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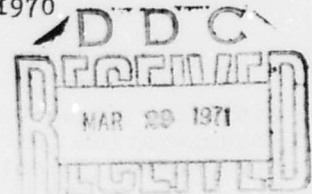
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FINAL SCIENTIFIC REPORT

PART I

ASSESSMENT OF REGIONAL HEAT LOSSES FOR VERIFICATION OF
MATHEMATICAL ANALOGUES OF THE HUMAN THERMAL SYSTEM

1 April 1966 - 30 November 1970



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Main outline of the studies carried out
under contract AF61(052)-936

The human thermal system comprises a complicated neural control system, which is linked by its interaction with other neural control systems to be dependent on the multidimensional physiological function of the organism. Passive effectors, muscles, cutaneous blood vessels and sweat glands connect the system into the complex geometry of the anatomy of the body, while at the same time the flow of blood maintains a continuous passive interaction between the different parts of the thermal system and the rest of the organism. In attempts at investigation of the dynamics of the thermal system as a whole in the intact organism, the existing possibilities of measuring technique are extremely limited. Only the skin temperatures, regional distribution of sweating and total heat production are measurable without difficulties. Monitoring of the central blood temperature, which occupies a key position, has met with difficulties. No chances exist for simultaneous measurement of the regional distributions of cutaneous circulation, other circulation and heat production. Transmission and handling of neural information are likewise completely inaccessible to measurement in the intact organism. In a situation of measuring technique such as this, assembly of information available in the literature and its supplementation by means of experimental measurements performed under controlled conditions constitute a central working method. As a result of the great variety of information of the dynamic processes to be integrated and of the quantitative nature of the synthetic process, the potential of the conventional verbal method of description remains highly unsatisfactory. On the other hand, excellent possibilities are furnished by the new methods of system analysis for the assembly of available

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information into mathematical models of varying scope. Comparison of the function of the model in the computer and the function of a test subject in the corresponding real situation enables existing experimental possibilities of measurement, even limited ones, to be usefully employed. By a method of trial and error, the function of the model may thus be made, step by step, to become a closer approximation of actual truth.

The aim of our work carried out in 1966-1970 under Contract HF61(052)-936, was to devise a mathematical model for the human thermal system. This principal aim necessitated the achievement of some separate, partial aims. The most important of these were the development of monitoring methods for the central blood temperature, development of a model for the heat exchange between blood and organ tissue, and finally development of techniques for programming the skin temperatures and for the measurement of heat losses of different body parts. These processes have been applied in the work aiming at elaboration of a mathematical model describing the passive heat losses of the organism and in its experimental verification. Furthermore, in the last phase of the contract period, a clarification of the control of thermoregulatory heat production in dynamic states was undertaken by the aid of this mathematical model. The subsequent phases of the research project were intended to concern the thermoregulatory control of skin circulation and of sweating.

The report to be presented comprises the major part of the work carried out in the years 1966-1970 under contract AF61(052)-936, and is subdivided into two studies. The 1st is contained in this volume and is titled:

1. Assessment of Regional Heat Losses for Verification of Mathematical Analogues of the Human Thermal System.

The 2nd is contained in a separate volume:

2. The Control of Human Thermoregulatory Heat
Production

The study concerning the heat exchange between blood and tissue was performed in the year 1968, and a manuscript presenting its stage at that time has been previously submitted under the title:

Heat Exchange Between Blood and Organ Tissue.

Work not included in the Contract, aiming at supplementation of the experimental part of the study concerning the heat exchange between blood and skeletal muscle, is in progress and will be published later.

The procedure which has been developed for employing sinusoidal signals in analysis of the heat transfer in the body was considered in a paper read at the symposium on the Physiological and Behavioral Temperature Regulation held in connection with the XXIV International Congress of Physiological Sciences in 1968, having the title:

Use of Sinusoidal Signals in Analysing Heat
Transfer in Man.

The paper is going to be published in the book:

Physiological and Behavioral Temperature
Regulation, Charles C. Thomas, Springfield,
Ill., U.S.A.

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ASSESSMENT OF REGIONAL HEAT LOSSES FOR VERIFICATION OF
MATHEMATICAL ANALOGUES OF THE HUMAN THERMAL SYSTEM

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Introduction

Elaboration of mathematical analogues for the human thermal system implies that various simplified, steady-state and dynamic, heat transfer situations are experimentally mastered with full repeatability. In a stable thermal environment, the high thermal time constants of peripheral tissues and individual structural and functional differences impede the achievement of comparable and reproducible stationary and dynamic heat transfer situations in the organism at rest and during exercise. However, by active governing of the skin temperature, the establishment of steady states can be greatly accelerated and the development of desired dynamic states is under control. The aim of our study was to develop a set of equipment for regional controlling of skin temperatures and equipment for measuring the external heat exchange of the different parts of the body. The methods and equipment have been applied in testing a simple mathematical model which was worked out to describe the passive part of human heat transfer system.

Methods

The purpose of the experimental study was to develop a set of equipment for local controlling of skin temperatures and equipment for measuring the external heat exchange of the different parts of the body. The aim was to use this equipment to clarify the dependence of the heat losses on regional differences in structure and function and to provide data for testing an elaborated mathematical model describing the function of the passive part of the human heat transfer system at rest and in exercise.

The purpose of the experimental study was to measure the heat losses from various parts of the human body in situations in which the cutaneous circulation and the thermoregulative heat production are at their minimum. Endeavours were made to accomplish this by a procedure which involved lowering of the skin temperature by steps and identifying, by following the heat losses and thermal conductance, at each energy level that skin temperature at which the decrease of conductance due to reduction of active heat transport turned into increase at the onset of thermoregulative heat production. The equipment described in the following was used in the study.

The control and calorimetry suit

For the purpose of programming the skin temperatures and for measurement of heat losses, a so-called control and calorimetry suit (CCS) was devised. The suit was made of neoprene foam rubber sheet 6 mm in thickness, with a plastic foam layer of 20 mm attached to its outer surface in order to reduce the heat losses. Desired skin temperatures were achieved by conducting water at given flow rates and at given temperatures into various parts of the suit. In each part of the body the water was distributed by means of a

system of perforated tubes to contact the skin as a spray. Rubber tubing of 4 mm I.D. and 5 mm O.D. was used for these tubes, perforated to give one spray jet per 2-5 cm² of skin area. The temperature of the water entering the suit was kept constant by letting it pass a heater installed in the flow pipe. This heater was controlled by an electronic controller. The maximum power rating of the heater was 5 kW. In the present experiments, the water with heater-stabilized temperature was divided into seven partial flows directed to the head, arms, chest, back and lower extremities. A flow meter was included in each branch line. For determination of the heat losses from various parts of the body, the entry and exit temperatures of the water were measured in each section. This enabled the heat losses to be calculated as the product of flow rate and temperature difference, by the formula $\dot{Q} = cV(T_{out} - T_{in})$. In order to obtain the exit temperatures, the water from each part of the body had to be collected in a special drain tube; these tubes had an S-shaped portion with a thermocouple inserted in its lower part. The S shape ensured that the thermocouple was immersed in water all the time. The water from the chest and back was collected in a common drain, and no separate observations were thus obtained for the ventral and dorsal sides of the trunk. The drains were located under the jaw, at the base of the abdomen, and at the extreme ends of the extremities. The temperature of the entering water in each section was measured with the aid of a thermocouple placed in the respective T fitting.

Other measurements

During the experiments, the skin temperature was measured on the chest and back, from an arm and a leg and from the head. For measurement of the head there were two separate thermocouples, one on the cheek and one on the forehead. For the other parts of the body 5 to

6 measuring points each were provided with thermocouples connected to indicate directly the respective mean temperature. Of internal temperatures, that in the oesophagus and that in the rectum (T_{oc} and T_r , respectively) were recorded in the experiments. The oxygen consumption was measured with a Spirolyt oxygen and carbon dioxide analyzer.

Muscular work

Experiments were performed at rest and during ergometric exercise of various levels. The CCS with its thermal insulation involved extra exertion of the test subject in the exercise experiments, and the ergometer settings do not represent the true load for this reason. On the strength of the oxygen consumption, the exercise levels were approximately equivalent to 1 000, 1 300 and 1 800 cal/min exercise. As a rule, the experimental procedure was to start the test run with comparatively high skin temperature. The temperature was then lowered after stationary state had been achieved. One to three different stationary states could be obtained in the course of one test run. The stationary state was judged by the oxygen consumption curve and by the various temperatures, in particular that of the oesophagus. Endeavours were made to choose for the lowest skin temperature a value at which the oxygen consumption was clearly elevated compared to the values found with higher skin temperatures but at which stationary state was still achievable.

Consideration of errors.

Accuracy of the experimental determinations

The heat losses from each part of the body were determined by the formula $\dot{Q} = cV(T_{out} - T_{in})$, where c is the specific heat of water, V is the rate of water flow applied to the body part in question (in g/min), and

$T_{out} - T_{in}$ is the temperature difference of the entering and exiting water. The relative total error in the heat loss value is then found by the formula

$$\frac{\Delta \dot{Q}}{\dot{Q}} = \frac{\Delta(cV)}{cV} + \frac{\Delta(T_{out} - T_{in})}{(T_{out} - T_{in})}$$

The accuracy of the flowmeters employed in the study was $\pm 5\%$. The accuracy of the equipment used for temperature measurement was $\pm 0.01^\circ\text{C}$, whereby the possible error in the temperature differences has to be estimated as $\pm 0.02^\circ\text{C}$. The flow rates used in heat loss determinations were such that the temperature differences varied within the range of 0.1 to 0.8°C , depending on the situation and on the part of the body concerned. In the majority of determinations the difference was between 0.3 and 0.6°C . On the basis of the extreme values, the relative accuracy of temperature measurement will thus be $\pm 20\%$ to $\pm 2.5\%$ and on the basis of the typical limits, $\pm 7\%$ to $\pm 3\%$. From this the accuracy in determination of heat losses is found to be $\pm 25\%$ to $\pm 7.5\%$ on the basis of the extreme values and $\pm 12\%$ to $\pm 9\%$ on the basis of the typical limits. At rest, it was necessary as a rule to be satisfied with temperature differences of 0.1 to 0.3°C in respect of all parts of the body; it follows that the accuracy of the results referring to resting condition is lower than that achieved in the experiments during exercise.

The conductance (in $\text{cal}/\text{min}/^\circ\text{C}$) was calculated by the formula $K = \dot{Q}/(T_{AR} - T_S)$. The temperature difference $T_{AR} - T_S$ can be considered to have been ascertained with at least $\pm 5\%$ accuracy. This accuracy corresponds in the determinations at rest, in which the temperature difference was in the order of $4-5^\circ\text{C}$, to $\pm 0.2^\circ\text{C}$ error in the average skin temperature. In the exercise situations with lower skin temperature the said accuracy of $\pm 5\%$ would permit a correspondingly higher absolute

error in average skin temperature, which would be about $\pm 0.8^{\circ}\text{C}$ at the lowest skin temperature employed in the study. In the exercise situations the accuracy of the temperature difference values can be assessed to amount to ± 2 to 3% . In typical instances, the accuracy of the conductance determinations will thus be ± 10 - 17% . When the conductances of different body parts of one and the same subject and which have been determined in the same experimental situation are mutually compared, the relative accuracy will be the same as that of the heat loss determination because in that case all conductances were calculated on the basis of the same temperature difference $T_{AR} - T_s$.

In the calculations, for each subject, of the conductances per unit body surface area (in $\text{cal}/\text{min}/^{\circ}\text{C}/\text{m}^2$) for the entire body and for its different parts, the limits of accuracy of the surface area determination introduce an additional error. However, this error has no effect when comparisons are made between different situations in which one and the same subject was placed.

In the preceding consideration comparatively wide error limits were found for the determinations of heat losses and of conductance. However, these limits represent the absolute limits which are only reached if all sources of error act simultaneously at their maximum effect and in the same direction. The probable error may be considerably less.

In order to verify the presence of thermal equilibrium in the test subjects, and also for detection of potential errors in measurement, the oxygen consumption of the subjects was also measured and on its basis the energy production was determined. The accuracy of this determination was about $\pm 3\%$. The respiratory losses were estimated to be about 10% at rest and about 5% during exercise (Caldwell 1969). Taking into account

also the contribution of external work in the exercise situations, a calculation of the thermal balance of the entire body could be made. In most instances these calculations indicated a rate of change of the entire heat content of the body less than $\pm 5\%$ of the heat production of the entire body. Test results in association of which the calculated rate exceeded the limits of $\pm 10\%$ of the heat production were not accepted as a rule.

It is to be noted, in spite of the good end results elicited in the heat balance calculations, that the possibility naturally exists that the errors incurred in the determinations relating to individual parts of the body have partly compensated for each other and the results found for a small part of the body may contain an appreciable relative error.

Experimental results and discussion

The aim of the experimental study was to develop equipment for regional controlling of skin temperatures and equipment for measuring the external heat exchange of the different parts of the body. This equipment was used to clarify the dependence of the heat losses on regional differences in structure and function and to provide data for testing a mathematical model elaborated to describe the behaviour of the passive part of the human heat transfer system at rest and during muscular exercise.

The aim was to measure the heat losses from different body parts in situations of minimum cutaneous circulation and thermoregulatory heat production. Achievement of this was endeavoured by lowering the skin temperature by steps and observing the heat losses as well as the thermal conductance, identifying on each energy level that skin temperature at which the decrease of thermal conductance caused by reduction of active heat transport turned into an increase as a result of thermoregulatory heat production.

Treatment of the results of measurement

Table 1 shows the dimensional data of the experimental subjects. Figures 1-6 and Tables 2-7 show the results obtained in experiments with three subjects and the magnitudes of heat losses and conductances calculated from them.

The average skin temperature T_s was found by computing the weighted mean of the different skin areas. The oesophageal temperature T_{oe} was employed to represent the central blood temperature T_{AR} . The rectal temperature T_r was also measured for comparison. The difference $T_{oe} - T_s$ was used in calculating the conductances.

The energy production was calculated from the oxygen consumption, using the RQ value of 0.86. The heat production was found by subtracting from the energy production the contribution of external work. This work in its turn was estimated by calculation from the energy production at high skin temperatures, assuming the efficiency of external work to be 20 %. Since the CCS suit involved extra exertion of the subject, the load rate consistent with the ergometer setting could not directly be used for the magnitude of the external work.

The heat losses of the different parts of the body were calculated by the formula $\dot{Q} = cV(T_{\text{out}} - T_{\text{in}})$. By addition of these partial heat losses the total heat losses from the skin were found. The respiratory losses were approximately calculated, assuming them to be about 10 % at rest and about 5 % during exercise, referred to the heat production (1).

Finally, the conductances (in cal/min/°C/m²) have been stated for the different body parts and for the entire body. These were obtained from the heat losses, dividing them by the temperature difference $T_{\text{oe}} - T_{\text{s}}$ and further by the area of the respective part of the body. The measured body surface areas and other data describing the test subjects can be seen in Table 1.

Fig. 3 shows the total conductances of the test subjects as functions of the skin temperature T_{s} , at rest and during exercise of various magnitudes.

Fig. 4 shows as a function of the skin temperature T_{s} , the conductances of different parts of the body of one test subject.

Table 7 is a compilation of the minimum conductance values for the different body parts of the test subjects at different energy levels. The minimum

conductances of one subject have been plotted over external work in Fig. 5.

Fig. 6 shows the minimum values of the total conductance of each test subject as functions of energy production, per m^2 body surface area; for the energy production the value measured at high skin temperature was used.

Heat losses

In the situation with minimum conductance, in three test subjects at rest on the average 39 % of the heat losses were from the trunk, 34 % from the lower extremities, 14 % from the upper extremities, and 13 % from the head, with skin temperatures of $31-32^{\circ}\text{C}$ (Tables 2-5, Figs. 1 and 2). During bicycle ergometer work, the heat loss from each body part increased in the first place as a linear function of work output (Figs. 1 and 2). The rate of increase was strongest in the lower extremities, which performed the work. At 1 800 cal/min external work load, the distribution of heat losses was as follows: lower extremities - 51 %, trunk - 24 %, upper extremities - 16 %, and head - 9 %, with $20-22^{\circ}\text{C}$ skin temperature. Fairly minor inter-subject dispersion of the results was noted. Transition from rest to work at 1 800 cal/min external work load, whereby the total heat production became 6.5-fold, caused the heat losses of head and trunk to increase by a factor of 3-4, while simultaneously those of the lower and upper extremities became about 9-fold.

Thermal conductances

When the skin temperature was lowered by steps, at each external work load the conductances of the entire body and of the different body parts were found to decrease powerfully at first and then, at a skin temperature level specific to each particular work load,

to attain a given minimum level (Figs. 3 and 4, Table 6). These changes of the thermal conductances express the cessation of effective, active heat transport in the surface layer with increasing passive heat losses. The abrupt lowering of conductance was only accompanied by minor increase of heat production. As a rule, the heat production had increased about 10 % when minimum conductance was attained. Thus, it seems as if these two thermal regulation systems, i.e., thermoregulatory heat production and cutaneous circulation, would in small degree be simultaneously active in the boundary region.

