

BOLT BERANEK AND NEWMAN INC

CONSULTING • DEVELOPMENT • RESEARCH

Report No. 1970

22 January 1971

THE UNSTEADY HYDRODYNAMICS AND CONTROL OF
HYDROFOILS NEAR A FREE SURFACE

Joseph I. Smullin
Erich K. Bender

Final Report — 1 August 1969 - 30 April 1970

Contract No. N00014-70-C-0032

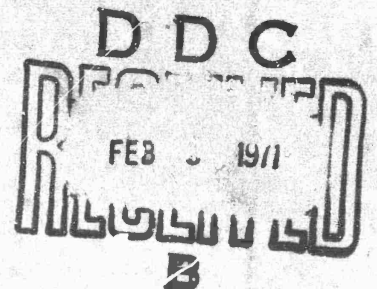
Department of Defense Distribution Statement No. 1

This research was sponsored by the Systems Development Office of the Naval Ship Research and Development Center under Naval Ship Systems Command Subproject S 46 06, Task 1700, Contract N00014-70-C-0032.

Submitted to:

Systems Development Office
Naval Ship Research and Development Center
Naval Ship Systems Command
Washington, D.C. 20007

Attention: Subprojects S-46-06
Task 1700



This document has been approved for public release and sale; its distribution is unlimited. Reproduction in whole or in part is permitted for any purpose of the United States Government.

Reproduced by
NATIONAL TECHNICAL
INFORMATION SERVICE
Springfield, Va. 22151

CAMBRIDGE

NEW YORK

CHICAGO

LOS ANGELES

86

AD 717681

Report No. 1970

22 January 1971

THE UNSTEADY HYDRODYNAMICS AND CONTROL OF
HYDROFOILS NEAR A FREE SURFACE

Joseph I. Smullin
Erich K. Bender

Bolt Beranek and Newman Inc.
50 Moulton Street
Cambridge, Massachusetts 02138

Final Report - 1 August 1969 - 30 April 1970

Contract No. N00014-70-C-0032

Department of Defense Distribution Statement No. 1

This research was sponsored by the Systems Development Office of the Naval Ship Research and Development Center under Naval Ship Systems Command Subproject S 46 06, Task 1700, Contract N00014-70-C-0032.

Submitted to:

Systems Development Office
Naval Ship Research and Development Center
Naval Ship Systems Command
Washington, D.C. 20007

Attention: Subprojects S-46-06
Task 1700

This document has been approved for public release and sale; its distribution is unlimited. Reproduction in whole or in part is permitted for any purpose of the United States Government.

ABSTRACT

In this Report we discuss (1) the development of a digital computer program to compute hydrofoil loads and (2) some aspects of hydrofoil control. The program computes the lift, pitching moment, and flap hinge moment on a two-dimensional hydrofoil with a trailing edge flap operating near a free surface with waves. The computational approach involves the numerical solution of an integral equation relating an upwash distribution to a kernel function and pressure distribution. The pressure distribution is expanded in a truncated Glauert series, the integration is carried out numerically using a Gaussian quadrature, and the coefficients of the Glauert series are evaluated by a minimum error collocation method.

The control problem investigated involves the positioning of a pivoted hydrofoil by means of a servo-controlled trim tab. When the foil is pivoted at its quarter chord and control is implemented solely by means of a servo tab, the system is virtually uncontrollable. However, by pivoting the foil off the quarter chord point or by augmenting the servo tab with a servo attached directly to the foil, the system can be controlled.

TABLE OF CONTENTS

	page
ABSTRACT	iii
LIST OF FIGURES	vii
CHAPTER 1. INTRODUCTION	1
2. UNSTEADY HYDRODYNAMICS	4
2.1 Hydrodynamic Coefficients	4
2.2 Hydrodynamic Analysis	6
3. COMPUTER PROGRAMS	19
3.1 Flap Program	20
3.2 Wave Program	21
3.3 Program Flow Charts and Print-Out Listings	21
3.4 Input-Output	22
3.5 Flap Sample Problem	26
3.6 Wave Sample Problem	27
3.7 Operation	27
3.8 Results	28
4. CONTROL PROBLEMS	41
4.1 Control	42
5. CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDY	45
ACKNOWLEDGEMENTS	46
REFERENCES	47

	page
APPENDIX A: KERNEL FUNCTION FOR TWO DIMENSIONAL FLOW WITH A FREE SURFACE	48
B: FLOW CHARTS FOR HYDRODYNAMIC PROGRAMS	50
C: PROGRAM TO COMPUTE THE HYDRODYNAMIC COEFFI- CIENTS FOR A FLAPPED FOIL OPERATING IN WAVES ..	54
D: PROGRAM TO COMPUTE HYDRODYNAMIC COEFFICIENT FOR A FOIL WITH AN OSCILLATING FLAP	60
E: SUBROUTINES	67

LIST OF FIGURES

	page
Figure 1. Foil Configurations	2
2. Hydrofoil Nomenclature	5
3. Hydrofoil Coordinates Used in Unsteady Hydro- dynamic Computation	7
4. Relation of Hydrofoil to Surface Waves	17
5. Output for Sample Problem of Hydrofoil With Oscillating Flap	24
6. Output for Sample Problems of Hydrofoil in Waves	25
7. $C_{M_{\beta V}}$ Operation in Waves	29
8. $C_{M_{\alpha V}}$ Operation in Waves	30
9. C_{L_V} Operation in Waves	31
10. C_{L_h} Heave Oscillation	32
11. $C_{M_{\dot{\alpha}h}}$ Heave Oscillation	33
12. $C_{M_{\beta h}}$ Heave Oscillation	34
13. $C_{L_{\alpha}}$ Pitch Oscillation	35
14. $C_{M_{\alpha\alpha}}$ Pitch Oscillation	36
15. $C_{M_{\beta\alpha}}$ Pitch Oscillation	37

	page
Figure 16. $C_{L\beta}$ Flap Oscillation	38
17. $C_{M_{\alpha\beta}}$ Flap Oscillation	39
18. $C_{M_{\beta\beta}}$ Flap Oscillation	40
19. Block Diagram of Hydrofoil Dynamics With A Servo Tab Comprising 10% of the Foil Chord	43

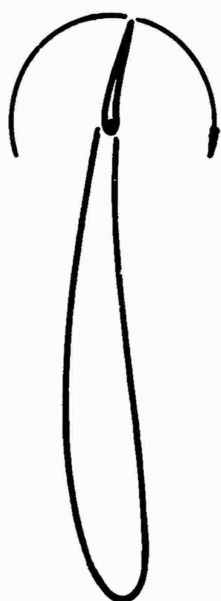
1. INTRODUCTION

One of the principal features of a hydrofoil craft with fully submerged foils is its ability to maneuver in a seaway with greater isolation from the sea surface than virtually every other type of (surface) vessel. A hydrofoil craft is able to "platform" or traverse the short waves of a rough sea without undergoing significant vertical motion and to "contour" over high-amplitude long wavelength swells. During turning maneuvers the craft leans or banks into the turn much like an aircraft or bicycle, thereby maintaining a shipboard environment in which the lateral loads are minimal. This is particularly desirable for personnel on board.

To control a hydrofoil craft, the foils must be continuously adjusted. When the craft is travelling a straight course at a constant mean height in waves, the orbital motion of the water results in a time-varying upwash and lift fluctuations on the foil. Typically, the foil angle of attack or the deflection of flaps is varied to counter the lift fluctuations due to wave motion. Foils or foil control surfaces must be similarly moved to contour over waves or to make coordinated (i.e., properly banked) turns.

Varying the foil angle of attack to control lift forces requires especially large, heavy, special-purpose servomechanisms. These servos require a great deal of power (as much as 800 hp) and high-pressure, high-volume hydraulic lines which are often hazardous. In order primarily to reduce the servomechanism requirements, we have undertaken a study of the unsteady hydrodynamics and control of hydrofoils.

The four hydrofoil configurations we studied are illustrated in Fig. 1. Lift on the basic foil, illustrated as configuration 1, is



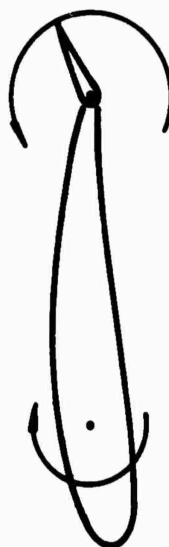
2. DIRECT FLAP CONTROL (FIXED FOIL)



1. DIRECT CONTROL (PIVOTED FOIL)



4. LEADING EDGE FLAP (PIVOTED FOIL)



3. TAB CONTROL (PIVOTED FOIL)

FIG.1 FOIL CONFIGURATIONS

controlled by varying the angle of attack of the entire foil. The foil in configuration 2 is rigidly attached to the craft and lift is controlled by deflecting a trailing-edge flap. Both of these configurations are presently used on hydrofoil craft. Configuration 3 uses a trailing-edge tab to rotate the foil about a pivot point. This configuration has the potential for very low power consumption but possesses certain inherent controllability problems. The foil in configuration 4 uses a leading-edge flap to rotate the foil. It has potentially low-power requirements and ease of control but is probably more susceptible to damage by impact with submerged debris.

The greatest part of this Report deals with the development of numerical methods and a digital computer program to determine the unsteady loads on a hydrofoil with a flap operating in waves in the vicinity of a free surface. This represents an extension of an earlier work [1] which did not include the effects of a flap. We do not explicitly develop results for a leading-edge flap; however, this configuration is simply the superposition of a rigid foil rotation plus a flap deflection. We also discuss a study of the control of hydrofoils, with emphasis on the tab-control system described by configuration 3.

2. UNSTEADY HYDRODYNAMICS

2.1 Hydrodynamic Coefficients

Assuming that the flow/foil interaction is linear (which is valid for surface wave heights that are small compared with the foil depth), the net forces and moments are equivalent to the sum of the loads generated by the upwash on a foil moving steadily at constant speed U through rough water plus the loads caused by the heave and rotary motion of a foil moving through calm water. The lift and moments due to upwash velocity V are given by

$$L_V = C_{LV}(k)V(k) \quad (1a)$$

$$M_{\alpha V} = C_{M_{\alpha}} V(k)V(k) \quad (1b)$$

$$M_{\beta V} = C_{M_{\beta}} V(k)V(k) \quad (1c)$$

where most of the variables are defined in Fig. 2 and k is the reduced frequency ($k = \omega b/U$), nondimensionalized by the semichord b and by U .

The forces and moments owing to the motion of the foil are

$$L = C_{L\alpha}(k)\alpha + C_{L\beta}(k)\beta + C_{Lh}(k)\dot{h} \quad (2a)$$

$$M_{\alpha} = C_{M_{\alpha\alpha}}(k)\alpha + C_{M_{\alpha\beta}}(k)\beta + C_{M_{\alpha h}}(k)\dot{h} \quad (2b)$$

$$M_{\beta} = C_{M_{\beta\alpha}}(k)\alpha + C_{M_{\beta\beta}}(k)\beta + C_{M_{\beta h}}(k)\dot{h} \quad (2c)$$

The purpose of our hydrodynamic analysis is to evaluate the twelve functions $C_{ij}(k)$ in Eqs. 1 and 2. This has been done by

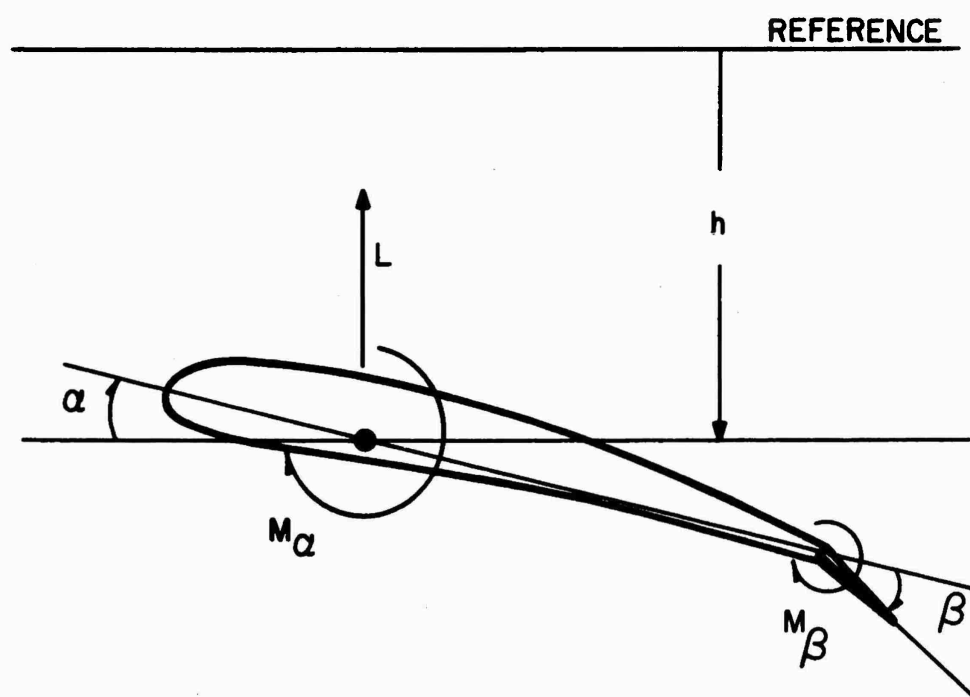


FIG.2 HYDROFOIL NOMENCLATURE

modifying an existing digital program [1], which computes the unsteady lift and moment on a two-dimensional foil near the surface, to include the effects of a flap and the upwash caused by surface waves.

2.2 Hydrodynamic Analysis

The unsteady pressure distribution over the surface of a hydrofoil is related to the upwash by an influence function, the kernel function [1], which is a complex function of reduced frequency, depth, and Froude number. For a foil which has its leading edge at $x = -1$ and its trailing edge at $x = +1$ as shown in Fig. 3, this relation can be written as:

$$V(x) = \frac{1}{2\pi} \int_{-1}^1 \Delta P(\xi) K(x-\xi) d\xi . \quad (3)$$

Here, ΔP is the pressure difference across the foil, V is the upwash nondimensionalized by the velocity U , and K is the kernel.

For a foil operating at finite depth, the kernel function must satisfy the boundary conditions of the free surface, as well as those of the foil itself. The free surface is modeled by including a virtual image of the foil pressure distribution on the opposite side of the surface. During operation at low Froude numbers, the moving pressure disturbance from the foil causes a large enough gravity wave to affect the kernel significantly. The complex kernel as a function of depth and Froude number for two-dimensional flow is developed in detail by Widnall [1]; the result is given here in Appendix A. For a rigid foil oscillating in heave or pitch, the upwash $V(x)$ is known, as it is simply the local vertical velocity of the foil. That is:

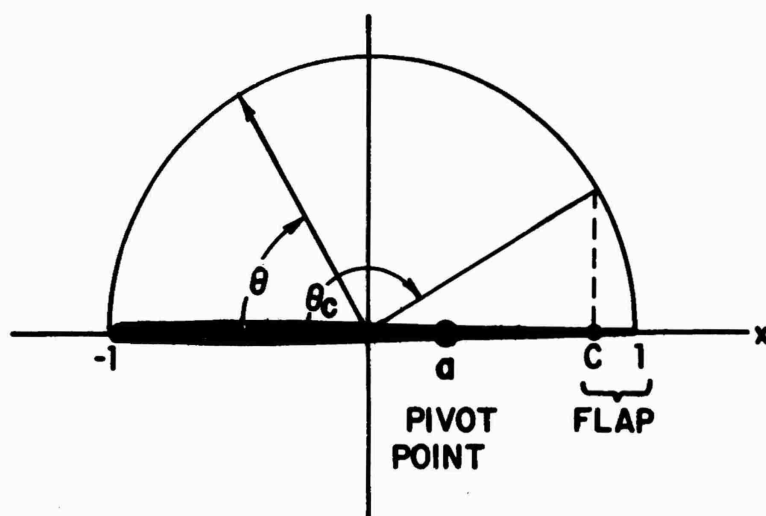


FIG.3 HYDROFOIL COORDINATES USED IN UNSTEADY HYDRODYNAMIC COMPUTATION

heave: $V(x) = \dot{h} = (\bar{h} e^{i\omega t})$

pitch: $V(x) = \alpha + \dot{\alpha}(x-a) \frac{b}{U} = (\bar{\alpha} e^{i\omega t}) [1 + ik(x-a)]$.

Here, $\dot{h}$ is nondimensionalized by U and a is the location of the foil pivot point (see Fig. 3). With the kernel and upwash known, Eq. 3 can be solved for $\Delta P(x)$.

Numerical technique for solving ΔP

For attached flow on the hydrofoil, the Kutta condition at the trailing edge must be satisfied. This boundary condition is satisfied by writing the pressure distribution as a Glauert series:

$$\Delta P = \sum_{n=1}^{\infty} p_n a_n(x) \quad (4)$$

where

$$a_n = \frac{\cot(\theta/2)}{\sin(n-1)\theta} \quad , \quad \begin{matrix} n = 1 \\ n \geq 2 \end{matrix}$$

and

$$x = -\cos\theta .$$

By combining Eqs. 3 and 4, the velocity may be written:

$$V(x) = \sum_{n=1}^{\infty} p_n \int_{-1}^{+1} a_n(\xi) K(x-\xi) d\xi . \quad (5)$$

If this series is truncated at NOLT terms* and the integration carried out, Eq. 5 can be written as

*We use somewhat cumbersome terms such as NOLT in this text to be consistent with expressions used in the computer program.

$$V(x) = \sum_{n=1}^{NOLT} p_n I_n(x) \quad (6)$$

where

$$I_n = \int_{-1}^{+1} a_n(\xi) K(x-\xi) d\xi .$$

If, in the numerical solution for ΔP , the velocity is specified at NP collocation points, there will be NP equations of the form of Eq. 6. Using a notation $A\{^B\}$, where $\{ \}$ is a matrix with A rows and B columns, this set of equations can be written

$$\begin{matrix} N \\ P \end{matrix} \{V\} = \begin{matrix} N \\ P \end{matrix} \begin{matrix} NOLT \\ \{I_n\} \end{matrix} \begin{matrix} N \\ L \\ T \end{matrix} \{p_n^1\} . \quad (7)$$

By making the number of collocation points, NP, larger than the number of terms in the Glauert series, NOLT, the set of equations is overdetermined and the pressure modes can be solved for by a method which minimizes the mean square error of the resultant downwash at the collocation points. The coefficients p_n which minimize the mean square error satisfy the following equation:

$$\frac{\partial}{\partial p_n} \left\{ \sum_{r=1}^{NP} \left[V(x_r) - \sum_{n=1}^{NOLT} p_n I_n(x_r) \right]^2 \right\} = 0 \quad \text{for } n = 1, 2, \dots, NOLT \quad (8)$$

The result [2] is

$$\{p_n\} = [\{\bar{I}_n\}^T \{I_n\}]^{-1} \{\bar{I}_n\}^T \{V\} \quad (9)$$

where $\{\bar{I}\}^T$ is the conjugate transpose of $\{I_n\}$.

The lift and moment acting on the foil can be found by integrating the pressure and its first moment over the chord of the foil. The lift and the moment about the foil quarter-chord in terms of the Glauert series are given by

$$C_L = \pi(p_1 + 1/2 p_2) \quad (10a)$$

$$C_M = \frac{\pi}{8} (-p_2 + p_3) . \quad (10b)$$

Foil with trailing-edge flap

Addition of a hinged trailing-edge flap to the hydrofoil expands the set of unknown force and moment parameters for unsteady motions to include the hinge moment due to pitch and heave and the effect of flap oscillation on lift, and moments at the quarter chord and flap hinge. (The geometry of the flapped hydrofoil is shown in Fig. 2.)

Hinge moment

The moment at the flap hinge can be computed by integrating the moment of each of the pressure modes over the flapped portion of the foil chord, $\theta_c \rightarrow \pi$. The result is:

$$\begin{aligned}
C_{M_\beta} &= p_1 \int_{\theta_c}^{\pi} \cot \frac{\theta}{2} (\cos \theta - \cos \theta_c) \sin \theta d\theta \\
&+ \sum_{n=2}^{NOLT} p_n \int_{\theta_c}^{\pi} \sin(n-1)\theta (\cos \theta - \cos \theta_c) \sin \theta d\theta \\
&= p_1 \int_{\theta_c}^{\pi} (1 + \cos \theta) \cos \theta d\theta \\
&- p_3 \left[\frac{\pi - \theta_c}{4} - \frac{\sin(4\theta_c)}{16} \right] + p_2 \frac{\sin^3(\theta_c)}{3} \\
&+ \sum_{n=4}^{NOLT} p_n \frac{1}{4} \left[\frac{\sin(n-3)\theta_c}{(n-3)} - \frac{\sin(n+1)\theta_c}{n+1} \right] \\
&+ \frac{1}{2} p_2 \cos \theta_c [\pi - \theta_c - \sin \theta_c \cos \theta_c] \\
&+ \sum_{n=3}^{NOLT} \frac{1}{2} \cos \theta_c p_n \left[\frac{\sin n\theta_c}{n} - \frac{\sin(n-2)\theta_c}{n-2} \right]. \quad (11)
\end{aligned}$$

Oscillating flap

A foil with an oscillating trailing-edge flap has a pressure distribution with a logarithmic singularity at the flap hinge, which is fit by the pressure series [3].

$$\frac{p}{\rho U^2} = \sum_{n=1}^{\infty} p_n a_n + p_c a_c, \quad (12)$$

where

$$p_c = \frac{\bar{\beta}}{\pi \sin \theta_c}$$

$\bar{\beta}$ = amplitude of angular deflection of flap

$$a_c = \sin \theta \ln(x-c)^2$$

and $\sum_{n=1}^{\infty} p_n a_n$ is the Glauert series.

