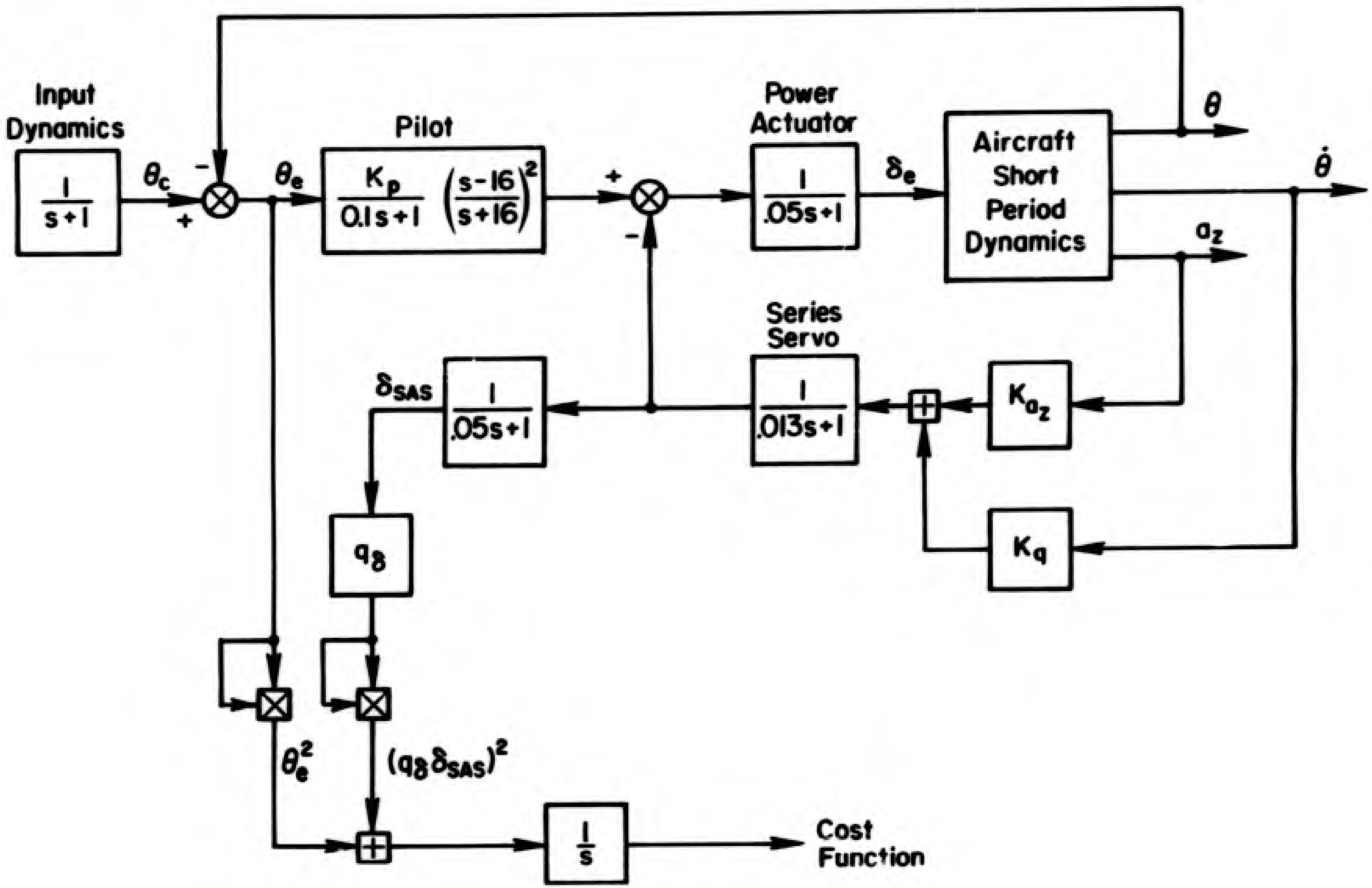
A. EXAMPLE PROBLEM

For an example application, the proposed design procedure was applied to the longitudinal axis of the F-4 airplane. Three different flight conditions were considered in the analysis. These correspond to a very low short-period frequency, a moderate value, and a relatively high frequency. The basic objectives of a SAS would be to augment the short-period frequency when it is low and to increase the short-period damping for the high-frequency case. To accomplish this the SAS was configured as normal acceleration and pitch rate feedbacks to the elevator. The accelerometer in all three cases was located at or very near the center of rotation for elevator inputs.

The complete structure of the resulting optimization problem is shown in Fig. 9. The actuator dynamics are the same as those in the current F-4, Ref. 6. The vehicle stability derivatives were also taken from Ref. 6 and are listed in Table VII. The table also indicates the relative cost of control, q_δ, which was used in the optimization. This parameter was set equal to the value of the pitch/elevator transfer function asymptote at 4 rad/sec. The results of the optimization are indicated in the next subsection.

B. RESULTS

The results of the three optimizations are summarized in Table VIII. Note that for the two subsonic flight conditions the feedbacks considerably augment the short-period frequency and damping. For the high dynamic pressure case the short-period frequency is already quite large and the optimal accelerometer gain is very low. In fact, this feedback has little effect on the short-period mode. The pitch rate feedback increases the damping and also significantly increases the frequency. The reason the pitch rate feedback increases the frequency is because of the significant actuator lags. A root locus plot of this feedback is shown in Fig. 10. Feedback gains higher than the optimal solution would increase the short-period



Free Parameters (to be optimized): K_{a_z}, K_q, K_p

Figure 9. Problem Structure for Examples

TABLE VII

EXAMPLE LONGITUDINAL CHARACTERISTICS

M	0.5	0.7	1.2
h (10^3 ft)	5	40	5
U_o (ft/sec)	549	678	1317
Z_w (sec^{-1})	−0.722	−0.278	−1.700
Z_{δ_e} (ft/sec^2/rad)	46.0	17.56	221
M_α (sec^{-2})	−3.78	−0.40	−50.2
$M_{\dot{\alpha}}$ (sec^{-1})	−0.285	−0.115	−0.670
M_q (sec^{-1})	−0.656	−0.255	−1.845
M_{δ_e} (sec^{-2})	10.84	4.16	52.1
$1/T_{\theta_2}$ (sec^{-1})	0.694	0.276	1.542
ζ_{sp}	0.403	0.472	0.289
ω_{sp} (rad/sec)	2.06	0.686	7.30
Center of rotation $Z_{\delta_e}/M_{\delta_e}$ (ft)	4.24	4.22	4.24
Accelerometer location x_a (ft)	4.39	4.07	4.24
Cost of control, q_δ	0.68	0.26	0.975

damping but would also significantly increase the frequency, which is already quite high. The optimal solution is apparently a reasonable compromise between increasing the damping and not overly increasing the frequency.

Step responses for the aircraft with and without the SAS are shown in Fig. 11. In all cases the input is a pilot step input of 1 deg of elevator. For the subsonic flight conditions, the SAS produces a

TABLE VIII

EXAMPLE OPTIMIZATION RESULTS

M	0.5	0.7	1.2
h (10^3 ft)	5	40	5
Accelerometer gain, K_{a_z} (rad/ft/sec^2)	−0.001168	−0.00703	-0.469×10^{-4}
Pitch rate gain, K_q (sec)	0.210	0.794	0.0636
Pilot gain, K_p	0.399	0.839	0.538
[ζ_{sp}, ω_{sp}]	[0.403, 2.06]	[0.472, 0.686]	[0.289, 7.30]
Closed-loop poles,* accelerometer loop closed	[0.224, 2.99](20.4)(76.9)	[0.060, 2.42](20.3)(77.1)	[0.256, 7.56](20.4)(77.0)
Closed-loop poles,* both SAS loops closed	[0.569, 3.54](16.9)(77.7)	[0.752, 3.00](14.9)(78.2)	[0.422, 8.96](15.6)(78.1)
Closed-loop poles,* manual and SAS loops closed	(0.224)[0.290, 3.28][0.827, 11.9] [0.975, 21.1](77.6)	(0.099)[0.376, 2.93][0.824, 11.7] [0.975, 20.4](78.1)	(0.571)[0.256, 5.76][0.549, 13.9] [0.952, 24.3](78.0)

*(p) denotes (s + p); [ζ, ω] denotes [$s^2 + 2\zeta\omega s + \omega^2$]

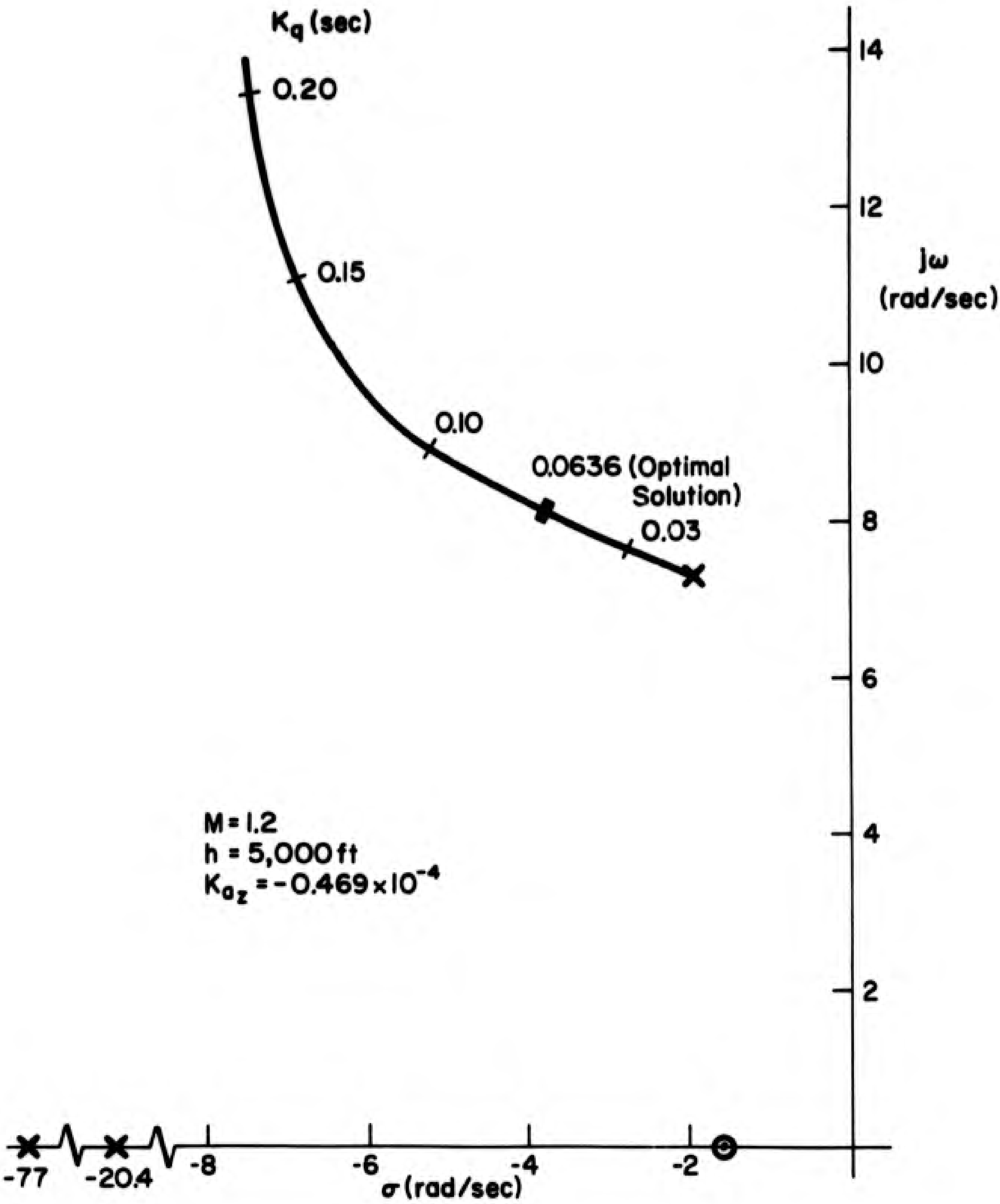


Figure 10. Pitch Damper Root Locus

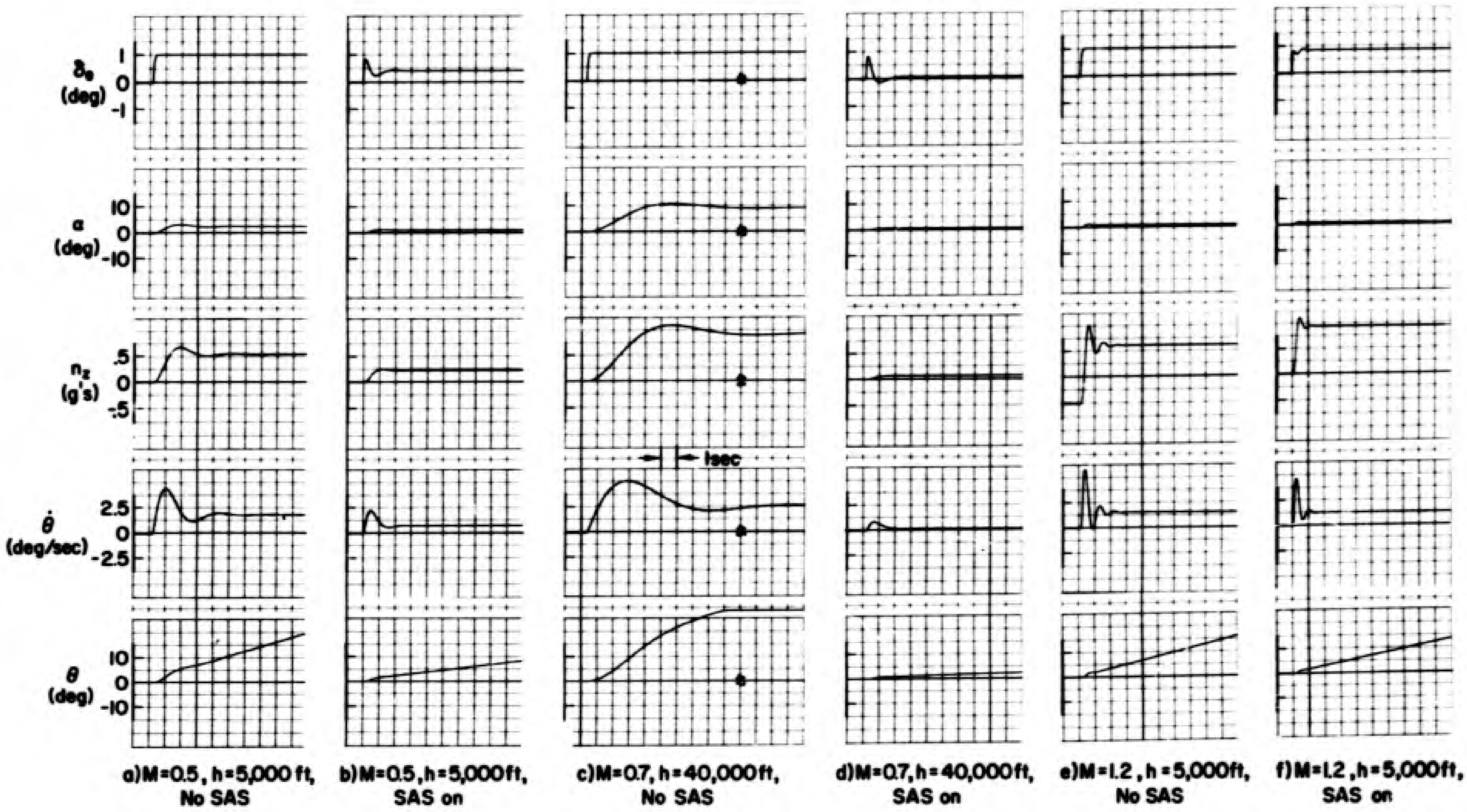


Figure 11. Step Responses

significant increase in frequency of response and damping. There is also a substantial reduction in response amplitudes because the SAS gains are in feedback paths, Fig. 9. This effect can be counteracted by adding a feedforward gain on the pilot inputs. For the supersonic condition the most noticeable effect of the SAS is the increase in damping.

