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A GENERAL THEORY OF PARACHUTE OPENING

by

Edward W. Ross, Jr.



January 1971

UNITED STATES ARMY
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Natick, Massachusetts, 01760



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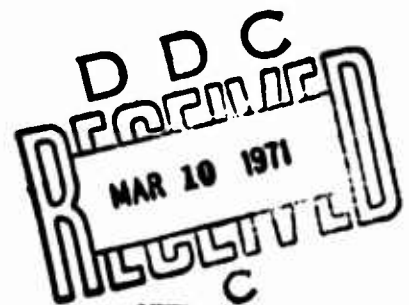
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FOREWORD

The design of parachutes cannot yet be done in a completely rational manner. One of the main reasons for this is that the opening process, during which the canopy experiences its most severe stresses, is not well understood. This report is an attempt to clarify our comprehension of parachute-opening and systematize the estimation of its main parameters.

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ABSTRACT

A simple mathematical model of the opening behavior of parachutes is presented. The model predicts the drag and velocity as functions of time and also gives an estimate of opening time. The model is very general and applicable to almost any parachute. The characteristics of a specific parachute enter the model by means of certain constants that have to be chosen. Values of these constants are found for flat, circular parachutes, partly by estimation and partly by numerical experimentation. The resulting model of the opening behavior of flat, circular canopies gives reasonable agreement with experiments for several parachutes of different sizes. Insofar as generality and flexibility are concerned, this model seems to be superior to present methods of predicting opening behavior of parachutes.

NOMENCLATURE

A	Maximum cross-sectional area of an ellipse
A_D	Drag Area
A_I	Effective net inlet area
A_p	Idealized projected area
a, b	Semi axes of an ellipse
C_D	Drag Coefficient based on frontal area
C_{g1}, C_{g2}	Constants in estimates of $\Theta(V)$, (21)
C_{m1}, C_{m2}	Constants in estimates of apparent mass, (19)
D	Drag
F	Function defined in (7)
g	Acceleration of gravity
H_D, H_I	Dimensionless functions introduced in (10)
h₀, h₁, h₂	Constants in the formula (35) for H_I(V)
m	Total effective mass of canopy-cargo system
m_a	Apparent mass of canopy
m_s	Mass of fully-opened canopy
m_o	Mass of cargo plus parachute
N	Number of mesh points in numerical calculations
P, Q	Functions introduced in (8) and (13)
R_o	Flat-circular radius of canopy
s	$u^2/2$, Function introduced in (5)
t	Time
t_i	Time at which inflation begins
t_f	Time at which full inflation is reached, $V = V_f$
t_s	Time at which $V = V_s$

u	Velocity of canopy and load
u_i, u_s	Velocities at start and end of inflation
u_∞	Velocity of steady descent
V	Volume of canopy
V_i, V_f	Initial and final volumes of canopy
V_s	$V_f - \Delta V$, (36)
$\delta_1, \delta_2, \delta_3, \delta_4$	Constants in estimate of H_D , (33)
ΔV	Increment of volume used in numerical integration
η	Function of ellipsoidal shape, (25)
θ	Angle between horizontal and parachute trajectory
λ	Constant specifying vent pull-down, (31)
ρ	Mass density of air
τ	Dimensionless time during steady descent, (40)
τ_s	Dimensionless time at start of full inflation.

1. Introduction

In attempting to improve the design of parachutes, especially with a view toward launching from a low altitude, it is desirable to understand and predict the behavior of parachutes during opening. Previous work on opening of parachutes ¹⁻⁵ has clarified a number of points, but the analyses have been either too specific (i.e. used experimental data for a single canopy), or contain questionable assumptions, or are simply too cumbersome for practical purposes. Since experimental results for parachutes are very inconsistent, it is pointless to develop a highly refined theory. For the present it appears that we need a relatively simple theory that gives roughly correct results for different kinds of canopies.

This paper will describe such a theory. The theory is formalized in a Fortran II program whose demands on the computer (as to speed and memory) are quite modest.

The procedure is to write the governing equations for parachute-opening in a form which is as precise as possible without containing any information that is specific to an individual parachute. The specific information is contained in certain input-functions which occur in the equations, but are left in general form at first. We assume that these functions depend on canopy volume, which is adopted as the independent variable in place of time. General solutions in the form of indefinite integrals are obtained for the period of canopy inflation. The solutions depend on the input-functions. Most of these can be estimated in advance, but the most important one, which prescribes the net flow rate into the canopy, is found by trying various functions

and choosing the one that gives the best agreement with experiments. For this purpose the theoretical predictions are compared with tests on three different, flat, circular canopies.

2. Analysis of Opening

During the opening period we assume that the direction of the canopy axis is approximately the same (Figure 1) as that of the canopy velocity vector, whose magnitude is called u . The governing equations are taken to be Conservation of Linear Momentum,

$$d(mu)/dt = m_0 g \sin \theta - D \quad (1)$$

Conservation of Mass,

$$dV/dt = A_I u \quad (2)$$

and the Fluid-Drag Hypothesis

$$D = (\rho/2) u^2 A_D \quad (3)$$

The various quantities are defined in the Nomenclature. The functions m , the total mass of the system, A_I , the effective net inlet area of the canopy, A_D , the drag area, and θ , the angle between horizontal and the velocity vector, contain the specific information for each parachute. Ultimately, we shall have to estimate each of these, but for the present we leave them in general form.

In adopting (1), (2) and (3) as our basic equations we are neglecting a number of factors, such as angular momentum and centrifugal force due to curvature of the trajectory. These factors are extraneous to those that are most typical of the opening period, and we wish to leave them aside, although we recognize that their effects are not always negligible in practice.

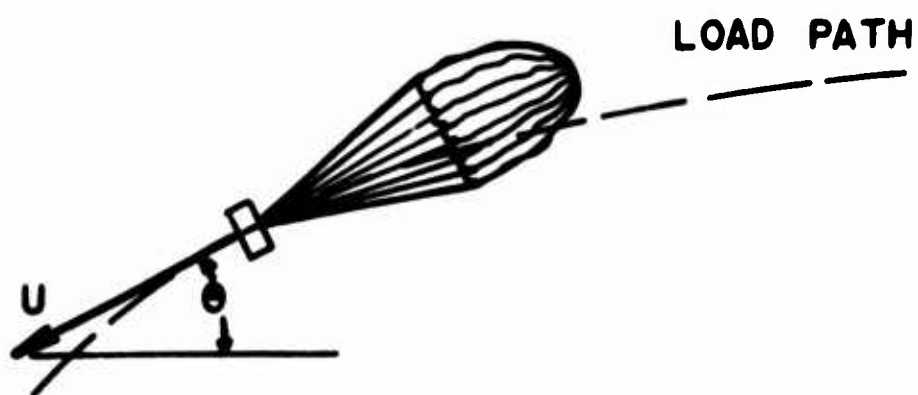


Figure 1: Parachute-Load System during Opening

The system (1), (2) and (3) can be reduced to a single equation by inserting (3) in (1), then dividing (1) by (2) and adopting the canopy volume, V , as independent variable.

$$u d(mu)/dV = (m_0 g \sin \theta / A_I) - (\rho A_D / A_I)(U^2/2) \quad (4)$$

In this form we assume that m , θ , A_I and A_D are functions of V .

If we set

$$s = U^2/2, \quad s_i = U_i^2/2$$

this can be reduced to a linear, ordinary differential equation of first order,

$$\begin{aligned} (ds/dV) + \left\{ (2/m)(dm/dV) + [\rho A_D / (m A_I)] \right\} s \\ = m_0 g \sin \theta / (m A_I) \end{aligned}$$

Such an equation may always be solved by finding a homogeneous solution and then using variation-of-parameters to find the particular solution. In view of the initial conditions the result is

$$s = \left\{ 1/F(V) \right\} \left\{ s_i + g m_0 \int_{V_i}^V \frac{F(V') \sin \theta(V') dV'}{m(V') A_I(V')} \right\} \quad (6)$$

where

$$F(V) = \left\{ m(V)/m(V_i) \right\}^2 e^{P(V)} \quad (7)$$

$$P(V) = \rho \int_{V_i}^V \left\{ A_D(V') / [m(V') A_I(V')] \right\} dV' \quad (8)$$

When $s(V)$ has been found, the other quantities of interest are determined by

$$D = \rho A_D S, \quad U = (2S)^{1/2} \quad (9)$$

$$t = t_1 + \int_{V_1}^V \frac{dV'}{U(V') A_I(V')}$$

Formulas (6) to (9) provide a general solution of the equations governing parachute opening, (1), (2) and (3). The only restriction on the validity of these equations is that the various functions must be such that the integrals can be evaluated.

