

COMPUTER PROGRAMS FOR CHARACTERISTIC MODES
OF BODIES OF REVOLUTION

by

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Electrical Engineering Department
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Contract No. F19628-68-C-0180

Project No. 5635

Task No. 563506

Work Unit No. 56350601

SCIENTIFIC REPORT NO. 10

January 1971

Contract Monitor: John F. McIlvenna
Microwave Physics Laboratory

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AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS 01730

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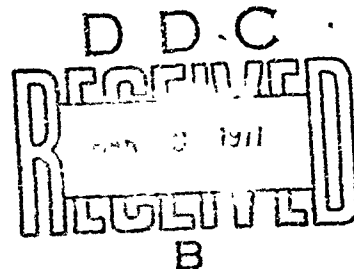
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ABSTRACT

Computer programs are given for calculating the characteristic currents and characteristic gain patterns of conducting bodies of revolution. Also given are computer programs for using these characteristic currents in aperture radiation and plane-wave scattering problems. Plot programs for use with a Calcomp plotter are included. Operating procedures and program details are discussed, and sample input-output data are given.

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I. INTRODUCTION

The general theory and method of computation of characteristic modes for conducting bodies of arbitrary shape is given in reference [1]. The notation of this report is consistent with that of [1], which should be referred to for further identification of the symbols used. The programs given here are those used for the numerical results presented in reference [1].

Six computer programs have been written. They are defined according to their function:

1. Calculate the generalized impedance matrix Z
2. Calculate the characteristic currents (eigencurrents).
3. Plot the eigencurrents.
4. Calculate and plot the gain patterns of the eigencurrents
5. Calculate and plot σ/λ^2 (scattering cross section divided by the square of the wavelength) for an axially incident plane wave
6. Calculate and plot the gain pattern for radiation from an axially symmetric excitation.

These programs are discussed and listed in the next six sections. Operating instructions and sample input-output data are also given.

II. GENERALIZED IMPEDANCE MATRIX

Program #1 calculates the elements of the generalized impedance matrix for a body of revolution, and is a slight modification of the computer program appearing in Appendix A of reference [1]. The subroutine LINEQ and its call statement 81 have been removed in order to obtain the generalized impedance matrix $\hat{Z}$ instead of the generalized admittance matrix. Except for NPHI which was taken to be 20 in all of our work, all the punched card data required by program #1 is explained in reference [2]. As on page 26 of reference [2], punched card data is read early in the main program according to

```

50  READ(1,51,END = 52) NN, NP, NPHI, BK
51  FORMAT(3I3, E14.7)
      READ(1,53)(RH(I), I = 1, NP)
      READ(1,53)(ZH(I), I = 1, NP)
53  FORMAT(10F8.4)

```

Here, BK is the propagation constant $k = \omega\sqrt{\mu\epsilon}$. An odd number NP of data points are taken from the generating curve C of the body of revolution. RH(I) and ZH(I) are respectively the distance ρ from the axis of the body of revolution and the corresponding z coordinate at the I^{th} data point. The first and NP^{th} data points are on the ends of C. If C closes upon itself, care must be taken to make the coordinates of the first data point equal to those of the NP^{th} . Program #1 writes the NM2 by NM2 impedance matrix

$$\hat{Z}_n = \begin{bmatrix} \hat{Z}_n^{tt} & \hat{Z}_n^{t\phi} \\ \hat{Z}_n^{\phi t} & \hat{Z}_n^{\phi\phi} \end{bmatrix} \quad \text{where } n = \text{NN} \quad (1)$$

on record 1 of direct access data set 6. NM2 is either NP-1 or NP-3 depending on whether or not C closes upon itself. If NN = 0, zeros are supplied for $\hat{Z}_0^{\phi t}$ and $\hat{Z}_0^{t\phi}$.

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```

//          (0034,EE,4,2),'MAUTZ,JOE',MSGLEVEL=1
// EXEC FORTGCLG,PARM,FORT='MAP'
//FORT,SYSIN DD *
      COMPLEX A3,A4,Z(1600),GS(40),G(5292),U
      DIMENSION RH(43),ZH(43),DH(43),TJ(20)
      DIMENSION SV(42),CV(42),ZS(42),R(42),ANG(40),AC(40),CSN(120)
      DIMENSION TP(80),T(80),TR(80),JK(4)
      REWIND 6
      II=(0.,1.)
50 READ(1,51,END=52) NN,NP,NPHI,BK
51 FORMAT(3I3,E14.7)
   READ(1,53)(RH(I),I=1,NP)
   READ(1,53)(ZH(I),I=1,NP)
53 FORMAT(10F8.4)
76 WRITE(3,54) NN,NP,NPHI,BK
54 FORMAT(1X// ' NN=',I3,' NP=',I3,' NPHI=',I3,' BK=',E14.7)
55 FORMAT(1X/ ' RH')
   WRITE(3,55)
   WRITE(3,46)(RH(I),I=1,NP)
46 FORMAT(1X,10F8.4)
   WRITE(3,56)
56 FORMAT(1X/ ' ZH')
   WRITE(3,46)(ZH(I),I=1,NP)
   IF((RH(1)-RH(NP)).NE.0..OR.(ZH(1)-ZH(NP)).NE.0.) GO TO 58
   RH(NP+1)=RH(2)
   ZH(NP+1)=ZH(2)
   RH(NP+2)=RH(3)
   ZH(NP+2)=ZH(3)
   NP=NP+2
58 DO 57 I=2,NP
   I2=I-1
   RR1=RH(I)-RH(I2)
   RR2=ZH(I)-ZH(I2)
   DH(I2)=SQRT(RR1*RR1+RR2*RR2)
   ZS(I2)=.5*(ZH(I)+ZH(I2))
   R(I2)=.5*(RH(I)+RH(I2))
   SV(I2)=RR1/DH(I2)
   CV(I2)=RR2/DH(I2)
57 CONTINUE
   KG=NP-1
   N=KG/2
   NM=N-1
   NM2=NM*2
   NM4=NM*4
   NZ=NM2*NM2
   NG=KG*KG
   M5=NN+2
   M6=NN+4
   FM=NN
   FM2=NN*NN
   PI=3.141593
   ETA=376.707
   DP=PI/NPHI
   CA=BK*BK*ETA
   CQ=ETA
   SS=0.
   DO 117 I=1,NM
   I1=2*(I-1)+1
   I2=I1+1
   SS=SS+DH(I1)+DH(I2)

```

```
COMPLEX Z(NZ)
DIMENSION RH(NP), ZH(NP), U(NZ), R(NZ), T2(NZ),
      A22(NZ), B(NZ), X(NZ), A(NZ), Y(NZ), T3(NZ),
      F1(NZ), EU(NM2), RU(NM2), AMD(NM2), LB(NM2),
      MB(NM2)
```

where

```
NM2 = NP - 3
```

```
NZ = NM2 * NM2
```

The above allocations are based upon the value of NP after execution of statement 145

```

      DO 13 K=1,NPHI
      K2=K+M4
      G(M2)=G(M2)+GS(K)*CSM(K2)
13  CONTINUE
68  CONTINUE
17  CONTINUE
16  CONTINUE
      DO 74 J=1,NM
      J2=2*(J-1)+1
      J3=J2+1
      J4=J3+1
      J5=J4+1
      J6=4*(J-1)+1
      J7=J6+1
      J8=J7+1
      J9=J8+1
      DEL1=DH(J2)+DH(J3)
      DEL2=DH(J4)+DH(J5)
      TP(J6)=DH(J2)/DEL1
      TP(J7)=DH(J3)/DEL1
      TP(J8)=-DH(J4)/DEL2
      TP(J9)=-DH(J5)/DEL2
      T(J6)=DH(J2)*DH(J2)/2./DEL1
      T(J7)=DH(J3)*(DH(J2)+DH(J3)/2.)/DEL1
      T(J8)=DH(J4)*(DH(J5)+DH(J4)/2.)/DEL2
      T(J9)=DH(J5)*DH(J5)/2./DEL2
74  CONTINUE
      DO 75 J=1,NM4
      TR(J)=T(J)
75  CONTINUE
115 IF((ZH(1)-ZH(NP-2)).EQ.0..AND.(RH(1)-RH(NP-2)).EQ.0.) GO TO 78
      IF(RH(1)) 77,23,77
77  DEL1=DH(1)+DH(2)
      TR(1)=DH(1)*(1.+(DH(2)+DH(1)/2.)/DEL1)
      TR(2)=DH(2)*(1.+(DH(2)/2.)/DEL1)
23  IF(RH(NP)) 79,78,79
79  J1=(N-2)*4+3
      J2=J1+1
      DEL2=DH(NP-2)+DH(KG)
      TR(J1)=DH(NP-2)*(1.+(DH(NP-2)/2.)/DEL2)
116 TR(J2)=DH(KG)*(1.+(DH(NP-2)+DH(KG)/2.)/DEL2)
78  DO 30 J=1,NM
      JL=(J-1)*NM2
      J3=(J-1)*4
      J1=2*(J-1)
      DO 31 I=1,NM
      L1=JL+I
      L2=L1+NM
      L3=NM*NM2+L1
      L4=L3+NM
      Z(L1)=0.
      Z(L2)=0.
      Z(L3)=0.
      Z(L4)=0.
      I1=2*(I-1)
      I3=(I-1)*4
      DO 70 JJ=1,4
      J2=J1+JJ
      J7=J3+JJ
      DO 71 II=1,4

```

6

```

I2=I1+I1
I7=I3+I1
J4=(J2-1)*KG+I2
J5=J4+NG
J6=J5+NG
SS=SV(I2)*SV(J2)
CC=CV(I2)*CV(J2)
A3=.5*(G(J6)+G(J4))
A4=.5*(G(J6)-G(J4))
Z(L1)=Z(L1)+(CA*T(I7)*T(J7)*(SS*A3+CC*G(J5))-C0*TP(I7)*TP(J7)*G(J5
1)))*U
Z(L2)=Z(L2)+CA*SV(J2)*TR(I7)*T(J7)*A4-FM*C0*G(J5)*TR(I7)*TP(J7)/R(
1I2)
Z(L3)=Z(L3)-CA*SV(I2)*T(I7)*TR(J7)*A4+FM*C0*G(J5)*TP(I7)*TR(J7)/R(
1J2)
Z(L4)=Z(L4)+(CA*A3-FM2*C0/R(I2)/R(J2)*G(J5))*TR(I7)*TR(J7)*U
71 CONTINUE
70 CONTINUE
31 CONTINUE
30 CONTINUE
WRITE(6)(Z(I),I=1,NZ)
88 FORMAT(1X,10G11.4)
JK(1)=1
JK(2)=NM
JK(3)=NM2*NM+1
JK(4)=JK(3)+NM
DO 93 J=1,4
K1=JK(J)
WRITE(3,24) J
24 FORMAT(1X/' Z',I1)
DO 92 I=1,NM
K2=K1+NM-1
96 WRITE(3,88)(Z(K),K=K1,K2)
K1=K1+NM2
92 CONTINUE
93 CONTINUE
GO TO 50
52 STOP
END

```

/*

```

//GO.FT06F001 DD DSN=EE0034.REV1,DISP=OLD,UNIT=2314,
// VOLUME=SER=SU0004,DCB=(RECFM=V,BLKSIZE=1800,LRECL=1796) X

```

//GO.SYSIN DD *

1 21 20 0.3141593E+00

0.0000 0.5000 1.0000 1.5000 2.0000 2.5000 3.0000 3.5000 4.0000 4.5000

5.0000 5.5000 6.0000 6.5000 7.0000 7.5000 8.0000 8.5000 9.0000 9.5000

10.0000

0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000

0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000

0.0000

/*

//

[illegible]

Year	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959	1960
Population	1,000,000	1,050,000	1,100,000	1,150,000	1,200,000	1,250,000	1,300,000	1,350,000	1,400,000	1,450,000	1,500,000
Area (sq. miles)	100	100	100	100	100	100	100	100	100	100	100
Population density (per sq. mile)	10,000	10,500	11,000	11,500	12,000	12,500	13,000	13,500	14,000	14,500	15,000

[illegible]

22

1960 2-0000 7.11.77 4.10.77 5.10.77 6.10.77 7.10.77 8.10.77 9.10.77

31.25	-317.1	25.01	834.5	35.07	799.6	32.64	107.5	29.78	53.31
26.47	28.44	22.39	15.67	19.17	5.89	15.46	6.2443		
36.73	817.6	36.77	-103.1	34.16	537.9	31.87	185.1	29.01	69.14
25.93	34.12	22.37	17.21	19.82	7.455	15.24	1.395		
35.03	402.9	34.07	430.4	32.63	-109.6	30.44	297.6	27.76	131.0
24.79	48.64	21.58	21.5	18.24	10.01	14.89	3.469		
32.71	119.1	31.73	146.3	30.37	293.5	28.48	-820.7	26.18	224.3
23.4	96.66	17.49	34.17	17.46	16.37	14.41	6.819		
29.8	55.49	23.01	67.43	27.74	132.6	26.13	221.6	24.01	-658.8
21.87	174.4	13.15	77.39	16.49	26.36	13.81	17.41		
26.48	26.52	24.79	34.71	24.79	48.79	23.38	99.34	21.79	177.5
19.64	-550.0	17.77	169.7	15.37	52.63	13.94	23.43		
22.91	14.51	22.44	17.25	21.59	23.12	20.47	36.30	19.14	77.87
17.74	149.4	15.77	-471.5	14.32	129.3	12.28	52.05		
19.14	5.831	15.03	7.478	18.25	10.54	17.46	16.41	16.49	28.43
15.36	62.00	14.31	128.3	12.49	-412.2	11.64	114.9		
15.77	23.48	15.27	1.47	14.96	3.454	14.41	6.840	13.81	12.44
13.77	24.48	12.27	52.33	11.67	114.1	10.33	-365.8		

22	37.55	43.56	35.62	37.31	31.31	34.91	27.24	24.87	21.81
-10.7.	15.39	22.22	9.853	21.02	5.029	36.83	-5.493		
-22.72	36.42	-22.51	36.33	163.6	30.74	48.25	26.47	27.42	21.20
-911.2	15.47	21.87	37.64	20.64	39.94	36.03	-5.208		
-23.11	34.71	-19.4	37.54	-18.05	29.42	91.83	25.20	35.79	20.22
-23.25	34.71	1.06	4.231	20.11	3.870	34.71	-4.748		
24.05	14.70	1.06	3.53	-157.4	27.62	32.59	23.50	63.12	18.89
-26.12	32.42	-1.9.5	8.712	19.52	3.726	33.05	-4.139		
29.43	14.87	22.34	27.73	-59.55	25.71	-88.97	21.46	-13.40	17.28
-49.53	20.63	51.52	6.777	19.46	31.33	3.567	-3.409	-54.12	15.46
47.81	12.73	25.41	26.65	-28.54	22.26	34.07	19.14		
-26.72	26.14	-27.15	7.353	21.13	3.358	30.01	-2.595		
-44.24	11.65	37.77	21.31	-14.30	19.29	-15.71	16.62	-20.06	13.48
-13.53	22.50	-13.73	21.31	29.01	3.120	30.58	-1.736		
-36.34	10.76	3.84	16.543	-5.73	16.19	-6.845	14.01	-8.577	11.43
-5.32	18.70	-5.572	7.55	1.322	2.837	38.27	-0.8873		
-12.41	05.05	-24.13	5.714	1.322	1.310	-1.864	11.39	-3.122	9.363
26.60	15.21	-	14.4	-1.9573	2.634	-	11.39		
-6.22	2.155	-	4.854	-12.52	2.634	-	11.39		

Z3

8

1930.	-37.59	498.2	-35.33	223.1	-34.64	75.53	-32.27	49.33	-29.39
26.87	-26.10	13.54	-27.55	5.281	-18.97	-1.6111E-01	-15.19		
-432.3	-35.46	230.3	-24.37	317.7	-32.64	1.13.3	-3.1.40	51.31	-27.70
27.59	-24.62	13.73	-21.23	5.547	-17.13	0.2142	-14.38		
-87.92	-31.59	-162.7	-3.13	79.84	-23.45	156.1	-27.42	59.26	-24.59
26.43	-22.24	14.25	-14.26	6.747	-16.17	0.945	-13.18		
-34.93	-27.26	-43.43	-23.49	-91.81	-25.23	33.41	-23.52	88.75	-21.46
33.92	-19.12	15.65	-14.61	6.917	-13.99	1.951	-11.37		
-24.86	-21.13	-27.41	-21.22	-35.87	-20.24	-63.11	-18.92	13.84	-17.30
54.01	-15.46	19.03	-13.43	3.546	-11.42	3.106	-9.351		
-22.71	-15.12	-23.18	-15.49	-24.65	-14.41	-29.87	-13.88	-47.87	-12.75
4.322	-11.46	34.87	-13.7	12.36	-3.652	4.983	-7.137		
-22.02	-9.873	-21.36	-9.635	-21.86	-3.248	-22.34	-8.727	-25.44	-8.091
-37.77	-7.351	0.9157E-1	-6.552	24.10	-5.717	8.437	-4.853		
-21.01	-4.047	-20.64	-3.336	-20.11	-3.886	-19.54	-3.749	-19.47	-3.579
-21.21	-3.378	-29.03	-3.143	-1.204	-2.874	18.77	-2.635		
-36.83	5.445	-36.50	5.132	-34.70	6.724	-33.15	4.116	-31.33	3.389
-30.23	2.577	-30.60	1.722	-39.24	0.8746	-24.45	1.7413E-01		

Z4

37.42	-3342.	35.09	-333.6	31.60	-65.38	27.06	-29.52	21.74	-22.40
15.94	-21.20	13.01	-2.94	4.285	-20.19	-4.763	-35.68		
35.03	-338.1	33.08	-224.4	29.91	-51.70	25.75	-23.53	20.86	-18.00
15.53	-17.73	13.07	-14.55	4.790	-17.76	-2.781	-31.94		
31.58	-65.92	29.90	-62.24	27.19	-34.13	23.64	-10.68	19.45	-9.961
14.87	-11.81	10.15	-14.26	5.579	-13.78	0.3489	-25.89		
27.05	-29.59	25.74	-23.23	23.64	-10.68	20.86	7.629	17.58	5.183
13.93	-2.828	10.24	-6.524	6.581	-4.355	4.383	-17.75		
21.74	-22.42	20.86	-14.61	19.45	-3.947	17.53	5.240	15.36	21.00
12.80	12.55	10.24	2.673	7.702	-1.465	9.005	-7.729		
15.94	-21.21	15.53	-17.73	14.87	-11.80	13.94	-2.803	12.89	12.62
11.64	27.31	16.27	17.38	8.838	7.349	13.86	4.079		
10.01	-20.95	10.07	-14.55	10.16	-13.26	10.25	-6.522	10.30	2.704
10.23	17.43	10.15	31.47	9.880	20.76	18.50	-8.19		
4.239	-21.20	4.794	-17.76	5.584	-13.78	6.580	-8.348	7.708	-1.453
8.343	7.372	3.893	2.31	10.73	32.27	22.74	37.63		
-4.757	-35.58	-2.775	-31.94	0.3568	-25.89	4.392	-17.74	9.016	-7.718
13.87	4.098	18.57	18.22	22.75	37.69	53.85	127.4		

III. EIGENCURRENTS

Program #2, which calculates the eigencurrents, accepts punched card data according to