A subjective judgment of the optimal solutions for these three cases is that the systems are at least reasonable and probably quite good. A comparison of the SAS/airplane characteristics with handling qualities specifications is given in the next subsection.

While the optimal solutions for the SAS feedbacks appear to be good, the results for the manual feedback are somewhat unusual. Asymptotic Bode plots of the manual loop closures are shown in Fig. 12. Admittedly these closures are not the type one would normally make in a pilot-vehicle analysis. The most drastic departures from the normal pilot model adjustment rules are the very low crossover frequencies. During our early investigations we did run across some cases in which the manual closures conform to the adjustment rules, that is, reasonably high crossover frequencies and K/s-like characteristics in the region of crossover. However, in all of these situations the SAS characteristics were very poor. In all cases the accelerometer feedback had the wrong sign, such that it reduced the short-period frequency rather than increased it, and, in addition, with both SAS loops closed the short-period mode was either overdamped or very nearly so. The objective of this project was to develop a SAS design procedure, not a technique for automated pilot-vehicle analysis. Therefore, given the choice between a problem formulation which gave good results on the SAS feedbacks or one which gave good results on the manual feedbacks, the choice was clear. The proposed procedure apparently has merit in terms of determining SAS characteristics even though the associated manual feedbacks are not realistic.

C. COMPARISON WITH EXISTING HANDLING QUALITIES SPECIFICATIONS

The optimal solutions were compared with two existing handling qualities specifications — the current military specification, Ref. 7, and the C*

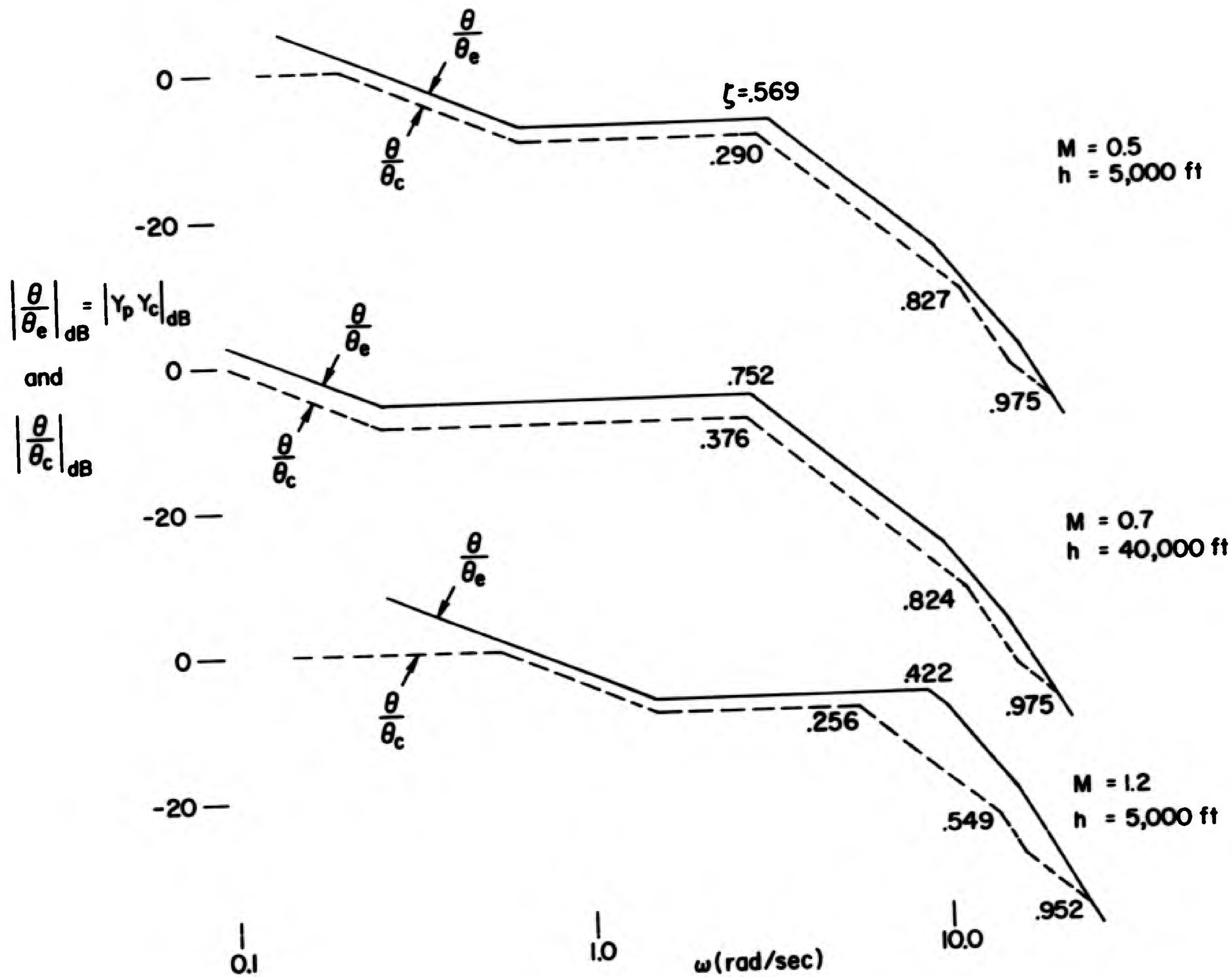


Figure 12. Manual Loop Closure Examples
(SAS Loops Closed)

criterion, Ref. 8. The comparison of the optimal results with the short-period requirements of the current military specification is shown in Table IX. The results are well within the specification limits.

TABLE IX

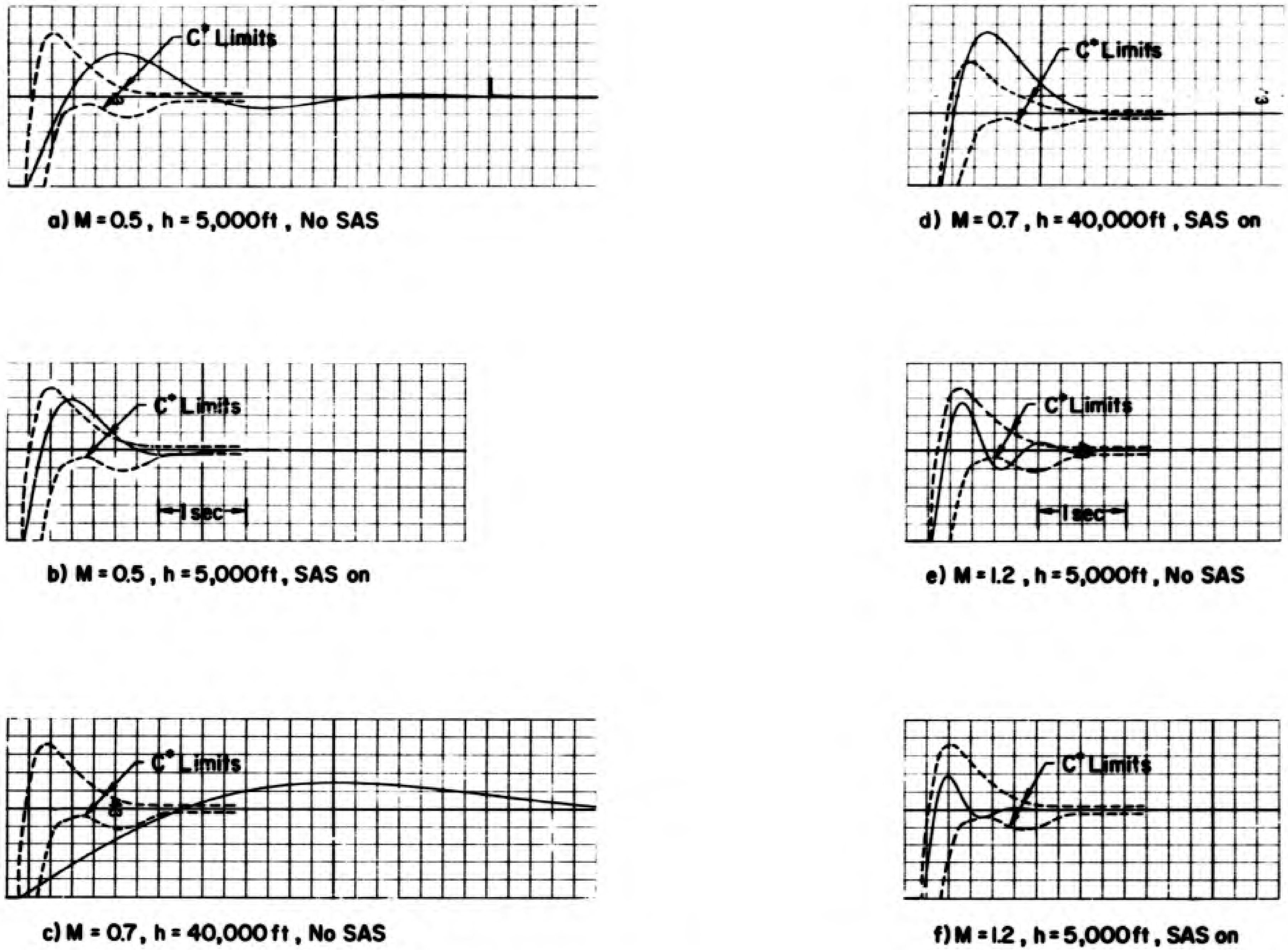
COMPARISON OF OPTIMIZATION RESULTS WITH MILITARY SPECIFICATION

M	0.5	0.7	1.2
h (10^3 ft)	5	40	5
n/α (g's/rad)	11.75	5.79	62.5
ω_{sp} (rad/sec)			
Airplane alone	2.06	0.686	7.30
Airplane + SAS	3.54	3.00	8.96
Spec* limits	1.81 — 6.50	1.27 — 4.57	4.18 — 15.0
ζ_{sp}			
Airplane alone	0.403	0.472	0.289
Airplane + SAS	0.569	0.752	0.422
Spec* limits	0.35 — 1.30	0.35 — 1.30	0.35 — 1.30

*Reference 7 requirements, Category A, Level 1.

The C* responses for the aircraft with and without the SAS are compared with the Ref. 8 limits in Fig. 13. For the two subsonic cases without the SAS, the limits are greatly exceeded because the short-period frequency is much too low. Supersonically, the limits are slightly exceeded because of inadequate damping.

Even with the SAS on, the M = 0.5 case exceeds the limits somewhat. The M = 0.7 case severely overshoots the boundaries. This is attributed to the low value of $1/T_{\theta_2}$ (0.276) which results in a very large pitch rate overshoot, see Fig. 11d. Because of this effect the C* criterion



a) M = 0.5, h = 5,000ft, No SAS

b) M = 0.5, h = 5,000ft, SAS on

c) M = 0.7, h = 40,000ft, No SAS

d) M = 0.7, h = 40,000ft, SAS on

e) M = 1.2, h = 5,000ft, No SAS

f) M = 1.2, h = 5,000ft, SAS on

Figure 13. C* Comparisons

cannot be met with elevator feedbacks alone. To do so requires use of a direct lift device which could augment $1/T_{\theta_2}$. For the M = 1.2 condition, the SAS brings the response just within the boundaries.

The fact that the optimal solutions do not meet the C* criterion is not really too significant because this criterion is overly conservative. To quote from Ref. 9, "The only point that can be made regarding these time history comparisons is to say that configurations satisfying the boundaries were indeed well-rated. However, some which exceeded the boundaries were equally well-rated." Failure to comply with the C* criterion should not be regarded as a deficiency of the proposed design procedure.

SECTION V

SUMMARY

A research program to utilize recent advances in pilot/vehicle analysis and modern control theory to provide a systematic SAS design procedure was conducted. Such a design procedure was developed and partially tested. It uses parameter optimization which, with the current state of the art, was judged superior to linear optimal control theory in this application.

The proposed design procedure has three steps:

1. Selection of essential feedbacks
2. Single case optimization
3. Multicase optimization

The selection of essential feedbacks is done by examining several key handling quality metrics. A discussion of these metrics and practical methods of modifying them by control crossfeeds or feedbacks is presented.

The single case optimizations are done for each of several combinations of flight condition and vehicle configuration. These results are used to estimate gain scheduling requirements and possible SAS modifications (e.g., necessity to add equalization). A discussion of the effects of the various elements in the optimization problem (such as the cost function) is presented and a definite problem formulation is proposed.

The final step, multicase optimization, involves optimizing over several combinations of flight condition and vehicle configurations. The optimization parameters would include the parameters in a gain scheduling algorithm (of prespecified form).

An example application of the proposed SAS design procedure through the second step, single case optimization, is presented to demonstrate the feasibility of this approach. Single case optimizations were performed for a longitudinal SAS for the F-4 aircraft at three very different flight conditions. All three used the same SAS form (pitch rate and normal acceleration feedback to the elevator), the identical problem formulation, and the same procedure for determining cost function weights. The resulting systems were judged quite satisfactory and, in fact, were well within the short-period requirements of the military handling qualities specification, Ref. 7.

While the success achieved for these simple examples is highly encouraging, we recognize the likelihood of some unforeseen difficulties in other situations. A complete substantiation of the procedure will require applying it to:

1. Problems where manual outer loop, path control is important, such as landing approach

2. Situations requiring SAS feedbacks to more than just the elevator, such as to the throttle and direct lift devices

3. Lateral/directional SAS design

In addition, the multicase optimization step should be tested.

Successful implementation of this design procedure could greatly reduce the required time and cost for SAS design. It should also result in better SAS designs and thereby better handling qualities for future aircraft. With this great potential, additional research on this design procedure is strongly recommended, especially as regards the items mentioned in the previous paragraph.