When the heat production was increased by steps by means of muscular work, the minimum values of conductance were found to increase distinctly, as functions of heat production, on transition from rest to exercise, and their rate of increase was later seen to become somewhat slower at transition to higher external work loads (Figs. 4 and 5, Table 7).

Considerable increase of the thermoregulatory heat production by further lowering the skin temperature resulted in the observation that the conductance increased only slightly from these minimum levels (Table 2-4, Figs. 3 and 4).

It was thus found that, compared to heat transport by the cutaneous circulation, the heat production caused by muscular work or by shivering had a considerably less marked effect on conductance. It is possible that the cooling of the superficial layers caused by lowering of the skin temperature, and the associated reduction of metabolism and circulation, cause a redistribution of metabolic heat production to centre on layers at greater depth, at the same time exerting a damping effect on the change of conductance.

The experimental conductances of individual parts of the body were calculated, using as temperature difference the difference between the temperatures of central blood and skin, $T_{AR}-T_s$. The conductance thus found is here called the central conductance of the body part in question. From the viewpoint of description of the state prevailing in each individual body part, it would be more appropriate to determine the regional conductance of the body part, using as temperature difference that between the local arterial blood of the body part and its skin, $T_{ar}-T_s$. If there is no countercurrent exchange in the blood vessels serving the body part, the two conductances are equal; but if countercurrent exchange is present, then the central conductance will be smaller than the regional conductance. Because the local arterial temperatures T_{ar} were not measured in the experiments, no regional conductances could be experimentally determined.

Fig. 4 presents the conductances of the different body parts of test subject S.M. at each work load/level. In this subject the thermoregulatory increase of conductance was most clearly evident. The conductances of the different body parts, with the exception of that of the lower extremities, which performed the work, are seen to behave rather uniformly as functions of the skin temperature. The conductances of the lower extremities show no ascending part of the graph at the two highest work loads; at the highest work level it seems that the minimum value was not even attained. This may be interpreted as indicating inhibition of thermoregulatory heat production in muscles performing work at higher rate.

At each work load level, distinct differences were noted between the minimum conductances of the different body parts (Table 7). At rest, the highest minimum conductance in all test subjects was that of the head, followed by trunk and lower extremities. The lowest

minimum conductances were found for the upper extremities. This succession of the body parts is readily understandable. The head has in its deeper tissues a powerful circulation, and even the cutaneous circulation was not easily reduced to a value as low as elsewhere in the body. The lower extremities, again, beat the upper extremities for the reason that the thighs constitute such a major portion of the lower extremities. The thighs represent a part of the body with rather great thickness, in which the countercurrent effect is still quite minor. At the work load levels of 1 300 and 1 800 cal/min all test subjects already presented higher minimum conductance in the lower extremities than in any other part of the body. This is due to the fact that in the extremity engaged in work, a more abrupt temperature gradient field is formed, whereby the heat losses from within to the surface of the skin increase. The conductances found for the lower extremities at 1 800 cal/min work load were about 3-fold, compared to those at rest. The conductances of the upper extremities, too, increased to 2-3-fold values. This change can even be considered surprisingly great. It suggests that the significance of countercurrent exchange at rest would be greater in the upper than in the lower extremities. To be sure, among other factors of influence it should be noted that the increase of conductance as a function of the increase of circulation in the muscular part of the body part is highly non-linear, as will be seen in the subsequent theoretical consideration. Thus, in work load situations, when the upper extremities are subjected even to minor load, their conductance increases relatively quite strongly. Moreover, the fat layer is thinner in the upper extremities, compared to the lower extremities, whereby the increased circulation of the muscle exerts a greater influence. The conductance of the upper extremities of test subject S.M. under work load was about 1.5 times that of other test subjects; in test subject K.K. also high conductances of the head

were often noted. This suggests individual differences in regulation of regional cutaneous circulation. The minimum conductance of the head showed only little increase with increasing work load level.

For comparisons concerning the minimum values of thermal conductances, previous studies associated with the state of rest are available in the first place. The situation is different as regards work load: although conductances have previously been determined also under work load, the experimental conditions have rarely been such as to justify any confidence in actual achievement of minimum conductance conditions. As a rule, previous investigators have expressed the conductance in units of $\text{kcal/m}^2/\text{hr}/^\circ\text{C}$, using in their calculations the temperature difference between rectum and average skin, $T_r - T_s$. Part of the experiments were carried out in water, whereby the skin temperature has been uniform; others were undertaken in air, implying non-uniform temperature distribution on the skin, and the average skin temperature was found by calculating the weighted mean of different local temperatures. Burton and Bazett found by their bath calorimeter at rest the following lowest conductance values (single points of measurement): 6.7, 7.6, 7.6, 7.9, 7.9, 8.7, 9.5, and 9.8. The mean curve presented on the basis of their test results by Gagge has its minimum at $9.2 \text{ kcal/m}^2/\text{hr}/^\circ\text{C}$, corresponding to $150 \text{ cal/min}/^\circ\text{C/m}^2$. In Gagge's publication also a mean curve from results obtained with DuBois-Hardy's calorimeter (Proc. Nat. Acad. Sci., Wash. 23:624, 1937) is presented, with the minimum $8.6 \text{ kcal/m}^2/\text{hr}/^\circ\text{C}$, corresponding to $140 \text{ cal/min}/^\circ\text{C/m}^2$. The lowest single points measured with this calorimeter were 7.9, 8.1, 8.5, 8.5, 8.7, and 9.2. In the experimental series of Wyndham and Atkins the lowest conductance value at rest, $9.2 \text{ kcal/m}^2/\text{hr}/^\circ\text{C}$, is found, which corresponds to $150 \text{ kcal/min}/^\circ\text{C/m}^2$.

If, in calculating the conductance, the temperature difference $T_{AR}-T_s$ is substituted for T_r-T_s , the minimum conductance values at rest may increase about 2-10 % because the temperature difference between interior temperatures and skin is then of the order of 4-5°C and because the rectal temperature may be 0.1-0.5°C higher than the blood temperature. If the temperature distribution of the skin is uniform, the minimum conductance will have somewhat higher values than in the case of natural, non-uniform temperature distribution (colder lower extremities). This is because in the lower extremities the minimum conductance per unit area is less than in the head and trunk. However, the quantitative influence of this effect is not thought to be very great; the difference between the conductances under normal distribution and uniform distribution would be of the order of 5 % at most.

In the present study, the minimum conductance of the entire body of the test subjects at rest was found to be 150 cal/min/°C/m², which agrees with the results obtained by other authors.

Summarizing the former, it can be noted that determination of regional heat losses improves the chances for quantitative understanding of the function of the heat transfer system. It brings substantial improvement in the elaboration of mathematical models for description of the thermal system of the organism or of its component parts. The method offers chances for determinations of the cutaneous circulation in steady-state as well as dynamic situations.

Mathematical model

The third aim of the study was to elaborate a simple mathematical model to describe the passive heat losses from various parts of the body depending on variations in structure and function. It is also intended to compare calculations based on this model with experimental results.

The mathematical model of passive heat losses of the body

For the mathematical treatment of its heat losses the body was divided into the following parts: head, trunk, upper arm, forearm, thigh, leg, and ankle + foot. Each of these body parts was represented by a cylinder, assumed to have two layers: an inner layer comprising the muscular tissue and bones, and an outer layer comprising the cutaneous and fat tissues. Since the consideration is restricted to the case in which the cutaneous circulation is minimal, it was considered possible to combine the skin and fat tissues: in this instance the circulation is of the same order in both, and their material constants are not greatly different either. In the inner as well as outer layers the homogeneous blood flows consistent with the situation under consideration were assumed. In the instance of an extremity performing mechanical work, for the perfusion of the interior part that of the muscular tissue may be used because, at least in the stationary case, the bone components at great depth have no significant bearing on the magnitude of the heat losses. The calculations are based on the anatomical dimensions stated in Table 8.

In the table, r_2 and r_1 denote the outer and inner diameters of the cylindrical body parts, and l denotes the length of the cylinder. The total body surface

area is 1.84 m^2 , and the total volume is 69.7 dm^3 .

The stationary temperature distributions in each cylindrical body part are found by starting from the heat balance equation of the tissue,

$$k \nabla^2 T_t + M_t + c_b \rho_b \frac{dV}{dv} (T_{ar} - T_v) = 0 \quad (1)$$

where the term $k \nabla^2 T_t$ represents the heat conduction, M_t the heat production (in $\text{cal}/\text{min.}/\text{cm}^3$) and the term $c_b \rho_b \frac{dV}{dv} (T_{ar} - T_v)$ the heat exchange between blood and tissue.

With the notations

$$\alpha = \frac{T_v - T_{ar}}{T_v - T_{ar}} \quad (2)$$

$$F = \frac{dV}{dv} \quad (F) = \frac{\text{cm}^3/\text{min.}}{\text{cm}^3} \quad (3)$$

$$M_t = EF \quad (E = \text{cal}/\text{cm}^3)$$

the equation is converted into

$$\nabla^2 T_t + \frac{EF}{k} + \frac{c_b \rho_b}{k} \alpha F (T_{ar} - T_t) = 0 \quad (4)$$

When axial conduction is neglected, the solution of the equation gives the temperatures in the inner and outer layer of each cylinder (Fig. 7) expressions of the following form (2):

$$T_1 = A_1 + B_1 I_1(\kappa_1 r)$$

$$T_2 = A_2 + B_2 I_0(\kappa_2 r) + C_2 K_0(\kappa_2 r)$$

where I_0 and K_0 are modified Bessel functions of the first and second kinds, respectively, of zero order, and

$$\kappa_1 = \sqrt{c_b \rho_b \alpha F_1 / k_1} \quad \kappa_2 = \sqrt{c_b \rho_b \alpha F_2 / k_2} \quad .$$

The coefficients A_1 and A_2 will have the values

$$A_1 = T_{ar} + \frac{E_1}{c_b \rho_b a} = T_{ar} + \Delta_1$$

$$A_2 = T_{ar} + \frac{E_1}{c_b \rho_b a} = T_{ar} + \Delta_2$$

The coefficients B_1 , B_2 and C_2 are found by solving the equations

$$A_1 + B_1 I_0(\kappa_1 r_1) = A_2 + B_2 I_0(\kappa_2 r_1) + C_2 K_0(\kappa_2 r_1)$$

$$\kappa_1 \kappa_1 B_1 I_1(\kappa_1 r_1) = \kappa_2 \kappa_2 B_2 I_1(\kappa_2 r_1) - \kappa_2 \kappa_2 C_2 K_1(\kappa_2 r_1)$$

$$A_2 + B_2 I_0(\kappa_2 r_2) + C_2 K_0(\kappa_2 r_2) = T_s$$

From these the following expression is found for B_2 :

$$B_2 = \frac{\kappa_1 \kappa_1 \frac{I_1(1,1)K_0(2,1)}{I_0(1,1)K_0(2,2)} + \kappa_2 \kappa_2 \frac{I_1(2,1)}{K_0(2,2)} (A_2 - T_s) + \kappa_1 \kappa_1 \frac{I_1(1,1)}{I_0(1,1)} (\Delta_1 - \Delta_2)}{\kappa_1 \kappa_1 \frac{I_1(1,1)}{I_0(1,1)} I_0(2,1) - \frac{K_0(2,1)I_0(2,2)}{K_0(2,2)} - \kappa_2 \kappa_2 I_1(2,1) + \frac{K_1(2,1)I_0(2,2)}{K_0(2,2)}} \quad (5)$$

C_2 and B_1 are then found from the expressions

$$C_2 = - \frac{A_2 - T_s}{K_0(2,2)} - \frac{I_0(2,2)}{K_0(2,2)} B_2 \quad (6)$$

$$B_1 = - \frac{\Delta_1 - \Delta_2}{I_0(1,1) I_0(1,1)} + \frac{1}{I_0(1,1)} \left(I_0(2,1) - \frac{K_0(2,1)I_0(2,2)}{K_0(2,2)} \right) B_2 - \frac{K_0(2,1)}{I_0(1,1)K_0(2,2)} (A_2 - T_s) \quad (7)$$

The following brief notations have been used in the expressions for the coefficients:

$$I_0(\kappa_1 r_1) = I_0(1,1) \text{ etc.}$$

If A is the surface area of the cylindrical body part, the following expression is found for the heat losses from the exterior surface:

$$\dot{Q} = -k_2 A \frac{dT_2}{dr}(r=r_2) = -k_2 \kappa_2 A (B_2 I_1(\kappa_2 r_2) - C_2 K_1(\kappa_2 r_2)) \quad (8)$$

Substituting in this expression for the coefficients B_2 and C_2 their expressions, the following form is obtained for the expression of the heat loss:

$$\dot{Q} = K_{RF}(T_{ar} - T_s) + \epsilon_1 \Delta_1 + \epsilon_2 \Delta_2 \quad (9)$$

Dividing this expression of the heat loss by the difference of the local arterial blood and skin temperatures, the local conductance of the cylinder is found to be

$$K_1 = K_{RF} + \frac{\epsilon_1 \Delta_1 + \epsilon_2 \Delta_2}{T_{ar} - T_s} = K_{RF} + K_{RM} \quad (10)$$

where K_{RF} is the contribution of the blood flow and K_{RM} is that due to metabolism.

For calculation of the countercurrent exchange occurring in the extremities, it is necessary to know the average venous blood temperature in each cylinder. Starting from the expression of the venous temperature in the tissue element, $T_v = \alpha T_t + (1 - \alpha)T_{ar}$, where T_t is the temperature of the tissue element, the average venous blood temperature in the inner layer is found to be

$$\bar{T}_{v1} = \frac{1}{\pi r_1^2} \int_0^{r_1} 2 \pi r (\alpha T_t + (1 - \alpha)T_{ar}) dr = (1 - \alpha)T_{ar} + \alpha A_1 + \frac{2 \alpha I_1(\kappa_1 r_1)}{\kappa_1 r_1} \cdot B_1 \quad (11)$$

and the average venous blood temperature in the outer layer,

$$\bar{T}_{v2} = \frac{1}{\pi(r_2^2 - r_1^2)} \int_{r_1}^{r_2} 2 \pi r (\alpha T_t + (1 - \alpha)T_{ar}) dr =$$

$$(1 - \alpha)T_{ar} + \alpha A_2 + \frac{2 \alpha}{\kappa_2(r_2^2 - r_1^2)} \left(r_2 I_1(\kappa_2 r_2) - r_1 I_1(\kappa_2 r_1) \right) B_2$$

$$+ \left(r_1 K_1(\kappa_2 r_1) - r_2 K_1(\kappa_2 r_2) \right) C_2 \quad (12)$$

For the average venous blood temperature of the entire cylinder one obtains then

$$\bar{T}_v = \frac{V_1}{V_1+V_2} \bar{T}_{v1} + \frac{V_2}{V_1+V_2} \bar{T}_{v2} \quad (13)$$

where V_1 and V_2 are the flow rates in the inner and outer layer of the cylinder, respectively.

Countercurrent exchange in the major vessels

In the estimation of the heat exchange taking place between the major arteries and veins the formula, known from technology, is employed which gives the heat exchange between two parallel, adjacent tubes:

$$\frac{dT}{dx} = \frac{a}{V} (T_1 - T_2) \quad (14)$$

where

$$a = \frac{2 \pi k/c}{\operatorname{ar} \cosh \frac{L^2 r_1^2 - r_2^2}{2r_1 r_2}} \quad (14a)$$

When both tubes carry the flow V , the change of temperature occurring in the tubes over the distance dx is dT if the temperature difference between the tubes is $T_1 - T_2$. The radii of the tubes are r_1 and r_2 , and L is the distance between the central axes of the tubes. The specific heat of the fluid in the tubes is c , and the thermal conductivity of the surrounding substance is k .

In estimating the countercurrent exchange, the dimensions stated in Table 9 were used for the blood vessels.

The table gives the values (a) calculated by formula (14a) from the anatomical dimensions (with the numerical values $\kappa = 0.001 \text{ cal/s/cm/}^\circ\text{C}$ and $c = 0.92$

cal/g/°C), and the effective (a) values for each body part. The latter were determined taking into account that in the upper extremity, distal to the elbow joint, there are two main arteries and the same is true in the lower extremity distal from the knee joint, and that all these arteries are accompanied by two adjacent veins. The effective (a) value for the thigh was obtained by combining the two main femoral arteries into one artery in such manner that the countercurrent exchange at rest is consistent with a two-vessel system. For the foot the same coefficient was assumed as for the lower leg, in lack of sufficiently accurate dimensions.

It should be noted in view of the actual situation that the arterial and venous temperatures are affected not only by countercurrent exchange but also by the continuous branching of the vessels. Progressing in the direction of flow, the flow rate in the artery continuously decreases, while the venous flow increases in the direction in which the blood flows. The continuous admixture of blood of another temperature to that flowing in the principal vein has an immediate effect on the venous temperature and, by mediation of countercurrent exchange, also on the temperature behaviour of the artery. The following is a mathematical consideration of the case in which the flow in the vessels is assumed to change linearly with distance along the vessel (for an extremity of cylindrical shape and functioning homogeneously in axial direction, Fig. 8).