A relation for the oscillating flap analogous to Eq. 5 is,

$$V = \sum_{n=1}^{\infty} p_n \int_{-1}^1 a_n K d\xi + p_c \int_{-1}^1 a_c K d\xi . \quad (13)$$

This can be written:

$$V^* = \sum_{n=1}^{\infty} p_n \int_{-1}^1 a_n K d\xi \quad (14)$$

with the "equivalent" upwash V^* ,

$$V^* = V - p_c \int_{-1}^1 a_c K d\xi , \quad (15)$$

which can be calculated directly since p_c is known. Using this equivalent upwash, the Glauert pressure modes can be computed in the same manner as was used for the plain foil, Eq. 9.

In order to find the equivalent upwash, the integration over the chord for the product of the flap hinge pressure mode times the kernel must be performed. This integral,

$$\int_{-1}^1 a_c K d\xi = \int_{-1}^1 \sin \theta \ln(\xi-c)^2 K d\xi , \quad (16)$$

has a logarithmic singularity at the hinge point which can be isolated by writing

$$\begin{aligned} \int_{-1}^1 a_c K d\xi &= \int_{-1}^1 (\sin\theta K - \sin\theta_c K_c) \ln(\xi-c)^2 d\xi \\ &+ \sin\theta_c K_c \int_{-1}^1 \ln(\xi-c)^2 d\xi . \end{aligned} \quad (17)$$

Writing the kernel as $K(x-\xi) = \frac{-1}{2\pi} \frac{1}{x-\xi} + \bar{K}(x-\xi)$, where $\bar{K}$ is a function of frequency, depth, and Froude number, and integrating in the second term, Eq. 17 becomes

$$\begin{aligned} \int_{-1}^1 a_c K d\xi &= \int_{-1}^1 H(\xi) d\xi + \sin\theta_c \left[\bar{K}_c - \frac{1}{2\pi(x-c)} \right] \\ &\times \left[-4 + 2c \ln\left(\frac{1+c}{1-c}\right) + 2 \ln(1-c)^2 \right] \end{aligned} \quad (18)$$

where

$$H(\xi) = \left\{ \sin\theta \left[\bar{K} - \frac{1}{2\pi(x-\xi)} \right] - \sin\theta_c \left[\bar{K}_c - \frac{1}{2\pi(x-c)} \right] \ln(\xi-c)^2 \right\}. \quad (19)$$

When the last singularity at $x=\xi$ is evaluated analytically, the equation can be written

$$\int_{-1}^1 a_c K d\xi = \int_{-1}^{x-\epsilon} N(\xi) d\xi + \int_{x+\epsilon}^{c-\epsilon} N(\xi) d\xi + \int_{c+\epsilon}^1 N(\xi) d\xi - Q(\xi) \quad \text{for } x < c \quad (20)$$

$$\int_{-1}^1 a_c K d\xi = \int_{-1}^{c-\epsilon} N(\xi) d\xi + \int_{c+\epsilon}^{x-\epsilon} N(\xi) d\xi + \int_{x+\epsilon}^1 N(\xi) d\xi - Q(\xi) \quad \text{for } x > c \quad (21)$$

where

$$N(\xi) = H(\xi) + \frac{\sin\theta}{2\pi(x-\xi)} \ln(x-c)^2 \quad (22)$$

$$Q(\xi) = -\frac{x}{4\pi} \ln(x-c)^2 + \sin\theta_c \left[\bar{K}_c - \frac{1}{2\pi(x-c)} \right] \\ \times \left[-4 + 2c \ln\left(\frac{1+c}{1-c}\right) + 2 \ln(1-c^2) \right]. \quad (23)$$

The integrals in Eqs. 20 and 21, having no singularities, may be evaluated by a simple numerical scheme.

The equivalent upwash, V^* as given in Eq. 15, may now be solved, using the values for the upwash due to flap oscillation:

$$V = \begin{matrix} 0 & -1 \leq x \leq c \\ -\beta - \dot{\beta}(x-c)b/U & c \leq x \leq 1 \end{matrix}.$$

The remainder of the solution for the p_n pressure modes is the same as developed in Eqs. 4 - 9.

The lift and moment coefficients are the same as those expressed in Eqs. 10 and 11, plus the effect of integrating the flap pressure mode, p_c (Eq. 12), over the foil. An equivalent set of equations is

$$C_L = \pi(p_1 + \frac{1}{2} p_2) + p_c \left[\int_{-1}^1 a_c d\xi \right] \quad (24a)$$

$$C_{M_\alpha} = -\frac{\pi}{8} (p_2 - p_3) - \frac{1}{2} p_c \left[\int_{-1}^1 a_c (\xi + \frac{1}{2}) d\xi \right] \quad (24b)$$

$$C_{M_\beta} = -\frac{1}{2} \sum_{n=1}^{\infty} p_n \left[\int_c^1 a_n \xi d\xi - c \int_c^1 a_n d\xi \right] \\ - \frac{1}{2} p_c \left[\int_c^1 a_c (\xi - c) d\xi \right] . \quad (24c)$$

The singularity in the lift coefficient integration can be isolated leaving a nonsingular integral (Eq. 24a) which can be evaluated numerically.

$$\int_{-1}^1 a_c d\xi = \int_{-1}^1 \sin\theta \ln(\xi-c)^2 d\xi \\ = \int_{-1}^1 (\sin\theta - \sin\theta_c) \ln(\xi-c)^2 d\xi + \sin\theta_c \int_{-1}^1 \ln(\xi-c)^2 d\xi \\ = \left[\int_{-1}^c + \int_{c+\epsilon}^1 \right] (\sin\theta - \sin\theta_c) \ln(\xi-c)^2 d\xi \\ + \sin\theta_c \left[-4 + 2c \ln\left(\frac{1+c}{1-c}\right) + 2 \ln(1-c^2) \right] . \quad (25)$$

The integral in Eq. 24c for the flap contribution to the quarter-chord moment is:

$$\begin{aligned}
 \int_{-1}^1 a_c \left(\xi + \frac{1}{2} \right) d\xi &= \int_{-1}^1 a_c (\xi - c) d\xi + \left(c + \frac{1}{2} \right) \int_{-1}^1 a_c d\xi \\
 &= \int_{-1}^{c-\epsilon} \sqrt{1-\xi^2} \ln(\xi-c)^2 (\xi-c) d\xi \\
 &\quad + \int_{c+\epsilon}^1 \sqrt{1-\xi^2} \ln(\xi-c)^2 (\xi-c) d\xi + \left(c + \frac{1}{2} \right) \int_{-1}^1 a_c d\xi ,
 \end{aligned}
 \tag{26}$$

and the contribution to the flap hinge moment is simply

$$\int_c^1 a_c (\xi - c) d\xi = \int_c^1 \sqrt{1-\xi^2} \ln(\xi-c)^2 (\xi-c) d\xi .
 \tag{27}$$

Unsteady lift in waves

Superposed on the hydrofoil steady velocity U are the vertical and horizontal components of the orbital motion of water associated with surface-wave propagation. Although the lift owing to the horizontal component of this orbital motion has been shown by O'Neill to be present for foils with a mean value of lift or camber [4], it is usually small and shall be neglected here. Let us evaluate the vertical component or upwash with respect to a foil moving in a seaway.

The upwash (nondimensionalized by U) at depth D associated with plane waves of amplitude A , traveling at an angle ϕ with respect to a stationary x', y' coordinate frame (see Fig. 4) is given by

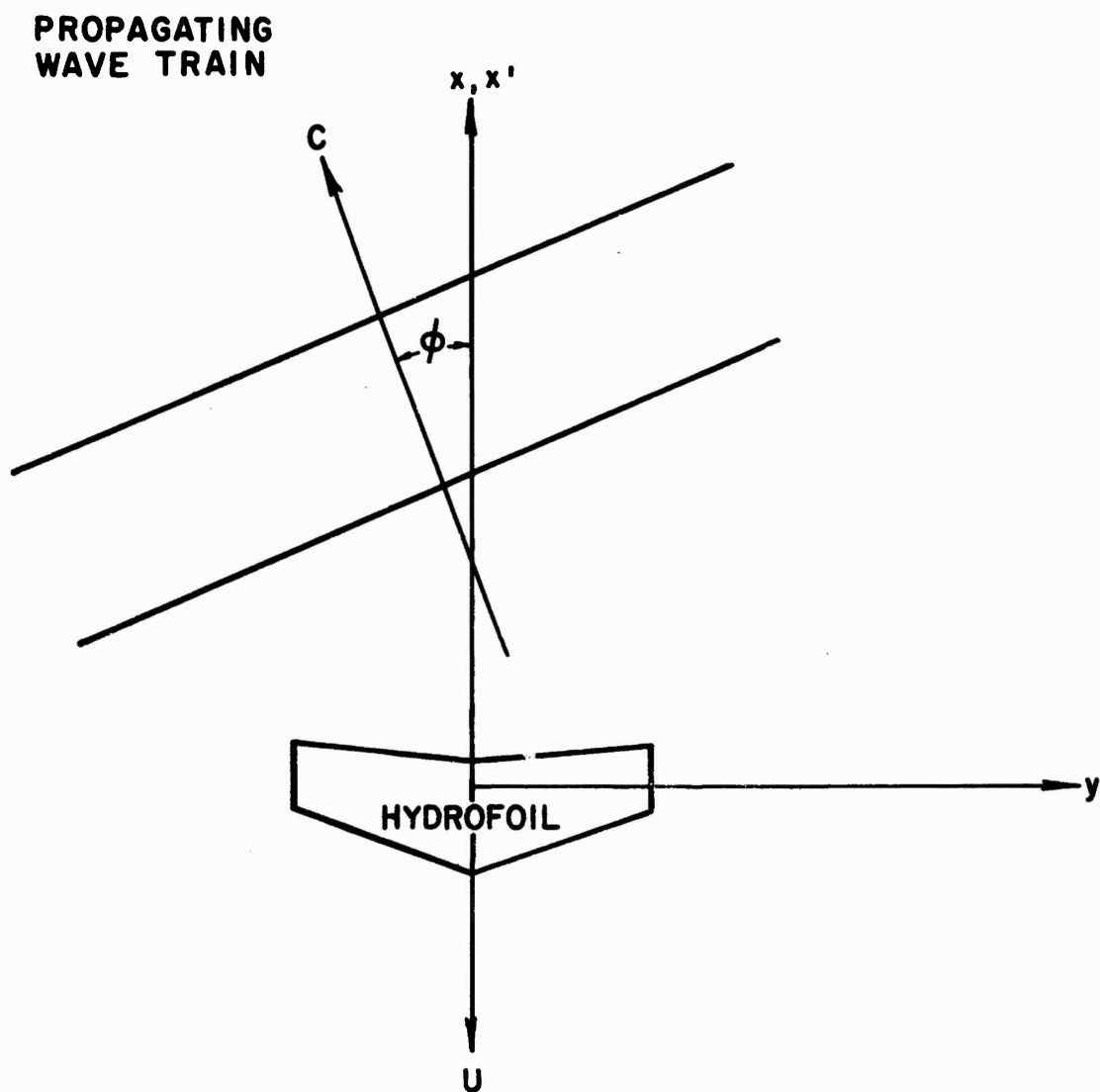


FIG.4 RELATION OF HYDROFOIL TO SURFACE WAVES

$$V(x',y',t) = \frac{A_0 \sqrt{k_w g}}{U} e^{-k_w D + i (\sqrt{k_w g} t - k_x x' + k_y y')} \quad (28)$$

where $k_{x'} = k_w \cos \phi$

$$k_{y'} = k_w \sin \phi$$

k_w is the wavenumber, $2\pi/\text{wavelength}$, and g is the gravitational constant.

To account for ship velocity through the water, we may define a codirectional coordinate system moving at the speed of the ship along the *negative* x axis with velocity U and write

$$x = + x' - Ut \quad (29)$$

Substituting this equation into Eq. 28 for $V(x',y',t)$, and neglecting y' variations gives

$$V(x',t) = \frac{A_0 \sqrt{k_w g}}{U} e^{-k_w D + i [(\sqrt{k_w g} + U k_w \cos \phi) t - k_w x' \cos \phi]} \quad (30)$$

The $kx' \cos \phi$ term describes the spatial variation of upwash which will depend on the angle of the foil with respect to the waves.

The lift associated with this upwash is solved in the same manner used for the heave and pitch upwash.

3. COMPUTER PROGRAMS

Two Fortran II programs for the calculation of the unsteady hydrodynamics of two-dimensional hydrofoils operating near a free surface are presented. The first program computes the unsteady lift, quarter chord moment, and flap hinge moment owing to foil pitch and heave, and to flap oscillations. The second program computes the unsteady lift and the moments at the flap hinge and the foil quarter chord for a hydrofoil platforming in waves. Both of the programs are based on an NSRDC program (see Ref. 1 for a description) which computes the unsteady hydrodynamics for a foil without a flap, oscillating in pitch and heave. Results are computed and plotted for specific cases and found to be in good agreement with theory.

The programs are written in Fortran II for use on the SDS 940 research computer in conjunction with the Bolt Beranek and Newman time sharing system.* The subroutines used are the same as those in the NSRDC version of the program by Widnall except for changes necessary in going from Fortran IV to II. These changes include the addition of an arctan function, THETA (X,Y) and, to reduce programming time and storage space, the substitution of a less sophisticated version of the exponential integral subroutine, EXPINT. In the subroutine for complex matrix inversion, CMPINV, the variable field, BLANK, has been broken down into its real number and interger components, PIVOT and INDEX. This was done because the SDS-940, unlike many other computers, allocates more space for the storage of real numbers than for intergers. The subroutine storage as changed, is still compatible with systems

*Number 1.85, using the 21 January 1969 Fortran operating system.

using the same amount of space for the storage of real and integer numbers.

3.1 Flap Program

Extending the NSRDC program to compute the flap hinge moment, foil pitch moment, and lift, owing to flap deflection, consists of two steps. The first is the integration of each term of the Glauert series over the flap to determine the moment about the hinge; the second step is the computation of the complex pressure distribution over the entire foil due to flap oscillation.

The integration for the hinge moment is done using the expression derived analytically in Eq. 11.

The computation of the pressure due to flap deflection, a_c in Eq. 12, is done in two separate steps. The first step is the calculation of the amplitude of the logarithmic pressure term for the flap (Eq. 12). The next step is the computation of the Glauert pressure series due to the equivalent upwash (Eq. 15). In order to calculate the lift and moment generated by the flap, a numerical integration of the flap mode must be done, as was seen in Eq. 20. This integration is done by a Gaussian quadrature [5] in the same manner as all the numerical integration in the original NSRDC program.

The program divides the specified number of collocation points evenly between the flapped and unflapped portions of the chord. In each portion of the foil the points are distributed sinusoidally in order to concentrate them at edges where the computed functions change most rapidly. The equations governing the distribution of collocation points are:

$$\begin{aligned}x_1 &= \frac{c-1}{2} - \left(\frac{1+c}{2} \right) \cos \frac{\pi x}{N} \quad \text{for } -1 < x < c \\x_1 &= \frac{c+1}{2} - \left(\frac{1-c}{2} \right) \cos \frac{\pi x}{N} \quad \text{for } c < x < 1 .\end{aligned} \quad (31)$$

3.2 Wave Program

To compute the hydrodynamic coefficients of a hydrofoil operating in waves, we have expanded the program to determine the upwash on the foil as a function of the input parameters: foil depth, Froude number, heading angle (see Fig. 4), and wavelength. This is done by application of Eq. 30, modified to give the phase of the coefficients relative to the maximum vertical wave velocity at a specified point on the foil chord (e.g., quarter chord point). The computations for this upwash are handled in exactly the same manner as those for heave and pitch in the original program. The integral for flap hinge moment as a function of hinge location (Eq. 11) has been added. The method of collocation point location is the same as used for the flapped foil.

3.3 Program Flow Charts and Print-Out Listings

Appendix B contains functional flow charts of the programs for hydrodynamic coefficients of a foil with an oscillating flap and for a foil operating in waves. There has been an attempt here to identify variables by their symbolic names in the program and their name from the theoretical development. Appendices C, D, and E contains listings of the two main programs plus a listing of the subroutines.

3.4 Input-Output

The inputs to the program are adapted to the BBN console-operated time sharing computer system. Some of the input that was introduced many times was stored on disk files for ease of operation. In a punch card system, this input would probably be more easily handled by reading the data from cards for each run. The inputs for the flap program, which follow the standard Fortran convention for interger and real number identification, are:

file NAME

(which reads from a file

NP, NOLT, IM, IFF, RF, FR, D)

The operator types in the values of the variables,

CFLAP, NFLP, NLE, NTE

using FORMAT (F 10.4, 3I 10).

The inputs to the wave program from file are the same as those for the flapped foil. The typed-in variables are,

WL, THET, FR, PPA, CFLAP, NLE, NTE

using

FORMAT (5 F10.4, 2I4).

The Froude number, FR, which had been read in from file was typed in again to facilitate easier variation of this parameter.

Table 1 lists and describes the inputs to the both programs. Figures 5 and 6 are sample runs of the FLAP and WAVE programs.

The underlined portions were typed by the operator.

TABLE 1. Input for Flap and Wave Programs

NP	total number of collocation points (even number, greater than 2 NOLT)
NOLT	number of terms in the Glauert Series
IM	determine whether or not virtual image on opposite side of free surface is accounted for in kernel. Set $IM = 0$ to exclude free surface effects of image. Set $IM =$ any integer other than zero to include free surface effects of virtual image.
IFF	$=0$: ($Fr = \infty$) Froude effect is not included in kernel $\neq 0$: Froude effect is included in kernel
RF	reduced frequency ($k = [\omega b]/[U_0]$) of oscillation (heave, pitch or flap)
FR	Froude number, $F = (U_0)/(\sqrt{gb})$
D	depth in semichords
CFLAP	fraction of the foil occupied by flap
NFLP	number of terms in the integration for flap pressure term
NLE	number of terms in the integration for I_n in the region $\xi < x$ (see Eq. 6)
NTE	number of terms in the integration for I_n in the region $\xi > x$ (see Eq. 6)
WL	wavelength in semichords
THET	angle (radians) of foil heading
PPA	coordinate on foil to which wave phase is related (e.g., the PPA for the quarter chord is -0.5)

```

1 IMAGE IFF      K      FROUDE NUM      DEPTH IN SEMCHDS      NP      NOLT
0              0      .2000      1.0000      20      6

FLAP CHORD      .25      FRACTION OF AIRFOIL CHORD
-.14550E 1      .37504E 0      -.15301E 1      .28404E 0      -.35105E 0      .20663E 0
.13395E- 3      -.39738E 0      .39955E- 1      -.79736E 0      -.74317E 0      -.15215E 0
.14821E- 3      .52333E- 4      -.19511E- 1      .74768E- 4      .24946E 0      .11459E 0
-.10429E- 3      .20587E- 4      -.10917E- 3      -.30692E- 5      -.27619E- 1      -.58644E- 1
-.54742E- 4      .13830E- 4      -.59580E- 4      .12334E- 4      -.13913E 0      .15656E- 1
-.23280E- 4      .57129E- 5      -.24219E- 4      .53858E- 5      .14798E 0      .18182E- 1
0 HEAVING OSCILLATION
CLR= -4.5709      CLI= .5540      CMK= .0000      CMI= .1561
CHR= .0257      CHI= .0059
0 COMPARISON WITH CORNELL HR= -.72749362 HI=,--.01182340
0 PITCHING OSCILLATION
CLR= -4.7441      CLI= -.3602      CMK= -.0234      CMI= .3132
CHR= .0248      CHI= .0201
FLAP OSCILLATION
CLR= -2.8148      CLI= .4102      CMK= .6462      CMI= .1047
CHR= .0528      CHI= .0100
-4.57087051      .55401701      .15607327      .201.001.0020 6
-4.74411224      -.36015309      .31315113      .201.001.0020 6

```

FIG. 5 OUTPUT FOR SAMPLE PROBLEM OF HYDROFOIL WITH OSCILLATING FLAP

1 IMAGE IFF 0 0
K FROUDE NUM DEPTH IN SEMCHDS NP NOLT
.400110000.0000 1.0000 20 6

FLAP CHORD .25 FRACTION OF AIRFOIL CHORD
WAVE LENGTH= 15.708 SEMI-CHORDS. HEADING= .000

-.12493E 1	.32670E 0	-.13803E 1	.22688E 0	-.72378E 0	.26071E 0
.11264E- 2	-.79565E 0	.16081E 0	-.15951E 1	.88772E- 3	.24788E- 2
.12775E- 2	.34434E- 4	-.77353E- 1	.30591E- 3	-.70198E- 4	.26125E- 4
-.17977E- 3	-.80877E- 5	-.18853E- 3	-.20852E- 3	-.10482E- 3	.31185E- 4
-.98315E- 4	.24245E- 4	-.11936E- 3	.15447E- 4	-.51934E- 4	.19133E- 4
-.37201E- 4	.13129E- 4	-.41007E- 4	.16225E- 4	-.22330E- 4	.70627E- 5

0 HEAVING OSCILLATION

CLR= -3.9230 CLI= -.2235 CMR= .0001 CMI= .3125
CHR= .0221 CHI= .0193
0 COMPARISON WITH CORNELL HR= -.62438072 HI= -.23559697
0 PITCHING OSCILLATION

CLR= -4.0837 CLI= -1.7929 CMR= -.0935 CMI= .6265
CHR= .0155 CHI= .0462

IN WAVES.

