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**FLUERICS: 29. TRANSIENT RESPONSE
OF SOME FLUERIC COMPONENTS TO STEP WAVES
AND APPLICATION TO MATCHING**

by
Jorma R. Keto

September 1970

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ABSTRACT

Experimental results are shown which verify predicted response characteristics of terminations of transmission lines. Idealized, simple theory is used to determine matching conditions and reflections when not matched. Orifice termination, branch circuits, and change of transmission-line cross section are considered.

Special apparatus and circuits are described for making the necessary measurements. These include (1) a step-wave generator, (2) a transducer adaptor, (3) a wave bridge, (4) a variable tap, (5) a probe shifter, (6) flow dividers, and (7) a specially modified solenoid valve.

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1. INTRODUCTION

This report is a compilation of basic concepts and simple, usually qualitative experiments directed toward the illustration and understanding of wave effects in fluid lines and elements. The analyses are not intended to supply a rigorous approach to generalized wave theory but to offer to the practical engineer ways of solving problems, due to reflections, arising in the development of high-speed flueric digital devices and circuits.

The time interval between the appearance of the command signal at the control-jet nozzle and the settling of the flow in the switched position has been defined as the response time for bistable elements.¹ Obviously, the shortest possible time interval in which this can occur is the time required for a pressure wave to propagate from the control nozzle to the outlet of the interaction region of the element. But this time cannot be the response time unless a fully settled final steady-state flow is established at the instant that the pressure wave passes the outlet. Consequently, no upstream moving waves within the outlet, as a result of reflections from downstream loads or couplings, can be tolerated if minimum response time is to be achieved.

In addition to the requirement of eliminating reflected waves from the outlet, predictable operation demands that no stray signals resulting from reflections appear at the control nozzle of a following stage. These reflections can occur from flow dividers or control inputs of other units in the case of fanout.

Finally, in contrast with applications requiring elimination of reflected waves, circuits exist which are based upon reflections. Among these are pulse-forming devices; single-shot devices; feedback, high-stability, and triggered oscillators.

In view of the need to understand the effect of various components upon wave phenomena, this report describes a number of simple experiments that were conducted and discusses comparisons that were made with simple theory to determine its relevance.

2. TERMINOLOGY

2.1 Definition of Terms

A simple wave is a fundamental wave in which the change of particle velocity and change of acoustic velocity are dependent functions of each other.

¹Katz, Winston, Howes, "The Response Time of a Bistable Fluid Amplifier to a Step Input," Second Fluid Amplification Symposium, Harry Diamond Laboratories, Washington, D. C. 20438, May 1964.

The pressure reflection coefficient is defined as $(p_1 - p_2) / (p_2 - p_0)$, where p_0 is the ambient pressure in a transmission line prior to being traversed by a step-wave of amplitude p_2 , and p_1 is the pressure following the reflection of the step-wave from a terminal of the transmission line (fig. 9).

The output mass flow coefficient is the mass flow transmitted at a terminal of a transmission line subjected to a step-wave, normalized to the mass flow resulting at a cross section of the transmission line due to the step wave.

The output power coefficient is the power transmitted through a termination of a transmission line normalized to the power being transmitted behind the incident wave.

The contact surface is the interface of adjacent regions of fluid which differ in property without flow across the interface.

2.2 Symbols

ρ = density, absolute

p = pressure

u = mean fluid particle velocity relative to stationary system

a = acoustic velocity relative to fluid

T = temperature, absolute

A = area

P = pressure, normalized to ambient

γ = ratio of specific heats $\frac{c_p}{c_v} = 1.4$ for air

U = velocity of fluid, normalized by ambient acoustic velocity

α = pressure reflection coefficient

Subscripts

o = ambient

s = source

Subscripts (cont'd)

n = nozzle exit plane

1,2,3 = location in space or time as specified

L = transmission line

3. APPARATUS

3.1 Initial Wave Circuit

The first experiments reported here were performed with the equipment used to determine previously reported transient behavior of bistable elements.² Figure 1 is a schematic of the wave-forming circuit. Pulsating signals were obtained by using a rotating slotted-drum generator and adjusting the lengths of 3/8-in. ID tubing to constant multiples of wave length consistent with the termination at the distal end. A reduced-amplitude signal was tapped from the main line by means of a standard reducing tubing tee which was modified by boring the straight-through head to 3/8-in. inside diameter and by inserting an orifice into the stem flush with the 3/8-in. bore. The stem was then bored up to the orifice to match the inside diameter of the 1/4-in. ID tubing. A quartz pressure transducer was mounted in an adapter (fig. 2) with an essentially constant internal cross-sectional area equal to that of the tubing. This was placed at the end of the 1/4-in. ID tubing as close to the component being tested as possible.

When single pulses were desired, a solenoid valve (fig. 3) was substituted for the slotted drum with a long length of 1/2-in. OD tubing connecting it to the source. The distal end of the main tube was now terminated open ended, and the distance from the open end to the tap, adjusted to give the pulse duration desired (the rarefaction reflected from the open end "cutting off" the positive step created by opening the solenoid valve).

3.2 Wave Bridge

Following the experiments on flow dividers, a wave bridge (fig. 4) was constructed in order to record the incident wave and to record the reflected pressure wave by subtracting the incident (reference) wave from the final pressure at the terminal of the component being tested (sect. 4.1.2). In order to obtain waves in the form of

²Keto, J. R., "Transient Behavior of Bistable Fluid Elements," Second Fluid Amplifier Symposium, Harry Diamond Laboratories, Washington, D. C. 20438, May 1964.

step functions with consistently fast rise times and flat tops, a sufficiently high source pressure is used to permit the wave to be nearly in shock by the time it arrives at the tap. The variable tap (fig. 5) is used to "tap off" a signal of the desired amplitude. A balanced zero reflection flow divider (fig. 6) is used to obtain identical signals in two lines. One of the lines goes to the component being tested with a quartz transducer located near its input terminal. The other line goes to a probe shifter (fig. 7) which permits a quartz transducer to be moved along the line to null the transport time of the two signals.

4. THEORETICAL ANALYSIS

4.1 General Considerations

4.1.1 Changes of Fluid State Relative to Switching of Bistable Elements

The introduction of a disturbance at any location in a subsonic flow field causes a perturbation that spreads by means of waves throughout the flow field with a local rate relative to the fluid determined by the local state of the fluid. The waves cause changes in state and velocity. Boundary conditions of the field and inhomogeneities within the flow itself cause refractions and reflections of the waves. The determination of the instantaneous flow field during a transient can therefore be extremely complex, especially within the interaction region of a multiport fluoric device. External to the device, the problem can be simplified by the consideration of these effects within fluid transmission lines coupled to components which introduce area changes to the flow.

The instantaneous conditions existing in the active output line of a bistable element a short time after switching are illustrated in figure 8. In this case, it is assumed that the line is sufficiently long so that changes of state can be considered to take place discontinuously and that the output switches from one steady state to another in a single step. The fluid states within the line are divided into the following: Region A is the portion that has not yet been subjected to any signal or pressure wave from the disturbance. The conditions are those existing prior to the switch and are derived from stagnation conditions other than the source of power for the element. The initial conditions here could result from flow entrained from the surrounding atmosphere before switching if vents exist downstream. Region B is the portion of fluid that was within the transmission line prior to the switch but which has been traversed by the compressional wave resulting from the switch. Region C is the fluid that originates from the source of power for the element. As this

fluid originates from a source (Region S) that usually is not at the same stagnation condition as the fluid in Regions A and B, discontinuities exist in temperature, density, and sound velocity at the interface (contact surface) of the two fluids. The significance of these two regions (B and C) to a particular application is related to the length of transmission line between the output terminal of the bistable element in question and the next component. The difference in the time of arrival of the wave front between A and B and the time of arrival of the contact surface between B and C at the next component downstream is the difference between the response time and the transport delay of the bistable element augmented by a time dependent upon the length of the transmission line. Furthermore, any subsequent waves, such as reflections of the initial wave from the next component, will be partially reflected at the contact surface giving rise to "noise" in the output of the element.

It should be noted that the contact surface discontinuities can in principle be eliminated, thus improving the response time and decreasing noise level. This can be accomplished by adjusting the effective stagnation temperature of the source of power for the element until the temperature discontinuity at the contact surface disappears.

4.1.2 Character and Superposition of Waves

The transient during the switching of a bistable element can be considered as existing from the time of the introduction of a disturbance into an existing steady-state flow until the re-establishment of a final steady-state flow, such as from before to after switching. Regions of steady flow may exist within areas of the element-transmission-line system, but unsteady flow will exist somewhere within that system during the transient. If the waves propagated during the transient are of small amplitude, the "principle of linear superposition" may be employed to resolve the flow field into fundamental components consisting of a steady flow plus harmonic waves of appropriate amplitude, frequency, phase, and direction. However, when waves of finite-amplitude are propagated during the transient, linear superposition can no longer be employed to obtain the unsteady flow field. Finite-amplitude waves can be thought of as successive pressure steps linked to form an approximation of the finite-amplitude wave. Thus, changes in parameters of the flow field are arrived at by progressing in a stepwise fashion through the small changes produced by the linked pressure changes that approximate the actual finite-amplitude wave.³

³Shapiro, A. H., "The Dynamics and Thermodynamics of Compressible Fluid Flow," Vol II, Ronald Press, New York, 1954, p. 915.

4.2 Orifice Termination of Transmission Line

4.2.1 General Characteristics

Consider a simple step-function wave traveling to the right along a line of uniform cross section. The fluid state ahead of the wave and the ambient are identical and stagnant; no contact-surface discontinuity is considered to exist. The wave arrives at an orifice (short nozzle) terminating the lines and reflects. The reflected wave (fig. 9) travels to the left into the steady-state flow which was established upstream by the initial wave and leaves behind it a new steady-state flow that feeds the jet issuing from the orifice into the ambient. The situation is defined by the following assumptions and equations where the subscripts refer to the regions of figure 9.

4.2.2 Assumptions

The theoretical considerations are based upon the following assumptions unless otherwise indicated:

- (a) Ideal gas
- (b) One-dimensional flow
- (c) Inviscid flow
- (d) Subsonic flow
- (e) Homenergetic flow
- (f) Inelastic walls

4.2.3 Equations Used

$$\text{Continuity: } \rho_1 A_1 u_1 = \rho_n A_n u_n \quad (1)$$

Conservation of energy:

$$a_1^2 + \frac{\gamma - 1}{2} u_1^2 = a_n^2 + \frac{\gamma - 1}{2} u_n^2 = a_0^2 \quad (2)$$

Simple wave:

(1) Right traveling

$$a_2 + a_0 = - \frac{\gamma - 1}{2} (u_2 - u_0) \quad (3)$$

(2) Reflection

$$a_1 - a_2 = - \frac{\gamma - 1}{2} (u_1 - u_2) \quad (4)$$

Acoustic speed

$$a = (\gamma RT)^{\frac{1}{2}} = \left(\frac{\gamma p}{\rho} \right)^{\frac{1}{2}} \quad (5)$$

Isentropic expansion

$$\frac{p}{\rho^\gamma} = \text{const} \quad (6)$$

$$\text{General gas: } \frac{p}{\rho} = RT \quad (7)$$

where: ρ = density, absolute
 A = area of cross section of flow
 u = mean fluid particle speed relative to stationary system
 a = acoustic speed
 γ = ratio of specific heats, $C_p/C_v = 1.4$ for air
 R = universal gas constant
 T = temperature, absolute
 p = pressure, absolute

subscripts 0, 1, 2, n refer to location as shown in figure 9.

The approximation that equations 3, 4, and 6 hold across the wave front is used in order to simplify the resulting equations. The errors this causes are small even at the highest pressures used.

(Notice that the equations of continuity of mass flow and energy do not hold across the wave front or a contact surface. Also, one-dimensional flow is assumed to exist everywhere, and no wave effects are considered external to the nozzle.)

By combining these equations and stating the results in normalized form, the following are obtained for $\gamma = 1.4$:

From equations 1, 2, 5, 6, and 7:

$$U_1 = S_n \sqrt{\frac{5(1 - P_1^{2/7})}{S_n^2 - P_1^{12/7}}} \quad (8)$$

From equations 1 and 6

$$U_n = \frac{P_1^{5/7} U_1}{S_n} \quad (9)$$

From equations 3, 4, 5, and 6 with $u_0 = 0$

$$U_2 = \frac{U + 5(P_1^{1/7} - 1)}{2} \quad (10)$$

From equations 3, 5, and 6 with $u_0 = 0$

$$P_2 = \left(\frac{U_2}{5} + 1 \right)^7 \quad (11)$$

$$\alpha = \frac{P_1 - P_2}{P_2 - 1} \quad (12)$$

where

$$U = u/a_0$$

$$P = p/p_0$$

$S_j = A_j/A_L$, area at location j normalized by transmission line area.

Subscripts 1, 2, n refer to location as shown in figure 9.

The above equations can be solved for various theoretical characteristic behaviors of the orifice-terminated transmission line. Some of these are plotted in figure 10a through 10e. All of these graphs have located on them a "match line" (or "match point") and a "sonic line." The match line specifies the locus of conditions that permits the steady-state plateau of a step-wave to be transmitted through the orifice without reflection. This will be discussed in greater detail in section 4.2.2. The sonic line is the condition at which the flow is sonic in the nozzle and therefore a boundary of the present discussion.

The pressure reflection coefficient (fig. 10a) of minus one exists for all values of amplitude of the incident wave (within the sonic limit) for a termination area ratio of one. This is the characteristic for a completely open-ended line in which case a rarefaction is reflected which is equal in magnitude to the incident wave. For a line that is completely closed at its end, a pressure

reflection coefficient exists that is greater than one. This results from the dynamic pressures that are present but not included in the definition since the pressure amplitudes have been defined by the static pressures only. The experimental values plotted on this curve will be discussed in section 5.1.2.

Figure 10b shows the idealized relation of the mass flow delivered at the nozzle, normalized in terms of the mass flow existing behind the initial pressure wave. Although area ratios greater than that required for a match produce increased mass flow over that initially available, one must remember that the reflected wave must be accounted for (see for example the mass flow oscillator described in reference 2).

The output power coefficient of figure 10c is the ratio of the total energy per unit time delivered at the nozzle to the total energy per unit time passing a section of the transmission line due to the initial wave. No consideration is given to the efficiency of utilization of the power delivered. An output power coefficient that is greater than 100 percent occurs during a mismatch for negative reflection coefficients in the case of reflection of a single step-wave. When a positive step-wave traverses a transmission line, the energy content per unit length of line is increased; when traversed by a negative step-wave, the energy content is decreased. A negative reflection from a termination will therefore increase the power delivered to the load by an amount equal to the time rate of decrease of the stored energy.

The flow velocity at the nozzle is shown in figure 10d as dependent on area ratio and the over-pressure of the initial wave normalized to ambient.

The relation of the mass flow and pressure reflection coefficients is shown in figure 10e.

4.2.4 Matching with an Orifice

The previous section indicated that a match condition exists in which the steady-state plateau of a step-wave will be transmitted through an orifice (short nozzle) without reflection. As there is no reflected wave, the state of the fluid in region 1 and 2 of figure 9 is identical. The matched case therefore requires a solution of steady-state flow from the state upstream of the nozzle to the state at the nozzle exit with due consideration of the flow upstream of the wave preceding it. In the general case, the flow upstream need not be restricted to region B (fig. 8) but can be flow from the source, as region C (fig. 8). As the pressure and velocity are the same for either side of the contact surface, one needs only to consider the difference in temperature. This can be arrived at from knowledge of the stagnation

temperature of the source relative to the stagnation temperature at source pressure resulting from isentropic compression from the ambient. This temperature factor shows up in the final steady state flow as a discontinuity between the temperature of the primary jet flow and the ambient into which it issues. The area ratio of the nozzle to the transmission line required for a match is given by

$$\frac{A_n}{A_L} = \sqrt{\frac{\frac{2}{\gamma-1} P_2 \frac{1}{\gamma} \left[P_2^{\frac{\gamma-1}{2\gamma}} - 1 \right]}{\frac{T_s}{T_o} P_s \frac{1-\gamma}{\gamma} \left[P_2^{\frac{\gamma-1}{\gamma}} - 1 \right] + \frac{2}{\gamma-1} \left[P_2^{\frac{\gamma-1}{2\gamma}} - 1 \right]^2}}$$

which is derived in appendix A.

This equation is shown graphically in figures 11a and 11b. The acoustic solution is also shown as a straight line tangent to the general equation in the log-log plot of figure 11b. The experimental values plotted are discussed in section 5.1.2.

In practice, the nozzle exit conditions will not be independent of the rate of establishment of the final steady-state conditions; consequently, transient reflections which have not been considered above may exist.

4.2.5 Matching with a Distributed Nozzle or Screen

The assumptions made on the basis of one-dimensional inviscid flow imply plane waves and identical behavior of all streamtubes within a transmission line. Thus one is led to the situation of each streamtube being individually matched by its corresponding nozzle. The ultimate, of course, would be an infinite number of infinitesimal nozzles uniformly distributed so that the total nozzle area would produce a match. Consequently, such devices as multiple-orifice plates, porous membranes, or screens are useful in situations where two-dimensional effects cause reflected pulses. These result when conditions of one-dimensional flow are not met, and a positive reflection from the wall of the nozzle leads the rarefaction wave from the opening. With the two-dimensional effect, both reflections will cause diverging wave fronts which eventually effectively merge into a plane wave, but in the interval of time before they have merged, the positive reflection front tends to steepen; whereas, the negative front sloughs off. Consequently, when they do finally merge, instead of cancelling each other throughout for a perfect match condition, a pip will remain.

Therefore, for highest fidelity in high-frequency matching, the distributed nozzle or screen is required.

4.3 Matching Transmission Lines

4.3.1 Flow Dividers

A flow divider for waves can be developed by isolating streamtubes from the incoming transmission line at any location as long as the wave "sees" a constant area throughout the region where the tubes diverge from each other. In practice, design and construction limitations may sometimes limit the frequency response due to two-dimensional effects or other compromises resulting in the development of noise.

It should be noted that, due to the streamtube identity, all outputs from flow dividers retain wave form and amplitude identical to the input wave. Obviously, the proportion of the power transmitted in each divided leg will be as the ratio of the area of the leg to that of the input line. And needless to say, flow dividers are not reversible inasmuch as the boundaries of the streamtube of any leg are lost to a reverse traveling wave at the junction with other streamtubes. The boundary and therefore reversibility could only be maintained if equal reverse traveling waves arrive at the junction from all legs in phase.

4.3.2 Incoming Transmission Line Larger Than Exit Line

This situation is simply that of a flow divider in which one output leg is the small exit line, and the other leg is terminated by a matching nozzle to an ambient reference. Various designs can be developed in axisymmetric form with the two transmission lines joined by an annular section containing a gap, orifices, porous material, or screening which can be continuously variable in area for matching to the amplitude of the incoming wave.

4.3.3 Incoming Transmission Line Smaller Than Exit Line

In this case, the geometry is the opposite of the previous section and cannot be treated by separation of streamtubes, as a flow divider is not reversible.

