

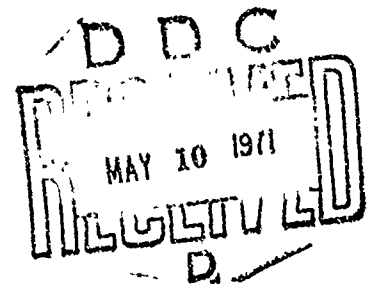
AD722713

DASA 2640
RRA-T7017
April 1971.

RRA RADIATION RESEARCH ASSOCIATES
Fort Worth, Texas

**COMPUTER PROCEDURE FOR CALCULATING
TIME DEPENDENT LIGHT SCATTERING IN
SPHERICAL-SHELL ATMOSPHERES**

D. G. Collins and M. B. Wells



Research Sponsored by
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Contract DASA01-68-C-0169

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ABSTRACT

This report describes three Monte Carlo procedures that were developed for the purpose of providing calculational tools that could be used to study the transport of visible and infrared light in spherical-shell atmospheres. These procedures, designated as FLASH, treat atmospheric scattering problems for plane-parallel, point, volume and spherical surface sources. The FLASH procedures consider the effect of Rayleigh and aerosol scattering and ozone, water vapor, CO_2 , and aerosol particle absorption on the transport of light. The procedures also consider the changes in polarization that occurs at each scattering event. The input formats of the codes allows one to describe the scattering and absorption properties of the atmosphere as a function of altitude. Provisions are incorporated for treating the reflection of light from ground and cloud surfaces with an albedo method. The output of the FLASH procedures gives the scattered and direct light intensities at each receiver position as a function of a polar and azimuthal angle and time. The FLASH procedures were written in FORTRAN-IV language for both the CDC-6600 and IBM Direct Couple System. The codes were verified through comparison with the results of other calculations of light transport in the atmosphere.

Also described are the MIE-II and SHINE procedures. The MIE-II procedure uses Mie theory to generate aerosol particle scattering and absorption coefficients and the four elements of the aerosol phase matrix for use as input data to the FLASH procedures. The SHINE procedure was developed to integrate the results of FLASH calculations for point monochromatic sources over an arbitrary wavelength and time dependent source spectrum.

FOREWORD

The machine procedures described in this report and a companion report (RRA-T7016) were developed under sponsorship of the Defense Atomic Support Agency for the CDC-6600 computer system at the Los Alamos Scientific Laboratory and the Direct Couple System at the Air Force Cambridge Research Laboratories. The DASA contract monitors were Lt. Col. H. C. Fitz, Jr. and Lt. Col. John Morgan. The authors wish to acknowledge the assistance of Dr. Henry Horak of LASL and Mr. Jack Cahill of AFCRL who provided assistance in the scheduling and running of the various procedures on the computers at their respective facilities. Several RRA staff members in addition to the authors assisted in the development of the TPART, FLASH, SHINE, and MIE-II procedures. These staff members include Mr. Walter Osborne, Mrs. Frances Hopper, Mr. W. G. Blittner, Dr. C. Thompson, and Mr. F. O. Leopard.

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I. INTRODUCTION

Several machine procedures were developed for the purpose of providing calculational tools for use in studying the propagation of visible and infrared radiation in both plane and spherical-shell atmospheres. These procedures consider the effect of polarization on scattered light. They can be used to compute the scattered and direct light intensities at receiver positions as a function of direction and time. This report describes the FLASH procedures which consider light transport in a spherical-shell atmosphere as well as the NIE-II and SHINE programs. A companion report (Ref. 1) describes the TPART-I and TPART-II procedures which treat atmospheric scattering of light in plane atmospheres for point and plane-parallel sources.

Both the TPART and FLASH procedures employ Monte Carlo methods to treat multiple scattering of light within the atmosphere. The Monte Carlo method is especially well suited to the computation of the time dependence of scattered radiation because the time elapsed during each of the photon histories is easily determined and thus a time record of the radiation arriving at each receiver position may be made.

The FLASH procedures utilize the backward Monte Carlo method to compute the scattered light intensity received within specified view angles at each of a set of point receivers located in a spherical shell atmosphere. Three separate versions of the FLASH procedure were developed. The first of these was designed to consider a point source of light located within the atmosphere. The second version considers the case of a point source at an infinite distance from the atmosphere, which is equivalent to a plane-parallel source of light incident to the top of the atmosphere. The third version considers scattering problems for either a spherical or a lense shaped volume source or a spherical surface source. The volume sources and the spherical surface source may be located at any position within the atmosphere.

A discussion of the backward Monte Carlo Method and the reasons for its use in the FLASH procedures are discussed in Section II. A description of the calculational methods used in the various versions of FLASH is given in Section III.

Two auxiliary programs were also developed. The MIE-II procedure provides data on the extinction, scattering, and absorption coefficients and the phase matrix for light scattering with aerosol particles. A procedure, denoted as SHINE, was developed for the purpose of using the output from the point source version of FLASH for several monochromatic wavelengths to compute the time distribution of the radiation at a receiver position for a given source spectrum, which is a function of wavelength and time of emission. The MIE-II and SHINE procedures are described in Section IV.

A number of calculations were run with the FLASH procedures to provide data for comparison with the results of other calculational methods for the purpose of validating the calculational methods used in the FLASH procedures. The results of some of these calculations are discussed in Section V.

II. BACKWARD MONTE CARLO METHOD

In the application of the Monte Carlo method to obtain the solution to an integral such as

$$\int_a^b f(x) dx, \quad (2.1)$$

the common practice is to express $f(x)$ as the product of two functions, $h(x)$, a density function, and $g(x)$, an estimating function. To evaluate the integral

$$\int_a^b h(x) g(x) dx, \quad (2.2)$$

the density function is normalized by multiplying by a constant, c , where c is given by

$$c = 1 / \int_a^b h(x) dx. \quad (2.3)$$

A random value of x is sampled from the normalized density function by evaluating the equation

$$RN = \int_a^{x_i} c h(x) dx, \quad (2.4)$$

where $a \leq x_i \leq b$ and RN is a random number drawn from a uniform distribution between 0 and 1. An estimate of the integral

$$\int_a^b f(x) dx \quad (2.5)$$

is obtained by averaging the estimator function

$$EST = g(x_i) / c \quad (2.6)$$

for the sample values x_i . An average of $g(x_i)/c$ for a large number,

N , of random values of x_i as given by

$$\frac{1}{N} \sum_{i=1}^N g(x_i) / c$$

is an estimate of the mean value of the integral.

An arbitrary division of $f(x)$ into density and estimating functions may result in an estimating function that produces extremely high values of $g(x)/c$ for some of the values of x sampled from the density function. In selecting the estimating function the objective should be to obtain a function that remains within reasonable limits for all of the sampled values. The portion of the integrand selected as the density function should be a function from which it is easy to obtain sample values. In some cases a better separation of the integrand into density and estimating functions can be achieved if the integral expression can be rewritten with a change of variables.

When using the backward Monte Carlo approach to calculate the scattered light intensity at a point receiver due to a point source or a volume source distributed within a spherical shell atmosphere, a change in variables in the integral form of the transport equation yields an integrand which is expressed in terms of integration variables defined about the receiver position rather than about the source position.

It will be shown that the integral expressed in terms of variables defined with respect to the receiver position allows the use of an estimating function that has less variation with the random values sampled from the density function than that which would be obtained when the variables of integration are expressed in terms of the source position.

A physical interpretation of the Monte Carlo solution of the integral expressed in terms of variables defined about the receiver

is that photon histories are started at the receiver and traced backward to the source position.

The backward approach is particularly advantageous for those problems in which the scattered intensity at the receiver is desired for only a portion of the total solid angle. The backward approach, in this case, will allow all samples to be taken within the solid angle of interest. For a volume source, the backward approach allows the same random path to be used for all points along a line passing through the source volume and thus should prove to be a much more economical means of computation than the forward approach.

Care must be exercised in the application of backward Monte Carlo (starting particle histories at the receiver and estimating the direct intensity at the first collision) to insure that the integral equation solved is equivalent to the actual physical situation. The approach taken was to write the integral equation which represents a direct interpretation of the single scattered intensity and then to change the variables of integration so that sampling of the particle's random path may be started at the receiver position. If the limits of integration on the forward form of the scattering integral are transposed in accordance with the change of variables for the backward calculation, the resulting equation should not only yield the same results as the original equation, but it should lend itself to a more economical solution.

2.1 Illustration of Backward Monte Carlo for Point and Volume Source

For the first illustration consider a spherical volume source $S(R, \theta, \phi)$ within which radiation is emitted isotropically at all positions. Also, suppose that at some position $(R_p, \theta = 0, \phi = 0)$ within or outside of the source volume one wishes to know the single scattered intensity from over the entire source. For simplicity assume that for attenuation purposes the source and receiver are embedded within an infinite homogeneous medium, Fig. 1. Then the equation for the single

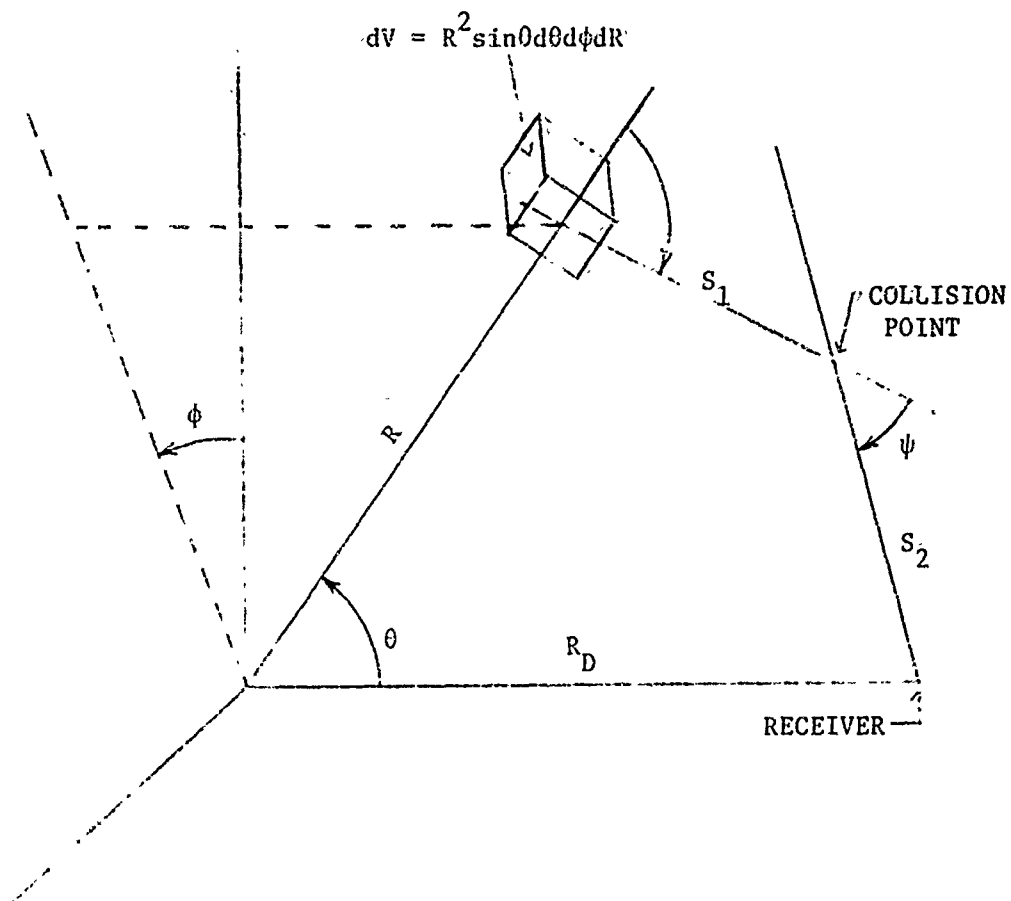


Fig. 1. Single Scattering Geometry

scattered intensity is given by the expression

$$\int_V \int_{\Omega} \int_{S_1} \frac{N(R, \theta, \phi)}{4\pi S_1^2} \Sigma_T e^{-\Sigma_T S_1} \frac{\Sigma_S}{\Sigma_T} \frac{\frac{d\Sigma}{d\Omega}(\psi)}{\Sigma_S} \frac{e^{-\Sigma_T S_2}}{S_2^2} S_1^2 dS_1 d\Omega dV, \quad (2.7)$$

where $N(R, \theta, \phi)$ is the source strength at the position (R, θ, ϕ) ,

$$\frac{\Sigma_T e^{-\Sigma_T S_1} dS_1}{4\pi S_1^2}$$

is the fraction of the source photons that have a single order collision at a distance S_1 from the source position and

$$\frac{\Sigma_S}{\Sigma_T} \frac{\frac{d\Sigma}{d\Omega}(\psi)}{\Sigma_S} \frac{e^{-\Sigma_T S_2}}{S_2^2}$$

is the fraction of those photons undergoing collisions that scatter into the receiver at $(R_D, 0, 0)$.

The single scatter centers are integrated over the entire volume $S_1^2 dS_1 d\Omega$ for each source volume element dV , where $d\Omega = \sin\theta d\theta d\phi$ (see Fig. 2) and $dV = R^2 \sin\theta d\theta d\phi dR$. In Equation (2.7) Σ_S is the scattering coefficient, Σ_T is the extinction coefficient, and

$$\frac{d\Sigma}{d\Omega}(\psi) / \Sigma_S$$

is the normalized phase function.

Equation (2.7) could be solved using the Monte Carlo method by sampling the source position (R, θ, ϕ) from a uniform distribution over the source volume, an initial direction from an isotropic distribution and the distance to the collision point from the function

$$\Sigma_T e^{-\Sigma_T S_1} dS_1.$$

This would leave for the estimating function,

$$N(R, \theta, \phi) = \frac{\Sigma_S}{\Sigma_T} \frac{\frac{d\Sigma}{d\Omega}(\psi)}{\Sigma_S} \frac{e^{-\Sigma_T S_2}}{S_2^2}$$

where S_2 is the distance from the collision point to the receiver position. The magnitude of the estimating function is highly dependent upon the value of S_2 since both $e^{-\Sigma_T S_2}$ and $1/S_2^2$ increase with decreasing values of S_2 . In fact, the estimating function approaches infinity as S_2 approaches zero.

In order to convert Equation (2.7) into an equation which can be solved by the backward Monte Carlo method, it is necessary to transform the variables of integration into a set of variables defined with respect to the receiver position.

A series of transformations are applied to transform Equation (2.7) into one expressed in terms of variables defined about the receiver position, Fig. 2. All transformations are of two basic types, a rotation or translation. The rotation of the coordinate system axis utilizes the spherical triangle relationships while the translation utilizes the law of cosines and the law of sines for a plane triangle.

First consider a single source volume element located at R, θ , and ϕ and join the center of the volume element with the receiver position (Fig. 2). The angular distribution of the source term expressed in terms of a polar angle γ measured about the radial through the volume element and an azimuthal angle α measured in a plane normal to the radial may be expressed in terms of an angle ϵ measured from the line joining the volume element and the receiver and η , an azimuthal angle measured in a plane normal to the line joining the volume element and the receiver.

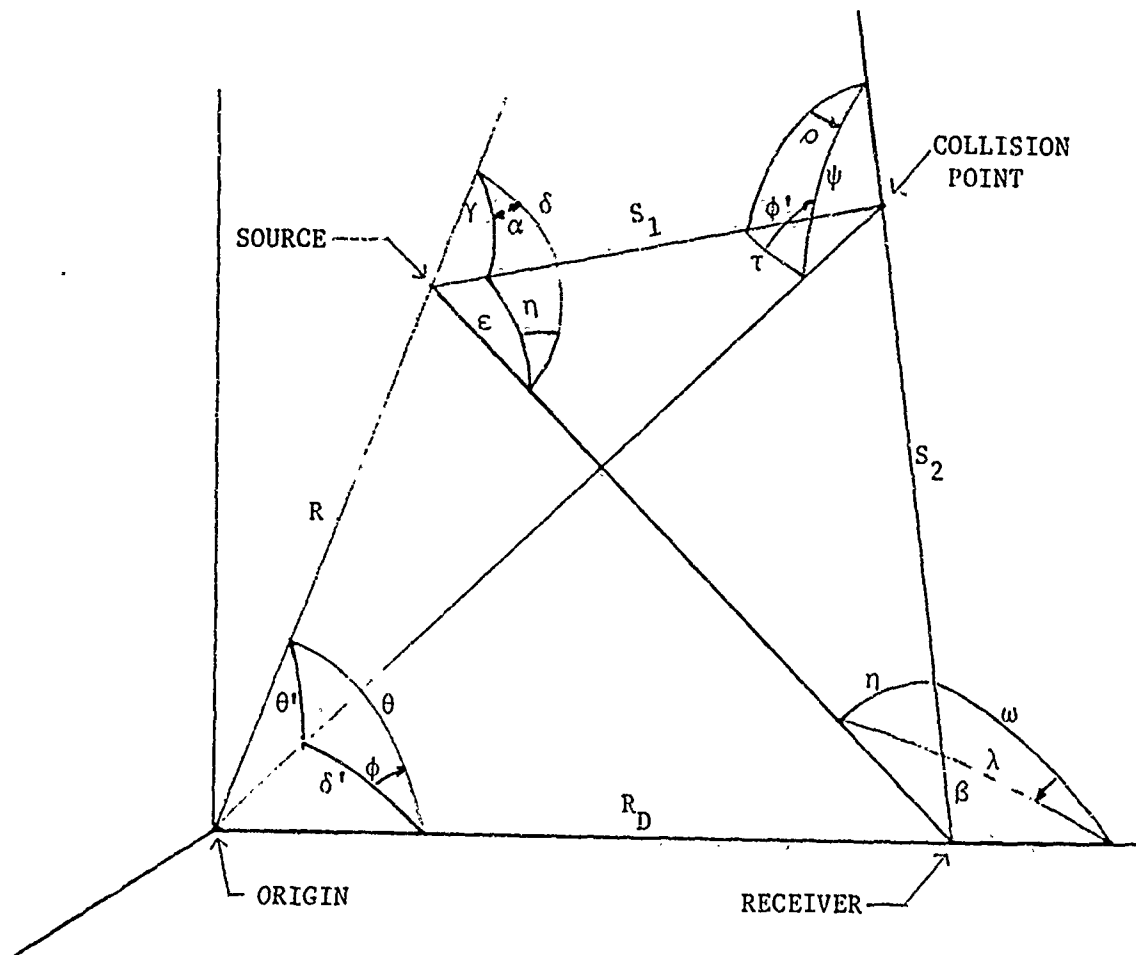


Fig. 2. Transformation Geometry

The angle δ between the radial through the volume element and the line joining the volume element and the receiver may be computed with the equation

$$\delta = \cos^{-1} \left[(R_D \cos \theta - R) / \sqrt{R^2 + R_D^2 - 2RR_D \cos \theta} \right], \quad (2.8)$$

and γ and α may both be expressed in terms of ϵ and η with the equations

$$\gamma = \cos^{-1} [\cos \epsilon \cos \delta + \sin \epsilon \sin \delta \cos \eta] \quad (2.9)$$

and

$$\alpha = \tan^{-1} [\sin \epsilon \sin \eta / (\cos \epsilon \sin \delta - \sin \epsilon \cos \delta \cos \eta)]. \quad (2.10)$$

Therefore we may transform Equation (2.7) into an equation expressed in terms of ϵ and η through the use of the following relation (Ref. 2):

$$\int \int f(\gamma, \alpha) d\gamma d\alpha = \int \int f(\gamma(\epsilon, \eta), \alpha(\epsilon, \eta)) \left| \frac{\delta(\gamma, \alpha)}{\delta(\epsilon, \eta)} \right| d\epsilon d\eta. \quad (2.11)$$

Solving for the partial derivatives in the Jacobian we obtain:

$$\frac{\delta\gamma}{\delta\epsilon} = \frac{\sin \epsilon \cos \delta - \cos \epsilon \sin \delta \cos \eta}{(1 - (\cos \epsilon \cos \delta + \sin \epsilon \sin \delta \cos \eta)^2)^{1/2}}, \quad (2.12)$$

$$\frac{\delta\gamma}{\delta\eta} = \frac{\sin \epsilon \sin \delta \sin \eta}{(1 - (\cos \epsilon \cos \delta + \sin \epsilon \sin \delta \cos \eta)^2)^{1/2}}, \quad (2.13)$$

$$\frac{\delta\alpha}{\delta\epsilon} = \frac{\sin \eta \sin \delta}{(\cos \epsilon \sin \delta - \sin \epsilon \cos \delta \cos \eta)^2 + (\sin \epsilon \sin \eta)^2}, \quad (2.14)$$

and

$$\frac{\delta\alpha}{\delta\eta} = \frac{\cos \epsilon \sin \epsilon \cos \eta \sin \delta - \sin^2 \epsilon \cos \delta}{(\cos \epsilon \sin \delta - \sin \epsilon \cos \delta \cos \eta)^2 + (\sin \epsilon \sin \eta)^2}. \quad (2.15)$$

Evaluating the Jacobian we obtain

$$\begin{vmatrix} \frac{\delta \gamma}{\delta \epsilon} & \frac{\delta \gamma}{\delta \eta} \\ \frac{\delta \alpha}{\delta \epsilon} & \frac{\delta \alpha}{\delta \eta} \end{vmatrix} = \frac{\sin \epsilon}{\sin \gamma} \quad (2.16)$$

We may rewrite Equation (2.7) as

$$\int_V \int_\eta \int_\epsilon \int_{S_1} \frac{N(R, \theta, \phi)}{4\pi S_1^2} \Sigma_T e^{-\Sigma_T S_1} \frac{\Sigma_S}{\Sigma_T} \frac{d\Sigma}{d\Omega}(\psi) \frac{e^{-\Sigma_T S_2}}{S_2^2} S_1^2 dS_1 \sin \epsilon d\epsilon d\eta dV. \quad (2.17)$$

Rather than expressing Equation (2.17) in terms of ϵ and S_1 we may rewrite it in terms of β and S_2 defined with respect to the receiver position (Fig. 2). Again we have an equation in terms of S_1 and ϵ which are both functions of S_2 and β :

$$S_1 = (A^2 - 2AS_2 \cos \beta + S_2^2)^{1/2} \quad (2.18)$$

and

$$\epsilon = \tan^{-1} \left(\frac{S_2 \sin \beta}{A - S_2 \cos \beta} \right), \quad (2.19)$$

where $A = \sqrt{R^2 + R_D^2 - 2RR_D \cos \theta}$ is the source-receiver separation distance.

Therefore we may rewrite Equation (2.17) by evaluating the partial derivatives $\delta S_1 / \delta S_2, \delta S_1 / \delta \beta, \delta \epsilon / \delta S_2, \delta \epsilon / \delta \beta$ and solving the Jacobian,

$$\begin{vmatrix} \frac{\delta S_1}{\delta S_2} & \frac{\delta S_1}{\delta \beta} \\ \frac{\delta \epsilon}{\delta S_2} & \frac{\delta \epsilon}{\delta \beta} \end{vmatrix}.$$

Evaluating the partial derivatives we obtain

$$\frac{\delta S_1}{\delta S_2} = \frac{S_2 - A \cos \beta}{(A^2 - 2AS_2 \cos \beta + S_2^2)^{1/2}}, \quad (2.20)$$

$$\frac{\delta S_1}{\delta \beta} = \frac{AS_2 \sin \beta}{(A^2 - 2AS_2 \cos \beta + S_2^2)^{1/2}}, \quad (2.21)$$

$$\frac{\delta \epsilon}{\delta S_2} = \frac{A \sin \beta}{A^2 - 2AS_2 \cos \beta + S_2^2}, \quad (2.22)$$

$$\text{and } \frac{\delta \epsilon}{\delta \beta} = \frac{AS_2 \cos \beta - S_2^2}{A^2 - 2AS_2 \cos \beta + S_2^2}. \quad (2.23)$$

The value of the Jacobian is

$$\begin{vmatrix} \frac{\delta S_1}{\delta S_2} & \frac{\delta S_1}{\delta \beta} \\ \frac{\delta \epsilon}{\delta S_2} & \frac{\delta \epsilon}{\delta \beta} \end{vmatrix} = \frac{S_2}{S_1}. \quad (2.24)$$

Noting that $S_1 \sin \epsilon = S_2 \sin \beta$ we may rewrite Equation (2.17) as

$$\int_V \int_{\eta} \int_{\beta} \int_{S_2} \frac{N(R, \theta, \phi)}{4\pi S_1^2} \Sigma_T e^{-\Sigma_T S_1} \frac{\Sigma_S}{\Sigma_T} \frac{\frac{d\Sigma}{d\Omega}(\psi)}{\Sigma_S} e^{-\Sigma_T S_2} \sin \beta d\beta d\eta dS_2 dV. \quad (2.25)$$

If there is a point source at (R, θ, ϕ) , the integral over the source volume V could be omitted and the backward Monte Carlo method can be used to solve Equation (2.25) by sampling β from $\sin \beta d\beta/2$, η from $d\eta/2\pi$, S_2 from $\Sigma_T e^{-\Sigma_T S_2} dS_2$ and evaluating the remainder of the expression under the integral,

$$N(R, \theta, \phi) \frac{\Sigma_S}{\Sigma_T} \frac{\frac{d\Sigma}{d\Omega}(\psi)}{\Sigma_S} \frac{e^{-\Sigma_T S_1}}{S_1^2} \quad (2.26)$$

for the sample values to obtain an estimate of the scattered intensity for a point isotropic source at (R, θ, ϕ) .