CLR= -2.2724	CLI= .8229	CMR= -.0004	CMI= -.0010
CHR= .0128	CHI= -.0047		
-3.92300816	-.22345817	.00005934	.31246508 .400.001.0020 6
-4.08365353	-1.79285942	-.09352663	.62652297 .400.001.0020 6

FIG. 6 OUTPUT FOR SAMPLE PROBLEMS OF HYDROFOIL IN WAVES

3.5 Flap Sample Problem

A sample problem using the FLAP program for a hydrofoil with a 25% flap, reduced frequency of 0.2, and no effect from the free surface ($IM=IFF=\emptyset$) is shown in Fig. 5. Twenty collocation points and six Glauert terms are used, while eight terms are used in the numerical integration ($NLE=NTE=NFLP=8$). The depth and Froude number are arbitrarily read in as 1.0, and will be set infinite because we have specified $IM=IFF=\emptyset$. The input read from the file is printed and identified. The fraction of the chord occupied by the flap is printed out after the data from the file. The six columns of numbers in exponential form which follow are the complex values for NOLT terms of the Glauert series. Here row number represents the order of the term and the columns are alternately the real and imaginary parts of the terms for heave, pitch and flap oscillations, in that order from left to right. Thus, for flap oscillation the amplitude of the third term in the Glauert series is $P_3 = .24946 + i.11459$. The lift and moment coefficients for the three cases follow. The R and I suffixes stand for the real and imaginary parts respectively, while L stands for the lift, M for quarter chord moment and H for the hinge moment. The "Comparison with Cornell" is the heave lift coefficient divided by 2π (i.e., the coefficient at zero frequency without free surface affects). The last two rows of output are a summary of the heave and then pitch output in the following order:

CLR, CLI, CMR, CMI, RD, D, NP, NOLT.

3.6 Wave Sample Problem

For the sample problem utilizing the WAVE program, we have taken a case with the foil operating at a very high Froude corresponding to a foil speed that is much larger than the wave speed. This condition approximates a foil moving through a stationary sinusoidal gust. Again the free surface effects have been ignored (IM and IFF are zero). The program computes values in waves based upon unity orbital velocity at the surface. Setting $D = \emptyset$ makes the amplitude of the sinusoidal upwash at the foil unity. The wavelength is 15.71 semi-chords while the heading, 0.0, is directly into the seas. The Froude number is 10^4 , phase is computed relative to the upwash at -0.5 (quarter chord), and the flap is 25% of the chord. The output format is the same as for the flap oscillation, except that the third set of coefficients is for operation in waves, and the wavelength and the heading are printed out.

3.7 Operation

For a given computation the operator must select the values of NOLT, NP, NFLP, NLE, and NTE. Running time is affected by all of these parameters, being most sensitive to the first two. The minimum number of Glauert coefficients, NOLT, is determined by the flap size and is critical for accuracy of the flap hinge moment. For flaps which are shorter than half of the chord, this minimum value will be

$$NOLT = 2\pi/(\pi - \theta_c).$$

(rounded up to an interger).

For flaps which are hinged forward of the mid chord NOLT must be at least three.

A general rule for NP is that it should be greater than $2 \times \text{NOLT}$. The numerically integrated functions are fairly smooth and values of eight for NFLP, NLE, and NTE gave good results. Values up to thirty-two may be used.

3.8 Results

We have computed the twelve hydrodynamic functions for representative values of depth and Froude number; the results are illustrated in Figs. 7 through 18. The functions are plotted in the complex plane with several values of reduced frequency indicated on each curve. Phase and amplitude may be found directly from the plots as the angle and magnitude of the vector from the origin to the point of interest on the curve.

Figures 8 and 9 show pitch and heave functions for foils in waves for $D = \infty$. Strictly speaking, there is no influence of waves at infinite depth; this notation merely means that the free surface effects have not been included but that the foil penetrates a sinusoidal gust. We have compared this solution with the Sears function [6] and have also compared the functions for foil motion at infinite depth with Theodorsen's function [7]. The agreement is within one percent.

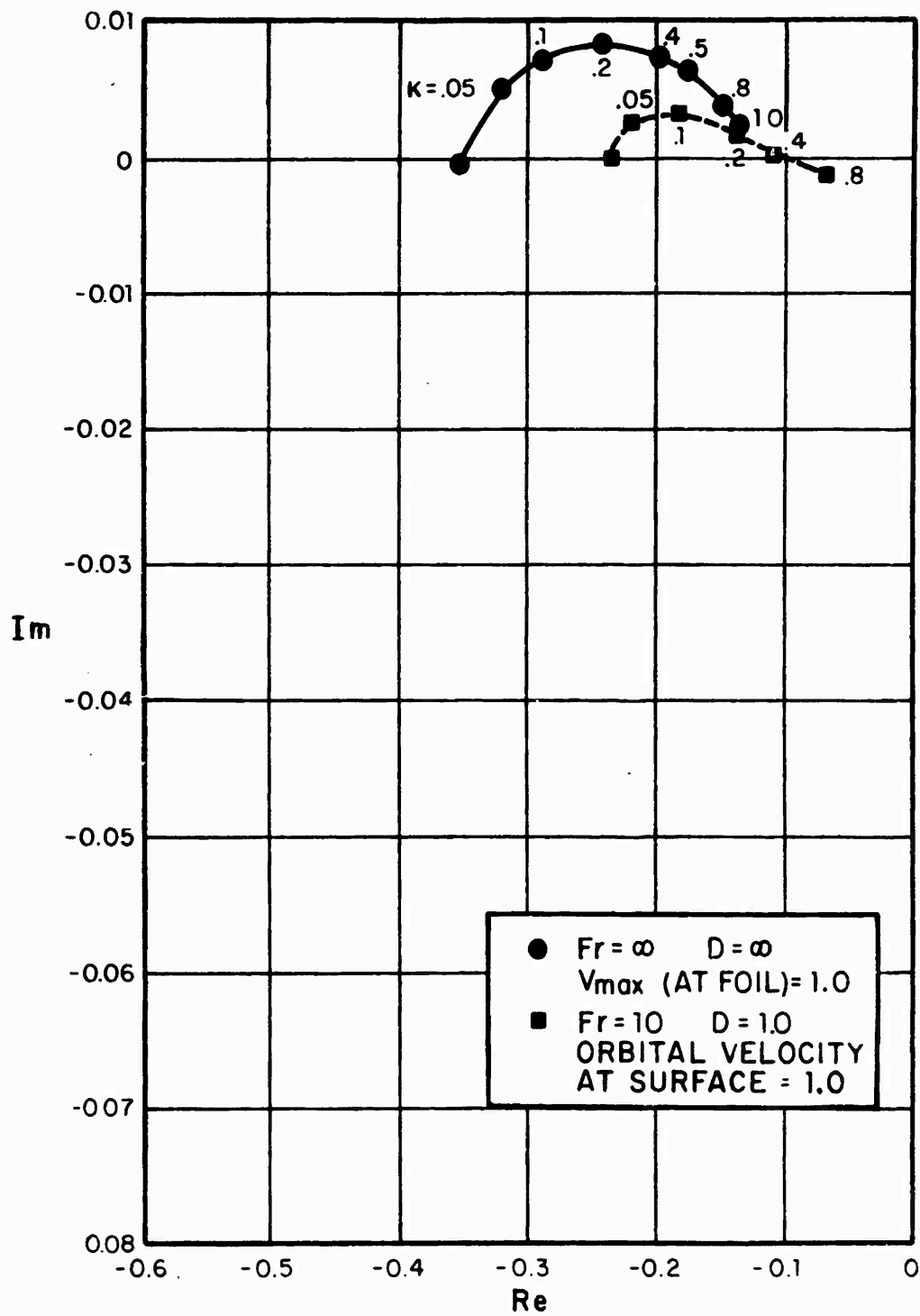
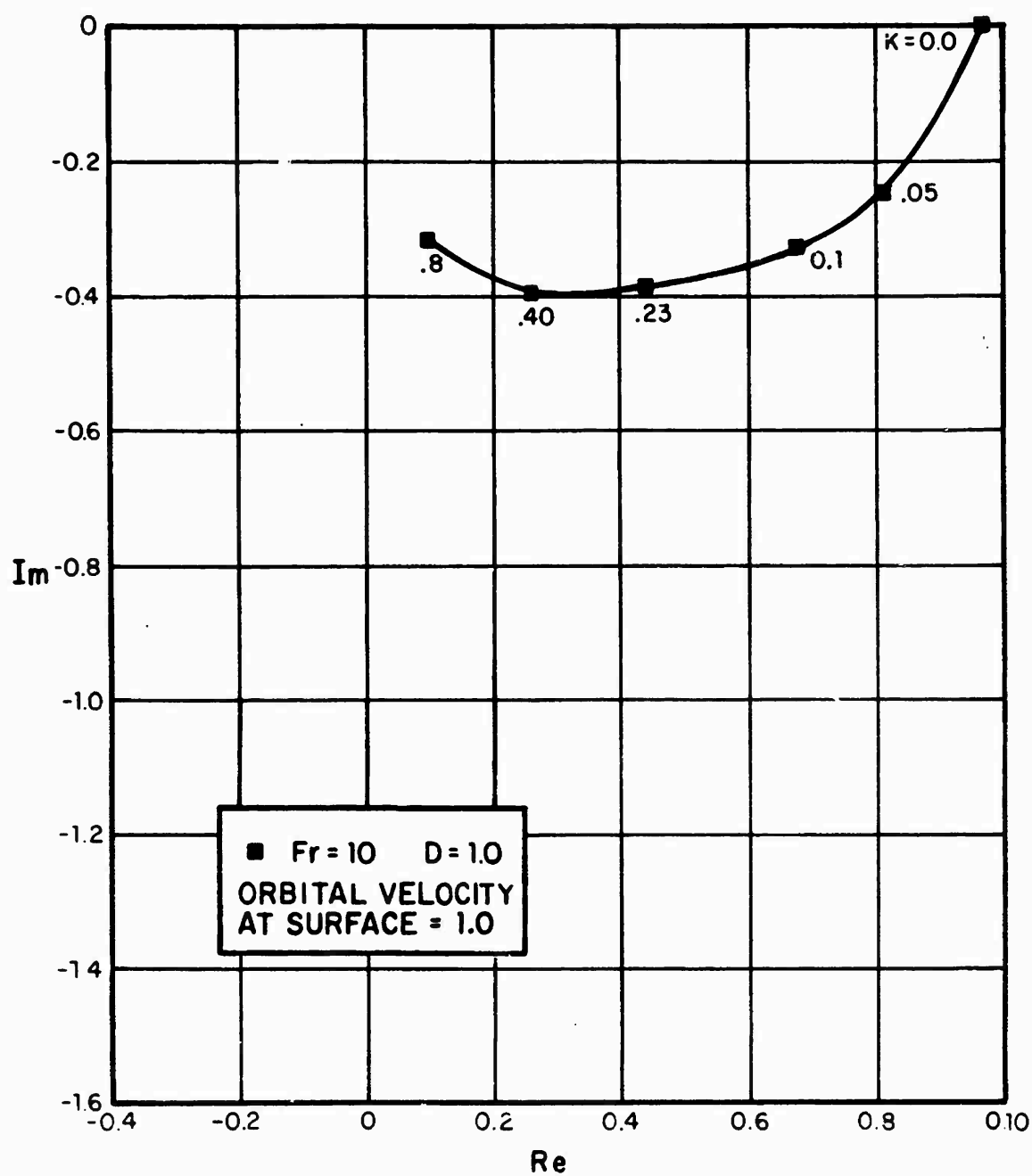
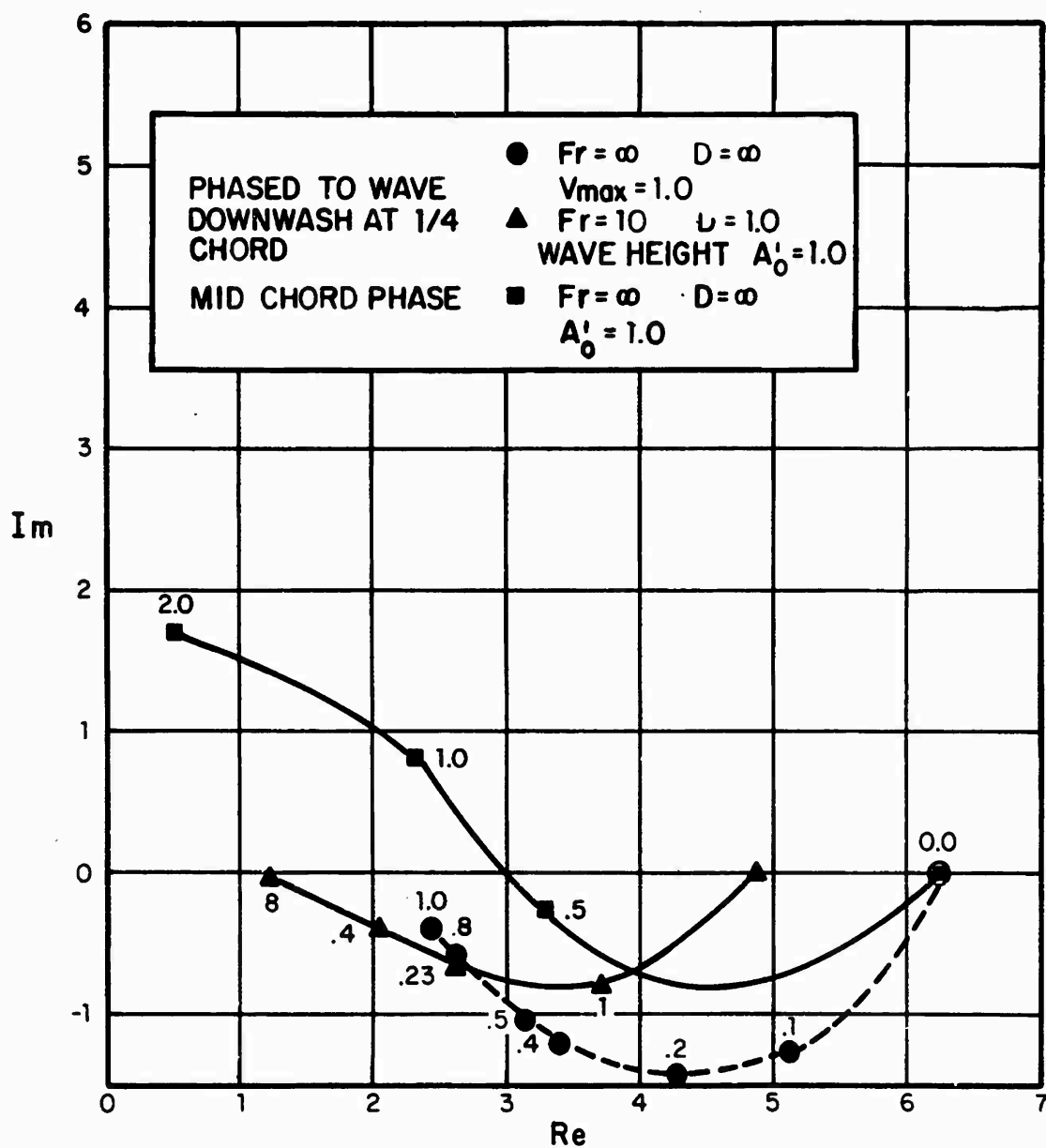
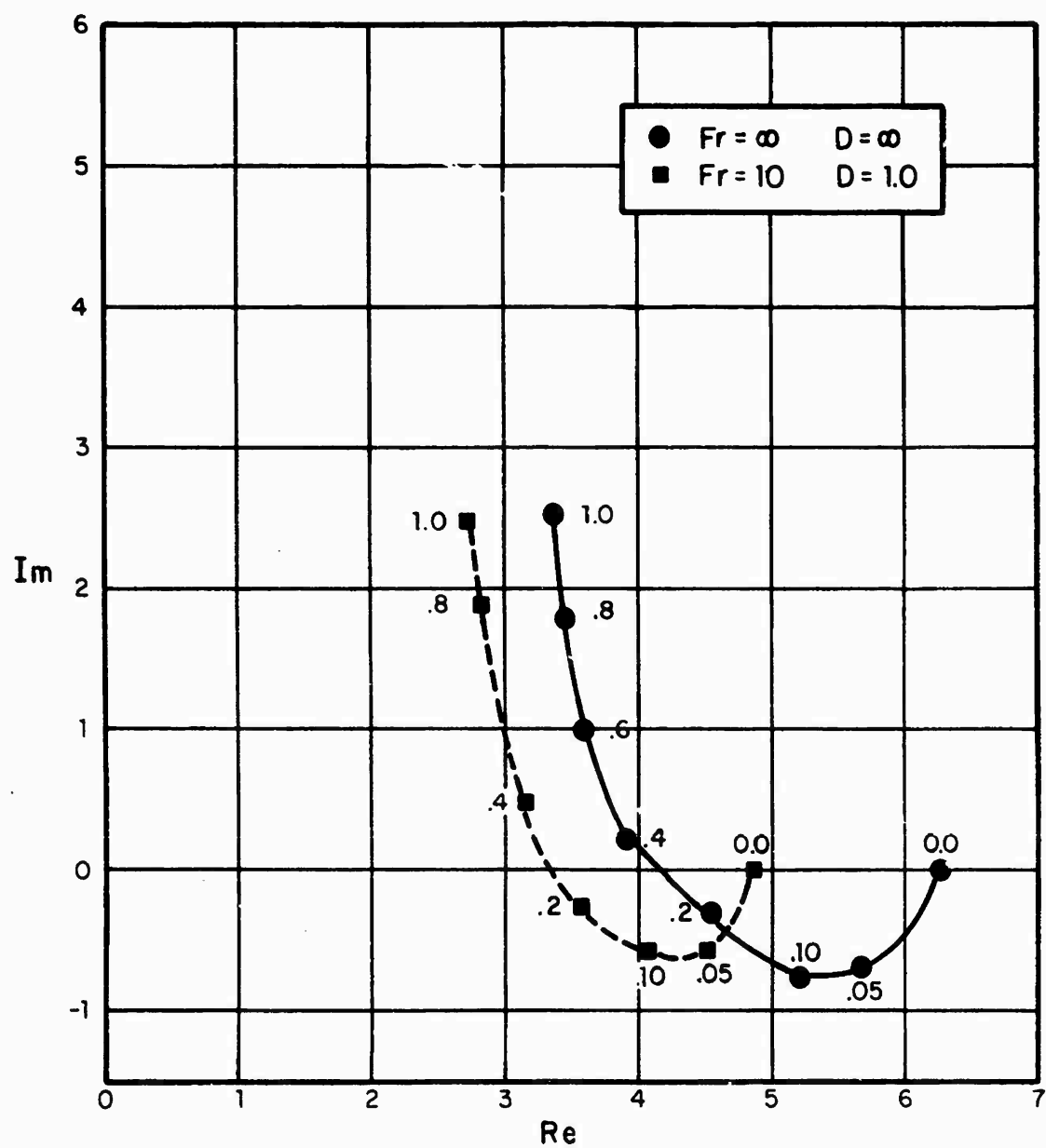
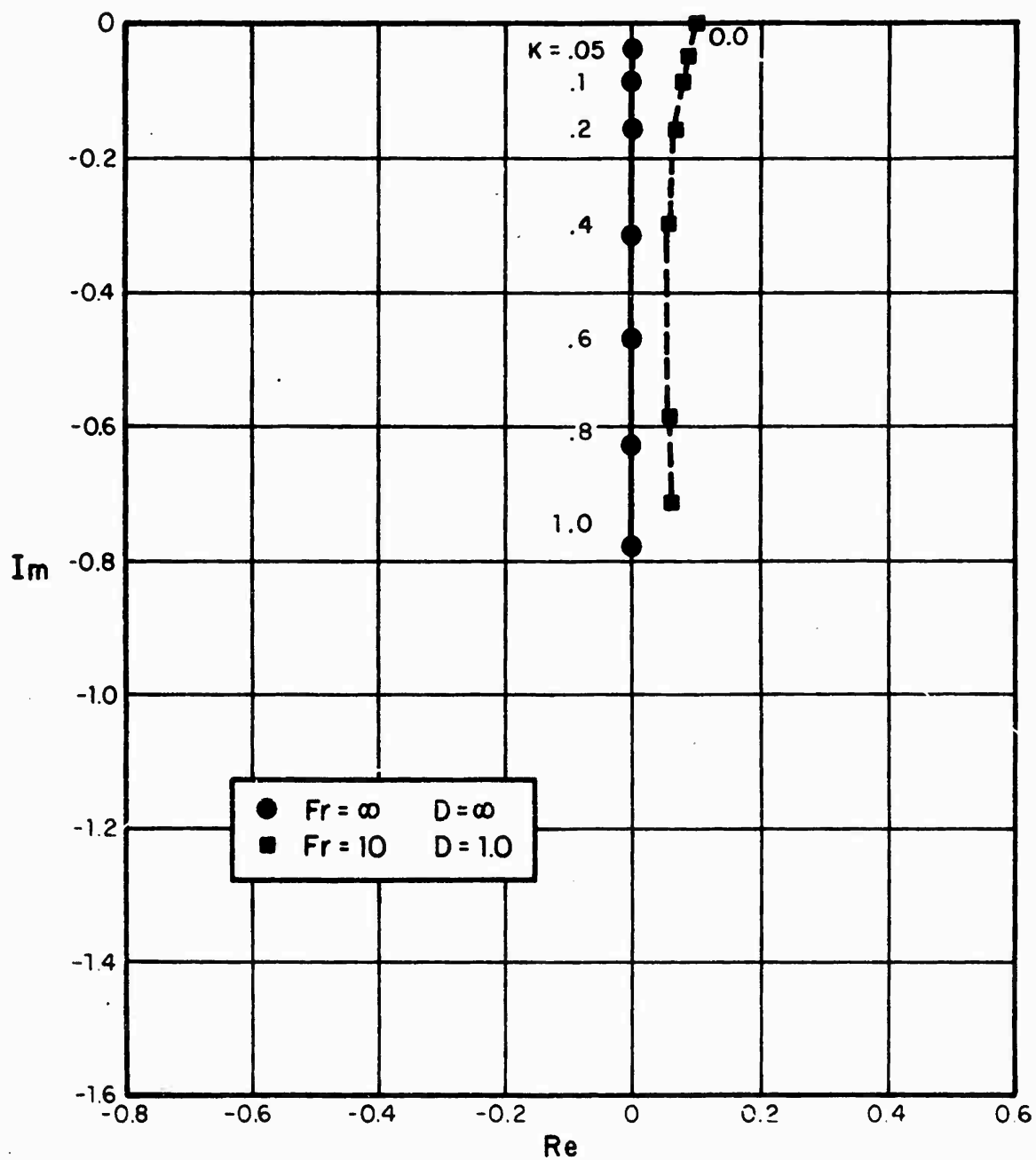


