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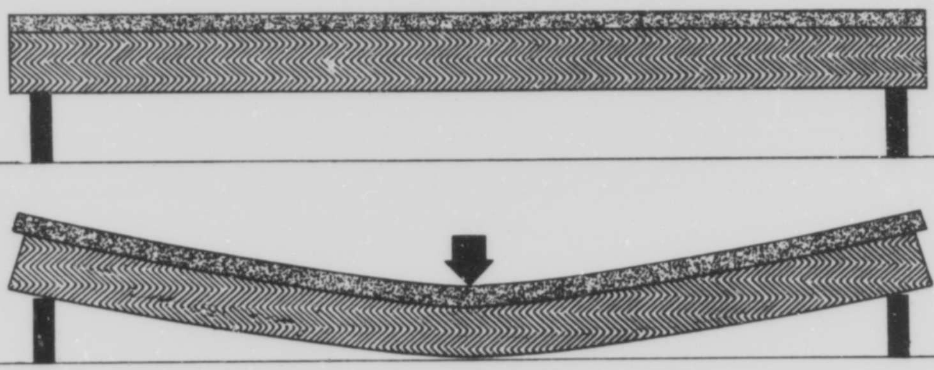
COMPOSITE BEAMS--

EFFECT OF ADHESIVE OR FASTENER RIGIDITY



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ABSTRACT

An analysis capable of predicting the deflections and stresses for composite beams with finite fastener rigidity is reported. Experimental research involving evaluation of several composites indicates applicability and limitations of the analysis.

Results of this study can be used for more efficient designs of structural components (thus resulting in a reduction in the amount of wood needed) in built-up panels to allow wider choice of materials for possible fabrication at the building site, or for other similar applications.

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COMPOSITE BEAMS--EFFECT OF ADHESIVE OR FASTENER RIGIDITY

by

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INTRODUCTION

Maximum utilization of materials in structural components can only be realized by rational design of composites wherein materials or portions of the cross sections of composites are held together rigidly. Fastening methods which are less than infinitely rigid will require more materials to increase dimensions to provide needed rigidity.

Built-up structural pieces with complete interaction, such as can be made by gluing the pieces together with a rigid adhesive, or with no interaction have been analyzed to determine their deflection and strength. However, pieces fastened with nails or mastics where there is only partial interaction have not been analyzed. If an analysis could be made, the additional stiffness provided by partial interaction could be taken into account in the design of structures.

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[1] Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

PAST WORK

A study by Newmark[2] dealt with incomplete interaction of composite steel and concrete T-beams consisting of two interacting elements. A theoretical analysis was developed which incorporated the load-slip characteristics of steel channel shear connectors. The degree of interaction for these beams was such that the theoretical values for complete and incomplete interaction differed only slightly. However, the test results generally agreed with the theory.

OBJECTIVE

This study was to determine a theoretical means of determining the deflections and stresses for composite wood beams assembled with adhesives or fasteners having finite rigidity, and to verify the theory experimentally. This would provide a means for rational design of various composites if the properties of the components, including fastener rigidity, were known.

THEORETICAL ANALYSIS

The mathematical analysis of sandwich constructions having stiff facings bonded to a core of low shear rigidity as derived in 1505A[3] is utilized to define parameters and arrive at theoretical expressions for deflections and stresses of composite beams assembled with adhesives or fastenings having finite rigidity.-

For a beam of composite construction, simply supported at the reactions and loaded at two points a distance <u>kL</u> from the reactions as shown in figure 1,

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[2] Newmark, N. M., Siess, C. P., and Viest, I. M. "Test and analysis of composite beams with incomplete interaction." Society for Experimental Stress Analysis, Vol. 9, No. 1, 1951.

[3] Norris, Charles B., Ericksen, Wilhelm S., and Kommers, Wm. J. "Flexural rigidity of a rectangular strip of sandwich construction--comparison between mathematical analysis and results of tests." Forest Prod. Lab. Rep. 1505A. May 1952.

Figure 1.--Beam loading. M 138 607

the midspan deflection is given by the following expression derived from the mathematical analysis:[3]

$$\Delta = \frac{k(3-4k^2)PL^3}{48(EI)} \left\{ 1 + \frac{6}{(3-4k^2)} \left[\frac{(EI)}{(EI)_u} - 1 \right] \left(\frac{2}{\alpha L} \right)^2 \left(1 - \frac{\sinh \alpha kL}{\alpha kL \cosh \frac{\alpha L}{2}} \right) \right\} \quad (1)$$

where Δ is midspan deflection
P is applied load
L is span length
k defines load position as shown in figure 1
(EI) is stiffness of composite beam as if parts were glued together with a rigid adhesive
$(EI)_u$ is stiffness of all beam parts as if unglued

$$\alpha^2 = \frac{h^2 S}{(EI) - (EI)_u} \left[\frac{(EI)}{(EI)_u} \right] \quad (2)$$

h is distance between centroids of principal moment-carrying members
S is shear load per unit span length to cause unit slip between principal moment-carrying members.[4]

[4]Values of S can be determined from the slope of a shear-slip curve for the type of fastening employed (adhesive or nails). The shear-slip curve can be obtained from a small shear specimen joined with the same adhesives or fastenings as the composite beam. The width of the specimen should be equal to that of a composite beam shear joint width for adhesives or should contain the same number of fasteners such as nails per width of beam shear joint as in the composite beam. An effective joint shear rigidity, γ, with units such as pounds per inch of length per inch of slip, can be obtained by dividing the slope of the shear-slip curve by the length of the shear specimen for adhesives or by the nail spacing in the composite beam. S is given by $S = \frac{n}{m} \gamma$, where n is the number of shear planes across the width of the composite beam and m is the number of shear planes through the depth of the beam. Use of this formula for S with beams of more than two layers may only give rough approximations because it assumes each slip-layer has the same amount of slip.