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APPENDIX A

OPTIMIZATION PROCEDURES

The basic purpose of this appendix is to review two different optimization procedures with regard to their applicability to flight control system (FCS) design. One procedure is based on conventional linear optimal control theory. This is referred to as free-controller optimization because the form of the controller is not restricted in any way and the optimal solution depends only on the given inputs, the controlled element characteristics, and the performance criterion. The other procedure is parameter optimization in which the controller form is prespecified and the controller parameters are optimized to minimize the performance criterion. In the limiting case where the prespecified form has the equivalent of all state variables fed back and a quadratic performance criterion is used, the results of the two forms of optimization merge, although the computational procedures remain different.

The remainder of this appendix considers each procedure in turn. Their advantages and disadvantages relative to the FCS design problem are discussed.

FREE-CONTROLLER OPTIMIZATION

Theory relevant to free-controller optimization has been burgeoning forth for more than a decade. The reason this subject receives so much attention is, of course, because it deals with selection of absolutes from within some restricted class. When a flight control system analyst is faced with a design problem, the considerable appeal of obtaining a unique, absolutely best design for a practical system often brings free-controller optimization to the fore. However, the word "practical" may assume much more significance in the overall scheme of things than was originally appreciated. The result is that free-controller optimization is often abandoned (perhaps even after it makes some contribution to the problem solution) for the more routine fixed-form optimization techniques, design through repeated analysis, comparative simulation, and the like.

Why is this the case? The nub of the matter in connection with aircraft flight control revolves around how far toward the ultimate goal state-of-the-art free-controller optimization techniques can carry the design process. It is a matter of historical fact that free-controller optimization has generally not been eminently successful when applied to aircraft flight control problems. There are some apparent exceptions, e.g., Refs. 10-12. But close inspection of these references reveals that their effectiveness in the application of the theory is largely because of the engineering acumen of the authors in applying the traditional skills of flight control art rather than because of the "magic" of free-controller optimization. Under these circumstances the free-controller optimization method is in actual fact employed as an alternative computational tool to some less esoteric computation tool.

Need this necessarily be the case? Our view of the answer is, "No." But we must hasten to add here that our efforts to make enlightened use of free-controller optimization without losing the "magic" have taught us much about its advantages and shortcomings. The following suggestions are the positive manifestations of our major criticisms.

- Physically meaningful performance indices must be used to retain the "magic" of free-controller optimization
- Realization of the optimal control from the given measurements with minimum order compensation is essential for practical implementation
- Consideration must be given to the fact that multiple uncorrelated inputs excite the closed-loop pilot/vehicle/automatic control/kinematics system
- Features known to be necessary for practical flight control systems must be produced. For example, washout for yaw rate-to-rudder feedback, or the washouts and/or integral bypass needed for windproofing in the landing approach.

Some reflection on these suggestions reveals that they can be largely met by the current state-of-the-art in free-controller optimization. It is somewhat strange therefore that this existing technology has not been fully utilized. However there are also some very important limitations of the current art which we shall explore later on.

The above preamble now sets the stage for some critical evaluations of the free-controller optimization techniques. We shall proceed by asking several questions and then consider the answer to each question in turn.

- What are the advantages and disadvantages of applying free-controller optimization to flight control problems?
- What are the characteristics of the flight control problem we wish to solve?
- What are the major approaches to free-controller optimization?
- Can fully manual, manual plus SAS, manual plus fly-by-wire, manual plus flight director, and fully automatic systems all be treated with equal effectiveness using free-controller cptimization?

Of course, the first question cannot really be answered until one has considered the other three questions. However, it will be discussed first to present an overall view of the problem. Consequently, the justifications for some of the remarks in the discussion on advantages and disadvantages of free-controller optimization will be given later.

Advantages and Disadvantages of Free-Controller Optimization

The major advantages of the free-controller optimization approach are given in the next few paragraphs. These are followed by additional paragraphs discussing the disadvantages.

Free-controller optimization provides the true optimal solution with respect to the performance index used. This solution is restricted only by the controlled element characteristics. In other words, the solution is independent of any controller restrictions.

For multiloop, and in particular for multicontrol-point problems, this approach implicitly provides simultaneous closure of all feedback loops and guarantees that the overall system will be stable.

There is no theoretical limit to the degree of controlled element complexity that can be handled by free-controller optimization methods.

When conventional methods are used the designer's mind is the first to bog down. With free-controller optimization the key limiting problem is a numerical one connected with the controlled element eigenvalue spread and the scaling of variables. Proper scaling can easily prevent this from being a limitation for the problem of interest.

Frequency domain techniques may be used. This means that many of the very powerful and familiar techniques can be used to understand, interpret, simplify the problem and to expedite computation. These techniques include root locus, Bode plots and multiloop analysis for the generation of root square locus plots and establishing the weights to be used in the performance index. Multiloop analysis and the equivalent systems concept can be used for aiding understanding and computation. For example, the performance index weights can be selected by requiring that the crossover frequencies in the root square locus be somewhat greater than the bandwidth of the model which is to be matched.

The sequential nature of the optimal control calculations can be advantageous in that it is possible to change portions of the problem without having to redo all the calculations. Let us assume the problem is structured according to the form discussed in Ref. 13; then the first step in the calculations, the root square locus, depends only on the airplane dynamics, the weights in the cost function, and the frequency weighting filters. The model dynamics only enter into the problem at the second step in the solution, that is, the solution of the Weiner-Hopf equations for the optimal control vectors. The final step is the determination of the controller. Various controller configurations can be evaluated without having to repeat the calculations from the first two steps.

Next consider the disadvantages.

The free-controller optimization techniques which are most fully developed do not lend themselves to the synthesis of systems which are optimal over multiple flight conditions.

Many real considerations cannot be adequately expressed in a form compatible with the theory. For example, nonlinearities such as inertial cross-coupling, control limits, aircraft stall and other nonlinear aerodynamic effects cannot be included.

There is no really effective way to impose constraints on some of the controller parameters. This is sometimes necessary for practical reasons. For example, a gain limitation might be necessary in order to keep structural feedback within reason.

A host of problems exist which all center about the sensitivity of the optimal design to off-nominal conditions. First off, it is possible in principle to introduce a cost for sensitivity into the problem formulation. However, this so greatly complicates the problem that it is essentially infeasible as a practical matter. A key problem is finding the minimal order optimal controller once the optimal control has been determined. It often turns out that system performance is very sensitive to parameter deviations from nominal when these minimal order controllers are used. Presently there is no alternative to crossed fingers for treating this problem. Last, it is sometimes necessary to approximate the optimal controller transfer functions. Very often this "practicalization" has disasterous effects on system performance and stability. Again, there is no alternative to just trying the practicalization and hoping. In connection with this, we should emphasize that the final design is rendered suboptimal in two ways. First, practicalization will cause the performance index to be greater than the absolute minimum determined for the ideal solution. Second, the practicalized controller which evolves is not even optimal for the number of parameters it contains.

It turns out to be very difficult to induce certain features in optimal controllers which are known to be very important in practice. These include the washout required in yaw rate-to-rudder feedback to prevent the rudder from fighting intentional steady turn rates, and the washouts and/or integral bypasses which are necessary in automatic landing approach systems. The reasons that these features are "difficult to induce" is painfully clear. First, the input and disturbance environment is inadequately modelled in the problem formulation. Second, the errors and other variables in the performance index do not represent the problem goals to the extent we often believe they do.

The inclusion of sensor and actuator dynamics greatly complicates the problem of determining the minimum order controller. The minimum order of compensation is increased, sometimes drastically because the plant order is greatly increased.

The Flight Control Problem

The flight control problem of interest is of the following form:

Given: A linear constant coefficient plant (possibly at several different operating points), a model of desired performance characteristics, a representation of typical system inputs, and a cost function (i.e., a performance index).

Find: The optimal controller generating the optimal control from the available measurements.

A key distinction in this problem statement is that it is the optimal controller rather than the optimal control which is the ultimate goal of the solution. Lack of a complete theory for obtaining the minimum controller(s) for generating the optimal control from the given measurements is currently one of the main impediments to the successful application of optimal control to practical flight control problems.

If free-controller optimization techniques are used, it is a virtual necessity to use an integral quadratic form as the performace index from the outset or to recast the problem into this form as is done in Ref. 14. This is because the theory pertaining to other, perhaps more general, forms is in so primitive a state in comparison to the linear-quadratic form as to be nearly useless for the flight control problem.

Another matter of importance is the multiplicity of flight conditions or operating points which characterize the flight control problem. In view of the current state-of-the-art in free-controller optimization, there are three ways to cope with this aspect of the problem. These are:

- Assume the plant parameters are either stochastically varying quantities or are constants picked at random from given probability distributions.
- Assume the plant is deterministic, but that the parameters vary with time in a way which is characteristic of a pseudo-mission covering the flight envelope.

- Assume the plant has constant coefficients; design the optimal controller at each of several flight conditions. Use engineering methods to determine how controller parameters should be scheduled with flight condition.

Each method has merit but the last method is our preference for the reasons which will be developed below.

Applicable Free-Controller Optimization Approaches

The theory for determining the optimal controllers for stochastic plants is the least mature of the candidates. Its application often gives rise to difficult mathematical and/or computational questions, especially when plants as complex as the multicontrol-point dynamics of an aircraft are considered. There is no aspect of the theory treating minimum complexity controllers. Also, the sticky matter of the proper representation of plant parameter covariances is usually avoided by assumption. This assumption should not be made for flight control applications because the resulting controller designs would be overly conservative in order to cope with stability problems that could not actually exist. Three elementary examples of this approach are given in Section V of Ref. 15. A more sophisticated theoretical treatment is given for a nearly trivial problem of the same class in Chapter III, Section 3b of Ref. 16. These examples tend to indicate that the flight control problem of interest is so complex in comparison to these examples that optimization by means of these techniques is infeasible as a practical matter.

When the plant is assumed to be time-varying but deterministic, matters are much better, especially if the cost function is a quadratic form. It is possible to compute the optimal state variable feedback gains using time domain techniques developed by Kalman (e.g., Ref. 17) and implemented [sometimes with difficulty — see Vaughn, Ref. 18, Fath, Ref. 19] by the automatic synthesis program, ASP, (Ref. 20) or the more recent Fortran IV version FASP (Ref. 21). These provide a solution to the problem in terms of state variable feedback. However, this is only the initial step towards the solution because many states of the system will not be available for feedback. These so-called "missing" states must be estimated in the most efficient way possible. The measurements available are of reasonably high quality in flight control with the result that Kalman filtering (Ref. 17) for state estimation does not qualify as efficient. A very recent development does qualify, however. This is an extension of the Luenberger observer theory for constant coefficient systems (Ref. 22) to time-varying systems by Ferguson and Rekasius (Ref. 23). Both of these observer theories lead to minimal order controllers for generating each component of the optimal control vector considered one at a time using the available measurements. At present, however, no theory exists for determining the overall minimal order controller for generating the optimal control vector from the given measurements. There is at this time a theory for a "reduced order" controller when the problem is constant coefficient (Ref. 24). This, and the fact that the designer tends to lose touch with the physics and numerical intricacies of the problem when time domain techniques are used recommends mildly against using the time domain approach to free-controller optimization. However, in favor of the time domain approach is the ability to deal effectively with some aspects of the multiple flight condition problem.

This brings us to discussion of the constant coefficient problem which is really a special case of the above. This specialization, however, enables us to bring a whole new cadre of mathematical techniques to bear on the problem. When the cost function is the integral over $t(0,\infty)$ of a constant quadratic function of the state and the control, frequency domain methods (Refs. 13, 15, and 25) can be used on the problem. These bring an ability to understand and interpret the physics of the problem which is unparalleled in time domain techniques. This is especially so when multiloop analysis techniques (Ref. 26) are employed to simplify computation and aid interpretation of results. An example of the synergistic combination of these techniques is given in Ref. 13. There, multiloop analysis and frequency domain optimal control techniques are used to show how performance index coefficients should be chosen for multicontrol-point model reference optimal control problems.

Application of frequency domain methods results leads to solution for the Laplace transform of the optimal control vector time history.* A key problem is to select a practical sensor complement and the minimum order controller (equalization) that can generate the optimal control from the sensor outputs. This is the same key problem that arises when time domain techniques are used.

An advantage in treating the constant coefficient problem is the availability of a more extensively developed theory for determining these minimal order optimal controllers. This capability is extremely important for applications to flight control. There are two techniques which are of interest here. The first is the Luenberger observer theory (Ref. 22). This theory enables one to find a family of minimal order filters any one of which can generate an asymptotic estimate of a single linear combination of the state. Since the optimal control can be equated to a linear combination of the state, the filters can generate an asymptotic approximation to the optimal control.

The order of the filter can be determined quite simply from the observability properties of the plant. If the observability index is ν, the minimum filter order is $\nu - 1$. The bounds on the filter order are

$$\frac{n}{m} - 1 \leq \nu - 1 \leq n - m$$

where n is the order of the plant and m is the number of linearly independent sensor outputs which may be used for feedback. The filter order can often be made equal to the lower bound by selecting the proper complement of sensors and placing them at appropriate locations and/or

*Frequency domain techniques have also been partially developed for the so-called semifree form optimization problem. See Chang (Ref. 27) and Rynaski, Whitbeck, and Dolbin (Ref. 28). Solution of this problem is directly for the optimal controller transfer functions. The theory is not yet capable of treating multiple control-point plants such as an aircraft.

orientations in the controlled element. (Exact placement is not usually important, qualitative placement is. For example, place a normal accelerometer near the center of rotation for elevator inputs.) As an example of how low an order the filter might be, consider a sixth order system in which three sensors are used. Then:

$$\min(\nu - 1) = \frac{n}{m} - 1 = \frac{6}{3} - 1 = 1$$

This means that only a first order filter is necessary to generate an asymptotic estimate of a single linear combination of the state from the given measurements if an appropriate complement of three sensors can be found.

The main shortcoming of this method revolves around the "asymptotic estimate." The asymptotic nature of the estimate does <u>not</u> affect the closed-loop stability of the system in which the filter is used but the resulting controller is a suboptimal one if a literal application of the theory is made. The Luenberger observer contains a number of arbitrary parameters. Reference 13 has recently shown for two examples that these arbitrary parameters can be selected to produce a single linear combination of the state exactly. One of these examples was the two control-point longitudinal flight control problem for landing approach. Reference 13 also shows how certain observability problems which arise when implicit model reference formulations of the free-controller optimization are used, can be resolved. Even more recently, the results obtained for the example in Ref. 13 have been shown to be obtainable in general by Ferguson and Rekasius in Ref. 23. Consequently a single linear combination of the state can always be generated <u>exactly</u> using a $\nu - 1$ order filter.

The second important technique is that recently published by Pearson and Ding (Ref. 24). This work strives in the direction of determining the overall minimal order optimal controller, but achieves a somewhat lesser, albeit important, result. The main theorem concerns obtaining a "reduced order compensator."