It is useful now to introduce three new functions in place of A_D and A_I . Foremost among these is the projected area, $A_p(V)$. Two other functions, H_D and H_I , are defined in terms of A_D and A_I as follows:

$$A_D(V) = C_D A_p(V) H_D(V) \quad (10)$$

$$A_I(V) = A_p(V) H_I(V)$$

We must have $H_I(V) > 0$ during opening or we cannot use V as the independent variable.

Then in terms of these functions we may write the solution during inflation in the following form:

$$P(V) = \rho C_D \int_{V_1}^V \frac{H_D(V') dV'}{m(V') H_I(V')} \quad (11)$$

$$F(V) = \left[\frac{m(V)}{m(V_1)} \right]^2 e^{P(V)} \quad (12)$$

$$Q(V) = g m_c \int_{V_1}^V \frac{F(V') \sin \theta(V') dV'}{m(V') H_I(V') A_p(V')} \quad (13)$$

$$s(V) = \{s_1 + Q(V)\} / F(V) \quad (14)$$

$$D(V) = \rho C_D H_D(V) A_p(V) s(V) \quad (15)$$

$$U(V) = [2s(V)]^{1/2} \quad (16)$$

$$t(V) = t_1 + \int_{V_1}^V \frac{dV}{U(V) H_I(V) A_p(V)} \quad (17)$$

Equations (11) to (17) determine the solution to the parachute opening problem, in the sense that we can find $u(V)$, $D(V)$ and $t(V)$ and, therefore, also $u(t)$, $D(t)$ and $V(t)$. Up to this point the model is quite general. In the next Section we suggest estimates of the input functions, $A_p(V)$, $m(V)$, $\theta(V)$, $H_D(V)$ and $H_I(V)$ for the case of a solid, flat canopy.

3. The Input Functions for a Solid, Flat Canopy

Here we wish to discuss the hitherto arbitrary functions, m , $\sin \theta$, A_p , H_D and H_I , which affect our solution.

The simplest of these to estimate is the total mass, m . We can write m as

$$m(V) = m_0 + \rho V + m_a(V) \quad (18)$$

Here m_0 is the constant mass of the cargo and parachute, ρV is the mass of the included air and $m_a(V)$ is the apparent mass due to the deceleration. The apparent mass is rather difficult to estimate and is often thought³ to be unimportant. We shall assume a general form suggested by an earlier⁴ treatment,

$$m_a(V) = C_{m1} \rho V (V/V_f)^{C_{m2}} \quad (19)$$

Here C_{m1} and C_{m2} are constants and V_f is the volume when the canopy is fully open. The final formula for m is therefore

$$m = m_0 + \rho V \left\{ 1 + C_{m1} (V/V_f)^{C_{m2}} \right\} \quad (20)$$

Usually we take $C_{m1} = .375$ and $C_{m2} = .333$.

The quantity $\theta(V)$ is difficult to evaluate for the usual opening conditions. Two extreme cases are easily handled, however. If the canopy velocity vector is horizontal during the opening, then $\sin \theta = 0$ and

$$Q \equiv 0$$

This is a very great simplification, and is usually accurate when opening occurs soon after launching. At the other extreme, if the velocity vector is vertical during opening, as in the case where a free fall precedes the opening, then $\sin \theta = 1$. The estimates for the intermediate cases are quite uncertain because θ really depends in part on the time and not merely (as we are assuming) on the volume. We shall adopt as a general form

$$\Theta = C_{g1} (V/V_S)^{C_{g2}} \quad (21)$$

whence $C_{g1} = 0$ implies horizontal opening, and $C_{g2} = 0$, $C_{g1} = \pi/2$ imply vertical opening. Usually we shall take $C_{g2} = 1$ and $C_{g1} = \pi/2$, so that θ varies linearly with volume, and the canopy attains a vertical orientation at the end of opening.

The relation, $A_p(V)$, between the projected area and the volume is another function that has to be estimated. Since $A_p(V)$ itself does not occur in (6) to (9), but is introduced in (10), we see that in (11) to (14) $A_p(V)$ always occurs in combination with $H_I(V)$ or $H_D(V)$. It is not, therefore, necessary that the estimate of $A_p(V)$ be extremely accurate, for inaccuracies in $A_p(V)$ can be corrected by changes in H_I and H_D . What is required is a simple, fairly good approximation that matches the general features of the true area-volume relation, but need not be accurate as to details.

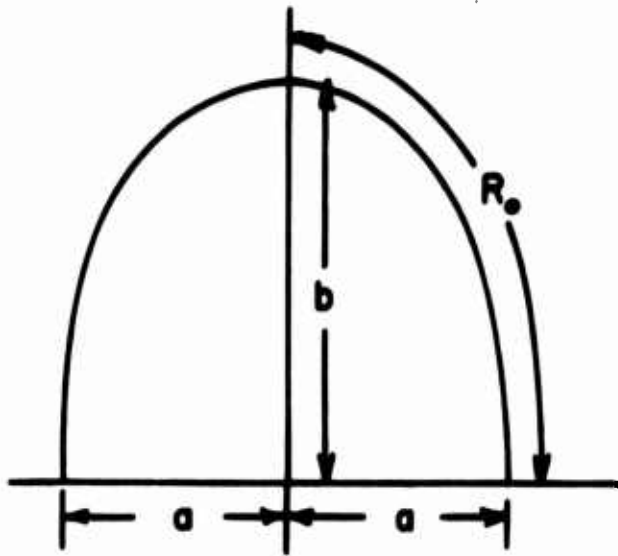


Figure 2a

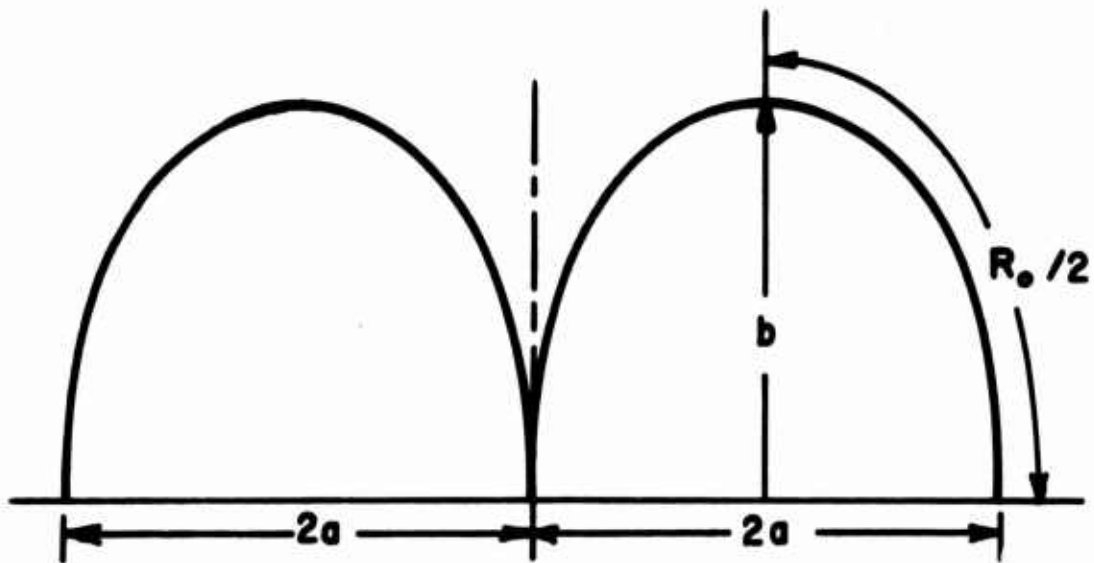


Figure 2b

Figure 2: Ellipses used in Estimating $A_p(V)$

With this in mind we derive $\Lambda_p(V)$ from an idealized picture of the opening canopy in which the canopy section (i.e. the section through the axis) is taken as a semi-ellipse whose arc-length is constant and equal to the flat circular diameter, $2R_0$, see Figure 2a. To find the relation between volume and area for this ellipsoid, we write the well-known formulas for volume, V , and cross-sectional area, A , of the ellipsoid with semi-axes a and b ,

$$V = (2/3)\pi a^2 b \quad (22)$$

$$A = \pi a^2 \quad (23)$$

Since the arc-length from vent to skirt is R_0 , we have

$$R_0 = b E(\eta) \quad (24)$$

$$\eta = 1 - (a/b)^2 \quad (25)$$

where $E(\eta)$ is the Complete Elliptic Integral of the Second Kind. The relation between A and V is found parametrically from (22) to (25),

$$\begin{aligned} V &= (2\pi/3)(1-\eta)[E(\eta)]^{-3} R_0^3 \\ A &= \pi(1-\eta)[E(\eta)]^{-2} R_0^2 \end{aligned} \quad (26)$$

where $\eta \approx 1$, ($a \ll b$) corresponds to the early stages of opening, $\eta = 0$ to a hemispherical shape and $\eta < 0$ to the oblate ($a > b$) ellipsoidal shape in the final phase. The implied relation between A and V is shown in Figure 3.