```

READ(1,7) NN, NP
7  FORMAT(2I3)
READ(1,39)(RH(I), I = 1, NP)
READ(1,39)(ZH(I), I = 1, NP)
39  FORMAT(10F8.4)

```

The variables NN, NP, RH, and ZH have the same meaning as in program #1. The generalized impedance matrix Z is read from the first record of direct access data set 6 according to

```

REWIND 6
READ(6)(Z(I), I = 1, NZ)

```

where NZ is either $(NP-1)^2$ or $(NP-3)^2$ depending on whether or not C closes upon itself. Program #2 writes the eigencurrents on record 2 of direct access data set 6.

The R and X in DO loop 11 are the matrices [R] and [X] appearing in (2-25) of [1] computed from $[Z_n]$ of (2-47) of [1]. The matrix [X] is stored columnwise, but the matrix [R] is stored according to the symmetric mode used in the IBM System/360 Scientific Subroutine Package [4]. Statement 130 invokes the eigenvalue and eigenvector subroutine EIGEN in the Scientific Subroutine Package. The subroutine EIGEN puts the eigenvalues μ of (2-26) of [1] ordered $\mu_1 \geq \mu_2 \geq \dots$ in the diagonal positions of R. The eigenvectors of the matrix [R] will appear in U. It has been observed that the eigenvectors obtained from EIGEN are normalized so that U is orthogonal in accordance with (2-26) of [1]. DO loop 104 stores the μ_i in EU. DO loop 75 puts the matrix [XU] appearing in (2-30) of [1] in T2. DO loop 78 puts the matrix [A] = $[\dot{U} X U]$ of (2-30) of [1] in A. Upon exit from DO loop 70, JM is the dimension of the submatrix $[\mu_{11}]$ in (2-28) of [1]. DO loop 73 puts the submatrix $[A_{22}]$ appearing in (2-30) of [1] in A_{22} . Since MINV is the matrix inversion subroutine from the

Scientific Subroutine Package [4], statement 128 inverts the matrix $[A_{22}]$. DO loop 81 puts $[A_{22}]^{-1} [\tilde{A}_{12}]$ in T3. DO loop 84 puts $[A_{12}][A_{22}]^{-1}[\tilde{A}_{12}]$ in B, using the symmetric mode of storage. Statement 129 finds the eigenvalues and eigenvectors of the matrix [B] appearing in (2-36) of [1]. DO loop 107 puts the eigenvalues λ of (2-36) of [1] in AMD. DO loop 91 puts $[\mu_{11}^{-1/2}y]$ of (2-37) of [1] in T2. Upon exit from DO loop 93, the matrix

$$\begin{bmatrix} [\delta] \\ -[A_{22}^{-1} \tilde{A}_{12}] \end{bmatrix} [\mu_{11}^{-1/2}y]$$

will be in T2.

The index J of DO loop 96 indicates the J^{th} eigencurrent. DO loop 98 puts the [I] of (2-37) of [1] in FI. Because $f_i(t)$ of (2-42) of [1] is defined by (30) of [3], statements 143 and 144 have to divide the elements of [I] by ρ in order to obtain the sinusoidal components of the eigencurrents. DO loop 137 sets the largest of these sinusoidal components equal to unity. At the time FI is printed, the eigencurrent (current per unit length) at the $(2*J+1)^{\text{th}}$ data point is given by

$$\begin{aligned} \vec{u}_t FI(J) + \vec{u}_\phi FI(J+NM) & \quad NN = 0 \\ \vec{u}_t FI(J) \cos n\phi + \vec{u}_\phi FI(J+NM) \sin n\phi & \quad NN = n \neq 0 \end{aligned}$$

When $NN = n \neq 0$, the alternate eigencurrent

$$\vec{u}_t FI(J) \sin n\phi - \vec{u}_\phi FI(J + NM) \cos n\phi$$

is also possible.

If $NP > 41$, some dimension statements must be changed. Minimum allocations are given by

```
COMPLEX Z(NZ)
DIMENSION RH(NP), ZH(NP), U(NZ), R(NZ), T2(NZ),
      A22(NZ), B(NZ), X(NZ), A(NZ), Y(NZ), T3(NZ),
      F1(NZ), EU(NM2), RU(NM2), AMD(NM2), LB(NM2),
      MB(NM2)
```

where $NM2 = NP - 3$
 $NZ = NM2 * NM2$

The above allocations are based upon the value of NP after execution of statement 145

Listing of Program #2

```

//      (0034,EE,2,2),'MAUTZ,JOE',MSGLEVEL=1
// EXEC SSPCLG,PARM,FORT='MAP'
//F001.SYSIN DD *
      COMPLEX Z(1444),U1,U2,U3
      DIMENSION RH(41),ZH(41),U(1444),R(1444),T2(1444),A22(1444),B(1444)
      DIMENSION X(1444),A(1444),Y(1444),T3(1444),FI(1444),EU(38),RU(38)
      DIMENSION AMD(38),LB(38),MB(38)
      EQUIVALENCE (R(1),T2(1),A22(1),B(1)),(X(1),A(1),Y(1))
      EQUIVALENCE (T3(1),FI(1)),(EU(1),AMD(1))
      READ(1,7) NN,NP
      7 FORMAT(2I3)
      WRITE(2,3) NN,NP
      3 FORMAT('1 NN NP'/1X,2I3)
      READ(1,39)(RH(I),I=1,NP)
      READ(1,39)(ZH(I),I=1,NP)
      39 FORMAT(10F8.4)
      WRITE(3,40)(RH(I),I=1,NP)
      40 FORMAT('0RH'/(1X,10F8.4))
      WRITE(3,41)(ZH(I),I=1,NP)
      41 FORMAT('0ZH'/(1X,10F8.4))
      145 IF((RH(1)-RH(NP)).EQ.0..AND.(ZH(1)-ZH(NP)).EQ.0.) NP=NP+2
      146 NM2=NP-3
      NM=NM2/2
      NZ=NM2*NM2
      REWIND 6
      READ(6)(Z(I),I=1,N7)
      U3=(0.,1.)
      S1=.25
      IF(NN.EQ.0) S1=.5
      U2=S1*U3
      J5=0
      DO 11 J=1,NM2
      J2=(J-1)*NM2
      DO 12 I=1,J
      J5=J5+1
      J3=J2+I
      J4=(I-1)*NM2+J
      IF(J.GT.NM.AND.J.LE.NM) GO TO 28
      U1=S1*(Z(J3)+Z(J4))
      GO TO 29
      28 U1=U2*(Z(J4)-Z(J3))
      29 R(J5)=U1
      X(J3)=AIMAG(U1)
      X(J4)=X(J3)
      12 CONTINUE
      11 CONTINUE
      130 CALL EIGEN(R,U,NM2,0)
      J1=0
      DO 104 J=1,NM2
      J1=J1+J
      FU(J)=R(J1)
      RU(J)=1./SQRT(ABS(EU(J)))
      104 CONTINUE
      WRITE(3,141)(EU(J),J=1,NM2)
      141 FORMAT('0EIGENVALUES OF THE MATRIX R'/(1X,7E11.4))
      DO 75 J=1,NM2
      J1=(J-1)*NM2
      DO 76 I=1,NM2
      J2=J1+I
      T2(J2)=0.

```

```

      J3=(I-1)*NM2
      DO 77 K=1,NM2
      K1=K+J3
      K2=K+J1
      T2(J2)=T2(J2)+X(K1)*U(K2)
77 CONTINUE
76 CONTINUE
75 CONTINUE
      DO 78 J=1,NM2
      J1=(J-1)*NM2
      DO 79 I=1,J
      J2=J1+I
      A(J2)=0.
      J3=(I-1)*NM2
      DO 80 K=1,NM2
      K1=K+J3
      K2=K+J1
      A(J2)=A(J2)+U(K1)*T2(K2)
80 CONTINUE
      J4=J3+J
      A(J4)=A(J2)
79 CONTINUE
78 CONTINUE
      X2=EU(1)*1.E-03
      DO 70 J=1,NM2
      JM=J-1
      IF(EU(J).LT.X2) GO TO 72
70 CONTINUE
72 JN=NM2-JM
      JM1=JM+1
      J1=0
      DO 73 J=JM1,NM2
      J2=(J-1)*NM2
      DO 74 I=JM1,NM2
      J1=J1+1
      J3=J2+I
      A22(J1)=A(J3)
74 CONTINUE
73 CONTINUE
128 CALL MINV(A22,JN,D,LB,MB)
      J1=0
      DO 81 J=1,JM
      J3=(J-1)*NM2+JM
      DO 82 I=1,JN
      J2=(I-1)*JN
      J1=J1+1
      T3(J1)=0.
      DO 83 K=1,JN
      K1=J2+K
      K2=J3+K
      T3(J1)=T3(J1)+A22(K1)*A(K2)
83 CONTINUE
82 CONTINUE
81 CONTINUE
      J2=0
      DO 84 J=1,JM
      J3=(J-1)*NM2
      J5=(J-1)*JN
      DO 85 I=1,J
      J2=J2+1

```

```

      J4=J3+I
      R(J2)=A(J4)
      J6=(I-1)*NM2+JM
      DO 86 K=1,JM
      K1=K+J6
      K2=K+J5
      R(J2)=R(J2)-A(K1)*I3(K2)
86 CONTINUE
      R(J2)=R(J2)*RU(J)*RU(I)
85 CONTINUE
84 CONTINUE
129 CALL EIGEN(R,Y,JM,0)
      J1=0
      DO 107 J=1,JM
      J1=J1+J
      AMD(J)=R(J1)
107 CONTINUE
      WRITE(3,58)(AMD(J),J=1,JM)
58 FORMAT('EIGENVALUES OF THE MATRIX R'/(1X,5E14.7))
      DO 91 J=1,JM
      J1=(J-1)*JM
      J4=(J-1)*NM2
      DO 92 I=1,JM
      J3=I+J4
      J2=I+J1
      T2(J3)=Y(J2)*RU(I)
92 CONTINUE
91 CONTINUE
      S1=0.
      DO 93 J=1,JM
      J1=(J-1)*NM2
      DO 94 I=1,JM
      J2=J1+I+JM
      T2(J2)=0.
      DO 95 K=1,JM
      K1=(K-1)*JN+I
      K2=K+J1
      T2(J2)=T2(J2)-T3(K1)*T2(K2)
95 CONTINUE
94 CONTINUE
93 CONTINUE
      DO 96 J=1,JM
      S1=0.
      J1=(J-1)*NM2
      DO 97 I=1,NM
      J2=J1+I
      J3=J2+NM
      J4=2*I+1
      FI(J2)=0.
      FI(J3)=0.
      DO 98 K=1,NM2
      K2=K+J1
      K1=(K-1)*NM2+I
      K3=K1+NM
      FI(J2)=FI(J2)+U(K1)*T2(K2)
      FI(J3)=FI(J3)+U(K3)*T2(K2)
98 CONTINUE
143 FI(J2)=FI(J2)/RH(J4)
144 FI(J3)=FI(J3)/RH(J4)
      S2=ABS(FI(J2))

```


Output of Program #2

```

NH 00P
1 21

RH
0.0 0.500 1.000 1.500 2.000 2.500 3.000 3.500 4.000 4.500
5.000 5.500 6.000 6.500 7.000 7.500 8.000 8.500 9.000 9.500
10.000

ZH
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0

EIGENVALUES OF THE MATRIX R
0.1753E+03 0.3901E+02 0.4532E+01 0.1545E+00 0.1460E+00 0.2000E+01 0.7320E+01
0.4387E+01 0.3325E+00 0.1770E+02 0.1249E+02 0.1000E+03 0.2170E+02 0.3226E+02
-0.4590E+02 -0.6731E+01 -0.1538E+00 -0.2073E+00

EIGENVALUES OF THE MATRIX S
0.2550E+03 0.1587E+04 0.2502E+02 -0.2644E+02

LAMBDA = 0.2560E+01
JT 0.2460 0.2443 0.2234 0.1000 0.1000 0.1385 0.1000 0.0777 0.0353
J0 -0.2470 -0.2449 -0.1517 -0.1340 0.0004 0.0202 0.2390 0.1371 1.0000

LAMBDA = -0.3780E+00
JT -0.9777 -0.9503 -1.9871 -0.7394 -1.0000 -0.5803 -0.4374 -0.2293 -0.2532
J0 0.9853 1.0000 0.9710 0.3912 0.8220 0.7562 0.6065 0.6163 0.6346

LAMBDA = -0.2544E+02
JT -0.9710 -0.7251 -0.4402 -0.1293 0.2206 0.5424 0.8021 0.9287 0.8733
J0 1.0000 0.7113 0.8536 0.0030 0.5752 0.4611 0.2459 0.2514 -0.3290

```

IV. EIGENCURRENT PLOTS

Program #3 which plots the eigencurrents accepts punched card data according to

```
      READ(1,90) NM2, JM
90    FORMAT(20I3)
```

In the previous program an NM2 by NM2 generalized impedance matrix has led to JM eigencurrents. More precisely, NM2 is defined by statement 146 of the previous program and JM is the number of numbers printed under the heading "Eigenvalues of the matrix B" in the output of the previous program. The eigencurrents are read from the second record of direct access data set 6 according to

```
      REWIND 6
      READ(6)
      NZ1 = NM2 * JM
      READ(6)(FI(I), I = 1, NZ1)
```

DO loop 93 prepares the vertical coordinate for plotting and DO loop 95 prepares the horizontal coordinate. The origin is at (1.,5.). The horizontal axis corresponds to the contour length variable. If the NF data points defining the contour C are not equally spaced, this correspondence may become nonlinear because DO loop 95 always supplies equally spaced horizontal coordinates. The index J of DO loop 94 indicates the J^{th} eigencurrent. Statements 18 and 19 plot the sinusoidal components of J_t and J_ϕ respectively.

Minimum allocations for FI and X are given by

```
      DIMENSION FI(NM2 * NM2), X(NM2/2).
```

Listing of Program #3

```

//          (0034,FF,2,2,,8), 'MAUTZ,JOE',MSGLEVEL=1
// EXEC FORTGCLG,PARM,FORT='MAP'
//FORT,SYSLIB DD *
    DIMENSION FI(1444),XP(4),YP(4),AREA(400),X(19)
    CALL PLOTS(AREA,400)
    READ(1,90) NM2,JM
90 FORMAT(20I3)
    WRITE(3,91) NM2,JM
91 FORMAT('0NM2 JM'/1X,2I3)
    NZ1=NM2/JM
    REWIND 6
    READ(6)
    READ(6)(FI(I),I=1,NZ1)
    NM=NM2/2
    XP(1)=1.
    XP(2)=6.
    YP(1)=5.
    YP(2)=5.
    XP(3)=1.
    XP(4)=1.
    YP(3)=3.
    YP(4)=7.
    DO 93 I=1,NZ1
        FI(I)=5.+FI(I)
93 CONTINUE
    S1=5./(NM+1)
    DO 95 J=1,NM
        X(J)=1.+J*S1
95 CONTINUE
    DO 94 J=1,JM
        J1=(J-1)*NM2+1
        J2=J1+NM
        CALL LINE(XP(1),YP(1),2,1,0,0)
        DO 96 K=2,6
            S1=8-K
            CALL SYMBOL(S1,5.,.14,13,0.,-1)
96 CONTINUE
        CALL LINE(XP(3),YP(3),2,1,0,0)
        DO 97 K=1,5
            S1=8-K
            CALL SYMBOL(1.,S1,.14,13,0.,-1)
97 CONTINUE
        CALL NUMBER(.76,5.93,.14,1.,0.,-1)
        CALL NUMBER(.76,4.93,.14,0.,0.,-1)
        CALL NUMBER(.64,3.93,.14,-1.,0.,-1)
18 CALL LINE(X(1),FI(J1),NM,1,4,1)
19 CALL LINE(X(1),FI(J2),NM,1,0,1)
    CALL PLOT(7.,0.,-3)
99 CONTINUE
    CALL PLOT(5.,0.,-3)
    STOP
    END
/*
//GO.FT06F001 DD DSNAME=EE0034,REVL1,DISP=OLD,UNIT=2314,
//          VOLUME=SER=SU0004,DCB=(RECFM=V,BLKSIZE=1800,LRECL=1796)
//GO.SYSLIB DD *
13 3
/*
//

```

V. GAIN EIGENPATTERNS

Program #4 which calculates and plots the gain patterns of the eigencurrents accepts punched card data according to