By an application of the first transformation on the angles β and η , Equation (2.25) can be written in terms of angles ω and λ defined with reference to the radial through the receiver position (see Fig. 2).

$$\int_{R_1}^{\infty} \int_{\phi_1}^{\phi_2} \int_{\theta_1}^{\theta_2} \int_{\lambda_1}^{\lambda_2} \int_{\omega_1}^{\omega_2} \int_{S_1}^{S_2} \frac{N(R, \theta, \phi)}{4\pi S_1^2} \Sigma_T e^{-\Sigma_T S_1} \frac{\Sigma_S}{\Sigma_T} \frac{\frac{dV}{d\psi}(\psi)}{L_S} e^{-\Sigma_T S_2} \sin\omega \, d\omega d\lambda dS_2 R^2 \sin\theta d\theta d\phi dR. \quad (2.27)$$

In Equation (2.27) the volume element dV has been replaced by $R^2 \sin\theta d\theta d\phi dR$.

The variables of integration in Equation (2.27), ω , λ , and S_2 are defined about the receiver position. The location of the source volume R , θ , ϕ is still defined in terms of the origin of the coordinate system. Three additional transformations are necessary to transform the variables of integration R , θ , and ϕ into a set of variables defined about the collision point. The three lines joining the origin with the source, collision, and receiver points form a spherical triangle with vertex at the origin (Fig. 2).

The angle δ' between the radial through the collision point and the radial through the receiver position is given by the expression

$$\delta' = \cos^{-1} \left[(R_D + S_2 \cos\omega) / \sqrt{R_D^2 + S_2^2 + 2R_D S_2 \cos\omega} \right]. \quad (2.28)$$

θ and ϕ , the polar and azimuthal angles of the source position, may be expressed in terms of θ' and ϕ' , the polar and azimuthal angles defined with respect to the radial through the collision point with the use of the equations

$$\theta = \cos^{-1} [\cos\theta' \cos\delta' - \sin\theta' \sin\delta' \cos\phi'] \quad (2.29)$$

and

$$\phi = \tan^{-1} [\sin\theta' \sin\phi' / (\sin\delta' \cos\theta' + \sin\theta' \cos\delta' \cos\phi')]. \quad (2.30)$$

Applying the first transformation to the angles θ and ϕ , Equation (2.27) may be expressed in terms of θ' and ϕ' :

$$\int_R \int_{\theta'} \int_{\phi'} \int_{\lambda} \int_{\omega} \int_{S_2} \frac{N(R, \theta', \phi')}{4\pi S_1^2} \Sigma_T e^{-\Sigma_T S_1} \frac{\Sigma_S}{\Sigma_T} \frac{\frac{d\Sigma}{d\Omega}(\psi)}{\Sigma_S} e^{-\Sigma_T S_2} \sin\omega d\omega d\lambda dS_2 \sin\theta' d\theta' d\phi' dR. \quad (2.31)$$

Next we may apply the second transformation to R and θ' to transform Equation (2.31) into an equation involving S_1 and τ , where S_1 is the distance from the source position to the collision point and τ is the angle at the collision between the line joining the source and collision points and the origin and the collision point. The resulting equation is:

$$\int_{S_1} \int_{\phi'} \int_{\tau} \int_{\lambda} \int_{\omega} \int_{S_2} \frac{N(S_1, \tau, \phi')}{4\pi} \Sigma_T e^{-\Sigma_T S_1} \frac{\Sigma_S}{\Sigma_T} \frac{\frac{d\Sigma}{d\Omega}(\psi)}{\Sigma_S} e^{-\Sigma_T S_2} \sin\omega d\omega d\lambda dS_2 \sin\tau d\tau d\phi' dS_1. \quad (2.32)$$

Then finally by applying the first transformation to change from variables τ and ϕ' to the angles ψ and ρ , Equation (2.32) becomes

$$\int_{S_1} \int_{\rho} \int_{\psi} \int_{\lambda} \int_{\omega} \int_{S_2} \frac{N(S_1, \psi, \rho)}{4\pi} \Sigma_T e^{-\Sigma_T S_1} \frac{\Sigma_S}{\Sigma_T} \frac{\frac{d\Sigma}{d\Omega}(\psi)}{\Sigma_S} e^{-\Sigma_T S_2} \sin\omega d\omega d\lambda dS_2 \sin\psi d\psi d\rho dS_1, \quad (2.33)$$

which can be solved with the Monte Carlo method by sampling ω from $\sin\omega d\omega/2$, λ from $d\lambda/2\pi$, S_2 from $\Sigma_T e^{-\Sigma_T S_2} dS_2$ and a scattering direction

(ψ, ρ) from $\frac{\frac{d\Sigma}{d\Omega}(\psi)}{\Sigma_S} \sin\psi d\psi d\rho$ and evaluating the estimator

$$\int_{S_1} N(S_1, \psi, \rho) \frac{\Sigma_S}{\Sigma_T} e^{-\Sigma_T S_1} dS_1 \quad (2.34)$$

for the sample values.

Note that the separation of Equation (2.33) into density and estimating functions gives an estimating function that fluctuates less with the values sampled from the density functions than the estimating function for Equation (2.7).

Expression (2.34) could be further separated into an estimating function and a density function for sampling the distance S_1 from the collision to the source position; but in many cases it is almost as easy to perform the integration as it is to evaluate the estimator for a single source point.

The main advantage of the backward approach is that for each photon history the contribution from all source points along the line passing through the source volume can be added. In addition, those source regions which have the greatest probability of contributing to the scattered intensity at the receiver are the ones sampled with the greatest frequency.

For some problems the size of the total source volume may be rather small in comparison with the distance from the center of the source volume to the collision centers. For this case the sampling of the direction after collision (ψ, ρ) can be restricted to the solid angle subtended by the source at the collision center and the weight of the photon adjusted to account for having sampled from only a portion of the solid angle.

2.2 Illustration of Backward Monte Carlo for a Plane Source

For the second illustration, consider an infinite plane parallel source which emits radiation normal to the source plane and is embedded within an infinite media that scatters radiation isotropically. Suppose that one wishes to know the average single scattered intensity at some distance Z_0 from the source plane, Fig. 3. The expression for the single scattered intensity may be written as follows,

$$SI = \int_0^{2\pi} \int_0^{\infty} \int_0^{\infty} S(\rho, \phi) \Sigma_T e^{-\Sigma_T Z} (1/4\pi) \frac{e^{-\Sigma_T r}}{r^2} \rho d\rho d\phi dZ \quad (2.35)$$

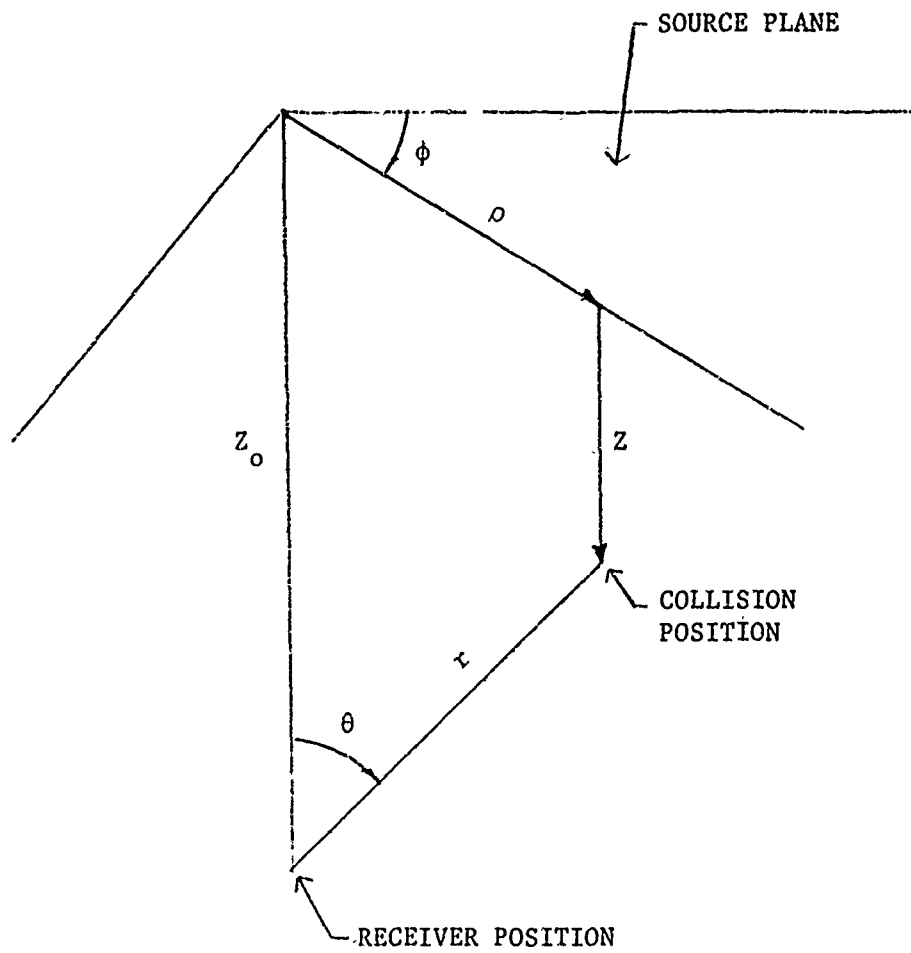


Fig. 3. Geometry for Plane Parallel Source and Receiver Embedded within an Infinite Medium

where $S(\rho, \phi)$ is the source strength at the position ρ, ϕ on the source plane, Σ_T is the extinction coefficient of the media, Z is the distance from the source plane to the collision point, and r is the distance from the collision to the receiver position. Equation (2.35) could be solved with the Monte Carlo method or some other method of numerical integration, but some decision must be made as to what the upper limit of the integral over the source radius will be and what $\Delta \rho$ values will yield the most efficient calculation. This is especially true of a Monte Carlo solution of Equation (2.35) because if one should select the position of the source photon from a uniform distribution over some finite portion of the plane, he could easily be concentrating his sampling at large radial distances that produce no significant single scattered intensity at the receiver position.

A better method for solving Equation (2.35) would be to change the variables of integration from ρ and Z to θ and r , where the relationship between the variables are as follows:

$$\begin{aligned}\rho &= r \sin \theta \\ Z &= Z_0 - r \cos \theta\end{aligned}\quad (2.36)$$

Since ρ and Z are both functions of r and θ , we may transform Equation (2.35) into an equation expressed in terms of r and θ through the use of the following relation:

$$\iint f(\rho, Z) d\rho dZ = \iint f(\rho(r, \theta), Z(r, \theta)) \left| \frac{\partial(\rho, Z)}{\partial(r, \theta)} \right| dr d\theta \quad (2.37)$$

where $\left| \frac{\partial(\rho, Z)}{\partial(r, \theta)} \right|$ is the absolute value of the Jacobian.

Solving for the partial derivative in the Jacobian we obtain:

$$\frac{\partial \rho}{\partial r} = \sin \theta \quad (2.38)$$

$$\frac{\partial \rho}{\partial \theta} = r \cos \theta \quad (2.39)$$

$$\frac{\partial Z}{\partial r} = -\cos\theta \quad (2.40)$$

and

$$\frac{\partial Z}{\partial \theta} = r \sin\theta. \quad (2.41)$$

Evaluating the Jacobian yields

$$\left| \begin{array}{cc} \frac{\partial \rho}{\partial r} & \frac{\partial \rho}{\partial \theta} \\ \frac{\partial Z}{\partial r} & \frac{\partial Z}{\partial \theta} \end{array} \right| = r. \quad (2.42)$$

Rewriting Equation (2.35) by substituting r and θ for ρ and Z gives:

$$SI = \int_0^{2\pi} \int_0^{\infty} \int_0^{\pi} S(\rho, \phi) e^{-\Sigma_T(Z_0 - r \cos\theta)} (1/4\pi) e^{-\Sigma_T r} \sin\theta d\theta dr d\phi. \quad (2.43)$$

Equation (2.43) may be solved using the Monte Carlo method by sampling a value of θ from $\sin\theta d\theta/2$, a value of ϕ from $d\phi/2\pi$, and a value of r from

$\Sigma_T e^{-\Sigma_T r} dr$ and evaluating the remainder of the expression under the integral

$$S(\rho(r, \theta), \phi) e^{-\Sigma_T(Z_0 - r \cos\theta)} \quad (2.44)$$

for the values sampled for r, θ , and ϕ .

Note that the integration of θ in Equation (2.43) is over a finite range and since the integral of

$$\Sigma_T e^{-\Sigma_T r} dr$$

has a value of one when integrated from 0 to infinity, values of r approaching infinity may be selected and thus values of $\rho = r \sin\theta$ are also unlimited.

Note also that the density function

$$\Sigma_T e^{-\Sigma_T r} dr$$

and the estimating function

$$e^{-\Sigma_T(Z_0 - r \cos \theta)}$$

both peak for small values of r indicating the sampling is concentrated near the area where the estimating function has its highest value. A situation of this nature produces less statistical fluctuation than would be expected for a Monte Carlo solution of Equation (2.35).

To check out the application of the backward approach for multiple scattering calculations, a test procedure was written using the backward approach to calculate the multiple scattered intensity at several distances from a plane-parallel normal incident radiation source embedded within an infinite homogeneous medium which has a scattering-to-total cross section ratio of 0.5 and within which scattering is isotropic. The results obtained from the test procedure are compared in Fig. 4 with results reported by Beach, et al. (Ref. 3) for the one-velocity neutron diffusion problem. The good agreement between the integrated scattered intensity obtained from the test procedure and from Beach's calculations for distances to 10 mean-free-paths from the source plane indicates that the backward Monte Carlo approach is feasible.

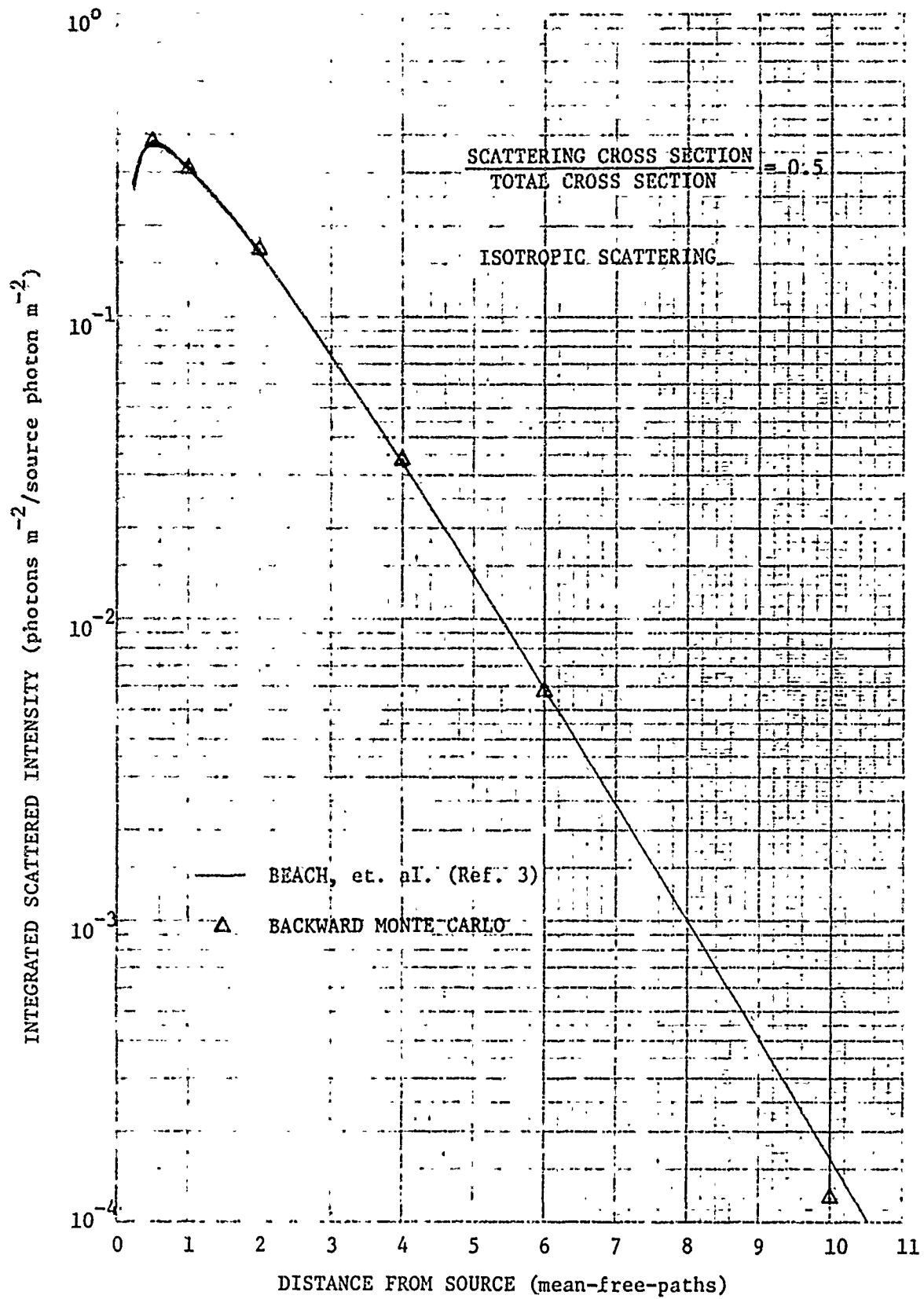


Fig. 4. Integrated Scattered Intensity Versus Distance from a Normal Incidence Plane Parallel Source in an Infinite Medium

III. CALCULATIONAL METHOD FOR SCATTERED LIGHT

All three versions of the FLASH program utilize the backward Monte Carlo approach to compute the scattered light intensity within specified solid angles of view at a set of point receiver positions. The portions of the three versions of FLASH which sample the distribution of the collision and reflection centers are identical, but the portions of the programs which estimates the direct intensity from the source which reaches each of the collision centers depends upon the type of source being considered. The estimating functions, Equation (2.26), (2.34), and (2.44), include a calculation of the direct intensity reaching a collision point from a point, volume or plane source. The methods common to all three programs for sampling the distribution of the collision and reflection centers within a spherical shell atmosphere are discussed first, then the methods used to evaluate the estimating functions are discussed separately for the point, plane and volume source programs.

3.1 Generation of Random Path

The initial intensity and polarization characteristics of the radiation at the receiver position are defined in terms of the polarization parameters $I_{\perp}$, $I_{\parallel}$, U , and V which are related to the Stokes parameter I , Q , U , and V by the relations

$$\begin{aligned} I &= I_{\perp} + I_{\parallel} \\ Q &= I_{\perp} - I_{\parallel} \\ U &= U \\ V &= V. \end{aligned} \tag{3.1}$$

These parameters are defined with reference to a plane containing the radial through the receiver position and the direction of propagation of the light incident at the receiver. The light emitted by the source is considered to be unpolarized; therefore, the polarization parameters are initialized to

$$\begin{aligned} I_{\perp} &= 0.5W \\ I_{\parallel} &= 0.5W \\ U &= 0.0 \\ V &= 0.0 \end{aligned} \tag{3.2}$$

where W is a weight parameter initially set equal to a unit intensity incident from the direction of propagation into the receiver position. Processes such as absorption that affect all polarization parameters in the same manner are included by an adjustment to the weight parameter while processes such as Rayleigh and aerosol scattering require individual adjustment of the polarization parameters. The product of the weight parameter times the sum of the first two polarization parameters gives the photon intensity at any position in the atmosphere. The degree of polarization at any position is given by

$$DP = \frac{((I_A - I_H)^2 + U^2 + V^2)^{1/2}}{(I_A + I_H)} \quad (3.3)$$

Since the atmospheric parameters which affect the transport of light all vary with altitude, it was necessary that the geometry of the FLASH program be capable of providing a means of defining the atmospheric conditions as a function of altitude.

3.1.1 Problem Geometry

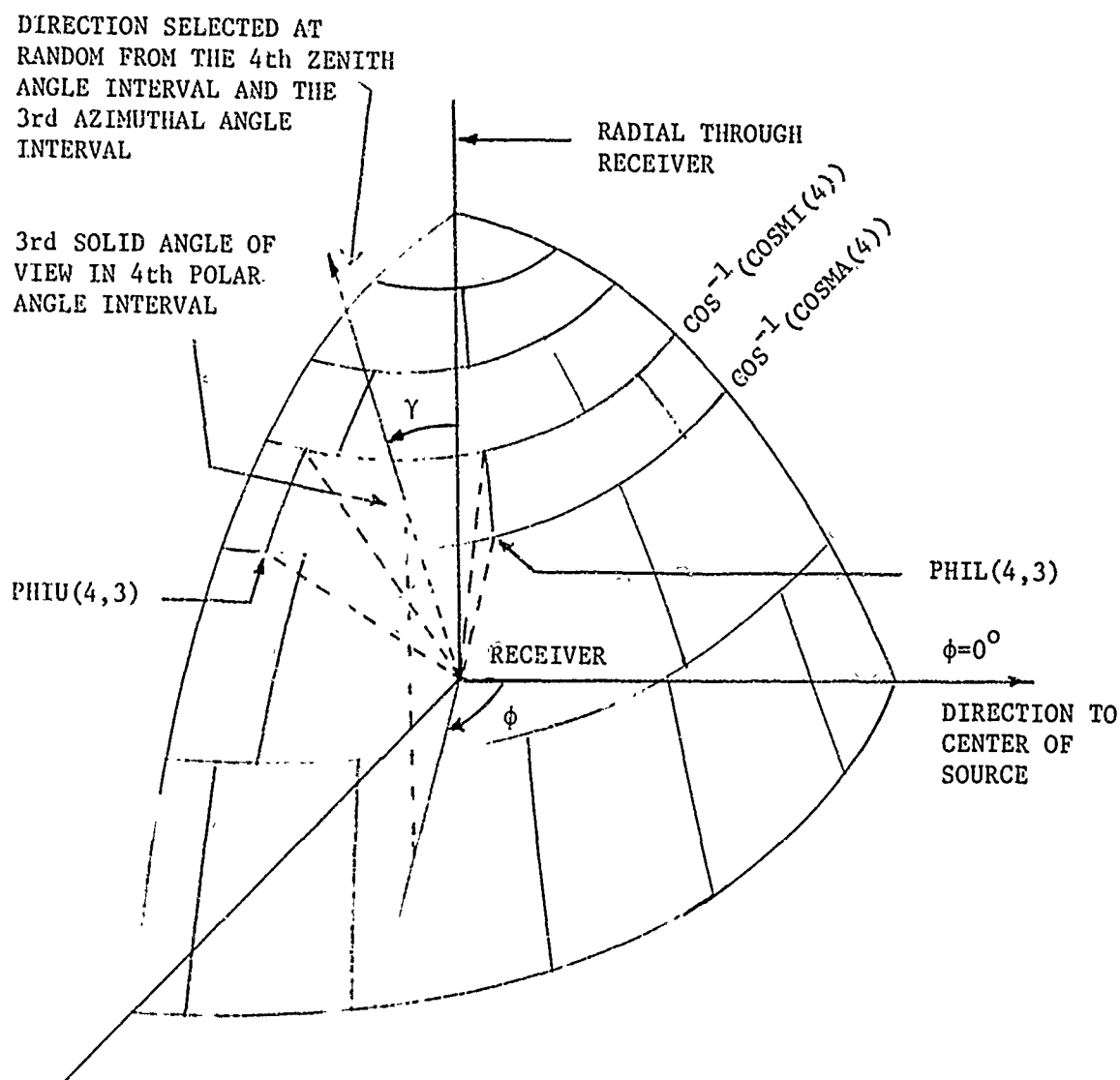
The geometry portion of the FLASH program provides for the division of a spherical-shell atmosphere into as many as 100 spherical-shell layers or atmospheric layers. The spherical-shell layers are defined by inputting the altitudes of the spherical surfaces separating the layers. The atmospheric conditions at each boundary are defined by inputting the optical distance from ground level to the boundary altitude, the ratio of the scattering-to-pseudo extinction coefficient at the boundary altitude, the ratio of the Rayleigh-to-scattering coefficient at the boundary altitude, and the index of refraction of the air just above the boundary altitude. The pseudo extinction coefficient is defined to be the sum of the Rayleigh, aerosol, and ozone coefficients. If the wavelength of the light being considered is in the range in which water vapor and CO_2 absorption is significant, provisions have been made for inputting amounts of the accumulated water vapor and accumulated CO_2 from ground level to the boundary altitude. If the water vapor and CO_2 concentrations are required, then the temperature and pressure for each boundary altitude are also required.