The parametric formulation, (26), of the area-volume relation is analytically inconvenient, so we shall construct an approximation that is both accurate and convenient. To do this, we first find the maximum value of V as given by (26), which is

$$V_{\max} = .6157 R_0^3 = V_g \quad (27)$$

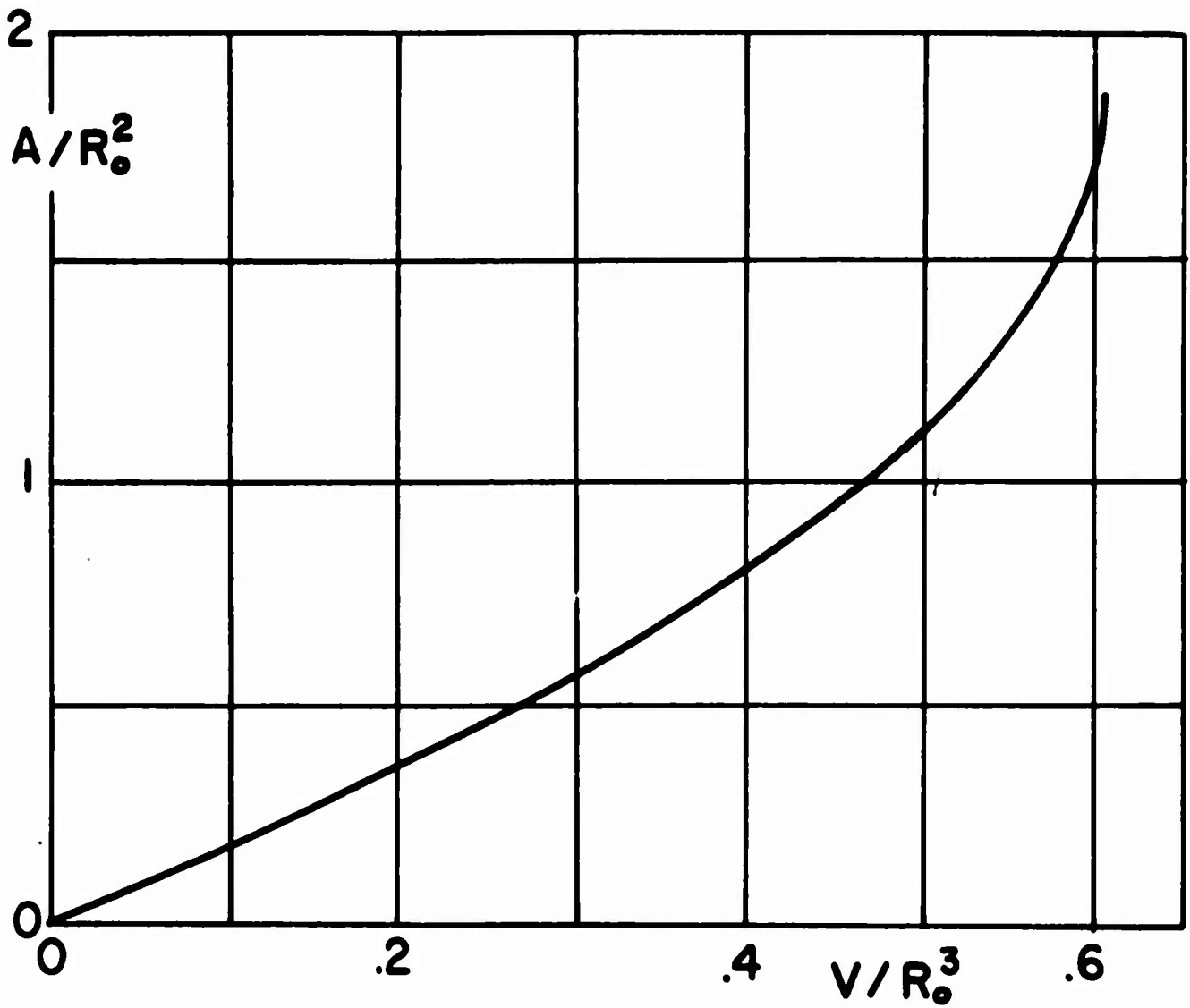


Figure 3: The Idealized Relation (26) between Area and Volume

and occurs at

$$\eta = -1.513 \quad \text{or} \quad A = 1.867 R_c^2 \quad (28)$$

Since $dV/dA = 0$ at this point, i.e. dA/dV is infinite, we have no hope of finding a polynomial approximation to $A(V)$ that is uniformly good for $0 \leq V \leq V_{\max}$. However, we can find a good, quadratic approximation to its inverse function, $V(A)$, by choosing the coefficients such that $V = 0$ when $A = 0$ and V attains its maximum, (27), at the A -value of (28). This approximate function $V(A)$ is then solved by the quadratic formula, and the resulting function is taken as the projected area, A_p .

$$A_p/R_c^2 = 1.867 - 2.399 \{ 6.057 - (V/R_c^3) \}^{1/2} \quad (29)$$

A slight modification of this formula permits us also to estimate roughly the area-volume relation for a canopy with a pulled-down vent. We assume that the vent is pulled down to the skirt during the whole opening period, and that the section through the axis consists of two semi-ellipses instead of one, see Figure 2b. Proceeding as before, we find instead of (26)

$$\begin{aligned} V &= (3\pi/16) \{ (2\pi/3)(1-\eta)[E(\eta)]^{-3} R_c^3 \} \\ A &= \pi(1-\eta)[E(\eta)]^{-2} \end{aligned} \quad (30)$$

We see that, if in (26) V is replaced by $16V/3\pi$, we obtain (30).

Thus, our approximate formula (29) can be generalized to

$$A_p/R_c^2 = 1.867 - 2.399 \{ 6.057 - \lambda(V/R_c^3) \}^{1/2} \quad (31)$$

where $\lambda = 1$ without a pulled-down vent and $\lambda = 1.7$ when the vent is pulled down to the skirt. The maximum volume (i.e. volume in the full-open configuration) is given by

$$V_f/R_o^3 = .6057/\lambda \quad (32)$$

This completes the estimation of A_p .

In estimating $H_D(V)$ we adopt the viewpoint that $C_D H_D(V)$ is the effective drag coefficient (based on projected area) at each stage of opening. We give C_D the constant value that it has for a fully open canopy and, therefore, regard $H_D(V)$ as a multiplier that corrects for the difference between the drag coefficient of a fully and partially open canopy. We shall adopt the following form for $H_D(V)$:

$$H_D(V) = \delta_1 \{ \delta_2 + \delta_3 (V/V_f)^{\delta_4} \} \quad (33)$$

For a solid, flat canopy the constants are given the values $C_D = 1.5$ and

$$\delta_1 = .5; \quad \delta_2 = \delta_3 = \delta_4 = 1$$

which means that the effective drag coefficient, $C_D H_D$, increases linearly from .75 at the start of opening to 1.5 for the fully open canopy. This estimate is based on the idea that in the early stages the effective drag coefficient is roughly that (unity) due to the impact pressure acting over the projected area, but reduced somewhat (to .75) because of losses through the vent and fabric.

The last of our functions, $H_I(V)$, is the most difficult to estimate. Because of (2) and (10)

$$H_I(V) = A_I(V)/A_p(V) \quad (34)$$

i.e. $H_I(V)$ is the ratio of the effective net inlet area to the projected area. Thus, $H_I(V)$ controls the rate at which air fills the canopy and so is affected by many factors such as the flow field ahead of the canopy, losses due to fabric porosity, flow out the vent, flapping of the gores, etc. It is not easy to find a rational estimate for $H_I(V)$, but certain aspects of its behavior are clear enough. First, if (as seems reasonable) the canopy reaches a steady state at the end of opening, the net flow rate into the canopy is then zero, hence

$$A_I(V_f) = 0 \quad \text{and} \quad H_I(V_f) = 0$$

Second, the most important single factor in determining the general shape of $H_I(V)$ is flow deflection around the canopy. In the early phases, because of the streamlined shape of the canopy, it seems plausible that relatively little deflection takes place. Later, as the shape becomes blunter, much more deflection occurs. This leads us to expect that the general behavior of $H_I(V)$ is to decrease with increasing V , but it is difficult to be more specific than this.