```

      READ(1,10) NN, NP, NT, NS, JM, BK, SCL
10    FORMAT (5I3, 2E14.7)
      READ(1,11)(AMD(I), I = 1, JM)
11    FORMAT (5E14.7)
      READ(1,15)(RH(I), I = 1, NP)
      READ(1,15)(ZH(I), I = 1, NP)
15    FORMAT (10F8.4)

```

Here, NN, NP, BK, RH, and ZH are the same as in program #1 and JM is the same as in program #3. The electric field and gain will be computed at polar angles $\theta_i = \frac{(i-1)\pi}{NT-1}$, $i=1,2,\dots, NT$ but will be printed only at $i=1, NS+1, 2*NS+1, \dots$. One inch will correspond to a gain of $1./SCL$ on the plot. The variable AMD appears under the heading "Eigenvalues of the Matrix B" in the output of program #2. The JM eigencurrents are read from record 2 of direct access data set 6 according to

```

      REWIND 6
      READ(6)
      READ(6)(FI(J), J = 1, NZ1)

```

where $NZ1 = NM2 * JM$. The variable NM2 is the same as that appearing in the first three programs.

In DO loop 40, $DH(I)$, $RS(I)$, $ZS(I)$, $SV(I)$, and $CV(I)$ are respectively the distance between the I^{th} and $(I+1)^{th}$ data point, and $\rho, z, \sin v$, and $\cos v$ midway between the I^{th} and $(I+1)^{th}$ data points. Here, v is the angle between $\vec{u}_t$ and the z axis. In DO loop 41, $T(4*(J-1) + I)$, $I=1,2,3,4$, is the amplitude of $\frac{T_J(t)}{\rho}$ (the J^{th} triangle function divided by the cylindrical coordinate ρ) multiplied by $DH(2*(J-1)+I)$ and evaluated midway between the $(2*(J-1)+I)^{th}$ and $(2*(J-1)+I+1)^{th}$ data points. Now T represents the functions $f_i^t(t)$ associated with J_t and TR those functions $f_i^\phi(t)$ associated with

J_ϕ although only $f_i(t)$ appears in (2-42) of [1], no distinction being made there between $f_i^t(t)$ and $f_i^\phi(t)$. If the surface S has no edges, $f_i^t(t) = f_i^\phi(t)$, but if S has an edge at either end of C , program #4 modifies the $f_i^\phi(t)$ nearest this edge so that $f_i^\phi = \frac{2}{\rho}$ at the edge in an attempt to account for the singularity of J_ϕ at the edge.

The subroutine PLANE is concerned with the integrals

$$R_n = \oint_S \vec{W}_i \cdot \vec{u}_m e^{-j\vec{k}_m \cdot \vec{r}} ds \quad (2)$$

appearing in (2-51) of [1]. The R integrals can be written as

$$R_n = \begin{bmatrix} R_n^{t\theta} & R_n^{\phi\theta} \\ R_n^{t\phi} & R_n^{\phi\phi} \end{bmatrix} \quad (3)$$

The row submatrices on the right hand side of (3) are defined according to which $\vec{W}_i$ and which $\vec{u}_m$ are used

Matrix element	$\vec{u}_m$	$\vec{W}_i$
$R_n^{t\theta}$	$\vec{u}_\theta$	$\vec{u}_t f_i(t) \cos n\phi$
$R_n^{\phi\theta}$	$\vec{u}_\theta$	$\vec{u}_\phi f_i(t) \sin n\phi$
$R_n^{t\phi}$	$\vec{u}_\phi$	$\vec{u}_t f_i(t) \cos n\phi$
$R_n^{\phi\phi}$	$\vec{u}_\phi$	$\vec{u}_\phi f_i(t) \sin n\phi$

If the coefficients I_i appearing in (2-51) of [1] are partitioned into column vectors I^t and I^ϕ corresponding to the $\vec{u}_t$ and $\vec{u}_\phi$ directed $\vec{W}_i$, the radiation field will be given by

$$\begin{bmatrix} E_\theta \\ E_\phi \end{bmatrix} = \frac{-j\omega\mu e^{-jkr}}{4\pi r} \begin{bmatrix} R_n^{t\theta} & R_n^{\phi\theta} \\ R_n^{t\phi} & R_n^{\phi\phi} \end{bmatrix} \begin{bmatrix} [I^t] \\ [I^\phi] \end{bmatrix} \quad (4)$$

Furthermore, it will be shown that

$$\begin{bmatrix} E_\theta \\ E_\phi \end{bmatrix} = \frac{-j\omega\mu e^{-jkr}}{4\pi r} \begin{bmatrix} \cos n\phi & 0 \\ 0 & \sin n\phi \end{bmatrix} \begin{bmatrix} \hat{R}_n^{t\theta} & -j\hat{R}_n^{\phi\theta} \\ j\hat{R}_n^{t\phi} & \hat{R}_n^{\phi\phi} \end{bmatrix} \begin{bmatrix} [I^t] \\ [I^\phi] \end{bmatrix} \quad (5)$$

where the caretted submatrices $\hat{R}_n$ are given by equations (77) and (81) of reference [3]. If the current has the polarization (2-43) of [1] instead of (2-42) of [1], the matrix $\begin{bmatrix} \cos n\phi & 0 \\ 0 & \sin n\phi \end{bmatrix}$ of (5) must be replaced by $\begin{bmatrix} \sin n\phi & 0 \\ 0 & -\cos n\phi \end{bmatrix}$. The submatrices R_n use sinusoidal $\vec{W}_i$ and depend upon the measurement azimuthal angle ϕ_m while the $\hat{R}_n$ use exponential functions and are evaluated at $\phi_m = 0$. Consider the contribution to E_θ from I_1^ϕ . According to (4),

$$E_\theta = \frac{-j\omega\mu e^{-jkr}}{4\pi r} I_1^\phi \int \rho dt \int_0^{2\pi} d\phi \vec{u}_\phi(\phi) f_1(t) \sin n\phi \cdot \vec{u}_\theta(\phi_m) \psi(\phi - \phi_m) \quad (6)$$

where

$$\psi(\phi - \phi_m) = e^{-j\vec{k}_m \cdot \vec{r}} \quad (7)$$

Since the integrand of (6) is periodic in ϕ with period 2π , ϕ_m can be added to ϕ without changing the value of the integral.

$$E_\theta = \frac{-j\omega\mu e^{-jkr}}{4\pi r} I_1^\phi \int \rho f_1(t) dt \int_0^{2\pi} d\phi \vec{u}_\phi(\phi + \phi_m) \cdot \vec{u}_\theta(\phi_m) \psi(\phi) \sin n(\phi + \phi_m) \quad (8)$$

But

$$\vec{u}_\phi(\phi + \phi_m) \cdot \vec{u}_\theta(\phi_m) = \vec{u}_\phi(\phi) \cdot \vec{u}_\theta(0) \quad (9)$$

so that

$$E_\theta = \frac{-j\omega\mu e^{-jkr}}{4\pi r} I_i^\phi \int_0^{\rho} f_i(t) dt \int_0^{2\pi} d\phi \vec{u}_\phi(\phi) \sin n(\phi + \phi_m) \cdot \vec{u}_\theta(0) \psi(\phi) \quad (10)$$

Now $\vec{u}_\theta(0)$ is the measurement plane wave coming from the direction $\phi = 0$. Alternatively, (10) can be obtained directly from (6) by reasoning that (6) should depend only on the phase difference between the current $\vec{J}$ and the direction ϕ_m from which the measurement plane wave comes. For instance, one should be able to turn both $\vec{J}$ and ϕ_m back by ϕ_m to obtain (10). Equation (10) leads to

$$E_\theta = \frac{-j\omega\mu e^{-jkr}}{4\pi r} I_i^\phi \frac{\hat{R}_n^{\phi\theta} - \hat{R}_{-n}^{\phi\theta}}{2j} \cos n\phi_m + \frac{\hat{R}_n^{\phi\theta} + \hat{R}_{-n}^{\phi\theta}}{2} \sin n\phi_m \quad (11)$$

Using the fact that $\hat{R}_n^{\phi\theta}$ is odd in n ,

$$E_\theta = \frac{-j\omega\mu e^{-jkr}}{4\pi r} I_i^\phi (-j \hat{R}_n^{\phi\theta} \cos n\phi_m) \quad (12)$$

in agreement with (5). The rest of (5) can be similarly verified.

The subroutine PLANE is essentially the same as the one appearing in Appendix B of reference [2]. For the L^{th} measurement polar angle $\theta = \text{THR}(L)$ and the J^{th} function $f_J(t)$ of (2-42), PLANE stores $\hat{R}_n^{t\theta}$, $-j\hat{R}_n^{\phi\theta}$, $j\hat{R}_n^{t\phi}$, and $\hat{R}_n^{\phi\phi}$ in $\text{VVR}(L1+J)$, $\text{VVR}(L1+J+NM)$, $\text{VVR}(L1+J+NM*2)$, and $\text{VVR}(L1+J+NM*3)$ respectively where $L1 = (L-1)*NM*4$ and $NM = NM2/2$.

DO loop 92 multiplies the eigencurrents by ρ to retrieve

$$[I] = \begin{bmatrix} [I^t] \\ [I^\phi] \end{bmatrix} \quad (13)$$

appearing in (2-25) of [1]. The i^{th} elements of $[I^t]$ and $[I^\phi]$ are the coefficients of the expansion functions $\vec{u}_t f_i(t) \cos n\phi$ and $\vec{u}_\phi f_i(t) \sin n\phi$ respectively for an eigencurrent of polarization (2-42) of [1].

The index KK of DO loop 81 indicates the KK^{th} eigencurrent. In DO loop 83, K=1 obtains E_θ and K=2 obtains E_ϕ . DO loop 82 obtains the NT polar angles θ . Inner DO loop 84 performs the actual matrix multiplication indicated by (5). The gains G_θ and G_ϕ are proportional to $|E_\theta|^2$ and $|E_\phi|^2$. The average of the total gain over the area of the radiation sphere is unity.

$$\int_0^\pi (G_\theta(\theta) + G_\phi(\theta)) \sin \theta d\theta = \begin{cases} 2 & \text{NN} = 0 \\ 4 & \text{NN} \geq 1 \end{cases} \quad (14)$$

DO loop 85 stores E_θ and G_θ in positions 1 to NT of E and G and E_ϕ and G_ϕ in positions NT+1 to 2*NT of E and G. The phase of E is normalized to $-je^{-jkr}$. The magnitude of E is normalized so that $|E_\theta|^2 = G_\theta$ and $|E_\phi|^2 = G_\phi$. Statements 34 to 86 are concerned with plotting the gains of the eigencurrents.

Minimum allocations are given by

```
COMPLEX VR(NT*NM2*2), E(NT*2)
COMMON RS(NP-1), ZS(NP-1), SV(NP-1), CV(NP-1), T(NM2*2), TR(NM2*2)
DIMENSION AMD(JM), RH(NP), ZH(NP), DH(NP-1), TH(NT), G(NT*2), SN(NT*2)
          CS(NT*2), FI(NM2*NM2), GX(NT*2), GY(NT*2)
DIMENSION BJ(3*(NP-1))
```

Here, NP is the value of NP after execution of statement 96 in the main program. Also, BJ appears in PLANE.

Listing of Program #4

```

//          (0034,EE,3,2,,5), 'MAUTZ,JOE',MSGLEVEL=1
// EXEC FORTGCLG,PARM.FORT='MAP'
//FORT.SYSIN DD *
      SUBRCUTINE PLANE(VVR,THR,NT)
      COMPLEX VVR(1),A5,A6,U
      COMMON U,RS(40),ZS(40),SV(40),CV(40),BK,NP,NN,T(80),TR(80)
      DIMENSION BJ(126),THR(1),FK(20)
      KG=NP-1
      NM=KG/2-1
      M2=NN+2
      A5=2.*3.141593*U**(NN+1)
      NV=NM*4
      FK(1)=1.
      DO 153 J=1,M2
      J1=J+1
      FK(J1)=FK(J)*J
153  CONTINUE
      DO 156 L=1,NT
      L1=(L-1)*NV
      CS=COS(THR(L))
      SN=SIN(THR(L))
      RCS=BK*CS
      DO 302 J=1,KG
      X=RS(J)*BK*SN
      J1=J
      I1=NN
      IF(I1) 303,304,303
304  I1=I1+1
      J1=J1+KG
303  DO 305 JJ=I1,M2
      IF(X-1.E-5) 1,1,2
      1 IF(JJ-1) 3,3,4
      3 BJ(J1)=1.
      GO TO 306
      4 BJ(J1)=0.
      GO TO 306
      2 RH=X/2.
      RH2=RH*RH
      RH3=RH**(JJ-1)
      BJ(J1)=RH3/FK(JJ)
      SS=BJ(J1)
      8 SST=SS*1.E-7
      DO 155 K=1,20
      SS=-SS*RH2/K/(K+JJ-1)
      RJ(J1)=RJ(J1)+SS
      IF(ABS(SS)-SST) 306,306,155
155  CONTINUE
      STOP 155
306  J1=J1+KG
305  CONTINUE
302  CONTINUE
      IF(NN) 307,308,307
308  DO 309 J=1,KG
      J1=J+KG
      BJ(J)=-BJ(J1)
309  CONTINUE
307  DO 300 J=1,NM
      J1=J+L1
      J2=J1+NM
      J3=J2+NM

```

```

J4=J3+NM
VVR(J1)=0.
VVR(J2)=0.
VVR(J3)=0.
VVR(J4)=0.
DO 301 I=1,4
  I1=2*(J-1)+I
  I4=4*(J-1)+I
  I2=I1+KG
  I3=I2+KG
  A6=(COS(ZS(I1)*BCS)+U*SIN(ZS(I1)*BCS))*A5
  BJ1=(BJ(I3)+BJ(I1))*0.5
  BJ2=(BJ(I3)-BJ(I1))*0.5
  VVR(J1)=VVR(J1)+A6*(CS*SV(I1)*BJ2+SN*CV(I1)*BJ(I2)*U)*T(I4)
  VVR(J2)=VVR(J2)+A6*CS*BJ1*TR(I4)
  VVR(J3)=VVR(J3)+A6*SV(I1)*BJ1*T(I4)
  VVR(J4)=VVR(J4)+A6*BJ2*TR(I4)
301 CONTINUE
300 CONTINUE
156 CONTINUE
  RETURN
  END
  COMPLEX U,U1,VR(5548),E(146)
  COMMON U,RS(40),ZS(40),SV(40),CV(40),BK,NP,NN,T(80),TR(80)
  DIMENSION AMD(38),RH(41),ZH(41),DH(40),TH(73),XP(2),YP(2),G(146)
  DIMENSION SN(73),CS(73),FI(1444),GX(146),GY(146),AREA(400)
  CALL PLOTS(AREA,400)
  READ(1,10) NN,NP,NT,NS,JM,BK,SCL
10 FORMAT(5I3,2E14.7)
  READ(1,11)(AMD(I),I=1,JM)
11 FORMAT(5E14.7)
  READ(1,15)(RH(I),I=1,NP)
  READ(1,15)(ZH(I),I=1,NP)
15 FORMAT(10F8.4)
  WRITE(3,33)
33 FORMAT('1 NN NP NT NS JM',6X,'BK',12X,'SCL')
  WRITE(3,12) NN,NP,NT,NS,JM,BK,SCL
12 FORMAT(1X,5I3,2E14.7)
  WRITE(3,5)(AMD(I),I=1,JM)
5 FORMAT('0AMD'/(1X,5E14.7))
  WRITE(3,16)(RH(I),I=1,NP)
16 FORMAT('0RH'/(1X,10F8.4))
  WRITE(3,18)(ZH(I),I=1,NP)
18 FORMAT('0ZH'/(1X,10F8.4))
  U=(0.,1.)
  PI=3.141593
  KL=1
  IF((RH(1)-RH(NP)).NE.0..OR.(ZH(1)-ZH(NP)).NE.0.) GO TO 96
  KL=0
  RH(NP+1)=RH(2)
  ZH(NP+1)=ZH(2)
  RH(NP+2)=RH(3)
  ZH(NP+2)=ZH(3)
  NP=NP+2
96 NM2=NP-3
  NM=NM2/2
  NM4=NM*4
  NT2=NT*2
  NZ1=NM2*JM
  REWIND 6

```