> Assume the controlled element is controllable and that it is observable with index ν. Let q be the smallest number of inputs which can control the state of the controlled element. Then a compensator of order $q(\nu - 1)$ can be constructed to obtain arbitrary pole placement.

A few remarks are in order here. Most obvious are the bounds on ζ, the overall minimum order compensator which we can currently construct. These are

$$\frac{n}{m} - 1 \leq \zeta \leq n - m$$

The lower bound is possible because of the prospect of a single point control plus the Luenberger theory. The upper bound is established by the fact that the entire state may be reconstructed using a $(n - m)$th order filter (see Ref. 13), and that arbitrary pole placement can be achieved by state variable feedback.

Treating the Manual/Automatic Controller Function Split

Flight control in its broadest sense encompasses systems ranging from fully manual to fully automatic with combinations of manual and automatic functions being of considerable importance. The specific flight control problem under discussion, a stability augmentation system, is in the final analysis a combination of manual and automatic functions. There are three key matters to be considered when applying free-controller optimization techniques to design such a system. These are discussed below.

The first is to consider whether the design process is to consider the pilot an integral system part, or to exclude the pilot from the system by definition. Our view is that for several compelling reasons the pilot should be considered an integral part of the system. To appreciate this consider the following. For design of a fully automatic system for some mission segment, it should be obvious that the design must include some outer loop which functions to regulate an error which is directly related to mission segment goals. This error or some quantity closely related to it, would form one of the most important terms in the performance index. Now reconsider the matter when part of the control function is accomplished manually. The pilot's participation always starts with the outer loops and progresses toward the most inner stabilization loops as control becomes more fully manual. The point is that if the pilot is not considered as an integral part of the system, only the inner loops can be designed. The inner loop design then bears only a tenuous relation to the outer loop (mission segment) goals. The resulting situation is undesirable, and very much analogous to the practice of trying to specify closed-loop handling qualities by specifying aircraft characteristics alone rather than closed-loop pilot-plus-aircraft characteristics which are responsive to the mission segment goal. This should make the need to treat the pilot as an integral part of the system quite clear.

This leads to the second point. This is, "How should the problem be formulated?" When the above viewpoint is adopted, a model reference approach is unsurpassed for its simplicity and effectiveness. The model amounts to a simple statement of the outer loop bandwidth required. A performance index consisting of the weighted sum of squares of the model-system error and the control variables is appropriate.* Selection of the weightings is not a crucial consideration in model reference formulations. The techniques governing their selection are given in Refs. 5 and 13. Furthermore, the model reference formulation is the only formulation using optimal control theory which has been shown to be capable of generating closed-loop controllers for the pure gain, pure rate, and pure acceleration controlled elements which are consistent with the controller designs developed using conventional control theory and which have long been recognized as "good", see Ref. 5.

Remaining for discussion is the third matter to be considered. This concerns dividing the controller functions between the pilot and the black boxes. Just how this is to be done depends upon whether the system is to be a stability augmentation type (SAS), flight director type, fly-by-wire type, or perhaps a combination of these. Once this matter of the pilot's

*Sometimes additional terms such as would be appropriate for limiting pitch overshoots, for example, might also be included.

role in the system is decided, the next consideration is to accommodate the physical constraints on possible pilot-black box-vehicle interaction. For example, information such as flight path error sensed by the pilot may not serve as an input to the black boxes. Then there are handling qualities constraints. The pilot's describing function amplitude ratio should be either pure gain-like or low-pass in nature for favorable pilot opinion. It is permissible to require some lead generation from the pilot and so on. Lastly, there is the requirement to keep the compensation in the black boxes "simple" (meaning low order).

It is at this point where the practitioner of free-controller optimization gets into some major difficulties. The existing theory offers some help, but, in itself, the theory is not enough; good measures of engineering judgment and luck are also needed.

One proceeds from this difficult point as follows. Determine which sensor outputs are available to the pilot and which are available to the automatic equipment. Next, determine which components of the optimal control are to be supplied by the pilot and which the automatic equipment will supply. This division is not necessarily just between elements of the optimal control vector, but can also be a division of individual optimal control vector elements as well. The only tool presently applicable to make these separations is engineering art. For flight director configurations this optimal control division process is trivial, for SAS configurations extraordinarily difficult, and for fly-by-wire configurations, somewhere in between. (It is of course completely unnecessary for fully automatic or fully manual configurations.)

Having done this the theory is again ready to help out. Either the Ferguson-Rekasius theory (Ref. 23) or the Pearson-Ding theory (Ref. 24) may be applied as is appropriate to synthesize <u>separate</u> compensators for the manual sensor-to-control paths and the automatic sensor-to-control paths.

This process would have to be repeated iteratively until a suitable optimal control division is found satisfying the handling qualities and black box complexity constraints. Failing this, it would be necessary to make engineering approximations to the optimal compensators which would satisfy these constraints. Clearly this is a difficult process at best.

Summary

The key step when using free-controller optimization techniques is that of converting the optimal solution into a minimal order controller which uses only the available measurements. Because this is *the* problem, the lion's share of the above discussion deals with this matter rather than with that of computing the optimal solution. In this day and age, computation of the optimal solution is a routine part of the overall solution, although this is not to deny the existence of numerical difficulties in certain circumstances.

To keep the "magic" in free-controller optimization, it is absolutely necessary to employ a performance index which is a function of physically meaningful error variables and control variables for the specific application. Much has been said criticizing the integral quadratic function as not accurately expressing design goals. But this is a rather nit-picking point when one considers that such an argument can only center around matters of secondary importance such as the choice of the function of the errors and the way in which the function is averaged. This is because the error variables and control variables are chosen to be design goal oriented in the first place. When a performance index as described above is employed it is possible to trade off physically meaningful error energies against one another and against physically meaningful control energies. This usually involves only a very few parameters, and both the *ab initio interpretation*, *and effect* of parametric twiddling is physically meaningful. On the other hand, when one merely manipulates the "Q" and "R" matrix elements as is so often done in the literature in order to achieve a pleasing pole-zero constellation or transient response, *only the effect* of the parametric twiddling is physically meaningful. In the latter usage, free-controller optimization becomes "just another algorithm." Its "magic" is lost. At that point, the free-controller optimization becomes the equal in philosophical elegance of "knob twiddling" on an analog computer, and is less effective for flight control problems to boot.

There are certain limitations to the practice of free-controller optimization even when applied in an enlightened manner. Chief among these in flight control applications arises when the pilot is a part of the controller. Then it is necessary to divide the control activity between the pilot and the black boxes in such a way that the physical constraints on pilot-black box-vehicle interaction are not violated. Some examples of these constraints are:

- Sensors unique to the pilot may not provide inputs to the automatic equipment, e.g., eyeball perception of flight path error.
- Pilot's control function (describing function) must be consistent with good handling qualities.
- In SAS systems all compensation supplied by the automatic equipment must occur in the feedback paths in distinction to the forward loop paths.
- In flight director or in fly-by-wire systems compensation supplied by the automatic equipment may occur in feedback and in forward paths.

What is more, the division of controller functions between the pilot and the black boxes must be done in a way requiring minimal order compensation from each. The tendency is, however, toward higher order compensation when this division is made because the human and inanimate sensory devices can no longer be used in a completely integrated way. Usually it will turn out that the required compensation needed in the black boxes is too complex to be completely practical. Similarly the compensation required of the pilot may not be consistent with good handling qualities. In the end both may be approximated.

There are other factors complicating the division of the controller functions between the pilot and the black boxes. For flight director and fly-by-wire systems, meeting the above constraints is merely a tradeoff exercise between handling qualities and system complexity. However, for SAS systems, all forward loop compensation must be supplied by the pilot. In this case there does not appear to be any way to reconcile the often conflicting requirements for good handling qualities with the reasonably complex forward loop compensation necessary for optimal control. Nor is this the end of the story. It turns out that the compensation required is sometimes unstable (by itself, but the overall closed-loop system is stable). This poses no problem in principle, but a very real problem in practice. Still other "practical" problems can be encountered because of undue system sensitivity to off-nominal conditions. Problems of this nature may arise because of a poor choice of sensor complement or even because of poor sensor location or orientation. These latter matters are not readily treated by the existing theory. Extreme sensitivity can be avoided by including costs for sensitivity in the performance index. But this is at the expense of greatly complicating the problem.

The point here is that free-controller optimization techniques can probably be applied to synthesis of a stability augmentation system, and practical results achieved if the key suggestions made in this appendix are followed. However, the final control system will wind up being a patch quilt of fixes and compromises instead of the hoped for clean optimal design. Furthermore, the level and number of skills required for routine practice of the technique will be higher and more numerous than those required by almost all other feasible approaches. This renders free-controller optimization a feasible technique, but hardly a practical one for flight control system synthesis as it currently stands. Hope remains for the future, of course.

PARAMETER OPTIMIZATION

Let us define what we mean by parameter optimization in terms of the types of problems which can be solved. In a parameter optimization problem the following are specified:

- Plant equations of motion
- Model equations of motion (if one is needed for the cost function)
- Cost function
- Controller form

One then uses an iterative computational procedure to determine the optimal values of certain variable parameters in the controller. If there are a sufficiently large number of variable parameters, this approach would result in the same solution as one obtained with free-controller optimization; however, the computational approach would be different.

The basic concept of parameter optimization is certainly a very old one but has received renewed interest in recent years due to the advent of large scale digital and hybrid computers. With this tremendous growth in computing capacity, it has become possible to solve practical design problems for very complex plants with numerous parameters to be optimized. This renewed interest has also sparked several significant developments of efficient computational algorithms. This combination of developments has brought parameter optimization to the status of a practical synthesis tool.

The key concern here is the advantages of parameter optimization relative to free-controller optimization as applied to flight control system design. We will first list these and then discuss each in turn. The primary advantages are:

1. The optimization can be done over several different flight conditions or vehicle configurations.

2. The final controller is truly optimized in the sense that the optimal values of all available controller parameters are determined.

3. From the outset, only practical controller configurations are considered.

4. The inclusion of actuator or sensor dynamics is straightforward and only increases the required computations.

5. Constraints on certain controller parameters, such as feedback gain or lead/lag ratio, can be specified.

6. Among the parameters which can be optimized are accelerometer location and rate gyro orientation.

7. Plant and controller nonlinearities can be included.

8. Nonquadratic cost functions can be utilized.

The ability to optimize over several flight conditions or vehicle configurations is a most significant advantage. This can be done by using an overall cost function which is the weighted sum of the cost functions for each individual flight condition. One can compute a single set of controller parameters which are optimal considering all the flight conditions, or one can allow a specified type of gain scheduling. If only the form of the gain scheduling is prespecified, the parameter optimization could include the optimization of the gain scheduling parameters. One obvious application of this capability is to optimize the control system over the altitude/Mach number envelope of the aircraft. The capability can also be utilized to optimize over a range of aircraft weights, c.g. locations, loading configurations, and normal accelerations. Since gain scheduling is normally a key practical problem in flight control system design, the ability to optimize over multiple flight conditions and vehicle configurations is a very significant advantage of the parameter optimization approach.

With parameter optimization the final form of the controller will be optimized because this form will be the one actually used in the final optimization calculation. Whenever, in the course of the design procedure, feedbacks are eliminated or compensations are simplified, the remaining system can be reoptimized. Thus, whatever the level of complexity the designer chooses for the final flight control system design, the system will be optimal in that an improvement in the cost function would require additional complexity.

Since the controller form is prespecified, only practical controller configurations need ever be considered. The prespecified controller would include only practical measurements and realistic compensations. However, additional feedbacks and compensations can be added to indicate the degree of performance improvement which is at least theoretically possible. This can be taken all the way to the free-controller equivalent for a particular flight condition.

The inclusion of actuator and sensor dynamics presents no basic difficulties in parameter optimization. If the optimization is done on a digital computer, the additional dynamics will simply increase computing time. If the computation is done on a hybrid computer with the system dynamics simulated on the analog portion, then the increase in dynamics simply requires the addition of a few more amplifiers.

Constraints on certain controller parameters can be included. One may wish to limit certain feedback gains to avoid structural feedback problems or control system saturation. Constraints can also be placed on compensation elements, such as restricting lead/lag ratios to realistic values. Any practical restrictions on any of the controller characteristics can be imposed on the optimization.

The location of accelerometers and the orientation of rate gyros can be an important design problem. These parameters can be among those which are optimized. This can be particularly useful in a multiple flight condition optimization where one must usually find the best compromise to cover the various conditions.

System nonlinearities can be included in the problem. This could include such things as nonlinear stick force characteristics, or control surface rate and position limits. One could include aerodynamic nonlinearities, such as when operating near the stall. Nonlinear dynamic problems can also be investigated. The optimization of the controller to minimize roll inertial cross coupling is an example.

Nonquadratic cost functions can be utilized. These can be used in conjunction with the normal quadratic forms or in place of them. Some items which are nonquadratic and might be included in the cost function are rise time, overshoot, roll angle in a specified time, or maximum sideslip angle. One could include in the cost function penalties for failure to meet certain items from the applicable handling qualities specification.

Having reviewed the advantages of parameter optimization, the next logical question is: What are its principal problems? For application to flight control design there are apparently two. The first is that the designer must always specify the form of the controller. This requires a certain level of capability on the part of the designer. He must have an adequate knowledge of handling qualities, be able to evaluate the basic handling quality deficiencies of the aircraft, and then to determine the essential feedbacks required to compensate for these deficiencies. It might be argued that this is really not a disadvantage for parameter optimization in that without these talents the designer is unlikely to ever come up with a reasonable control system design, whether he uses free-controller or parameter optimization. Furthermore, the handling qualities state of the art has advanced sufficiently in recent years so that analytical evaluation of the aircraft's characteristics is a relatively straightforward proposition. Likewise, the selection of the types of feedbacks required, as well as the form of the compensation, is not that difficult. The costly, time-consuming portion of the design effort is really in the determination of the proper numerical values for the controller parameters, especially when considering operation over the entire flight envelope. Thus, parameter optimization provides the designer with assistance in the area in which he needs it the most.

The second apparent problem with parameter optimization is the rather large computing requirement, although this may not be much more severe than for free-controller optimization. The problem is partially alleviated by recent developments in efficient parameter optimization algorithms (Ref. 29 provides a fairly complete review of available algorithms). While these have been developed primarily outside the aerospace industry, they have direct application to flight control design. The most significant advance which tends to reduce the problem of excessive computation is the development of the modern hybrid computer. This is an ideal computer for the parameter optimization problem. The dynamics of the system can be simulated on the analog portion, with a much compressed time scale, while the digital portion controls the optimization logic. In this manner a very complex system can be optimized with respect to a large number of parameters in a matter of a few minutes. It is worth noting that at least one commercial computing service offers a hybrid computer complete with parameter optimization software.