Because a rarefaction is reflected from a direct mating of a smaller line to a larger one, a constriction is required at the junction. However, if one considers the use of a convergent-divergent nozzle with isentropic flow, the pressure-velocity relationship of the wave in the area upstream of the nozzle will be re-established downstream of the nozzle throat where an equal area exists.

As no energy change is assumed in the flow, a convergent-divergent nozzle can only be used to connect two lines of equal area without continuous reflection. The conservation of energy and mass flow demands, in order to maintain the pressure-velocity relationship required for a wave, that the downstream pressure, velocity, and density be less than that upstream of the junction. This can only be accomplished by a throttling process with flow separation and mixing. This is illustrated in figure 12.

When the area ratio of the transmission lines is known, the relation of the downstream pressure to that upstream as determined from conservation of mass flow combined with the simple wave equation is:

$$P_3^{-\gamma} \left(P_3^{\frac{\gamma-1}{2\gamma}} - 1 \right) = \frac{A_2}{A_3} P_2^{-\gamma} \left(P_2^{\frac{\gamma-1}{2\gamma}} - 1 \right)$$

Once P_3 is known, the problem reduces to one of terminating the upstream transmission line for steady-state flow with a nozzle exhausting the flow from the upstream state to P_3 isentropically. The equation for matching with an orifice (sect. 4.2.2) becomes modified for the conditions above to

$$\frac{A_n}{A_L} = \frac{\frac{2}{\gamma-1} P_2^{\frac{1}{\gamma}} \left[P_2^{\frac{\gamma-1}{2\gamma}} - 1 \right]}{\frac{T_s}{T_0} P_s^{\frac{1-\gamma}{\gamma}} \left[\left(\frac{P_2}{P_s} \right)^{\frac{\gamma-1}{\gamma}} + 1 \right] + \frac{2}{\gamma-1} \left[P_2^{\frac{\gamma-1}{2\gamma}} - 1 \right]^2}$$

The same discussion as is given in the appendix (sect. A.2 and A.3) applies to the term $\frac{T_s}{T_0} \left(P_s \right)^{\frac{1-\gamma}{\gamma}}$

As in the case of the termination with an orifice, four regions exist in time. The first is the transient during which the wave-established flow is upstream of the nozzle and a steady-state condition has not yet been established downstream; the second is with wave-established flow upstream and established steady-state flow downstream; third, with the flow upstream that existing behind a contact surface and re-adjusting transient flow downstream; finally, with source-originated flow upstream and steady-state flow downstream.

5. EXPERIMENTAL RESULTS

5.1 Transmission Line Terminated with Orifice

5.1.1 Qualitative Evaluation of Orifice Matching

A series of quick experiments were performed to illustrate the effect of orifice-type terminations on pressure waves in transmission lines. The equipment described in section 3.1 was used with the orifice or termination being the "component under test" of figure 1. A quartz transducer was used to measure the effect when a finite amplitude wave of about 7 kN/m^2 to 14 kN/m^2 gage (1 to 2 psig) arrived at the termination. The orifice was a square-edged hole drilled into a disc. Figure 13a shows the incident pulse used for the remainder of figure 13 as measured when the line was made continuous. In figure 13b, the line of the wave-forming circuit was completely closed off. A positive amplitude reflection occurs which is delayed on the trace due to the location of the transducer from the closed end. The irregularity of the reflection is due to undetermined experimental errors that are of no interest at this time. Multiple reflections exist due to the small orifice used in the T at the other end of the line. The trace in figure 13c shows reflections nearly eliminated by the use of an appropriately sized orifice. The final trace in figure 13d shows the negative amplitude reflection due to the transmission line of the circuit having been left open-ended.

The orifice was then modified into a short nozzle by turning a smooth bell mouth on the upstream entrance, and its ability to match various shapes was determined. The results are shown in figure 14a - 14d. The match is greatly improved, although some distortion does occur, especially in figure 14c.

Inasmuch as the match only occurs theoretically for a given amplitude with a fixed orifice size, one would expect that the reflections occurring from the mismatched portions of the wave would cause considerable distortion. From the appearances of the waves in figure 14, the distortion seems slight. To further observe the effect of amplitude, a cyclic wave was matched by means of a short nozzle, with the results of figure 15a. When the amplitude of the incoming wave was increased slightly for the same nozzle, a distortion was observed, as in figure 15b. By substituting a large nozzle, a wave-form more nearly resembling the original was achieved. The exact analysis of the distortion in figure 15b cannot be made due to the fact that it is the composite result of repetitive reflections within the circuit used. It is sufficient to state that for the least noticeable distortion, a match should be made for the peak amplitude. It is obvious that even through a greater reflection coefficient would then

exist for the smaller amplitude portions, the net distortion relative to the peak amplitude is diminished.

5.1.2 Comparison of Theory and Experiment, Step Wave

In order to conduct a quantitative comparison of experimental results with the theory of section 4.2.1, two experiments were conducted. Each of these was made by measuring the amplitude p_2 of a step pressure wave with the transducer located slightly upstream of a short nozzle or orifice, and measuring the final pressure p_1 after reflection. In the first experiment, the pressure reflection coefficient was calculated from the measurements for various amplitude initial waves and compared with the theoretical curves. This was done for a completely closed line and one terminated with a short nozzle of an area ratio of 0.178. The results are plotted on figure 10a. The experimental data is in excellent agreement with the theoretical results for the nozzle. Some greater randomness is seen in the results for the blocked line; however, the results are not in disagreement by any greater relative error than for the nozzle.

The second experiment was made with short nozzles and with square-edge orifices with a length approximately equal to the diameter. The incoming step-wave amplitude was adjusted and measured when no reflection could be detected for a number of different orifice diameters for each of three transmission-line diameters. The results are plotted on figures 11a and 11b. The nozzle gave experimental results that are random about the theoretical results (fig. 11a), but the results for the orifice in figure 11b are consistently displaced from the theoretical. If one assumes that the upstream edge of the orifice is sharp and no subsequent flow attachment takes place within the orifice, a discharge coefficient of about 0.6 would be required in the continuity equation to correct for jet contraction. A dashed line on figure 11b is the location of theoretical match for a discharge coefficient of 0.6. If, however, instead of a sharp edge on the upstream entrance of the orifice, a radius of 10 percent of the orifice diameter exists, the discharge coefficient becomes 0.8. Likewise, if the flow reattaches within the orifice, the discharge coefficient becomes about 0.8. This coefficient would be approximately the correct mean for the data. Probably both the slight chamfer or rounding of the edge plus reattachment are responsible. In theory, a discontinuous wave front would initially establish fully attached flow within the orifice tube with a time-dependent growth of separation. The degree of separation for this experiment was not determined.

5.1.3 Comparison of Theory and Experiment, Ramp Wave

In view of the fact that a step pressure wave was used in the previous quantitative experiments to determine amplitude

dependence only, the question arises as to the error introduced by ignoring the time dependence caused by the reflected wave traveling back into a time-dependent forward wave. The time correction can be treated theoretically but can become quite complex and time consuming. From the applied engineering approach, a simple although approximate technique is desired. The time distortion should be slight during the beginning of the ramp since relatively low Mach numbers exist in the flow developed by the wave.

In order to have a reference wave for this experiment, the circuit of figure 1 was modified to approximate the wave bridge of figure 4 by dividing the output at the restricted T into two identical transmission lines. The lines were balanced by shortening one until the times of arrival of the generated signal at a transducer in each line were identical. This was accomplished accurately by having the transducers mounted in identical fittings which permitted a reading to be made close to the end of the fitting, where orifices or matching continuing lines could be attached. This fitting was then closed by a "zero" area orifice, and the difference in transducer readings was examined on an oscilloscope as the longer line was shortened.

The results shown in figure 16 were obtained by keeping the reference leg closed and substituting various area nozzles as terminations in the other leg. Only those readings were used in which no variation of the signal in the reference leg was noted. The amplitude P_2 of the incoming signal was calculated using equations (10) and (11) (sect. 4.2.1) for various points in time from the reference curve and barometric pressure. The value of the reflection coefficient was then taken from figure 10a for each point for the different area ratios. Knowing P_2 and the reflection coefficient permitted a new P_1 to be determined for each point in time for each area-ratio curve. The comparison of the calculated points with the experimental results shows that in the region investigated ($U_2 \leq \text{Mach } 0.13$) errors due to lack of corrections in time are not great.

A recording of the waveform for P_2 was made by using a continuous transmission line in place of a nozzle with the same stagnation reference signal as in figure 16. This is shown in figure 17 along with theoretical values of P_2 calculated from the pressure upstream of a 0.254 area ratio nozzle, for which data is also plotted. The theory predicts the waveform of P_2 nicely but shows a predicted value that decreases from the measured P_2 as time increases. However, in figure 16, the theoretical prediction of the area-ratio curve from the stagnation pressure is in excellent agreement. These facts lead one to

believe that the experimental curve for the continuous line contains a factor that is not as prominent in the theoretical values derived from the stagnation reference curve, or the experimental nozzle curve. Further experimentation revealed the fact that the series-distributed resistance due to viscosity causes the pressure to increase with time at any point in the continuous line. This apparent increase in resistance as the flow fills more and more of the line is not present in the theory nor in the case of flow through a matching nozzle.

5.2 Branch Circuits

5.2.1 Evaluation of Directionality of a T Branch

The effect of turns and branches upon waves was investigated by examining directional effects in a standard T.

The circuit of figure 1 was used to develop a step pressure wave. The wave transmitted by the device tested was measured in its stagnation state by a transducer indicated in the diagrams of figure 18a-e by "transmitted." The reflected wave superimposed on the incident wave was measured by the transducer marked "reflected", after having been delayed by the time required for the reflection to arrive back to the transducer. The incident wave does not show on the oscillograph trace marked "reflected" as it occurred off the picture to the left. The superposition of incident and reflected waves offer difficulties inasmuch as we are interested in observing qualitatively the effects of directionality in the T.

A straight section of tubing was initially used to obtain a reference for comparison as shown in figure 18a. Following this, a T for the plastic tubing was used with equal-length exit branches arranged so that the distances between the transducers and the T were kept invariable.

When an unmodified standard T with an inside diameter less than that of the tubing was used (fig. 18b), the reflected signal shows a short step up and then two downward steps or falls preceding a valley which is followed by a positive step. The first step up is due to the inlet of the T, the first fall from the increase of area caused by the branching into two paths, and the following fall from the enlargement between the termination of the bore of the T and the tubing. The valley following the second fall is at a lower amplitude than the wave ahead of the first rise due to the total area of the two branches being larger than the single tube entering the stem of the T. The positive step following the valley is the sum of the reflections from the closed ends of the equal-length branches. In the case of the transmitted signal, the first rise is the arrival of the initial

transmitted wave; the second rise is a reflection from the closed end of the opposite branch. The reflection from the opposite branch has suffered an attenuation and a stepping of the rise from the presence of the T. When the bore of the T is matched to the tubing, as in figure 18c, the intermediate reflections are eliminated, and the only reflection from the T is from the branching which is now fully achieved at the intersection of the T.

Figures 18d and 18e show measurements taken when the signal approaches the same T from a different leg. Comparison of the results shows no discernable evidence of any directional effects in a T to a wave alone, i.e., when dc is not permitted to be established.

5.2.2 Flow Divider

As was shown above, the T with all legs equal in area cannot be used as a flow divider without a rarefaction wave being reflected upstream. The theory predicts that if a constant area is presented to the wave fronts, no reflection can occur if wall effects are negligible. As a result, flow dividers were designed on the basis of constant area as shown in figure 6. Unfortunately, plastic tubing, such as is used in practice, is not standardized by multiples of cross-sectional area. Consequently, special metal tubing was made to supply the input of the divider in order that the multiple outputs could be matched to standard tubing. (Standard tubing can be used throughout with slight reflection and a loss of power by the use of an orifice-matching bleed designed into the divider by applying the principles of section 4.3.2). It should be remembered that these flow dividers are not reversible. Flow dividers with fanouts of 2x, 4x, and 8x were made and tested, as illustrated below, with consistent results.

The testing method for the divider was identical to that for the T. A reference was taken with the input line continued by a length of the same bore tubing, figure 19a. The flow divider was then substituted, with equal length closed matching-area output tubes, with a transducer being the closure of one. The secondary step that can be seen in the transmitted signal for all of the tests is not related to the divider, but was caused by a reflection from the T with a restricted stem of the wave-forming circuit (fig. 1) that was used since the special input tubing had to be made in short lengths. It will be noted from figure 19b that no undesirable reflections occur from the junction of the input and fanout tubes. The location of the junction was marked on the figure by sliding the divider away from the input tube by about 1 mm at point A. Any reflection caused by the junction would show up as noise in the same relative location in the absence of the gap. Also, unlike the T, the flow divider transmits the pressure wave without any attenuation, as can be seen by comparing the signals with

the flow divider to the reference signals. Although the flow divider is not reversible, no adverse effects to the reflected waves occur in this test because each output leg supplies an identical reflection to the junction simultaneously.

5.3 Area Changes in Continuous Lines

5.3.1 Transducer Adapter

Methods of probing for experimental data such as using in-line transducers should not introduce reflections into the system. Due to the method of making the transducer adapter by swaging round brass tubing into a transition leading into a rectangular section for flush mounting the transducer, difficulty occurs in maintaining constant area. A silicon rubber cast of the interior indicated an apparent bulge in the transition, thus calling for an experimental test.

For this test the wave bridge, having become possible as a result of the flow divider, was used. The top trace in figure 20 shows a section of the plateau of the input step function ($p_2 - p_0$); the second trace, a section of the plateau ($p_1 - p_0$) upstream of the adaptor; the bottom trace is the difference ($p_1 - p_2$) enlarged by a factor of ten to show the reflected pressure wave. Sections of the reflected wave have been identified with corresponding portions of the adaptor. The results indicate not only that bulges do exist at the transitions, but also, that the rectangular section has slightly too large an area.

5.3.2 Matching Larger Transmission Line to a Smaller One

A qualitative test was made to determine the effect of matching by means of an orifice bleed in the case of a larger transmission line mating a smaller one. The result, as determined by use of the wave bridge, is shown in figures 21 and 22. Comparisons were also made under the same conditions for a continuous line, a nozzle-terminated line, and an orifice-matched smaller to larger transmission line.

A comparison to the experiment conducted in figure 21c was made by calculating the reflected wave on the basis of the experimental incident wave. The comparison is shown in figure 23.

5.3.3 Matching a Smaller Transmission Line into a Larger One

The results of the use of an orifice for matching in this instance is shown in figure 21d and compared to other situations with the same incident wave in the same first transmission line. The upstream and downstream effects are shown under matched and mismatched conditions when a convergent-divergent nozzle is used. The reason for the oscillatory reflected signal in the case of a mismatch has not been

thoroughly investigated at this time; however, it is known that a flow that was initially attached, separated, and the point of separation moved upstream. This means that a reflection occurred when the separation took place.

6. CONCLUSION

Simplified idealized wave theory which neglects friction and amplitude-time distortion can be applied to determine reflections and changes of state during, and resulting from, short-duration transient pulses and wave fronts. The errors caused by the neglected factors may increase with time, depending upon the specific situation.

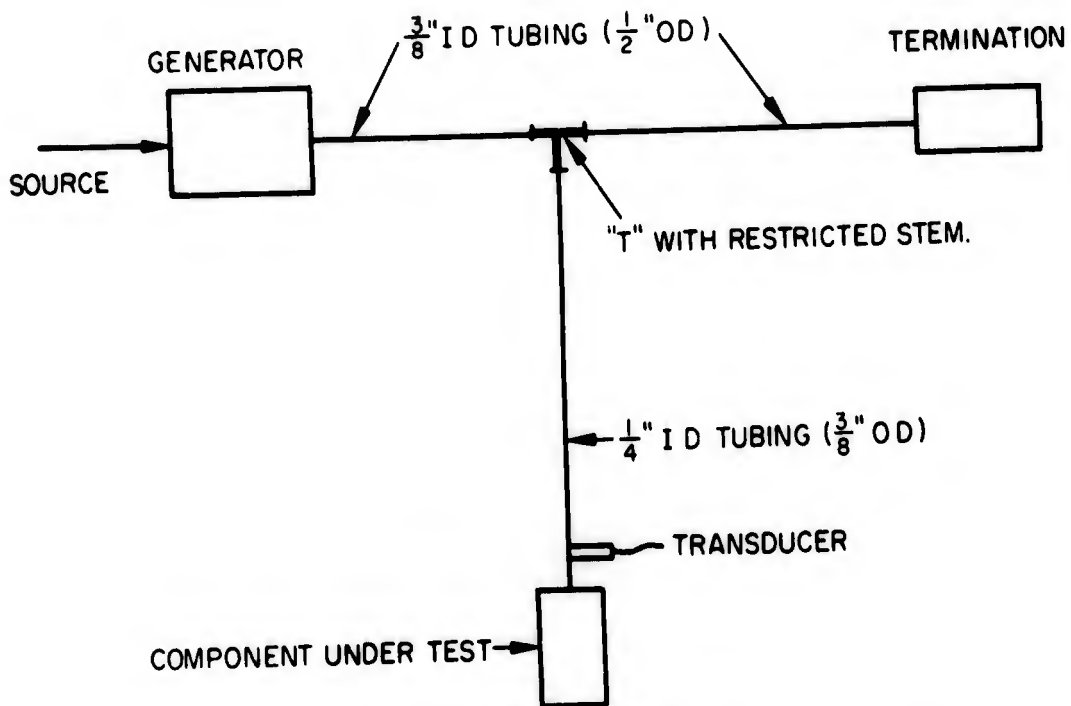
The size of short nozzles required to terminate transmission lines with minimum reflections without the steady-state resistance of continuous nonporous transmission lines can be predicted on the basis of simple theory.

Area changes in flow paths, even though slight, may cause undesirable noise from wave reflections during rapid changes of flow.

Branch T's show no directional effects to wave transmission; a wave turning a sharp corner is not noticeably different from one proceeding straight in the T. Differences result from growth of steady-state flow effects such as momentum of the d-c flow, flow separation, boundary layers, entrainment, etc.

Constant-total-area flow dividers can be used in branch circuits to eliminate reflections.

Transmission lines of different areas can be matched by the use of an orifice (short nozzle).



THE VARIOUS LENGTHS OF TUBING MUST BE SUFFICIENT TO PRODUCE A STEP WAVE OF A DURATION ADEQUATE FOR THE EXPERIMENT.

Figure 1. Wave-forming circuit.