```

      READ(6)
      READ(6)((FI(J),J=1,NZ1)
      DO 40 I=2,NP
      I2=I-1
      RR1=RH(I)-RH(I2)
      RR2=ZH(I)-ZH(I2)
      DH(I2)=SQRT(RR1*RR1+RR2*RR2)
      RS(I2)=.5*(RH(I)+RH(I2))
      ZS(I2)=.5*(ZH(I)+ZH(I2))
      SV(I2)=RR1/DH(I2)
      CV(I2)=RR2/DH(I2)
40  CONTINUE
      DO 41 J=1,NM
      J2=2*(J-1)+1
      J3=J2+1
      J4=J3+1
      J5=J4+1
      J6=4*(J-1)+1
      J7=J6+1
      J8=J7+1
      J9=J8+1
      DEL1=DH(J2)+DH(J3)
      DEL2=DH(J4)+DH(J5)
      T(J6)=DH(J2)*DH(J2)/2./DEL1
      T(J7)=DH(J3)*(DH(J2)+DH(J3)/2.)/DEL1
      T(J8)=DH(J4)*(DH(J5)+DH(J4)/2.)/DEL2
      T(J9)=DH(J5)*DH(J5)/2./DEL2
41  CONTINUE
      DO 97 J=1,NM4
      TR(J)=T(J)
97  CONTINUE
      IF(KL.EQ.0) GO TO 98
      IF(RH(1)) 23,24,23
23  DEL1=DH(1)+DH(2)
      TR(1)=DH(1)*(1.+(DH(2)+DH(1)/2.)/DEL1)
      TR(2)=DH(2)*(1.+(DH(2)/2.)/DEL1)
24  IF(RH(NP)) 26,98,26
26  J1=(NM-1)*4+3
      J2=J1+1
      DEL2=DH(NP-2)+DH(NP-1)
      TR(J1)=DH(NP-2)*(1.+(DH(NP-2)/2.)/DEL2)
      TR(J2)=DH(NP-1)*(1.+(DH(NP-2)+DH(NP-1)/2.)/DEL2)
98  DEL=PI/(NT-1)
      C2=SCL*4./DEL
      IF(NN.EQ.0) C2=C2*.5
      DO 43 J=1,NT
      TH(J)=(J-1)*DEL
      SN(J)=SCL*SIN(TH(J))
      CS(J)=SCL*COS(TH(J))
43  CONTINUE
      XP(1)=2.
      XP(2)=8.
      YP(1)=5.
      YP(2)=5.
      CALL PLANE(VR,TH,NT)
      C1=180./PI
      DO 31 J=1,NT
      TH(J)=TH(J)*C1
31  CONTINUE
      DO 92 J=1,JM

```

```

J1=(J-1)*NM2
DO 93 I=1,NM
J2=J1+I
J3=J2+NM
J4=2*I+1
FI(J2)=FI(J2)*RH(J4)
FI(J3)=FI(J3)*RH(J4)
93 CONTINUE
92 CONTINUE
DO 81 K=1,JM
K1=(KK-1)*NM2
S2=0.
DO 83 K=1,2
K2=(K-1)*NT
K3=(K-1)*NM2
DO 82 J=1,NT
J1=(J-1)*NM4+K3
U1=0.
DO 84 I=1,NM2
J3=I+J1
J4=I+K1
U1=U1+VR(J3)*FI(J4)
84 CONTINUE
J5=J+K2
E(J5)=U1
S1=CABS(U1)
G(J5)=S1*S1
S2=S2+G(J5)*SN(J)
82 CONTINUE
83 CONTINUE
S3=SQRT(C2/S2)
DO 85 J=1,NT2
E(J)=E(J)*S3
S2=CABS(E(J))
G(J)=S2*S2
85 CONTINUE
34 CALL LINE(XP,YP,2,1,0,0)
DO 77 J=1,7
S1=9-J
CALL SYMBOL(S1,5.,.14,13,0.,-1)
77 CONTINUE
CALL LINE(YP,XP,2,1,0,0)
DO 78 J=1,7
S1=9-J
CALL SYMBOL(5.,S1,.14,13,90.,-1)
78 CONTINUE
DO 86 K=1,2
K2=(K-1)*NT
DO 87 J=1,NT
J1=J+K2
S1=G(J1)*SN(J)
GX(J)=5.+S1
GY(J)=5.+G(J1)*CS(J)
J2=NT2-J+1
GX(J2)=5.-S1
GY(J2)=GY(J)
87 CONTINUE
CALL LINE(GX,GY,NT2,1,0,0)
86 CONTINUE
WRITE(3,27) AMD(KK)

```


NN NP NT NS JM BK SCL
1 21 73 4 3 0.3141593E 00 .5000000E 00

AMD

0.2559665E 01-0.4785684E-02-0.26+4527E 02

RH

0.0 0.5000 1.0000 1.5000 2.0000 2.5000 3.0000 3.5000 4.0000 4.5000
5.0000 5.5000 6.0000 6.5000 7.0000 7.5000 8.0000 8.5000 9.0000 9.5000
10.0000

ZH

0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0

ELECTRIC FIELD AND GAIN OF EIGENCURRENT FOR LAMBDA = 0.2559665E 01

E	REAL(E0)	IMAG(E0)	GAIN0	REAL(E1)	IMAG(E1)	GAIN1
0.0	-0.1304E 01	0.0	0.1699E 01	0.1304E 01	0.0	0.1699E 01
10.0	-0.1243E 01	0.0	0.1546E 01	0.1134E 01	0.0	0.1287E 01
20.0	-0.1080E 01	0.0	0.1165E 01	0.6803E 00	0.0	0.4628E 00
30.0	-0.8558E 00	0.0	0.7324E 00	0.7477E-01	0.0	0.5591E-02
40.0	-0.6218E 00	0.0	0.3867E 00	-0.5331E 00	0.0	0.2842E 00
50.0	-0.4158E 00	0.0	0.1729E 00	-0.1036E 01	0.0	0.1739E 01
60.0	-0.2558E 00	0.0	0.6542E-01	-0.1391E 01	0.0	0.1935E 01
70.0	-0.1416E 00	0.0	0.2005E-01	-0.1607E 01	0.0	0.2582E 01
80.0	-0.6132E-01	0.0	0.3822E-02	-0.1716E 01	0.0	0.2944E 01
90.0	-0.1058E-06	0.0	0.1119E-13	-0.1748E 01	0.0	0.3757E 01
100.0	0.6182E-01	0.0	0.3822E-02	-0.1716E 01	0.0	0.2944E 01
110.0	0.1416E 00	0.0	0.2005E-01	-0.1607E 01	0.0	0.2582E 01
120.0	0.2558E 00	0.0	0.6542E-01	-0.1391E 01	0.0	0.1935E 01
130.0	0.4158E 00	0.0	0.1729E 00	-0.1036E 01	0.0	0.1739E 01
140.0	0.6218E 00	0.0	0.3867E 00	-0.5331E 00	0.0	0.2842E 00
150.0	0.8558E 00	0.0	0.7324E 00	0.7477E-01	0.0	0.5591E-02
160.0	0.1080E 01	0.0	0.1165E 01	0.6803E 00	0.0	0.4628E 00
170.0	0.1243E 01	0.0	0.1546E 01	0.1134E 01	0.0	0.1287E 01
180.0	0.1304E 01	0.0	0.1699E 01	0.1304E 01	0.0	0.1699E 01

ELECTRIC FIELD AND GAIN OF EIGENCURRENT FOR LAMBDA = 0.4785684E-02

E	REAL(E0)	IMAG(E0)	GAIN0	REAL(E1)	IMAG(E1)	GAIN1
0.0	-0.2266E 01	0.0	0.5136E 01	0.2266E 01	0.0	0.5136E 01
10.0	-0.2176E 01	0.0	0.4736E 01	0.2180E 01	0.0	0.4752E 01
20.0	-0.1929E 01	0.0	0.3722E 01	0.1943E 01	0.0	0.3776E 01
30.0	-0.1584E 01	0.0	0.2510E 01	0.1614E 01	0.0	0.2607E 01
40.0	-0.1210E 01	0.0	0.1465E 01	0.1263E 01	0.0	0.1595E 01
50.0	-0.8632E 00	0.0	0.7452E 00	0.9445E 00	0.0	0.8921E 00
60.0	-0.5722E 00	0.0	0.3275E 00	0.6923E 00	0.0	0.4793E 00
70.0	-0.3416E 00	0.0	0.1167E 00	0.5167E 00	0.0	0.2670E 00
80.0	-0.1592E 00	0.0	0.2501E-01	0.4154E 00	0.0	0.1725E 00
90.0	-0.2768E-06	0.0	0.7661E-13	0.3825E 00	0.0	0.1463E 00
100.0	0.1582E 00	0.0	0.2501E-01	0.4154E 00	0.0	0.1725E 00
110.0	0.3416E 00	0.0	0.1167E 00	0.5167E 00	0.0	0.2670E 00
120.0	0.5722E 00	0.0	0.3275E 00	0.6923E 00	0.0	0.4793E 00
130.0	0.8632E 00	0.0	0.7452E 00	0.9445E 00	0.0	0.8921E 00
140.0	0.1210E 01	0.0	0.1465E 01	0.1263E 01	0.0	0.1595E 01
150.0	0.1584E 01	0.0	0.2510E 01	0.1614E 01	0.0	0.2607E 01
160.0	0.1929E 01	0.0	0.3722E 01	0.1943E 01	0.0	0.3776E 01
170.0	0.2176E 01	0.0	0.4736E 01	0.2180E 01	0.0	0.4752E 01
180.0	0.2266E 01	0.0	0.5136E 01	0.2266E 01	0.0	0.5136E 01

ELECTRIC FIELD AND GAIN OF EIGENCURRENT FOR LAMBDA = 0.2645F P2

P	REAL(E0)	IMAG(E0)	GAINP	REAL(E7)	IMAG(E7)	GAINP
0.0	0.1836E 01	0.0	0.3371E 01	-0.1836E 01	0.0	0.3371E 01
10.0	0.1503E 01	0.0	0.2260E 01	-0.1723E 01	0.0	0.2969E 01
20.0	0.6433E 00	0.0	0.4139E 00	-0.1419E 01	0.0	0.2014E 01
30.0	-0.4076E 00	0.0	0.1661E 00	-0.1011E 01	0.0	0.1021E 01
40.0	-0.1289E 01	0.0	0.1661E 01	-0.5953E 00	0.0	0.3544E 00
50.0	-0.1766E 01	0.0	0.3120E 01	-0.2446E 00	0.0	0.5984E 01
60.0	-0.1780E 01	0.0	0.3169E 01	0.1034E -01	0.0	0.1059E 01
70.0	-0.1403E 01	0.0	0.1968E 01	0.1716E 00	0.0	0.2945E 01
80.0	-0.7642E 00	0.0	0.5840E 00	0.2568E 00	0.0	0.6593E 01
90.0	-0.1412E -05	0.0	0.1992E -11	0.2829E 00	0.0	0.8005E 01
100.0	0.7642E 00	0.0	0.5840E 00	0.2568E 00	0.0	0.6593E 01
110.0	0.1403E 01	0.0	0.1968E 01	0.1716E 00	0.0	0.2945E 01
120.0	0.1780E 01	0.0	0.3169E 01	0.1034E -01	0.0	0.1059E 01
130.0	0.1766E 01	0.0	0.3120E 01	-0.2446E 00	0.0	0.5984E 01
140.0	0.1289E 01	0.0	0.1661E 01	-0.5953E 00	0.0	0.3544E 00
150.0	0.4076E 00	0.0	0.1661E 00	-0.1011E 01	0.0	0.1021E 01
160.0	-0.6433E 00	0.0	0.4139E 00	-0.1419E 01	0.0	0.2014E 01
170.0	-0.1503E 01	0.0	0.2260E 01	-0.1723E 01	0.0	0.2969E 01
180.0	-0.1836E 01	0.0	0.3371E 01	-0.1836E 01	0.0	0.3371E 01

VI. SCATTERING CROSS SECTIONS

Program #5 which calculates and plots σ/λ^2 for an axially incident plane wave accepts punched card data according to

```

      READ (1,10) NP, NT, NS, JM, BK
10   FORMAT (4I3, E14.7)
      READ (1,11)(AMD(I), I = 1, JM)
11   FORMAT (5E14.7)
      READ (1,15) (RH(I), I = 1, NP)
      READ (1,15)(ZH(I), I = 1, NP)
15   FORMAT (10F8.4)
      READ (1,50)(L(I), I = 1, JM)
50   FORMAT (20I3)

```

The variables NP, NT, NS, JM, BK, RH and ZH are the same as those in program #4. The variable AMD appears under the heading "eigenvalues of the matrix B" in the output of program #2. The $L(I)^{th}$ eigencurrent is the I^{th} eigencurrent to be considered in the modal expansion of the scattered field. The variable $L(I)$ is necessary because it is desirable to perform the modal expansion by adding eigencurrents in order of increasing $|\lambda|$. (Program #2 has ordered the eigencurrents in order of increasing λ .) For instance, if λ_6 corresponds to the smallest $|\lambda|$ and λ_5 corresponds to the next smallest $|\lambda|$, then $L(1) = 6$ and $L(2) = 5$. The impedance matrix and eigencurrents are read from records 1 and 2 of direct access data set 6 according to

```

      REWIND 6
      READ(6)(Z(I), I = 1, NZ)
      READ(6)(FI(I), I = 1, NZ1)

```

where

```

      NZ = NM2*NM2
      NZ1 = NM2*JM

```

and NM2 is either NP-1 or NP-3 depending on whether or not the generating curve C closes upon itself.

The subroutine PLANE is the same as in program #4. Much of the logic before statement 7 in the main program is devoted to preparing the input data for plane and is thus the same as in program #4.

DO loop 80 modifies the impedance matrix according to (2-47) of [1]. DO loop 19 obtains the matrix X appearing in (2-15) of [1]. Statement 6 inverts the impedance matrix to obtain the admittance matrix Y.

DO loop 85 calculates the scattered field by inserting

$$\begin{bmatrix} I^t \\ I^\phi \end{bmatrix} = [I] = [Y] \begin{bmatrix} R_1^{t\theta} & -jR_1^{\phi\theta} \end{bmatrix} \quad (15)$$

into (5) for $n=1$. Equation (15) is possible because plane wave excitation and measurement coefficients have the same form. Notice that (1-48) of [1] is written using the eigencurrents as a basis while (15) is written with the expansion functions (2-42) in mind. The excitation of a $\vec{u}_\theta$ polarized plane wave incident in the plane $\phi = 0$ gives rise to expansion functions (2-42) of [1] while the excitation of a $\vec{u}_\phi$ polarized plane wave in the plane $\phi = 0$ gives rise to expansion functions (2-43) of [1]. In DO loop 85, K5=1 obtains plane wave excitation from the direction ($\theta=\pi, \phi=0$) and K5=2 that from the direction ($\theta=0, \phi=0$). DO loop 82 stores the column matrix [I] of (15) in E3. DO loop 103 stores E_θ , E_ϕ , $(\sigma/\lambda^2)_\theta$, and $(\sigma/\lambda^2)_\phi$ in E(J), E(J+NT), SIG(J), and SIG(J+NT) where J indicates the Jth measurement polar angle θ . Here, E_θ is the θ component of the far zone scattered field in the plane $\phi = 0$ and E_ϕ is the ϕ component of the scattered field in the plane $\phi = \frac{\pi}{2}$. The phase of the scattered field is normalized to $-je^{-jkr}$. The magnitudes $|E_\theta|$ and $|E_\phi|$ are such that

$$(\sigma/\lambda^2)_\theta = |E_\theta|^2 \quad (16)$$

$$(\sigma/\lambda^2)_\phi = |E_\phi|^2$$

The constant C1 used to normalize the scattered field is given by

$$C1 = \left(\frac{\omega^2 \mu^2}{4\pi \lambda^2} \right)^{1/2} \quad (17)$$

The factor $\omega^2 \mu^2 / (4\pi \lambda^2)$ appears in (1-51) of [1]. The logic between statements 119 and 109 finds SCL(K5) so that the maximum value of SIG(J)*SCL(K5) for $J = 1, 2, \dots, 2*NT$ is always between 1.2 and 3.0. DO loop 106 prepares horizontal and vertical coordinates E5 and E6 suitable for plotting σ/λ^2 .

DO loop 48 multiplies the eigencurrents by ρ to retrieve the coefficients [I] of the expansion functions (2-42) of [1]. The matrix [I] is stored in FI with subscripts (J-1)*NM2+1 to J*NM2 denoting the J^{th} eigencurrent. The admittance matrix [Y] appearing in (15) will be expressed in terms of [I] whose columns are the expansion coefficients of the eigencurrents. For the moment, assume that there are NM2 eigencurrents so that [I] is a square matrix. Equation (2-18) of [1] can be multiplied by $[\tilde{I}]$ to yield

$$[\tilde{I}][Z][I] = [\tilde{I}][X][I] (j + 1/\lambda) \quad (18)$$

Because of the orthogonality relationships (2-24), of [1], the right side of (18) is a diagonal matrix. Equation (18) can be inverted to obtain

$$[\tilde{I}]^{-1}[Y][I]^{-1} = \frac{1}{[\tilde{I}][X][I] (j + 1/\lambda)} \quad (19)$$

which leads to

$$[Y] = [I] \frac{1}{[\tilde{I}][X][I] (j + 1/\lambda)} [\tilde{I}] \quad (20)$$

Equation (15) is introduced into (5) with the result

$$\begin{bmatrix} E_\theta \\ E_\phi \end{bmatrix} = \frac{-j\omega\mu e^{-jkr}}{4\pi r} \begin{bmatrix} \cos \phi & 0 \\ 0 & \sin \phi \end{bmatrix} \begin{bmatrix} \hat{R}_1^{t\theta} & -j\hat{R}_1^{\phi\theta} \\ j\hat{R}_1^{t\phi} & \hat{R}_1^{\phi\phi} \end{bmatrix} [Y] \begin{bmatrix} \hat{R}_1^{t\theta} & -j\hat{R}_1^{\phi\theta} \end{bmatrix} \quad (21)$$

where $[Y]$ is given by (20). Equation (21) is a specialization of (1-37) of [1]. Alternatively, (21) could have been obtained by finding the matrix

$$\begin{bmatrix} [I^t] \\ [I^v] \end{bmatrix} \text{ associated with } \vec{J} \text{ of (1-30) of [1] and inserting this matrix into (5).}$$

Evidently, the use of $JM < NM^2$ terms of the sum (1-37) of [1] is equivalent to using only JM columns of $[I]$ in (20). DO loop 21 stores the diagonal matrix

$$\frac{1}{[\tilde{I}][X][I] (j + 1/\lambda)} \text{ appearing in (20) in E3. The inner DO loop 27 stores}$$

$$[I] \frac{1}{[\tilde{I}][X][I] (j + 1/\lambda)} \text{ in T3.}$$

The index K of DO loop 29 indicates that K columns of $[I]$ are being used. DO loop 30 adds the contribution of the $L(K)^{th}$ column of $[I]$ to the admittance matrix $[Y]$. In DO loop 65, $K4 = 1$ obtains the plane wave incident from $(\theta = \pi, \phi = 0)$ and $K4 = 2$ that from $(\theta = 0, \phi = 0)$. Inner DO loop 45 stores $\{Y\}[\hat{R}_1^{t\theta} \quad -j\hat{R}_1^{t\phi}]$ in E3. In DO loop 66, $K5 = 1$ obtains E_θ and $K5 = 2$ obtains E_ϕ . DO loop 44 calculates E_θ , E_ϕ , $(\sigma/\lambda^2)_\theta$, and $(\sigma/\lambda^2)_\phi$ and stores them in E and SIG. The quantity σ/λ^2 computed using the modal approximation to $[Y]$ is plotted as symbols (X for θ polarization and $\square$ for ϕ polarization) and the quantity σ/λ^2 computed using $[Y]$ obtained by inverting the impedance matrix is plotted as a solid curve in the latter part of DO loop 66. The logic following statement 66 is devoted to drawing the axes for the plot and printing the quantities E_θ , E_ϕ , $(\sigma/\lambda^2)_\theta$ and $(\sigma/\lambda^2)_\phi$. The scattered field components E_θ , E_ϕ that are printed lack the phase factor $-je^{-jkr}$ and are normalized according to (16).

Minimum allocations are given by