In addition to the division of the atmosphere into atmospheric layers, the FLASH programs provide for the division of the atmosphere into as many as 10 aerosol scattering regions within each of which the aerosol size distribution remains constant even though the number density may vary. An aerosol scattering region is defined in the program input by listing the altitudes of the upper and lower spherical surfaces bounding the region. A phase matrix number is also assigned to each aerosol scattering region indicating which of the normalized aerosol phase matrices in the problem input pertains to the aerosol size distribution within that region. The receiver positions are all at the same altitude but at different polar positions within the zero azimuthal plane. The polar position angle is defined by the polar angle between the radials through the receiver and the center of the source.

3.1.2 Receiver View Angles

A set of view angles (solid angle intervals) at a receiver position are defined in the input data for the FLASH programs by listing the angle bounds for a set of polar angle intervals and the angle bounds for a set azimuthal angle intervals within each of the polar angle intervals (Fig. 5). The polar angles are defined with respect to the radial passing through the receiver position and the azimuthal angles are defined in a tangent plane with the zero azimuth toward a radial through the center of the source. The sizes of the receiver view angles defined in this manner are completely arbitrary and may even be reduced to a set of discrete directions.

It is required that the total number of photon histories run for any given FLASH problem be equally divisible by the number of the receiver polar angle intervals and the number of histories within each receiver polar angle interval be equally divisible by the number of receiver azimuthal angle intervals defined for the particular receiver polar angle interval.



The FLASH programs are designed to systematically sample backward photon histories from the receiver solid angles of view and to printout the scattered intensity for each receiver solid angle of view before proceeding to the next.

At the start of each history before the history counter NHIST is incremented, the index of the zenith angle interval, LA, and the azimuthal interval, LAZ, within the zenith angle interval are determined with the following expressions

$$\begin{aligned} LA &= 1 + NPVA \cdot NHIST / NHMAX \\ JAZ &= NHIST - (LA-1) \cdot NHMAX / NPVA \\ LAZ &= 1 + NAZ(LA) \cdot JAZ \cdot NPVA / NHMAX \end{aligned} \quad (3.4)$$

where NHMAX is the total number of histories to be run, NPVA is the number of zenith angle intervals used to define the solid angles of view, and NAZ(LA) is the number of solid angles of view in the LAth zenith angle interval.

For each photon history the direction of the light entering the receiver solid angle of view is sampled from a uniform distribution within the solid angle of view. The cosine of γ , the zenith polar angle of the light's direction as it enters the view angle, is given by

$$\cos \gamma = \text{COSMI}(LA) - \text{RN}(\text{COSMI}(LA) - \text{COSMA}(LA)) \quad (3.5)$$

where RN is a random number selected from a uniform distribution between 0.0 and 1.0, and COSMI(LA) and COSMA(LA) are the cosines of the lower and upper bounds of the zenith angles bounding the LAth zenith angle interval. The azimuthal direction, ϕ , of the light entering the solid angle of view is given by

$$\phi = \text{PHIL}(LA, LAZ) + \text{RN}(\text{PHIU}(LA, LAZ) - \text{PHIL}(LA, LAZ)) \quad (3.6)$$

where RN is a random number, and PHIL(LA, LAZ) and PHIU(LA, LAZ) are the lower and upper azimuthal angle bounds of the LAZth receiver solid angle of view in the LAth zenith angle interval.

3.1.3 Distance to Collision

Once the initial direction of the photon's backward path is determined, the optical distance through each of the atmospheric layers that

the photon traverses along a path to the upper or lower surface of the atmosphere is computed. The path followed by the photon is bent at each boundary between the atmospheric layers due to the change in the index of refraction from one layer to the next. If the photon's path intersects the ground surface before reaching the upper boundary of the atmosphere, the optical distance between collisions is chosen from the exponential distribution

$$\rho = -\ln(1-R) \quad (3.7)$$

where ρ is the sampled path length and R is a random number chosen from a uniform distribution between 0.0 and 1.0. The random path length is then compared with the total optical distance computed for the photon's path from the receiver to the point where the path intersects the ground surface. If the sampled pathlength is greater than the optical distance to the ground intersection, a reflection is forced to occur at the intersection point. If the random pathlength is less than the optical distance to the ground intersection, a collision is considered to occur at the point on the photon's path where the optical distance is equal to the sample pathlength.

If the photon's path intersects the upper boundary of the earth's atmosphere before intersecting the ground surface, the random pathlength between collisions is chosen from a truncated exponential distribution, thus forcing a collision before the photon escapes the atmosphere. The random path is chosen with the expression

$$\rho = -\ln(1-R(1-e^{-\rho_{\max}})), \quad (3.8)$$

where R is a random number chosen from a uniform distribution between 0.0 and 1.0 and $\rho_{\max}$ is the optical distance along the photon's path from the receiver to the top of the atmosphere. When the collision is forced to occur before the photon escapes the atmosphere, the particle weight is adjusted to remove the bias introduced by forcing the collision with the expression

$$W = W(1 - e^{-\rho_{\max}}), \quad (3.9)$$

where W is the photon weight.

3.1.3.1 Sampling from Linear Distributions. An option has been incorporated in the point source version of FLASH to permit use of biased sampling on the distances between photon collision when the extension of the photon path intersects the upper boundary of the atmosphere. The option provides for sampling the distances between collisions from a truncated exponential distribution, from a linear distribution of the optical distance from the collision to the upper limit of the atmosphere or from a linear distribution of the geometric distance from a collision to the upper limit of the atmosphere. If the distance to the next collision is selected from the linear distribution of the geometric distance, then

$$D = R S_{\max} \quad (3.10)$$

where D is the distance to the collision, R is a random number between 0 and 1, and $S_{\max}$ is the distance from the present collision to the upper atmospheric bound. When this biased selection of the photon distance between collisions is applied, the photon weight is adjusted by multiplying the factor

$$\frac{\rho_i}{S_i} S_{\max} e^{-\rho} \quad (3.11)$$

where ρ is the optical distance corresponding to D , ρ_i/S_i is the extinction coefficient in the i th atmospheric layer containing the new collision position.

If the optical distance to the next collision is selected from a linear distribution of optical distances, then

$$\rho = R \rho_{\max} \quad (3.12)$$

where ρ is the optical distance to the next collision, R is a random number, and $\rho_{\max}$ is the optical distance to the upper atmospheric bound. For this method of selecting the optical distance between collisions, the weight adjustment factor is given by

$$\rho_{\max} e^{-\rho} \quad (3.13)$$

Both of the methods for biasing the sampling of the optical distance

between collisions for those photon paths that extend to the upper atmospheric bound should place a larger number of photon collisions at a higher altitude, thus giving the photon a better chance to traverse large horizontal distances because of the thinner atmosphere at the higher altitudes. The selection of collision positions from a linear distribution of the distance should place more collisions at the higher altitudes than selection of the collision position from a linear distribution of optical distances.

3.1.4 CO₂ and Water Vapor Absorption

If the wavelength of light being considered is one for which carbon dioxide absorption is significant, the CO₂ transmission is computed for a finite wavelength interval by first computing the equivalent CO₂ thickness between collisions for some standard temperature T_0 and pressure P_0 with the equation

$$c = \sum_i X_i \frac{P_i}{P_0} \frac{T_0}{T_i} \quad (3.14)$$

where X_i is the product of the CO₂ density times the distance traversed in the i th atmospheric layer, P_i is the pressure in the i th atmospheric layer, and T_i is the temperature in the i th atmospheric layer. The CO₂ transmission is computed in FLASH with the expression (Refs. 4 and 5)

$$T(\lambda) = e^{-1.83(c \cdot k(\lambda))^{0.73}} \quad (3.15)$$

The wavelength-dependent parameter $k(\lambda)$ is required as input to FLASH and must be determined from some known data for a given P_0 and T_0 . Two sources of data frequently used are the measurements of D. Burch (Ref. 6) and the calculations of Skull, et. al. (Ref. 7). Table I lists values of $k(\lambda)$ determined from Skull, et. al. for 0.05 μ wavelength intervals from $\lambda = 1.3\mu$ to 19.6 μ for $T_0 = 300^\circ\text{K}$ and $P_0 = 1 \text{ atm}$.

In the FLASH programs, it is assumed that the effect of Rayleigh and aerosol scattering and ozone absorption for a small finite wavelength interval can be represented with monochromatic light for a wavelength within the interval, since the scattering and absorption coefficients for these processes vary fairly smoothly with wavelength.

Table I. Absorption Coefficient, $k(\lambda)$, for CO_2 as Evaluated
from Data for $T_0 = 300^\circ\text{K}$, $P_0 = 1$ Atmosphere, and $\Delta\lambda = 0.05$

WAVELENGTH (MICRONS)	ABSORPTION COEFFICIENT
0.1300000E 01	0.8779655E-07
0.1350000E 01	0.1400175E-05
0.1400000E 01	0.3394442E-08
0.1450000E 01	0.2997619E-05
0.1500000E 01	0.0000000E 00
0.1550000E 01	0.6925429E-07
0.1600000E 01	0.2343412E-04
0.1650000E 01	0.1530664E-05
0.1700000E 01	0.0000000E 00
0.1750000E 01	0.0000000E 00
0.1800000E 01	0.8779654E-08
0.1850000E 01	0.0000000E 00
0.1900000E 01	0.8779654E-08
0.1950000E 01	0.2441837E-03
0.2000000E 01	0.1626659E-02
0.2050000E 01	0.6670115E-03
0.2100000E 01	0.2737666E-03
0.2150000E 01	0.2272307E-07
0.2200000E 01	0.8779654E-08
0.2250000E 01	0.0000000E 00
0.2300000E 01	0.0000000E 00
0.2350000E 01	0.0000000E 00
0.2400000E 01	0.0000000E 00
0.2450000E 01	0.0000000E 00
0.2500000E 01	0.3394442E-08
0.2550000E 01	0.2997619E-06
0.2600000E 01	0.1081474E-04
0.2650000E 01	0.1260083E-03
0.2700000E 01	0.1103279E 00
0.2750000E 01	0.4902803E-01
0.2800000E 01	0.2441837E-01

Table T. (Cont.)

WAVELENGTH (MICRONS)	ABSORPTION COEFFICIENT
0.2850000E 01	0.1266498E-02
0.2900000E 01	0.7978218E-04
0.2950000E 01	0.8744977E-02
0.3000000E 01	0.8006388E-07
0.3050000E 01	0.0000000E 00
0.3100000E 01	0.0000000E 00
0.3150000E 01	0.0000000E 00
0.3200000E 01	0.0000000E 00
0.3250000E 01	0.0000000E 00
0.3300000E 01	0.0000000E 00
0.3350000E 01	0.0000000E 00
0.3400000E 01	0.0000000E 00
0.3450000E 01	0.4901436E-07
0.3500000E 01	0.1941039E-06
0.3550000E 01	0.5272262E-06
0.3600000E 01	0.1105134E-05
0.3650000E 01	0.2238255E-05
0.3700000E 01	0.4528536E-05
0.3750000E 01	0.9508306E-05
0.3800000E 01	0.2159536E-04
0.3850000E 01	0.3389235E-04
0.3900000E 01	0.7301785E-04
0.3950000E 01	0.1006724E-03
0.4000000E 01	0.2686753E-03
0.4050000E 01	0.8386594E-03
0.4100000E 01	0.1311917E-02
0.4150000E 01	0.8754916E-02
0.4200000E 01	0.8987492E 00
0.4250000E 01	0.5987970E 01
0.4300000E 01	0.5149409E 01
0.4350000E 01	0.7435272E 00
0.4400000E 01	0.1108793E 00
0.4450000E 01	0.3338004E-01
0.4500000E 01	0.1725388E-02
0.4550000E 01	0.3678403E-03
0.4600000E 01	0.1597013E-03
0.4650000E 01	0.9785719E-04
0.4700000E 01	0.2582526E-03
0.4750000E 01	0.2505060E-03
0.4800000E 01	0.3358310E-03
0.4850000E 01	0.3613991E-03
0.4900000E 01	0.6904131E-04
0.4950000E 01	0.3965552E-06
0.5000000E 01	0.1626660E-04
0.5050000E 01	0.6998770E-05
0.5100000E 01	0.1733137E-04
0.5150000E 01	0.3839011E-04
0.5200000E 01	0.5438877E-04

NOT REPRODUCIBLE

Table I. (Cont.)

WAVELENGTH (MICRONS)	ABSORPTION COEFFICIENT
0.5250000E 01	0.2356702E-04
0.5300000E 01	0.2421292E-05
0.5350000E 01	0.8760352E-06
0.5400000E 01	0.3322616E-06
0.5450000E 01	0.2083803E-06
0.5500000E 01	0.1029231E-06
0.5550000E 01	0.3086996E-07
0.5600000E 01	0.0000000E 03
0.5650000E 01	0.0000000E 00
0.5700000E 01	0.0000000E 00
0.5750000E 01	0.0000000E 00
0.5800000E 01	0.0000000E 00
0.5850000E 01	0.0000000E 00
0.5900000E 01	0.0000000E 00
0.5950000E 01	0.0000000E 00
0.6000000E 01	0.0000000E 00
0.6050000E 01	0.0000000E 00
0.6100000E 01	0.0000000E 00
0.6150000E 01	0.0000000E 00
0.6200000E 01	0.0000000E 00
0.6250000E 01	0.0000000E 00
0.6300000E 01	0.0000000E 00
0.6350000E 01	0.0000000E 00
0.6400000E 01	0.0000000E 00
0.6450000E 01	0.0000000E 00
0.6500000E 01	0.0000000E 00
0.6550000E 01	0.0000000E 00
0.6600000E 01	0.0000000E 00
0.6650000E 01	0.0000000E 00
0.6700000E 01	0.0000000E 00
0.6750000E 01	0.0000000E 00
0.6800000E 01	0.0000000E 00
0.6850000E 01	0.0000000E 00
0.6900000E 01	0.0000000E 00
0.6950000E 01	0.0000000E 00
0.7000000E 01	0.0000000E 00
0.7050000E 01	0.0000000E 00
0.7100000E 01	0.0000000E 00
0.7150000E 01	0.0000000E 00
0.7200000E 01	0.0000000E 00
0.7250000E 01	0.0000000E 00
0.7300000E 01	0.0000000E 00
0.7350000E 01	0.0000000E 00
0.7400000E 01	0.0000000E 00
0.7450000E 01	0.0000000E 00
0.7500000E 01	0.0000000E 00
0.7550000E 01	0.0000000E 00
0.7600000E 01	0.0000000E 00

NOT REPRODUCIBLE

Table I. (Cont.)

WAVELENGTH (MICRONS)	ABSORPTION COEFFICIENT
0.7650000E 01	0.0000000E 00
0.7700000E 01	0.0000000E 00
0.7750000E 01	0.0000000E 00
0.7800000E 01	0.0000000E 00
0.7850000E 01	0.0000000E 00
0.7900000E 01	0.0000000E 00
0.7950000E 01	0.0000000E 00
0.8000001E 01	0.0000000E 00
0.8050001E 01	0.0000000E 00
0.8100000E 01	0.0000000E 00
0.8150001E 01	0.0000000E 00
0.8200000E 01	0.0000000E 00
0.8250001E 01	0.0000000E 00
0.8300001E 01	0.0000000E 00
0.8350000E 01	0.0000000E 00
0.8400001E 01	0.0000000E 00
0.8450000E 01	0.0000000E 00
0.8500001E 01	0.0000000E 00
0.8550001E 01	0.0000000E 00
0.8600000E 01	0.0000000E 00
0.8650001E 01	0.0000000E 00
0.8700000E 01	0.0000000E 00
0.8750001E 01	0.0000000E 00
0.8800001E 01	0.0000000E 00
0.8850000E 01	0.0000000E 00
0.8900001E 01	0.0000000E 00
0.8950000E 01	0.0000000E 00
0.9000001E 01	0.3394442E-08
0.9050001E 01	0.2997619E-06
0.9100000E 01	0.9785719E-05
0.9150001E 01	0.3570925E-04
0.9200000E 01	0.1035291E-03
0.9250001E 01	0.1589655E-03
0.9300001E 01	0.1567705E-03
0.9350000E 01	0.1058447E-03
0.9400001E 01	0.4333821E-04
0.9450000E 01	0.8753051E-04
0.9500001E 01	0.1087775E-03
0.9550001E 01	0.8335441E-04
0.9600000E 01	0.7579180E-04
0.9650001E 01	0.4535445E-04
0.9700000E 01	0.2545861E-04
0.9750001E 01	0.1371405E-04
0.9800001E 01	0.6752478E-05
0.9850000E 01	0.5051162E-05
0.9900001E 01	0.4142590E-05
0.9950000E 01	0.4256913E-05
0.1000000E 02	0.2390429E-05

NOT REPRODUCIBLE

Table I. (Cont.)

WAVELENGTH (MICRONS)	ABSORPTION COEFFICIENT
0.1005000E 02	0.4969214E-05
0.1010000E 02	0.1035291E-04
0.1015000E 02	0.1876487E-04
0.1020000E 02	0.2597695E-04
0.1025000E 02	0.2511436E-04
0.1030000E 02	0.2921474E-04
0.1035000E 02	0.1553176E-04
0.1040000E 02	0.9618783E-05
0.1045000E 02	0.1255687E-04
0.1050000E 02	0.2319842E-04
0.1055000E 02	0.2756806E-04
0.1060000E 02	0.4728143E-04
0.1065000E 02	0.2132561E-04
0.1070000E 02	0.1901111E-04
0.1075000E 02	0.1035028E-04
0.1080000E 02	0.7480940E-05
0.1085000E 02	0.6951375E-05
0.1090000E 02	0.6716620E-05
0.1095000E 02	0.5300395E-05
0.1100000E 02	0.5258501E-05
0.1105000E 02	0.5990043E-05
0.1110000E 02	0.1075993E-04
0.1115000E 02	0.1819854E-04
0.1120000E 02	0.3309651E-04
0.1125000E 02	0.6429261E-04
0.1130000E 02	0.8208052E-04
0.1135000E 02	0.1093692E-03
0.1140000E 02	0.1029544E-03
0.1145000E 02	0.1012403E-03
0.1150000E 02	0.8335441E-04
0.1155000E 02	0.6892017E-03
0.1160000E 02	0.1235742E-02
0.1165000E 02	0.2223174E-03
0.1170000E 02	0.1538729E-03
0.1175000E 02	0.1190727E-03
0.1180000E 02	0.1012403E-03
0.1185000E 02	0.7678022E-04
0.1190000E 02	0.6559588E-04
0.1195000E 02	0.5133671E-04
0.1200000E 02	0.4295300E-04
0.1205000E 02	0.4647041E-04
0.1210000E 02	0.3880702E-04
0.1215000E 02	0.4193548E-04
0.1220000E 02	0.4167720E-04
0.1225000E 02	0.4180560E-04
0.1230000E 02	0.5258501E-04
0.1235000E 02	0.1265939E-02
0.1240000E 02	0.1589555E-03

NOT REPRODUCIBLE

Table I. (Cont.)

WAVELENGTH (MICRONS)	ABSORPTION COEFFICIENT
0.1245000E 02	0.8170612E-04
0.1250000E 02	0.8133299E-04
0.1255000E 02	0.7838528E-04
0.1260000E 02	0.1901111E-03
0.1265000E 02	0.3704150E-03
0.1270000E 02	0.2278781E-03
0.1275000E 02	0.1311916E-03
0.1280000E 02	0.1117531E-03
0.1285000E 02	0.1764354E-03
0.1290000E 02	0.2250854E-03
0.1295000E 02	0.3007960E-03
0.1300000E 02	0.3404556E-03
0.1305000E 02	0.4977070E-03
0.1310000E 02	0.7911591E-03
0.1315000E 02	0.1041055E-02
0.1320000E 02	0.1567705E-02
0.1325000E 02	0.2335387E-02
0.1330000E 02	0.3411507E-02
0.1335000E 02	0.3650892E-02
0.1340000E 02	0.5051771E-02
0.1345000E 02	0.6445174E-02
0.1350000E 02	0.1405657E-01
0.1355000E 02	0.1299119E-01
0.1360000E 02	0.1125427E-01
0.1365000E 02	0.1116187E-01
0.1370000E 02	0.1088792E-01
0.1375000E 02	0.9139372E-02
0.1380000E 02	0.1874006E-01
0.1385000E 02	0.5550154E-01
0.1390000E 02	0.1166223E 00
0.1395000E 02	0.4319116E-01
0.1400000E 02	0.3259720E-01
0.1405000E 02	0.3235000E-01
0.1410000E 02	0.3582930E-01
0.1415000E 02	0.5128081E-01
0.1420000E 02	0.6669726E-01
0.1425000E 02	0.9588295E-01
0.1430000E 02	0.1072939E 00
0.1435000E 02	0.1636070E 00
0.1440000E 02	0.1798215E 00
0.1445000E 02	0.2224865E 00
0.1450000E 02	0.2712863E 00
0.1455000E 02	0.3100085E 00
0.1460000E 02	0.3486278E 00
0.1465000E 02	0.4105408E 00
0.1470000E 02	0.4283929E 00
0.1475000E 02	0.4339689E 00
0.1480000E 02	0.4053934E 00

Table I. (Cont.)

WAVELENGTH (MICRONS)	ABSORPTION COEFFICIENT
0.1485000E 02	0.5128947E 00
0.1490000E 02	0.8210817E 00
0.1495000E 02	0.1565551E 01
0.1500000E 02	0.1788073E 01
0.1505000E 02	0.9568034E 00
0.1510000E 02	0.5425727E 00
0.1515000E 02	0.3542298E 00
0.1520000E 02	0.5227743E 00
0.1525000E 02	0.3570750E 00
0.1530000E 02	0.3856865E 00
0.1535000E 02	0.4302399E 00
0.1540000E 02	0.4594240E 00
0.1545000E 02	0.4283929E 00
0.1550000E 02	0.3124123E 00
0.1555000E 02	0.2583352E 00
0.1560000E 02	0.1881040E 00
0.1565000E 02	0.1648553E 00
0.1570000E 02	0.1186845E 00
0.1575000E 02	0.1039808E 00
0.1580000E 02	0.7838529E-01
0.1585000E 02	0.5708245E-01
0.1590000E 02	0.4837132E-01
0.1595000E 02	0.3358310E-01
0.1600000E 02	0.2905132E-01
0.1605000E 02	0.7557316E-01
0.1610000E 02	0.4677124E 00
0.1615000E 02	0.9099273E-01
0.1620000E 02	0.8473089E-01
0.1625000E 02	0.3667356E-01
0.1630000E 02	0.1527305E-01
0.1635000E 02	0.1053011E-01
0.1640000E 02	0.9464394E-02
0.1645000E 02	0.8861066E-02
0.1650000E 02	0.9019415E-02
0.1655000E 02	0.9179592E-02
0.1660000E 02	0.9797029E-02
0.1665000E 02	0.1167693E-01
0.1670000E 02	0.1172459E-01
0.1675000E 02	0.1177240E-01
0.1680000E 02	0.9019415E-02
0.1685000E 02	0.7164889E-02
0.1690000E 02	0.4429512E-02
0.1695000E 02	0.3452065E-02
0.1700000E 02	0.3017181E-02
0.1705000E 02	0.2686753E-02
0.1710000E 02	0.2481427E-02
0.1715000E 02	0.1934277E-02
0.1720000E 02	0.1664197E-02

Table 1. (Cont.)

WAVELENGTH (MICRONS)	ABSORPTION COEFFICIENT
0.1725000E 02	0.1419217E-02
0.1730000E 02	0.9729906E-03
0.1735000E 02	0.8900486E-03
0.1740000E 02	0.7514872E-03
0.1745000E 02	0.5090506E-03
0.1750000E 02	0.4193548E-03
0.1755000E 02	0.3335057E-03
0.1760000E 02	0.2671213E-03
0.1765000E 02	0.1688654E-03
0.1770000E 02	0.1195214E-03
0.1775000E 02	0.2011937E-03
0.1780000E 02	0.1846114E-03
0.1785000E 02	0.1580897E-03
0.1790000E 02	0.1391672E-03
0.1795000E 02	0.1035433E-03
0.1800000E 02	0.3084097E-04
0.1805000E 02	0.8320988E-03
0.1810000E 02	0.7514872E-04
0.1815000E 02	0.8133299E-04
0.1820000E 02	0.9059284E-04
0.1825000E 02	0.1186845E-03
0.1830000E 02	0.1481035E-03
0.1835000E 02	0.1364675E-03
0.1840000E 02	0.1134721E-03
0.1845000E 02	0.9923773E-04
0.1850000E 02	0.5147723E-04
0.1855000E 02	0.4245435E-04
0.1860000E 02	0.3219890E-04
0.1865000E 02	0.3174334E-04
0.1870000E 02	0.3129070E-04
0.1875000E 02	0.3084097E-04
0.1880000E 02	0.2972932E-04
0.1885000E 02	0.2972932E-04
0.1890000E 02	0.2885297E-04
0.1895000E 02	0.2798807E-04
0.1900000E 02	0.2546174E-04
0.1905000E 02	0.3238431E-04
0.1910000E 02	0.1996053E-04
0.1915000E 02	0.1830816E-04
0.1920000E 02	0.1653806E-04
0.1925000E 02	0.1500431E-04
0.1930000E 02	0.1434073E-04
0.1935000E 02	0.1320638E-04
0.1940000E 02	0.1273075E-04
0.1945000E 02	0.1164566E-04
0.1950000E 02	0.2097195E-04
0.1955000E 02	0.2027497E-04
0.1960000E 02	0.1951005E-04

However, since CO_2 absorption varies quite radically with wavelength even within small intervals, up to 20 values of $k(\lambda)$ may be input representing twenty smaller intervals within some larger interval and the average of the CO_2 transmission for the smaller intervals is computed to give the CO_2 transmissions for the larger interval.