Because of the uncertainty in estimating $H_I(V)$, it is merely assumed that $H_I(V)$ can be approximated by a cubic polynomial of the form

$$H_I(V) = \left(1 - \frac{V}{V_f}\right) \left\{ h_0 + h_1 \left(\frac{V}{V_f}\right) + h_2 \left(\frac{V}{V_f}\right)^2 \right\} \quad (35)$$

The values of the constants h_0 , h_1 and h_2 were left to be determined by comparison with experiments.

4. Numerical Treatment

Equations (11) to (17) are taken as the basis of a Fortran computer program in which all quantities are evaluated successively at $V = V_1, V_1 + \Delta V, V_1 + 2\Delta V \dots$, where $\Delta V = (V_f - V_1)/N$,

and $N = 30$ usually. At each step in V the functions $m(V)$, $\theta(V)$, $A_p(V)$, $H_D(V)$ and $H_I(V)$ are computed from (20), (21), (31), (33) and (35), respectively, then the integrals (11), (13) and (17) are evaluated by Simpson's Rule and finally $D(V)$ and $U(V)$ are calculated from (14), (15) and (16).

Since $H_I(V_f) = 0$, the integrands in (11), (13) and (17) become singular at $V = V_f$ and the process has to be terminated one step earlier, i.e. at

$$V = V_s = V_f - \Delta V$$

In particular, this means that we cannot calculate

$$t_f = t(V_f)$$

but, must take the last obtainable value,

$$t_s = t(V_s)$$

as our measure of opening time. This somewhat unsatisfactory procedure is made necessary because we are using V as independent variable. At full inflation and thereafter ($t \geq t_f$) V remains constant and cannot be used as the independent variable.

If we wish to see what happens to the system after opening is completed, we may use time as the independent variable for $t \geq t_s$ and continue the calculation, setting M , θ and A_p equal to the values at $V = V_s$. In this phase of the process, Equation (1) is easily solvable in closed form,

$$U = U_{\infty} \left\{ \frac{U_s + \tanh(\tau - \tau_s)}{1 + U_s \tanh(\tau - \tau_s)} \right\} \quad (38)$$

where

$$U_{\infty}^2 = 2g m_0 / \{\rho A_D(V_s)\} \quad (39)$$

$$\tau = t \left\{ g A_D(V_s) \rho m_0 / (2 m_s^2) \right\}^{1/2} \quad (40)$$

and τ_s and u_s are the values of τ and u when $V = V_s$. The program carries out this calculation.

The very early stage of opening requires some special assumptions. During this period air works its way into the gores and opens them from the skirt to the vent. The area-volume relation, (31), derived from the assumption of an elliptical section, is not accurate in this period, and the time required for this initial filling is very uncertain. In this treatment we by-pass these difficulties by starting the calculations at the end of this period and making certain rough assumptions about the events preceding the start of the calculation. These are as follows: first, the speed remains constant during this period; second, the elapsed time in this period is

$$t_i = (3/2) R_o / u_i \quad (41)$$

and, third, the volume, V_i , at the end of this period is calculated from the approximate formula,

$$V_i = .151 V_f e^{-.056 R_o} \quad (42)$$

This formula was derived by calculating the volume of the gores when the vent is open, the lines straight from the vent to the confluence point and the gores wide open. This calculation was carried through for several different parachutes and the results approximated by the formula (42).

Several tests were run in which the effects of doubling or halving the number of mesh points were investigated. The final choice, $N = 30$, gives good (but not excessive) accuracy and short running times.

5. Calculation and Results

In this section we discuss first the determination of the function H_I for solid flat canopies under standard conditions, and then describe the predictions

of the theory when various parameter-changes are introduced.

The function $H_1(V)$ is specified by the constants h_0 , h_1 and h_2 in (35). To find these constants, many trial cases were run with various values of h_0 , h_1 and h_2 and for various canopy sizes and launch conditions. Some guidance was obtained from tests⁵ on flow fields around model parachutes, but much exploration was still necessary. The criterion for an acceptable set of constants was that it had to give at least rough agreement with the following set of experimental results.

(i) For the C-9 canopy with 28 ft. diameter, $u_1 = 170$ ft./sec., $m_0 = 7$ lb.-sec./ft. (i.e. load = 225 lbs.), the opening time⁴ is about one second and the maximum drag¹ is about 1200 lbs.

(ii) For the G-12 canopy with 64 ft. diameter, recent unpublished tests with $u_1 = 220$ ft./sec., $m_0 = 70$ lb.-sec./ft. (i.e. load = 2250 lbs.) gave a maximum drag of about 9000 lbs. No estimates of opening time were made from these tests.

(iii) For the G-11-A canopy with 100 ft. diameter, $u_1 = 200$ ft./sec., $m_0 = 120$ lb.-sec./ft. (i.e. load = 3860 lbs.) the opening time is in the range 6-8 seconds and the maximum drag¹ is 8000-9000 lbs. Some care is necessary in interpreting these results because the canopy includes a reefing line which retards the early phases of opening and allows some deceleration at nearly constant volume. Also, the aircraft speed is greater than that at which opening begins because the canopy and load are launched with a negative velocity relative to the airplane. These results are probably appropriate for an aircraft velocity of about 250 ft./sec. or 150 kts.

Eventually it was found that the values

$$h_0 = h_1 = .35, \quad h_2 = 0$$

or

$$H_I(V) = .35 [1 - (V/V_f)^2] \quad (42)$$

give the results summarized in Table 1 and shown in Figures 4, 5 and 6. The results for maximum drag are roughly correct for the three canopies, and the opening times for the C-9 and G-11 are also in the right range. Accordingly, we assume henceforth that (42) defines the "standard" behavior of the function $H_I(V)$, at least for solid, flat canopies.

If we compare the drag curves qualitatively for the three canopies under the conditions analyzed, we see that the general shapes of the C-9 and G-12 curves are similar, but for the G-11 maximum drag occurs distinctly earlier than for the others.

It is profitable next to inquire whether the drag and opening time are morbidly sensitive to changes in $H_I(V)$. This is examined by introducing variations of various kinds about the standard function $H_I(V)$ for the C-9 (see Figure 7) and seeing what are the resulting changes in drag and opening time (Figure 8). In Figure 7, curve B is the basic curve, given by (42), and EP and LP are pulses in the curve B for H_I . The general conclusion is that the results are not excessively sensitive to changes in $H_I(V)$ as long as $H_I(V) \neq 0$. If $H_I(V) \approx 0$ somewhere, the opening process almost stops, and the results can be radically different from cases where $H_I(V) \neq 0$.

The specific trends in these results are as follows: (a) Increasing $H_I(V)$ causes an increase in drag and a decrease in opening time. (b) The

TABLE 1

<u>Canopy Name</u>	<u>Diam ft.</u>	<u>m_o lb.sec²/ft⁴</u>	<u>u(1) ft/sec</u>	<u>D_{max} lbs.</u>	<u>t_s sec.</u>	<u>t(D_{max}) sec.</u>	<u>Opening Shock Factor</u>
C-9	28	7	170	1,400	1.15	.51	6.2
G-12	64	70	220	9,050	2.60	1.28	4.0
G-11	100	120	220	8,460	6.59	2.08	2.2

Summary of Basic, Computational Results for Three Parachutes

TABLE 2

<u>Canopy Name</u>	<u>Diam ft.</u>	<u>$\theta(V)$ radians</u>	<u>D_{max} lbs.</u>	<u>t_s secs.</u>
C-9	28	0	1290	1.25
C-9	28	variable	1400	1.15
C-9	28	$\pi/2$	1530	1.12
G-11	100	0	7920	11.14
G-11	100	variable	8460	6.59