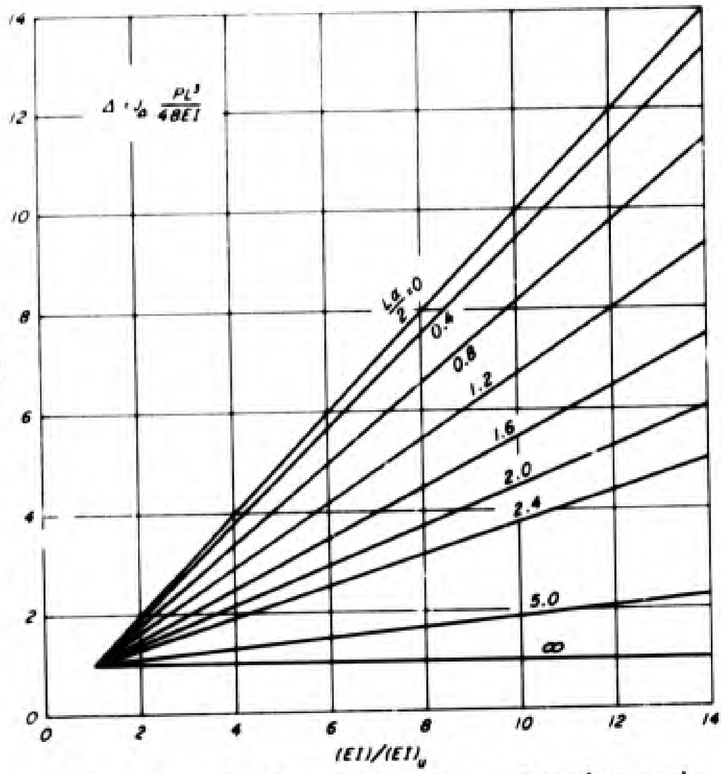

Figure 2.--Design chart for determining J_Δ for computing deflection of beam under midspan load. M 138 598

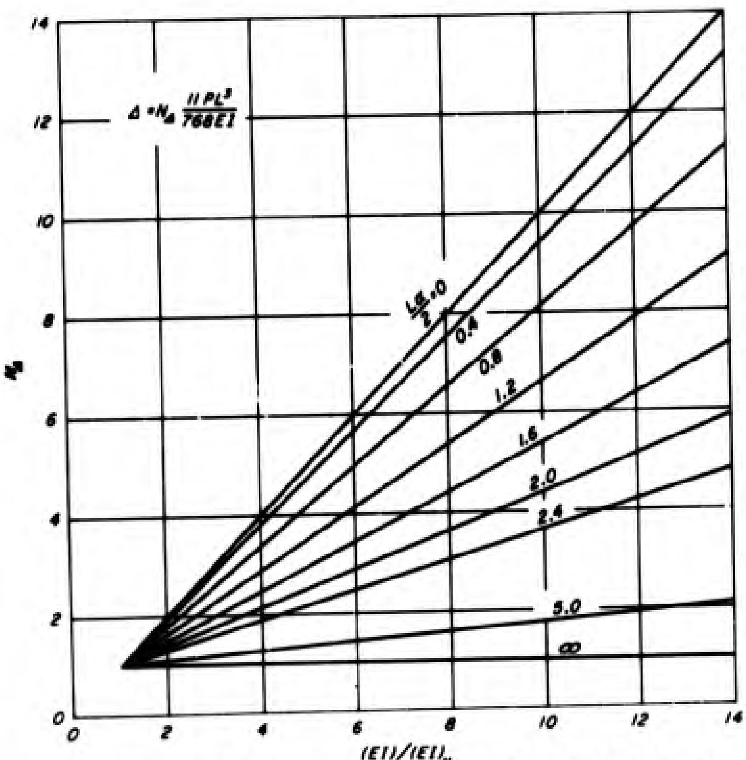

Figure 3.--Design chart for determining N_Δ for computing deflection of beam loaded at quarter points. M 138 599

Formula (1) for several familiar loadings becomes:

For k = 1/2 (midspan loading) $\Delta = J_\Delta \frac{PL^3}{48(EI)}$

where

$$J_\Delta = 1 + 3\left[\frac{(EI)}{(EI)_u} - 1\right]\left(\frac{2}{\alpha L}\right)^2\left(1 - \frac{\tanh\frac{\alpha L}{2}}{\frac{\alpha L}{2}}\right) \quad (3)$$

For k = 1/4 $\Delta = N_\Delta \frac{11\ PL^3}{768(EI)}$

where

$$N_\Delta = 1 + \frac{24}{11}\left[\frac{(EI)}{(EI)_u} - 1\right]\left(\frac{2}{\alpha L}\right)^2\left(1 - \frac{\sinh\frac{\alpha L}{4}}{\frac{\alpha L}{4}\cosh\frac{\alpha L}{2}}\right) \quad (4)$$

For a beam, simply supported at the reactions, and loaded with a uniformly distributed load, the midspan deflection is given by:[5]

$$\Delta = K_\Delta \frac{5\ WL^3}{384(EI)}$$

where

$$K_\Delta = 1 + \frac{12}{5}\left[\frac{(EI)}{(EI)_u} - 1\right]\left(\frac{2}{\alpha L}\right)^2\left[1 - 2\left(\frac{2}{\alpha L}\right)^2\left(1 - \frac{1}{\cosh\frac{\alpha L}{2}}\right)\right] \quad (5)$$

where $\underline{W}$ is the total load uniformly distributed.

Graphs of J_Δ, N_Δ, and K_Δ are shown as families of straight lines having slopes dependent upon the parameter $\left(\frac{\alpha L}{2}\right)$ in figures 2, 3, and 4.

The shear stress is maximum at the reactions and is assumed to be constant throughout the thickness of an adhesive joint and inner members in a composite beam. For beams under concentrated loads, figure 1, the shear stress at the reactions is given by the formula:

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[5] This case was not covered in 1505A. A derivation based on that reference is given in appendix A.

$$f_s = \frac{P}{2hb}\left[1 - \frac{(EI)_u}{(EI)}\right]\left[1 - \frac{\cosh \frac{\alpha(1-2k)L}{2}}{\cosh \frac{\alpha L}{2}}\right] \quad (6)$$

where $\underline{b}$ is the width of the joined surface.

For k = 1/2 (midspan loading) $f_s = J_\delta \frac{P}{hb}$

where

$$J_\delta = \frac{1}{2}\left[1 - \frac{(EI)_u}{(EI)}\right]\left(1 - \frac{1}{\cosh \frac{\alpha L}{2}}\right) \quad (7)$$

For k = 1/4 $f_s = N_\delta \frac{P}{hb}$

where

$$N_\delta = \frac{1}{2}\left[1 - \frac{(EI)_u}{(EI)}\right]\left(1 - \frac{\cosh \frac{\alpha L}{4}}{\cosh \frac{\alpha L}{2}}\right) \quad (8)$$

For beams under uniformly distributed load, the shear stress at the reaction is

$$f_s = K_\delta \frac{W}{hb}$$

where

$$K_\delta = \frac{1}{2}\left[1 - \frac{(EI)_u}{(EI)}\right]\left(1 - \frac{\tanh \frac{\alpha L}{2}}{\frac{\alpha L}{2}}\right) \quad (9)$$

Graphs of J_δ, N_δ, and K_δ are shown in figures 5, 6, and 7.