At this point the choice of free-controller or parameter optimization for application in a flight control system design procedure should be clear. The additional capabilities of parameter optimization overwhelmingly favor its selection. It can be used to obtain free controller solutions although possibly at the expense of additional computational requirements. However, it can also be applied to a variety of problems for which the free-controller optimization is inadequate. On balance, for the present, parameter optimization is clearly the better choice for a practical design procedure.

APPENDIX B

PARAMETER OPTIMIZATION PROGRAM

INTRODUCTION

Given a system defined by:

$$[A(s)][X(s)] = [B(s)] \tag{B-1}$$

where $[A(s)]$ is an $n \times n$ matrix of elements which are quadratic functions of the Laplace variable, s, $[X(s)]$ is an $n \times 1$ matrix of state variables, and $[B(s)]$ is an $n \times 1$ matrix of quadratic elements specifying the system input, and a quadratic cost function:

$$CF = \sum_{i=1}^{n} q_i^2 \int_0^\infty x_i^2(t)\, dt \tag{B-2}$$

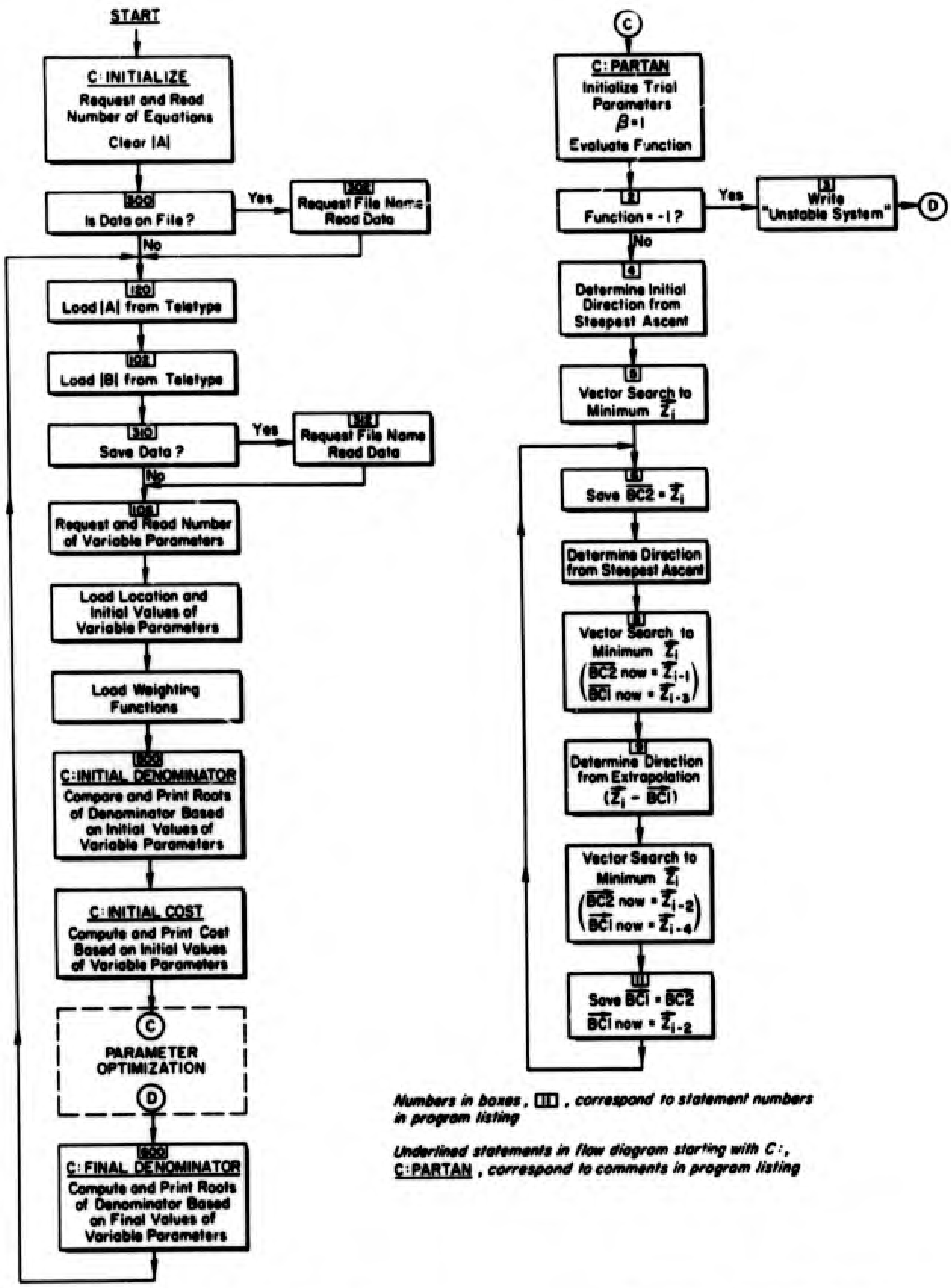
This program will determine the values of specified variable coefficients of the [A] matrix which result in a minimum value of the cost function.

The program was written in Batch Fortran language for use on the Tymshare computer system.

PROGRAM DESCRIPTION

The program consists of two parts: (1) an input/output and user control section, and (2) the parameter optimization section. The latter consists of four subsections: (1) parameter optimization control, (2) function evaluation, (3) steepest descent, and (4) vector search. Flow diagrams for these sections are shown in Fig. B-1. The program also uses eight Fortran subroutines which will be described subsequently.

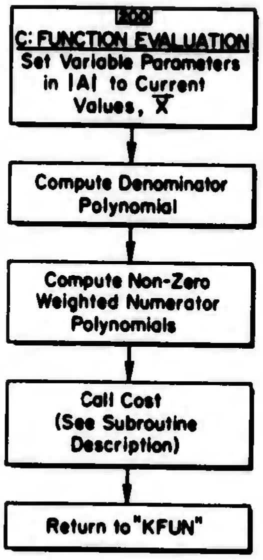
The input/output and user control section, Fig. B-1a, is discussed under program use. The parameter optimization algorithm used in this program is a version of the parallel tangents (partan) method. With minor exceptions the version used here is that of Ref. 30. Basically the method is a sequence of vector searches. After the first few steps, the search direction alternates between the steepest ascent (or descent)



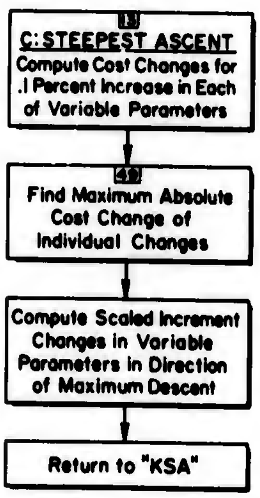
a) Input/Output and User Control

b) Parameter Optimization Control

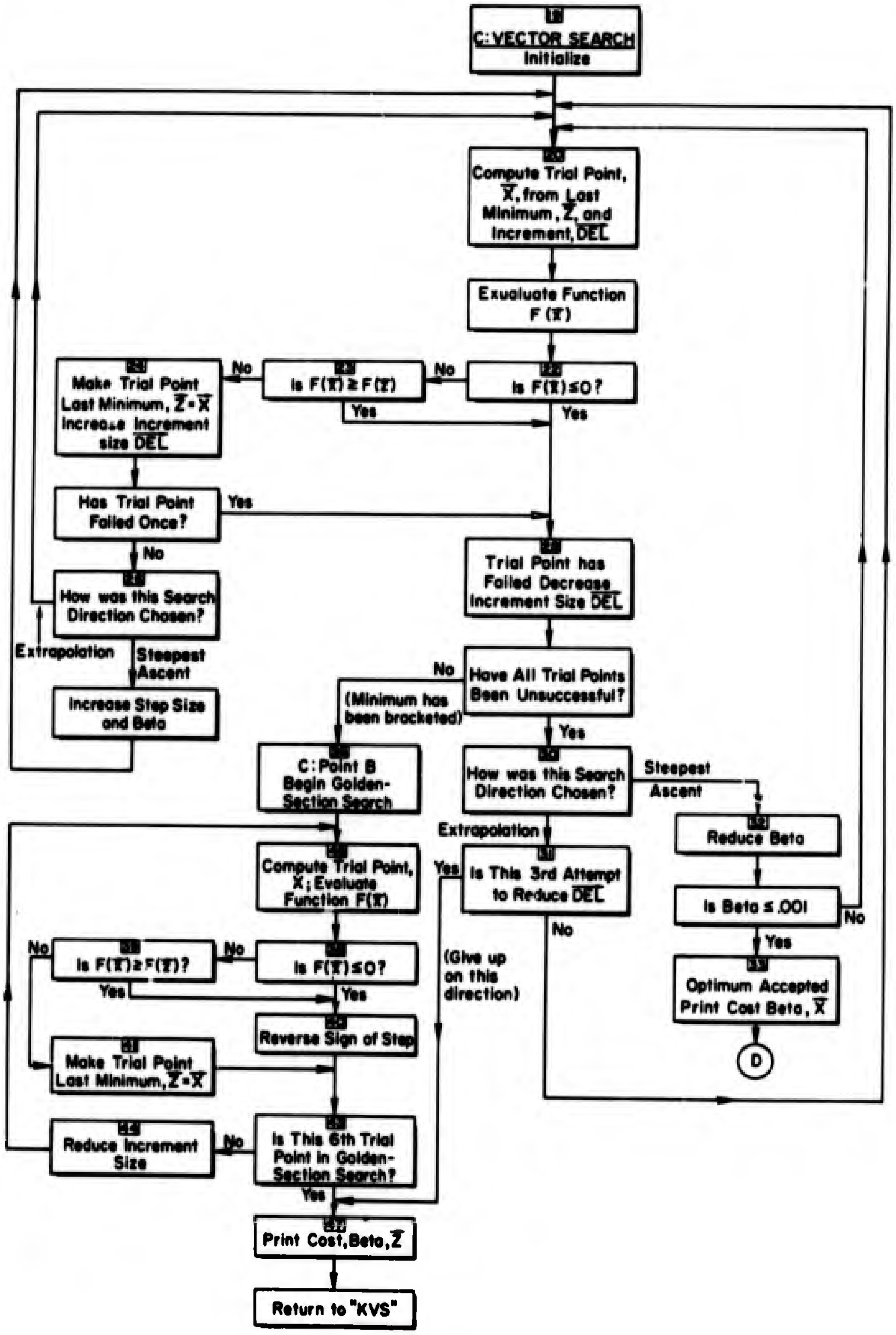
Figure B-1. Flow Diagram of Parameter Optimization Program



c) Function Evaluation



d) Steepest Ascent



e) Vector Search

Figure B-1. (Concluded)

and an extrapolated direction, see Fig. B-2. The extrapolated direction is generally a line through the last maximum (or minimum) point, Z_i, and the third prior point, Z_{i-3}. The parameter optimization control section (Fig. B-1b) after checking for initial system stability and initialization, accomplishes the alternation between steepest ascent direction and extrapolated direction. It also keeps track of the third prior extreme and computes the extrapolation direction and initial step size.

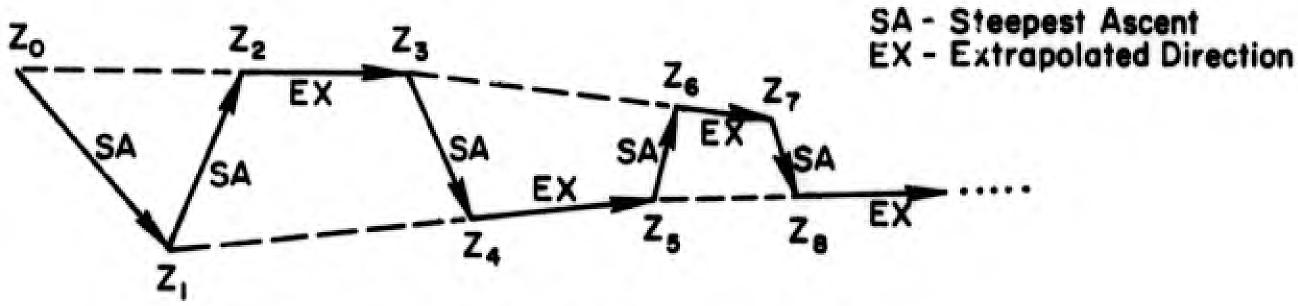


Figure B-2. Search Sequence

The function evaluation subsection is shown in Fig. B-1c. Entered with the coordinates of a trial point, $\overleftarrow{x}$, the section returns the value of the cost function, CF. As this section is entered from several places in the program, as are the remaining two sections, "assigned go to" commands are used in the program. In evaluating the cost function, use is made of Parseval's theorem by which:

$$\int_0^\infty x_1^2(t)\,dt = \frac{1}{2\pi j}\int_{-j\infty}^{+j\infty} x_1(s)x_1(-s)ds \qquad \text{(B-3)}$$

The polynomial functions of s are computed (making use of Fortran subroutines) and the integral is evaluated using the subroutine COST.

To determine the direction of steepest ascent (or descent), the cost function differences due to a small percentage change in each parameter are computed. The initial step size is set so that the fractional change in the parameter which had the largest effect on the cost function is BETA. This is done in the steepest ascent section, Fig. B-1d.

The last section is the vector search; the flow diagram is shown in Fig. B-1e. First, the step size is increased or decreased until the cost function minimum* is bracketed. This is accomplished when "Point B" in the flow diagram is reached. Then a golden-section technique is used, see Ref. 30.

The value of BETA, which is initially 0.1, is changed only during a steepest ascent search. If the initial step size improves the cost function, the step size and BETA are multiplied by $(1+\sqrt{5})/2 \doteq 1.618$. This continues until a degradation in the cost function is reached. If the initial step size degrades the cost function, the step size and BETA are divided by $(3+\sqrt{5})/2 = [(1+\sqrt{5})/2]^2 \doteq 2.618$. This continues until a cost function improvement is achieved or BETA is reduced below the cutoff limit of 10^{-3}. When BETA is less than the cutoff the optimization is terminated.

The following briefly describes each of the Fortran subroutines used by the main program.

1. SUBROUTINE POLYEL (NDEG, POLY, NPC)

This routine factors a given polynomial, p(s), and prints the roots. An option is available by which a printout of the polynomial coefficients also may be obtained. The inputs are:

a. NDEG an integer scalar variable greater than 1 and less than 15 which defines the order of the polynomial

b. POLY a floating point one dimensional array variable which contains the polynomial coefficients; these are assumed to be arranged as:

$$p(s) = \sum_{I=0}^{NDEG} POLY(I)s^I$$

The corresponding variable in the using program's call statement must be dimensioned from 0 to 14.

*This program is written to minimize the cost function; however, the term, steepest ascent, is sometimes used because of its general usage in the literature.

c. NPC an integer scalar variable which is used as a control and must be either 0 or 1. If 1 the polynomial coefficients are printed starting with the constant term of the polynomial. If 0 this printout is suppressed.