FLASH uses a modification of the expression developed by Howard et. al (Ref. 8) fitting the Goody model to their water vapor absorption data to calculate the water vapor transmission. The transmission at wavelength λ , $T(\lambda)$, is given by

$$T(\lambda) = \exp \sum_i \frac{-1.97 W_i / W_o(\lambda)}{\left(1 + 6.57 W_i / W_o(\lambda) \frac{P_o}{P_i} \left(\frac{T_i}{T_o}\right)^{1/2}\right)^{1/2}} \quad (3.16)$$

where W_i is the product of the water vapor density times the distance traversed in the i th atmospheric layer, P_i is the pressure in the i th atmospheric layer and $W_o(\lambda)$ is the vapor length necessary to reduce the transmission to 50% at the pressure P_o and temperature T_o . The water vapor transmission for some finite wavelength interval is then computed by averaging the transmission for as many as 20 subintervals as is the CO_2 transmission. The photon weight at each collision is multiplied by the average CO_2 and water vapor transmissions along the path between collisions to account for the water vapor and CO_2 absorption along that path.

Values of $W_o(\lambda)$ for the case where $T_o = 300^\circ\text{K}$ and $P_o = 1$ atmosphere were derived from the water vapor transmission data reported by Wyatt, Skull, and Plass in Ref. 9 for the interval between 1.05μ and 10.0μ in 0.05μ steps. These data are tabulated in Table II with W_o in pr. cm.

3.1.5 Direction Before Collision

For each collision or reflection that occurs during the course of tracing the life history of the particle, the angle γ_n between the particle's direction before collision and a radial through the collision

Table II. Values of W_0 for $T_0=300^\circ\text{K}$ and
 $P_0=1$ Atmosphere in Wavelength Intervals of 0.05μ

WAVELENGTH (MICRONS)	W(ZERO) (PR. CM.)
0.1050000E 01	0.1364798E 04
0.1100000E 01	0.2132600E 02
0.1150000E 01	0.3142313E 01
0.1200000E 01	0.5185620E 02
0.1250000E 01	0.6373659E 03
0.1300000E 01	0.2798141E 01
0.1350000E 01	0.1337385E 00
0.1400000E 01	0.8627985E-01
0.1450000E 01	0.2439766E 00
0.1500000E 01	0.7434676E 01
0.1550000E 01	0.4907541E 01
0.1600000E 01	0.9864713E 01
0.1650000E 01	0.3983914E 02
0.1700000E 01	0.6624195E 02
0.1750000E 01	0.2413114E 02
0.1800000E 01	0.2798141E 00
0.1850000E 01	0.5246718E-01
0.1900000E 01	0.3431032E-01
0.1950000E 01	0.2555008E 00
0.2000000E 01	0.2705231E 01
0.2050000E 01	0.1001521E 02
0.2100000E 01	0.4782930E 01
0.2150000E 01	0.4474844E 01
0.2200000E 01	0.4077195E 01
0.2250000E 01	0.2074389E 02
0.2300000E 01	0.2738205E 02
0.2350000E 01	0.1879797E 02
0.2400000E 01	0.5975355E 01
0.2450000E 01	0.3736459E 01
0.2500000E 01	0.1609214E 00
0.2550000E 01	0.1399988E-01
0.2600000E 01	0.3172497E-02
0.2650000E 01	0.7630715E-02
0.2700000E 01	0.3819842E-02
0.2750000E 01	0.9816007E-02
0.2800000E 01	0.1152515E-01
0.2850000E 01	0.4034963E-01
0.2900000E 01	0.9239579E-01
0.2950000E 01	0.2913515E 00
0.3000000E 01	0.5513478E 00
0.3050000E 01	0.6593292E-01
0.3100000E 01	0.1731397E 00
0.3150000E 01	0.3319456E 00
0.3200000E 01	0.1292448E 00
0.3250000E 01	0.1038071E 01
0.3300000E 01	0.1934875E 00
0.3350000E 01	0.2957798E 01
0.3400000E 01	0.3632298E 01

Table II. (Cont.)

WAVELENGTH (MICRONS)	W(ZERO) (PR. CM.)
0.3450000E 01	0.9817541E 01
0.3500000E 01	0.1669469E 02
0.3550000E 01	0.1270295E 02
0.3600000E 01	0.9106810E 02
0.3650000E 01	0.2954507E 02
0.3700000E 01	0.5853565E 02
0.3750000E 01	0.4948389E 02
0.3800000E 01	0.3179620E 02
0.3850000E 01	0.9986743E 02
0.3900000E 01	0.2383577E 03
0.3950000E 01	0.1393616E 03
0.4000000E 01	0.1584671E 03
0.4050000E 01	0.1331901E 03
0.4100000E 01	0.8944226E 02
0.4150000E 01	0.8053956E 02
0.4200000E 01	0.7593930E 02
0.4250000E 01	0.6521984E 02
0.4300000E 01	0.5914692E 02
0.4350000E 01	0.3211000E 02
0.4400000E 01	0.2031153E 02
0.4450000E 01	0.1619105E 02
0.4500000E 01	0.1904271E 02
0.4550000E 01	0.1322725E 02
0.4600000E 01	0.1072237E 02
0.4650000E 01	0.2213256E 01
0.4700000E 01	0.3235857E 01
0.4750000E 01	0.1953745E 01
0.4800000E 01	0.1669469E 01
0.4850000E 01	0.2403700E 00
0.4900000E 01	0.1778163E 00
0.4950000E 01	0.1360278E 00
0.5000000E 01	0.5058449E 00
0.5050000E 01	0.1505841E 01
0.5100000E 01	0.6739373E-01
0.5150000E 01	0.2555400E-01
0.5200000E 01	0.2206347E-01
0.5250000E 01	0.1985114E 00
0.5300000E 01	0.4056031E-01
0.5350000E 01	0.1451926E-01
0.5400000E 01	0.7752688E-01
0.5450000E 01	0.1547680E-01
0.5500000E 01	0.5653657E-01
0.5550000E 01	0.9754454E-02
0.5600000E 01	0.1904645E-01
0.5650000E 01	0.2651323E-02
0.5700000E 01	0.1337385E-01
0.5750000E 01	0.1454753E-01
0.5800000E 01	0.3125744E-01

Table II. (Cont.)

WAVELENGTH (MICRONS)	W(ZERO) (PR. CM.)
0.5850000E 01	0.6948524E-02
0.5900000E 01	0.3646725E-02
0.5950000E 01	0.5810275E-02
0.6000000E 01	0.3943010E-02
0.6050000E 01	0.1325711E-02
0.6100000E 01	0.5292978E-02
0.6150000E 01	0.9450189E-02
0.6200000E 01	0.7155233E-02
0.6250000E 01	0.9605598E-01
0.6300000E 01	0.2352232E 00
0.6350000E 01	0.9330084E-02
0.6400000E 01	0.4081000E-02
0.6450000E 01	0.8600829E-02
0.6500000E 01	0.1748569E-02
0.6550000E 01	0.3869750E-02
0.6600000E 01	0.4807399E-02
0.6650000E 01	0.4219288E-02
0.6700000E 01	0.4219288E-02
0.6750000E 01	0.1830322E-01
0.6800000E 01	0.1171621E-01
0.6850000E 01	0.5565326E-02
0.6900000E 01	0.1714174E-01
0.6950000E 01	0.8399252E-01
0.7000000E 01	0.2166749E-01
0.7050000E 01	0.8337099E-02
0.7100000E 01	0.5856414E-01
0.7150000E 01	0.1095852E-01
0.7200000E 01	0.2383481E-01
0.7250000E 01	0.4429046E-01
0.7300000E 01	0.8399252E-01
0.7350000E 01	0.4614762E-01
0.7400000E 01	0.3338937E 00
0.7450000E 01	0.9332983E-01
0.7500000E 01	0.3443685E 00
0.7550000E 01	0.5041576E 00
0.7600000E 01	0.9864714E-01
0.7650000E 01	0.5108655E 00
0.7700000E 01	0.1063807E 01
0.7750000E 01	0.4835613E 00
0.7800000E 01	0.5385868E 00
0.7850000E 01	0.1191740E 01
0.7900000E 01	0.3603538E 00
0.7950000E 01	0.1299852E 01
0.8000001E 01	0.3063640E 01
0.8050001E 01	0.1731396E 01
0.8100000E 01	0.8046073E 01
0.8150001E 01	0.2214522E 01
0.8200000E 01	0.2898895E 01

Table II: (Cont.)

WAVELENGTH (MICRONS)	W(ZERO) (PR. CM.)
0.8250001E 01	0.6596128E 01
0.8300001E 01	0.9098724E 01
0.8350000E 01	0.7919357E 01
0.8400001E 01	0.9049774E 01
0.8450000E 01	0.5945405E 01
0.8500001E 01	0.5321359E 01
0.8550001E 01	0.5653656E 01
0.8600000E 01	0.1037592E 02
0.8650001E 01	0.1617764E 02
0.8700000E 01	0.1017689E 02
0.8750001E 01	0.2454726E 02
0.8800001E 01	0.1221207E 02
0.8850000E 01	0.3784315E 02
0.8900001E 01	0.1942178E 02
0.8950000E 01	0.2788205E 02
0.9000001E 01	0.1963503E 02
0.9050001E 01	0.2097445E 02
0.9100000E 01	0.2306256E 02
0.9150001E 01	0.3784315E 02
0.9200000E 01	0.4281512E 02
0.9250001E 01	0.1247191E 03
0.9300001E 01	0.5694725E 02
0.9350000E 01	0.3983914E 02
0.9400001E 01	0.2958184E 02
0.9450000E 01	0.5297365E 02
0.9500001E 01	0.9132691E 00
0.9550001E 01	0.6471716E 02
0.9600000E 01	0.7469561E 02
0.9650001E 01	0.7918428E 02
0.9700000E 01	0.7171299E 02
0.9750001E 01	0.9273889E 02
0.9800001E 01	0.7229554E 02
0.9850000E 01	0.4672447E 02
0.9900001E 01	0.5612033E 02
0.9950000E 01	0.1135203E 03
0.1000000E 02	0.9896778E 02

point is computed (Fig. 6). The angle α_n between the spherical projection of the particle's direction before collision and the spherical projection of the line joining the collision and receiver points is also calculated. In addition to the direction angles with respect to the radial through the collision or reflection point, the displacement from the receiver position is computed in terms of the two position angles θ_n and ϕ_n . Figure 6 shows the displacement angles θ_n and ϕ_n where θ_n is the angle between the radial through the receiver and the radial through the nth collision or reflection point and ϕ_n is the angle at the receiver between the zero azimuthal and the spherical projection of a line joining the receiver and the nth collision point. The cosine of the displacement angle θ_n is given by the expression

$$\cos\theta_n = \cos\theta_{n-1}\cos D\theta - \sin\theta_{n-1}\sin D\theta\cos\alpha_{n-1} \quad (3.17)$$

where θ_{n-1} is the displacement angle at the previous collision or reflection, $D\theta$ is the angle at the earth's center subtending the path length between the nth and the (n-1)th collisions, and α_{n-1} is the angle between the spherical projection of the pathlength between the (n-1)th collision and the spherical projection of the line joining the (n-1)th collision position and the receiver position. α_n is the angle between the spherical projection of the pathlength between the (n-1) and nth collisions and the spherical projection of the line joining the nth collision with the receiver position. $\cos\alpha_n$ is given by the expression

$$\cos\alpha_n = (\cos\theta_{n-1} - \cos D\theta \cos\theta_n) / (\sin\theta_{n-1} \sin D\theta), \quad (3.18)$$

where the angles θ_{n-1} , θ_n , and $D\theta$ are defined as above.

3.1.6 Effects of Changes in Index of Refraction

The direction that a photon travels between collisions is modified as it passes from one atmospheric layer to the next by the change in the index of refraction. If γ is the angle between the photon's direction and the radial through the point of intersection with an atmospheric layer boundary, the angle is modified by the expression $\gamma' = \sin^{-1}((n/n') \sin\gamma)$ where γ' is the angle the photon direction makes with the radial through the intersection point after crossing the boundary and n and n'

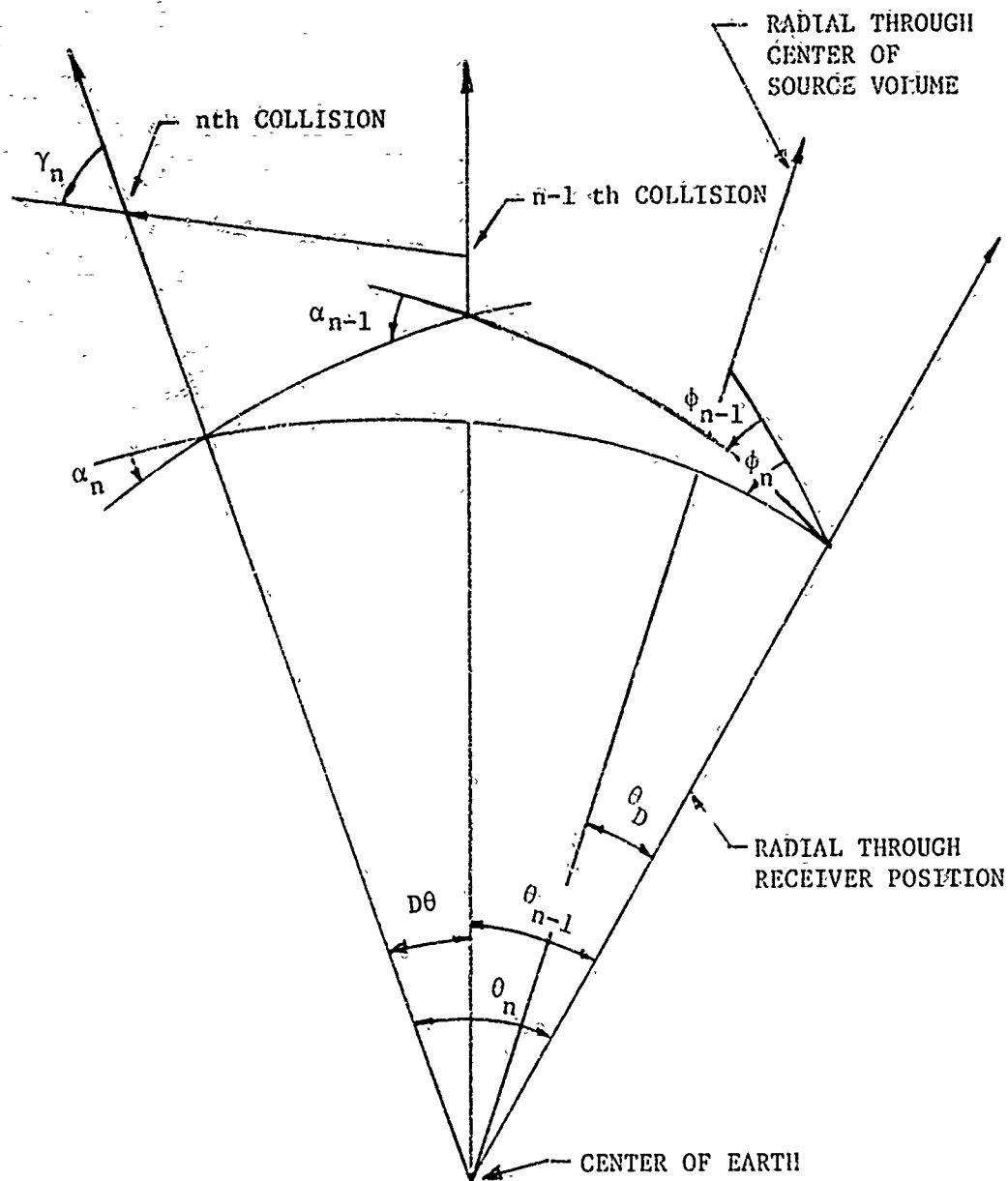


Fig. 6. Displacement Angles

are the indices of refraction for the original and new atmospheric layers. In the case that $\sin y$ is approximately 1.0 and $n/n' > 1$ so that $\sin y'$ becomes greater than 1.0, $\sin y'$ is set equal to 1.0 and the photon leaves the spherical surface in a tangential plane.

3.1.7 Selection of the Scattering Event

Each time a collision or reflection position is determined the FLASH program locates the atmospheric layer containing the collision center and interpolates between the scattering-to-pseudo extinction coefficient ratios and the Rayleigh-to-scattering coefficient ratios for the bounds of the layer to find the scattering-to-pseudo extinction coefficient ratio and Rayleigh-to-scattering coefficient ratio for the collision altitude. The photon weight is multiplied by the scattering-to-pseudo extinction coefficient ratio to account for the fact that no ozone or aerosol absorption of the photon is allowed. The type of scattering event is determined to be either a Rayleigh or an aerosol scattering event by comparing a random number with the ratio of the Rayleigh-to-scattering coefficient. If the random number is less than the ratio, the scattering event is assumed to be a Rayleigh event, otherwise, it is assumed to be an aerosol scattering event.

3.1.8 Selection of Scattering Angles

The scattering angle, which determines the photon's direction after a collision, is selected from the scattering distribution for unpolarized light. The polarization parameters after scattering are adjusted to correct for the fact that the scattering angle was selected from the distribution for unpolarized light instead of the distribution for polarized light.

The type of scattering event is chosen to be a molecular or aerosol event with the frequency at which each event is selected corresponding to the ratio of the event-to-the total scattering coefficient at the collision altitude. For either process, the orientation of the scattering plane is selected from a uniform distribution between 0 and 2π . The polarization parameters characterizing the radiation before

scattering are rotated into the scattering plane using the expressions:

$$\begin{aligned}
 I'_{||} &= I_{||} \cos^2 \phi_r + I_{\perp} \sin^2 \phi_r + 0.5U \sin 2\phi_r \\
 I'_{\perp} &= I_{||} \sin^2 \phi_r + I_{\perp} \cos^2 \phi_r - 0.5U \sin 2\phi_r \\
 U' &= I_{\perp} \sin 2\phi_r = I_{||} \sin 2\phi_r + U \cos 2\phi_r \\
 V' &= V
 \end{aligned} \tag{3.19}$$

where $I'_{||}$, $I'_{\perp}$, U' , V' are the polarization parameters in the scattering plane, $I_{||}$, $I_{\perp}$, U , and V are the polarization parameters in the reference plane before rotation and ϕ_r is the angle between the reference plane and the scattering plane.

If the Rayleigh process is chosen for the scattering event, the scattering angle is chosen from the density function,

$$P(\psi) = 3/8 (1 + \cos^2 \psi) \sin \psi d\psi \tag{3.20}$$

and the polarization parameters are adjusted with the following equations

$$\begin{aligned}
 I''_{||} &= 2 \cos^2 \psi I'_{||} / (I'_{||} + I'_{\perp}) \\
 I''_{\perp} &= 2 I'_{\perp} / (I'_{||} + I'_{\perp}) \\
 U'' &= 2 U' \cos \psi / (I'_{||} + I'_{\perp}) \\
 V'' &= 2 V' \cos \psi / (I'_{||} + I'_{\perp})
 \end{aligned} \tag{3.21}$$

where $I''_{||}$, $I''_{\perp}$, U'' , and V'' are the polarization parameters after scattering through the angle ψ and $I'_{||}$, $I'_{\perp}$, U' and V' are the polarization parameters in the scattering plane before scattering.

If the Mie process is chosen for the scattering event, the scattering angle is chosen from the density function,

$$\frac{2\pi(R_{\perp} i_1(\psi) + R_{||} i_2(\psi))}{\sigma_s} \sin \psi d\psi \tag{3.22}$$

where $R_{\perp} = \frac{I'_{\perp}}{I'_{\perp} + I'_{||}}$ and $R_{||} = \frac{I'_{||}}{I'_{\perp} + I'_{||}}$ and $i_1(\psi)$ and $i_2(\psi)$ are the elements of the aerosol phase matrix relating to the perpendicular and parallel components of the scattered intensity as computed with Mie theory. The aerosol phase matrix is then used to adjust the polarization parameters

through the following equations to yield the value of the polarization parameters after scattering:

$$\begin{aligned}
 I''_{||} &= 2I'_H i_2(\psi) / (i_1(\psi) + i_2(\psi)) \\
 I''_{\perp} &= 2I'_V i_1(\psi) / (i_1(\psi) + i_2(\psi)) \\
 U'' &= 2(U' i_3(\psi) - V' i_4(\psi)) / (i_1(\psi) + i_2(\psi)) \\
 V'' &= 2(U' i_4(\psi) + V' i_3(\psi)) / (i_1(\psi) + i_2(\psi)).
 \end{aligned}
 \tag{3.23}$$

The parameters $I''_{||}$, $I''_{\perp}$, U'' and V'' are defined in the plane of scatter. The angle ϕ'_r between the scattering plane and the new reference plane containing the direction of propagation after scatter and the radial vector is calculated with the trigonometric relationships for spherical triangles. The four parameters are then rotated through this angle using the expressions for rotating the parameters given in Equation (3.19).

3.1.8.1 Biased Sampling of Scattering Angles. A method was incorporated into the point source version of the FLASH program to improve the application of the program to problems having receivers at large horizontal distances from the source. For this method the polar and azimuthal angles of the photon's direction after scattering are selected from input distributions peaked toward the source location rather than from the Rayleigh or aerosol phase function.

The biased distribution used for the selection of the absolute value of the cosine of the photons zenith direction after collision is given by the expression

$$\frac{a \cdot e^{-a |\cos \gamma|}}{1.0 - e^{-a}}, \tag{3.24}$$

where $|\cos \gamma|$ is the absolute value of the cosine of the zenith direction after scattering (see Fig. 7) and a is an input parameter which controls the extent of the biasing. The absolute value of the zenith angle is sampled from the density function, then the sign of the cosine is determined at random with a 50% probability of being either positive or negative. When the zenith direction is sampled from the biased distribution,

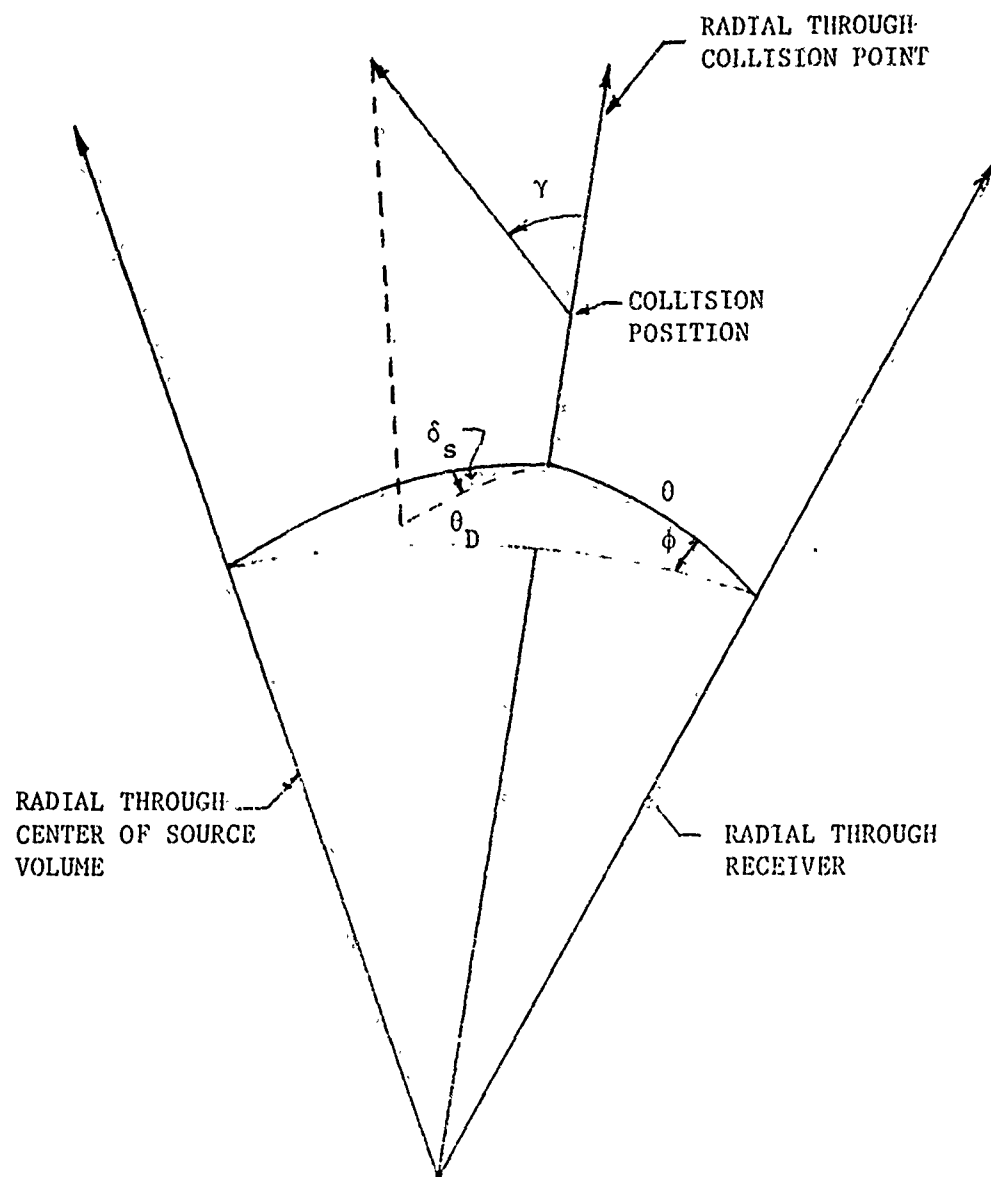


Fig. 7. Geometry for Biased Selection of Photon Direction After Collision

the photon weight is adjusted by the factor

$$\frac{2(1.0 - e^{-a})}{a \cdot e^{-a} |\cos \gamma'|}, \quad (3.25)$$

where $|\cos \gamma'|$ is the absolute value of the cosine of the angle sampled from the biased distribution.