The shear slip between principal moment-carrying members of the composite beam is given by the formula:

$$\delta = \frac{f_s b}{S} \quad (10)$$

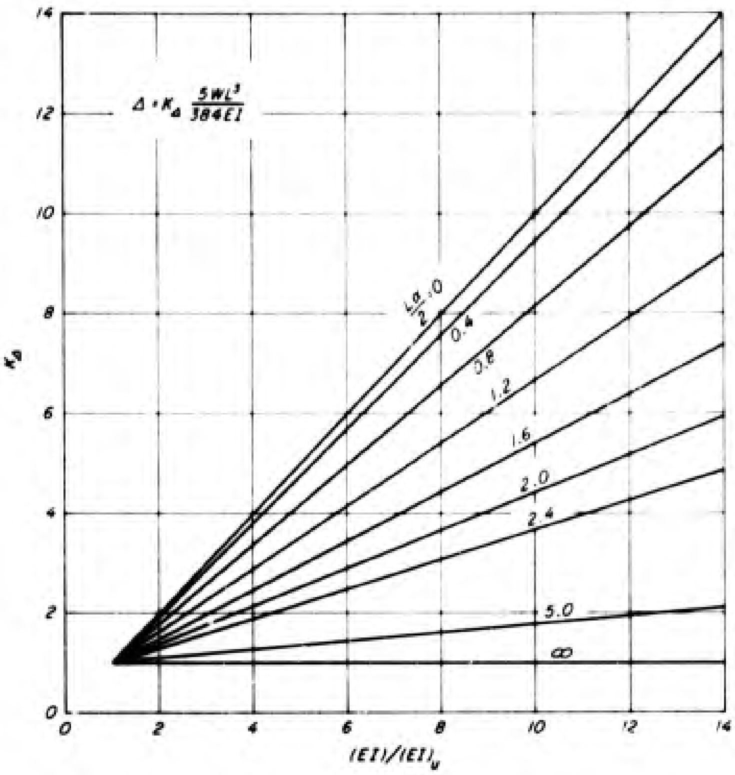

Figure 4.--Design chart for determining $\underline{K_\Delta}$ for computing deflection of beam under uniformly distributed load. M 138 605

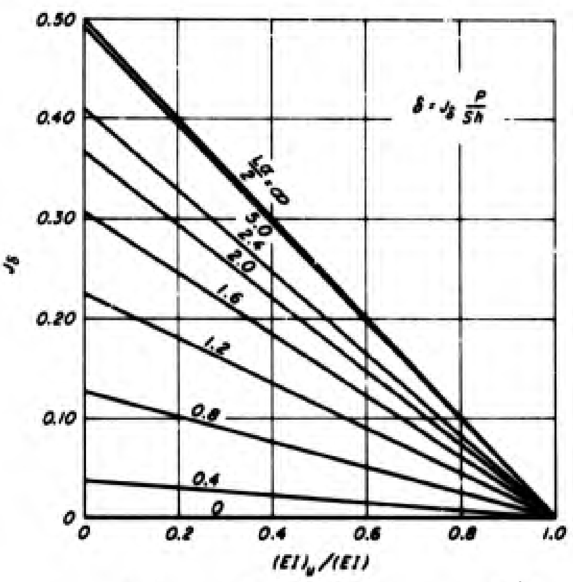

Figure 5.--Design chart for determining $\underline{J_\delta}$ for computing shear-slip of beam under midspan load. M 138 601

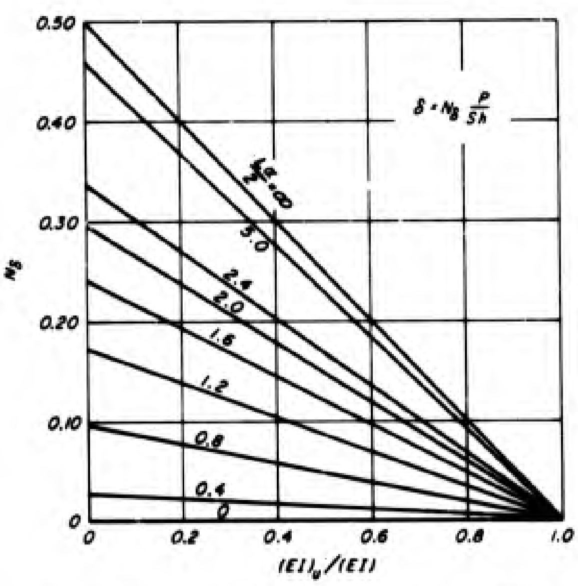

Figure 6.--Design chart for determining $\underline{N_\delta}$ for computing shear-slip of beam under quarter point load. M 138 600

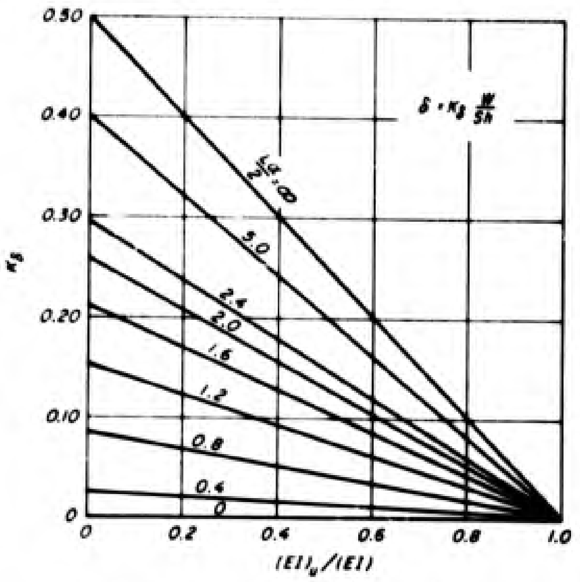

Figure 7.--Design chart for determining K_δ for computing shear-slip of beam under uniformly distributed load. M 138 604