Computational Results showing the Influence of
Gravity on Opening of Parachutes

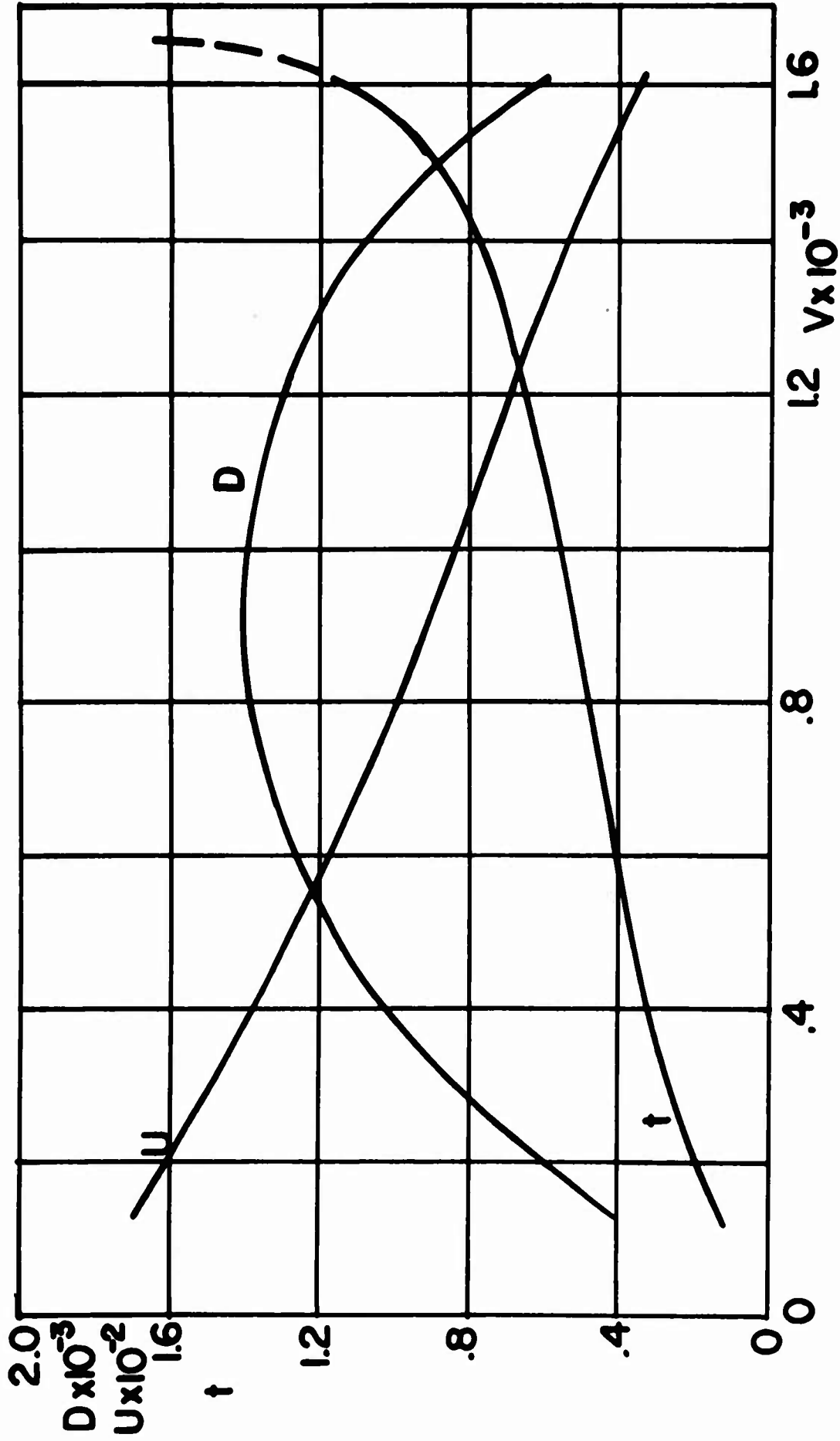


Figure 4: Drag (D), Velocity (u) and time (t) as Functions of Volume (V) during Opening of C-9 Canopy, with $u_1 = 170$ ft./sec. and system Weight = 225 lbs. at sea Level

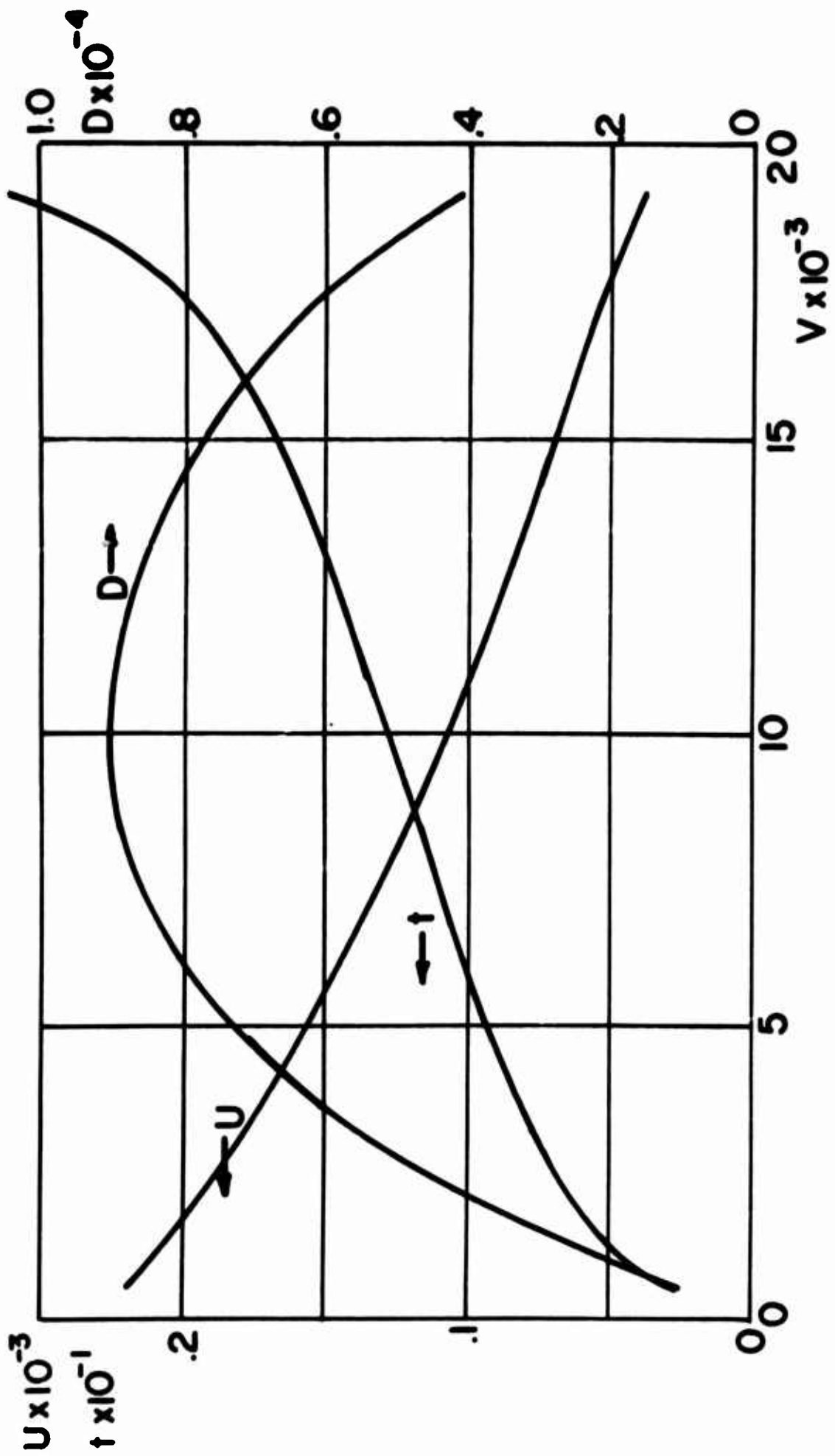


Figure 5: Drag (D), Velocity (u) and Time (t) as Functions of Volume (V) during Opening of G-12 Canopy, with $u_i = 220$ ft./sec. and system Weight = 2,250 lbs. at sea Level