FIG. 7 $CM_{\beta v}$ OPERATION IN WAVES 25% FLAP, PHASED TO 1/4 CHORD
DOWN WASH

FIG. 8 $C_{M_{\alpha v}}$ OPERATION IN WAVES (PHASED TO 1/4 CHORD DOWNWASH)

FIG. 9 C_{LV} OPERATION IN WAVES

FIG. 10 C_{Lh} HEAVE OSCILLATION

FIG. 11 $C_{M_{\alpha} \dot{h}}$ HEAVE OSCILLATION

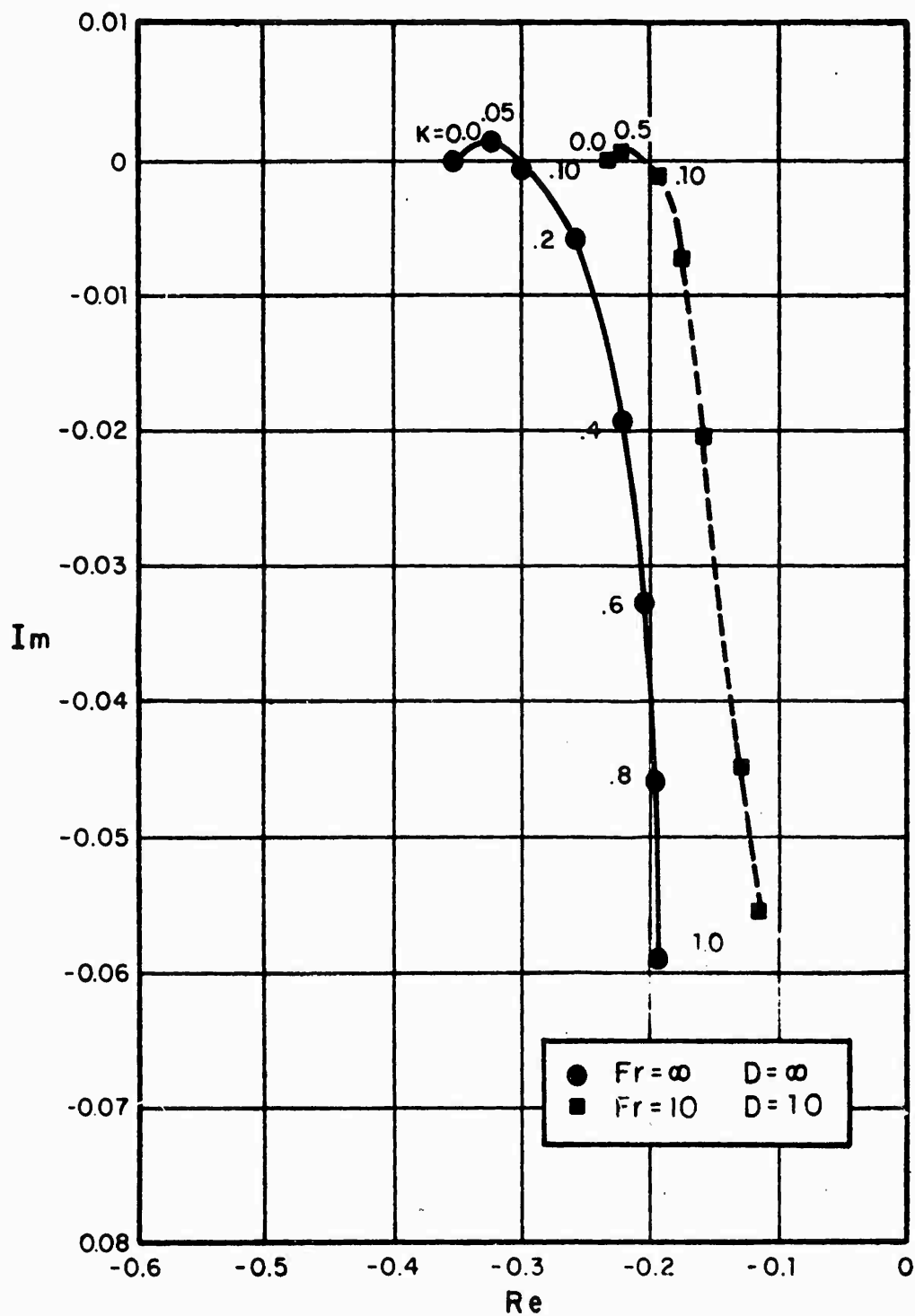
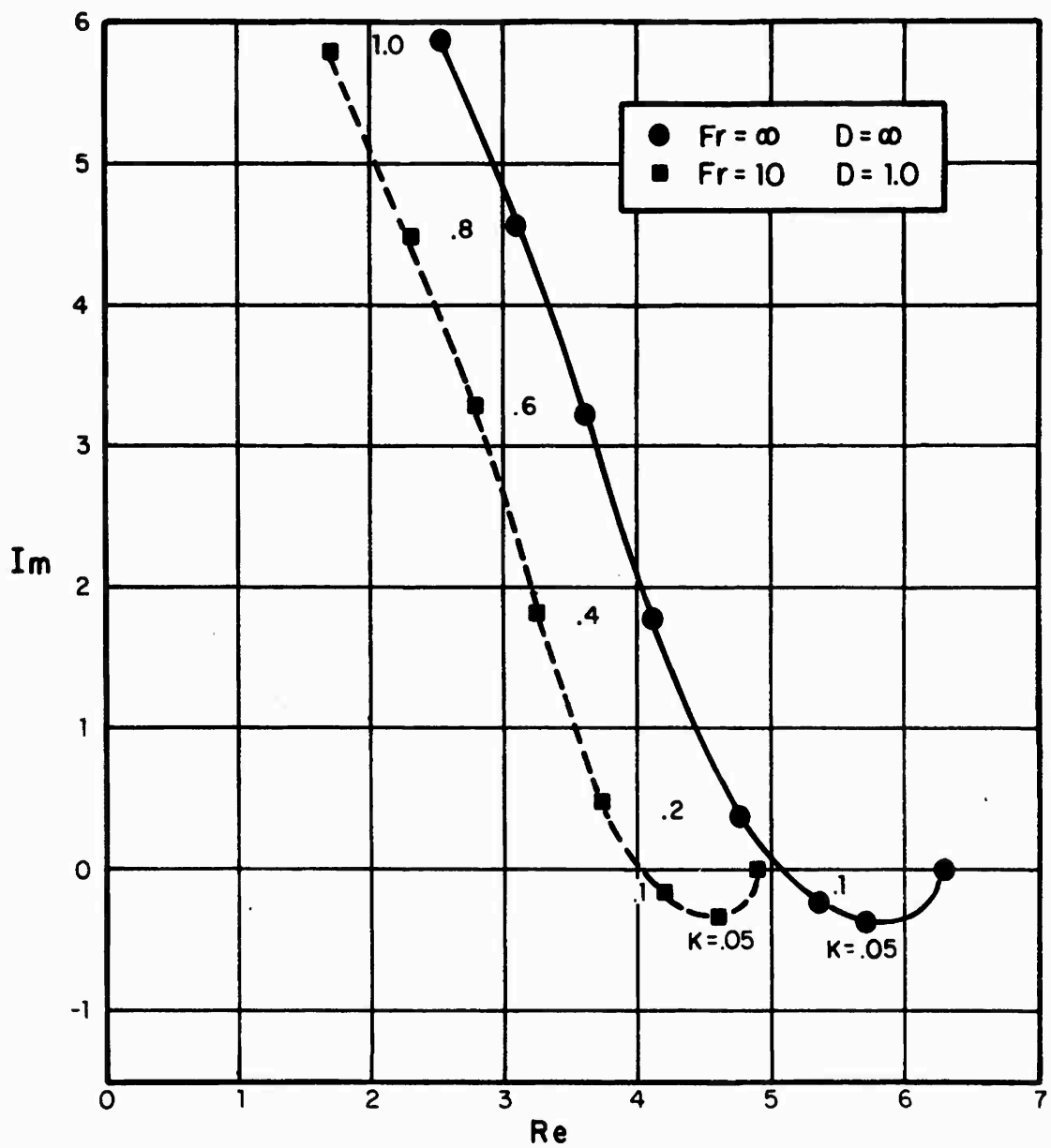
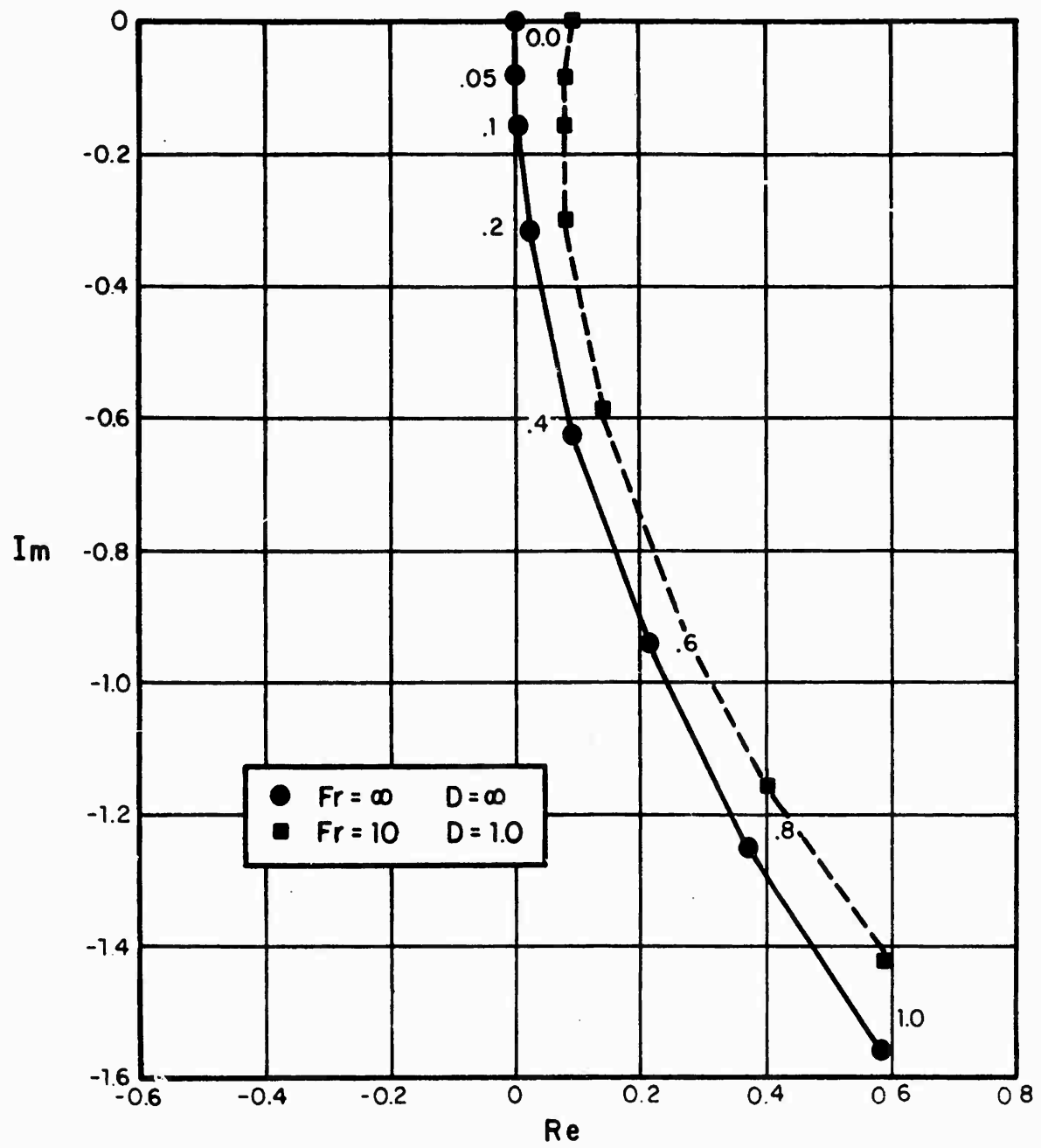


FIG. 12 $C_{M\beta h}$ HEAVE OSCILLATION 25% FLAP

FIG. 13 $C_{L\alpha}$ PITCH OSCILLATION

FIG. 14 C_{M_α} PITCH OSCILLATION

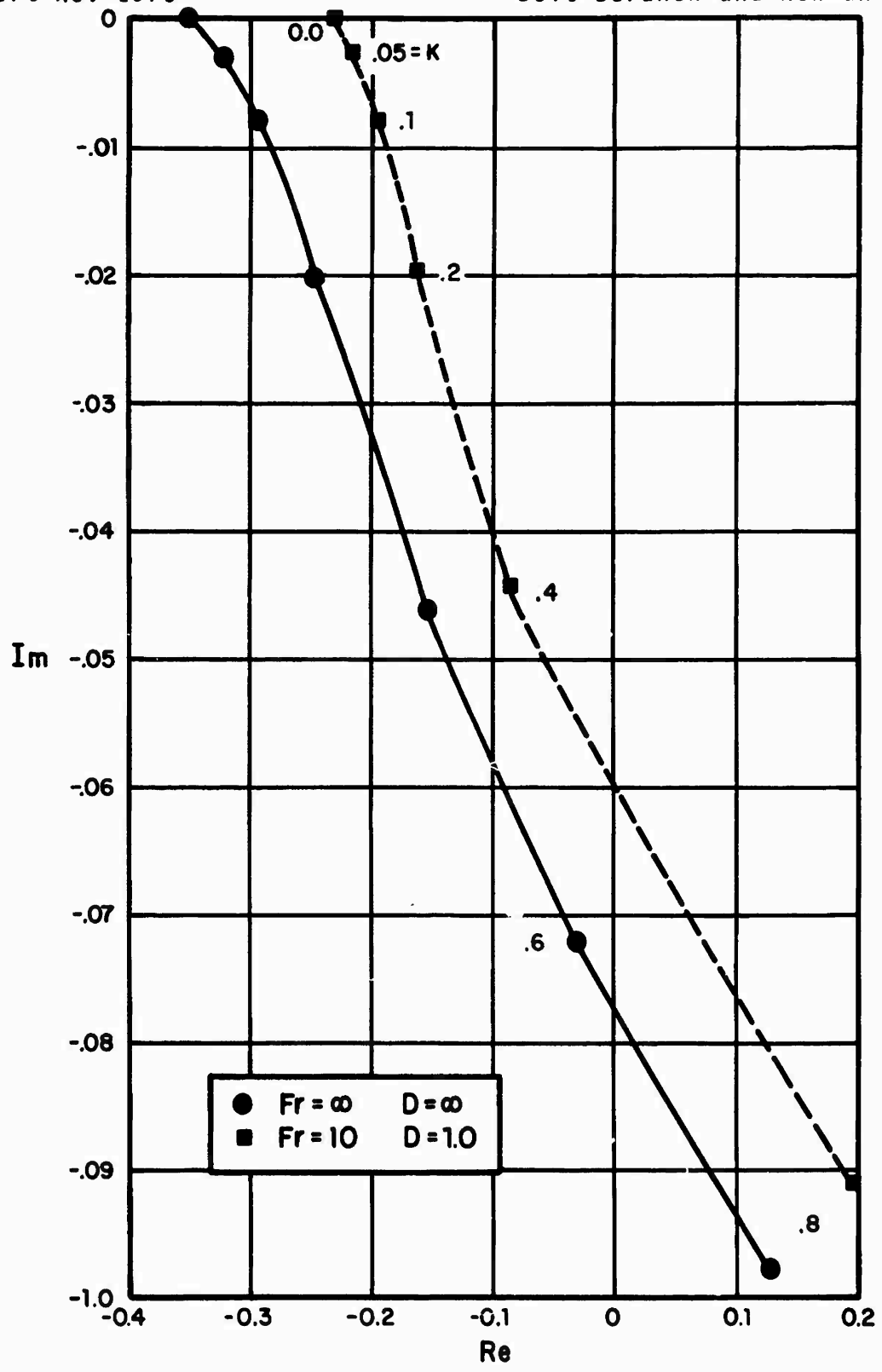
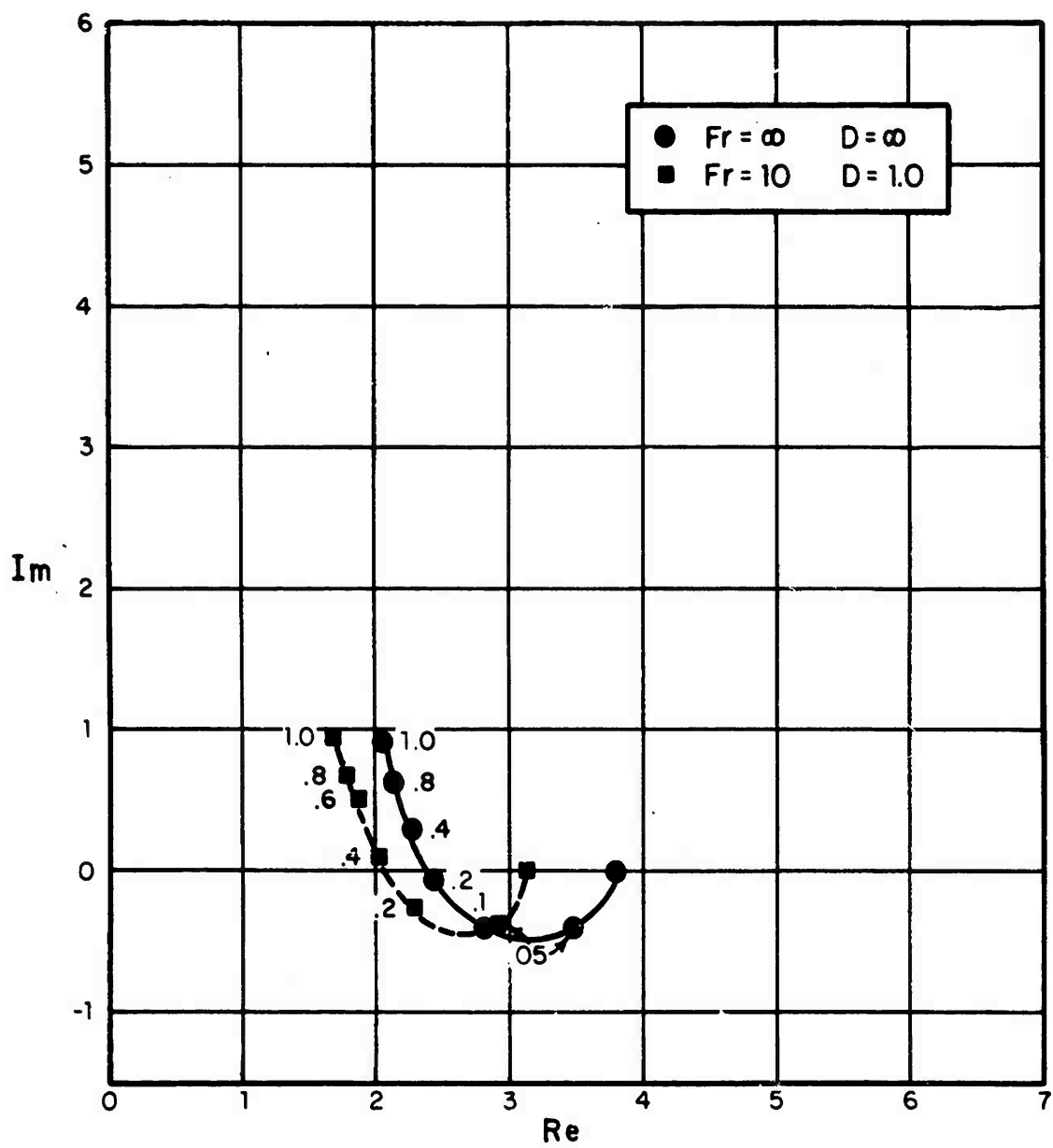
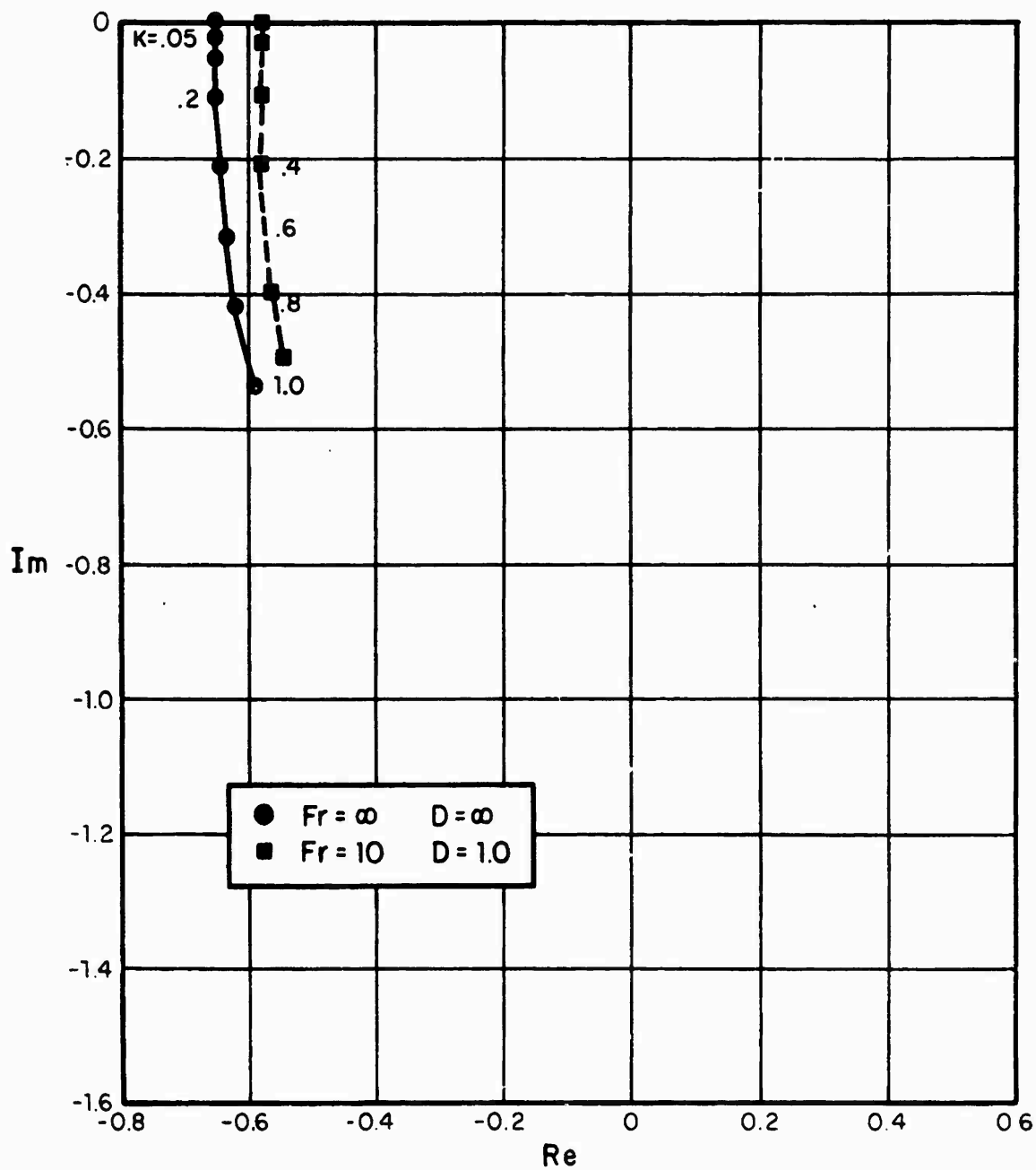