Maximum compression and tension stresses occur at the surfaces of the beam and at the load points for beams with two concentrated loads, each a distance kL from each reaction, and at midspan for beams with load uniformly distributed. These maximum stresses are given by the following formulas:

For beam under concentrated loads--

$$f_c = \frac{PkLE_c h}{4(EI)} \left\{ \frac{2(EI)}{E_c A_c h^2}\left[1 - \frac{(EI)_u}{(EI)}\right] + \frac{t_c}{h} - \left[\frac{(EI)}{(EI)_u} - 1\right]\left[\frac{2(EI)}{E_c A_c h^2}\left(2 - \frac{(EI)_u}{(EI)}\right) + \frac{t_c}{h}\right] \times \frac{\cosh \frac{\alpha(1-2k)L}{2} \sinh \alpha kL}{\alpha kL \cosh \frac{\alpha L}{2}} \right\} \qquad (11)$$

$$f_t = \frac{PkLE_t h}{4(EI)} \left\{ \frac{2(EI)}{E_t A_t h^2}\left[1 - \frac{(EI)_u}{(EI)}\right] + \frac{t_t}{h} - \left[\frac{(EI)}{(EI)_u} - 1\right]\left[\frac{2(EI)}{E_t A_t h^2}\left(2 - \frac{(EI)_u}{(EI)}\right) + \frac{t_t}{h}\right] \times \frac{\cosh \frac{\alpha(1-2k)L}{2} \sinh \alpha kL}{\alpha kL \cosh \frac{\alpha L}{2}} \right\} \qquad (12)$$

For beams under uniformly distributed loads--

$$f_c = \frac{WLE_c h}{16(EI)} \left\{ \frac{t_c}{h} + \frac{2(EI)}{E_c A_c h^2} \left[1 - \frac{(EI)_u}{(EI)} \right] + \left[\frac{(EI)}{(EI)_u} - 1 \right] \left[\frac{t_c}{h} - \frac{2(EI)_u}{E_c A_c h^2} \right] \frac{8}{(\alpha L)^2} \times \left(1 - \frac{1}{\cosh \frac{\alpha L}{2}} \right) \right\} \quad (13)$$

$$f_t = \frac{WLE_t h}{16(EI)} \left\{ \frac{t_t}{h} + \frac{2(EI)}{E_t A_t h^2} \left[1 - \frac{(EI)_u}{(EI)} \right] + \left[\frac{(EI)}{(EI)_u} - 1 \right] \left[\frac{t_t}{h} - \frac{2(EI)_u}{E_t A_t h^2} \right] \frac{8}{(\alpha L)^2} \times \left(1 - \frac{1}{\cosh \frac{\alpha L}{2}} \right) \right\} \quad (14)$$

where subscripts $\underline{c}$ and $\underline{t}$ denote quantities pertaining to the principal compression and tension components, respectively. $\underline{A}$ denotes cross sectional area and $\underline{E}$ is the modulus of elasticity of these components.

EXPERIMENTAL VERIFICATION

Description of Specimens

The composite beams consisting of T-beams, stressed-skin panels, box beams, and laminated beams used to verify the theoretical expressions are described in table 1. Variation in shear stiffness of the joints between members was obtained by using different nailings or adhesives as described in column 2 of table 1. The plywood faces for panels 1 through 11 were one piece, obtained by gluing two 2-foot-wide pieces end to end with a 1:16 slope scarf joint where necessary. All lumber was dry material with a moisture content of 12 to 15 percent.

All nails were driven in prebored holes, 90 percent of the nail diameter. A gap of about 1/32 inch was left between elements of all nailed beams to eliminate friction. This simulates actual use conditions that may prevail after changes in moisture content of the material.

A controlled glueline thickness of 1/32 inch was used with all beams constructed with construction mastic, except for plywood panel No. 11 which was assembled by using ordinary nail-gluing techniques which resulted in irregular glueline thickness and width. Nails were removed before testing.

The length of all beams was slightly longer than the indicated spans in table 1.

Test Procedure

The beams were loaded with line loads across their width at midspan or quarter span as indicated in table 1. Deflections at midspan were measured with dial gages, reading to the nearest 0.001 inch, mounted on yokes which were suspended from the beams at the reactions. Shear slip between the principal moment-carrying members was measured with a dial gage, reading to the nearest 0.0001 inch, mounted on the beam at the reaction.

Data were collected in the form of load-deflection and load-shear slip curves. Before obtaining these curves for nailed beams, the beams were subjected to two to three cycles of preloading. This has been found to result in linear load-shear slip curves. The level of load was limited to that which would not cause a slip greater than 0.012 inch at the highest stressed nail (directly over the reaction).

Face strain, 6 inches from midspan, was measured with a Tuckerman strain gage on plywood panel No. 8 and with an electrical-resistance strain gage on the five-layer laminated beam, No. 22.

Data for computing S values for nailed joints were based on previous theoretical considerations of the lateral resistance of nailed joints.[6] The values of the load-slip curve slopes for nailed joints were computed for Douglas-fir having an average specific gravity[7] of 0.45 and for white pine having an average specific gravity[8] of 0.38.

Shear load-slip data for computation of S values were obtained for joints with construction mastic adhesives by testing small shear specimens. These specimens were 5 to 6 inches in length, about 1-1/2 inches wide, and had an adhesive layer controlled to a thickness of 1/32 inch by using wood veneer shims 1/8 inch wide at the specimen edges. The specimen was made 2 inches wide and the shims were sawn off when the specimen was cut to 1-1/2-inch width after proper curing of the adhesive. The shear test apparatus with a dial gage mounted on the specimen for measuring shear slip is shown in figure 8. A shear stress-slip curve obtained for a construction mastic adhesive is shown in figure 9. The curve does not have a linear portion during initial loading; however, on second and subsequent loadings a short linear portion that appeared at low slip was used to compute an S value. In using test data to compute S, it should be recalled that the rigidity of the adhesive joint will vary inversely as the adhesive thickness.

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[6] Wilkinson, Thomas Lee. "Theoretical lateral resistance of nailed joints." (Submitted to ASCE for publication.)

[7] U.S. Forest Service. Western Wood Density Survey, Report No. 1. U.S. Forest Service Res. Pap. FPL 27. July 1965.

[8] Forest Products Laboratory. Wood Handbook. U.S.D.A. Agr. Handb. 72, 1955.

