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Final Report

Contract No. N00014-67-C-0424

January 1971

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Final Report on Acoustic Radiation Problems

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Abstract

In this final report we review the work which was done under Contract No. N00014-67-C-0424 on the analysis of noise radiated from submarines and other acoustic field problems. The review is followed by a discussion of possible applications and extensions of the work which might have value in naval problems. A complete list of publications is included.

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### Introduction

Contract No. N00014-67-C-0424 with the Acoustics Programs Branch of the Office of Naval Research extended from May 1967 to January 1971. The main topic was the analysis of noise measurements made in the near field of a submarine. The initial goal was (1) prediction of the far field pattern of the noise by analysis of the near field measurements, but as the work progressed we included (2) near field to far field prediction in situations of interest for transducer calibration (3) noise source localization from analysis of near field measurements and (4) acoustic boundary value problems for complex shapes and mixed boundary conditions. During the latter part of the contract additional support was provided by the Naval Underwater Systems Center (N London Laboratory) for work on (5) generalization of transducer array analysis to include flexing transducer heads, (6) study of transducer head vibrations, (7) estimates of impedance of interstitial gaps between transducers in an array and (8) study of a new mathematical model for the free-flooding ring transducer.

In this final report we will not present all the details of the work, since these have been described in 2 articles in the Journal of the Acoustical Society, 3 papers presented at Acoustical Society Meetings, 2 papers presented at Navy Underwater Acoustics Symposia, 7 Scientific Reports, 4 Technical Memoranda and 6 Programming Memoranda. All these publications are listed in the last section of this report. Here we will give an overall picture of the work with a discussion of possible future applications in the Navy.

### Review

#### Near Field to Far Field Prediction

The problem of predicting the far field from noise measurements made in the near field of a submarine is complicated mainly by the difficulty of making the measurements at locations which would be convenient for analysis. If measurements could be made at closely spaced positions on a closed surface surrounding the submarine there would be no basic obstacle to determining the far field accurately. If in addition the closed surface had a mathematically convenient shape the problem would become relatively easy. In

practice, however, such measurements are not feasible for a submarine underway. Special hydrophone arrays have been designed and tested for approximating a closed surface of measurements, but such measurements have not yet been made with complete success.

It was evident early in our considerations of this problem that it would be a long time before the measurement techniques could be developed to the point where they would provide all the measurements required for the usual methods of analysis. We decided, therefore, to concentrate on a different method in which some results could be obtained regardless of the shape of the surface on which the measurements were made or the extent to which they encompassed the submarine. This could be done by use of the method of boundary collocation, a method which had been discussed in the applied mathematics literature but had seldom been used in acoustic field problems.

The essence of the boundary collocation method is the expression of the field under study as a finite series expansion of  $N$  terms containing  $N$  unknown coefficients and  $N$  known functions. These functions should fit the field under study; for example, in a sound field they should be solutions of the scalar wave equation. Now, if we have  $N$  measurements of this field made at  $N$  different points we can write out the series expansion  $N$  times, once for each measurement. We then have  $N$  linear equations which can be solved for the  $N$  unknown coefficients. These coefficients are put back in the finite series giving an analytical expression for the field at any other point. If  $N$  is large enough and if the  $N$  measurement points surround the source well enough this expression will be accurate everywhere. But even if  $N$  is small and if the points are poorly distributed some results can be obtained which may have some usefulness.

Boundary collocation is simpler and more direct than any other method for the near field to far field problem. The solution only involves evaluation of known functions at certain points and solution of linear equations. No integrations are required. The computations are also convenient because if  $p$  represents a column vector of near field measurements,  $b$  a column vector of unknown coefficients, and  $A$  a matrix in which the

elements are the known functions at the measurement points we have

$$p = Ab,$$

and

$$b = A^{-1}p.$$

Thus, once the inverse,  $A^{-1}$ , is computed for a given number of hydrophones at given locations it can be used to determine coefficients for different sets of measurements with only matrix multiplication required.

The most important part of the development of the collocation methods was their testing. This method, or any other method, can predict the far field accurately if given enough data. The important question concerns the accuracy which can be obtained when the data is limited both in number and distribution of measurements. To answer this question we simulated actual situations by constructing simple theoretical source distributions from which exact near field values could be calculated to simulate near field measurements. Exact far field values could also be calculated for comparison with the approximate far field values derived from the prediction method.