The printout will list the first (real) and second order (complex) roots.*
For:

$$p(s) = (s+a)(s+b)[s^2 + 2\zeta\omega_n s + \omega_n^2]$$

the printout will be

$$\begin{array}{llll} a & & & \\ b & & & \\ \zeta & \omega_n & \zeta\omega_n & \omega_n\sqrt{1-\zeta^2} \end{array}$$

where one number on a line indicates a first order root and four numbers on a line indicate a second order root.

2. SUBROUTINE DET (A, N, D)

This routine evaluates and returns the value of a numerical determinant. The inputs are:

a. A a floating point two dimensional array variable containing the elements of the determinant:

$$D = \begin{vmatrix} A(1,1) & \cdots\cdots & A(1,J) & \cdots\cdots & A(1,N) \\ \vdots & & & & \vdots \\ A(I,1) & & & & \vdots \\ \vdots & & & & \vdots \\ A(N,1) & \cdots\cdots & \cdots\cdots & \cdots\cdots & A(N,N) \end{vmatrix}$$

The corresponding variable in the using program's call statement must be dimensioned from 1 to 14 in both directions of the array. (Higher order determinants may be evaluated by increasing the dimensions of A.)

---

*Actually the negative of the real roots are printed.

b. N   an integer scalar variable equal to the order of the determinant to be evaluated.

The output returned by this routine is D, a floating point scalar variable, which is the value of the determinant.

3. SUBROUTINE COST (ND, D, NC, C, Q, CF)

This routine evaluates the cost function integral:

$$CF = \sum_{i=1}^{NC} \frac{q_i^2}{2\pi j} \int_{-j\infty}^{+j\infty} \left| \frac{C_i(s)}{D(s)} \right|^2 ds$$

The method of Ref. 31 is used. The inputs are:

a. D   a floating point one dimensional array variable which contains the polynomial coefficients of:

$$D(s) = \sum_{J=0}^{ND} D(J)s^J$$

The corresponding variable in the using program's call statement must be dimensioned from 0 to 14.

b. ND   an integer scalar variable greater than 1 and less than 15 which defines the order of the polynomial D(s).

c. NC   an integer scalar variable greater than 0 and less than 8 which defines the number of terms to be included in the cost function integral.

d. C   a floating point two dimensional array variable which contains the polynomial coefficients of the polynomials:

$$C_i(s) = \sum_{J=0}^{ND-1} C(J,I)s^J$$

Note that the order of all C(s) polynomials must be at least one less than D(s). The corresponding variable in the using program's call statement must be dimensioned from 0 to 14 in the first direction and 1 to 7 in the second direction.

e. Q   a floating point one dimensional array variable which contains the weights, q_i, on the various terms in the cost function. The corresponding variable in the using program's call statement must be dimensioned from 1 to 7.

The output returned by this routine is CF, a floating point scalar variable, which is the value of the integral. The routine checks D(s) for nonnegative roots (i.e., system instability) and returns a cost function of -1.0 if this condition exists.

4. SUBROUTINE DET7 (E, P7)
   SUBROUTINE DET6 (D, P6)
   SUBROUTINE DET5 (C, P5)
   SUBROUTINE DET4 (B, P4)
   SUBROUTINE DET3 (A, P)

This is a group of subroutines intended to be used together to obtain the coefficients of the polynomial resulting from the expansion of a determinant with complex elements. Determinants up to seventh order with second order elements may be evaluated. If a seventh order determinant is to be evaluated, DET7 is called, which in turn will call DET6, which in turn will call DET5, etc., until the basic computation is done by DET3. If the determinant to be evaluated is less than seventh order this chain may be entered in the appropriate place, i.e., if a fifth order determinant is to be evaluated the using program should call DET5.

The input to all these routines (E,D,C,B,A) is a floating point three dimensional array of the determinant elements:

$$P7(s) = \begin{bmatrix} E(1,1,2)s^2 + E(1,1,1)s + E(1,1,0) & E(1,2,2)s^2 + E(1,2,1)s + E(1,2,0) \cdots \\ E(2,1,2)s^2 + E(2,1,1)s + E(2,1,0) & \\ \vdots & \end{bmatrix}$$

The first index on the array indicates the row of the element, the second index indicates the column, and the third index indicates the order of the complex variable associated with the specified coefficient. <u>All</u> corresponding arrays in the using program's call statement (for all of these subroutines) should be dimensioned from 1 to 7 in the first two directions and 0 to 2 in the third direction. The subroutines assume that the pertinent data for a determinant of lower order than 7 is in the upper left hand corner of the array.

The output returned by these subroutines (P7, P6, P5, P4, P) is a floating point one dimensional array of polynomial coefficients arranged such that

$$P(s) = \sum_{I=0}^{2N} P(I)s^{I}$$

where N is the order of the determinant. The order of the array is dependent on the initial subroutine called; for DET7, a fourteenth order polynomial will be returned (the higher order, or all, coefficients may be zero); for DET6, a twelfth order polynomial will be returned; for DET5, a tenth order, etc. The dimensioning of the corresponding variables in the using program's call statement should also be dependent on which routine is initially entered. For DET7, the variable corresponding to P7 should be dimensioned from 0 to 14; for DET6, the variable corresponding to P6 should be dimensioned from 0 to 12; etc.

PROGRAM USE

The first step is to convert the equations defining the system and input to the form of Eq. B-1. The program requires:

1. The number of equations must be 7 or less.
2. Up to 10 parameters may be optimized, all of which must appear as unique coefficients of the [A] matrix.
3. With the initial values of the variable parameters, the roots of the determinant of A must all have negative real parts. No zero valued roots are allowed.
4. The response of all variables included in the cost function must be finite at all times. The numerator polynomial of all variables in the cost function must be one order less than the denominator polynomial (see Eq. B-3).

A listing of the program is given on pp. 99-112 under the heading Program Listing. Reference to the flow diagram of Fig. B-1a may aid in understanding the following description.

Upon execution the program will request the number of equations:

THE NO. OF EQ. = .

The user must type a single integer from 3 to 7 followed by a comma.*

The program next asks:

DATA ON FILE ?

If the user types YES the program will request:

FILE NAME ?

The user must enter the file name.

If the user types NO the program requests the data for the [A] matrix:

LOAD A
I,J,K,A(I,J,K)

The user enters the coefficients of the elements of the [A] matrix, one to a line; I and J being respectively the row and column of the element, K being the order of s corresponding to the coefficient, and A(I,J,K) the floating point value of the coefficient. Only nonzero coefficients need to be entered. After entering all data on the [A] matrix the user types a single zero to terminate this part of the input.

The program next requests data for [B];

LOAD B
I,K,B(I,K)

---

*All inputs are terminated by a carriage return.

The user enters the coefficients of [B], one to a line; I is the row of the element, K the order of the coefficient, and B(I,K) its value. This input is terminated by a single zero.

The program next gives the user the option of saving the data:

SAVE DATA ?

If the user enters YES, a file name is requested and the data is available for subsequent running of the program. If the initial data had been loaded from a file, the user would have the option of changing or adding selected coefficients when asked to LOAD A and LOAD B.

If the user does not want to retain the data, NO should be entered.

The number of variable parameters is requested; the user should enter a single integer from 1 to 10 followed by a comma.

The user is requested to

LOAD LOCATION AND INITIAL VALUES
I,J,K,A(I,J,K)

of the variable parameters. The input is in the same format as for the basic [A] data. The program expects the number to be entered as previously specified; it will automatically terminate the inputs.

The program next requests the weighting functions:

LOAD Q(I),I=1 TO n

where n is the number of equations. These are entered one to a line. Note that the cost function is written in terms of q_i^2 while q_i is to be entered. n weighting functions must be entered, some of which may be zero.

To provide a check on the input the roots of the system denominator will be printed; one number to a line indicating a first-order root (the negative of the root is printed), four numbers to a line indicating a second-order root, the four numbers being

$$\zeta \ , \ \omega_n \ , \ \zeta\omega_n \ , \ \omega_n\sqrt{1-\zeta^2}$$

The initial value of the cost function is printed. The program then enters the optimization routine and at each point where the direction of search is changed the following is typed:

```
COST= xxxxx        BETA= xxxxxx
X1       X2    ...     Xm
```

The X's are the current values of the variable parameters.

When the program has accepted an optimum (BETA $< 10^{-3}$) it will type the final cost and the values of the variable parameters and then the roots of the denominator of the final system.

The program will then return to the input mode requesting [A], [B], variable parameters, and Q. [A] and [B] from the previous case are retained; only changes need be entered. If desired no changes to [A] or [B] are necessary and the program can be rerun with only changes to the weighting functions. Note that after each case the A matrix contains not only the fixed elements but also the final values of the variable parameters.

EXAMPLE

An optimum regulator problem will be used as an example. The problem selected is very simple so that a literal solution is readily available for checking the program results. The specific details of the problem are:

1. Aircraft short-period equations of motion with $M_{\dot{\alpha}} = Z_{\delta_e} = 0$
2. Cost function $= \int_0^\infty [\delta_e(t)^2 + q_\alpha^2 \, \alpha(t)^2]dt$
3. Control law, $\delta_e = -(K_\alpha \alpha + K_q q)$

The complete equations of motion are then:

$$\begin{bmatrix} s - Z_w & -1 & 0 \\ -M_\alpha & s - M_q & -M_{\delta_e} \\ K_\alpha & K_q & 1 \end{bmatrix} \begin{Bmatrix} \alpha \\ q \\ \delta_e \end{Bmatrix} = \begin{Bmatrix} \alpha(0) \\ q(0) \\ 0 \end{Bmatrix}$$

The optimal closed-loop roots are found from the root square locus, i.e.,

$$(s^2 + 2\zeta_{sp}\omega_{sp}s + \omega_{sp}^2)(s^2 - 2\zeta_{sp}\omega_{sp}s + \omega_{sp}^2) + q_\alpha^2 M_{\delta_e}^2 = (s^2 + 2\zeta_{sp}'\omega_{sp}'s + \omega_{sp}'^2) \times (s^2 - 2\zeta_{sp}'\omega_{sp}'s + \omega_{sp}'^2)$$

or

$$\omega_{sp}'^2 = [\omega_{sp}^4 + q_\alpha^2 M_{\delta_e}^2]^{1/2}$$

$$2\zeta_{sp}'\omega_{sp}' = \left[2(\omega_{sp}'^2 - \omega_{sp}^2) + (2\zeta_{sp}\omega_{sp})^2\right]^{1/2}$$

where

$$\omega_{sp}^2 = Z_w M_q - M_\alpha$$

$$2\zeta_{sp}\omega_{sp} = -Z_w - M_q$$

The closed-loop roots can also be written in terms of the feedback gains, i.e.,

$$s^2 + 2\zeta_{sp}'\omega_{sp}'s + \omega_{sp}'^2 = s^2 + 2\zeta_{sp}\omega_{sp}s + \omega_{sp}^2 + K_\alpha M_{\delta_e} + K_q M_{\delta_e}(s - Z_w)$$

Thus the optimal gains are

$$K_q = \frac{1}{M_{\delta_e}}(2\zeta_{sp}'\omega_{sp}' - 2\zeta_{sp}\omega_{sp})$$

$$K_\alpha = \frac{1}{M_{\delta_e}}(\omega_{sp}'^2 - \omega_{sp}^2) + Z_w K_q$$

For the example, the following numerical values were used:

$$Z_w = M_q = -0.5 \text{ sec}^{-1}$$

$$M_\alpha = -0.75 \text{ sec}^{-2}$$

$$M_{\delta_e} = 1 \text{ sec}^{-2}$$

$$q_\alpha = \sqrt{8} \doteq 2.8284271$$

This results in the following values of the other parameters:

$$\omega_{sp} = 1$$

$$2\zeta_{sp}\omega_{sp} = 1$$

$$\zeta_{sp} = 0.5$$

$$\omega'_{sp} = \sqrt{3} \doteq 1.73205$$

$$2\zeta'_{sp}\omega'_{sp} = \sqrt{5} \doteq 2.23607$$

$$\zeta'_{sp} = \frac{\sqrt{15}}{6} \doteq 0.645497$$

$$K_q = \sqrt{5} - 1 \doteq 1.23607$$

$$K_\alpha = \frac{5 - \sqrt{5}}{2} \doteq 1.38197$$

The actual computer output (for $\alpha(0) = 0$, $q(0) = 1$) is shown in Fig. B-3. The initial values of the gains were set at approximately twice the optimal solutions. The computed closed-loop damping ratio and frequency agree with the theoretical values within the accuracy of the printout. The errors in the optimized gains are 0.11 percent for K_α and 0.01 percent for K_q. These are compatible with the program cutoff of 0.1 percent.

```
 THE NO. OF EQ. = 3,

DATA ON FILE ?N

LOAD A
I,J,K,A(I,J,K)

1,1,1,1.
1,1,0,.5
1,2,0,-1.
2,1,0,.75
2,2,1,1.
2,2,0,.5
2,3,0,-1.
3,3,0,1.
0

LOAD B
I,K,B(I,K)

2,0,1.
0

SAVE DATA ?N
THE NO. OF VARIABLE PARAMETERS IS 2,

LOAD LOCATION AND INITIAL VALUES
I,J,K,A(I,J,K)

3,1,0,2.8
3,2,0,2.5

LOAD Q(I),I=1 TO  3
2.8284271
0
1.


        .779      2.247      1.750      1.410

 INITIAL COST =  .158317E  1

COST= .128274E  1       BETA= .423580E  0
 .228533E  1    .113200E  1
COST= .123685E  1       BETA= .685353E  0
 .145855E  1    .118541E  1
COST= .123685E  1       BETA= .685353E  0
 .145855E  1    .118541E  1
COST= .123622E  1       BETA= .381949E- 1
 .142730E  1    .123729E  1
COST= .123608E  1       BETA= .381949E- 1
 .138181E  1    .124288E  1
COST= .123607E  1       BETA= .557270E- 2
 .138041E  1    .123595E  1
COST= .123607E  1       BETA= .557270E- 2
 .138041E  1    .123595E  1
COST= .123607E  1       BETA= .813067E- 3
 .138041E  1    .123595E  1

        .645      1.732      1.117      1.324

 LOAD A
 I,J,K,A(I,J,K)
```