```
COMPLEX Z(NM2*NM2), Y(NM2*NM2), VR(2*NT*NM2), E3(NM2), E(NT*2), T3(NM2*JM)
DIMENSION AND(JM), RH(NP), ZH(NP), FI(NM2*JM),
          DH(NP-1), TH(NT), X(NM2*NM2), SIG(NT*2),
          L(JM), SN(NT), CS(NT), E9(NT), E10(NT),
          E5(NT*8), E6(NT*8)
```

```
COMMON RS(NP-1), ZS(NP-1), SV(NP-1), T(NM2*2), TR(NM2*2)
DIMENSION BJ(3* (NP-1))
DIMENSION LR(NM2)
```

Here, NP is its value after execution of statement 90 in the main program.
Note that BJ appears in PLANE and LR in LINEQ.

36 Listing of Program #5

```
//      (0034,EE,4,2,,7),'MAUTZ,JDE',MSGLEVEL=1
// EXEC FORTGCLG,PARM.FORT='MAP'
//FORT.SYSIN DD *
      SUBROUTINE PLANE(VVR,THR,NT)
      COMPLEX VVR(1),A5,A6,U
      COMMON U,RS(40),ZS(40),SV(40),CV(40),BK,NP,NN,T(80),TR(80)
      DIMENSION BJ(126),THR(1),FK(20)
      KG=NP-1
      NM=KG/2-1
      M2=NM+2
      A5=2.*3.141593*(1)*(NN+1)
      NV=NM*4
      FK(1)=1.
      DO 153 J=1,M2
      J1=J+1
      FK(J1)=FK(J)*J
153 CONTINUE
      DO 156 L=1,NT
      L1=(L-1)*NV
      CS=COS(THR(L))
      SN=SIN(THR(L))
      RCS=BK*CS
      DO 302 J=1,KG
      X=RS(J)*BK*SN
      J1=J
      I1=NN
      IF(I1) 303,304,303
304 I1=I1+1
      J1=J1+KG
303 DO 305 JJ=I1,M2
      IF(X-1.E-5) 1,1,2
      1 IF(JJ-1) 3,3,4
      3 BJ(J1)=1.
      GO TO 306
      4 BJ(J1)=0.
      GO TO 306
      2 RH=X/2.
      RH2=RH*RH
      RH3=RH*(JJ-1)
      BJ(J1)=RH3/FK(JJ)
      SS=BJ(J1)
      8 SST=SS*1.E-7
      DO 155 K=1,20
      SS=-SS*RH2/K/(K+JJ-1)
      BJ(J1)=BJ(J1)+SS
      IF(ABS(SS)-SST) 306,306,155
155 CONTINUE
      STOP 155
306 J1=J1+KG
305 CONTINUE
302 CONTINUE
      IF(NN) 307,308,307
308 DO 309 J=1,KG
      J1=J+2*KG
      BJ(J)=-BJ(J1)
309 CONTINUE
307 DO 300 J=1,NM
      J1=J+L1
      J2=J1+NM
      J3=J2+NM
```

```

J4=J3+NM
VVR(J1)=0.
VVR(J2)=0.
VVR(J3)=0.
VVR(J4)=0.
DO 301 I=1,4
I1=2*(J-1)+I
I4=4*(J-1)+I
I2=I1+KG
I3=I2+KG
A6=(COS(ZS(I1)*BCS)+U*SIN(ZS(I1)*BCS))*A5
BJ1=(BJ(I3)+BJ(I1))*0.5
BJ2=(BJ(I3)-BJ(I1))*0.5
VVR(J1)=VVR(J1)+A6*(CS*SV(I1)*BJ2+SN*CV(I1)*BJ(I2)*U)*T(I4)
VVR(J2)=VVR(J2)+A6*CS*BJ1*TR(I4)
VVR(J3)=VVR(J3)+A6*SV(I1)*BJ1*T(I4)
VVR(J4)=VVR(J4)+A6*BJ2*TR(I4)
301 CONTINUE
300 CONTINUE
156 CONTINUE
RETURN
END
SUBROUTINE LINEQ(LL,C)
COMPLEX C(1),STOR,STO,ST,S
DIMENSION LR(58)
DO 20 I=1,LL
LR(I)=I
20 CONTINUE
M1=0
DO 18 M=1,LL
K=M
DO 2 I=M,LL
K1=M1+I
K2=M1+K
IF(CABS(C(K1))-CABS(C(K2))) 2,2,6
6 K=I
2 CONTINUE
LS=LR(M)
LR(M)=LR(K)
LR(K)=LS
K2=M1+K
STOR=C(K2)
J1=0
DO 7 J=1,LL
K1=J1+K
K2=J1+M
STO=C(K1)
C(K1)=C(K2)
C(K2)=STO/STOR
J1=J1+LL
7 CONTINUE
K1=M1+M
C(K1)=1./STOR
DO 11 I=1,LL
IF(I-M) 12,11,12
12 K1=M1+I
ST=C(K1)
C(K1)=0.
J1=0
DO 10 J=1,LL

```

```

      K1=J1+I
      K2=J1+M
      C(K1)=C(K1)-C(K2)*ST
      J1=J1+LL
10  CONTINUE
11  CONTINUE
      M1=M1+LL
18  CONTINUE
      J1=0
      DO 9 J=1,LL
        IF(J-LR(J)) 14,8,14
14  LRJ=LR(J)
      J2=(LRJ-1)*LL
21  DO 13 I=1,LL
      K2=J2+I
      K1=J1+I
      S=C(K2)
      C(K2)=C(K1)
      C(K1)=S
13  CONTINUE
      LR(J)=LR(LRJ)
      LR(LRJ)=LRJ
      IF(J-LR(J)) 14,8,14
      J1=J1+LL
      9  CONTINUE
      RETURN
      END
      COMPLEX U,U1,Z(1444),Y(1444),VR(5548),E3(38),E(146),T3(1444)
      DIMENSION AMD(38),RH(41),ZH(41),FI(1444),DH(40),TH(73),X(1444)
      DIMENSION SIG(146),L(38),AREA(400),SN(73),CS(73),E9(73),E10(73)
      DIMENSION XP(2),YP(2),E5(584),E6(584),SCL(2)
      EQUIVALENCE (Z(1),Y(1))
      COMMON U,RS(40),ZS(40),SV(40),CV(40),BK,NP,NN,T(80),TR(80)
      CALL PLOTS(AREA,400)
      READ(1,10) NP,NT,NS,JM,BK
10  FORMAT(4I3,E14.7)
      READ(1,11)(AMD(I),I=1,JM)
11  FORMAT(5E14.7)
      READ(1,15)(RH(I),I=1,NP)
      READ(1,15)(ZH(I),I=1,NP)
15  FORMAT(10F8.4)
      READ(1,50)(L(I),I=1,JM)
50  FORMAT(20I3)
      WRITE(3,9)
      9  FORMAT('1 NP NT NS JM      BK')
      WRITE(3,12) NP,NT,NS,JM,BK
12  FORMAT(1X,4I3,E14.7)
      WRITE(3,13)(AMD(I),I=1,JM)
13  FORMAT('0AMD'/(1X,5E14.7))
      WRITE(3,16)(RH(I),I=1,NP)
16  FORMAT('0RH'/(1X,10F8.4))
      WRITE(3,127)(ZH(I),I=1,NP)
127 FORMAT('0ZH'/(1X,10F8.4))
      WRITE(3,51)(L(I),I=1,JM)
51  FORMAT('0L'/(1X,20I3))
      PI=3.141593
      ETA=376.730
      U=(0.,1.)
      C1=BK*BK*E1A/(4.*SORT(PI**3))
      KL=1

```

```

IF((RH(1)-RH(NP)).NE.0..OR.(ZH(1)-ZH(NP)).NE.0.) GO TO 90
KL=0
RH(NP+1)=RH(2)
ZH(NP+1)=ZH(2)
RH(NP+2)=RH(3)
ZH(NP+2)=ZH(3)
NP=NP+2
90 NM2=NP-3
NZ=NM2*NM2
NZ1=NM2*JM
RFWIND 6
READ(6)(Z(I),I=1,NZ)
READ(6)(FI(I),I=1,NZ1)
NM=NM2/2
NM4=NM*4
NT2=NT*2
NT3=NT-NS
NT4=NS+1
DO 40 I=2,NP
I2=I-1
RR1=RH(I)-RH(I2)
RR2=ZH(I)-ZH(I2)
DH(I2)=SQRT(RR1*RR1+RR2*RR2)
RS(I2)=.5*(RH(I)+RH(I2))
ZS(I2)=.5*(ZH(I)+ZH(I2))
SV(I2)=RR1/DH(I2)
CV(I2)=RR2/DH(I2)
40 CONTINUE
DO 41 J=1,NM
J2=2*(J-1)+1
J3=J2+1
J4=J3+1
J5=J4+1
J6=4*(J-1)+1
J7=J6+1
J8=J7+1
J9=J8+1
DEL1=DH(J2)+DH(J3)
DEL2=DH(J4)+DH(J5)
T(J6)=DH(J2)*DH(J2)/2./DEL1
T(J7)=DH(J3)*(DH(J2)+DH(J3)/2.)/DEL1
T(J8)=DH(J4)*(DH(J5)+DH(J4)/2.)/DEL2
T(J9)=DH(J5)*DH(J5)/2./DEL2
41 CONTINUE
DO 91 J=1,NM4
TR(J)=T(J)
91 CONTINUE
IF(KL.EQ.0) GO TO 95
IF(RH(1))93,94,93
93 DEL1=DH(1)+DH(2)
TR(1)=DH(1)*(1.+(DH(2)+DH(1)/2.)/DEL1)
TR(2)=DH(2)*(1.+(DH(2)/2.)/DEL1)
94 IF(RH(NP))96,95,96
96 J1=(NM-1)*4+3
J2=J1+1
DEL2=DH(NP-2)+DH(NP-1)
TR(J1)=DH(NP-2)*(1.+(DH(NP-2)/2.)/DEL2)
TR(J2)=DH(NP-1)*(1.+(DH(NP-2)+DH(NP-1)/2.)/DEL2)
95 DEL=PI/(NT-1)
DO 43 J=1,NT

```

49

```

      TH(J)=(J-1)*DEL
      SN(J)=SIN(TH(J))
      CS(J)=COS(TH(J))
44 CONTINUE
      NN=1
      7 CALL PLANE(VR,TH,NT)
      DEL1=180./PI
      DO 128 J=1,NT
      TH(J)=TH(J)*DEL1
128 CONTINUE
      XP(1)=2.
      XP(2)=8.
      YP(1)=5.
      YP(2)=5.
      U1=.5*U
      DO 80 J=1,NM
      J1=(J-1)*NM2+NM
      J4=(J+NM-1)*NM2
      DO 81 I=1,NM
      J2=J1+I
      J3=J4+I
      Z(J2)=U1*Z(J2)
      Z(J3)=-U1*Z(J3)
      J5=J2-NM
      J6=J3+NM
      Z(J5)=.5*Z(J5)
      Z(J6)=.5*Z(J6)
81 CONTINUE
80 CONTINUE
      DO 19 J=1,NM2
      J2=(J-1)*NM2
      DO 20 I=1,J
      J3=J2+I
      J4=(I-1)*NM2+J
      U1=.5*(Z(J3)+Z(J4))
      X(J3)=AIMAG(U1)
      X(J4)=X(J3)
20 CONTINUE
19 CONTINUE
      6 CALL LINEQ(NM2,Z)
      DO 85 K5=1,2
      J5=(2-K5)*(NT-1)*NM4
      DO 82 I=1,NM2
      E3(I)=0.
      DO 83 KK=1,NM2
      J3=I+(KK-1)*NM2
      J4=J5+KK
      E3(I)=E3(I)+Z(J3)*VR(J4)
83 CONTINUE
82 CONTINUE
      S2=0.
      DO 103 K6=1,2
      J5=(K6-1)*NM2
      J8=(K6-1)*NT
      DO 84 J=1,NT
      J7=J+J8
      U1=0.
      J1=(J-1)*NM4+J5
      DO 88 I=1,NM2
      J2=J1+I

```

```

      U1=U1+VR(J2)*E3(I)
88 CONTINUE
      E(J7)=C1*U1
      S1=CABS(E(J7))
      SIG(J7)=S1*S1
      IF(SIG(J7).GT.S2) S2=SIG(J7)
84 CONTINUE
103 CONTINUE
      WRITE(3,5)
      5 FORMAT('SCATTERED FIELD AND SCATTERING CROSS SECTION/WAVELENGTH S
      QUARED')
      S1=(2-K5)*180.
      WRITE(3,124) S1
124 FORMAT(' BY MATRIX INVERSION, INCIDENCE FROM 0=',F4.0,' 0=0')
      2 WRITE(3,118)
118 FORMAT('+',36X,'-',7X,'/')
      WRITE(3,39)
      39 FORMAT('0 0 REAL(E0) IMAG(E0) SO/(LAM)**2 REAL(F0)
      1IMAG(E0) SO/(LAM)**2')
      WRITE(3,117)
117 FORMAT('+ -',11X,'-',11X,'- -',18X,'/',11X,'/' '/')
      DO 37 J=1,NT,NS
      J1=J+NT
      WRITE(3,38) TH(J),E(J),SIG(J),E(J1),SIG(J1)
      38 FORMAT(1X,F6.1,6E12.4)
      37 CONTINUE
119 J1=10+ALOG10(S2)
      S3=.1**(J1-10)
      S4=S2*S3
      IF(S4-1.5) 110,110,111
110 SCL(K5)=2.*S3
      GO TO 112
111 IF(S4-3.) 113,113,114
113 SCL(K5)=S3
      GO TO 112
114 IF(S4-6.) 115,115,116
115 SCL(K5)=.5*S3
      GO TO 112
116 SCL(K5)=.2*S3
112 S5=1./SCL(K5)
      WRITE(3,109) S5
109 FORMAT('ONE INCH CORRESPONDS TO SIGMA/(LAMBDA)**2=',E11.4)
      DO 106 K6=1,2
      J1=(K6-1)*NT
      J3=NT2*((K5-1)*2+K6-1)
      DO 107 J=1,NT
      J7=J1+J
      J8=J+J3
      J9=J3+NT2-J+1
      S2=SIG(J7)*SCL(K5)
      S1=SN(J)*S2
      E5(J8)=5.+S1
      E5(J9)=5.-S1
      E6(J8)=5.+CS(J)*S2
      E6(J9)=E6(J8)
107 CONTINUE
106 CONTINUE
      85 CONTINUE
      DO 48 J=1,JM
      J1=(J-1)*NM2

```