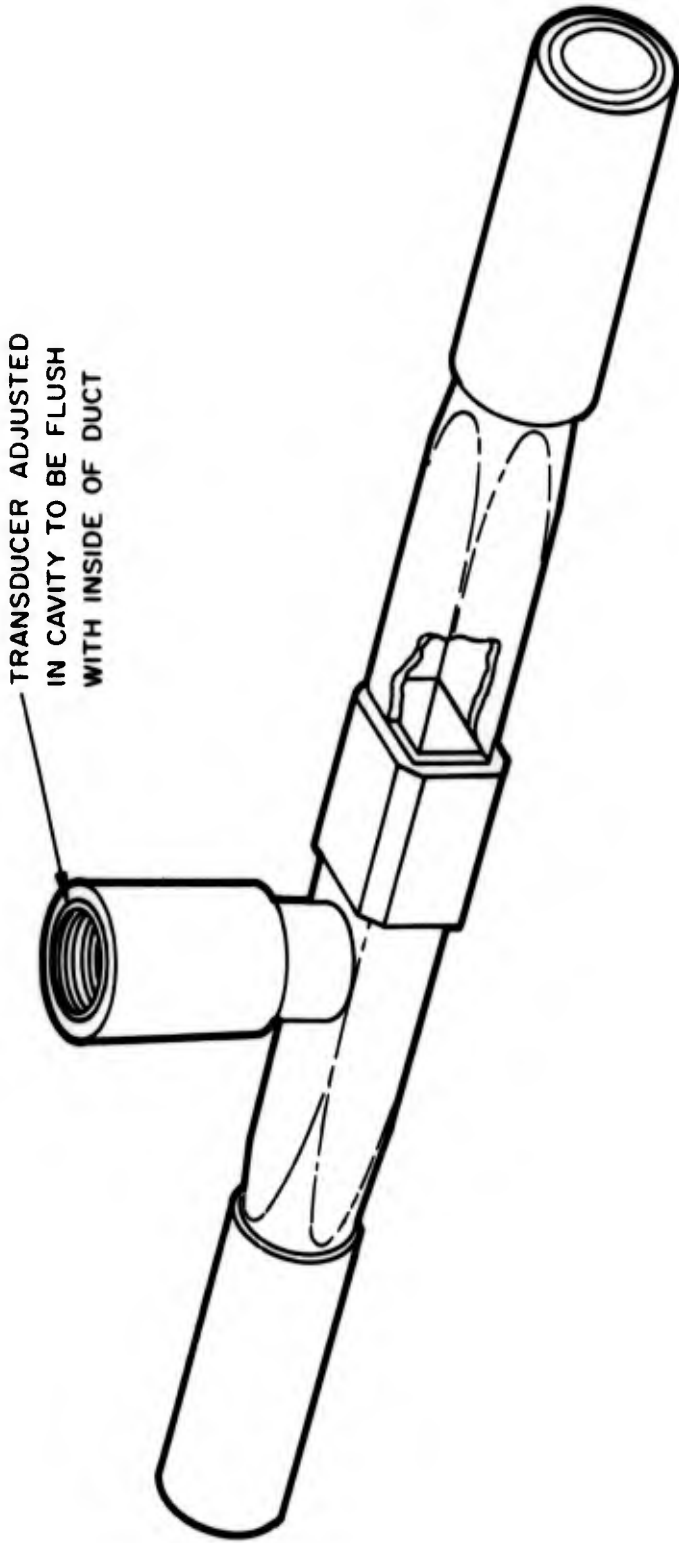


Figure 2. Transducer adapter. The adapter is constructed by swaging a constant cross-sectional-area transition from round to rectangular.

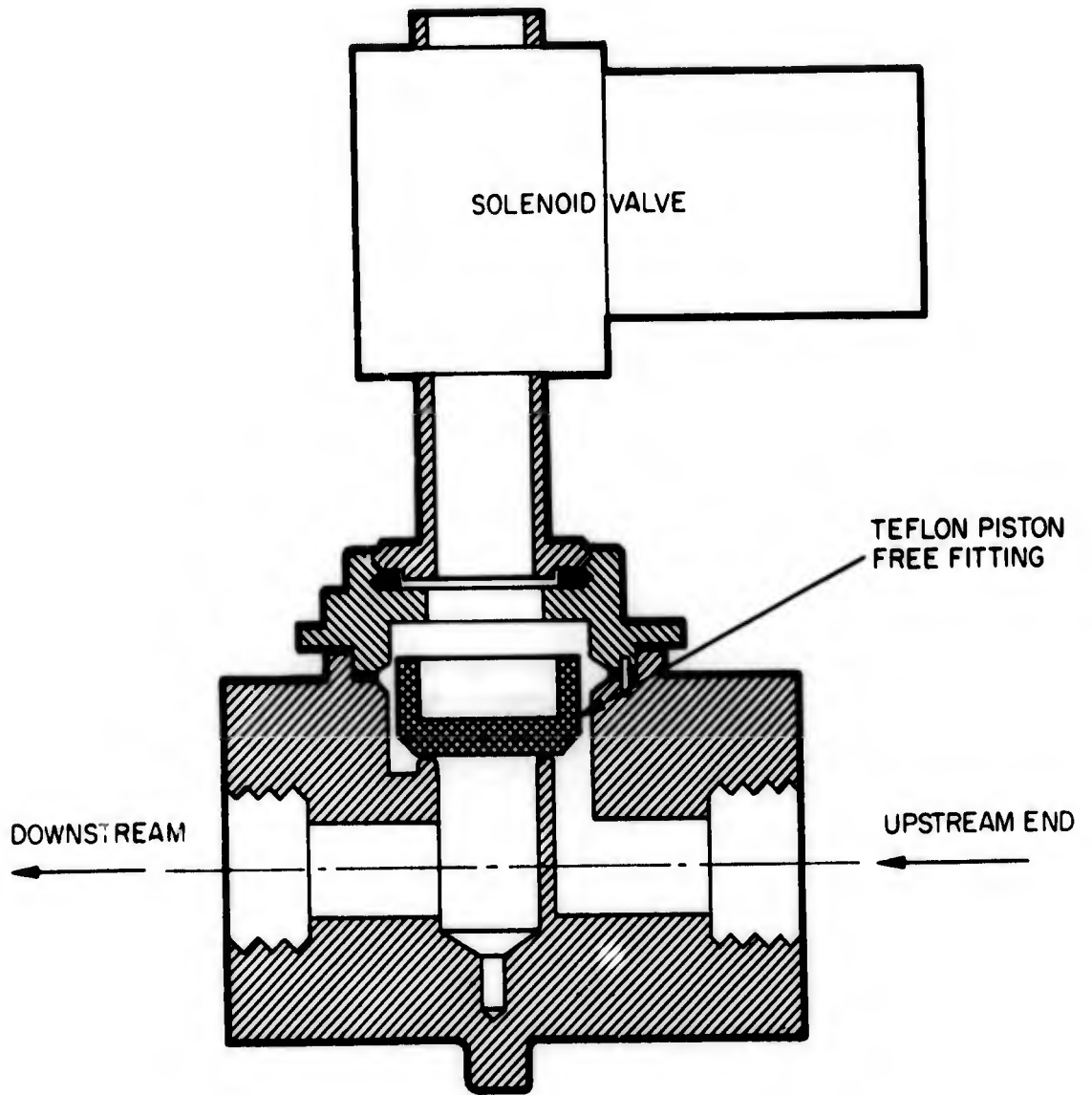


Figure 3. Quick-opening solenoid valve.

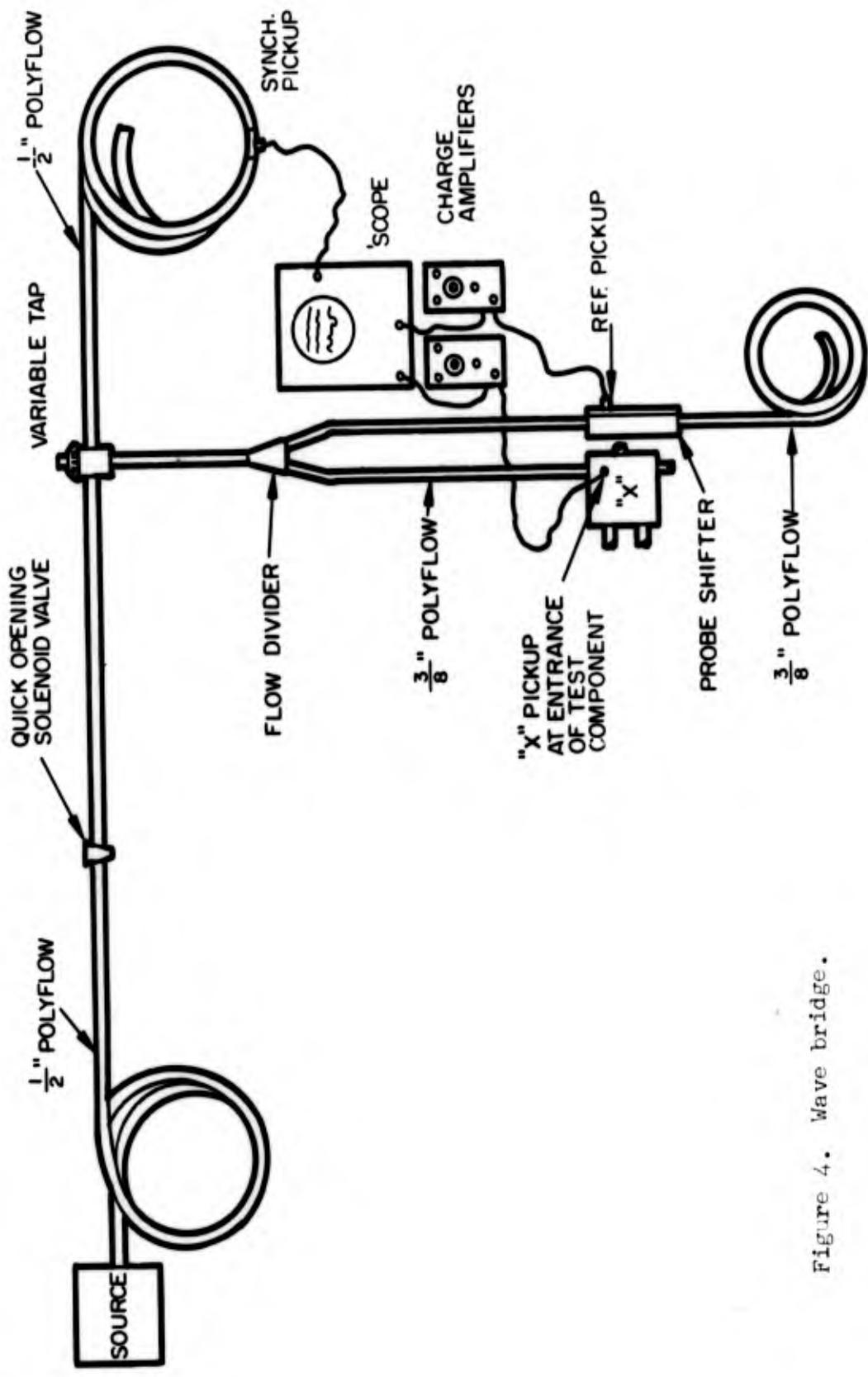


Figure 4. Wave bridge.

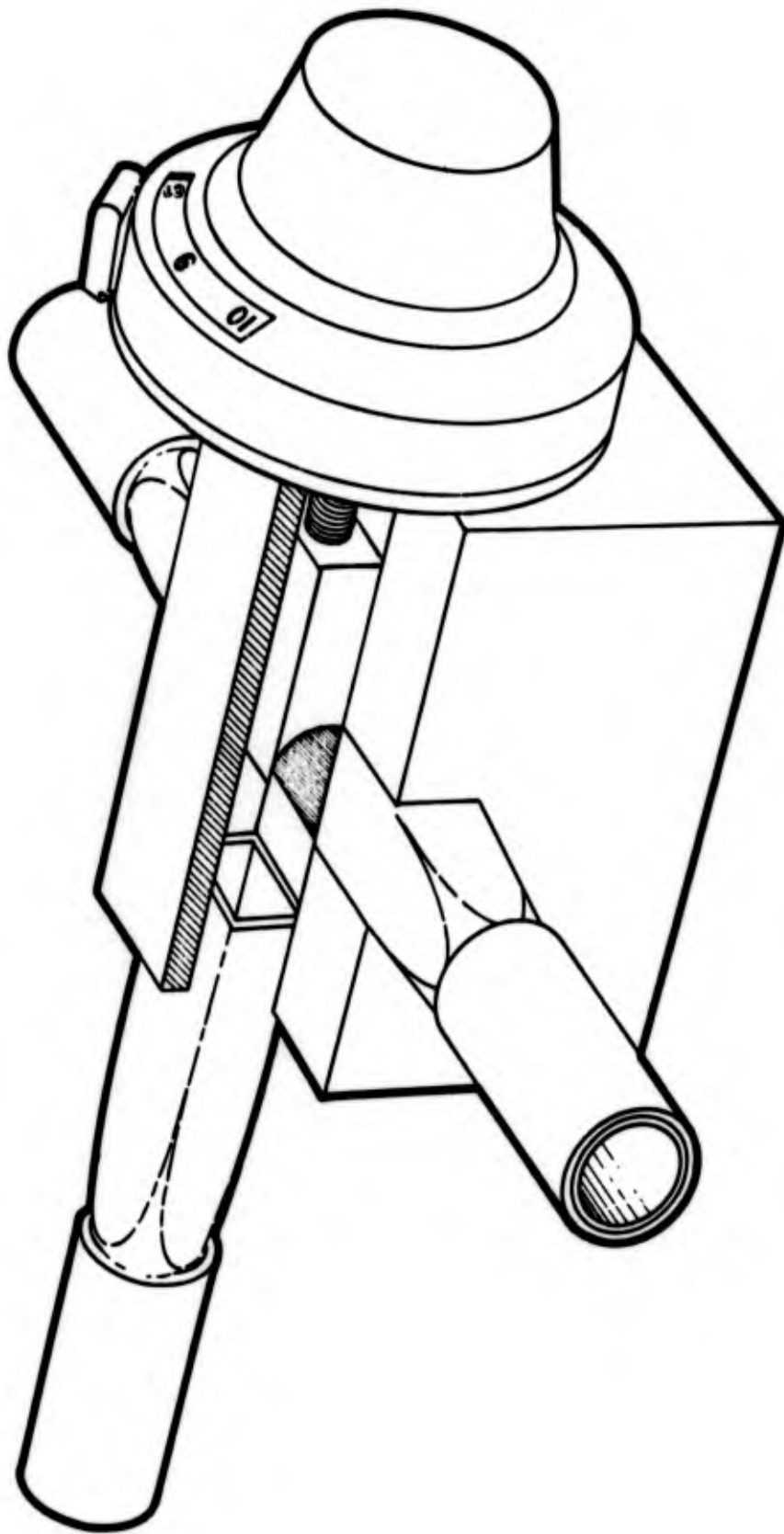


Figure 5. Variable tap. Although constant-area transitions are used, this tap is not a constant-matched impedance device.

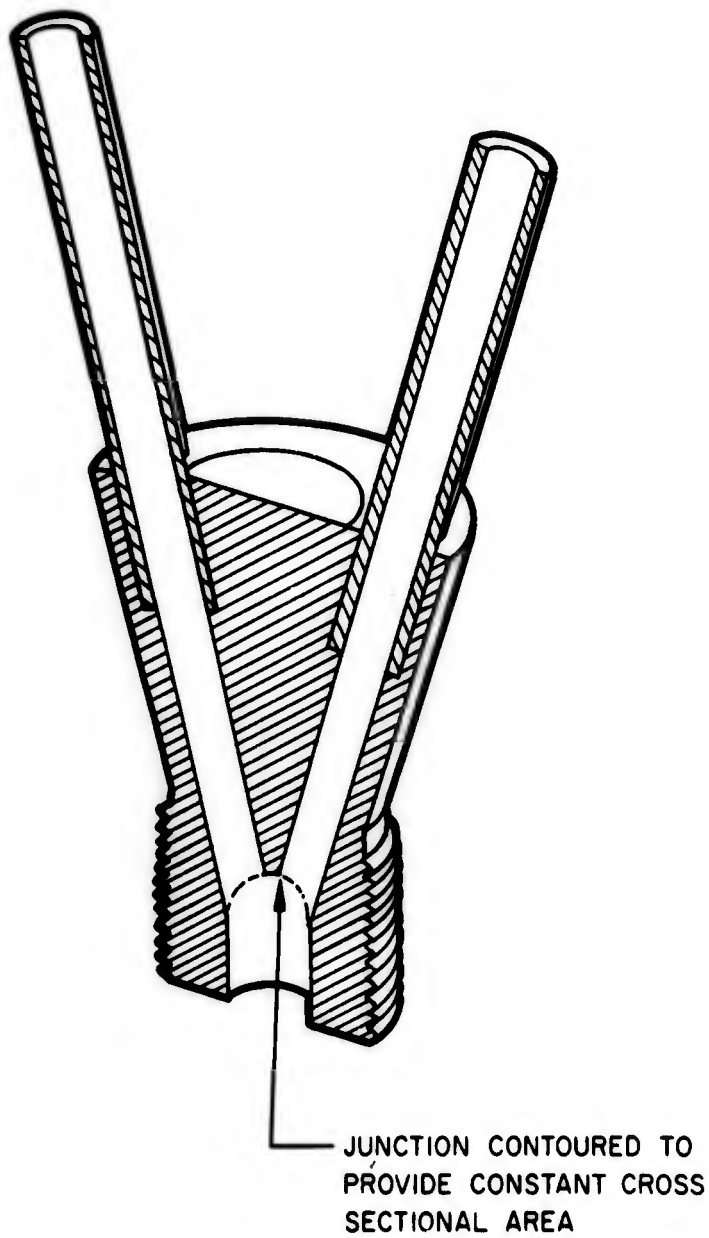


Figure 6a. Flow divider, 2X.

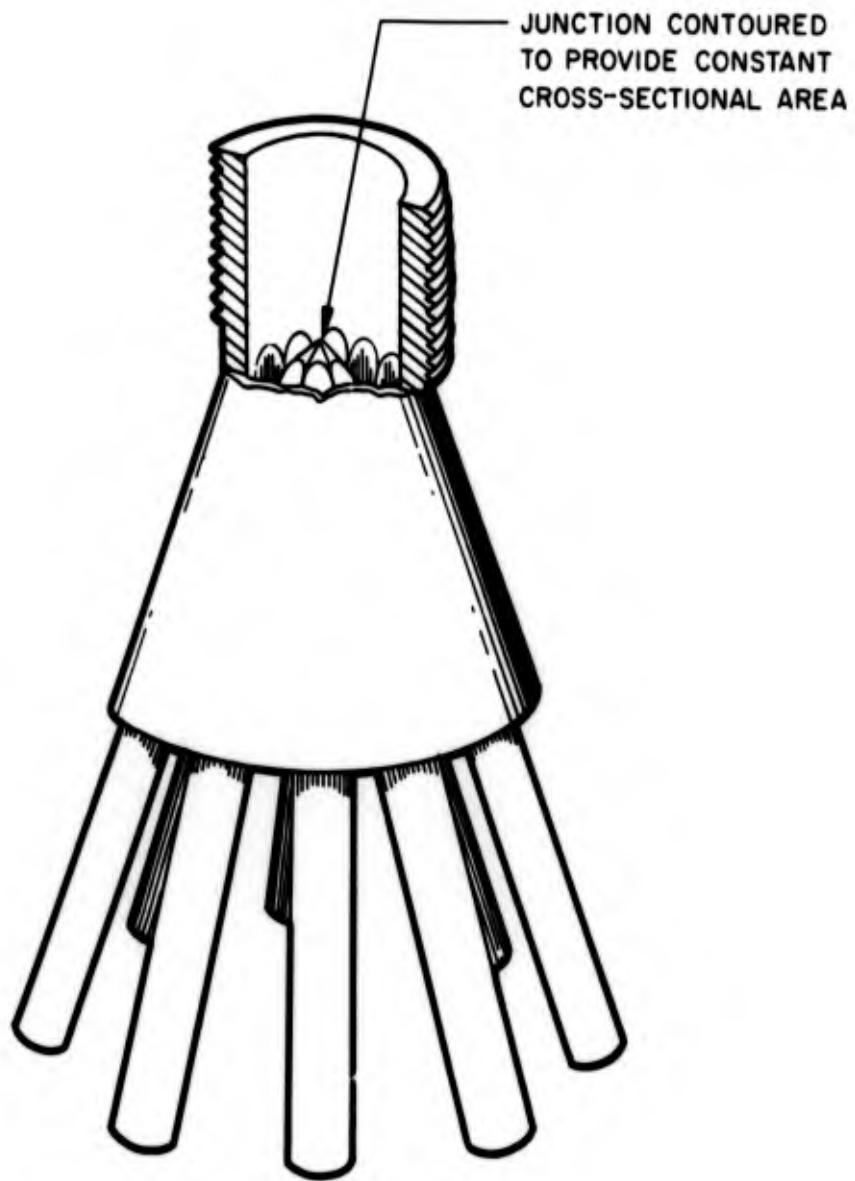


Figure 6b. Flow divider, 8X.