Table 1.--Construction and properties of composite beams

Construction	Fastening	Properties: S	Properties: (EI)	Properties: $(EI)_u$	$\frac{(EI)}{(EI)_u}$	Loading	Span	J_Δ or N_Δ	Load-deflection ratio: Exp.	Load-deflection ratio: Theo.	Load shear slip[1] ratio: Exp.	Load shear slip[1] ratio: Theo.	Load-facing strain ratio 6 in. from center: Exp.	Load-facing strain ratio 6 in. from center: Theo.
(1)	(2)	(3)			(4)	(5)	(6)	(7)	(8)		(9)		(10)	
		Lb.-in.^{2}	Million lb.-in.^{2}	Million lb.-in.^{2}			In.							
1 Face--1/2-inch Douglas-fir plywood, 32 inches wide Stringers--nominal 2 by 4 Douglas-fir	6d nails, 6 inches on center	2,720	57.2	19.2	2.98	mid span	90	2.23	1,490	1,690	25,000	35,200	--	---
2 Faces--1/2-inch Douglas-fir plywood, 32 inches wide: Stringers--nominal 2 by 4 Douglas-fir	do.	1,360	170.9	19.6	8.72	do.	90	3.89	2,140	2,800	15,400	20,200	--	--
3 Do.	6d nails, 3 inches on center	2,720	170.9	19.6	8.72	do.	90	3.58	2,720	3,140	25,500	30,900	--	--
4 Face--1/2-inch Douglas-fir plywood, 24 inches wide Stringers--nominal 2 by 4 Douglas-fir	6d nails, 6 inches on center	2,720	45.8	14.8	3.09	do.	90	2.13	1,500	1,350	37,000	28,200	--	--
5 Faces--1/2-inch Douglas-fir plywood, 24 inches wide: Stringers--nominal 2 by 4 Douglas-fir	do.	1,360	156.5	15.3	10.23	do.	90	5.09	2,100	2,030	22,500	17,900	--	--
6 Do.	6d nails, 3 inches on center	2,720	156.5	15.3	10.23	do.	90	3.64	2,560	2,840	27,200	28,600	--	--
7 Face--3/4-inch Douglas-fir plywood, 24 inches wide Stringers--nominal 2 by 6 Douglas-fir	8d nails, 12 inches on center	1,840	223.0	87.0	2.56	do.	144	1.94	1,800	1,850	43,500	37,600	--	--
8 Do.	8d nails, 6 inches on center	3,680	223.0	87.0	2.56	do.	144	1.67	1,960	2,140	58,000	[illegible]3,900	5.85 x 10^6	6.80 x 10^6
9 Faces--3/4-inch Douglas-fir plywood, 24 inches wide: Stringers--nominal 2 by 6 Douglas-fir	do.	1,840	488.0	88.0	5.55	do.	144	2.76	2,780	2,840	37,400	37,000	--	--
10 Do.	8d nails, 3 inches on center	3,680	488.0	88.0	5.55	do.	144	2.11	3,630	3,710	60,100	55,600	--	--
11 Do.	Construction mastic	6,130	488.0	88.0	5.55	do.	144	1.74	4,390	4,530	93,500	96,000	--	--
12 3/4 by 1-1/2-inch Douglas-fir	do.	3,100	1.04	.24	4,34	do.	52	1.92	190	186	7,500	7,340	--	--
" 13 Do.	do.	3,100	1.04	.24	4.34	quarter span	52	1.87	297	278	9,000	8,790	--	--
" 14 Do.	do.	3,100	1.04	.24	4.34	mid span	26	3.00	1,050	950	12,000	12,600	--	--
" 15 Do.	do.	3,100	1.04	.24	4.34	quarter span	26	2.96	1,420	1,400	15,400	16,250	--	--
16 Do.	6d nails, 2 inches on center	5,280	.93	.23	4.04	mid span	46	1.67	290	276	--	--	--	--
" 17 Do.	6d nails, 4 inches on center	2,640	.93	.23	4.04	do.	46	2.02	220	228	--	--	--	--
" 18 Do.	6d nails, 2 inches on center	5,280	.93	.23	4.04	quarter span	46	1.62	450	410	--	--	--	--
" 19 Do.	6d nails, 4 inches on center	2,640	.93	.23	4.04	do.	46	1.98	330	338	--	--	--	--
20 Stringers--nominal 2 by 4 Douglas-fir Webs--1/2-inch Douglas-fir plywood, 9-1/2 inches deep	6d nails, 6 inches on center	1,360	427.2	111.9	3.82	mid span	90	2.53	11,160	11,150	44,800	52,700	--	--
" 21 Do.	6d nails, 3 inches on center	2,720	427.2	111.9	3.82	do.	90	2.05	12,270	13,700	85,400	78,400		
22 0.75 by 3.94-inch Douglas-fir (5 layers)	Construction mastic	1,940	26.0	.94	27.65	do.	90	2.68	709	671	21,300	12,600	0.455 x 10^6	0.194 x 10^6
23 Do.	do.	1,940	26.0	.94	27.65	quarter span	90	2.42	1,160	1,080	26,300	13,000	--	--
24 1-3/8- by 5-1/2-inch white pine (4 layers)	Two rows of 8d nails, spaced 6 inches on center	710	114.0	7.5	15.20	mid span	68	8.72	2,150	2,000	600	420	--	--

[1]Slip is the movement at the reaction between the top and bottom pieces.