```

      DO 18 I=1,NM
        J2=J1+I
        J4=J2+NM
        J4=2*I+1
        F1(J2)=F1(J2)*RH(J4)
        F1(J3)=F1(J3)*RH(J4)
18    CONTINUE
48    CONTINUE
      DO 21 J=1,JM
        J1=(J-1)*NM2
        S1=0.
        DO 22 I=1,NM2
          S2=0.
          J4=(I-1)*NM2
          DO 23 K=1,NM2
            J3=J1+K
            J2=J4+K
            S2=S2+X(J2)*F1(J3)
23    CONTINUE
          J2=J1+I
          S1=S1+S2/F1(J2)
22    CONTINUE
        E3(J)=1./S1/(1+1./AMD(J))
        DO 27 I=1,NM2
          J2=J1+I
          T3(J2)=F1(J2)*E3(J)
27    CONTINUE
21    CONTINUE
      DO 32 J=1,N7
        Y(J)=0.
32    CONTINUE
      DO 29 K=1,JM
52    J3=(L(K)-1)*NM2
        DO 30 J=1,NM2
          J1=(J-1)*NM2
          J5=J3+J
          DO 31 I=1,J
            J2=J1+I
            J4=J3+I
            Y(J2)=Y(J2)+F1(J5)*T3(J4)
            J6=(I-1)*NM2+J
            Y(J6)=Y(J2)
31    CONTINUE
30    CONTINUE
        DO 65 K4=1,2
          J8=(2-K4)*(NT-1)*NM4
          DO 45 I=1,NM2
            F3(I)=0.
            DO 46 KK=1,NM2
              J3=I+(KK-1)*NM2
              J4=J8+KK
              F3(I)=E3(I)+Y(J3)*VR(J4)
46    CONTINUE
45    CONTINUE
          DO 66 K5=1,2
            K3=(K4-1)*2+K5-1)*NT2
            J7=(K5-1)*NT
            K2=(2-K5)*4
            J5=(K5-1)*NM2
            DO 44 J1=1,NT*NS

```


Output of Program #5

NP NT NS JM RK
 21 73 4 3 0.3141593E 01

AMU

0.2559665E 01-0.8785644E-02-0.2644527E 02

RH

0.0 0.5000 1.0000 1.5000 2.0000 2.5000 3.0000 3.5000 4.0000 4.5000
 5.0000 5.5000 6.0000 6.5000 7.0000 7.5000 8.0000 8.5000 9.0000 9.5000
 10.0000

ZH

0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
 0.0

L

2 1 3

SCATTERED FIELD AND SCATTERING CROSS SECTION/WAVELENGTH SQUARED
 BY MATRIX INVERSION, INCIDENCE FROM $\theta=130^\circ$, $\phi=0$

θ	REAL(E θ)	IMAG(E θ)	S θ /(λ AM)**2	REAL(E ϕ)	IMAG(E ϕ)	S ϕ /(λ AM)**2
0.0	-0.3024E 01	0.2301E 00	0.9198E 01	0.3024E 01	-0.2301E 00	0.9198E 01
10.0	-0.2903E 01	0.2290E 00	0.8478E 01	0.2897E 01	-0.1925E 00	0.8429E 01
20.0	-0.2570E 01	0.2243E 00	0.6655E 01	0.2550E 01	-0.9186E-01	0.6510E 01
30.0	-0.2106E 01	0.2130E 00	0.4480E 01	0.2071E 01	0.4138E-01	0.4289E 01
40.0	-0.1604E 01	0.1929E 00	0.2611E 01	0.1561E 01	0.1736E 00	0.2468E 01
50.0	-0.1140E 01	0.1636E 00	0.1327E 01	0.1105E 01	0.2812E 00	0.1301E 01
60.0	-0.7531E 00	0.1272E 00	0.5833E 00	0.7484E 00	0.3552E 00	0.6863E 00
70.0	-0.4479E 00	0.8638E-01	0.2081E 00	0.5029E 00	0.3988E 00	0.4120E 00
80.0	-0.2069E 00	0.4351E-01	0.4469E-01	0.3627E 00	0.4200E 00	0.3079E 00
90.0	-0.3617E-06	0.7842E-07	0.1370E-12	0.3175E 00	0.4261E 00	0.2824E 00
100.0	0.2069E 00	-0.4351E-01	0.4469E-01	0.3627E 00	0.4200E 00	0.3079E 00
110.0	0.4479E 00	-0.8638E-01	0.2081E 00	0.5029E 00	0.3988E 00	0.4120E 00
120.0	0.7531E 00	-0.1272E 00	0.5833E 00	0.7484E 00	0.3552E 00	0.6863E 00
130.0	0.1140E 01	-0.1636E 00	0.1327E 01	0.1105E 01	0.2812E 00	0.1301E 01
140.0	0.1604E 01	-0.1929E 00	0.2611E 01	0.1561E 01	0.1736E 00	0.2468E 01
150.0	0.2106E 01	-0.2130E 00	0.4480E 01	0.2071E 01	0.4138E-01	0.4289E 01
160.0	0.2570E 01	-0.2243E 00	0.6655E 01	0.2550E 01	-0.9186E-01	0.6510E 01
170.0	0.2903E 01	-0.2290E 00	0.8478E 01	0.2897E 01	-0.1925E 00	0.8429E 01
180.0	0.3024E 01	-0.2301E 00	0.9198E 01	0.3024E 01	-0.2301E 00	0.9198E 01

ONE INCH CORRESPONDS TO SIGMA/(λ ABDA)**2= 0.5000E 01

SCATTERED FIELD AND SCATTERING CROSS SECTION/WAVELENGTH SQUARED
 BY MATRIX INVERSION, INCIDENCE FROM $\theta=0^\circ$, $\phi=0$

θ	REAL(E θ)	IMAG(E θ)	S θ /(λ AM)**2	REAL(E ϕ)	IMAG(E ϕ)	S ϕ /(λ AM)**2
0.0	0.3024E 01	-0.2301E 00	0.9198E 01	-0.3024E 01	0.2301E 00	0.9198E 01
10.0	0.2903E 01	-0.2290E 00	0.8478E 01	-0.2897E 01	0.1925E 00	0.8429E 01
20.0	0.2570E 01	-0.2243E 00	0.6655E 01	-0.2550E 01	0.9186E-01	0.6510E 01
30.0	0.2106E 01	-0.2130E 00	0.4480E 01	-0.2071E 01	0.4138E-01	0.4289E 01
40.0	0.1604E 01	-0.1929E 00	0.2611E 01	-0.1561E 01	0.1736E 00	0.2468E 01
50.0	0.1140E 01	-0.1636E 00	0.1327E 01	-0.1105E 01	0.2812E 00	0.1301E 01
60.0	0.7531E 00	-0.1272E 00	0.5833E 00	-0.7484E 00	0.3552E 00	0.6863E 00
70.0	0.4479E 00	-0.8638E-01	0.2081E 00	-0.5029E 00	0.3988E 00	0.4120E 00
80.0	0.2069E 00	-0.4351E-01	0.4469E-01	-0.3627E 00	0.4200E 00	0.3079E 00
90.0	0.3617E-06	-0.7842E-07	0.1370E-12	-0.3175E 00	0.4261E 00	0.2824E 00
100.0	-0.2069E 00	0.4351E-01	0.4469E-01	-0.3627E 00	-0.4200E 00	0.3079E 00
110.0	-0.4479E 00	0.8638E-01	0.2081E 00	-0.5029E 00	-0.3988E 00	0.4120E 00

120.0	-0.7531E-01	0.1272E-01	0.5833E-01	-0.7484E-01	-0.3552E-01	0.4863E-01
130.0	-0.1140E-01	0.1636E-01	0.1327E-01	-0.1105E-01	-0.2812E-01	0.1331E-01
140.0	-0.1604E-01	0.1729E-01	0.2611E-01	-0.1561E-01	-0.1736E-01	0.2468E-01
150.0	-0.2106E-01	0.2137E-01	0.4480E-01	-0.2071E-01	-0.4138E-01	0.4249E-01
160.0	-0.2570E-01	0.2243E-01	0.6655E-01	-0.2550E-01	-0.9186E-01	0.6510E-01
170.0	-0.2903E-01	0.2290E-01	0.8478E-01	-0.2897E-01	-0.1925E-01	0.8429E-01
180.0	-0.3024E-01	0.2301E-01	0.9193E-01	-0.3024E-01	0.2311E-01	0.9193E-01

ONE INCH CORRESPONDS TO SIGMA/(LAMBDA)**2= 0.5000E-01

1 MODE SCATTERED FIELD AND SCATTERING CROSS SECTION/WAVELENGTH SQUARED
INCIDENCE FROM $\theta=180^\circ$, $\lambda=0$

θ	REAL(E θ)	IMAG(E θ)	S θ /(LAMBDA)**2	REAL(E λ)	IMAG(E λ)	S λ /(LAMBDA)**2
0.0	-0.2895E-01	-0.2543E-01	0.8381E-01	0.2895E-01	0.2543E-01	0.8381E-01
10.0	-0.2780E-01	-0.2442E-01	0.7728E-01	0.2784E-01	0.2446E-01	0.7753E-01
20.0	-0.2464E-01	-0.2165E-01	0.6073E-01	0.2482E-01	0.2131E-01	0.6161E-01
30.0	-0.2023E-01	-0.1773E-01	0.4095E-01	0.2062E-01	0.1812E-01	0.4253E-01
40.0	-0.1546E-01	-0.1358E-01	0.2390E-01	0.1613E-01	0.1417E-01	0.2602E-01
50.0	-0.1103E-01	-0.9687E-02	0.1216E-01	0.1206E-01	0.1060E-01	0.1456E-01
60.0	-0.7309E-02	-0.6422E-02	0.5343E-02	0.8343E-02	0.7759E-02	0.7820E-02
70.0	-0.4363E-02	-0.3833E-02	0.1904E-02	0.6600E-02	0.5799E-02	0.4357E-02
80.0	-0.2020E-02	-0.1775E-02	0.4081E-03	0.5305E-02	0.4661E-02	0.2915E-02
90.0	-0.3535E-06	-0.3106E-08	0.1250E-12	0.4885E-02	0.4292E-02	0.2387E-02
100.0	0.2020E-02	0.1775E-02	0.4081E-03	0.5305E-02	0.4661E-02	0.2915E-02
110.0	0.4363E-02	0.3833E-02	0.1904E-02	0.6600E-02	0.5799E-02	0.4357E-02
120.0	0.7309E-02	0.6422E-02	0.5343E-02	0.8343E-02	0.7759E-02	0.7820E-02
130.0	0.1103E-01	0.9687E-02	0.1216E-01	0.1206E-01	0.1060E-01	0.1456E-01
140.0	0.1546E-01	0.1358E-01	0.2390E-01	0.1613E-01	0.1417E-01	0.2602E-01
150.0	0.2023E-01	0.1778E-01	0.4095E-01	0.2062E-01	0.1812E-01	0.4253E-01
160.0	0.2464E-01	0.2165E-01	0.6073E-01	0.2482E-01	0.2131E-01	0.6161E-01
170.0	0.2780E-01	0.2442E-01	0.7728E-01	0.2784E-01	0.2446E-01	0.7753E-01
180.0	0.2895E-01	0.2543E-01	0.8381E-01	0.2895E-01	0.2543E-01	0.8381E-01

1 MODE SCATTERED FIELD AND SCATTERING CROSS SECTION/WAVELENGTH SQUARED
INCIDENCE FROM $\theta=0^\circ$, $\lambda=0$

θ	REAL(E θ)	IMAG(E θ)	S θ /(LAMBDA)**2	REAL(E λ)	IMAG(E λ)	S λ /(LAMBDA)**2
0.0	0.2895E-01	0.2543E-01	0.8381E-01	-0.2895E-01	-0.2543E-01	0.8381E-01
10.0	0.2780E-01	0.2442E-01	0.7728E-01	-0.2784E-01	-0.2446E-01	0.7753E-01
20.0	0.2464E-01	0.2165E-01	0.6073E-01	-0.2482E-01	-0.2131E-01	0.6161E-01
30.0	0.2023E-01	0.1773E-01	0.4095E-01	-0.2062E-01	-0.1812E-01	0.4253E-01
40.0	0.1546E-01	0.1358E-01	0.2390E-01	-0.1613E-01	-0.1417E-01	0.2602E-01
50.0	0.1103E-01	0.9687E-02	0.1216E-01	-0.1206E-01	-0.1060E-01	0.1456E-01
60.0	0.7309E-02	0.6422E-02	0.5343E-02	-0.8343E-02	-0.7759E-02	0.7820E-02
70.0	0.4363E-02	0.3833E-02	0.1904E-02	-0.6600E-02	-0.5799E-02	0.4357E-02
80.0	0.2020E-02	0.1775E-02	0.4081E-03	-0.5305E-02	-0.4661E-02	0.2915E-02
90.0	0.3535E-06	0.3106E-08	0.1250E-12	-0.4885E-02	-0.4292E-02	0.2387E-02
100.0	-0.2020E-02	-0.1775E-02	0.4081E-03	-0.5305E-02	-0.4661E-02	0.2915E-02
110.0	-0.4363E-02	-0.3833E-02	0.1904E-02	-0.6600E-02	-0.5799E-02	0.4357E-02
120.0	-0.7309E-02	-0.6422E-02	0.5343E-02	-0.8343E-02	-0.7759E-02	0.7820E-02
130.0	-0.1103E-01	-0.9687E-02	0.1216E-01	-0.1206E-01	-0.1060E-01	0.1456E-01
140.0	-0.1546E-01	-0.1358E-01	0.2390E-01	-0.1613E-01	-0.1417E-01	0.2602E-01
150.0	-0.2023E-01	-0.1778E-01	0.4095E-01	-0.2062E-01	-0.1812E-01	0.4253E-01
160.0	-0.2464E-01	-0.2165E-01	0.6073E-01	-0.2482E-01	-0.2131E-01	0.6161E-01
170.0	-0.2780E-01	-0.2442E-01	0.7728E-01	-0.2784E-01	-0.2446E-01	0.7753E-01
180.0	-0.2895E-01	-0.2543E-01	0.8381E-01	-0.2895E-01	-0.2543E-01	0.8381E-01

2 MODE SCATTERED FIELD AND SCATTERING CROSS SECTION/WAVELENGTH SQUARED
INCIDENCE FROM $\theta=180^\circ$, $\theta=0$

θ	REAL(E θ)	IMAG(E θ)	S θ /(L λ)**2	REAL(E θ)	IMAG(E θ)	S θ /(L λ)**2
0.0	-0.3022E 01	0.2994E 00	0.9221E 01	0.3022E 01	-0.2994E 00	0.9221E 01
10.0	-0.2901E 01	0.2854E 00	0.8496E 01	0.2895E 01	-0.2582E 00	0.8446E 01
20.0	-0.2569E 01	0.2473E 00	0.6653E 01	0.2548E 01	-0.1477E 00	0.6516E 01
30.0	-0.2107E 01	0.1955E 00	0.4477E 01	0.2069E 01	-0.5127E -03	0.4283E 01
40.0	-0.1607E 01	0.1414E 00	0.2601E 01	0.1561E 01	0.1470E 00	0.2459E 01
50.0	-0.1143E 01	0.9392E -01	0.1315E 01	0.1106E 01	0.2687E 00	0.1295E 01
60.0	-0.7558E 00	0.5731E -01	0.5746E 00	0.7489E 00	0.3543E 00	0.6864E 00
70.0	-0.4501E 00	0.3144E -01	0.2036E 00	0.5036E 00	0.4061E 00	0.4186E 00
80.0	-0.2680E 00	0.1363E -01	0.4347E -01	0.3635E 00	0.4322E 00	0.3189E 00
90.0	-0.3638E -06	0.2325E -07	0.1329E -12	0.3183E 00	0.4399E 00	0.2949E 00
100.0	0.2080E 00	-0.1363E -01	0.4346E -01	0.3635E 00	0.4322E 00	0.3189E 00
110.0	0.4501E 00	-0.3144E -01	0.2036E 00	0.5036E 00	0.4061E 00	0.4186E 00
120.0	0.7558E 00	-0.5731E -01	0.5746E 00	0.7489E 00	0.3543E 00	0.6864E 00
130.0	0.1143E 01	-0.9392E -01	0.1315E 01	0.1106E 01	0.2687E 00	0.1295E 01
140.0	0.1607E 01	-0.1414E 00	0.2601E 01	0.1561E 01	0.1470E 00	0.2459E 01
150.0	0.2107E 01	-0.1955E 00	0.4477E 01	0.2069E 01	-0.5123E -03	0.4283E 01
160.0	0.2569E 01	-0.2473E 00	0.6663E 01	0.2548E 01	-0.1477E 00	0.6515E 01
170.0	0.2901E 01	-0.2854E 00	0.8496E 01	0.2895E 01	-0.2582E 00	0.8446E 01
180.0	0.3022E 01	-0.2994E 00	0.9221E 01	0.3022E 01	-0.2994E 00	0.9221E 01

2 MODE SCATTERED FIELD AND SCATTERING CROSS SECTION/WAVELENGTH SQUARED
INCIDENCE FROM $\theta = 0^\circ$, $\phi = 0^\circ$

θ	REAL(E θ)	IMAG(E θ)	S θ /(L λ)**2	REAL(E θ)	IMAG(E θ)	S θ /(L λ)**2
0.0	0.3022E 01	-0.2994E 00	0.9221E 01	-0.3022E 01	0.2994E 00	0.9221E 01
10.0	0.2901E 01	-0.2854E 00	0.8496E 01	-0.2895E 01	0.2582E 00	0.8446E 01
20.0	0.2569E 01	-0.2473E 00	0.6663E 01	-0.2548E 01	0.1477E 00	0.6516E 01
30.0	0.2107E 01	-0.1955E 00	0.4477E 01	-0.2069E 01	0.5127E -03	0.4283E 01
40.0	0.1507E 01	-0.1414E 00	0.2601E 01	-0.1561E 01	0.1470E 00	0.2459E 01
50.0	0.1143E 01	-0.9392E -01	0.1315E 01	-0.1106E 01	0.2687E 00	0.1295E 01
60.0	0.7558E 00	-0.5731E -01	0.5746E 00	-0.7489E 00	0.3543E 00	0.6864E 00
70.0	0.4501E 00	-0.3144E -01	0.2036E 00	-0.5036E 00	0.4061E 00	0.4186E 00
80.0	0.2680E 00	-0.1363E -01	0.4347E -01	-0.3635E 00	0.4322E 00	0.3189E 00
90.0	0.3638E -06	-0.2325E -07	0.1329E -12	-0.3183E 00	0.4399E 00	0.2949E 00
100.0	-0.2080E 00	0.1363E -01	0.4346E -01	-0.3635E 00	0.4322E 00	0.3189E 00
110.0	-0.4501E 00	0.3144E -01	0.2036E 00	-0.5036E 00	0.4061E 00	0.4186E 00
120.0	-0.7558E 00	0.5731E -01	0.5746E 00	-0.7489E 00	0.3543E 00	0.6864E 00
130.0	-0.1143E 01	0.9392E -01	0.1315E 01	-0.1106E 01	0.2687E 00	0.1295E 01
140.0	-0.1607E 01	0.1414E 00	0.2601E 01	-0.1561E 01	0.1470E 00	0.2459E 01
150.0	-0.2107E 01	0.1955E 00	0.4477E 01	-0.2069E 01	0.5123E -03	0.4283E 01
160.0	-0.2569E 01	0.2473E 00	0.6663E 01	-0.2548E 01	0.1477E 00	0.6515E 01
170.0	-0.2901E 01	0.2854E 00	0.8496E 01	-0.2895E 01	0.2582E 00	0.8446E 01
180.0	-0.3022E 01	0.2994E 00	0.9221E 01	-0.3022E 01	0.2994E 00	0.9221E 01

3 MODE SCATTERED FIELD AND SCATTERING CROSS SECTION/WAVELENGTH SQUARED
INCIDENCE FROM $\theta = 180^\circ$, $\phi = 0^\circ$

θ	REAL(E θ)	IMAG(E θ)	S θ /(L λ)**2	REAL(E θ)	IMAG(E θ)	S θ /(L λ)**2
0.0	-0.3024E 01	0.2268E 00	0.9199E 01	0.3024E 01	-0.2269E 00	0.9199E 01
10.0	-0.2903E 01	0.2259E 00	0.8479E 01	0.2897E 01	-0.1931E 00	0.8431E 01
20.0	-0.2570E 01	0.2219E 00	0.6656E 01	0.2550E 01	-0.9159E -01	0.6513E 01
30.0	-0.2105E 01	0.2116E 00	0.4481E 01	0.2071E 01	0.3964E -01	0.4291E 01
40.0	-0.1605E 01	0.1923E 00	0.2612E 01	0.1562E 01	0.1705E 00	0.2469E 01
50.0	-0.1140E 01	0.1637E 00	0.1327E 01	0.1106E 01	0.2784E 00	0.1301E 01
60.0	-0.7532E 00	0.1277E 00	0.5836E 00	0.7489E 00	0.3539E 00	0.6861E 00
70.0	-0.4480E 00	0.8690E -01	0.2082E 00	0.5034E 00	0.3993E 00	0.4129E 00
80.0	-0.2069E 00	0.4384E -01	0.4473E -01	0.3631E 00	0.4220E 00	0.3100E 00
90.0	-0.3617E -06	0.7905E -07	0.1371E -12	0.3179E 00	0.4287E 00	0.2849E 00
100.0	0.2069E 00	-0.4384E -01	0.4473E -01	0.3631E 00	0.4220E 00	0.3100E 00
110.0	0.4480E 00	-0.8690E -01	0.2082E 00	0.5034E 00	0.3993E 00	0.4129E 00

120.0	0.7532E 01	-0.1277E 00	0.5836E 00	0.7489E 00	0.3539E 01	0.6861E 00
130.0	0.1140E 01	-0.1637E 00	0.1327E 01	0.1106E 01	0.2784E 00	0.1301E 01
140.0	0.1605E 01	-0.1923E 00	0.2612E 01	0.1562E 01	0.1705E 00	0.2469E 01
150.0	0.2106E 01	-0.2116E 00	0.4481E 01	0.2071E 01	0.3944E-01	0.4291E 01
160.0	0.2570E 01	-0.2219E 00	0.6656E 01	0.2550E 01	0.9159E-01	0.6513E 01
170.0	0.2903E 01	-0.2259E 00	0.8479E 01	0.2897E 01	0.1901E 00	0.8431E 01
180.0	0.3024E 01	-0.2268E 00	0.9199E 01	0.3024E 01	0.2268E 00	0.9199E 01

3 MODE SCATTERED FIELD AND SCATTERING CROSS SECTION/WAVELENGTH SQUARED
INCIDENCE FROM $\theta = 0^\circ, 7^\circ$

θ	REAL(E ₀)	IMAG(E ₀)	SE/(LAM)**2	REAL(E ₀)	IMAG(E ₀)	SE/(LAM)**2
0.0	0.3024E 01	-0.2268E 00	0.9199E 01	-0.3024E 01	0.2268E 00	0.9199E 01
10.0	0.2903E 01	-0.2259E 00	0.8479E 01	-0.2897E 01	0.1901E 00	0.8431E 01
20.0	0.2570E 01	-0.2219E 00	0.6656E 01	-0.2550E 01	0.9159E-01	0.6513E 01
30.0	0.2106E 01	-0.2116E 00	0.4481E 01	-0.2071E 01	0.3944E-01	0.4291E 01
40.0	0.1605E 01	-0.1923E 00	0.2612E 01	-0.1562E 01	0.1705E 00	0.2469E 01
50.0	0.1140E 01	-0.1637E 00	0.1327E 01	-0.1106E 01	0.2784E 00	0.1301E 01
60.0	0.7532E 00	-0.1277E 00	0.5836E 00	-0.7489E 00	0.3539E 01	0.6861E 00
70.0	0.4480E 00	-0.3690E-01	0.2082E 00	-0.5034E 00	0.3993E 00	0.4129E 00
80.0	0.2909E 00	-0.4384E-01	0.4473E-01	-0.3631E 00	0.4220E 00	0.3100E 00
90.0	0.3617E-06	-0.7905E-07	0.1371E-12	-0.3179E 00	0.4287E 00	0.2849E 00
100.0	-0.2909E 00	0.4384E-01	0.4473E-01	-0.3631E 00	-0.4220E 00	0.3100E 00
110.0	-0.4480E 00	0.3690E-01	0.2082E 00	-0.5034E 00	-0.3993E 00	0.4129E 00
120.0	-0.7532E 00	0.1277E 00	0.5836E 00	-0.7489E 00	-0.3539E 00	0.6861E 00
130.0	-0.1140E 01	0.1637E 00	0.1327E 01	-0.1106E 01	-0.2784E 00	0.1301E 01
140.0	-0.1605E 01	0.1923E 00	0.2612E 01	-0.1562E 01	-0.1705E 00	0.2469E 01
150.0	-0.2106E 01	0.2116E 00	0.4481E 01	-0.2071E 01	-0.3944E-01	0.4291E 01
160.0	-0.2570E 01	0.2219E 00	0.6656E 01	-0.2550E 01	0.9159E-01	0.6513E 01
170.0	-0.2903E 01	0.2259E 00	0.8479E 01	-0.2897E 01	0.1901E 00	0.8431E 01
180.0	-0.3024E 01	0.2268E 00	0.9199E 01	-0.3024E 01	0.2268E 00	0.9199E 01

VII. GAIN PATTERNS

Program #6 which calculates and plots the gain pattern for radiation from axially symmetric excitation on a surface of revolution accepts punched card data according to