The simplest way to construct theoretical source distributions is by use of point sources. For coherent sources this gives completely familiar distributions, such as linear arrays of equally spaced sources. Similar geometrical arrangements of partially coherent sources give partially coherent distributions which are much less familiar. In fact, we found many aspects of such sources worth considerable study in order to gain insight into noise producing mechanisms and noise propagation<sup>2,16\*</sup>.

As a first step we tested the collocation method for coherent sound where pressure amplitude and phase is measured, the case of interest for transducer calibration<sup>1,5,8,15</sup>. It was found that reasonably accurate far field predictions could be made with distributions of near field measurement points which departed considerably from completely surrounding the source.

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\* These numbers refer to the publications listed in the last section of this report.

The distributions of measurement points used were geometrically similar to those which were planned in the submarine case. This encouraged us to extend the testing of the method to the noise case where the crosspower spectral density was simulated<sup>1,4,9</sup>. These tests were also successful for various degrees of coherence going from the completely incoherent case to the completely coherent case where the results based on simulated crosspower measurements were shown to be the same as those based on simulated pressure amplitude and phase measurements. Item 24 in the list of publications contains FORTRAN listings of the programs used for this testing as well as all other programs used in the work described below.

Another extension of the collocation methods became apparent when J. E. Greenspon of J. G. Engineering Research Associates suggested to us that it might be possible to determine the expansion coefficients from auto power, or amplitude, measurements alone. This has the great advantage of eliminating both the need for simultaneous measurements in different locations and the data processing required to obtain the crosspower. This approach, which we called the amplitude method, was also tested<sup>10,23</sup>. It was found that successful far field predictions could be made, although many more measurements were required for accuracy comparable to that which can be obtained from crosspower measurements. The amplitude method can also be applied to a coherent sound field<sup>12,20</sup> where the special properties of the coherent field can be used. In this case the far field pattern can be predicted from near field measurements of pressure amplitude alone. Phase measurements are not required.

The next logical step in testing far field prediction methods would be to use actual near field measurements made under laboratory conditions where the far field could also be measured. Then the measured and predicted far fields could be compared. The testing described above using simulated near field measurements is unrealistic in that it includes no errors. In practice errors exist in the pressure measurements, more errors may be introduced in the data processing to obtain the crosspower and errors exist in the locations of the hydrophones. In one case we arbitrarily introduced some errors in our simulated near field values and found that the far field

predictions were strongly degraded in those directions not adequately covered by the measurements. However, it is difficult to make such simulated errors realistic, and actual experimental tests are needed. Unfortunately, no provision for such experimental tests was made in any part of this project.

The final step in our near field to farfield prediction work on this contract was to apply the collocation methods to measurements made in the near field of the USS Albacore<sup>17</sup>. There were not enough measurements, they were not adequately distributed around the submarine, and only autopower values were available at the time. Thus it was necessary to use the amplitude method with insufficient data. Some far field patterns were obtained, but there was no way to check their validity.

#### Source Localization

During the development of the collocation methods for far field prediction it became evident that we could also calculate the field at points closer to the submarine than the locations where the measurements were made. It might then be possible to determine the positions in the submarine where the dominant noise producing mechanisms were located. Although this could also be done with other methods of analysis, collocation offers some especially convenient possibilities.

There are many different functions which can be used in the finite expansions involved in the collocation method. One type of function which we used extensively was point source wave functions<sup>1,17,22</sup>. Their use in collocation is equivalent to fitting the measured sound field to the field produced by a collection of point sources. If we use a smaller number of point sources than the number of measurements available it is possible to vary the positions of the sources to get the best fit. The positions thus found indicate where the specific noise mechanisms are located. This aspect of the analysis of near field measurements originated during discussions with Marvin Blizard of ONR and Quentin Dolachek of NSRDC. Similarly, when using spherical wave functions in the expansions it is possible to use the near field measurements in such a way that the best fitting origin of the coordinate system is found.

During the analysis of the Albacore data this source localization procedure was carried out using both point source wave functions and spherical wave functions. Both methods gave results in fair agreement with each other and indicated source positions close to expected noise mechanisms<sup>17</sup>.

#### Boundary Value Problems

It was convenient to evaluate our first work with the collocation method by application to familiar boundary value problems. We found that collocation was an effective approach to such problems, and, more importantly, we found that problems which would be very difficult or impossible by standard methods could be attacked by collocation with no difficulty. For example, difficult boundary shapes such as a finite cylinder can be handled almost as easily as simple boundaries such as a sphere. Problems with mixed boundary conditions, which are impossible by standard orthogonal function expansion methods, can be easily handled by collocation. When these possibilities arose we extended our studies somewhat by applying collocation to certain radiation problems of longstanding interest in underwater sound<sup>1,3,11,19</sup>.