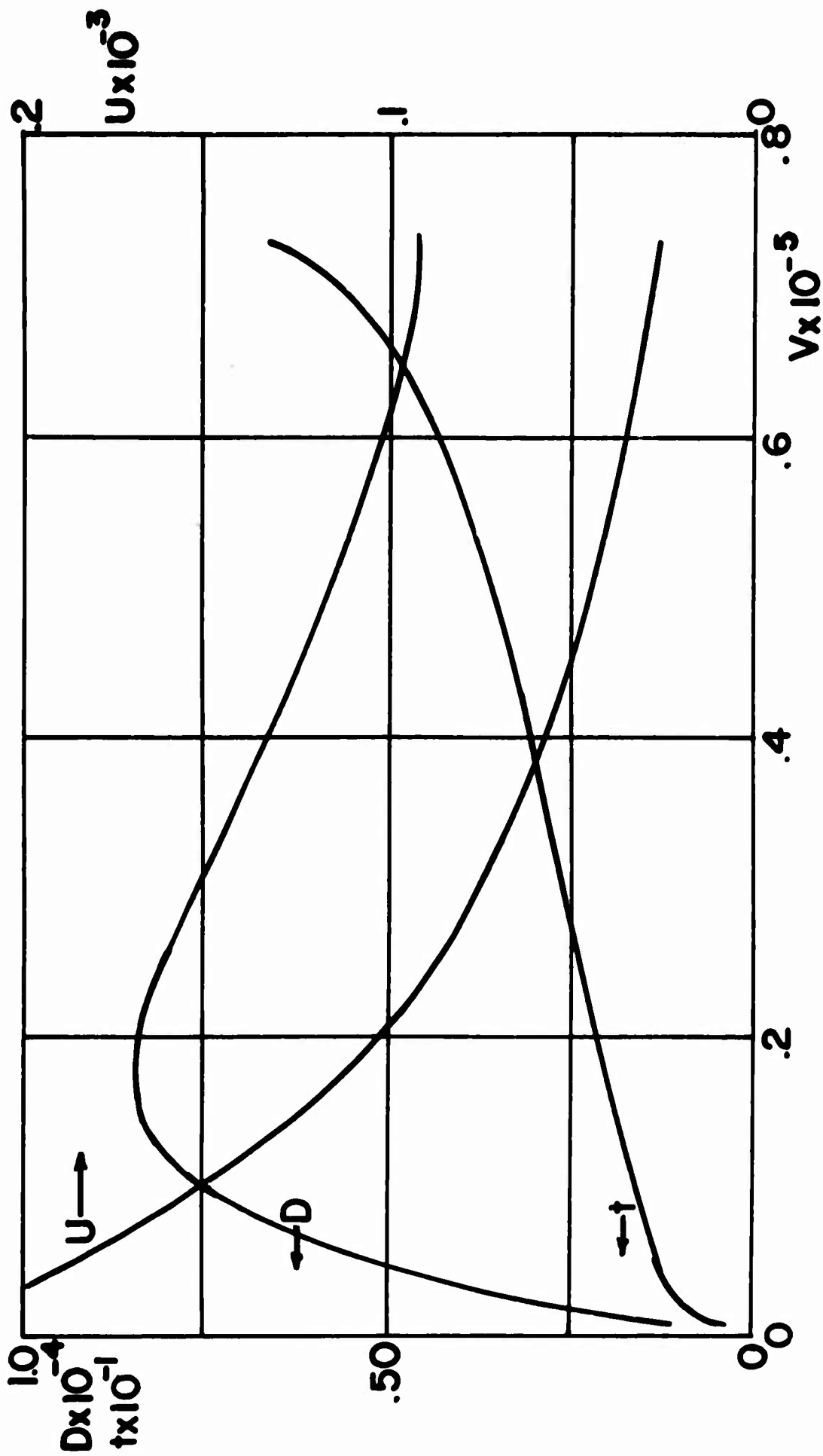


Figure 6: Drag (D), Velocity (u) and time (t) as Functions of Volume (V) during Opening of G-11-A Canopy. with $u_f = 220$ ft./sec. and system Weight = 3,860 lbs. at sea Level