```

      READ(1,10) NP, NT, NS, JM, NC, NV, BK, SCL
10    FORMAT(6I3, 2E14.7)
      READ(1,11)(AMD(I), I = 1, JM)
11    FORMAT (5E14.7)
      READ(1,15)(RH(I), I = 1, NP)
      READ(1,15)(ZH(I), I = 1, NP)
15    FORMAT (10F8.4)
      READ(1,50)(L(I), I = 1, NC)
50    FORMAT (20I3)
      READ(1,57)(V(I), I = 1, NV)
57    FORMAT (7E11.4)
      READ(1,50)(LV(I), I = 1, NV)

```

The variables NP, NT, NS, JM, BK, AMD, RH, ZH, and L are the same as in program #5 except that L now applies only to the $\vec{u}_t$ directed eigencurrents. The output of program #2 characterizes each eigencurrent by NM2 numbers. For the axially symmetric mode, either the first NM2/2 numbers (corresponding to J_t) or the last NM2/2 numbers (corresponding to J_ϕ) are supposed to be zero. Thus L(I) indicates that the L(I)th eigencurrent (only $\vec{u}_t$ directed eigencurrents being considered) to come out of program #2 will be the Ith current to be added to the modal expansion. There are $NV \leq NM$ axially symmetric slots on the body of revolution. The voltage across the Ith slot is V(I). Note that V(I) is complex. The Ith slot is located at the $(2*LV(1)+1)^{th}$ data point ($\rho = RH$, $z = ZH$) on the generating curve C. Accordingly, the Ith slot occurs at the peak of the LV(I)th triangle function of LV(1). The function f_i appears in (2-42) of [1]. For the plots, one inch corresponds to a gain of 1./SCL.

The impedance matrix and eigencurrents are read from the third and fourth records of direct access data set 6 according to

```
REWIND 6
READ (6)
READ (6)
READ (6) (Z(I), I = 1, NZ)
READ (6) (FI(I), I = 1, NZ1)
```

Only half of the impedance matrix is read because only $[Z_o^{tt}]$ of (2-46) of [1] is needed. Program #1 has stored the impedance matrix on the third record of data set 6 columnwise in the block diagonal form given by

$$[Z_o] = \begin{bmatrix} [Z_o^{tt}] & [0] \\ [0] & [Z_o^{\phi\phi}] \end{bmatrix} \quad (22)$$

The subroutine PLANE is similar to the one compiled with program #5. At present, only $\hat{R}_o^{t\theta}$ of (5) is needed because a slot voltage, corresponding to a $\vec{u}_t$ directed axially symmetric aperture electric field, induces only a $\vec{u}_t$ directed electric current which radiates only a u_θ directed far field.

DO loop 60 suppresses the lower left zero submatrix on the right hand side of (22). The net result of DO loops 71 and 73 is to arrange the eigencurrents (FI) and their eigenvalues (AMD) in the order dictated by L and to suppress intervening zeros from FI. DO loop 62 obtains the matrix [X] appearing in (2-15) of [1] by taking the imaginary part of the average of corresponding off diagonal elements of the impedance matrix. Statement 79 inverts the impedance matrix to obtain the admittance matrix. DO loop 82 obtains the column matrix $[I_o^t]$ associated with the electric current by pre-multiplying the excitation column matrix by the admittance matrix. The elements of the excitation column matrix of (27) of [3] are actually 2π times the slot voltages, but the factor 2π is inconsequential as far as that gain is concerned. DO loop 82 stores $[I_o^t]$ in E3. DO loops 84 and 97 compute the radiation field $E_\theta \sim R_o^{t\theta} [I_o^t]$ and gain G_θ and store them in E and G. The radiation field and gain are normalized so that

$$\int_0^\pi G_\theta \sin \theta \, d\theta = 2 \quad (23)$$

$$|E_\theta|^2 = G_\theta$$

The phase factor $-je^{-jkr}$ is suppressed from E_θ . DO loop 99 prepares horizontal and vertical components E5 and E6 suitable for plotting the gain. DO loop 48 multiplies the eigencurrents by ρ to retrieve the column matrix $[I_o^t]$ associated with each eigencurrent. The index J of DO loop 48 indicates the Jth eigencurrent. DO loop 21 stores the matrix $[I] \frac{1}{[\tilde{I}][X][I] (j + 1/\lambda)}$ appearing in (20) in T3.

The index K of DO loop 29 indicates that inner DO loop 31 will add the contribution from the Kth eigencurrent to the admittance matrix (20). DO loop 44 premultiplies $[I_o^t]$ stored in E3 by $\hat{R}_o^{t\theta}$ stored in VR to obtain the radiation field and the gain. The radiation field E_θ and gain G_θ , normalized according to (23), are stored in E and G. In order not to mask a possible discrepancy in the amplitude of the approximate pattern obtained by superimposing a few eigenfields, the present normalization uses the value of the pattern integral (23) previously computed from the admittance matrix obtained by inverting the impedance matrix.

Minimum allocations are given by

```

COMPLEX Z(NM2*NM), Y(NM2*NM), VR(NT*NM), E3(NM),
      T3(NM*NM), E(NT), V(NV)
DIMENSION AMD(JM), RH(NP), ZH(NP), FI(NM2*JM),
      DH(NP-1), TH(NT), T2(NM*NM), X(NM*NM),
      G(NT), L(NC), LV(NV), SN(NT), CS(NT), GX(NT),
      GY(NT), E5(NT*2), E6(NT*2)
COMMON RS(NP-1), ZS(NP-1), SV(NP-1), CV(NP-1), T(NM*4)
DIMENSION BJ(3*(NP-1))
DIMENSION LR(NM)

```

where

```

NM2 = NP - 3
NM = NM2/2

```

Here, NP is its value after execution of statement 90 in the main program. Note that BJ appears in PLANE and LR in LINEQ.

```

//          (0034,EE,3,2,,6),'MAUTZ,JOE',MSGLEVEL=1
// EXEC FORTGCLG,PARM.FORT='MAP'
//FORT.SYSIN DD *
      SUBROUTINE PLANE(VVR,THR,NT)
      COMPLEX VVR(1),A5,A6,U
      COMMON U,RS(40),ZS(40),SV(40),CV(40),BK,NP,T(80)
      DIMENSION BJ(126),THR(1),FK(20)
      NN=0
      KG=NP-1
      NM=KG/2-1
      M2=NN+2
      A5=2.*3.141593*U**(NN+1)
      FK(1)=1.
      DO 153 J=1,M2
      J1=J+1
      FK(J1)=FK(J)*J
153 CONTINUE
      DO 156 L=1,NT
      L1=(L-1)*NM
      CS=COS(THR(L))
      SN=SIN(THR(L))
      BCS=BK*CS
      DO 302 J=1,KG
      X=RS(J)*BK*SN
      J1=J
      I1=NN
      IF(I1) 303,304,303
304 I1=I1+1
      J1=J1+KG
303 DO 305 JJ=I1,M2
      IF(X-1.E-5) 1,1,2
      1 IF(JJ-1) 3,3,4
      3 BJ(J1)=1.
      GO TO 306
      4 BJ(J1)=0.
      GO TO 306
      2 RH=X/2.
      RH2=RH*RH
      RH3=RH**(JJ-1)
      RJ(J1)=RH3/FK(JJ)
      SS=BJ(J1)
      8 SST=SS*1.E-7
      DO 155 K=1,20
      SS=-SS*RH2/K/(K+JJ-1)
      RJ(J1)=RJ(J1)+SS
      IF(ABS(SS)-SST) 306,306,155
155 CONTINUE
      STOP 155
306 J1=J1+KG
305 CONTINUE
302 CONTINUE
      IF(NN) 307,308,307
308 DO 309 J=1,KG
      J1=J+2*KG
      BJ(J)=-BJ(J1)
309 CONTINUE
307 DO 300 J=1,NM
      J1=J+L1
      VVR(J1)=0.
      DO 301 I=1,4

```

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```

      I1=2*(J-1)+1
      I4=4*(J-1)+1
      I2=I1+KG
      I3=I2+KG
      A6=(COS(ZS(I1)*BCS)+U*SIN(ZS(I1)*BCS))*A5
      BJ2=(BJ(I3)-BJ(I1))*0.5
      VVR(J1)=VVR(J1)+A6*(CS*SV(I1)*BJ2+SN*CV(I1)*BJ(I2)*U)*T(I4)
301 CONTINUE
300 CONTINUE
156 CONTINUE
      RETURN
      END
      SUBROUTINE LINE7(LL,C)
      COMPLEX C(1),STOR,STO,ST,S
      DIMENSION LR(58)
      DO 20 I=1,LL
      LR(I)=I
20 CONTINUE
      M1=0
      DO 18 M=1,LL
      K=M
      DO 2 I=M,LL
      K1=M+I
      K2=M1+K
      IF(CABS(C(K1))-CABS(C(K2))) 2,2,6
6 K=I
2 CONTINUE
      LS=LR(M)
      LR(M)=LR(K)
      LR(K)=LS
      K2=M1+K
      STOR=C(K2)
      J1=0
      DO 7 J=1,LL
      K1=J1+K
      K2=J1+M
      STO=C(K1)
      C(K1)=C(K2)
      C(K2)=STO/STOR
      J1=J1+LL
7 CONTINUE
      K1=M1+M
      C(K1)=1./STOR
      DO 11 I=1,LL
      IF(I-M) 12,11,12
12 K1=M1+I
      ST=C(K1)
      C(K1)=0.
      J1=0
      DO 10 J=1,LL
      K1=J1+I
      K2=J1+M
      C(K1)=C(K1)-C(K2)*ST
      J1=J1+LL
10 CONTINUE
11 CONTINUE
      M1=M1+LL
18 CONTINUE
      J1=0
      DO 9 J=1,LL