Assume that the temperatures in the artery and vein at the point x are $T_{ar}(x)$ and $T_v(x)$, respectively, and the flow rate in both at the point x is $V(x) = V_0 - vx$, where V_0 is the flow at the initial end of the vessels. The temperature of the venous blood arriving at the point x from the tissue is expressed as

$T_{vt}(x) = (1-b)T_{ar}(x) + bT_s + D$, where T_s is the skin temperature (assumed to be constant) and the term D

is due to heat production in the tissue.

The following equations are then valid for the arterial and venous blood temperatures:

$$\frac{dT_{ar}(x)}{dx} = \frac{a}{V(x)} \cdot (T_v(x) - T_{ar}(x)) \quad (15)$$

$$\frac{dT_v(x)}{dx} = \frac{a}{V(x)} (T_v(x) - T_{ar}(x)) + \frac{v}{V(x)} (T_v(x) - T_{vt}(x)) \quad (16)$$

From these, the differential equation is obtained for arterial temperature:

$$T_{ar}'' - \frac{2v}{V} T_{ar}' - \frac{abv}{v^2} T_{ar} = -\frac{abv}{v^2} T_s - \frac{av}{v^2} D$$

The temperature in the vein is found by the equation

$$T_v = T_{ar} + \frac{v}{a} T_{ar}'$$

The complete solutions will be the following expressions for the temperatures:

$$T_{ar}(x) = T_s + D/b + C_1(V_0 - vx)^{r_1} + C_2(V_0 - vx)^{r_2} \quad (17)$$

$$T_v(x) = T_s + D/b + C_1\left(1 - \frac{vr_1}{a}\right)(V_0 - vx)^{r_1} + C_2\left(1 - \frac{vr_2}{a}\right)(V_0 - vx)^{r_2} \quad (18)$$

$$r_1 = -0.5 + \sqrt{\frac{ab}{v} + 0.25} \quad r_2 = -0.5 - \sqrt{\frac{ab}{v} + 0.25}$$

The coefficients C_1 and C_2 depend on the initial conditions. Assuming that the length of the extremity is x_1 and that the flow in the vessels ceases at the other end ($V_0 = vx_1$) (that is, there is no second cylinder in continuation of the cylinder under consideration, which would be served by blood flow through the same vessels), the boundary conditions may be expressed in the form

$$T_{ar}(0) = T_{AR}$$

$$T_v(x_1) = T_{vt}(x_1)$$

For the arterial blood temperature the following expression is then found:

$$T_{ar} = \left(\frac{V_0 - vx}{V_0}\right)^{r_1} \cdot T_{AR} + \left(1 - \left(\frac{V_0 - vx}{V_0}\right)^{r_1}\right) \cdot T_s + \left(1 - \left(\frac{V_0 - vx}{V_0}\right)^{r_1}\right) \cdot D/b \quad (19)$$

Integration yields for the average arterial blood temperature of the extremity:

$$\bar{T}_{ar} = \frac{1}{r_1+1} T_{AR} + \frac{r_1}{r_1+1} T_s + \frac{r_1}{r_1+1} D/b \quad (20)$$

Countercurrent exchange in an extremity comprising two consecutive cylinders

If the extremity is composed of two cylinders one after the other, different boundary conditions have to be applied to the first of the cylinders because the flow does not become zero at its distal end. For the second cylinder, however, the solution is obtained from the preceding formula (19) if the temperature T_0 is substituted for T_{AR} (Fig. 9).

The coefficient C_1 and C_2 of the first cylinder and the temperatures T_0 and T_1 can be solved from the equations

$$T_{AR} = T_s + D/b + C_1 V_0^{r_1} + C_2 V_0^{r_2}$$

$$T_0 = T_s + D/b + C_1 (V_0 - v_1 x_1)^{r_1} + C_2 (V_0 - v_1 x_1)^{r_2}$$

$$T_1 = T_s + D/b + C_1 \left(1 - \frac{v_1 r_1}{a_1}\right) (V_0 - v_1 x_1)^{r_1} + C_2 \left(1 - \frac{v_1 r_2}{a_1}\right) (V_0 - v_1 x_1)^{r_2}$$

$$T_0 - T_1 = \frac{v_2 r_2}{a_2} (T_0 - T_s - D/b)$$

The solution is the following expression for the arterial blood temperature in the first cylinder:

$$T_{ar}(x) = T_s + D/b + \left(\frac{1}{G_1} \left(\frac{v_0 - v_1 x}{v_0} \right)^{r_1} + \frac{1}{G_2} \left(\frac{v_0 - v_1 x}{v_0} \right)^{r_2} \right) (T_{AR} - T_s - D) \quad (21)$$

where

$$G_1 = 1 - \frac{v_2^{r_3} a_1 - v_1^{r_1} a_2}{v_2^{r_3} a_1 - v_1^{r_2} a_2} \left(\frac{v_0 - v_1 x_1}{v_0} \right)^{r_1 - r_2}$$

and

$$G_2 = 1 - \frac{v_2^{r_3} a_1 - v_1^{r_2} a_2}{v_2^{r_3} a_1 - v_1^{r_1} a_1} \cdot \left(\frac{v_0 - v_1 x_1}{v_0} \right)^{r_2 - r_1}$$

The average arterial blood temperature will have the expression

$$\begin{aligned} \bar{T}_{ar} = T_s + D/b + (T_{AR} - T_s - D/b) & \left(\frac{1}{G_1} \frac{1}{v_1 x_1 (r_1 + 1)} \left[v_0 - \left(\frac{v_0 - v_1 x_1}{v_0} \right)^{r_1} (v_0 - v_1 x_1) \right] \right. \\ & \left. + \frac{1}{G_2} \frac{1}{v_1 x_1 (r_2 + 1)} \left[v_0 - \left(\frac{v_0 - v_1 x_1}{v_0} \right)^{r_2} (v_0 - v_1 x_1) \right] \right) \quad (22) \end{aligned}$$

Practical countercurrent exchange model

Since in the model, in each cylinder, supply of arterial blood at uniform temperature to the tissue everywhere has been assumed, the theory of countercurrent exchange presented in the former is replaced with a model in which there is, instead of continuous branching, only one branch for each cylinder. Fig. 10 shows schematic diagrams of the models both for the upper and lower extremity.

For the effective length of the heat exchange paths such lengths have been chosen which produce in the model a countercurrent effect equal in magnitude to that in the model of a continuously branching vascular system presented above. By this procedure the heat

exchange coefficients stated in Table 10 were obtained.

Complete solution for an extremity

It is desired to express the heat losses of all the parts of the body in terms of the temperature difference $T_{AR} - T_s$. This implies that the local arterial blood temperatures T_{ar} have to be expressed as functions of the central blood temperature T_{AR} and skin temperature T_s . In the case of the lower extremity (consisting of three consecutive cylinders) the following system of equations has to be solved (with notations as indicated in Fig. 11):

$$T_{v1} = (1-b)T_{ar1} + b_1T_s + c_1 \Delta_1 + d_1 \Delta_2$$

$$T_{v2} = (1-b_2)T_{ar2} + b_2T_s + c_2 \Delta_2 + d_2 \Delta_2$$

$$T_{v3} = (1-b_3)T_{ar3} + b_3T_s + c_3 \Delta_2 + d_3 \Delta_2$$

$$T_{AR} = T_{ar1} + (a_1x_1/V_0)(T_{AR} - T_{v8})$$

$$T_{ar1} = T_{ar2} + \left[a_2x_2/(V_2+V_3) \right] \cdot (T_{ar2} - T_{v8})$$

$$T_{ar2} = T_{ar3} + (a_3x_3/V_3) \cdot (T_{ar3} - \bar{T}_{v3})$$

$$T_{v7} = (V_1/V_0) \cdot \bar{T}_{v1} + (V_2+V_3)/V_0 \cdot T_{v6}$$

$$T_{v5} = \left[V_2/(V_2+V_3) \right] \cdot \bar{T}_{v2} + \left[V_3/(V_2+V_3) \right] \cdot T_{v4}$$

$$T_{ar2} - T_{ar3} = T_{v4} - \bar{T}_{v3}$$

$$T_{ar1} - T_{ar2} = T_{v6} - T_{v5}$$

$$T_{AR} - T_{ar1} = T_{v8} - T_{v7}$$

Solution of this system yields the arterial blood temperatures for each cylinder in the form

$$T_{ar} = \beta T_{AR} + (1-\beta)T_s + e_1 \Delta_1 + e_2 \Delta_2 \quad (23)$$

where the terms $e_1 \Delta_1$ and $e_2 \Delta_2$ are due to the heat production in the inner and outer layers of the cylinders, respectively. When the expression of the arterial blood temperature in the cylinder is substituted in that of the heat loss from the cylinder (9), the expression for the heat loss of the cylinder becomes

$$\dot{Q} = \beta \cdot K_{RF}(T_{AR}-T_s) + (g_1 + K_{RF}e_1) \Delta_1 + (g_2 + K_{RF}e_2) \Delta_2 \quad (24)$$

When the expression of the loss is divided by the temperature difference $T_{AR}-T_s$, the conductance of the cylinder is found to be

$$K = \beta K_{RF} + \frac{(g_1 + K_{RF}e_1) \Delta_1 + (g_2 + K_{RF}e_2) \Delta_2}{T_{AR} - T_s} = K_F + K_M \quad (25)$$

It is thus possible to distinguish in the conductance two components: the component caused by flow, K_F , and the component due to metabolism, K_M . The latter is dependent on the magnitude of the temperature difference $T_{AR}-T_s$. In its magnitude, however, the metabolic component is small in comparison with the flow-induced component K_F ; as a consequence, the total conductance is only little dependent on the temperature difference $T_{AR}-T_s$ for which the conductance is calculated.

Determination of the conductance for the upper extremity follows the same principle as that for the lower extremity. However, since the model of the upper extremity only comprises two consecutive cylinders, there is only a system of seven simultaneous equations to be solved instead of 11 for the lower extremity. Since the model does not presuppose any countercurrent exchange in the head nor in the trunk, the expression of the regional conductance for these parts of the body (10) also represents their central conductance.

Criticism of the mathematical model

The criticism of the model may be divided into three parts: (1) idealization of the anatomical structure, (2) the thermal equilibrium equation of tissue, and (3) countercurrent heat exchange of the major blood vessels.

In the model, the different parts of the body were approximated by cylinders, which were assumed to comprise two internally homogeneous, and homogeneously functioning, layers. In actual truth the body parts are not cylindrical, and their interior structure is not cylindrically symmetric. The bones of the extremities are often rather asymmetrically located in the cross-section of the extremity, and the same is true for muscles. Since, furthermore, different muscles have different duties, the assumption of uniform distribution of circulation in a body part performing work is not valid in exercise situations. If in the extremity performing work inhomogeneous perfusion in fact prevails, the consequence will be that the conductance of the total extremity remains at least somewhat lower than its value in the homogeneous case. This is due to the circumstance that the rate of increase of conductance with perfusion is lower as perfusion is higher. The local increase of conductance at the site of the muscle engaged in work then remains less than the decrease in the tissues which are in a state closer to rest. In addition, the thickness of the fat layer may be different in different parts of the extremity. To be quite exact, the effective fat layer of the extremity would then have to be determined by assigning to each area a weight according to its conductance. Experimentally, however, in lower extremities, engaged in work, results in fairly good agreement with theoretical calculation were obtained. Since in the case of a lower extremity performing work the assumption of point source character of the heat

exchange can be considered fairly well appropriate, and since in the situation in question countercurrent exchange can have no significance whatsoever in view of the great flow rates involved, the conclusion seems justified that the idealization of structure which was undertaken in the model has not caused any appreciable error. In the case of an extremity at rest, in which the inner function is more nearly homogeneous, the idealization can be considered even better justified.

The critical point in the heat equilibrium equation of tissues is the assumption of point source character of the heat exchange between blood and tissue. This assumption is rather well valid when the perfusion of the tissue is very high. In tissue at rest, however, the heat exchange may take place over a distance of several centimetres. The assumption of constant arterial blood temperature throughout the body part then also ceases to be valid; the blood temperature acquires, instead, a given, radial distribution. In that case the assumption of point source character yields, in principle, too high values for the conductance. If, again, the theory were corrected by using an effective arterial temperature which would give the correct conductance, it would follow that the model leads to an internal temperature value of the body part which is too low. Comparisons of experimental and theoretical results suggested that the error introduced by the point source assumption might be in the order of 15-20 % at rest. However, the magnitude of this error should be taken with great reservations, because there is no certainty as to how well the tissue perfusions applied in the theory and the actual perfusions prevailing in the tests were mutually consistent. The assumption of lacking axial conduction, which was made in solving the tissue heat balance equation, has under no circumstances any practical significance.

In the heat exchange of the major vessels leading to the different body parts, the heat exchange of the blood with surrounding tissues was not taken into account, but rather only the countercurrent exchange between major vessels. The results of the countercurrent calculation can be considered correct in respect of their order of magnitude at the most, seeing how inaccurate were the available data on blood vessel dimensions and on the mutual spacing of blood vessels. In actual truth there are several main blood vessels in each body part, and it is difficult to substitute one single effective blood vessel for these. In the model no account was taken of the fact that part of the blood returns by superficial veins, whereby the flow in the deep veins may be smaller than that in the corresponding arteries. However, the superficial veins are not in operation in the situation with minimal cutaneous circulation. If, again, the flow in the body part is very high, then the significance of countercurrent exchange is negligible at all events.

The countercurrent model which was employed can be considered, in principle, rather well able to represent the countercurrent exchange of major vessels, if only the numerical values can be properly determined.

Applications of the mathematical model

In an application of the model presented above, it was used to elicit the minimum conductances of the different parts of the body, and also of the entire body, of a person at rest and during ergometric exercise of various levels. Moreover, as a separate study for the lower leg, its conductance is examined as a function of blood flow rate and, furthermore, its conductance as a function of the thickness of the fat layer, at rest and during exercise.

The following numerical values were used in these applications:

- Thermal conductivity of the inner layer,
 $k_1 = 0.0011 \text{ cal/s/cm/}^\circ\text{C}$,
- Thermal conductivity of the outer layer,
 $k_2 = 0.0005 \text{ cal/s/cm/}^\circ\text{C}$.

In some special considerations the skin was separately taken into account, for which the value $k = 0.0007 \text{ cal/s/cm/}^\circ\text{C}$ was used. For the specific heat of blood the value $c_b = 0.92 \text{ cal/g/}^\circ\text{C}$ was used and for its density, $\rho_b = 1.0 \text{ g/cm}^3$.

The heat exchange coefficient between blood and tissue was taken to be $\alpha = 0.50$ everywhere.

In Table 12 the results referring to the human body at rest have been presented. The calculations are based on the anatomical dimensions given in Table 8. Perfusion in the amount of 2 % to the outer layers of all parts of the body ($F_2 = 0.02 \text{ cm}^3/\text{min}/\text{cm}^3$) was also assumed to take place in the inner layers of the body parts excepting the head, where 10 % perfusion ($F_1 = 0.10 \text{ cm}^3/\text{min}/\text{cm}^3$) was assumed. The temperature increment due to metabolism in the tissues was taken to be $\Delta_1 = \Delta_2 = 0.45^\circ\text{C}$. This corresponds to the tissue heat production/circulation ratio (caloric value) $E_1 = E_2 = 0.21 \text{ cal/g}$. The flow rates and the temperature increments employed have also been shown in Table 11. In Table 12 the expressions are given for the temperature of the arterial blood T_{ar} arriving in each body part, for the average exiting venous blood temperature \bar{T}_v , for the conductance K_R , which was calculated on the basis of the local temperature difference $T_{ar} - T_s$, and for the conductance K , calculated on the basis of the central temperature difference $T_{AR} - T_s$. The last column contains the values of central conductance for the temperature difference $T_{AR} - T_s = 4^\circ\text{C}$. Addition gave for the entire body the minimum conductance of 318 $\text{cal}/\text{min}/^\circ\text{C}$, or 173 $\text{cal}/\text{min}/^\circ\text{C}/\text{m}^2$.

K_R calculated on the basis of the local temperature difference $T_{AR}-T_S$, and for the conductance K calculated on the basis of the central temperature difference $T_{AR}-T_S$.

The last column contains the values of the central conductance for the temperature difference $T_{AR}-T_S = 10^\circ\text{C}$. Addition yielded for the conductance of the entire body the value of 488 cal/min/ $^\circ\text{C}$, or 265 cal/min/ $^\circ\text{C}/\text{m}^2$.

In Fig. 12 the increase of the conductance of the entire body has been plotted over the rate of ergometric work. Use was made in the plotting of this graph, in addition to the data presented above, of the value 240 cal/min/ $^\circ\text{C}/\text{m}^2$ elicited with 300 kpm/min ergometric load rate. It is seen that the conductance increases initially at a steep rate from rest to even light exercise. Its rate of increase becomes less with increasing intensity of exercise.

As a special application, in Fig. 13, the increase of the main component K_{RF} of the regional conductance of the thigh with increasing flow in the inner part has been displayed. Here, as in the analysis with reference to the entire body, the thickness of the outer layer was taken to be 7 mm.