Figure B-3. Example Printout

PROGRAM LISTING

```
C: PARAMETER OPTIMIZATION
DIMENSION X(10),Z(10),BC1(10),BC2(10),DEL(10),DF(10)
DIMENSION A(7,7,0/2),B(7,0/2),II1(10),JJ1(10),KK1(10),Q(7)
,D(0/14),A1(7,7,0/2),P(0/14),C(0/14,7),Q1(7)
C: INITIALIZE
WRITE (1,160)
160 FORMAT ($ THE NO. OF EQ. = $)
READ (0,161) NEQ
161 FORMAT (I3/)
DO 100 I1=1,NEQ
DO 100 J1=1,NEQ
DO 100 K1=0,2
100 A(I1,J1,K1)=0
DO 104 I1=1,NEQ
DO 104 K1=0,2
104 B(I1,K1)=0
300 WRITE (1,400)
400 FORMAT (/$DATA ON FILE ?$)
IY =1HY
IN =1HN
READ (0,401) IANS
401 FORMAT (A1/)
IF (IANS-IY) 301,302,301
301 IF(IANS-IN) 300,120,200
302 WRITE(1,402)
402 FORMAT(/$FILE NAME ?$)
OPEN (2,INPUT,BINARY,//)
303 READ (2) I1,J1,K1,AT
IF(I1) 304,304,305
305 A(I1,J1,K1) = AT
GO TO 303
304 READ (2) I1,K1,BT
IF (I1) 307,307,306
306 B(I1,K1) = BT
GO TO 304
307 CLOSE (2)
GO TO 120
120 WRITE (1,150)
150 FORMAT(/$LOAD A$,/,$I,J,K,A(I,J,K)$//)
101 READ (0,163) I1,J1,K1,AT
163 FORMAT(3I3,F14.6/)
IF(I1) 102,102,103
103 A(I1,J1,K1)=AT
GO TO 101
102 WRITE (1,151)
151 FORMAT(/$LOAD B$,/,$I,K,B(I,K)$//)
105 READ(0,164) I1,K1,BT
164 FORMAT(2I3,F14.6/)
IF (I1) 310,310,107
107 B(I1,K1)=BT
GO TO 105
310 WRITE (1,403)
403 FORMAT (/$SAVE DATA ?$)
READ (0,401) IANS
IY =1HY
IN =1HN
```

```
IF(IANS-IY) 311,312,311
311 IF(IANS-IN) 310,106,310
312 WRITE (1,402)
OPEN (2,OUTPUT,BINARY,//)
DO 313 I1=1,NEQ
DO 313 J1=1,NEQ
DO 313 K1=0,2
313 WRITE(2) I1,J1,K1,A(I1,J1,K1)
ZERO =0.
IZERO =0
WRITE (2) IZERO,IZERO,IZERO,ZERO
DO 314 I1 =1,NEQ
DO 314 K1 =0,2
314 WRITE (2) I1,K1,B(I1,K1)
WRITE (2) IZERO,IZERO,ZERO
CLOSE (2)
GO TO 106
106 WRITE (1,165)
165 FORMAT ($THE NO. OF VARIABLE PARAMETERS IS $)
READ(0,161) NZ
WRITE (1,152)
152 FORMAT (/,$LOAD LOCATION AND INITIAL VALUES $,/,$I,J,K
,A(I,J,K)$//)
DO 109 NZ1=1,NZ
109 READ(0,163) II1(NZ1),JJ1(NZ1),KK1(NZ1),Z(NZ1)
WRITE (1,153) NEQ
153 FORMAT(/,$LOAD Q(I),I=1 TO $,I2,/)
DO 112 I1=1,NEQ
112 READ(0,166) Q(I1)
166 FORMAT(F12.5/)
C: INITIAL DENOMINATOR
500 DO 501 NZ1=1,NZ
I1=II1(NZ1)
J1=JJ1(NZ1)
K1=KK1(NZ1)
501 A(I1,J1,K1)=Z(NZ1)
INE=NEQ-2
GO TO (503,504,505,506,507),INE
503 CALL DET3[A,D]
GO TO 502
504 CALL DET4[A,D]
GO TO 502
505 CALL DET5[A,D]
GO TO 502
506 CALL DET6[A,D]
GO TO 502
507 CALL DET7[A,D]
502 ND=2*NEQ
CALL POLYEL[ND,D,0]
C: INITIAL COST
DO 508 NUM=1,NEQ
IF (Q(NUM)) 508,508,511
511 DO 509 I1=1,NEQ
DO 509 J1=1,NEQ
DO 509 K1=0,2
509 A1(I1,J1,K1)=A(I1,J1,K1)
```

```
DO 510 I1=1,NEQ
DO 510 K1=0,2
510 A1(I1,NUM,K1)=B(I1,K1)
GO TO (513,514,515,516,517),INE
513 CALL DET3[A1,P]
GO TO 512
514 CALL DET4[A1,P]
GO TO 512
515 CALL DET5[A1,P]
GO TO 512
516 CALL DET6[A1,P]
GO TO 512
517 CALL DET7[A1,P]
512 DO 521 IC=0,ND-1
521 C(IC,NUM)=P(IC)
508 CONTINUE
NC=0
DO 522 NUM=1,NEQ
IF(Q(NUM)) 522,522,524
524 NC=NC+1
Q1(NC)=Q(NUM)
DO 525 IC=0,ND-1
525 C(IC,NC)=C(IC,NUM)
522 CONTINUE
CALL COST[ND,D,NC,C,Q1,F]
WRITE (1,404) F
404 FORMAT(/$INITIAL COST = $,E12.6/)
C: PARTAN
DO 1 I=1,NZ
X(I)=Z(I)
1 BC1(I)=Z(I)
BETA=.1
ASSIGN 2 TO KFUN
GO TO 200
2 IF (F) 3,3,4
3 WRITE (1,167)
167 FORMAT($UNSTABLE SYSTEM$)
GO TO 600
STOP
4 FB=F
ASSIGN 5 TO KSA
GO TO 13
5 KK=1
ASSIGN 6 TO KVS
GO TO 19
6 DO 7 I=1,NZ
7 BC2(I)=Z(I)
ASSIGN 8 TO KSA
GO TO 13
8 KK=1
ASSIGN 9 TO KVS
GO TO 19
9 KK=0
DO 10 I=1,NZ
10 DEL(I)=(Z(I)-BC1(I))/3.
ASSIGN 11 TO KVS
```

```
GO TO 19
11 DO 12 I=1,NZ
12 BC1(I)=BC2(I)
GO TO 6
C: STEEPEST ASCENT
13 ASSIGN 14 TO KFUN
I=0
15 I=I+1
X(I)=Z(I)*1.001
GO TO 200
14 X(I)=Z(I)
 DF(I)=F-FB
IF (I-NZ) 15,49,49
49 DFMAX=0.
DO 16 I=1,NZ
IF (ABS[DF(I)]-DFMAX) 16,16,17
17 DFMAX=ABS[DF(I)]
16 CONTINUE
DO 18 I=1,NZ
18 DEL(I)=-BETA*Z(I)*DF(I)/DFMAX
GO TO KSA
C: VECTOR SEARCH
19 S=0.
KKK=-3
TEST=0.
20 DO 21 I=1,NZ
21 X(I)=Z(I)+DEL(I)
ASSIGN 22 TO KFUN
GO TO 200
22 IF (F) 28,28,23
23 IF (F-FB) 24,28,28
24 FB=F
S=1.
DO 25 I=1,NZ
Z(I)=X(I)
25 DEL(I)=DEL(I)*1.618
IF (TEST) 26,26,28
26 IF (KK) 20,20,27
27 BETA=BETA*1.618
GO TO 20
28 TEST=1.
DO 29 I=1,NZ
29 DEL(I)=DEL(I)/2.618
IF (S) 30,30,36
30 IF (KK) 31,31,32
31 KKK=KKK+1
IF (KKK) 20,47,47
32 BETA=BETA/2.618
IF (BETA-.001) 33,33,20
33 WRITE (1,34) FB,BETA
WRITE (1,35) (Z(I), I=1,NZ)
34 FORMAT (/5HCOST=,E12.6,5X,5HBETA=,E12.6/)
35 FORMAT (5(E12.6,2X)/)
GO TO 600
STOP
C: POINT B
```

```
36 II=1
46 DO 37 I=1,NZ
37 X(I)=Z(I)+S*DEL(I)
ASSIGN 38 TO KFUN
GO TO 200
38 IF (F) 40,40,39
39 IF(F-FB) 41,40,40
40 S=-S
GO TO 43
41 FB=F
DO 42 I=1,NZ
42 Z(I)=X(I)
43 IF (II-5) 44,47,47
44 II=II+1
DO 45 I=1,NZ
45 DEL(I)=DEL(I)/1.618
GO TO 46
47 DO 48 I=1,NZ
48 X(I)=Z(I)
WRITE (1,34) FB,BETA
WRITE (1,35) (Z(I),I=1,NZ)
GO TO KVS
C: FUNCTION EVALUATION
200 DO 201 NZ1=1,NZ
I1=II1(NZ1)
J1=JJ1(NZ1)
K1=KK1(NZ1)
201 A(I1,J1,K1)=X(NZ1)
INE=NEQ-2
GO TO (203,204,205,206,207),INE
203 CALL DET3[A,D]
GO TO 202
204 CALL DET4[A,D]
GO TO 202
205 CALL DET5[A,D]
GO TO 202
206 CALL DET6[A,D]
GO TO 202
207 CALL DET7[A,D]
202 ND=2*NEQ
DO 208 NUM=1,NEQ
IF (Q(NUM)) 208,208,211
211 DO 209 I1=1,NEQ
DO 209 J1=1,NEQ
DO 209 K1=0,2
209 A1(I1,J1,K1)=A(I1,J1,K1)
DO 210 I1=1,NEQ
DO 210 K1=0,2
210 A1(I1,NUM,K1)=B(I1,K1)
GO TO (213,214,215,216,217),INE
213 CALL DET3[A1,P]
GO TO 212
214 CALL DET4[A1,P]
GO TO 212
215 CALL DET5[A1,P]
GO TO 212
```

```
216 CALL DET6[A1,P]
GO TO 212
217 CALL DET7[A1,P]
212 DO 221 IC=0,ND-1
221 C(IC,NUM)=P(IC)
208 CONTINUE
NC=0
DO 222 NUM=1,NEQ
IF(Q(NUM)) 222,222,224
224 NC=NC+1
Q1(NC)=Q(NUM)
DO 225 IC=0,ND-1
225 C(IC,NC)=C(IC,NUM)
222 CONTINUE
CALL COST[ND,D,NC,C,Q1,F]
GO TO KFUN
C: FINAL DENOMINATOR
600 DO 601 NZ1=1,NZ
I1=II1(NZ1)
J1=JJ1(NZ1)
K1=KK1(NZ1)
601 A(I1,J1,K1)=X(NZ1)
INE=NEQ-2
GO TO (603,604,605,606,607),INE
603 CALL DET3[A,D]
GO TO 602
604 CALL DET4[A,D]
GO TO 602
605 CALL DET5[A,D]
GO TO 602
606 CALL DET6[A,D]
GO TO 602
607 CALL DET7[A,D]
602 ND=2*NEQ
CALL POLYEL[ND,D,O]
GO TO 120
END
SUBROUTINE DET [A,N,D]
DIMENSION A(14,14)
D=1.
DO 9 K=1,N-1
BIG=A(K,K)
JM=K
DO 2 I=K+1,N
IF (ABS[BIG]-ABS[A(K,I)]) 1,2,2
1 BIG=A(K,I)
JM=I
2 CONTINUE
IF (JM-K) 3,5,3
3 DO 4 I=K,N
TEMP=A(I,K)
A(I,K)=A(I,JM)
4 A(I,JM)=TEMP
D=-D
5 D=D*BIG
IF (D) 7,6,7
```

```
6 RETURN
7 DO 9 I=K+1,N
IF (A(I,K)) 8,9,8
8 DO 9 J=K+1,N
A(I,J)=A(I,J)-A(I,K)*A(K,J)/BIG
9 CONTINUE
D=D*A(N,N)
RETURN
END
SUBROUTINE COST [ND,D,NC,C,Q,CF]
DIMENSION DD(14,14),A(14,14),D(0/14),C(0/14,7),Q(7)
1 IF (D(ND)) 3,2,5
2 ND=ND-1
GO TO 1
3 DO 4 I=0,ND
4 D(I)=-D(I)
5 IF (D(ND-1)) 6,6,7
6 CF=-1.
RETURN
7 DO 8 I=1,ND
DO 8 J=1,ND
8 DD(I,J)=0.
DO 9 I=0,ND,2
DO 9 J=1,ND,2
9 DD((1+I+J)/2,J)=D(I)
DO 10 I=1,ND,2
DO 10 J=2,ND,2
10 DD((1+I+J)/2,J)=D(I)
DO 12 NH=2,ND
DO 11 I=1,NH
DO 11 J=1,NH
11 A(I,J)=DD(I+ND-NH,J+ND-NH)
CALL DET [A,NH,H]
IF (H) 6,6,12
12 CONTINUE
DO 13 I=1,ND
13 DD(I,ND)=0.
DO 17 I=1,NC
DO 15 M=0,ND-1,2
X=0.
DO 14 K=0,M
14 X=X+(-1.)**K*C(K,I)*C(M-K,I)
15 DD(1+M/2,ND)=DD(1+M/2,ND)+.5*X*Q(I)**2
M1=M
DO 17 M=M1,2*ND-2,2
X=0.
DO 16 K=M-ND+1,ND-1
16 X=X+(-1.)**K*C(K,I)*C(M-K,I)
17 DD(1+M/2,ND)=DD(1+M/2,ND)+.5*X*Q(I)**2
CALL DET [DD,ND,X]
CF=-X*(-1.)**ND/(H*D(ND))
RETURN
END
       SUBROUTINE DET7[E,P7]
       DIMENSION E(7,7,0/2),P7(0/14),D(7,7,0/2),P6(0/12)
       DO 1 I0=0,14
```

```
    1  P7(IO)=0.
       DO 2 I=1,7
C:     TEST FOR ZERO ELEMENT
       IF(E(I,7,0)) 3,4,3
    4  IF(E(I,7,1)) 3,5,3
    5  IF(E(I,7,2)) 3,2,3
    3  DO 6 L=1,6
       DO 6 M=1,6
       DO 6 N=0,2
       IF(L-I) 7,8,8
    7  L1=L
       GO TO 6
    8  L1=L+1
    6  D(L,M,N)=E(L1,M,N)
       CALL DET6[D,P6]
       S=(-1.)**(I+1)
       P7(0)=P7(0)+S*E(I,7,0)*P6(0)
       P7(1)=P7(1)+S*(E(I,7,0)*P6(1)+E(I,7,1)*P6(0))
       DO 9 IO=2,12
    9  P7(IO)=P7(IO)+S*(E(I,7,0)*P6(IO)+E(I,7,1)*P6(IO-1)+
E(I,7,2)*P6(IO-2))
       P7(13)=P7(13)+S*(E(I,7,1)*P6(12)+E(I,7,2)*P6(11))
       P7(14)=P7(14)+S*E(I,7,2)*P6(12)
    2  CONTINUE
       RETURN
       END
       SUBROUTINE DET6[D,P6]
       DIMENSION D(7,7,0/2),P6(0/12),C(7,7,0/2),P5(0/10)
       DO 1 IO=0,12
    1  P6(IO)=0.
       DO 2 I=1,6
C:     TEST FOR ZERO ELEMENT
       IF(D(I,6,0)) 3,4,3
    4  IF(D(I,6,1)) 3,5,3
    5  IF(D(I,6,2)) 3,2,3
    3  DO 6 L=1,5
       DO 6 M=1,5
       DO 6 N=0,2
       IF(L-I) 7,8,8
    7  L1=L
       GO TO 6
    8  L1=L+1
    6  C(L,M,N)=D(L1,M,N)
       CALL DET5[C,P5]
       S=(-1.)**I