#### Transducer Problems

Support was provided by NUSC-NL to study specific transducer and array problems. The main problem was to generalize the usual methods of array analysis to include transducers with non-fixed velocity distributions. This generalization was required particularly to account for the effects of transducer head flexing. A normal modes approach to this problem was taken and a general method of analysis was developed<sup>6,13</sup> which was then applied by D. T. Porter at the New London Laboratory to a specific array.

This method of array analysis focused attention on the need for normal mode functions for practical vibrators which could reasonably approximate transducer heads. A particular problem of immediate interest was the rectangular elastic plate with free edges, a problem with no known analytical solution and very limited numerical solutions. As a result of our experience on the other parts of the contract we attacked this problem by boundary collocation. After some preliminary studies to find the appropriate form

for the known functions in the expansion we were able to obtain some approximate solutions quite easily. For squares these solutions agreed well with earlier numerical calculations. The semi-analytical nature of the collocation method made it possible to easily calculate the normal mode functions as a function of position. The resonant frequencies and normal mode functions for a particular transducer head were compared with experimental results obtained by J. M. Powers at NUSC-NL<sup>7</sup>.

The theoretical radiation problem posed by the free-flooding ring transducer has long been of interest in underwater sound. Various models have been proposed which are of some help as a guide in transducer design, but none are completely satisfactory. Work with the collocation method in spherical coordinates suggested another quite different approach to a free-flooding ring model. This approach was developed, and the results for far field directivity patterns were compared with measurements on different transducers with reasonably good agreement<sup>14</sup>. A computer program based on this model was provided to NUSC-NL<sup>21</sup>.

The problem of the effects on array behaviour of interstices between transducers was also studied. The particular aspect which we studied was the construction of models for determining the input impedance to the interstices. These impedance values can be used in existing programs at NUSC-NL to study the effects of interstices on array behaviour. Again we used collocation to get approximate solutions to interior mixed boundary value problems which were used as models for the interstices<sup>18</sup>.

#### Discussion and Recommendations

In this section we will try to indicate the most fruitful lines of extension of the work under this contract and the areas of most usefulness to the Navy.

1) Most of our near field to far field predictions by collocation were done in two dimensions corresponding to the assumption of an axis of symmetry through the submarine. The programs developed in this work which use point source wave functions can be applied to three dimensional sources,

and this should be done both with simulated and real measurements.

Another extension which might be important is use of cylindrical wave functions in the collocation expansions. These functions best suit the geometry of the hydrophone measuring arrays under development, and thus fewer measurements would be needed for a given accuracy. However, the computations are somewhat more difficult with cylindrical wave functions than with the spherical and point source wave functions which we have used.

2) Before putting much effort into extensions such as those just mentioned we feel that it would be important to carry out some experimental confirmations of this method and some experimental comparisons of different methods. This could be done in any situation where both the far field and the near field of a source could be measured. The near field measurements would be used to obtain predicted far field values which would be compared with the measured far field values. Any type of source could be used. The more complexities the source had, the more meaningful the test would be. However, if it was found that the predicted and measured far fields did not agree well, it would be necessary to turn to simpler sources in order to track down the reasons for the failure. Thus a systematic progression of experiments from rather simple sources (coherent, single frequency, etc) to more complex sources that bore some resemblance to a submarine (a model submarine, possibly) would probably be the most efficient approach. In such experiments it would, of course, also be possible to test the source localization capabilities of the collocation methods.

3) Another possibility for obtaining experimental data before the planned hydrophone arrays are fully developed is to use existing transducer arrays on submarines. A specific example of this was discussed by G. L. Mohnkern in Paper 19F2 of the 28<sup>th</sup> Navy Symposium on Underwater Acoustics on Nov. 19, 1970. He used the individual transducer elements of existing spherical arrays on submarines and made noise measurements inside the sonar dome. Then the cross powers between different pairs of hydrophones with different spacings and different angular orientations were calculated. Mohnkern was usually measuring ambient noise, but under conditions of high speed and low sea state it should be possible to measure primarily noise radiated from the submarine. Under these conditions approximate far field

predictions might be made by the collocation method in a limited range of directions.