Fig. 14 shows the principal component K_{RF} of the regional conductance of the thigh as a function of the thickness of the fat layer, at rest and during exercise. The inner part of the thigh was assumed to be constant in size and the fat layer was increased in the outward direction, whereby simultaneously the skin area of the thigh increased. It is seen that at complete rest, when the same perfusion of 2 % was assumed both for the fat layer and the muscle layer, the thickness of the fat layer has no practical effect. In the situation consistent with 600 kpm/min

Table 13 contains the results from analysis of a subject performing ergometric exercise in the amount of 600 kpm/min (1400 cal/min).

The load imposed by the work was imagined to be homogeneously distributed over the muscular parts of the lower extremities. The blood flow in the lower extremities was calculated, taking the efficiency of the muscles to be 30 % and the amount of energy derivable from the blood (caloric value), 0.80 cal/g. On these assumptions the perfusion of the muscle was found to be $F = 0.40 \text{ cm}^3/\text{min}/\text{cm}^3$. Since the bones of the thigh and lower leg are located at great depth and cannot affect the heat losses in the stationary state, this value of muscle perfusion was used in the case of thigh and lower leg for the perfusion of the inner part ($F_1 = 0.40 \text{ cm}^3/\text{min}/\text{cm}^3$). In the calculation of the losses. In the calculations of countercurrent exchange, however, the true flows were naturally taken into account. In the instance of the foot, for the perfusion of the inner part a weighted perfusion mean ($F_1 = 0.09 \text{ cm}^3/\text{min}/\text{cm}^3$) was used. The inner part perfusion value applied for the upper extremities was $F_1 = 0.05 \text{ cm}^3/\text{min}/\text{cm}^3$, for the head $F_1 = 0.10 \text{ cm}^3/\text{min}/\text{cm}^3$, and for the trunk $F_1 = 0.04 \text{ cm}^3/\text{min}/\text{cm}^3$.

In all the outer layers the perfusion of $F_2 = 0.02 \text{ cm}^3/\text{min}/\text{cm}^3$ was placed. The temperature increment due to heat production used for the inner parts of the thigh and lower leg was $\Delta_1 = 1.25^\circ\text{C}$, for the inner part of the foot $\Delta_1 = 0.55^\circ\text{C}$, and for the inner parts of the upper extremities $\Delta_1 = 0.90^\circ\text{C}$. Elsewhere the value of $\Delta = 0.45^\circ\text{C}$ was used. These values have also been stated in Table 11.

In Table 13 the expressions are given for the temperature T_{ar} of the arterial blood arriving in each part of the body, for the average temperature T_v of the venous blood leaving the body part, for the conductance

rate of exercise, however, the effect is already quite dramatic.

Theoretical results

For determination of the minimum conductances of the body and of its different parts, the body was approximated by a set of cylinders corresponding to the following parts: head, trunk, upper arm, forearm, thigh, lower leg, and ankle + foot. In each cylinder two layers were assumed, the outer layer comprising the skin and the fat layer, while the inner one comprised the muscle and bone. In the outer layer, a small metabolic circulation was assumed and in the inner layer, a circulation consistent with each particular situation (i.e., rest or exercise). When the radial temperature distribution field in a cylindrical body part in steady-state condition was analytically determined, an expression was found on this basis for the heat losses through the surface of the cylinder. Dividing the heat loss expression by the difference between arterial blood temperature and skin temperature of the body part in question, an expression was found for the local conductance of the body part. When again the same heat loss expression was divided by the difference of the temperatures of central blood and skin, the central conductance of the body part was found. In order to find the local arterial blood temperature of each cylinder in the case of the extremity cylinders, a model had to be devised for the countercurrent exchange of the major blood vessels. On the basis of the model, effective countercurrent exchange coefficients were determined for each cylinder, which could be employed for determining the local arterial blood temperature of each cylinder. The conductance of the whole body could be calculated by adding the central conductances of the individual body parts. Such solutions were performed for the state of rest as well as for exercise conditions.

This yielded a result, according to which the conductance of the total body increased from the resting value of $173 \text{ cal/min/}^{\circ}\text{C/m}^2$ so that at 300 kpm/min (700 cal/min) bicycle ergometer work the conductance was $240 \text{ cal/min/}^{\circ}\text{C/m}^2$ and at work at the rate of 600 kpm/min (1400 cal/min), $265 \text{ cal/min/}^{\circ}\text{C/m}^2$. It is thus seen that the conductance at first increases at a high rate on transition from complete rest to light work. The increase becomes continuously slower with increasing intensity of work. The increase of conductance is caused by increase of the conductance of the working extremities. It is equivalent in its character to the increase of overall total conductance, but the relative changes are greater. For the thigh, for instance, the conductance of $165 \text{ cal/min/}^{\circ}\text{C/m}^2$ was found at rest, and at work at the rate of 600 kpm/min the value of $311 \text{ cal/min/}^{\circ}\text{C/m}^2$ was obtained, that is about twice that at rest. For the conductances of the different body parts at rest, the following values were found: head - 243, trunk - 200, lower extremity - 147, and upper extremity - $125 \text{ cal/min/}^{\circ}\text{C/m}^2$. In the exercise situation (600 kpm/min), the conductance of the lower extremity was found to be $354 \text{ cal/min/}^{\circ}\text{C/m}^2$ and that of the upper extremity, $211 \text{ cal/min/}^{\circ}\text{C/m}^2$. The conductance of an extremity engaged in work was found to be strongly dependent on the thickness of the fat layer. For the conductance of the thigh at 600 kpm/min work load, with 2 mm fat layer a value about twice that corresponding to 10 mm fat layer was obtained. At complete rest, on the contrary, when the circulation in the inner layer was assumed to be the same as in the outer layer, the fat layer of the thigh had no effect of practical significance on the conductance. However, the effect manifests itself strongly with increasing circulation in the inner part.

Comparison of theoretical and experimental results

The theoretically derived change of the minimum conductance of the total body, as a function of the external exertion level, was found to conform to the experimental determinations, as can be seen from Fig. 6. The experimental and theoretical results can be said to be largely equivalent in all instances except in that of the arms at rest, where the experimental results were less than 2/3 of those found by theoretical approach. The discrepancy is most probably due to the fact that the countercurrent exchange according to the theory is too low in the case of the upper extremities. Another observation suggesting the same is that experimentally a surprisingly great increase was noted in the minimum conductance of the arms on transition from rest to work. Accordingly, the countercurrent exchange in the upper extremities would possess relatively greater significance than that in the lower extremities. Another circumstance, under the influence of which the theory might be expected to yield excessive conductance values at rest, is the assumption of point source character of the heat exchange between tissue and circulation, which is part of the theory. This assumption is not valid at low perfusion rates; on the contrary, the heat exchange of the blood is accomplished over a distance of several centimetres. For the trunk and lower extremities, at rest, conductances were theoretically found which exceeded the experimentally determined values by 15-20 % on the average. Since at small flow rates, according to the theory, the thickness of the fat layer can have no noteworthy significance, the discrepancy has to be attributed to factors associated with circulation and with its heat exchange. If it is considered that the tissue perfusion $F = 0.02 \text{ cm}^3/\text{min}/\text{cm}^3$ employed in the theoretical calculations is consistent with the true perfusion in the experimental situations, the difference of 15-20 %, which was found, would have to be

accounted to the point source assumption of the heat exchange of circulation. If it is assumed that the true perfusion was higher than the said value of $0.02 \text{ cm}^3/\text{min}/\text{cm}^3$, this would imply that the point source exchange assumption would have to be considered to have caused a correspondingly greater error, and vice versa.

In the case of the head, the calculated value remained slightly lower than that found by experiment. However, the difference has no practical significance, seeing that assumption of a fat layer on the head 1 mm less in thickness would already reverse the situation.

In the exercise situation, the theoretical model can be said to be well consistent with the experimental results. In all instances except that of the lower extremities, comparisons are impeded by the fact that it is hard to assess exactly how great perfusions should be assigned to the muscles of different body parts.

Summary

1. For the purpose of programming the skin temperatures of the head, trunk and extremities and of continuous measurement of their separate heat losses, an exposure and measuring device was elaborated, which is called the control and calorimetry suit (CCS).
2. The method involving use of the CCS was applied in determinations of the passive heat losses of different parts of the human body at rest and during exercise. By means of lowering the skin temperature by consecutive steps and observing the heat losses and thermal conductance, that skin temperature value was identified on each energy level at which the decrease of conduction resulting from reduction of active heat transport turned into increase at onset of thermoregulative heat production. The heat losses at this limiting value were considered to be passive in character.
3. A simple mathematical model was devised to describe the passive heat losses from different parts of the body depending on variations of structure and function. The degree of agreement between theoretical calculations and experimental observations was judged to be encouraging.

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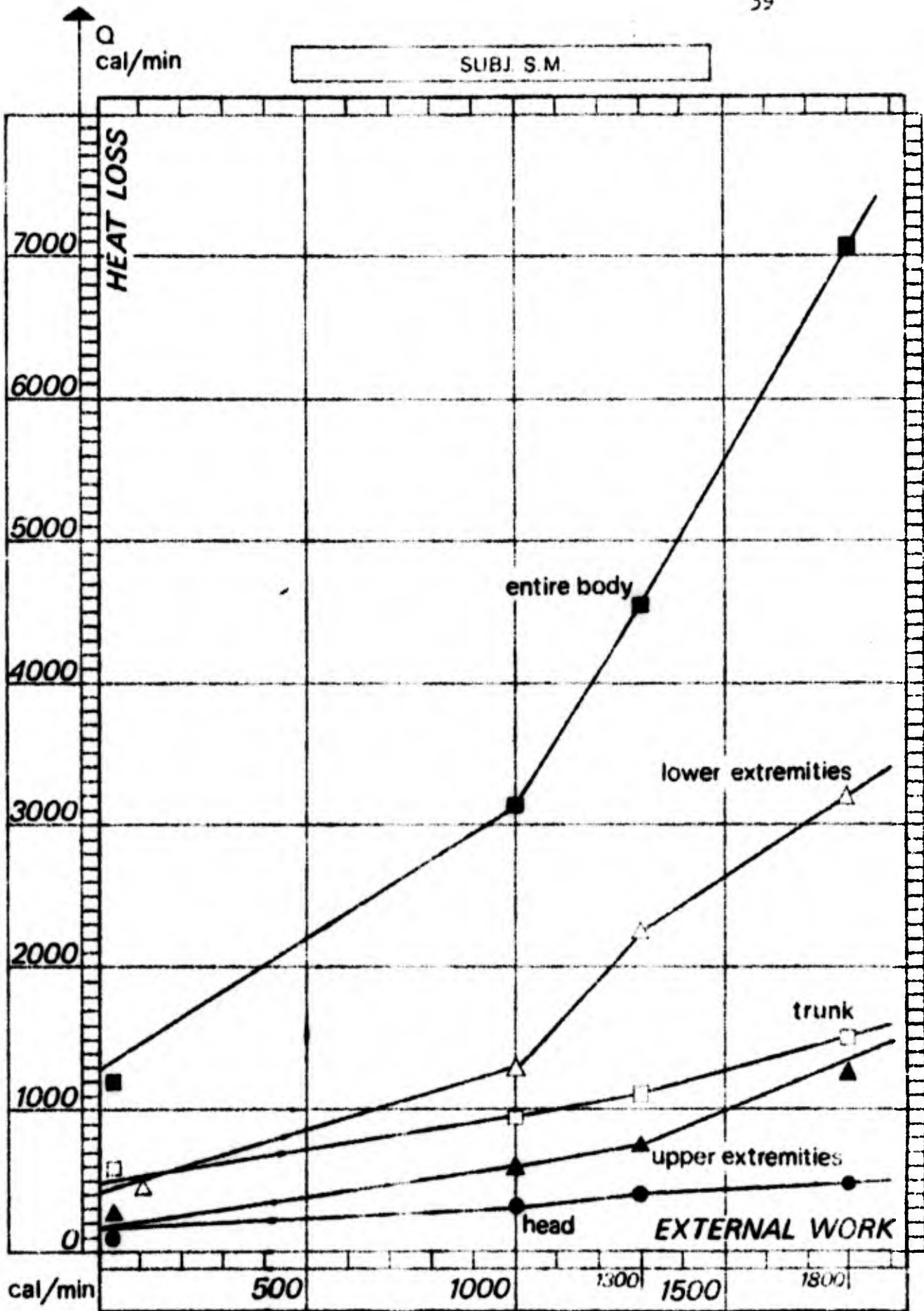


Fig. 1. Heat losses from the different body parts of Subject S.M., as functions of external work load.

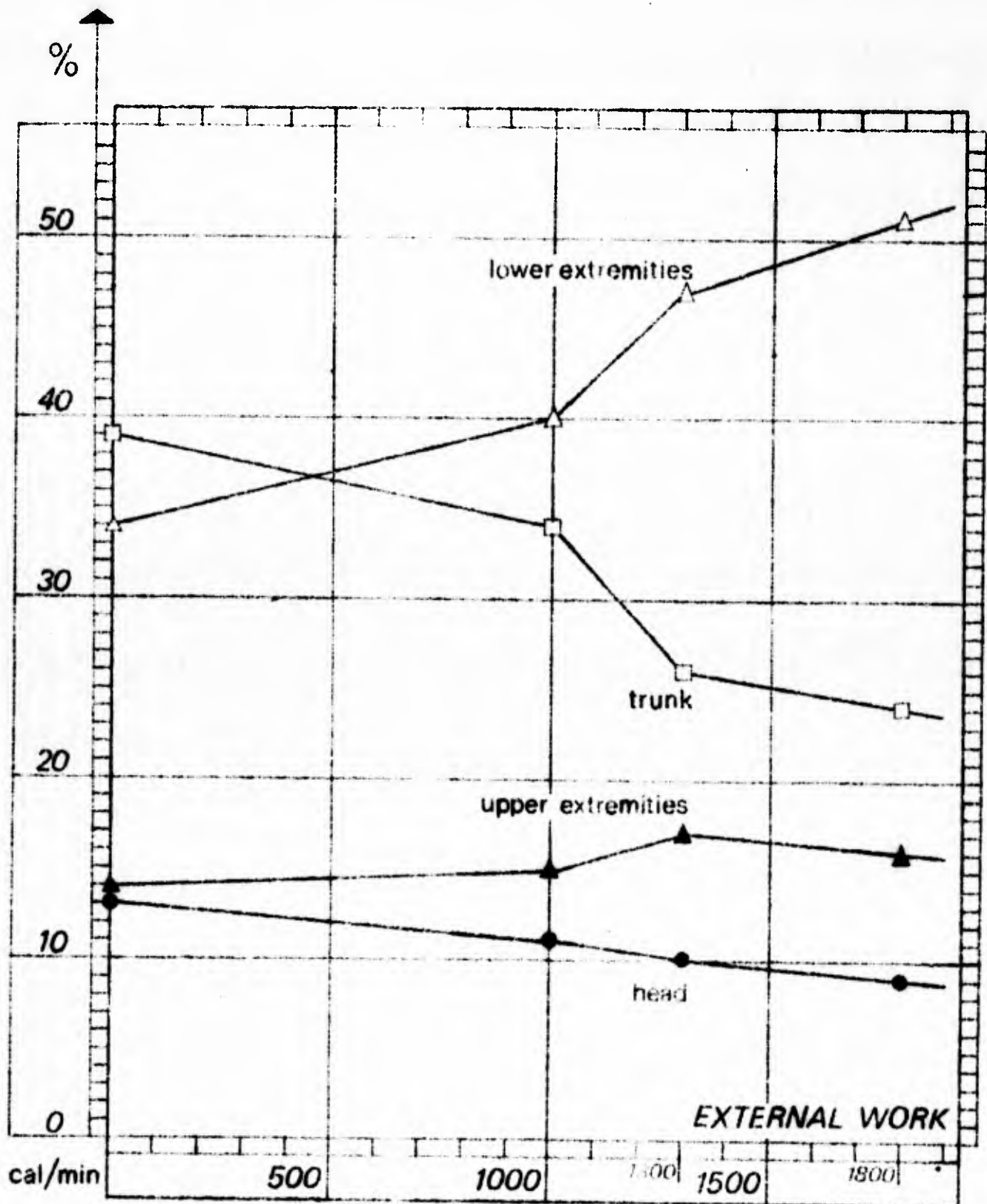


Fig. 2. Percentage distribution of heat losses between the different body parts, as function of external work load (mean of all test subjects).