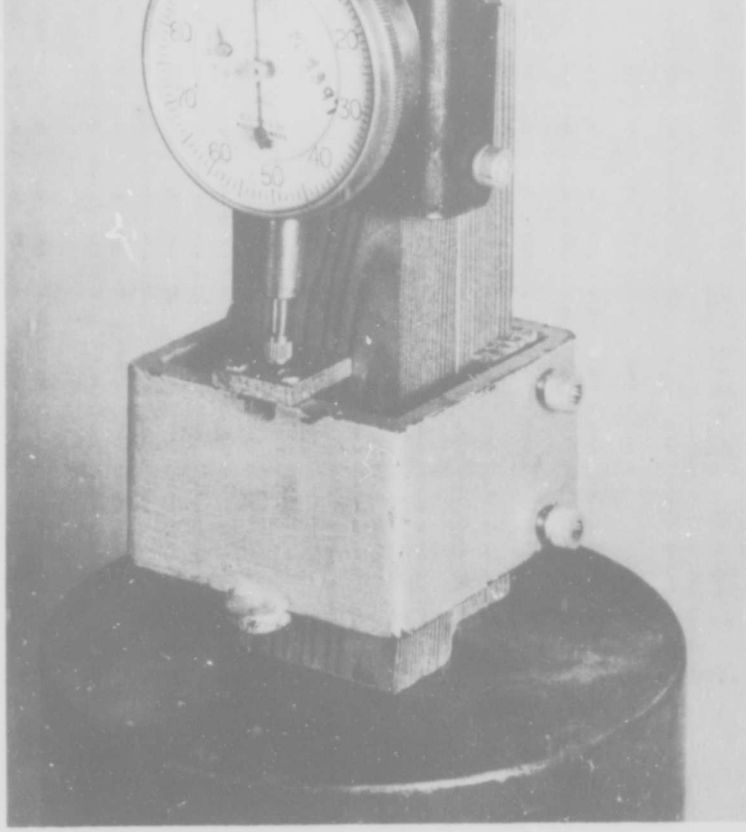

Figure 8.--Shear test apparatus for obtaining load-slip data for construction mastic adhesive. M 138 260

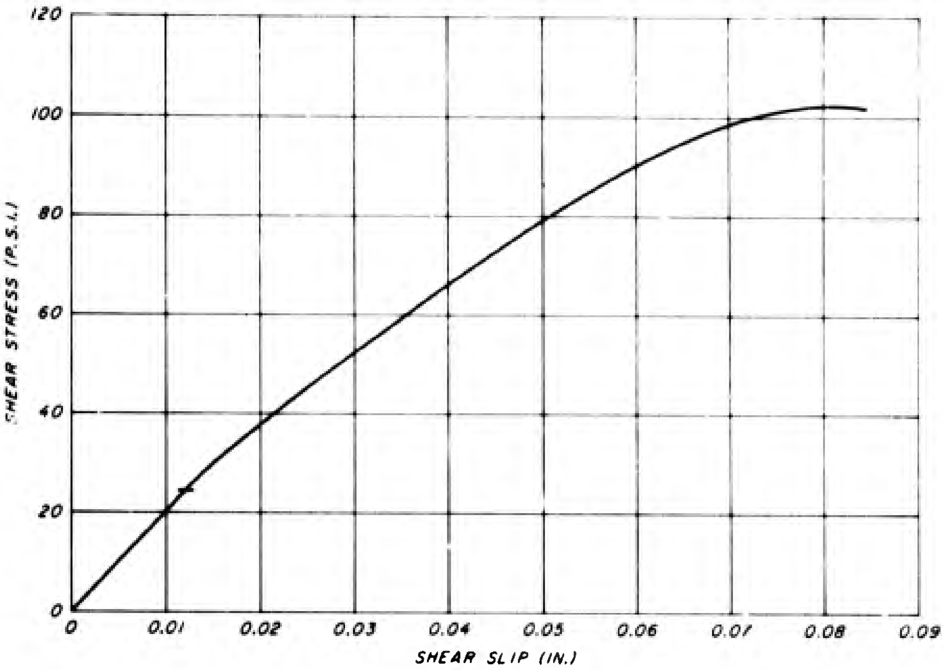

Figure 9.--Typical shear stress-slip curve for a construction mastic 1/32 inch thick. M 138 606

The rectangular beams No. 12-19 and 22, 23, and 24 were square in cross section and their stiffness as if glued with a rigid adhesive, $\underline{(EI)}$, was determined by loading the beams with the laminations vertical. For the other constructions, the stiffness of the individual elements was measured before assembling the beams; from the dimensions of the components and their elastic moduli, the stiffness as if rigidly glued, $\underline{(EI)}$, was computed with basic mechanics theory.

Results

Composite beam properties and experimental and theoretical results are given in table 1. Details of the composite beam constructions and properties are given in columns 1, 2, and 3.

Ratios of $(EI)/(EI)_u$ for the various constructions as given in column 4 ranged from 2.56 to 27.65, thus covering a wide variety of composites with which to check the effectiveness of joint rigidities. Column 7 gives values of $\underline{J}_\Delta$ or $\underline{N}_\Delta$ which range from 1.62 to 8.72. These computed values of $\underline{J}_\Delta$ or $\underline{N}_\Delta$ are the ratios of beam deflections with joint slip to deflections of beams with rigid joints. Thus the computed deflections of the composite beams were from 1.62 to 8.72 times the deflections of the beams if rigidly glued or somewhat less than for beams with no glue as indicated by the larger values of the ratios of $(EI)/(EI)_u$ in column 4.

Load-deflection ratios tabulated in column 8 show there was good agreement between theoretical and experimental deflection for the beams evaluated. A graphical representation of theoretical and experimental load-deflection ratios in figure 10 shows good agreement between experimental and theoretical values. This is probably the most important property to be able to predict for composite beams, as it will quite often govern their design in many structures. Fair agreement was found between theoretical and experimental load-shear slip ratios tabulated in column 9 and shown in the graph of figure 11. The notable exceptions were the four- and five-layer laminated beams. This lack of agreement is probably due to the assumption of constant shear stress throughout the thickness of all inner members.

Comparison of theoretical and experimental face strain, table 1, shows good agreement for the plywood panel and poor agreement for the five-layer laminated beam. This poor agreement is again probably due to the assumption of constant shear stress throughout the inner members.

The general agreement between experimental and theoretical values enables use of the theory to predict effects of time, temperature, moisture, etc., on composites by evaluating basic properties E and S as affected by these same variables of time, temperature, moisture, etc. Thus evaluation of E and S can be determined on small specimens rather than evaluating large composites to assess the effects of these variables.