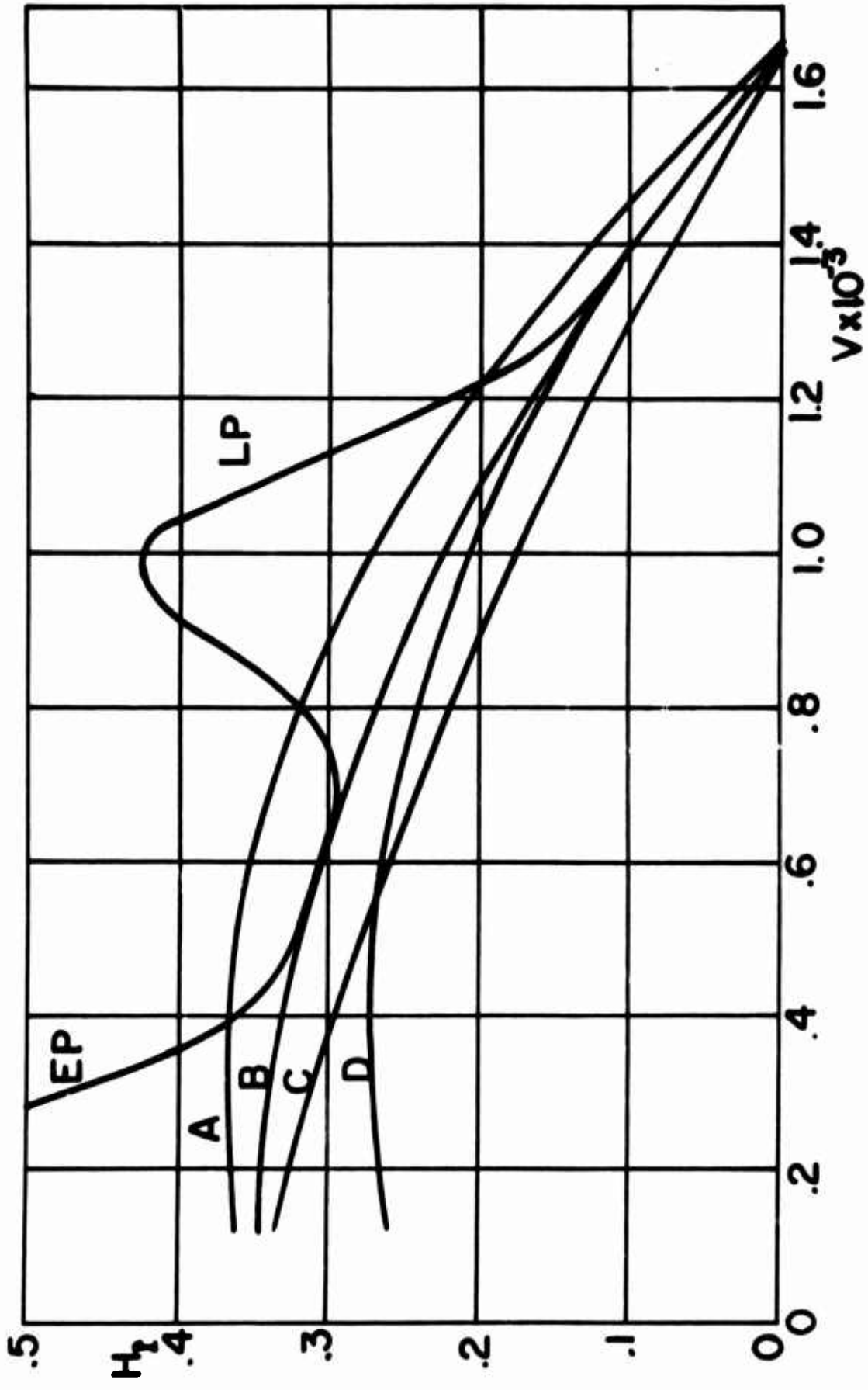


Figure 7: Various Functions $H_1(V)$

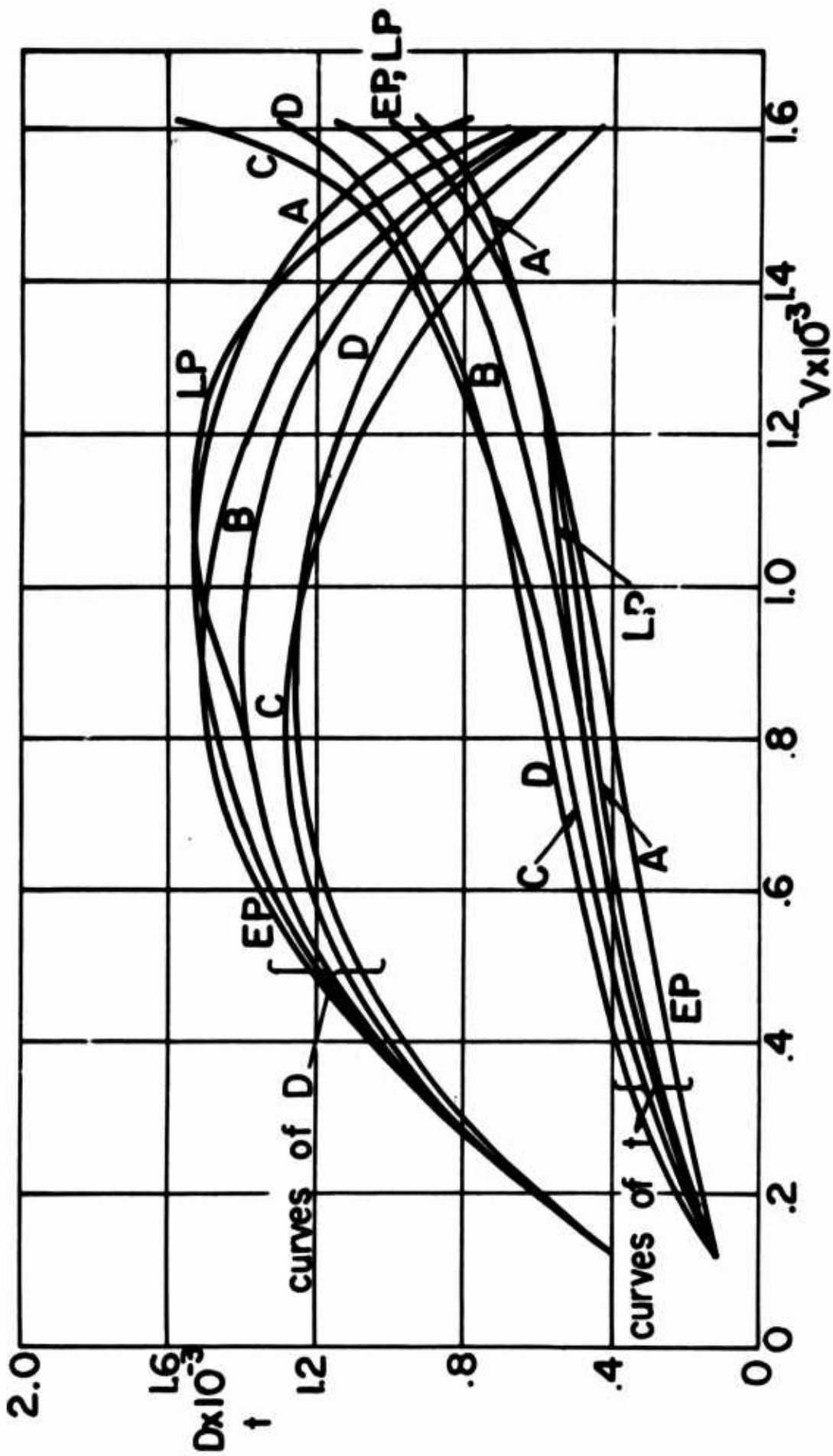


Figure 8: Drag (D) and Time (t) as functions of V for the six functions $H_1(V)$ shown in Figure 7

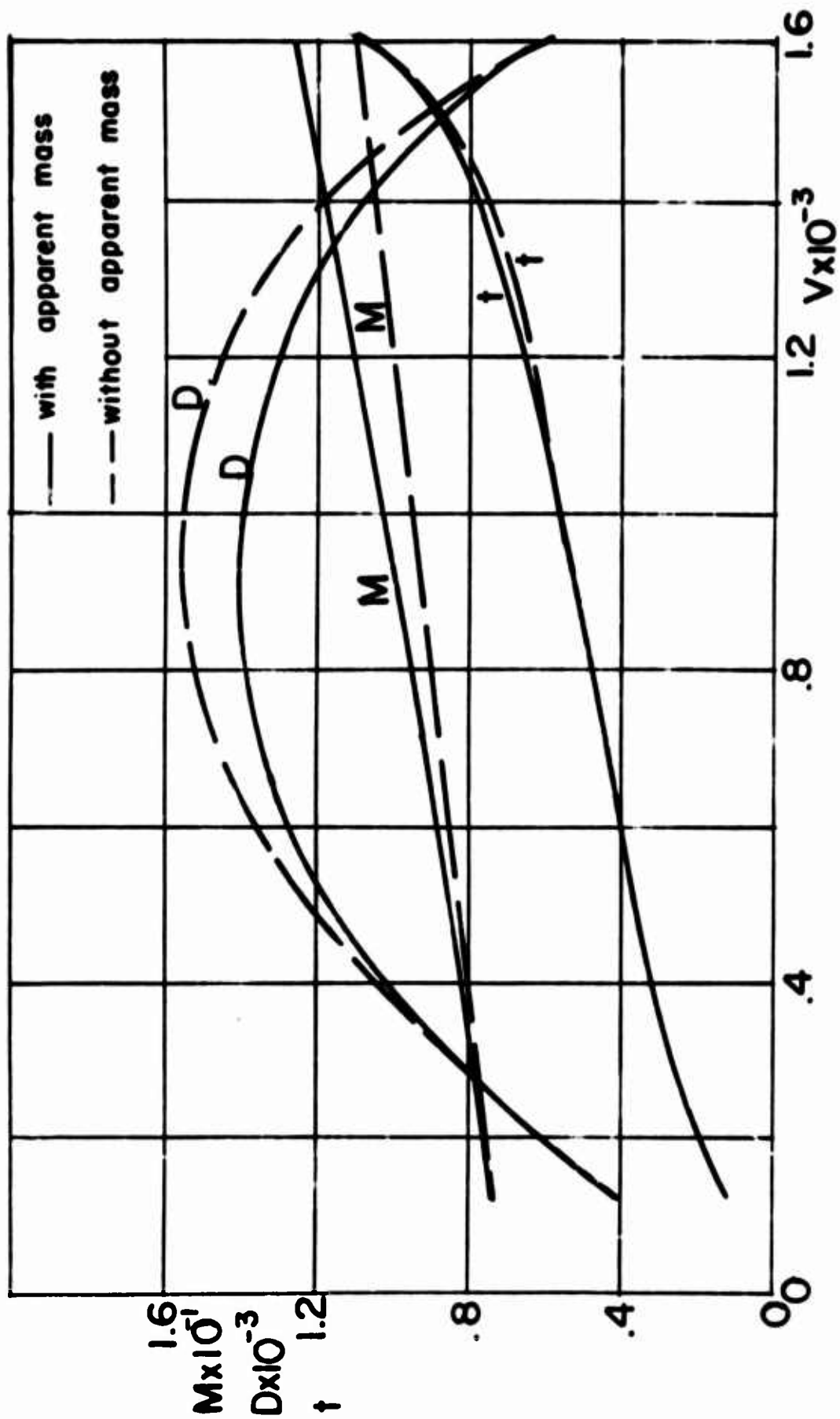


Figure 9: The Effect of Apparent Mass, Drag (D), Mass (M) and Time (τ) as Functions of V