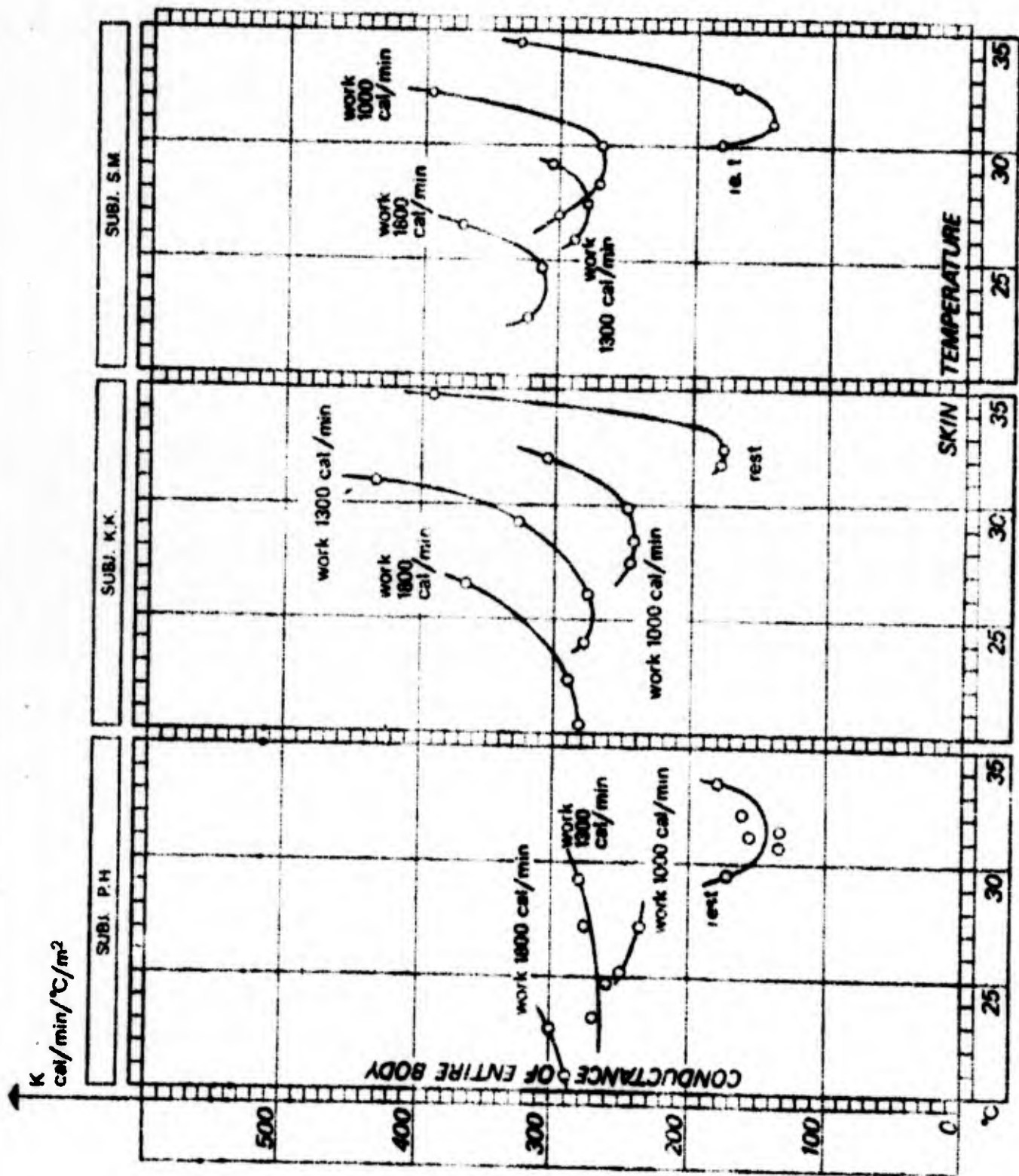


Fig. 3. Total conductances of Subjects P.H., K.K. and S.M. as functions of the skin temperature T_s , at rest and during exercise at various work loads.

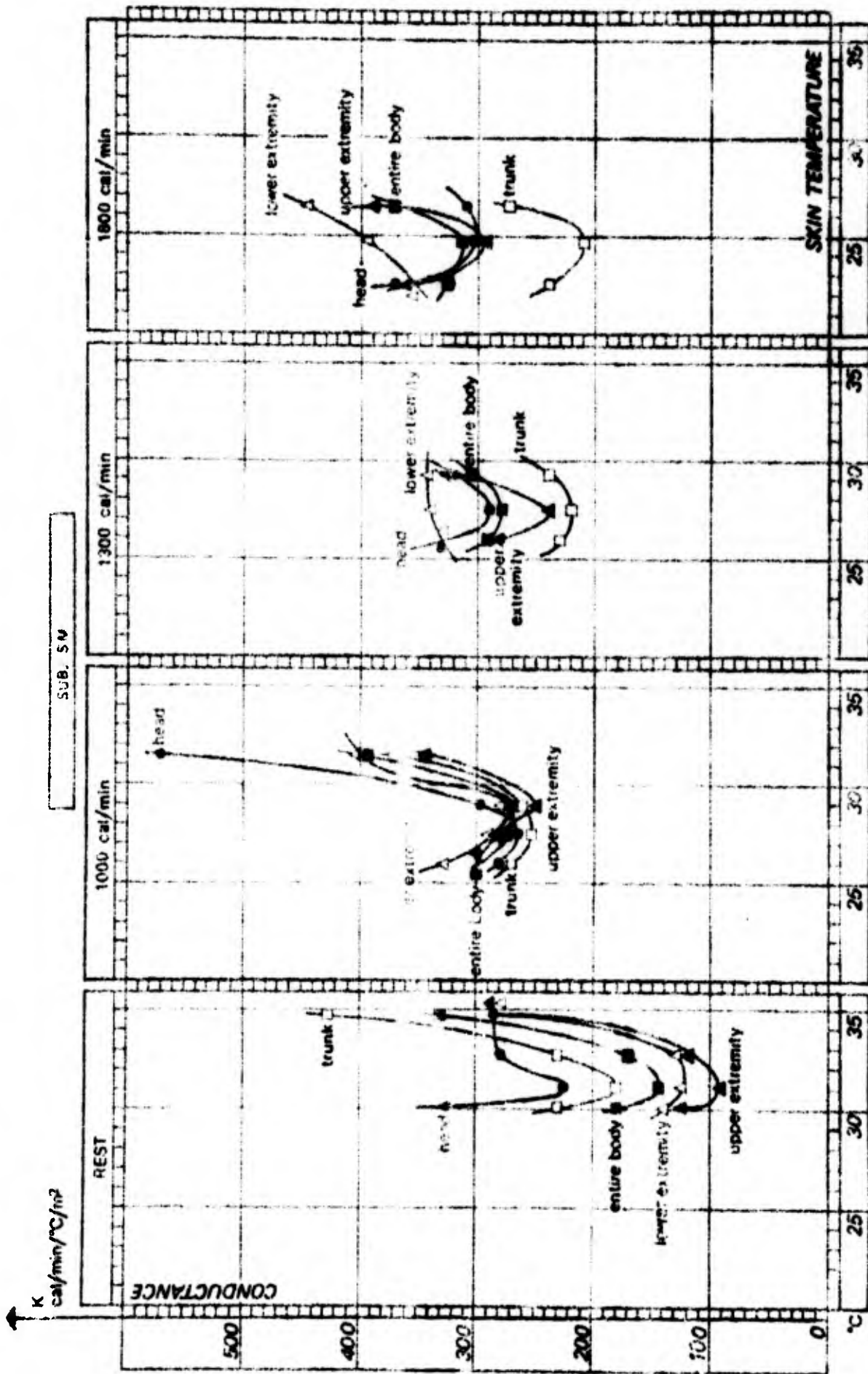


Fig. 4. Conductances of the different body parts of Subject S.M. as function of the skin temperature T_s , at rest and during exercise at various work loads.

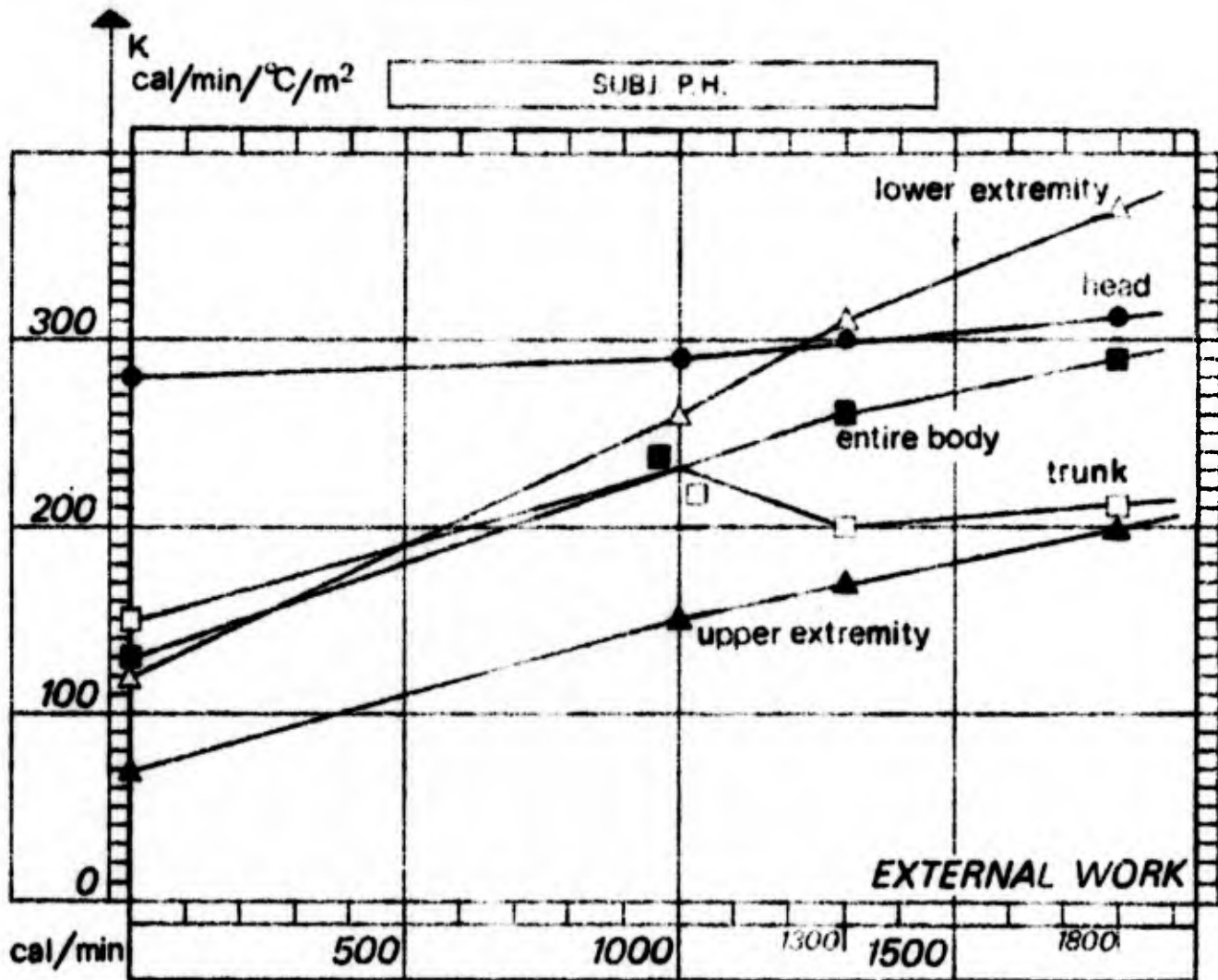


Fig. 5. Values of the minimum conductances of Subject P.H. found for the different body parts, as functions of external work load.

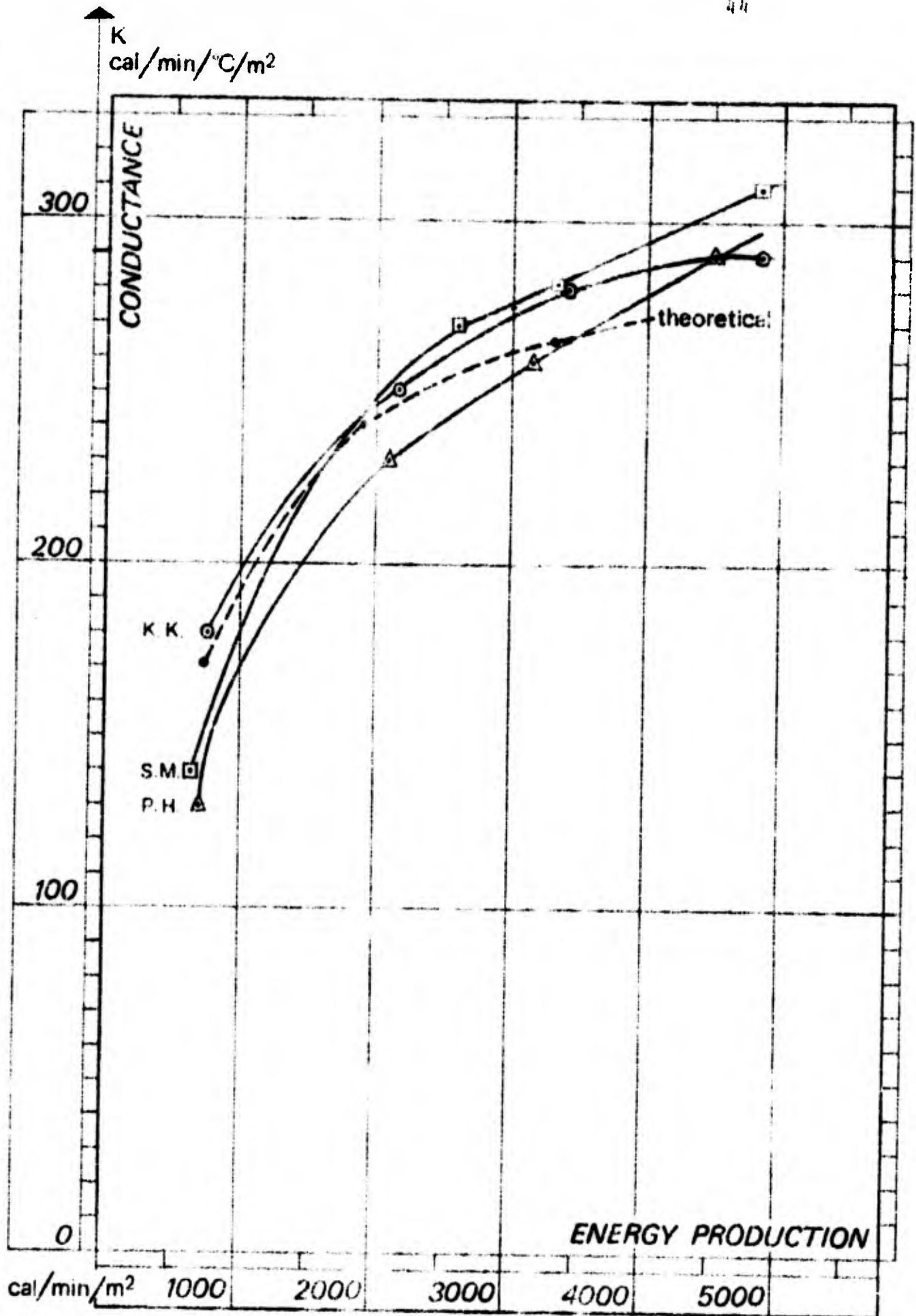


Fig. 6. Experimentally and theoretically found dependence of the minimum conductance of the entire body on the energy production per unit area.

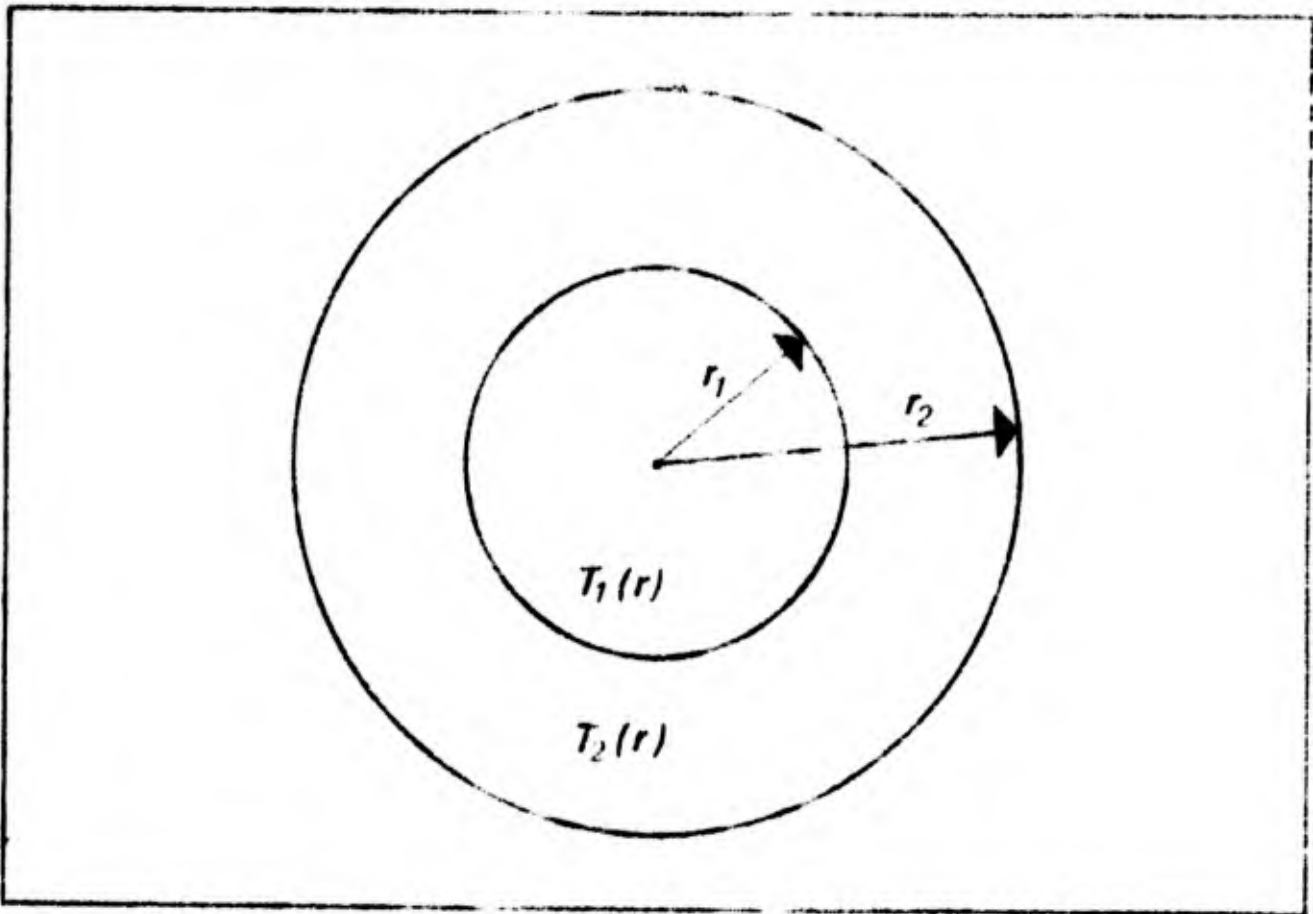


Fig. 7. The cross section of a cylindrical body part.