4) The work reported by Mohnkern should also remind us that ambient noise might often be a confusing factor in the attempt to measure submarine radiated noise. Available information about the spatial coherence of ambient noise might help in eliminating this part of the problem. It should also be remembered that the surface has been found to reflect screw-noise to the bow region of a submarine. Thus the surface acting as a passive reflector, as well as a noise source, could also confuse the interpretation of the measurements.

5) Collocation methods can be used to solve many difficult acoustic radiation problems, especially those involving mixed boundary conditions and boundary shapes which are not susceptible to the usual mathematical methods. A specific problem area where more work needs to be done, and where collocation could be applied, is that of reflectors and transducer-reflector combinations. Such problems are of particular interest for low frequency projectors where it is difficult to get directivity from a transducer without using it in combination with a reflector.

6) Methods for calibration of large transducers in tanks or transducers installed on a ship might be developed by using collocation. Both situations require near to far field prediction, and there are usually conditions which limit the number of measurements and the extent to which the measurements surround the transducer. These are the conditions where collocation can be used more conveniently than most methods.

7) The amplitude version of near to far field prediction is worth further basic study and further application to specific problems. It seems surprising at first that amplitude measurements alone contain enough information to determine almost all aspects of a sound field. But when we recall that the measured amplitude values depend on the amplitude and phase of all the contributions from the various parts of the source it becomes more reasonable. The source phase relations are contained in the field amplitude values, and it is only necessary to disentangle them by appropriate analysis. In

this respect there is some similarity to holography, where the disentangling to get the phase information is accomplished in the reconstruction process.

Amplitude methods are very attractive for near to far field predictions in partially coherent noise fields, because of the great saving in measurement instrumentation and data processing when the cross power spectral density is not required. There is also an advantage in coherent field problems, such as transducer calibration, where phase measurements are not required.

8) A more speculative possibility was suggested by our study of partially coherent fields. This would consist of trying to do in acoustics the equivalent of determining the size of a star by measuring the coherence of its light transmitted to earth. By making an assumption, or having prior knowledge, about the coherence of a noise source it might be possible to determine the size of the source if its range was known (and not too great) by making coherence measurements of the transmitted sound. Alternatively, the range might be found if the size of the source could be estimated by other means. Analysis of this kind would be complicated in the underwater sound case by the random fluctuations introduced into the signal by transmission through the ocean. However, it might be worth exploring as an additional approach to target classification.

9) The general method of transducer array analysis developed under this contract should be used to study the questions which arise as attempts are made to improve transducer arrays. This method does not make the fixed velocity distribution assumption which has usually been made in the past. It can therefore be applied to transducers with flexing heads and to arrays in which the degree of head flexing varies from one transducer to another. The most interesting cases to study would be hypothetical arrays related to actual applications, in which various transducer parameters could be varied to produce different degrees of head flexing. Both desirable and undesirable effects may occur, and it should be possible by such studies to determine the feasibility of optimizing the amount of head flexing.

10) Many mechanical vibration problems, such as the rectangular plate studied here in connection with transducer head flexing, can be successfully

attacked by the collocation method. The solutions to this particular problem should be extended and other similar problems should be solved. Such solutions are especially useful in problems involving fluid-structural interactions.

Publications under Contract N00014-67-C-0424

1. J. L. Butler, "Solution of Acoustical Radiation Problems by Boundary Collocation", J. Acoust. Soc. Am. 48 325 (1970)
2. J. L. Butler and C. H. Sherman, "Acoustic Radiation from Partially Coherent Line Sources", J. Acoust. Soc. Am. 47 1290 (1970)
3. J. L. Butler, "A New Method for Acoustical Radiation Problems with Mixed Boundary Values", presented at 78th meeting of ASA, J. Acoust. Soc. Am. 47 (A) 55 (1970)
4. J. L. Butler, "Farfield Prediction from Nearfield Cross-Power Measurements", presented at 76th meeting of the ASA, J. Acoust. Soc. Am. 45 (A) 338 (1969)
5. J. L. Butler, "Series Expansion Prediction of the Farfield from the Nearfield", presented at 75th meeting of the ASA, J. Acoust. Soc. Am. 44 (A) 350 (1968)
6. C. H. Sherman, "General Analysis of Transducer Arrays", presented at 28th Navy Symposium on Underwater Acoustics, Nov. 1970, UNCLASSIFIED
7. J. M. Powers (NUSC-NL) and C. H. Sherman, "Experimental and Theoretical Analysis of Transducer Head Vibrations" (U), presented at 28th Navy Symposium on Underwater Acoustics, Nov. 1970, CONFIDENTIAL-NOFORN
8. J. L. Butler, "A Series Expansion Method for the Prediction of the Far Field from Near Field Measurements", PML Scientific Report No. 1, July 1968
9. J. L. Butler and D. A. Moran, "Numerical Evaluation of the Series Expansion Method for the Prediction of the Far Field Intensity from Near Field Cross-Power Measurements", PML Scientific Report No. 2, August 1968