       P6(0)=P6(0)+S*D(I,6,0)*P5(0)
       P6(1)=P6(1)+S*(D(I,6,0)*P5(1)+D(I,6,1)*P5(0))
       DO 9 IO=2,10
    9  P6(IO)=P6(IO)+S*(D(I,6,0)*P5(IO)+D(I,6,1)*P5(IO-1)+
D(I,6,2)*P5(IO-2))
       P6(11)=P6(11)+S*(D(I,6,1)*P5(10)+D(I,6,2)*P5(9))
       P6(12)=P6(12)+S*D(I,6,2)*P5(10)
    2  CONTINUE
       RETURN
       END
       SUBROUTINE DET5[C,P5]
```

```
      DIMENSION C(7,7,0/2),P5(0/10),B(7,7,0/2),P4(0/8)
      DO 1 IO=0,10
    1 P5(IO)=0.
     DO 2 I=1,5
C:  TEST FOR ZERO ELEMENT
      IF(C(I,5,0)) 3,4,3
    4 IF(C(I,5,1)) 3,5,3
    5 IF(C(I,5,2)) 3,2,3
    3 DO 6 L=1,4
      DO 6 M=1,4
      DO 6 N=0,2
      IF(L-I) 7,8,8
    7 L1=L
      GO TO 6
    8 L1=L+1
    6 B(L,M,N)=C(L1,M,N)
      CALL DET4[B,P4]
      S=(-1.)**(I+1)
      P5(0)=P5(0)+S*C(I,5,0)*P4(0)
      P5(1)=P5(1)+S*(C(I,5,0)*P4(1)+C(I,5,1)*P4(0))
      DO 9 IO=2,8
    9 P5(IO)=P5(IO)+S*(C(I,5,0)*P4(IO)+C(I,5,1)*P4(IO-1)+
 C(I,5,2)*P4(IO-2))
      P5(9)=P5(9)+S*(C(I,5,1)*P4(8)+C(I,5,2)*P4(7))
      P5(10)=P5(10)+S*C(I,5,2)*P4(8)
    2 CONTINUE
      RETURN
      END
      SUBROUTINE DET4[B,P4]
      DIMENSION B(7,7,0/2),P4(0/8),A(7,7,0/2),P(0/6)
      DO 1 IO=0,8
    1 P4(IO)=0.
      DO 2 I=1,4
C:  TEST FOR ZERO ELEMENT
      IF(B(I,4,0)) 3,4,3
    4 IF(B(I,4,1)) 3,5,3
    5 IF(B(I,4,2)) 3,2,3
    3 DO 6 L=1,3
      DO 6 M=1,3
      DO 6 N=0,2
      IF(L-I) 7,8,8
    7 L1=L
      GO TO 6
    8 L1=L+1
    6 A(L,M,N)=B(L1,M,N)
      CALL DET3[A,P]
      S=(-1.)**I
      P4(0)=P4(0)+S*B(I,4,0)*P(0)
      P4(1)=P4(1)+S*(B(I,4,0)*P(1)+B(I,4,1)*P(0))
      DO 9 IO=2,6
    9 P4(IO)=P4(IO)+S*(B(I,4,0)*P(IO)+B(I,4,1)*P(IO-1)+B(
 I,4,2)*P(IO-2))
      P4(7)=P4(7)+S*(B(I,4,1)*P(6)+B(I,4,2)*P(5))
      P4(8)=P4(8)+S*B(I,4,2)*P(6)
    2 CONTINUE
      RETURN
```

```
        END
        SUBROUTINE DET3[A,P]
        DIMENSION A(7,7,0/2),D(0/2,0/2,0/2),P(0/6)
        DO 1 L=0,2
        DO 1 M=0,2
        DO 1 N=0,2
    1   D(L,M,N)=A(1,1,L)*A(2,2,M)*A(3,3,N)+A(1,2,M)*A(2,3,
N)*A(3,1,L)+A(1,3,N)*A(3,2,M)*A(2,1,L)-A(1,3,N)*A(2,2,M)*A
(3,1,L)-A(1,2,M)*A(2,1,L)*A(3,3,N)-A(1,1,L)*A(3,2,M)*A(2,3
,N)
        P(0)=D(0,0,0)
        P(1)=D(0,0,1)+D(0,1,0)+D(1,0,0)
        P(2)=D(2,0,0)+D(0,2,0)+D(0,0,2)+D(1,1,0)+D(1,0,1)+D
(0,1,1)
        P(3)=D(1,1,1)+D(1,2,0)+D(1,0,2)+D(0,1,2)+D(0,2,1)+D
(2,0,1)+D(2,1,0)
        P(4)=D(2,2,0)+D(2,0,2)+D(0,2,2)+D(1,1,2)+D(1,2,1)+D
(2,1,1)
        P(5)=D(2,2,1)+D(2,1,2)+D(1,2,2)
        P(6)=D(2,2,2)
        RETURN
        END
SUBROUTINE POLYEL[NDEG,POLY,NPC]
DIMENSION POLY(0/14),A(15),B(16),C(3),F(2),ROOTS(30),CHECK(16),
ROOT(15,2)
TYPE 600
        IF(NPC) 501,500,501
  501   TYPE 502
  502   FORMAT($POLYNOMIAL COEFFICIENTS$/)
        TYPE 503,(POLY(I),I=0,NDEG)
  503   FORMAT(E14.6)
TYPE 600
600 FORMAT(//)
  500 P=0
Q=0
IERROR=0
DEG=FLOAT[NDEG]
NC=ABS[DEG]+1.
IF(DEG)199,1,1
1 CTEST=1.E+8
C:IGNORE CASES OUT OF RANGE
IF(NC-36)2,2,29
2 DO 3 I=1,NC
3 A(I)=POLY(NC-I)
C:MAKE LEADING COEFFICIENT NON ZERO
4 IF(A(1)) 7,5,7
5 NC=NC-1
DO 6 I=1,NC
6 A(I)=A(I+1)
IF(NC-36)4,4,199
7 SCALE=A(1)
N=NC-1
C:REMOVE ZERO ROOTS
8 IF(A(N+1))10,9,10
9 ROOTS(2*N)=0
ROOTS(2*N-1)=0
```

```
N=N-1
GO TO 8
C:DIVIDE COEFFICIENTS BY THEIR GEOMETRIC MEAN
10 TEMP=0
DO 12 I=1,NC
IF(A(I))11,12,11
11 TEMP=TEMP+ALOG[ABS[A(I)]]
12 CONTINUE
TEMP=EXP[2.302585*TEMP/NC]
DO 13 I=1,NC
13 A(I)=A(I)/TEMP
REV=1
GATE=0
C:GET NEXT FACTOR
20 CNVRG=CTEST
IF(N) 101,101,21
21 IF(N-2)22,39,40
22 R=-A(2)/A(1)
C:LINEAR FACTOR CONVERGED
23 DO 38 I=1,N
38 A(I+1)=A(I+1)+R*A(I)
IF(REV)24,25,25
24 R=1./R
25 ROOTS(2*N-1)=R
ROOTS(2*N)=0
N=N-1
GATE=1
GO TO 20
39 P=A(2)/A(1)
Q=A(3)/A(1)
C:QUADRATIC FACTOR CONVERGED
26 IF(REV)27,28,28
27 Q=1./Q
P=P*Q
28 DISCR=P*P/4.-Q
IF(DISCR)29,30,30
29 ROOTS(2*N-1)=-P/2.
ROOTS(2*N-3)=-P/2.
ROOTS(2*N)=SQRT[-DISCR]
ROOTS(2*N-2)=-ROOTS(2*N)
GO TO 33
30 TEMP=SQRT[DISCR]
IF(P)31,32,32
C:AVOID SIGNIFICANCE LOSS IN SUBTRACTION
31 TEMP=-TEMP
32 ROOTS(2*N-3)=-P/2.-TEMP
ROOTS(2*N-1)=Q/ROOTS(2*N-3)
ROOTS(2*N-2)=0
ROOTS(2*N)=0
33 N=N-2
GATE=0
IF(N) 20,20,133
133 DO 34 I=1,N
34 A(I+1)=B(I+2)
GO TO 20
C:ELLENBERGER'S TEST FOR WHETHER TO REVERSE COEFFICIENTS
```

```
40 IF(ABS[A(2)/A(1)]-ABS[A(N)/A(N+1)])44,44,41
C:REVERSE COEFFICIENTS TO AVOID ACCURACY LOSS IN REDUCED P
OLYNOMIAL
41 REV=-REV
L=(N+1)/2
DO 42 I=1,L
TEMP=A(N+2-I)
A(N+2-I)=A(I)
42 A(I)=TEMP
IF(CTEST/1000.-CNVRG) 45,45,43
43 TEMP=PSAVE
PSAVE=P
P=TEMP
TEMP=QSAVE
QSAVE=Q
Q=TEMP
TEMP=RSAVE
RSAVE=R
R=TEMP
GO TO 48
44 IF(GATE*(P*P/4.-Q))37,45,45
C:COMPUTE FIRST GUESS
45 IF(A(N-1))47,46,47
46 P=-1.
Q=-1.
GO TO 37
47 Q=A(N+1)/A(N-1)
P=(A(N)-Q*A(N-2))/A(N-1)
37 R=0
C:START SYNTHETIC DIVISIONS.
48 DO 70 L=1,50
B(1)=0
C(2)=0
C(3)=0
B(2)=A(1)
F(2)=A(1)
E=A(1)
DO 49 I=1,N
E=A(I+1)+R*E
F(1)=F(2)
F(2)=E+R*F(1)
B(I+2)=A(I+1)-P*B(I+1)-Q*B(I)
C(1)=C(2)
C(2)=C(3)
49 C(3)=B(I+1)-P*C(2)-Q*C(1)
C:TEST QUADRATIC FOR CONVERGENCE
IF(A(N)*B(N+1))50,51,50
50 IF(ABS[A(N)/B(N+1)]-CNVRG)53,51,51
51 B(N+2)=A(N+1)-Q*B(N)
IF(B(N+2))52,26,52
52 IF(ABS[A(N+1)/B(N+2)]-CNVRG)53,26,26
C:TEST LINEAR FOR CONVERGENCE
53 IF(E)54,23,54
54 IF(CNVRG-ABS[A(N+1)/E])23,23,55
55 IF(F(1))57,56,57
56 R=R+1
```

```
GO TO 58
C:COMPUTE NEWTON CORRECTION
57 R=R-E/F(1)
C:COMPUTE BAIRSTOW CORRECTION
58 C(3)=-P*C(2)-Q*C(1)
TEMP=C(2)*C(2)-C(3)*C(1)
IF(TEMP) 60,59,60
59 Q=Q*(Q+1.)
P=P-2.
GO TO 70
60 P=P+(B(N+1)*C(2)-B(N+2)*C(1))/TEMP
Q=Q-(B(N+1)*C(3)-B(N+2)*C(2))/TEMP
70 CONTINUE
C:REDUCE CONVERGENCE REQUIREMENT
IF(CNVRG-CTEST)72,71,71
71 SWTCH=1
CNVRG=CNVRG/100.
GO TO 48
72 SWTCH=-SWTCH
IF(CTEST/1000.-CNVRG)73,73,74
73 PSAVE=P
QSAVE=Q
RSAVE=R
74 IF(SWTCH)41,41,75
75 CNVRG=CNVRG/100.
GO TO 41
C:EXPAND ROOTS TO POLYNOMIAL
101 B(NC)=SCALE
GO TO 103
102 B(NC)=POLY(1)
IF(NC-36)103,199,199
103 B(NC+1)=0
B(NC+2)=0
N=0
104 L=NC-N
I=2*N+1
IF(I-L)109,110,110
109 IF(ROOTS(I+1))105,107,105
105 P=-2.*ROOTS(I)
Q=ROOTS(I)*ROOTS(I)+ROOTS(I+1)*ROOTS(I+1)
B(L-2)=B(L)
B(L-1)=P*B(L)+B(L+1)
DO 106 I=L,NC
106 B(I)=Q*B(I)+P*B(I+1)+B(I+2)
N=N+2
GO TO 104
107 R=-ROOTS(I)
B(L-1)=B(L)
DO 108 I=L,NC
108 B(I)=R*B(I)+B(I+1)
N=N+1
GO TO 104
110 DO 111 I=1,NC
111 CHECK(I)=B(I)
GO TO 200
199 IERROR=1
```

```
200 DO 201 I=1,NC-1
ROOT(I,1)=ROOTS(2*I-1)
201 ROOT(I,2)=ROOTS(2*I)
K=0
I=0
214 I=I+1
IF(ROOT(I,2))202,208,208
202 IDELT=(N-1)-I
DO 204 J=0,IDELT
ROOT(J+I,1)=ROOT((J+I)+1,1)
204 ROOT(J+I,2)=ROOT((J+I)+1,2)
ROOT(N,1)=0
ROOT(N,2)=0
K=K+1
N=N-1
I=I-1
208 IF(I-N)214,213,213
213 NS=NC-K-1
DO 211 I=1,NS
IF(ROOT(I,2))210,209,210
209 ROOT(I,1)=-ROOT(I,1)
 IF(ABS[ROOT(I,1)]-.001)212,212,224
212 WRITE(1,125) ROOT(I,1)
125 FORMAT(2XE11.4/)
GO TO 211
224 WRITE(1,127) ROOT(I,1)
127 FORMAT(2XF10.3/)
GO TO 211
210 RE=-ROOT(I,1)
AIM=ROOT(I,2)
OMEGA=SQRT[AIM**2+RE**2]
ZETA= RE/OMEGA
IF(ABS[OMEGA]-.001)215,215,217
217 IF(ABS[ZETA]-.001)215,215,218
215 WRITE(1,126)ZETA,OMEGA,RE,AIM
126 FORMAT(2X,4(E11.4)/)
GO TO 211
218 WRITE(1,128)ZETA,OMEGA,RE,AIM
128 FORMAT(2X,4(F10.3)/)
211 CONTINUE
RETURN
END
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13. ABSTRACT

A systematic procedure for the design of aircraft stability augmentation systems is presented. The key features of this procedure are the selection of essential feedbacks from an examination of several handling quality metrics and the use of parameter optimization techniques to determine the numerical values of the SAS parameters. The optimization problem is structured to include both manual and SAS feedbacks. The cost function includes pilot tracking errors and SAS control deflections. A method of selecting the relative weighting is presented.

The feasibility of this procedure is demonstrated by applying it to the longitudinal axis of the F-4 aircraft. Three widely different flight conditions are selected. For all three, the same SAS form (pitch rate and normal acceleration feedbacks to the elevator), the identical problem formulation, and the same method of selecting the cost function weights are used. The resulting systems are judged quite satisfactory and well within the short-period requirements of the current military handling qualities specification.

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