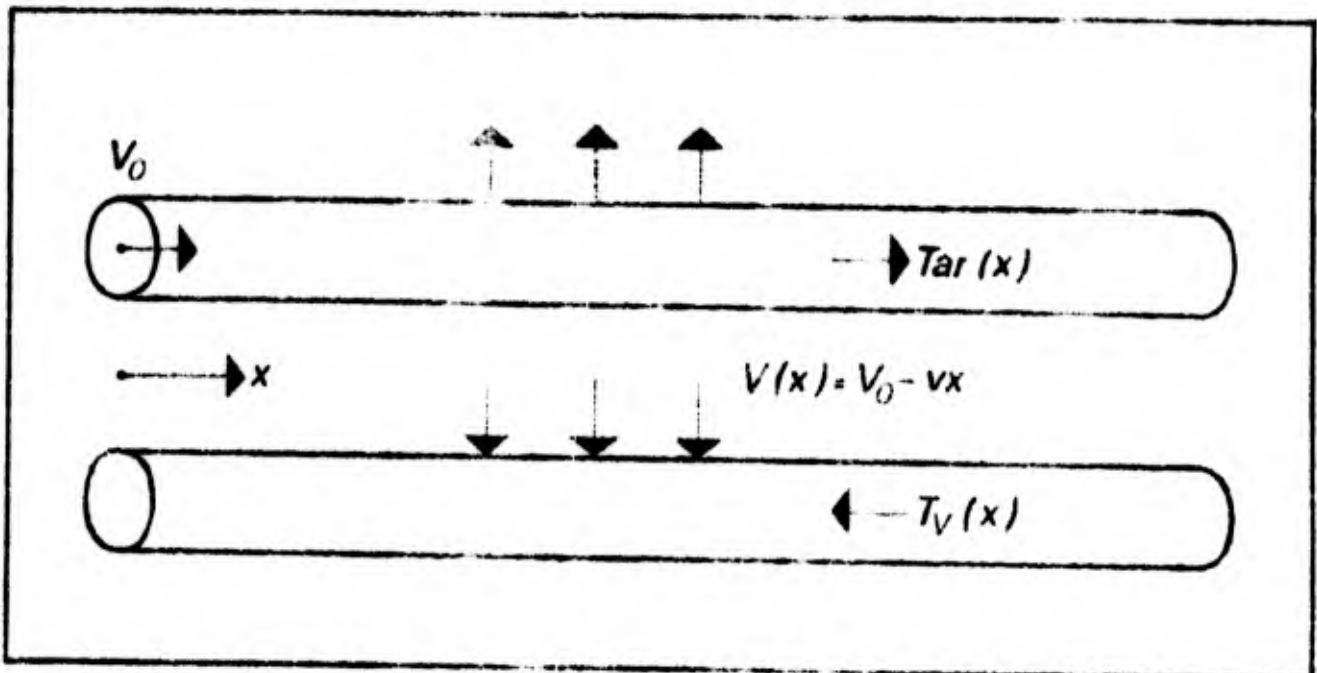


Fig. 8. The countercurrent heat exchange in major blood vessels.

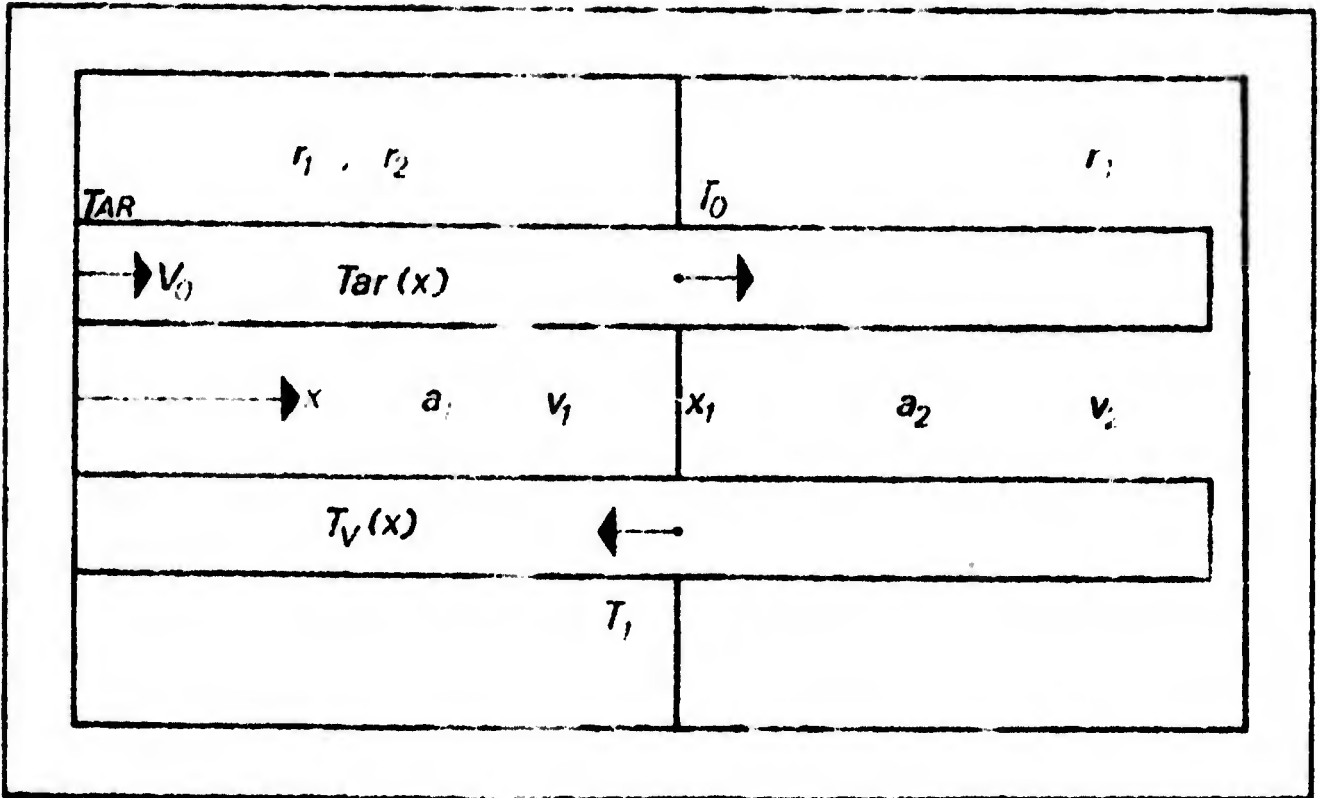


Fig. 9. The countercurrent heat exchange in an extremity comprising two consecutive cylinders.

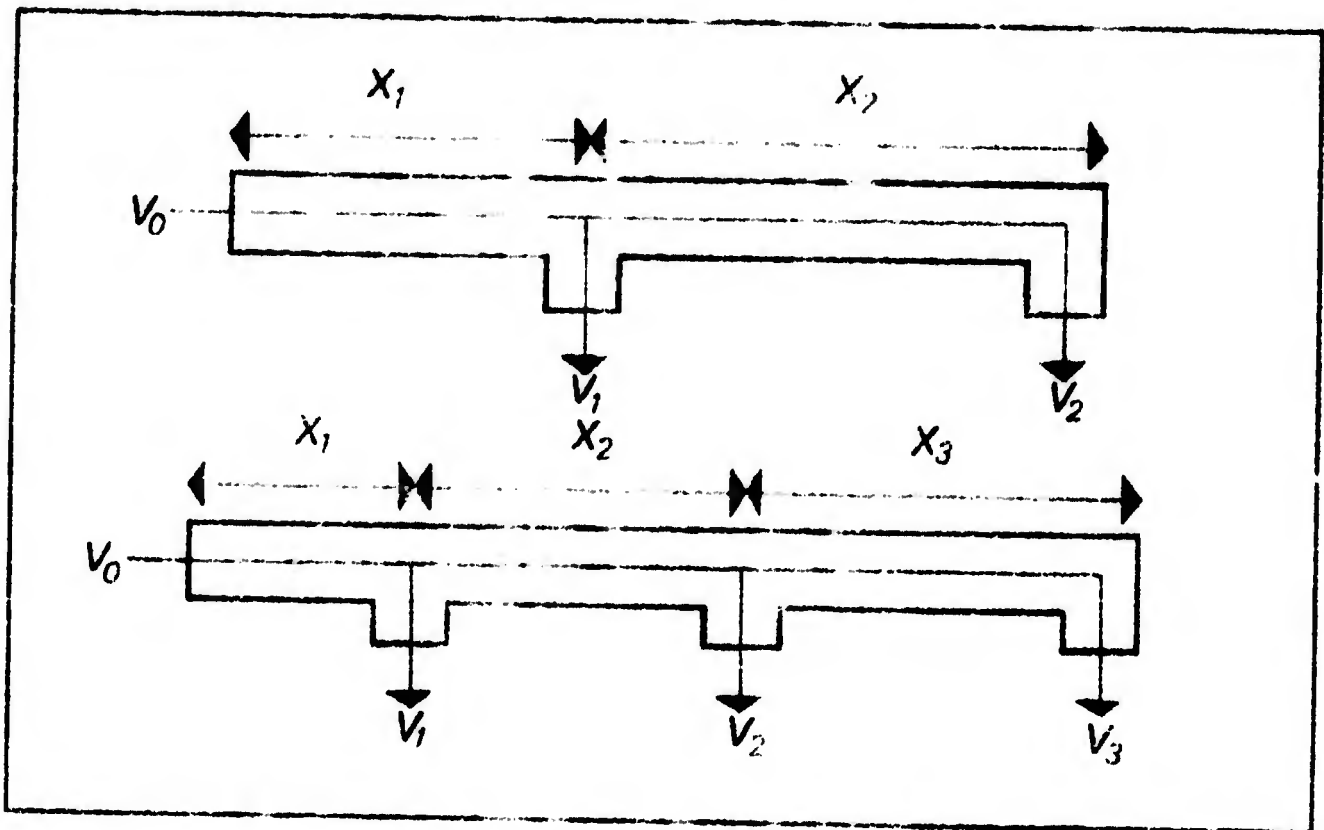


Fig. 10. The practical countercurrent exchange models of upper and lower extremity.

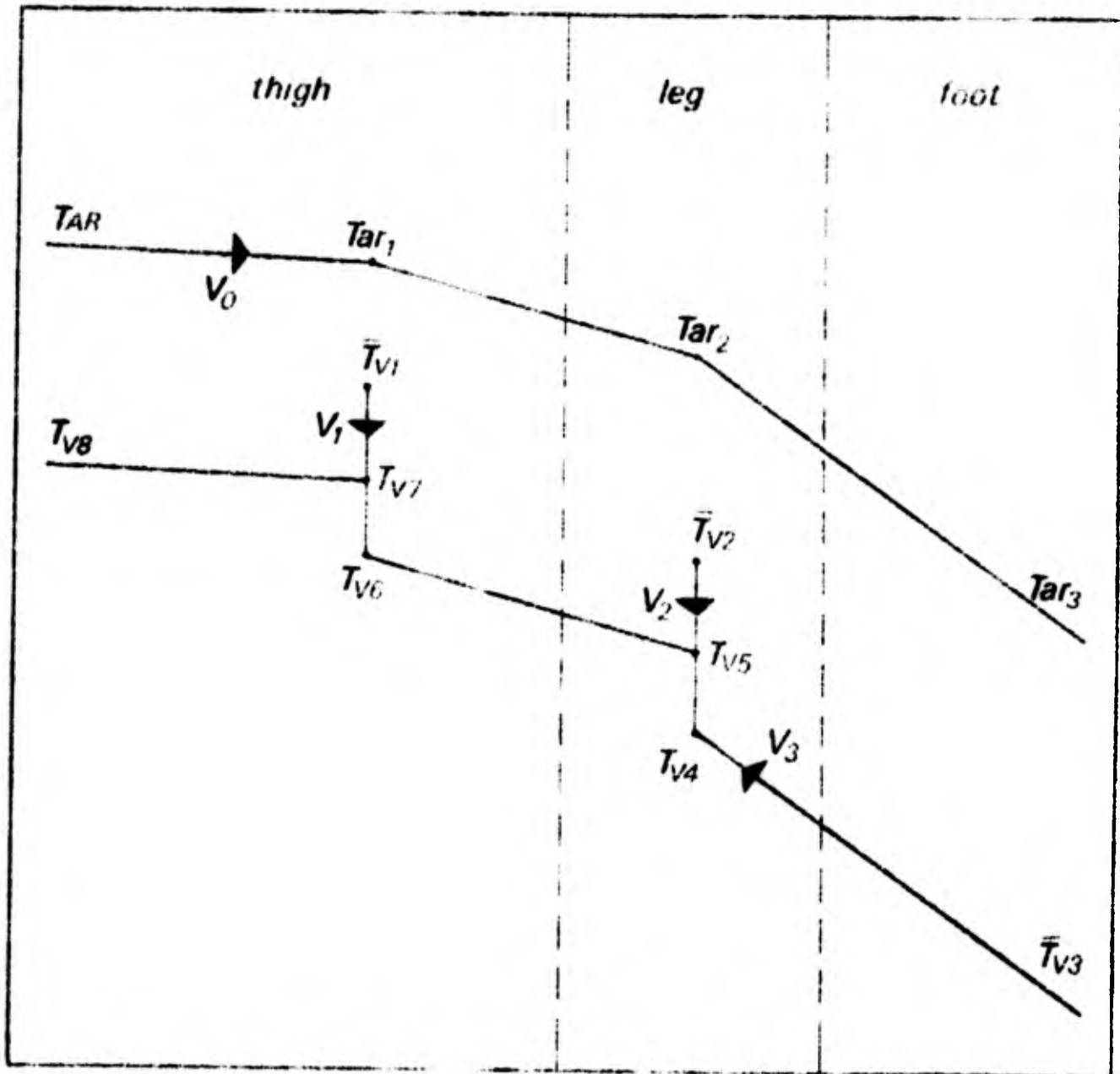


Fig. 11. The temperature distribution in the major blood vessels of the lower extremity, on the basis of the countercurrent exchange model.