FIG. 15 $C_{M\beta\alpha}$ PITCH OSCILLATION

FIG. 16 $C_{L\beta}$ FLAP OSCILLATION. 25% FLAP

FIG. 17 $C_{M_{\alpha}\beta}$ FLAP OSCILLATION

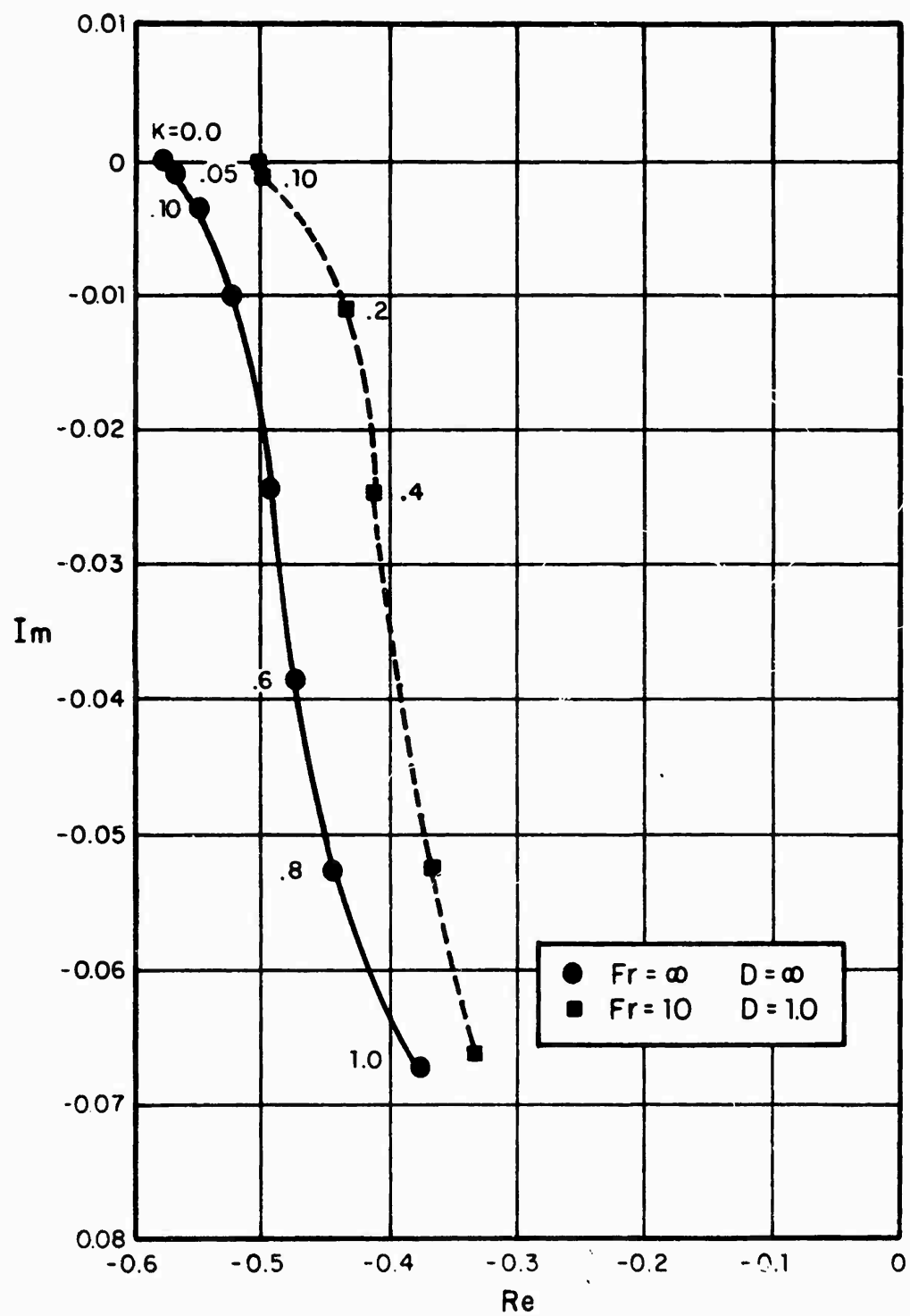


FIG. 18 $C_{M\beta}$ FLAP OSCILLATION 25% FLAP

4. CONTROL PROBLEMS

Freely pivoted foils, controlled by trailing edge servo tabs, exhibit the undesirable feature of initially developing lift in the direction opposite to that commanded. For example, when the tab is deflected downward, the lift is upward until the foil rotates sufficiently far to generate negative lift. This behavior leads one to suspect that the control of such foils may not be particularly straightforward and that there may, in fact, be some fundamental control system limitations affecting the suitability of pivoted tab-controlled foils for hydrofoil craft. Consequently, we have very briefly examined this problem to determine its nature and to outline a possible course of investigation.

We have conducted this study concurrently with the development of the hydrodynamics program discussed above. Consequently, the results of the program were not available for the control system study and we use Theodorsen's equations for unsteady two-dimensional flow without free-surface effects [7]. Nondimensionalizing the equations for lift and foil moment, utilizing Newton's laws for the rigid body response of the foil and ship, and putting the equations into Laplace transform notation gives

$$\begin{aligned}
 -[(m+1)\phi+2]\phi h &= [-a\phi^2+2(1-a)\phi+2]\alpha \\
 &\quad -\pi^{-1}[T_1\phi^2+(T_4-T_{11})\phi-2T_{10}]\beta
 \end{aligned} \tag{32}$$

and

$$\begin{aligned}
 -M_S + \left[\left(I + \frac{1}{8} + a^2 \right) \phi^2 - a(1-2a)\phi - 2 \left(a + \frac{1}{2} \right) \right] \alpha \\
 = [a\phi+(2a+1)]\phi h + \frac{1}{\pi} \{ [T_7+(c-a)T_1]\phi^2 \\
 + [aT_{11}-T_1+T_8+(c-a)T_4]\phi + [2aT_{10}-T_4] \} \beta .
 \end{aligned} \tag{33}$$

Here m is the ship mass per unit span of the foil divided by $\pi \rho b^2$, ϕ is the Laplace operator nondimensionalized by b/U , a is the foil pivot position varying linearly from -1 at the leading edge to $+1$ at the trailing edge, M_s is a nondimensional moment applied directly to the foil by a servomechanism, I is the foil moment of inertia per unit span divided by $\pi \rho b^4$, and T 's are functions of the tab pivot point defined by c .

For a flap comprising 10% of the foil chord, the coefficients of β in Eqs. 32 and 33 may be approximated by

$$+ \frac{1}{2\pi} (\phi+5) \quad \text{for Eq. (32)}$$

$$+ \frac{a}{2\pi} (\phi+5) \quad \text{for Eq. (33)}$$

We also may assume that I is negligible.

4.1 Control

The above equations, in block diagram form, are shown in Fig. 19. Lifts and moments owing to foil and tab angles and to vertical motion are indicated. In addition, a feedback gain K is shown, representing a loop in which a moment proportional to vertical position is applied directly to the foil. The transfer function for this system is

$$\frac{h}{b\beta} = - \frac{\frac{1}{16\pi} (\phi+5) [\phi^2 + 8a\phi - 8]}{\left\{ \left[\frac{1}{8} (m+1) + ma^2 \right] \phi^4 + \left[\frac{1}{4} - ma(1-2a) \right] \phi^3 \right.} \\ \left. + [1 - m(2a+1) - aK] \phi^2 + 2K(1-a)\phi + 2K \right\} . \quad (34)$$

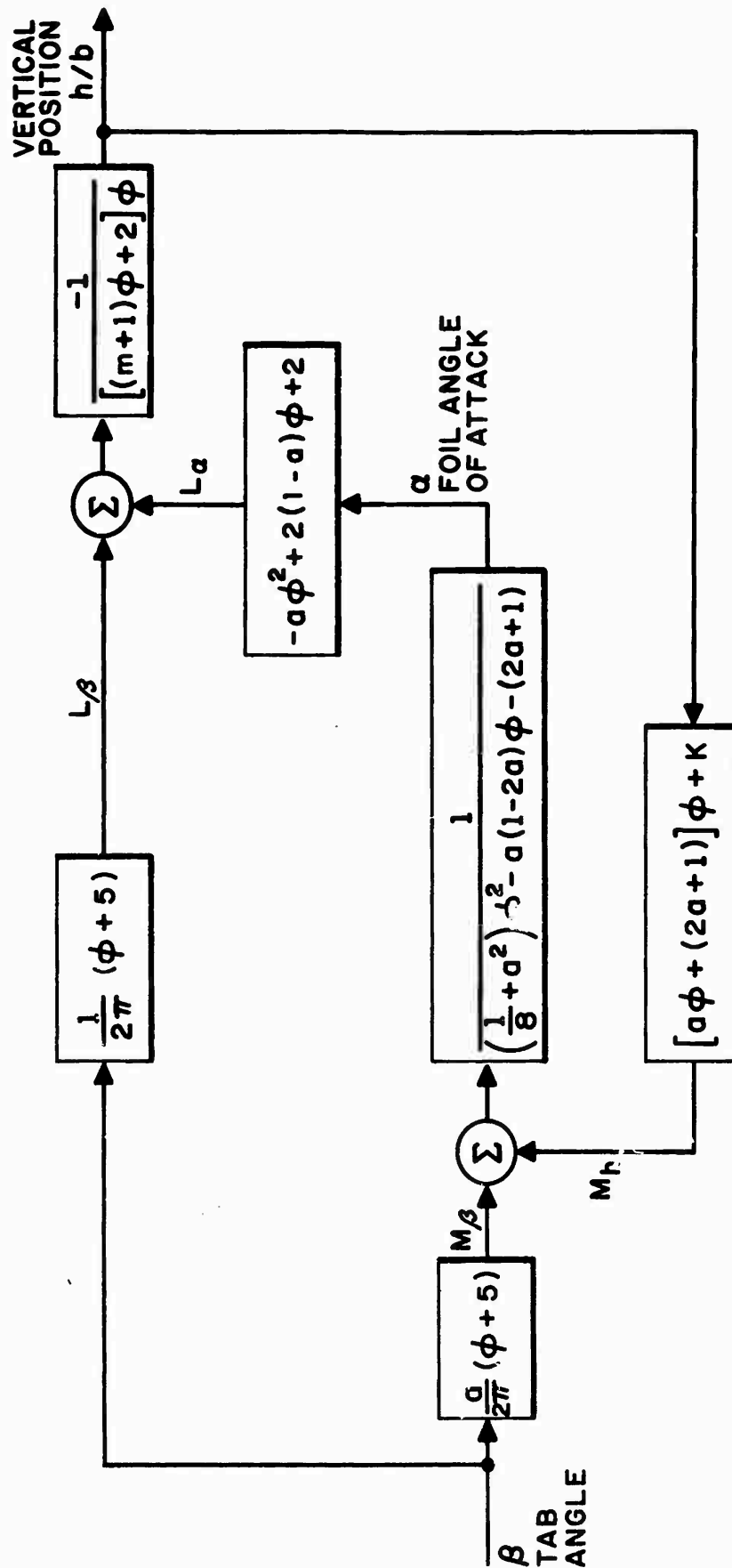


FIG. 19 BLOCK DIAGRAM OF HYDROFOIL DYNAMICS WITH A SERVO TAB COMPRISING 10% OF THE FOIL CHORD

There are several interesting aspects of Eq. 34. First, the quadratic polynomial in the numerator always has one positive and one negative real root (i.e., open loop zeroes). Thus a root locus always has a branch ending in the right half plane, despite the form of compensation network employed. Secondly, if K is zero, Eq. 34 will have two poles at the origin. This means that an impulse in β will result in a steady velocity in h . Thus, a momentary deflection of the tab causes a transient that does *not* end with the foil realigning itself with the inflow vector $\vec{U}$. Instead, the foil becomes aligned with the vector sum of U and $\vec{h}$, defining an "effective" angle of attack of zero. If, in addition to K being zero, m is very large (corresponding in the limit as $m \rightarrow \infty$ to a fixed pivot point) the denominator of Eq. 34 becomes

$$m\phi^2 \left[\left(\frac{1}{8} + a^2 \right) \phi^2 - a(1-2a)\phi - (2a+1) \right]$$

which is always stable for $a < -1/2$ and unstable for $a > -1/2$ (corresponding to pivoting the foil upstream and downstream respectively of the quarter chord point).

If we assume $a = -1/2$ and that $m = 1$ (a good approximation for typical hydrofoil craft) and apply Routh's stability criterion to the denominator of Eq. 34, all roots will lie in the left half plane (off the real axis) for $0 < K < 1/4$. Thus, we see that by choosing an appropriate value of feedback gain the open loop transfer function can be made controllable.

5. CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDY

In this Report we have developed a program to compute the unsteady hydrodynamic lift and moment functions for a two-dimensional hydrofoil operating near a free surface, and have briefly investigated the control of hydrofoils. In several example computations, utilizing the developed program, we found that the effect of the free surface is to modify the magnitude of phase of the load on the foil. Lift due to heave and pitch oscillations, for example, decrease with decreasing depth as one might expect. The control studies indicate certain limitations in controlling foil lift by means of servo tabs.

These studies are largely preliminary to the primary objective of designing hydrofoil systems that have good response characteristics and low power consumption. Based on these results there are several control-system studies that ought to be undertaken as well as refinements in the hydrodynamic analysis. They are to:

1. Design a hydrofoil control system configuration that maximizes bandwidth and minimizes control power. This involves the choice of appropriate feedback loops, compensation functions, and foil parameter values.
2. Extend the unsteady hydrodynamic analysis to three dimensional hydrofoils. This would be particularly suitable to foils of low aspect ratio.

ACKNOWLEDGEMENTS

This Report reflects not only the efforts of the authors but also the contributions of several other people. We are grateful to Professor Sheila Widnall and Mr. Wayne Johnson of the Massachusetts Institute of Technology for their contributions to the hydrodynamic analysis of a hydrofoil with a trailing-edge flap. We also appreciate the helpful comments of Mr. William C. O'Neill, the technical monitor of this project, and Mr. Peter Besch; both are of the Naval Ship Research and Development Center.

The work reported here was sponsored by Systems Development Office of the Naval Ship Research and Development Center under the Naval Ship Systems Command.

REFERENCES

1. S.E. Widnall, "Unsteady Loads on Hydrofoils Including Free Surface Effects and Cavitation", Massachusetts Institute of Technology Fluid Dynamics Research Report 64-2, June 1964.
2. F.B. Hildebrand, *Methods of Applied Mathematics*. Prentice-Hall, New Jersey, 1961.
3. M. Landahl, "Pressure-Loading Functions for Oscillating Wings With Control Surfaces", *AIAA Journal*, 6, No. 2, pp. 345-348, February 1968.
4. W.C. O'Neill, "Unsteady Lift and Hinge Moment Characteristics of the AG(EH) Main Foil and Strut Assembly", NSRDC Report 2805, July 1968.
5. M. Abramowitz and A. Stegun, *Handbook of Mathematical Functions*. Dover, New York, 1965.
6. W.R. Sears, "Some Aspects of Non-Stationary Airfoil Theory and Its Practical Application", *J. Aeronautical Sciences*, 3, No. 8, pp. 104-108, January 1941.
7. T. Theodorsen, "General Theory of Aerodynamic Instability and the Mechanism of Flutter", N.A.C.A Report 496, 1934.