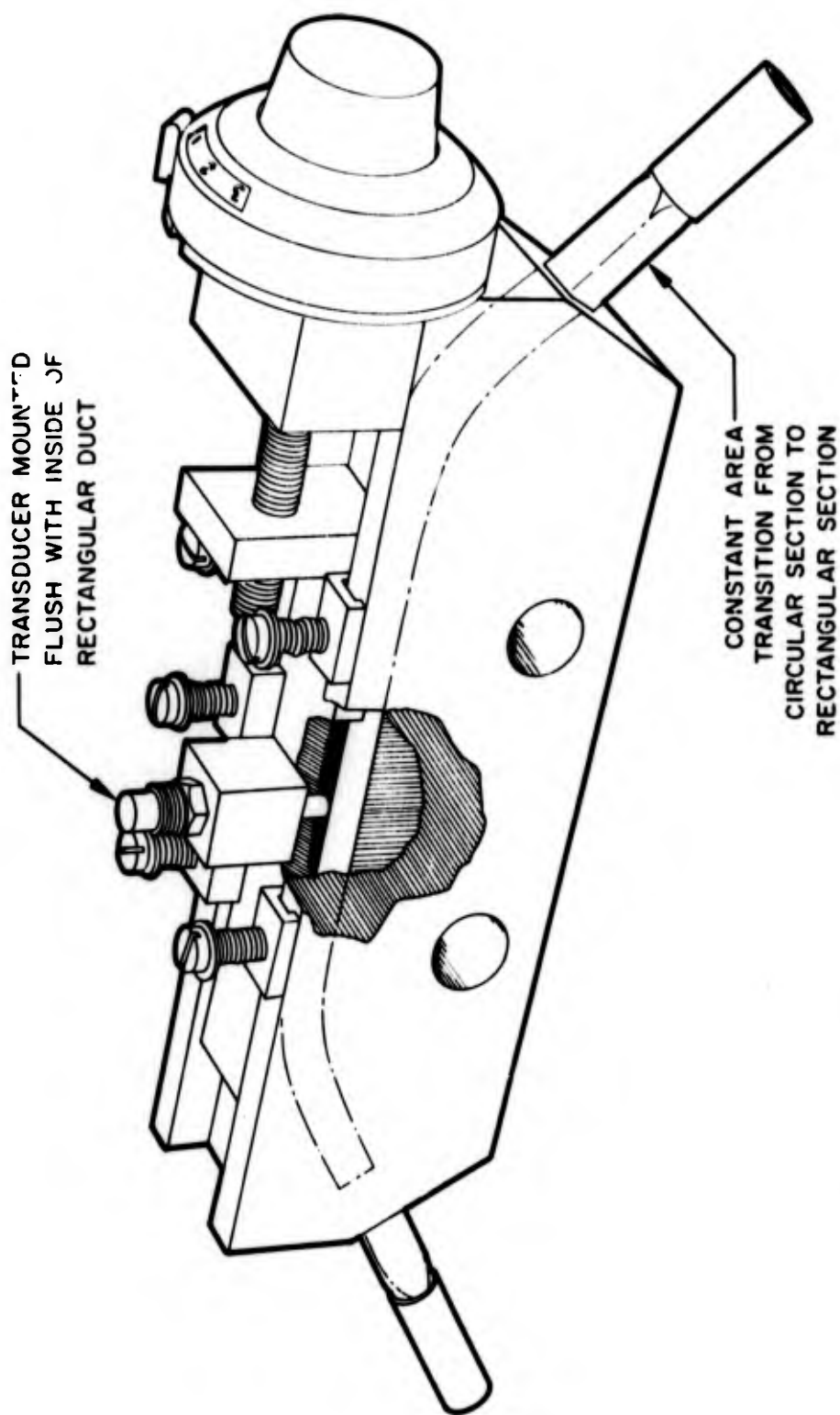


Figure 7. Probe shifter.

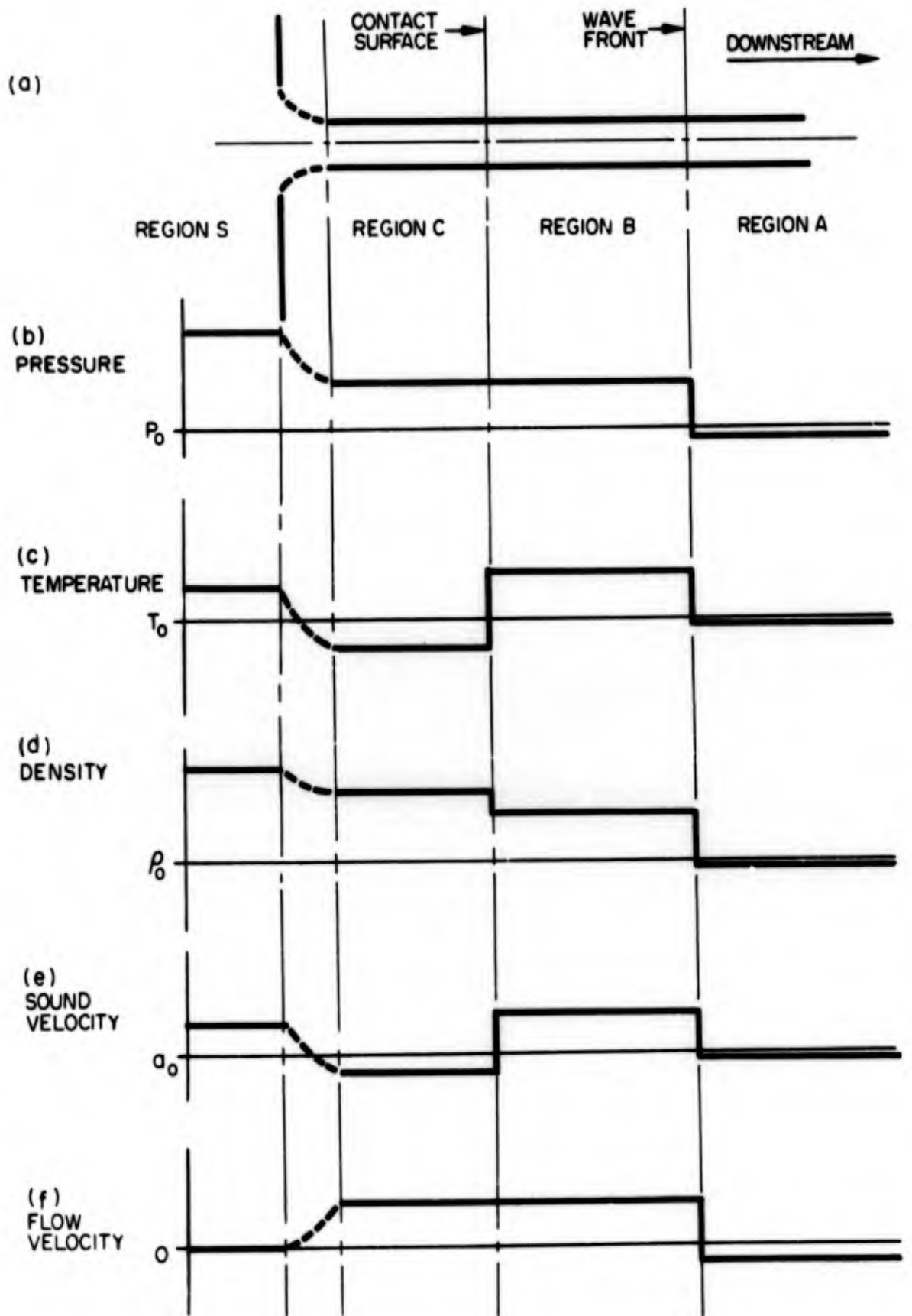


Figure 8. Fluid conditions in fluid transmission line following sudden change of source.

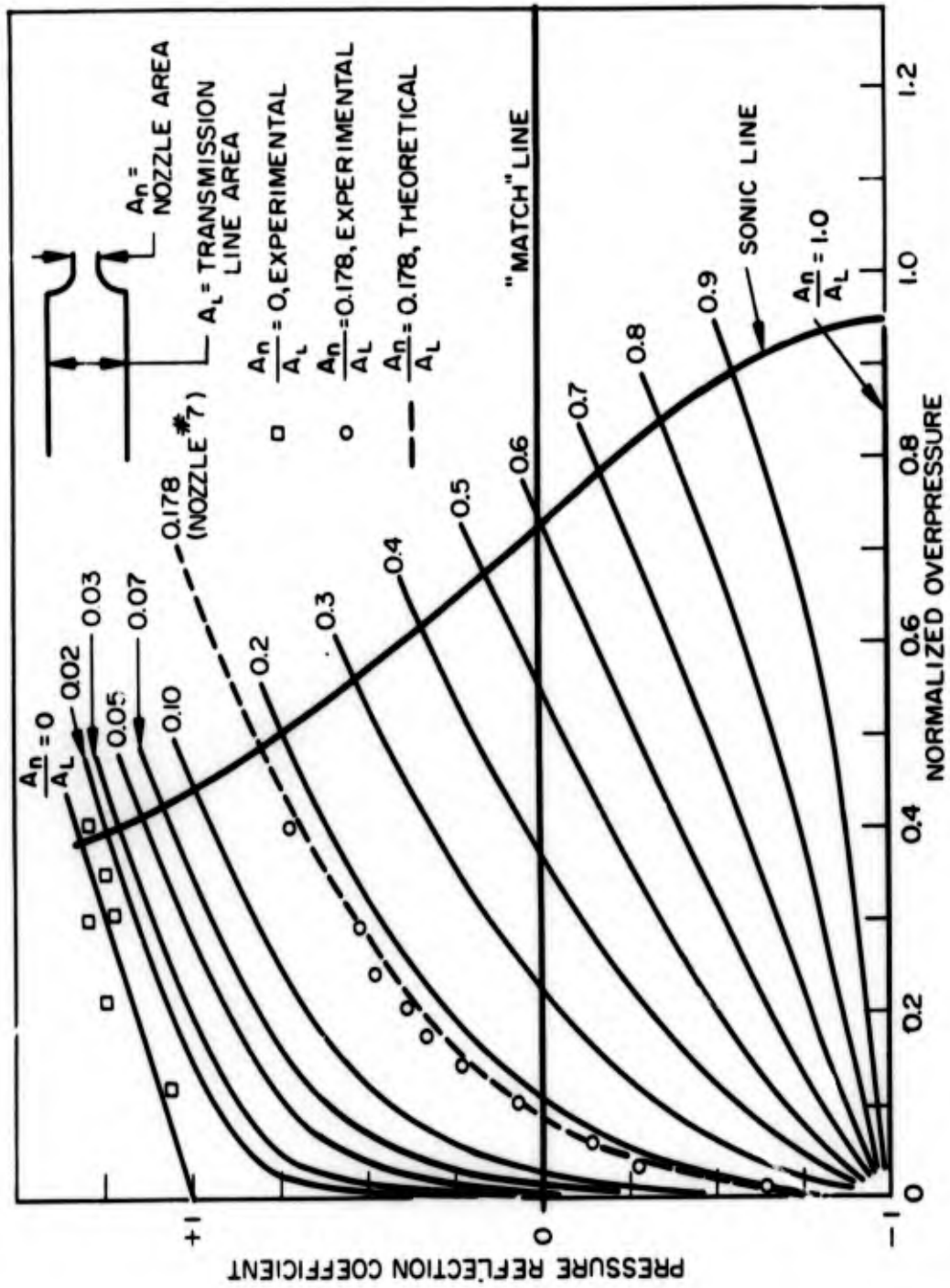


Figure 10a. Orifice termination of transmission line: pressure reflection coefficient versus normalized overpressure.

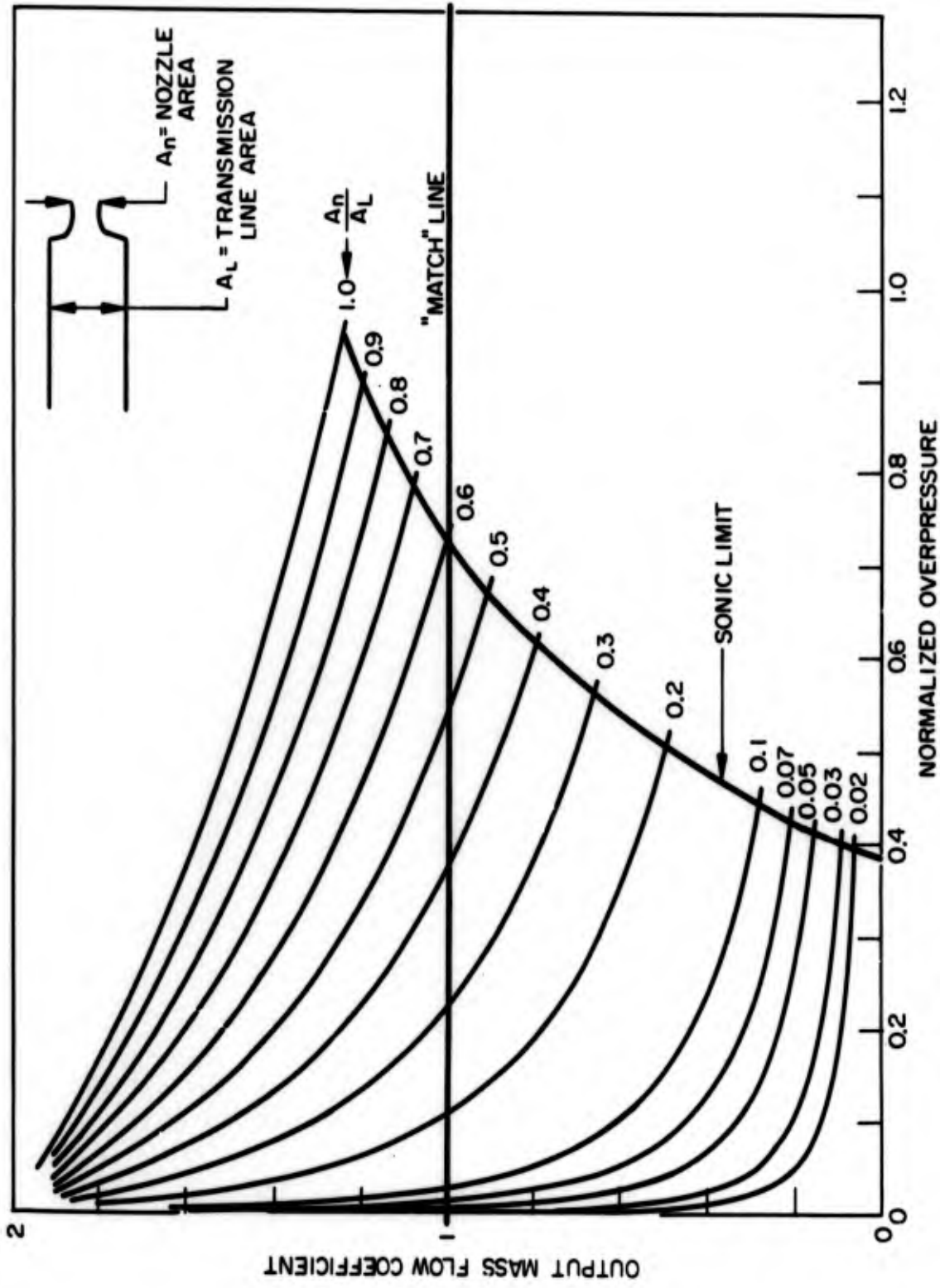


Figure 10b. Orifice termination of transmission line: output mass flow coefficient versus normalized overpressure.

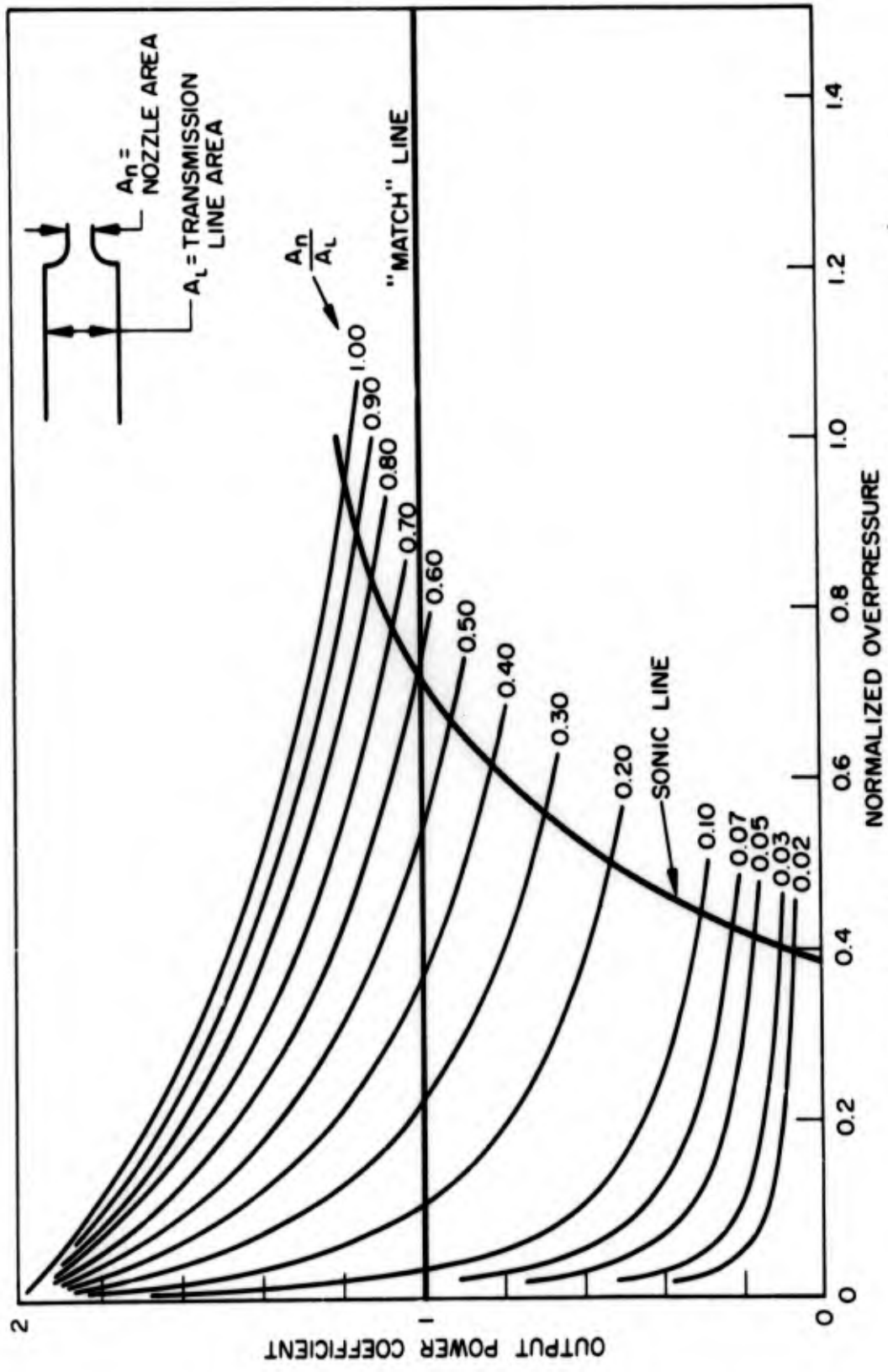


Figure 10c. Orifice termination of transmission line: output power coefficient versus normalized overpressure.