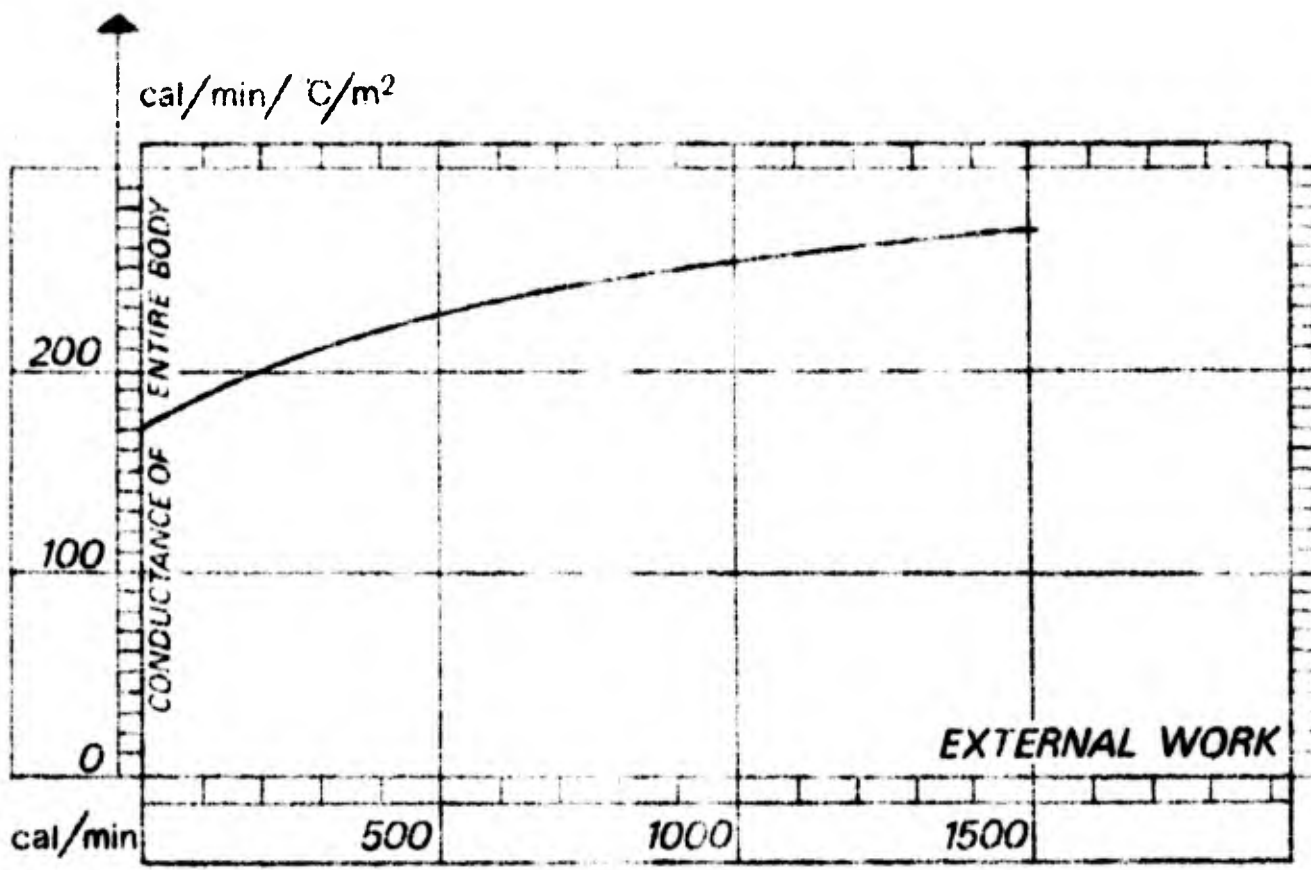


Fig. 12. The theoretical conductance of the entire body, as a function of external work load.

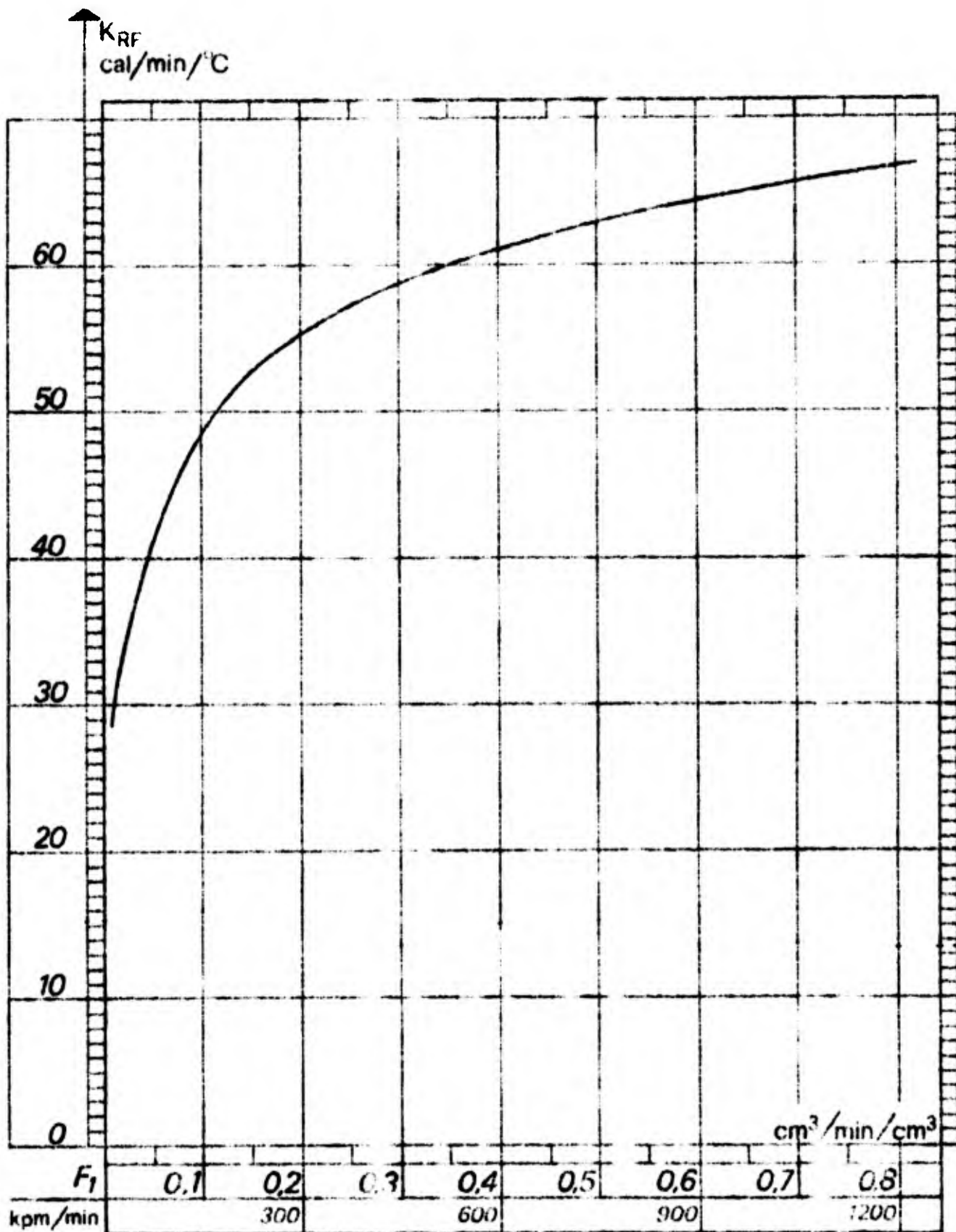


Fig. 13. The theoretical conductance of the thigh, as a function of ergometric work load.

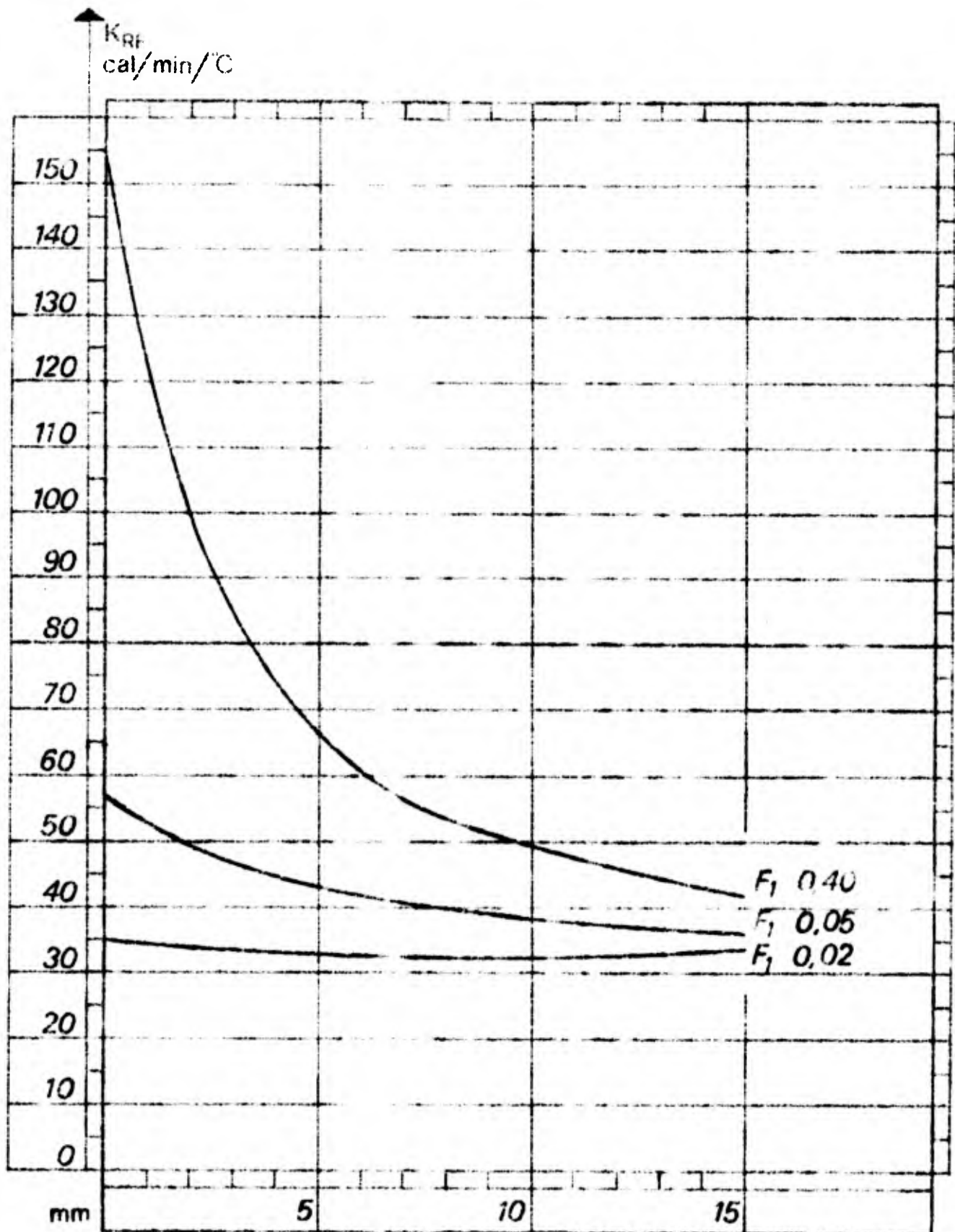


Fig. 14. The theoretical conductance of the thigh, as a function of the thickness of the fat layer at various flows in the inner layer.

TABLE 1. Dimensional data of the test subjects

	P.H.	K.K.	S.M.
Age (y.)	23	25	25
Length (cm)	189.5	171.5	177
Weight (kg)	70	69	68
Surface area, head (m ²)	0.15	0.15	0.15
- " - trunk (m ²)	0.67	0.63	0.55
- " - upper extr. (m ²)	0.19	0.17	0.18
- " - lower extr. (m ²)	0.39	0.36	0.36
- " - body (m ²)	1.98	1.84	1.78
Total fat tissue (kg)	8.2	8.7	8.2
Thickness of sub-			
cutaneous fat tissue, head (mm)	3.2	3.2	3.2
- " - trunk (mm)	4.5	5.5	5.5
- " - upper extr. (mm)	3.2	2.2	2.7
- " - lower extr. (mm)	4.2	5.6	4.8

TABLE 2. Test results obtained with subject P.H.

		At rest					
Temperatures (°C)	T_s	33.5	32.1	31.5	31.3	30.7	29.5
	T_{oe}	36.8	36.3	36.1	35.9	35.9	36.1
	T_r		36.8	36.3	36.1	36.2	36.0
	$T_{oe} - T_s$	3.3	4.2	4.6	4.6	5.2	6.6
Productions (cal/min.)	Energy	1390	1420	1370	1560	1480	2340
	heat	1390	1420	1370	1560	1480	2340
Losses (cal/min.) (%)	Head	180	200	190	200	210	270
	Trunk	480	450	450	630	540	900
	Upper extr.	100	180	120	170	150	240
	Lower extr.	410	500	440	450	470	840
	Entire body	1170	1330	1200	1450	1370	2250
	Respir. loss	150	150	150	150	150	150
	Head	15 %	15 %	16 %	14 %	15 %	12 %
	Trunk	41 %	34 %	37 %	43 %	40 %	40 %
	Upper extr.	9 %	13 %	10 %	12 %	11 %	11 %
	Lower extr.	35 %	38 %	37 %	31 %	34 %	37 %
Balance (cal/min.)	Rate of change of heat content of the body	+ 70	- 60	+ 20	+ 10	+ 40	- 60
Conductances (cal/min./ °C/m ²)	Head	380	330	280	290	280	280
	Trunk	220	160	150	210	160	210
	Upper extr.	80	110	70	100	80	90
	Lower extr.	160	150	130	130	120	170
	Entire body	180	160	130	160	130	170

TABLE 2. Test results obtained with subject P.H. (contin.)

		Exercise 1.000 cal/min.		Exercise 1.300 cal/min.			
Temper- atures (°C)	T_s	27.4	25.3	29.3	27.3	24.8	23.3
	T_{oe}	36.2	36.0	37.3	36.7	36.4	36.3
	T_r	36.5	36.0	37.8	37.3	-	36.7
	$T_{oe}-T_s$	8.8	10.7	8.0	9.4	11.6	13.0
Produc- tions (cal/min.)	Energy	5340	6260	-	6080	7300	8370
	heat	4340	5260		4780	6000	7070
Losses (cal/min.) (%)	Head	380	460	380	470	590	570
	Trunk	1360	2040	1080	1480	1680	2200
	Upper extr.	510	620	610	640	760	960
	Lower extr.	1820	2160	2360	2560	2860	3160
	Entire body	4070	5280	4430	5150	5890	6890
	Respir. loss	200	200	250	250	250	250
	Head	9 %	9 %	9 %	9 %	10 %	8 %
	Trunk	33 %	38 %	24 %	29 %	29 %	32 %
	Upper extr.	13 %	12 %	14 %	12 %	13 %	14 %
	Lower extr.	45 %	41 %	53 %	50 %	48 %	46 %
Balance (cal/min.)	Rate of change of heat content of the body	+ 70	-220			-140	- 70
Conduct- ances (cal/min./ °C/m ²)	Head	290	290	320	340	350	310
	Trunk	230	290	200	240	220	250
	Upper extr.	150	150	200	180	170	190
	Lower extr.	270	260	380	350	320	310
	Entire body	230	250	280	280	260	270

TABLE 2. Test results obtained with subject P.H. (contin.)

		Exercise 1.000 cal/min.		Exercise 1.300 cal/min.			
Temper- atures (°C)	T _s	27.4	25.3	29.3	27.3	24.8	23.3
	T _{oe}	36.2	36.0	37.3	36.7	36.4	36.3
	T _r	36.5	36.0	37.8	37.3	-	36.7
	T _{oe} -T _s	8.8	10.7	8.0	9.4	11.6	13.0
Produc- tions (cal/min.)	Energy	5340	6260	-	6080	7300	8370
	heat	4340	5260		4780	6000	7070
Losses (cal/min.)	Head	380	460	380	470	590	570
	Trunk	1360	2040	1080	1480	1680	2200
	Upper extr.	510	620	610	640	760	960
	Lower extr.	1820	2160	2360	2560	2860	3160
	Entire body	4070	5280	4430	5150	5890	6890
(%)	Respir. loss	200	200	250	250	250	250
	Head	9 %	9 %	9 %	9 %	10 %	8 %
	Trunk	33 %	38 %	24 %	29 %	29 %	32 %
	Upper extr.	13 %	12 %	14 %	12 %	13 %	14 %
	Lower extr.	45 %	41 %	53 %	50 %	48 %	46 %
Balance (cal/min.)	Rate of change of heat content of the body	+ 70	-220			-140	- 70
Conduct- ances (cal/min./ °C/m ²)	Head	290	290	320	340	350	310
	Trunk	230	290	200	240	220	250
	Upper extr.	150	150	200	180	170	190
	Lower extr.	270	260	380	350	320	310
	Entire body	230	250	280	280	260	270

TABLE 2. Test results obtained with subject P.H. (contin.)

		Exercise 1.800 cal/min.	
Temperatures (°C)	T _s	22.7	20.8
	T _{oe}	36.7	35.8
	T _r	37.4	-
	T _{oe} -T _s	14.0	15.0
Productions (cal/min.)	Energy	10680	10640
	heat	8880	8840
Losses (cal/min.)	Head	750	690
	Trunk	2000	2240
	Upper extr.	1120	1270
	Lower extr.	4450	4330
	Entire body	8320	8530
(%)	Respir. loss	300	300
	Head	9 %	8 %
	Trunk	24 %	26 %
	Upper extr.	13 %	15 %
	Lower extr.	54 %	51 %
Balance (cal/min.)	Rate of change of heat content of the body	+260	+ 10
Conductances (cal/min./ °C/m ²)	Head	370	310
	Trunk	220	220
	Upper extr.	210	220
	Lower extr.	410	370
	Entire body	300	290

TABLE 3. Test results obtained with subject K.K.