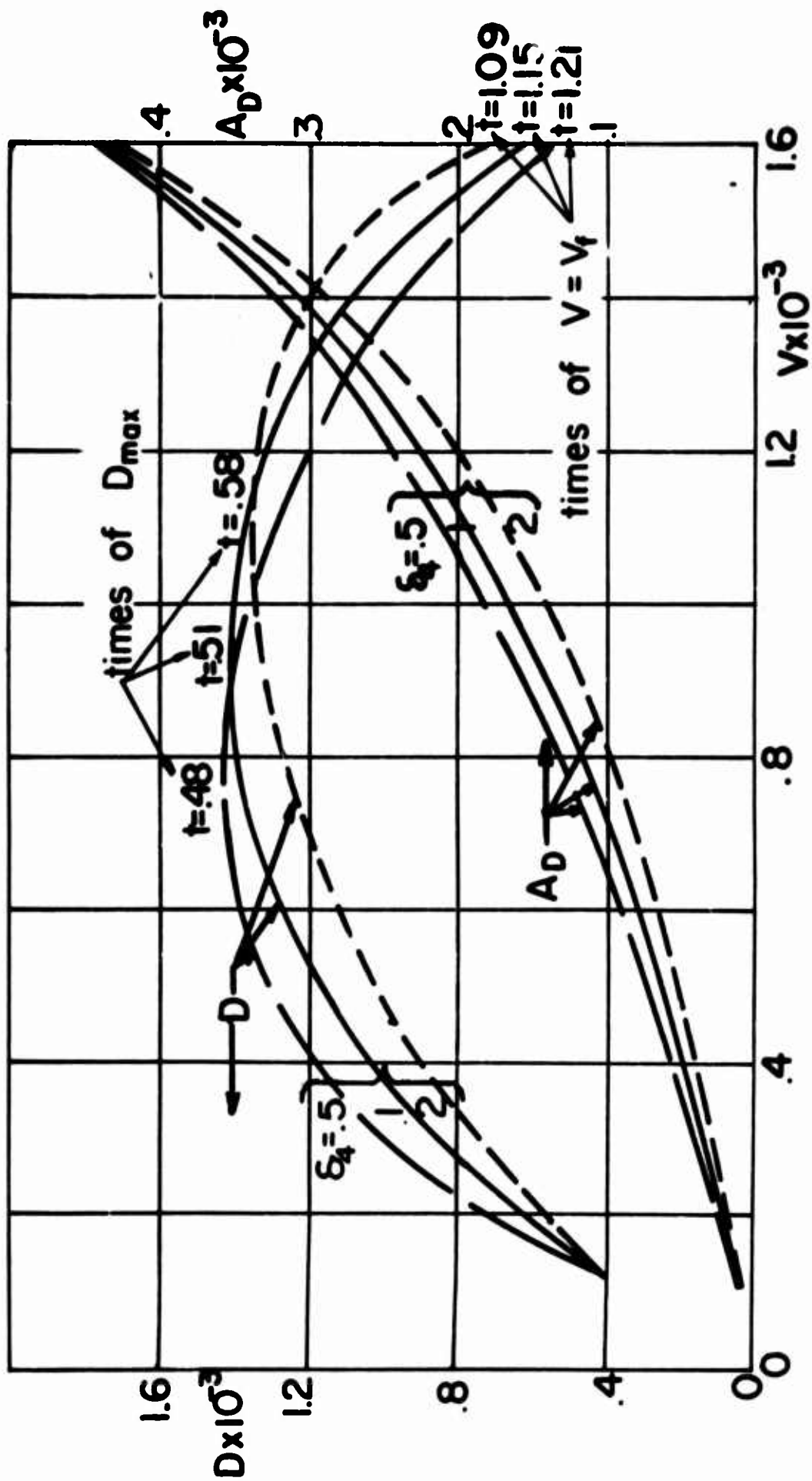


Figure 10: Drag (D) and Drag Area (A_D) as functions of V for $\delta_4 = .5, 1$ and 2

effect on the drag of pulses in H_I is greater when the pulse occurs later in the opening (assuming the pulse produces the same percentage increase in H_I at both stages).

The effect of apparent mass can be studied by comparing the basic results for the C-9 with those for the case where apparent mass is wholly neglected, i.e. in (20) we take $C_{ml} = 0$. Figure 9 shows the comparison. There is little effect on opening time, but omitting apparent mass produces about a 10% increase in H_{max} .

The influence on the solution of changes in H_I (or drag area, A_D) is seen in Figure 10, where results are shown for three values of δ_4 , i.e.

$$\delta_4 = .5, 1 \text{ and } 2$$

We see that increasing the drag area increases both the maximum drag and opening time and causes the drag maximum to occur earlier.

To obtain an estimate of the effect of gravity on the solution, we examine what happens to the C-9 canopy in the simple, extreme cases where $\theta(V) = 0$ and $\theta(V) = \pi/2$. The results are displayed in Table 2. We see that the maximum drag is about 8% less in the former case and 9% greater in the latter case than the standard value. Likewise, opening times are only slightly affected. For the G-11 the case $\theta \equiv 0$ produces only a small decrease in drag, but a long opening time (11.1 seconds) compared with the standard value (6.6 sec.).

The initial velocity, $u(1)$, has a profound effect on the maximum drag and a smaller, but still substantial effect on opening time. In Table 3 we see that for both the C-9 and G-11, the maximum drag is roughly proportioned to $[u(1)]^2$ and the opening time to $[u(1)]^{-1}$. Both these results conform to our intuitive expectations.

TABLE 3

<u>Canopy name</u>	<u>Diam ft.</u>	<u>m_0 lb. sec²/ft⁴</u>	<u>u(1) ft/sec</u>	<u>alt. ft.</u>	<u>D_{max} lbs.</u>	<u>t_s secs.</u>	<u>t(D_{max})/t_s</u>
C-9	28	7	250	0	2,930	.82	.42
C-9	28	10	170	0	2,060	.97	.59
C-9	28	7	170	0	1,400	1.15	.44
C-9	28	7	170	15,000	1,460	.91	.63
G-11	100	100	300	0	15,200	5.5	.26
G-11	100	150	220	0	11,000	5.8	.38
G-11	100	100	220	0	8,460	6.6	.32
G-11	100	100	220	15,000	9,400	5.0	.47

Computational Results showing the Influence of Launch Velocity,
System Mass and Altitude on Opening of Parachutes

TABLE 4

<u>m_0 lb/sec²/ft⁴</u>	<u>u(1) ft/sec</u>	<u>D_{max}^(t) lbs</u>	<u>D_{max}^(e) lbs</u>	<u>t_s^(t) sec</u>	<u>t_s^(e) sec</u>
.0165	50	3.31	9.4	.42	.215
.0165	70	5.53	16.1	.31	.145
.0165	85	7.68	21.5	.26	.121

Comparison of Theoretical (^(t)) and Experimental (^(e)) Results
for the Opening of Parachute Models

Increases in load, m_0 , cause changes of the same sort as increases in v_0 (1), i.e. maximum drag is increased and opening time decreased. These results are also displayed in Table 3. In addition, it appears that increases in m_0 cause the maximum drag to occur later in the opening sequence.

The effect of altitude on the opening behavior is shown in Table 3. For the C-9, if we compare identical tests at sea level and 15,000 ft., we find the latter causes about a 4% increase in drag, a 20% decrease in opening time and a substantial shift (delay) in the occurrence of maximum drag. For the G-11 the results are roughly similar except that the increase in drag is 12% instead of 4%.

Finally, it is interesting to see what happens when we apply our theory to model tests. Here we compare our results with tests of Heinrich and Noreen⁴ on three foot diameter models of a C-9 canopy in a vertical launch with a load of .53 lbs., see Table 4. Clearly there is a large discrepancy between the theory and the model tests. The theory rather consistently estimates D_{max} and t_s as 35% and 210% of their measured values, respectively.

6. Discussion

The opening model proposed in this paper gives fairly satisfactory results for drag and opening time of three canopies that cover the range of those in common use. Also, it predicts successfully a number of qualitative trends that are observable in experiments, such as the drag increases and opening time decreases that occur when the load, launch velocity or altitude are increased.

We may make several comments about the specific results obtained. First, the shapes of the drag curves are not wholly satisfactory because the drag

maxima occur somewhat earlier than is usually observed in tests, especially for the C-9. Second, because of the use of V as independent variable, it is inconvenient to reproduce in our model the behavior at the end of opening. It has been observed⁴ that solid, flat canopies usually "over-inflate," by which is meant that A_p increases past the equilibrium value (given approximately by (28)), into the region where V decreases, before attaining its equilibrium value. In the interests of simplicity this phenomenon is not incorporated into the model. The resulting inaccuracy tends to shift the drag maximum to an earlier time and may partially account for the unsatisfactory shape of the drag curves.

We now make some general comments concerning the parachute-opening problem. First, we wish to emphasize the difficulties in the definition of "opening-time" or "filling-time." There is a large element of uncertainty in our predictions of opening-time due to substituting t_s for t_f . In a sense this is a feature of the model which accords well with reality, because much uncertainty also attends experimental estimates of t_f .

Second, the contrast between the satisfactory results for canopies with diameters in the range 28 to 100 ft. (i.e. full-size canopies) and the very unsatisfactory results for model canopies requires some comment. It is possible of course that this is a deficiency of the theory. However, with the chosen constants the theory gives results that are satisfactory for canopies where the largest diameter is 3.6 times the smallest. The diameter ratio of the model to full-scale C-9 is

$$28/3 = 9.3$$

which is only 2.6 times the scale range among the full-scale canopies. It seems unlikely, if the flow fields are similar, that the theory would be so

poor for the model canopies. We infer, therefore, that probably the flow field around the model is substantially different from that around the full-scale canopy. This could be caused by differing effects of porosity or bending stiffness in the model and full-scale.

Indeed, a very interesting, recent investigation⁶ of the effect of flexibility on parachute model tests shows that moderate increases in flexibility of the models causes sizeable increases in measured opening times and decreases in drag forces. For example, the flexible model⁶ gave opening times of .35, .21 and .14 seconds for snatch velocities of 50, 70 and 85 ft./sec. respectively, instead of the previous values (Table 4) .215, .145, .121. The agreement with the theoretical opening times of Table 4 is better but still rather poor, but Table 1⁶ shows that even the flexible model is still much stiffer than the full-scale 28-foot canopy. This suggests that a considerable portion of the discrepancy between theory and experiment for small models may be attributable to differences in flexibility.

In closing we emphasize that despite its crudities the mathematical model of parachute opening given here (i.e. Equations (11) to (17) together with (20), (21), (31), (33), (35) and (42)) provides a relatively simple method of estimating the drag histories and opening times for flat, circular canopies. The results agree reasonably well with experiments on three canopies, and the computer program that embodies the model is both short and convenient. We anticipate that with suitable changes in the constants of (33) and (35), decent results may also be obtained for ribbon canopies and canopies with pulled-down vents. Although the model must be employed with caution, it appears to constitute an improvement over present procedures insofar as generality and flexibility are concerned.

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