APPENDIX A: KERNEL FUNCTION FOR TWO DIMENSIONAL FLOW
WITH A FREE SURFACE [2]

$$\begin{aligned}
 K = & - \frac{x}{2\pi(x^2 + 4d^2)} - \frac{ik}{4\pi} e^{-q_0} \left[\text{Ei}(q_0) + \pi i \left(1 + \frac{|x|}{x} \right) \right] \\
 & - \frac{ik}{4\pi} e^{q_0} \text{Ei}(-q_0^*) + \frac{ia_1}{4\pi} e^{-q_1} \left[\text{Ei}(q_1) + \pi i \left(1 + \frac{|x|}{x} \right) \right] \\
 & + \frac{ia_2}{4\pi} e^{-q_2} \left[\text{Ei}(q_2) + \pi i \left(1 + \frac{|x|}{x} \right) \right] + \frac{ia_3}{4\pi} e^{q_3} \text{Ei}(-q_3) \\
 & + \frac{ia_4}{4\pi} e^{q_4} \text{Ei}(-q_4) ,
 \end{aligned}$$

where a star denotes complex conjugate and

$$a_1 = k[1 + f^{-1} + (3 + f^{-1})(4f + 1)^{-\frac{1}{2}}]$$

$$a_2 = k[1 + f^{-1} - (3 + f^{-1})(4f + 1)^{-\frac{1}{2}}]$$

$$a_3 = k[1 - f^{-1} + i(3 - f^{-1})(4f - 1)^{-\frac{1}{2}}]$$

$$a_4 = k[1 - f^{-1} - i(3 - f^{-1})(4f - 1)^{-\frac{1}{2}}]$$

$$q_0 = k(2d + ix)$$

$$q_1 = -s_1(2d + ix)$$

$$q_2 = -s_2(2d + ix)$$

$$q_3 = -s_3(2d - ix)$$

$$q_4 = -s_4(2d - ix)$$

$$f = kF_r^2 = \frac{\omega U}{g}$$

$$Ei(q) = \int_{-\infty}^q \frac{e^t}{t} dt .$$

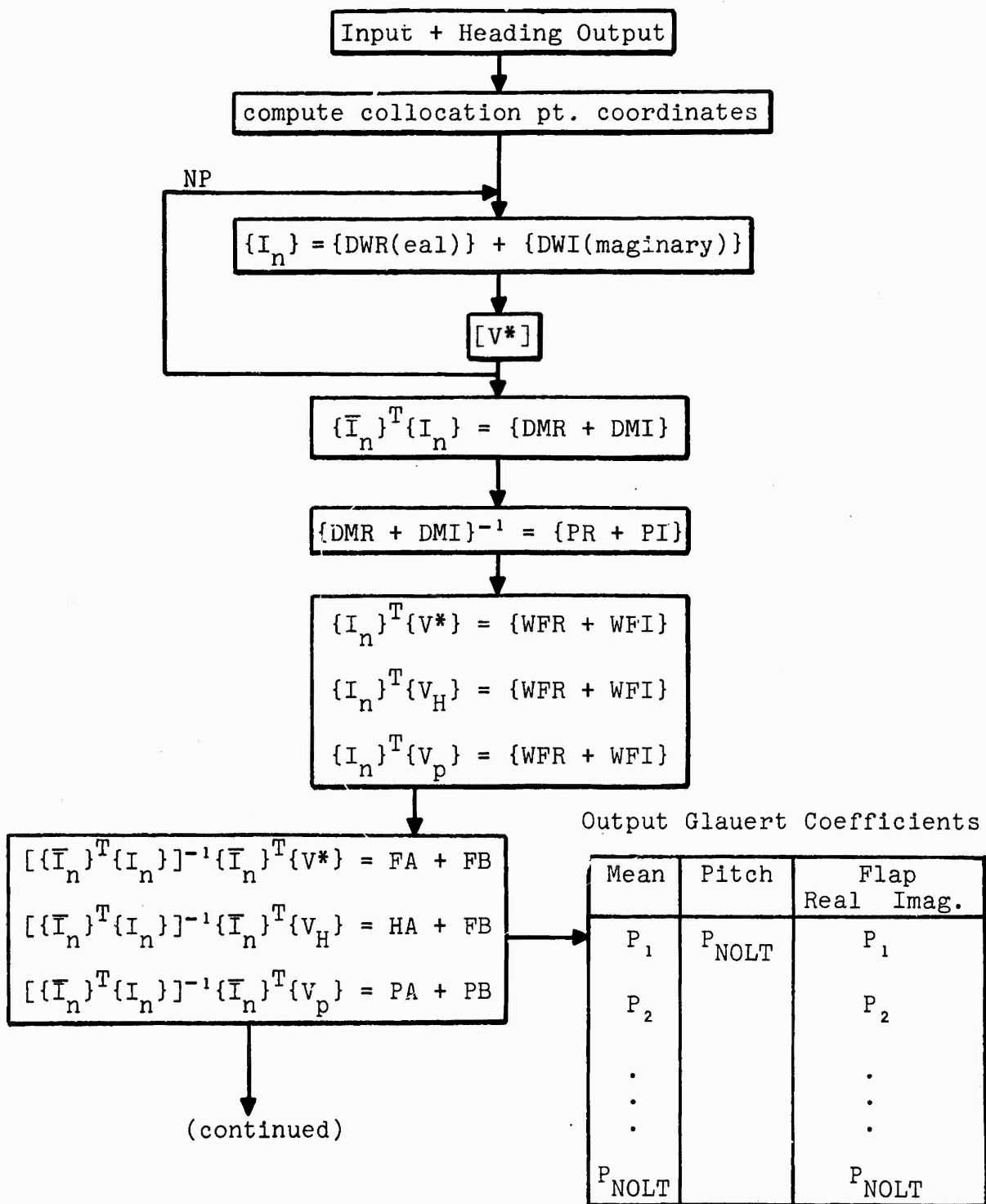
Report No. 1970

Bolt Beranek and Newman Inc.

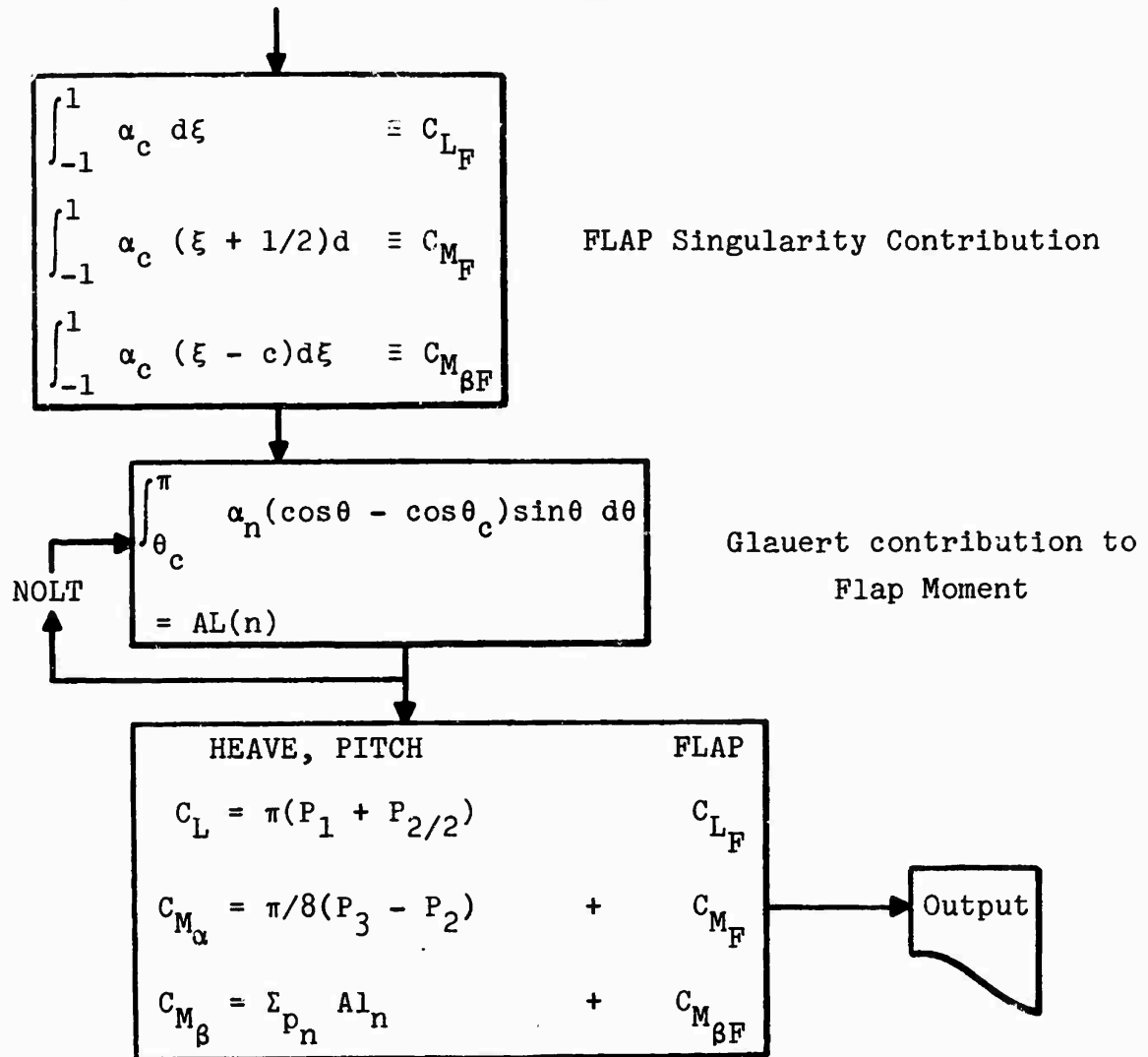
APPENDIX B

Flow Charts for Hydrodynamic Programs

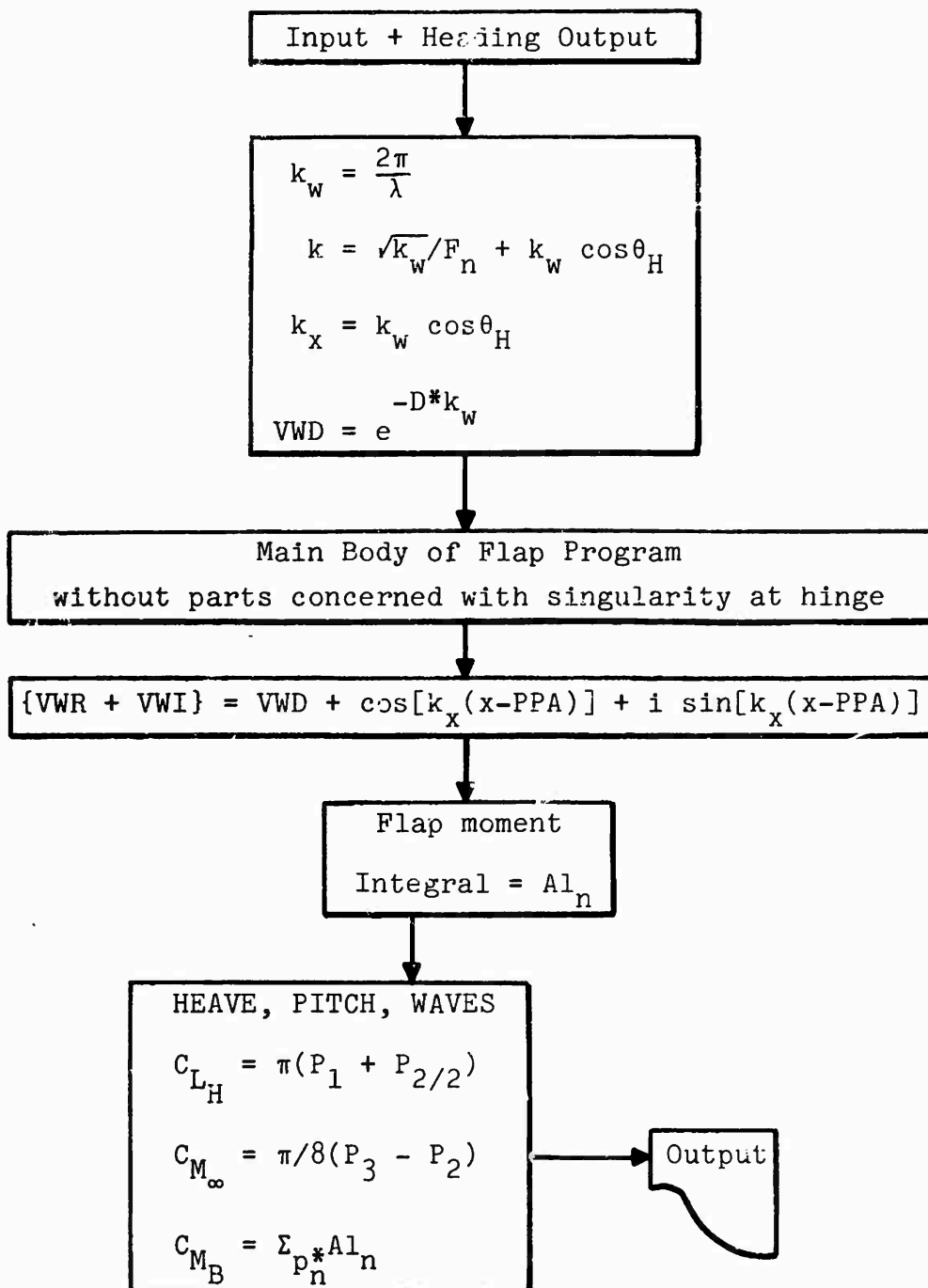
FUNCTIONAL FLOW CHART OF FLAP PROGRAM
(using notation from text and program)



FUNCTIONAL FLOW CHART OF FLAP PROGRAM (continued)



FLOW CHART OF MODIFICATIONS FOR HYDRODYNAMICS IN WAVES



APPENDIX C: PROGRAM TO COMPUTE THE HYDRODYNAMIC COEFFICIENTS
FOR A FLAPPED FOIL OPERATING IN WAVES

$$C_{L_h}, C_{L_P}, C_{L_V}$$

$$C_{M_h}, C_{M_{\alpha P}}, C_{M_{\alpha V}}$$

$$C_{M_{\beta h}}, C_{M_{\beta P}}, C_{M_{\beta V}}$$

```

C      TWO DIMENSIONAL PROGRAMME FOR FOIL IN WAVES
      DIMENSION AL(10),CR(10),CI(10),GR(32),GI(32),THETA(32),GX(32),
      1XO(32),DWR(20,6),DWI(20,6),DMR(6,6),DMI(6,6),X(100),AM(10),
      2PA(10),PB(10),HA(10),HB(10),PR(6,6),PI(6,6)
      3,WPR(10),WPI(10),WHR(10),CPVC(10),BANGL(20),NLE1(20),NTE1(
      420),WHI(10),WWR(10),WWI(10),WA(10),WB(10)
      COMMON XO,GR,GI,PO2,T,JEST,IM,IFF,D,FR,NT,S1,S2,S3R,S3I,S4R,S4I,
      1A1,A2,A3R,A3I,A4R,A4I
      PO2=1.57079633
      TI=-.5
      TYPE 1001
1001  FORMAT(/$ INPUT DATA FROM FILE:$)
      CALL OPENR(5)
      CALL READB(5,NP)
999   FORMAT(I5)
998   FORMAT(F10.4)
1     CALL READB(5,NOLT)
      CALL READB(5,NT)
      CALL READB(5,JEST)
      CALL READB(5,IM)
      CALL READB(5,IFF)
      CALL READB(5,RF)
      CALL READB(5,FR)
      CALL READB(5,D)
      CLOSE
      ACCEPT 996,WL,THET,FR,PPA,CFLAP,NIE,NTE
996   FORMAT(5F10.4,2I4)
      XKW=4.*PO2/WL
      RF=SQRT(XKW)/FR+XKW*COS(THET)
      WK=XKW*COS(THET)
      VWD=EXP(-XKW*D)
3     FORMAT (5I5,3F10.5)
      IF(NOLT)501,501,500
500   TYPE 2
2     FORMAT(/62H1 IMAGE IFF      K      FRCUDE NUM      DEPTH IN SEMCHDS      NP
1      NOLT      )
      TYPE 5,IM,IFF,RF,FR,D,NP,NOLT
5     FORMAT(/2I5,3F10.4,12X,2I3)
      XFLAP=1.-2.*CFLAP
      NP2=NP/2
      PI2=4.*PO2
      DO 31 J=1,NP2
      DX=COS(J*PI2/(NP+1.))
      X(J)=(-(XFLAP+1.)*DX+XFLAP-1.)/2.
      I=NP+1-J
31    X(I)=((-XFLAP+1.)*DX+XFLAP+1.)/2.
      DO 32 L=1,NP
      BANGL(L)=ANGL (-X(L),SORT(1.-X(L)*X(L)))/(2.*PO2)
      NLE1(L)=NLE

```

```

52   NTE1(L)=NTE
      TFLAP=ANGL (-XFLAP.SORT(1,-XFLAP*XFLAP))
      TYPE 9001,CFLAP
9001  FORMAT(//,$   FLAP CHORD$,F8.2,5X,$FRACTION OF AIRFOIL CHORD$)
      SNTF=SIN(TFLAP)
      TYPE 51,WL,THET
51   FORMAT(/,$WAVE LENGTH=$,F8.3,$ SEMI-CHORDS.  HEADING=$,F8.3,/)
      DO 100 J=1,NP
      ANGLE=BANG1(J)
      NLE=NLE1(J)
      NTE=NTE1(J)
4    FORMAT(F10.4,2I10)
      THMAX=PO2*2.*ANGLE
      X(J)=-COS(THMAX)
      DO 6 I=1,NLE
      THETA(I)=THMAX/2.*(1.-GN(I,NLE))
6    XO(I)=X(J)+COS(THETA(I))
      DO 7 K=1,NOLT
      CR(K)=0.0
7    CI(K)=0.0
      CALL KER2(RF,NLE)
      DO 10 I=1,NLE
      AL(1)=COS(THETA(I)/2.)/SIN(THETA(I)/2.)
      DO 71 N=2, NOLT
71   AL(N)=SIN((N-1)*THETA(I))
      CW=WN(I,NLE)*THMAX/2.*AL(2)
      DO 8 K=1,NOLT
      CR(K)=AL(K)*CW*GR(I)+CR(K)
8    CI(K)=AL(K)*CW*GI(I)+CI(K)
      IF (J-5) 13,9,10
9    IF(JEST) 80,10,80
80   TYPE 22,XO(I),GR(I),GI(I),CW,(AL(K),K=1,3),(CR(K),K=1,3)
10   CONTINUE
      DO 15 I=1,NTE
      THETA(I)=THMAX+(PO2-THMAX/2.)*(1.-GN(I,NTE))
15   XO(I)=X(J)+COS(THETA(I))
      CALL KER2(RF,NTE)
      DO 25 I=1,NTE
      AL(1)=COS(THETA(I)/2.)/SIN(THETA(I)/2.)
      DO 151 N=2, NOLT
151  AL(N)=SIN((N-1)*THETA(I))
      CW=WN(I,NTE)*(PO2-THMAX/2.)*SIN(THETA(I))
      DO 20 K=1,NOLT
      CR(K)=AL(K)*CW*GR(I)+CR(K)
20   CI(K)=AL(K)*CW*GI(I)+CI(K)
      IF (JEST) 301,25,301
301  IF (J-5) 25,21,25
21   TYPE 22,XO(I),GR(I),GI(I),CW,(AL(K),K=1,3),(CR(K),K=1,3)
22   FORMAT(//5F12.4,/,5E12.4,/)

```

```

25    CONTINUE
      CPVC(1)=TI
      DO 251 N=2, NOLT
251   CPVC(N)=TI*COS((N-1)*THMAX)
      DO 26 K=1, NOLT
      DWR(J,K)=CR(K)+CPVC(K)
26    DWI(J,K)=CI(K)
      IF(J-5)100,27,100
27    IF(JEST)28,100,28
28    TYPE 29,(CPVC(I),I=1,3)
29    FORMAT(/3E20.8)
100   CONTINUE
      DO 101 I=1, NOLT
      DO 101 K=1, NOLT
      DMR(I,K)=0.0
101   DMI(I,K)=0.0
      DO 110 I=1, NOLT
      DO 110 K=1, NOLT
      DO 110 J=1, NP
      DMR(I,K)=DWR(J,I)*DWR(J,K)+DWI(J,I)*DWI(J,K)+DMR(I,K)
109   DMI(I,K)=-DWI(J,I)*DWR(J,K)+DWR(J,I)*DWI(J,K)+DMI(I,K)
110   CONTINUE
      CALL CMPINV(DMR,DMI,PR,PI,NOLT,INDEX1)
      IF(JEST) 111,115,111
111   TYPE 112,((DMR(I,J),DMI(I,J),J=1,NOLT),I=1,NOLT)
      TYPE 112,((PR(I,J),PI(I,J),J=1,NOLT),I=1,NOLT)
112   FORMAT(/3E20.8,/,3E20.8,/)
      TYPE 112,((DWR(J,K),DWI(J,K),K=1,NOLT),J=1,NP)
115   VPR=1.0
      VHR=1.0
      DO 150 I=1, NOLT
      WPR(I)=0.0
      WPI(I)=0.0
      WHR(I)=0.0
      WHI(I)=0.0
      WWR(I)=0.0
      WWI(I)=0.0
      DO 140 J=1, NP
      VPI=(X(J)+.5)*RF
      VWR=VWD*COS(WK*(X(J)-PPA))
      VWI=-VWD*SIN(WK*(X(J)-PPA))
      WHR(I)=WHR(I)+VHR*DWR(J,I)
      WHI(I)=WHI(I)-VHR*DWI(J,I)
      WPR(I)=WPR(I)+VPR*DWR(J,I)+VPI*DWI(J,I)
      WPI(I)=WPI(I)-VPR*DWI(J,I)+VPI*DWR(J,I)
      WWR(I)=WWR(I)+VWR*DWR(J,I)+VWI*DWI(J,I)
140   WWI(I)=WWI(I)-VWR*DWI(J,I)+VWI*DWR(J,I)
      IF (JEST) 141,150,141
141   TYPE 142,WHR(I),WHI(I),WPR(I),WPI(I)
142   FORMAT(/4E18.8)