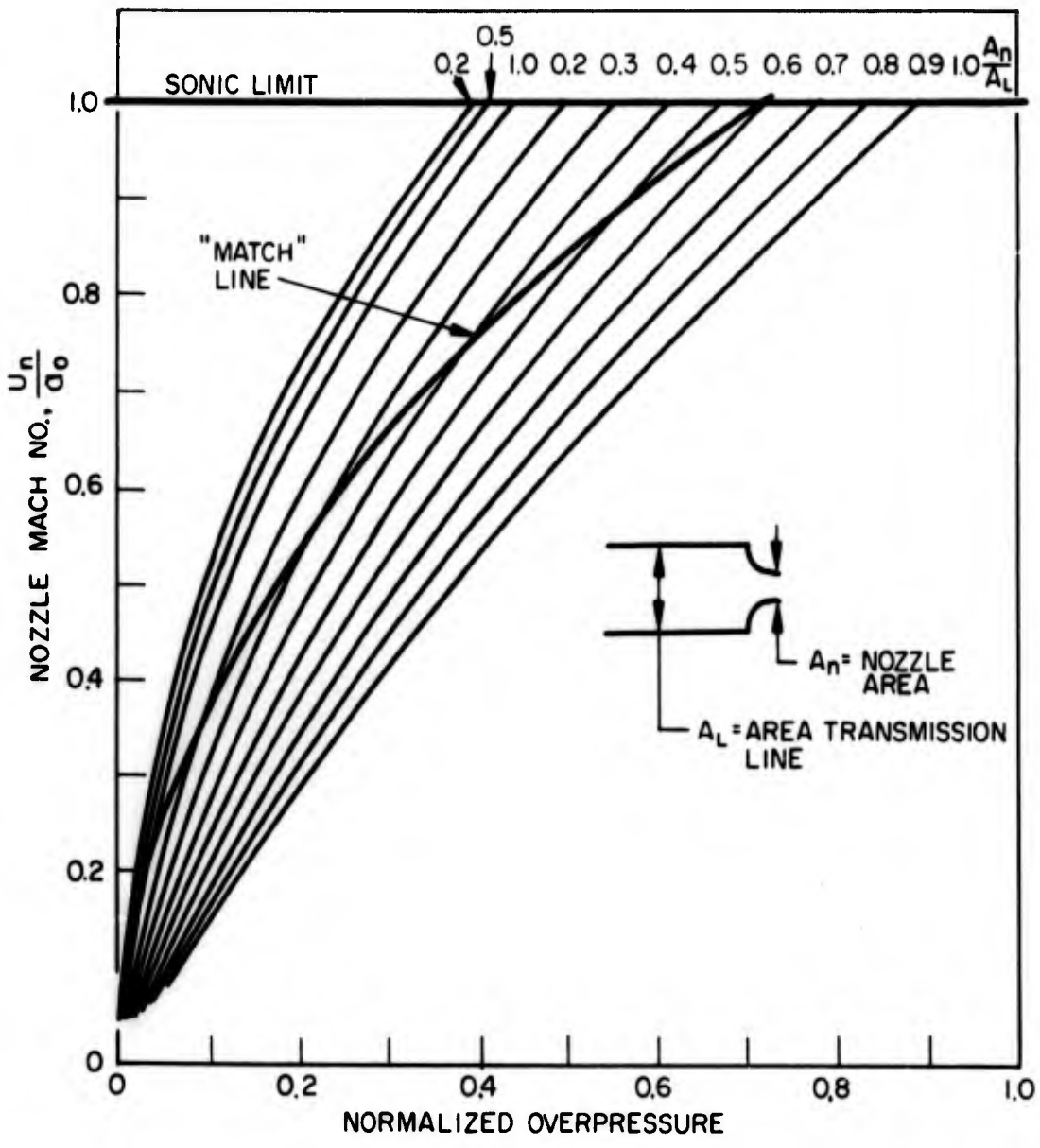


Figure 10d. Orifice termination of transmission line: nozzle Mach number versus normalized overpressure.

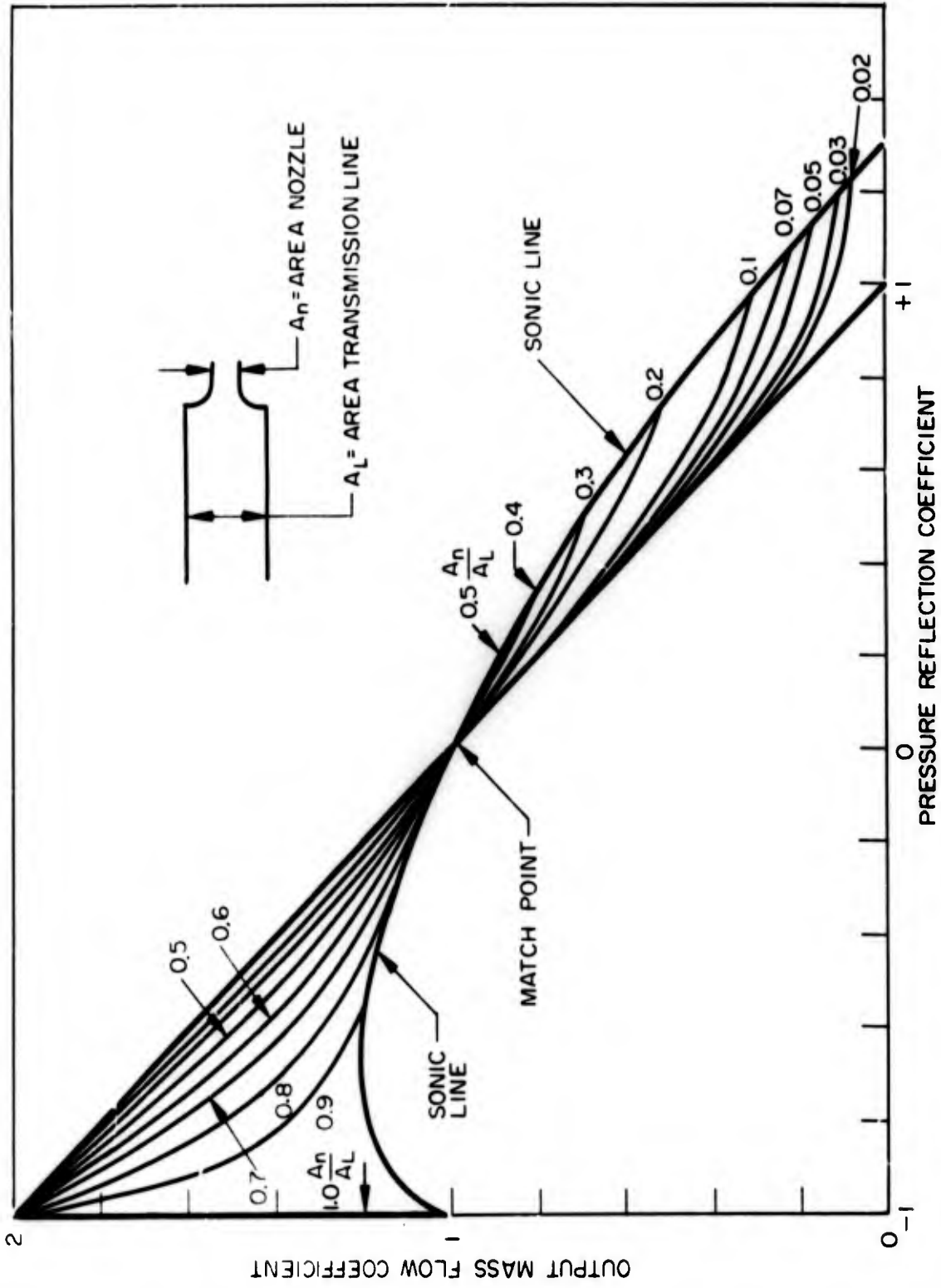


Figure 10e. Orifice termination of transmission line: output mass flow coefficient versus pressure reflection coefficient.

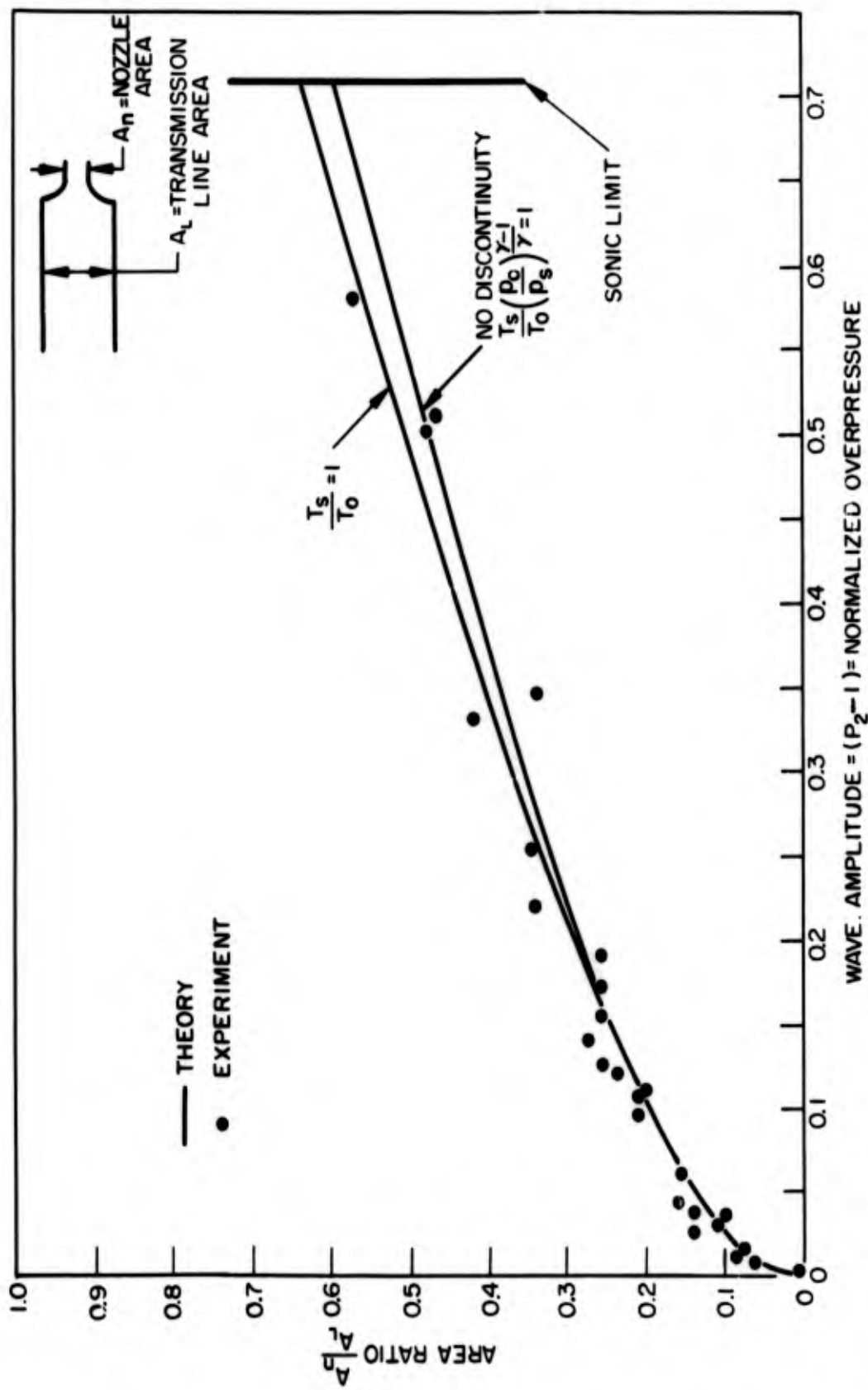


Figure 11a. Orifice-terminated transmission line: area ratio versus wave amplitude for a matching termination for a step function (linear plot).

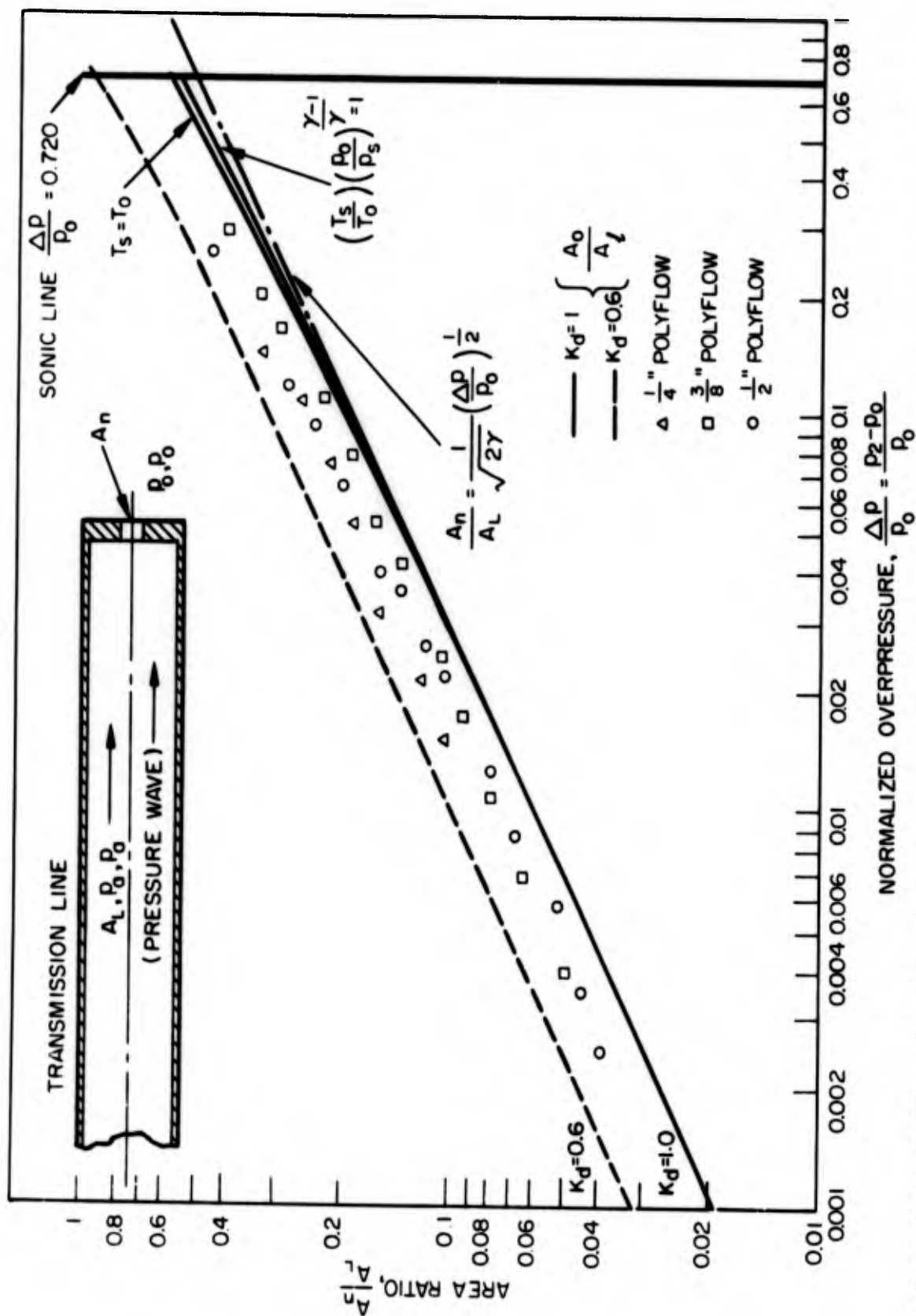


Figure 11b. Orifice-terminated transmission line: area ratio versus wave amplitude for a matching termination for a step function (log-log plot).

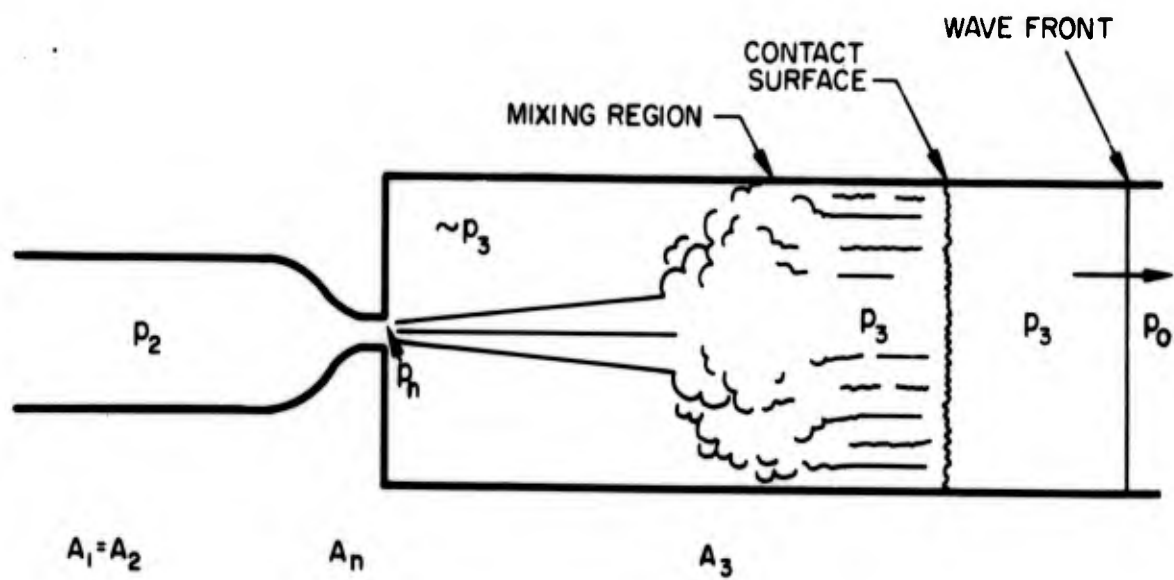


Figure 12. Matching smaller transmission line into a larger one.