The biased distribution for the selection of the absolute value of the azimuthal direction after collision is given by the expression

$$\frac{Z \cdot e^{-Z|\delta_s|}}{1.0 - e^{-\pi Z}}, \quad (3.26)$$

where $|\delta_s|$ is the absolute value of the angle between the spherical projection of the photons direction after collision and the spherical projection of the direction to the center of the source volume from the collision position, and Z is an input parameter which controls the extent of biasing in the direction toward the source. The weight adjustment factor for azimuthal biasing is

$$\frac{2(1.0 - e^{-\pi Z})}{Z e^{-Z|\delta'_s|}} \quad (3.27)$$

where $|\delta'_s|$ is the absolute value of the azimuthal angle sampled from the biased distribution (Equation (3.26)).

The sign of the azimuthal angle sampled is chosen to be either positive or negative, each with a 50% probability.

Variation of the input parameters a and Z controls the amount of biasing in the horizontal and azimuthal directions, respectively. If a zero or negative value is input for a , there is no biasing of the zenith direction after collision, and likewise, if a zero or negative value is input for Z , there is no biasing of the azimuthal direction after collision.

3.1.9 Ground and Cloud Reflection

If the photon is reflected from the ground or a cloud surface, the FLASH programs select the direction after reflection from a reflection

distribution rather than from the Rayleigh or aerosol phase matrix.

Once it has been determined that the photon intersected a reflection surface, the indices of refraction for the two materials on either side of the reflection boundary are checked to determine whether specular reflection is possible. If both indices are the same, only diffuse reflection is possible; but if they are different, the reflection coefficients for the parallel and perpendicular components of the specular reflectance are computed with the expressions:

$$\begin{aligned} r_{||} &= (N_I \cos \gamma_R + N_R \cos \gamma_I) / (N_I \cos \gamma_R - N_R \cos \gamma_I) \\ r_{\perp} &= (-N_I \cos \gamma_I + N_R \cos \gamma_R) / (-N_I \cos \gamma_I + N_R \cos \gamma_R), \end{aligned} \quad (3.28)$$

where N_I and N_R are the indices of refraction for the materials containing the incident and refracted rays, respectively, and γ_I and γ_R are the angle of incidence and the angle of refraction, respectively. The probability of specular reflectance is given by

$$PR = I_{||} r_{||}^2 + I_{\perp} r_{\perp}^2. \quad (3.29)$$

A random number is generated and if the random number is less than PR, the reflection is assumed to be specular. If the random number is greater than PR, reflection is assumed to be diffuse.

If the reflection is determined to be specular, the angle of incidence is taken to be the angle of reflection and the polarization parameters for the reflected photon is given by the expressions

$$\begin{aligned} I'_{||} &= r_{||}^2 I_{||} / PR \\ I'_{\perp} &= r_{\perp}^2 I_{\perp} / PR \\ U' &= -r_{\perp} r_{||} U / PR \\ V' &= -r_{\perp} r_{||} V / PR \end{aligned} \quad (3.30)$$

If the reflection event is determined to be diffuse, then the reflection angle can be sampled from an isotropic or cosine distribution or from an arbitrary distribution defined by a set of tabulated input data. Diffuse reflected light is assumed to be unpolarized so the polarization parameters for the diffuse reflected photons are given by

$$I'_{\parallel} = A(I_{\perp} + I_{\parallel})/2.0$$

$$I'_{\perp} = A(I_{\perp} + I_{\parallel})/2.0$$

$$U' = 0.0$$

(3.31)

$$V' = 0.0,$$

where $I_{\parallel}$ and $I_{\perp}$ are the parallel and perpendicular components of the intensity associated with the incident photon, and A is the diffuse albedo for the reflection surface.

3.2 Evaluation of the Estimating Function for the Scattered Intensity

The integral equation that represents the multiple scattered intensity at a receiver position can be divided into two parts. The first part is a probability density Function that describes the backward path of a photon from the receiver position to the nth collision position. The other part is an estimating function. This function, when evaluated with the parameters describing the type of collision and the position of the nth collision, gives an estimate of the scattered intensity at the receiver position that results from a photon that has undergone n collision events.

The form of the estimating function depends on the type of source being considered and on whether the collision occurred in the atmosphere or at a reflection surface. The following sections describe how the estimating function is evaluated for a point source, a plane parallel source, and a volume or spherical surface source.

3.2.1 Estimating Function for a Point Source

Once the position and type of an atmospheric collision event and the direction angles from the previous position to the collision position along the backward path has been determined, then the direction that the photon must take to be headed toward the source is calculated. The cosine of the angle between the radials through the source and collision positions is computed using the equation

$$\cos\beta = \cos\theta\cos\theta_D + \sin\theta\sin\theta_D\cos\phi \quad (3.32)$$

where θ and ϕ are the polar and azimuthal displacement of the collision from the receiver position located on the $\phi = 0^\circ$ azimuthal plane at an angle of θ_D from the radial through the source point (Fig. 8).

The distance from the collision to the source is given by

$$D = \sqrt{H_s^2 + H_c^2 - 2H_sH_c\cos\beta} \quad (3.33)$$

where H_s is the source altitude, and H_c is the collision altitude. The cosine and sine of the zenith direction of the photon's path from the

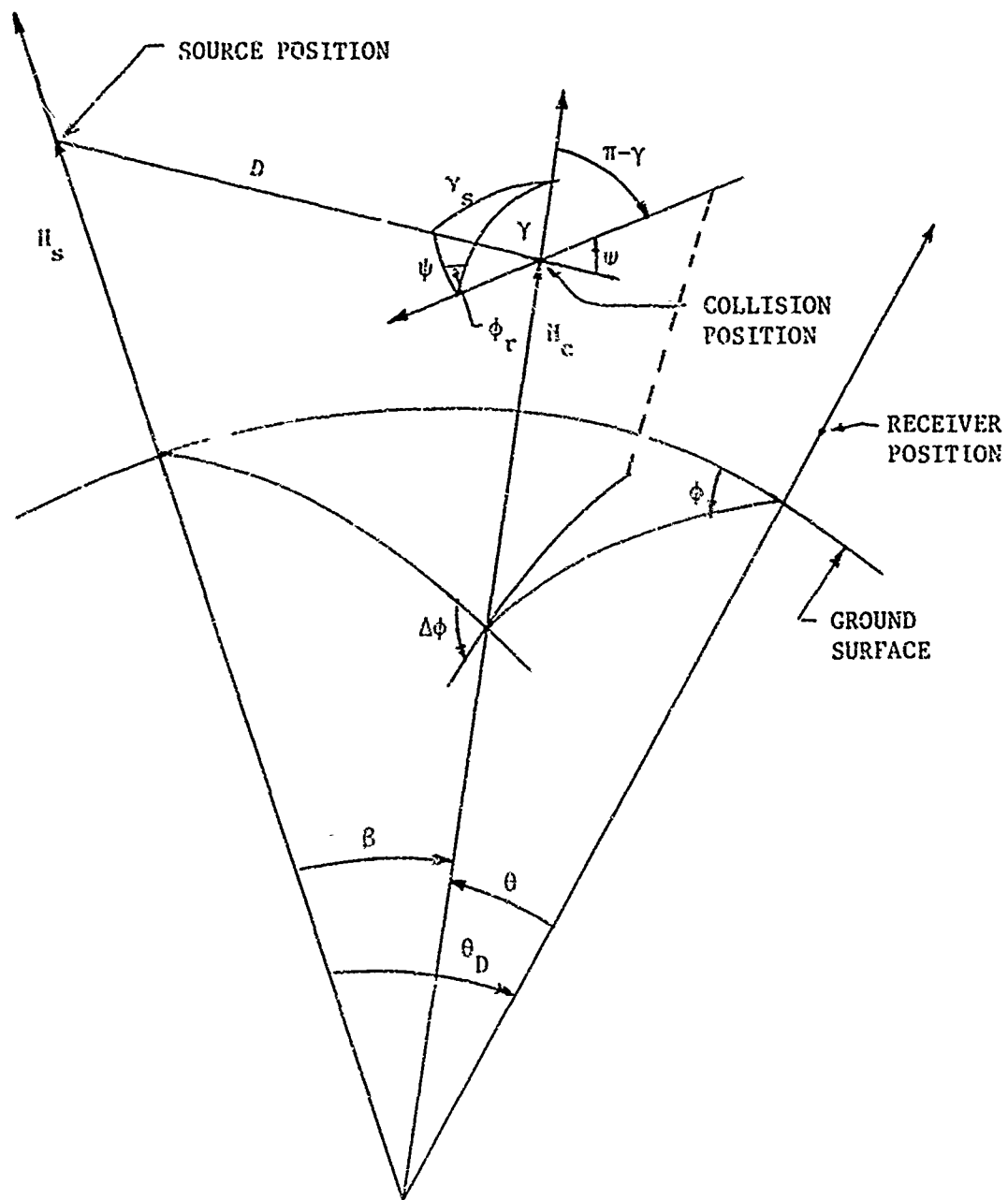


Fig. 8. Geometry for Point Source Estimating Function

collision to the source are given by the expressions

$$\cos \gamma_s = (H_s \cos \beta - H_c) / D \quad (3.34)$$

$$\sin \gamma_s = H_s \sin \beta / D. \quad (3.35)$$

Next the scattering angle ψ between the direction of propagation before collision along the photons backward path and the direction toward the source position is computed (see Fig. 8). The cosine of the scattering angle is given by the expression

$$\cos \psi = \cos \gamma_s \cos \gamma + \sin \gamma_s \sin \gamma \cos \Delta \phi \quad (3.36)$$

where γ is the zenith angle of the photon's direction before collision along the backward path and $\Delta \phi$ is the angle between the spherical projections of the photon's direction before collision and the line joining the collision and source position.

In order to determine the magnitude of the four polarization parameters in the direction toward the source position the polarization parameters are first rotated into the plane of scatter containing the direction of propagation before collision along the backward path and the direction from the collision position to the source point. The sine and cosine of the rotation angle are given by the equations

$$\begin{aligned} \sin \phi_r &= \sin \gamma_s \sin \Delta \phi / \sin \psi \\ \cos \phi_r &= (\cos \gamma_s - \cos \psi \cos \gamma) / \sin \psi \sin \gamma. \end{aligned} \quad (3.37)$$

The polarization parameters I_L , I_H , U , and V characterizing the radiation before scattering along the backward path are rotated into the plane of scatter with the expressions

$$\begin{aligned} I'_L &= I_H \sin^2 \phi_r + I_L \cos^2 \phi_r - 0.5 U \sin 2 \phi_r \\ I'_H &= I_H \cos^2 \phi_r + I_L \sin^2 \phi_r + 0.5 U \sin 2 \phi_r \\ U' &= (I_L - I_H) \sin 2 \phi_r + U \cos 2 \phi_r \\ V' &= V. \end{aligned} \quad (3.38)$$

If the collision occurred in the atmosphere and the Rayleigh process was chosen for the scattering event, then the polarization parameters in the direction toward the source are given by the

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expressions

$$\begin{aligned}
 I_1'' &= \frac{3}{16\pi} I_1' \\
 I_2'' &= \frac{3}{16\pi} \cos^2 \psi I_1' \\
 U'' &= \frac{3}{16\pi} \cos \psi U' \\
 V'' &= \frac{3}{16\pi} \cos \psi V'.
 \end{aligned}
 \tag{3.39}$$

If the scattering process was selected as an aerosol event, then the polarization parameters in the direction toward the source point are given by

$$\begin{aligned}
 I_1'' &= i_1(\psi) I_1' \\
 I_2'' &= i_2(\psi) I_1' \\
 U'' &= i_3(\psi) U' - i_4(\psi) V' \\
 V'' &= i_4(\psi) U' + i_3(\psi) V'.
 \end{aligned}
 \tag{3.40}$$

The optical distance along the photon path from the source to the collision position is computed and if the wavelength is one for which water vapor or CO_2 absorption is significant, then the CO_2 and water vapor transmissions, T_{CO_2} and T_W along the path are computed.

When the collision position is in the atmosphere, the estimate of the scattered intensity at the receiver position in the view angle used to start the backward photon path is given by the expression

$$W \frac{\Sigma_s}{\Sigma_{\text{ext}}} T_{\text{CO}_2} T_W (I_1'' + I_2'') e^{-\rho/D^2}
 \tag{3.41}$$

where W is the photon weight before collision along the backward path, $\Sigma_s / \Sigma_{\text{ext}}$ is the ratio of the scattering to pseudo extinction coefficient at the collision altitude, T_{CO_2} and T_W are the CO_2 and water vapor transmission for the collision position to the source position, ρ is the optical distance from the collision to the source, and D is the geometric distance from the collision to the source.

For each collision position at which a reflection occurs along the photon's backward path, the geometric and optical distances along the path from the reflection position to the source position are computed with the formulae previously given in this section for the case where the collision occurs within the atmosphere." The water vapor and CO_2 transmission are also computed along the path from the reflection position to the source position if these absorption processes are significant. The values of the polarization parameters in the direction toward the receiver depend on whether the reflection is determined to be specular or diffuse.

If specular reflection is being considered and if $\gamma_s = \pi - \gamma$ (see Fig. 8), then an estimate is made of the scattered intensity at the receiver that results from specular reflection. The polarization parameters in the direction toward the source position after specular reflection are given by

$$\begin{aligned} I''_{us} &= I_u r_u^2 \\ I''_{ls} &= I_l r_l^2 \\ U''_s &= -U r_u r_l \\ V''_s &= -V r_u r_l \end{aligned} \quad (3.42)$$

where r_u and r_l are defined by Equation (3.28). If specular reflection is not possible the polarization parameter I''_{us} , I''_{ls} , U''_s and V''_s are set to zero.

The polarization parameters after a diffuse reflection along the backward path are given by the following expressions

$$\begin{aligned} I''_{ud} &= r(\gamma_s)(I_l + I_u)/2.0 \\ I''_{ld} &= r(\gamma_s)(I_l + I_u)/2.0 \\ U''_d &= 0.0 \\ V''_d &= 0.0 \end{aligned} \quad (3.43)$$

where $r(\gamma_s)$ is the normalized reflection distribution for diffuse reflection evaluated at γ_s , the angle between the radial through the reflection position and the line joining the reflection and source points. I_u , I_l , U , and V are the polarization parameters for the intensity before reflection along the photon's backward path.

The estimate of the scattered intensity at the receiver position in the view angle used to start the backward photon path for a diffuse reflection event is given by

$$W T_{\text{CO}_2} T_W D^{-2} e^{-\rho} [A(I''_{\text{id}} + I''_{\text{ud}}) + (I''_{\text{is}} + I''_{\text{us}})] \quad (3.44)$$

where A is the diffuse albedo of the reflection surface. In Equation (3.44), ρ is the optical distance from the reflection position to the source position and D is the geometric distance, W is the photon weight before reflection, and T_{CO_2} and T_W are the CO_2 and water vapor transmission along the path between the reflection point and the source position.

In the point source version of FLASH, the estimates of the scattered intensity in a given receiver solid angle of view is recorded as a function of the time after arrival of the uncollided intensity or the minimum time required for light to travel from the point source to the receiver position when the receiver is located over-the-horizon from the source position. This minimum time is taken to be the zero retarded time and the time interval in which an estimate of the scattered intensity is recorded is the difference between the actual time required for the photon to travel the random path from the receiver to the source minus the minimum time.

The estimating functions defined by Equations (3.41) and (3.44) are for a point isotropic source emitting 1.0 photon per steradian uniformly in all directions. The total source strength is then 4π photons.

3.2.2 Estimating Functions for a Plane Source

Once the position of a collision event in the atmosphere relative to a receiver position has been determined and the direction angles of the photon path before collision have been calculated, then the direction that the photon must take in order to intersect the source plane at an angle of 90 degrees must be determined. Due to the bending of the photon's

path by the change in the atmospheric index of refraction with altitude, an iterative process was developed to determine the initial direction to start the photon path so that it will intersect the source plane at a right angle. The iterative process is described below.

The position of the collision relative to the earth's radial normal to the source plane (see Fig. 9) is computed with the expression:

$$\cos \beta = \cos \theta \cos \theta_D + \sin \theta \sin \theta_D \cos \phi \quad (3.45)$$

where β is the angle between the radial through the collision point and the radial normal to the source plane, θ and ϕ are the polar and azimuthal displacement angles from the receiver located at the polar position θ_D on the zero azimuthal plane. If it were not for the bending of the photon path, a light ray emitted normal to the source plane so that it passes through the collision center would intersect the radial through the collision center at the angle β (Fig. 9). The iterative process used to determine the angle of intersection between the radial and the curved path from the source plane to the collision position first starts the photon path from the collision position toward the source plane in the direction defined by the angle β . The angle of intersection of the curved path with the source plane is then determined. The difference between the angle of intersection and 90 degrees, ϵ , is computed and this difference is tested to determine whether the angle of intersection is sufficiently close to 90 degrees to be considered an intersection at a right angle. If ϵ is too large it is subtracted from the angle β and a photon path from the collision point is started in the direction of $\beta' = \beta - \epsilon$. The angle of intersection of the second photon path and the source plane is computed and tested as discussed above. This process is repeated until the angle between the intersection of the photon's path and the source plane is within an acceptable interval about 90 degrees.

After computing the photon's initial direction along a curved path which is incident normal to the source plane, the cosine of the scattering angle between this path and the photon's direction before

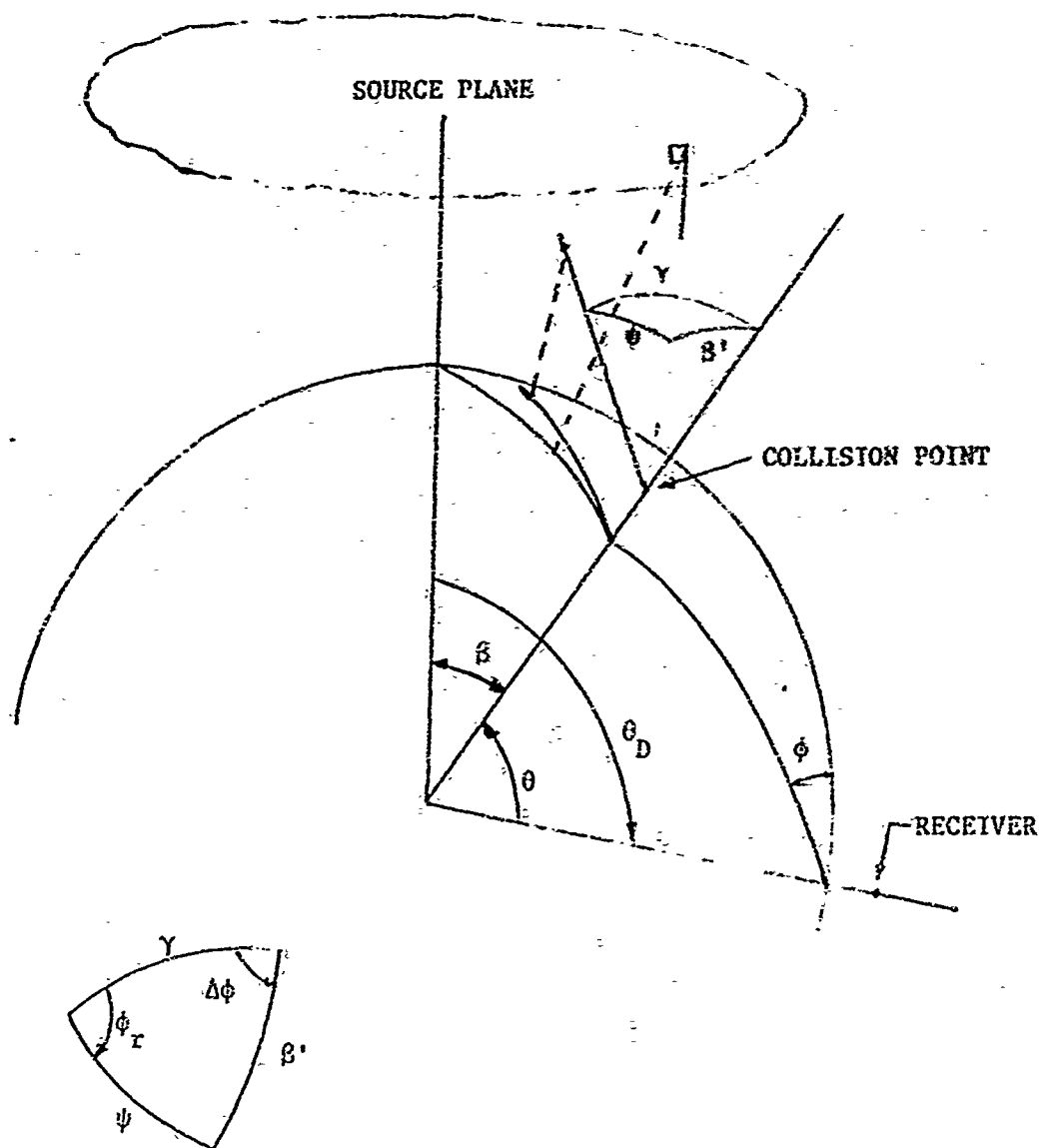


Fig. 9. Geometry for Calculation of Scattering Angle between Direction to Source Plane and Direction before Collision

collision along its backward path is computed with the equation

$$\cos\phi = \cos\beta' \cos\gamma + \sin\beta' \sin\gamma \cos\Delta\phi \quad (3.46)$$

where β' is the angle between the curved path to the source plane and a radial through the collision point, γ is the zenith angle of the photon's direction before collision along its backward path, and $\Delta\phi$ is the angle between the spherical projections of the two directions.

The sine and cosine of the rotation angle ϕ_r between the plane containing the radial through the collision point and the direction before collision and the plane containing the direction to the source plane, as defined by the angle β' , and the direction before collision along the backward path are given by Equation (3.37) with the angle γ_s replaced by the angle β' . The polarization parameters characterizing the radiation before collision along the backward path are rotated into the plane of scatter with use of Equation (3.38). They then are modified by use of Equation (3.39) or Equation (3.40) depending on whether the collision event was selected to be a Rayleigh or aerosol scattering event, respectively.

An estimate of the scattered light intensity at a receiver position when the collision occurred in the atmosphere is given by the expression:

$$W \frac{\Sigma_s}{\Sigma_{\text{ext}}} T_{\text{CO}_2} T_W (I_1'' + I_2'') e^{-\rho} \quad (3.47)$$

where W is the photon weight before scattering, $\Sigma_s/\Sigma_{\text{ext}}$ is the ratio of the scattering to the pseudo extinction coefficient at the collision altitude, T_{CO_2} and T_W are the carbon dioxide and water vapor transmission from the collision to the source plane, I_1'' and I_2'' are the perpendicular and parallel components of the polarization parameters after scattering in the direction of the source plane, and ρ is the optical distance along the curved path from the collision to the source plane.