```

```

150  CONTINUE
      DO 200 I=1,NOLT
        PA(I)=0.0
        PB(I)=0.0
        HA(I)=0.0
        HB(I)=0.0
        WA(I)=0.0
        WB(I)=0.0
      DO 200 J=1,NOLT
        PA(I)=PA(I)+PR(I,J)*WPR(J)-PI(I,J)*WPI(J)
        PB(I)=PB(I)+PP(I,J)*WPI(J)+PI(I,J)*WPR(J)
        HA(I)=HA(I)+PR(I,J)*WHR(J)-PI(I,J)*WHI(J)
        HB(I)=HB(I)+PR(I,J)*WHI(J)+PI(I,J)*WHR(J)
        WA(I)=WA(I)+PR(I,J)*WWP(J)-PI(I,J)*WWI(J)
        WB(I)=WB(I)+PR(I,J)*WWI(J)+PI(I,J)*WWP(J)
      TYPE 201,(HA(I),HB(I),PA(I),PB(I),WA(I),WB(I),I=1,NOLT)
201  FORMAT(/6E12.5)
      AL(1)=PO2*2.-TFLAP-SNTF
      AL(2)=PO2-.5*TFLAP-.5*XFLAP*SNTF
      DO 35 N=3, NOLT
35   AL(N)=.5*(SIN(N*TFLAP)/N-SIN(N-2)*TFLAP)/(N-2)
      AM(1)=-PO2+.5*TFLAP+SNTF-.5*SNTF*XFLAP
      AM(2)=(SNTF**3)/3.
      AM(3)=-.5*PO2+.25*TFLAP-.0625*SIN(4.*TFLAP)
      DO 36 N=4, NOLT
36   AM(N)=.25*(SIN(N-3)*TFLAP)/(N-3)-SIN((N+1)*TFLAP)/(N+1)
      DO 1102 I=1,NOLT
1102 AL(I)=AM(I)-XFLAP*AL(I)
      CMHR=PO2/4.*(HA(3)-HA(2))
      CMHI=PO2/4.*(HB(3)-HB(2))
      CLHR=2.*PO2*(HA(1)+.5*HA(2))
      CLHI=2.*PO2*(HB(1)+.5*HB(2))
      CHHR=0.
      CHHI=0.
      DO 1201 I=1,NOLT
1201 CHHR=CHHR-.5*HA(I)*AL(I)
      CHHI=CHHI-.5*HB(I)*AL(I)
      TYPE 300
300  FORMAT(/30H0 HEAVING OSCILLATION )
      TYPE320,CLHR,CLHI,CMHR,CMHI,CHHR,CHHI
320  FORMAT(/SCLR=%F8.4,$ CLH=%F8.4,4X,$CMR=%F8.4,4X,
1$CMI=%F8.4,$/SCHR=%F8.4,4X,$CHI=%F8.4)
      CALL COMPR(CLHR,CLHI,RF)
      TYPE 350
350  FORMAT(/30H0 PITCHING OSCILLATION )
      CMPR=PO2/4.*(PA(3)-PA(2))
      CMPI=PO2/4.*(PB(3)-PB(2))
      CLPR=2.*PO2*(PA(1)+.5*PA(2))
      CLPI=2.*PO2*(PB(1)+.5*PB(2))
      CHPR=0.
      CHPI=0.
      DO 1202 I=1,NOLT
1202 CHPR=CHPR-.5*PA(I)*AL(I)

```

```
1202 CHPI=CHPI-.5*PB(I)*AL(I)
      TYPE 320,CLPR,CLPI,CMPR,CMPI,CHPR,CHPI
      CMWR=PO2/4.*(WA(3)-WA(2))
      CMWI=PO2/4.*(WB(3)-WB(2))
      CLWR=2.*PO2*(WA(1)+.5*WA(2))
      CLWI=2.*PO2*(WB(1)+.5*WB(2))
      CWHR=0.
      CWHI=0.
      DO 1204 I=1,NOLT
      CWHR=CWHR-.5*WA(I)*AL(I)
1204 CWHI=CWHI-.5*WB(I)*AL(I)
      TYPE 351
351  FORMAT(//SIN WAVES.$)
      TYPE 320,CLWP,CLWI,CMWP,CMWI,CWHR,CWHI
360  FORMAT(//10H CL REAL=,5XF12.8,10H CL IMAG=,5XF12.8,/,
110H CM REAL=,5XF12.8,10H CM IMAG=5XF12.8,/)
      TYPE 997, CLPR,CLPI,CMWR,CMWI,RF,FR,D,NP,NOLT
      TYPE 997, CLPR,CLPI,CMPR,CMPI,RF,FR,D,NP,NOLT
997  FORMAT(/4F14.8,3F4.2,2I2)
41   CONTINUE
      GO TO 1
501  CONTINUE
      STOP
      END
```

APPENDIX D: PROGRAM TO COMPUTE HYDRODYNAMIC COEFFICIENT
FOR A FOIL WITH AN OSCILLATING FLAP.

$$C_{L_h} , C_{L_P} , C_{L_B}$$

$$C_{M_{\alpha h}} , C_{M_{\alpha P}} , C_{M_{\alpha B}}$$

$$C_{M_{\beta h}} , C_{M_{\beta P}} , C_{M_{\beta B}}$$

```

C      TWO DIMENSIONAL PROGRAMME
      DIMENSION VFR(20),VFI(20),FKR(20),FKI(20),TT(4),WFR(10)
1,WFI(10),FA(10),FB(10),AM(10)
      DIMENSION AL(10),CR(10),CI(10),GR(32),GI(32),THETA(32),GX(32),
1XO(32),DWR(20,6),DWI(20,6),DMR(6,6),DMI(6,6),X(20)
2PA(10),PB(10),HA(10),HB(10),PR(6,6),PI(6,6)
3,WPR(10),WPI(10),WHR(10),CPVC(10),BANGL(20),NLE1(20),NTE1(
420),WHI(10)
      COMMON XO,GR,GI,PO2, T,JFST,IM,IFF,D,FR,NT,S1,S2,S3R,S3I,S4R,S4I,
1A1,A2,A3R,A3I,A4R,A4I
      PO2=1.57079633
      TI=-.5
      TYPE 1001
1201  FORMAT(/$ INPUT DATA FROM FILE:$)
      CALL OPENR(5)
      CALL READB(5,NP)
999   FORMAT(I5)
998   FORMAT(F10.4)
1     CALL READB(5,NOLT)
      CALL READB(5,NT)
      CALL READB(5,JEST)
      CALL READB(5,IM)
      CALL READB(5,IFF)
      CALL READB(5,RF)
      CALL READB(5,FR)
      CALL READB(5,D)
      CLOSE
3     FORMAT (5I5,3F10.5)
      ACCEPT 4 ,CFLAP,NFLP,NLE,NTE
      IF(NOLT) 501,501,500
500   TYPE 2
2     FORMAT(/62H1 IMAGE IFF      K      FROUDE NUM      DEPTH IN SEMCHDS      NP
1      NOLT      )
      TYPE 5,IM,IFF,RF,FR,D,NP,NOLT
5     FORMAT(/2I5,3F10.4,12X,2I3)
      XFLAP=1.-2.*CFLAP
      NP2=NP/2
      PI2=4.*PO2
      DO 31 J=1,NP2
      DX=COS(J*PI2/(NP+1.))
      X(J)=(-(XFLAP+1.)*DX+XFLAP-1.)/2.
      I=NP+1-J
31    X(I)=((-XFLAP+1.)*DX+XFLAP+1.)/2.
      DO 32 L=1,NP
      BANGL(L)=ANGL (-X(L),SORT(1.-X(L)*X(L)))/(2.*PO2)
      NLE1(L)=NLE
32    NTE1(L)=NTE
      TFLAP=ANGL (-XFLAP,SORT(1.-XFLAP*XFLAP))
      TYPE 9001,CFLAP

```

```

0001  FORMAT(///%   FLAP CHORDS,P02.5X,$FRACTION OF AIRFOIL CHORDS)
      SNTF=SIN(TPLAP)
      DO 1002 I=1,NP
1002  XO(I)=-COS(P02*2.*BANGL(I))-XFLAP
      CALL KER2(RF,NP)
      DO 1003 I=1,NP
      FKI(I)=GI(I)*SNTF
1003  FKP(I)=(GR(I)-1./(4.*P02*XO(I)))*SNTF
      CNST=-4.+2.*ALOG(1.-XFLAP**2)+2.*XFLAP*ALOG((1.+XFLAP)/(1.-XFLAP))
      PC=+1./(4.*P02*SNTF)
      DO 100 J=1,NP
      ANGLE=BANGL(J)
      NLF=NLE1(J)
      NTE=NTE1(J)
4     FORMAT(F10.4,3I10)
      THMAX=P02*2.*ANGLE
      X(J)=-COS(THMAX)
      DO 6 I=1,NLE
      THETA(I)=THMAX/2.*(1.-GN(I,NLE))
6     XO(I)=X(J)+COS(THETA(I))
      DO 7 K=1,NOLT
      CR(K)=0.0
7     CI(K)=0.0
      CALL KER2(RF,NLE)
      DO 10 I=1,NLE
      AL(1)=COS(THETA(I)/2.)/SIN(THETA(I)/2.)
      DO 71 N=2, NOLT
71    AL(N)=SIN((N-1)*THETA(I))
      CW=WN(I,NLF)*THMAX/2.*AL(2)
      DO 8 K=1,NOLT
      CR(K)=AL(K)*CW*GR(I)+CR(K)
8     CI(K)=AL(K)*CW*GI(I)+CI(K)
      IF (J-5) 13,9,10
9     IF (JEST) 80,10,80
80    TYPE 22,XO(I),GR(I),GI(I),CW,(AL(K),K=1,3),(CR(K),K=1,3)
10    CONTINUE
      DO 15 I=1,NTE
      THETA(I)=THMAX+(P02-THMAX/2.)*(1.-GN(I,NTE))
15    XO(I)=X(J)+COS(THETA(I))
      CALL KER2(RF,NTE)
      DO 25 I=1,NTE
      AL(1)=COS(THETA(I)/2.)/SIN(THETA(I)/2.)
      DO 151 N=2, NOLT
151   AL(N)=SIN((N-1)*THETA(I))
      CW=WN(I,NTE)*(P02-THMAX/2.)*SIN(THETA(I))
      DO 20 K=1,NOLT
      CR(K)=AL(K)*CW*GR(I)+CR(K)
20    CI(K)=AL(K)*CW*GI(I)+CI(K)
      IF (JEST) 301,25,301
301   IF (J-5) 25,21,25

```

```

21  TYPE 22,XO(I),GR(I),GI(I),CW,(AL(K),K=1,3),(CR(K),K=1,3)
22  FORMAT(/5E12.4/,5E12.4,/)
25  CONTINUE
    CPVC(1)=TI
    DO 251 N=2, NOLT
251  CPVC(N)=TI*COS((N-1)*THMAX)
    DO 26 K=1,NOLT
    DWR(J,K)=CR(K)+CPVC(K)
26  DWI(J,K)=CI(K)
    IF(J-5)291,27,291
27  IF(JEST)28,291,28
28  TYPE 29,(CPVC(I),I=1,3)
29  FORMAT(/3E20.8)
291  ALNXC=ALOG((X(J)-XFLAP)**2)
    VFR(J)=0.
    VFI(J)=0.
    TT(1)=0.
    TT(2)=THMAX
    TT(3)=TFLAP
    TT(4)=PO2*2.
    IF (TT(3)-TT(2)) 1011,1012,1012
1011 VFP(J)=1.
    VFI(J)=(X(J)-XFLAP)*RF
    TT(2)=TFLAP
    TT(3)=THMAX
1012 CONTINUE
    DO 1020 K=1,3
    DO 1021 I=1,NFLP
    TH=TA(I)=TT(K)+.5*(TT(K+1)-TT(K))*(1.-GN(I,NFLP))
1021 XO(I)=X(J)+COS(THETA(I))
    CALL KER2(RF,NFLP)
    DO 1022 I=1,NFLP
    SN=SIN(THETA(I))
    CW=WN(I,NFLP)*.5*(TT(K+1)-TT(K))*SN
    ALNZC=ALOG((XFLAP+COS(THETA(I)))**2)
    VFR(J)=VFR(J)-PC*CW*((SN*(GR(I)-1.)/(PO2*4.*XO(I)))
1-FKR(J))*ALNZC+SN/(PO2*4.*XO(I))*ALNXC)
1022 VFI(J)=VFI(J)-PC*CW*((SN*GI(I)-FKI(J))*ALNZC)
1020 CONTINUE
    VFR(J)=VFR(J)-PC*(-.5*X(J)*ALNXC +FKR(J)*CNST)
    VFI(J)=VFI(J)-PC*(FKI(J)*CNST)
100  CONTINUE
    DO 101 I=1,NOLT
    DO 101 K=1,NOLT
    DMR(I,K)=0.0
101  DMI(I,K)=0.0
    DO 110 I=1,NOLT
    DO 110 K=1,NOLT
    DO 110 J=1,NP
    DMR(I,K)=DWR(J,I)*DWR(J,K)+DWI(J,I)*DWI(J,K) +DMR(I,K)

```

```

109 DMI(I,K)=-DWI(J,I)*DWR(J,K)+DWR(J,I)*DWI(J,K)+DMI(I,K)
110 CONTINUE
    CALL CMPINV(DMR,DMI,PR,PI,NOLT,INDEX1)
    IF(JEST) 111,115,111
111 TYPE 112,((DMR(I,J),DMI(I,J),J=1,NOLT),I=1,NOLT)
    TYPE 112,((PR(I,J),PI(I,J),J=1,NOLT),I=1,NOLT)
112 FORMAT(/3E20.8,/,3E20.8,/)
    TYPE 112,((DWR(J,K),DWI(J,K),K=1,NOLT),J=1,NP)
115 VPR=1.0
    VHR=1.0
    DO 150 I=1,NOLT
        WFR(I)=0.
        WFI(I)=0.
        WPR(I)=0.0
        WPI(I)=0.0
        WHR(I)=0.0
        WHI(I)=0.0
        DO 140 J=1,NP
            VPI=(X(J)+.5)*RF
            WFR(I)=WFR(I)+VFR(J)*DWR(J,I)+VFI(J)*DWI(J,I)
            WFI(I)=WFI(I)-VFR(J)*DWI(J,I)+VFI(J)*DWR(J,I)
            WHR(I)=WHR(I)+VHR(J)*DWR(J,I)
            WHI(I)=WHI(I)-VHR(J)*DWI(J,I)
            WPR(I)=WPR(I)+VPR(J)*DWR(J,I)+VPI(J)*DWI(J,I)
140 WPI(I)=WPI(I)-VPR(J)*DWI(J,I)+VPI(J)*DWR(J,I)
            IF (JEST) 141,150,141
141 TYPE 142,WHR(I),WHI(I),WPR(I),WPI(I)
142 FORMAT(/4E18.8)
150 CONTINUE
    DO 200 I=1,NOLT
        FA(I)=0.
        FB(I)=0.
        PA(I)=0.0
        PB(I)=0.0
        HA(I)=0.0
        HB(I)=0.0
        DO 200 J=1,NOLT
            FB(I)=FB(I)+PR(I,J)*WFI(J)+PI(I,J)*WFR(J)
            FA(I)=FA(I)+PR(I,J)*WFR(J)-PI(I,J)*WFI(J)
            PA(I)=PA(I)+PR(I,J)*WPR(J)-PI(I,J)*WPI(J)
            PB(I)=PB(I)+PR(I,J)*WPI(J)+PI(I,J)*WPR(J)
            HA(I)=HA(I)+PR(I,J)*WHR(J)-PI(I,J)*WHI(J)
200 HB(I)=HB(I)+PR(I,J)*WHI(J)+PI(I,J)*WHR(J)
    TYPE 201, (HA(I),HB(I),PA(I),PB(I),FA(I),FB(I),I=1,NOLT)
201 FORMAT (/,6E12.5)
    AFL=0.
    AFM=0.
    AFF=0.
    DO 1101 I=1,NFLP
        THTR=.5*TFLAP*(1.-GN(I,NFLP))

```

```

      SN=SIN(THTA)
      CS=COS(THTA)
      ALNXC=ALOG((XFLAP+CS)**2)
      CW=WN(I,NFLP)*.5*TFLAP*SN
      AFL=AFL+CW*(SN-SNTF)*ALNXC
      AFM=AFM+CW*SN*ALNXC*(-CS-XFLAP)
      THTA=TFLAP+(PO2-TFLAP*.5)*(1.-CN(I,NFLP))
      SN=SIN(THTA)
      CS=COS(THTA)
      ALNXC=ALOG((XFLAP+CS)**2)
      CW=WN(I,NFLP)*(PO2-TFLAP*.5)*SN
      AFL=AFL+CW*(SN-SNTF)*ALNXC
      AFF=AFF+CW*SN*ALNXC*(-CS-XFLAP)
1101  CONTINUE
      AFL=AFL+SNTF*CNST
      AFM=(AFM+AFF)+(XFLAP+.5)*AFL
      AL(1)=PO2*2.-TFLAP-SNTF
      AL(2)=PO2-.5*TFLAP-.5*XFLAP*SNTF
      DO 35 N=3, NOLT
35    AL(N)=.5*(SIN(N*TFLAP)/N-SIN(N-2)*TFLAP)/(N-2)
      AM(1)=-PO2+.5*TFLAP+SNTF-.5*SNTF*XFLAP
      AM(2)=(SNTF**3)/3.
      AM(3)=-.5*PO2+.25*TFLAP-.0625*SIN(4.*TFLAP)
      DO 36 N=4, NOLT
36    AM(N)=.25*(SIN(N-3)*TFLAP)/(N-3)-SIN((N+1)*TFLAP)/(N+1)
      DO 1102 I=1,NOLT
1102  AL(I)=AM(I)-XFLAP*AL(I)
      CMHR=PO2/4.*(HA(3)-HA(2))
      CMHI=PO2/4.*(HB(3)-HB(2))
      CLHR=2.*PO2*(HA(1)+.5*HA(2))
      CLHI=2.*PO2*(HB(1)+.5*HB(2))
      CHHR=0.
      CHHI=0.
      DO 1201 I=1,NOLT
1201  CHHR=CHHR-.5*HA(I)*AL(I)
      CHHI=CHHI-.5*HB(I)*AL(I)
      TYPE 300
300   FORMAT(/30H0      HEAVING      OSCILLATION      )
      TYPE320,CLHR,CLHI,CMHR,CMHI,CHHR,CHHI
320   FORMAT(/$CLR=$,F8.4,$      CLI=$,F8.4,4X,$CMR=$,F8.4,4X,
1$CMI=$,F8.4,$CHR=$,F8.4,4X,$CHI=$,F8.4)
      CALL COMPR(CLHR,CLHI,RF)
      TYPE 350
350   FORMAT(/30H2      PITCHING OSCILLATION      )
      CMPR=PO2/4.*(PA(3)-PA(2))
      CMPI=PO2/4.*(PB(3)-PB(2))
      CLPR=2.*PO2*(PA(1)+.5*PA(2))
      CLPI=2.*PO2*(PB(1)+.5*PB(2))
      CHPR=0.
      CHPI=0.
      DO 1202 I=1,NOLT
      CHPR=CHPR-.5*PA(I)*AL(I)

```

```

1202 CHPI=CHPI-.5*PR(I)*AL(I)
      TYPE 320,CLPE,CLPI,CMPE,CMPI,CMPE,CHPI
      TYPE 1301
1301 FORMAT(/,$          FLAP OSCILLATIONS)
      CMPE=PO2/4.*(FA(3)-FA(2))-.5*PC*AFH
      CMFI=PO2/4.*(FB(3)-FB(2))
      CLPE=2.*PO2*(FA(1)+.5*FA(2))+PC*AFL
      CLFI=2.*PO2*(FB(1)+.5*FB(2))
      CMPE=-.5*PC*AFH
      CMFI=0.
      DO 1203 I=1,NOLT
      CMPE=CMPE-.5*FA(I)*AL(I)
1203 CMFI=CMFI-.5*FB(I)*AL(I)
      TYPE 320, CLPE,CLFI,CMPE,CMFI,CMPE,CHPI
      TYPE 997,   CLPE,CLFI,CMPE,CMFI,RE,FR,D,NP,NOLT
      TYPE 997,   CLPE,CLFI,CMPE,CMFI,RE,FR,D,NP,NOLT
997  FORMAT(/4F14.8,3F4.2,2I2)
      GO TO 1
501  CONTINUE
      STOP
      END