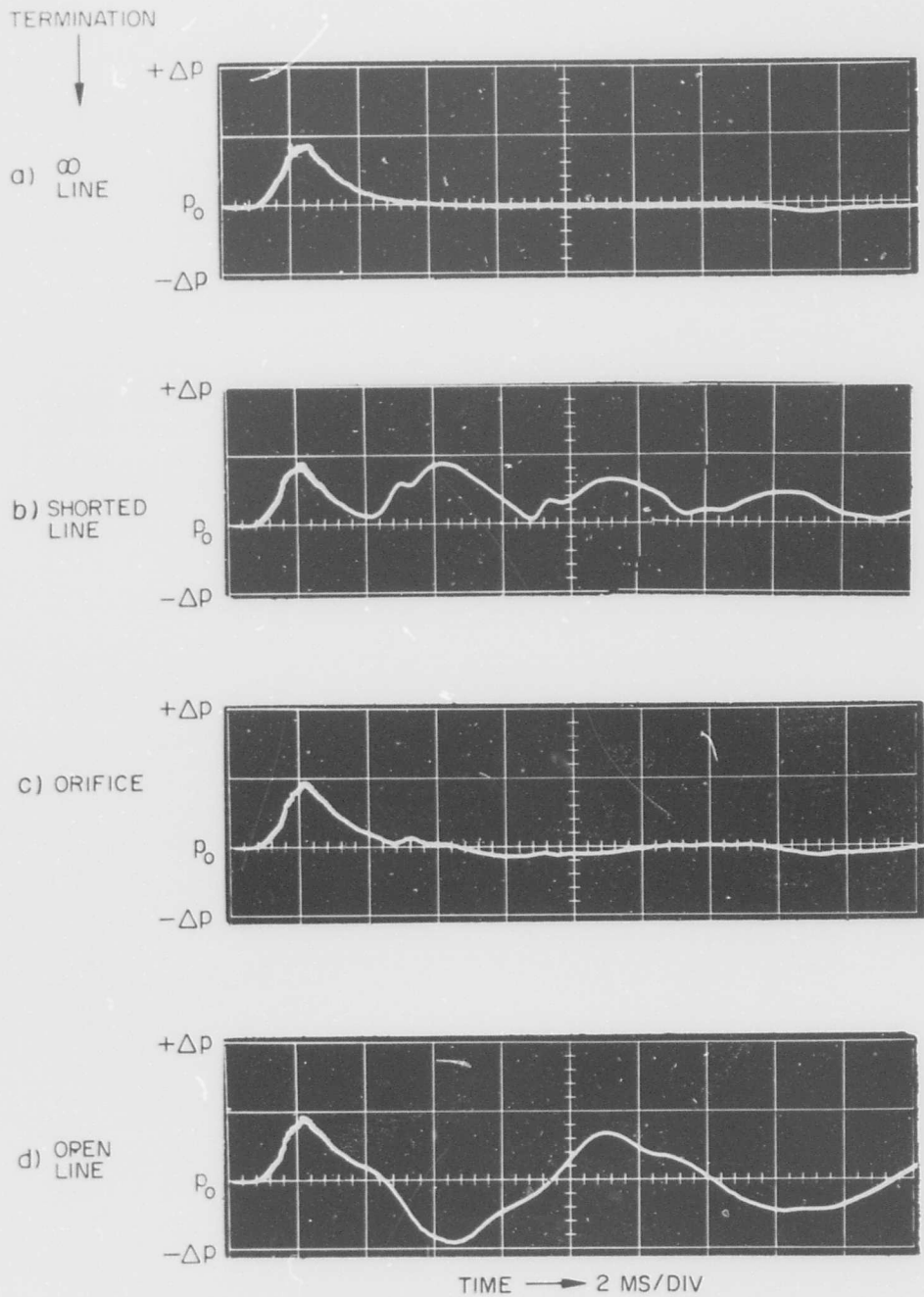
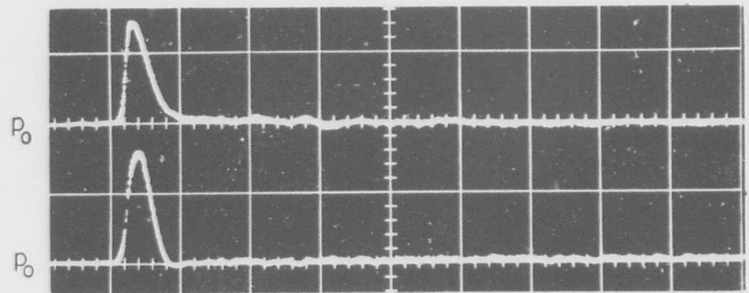


Figure 13. Experimental results of matching with an orifice. (Note: the symbol ∞ signifies a continuous line.)

TERMINATION

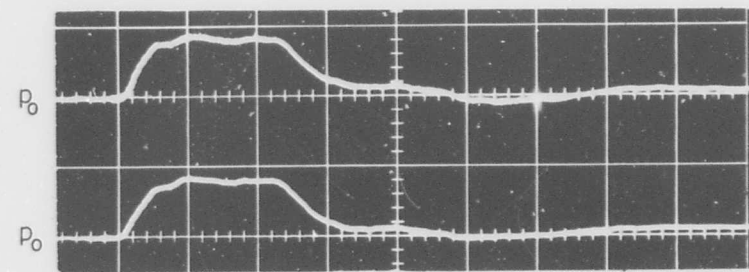


∞ LINE
(SIGNAL a)



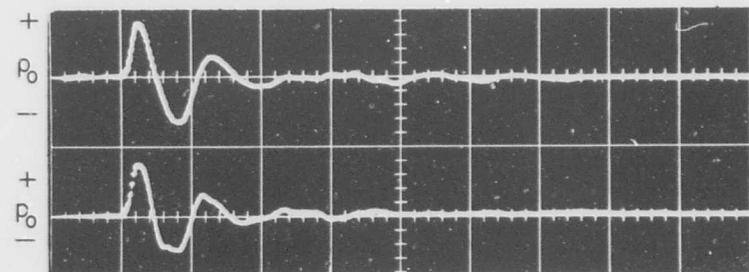
NOZZLE

∞ LINE
(SIGNAL b)



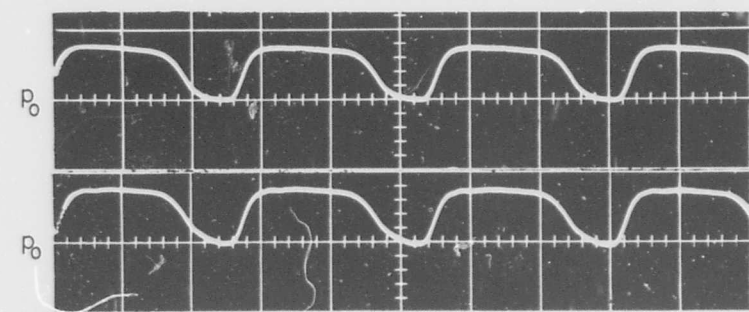
NOZZLE

∞ LINE
(SIGNAL c)



NOZZLE

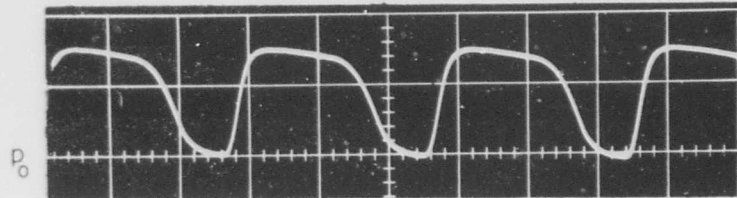
∞ LINE
(SIGNAL d)



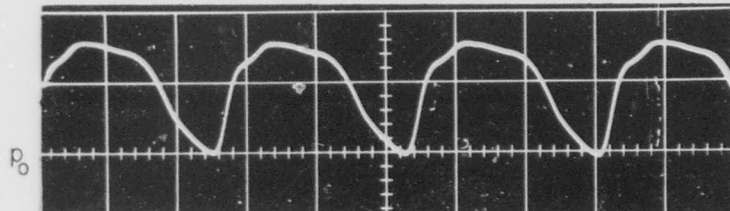
NOZZLE

TIME → 2 MS/DIV

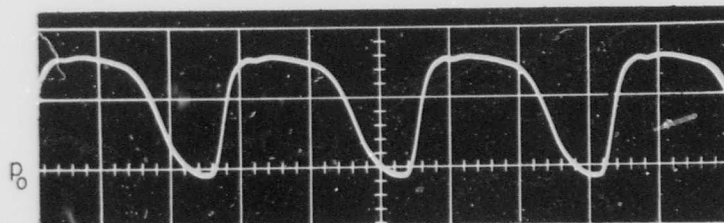
Figure 14. Experimental results of matching with a nozzle. (Note: the symbol ∞ signifies a continuous line.)



(a) NOZZLE MATCHED TO PRESSURE WAVE



(b) PRESSURE WAVE AMPLITUDE INCREASED, SAME NOZZLE AS (a)



(c) NOZZLE OPENED TO IMPROVE MATCH TO INCREASED PRESSURE AMPLITUDE

TIME → 2 MS/DIV.

Figure 15. Effect of wave pressure amplitude on nozzle match.

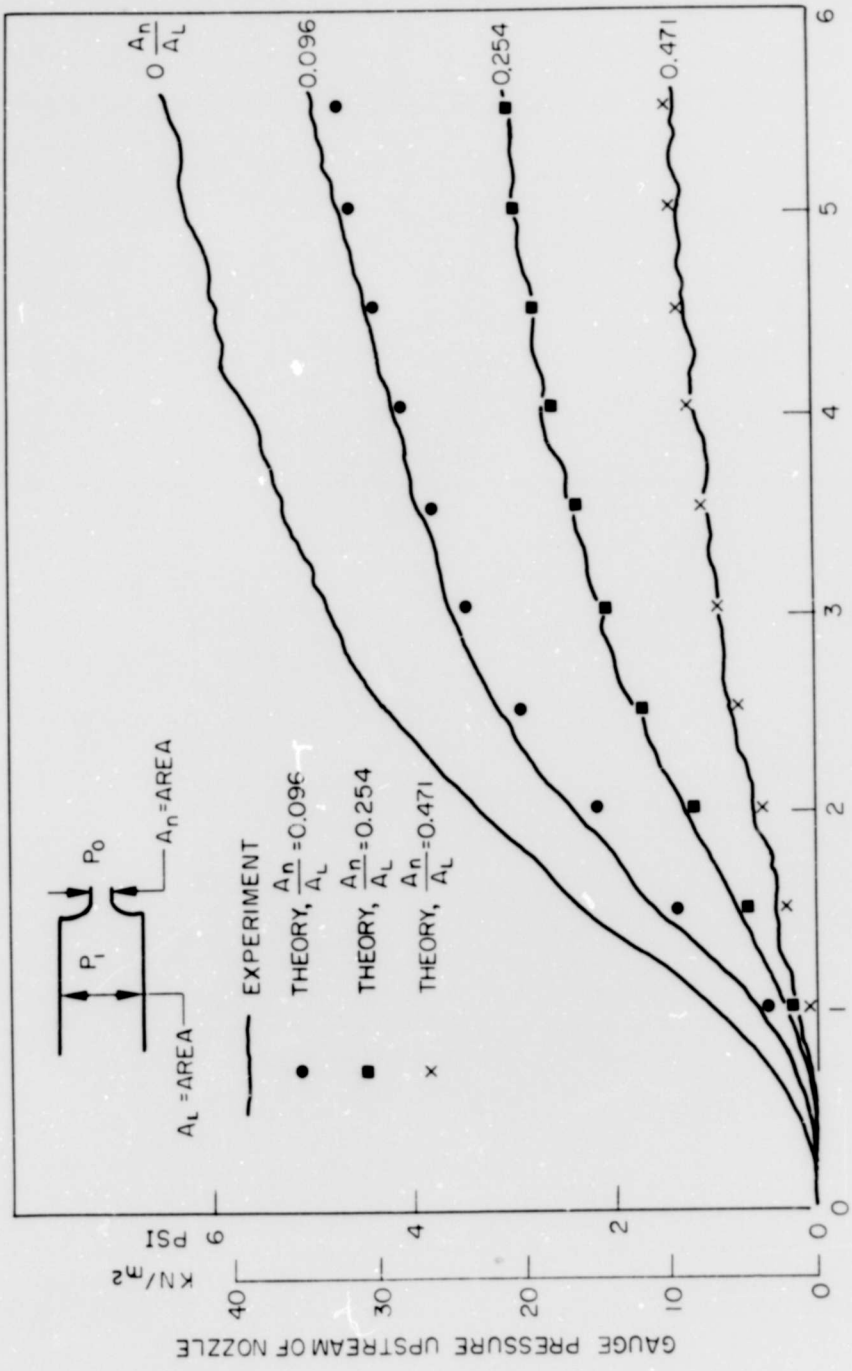


Figure 16. Matching with a nozzle to a rising wave front; theory based on stagnation pressure.

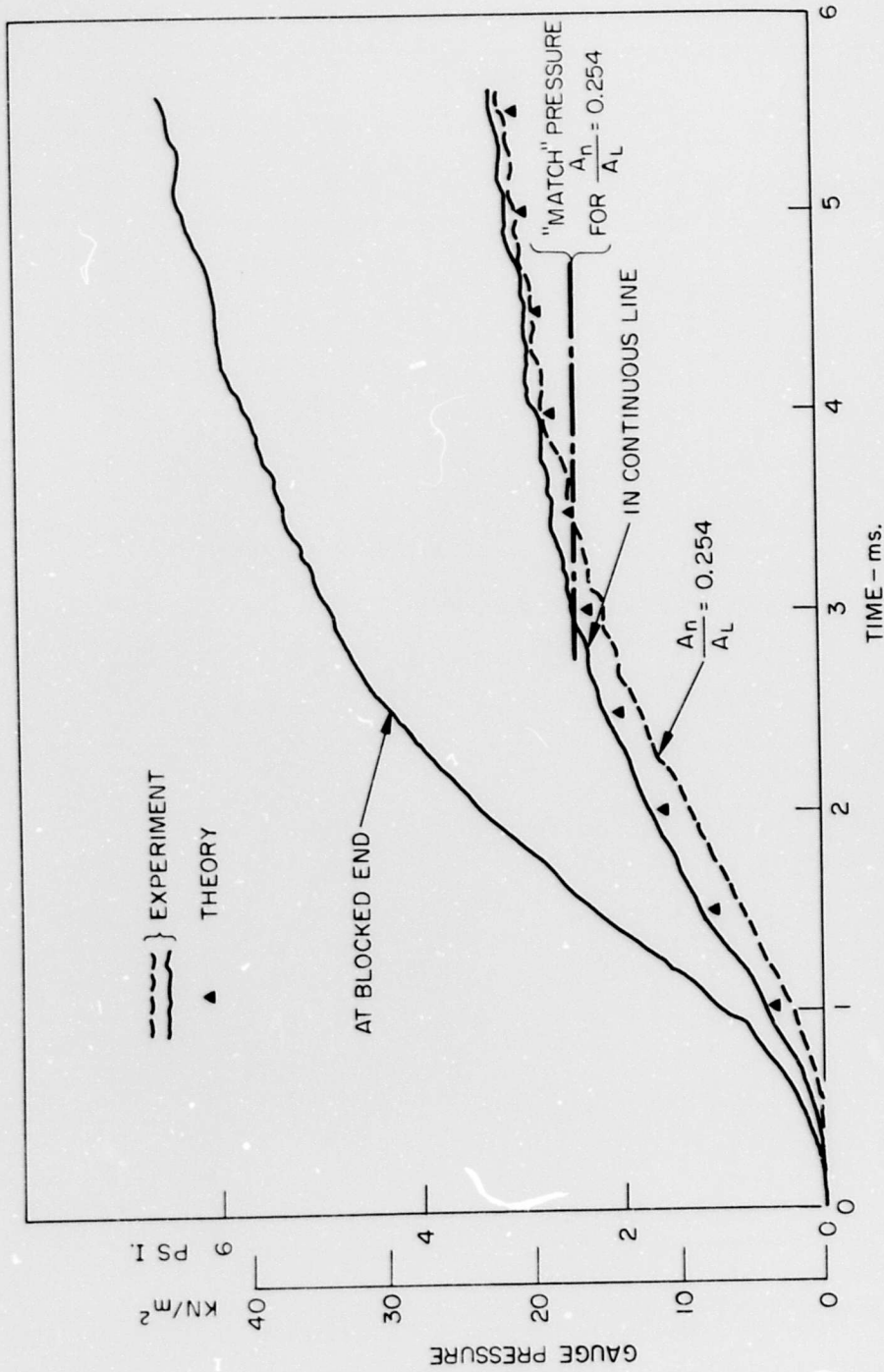
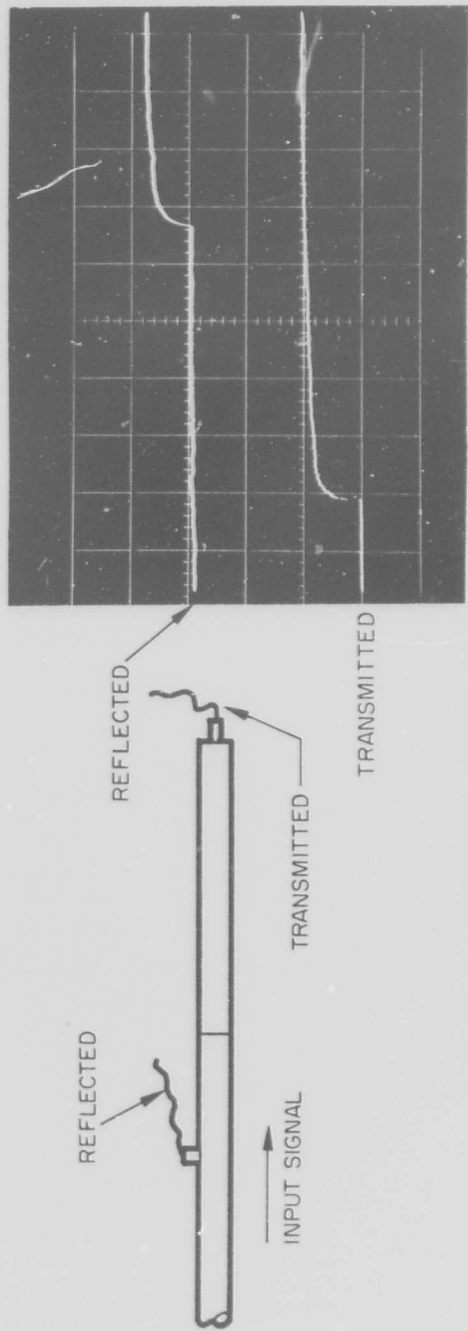


Figure 17. Matching with a nozzle to a rising wave front; signal for continuous line predicted theoretically from matched nozzle data.