```

```

      IF(J-LR(J)) 14,P,14
14  LRJ=LR(J)
      J2=(LRJ-1)*LL
21  DO 13 I=1,LL
      K2=J2+1
      K1=J1+1
      S=C(K2)
      C(K2)=C(K1)
      C(K1)=S
13  CONTINUE
      LR(J)=LR(LRJ)
      LR(LPJ)=LRJ
      IF(J-LR(J)) 14,R,14
      J1=J1+LL
9  CONTINUE
      RETURN
      END
      COMPLEX U,U1,Z(722),Y(1444),VR(1387),E3(19),T3(361),F(73),V(19)
      DIMENSION AMD(38),RH(41),ZH(41),FI(1444),DH(40),TH(73),T2(361)
      DIMENSION X(361),G(73),L(19),LV(19),AREA(400),SN(73),CS(73)
      DIMENSION GX(73),GY(73),XP(2),YP(2),E5(146),E6(146)
      EQUIVALENCE(T2(1),X(1)),(Z(1),Y(1))
      COMMON U,RS(40),TS(40),SV(40),CV(40),BK,NP,T(80)
      CALL PLOTS(AREA,400)
      READ(1,10) NP,NT,NS,JM,NC,NV,BK,SCL
10  FORMAT(6I3,2E14.7)
      READ(1,11)(AMD(I),I=1,JM)
11  FORMAT(5E14.7)
      READ(1,15)(RH(I),I=1,NP)
      READ(1,15)(ZH(I),I=1,NP)
15  FORMAT(10F8.4)
      READ(1,50)(L(I),I=1,NC)
50  FORMAT(20I3)
      READ(1,57)(V(I),I=1,NV)
57  FORMAT(7E11.4)
      READ(1,50)(LV(I),I=1,NV)
      WRITE(3,37)
37  FORMAT('0 NP NT NS JM NC NV',6X,'BK',12X,'SCL')
      WRITE(3,36) NP,NT,NS,JM,NC,NV,BK,SCL
38  FORMAT(1X,6I3,2E14.7)
      WRITE(3,5)(AMD(I),I=1,JM)
5  FORMAT('0AMD'/(1X,5E14.7))
      WRITE(3,39)(RH(I),I=1,NP)
39  FORMAT('0RH'/(1X,10F8.4))
      WRITE(3,9)(ZH(I),I=1,NP)
9  FORMAT('0ZH'/(1X,10F8.4))
      WRITE(3,8)(L(I),I=1,NC)
8  FORMAT('0L'/(1X,20I3))
      WRITE(3,7)(V(I),I=1,NV)
7  FORMAT('0V'/(1X,7E11.4))
      WRITE(3,6)(LV(I),I=1,NV)
6  FORMAT('0LV'/(1X,20I3))
      PI=3.141593
      U=(0.,1.)
      IF((RH(1)-RH(NP)).NE.0..OR.(ZH(1)-ZH(NP)).NE.0.) GO TO 90
      RH(NP+1)=RH(2)
      ZH(NP+1)=ZH(2)
      RH(NP+2)=RH(3)
      ZH(NP+2)=ZH(3)
      NP=NP+2

```

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```

90 NM2=NP-3
   NM=NM2/2
   NZ=NM2*NM
   NZ1=NM2*JM
   RFWIND 6
   READ(6)
   READ(6)
   READ(6)(Z(I),I=1,NZ)
   READ(6)(FI(I),I=1,NZ1)
   NT2=NT*2
   NT3=NT-NS
   NT4=NS+1
   DO 40 I=2,NP
     I2=I-1
     RR1=RH(I)-RH(I2)
     RR2=ZH(I)-ZH(I2)
     DH(I2)=SQRT(RR1*RR1+RR2*RR2)
     RS(I2)=.5*(RH(I)+RH(I2))
     ZS(I2)=.5*(ZH(I)+ZH(I2))
     SV(I2)=RR1/DH(I2)
     CV(I2)=RR2/DH(I2)
40  CONTINUE
   DO 41 J=1,NM
     J2=2*(J-1)+1
     J3=J2+1
     J4=J3+1
     J5=J4+1
     J6=4*(J-1)+1
     J7=J6+1
     J8=J7+1
     J9=J8+1
     DEL1=DH(J2)+DH(J3)
     DEL2=DH(J4)+DH(J5)
     T(J6)=DH(J2)*DH(J2)/2./DEL1
     T(J7)=DH(J3)*(DH(J2)+DH(J3)/2.)/DEL1
     T(J8)=DH(J4)*(DH(J5)+DH(J4)/2.)/DEL2
     T(J9)=DH(J5)*DH(J5)/2./DEL2
41  CONTINUE
   DEL=PI/(NT-1)
   DO 43 J=1,NT
     TH(J)=(J-1)*DEL
     SN(J)=SCL*SIN(TH(J))
     CS(J)=SCL*COS(TH(J))
43  CONTINUE
   CALL PLANE(VR,TH,NT)
   XP(1)=2.
   XP(2)=8.
   YP(1)=5.
   YP(2)=5.
   DEL1=180./PI
   DO 64 J=1,NT
     TH(J)=TH(J)*DEL1
64  CONTINUE
   J1=0
   DO 60 J=1,NM
     J2=(J-1)*NM2
     DO 61 I=1,NM
       J1=J1+1
       J3=J2+I
       Z(J1)=Z(J3)

```

```

61 CONTINUE
60 CONTINUE
  J1=0
  DO 71 J=1,NC
    J2=(L(J)-1)*NM2
    DO 72 I=1,NM
      J1=J1+1
      J3=J2+I
      T2(J1)=F1(J3)
72 CONTINUE
    J2=L(J)
    E5(J)=AMD(J2)
71 CONTINUE
  J1=0
  DO 73 J=1,NC
    DO 74 I=1,NM
      J1=J1+1
      F1(J1)=T2(J1)
74 CONTINUE
    AMD(J)=E5(J)
73 CONTINUE
  DO 62 J=1,NM
    J2=(J-1)*NM
    DO 63 I=1,J
      J1=J2+I
      J3=(I-1)*NM+J
      X(J1)=.5*A[MAG(Z(J1)+Z(J3))
      X(J3)=X(J1)
63 CONTINUE
62 CONTINUE
70 CALL LINEQ(NM,Z)
  DO 82 I=1,NM
    E3(I)=0.
    DO 65 K=1,NV
      K1=I+(LV(K)-1)*NM
      E3(I)=E3(I)+7(K1)*V(K)
65 CONTINUE
82 CONTINUE
  S2=0.
  J2=0
  DO 84 J=1,NT
    U1=0.
    DO 88 I=1,NM
      J2=J2+1
      U1=U1+VR(J2)*E3(I)
88 CONTINUE
    E(J)=U1
    S1=CABS(U1)
    G(J)=S1*S1
    S2=S2+G(J)*SN(J)
84 CONTINUE
  S5=2.*SCL/S2/DFL
  S6=SQRT(S5)
  DO 97 J=1,NT
    E(J)=E(J)*S6
    G(J)=G(J)*S6
97 CONTINUE
  WRITE(3,66)
66 FORMAT('RADIATION FIELD AND GAIN BY MATRIX INVERSION')
  WRITE(3,67)

```

```

67 FFORMAT('O O REAL(E0) IMAG(E0) GAINO')
WRITE(7,68)
68 FFORMAT(' + -',11X,' -',11X,' -',10X,' -')
DO 69 J=1,NT,NS
WRITE(3,70) TH(J),F(J),G(J)
70 FFORMAT(1X,F6.1,3E12.4)
69 CONTINUE
DO 99 J=1,NT
S1=SN(J)*G(J)
J9=NT2-J+1
E5(J)=5.+S1
E5(J9)=5.-S1
E6(J)=5.+CS(J)*G(J)
E6(J9)=E6(J)
99 CONTINUE
J1=0
DO 48 J=1,NC
DO 18 I=1,NM
J1=J1+1
J4=2*I+1
FI(J1)=FI(J1)*RH(J4)
18 CONTINUE
48 CONTINUE
DO 21 J=1,NC
J1=(J-1)*NM
S1=0.
DO 22 I=1,NM
S2=0.
J4=(I-1)*NM
DO 23 K=1,NM
J2=J4+K
J3=J1+K
S2=S2+X(J2)*FI(J3)
23 CONTINUE
J3=J1+I
S1=S1+S2*FI(J3)
22 CONTINUE
U1=1./S1/(U+1./AMD(J))
DO 27 I=1,NM
J2=J1+I
T3(J2)=FI(J2)*U1
27 CONTINUE
21 CONTINUE
NZ2=NM*NM
DO 32 J=1,NZ2
Y(J)=0.
32 CONTINUE
DO 29 K=1,NC
J3=(K-1)*NM
J1=0
DO 31 I=1,NM
J4=J3+I
E3(I)=0.
DO 75 J=1,NV
J2=J3+LV(J)
J1=J1+1
Y(J1)=Y(J1)+T3(J4)*FI(J2)
E3(I)=E3(I)+Y(J1)*V(J)
75 CONTINUE
31 CONTINUE

```

```

      DO 44 J=1,NT,NS
      U1=0.
      J1=(J-1)*NM
      DO 47 I=1,NM
      J2=J1+I
      U1=U1+VR(I,J2)*E3(I)
47  CONTINUE
      E(J)=U1*S6
      S1=CABS(E(J))
      G(J)=S1*S1
      GX(J)=5.+SN(J)*G(J)
      GY(J)=5.+CS(J)*G(J)
44  CONTINUE
      WRITE(3,76) K
76  FORMAT('0',I3,' MODE RADIATION FIELD AND GAIN')
      WRITE(3,67)
      WRITE(3,68)
      DO 80 J=1,NT,NS
      WRITE(3,70) TH(J),E(J),G(J)
80  CONTINUE
      CALL LINE(XP,YP,2,1,0,0)
      DO 77 J=1,7
      S1=9-J
      CALL SYMBOL(S1,5.,.14,13,0.,-1)
77  CONTINUE
      CALL LINE(YP,XP,2,1,0,0)
      DO 78 J=1,7
      S1=9-J
      CALL SYMBOL(5.,S1,.14,13,90.,-1)
78  CONTINUE
      CALL LINE(E5,E6,NT2,1,0,0)
      DO 86 J=1,NT,NS
      CALL SYMBOL(GX(J),GY(J),.07,4,0.,-1)
86  CONTINUE
      DO 89 J=NT4,NT3,NS
      J1=NT-J+1
      S1=10.-GX(J1)
      CALL SYMBOL(S1,GY(J1),.07,4,0.,-1)
89  CONTINUE
      CALL PLOT(7.,0.,-3)
29  CONTINUE
      CALL PLOT(6.,0.,-3)
      STOP
      END

/*
//GO.FT06F001 DD DSN=EE0034.REV1,DISP=OLD,UNIT=2314, X
//              VOLUME=SER=SU0004,DCB=(RECFM=V,BLKSIZE=1800,LRECL=1796)
//GO.SYSIN DD *
21 73  4  5  3  1 0.3141593E+00 0.5030000E+00
0.3359053E+02 0.6264798E+00-0.1842634E+01-0.1062163E+04-0.1210105E+05
0.0000 0.5000 1.0000 1.5000 2.0000 2.5000 3.0000 3.5000 4.0000 4.5000
5.0000 5.5000 6.0000 6.5000 7.0000 7.5000 8.0000 8.5000 9.0000 9.5000
10.0000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000
3  4  5
0.1000E+01 0.0000E+00
5
/*

```

58 Output of Program #6

NP NT NS JM NO NV RCL SOL
21 73 4 5 3 1 1.5141581E 0 1.5000000E 00

AND
0.3359052E 02 0.6264797E 00 -0.1842633E 01 -0.1062163E 04 -0.1210105E 05

RH
0.0 -0.5000 1.000 1.500 2.000 2.500 3.000 3.500 4.000 4.500
5.000 5.500 6.000 6.500 7.000 7.500 8.000 8.500 9.000 9.500
10.000

ZH
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0

L
3 4 5

V
0.1000E 01 0.0

LV
5

RADIATION FIELD AND GAIN BY MATRIX INVERSION

A	REAL(EH)	IMAG(EH)	GAINH
0.0	0.0	0.0	0.0
10.0	-0.5429E 00	0.3129E 00	0.3926E 00
20.0	-0.9745E 00	0.5551E 00	0.1259E 01
30.0	-0.1223E 01	0.5866E 00	0.1967E 01
40.0	-0.1273E 01	0.7501E 00	0.2111E 01
50.0	-0.1157E 01	0.5215E 00	0.1726E 01
60.0	-0.9292E 00	0.4472E 00	0.1101E 01
70.0	-0.6397E 00	0.3285E 00	0.5171E 00
80.0	-0.3239E 00	0.1640E 00	0.1318E 00
90.0	-0.5844E -06	0.2944E -06	0.4221E -12
100.0	0.3239E 00	-0.1640E 00	0.1318E 00
110.0	0.6397E 00	-0.3285E 00	0.5171E 00
120.0	0.9292E 00	-0.4472E 00	0.1101E 01
130.0	0.1157E 01	-0.5215E 00	0.1726E 01
140.0	0.1273E 01	-0.7501E 00	0.2111E 01
150.0	0.1223E 01	-0.5866E 00	0.1967E 01
160.0	0.9745E 00	-0.5551E 00	0.1259E 01
170.0	0.5429E 00	-0.3129E 00	0.3926E 00
180.0	0.0	0.0	0.0

1 MODE RADIATION FIELD AND GAIN

59

θ	REAL(E _θ)	IMAG(E _θ)	GAIN _θ
0.0	0.0	0.0	0.0
10.0	-0.5762E 00	0.3127E 00	0.4299E 00
20.0	-0.1024E 01	0.5557E 00	0.1357E 01
30.0	-0.1255E 01	0.6863E 00	0.2070E 01
40.0	-0.1289E 01	0.6998E 00	0.2152E 01
50.0	-0.1145E 01	0.6212E 00	0.1696E 01
60.0	-0.8974E 00	0.4970E 00	0.1042E 01
70.0	-0.6051E 00	0.3284E 00	0.4739E 00
80.0	-0.3020E 00	0.1639E 00	0.1181E 00
90.0	-0.5422E-06	0.2943E-06	0.3806E-12
100.0	0.3220E 00	-0.1639E 00	0.1181E 00
110.0	0.6051E 00	-0.3284E 00	0.4739E 00
120.0	0.8974E 00	-0.4970E 00	0.1042E 01
130.0	0.1145E 01	-0.6212E 00	0.1696E 01
140.0	0.1289E 01	-0.6998E 00	0.2152E 01
150.0	0.1255E 01	-0.6863E 00	0.2070E 01
160.0	0.1024E 01	-0.5557E 00	0.1357E 01
170.0	0.5762E 00	-0.3127E 00	0.4299E 00
180.0	0.0	0.0	0.0

2 MODE RADIATION FIELD AND GAIN

θ	REAL(E _θ)	IMAG(E _θ)	GAIN _θ
0.0	0.0	0.0	0.0
10.0	-0.5422E 00	0.3127E 00	0.3918E 00
20.0	-0.9735E 00	0.5557E 00	0.1256E 01
30.0	-0.1222E 01	0.6862E 00	0.1964E 01
40.0	-0.1273E 01	0.6997E 00	0.2109E 01
50.0	-0.1157E 01	0.6212E 00	0.1725E 01
60.0	-0.9296E 00	0.4970E 00	0.1101E 01
70.0	-0.6402E 00	0.3284E 00	0.5178E 00
80.0	-0.3243E 00	0.1639E 00	0.1320E 00
90.0	-0.5852E-06	0.2943E-06	0.4290E-12
100.0	0.3243E 00	-0.1639E 00	0.1320E 00
110.0	0.6402E 00	-0.3284E 00	0.5173E 00
120.0	0.9296E 00	-0.4970E 00	0.1101E 01
130.0	0.1157E 01	-0.6212E 00	0.1725E 01
140.0	0.1273E 01	-0.6997E 00	0.2109E 01
150.0	0.1222E 01	-0.6862E 00	0.1964E 01
160.0	0.9735E 00	-0.5557E 00	0.1256E 01
170.0	0.5422E 00	-0.3127E 00	0.3918E 00
180.0	0.0	0.0	0.0

3 MODE RADIATION FIELD AND GAIN

θ	REAL(E _θ)	IMAG(E _θ)	GAIN _θ
0.0	0.0	0.0	0.0
10.0	-0.5416E 00	0.3127E 00	0.3912E 00
20.0	-0.9726E 00	0.5557E 00	0.1255E 01
30.0	-0.1221E 01	0.6862E 00	0.1962E 01
40.0	-0.1272E 01	0.6997E 00	0.2109E 01
50.0	-0.1155E 01	0.6212E 00	0.1726E 01
60.0	-0.9301E 00	0.4970E 00	0.1102E 01
70.0	-0.6407E 00	0.3284E 00	0.5183E 00
80.0	-0.3245E 00	0.1639E 00	0.1322E 00
90.0	-0.5857E-06	0.2943E-06	0.4296E-12
100.0	0.3245E 00	-0.1639E 00	0.1322E 00
110.0	0.6407E 00	-0.3284E 00	0.5183E 00
120.0	0.9301E 00	-0.4970E 00	0.1102E 01
130.0	0.1158E 01	-0.6212E 00	0.1726E 01
140.0	0.1272E 01	-0.6997E 00	0.2109E 01
150.0	0.1221E 01	-0.6862E 00	0.1962E 01
160.0	0.9726E 00	-0.5557E 00	0.1255E 01
170.0	0.5417E 00	-0.3127E 00	0.3912E 00
180.0	0.0	0.0	0.0

REFERENCES

- [1] R. F. Harrington and J. R. Mautz, "Theory and Computation of Characteristic Modes for Conducting Bodies," Report AFCRL-70-0657, Contract F19628-68-C-0180 between Syracuse University and Air Force Cambridge Research Laboratories, December 1970.
- [2] R. F. Harrington and J. R. Mautz, "Radiation and Scattering from Bodies of Revolution, Report AFCRL-69-0305, Contract F19628-67-C-0233 between Syracuse University and Air Force Cambridge Research Laboratories, July 1969.
- [3] J. R. Mautz and R. F. Harrington, "Radiation and Scattering from Bodies of Revolution," Appl. Sci. Res., vol. 20, June 1969, pp. 405-435.
- [4] IBM System/360 Scientific Subroutine Package (360A-CM-03X) Version III, Programmer's Manual.

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13. ABSTRACT Computer programs are given for calculating the characteristic currents and characteristic gain patterns of conducting bodies of revolution. Also given are computer programs for using these characteristic currents in aperture radiation and plane-wave scattering problems. Plot programs for use with a Calcomp plotter are included. Operating procedures and program details are discussed, and sample input-output data are given.		

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