```

APPENDIX E: SUBROUTINES

```

SUBROUTINE COMPR(CR,CI,RF)
  PI=3.1415193
  COR=CR/(2.*PI)
  COI=CI/(2.*PI)-RF/2.
  TYPE 10, COR,COI
10  FORMAT(/30HO COMPARISON WITH CORNELL HR= ,F10.8,5H HI=,F10.8)
  RETURN
  END

SUBROUTINE KER2(RF,NCP)
  DIMENSION GR(32),GI(32),XO(32)
  COMMON XO,GR,GI,PO2,X,JEST,IM,IFF,D,FR,NT,S1,S2,S3R,S3I,S4R,S4I,
1A1,A2,A3R,A3I,A4R,A4I
  CM3R(A,B,C,D,P,Q) =A*C*P-A*D*Q-C*B*Q-P*B*D
  CM3I(A,B,C,D,P,Q) =A*C*Q+P*C*B+A*P*D-B*D*Q
  PI2=PO2*4.
3  DO 10 I=1,NCP
  XK=XO(I)*RF
  AB=(1.+ABS(XK)/XK)*PO2
  DK=2.*D*RF
  SINK=SIN(XK)
  COSK=COS(XK)
  EXPK=EXP(DK)
  CALL EXPINT(NT,0.0,XK,ER,EI)
  GR(I)=RF/PI2*CM3R(0.0,1.0,COSK,-SINK,ER,EI+AB)
  GI(I)=RF/PI2*CM3I(0.0,1.0,COSK,-SINK,ER,EI+AB)
  IF(IM) 4,10,4
4  IF(IFF) 6,5,6
5  CALL EXPINT(NT,DK,XK,ER,EI)
  GR(I)=GR(I)-XO(I)/(PI2*(XO(I)**2+4.*D**2))+RF/(2.*PI2)/EXPK
1*CM3R(0.0,1.0,COSK,-SINK,ER,EI+2.*AB)
  GI(I)=GI(I)+RF/(2.*PI2)/EXPK*CM3I(0.0,1.0,COSK,
1-SINK,ER,EI+2.*AB)
  CALL EXPINT(NT,-DK,XK,ER,EI)
  GR(I)=GR(I)+EXPK/(PI2*2.)*CM3R(0.0,RF,COSK,
1-SINK,ER,EI)
  GI(I)=GI(I)+EXPK/(PI2*2.)*CM3I(0.0,RF,COSK,
1-SINK,ER,EI)
  GO TO 10
6  CALL EXPINT(NT,DK,XK,ER,EI)
  GR(I)=GR(I)-XO(I)/(PI2*(XO(I)**2+4.*D**2))-RF/(2.*PI2)/EXPK
1*CM3R(0.0,1.0,COSK,-SINK,ER,EI+2.*AB)
  GI(I)=GI(I)-RF/(2.*PI2)/EXPK*CM3I(0.0,1.0,COSK,
1-SINK,ER,EI+2.*AB)
  CALL EXPINT(NT,-DK,XK,ER,EI)
  GR(I)=GR(I)-EXPK/(PI2*2.)*CM3R(0.0,RF,COSK,
1-SINK,ER,EI)
  GI(I)=GI(I)-EXPK/(PI2*2.)*CM3I(0.0,RF,COSK,
1-SINK,ER,EI)
  CALL WAVES(RF,I)

```

```

10  CONTINUE
    RETURN
    END
    SUBROUTINE WAVES(RF,I)
    DIMENSION XO(32),GR(32),GI(32)
    COMMON XO,GR,GI,PO2,X,JEST,IM,IFF,D,FR,NT,S1,S2,S3R,S3I,S4R,S4I,
1A1,A2,A3R,A3I,A4R,A4I
    CM3R(A,B,C,D,P,Q)=A*C*P-A*D*Q-C*B*Q-P*B*D
    CM3I(A,B,C,D,P,Q)=A*C*Q+P*C*B+A*P*D-B*D*Q
    PI4=PO2*8.
    DT=2.*D
    EX=XO(I)
    ABX=PO2*2.*(1.+ABS (EX)/EX)
2    F=RF*FR**2
    S1=-RF*(1.+1./(2.*F)*(1.+SQRT (4.*F+1.)))
    S2=-RF*(1.+1./(2.*F)*(1.-SQRT (4.*F+1.)))
    A1=RF*(1.+1./F+(3.+1./F)*1./SQRT (4.*F+1.))
    A2=RF*(1.+1./F-(3.+1./F)*1./SQRT (4.*F+1.))
    CALL EXPINT(NT,-S1*DT,-S1*EX,ER,EI)
    GR(I)=GR(I)+EXP (S1*DT)/PI4*CM3R (0.0,A1,  COS (S1*XO(I)),
1SIN (S1*XO(I)),ER,EI+ABX)
    GI(I)=GI(I)+EXP (S1*DT)/PI4*CM3I (0.0,A1,  COS (S1*XO(I)),
1SIN (S1*XO(I)),ER,EI+ABX)
    CALL EXPINT(NT,-S2*DT,-S2*EX,ER,EI)
    GR(I)=GR(I)+EXP (S2*DT)/PI4*CM3R (0.0,A2,  COS (S2*XO(I)),
1SIN (S2*XO(I)),ER,EI+ABX)
    GI(I)=GI(I)+EXP (S2*DT)/PI4*CM3I (0.0,A2,  COS (S2*XO(I)),
1SIN (S2*XO(I)),ER,EI+ABX)
    IF (F-.25) 10,100,20
100 STOP
10  R=1.-4.*F
    RAD =SQRT (R)
    S3= -RF*(1.-.5/F*(1.-RAD))
    S4= -RF*(1.-.5/F*(1.+RAD))
    A3= RF*(1.-1./F-(3.-1./F)/RAD)
    A4= RF*(1.-1./F+(3.-1./F)/RAD)
    Q3R=DT*S3
    Q3I=-EX*S3
    Q4R=DT*S4
    Q4I=-EX*S4
    ABXM=(1.-ABS (EX)/EX)*PO2*2.
    CALL EXPINT(NT,Q3R,Q3I,ER,EI)
    GR(I)=GR(I)+EXP (-Q3R)/PI4*CM3R (0.0,A3,COS (-Q3I),SIN (-Q3I),
1ER,EI+ABXM)
    GI(I)=GI(I)+EXP (-Q3R)/PI4*CM3I (0.0,A3,COS (-Q3I),SIN (-Q3I),
1ER,EI+ABXM)
    CALL EXPINT(NT,Q4R,Q4I,ER,EI)
    GR(I)=GR(I)+EXP (-Q4R)/PI4*CM3R (0.0,A4,COS (-Q4I),SIN (-Q4I),
1ER,EI-ABX)
    GI(I)=GI(I)+EXP (-Q4R)/PI4*CM3I (0.0,A4,COS (-Q4I),SIN (-Q4I),
1ER,EI-ABX)
    GO TO 99

```

```

20  S3R=-RF*(1.-1./(2.*F))
    S3I=-RF*(1./(2.*F)*SQRT (4.*F-1.))
    S4R=S3R
    S4I=-S3I
    A3R=RF*(1.-1./F)
    A3I=RF*(3.-1/F)*1./SQRT (4.*F-1.)
    A4R=A3R
    A4I=-A3I
    Q3R=-S3R*DT-S3I*XO(I)
    Q3I=S3R*XO(I)-S3I*DT
    CALL EXPINT(NT,-Q3R,-Q3I,ER,EI)
    IF (F-.5) 21,22,22
21  AB=PO2*2.*(1.-ABS (Q3I)/Q3I)
    GO TO 23
22  AB=0.
23  GR(I)=GR(I)+EXP (Q3R)/PI4*CM3R (-A3I,A3R,COS (Q3I),SIN (Q3I),ER,
1  EI+AB)
    GI(I)=GI(I)+EXP (Q3R)/PI4*CM3I (-A3I,A3R,COS (Q3I),SIN (Q3I),ER,
1  EI*AB)
    Q4R=-S4R*DT-S4I*XO(I)
    Q4I=S4R*XO(I)-S4I*DT
    CALL EXPINT(NT,-Q4R,-Q4I,ER,EI)
    IF (F-.5) 24,25,25
24  AB=PO2*2.*(1.+ABS (Q4I))/Q4I
    GO TO 26
25  AB=0.
26  GI(I)=GI(I)+EXP (Q4R)/PI4*CM3I (-A4I,A4R,COS (Q4I),SIN (Q4I),ER,
1  EI-AB)
    GR(I)=GR(I)+EXP (Q4R)/PI4*CM3R (-A4I,A4R,COS (Q4I),SIN (Q4I),ER,
1  EI+AB)
99  RETURN
END
C    COMPLEX MATRIX INVERSION

    SUBROUTINE CMPINV(A,B,C,D,N,INDEX1)

    DIMENSION A(6,6),B(6,6),C(6,6),D(6,6),PIVOT(25),INDEX(50),
1IPIVOT(25),SPACE(200)
    COMMON SPACE,PIVOT,INDEX ,IPIVOT

C    SAVE A AND INVERT -A.

5    M=N
7    L=1
10   DO 30 I=1,M
20   DO 30 J=1,M
30   D(I,J)=-A(I,J)
40   CALL MATINV (D,M,DUMMY,0,DUMMY)

```

```

C      CHECK IF A WAS NON-SINGULAR.

50     DO 70 I=1,M
60     IF (IPIVOT(I)-1) 250, 70, 250
70     CONTINUE

C      COMPUTE C=INVERSE OF (A+BA(INVERSE)B).

80     CALL SQMULT (B,D,D,M)
90     CALL SQMULT (D,B,C,M)
100    DO 120 I=1,M
110    DO 120 J=1,M
120    C(I,J)=A(I,J)-C(I,J)
130    CALL MATINV (C,M,DUMMY,0,DUMMY)

C      CHECK THAT C EXISTS.

140    DO 170 I=1,M
150    IF (IPIVOT(I)-1) 155, 170, 155

C      INVERSE DOES NOT EXIST, SET SIGNAL AND RETURN.
155    INDEX1=2
160    RETURN
170    CONTINUE

C      COMPUTE D=-CBA(INVERSE).

180    CALL SQMULT (C,D,D,M)
220    GO TO (230,330), L

C      SUCCESSFUL INVERSION

230    INDEX1=1
240    RETURN

C      A IS SINGULAR, SO INTERCHANGE A AND B AND TRY AGAIN.
250    DO 300 I=1,M
280    DO 300 J=1,M
290    DUMMY=A(I,J)
      A(I,J)=B(I,J)
300    B(I,J)=DUMMY
305    IF (L-2) 310, 370, 370
310    L=2
      GO TO 10

C      INTERCHANGE A AND B, C AND D WITH CHANGED SIGNS.

```

```

330 DO 350 I=1,M
340 DO 350 J=1,M
    DUMMY=A(I,J)
    A(I,J)=B(I,J)
    B(I,J)=DUMMY
    DUMMY=-C(I,J)
    C(I,J)=-D(I,J)
350 D(I,J)=DUMMY
360 GO TO 230

```

```

C A AND B BOTH SINGULAR. CANNOT FIND INVERSE. SET SIGNAL AND RETURN.
370 INDEX1=3
380 RETURN
    END

```

```

C MATRIX INVERSION WITH ACCOMPANYING SOLUTION OF LINEAR EQUATIONS

SUBROUTINE MATINV(A,N,B,M,DETERM)

```

```

    DIMENSION IPIVOT(25),A(6,6),B(25,1),INDEX(25,2),PIVOT(25),
1 SPACE(200)
    COMMON SPACE,PIVOT,INDEX,IPIVOT
    EQUIVALENCE (IHOW,JROW), (ICOLUM,JCOLUM), (AMAX, T, SWAP)

```

```

C INITIALIZATION

```

```

10 DETERM=1.0
15 DO 20 J=1,N
20 IPIVOT(J)=0
30 DO 550 I=1,N

```

```

C SEARCH FOR PIVOT ELEMENT

```

```

40 AMAX=0.0
45 DO 105 J=1,N
50 IF (IPIVOT(J)-1) 60, 105, 60
60 DO 100 K=1,N
70 IF (IPIVOT(K)-1) 80, 100, 740
80 IF (ABS (AMAX)-ABS (A (J,K))) 85, 100, 100
85 IROW=J
90 ICOLUM=K
95 AMAX=A(J,K)
100 CONTINUE
105 CONTINUE
110 IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1

```

```

C INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL

```

```

130 IF (IROW-ICOLUM) 140, 260, 140
140 DETERM=-DETERM
150 DO 200 L=1,N
160 SWAP=A(IROW,L)
170 A(IROW,L)=A(ICOLUM,L)
200 A(ICOLUM,L)=SWAP
205 IF(M) 260, 260, 210
210 DO 250 L=1, M
220 SWAP=B(IROW,L)
230 B(IROW,L)=B(ICOLUM,L)
250 B(ICOLUM,L)=SWAP
260 INDEX(I,1)=IROW
270 INDEX(I,2)=ICOLUM
310 PIVOT(I)=A(ICOLUM,ICOLUM)
320 DETERM=DETERM*PIVOT(I)

```

C DIVIDE PIVOT ROW BY PIVOT ELEMENT

```

330 A(ICOLUM,ICOLUM)=1.0
340 DO 350 L=1,N
350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT(I)
355 IF(M) 380, 380, 360
360 DO 370 L=1,M
370 B(ICOLUM,L)=B(ICOLUM,L)/PIVOT(I)

```

C REDUCE NON-PIVOT ROWS

```

380 DO 550 L1=1,N
390 IF(L1-ICOLUM) 400, 550, 400
400 T=A(L1,ICOLUM)
420 A(L1,ICOLUM)=0.0
430 DO 450 L=1,N
450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
455 IF(M) 550, 550, 460
460 DO 500 L=1,M
500 B(L1,L)=B(L1,L)-B(ICOLUM,L)*T
550 CONTINUE

```

C INTERCHANGE COLUMNS

```

600 DO 710 I=1,N
610 L=N+1-I
620 IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630
630 JROW=INDEX(L,1)
640 JCOLUMN=INDEX(L,2)
650 DO 705 K=1,N
660 SWAP=A(K,JROW)
670 A(K,JROW)=A(K,JCOLUMN)
700 A(K,JCOLUMN)=SWAP
705 CONTINUE
710 CONTINUE
740 RETURN
END

```

C SQUARE MATRIX MULTIPLICATION

SUBROUTINE SQMULT(A,B,C,N)

```

DIMENSION A(6,6),B(6,6),C(6,6),D(6,6),COLUMN(25),
1 SPACE(200)
COMMON SPACE,COLUMN

```

```

5 M=N
10 DO 50 J=1,M
20 DO 25 K=1,M
25 COLUMN(K)=B(K,J)
30 DO 50 I=1,M
35 C(I,J)=0.0
40 DO 50 K=1,M
50 C(I,J)=C(I,J)+A(I,K)*COLUMN(K)
60 RETURN
END

```

```

FUNCTION GN(I,N)
DIMENSION G(32)
1 IF(N-6) 2,60,2
2 IF(N-8) 3,8,3
3 IF(N-10) 4,10,4
4 IF (N-16) 6,16,6
6 IF (N-32) 7,32,7
60 G(1)=.9324695142
G(2)=.6612093865
G(3)=.238619161
IF (I-3) 61,61,62
61 GN=G(I)
GO TO 7
62 M=7-I
GN=-G(M)
GO TO 7

```

```

8      G(1)=.9602808565
      G(2)=.7966664714
      G(3)=.5255324099
      G(4)=.1834346425
      IF (I-4) 9,9,89
9      GN=G(I)
      GO TO 7
89     M=9-I
      GN=-G(M)
      GO TO 7
10     G(1)=.973906529
      G(2)=.8650633667
      G(3)=.6794095683
      G(4)=.43339539
      G(5)=.1488743389
      IF (I-5) 11,11,12
11     GN=G(I)
      GO TO 7
12     M=11-I
      GN=-G(M)
      GO TO 7
16     G(1)=.989400935
      G(2)=.944575
      G(3)=.8656312
      G(4)=.7554044
      G(5)=.617862
      G(6)=.45801678
      G(7)=.2816035
      G(8)=.0950125
      IF (I-8) 17,17,18
17     GN=G(I)
      GO TO 7
18     M=17-I
      GN=-G(M)
      GO TO 7
32     G(1)=.9972639
      G(2)=.985611512
      G(3)=.9647623
      G(4)=.9305
      G(5)=.89632116
      G(6)=.849367614
      G(7)=.7944838
      G(8)=.732182119
      G(9)=.663044261
      G(10)=.587715757
      G(11)=.506899909
      G(12)=.42135128
      G(13)=.331868602
      G(14)=.239287362
      G(15)=.144471962
      G(16)=.048307666
      IF (I-16) 33,33,34

```

```

33      GN=G(I)
        GO TO 7
34      M=33-I
        GN=-G(M)
        GO TO 7
7       RETURN
        END
        FUNCTION WN(I,N)
        DIMENSION W(32)
1       IF(N-6) 2,60,2
2       IF (N-8) 3,8,3
3       IF (N-10) 4,10,4
4       IF(N-16) 6,16,6
6       IF(N-32) 7,32,1
60      W(1)=.1713244924
        W(2)=.3607615731
        W(3)=.4679139346
        IF (I-3) 61,61,62
61      WN=W(I)
        GO TO 7
62      M=7-I
        WN=W(M)
        GO TO 7
8       W(1)=.1012285303
        W(2)=.2223810345
        W(3)=.3137066459
        W(4)=.3626837834
        IF (I-4) 9,9,89
9       WN=W(I)
        GO TO 7
89      M=9-I
        WN=W(M)
        GO TO 7
10      W(1)=.0666713443
        W(2)=.1494513492
        W(3)=.2192863625
        W(4)=.2692667193
        W(5)=.2955242247
        IF (I-5) 11,11,12
11      WN=W(I)
        GO TO 7

```

```

12      M=11-I
        GO TO 7
16      W(1)=.027152459
        W(2)=0.062253524

        W(3)=.0951585117
        W(4)=.1246289713
        W(5)=.1495959888
        W(6)=.1691565194
        W(7)=.182663415
        W(8)=.1894506105
        IF (I-8) 17,17,18
17      WN=W(I)
        GO TO 7
18      M=17-I
        WN=W(M)
        GO TO 7
32      W(1)=.00701861
        W(2)=.016274395
        W(3)=0.02539201
        W(4)=.034273863
        W(5)=.042835898
        W(6)=.050998059
        W(7)=.058684093
        W(8)=.065822223
        W(9)=.072345794
        W(10)=.078193896
        W(11)=.083311924
        W(12)=.087652093
        W(13)=.091173879
        W(14)=.0938444
        W(15)=.093872
        W(16)=.099654009
        IF (I-16) 33,33,34
33      WN=W(I)
        GO TO 7
34      M=33-I
        WN=W(M)
        GO TO 7
7       RETURN
        END
C      MODIFIED PROGRAM FOR EXPONENTIAL INTEGRAL
        SURROUTINE EXPINT(NT,X,Y,ER,EI)
        R=X**2+Y**2
1       PI=3.14159265

```

```

50  SUMR=X
    SUMI=Y
    TEMR=X
    TEMI=Y
    EN=1.0
53  EN=EN+1.0
    FACT=(EN-1.0)/(EN**2)
    TERMR=(TEMR*X-TEMI*Y)*FACT
    TERMI=(TEMI*X+TEMR*Y)*FACT
    TEMR=TERMR
    TEMI=TERMI
    SUMR=SUMR+TERMR
    SUMI=SUMI+TERMI
    IF(R*FACT**2-.001) 60,60,53
60  ER=.5772157+.5*ALOG(R)+SUMR
    EI=-PI+ ANGL(X,Y)+SUMI
    NT=EN
    RETURN
    END
    FUNCTION ANGL(X,Y)
C    ANGL BETWEEN 0. AND 2.
    IF (Y) 10,20,30
10  ANGL=ATAN(-X/Y)+4.721388975
    RETURN
20  IF(X) 21,22,22
21  ANGL=3.14159265
    RETURN
22  ANGL=0.
    RETURN
30  ANGL=ATAN(-X/Y)+1.57079633
    RETURN
    END
    STOP

```

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Bolt Beranek and Newman Inc. 50 Moulton Street Cambridge, Massachusetts 02138		2a. REPORT SECURITY CLASSIFICATION Unclassified	
		2b. GROUP	
3. REPORT TITLE THE UNSTEADY HYDRODYNAMICS AND CONTROL OF HYDROFOILS NEAR A FREE SURFACE			
4. DESCRIPTIVE NOTES (Type of report and, inclusive dates) Final Report — 1 August 1969 — 30 April 1970			
5. AUTHOR(S) (First name, middle initial, last name) Joseph I. Smullin Erich K. Bender			
6. REPORT DATE 4 June 1970		7a. TOTAL NO. OF PAGES 88	7b. NO. OF REFS 7
8a. CONTRACT OR GRANT NO. N00014-70-C-0032		9a. ORIGINATOR'S REPORT NUMBER(S) BBN Report No. 1970	
b. PROJECT NO.			
c.		9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
d.			
10. DISTRIBUTION STATEMENT This document has been approved for public release and sale; its distribution is unlimited.			
11. SUPPLEMENTARY NOTES → A discussion is given of		12. SPONSORING MILITARY ACTIVITY Naval Ship Research and Development Center, Department of the Navy Washington, D.C. 20007	
13. ABSTRACT In this Report we discuss (1) the development of a digital computer program to compute hydrofoil loads and (2) some aspect of hydrofoil control. The program computes the lift, pitching moment, and flap hinge moment on a two-dimensional hydrofoil with a trailing edge flap operating near a free surface with waves. The computational approach involves the numerical solution of an integral equation relating an upwash distribution to a kernel function and pressure distribution. The pressure distribution is expanded in a truncated Glauert series; the integration is carried out numerically using a Gaussian quadrature; and the coefficients of the Glauert series are evaluated by a minimum error collocation method. The control problem investigated involves the positioning of a pivoted hydrofoil by means of a servo-controlled trim tab. When the foil is pivoted at its quarter chord point and control is implemented solely by means of a servo tab, the system is virtually uncontrollable. However, by pivoting the foil off the quarter chord point or by augmenting the servo tab with a servo attached directly to the foil, the system can be controlled. () ←			

DD FORM 1473

1 NOV 65

(PAGE 1)

S/N 0101-807-6811

UNCLASSIFIED

Security Classification

A-31408