REFLECTED @ 6.9 kN/m² PER DIV; 0.5 MS. PER DIV
 TRANSMITTED @ 13.8 kN/m² PER DIV; 0.5 MS. PER DIV

Figure 18a. Determination of directionality of a T, reference set-up.

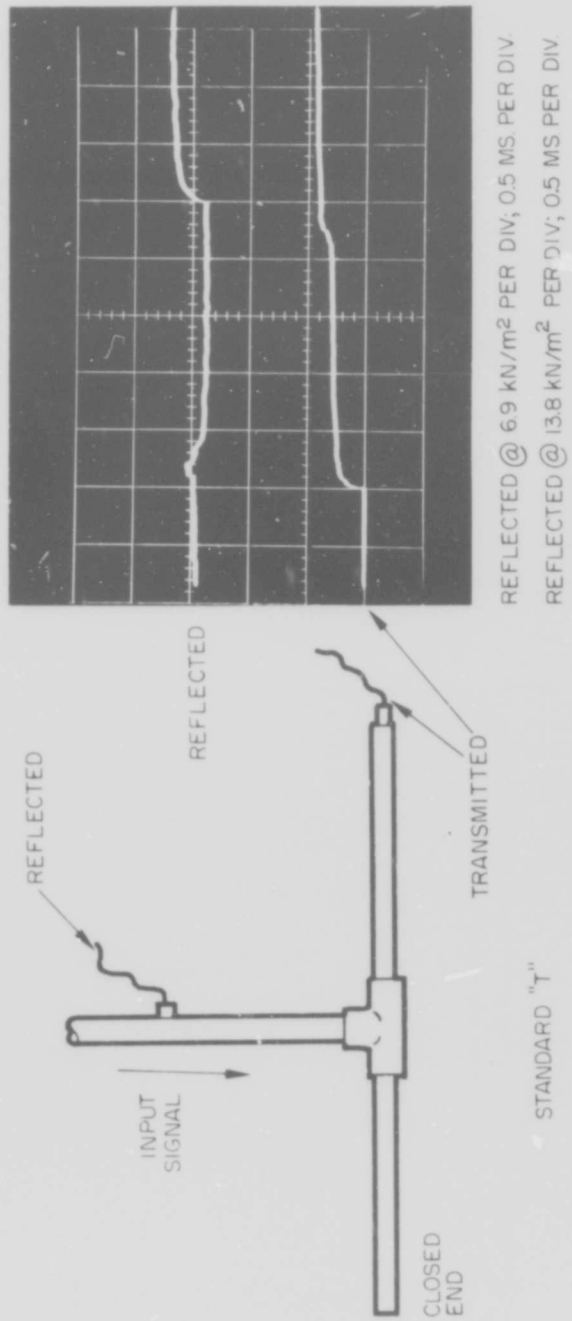


Figure 1b. Determination of directionality of a T, standard T.

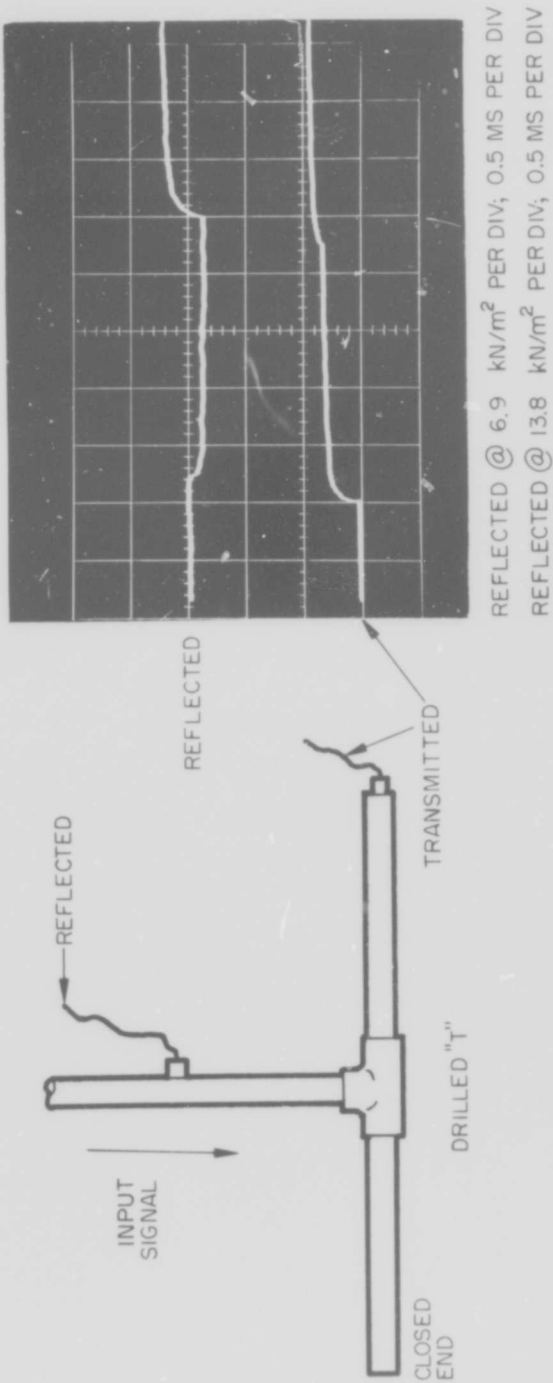


Figure 18c. Determination of directionality of a T, with bore matched to tubing.

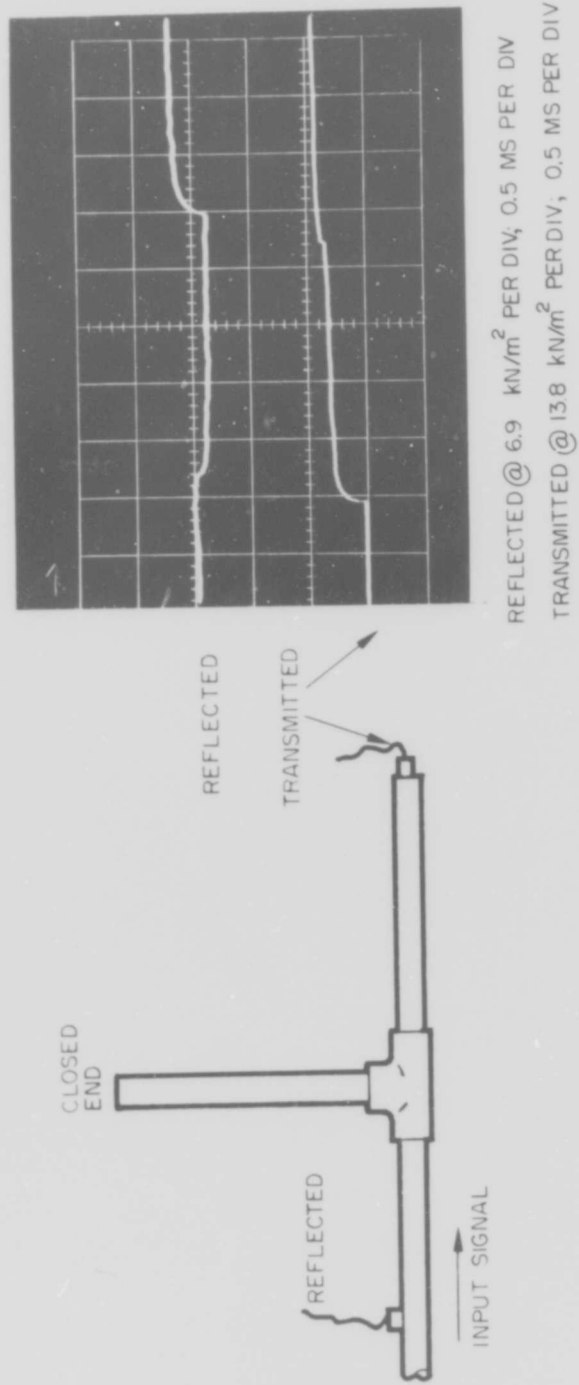


Figure 18d. Determination of directionality of T, bore matched to tubing with different input terminal.

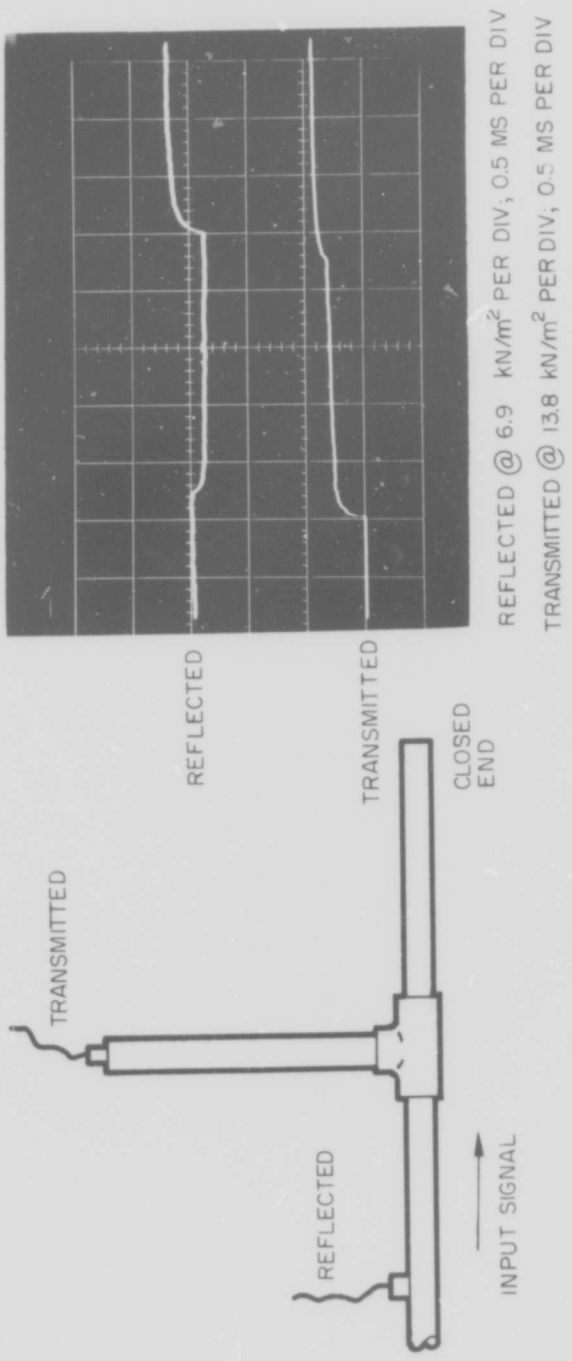


Figure 18e. Determination of directionality of a T, bore matched to tubing, alternate transmitted signal measured.

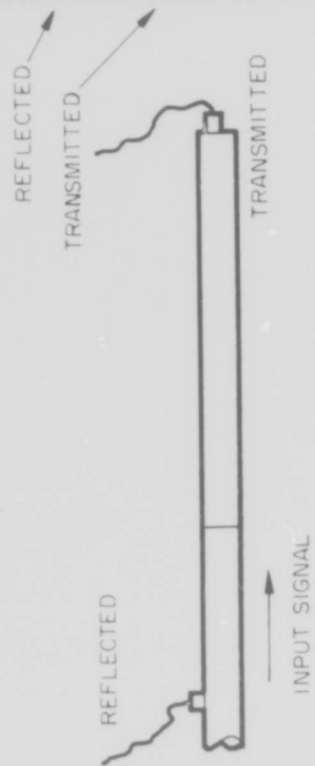
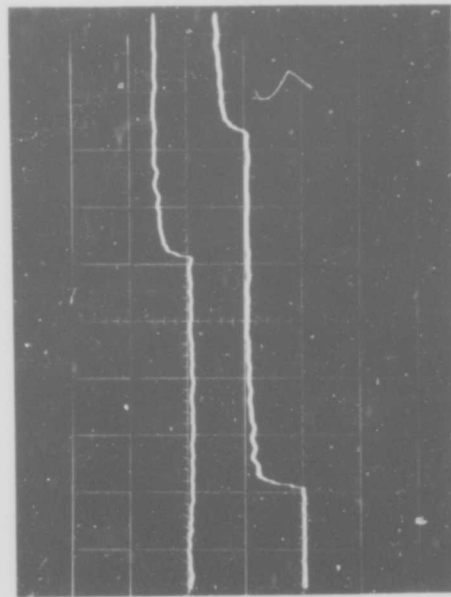


Figure 19a. Test of flow divider, reference set-up.

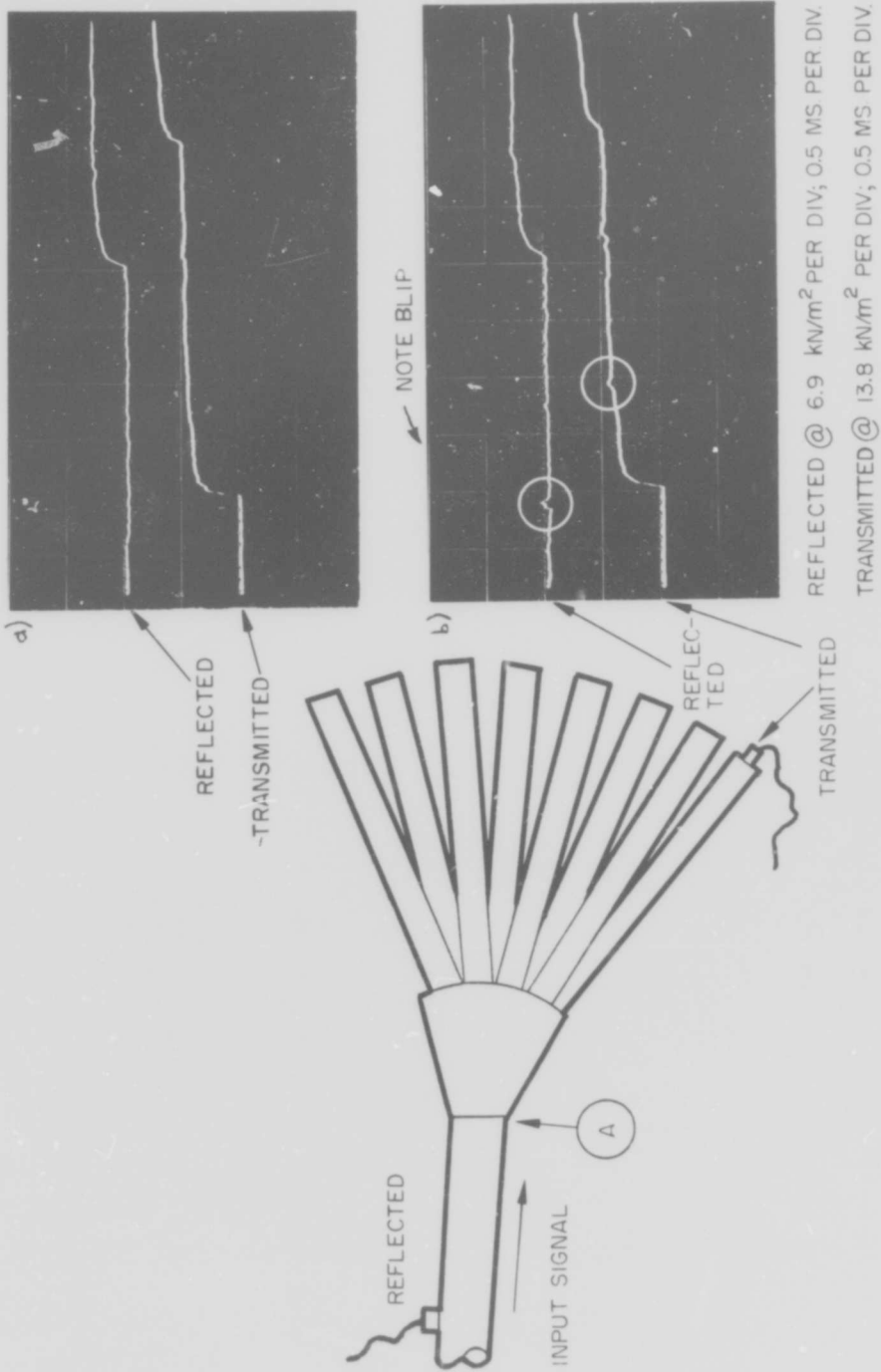


Figure 104. Test of fanout divider, fanout of eight.

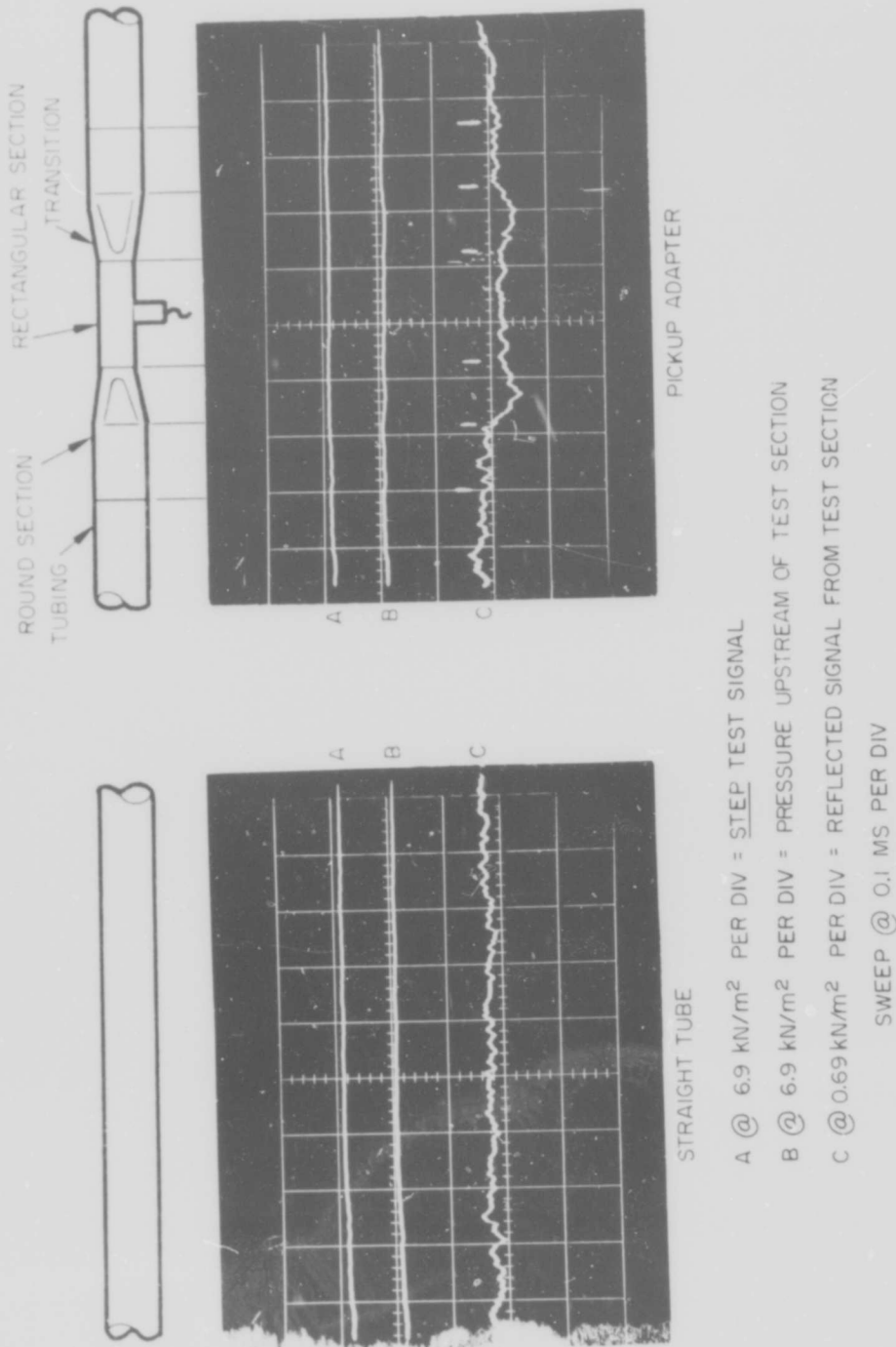


Figure 20. Test of transducer adapter.