10. J. L. Butler, "A Method for the Prediction of the Farfield from Nearfield Measurements of the Amplitude Alone", PML Scientific Report No. 3, May 1969
11. J. L. Butler, "A New Method for Acoustical Radiation Problems with Mixed Boundary Values", PML Scientific Report No. 4, September 1969
12. J. L. Butler, "A Method for the Prediction of the Farfield from Nearfield Amplitude Measurements of a Coherent Sound Field", PML Scientific Report No. 5, October 1969
13. C. H. Sherman, "General Transducer Array Analysis", PML Scientific Report No. 6, February 1970
14. J. L. Butler, "A Simple Mathematical Model for the Acoustic Radiation from a Free-Flooding Ring Transducer", PML Scientific Report No. 7, July 1970
15. J. L. Butler, "A Series Expansion Method for the Prediction of the Far Field from Near Field Measurements", PML Tech. Memo. No. 1, December 1967
16. J. L. Butler and C. H. Sherman, "Point Source Noise Fields", PML Tech. Memo. No. 2, April 1968
17. J. L. Butler, "Farfield Prediction and Source Localization from Pressure Amplitude Measurements Made in the Nearfield of the USS Albacore", (U) PML Tech. Memo. No. 3, September 1970, CONFIDENTIAL
18. C. H. Sherman, "Impedance of Interstitial Gaps in Transducer Arrays", PML Tech. Memo. No. 4 November 1970
19. R. P. Gabriel, "Program for Acoustical Mixed Boundary Value Problems (MBVP)", PML Prog. Memo. No. 1, June 1969
20. J. E. Martine, "Nonlinear Programming for a Study of the Farfield Prediction from Nearfield Amplitude Values of a Coherent Acoustical Field", PML Prog. Memo. No. 2, August 1969

21. R. Lurvey, "Program for Acoustical Free-Flooded Cylinder Problem Using Spherical Wave Functions", PML Prog. Memo No. 3, November 1969
22. R. P. Gabriel, "Programs for Point Source Amplitude Method", PML Prog. Memo. No. 4, March 1970
23. R. Lurvey, L. Calabi, R. P. Gabriel, "Programs for the Prediction of the Farfield from Nearfield Measurements of the Amplitude Alone", PML Prog. Memo. No. 5, August 1970
24. PML Staff, "Collection of Programs on ONR Nearfield to Farfield Studies", PML Prog. Memo. No. 6, August 1970

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Parke Mathematical Laboratories, Incorporated One River Road Carlisle, Massachusetts 01741		2a. REPORT SECURITY CLASSIFICATION Unclassified	
		2b. GROUP	
3. REPORT TITLE Final Report on Acoustic Radiation Problems			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Scientific Final (1 May 1967 - 31 January 1971)			
5. AUTHOR(S) (First name, middle initial, last name) Charles H. Sherman John L. Butler			
6. REPORT DATE January 1971		7a. TOTAL NO. OF PAGES 13	7b. NO. OF REFS 24
8a. CONTRACT OR GRANT NO. N00014-67-C-0424		9a. ORIGINATOR'S REPORT NUMBER(S) Final Report 0424	
b. PROJECT, TASK, WORK UNIT NOS. NR-185-306			
c. DOD ELEMENT		9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
d. DOD SUBELEMENT			
10. DISTRIBUTION STATEMENT 1-Distribution of this document is unlimited. It may be released to the Clearinghouse, Department of Commerce, for sale to the general public.			
11. SUPPLEMENTARY NOTES		12. SPONSORING MILITARY ACTIVITY Office of Naval Research (ATTN: Code 468) Navy Department Washington, D. C., 20360	
13. ABSTRACT In this final report we review the work which was done under Contract No. N00014-67-C-0424 on the analysis of noise radiated from submarines and other acoustic field problems. The review is followed by a discussion of possible applications and extensions of the work which might have value in naval problems. A complete list of publications is included.			

DD FORM 1473  
1 NOV 65

UNCLASSIFIED

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Noise Submarine Acoustic Fields Boundary Value Problems Transducer Radiation						