When a reflection occurs, the angle β' between the curved path from the reflection point and the source plane and a radial through the reflection point is computed. The optical distance ρ from the reflection point to the source plane in the direction defined by β' is then computed. The polarization parameters in the direction β' are then computed in a manner described in the previous section for specular and

volume from the collision position, $I_{\perp}'$ and $I_{\parallel}'$ are the perpendicular and parallel components of the polarization parameters in the direction selected through the source volume and A is the diffuse albedo for the reflection surface. The parameter $DI(t)$ is the time dependent direct intensity at the collision position from sources laying along the direction selected through the source volume. The methods used to obtain the time dependent parameter $DI(t)$ depends on whether the source is a spherical volume source, a lense shaped volume source, or a spherical surface source. The time dependence of the estimate of the scattered intensity as given by Equations (3.49) and (3.50) is the same as the time dependence of the direct intensity ($DI(t)$) except that the scattered intensity is delayed by the time, t_c , required to traverse the random path from the receiver position to the collision position.

The methods used in FLASH to compute the solid angle Ω , to sample a direction from the collision position through the source in the solid angle Ω , and to evaluate the direct intensity, $DI(t)$, at the collision position are outlined in the following two sections.

Once the limits of the solid angle, Ω , subtended by the volume source is determined and a random direction is sampled from within the solid angle, then the polar and azimuthal scattering angles, ψ and $\Delta\phi$, through which the photon must be scattered or the polar reflection angle through which the photon is reflected from the reflection surface can be calculated.

For a collision in the atmosphere, the polarization parameters are first rotated, using Equation (3.38), through the angle ϕ_r , which is given by Equation (3.37). The polarization parameters $I_{\perp}'$ and $I_{\parallel}'$ in the direction selected through the source volume after the collision are given either by Equation (3.39) if the scattering process was selected to be a Rayleigh scattering event, by Equation (3.40) if the scattering process was selected to be an aerosol scattering event, by Equation (3.42) if the scattering process was selected to be a diffuse reflection, or by Equation (3.43) if specular reflection is possible.

3.2.3.1 Spherical Surface and Volume Sources. A spherical surface or volume source is assumed to be located with its center on the polar axis and to have maximum radius of $SR_{\max}$ (see Fig. 11). The cosine of the angle between the radials through the center of the source and the collision position is computed with the expression

$$\cos\beta = \cos\theta_c \cos\theta_D + \sin\theta_c \sin\theta_D \cos\phi \quad (3.51)$$

where θ and ϕ are the polar and azimuthal displacement of the collision from the receiver position located on the 0.0 azimuthal plane at an angle of θ_D from the radial through the center of the source volume.

The geometric distance from the collision to the center of the source is given by

$$D = \sqrt{H_s^2 + H_c^2 - 2H_s H_c \cos\beta} \quad (3.52)$$

The distance D is compared with the maximum radius of the spherical source to determine whether the collision is within the source bounds or outside of the source bounds. If the collision is within the spherical source, then $\Omega = 4\pi$ since the source subtends 4π steradians at the collision position. If the collision is outside the source bounds, the solid angle subtended by the source is computed with the expression

$$\Omega = 2\pi(1.0 - \cos\delta_{\max}) \quad (3.53)$$

where

$$\delta_{\max} = \tan^{-1}(SR_{\max} / \sqrt{D^2 - SR_{\max}^2}). \quad (3.54)$$

In order to sample a direction within the solid angle subtended by the spherical source, the cosine of an angle, δ , is sampled from a uniform distribution of cosines between 1.0 and $\cos\delta_{\max}$, and an azimuthal angle ϕ' is sampled from a uniform distribution between 0 and 2π . The angle δ is the angle between the direction sampled within the solid angle Ω subtended by the spherical source and the line joining the collision to the source center and ϕ' is the angle between a vertical plane containing the line joining the collision and source center and the plane containing the sampled direction and the line joining the collision to the source center (Fig. 11). The direction angles γ_s, α_s of the sampled

volume from the collision position, $I'_\perp$ and $I''_\parallel$ are the perpendicular and parallel components of the polarization parameters in the direction selected through the source volume and A is the diffuse albedo for the reflection surface. The parameter $DI(t)$ is the time dependent direct intensity at the collision position from sources laying along the direction selected through the source volume. The methods used to obtain the time dependent parameter $DI(t)$ depends on whether the source is a spherical volume source, a lense shaped volume source, or a spherical surface source. The time dependence of the estimate of the scattered intensity as given by Equations (3.49) and (3.50) is the same as the time dependence of the direct intensity ($DI(t)$) except that the scattered intensity is delayed by the time, t_c , required to traverse the random path from the receiver position to the collision position.

The methods used in FLASH to compute the solid angle Ω , to sample a direction from the collision position through the source in the solid angle Ω , and to evaluate the direct intensity, $DI(t)$, at the collision position are outlined in the following two sections.

Once the limits of the solid angle, Ω , subtended by the volume source is determined and a random direction is sampled from within the solid angle, then the polar and azimuthal scattering angles, ψ and $\Delta\phi$, through which the photon must be scattered or the polar reflection angle through which the photon is reflected from the reflection surface can be calculated.

For a collision in the atmosphere, the polarization parameters are first rotated, using Equation (3.38), through the angle ϕ_r , which is given by Equation (3.37). The polarization parameters $I'_\perp$ and $I''_\parallel$ in the direction selected through the source volume after the collision are given either by Equation (3.39) if the scattering process was selected to be a Rayleigh scattering event, by Equation (3.40) if the scattering process was selected to be an aerosol scattering event, by Equation (3.42) if the scattering process was selected to be a diffuse reflection, or by Equation (3.43) if specular reflection is possible.

3.2.3.1 Spherical Surface and Volume Sources. A spherical surface or volume source is assumed to be located with its center on the polar axis and to have maximum radius of $SR_{\max}$ (see Fig. 11). The cosine of the angle between the radials through the center of the source and the collision position is computed with the expression

$$\cos \delta = \cos \theta \cos \theta_D + \sin \theta \sin \theta_D \cos \phi \quad (3.51)$$

where θ and ϕ are the polar and azimuthal displacement of the collision from the receiver position located on the 0.0 azimuthal plane at an angle of θ_D from the radial through the center of the source volume.

The geometric distance from the collision to the center of the source is given by

$$D = \sqrt{H_s^2 + H_c^2 - 2H_s H_c \cos \delta} \quad (3.52)$$

The distance D is compared with the maximum radius of the spherical source to determine whether the collision is within the source bounds or outside of the source bounds. If the collision is within the spherical source, then $\Omega = 4\pi$ since the source subtends 4π steradians at the collision position. If the collision is outside the source bounds, the solid angle subtended by the source is computed with the expression

$$\Omega = 2\pi(1.0 - \cos \delta_{\max}) \quad (3.53)$$

where

$$\delta_{\max} = \tan^{-1}(SR_{\max} / \sqrt{D^2 - SR_{\max}^2}). \quad (3.54)$$

In order to sample a direction within the solid angle subtended by the spherical source, the cosine of an angle, δ , is sampled from a uniform distribution of cosines between 1.0 and $\cos \delta_{\max}$, and an azimuthal angle ϕ' is sampled from a uniform distribution between 0 and 2π . The angle δ is the angle between the direction sampled within the solid angle Ω subtended by the spherical source and the line joining the collision to the source center and ϕ' is the angle between a vertical plane containing the line joining the collision and source center and the plane containing the sampled direction and the line joining the collision to the source center (Fig. 11). The direction angles γ_s, α_s of the sampled

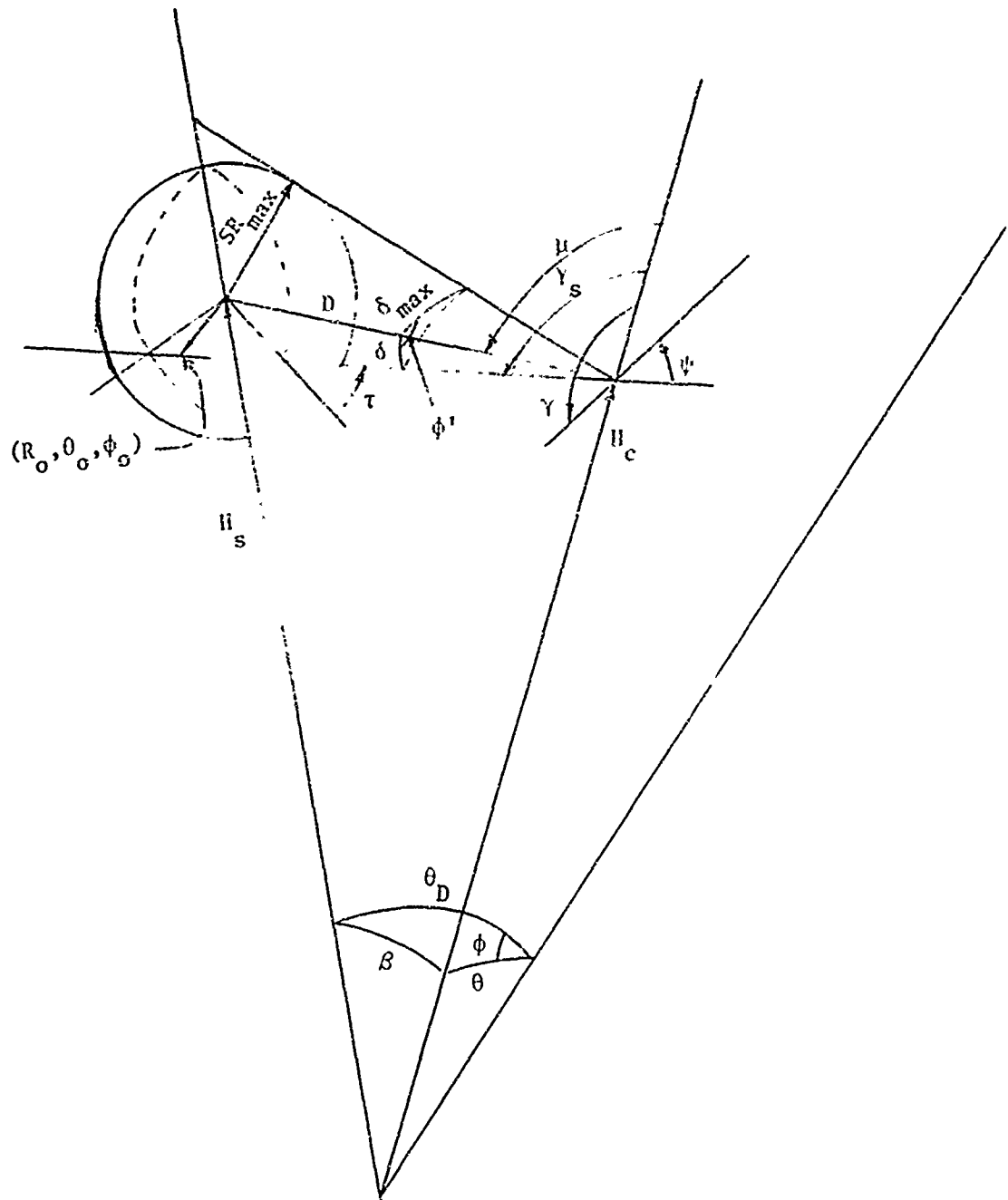


Fig. 11. Source and Collision Geometry for Spherical Volume and Surface Sources

direction at the collision position (see Fig. 11) are given by

$$\cos \gamma_s = \cos \delta \cos \mu + \sin \delta \sin \mu \cos \phi' \quad (3.55)$$

$$\sin \gamma_s = (1 - \cos^2 \gamma_s)^{1/2} \quad (3.56)$$

$$\cos \alpha_s = (\cos \delta - \cos \gamma_s \cos \mu) / (\sin \gamma_s \sin \mu) \quad (3.57)$$

$$\sin \alpha_s = \sin \delta \sin \phi' / \sin \gamma_s \quad (3.58)$$

where μ is the angle between the radial through the collision point and the line joining the collision and the center of the source region.

After a direction is sampled from within the solid angle subtended by the spherical source, the time dependent direct intensity at the collision from all source points lying along the line is computed. For the spherical surface source the direct intensity at the collision is given by

$$DI(t = t_e + t_v) = S(t_e) T_W T_{CO_2} e^{-\rho} \quad (3.59)$$

where $S(t_e)$ is the intensity emitted per steradian per unit area within a time interval whose upper limit is t_e from the spherical surface, T_W is the water vapor transmission from the spherical surface to the collision position, T_{CO_2} is the carbon dioxide transmission from the spherical surface to the collision position and ρ is the optical distance from the spherical surface to the collision position. The source intensity $S(t_e)$ is assumed to be independent of the direction of emission. t_v is the time required for light to traverse the distance between the surface of the source sphere and the collision position in the direction defined by the angles γ_s and α_s .

For the spherical volume source the direct intensity at the collision is given by the integral

$$DI(t) = \int_{S_1}^{S_2} N(R_o, t_e) e^{-\rho(R_o, \theta_o, \phi_o)} T_W(R_o, \theta_o, \phi_o) T_{CO_2}(R_o, \theta_o, \phi_o) dS \quad (3.60)$$

where $N(R_o, t_e)$ is the number of source photons emitted per steradian per unit volume at radial distance R_o in the source sphere in a time interval whose upper limit is t_e and $\rho(R_o, \theta_o, \phi_o)$ is the optical distance from a point along the direction through the source volume located at R_o, θ_o, ϕ_o to the collision point. The time t is the sum of the time t_e and the time t_ℓ required for the photon to travel the geometric distance defined by the optical distance $\rho(R_o, \theta_o, \phi_o)$.

The parameters $T_W(R_o, \theta_o, \phi_o)$ and $T_{CO_2}(R_o, \theta_o, \phi_o)$ as used in Equation (3.60) are the water vapor and carbon dioxide transmissions from a source point at R_o, θ_o, ϕ_o to the collision point, respectively. In FLASH the integral in Equation (3.60) is replaced by a sum of a series of integrals over segments of the lines passing through the source volume. Thus, the time dependent parameter $DI(t)$ is given by

$$DI(t) = \sum_{i=1}^{NS} \int_{S_{imax}}^{S_{imin}} N(R_o, t_e) e^{-\rho(R_o, \theta_o, \phi_o)} T_W(R_o, \theta_o, \phi_o) T_{CO_2}(R_o, \theta_o, \phi_o) dS \quad (3.61)$$

where the NS segments $S_{imax} - S_{imin}$ are chosen such that the source term and the extinction coefficient are constant within any one segment. If it can be assumed that the water vapor and carbon dioxide transmissions can be represented with the average value of the transmissions from the end points, S_{imax} and S_{imin} , of the i th segment of the line through the source volume then the integration of $N(R_o, t_e) e^{-\rho(R_o, \theta_o, \phi_o)}$ over S for a single segment may be performed analytically with the following equation:

$$\int_{S_{imin}}^{S_{imax}} N(R_o, t_e) e^{-\rho(R_o, \theta_o, \phi_o)} dS = \frac{N(R_o, t_e)}{\Sigma_i} \left[e^{-\rho_{imin}} - e^{-\rho_{imax}} \right] \quad (3.62)$$

In Equation (3.62), $N(R_o, t_e)$ is the source strength per unit volume per steradian in the time interval whose upper bound is t_e for the source region in which the segment $S_{imax} - S_{imin}$ lies, ρ_{imin} and ρ_{imax} are the optical distances from the collision point to the near and far ends of the line segment, and Σ_i is the pseudo extinction coefficient of the

atmospheric layer containing the segment. The time that is required for the light to travel the distance between the midpoint of the line segment, $S_{imax} - S_{imin}$, and the collision position is denoted as t_ℓ .

The direct intensity in the time interval $t = t_e + t_\ell$ at the collision position that arrives from the spherical source volume along the direction defined by the angles δ and ϕ' (Fig. 11) is given by

$$DI(t=t_e+t_\ell) = \sum_{i=1}^{NS} \frac{N(R_o, t_e)}{\Sigma_i} T_{iW} T_{iCO_2} \left[e^{-\rho_{imin}} e^{-\rho_{imax}} \right] \quad (3.63)$$

where the sum is over all the line segments between the intercepts of the spherical bounds of the source region and the intercept of the atmospheric layer bounds.

Equation (3.49) or Equation (3.50) is used, depending on whether the collision was an atmospheric scattering or a reflection event, to compute the estimate of the time dependent intensity at the receiver position in the receiver solid angle of view for which the photons backward path was started. The estimate given by either Equations (3.49) or (3.50) for a spherical surface or a spherical volume source are recorded as a function of the time $t+t_c$ which includes the emission time t_e and t_c , the time required for the photon to travel the random path between the receiver and the source.

3.2.3.2 Lense Type Volume Source. For the lense type volume source, the radiation source is assumed to be distributed within a volume bounded by upper and lower spherical surfaces, and a conical surface with vertex at the center of the earth and axis through the center of the source region (Fig. 10). This volume is further divided into annular ring elements bounded by spherical and conical surfaces within the outer source boundaries, so that the source distribution may vary as a function of radial and polar position.

In tracing photon histories by the backward method, an estimate of the direct intensity at each collision point is made by choosing a direction through the source volume and integrating the source contribution along this line.

When the collision point lies within the source volume, the source volume subtends a solid angle of 4π steradians about the collision point. In this case $\Omega = 4\pi$ and a direction through the source volume is obtained by sampling from an isotropic angular distribution. When the collision center is outside of the source volume, the solid angle subtended by the source volume from the collision position has to be calculated. The direction through the source volume from the collision position can then be sampled from within this solid angle.

Three methods were developed for determining the solid angle subtended by the source volume when the collision center is outside of the source volume. These sampling methods depend on whether the collision position lies above or below the source volume or outside of the cone containing the source volume.

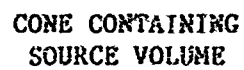
For the case where the collision center lies outside the cone containing the source volume, a plane containing the radial through the collision center and intersecting the source volume is selected. This is accomplished by first calculating the angle between a plane containing the radial through the collision center and the axis of the cone containing the source volume and a plane tangential to the cone and containing the radial through the collision center (Fig. 12). This angle is given by

$$\phi_1 = \tan^{-1}(\sin\theta_S / \sqrt{\cos^2\theta_S - \cos^2\beta}) , \quad (3.64)$$

where θ_S is the half angle of the cone containing the source volume and β is the angle between the axis of this cone and the radial through the collision center. The angle α_S between the arc joining the axis of the source volume cone and the collision center and the spherical projection of the direction through the source volume is computed with the expression

$$\alpha_S = RN\phi_1, \quad (3.65)$$

where RN is a random number. The geometrical cross section of the



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source volume in the plane containing the spherical projection of the direction through the source volume lies between the two angles γ_l and γ_u measured from the radial through the collision center (Fig. 13). The values of γ_l and γ_u are the minimum and maximum of the four expressions:

$$\gamma_1 = \cos^{-1} \left[(R_{Smax} \cos \delta_1 - R_c) / \sqrt{R_{Smax}^2 + R_c^2 - 2R_{Smax} R_c \cos \delta_1} \right] \quad (3.64)$$

$$\gamma_2 = \cos^{-1} \left[(R_{Smin} \cos \delta_1 - R_c) / \sqrt{R_{Smin}^2 + R_c^2 - 2R_{Smin} R_c \cos \delta_1} \right] \quad (3.65)$$

$$\gamma_3 = \cos^{-1} \left[(R_{Smax} \cos \delta_2 - R_c) / \sqrt{R_{Smax}^2 + R_c^2 - 2R_{Smax} R_c \cos \delta_2} \right] \quad (3.66)$$

$$\gamma_4 = \cos^{-1} \left[(R_{Smin} \cos \delta_2 - R_c) / \sqrt{R_{Smin}^2 + R_c^2 - 2R_{Smin} R_c \cos \delta_2} \right] \quad (3.67)$$

where R_{Smax} and R_{Smin} are the maximum and minimum radii of the spherical surface bounding the source volume, R_c is the radial distance to the collision center,

$$\cos \delta_1 = \frac{\cos \theta_S \cos \beta + \sin \theta_S \sin \beta \cos \alpha_S}{\cos^2 \theta_S + \sin^2 \theta_S \cos^2 \alpha_S} \sqrt{\sin^2 \theta_S - \sin^2 \beta \sin^2 \alpha_S}, \quad (3.68)$$

and

$$\cos \delta_2 = \frac{\cos \theta_S \cos \beta - \sin \theta_S \sin \beta \cos \alpha_S}{\cos^2 \theta_S + \sin^2 \theta_S \cos^2 \alpha_S} \sqrt{\sin^2 \theta_S - \sin^2 \beta \sin^2 \alpha_S}.$$

The angle that the direction through the source volume makes with the radial vector is chosen between the values of γ_l and γ_u using the expression

$$\cos \gamma_S = \cos \gamma_u - RN(\cos \gamma_u - \cos \gamma_l) \quad (3.70)$$

where RN is a random number. The solid angle Ω subtended by the source volume from the collision point when the collision point lies outside the cone containing the source volume is given by

$$\Omega = \pi [1 - \cos \gamma_u - \cos \gamma_l]. \quad (3.71)$$

If the collision point is above or below the source volume, the angles α_s is selected from a uniform distribution between 0 and 2π and one of the limits of the polar angle interval γ_l or γ_u becomes 1.0 or -1.0, depending on whether the collision point is above or below the source point, respectively. The solid angle Ω is then given by

$$\Omega = 2\pi(\cos\gamma_u - \cos\gamma_l). \quad (3.72)$$

If the ground surface hides a portion of the source volume as viewed from the collision point, the limits of the solid angle from which the direction through the source volume is sampled are adjusted to give a solid angle Ω at the receiver that views only that position of the source volume that is not hidden. If the entire source volume is hidden, then $\Omega = 0.0$ and the program skips the calculation of the direct intensity for that collision center and begins tracing the photon to the next collision center along the photon's backward path.

The time dependent direct intensity at a collision position from source volumes lying along the direction selected in the solid angle Ω is evaluated with the use of the equation

$$DI(t=t_e+t_l) = \sum_{i=1}^{NS} \frac{N_i(t_e)}{\Sigma_i} \bar{T}_{iW} \bar{T}_{iCO_2} (e^{-\rho_{imin}} - e^{-\rho_{imax}}) \quad (3.73)$$

where $N_i(t_e)$ is the source strength, per unit volume per steradian in the time interval whose upper bound is the time t_e , in the annular ring volume element containing the i th segment of the line from the collision point through the lense type source volume; Σ_i is the pseudo-extinction coefficient for the annular ring volume element containing the i th segment; $\bar{T}_{iW}$ and $\bar{T}_{iCO_2}$ are the average values of the water vapor and CO_2 transmission between the receiver position and the near and far ends of the i th line segment; ρ_{imin} and ρ_{imax} are the optical distances from the collision point to the near and far ends of the i th segment; and NS is the number of line segments into which the distance between the points at which the line from the collision point intersected the outside surfaces of the lense type source volume is divided.

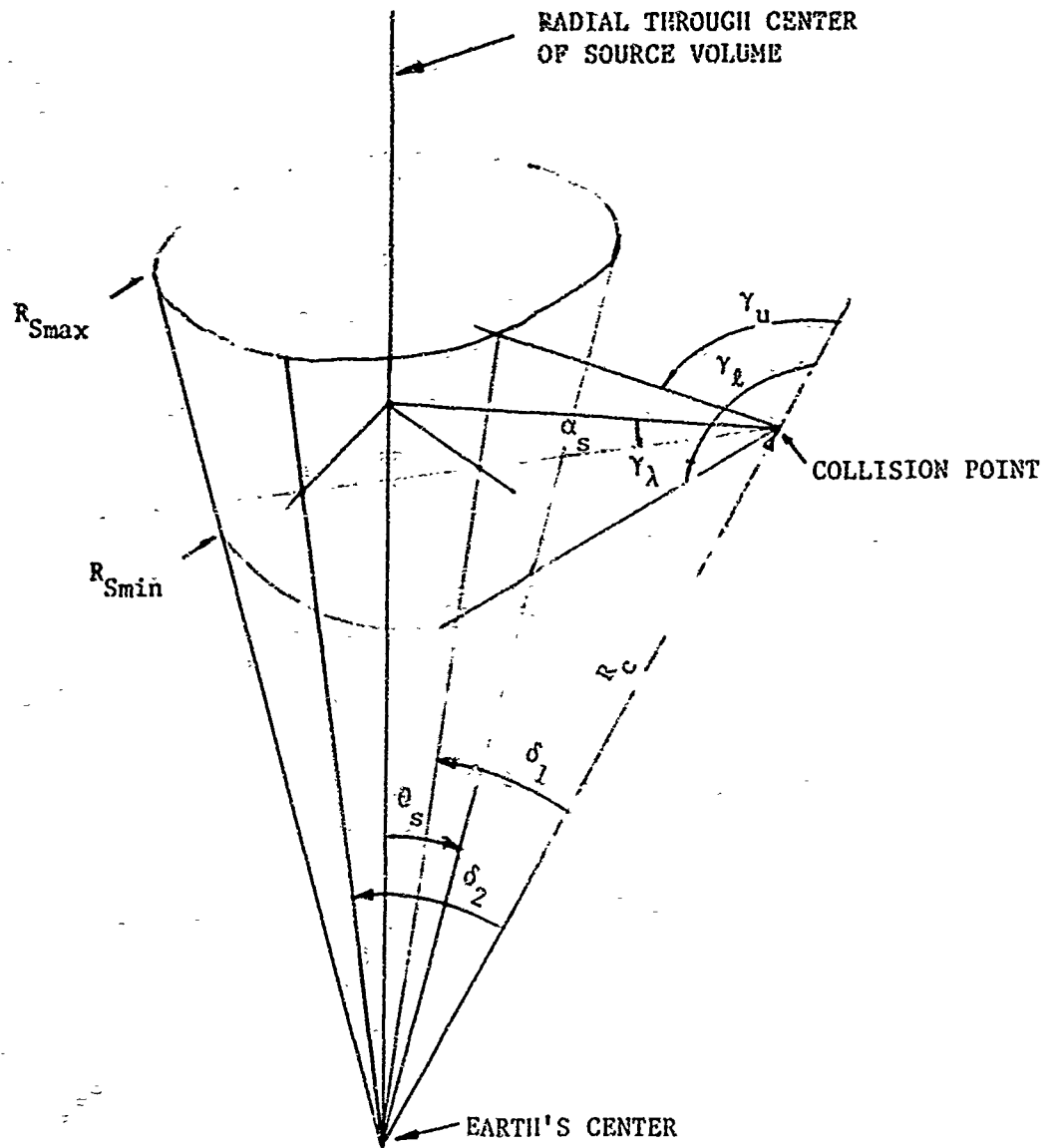


Fig. 13. Upper and Lower Limits of Polar Angle Subtended by Geometrical Cross Section of Source Volume

Once the solid angle, Ω , subtended by the source volume at the collision point and the direct intensity, $DI(t)$, are determined, then Equation (3.49) or Equation (3.50) is used, depending on whether the collision was an atmosphere scattering or a reflection event, to compute the estimate of the time dependent intensity at the receiver position in the solid angle of view for which the photons backward path was started.