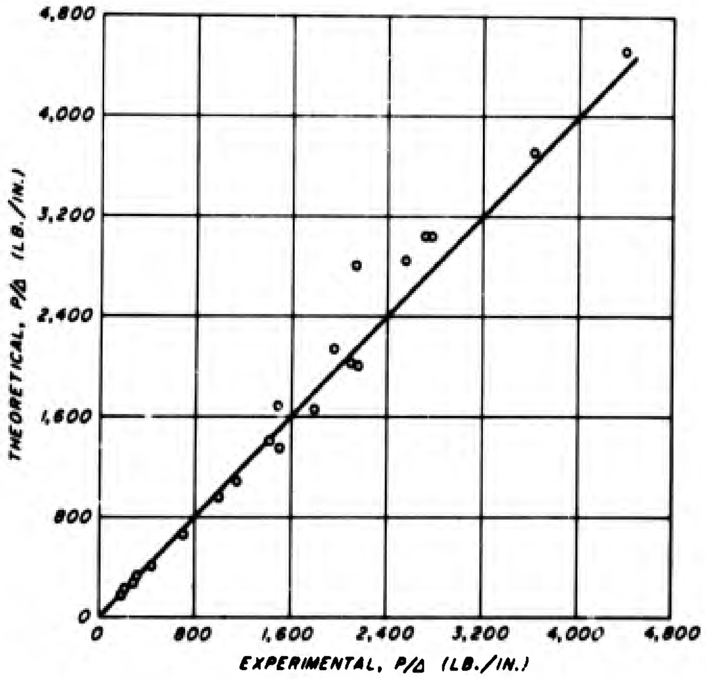

Figure 10.--Comparison of theoretical and experimental load-deflection ratios.

M 138 603

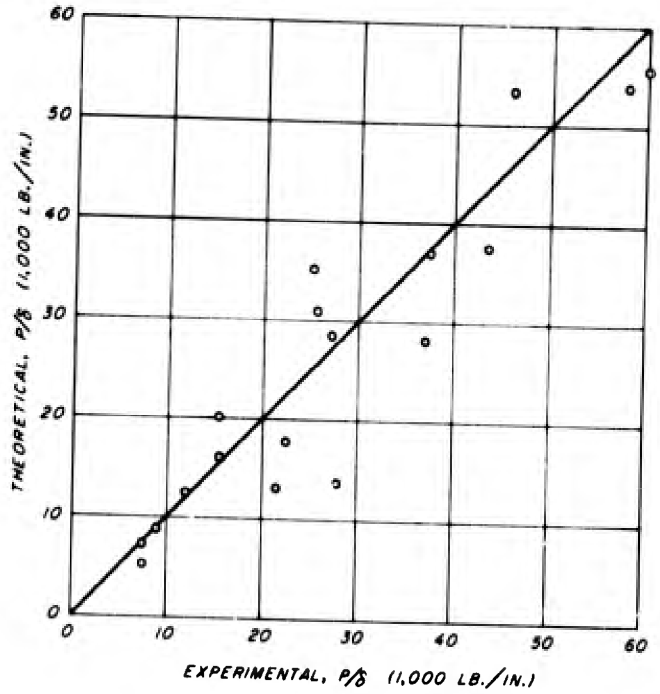

Figure 11.--Comparison of theoretical and experimental load-shear slip ratios.

M 138 602

CONCLUSIONS

The good agreement of experiment and theory for deflection and shear-slip of composite beams of two or three continuous layers demonstrates that the theory presented can be used for rational design of members with finite fastener rigidity.

APPENDIX A

Derivation of Formulas For Beam Under Uniformly Distributed Load*

For a simple supported beam under a total load W that is uniformly distributed along a beam of length L, the bending moment at a point located a distance x from a reaction is given by:

$$M = \frac{W}{2} x - \frac{W}{2L} x^2 \tag{A1}$$

and the shear load by:

$$V = \frac{W}{2} - \frac{W}{L} x \tag{A2}$$

The analysis given in the appendix of FPL 1505A derives the differential equation for the shear stress, τ, to be:

$$\frac{d^2\tau}{dx^2} - \alpha^2\tau = -\frac{\alpha^2}{h}\left[1 - \frac{(EI)_u}{(EI)}\right] V \tag{A3}$$

with terms defined in the Theoretical Analysis section of this report.

Substitution of (A2) into (A3) results in:

$$\frac{d^2\tau}{dx^2} - \alpha^2\tau = -\frac{\alpha^2}{h}\left[1 - \frac{(EI)_u}{(EI)}\right]\left[\frac{W}{2} - \frac{W}{L} x\right] \tag{A4}$$

which has a solution:

$$\tau = A \cosh \alpha x + B \sinh \alpha x + \frac{1}{h}\left[1 - \frac{(EI)_u}{(EI)}\right]\left(\frac{W}{2} - \frac{W}{L} x\right) \tag{A5}$$

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*Derived from FPL Report 1505A by Norris, Ericksen, and Kommers (footnote 2).

$$\text{at } x = 0 \qquad \frac{d\tau}{dx} = 0, \text{ therefore, } B = \frac{W}{\alpha L h}\left[1 - \frac{(EI)_u}{(EI)}\right] \tag{A6}$$

$$\text{at } x = \frac{L}{2} \qquad \tau = 0, \text{ therefore, } A = -\frac{W}{\alpha L h}\left[1 - \frac{(EI)_u}{(EI)}\right] \tanh \frac{\alpha L}{2} \tag{A7}$$

and substitution of (A6) and (A7) into (A5) results in:

$$\tau = \frac{W}{h}\left[1 - \frac{(EI)_u}{(EI)}\right]\left[\frac{1}{\alpha L} \sinh \alpha x - \frac{1}{\alpha L} \tanh \frac{\alpha L}{2} \cosh \alpha x + \frac{1}{2} - \frac{x}{L}\right] \tag{A8}$$

The analysis in the appendix of FPL 1505A leads to the differential equation for the deflection, Δ:

$$\frac{d^2\Delta}{dx^2} - \frac{h}{\alpha^2 (EI)_u} \frac{d\tau}{dx} = -\frac{M}{(EI)} \tag{A9}$$

which after integration and evaluation of integration constants, results in:

$$\Delta = \frac{WL^3}{24(EI)}\left[\left(\frac{x}{L}\right)^4 - 2\left(\frac{x}{L}\right)^3 + \left(\frac{x}{L}\right)\right] + \frac{WL^3}{(\alpha L)^2 (EI)}\left[\frac{(EI)}{(EI)_u} - 1\right] \times$$

$$\left[\frac{1}{(\alpha L)^2} \cosh \alpha x - \frac{1}{(\alpha L)^2} \tanh \frac{\alpha L}{2} \sinh \frac{\alpha L}{2} + \frac{x}{2L} - \frac{x^2}{2L^2} - \frac{1}{(\alpha L)^2}\right] \tag{A10}$$

and evaluation of formula (A10) at midspan leads eventually to formula (5).