		At rest			Exercise 1.000 cal/min.			
Temperatures (°C)	T _s	34.8	32.6	31.9	32.0	29.9	28.4	27.5
	T _{oe}	36.9	36.6	36.2	36.8	36.8	36.6	36.3
	T _r	37.4	-	-	-	37.2	36.9	36.7
	T _{oe} -T _s	2.1	4.0	4.3	4.8	6.9	8.2	8.8
Productions (cal/min.)	Energy	1740	1390	-	3910	4030	4690	5330
	heat	1740	1390	-	2910	3030	3690	4330
Losses (cal/min.)	Head	100	190	190	180	520	560	590
	Trunk	480	480	600	1120	1200	1280	1600
	Upper extr.	500	150	130	470	530	500	570
	Lower extr.	430	480	500	930	1020	1430	1370
	Entire body	1510	1300	1420	2700	3270	3770	4130
(%)	Respir. loss	150	150	150	200	200	200	200
	Head	7 %	15 %	14 %	7 %	16 %	15 %	14 %
	Trunk	32 %	37 %	42 %	41 %	37 %	34 %	39 %
	Upper extr.	33 %	11 %	9 %	17 %	16 %	13 %	14 %
	Lower extr.	28 %	37 %	35 %	35 %	31 %	38 %	33 %
Balance (cal/min.)	Rate of change of heat content of the body	+ 80	- 60		+ 10	-440	-280	± 0
Conductances (cal/min. °C/m ²)	Head	320	310	290	250	490	450	440
	Trunk	360	190	220	370	280	250	290
	Upper extr.	720	120	90	300	230	190	200
	Lower extr.	290	170	160	270	210	240	220
	Entire body	390	180	180	310	250	250	260

TABLE 3. Test results obtained with subject K.K. (contin.)

		Exercise 1.300 cal/min.				Exercise 1.800 cal/min.		
Temperatures (°C)	T_s	31.0	29.3	26.1	24.0	26.5	22.4	20.4
	T_{oe}	37.0	37.4	37.3	37.1	36.6	37.5	36.9
	T_r	-	37.8	37.8	37.4	-	37.9	37.2
	$T_{oe} - T_s$	6.0	8.5	11.1	13.1	10.1	15.1	16.5
Productions (cal/min.)	Energy	6420	6600	6770	7300	8870	9940	10860
	heat	5120	5300	5470	6000	7070	8140	9060
Losses (cal/min.) (%)	Head	280	600	700	810	750	770	1000
	Trunk	1360	1200	1320	1640	2000	2000	2080
	Upper extr.	1130	1030	720	910	990	990	1050
	Lower extr.	2000	2260	2880	3340	3080	4250	4500
	Entire body	4770	5090	5620	6700	6820	8010	8630
	Respir. loss	250	250	250	250	300	300	300
	Head	6 %	12 %	12 %	12 %	11 %	10 %	12 %
	Trunk	29 %	24 %	24 %	24 %	29 %	25 %	24 %
	Upper extr.	24 %	20 %	13 %	14 %	15 %	12 %	12 %
	Lower extr.	41 %	44 %	51 %	50 %	45 %	53 %	52 %
Balance (cal/min.)	Rate of change of heat content of the body	+100	- 40	-400	-950	- 50	-170	+130
Conductances (cal/min. °C/m ²)	Head	310	470	420	410	490	340	400
	Trunk	360	220	190	200	320	210	200
	Upper extr.	570	370	200	210	300	200	190
	Lower extr.	470	370	360	360	430	390	380
	Entire body	430	330	280	280	370	290	290

TABLE 4. Test results obtained with subject S.M.

		At rest				Exercise 1.000 cal/min.			
Temperatures (°C)	T_s	34.8	32.8	31.1	30.1	32.2	30.0	28.4	27.0
	T_{oe}	36.2	36.6	36.1	35.8	36.5	36.5	36.3	36.1
	T_r	36.5	36.7	36.3	36.0	-	36.6	36.4	36.2
	$T_{oe} - T_s$	1.4	3.8	5.0	5.7	4.3	6.6	8.0	9.2
Productions (cal/min.)	Energy	1170	1350	1400	2080	4680	4650	5430	6380
	heat	1170	1350	1400	2080	3680	3650	4430	5380
Losses (cal/min.)	Head	60	160	170	280	370	300	320	390
	Trunk	330	480	500	720	880	960	1120	1360
	Upper extr.	140	160	160	250	520	590	800	980
	Lower extr.	290	350	450	590	1250	1290	1650	2190
	Entire body	820	1150	1280	1840	3020	3140	3890	4920
(%)	Respir. loss	150	150	150	150	200	200	200	200
	Head	7 %	14 %	13 %	15 %	12 %	10 %	8 %	8 %
	Trunk	40 %	42 %	39 %	39 %	29 %	30 %	29 %	28 %
	Upper extr.	17 %	14 %	13 %	14 %	17 %	19 %	21 %	20 %
	Lower extr.	35 %	30 %	35 %	32 %	42 %	41 %	42 %	44 %
Balance (cal/min.)	Rate of change of heat content of the body	+200	+ 50	- 30	+ 90	+460	+310	+340	+260
Conductances (cal/min./ °C/m ²)	Head	280	280	230	330	570	300	270	280
	Trunk	430	230	180	230	370	260	260	270
	Upper extr.	280	120	90	120	350	250	290	300
	Lower extr.	290	130	120	140	400	270	290	330
	Entire body	330	170	140	180	400	270	270	300

TABLE 4. Test results obtained with subject S.M. (contin.)

		Exercise 1.300 cal/min.			Exercise 1.800 cal/min.		
Temper- atures (°C)	T_s	29.3	27.5	26.0	26.5	24.8	22.5
	T_{oe}	37.2	36.7	36.1	36.4	37.5	37.0
	T_r	37.3	37.0	36.1	-	-	37.6
	$T_{oe} - T_s$	7.9	9.2	10.1	9.9	12.7	14.5
Produc- tions (cal/min.)	Energy	6420	5480	6260	8600	9070	10200
	heat	5120	4180	4960	6800	7270	8400
Losses (cal/min.)	Head	380	400	510	470	580	810
	Trunk	1040	1120	1280	1520	1480	1920
	Upper extr.	900	770	1010	1360	1350	1860
	Lower extr.	1960	2260	2400	3200	3650	3750
	Entire body	4280	4550	5200	6550	7060	8340
(%)	Respir. loss	250	250	250	300	300	300
	Head	9 %	9 %	10 %	7 %	8 %	10 %
	Trunk	24 %	24 %	25 %	23 %	21 %	23 %
	Upper extr.	21 %	17 %	19 %	21 %	19 %	22 %
	Lower extr.	46 %	50 %	46 %	49 %	52 %	45 %
Balance (cal/min.)	Rate of change of heat content of the body	+590	-680	-490	- 50	- 90	-240
Conduct- ances (cal/min./ °C/m ²)	Head	320	290	330	310	310	370
	Trunk	240	220	230	280	210	240
	Upper extr.	330	240	290	390	300	370
	Lower extr.	340	340	330	450	400	360
	Entire body	310	280	290	370	310	320

TABLE 5. Division of heat loss between different body parts at rest and exercise

Subject	Body part	Rest	Work 1000 cal/min.	Work 1300 cal/min.	Work 1800 cal/min.
P.H.	Head	14 %	9 %	9 %	8 %
	Trunk	40 %	36 %	29 %	25 %
	Upper extr.	11 %	12 %	13 %	14 %
	Lower extr.	35 %	43 %	49 %	53 %
K.K.	Head	12 %	13 %	11 %	11 %
	Trunk	37 %	37 %	25 %	26 %
	Upper extr.	17 %	15 %	18 %	13 %
	Lower extr.	34 %	35 %	46 %	50 %
S.M.	Head	12 %	10 %	9 %	8 %
	Trunk	40 %	29 %	24 %	22 %
	Upper extr.	15 %	19 %	19 %	21 %
	Lower extr.	33 %	42 %	48 %	49 %
Mean	Head	13 %	11 %	10 %	9 %
	Trunk	39 %	34 %	26 %	24 %
	Upper extr.	14 %	15 %	17 %	16 %
	Lower extr.	34 %	40 %	47 %	51 %

TABLE 6. Experimental heat losses for the different body parts at rest and during exercise of various levels and values found theoretically

Part of body	Subject	Heat losses (in cal/min.) during external work at rate (approx.) (in cal/min.)			
		0	1000	1300	1800
Head	P.H.	190	380	590	690
	K.K.	190	560	700	1000
	S.M.	200	300	400	580
	Mean	195	415	465	755
	Theor. value	149		350	
Upper extremity	P.H.	60	255	380	635
	K.K.	75	250	360	525
	S.M.	75	295	385	675
	Mean	70	265	375	610
	Theor. value	85		350	
Trunk	P.H.	450	1360	1680	2240
	K.K.	480	1280	1320	2080
	S.M.	510	960	1120	1480
	Mean	480	1200	1370	1930
	Theor. value	507		1210	
Lower extremity	P.H.	220	910	1430	2170
	K.K.	240	715	1440	2250
	S.M.	235	645	1130	1825
	Mean	230	750	1330	2080
	Theor. value	223		1310	
Entire body	P.H.	1200	4070	5890	8530
	K.K.	1300	3770	5620	8630
	S.M.	1330	3140	4550	7060
	Mean	1280	3660	5350	8070
	Theor. value	1270		4870	

TABLE 7. Experimental minimum conductances for the different body parts at rest and during exercise of various levels, and values found theoretically

Part of body	Subject	Minimum conductance (in cal/min./°C/m ²) during external work at rate (approx.) (in cal/min.)			
		0	1000	1300	1800
Head	P.H.	280	290	300	310
	K.K.	290	250	310	330
	S.M.	230	260	290	300
	Mean	270	270	300	310
	Theor. value	248		233	
Upper extremity	P.H.	70	150	170	200
	K.K.	90	180	200	190
	S.M.	90	250	240	300
	Mean	80	190	200	230
	Theor. value	128		211	
Trunk	P.H.	150	230	200	210
	K.K.	190	250	190	200
	S.M.	180	250	220	210
	Mean	170	240	200	210
	Theor. value	205		195	
Lower extremity	P.H.	120	260	310	370
	K.K.	160	210	360	380
	S.M.	120	270	330	360
	Mean	130	250	330	370
	Theor. value	151		354	
Entire body	P.H.	130	230	260	290
	K.K.	180	250	280	290
	S.M.	140	270	280	310
	Mean	150	250	270	300
	Theor. value	173		265	

TABLE 8. Theoretical model of the body. Dimensions of the cylinders representing the different parts of the body

Part of body	Surface area A (cm ²)	Cylinder		Cylinder length l (cm)	Volume (cm ³)			
		outer radius r ₂ (cm)	inner radius r ₁ (cm)		outer layer	inner layer		
						muscle	other	total
Head	1500	7.0	6.5	34	720			4510
Trunk	6200	11.3	10.7	37	3500			31300
Upper-arm	780	4.8	4.1	26	510			1370
Fore-arm	880	3.5	3.2	40	250			1290
Thigh	1960	7.8	7.1	40	1340	5600	720	6320
Lower leg	980	5.2	4.7	30	460	1500	580	2080
Ankle + foot	750	3.3	3.1	36	140	200	890	1090

TABLE 9. Assumed dimensions of the major extremity vessels, and counter-current exchange coefficients calculated on their basis for each vessel, and effective coefficients thence found for each extremity

	Inner radius of artery r_{ar} (mm)	Inner radius of vein r_v (mm)	Distance between vessels h (mm)	Countercurrent exchange coefficient a (g/min/cm)	
				individual vessel	entire extremity
Upper arm	1.5	2.0	4.9	0.25	0.25
Forearm	1.1	0.6	2.35	0.25	1.0
Thigh, ant. part	2.75	7.0	11.5	0.34	
Thigh, post. part	2.0	2.0	5.8	0.24	
Thigh, total					0.60
Lower leg	2.5	2.75	6.2	0.39	1.5
Foot					1.5

TABLE 10. Countercurrent exchange coefficients for the extremities, consistent with the major vessel countercurrent exchange model employed (see Fig. 4)

	Effective lengths			Countercurrent exchange coefficients		
	x_1 (cm)	x_2 (cm)	x_3 (cm)	a_1x_1 (g/min.)	a_2x_2 (g/min.)	a_3x_3 (g/min.)
Upper extr.	16	30		4	30	
Lower extr.	25	20	40	15	30	60

TABLE 11. Perfusions (F_1 and F_2) of the inner and outer layers of the different body parts, and temperature increments, employed in applying the theory in the case of a test subject at rest and during bicycle ergometer exercise at 1400 cal/min. load rate.

	At rest				Exercise 1400 cal/min.			
	F_1 (l/min.)	F_2 (l/min.)	Δ_1 (°C)	Δ_2 (°C)	F_1 (l/min.)	F_2 (l/min.)	Δ_1 (°C)	Δ_2 (°C)
Head	0.10	0.02	0.45	0.45	0.10	0.02	0.45	0.45
Trunk	0.02	"-	"-	"-	0.04	"-	"-	"-
Upper extr.	"-	"-	"-	"-	0.05	"-	0.90	"-
Thigh, lower leg	"-	"-	"-	"-	0.40	"-	1.25	"-
Foot	"-	"-	"-	"-	0.09	"-	0.55	"-

TABLE 12. Human body at rest. Expressions, for different parts of the body, of local arterial blood temperature T_{ar} , average venous blood temperature T_v , local conductance K_r and central conductance K , and numerical values of the conductance K for temperature difference $T_{ar}-T_s=4^{\circ}\text{C}$

	T_{ar}	T_v	K_r	K	K $T_{ar}-T_s=4^{\circ}\text{C}$
Head	T_{ar}	-	$33.4+15.0/(T_{ar}-T_s)$	$33.4+15.0/(T_{ar}-T_s)$	37.2
Trunk	T_{ar}	-	$114+51.3/(T_{ar}-T_s)$	$114+51.3/(T_{ar}-T_s)$	166.8
Upper arm	$0.980T_{ar}+0.020T_s+0.00$	$0.655T_{ar}+0.345T_s+0.07$	$11.1+5.0/(T_{ar}-T_s)$	$10.9+5.0/(T_{ar}-T_s)$	18.2
Forearm	$0.708T_{ar}+0.292T_s+0.04$	$0.605T_{ar}+0.395T_s+0.05$	$10.8+4.9/(T_{ar}-T_s)$	$7.7+5.3/(T_{ar}-T_s)$	9.0
Entire upper extr.					
Thigh	$0.981T_{ar}+0.019T_s+0.01$	$0.768T_{ar}+0.232T_s+0.12$	$32.3+14.5/(T_{ar}-T_s)$	$18.6+10.3/(T_{ar}-T_s)$	21.2
Lower leg	$0.860T_{ar}+0.120T_s+0.02$	$0.683T_{ar}+0.317T_s+0.08$	$14.9+6.7/(T_{ar}-T_s)$	$57.7+14.7/(T_{ar}-T_s)$	35.4
Foot	$0.424T_{ar}+0.576T_s+0.04$	$0.555T_{ar}+0.445T_s+0.02$	$10.1+4.5/(T_{ar}-T_s)$	$13.1+7.1/(T_{ar}-T_s)$	14.9
Entire lower extr.				$4.2+4.9/(T_{ar}-T_s)$	5.4
Entire body				$49.0+26.7/(T_{ar}-T_s)$	55.7
					517.8
					173

TABLE 13. Human body during bicycle ergometer exercise at 1400 cal/min. load rate. Expressions, for different parts of the body, of local arterial blood temperature T_{ar} , average venous blood temperature T_v , local conductance K_r and central conductance K_c , and numerical values of the conductance K for temperature difference $T_{ar}-T_s=10^\circ C$

	Local arterial blood temperature T_{ar}	Average venous blood temperature T_v	Local conductance K_r	Central conductance K_c	K ($T_{ar}-T_s=10^\circ C$) cal/min./ $^\circ C$
Head			$33.4+15.0/(T_{ar}-T_s)$	$33.4+15.0/(T_{ar}-T_s)$	34.9
Trunk	T_{ar}		$116+52.2/(T_{ar}-T_s)$	$116+52.2/(T_{ar}-T_s)$	121
Upper arm	T_{ar}	$0.730T_{ar}+0.220T_s+0.25$	$15.6+12.6/(T_{ar}-T_s)$	$15.6+12.6/(T_{ar}-T_s)$	16.9
Forearm	$0.891T_{ar}+0.119T_s+0.04$	$0.699T_{ar}+0.301T_s+0.18$	$18.7+15.9/(T_{ar}-T_s)$	$16.5+17.3/(T_{ar}-T_s)$	16.2
Entire upper extr.				$32.1+29.9/(T_{ar}-T_s)$	35.1
Thigh	T_{ar}	$0.975T_{ar}+0.025T_s+0.59$	$61.0+70.0/(T_{ar}-T_s)$	$61.0+70.0/(T_{ar}-T_s)$	69.0
Lower leg	T_{ar}	$0.952T_{ar}+0.048T_s+0.56$	$36.6+43.2/(T_{ar}-T_s)$	$36.6+43.2/(T_{ar}-T_s)$	41.7
Foot	$0.885T_{ar}+0.115T_s+0.08$	$0.747T_{ar}+0.253T_s+0.14$	$23.1+12.5/(T_{ar}-T_s)$	$20.4+14.4/(T_{ar}-T_s)$	21.8
Entire lower extr.				$118+127.6/(T_{ar}-T_s)$	130.7
Entire body					488
					233
					195
					216
					207
					211
					347
					417
					290
					354
					265