For the lense shaped volume source version of FLASH, the estimates given by Equations (3.49) and (3.50) are recorded as a function of the elapsed time $t+t_c$.

3.3 Variation of the Scattered Intensity with the Ground Albedo

The estimates of scattered intensity in a given receiver solid angle of view and a given print time interval are recorded as a function of the order of reflection so that they may be converted to data for the same atmospheric model but with a different value of the diffuse albedo for the ground surface. The scattered intensity $I_n(t_i, A)$ at the receiver in time interval t_i for the case where the ground albedo was A that resulted from photons which had reflected n times from the ground surface are converted to give the scattered intensity for a different surface albedo, A' , with use of the equation

$$I(t_i, A') = \sum_{n=0}^{n_{\max}} I_n(t_i, A) (A'/A)^n. \quad (3.74)$$

The summation is from $n=0$, to include the intensity recorded before any reflection occurs, to $n_{\max}$, the maximum number of reflections allowed during the course of running a problem.

3.4 Direct Intensity Calculations

In addition to calculating the scattered light intensity, all three versions of the FLASH program compute the direct light intensity at each receiver position. The direct intensity from the point and plane sources reaches each receiver in a discrete direction, but the direct intensity from a volume source or the spherical surface source may be distributed over a finite sized solid angle interval. Therefore, the direct intensity is given by a single value for each receiver

position for the point and plane source versions of the FLASH program and as a function of time and direction for the volume and spherical surface source version of FLASH.

In the volume source version of the FLASH program, computations of the direct intensity at a receiver position is very similar to the methods described in Section 3.2.3 for evaluating the direct intensity parameter DI in the estimating function for the scattered intensity from a collision position.

3.4.1 Point Sources

In the point source version of FLASH the direct intensity is computed with the expression

$$DI = T_{CO_2} T_W D^{-2} e^{-\rho} \quad (3.75)$$

where T_{CO_2} and T_W are the carbon dioxide and water vapor transmissions, computed as discussed in Section 3.1.4, along the line joining the source and receiver position, and ρ is the pseudo optical distance and D is the geometric distance from the source to the receiver position.

The geometric distance D is computed with the expression

$$D = \sqrt{H_S^2 + H_D^2 - 2H_S H_D \cos \theta_D} \quad (3.76)$$

where H_S and H_D are the radial positions of the source and receiver points and θ_D is the angle between the radicals through the source and receiver positions.

In the routine for computing the direct intensity a check is made to determine whether the source is over-the-horizon from the receiver. If the source is over the horizon from the receiver, the direct intensity is set equal to zero, but a minimum time for radiation to traverse the distance from the source to the receiver is computed. The minimum time is computed as the sum of the times required to traverse the three distances, D_1 , D_2 , D_3 shown in Fig. 14. The distance D_1 is given by

$$D_1 = \sqrt{H_S^2 - H_0^2} \quad (3.77)$$

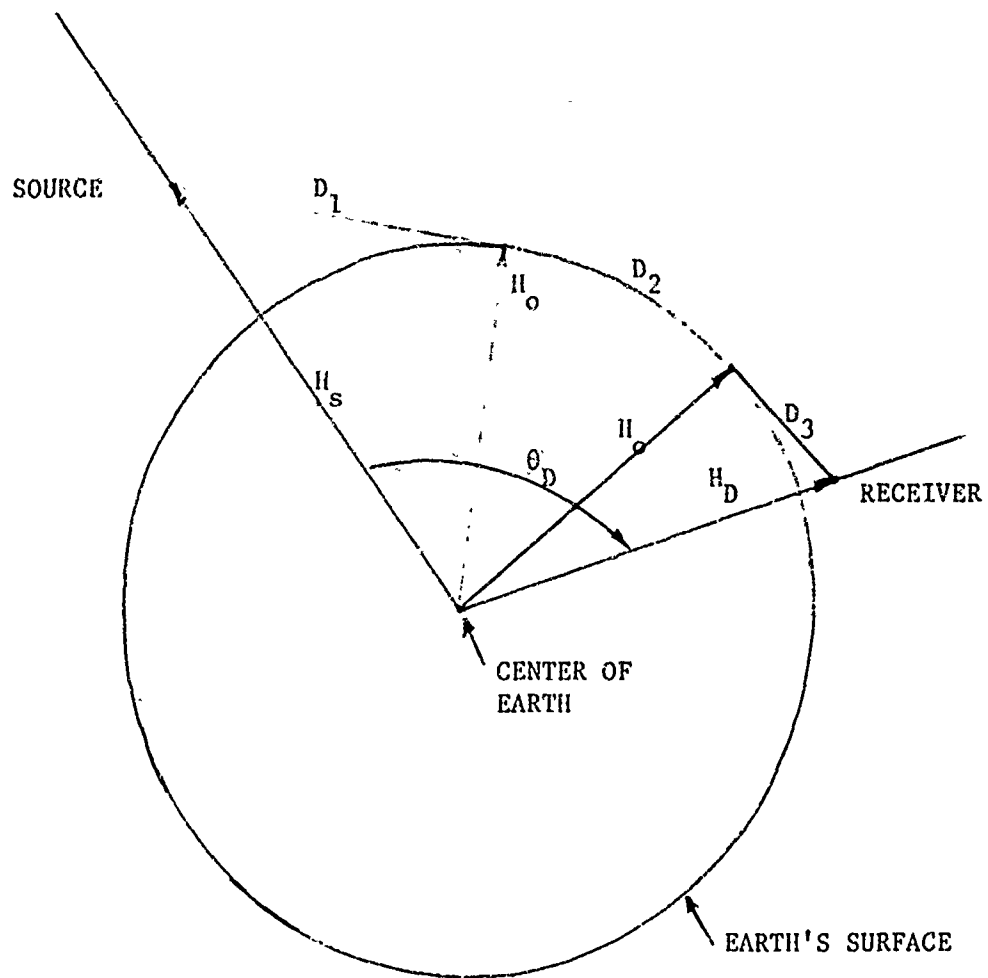


Fig. 14. Minimum Over-the-horizon Distance

where H_0 is the earth's radius and H_S is the radial distance to the source. The distances D_3 and D_2 are given by the expressions

$$D_3 = \sqrt{H_D^2 - H_0^2} \quad , \quad (3.78)$$

and

$$D_2 = H_0 \left[\theta_D - \tan^{-1} \left(\frac{D_1}{H_0} \right) - \tan^{-1} \left(\frac{D_3}{H_0} \right) \right] \quad (3.79)$$

where θ_D is the angle between the radials through the source and receiver positions and H_D is the radial distance to the receiver position.

If the source is not over-the-horizon from the receiver, the minimum time to the receiver position is just the time required to traverse the straight line distance D .

3.4.2 Plane Parallel Sources

In the plane parallel source version of FLASH the direct intensity is given by

$$DI = T_{CO_2} T_W e^{-\rho} \quad (3.80)$$

where T_{CO_2} and T_W are the carbon dioxide and water vapor transmissions along a curved path from the receiver to the source plane and ρ is the optical distance along that same path. The curved path is determined with the iterative process discussed in Section 3.2.2 where the initial direction of the path leaving the receiver position is taken to be equal to θ_D . Once the curved path is determined a check is made to determine if the path intersects the earth's surface. If the path does intersect the earth's surface the direct intensity is set to zero, otherwise, it is computed with Equation (3.80).

3.4.3 Volume and Spherical Surface Sources

In the volume and spherical surface source version of FLASH the direct intensity is computed at the center of each of the view angles defined in the problem input data.

The direct intensity from the spherical surface source as a function of time is computed for each view angle with the equation

$$DI(t = t_e + t_l) = S(t_e) T_W T_{CO_2} e^{-\rho} \quad (3.81)$$

where $S(t_e)$ is the intensity emitted per km^2 in a given time interval for which t_e is the upper limit of the time interval, T_W , T_{CO_2} and ρ are the water vapor and carbon dioxide transmissions and optical distance from the receiver along the line through the center of the view angle to the surface of the spherical surface source, and t_l is the time required for the photons to travel the distance between the point where the direction to the source intersect the spherical surface and the receiver position. If the line through the center of a view angle does not intersect the spherical surface source, the direct intensity within that view angle is set to zero.

The direct intensities at a receiver position as a function of time for both the spherical volume source and the lense shape volume source are computed for a direction through the center of each of the receiver view angles defined in the program input data using Equation (3.60). In Equation (3.60) the integration from S_1 to S_2 is along the line through the center of the view angle from the nearest point of intersection with the source volume to the farthest point of intersection.

The method of solving Equation(3.60), which is discussed in Section 3.2.3 and Section 3.2.3.2 for spherical and lense type volume sources, is utilized to obtain the direct intensity as a function of time at the center of each of the view angles defined in the problem input.

3.5 FLASH Program Output

The FLASH programs are designed to compute the direct and scattered light intensities within finite solid angles of view for each receiver position. The scattered intensities are recorded as a function of the order of reflection from the ground surface and as a function of the order of collision for each view angle. In the point source version of FLASH, the scattered intensities are recorded as a function of the

time of arrival at the receiver position for each of the view angles. The plane source version of FLASH does not compute the time dependence of the scattered intensities. Both the point and plane source versions of FLASH integrate the scattered intensities as a function of direction over all of the solid angles of view defined in the input data to yield a set of "integrated intensities."

3.5.1 Units of Point Source FLASH Results

The units of the direct and scattered intensities computed for each view angle with the point source version of FLASH are photons km^{-2} ster $^{-1}$ /source photon ster $^{-1}$. The time dependent intensities are not given on a per unit time basis but are in units of photons km^{-2} ster $^{-1}$ /source photon ster $^{-1}$ within a designated time interval. The integrated intensities are in units of photons km^{-2} /source photon ster $^{-1}$ and the integrated time dependent intensities are in units of photons km^2 /source photon ster $^{-1}$ within the designated time interval.

3.5.2 Units of Plane Source FLASH Results

The units of the scattered intensities computed for each view angle with the plane source version of FLASH are photons km^{-2} ster $^{-1}$ /source photon km^{-2} at the source plane. The plane source version of FLASH also integrates the scattered intensity over all of the defined receiver view angles to give the integrated intensity in units of photons km^{-2} /source photon km^{-2} .

3.5.3 Units of Spherical Surface Source Flash Results

The direct and scattered intensities computed for the spherical surface source are not normalized to a unit source strength but are computed for the surface source defined in the problem input. The source term input to define the spherical surface source should be in units of photons km^{-2} ster $^{-1}$ within the time interval specified in the problem input data. The source is assumed to be uniformly distributed over the surface of the sphere and to radiate isotropically in the outward direction. The units of the direct and scattered intensities computed for each receiver view angle computed are photons km^{-2} ster $^{-1}$ for the defined

source. The units of the time dependent intensities is photons km^{-2} ster^{-1} within the time intervals specified for the source in the problem input data.

The intensities integrated over both time and all receiver view angles are in units of photons km^{-2} . The time dependent integrated intensity is given in units of photons km^{-2} in each of the time intervals for which the source is defined in the problem input.

3.5.4 Units of Volume Source FLASH Results

The output of the volume source FLASH program are not normalized to a unit source strength as in the plane and point source versions. The source strength as a function of position and time within the spherical or lense shaped volume source is required as input to the volume source FLASH program. For both the spherical and lense shaped volume sources, the source term should be given in units of photons km^{-3} ster^{-1} within a specified time interval. The direct and scattered intensities computed with the volume source FLASH program is then the number of photons km^{-2} ster^{-1} for the source defined with the problem input. The time dependent intensities given for each receiver solid angle of view are the number of photons km^{-2} ster^{-1} reaching the receiver within the specified time interval.

The volume source FLASH program also computes the intensity integrated over the receiver view angles defined by the problem input. These data are in units of photons km^{-2} for the time integrated data and photon km^{-2} within a given time interval for the time dependent data, where the data in both instances pertain to the particular source defined in the input data.

IV. AUXILIARY PROGRAMS

Two auxiliary programs have been prepared to aid in the utilization of the FLASH programs. The MIE-II program was provided as an aid in the preparation of the aerosol scattering, absorption, and extinction coefficients and the aerosol phase matrix required as input to FLASH. The SHINE program was developed to combine the results from several point source FLASH problems for different wavelengths with a time and wavelength dependent source term to yield the time dependent scattered intensity at a receiver for a point source emitting light over a finite wavelength interval.

4.1 MIE-II Program

The following two sections describe the equations used in MIE-II to compute scattering and absorption cross section data based on Mie theory for single size particles and to integrate the Mie data over arbitrary aerosol particle size distributions to obtain macroscopic aerosol cross section data.

4.1.1 The Scattering of Single Particles

Consider light with wavelength λ propagating in the region of a dielectric sphere of radius r and complex index of refraction $m = n - in'$. The resulting scattered radiation field is defined by the Mie theory to have vector components of the electric field perpendicular to the direction of propagation with magnitudes

$$S_1(r, \lambda, m, \theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} [a_n \pi_n + b_n \tau_n] , \quad (4.1)$$

and

$$S_2(r, \lambda, m, \theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} [b_n \pi_n + a_n \tau_n] , \quad (4.2)$$

where S_1 and S_2 are the complex amplitude functions of the scattered wave. The quantities a_n and b_n are the Mie coefficients and π_n and τ_n are angular dependent functions of Legendre polynomials. The explicit

dependence of these and following relationships upon a specific pair of r and λ values may be removed by defining the variable x , the size parameter, by

$$x = 2\pi r/\lambda \quad . \quad (4.3)$$

The Mie coefficients may then be written in the form given by Van de Hulst (Ref. 11)

$$a_n = \frac{\psi'_n(mx)\psi_n(x) - m\psi_n(mx)\psi'_n(x)}{\psi'_n(mx)\zeta_n(x) - m\psi_n(mx)\zeta'_n(x)} \quad , \quad (4.4)$$

and

$$b_n = \frac{m\psi'_n(mx)\psi_n(x) - \psi_n(mx)\psi'_n(x)}{m\psi'_n(mx)\zeta_n(x) - \psi_n(mx)\zeta'_n(x)} \quad (4.5)$$

with $\psi_n(x) = xj_n(x)$

and $\zeta_n(x) = xh_n^{(2)}(x)$,

where $j_n(x)$ are the spherical Bessel functions, $h_n^{(2)}(x)$ are spherical Hankel functions of the second kind, and the primes indicate differentiation with respect to the argument. The angle-dependent coefficients, π_n and τ_n , are written

$$\pi_n = \frac{\partial}{\partial \cos\theta} P_n(\cos\theta) \quad (4.6)$$

and

$$\tau_n = \pi_n \cos\theta - (\sin^2\theta) \frac{\partial \pi_n}{\partial \cos\theta} \quad , \quad (4.7)$$

where θ is the scattering angle, $P_n(\cos\theta)$ are Legendre polynomials, and the symbol ∂ indicates partial differentiation.

It can be seen that the amplitude functions may be computed to arbitrary accuracy by evaluating the convergent series in n to the required number of terms. However, the Mie coefficients are difficult to evaluate in the form shown in Equations (4.4) and (4.5). The calculations

may be simplified by expressing a_n and b_n as functions involving only ordinary Bessel functions of order $\pm n \pm 1/2$, $J_{\pm n \pm 1/2}(z)$, and their derivatives, where $z = x$ or $z = mx$. This may be done by the application of the following identities:

$$j_n(z) = \left(\frac{\pi}{2z}\right)^{1/2} J_{n+1/2}(z) ,$$

$$h_n^{(2)}(z) = \left(\frac{\pi}{2z}\right)^{1/2} [J_{n+1/2}(z) - i N_{n+1/2}(z)] ,$$
(4.8)

$$N_{n+1/2}(z) = \left(\frac{2z}{\pi}\right)^{1/2} n_n(z) ,$$

and

$$n_n(z) = (-1)^{n+1} \left(\frac{\pi}{2z}\right)^{1/2} J_{-n-1/2}(z) .$$

After performing these substitutions and taking the required derivatives, Equations (4.4) and (4.5) may be algebraically altered and written in a form previously reported by Deirmendjian (Ref. 12);

$$a_n = \frac{\left[\frac{A_n}{m} + \frac{n}{x}\right] J_{n+1/2}(x) - J_{n-1/2}(x)}{\left[\frac{A_n}{m} + \frac{n}{x}\right] [J_{n+1/2}(x) + i(-1)^n J_{-n-1/2}(x)] - J_{n-1/2}(x) + i(-1)^n J_{-n+1/2}(x)} ,$$
(4.9)

and

$$b_n = \frac{[mA_n + \frac{n}{x}] J_{n+1/2}(x) - J_{n-1/2}(x)}{[mA_n + \frac{n}{x}] [J_{n+1/2}(x) + i(-1)^n J_{-n-1/2}(x)] - J_{n-1/2}(x) + i(-1)^n J_{-n+1/2}(x)} ,$$
(4.10)

where

$$A_n = A_n(mx) = -\frac{n}{mx} + \frac{J_{n-1/2}(mx)}{J_{n+1/2}(mx)} .$$
(4.11)

The form of these expressions for a_n and b_n is particularly convenient since the portion of the coefficients containing Bessel functions of complex argument is restricted to the A_n terms, which may be calculated separately. This is done through the simple recursion formula

$$A_n(mx) = -\frac{n}{mx} + \frac{1}{\frac{n}{mx} - A_{n-1}(mx)}, \quad (4.12)$$

where

$$A_0(mx) = \frac{J_{-1/2}(mx)}{J_{+1/2}(mx)} = \cot(mx). \quad (4.13)$$

Again capitalizing upon the affinity of Bessel functions for recursion formulae, Equations (4.9) and (4.10) may be written

$$a_n = \frac{\left[\frac{n}{m} + \frac{n}{x}\right] \operatorname{Re}(w_n) - \operatorname{Re}(w_{n-1})}{\left[\frac{n}{m} + \frac{n}{x}\right] w_n - w_{n-1}} \quad (4.14)$$

and

$$b_n = \frac{\left[mA_n + \frac{n}{x}\right] \operatorname{Re}(w_n) - \operatorname{Re}(w_{n-1})}{\left[mA_n + \frac{n}{x}\right] w_n - w_{n-1}}, \quad (4.15)$$

where

$$w_n(x) = \frac{2n-1}{x} w_{n-1} - w_{n-2}, \quad (4.16)$$

and

$$\begin{aligned} w_0(x) &= \sin x + i \cos x, \\ w_1(x) &= \cos x - i \sin x. \end{aligned} \quad (4.17)$$

Thus a_n and b_n may be computed by evaluating A_n and w_n separately, with the only preliminary computations being $\cos x$, $\sin x$ and $\cot mx$.

Likewise, the angular-dependent functions, π_n and τ_n , may be determined by recursion formulae with the relations

$$\pi_n(\theta) = \left(\frac{2n-1}{n-1}\right)\pi_{n-1}(\theta)\cos\theta - \left(\frac{n}{n-1}\right)\pi_{n-2}(\theta) \quad , \quad (4.18)$$

and

$$\tau_n(\theta) = \cos\theta[\pi_n(\theta) - \pi_{n-2}(\theta)] - (2n-1)\sin^2\theta[\pi_{n-1}(\theta)] + \tau_{n-2}(\theta) \quad , \quad (4.19)$$

where

$$\begin{aligned} \pi_0(\theta) &= 0 \quad , & \tau_0(\theta) &= 0 \quad , \\ \pi_1(\theta) &= 1 \quad , & \tau_1(\theta) &= \cos\theta \quad , \\ \pi_2(\theta) &= 3\cos\theta \quad , & \tau_2(\theta) &= 3(1 - 2\sin^2\theta) \quad . \end{aligned}$$

The extinction and scattering cross sections for light of wavelength λ incident on a sphere of radius r are calculated by evaluating the convergent series

$$\sigma_{\text{ext}}(\lambda, r, m) = \frac{\lambda^2}{2\pi} \sum_{n=1}^{\infty} (2n+1) \text{Re}(a_n + b_n) \quad (4.20)$$

and

$$\sigma_{\text{sc}}(\lambda, r, m) = \frac{\lambda^2}{2\pi} \sum_{n=1}^{\infty} (2n+1) (|a_n|^2 + |b_n|^2) \quad . \quad (4.21)$$

Explicit dependence upon a particular pair of r and λ values may be eliminated by defining efficiency factors, Q_{ext} and Q_{sc} , by

$$Q_{\text{ext}}(x, m) = \sigma_{\text{ext}}(r, \lambda, m) / \pi r^2$$

and

$$Q_{\text{sc}}(x, m) = \sigma_{\text{sc}}(r, \lambda, m) / \pi r^2 \quad ,$$

or

$$Q_{\text{ext}}(x, m) = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1) \operatorname{Re}(a_n + b_n) \quad (4.22)$$

and

$$Q_{\text{sc}}(x, m) = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1) (|a_n|^2 + |b_n|^2) \quad (4.23)$$

In order to fully describe the scattered radiation, the following four intensity functions must also be calculated:

$$i_1(x, m, \theta) = |S_1|^2, \quad (4.24)$$

$$i_2(x, m, \theta) = |S_2|^2, \quad (4.25)$$

$$i_3(x, m, \theta) = \operatorname{Re}(S_1 S_2^*) \quad (4.26)$$

and

$$i_4(x, m, \theta) = -\operatorname{Im}(S_1 S_2^*) \quad (4.27)$$

where the * indicates the complex conjugate. Since S_1 and S_2 , defined in Equations (4.1) and (4.2), are the magnitude of the electric amplitude perpendicular and parallel, respectively, to the plane of scattering, the quantities i_1 and i_2 are proportional to the intensity of the light scattered per steradian, at some particular scattering angle, perpendicular and parallel to the plane of scattering. The quantities i_3 and i_4 are related to the Stokes' parameters defining the ellipticity and plane of polarization of the scattered light.

At a point in the radiation region of the scattered field, assuming azimuthal symmetry, the intensity of the scattered light is given by

$$I = \frac{F(\theta)}{k^2 R^2} I_0 \quad (4.28)$$

where I_0 is the intensity of the incident radiation, $F(\theta)$ is a function of direction only, $k = 2\pi/\lambda$, and R is here the distance from the

scattering center. For incident light polarized perpendicularly to the plane of scattering

$$F(\theta) = i_1 ; \quad (4.29)$$

for parallel polarization

$$F(\theta) = i_2 ; \quad (4.30)$$

and for natural light

$$F(\theta) = \frac{i_1 + i_2}{2} . \quad (4.31)$$

4.1.2 Volume Scattering

To obtain the four elements of the aerosol phase matrix, i_1 , i_2 , i_3 , and i_4 , the volume scattering function, and the macroscopic scattering, absorption, and extinction cross sections for a given aerosol particle size distribution it will be necessary to integrate the microscopic scattering data described in the previous section for a given index of refraction over the aerosol size distribution.