APPENDIX B

Sample Calculation

Determine the deflection and shear slip at the reaction for a stressed-skin panel 48 inches wide having 3/4-inch Douglas-fir plywood faces on three Douglas-fir stringers each 1-1/2 inches wide and 3-1/2 inches deep. The beam is subjected to a concentrated load of P pounds at the middle of its 92-1/2-inch span. The plywood is nailed to the stringers with eightpenny common nails, spaced 8 inches on center. The following properties are known:

(1) Slope of the load-slip curve for an eightpenny nail--16,800 lb. per in.

(2) Modulus of elasticity of Douglas-fir--E_ℓ = 1,800,000 lb. per in.^{2}

(3) Bending stiffness of 3/4-inch plywood--$(EI)_p = 0.023\ E_\ell$ lb.-in.^{2} per in. of width, and

(4) Extensional stiffness of 3/4-inch plywood--$(EA)_p = 0.42\ E_\ell$ lb.-in.^{2} per in. of width.

Solution:

From footnote 4, $S = \frac{n}{m}\gamma = \frac{3}{2}\frac{16,800}{8} = 3,150$ lb. per in.^{2} (B1)

Stringer bending stiffness $(EI)_s = \frac{(1.5)(3.5)^3}{12}(1.8 \times 10^6) = 9.65 \times 10^6$ lb.-in.^{2} for each stringer (B2)

Plywood bending stiffness $(EI)_p = (0.023)(48)(1.8 \times 10^6) = 1.99 \times 10^6$ lb.-in.^{2} for each skin. (B3)

$$(EI)_u = \left[3(9.65) + 2(1.99)\right] \times 10^6 = 32.93 \times 10^6 \text{ lb.-in.}^2 \quad \text{(B4)}$$

$$(EI) = \frac{1}{2}(EA)_p h^2 + (EI)_u = \frac{(0.42)(48)(1.8 \times 10^6)(4.25)^2}{2} + 32.93 \times 10^6$$

$$= 327.0 \times 10^6 + 32.93 \times 10^6 = 359.9 \times 10^6 \text{ lb.-in.}^2 \quad \text{(B5)}$$

$$\frac{(EI)}{(EI)_u} = \frac{359.9 \times 10^6}{32.93 \times 10^6} = 10.94 \qquad \text{(B6)}$$

$$\frac{(EI)_u}{(EI)} = 0.091 \qquad \text{(B7)}$$

From formula (2),

$$\alpha^2 = \frac{h^2 S}{(EI) - (EI)_u}\left[\frac{(EI)}{(EI)_u}\right] = \frac{(4.25)^2 (3150)}{327.0 \times 10^6}(10.94) = 1,880 \times 10^{-6} \text{ in.}^{-2} \qquad \text{(B8)}$$

$$\alpha = 0.0433 \text{ in.}^{-1} \qquad \text{(B9)}$$

$$\frac{\alpha L}{2} = \frac{(0.0433)(92.5)}{2} = 2.00 \qquad \text{(B10)}$$

From figure 2, $J_\Delta = 4.85$ and from figure 5, $J_\delta = 0.33$.

The midspan deflection is

$$\Delta = J_\Delta \frac{PL^3}{48EI} = (4.85)\frac{P(92.5)^3}{(48)(359.9 \times 10^6)} = 0.000222P \text{ in.} \qquad \text{(B11)}$$

which is 4.85 times the deflection of the panel if it were glued with a rigid adhesive.

The shear-slip between the two plywood faces at the reaction is:

$$\delta = J_\delta \frac{P}{Sh} = 0.33 \frac{P}{(3150)(4.25)} = 0.0000247P \text{ in.} \qquad \text{(B12)}$$

For a proportional limit slip of 0.012 in. per nail, the maximum shear slip between the plywood faces would be 0.024 inches, because of two shear planes, and the load would be

$$0.0000247P = 0.024$$

$$P = 970 \text{ lb.}$$

and at this load the deflection would be

$$\Delta = 0.000222(970) = 0.216 \text{ in.}$$

with resulting span-deflection ratio of 428. Thus, the panel is amply stiff provided the load does not exceed 970 pounds.

APPENDIX C

Notation

$A_{c,t}$ - cross-sectional area of principal moment-carrying member; subscripts c denotes compression and t tension.

b - width of joined surface.

$E_{c,t}$ - modulus of elasticity of principal moment-carrying member; subscripts c denotes compression and t tension.

(EI) - stiffness of composite beam as if parts were glued together with a rigid adhesive.

$(EI)_u$ - stiffness of all beam parts as if unglued.

$f_{c,t}$ - maximum compression or tension stress at the surface of the beam; subscripts c denotes compression and t tension.

f_s - maximum shear stress.

h - distance between centroids of principal moment-carrying members.

J_Δ - function defined by formula (3).

J_δ - function defined by formula (7).

K_Δ - function defined by formula (5).

K_δ - function defined by formula (9).

k - constant defining load position.

L - span length.

M - bending moment.

m - number of shear planes through the depth of the beam.

N_Δ - function defined by formula (4).

N_δ - function defined by formula (8).

n - number of shear planes across the width of the beam.

P - concentrated load.

$\underline{S}$ - shear load per unit span length to cause unit slip between principal moment members. (For details see footnote 4.)

$\underline{t_{c,t}}$ - thickness of principal moment-carrying member; subscripts $\underline{c}$ denotes compression and $\underline{t}$ tension.

$\underline{V}$ - shear load.

$\underline{W}$ - total load, uniformly distributed.

$\underline{x}$ - variable distance from a reaction.

$\underline{\alpha}$ - function defined by formula (2).

$\underline{\Delta}$ - beam deflection.

$\underline{\delta}$ - shear slip between principal moment-carrying members.

$\underline{\tau}$ - shear stress.

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* Conducting forest and range research at over 75 locations ranging from Puerto Rico to Alaska to Hawaii.

* Participating with all State forestry agencies in cooperative programs to protect, improve, and wisely use our Country's 395 million acres of State, local, and private forest lands.

* Managing and protecting the 187-million acre National Forest System.

The Forest Service does this by encouraging use of the new knowledge that research scientists develop; by setting an example in managing, under sustained yield, the National Forests and Grasslands for multiple use purposes; and by cooperating with all States and with private citizens in their efforts to achieve better management, protection, and use of forest resources.

Traditionally, Forest Service people have been active members of the communities and towns in which they live and work. They strive to secure for all, continuous benefits from the Country's forest resources.

For more than 60 years, the Forest Service has been serving the Nation as a leading natural resource conservation agency.