Incident wave
Reflected wave

b. 1/4 ID tubing, nozzle terminated



Incident wave
Reflected wave

d. 1/4 ID into 3/8 ID tubing, orifice matched.



Incident wave
Reflected wave

a. 1/4 ID tubing, continuous



Incident wave
Reflected wave

c. 1/4 ID into .172 ID tubing, bleed matched.

Figure 21. Comparison of matching of transmission lines.

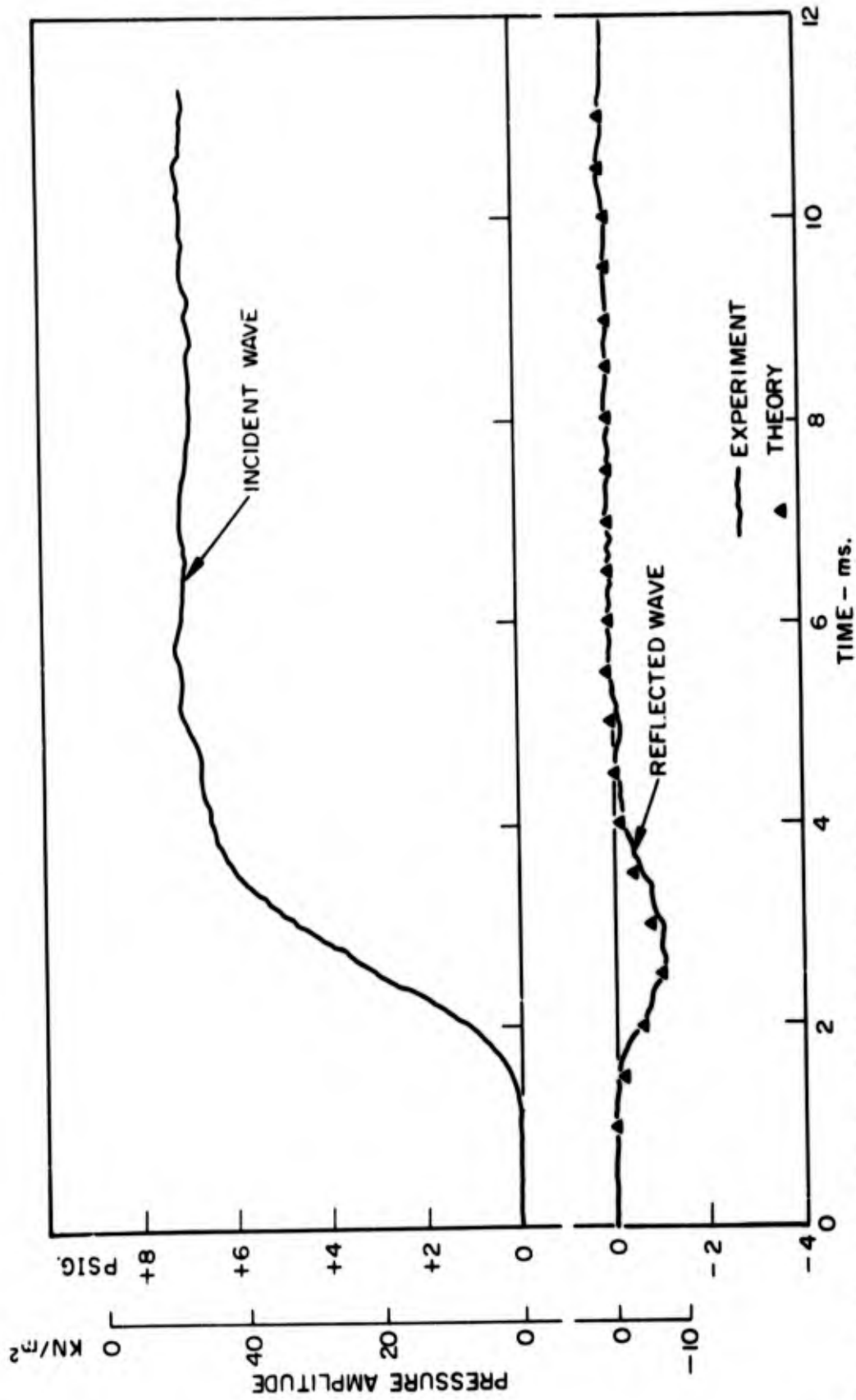


Figure 22. Comparison of experiment with theory for matching larger transmission line into a smaller one.

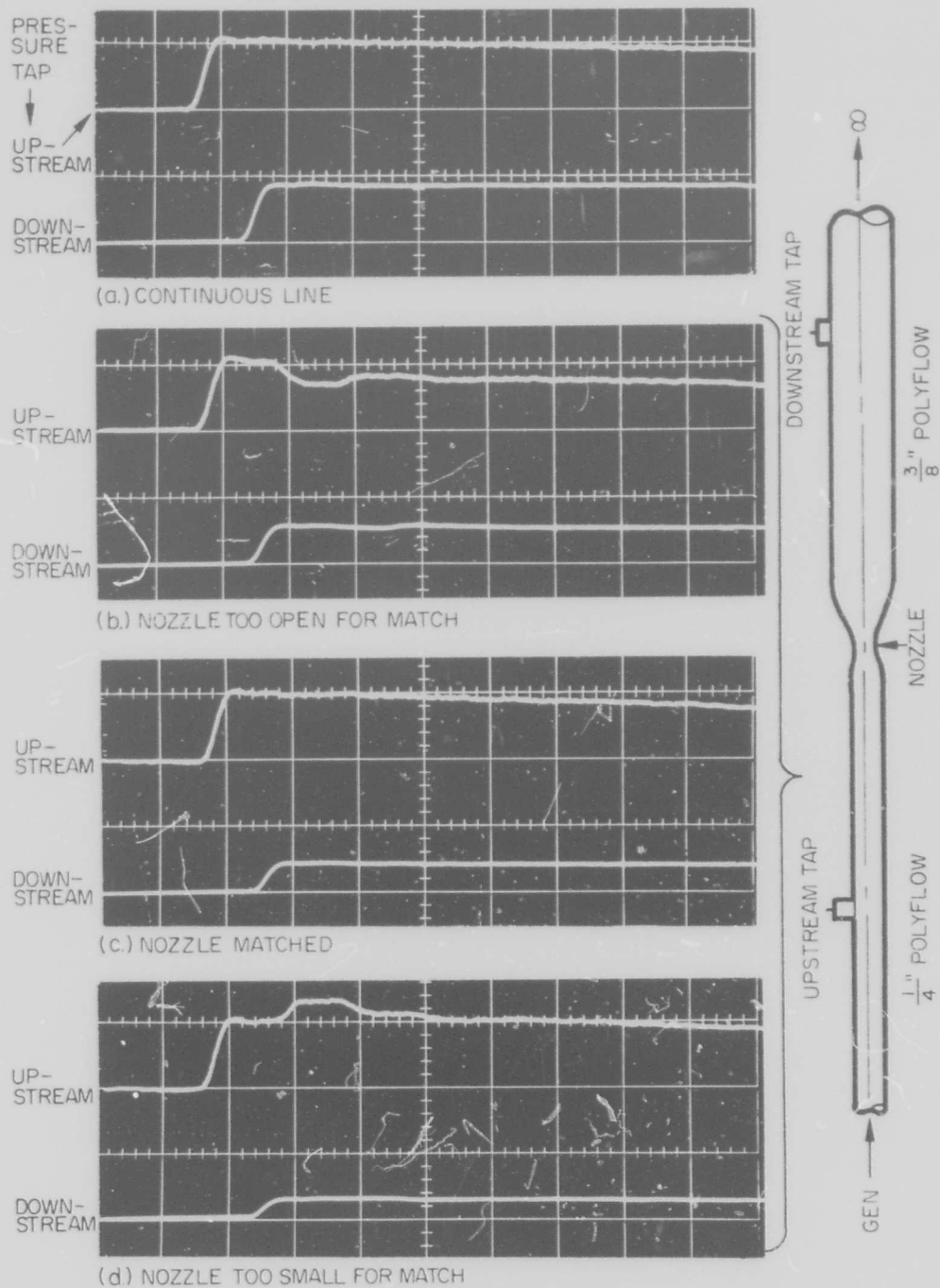
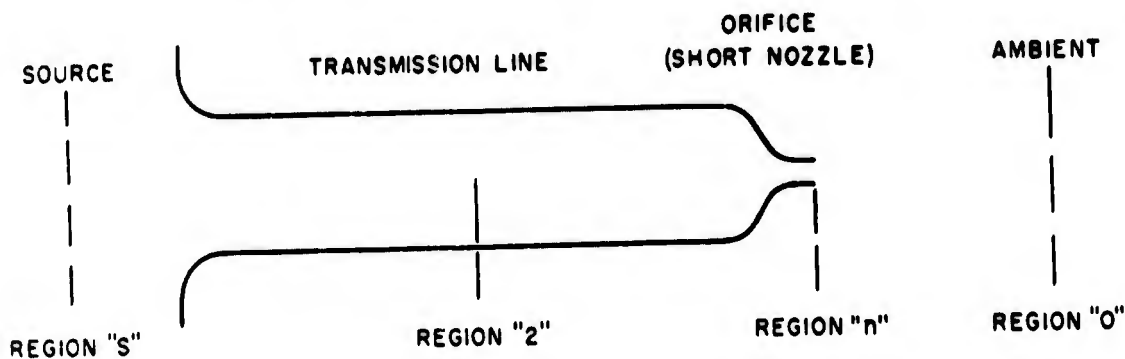


Figure 23. Matching a smaller transmission line into a larger one.

APPENDIX A.—Determination of Area Ratio for Orifice Matching



Known:	p_s	p_2	p_n	=	p_o
Conditions:	T_s	$A_2 = A_L$	A_n		T_o

The problem assumes that a pressure wave in the form of a step function has traveled down the transmission line which was initially at ambient conditions. The wave front has passed through the nozzle, so that the plateau behind it exists as a steady state throughout the transmission line. As the wave had its origin due to the establishment of flow from some source S which may not be at a state arrived at by isentropic compression from the ambient, a contact surface exists between that flow and the flow caused by the wave preceding it. A discontinuity exists at this surface in temperature and density but not in pressure or flow velocity (fig. 8). Wave reflections at the transmission line terminations due to the discontinuities of the contact surface and wave front have been ignored. The solution seeks only to determine the nozzle size that will sustain the same upstream flow state as would exist if the line were continuous and is not concerned with the transient reflections. This is the flow which exists in region 2, whether derived from the source or the ambient.

The source pressure, p_s ; pressure wave amplitude, p_2 ; ambient pressure, p_o ; source temperature, T_s ; ambient temperature, T_o ; and transmission line cross-sectional area $A_2 = A_L$ have been assumed as known measurements. The nozzle exit pressure, p_n , is assumed as equal to ambient.

The flow velocity at 2 is determined by the simple wave relation:

$$a_2 - a_o = \frac{\gamma - 1}{2} [u_2 - u_o] ; u_o = 0$$

from which

$$u_2 = \frac{2 a_o}{\gamma - 1} \left[\left(\frac{p_2}{p_o} \right)^{\frac{\gamma - 1}{2\gamma}} - 1 \right] \quad (A-1)$$

as

$$\frac{a_2}{a_o} = \left(\frac{p_2}{p_o} \right)^{\frac{\gamma - 1}{2\gamma}}$$

From region 2 to region n, energy is conserved:

$$a_2^2 + \frac{\gamma - 1}{2} u_2^2 = a_n^2 + \frac{\gamma - 1}{2} u_n^2$$

or

$$u_n = \sqrt{\frac{2}{\gamma - 1} \left[a_2^2 - a_n^2 + \frac{\gamma - 1}{2} u_2^2 \right]}$$

Substituting equation (1), and $\frac{a_2}{a_n} = \left(\frac{p_2}{p_n} \right)^{\frac{\gamma - 1}{2\gamma}}$ gives

$$u_n = a_o \sqrt{\frac{2}{\gamma - 1} \left[\left(\frac{a_n}{a_o} \right)^2 \left[\left(\frac{p_2}{p_n} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] + \frac{2}{\gamma - 1} \left[\left(\frac{p_2}{p_o} \right)^{\frac{\gamma - 1}{2\gamma}} - 1 \right]^2 \right]} \quad (A-2)$$

From conservation of flow:

$$\rho_2 A_2 u_2 = \rho_n A_n u_n$$

$$\frac{A_n}{A_2} = \frac{\rho_2}{\rho_n} \frac{u_2}{u_n} = \left(\frac{p_2}{p_n} \right)^{\frac{1}{\gamma}} \left(\frac{u_2}{u_n} \right)$$

Substituting equations (1) and (2) gives:

$$\frac{A_n}{A_2} = \frac{\sqrt{\frac{2}{\gamma - 1} \left(\frac{p_2}{p_o} \right)^{\frac{1}{\gamma}} \left[\left(\frac{p_2}{p_o} \right)^{\frac{\gamma - 1}{2\gamma}} - 1 \right]}}{\sqrt{\left(\frac{a_n}{a_o} \right)^2 \left[\left(\frac{p_2}{p_n} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] + \frac{2}{\gamma - 1} \left[\left(\frac{p_2}{p_o} \right)^{\frac{\gamma - 1}{2\gamma}} - 1 \right]^2}} \quad (A-3)$$

This is a generalized expression for matching a nozzle to a transmission line. The ratio a_n/a_o depends upon the source from which the flow at 2 is derived.

A.1 Case I: Flow Due to Wave

The flow at 2 is established by the initial wave compressing the ambient fluid in the line isentropically from p_o to p_2 . This flow then expands from p_2 to $p_n = p_o$ isentropically with the nozzle, so $\frac{a_n}{a_o} = 1$. Then

$$\frac{A_n}{A_L} = \frac{\frac{2}{\gamma-1} \left(\frac{p_2}{p_o}\right)^{\frac{1}{\gamma}} \left[\left(\frac{p_2}{p_o}\right)^{\frac{\gamma-1}{2\gamma}} - 1 \right]}{\left[\left(\frac{p_2}{p_o}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right] + \frac{2}{\gamma-1} \left[\left(\frac{p_2}{p_o}\right)^{\frac{\gamma-1}{2\gamma}} - 1 \right]^2} \quad (A-4)$$

A.2 Case II: Flow Due to Source

When the flow at 2 is upstream of the contact surface, the fluid is expanding isentropically from the source S through the nozzle to $p_n = p_o$. Then,

$$\frac{a_n}{a_s} = \left(\frac{p_n}{p_s}\right)^{\frac{\gamma-1}{2\gamma}} = \left(\frac{p_o}{p_s}\right)^{\frac{\gamma-1}{2\gamma}}$$

and

$$\left(\frac{a_n}{a_o}\right)^2 = \left(\frac{a_s}{a_o}\right)^2 \left(\frac{p_o}{p_s}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{T_s}{T_o}\right) \left(\frac{p_o}{p_s}\right)^{\frac{\gamma-1}{\gamma}}$$

Equation (3) then reduces to:

$$\frac{A_n}{A_L} = \frac{\frac{2}{\gamma-1} \left(\frac{p_2}{p_o}\right)^{\frac{1}{\gamma}} \left[\left(\frac{p_2}{p_o}\right)^{\frac{\gamma-1}{2\gamma}} - 1 \right]}{\left(\frac{T_s}{T_o}\right) \left(\frac{p_o}{p_s}\right)^{\frac{\gamma-1}{\gamma}} \left[\left(\frac{p_2}{p_o}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right] + \frac{2}{\gamma-1} \left[\left(\frac{p_2}{p_o}\right)^{\frac{\gamma-1}{2\gamma}} - 1 \right]^2} \quad (A-5)$$

Obviously, if T_s results from isentropic compression from T_o , the term $\left(\frac{T_s}{T_o}\right)\left(\frac{p_o}{p_s}\right)^{\frac{\gamma-1}{\gamma}}$ becomes equal to unity and equation (A-5) is identical to (A-4). This simply states that all discontinuities have disappeared at the contact surface. However, in usual practice $\frac{T_s}{T_o} \neq \frac{p_s}{p_o}^{\frac{\gamma-1}{\gamma}}$. More often the source temperature is nearly the same as the ambient. When equal, the terms

$$\frac{T_s}{T_o} = 1 \quad \text{and} \quad \left(\frac{p_s}{p_o}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_2}{p_o}\right)^{\frac{\gamma-1}{\gamma}} + \frac{2}{\gamma-1} \left[\left(\frac{p_2}{p_o}\right)^{\frac{\gamma-1}{2\gamma}} - 1 \right]^2$$

A.3 Acoustic Solution

In the acoustic case, when changes of overpressure and density are considered very small, the relation for a simple wave is given by¹

$$(p_2 - p_o) = \rho_o a_o u_2 \quad (\text{A-6})$$

and

$$a_o = \sqrt{\frac{\gamma p_o}{\rho_o}} \quad (\text{A-7})$$

As the change in density is very slight, the flow in the nozzle can be considered incompressible. Bernoulli's equation then gives

$$u_n = \sqrt{2 \left(\frac{p_2 - p_o}{\rho_o} \right)^{\frac{1}{2}}} \quad (\text{A-8})$$

The area ratio is then determined from the conservation of flow

$$\frac{A_n}{A_L} = \frac{u_2}{u_n}$$

$$\frac{A_n}{A_L} = \frac{1}{\sqrt{2\gamma}} \left(\frac{p_2 - p_o}{p_o} \right)^{\frac{1}{2}}$$

¹Marks, "Mechanical Engineers Handbook," 5th Ed., McGraw-Hill Book Company, New York, 1951.

after substitution of (A-6), (A-7), and (A-8). Notice that this gives a straight line of a slope of 1/2 on a log-log plot, which is in agreement with the general equation for values of $\left(\frac{p_2 - p_0}{p_0} \right) < .01$.

The equation is also verified at a value of $\frac{1}{\sqrt{2\gamma}}$ for $\left(\frac{p_2 - p_0}{p} \right) = 1$

coinciding with the general solution with the value of the extended tangent on the log-log plot for low values of $\left(\frac{p_2 - p_0}{p_0} \right)$.

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