Consider a medium containing N particles per cm^3 with the size distribution of the particles characterized by the function $n(r)$ such that

$$N = \int_{r_{\min}}^{r_{\max}} n(r) dr , \quad (4.32)$$

where $r_{\min}$ and $r_{\max}$ are the minimum and maximum radii in the particle concentration. The volume scattering cross section is thus obtained by the expression

$$\Sigma_{sc}(m, \lambda) = \int_{r_{\min}}^{r_{\max}} \sigma_{sc}(m, \lambda, r) n(r) dr , \quad (4.33)$$

where $\Sigma_{sc}(m, \lambda)$ is the volume scattering cross section (or attenuation coefficient), $\sigma_{sc}(m, \lambda, r)$ are the microscopic cross section defined in

Equation (4.21) and $n(r)$ is the size distribution function. Since the MIE-II program calculates all of the microscopic parameters as a function of the size parameter, $x = kr$, Equation (4.33) may be written in terms of the efficiency factors defined in Equation (4.23) as

$$\Sigma_{sc}(m, \lambda) = \int_{r_{min}}^{r_{max}} \pi r^2 Q_{sc}(m, r/\lambda) n(r) dr \quad (4.34)$$

Likewise the volume extinction cross section is written

$$\Sigma_{ext}(m, \lambda) = \int_{r_{min}}^{r_{max}} \pi r^2 Q_{ext}(m, r/\lambda) n(r) dr \quad (4.35)$$

The four elements of the phase matrix for an aerosol size distribution characterized by the function $n(r)$ are given by the following integrals:

$$Y_1(\theta, \lambda, m) = (2\pi/k^2) \int_{r_{min}}^{r_{max}} i_1(r/\lambda, m, \theta) n(r) dr \quad (4.36)$$

$$Y_2(\theta, \lambda, m) = (2\pi/k^2) \int_{r_{min}}^{r_{max}} i_2(r/\lambda, m, \theta) n(r) dr \quad (4.37)$$

$$Y_3(\theta, \lambda, m) = (2\pi/k^2) \int_{r_{min}}^{r_{max}} i_3(r/\lambda, m, \theta) n(r) dr \quad (4.38)$$

and

$$Y_4(\theta, \lambda, m) = (2\pi/k^2) \int_{r_{min}}^{r_{max}} i_4(r/\lambda, m, \theta) n(r) dr \quad (4.39)$$

If we define $\Sigma_j(\lambda, m)$ by the integral equation

$$\Sigma_j(\lambda, m) = 2\pi \int_0^\pi Y_j(\theta, \lambda, m) \sin\theta d\theta \quad (4.40)$$

for $j = 1, 2, 3$, and 4 , then $Y_j(\theta, \lambda, m)$ may be normalized so that its integral over 4π steradians is 1.0 by

$$I_j(\theta, \lambda, m) = Y_j(\theta, \lambda, m) / \Sigma_j(\lambda, m) \quad (4.41)$$

The parameter $I_j(\theta, \lambda, m)$ is denoted as the normalized form of the j th element of the aerosol phase matrix. The volume scattering function for unpolarized light is defined as

$$Y(\theta, \lambda, m) = 1/2 [Y_1(\theta, \lambda, m) + Y_2(\theta, \lambda, m)] \quad (4.42)$$

The volume scattering function may be interpreted as the intensity per steradian of radiation of wavelength λ being scattered in the direction θ , if the amount of radiation incident on the volume element is set to 1. The integration of $Y(\theta, \lambda, m)$ over 4π solid angle is therefore equivalent to the total scattered radiation, i.e., the volume scattering coefficient. This leads to the relation

$$\Sigma_s(\lambda, m) = 2\pi \int_0^\pi Y(\theta, \lambda, m) \sin\theta d\theta \quad (4.43)$$

The probability P_i that an unpolarized photon will be scattered into a given angle θ_i is given by the relation

$$P_i(\lambda, m) = \frac{2\pi}{\Sigma_s(\lambda, m)} \int_0^{\theta_i} Y(\theta, \lambda, m) \sin\theta d\theta \quad (4.44)$$

The MIE-II procedure evaluates the above equation for the upper limits, θ_i , where the probabilities P_i are fractional values placed at equal intervals on the interval from 0.0 to 1.0.

Values of the normalized aerosol phase matrix as given by Equation (4.41) and the cosines of the scattering angle, θ , as given by Equation (4.44) for equal intervals of the probability function P are punched on cards by the MIE-II program in the format required for input into the various versions of the FLASH programs.

4.2 SHINE Program

The SHINE program utilizes the direct and scattered light intensities stored on magnetic tape by the point source version of FLASH for each of a set of monochromatic point isotropic sources. The program determines the time dependent radiation intensity at a set of receiver positions for an isotropic source emitting radiation over a finite wavelength interval. The SHINE program requires that the radiation emitted from a point isotropic source be input as a function of time and wavelength. For each source wavelength, the time distribution of the radiation emitted at that wavelength is multiplied by the fraction of the emitted radiation that will reach the receiver for a given retarded time and the entire distribution is shifted by the retarded time and accumulated into a set of time intervals used to record the time dependence of the radiation at the receiver. The radiation computed for each of the record time intervals as a function of wavelength is integrated over wavelength to give the time distribution of the radiation at a receiver for the source spectrum.

4.2.1 Computational Methods Used in the SHINE Program

In order to determine the intensity at a given receiver as a function of time and wavelength, SHINE is designed to solve the following equation:

$$I(\lambda, T) = \int_0^T S(\lambda, t) TR(\lambda, T-t) dt \quad (4.45)$$

where $S(\lambda, t)$ is the radiation emitted per unit wavelength per unit time per steradian at the time t and $TR(\lambda, T-t)$ is the fraction of the radiation of wavelength λ emitted per steradian that reaches the receiver after an elapse of time equal to $T-t$. The fraction of the radiation emitted per steradian that reaches the receiver as a function of time is given in terms of retarded time; that is, the time after the arrival of the direct intensity. Therefore, $TR(\lambda, t=0)$ represents the direct intensity and $TR(\lambda, t>0)$ represents the integrated scattered intensity at the receiver position for a point isotropic source of wavelength λ that emits 1 photon

per steradian. By integrating Equation (4.45) over all emission times from 0 to T, the intensity of the light of wavelength λ arriving per unit time at the receiver at time T is determined.

Integrating $I(\lambda, T)$ with respect to wavelength gives the total amount of radiation received per unit time at the time T for the source spectrum.

$$I(T) = \int_{\lambda_{\min}}^{\lambda_{\max}} I(\lambda, T) d\lambda \quad (4.46)$$

where the integral is over the wavelength interval for which the source term is defined. The intensity per unit time at the receivers as a function of wavelength and time as given in Equation (4.45) and the intensity per unit time integrated over wavelength as a function of time given in Equation (4.46) are both expressed in terms of retarded time, since the fraction of the emitted radiation reaching the receivers is expressed in terms of retarded time. However, since the source emits radiation over a period of time, the retarded time of zero is the time when the direct intensity from the initial emission from the source begins to reach the receiver.

In order to solve Equation (4.45), the source term $S(\lambda, t)$ is read from card input into the SHINE program. The fraction of the emitted radiation that will reach the receiver is computed from the data stored on library tape by the point source FLASH program.

Each point source FLASH problem produces the fraction of the emitted radiation per steradian at a single wavelength that will reach each of the receiver positions. The library tape contains the results of several point source FLASH problems for different wavelengths. The data written on the library tape for each point source FLASH problem contains the following information:

An initial record for each run contains:

NPROB - The FLASH problem number used to identify the FLASH problem for the given atmospheres, set of source-receiver positions and wavelength.

- ATMO - An eight-character alphanumeric symbol identifying the atmosphere used in the FLASH calculation.
- WAVEL - The wavelength in microns of the monochromatic light source used for the FLASH problem.
- NRECV - The number of receivers for which scattered intensities were calculated.
- NPTIM - The number of upper bounds of the print time intervals used to record the time dependent scattered intensities in the FLASH calculation.
- PTIM(I) - The upper bounds of the print time intervals (seconds) used in the FLASH calculation.

Following the initial record, a separate record is written on the library tape for each receiver position considered in the problem. Each of these records contain the following information:

- RADS - The radial distance from the earth's center to the source altitude (kilometers).
- RADD(I) - The radial distance from the earth's center to the ith receiver altitude (kilometers).
- THED(I) - The angle at the earth's center between the radial through the source position and the radial through the ith receiver position (degrees).
- DIRI(I) - The fraction of the one source photon emitted per steradian by a point isotropic source that reaches the ith receiver without undergoing a collision (photons $\text{km}^{-2}/\text{source photon ster}^{-1}$).
- ANS(I,J) - The fraction of the one source photon emitted per steradian by a point isotropic source that scatters into the receiver within the jth print time interval for the ith ground albedo (photons $\text{km}^{-2}/\text{source photon ster}^{-1}$).

The method used in the SHINE program to obtain the solution to Equation (4.45) is given in the following paragraphs. The wavelengths SWAV(I), used in defining the source term are read from cards. Next, the time values, STB(J), for which the source term is to be input, are read from cards. Then, the source strength SPS(J,I) for each time and wavelength is read from cards. The source term, SPS(J,I) should be in

units denoting the radiation emitted per second per micron per steradian at the time $STB(J)$ and wavelength $SWAV(I)$. The SHINE program computes a delta wavelength interval for each of the source wavelengths by taking the interval between the midpoints on either side of an input wavelength. The wavelength interval for the first input wavelength is taken to be the smaller of twice the interval between the first wavelength and the midpoint between the first two input wavelengths, or the interval from 0.0 to the midpoint between the first two wavelengths input. The wavelength interval for the last wavelength input is taken to be twice the interval between the midpoint of the last two wavelengths input and the last wavelength input.

The wavelengths that are used to define the source term in SHINE do not have to correspond to the set of wavelengths used for the point source FLASH problems. Therefore, for each source wavelength used in SHINE, the library tape is searched to locate the two point source FLASH runs for wavelengths that bracket the source wavelength. Then a linear interpolation of the point source FLASH data is performed with respect to wavelength to obtain the fraction of the emitted radiation transmitted to the receiver for the source wavelength.

The interpolated data from the FLASH produced tape represents the second factor in Equation (4.45), $TR(T-t)$. However, since both the source term and the fraction of the radiation transmitted are in tabulated form, it is not feasible to integrate analytically. In order to perform the integration, it is assumed that the fraction of the emitted radiation that reaches the receiver before scattering is represented with a delta function at retarded time $t=0$, and the radiation transmitted after having scattered is represented with a set of delta functions located at the centers of the print time intervals used in the FLASH problems. It is further assumed that the source strength for a given wavelength as a function of time may be fitted with a power law curve between each two source time intervals. To obtain the solution to Equation (4.45) for a set of record time intervals (time intervals for recording the scattered intensity), the source term is multiplied by the portion of the emitted radiation that is transmitted directly and that which is transmitted for each print time interval.

For each FLASH print time interval the product of the source term times the fraction of the emitted radiation that reaches the receiver in that time interval is shifted on the time scale by a time equivalent to the midpoint of the FLASH print time interval for which the fraction received is given. The total amount of radiation received within each of the record time intervals is obtained by integrating the product curve over the record time interval and adding the results of the integration to the radiation already stored in that record time interval from the integration of the product curves for the earlier FLASH print times. After accumulating the radiation into the record time intervals for all of the FLASH print times, the total intensity at the source wavelength received at the receiver in each record time interval is divided by the width of the interval and plotted as a function of time.

V. VALIDATION OF THE FLASH PROCEDURES

Each of the three versions of the FLASH program have been thoroughly checked out through comparisons that were made between the results of the programs with those from other calculational methods. Some of the results of these comparisons are discussed in the following Sections.

5.1 Validation of the Point Source Version of Flash

The point source version of the FLASH program was validated through comparisons made with the POLO (Ref. 13) and TPART-I (Ref. 2) programs. The POLO program is a forward Monte Carlo program designed to treat a spherical-shell atmospheric geometry. All three programs are capable of treating inhomogeneous atmospheres and should produce similar results for source and receivers located such that the source-receiver separation distance is small in comparison with the earth radius.

One such test problem involved a 0.43μ wavelength point isotropic source located at 30 km altitude in the Elterman 1968 model atmosphere (Ref. 14) with ground level receivers located at horizontal ranges of 25, 50, 75, and 100 km. The ground surface was assumed to be a Lambert reflecting surface. The integrated single scattered light intensity that would be seen with an omnidirectional receiver is compared in Fig. 15 with the integrated single scattered intensities as given by the POLO and TPART-I calculations. The data in Fig. 15 are for a ground albedo of 0.8. The integrated total scattered intensity for surface albedo values of 0.0 and 0.8 are compared in Fig. 16 with similar results from POLO and TPART-I. It is seen in Figs. 15 and 16 that the results obtained from the point source version of FLASH are in good agreement with the results given by both POLO and TPART-I.

Fig. 17 shows a comparison of the time distribution of the integrated scattered intensity at the receiver which was positioned at a horizontal range of 25 km. All of the photons emitted by the source were assumed to be emitted instantaneously. The surface albedo was 0.8 for the data shown in Fig. 16.

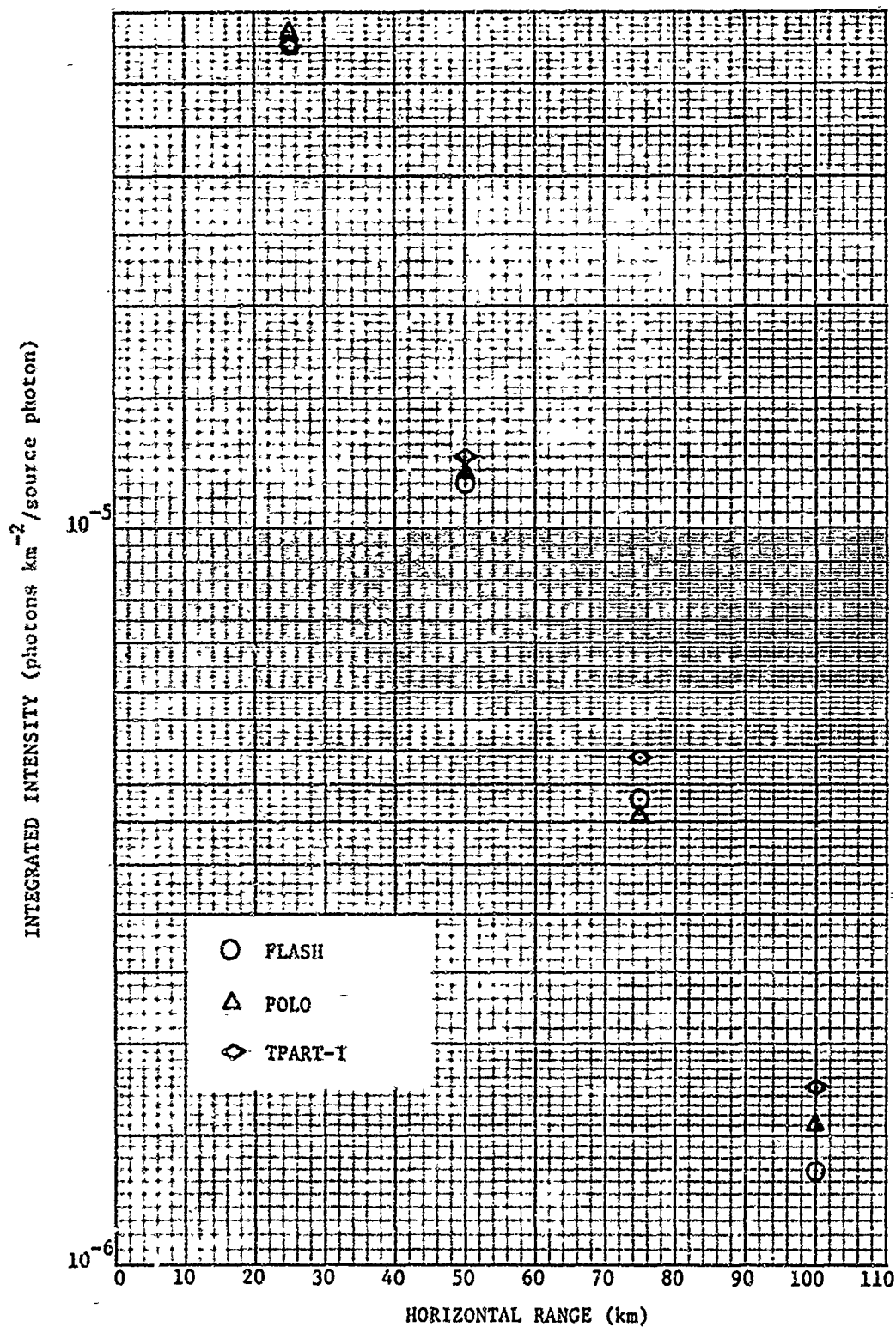


Fig. 15. Integrated Single Scattered Light Intensity Versus Horizontal Range at Ground Level for a Point Isotropic 0.43μ Wavelength Source at 30 km Altitude: Ground Albedo = 0.8

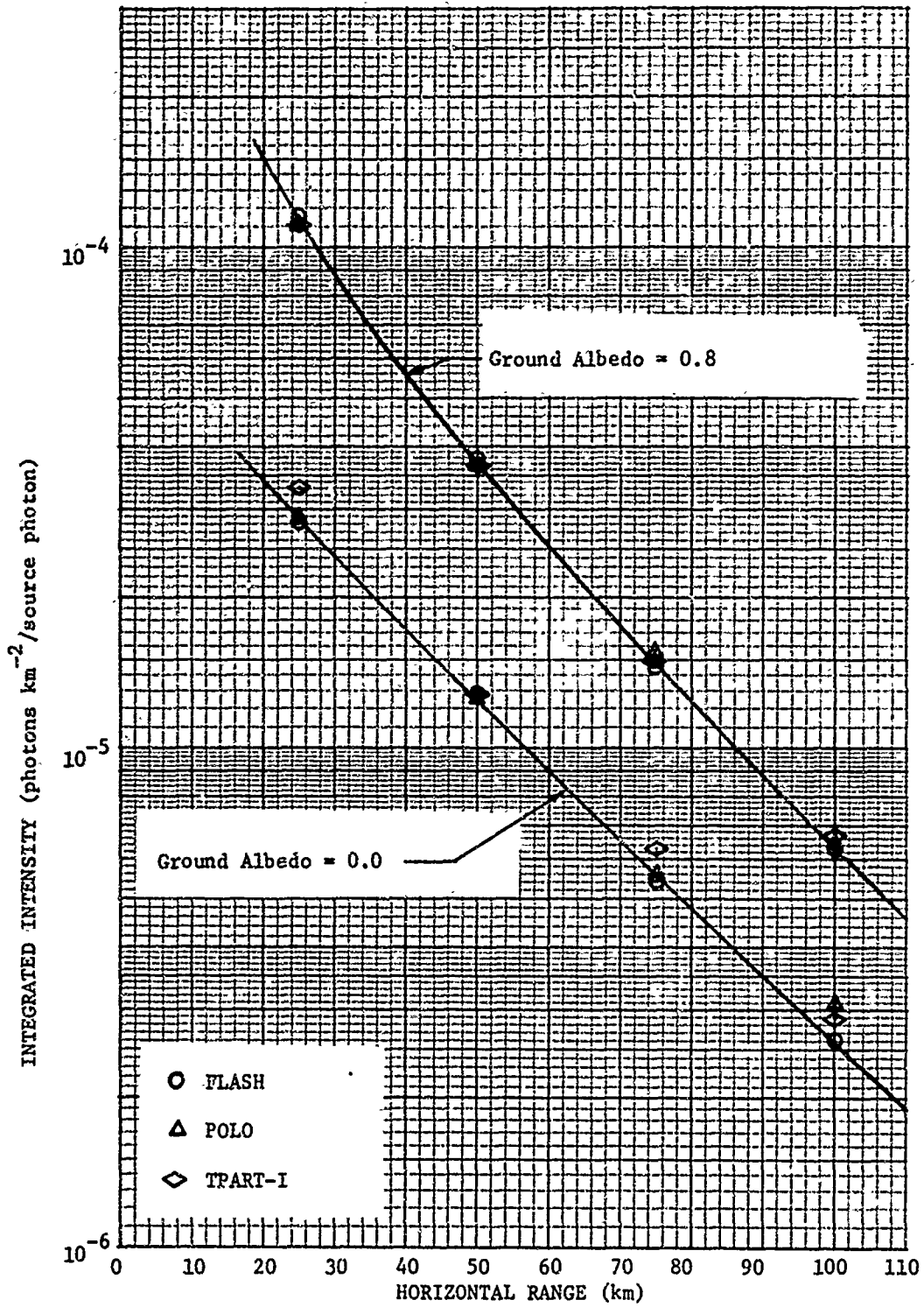


Fig. 16. Integrated Scattered Intensity Versus Horizontal Range at Ground Level for a Point Isotropic 0.43μ Wavelength Source at 30 km Altitude

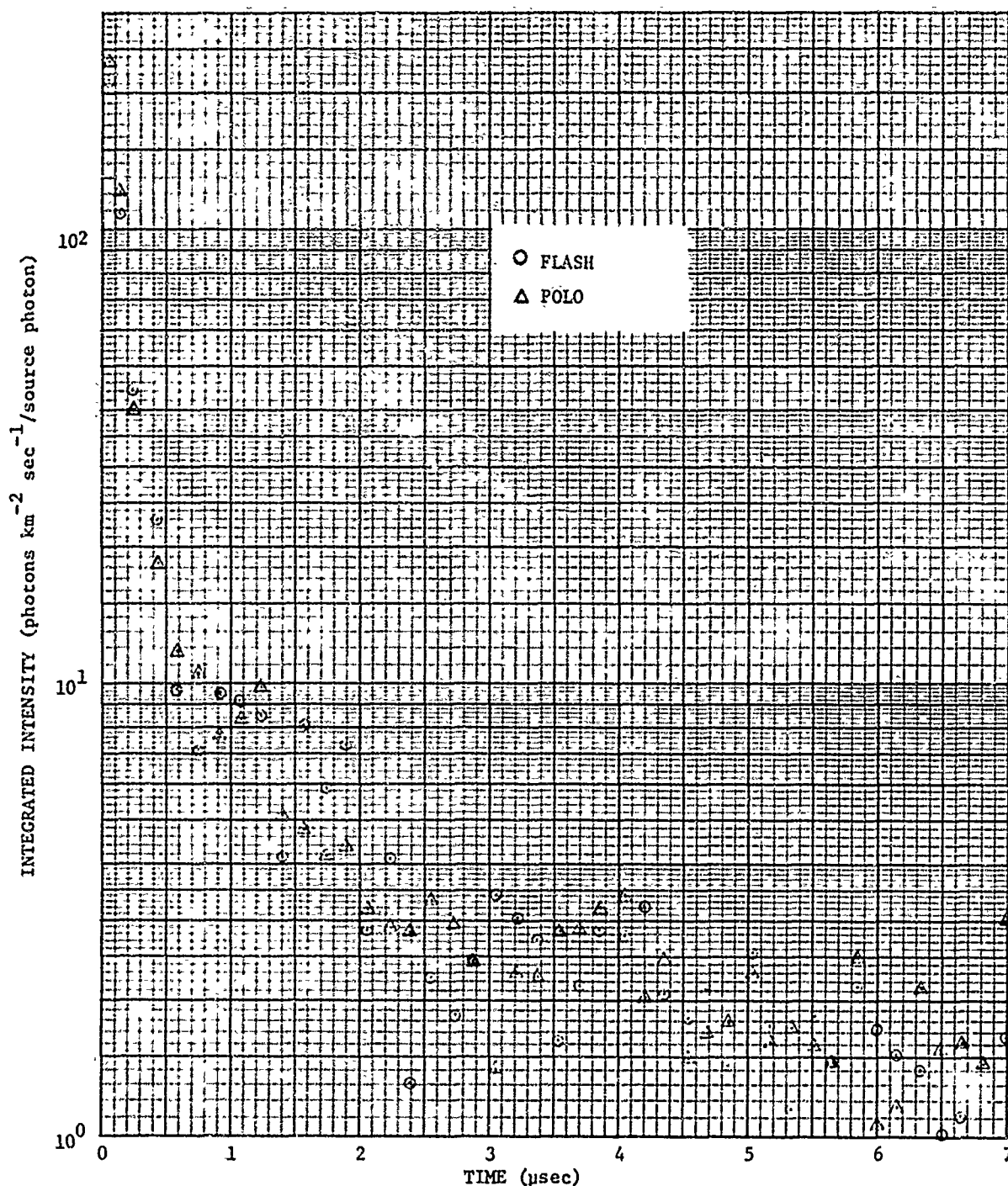


Fig. 17. Time Distribution of the Integrated Scattered Light Intensity at a Ground Level Receiver Located 25 km Horizontal Range for a Point Isotropic 0.43μ Wavelength Source at 30 km Altitude: Ground Albedo = 0.8

The time distribution of the integrated scattered intensity is given in terms of retarded time rather than real time. The retarded time is defined as the time elapsed as the photon travels its random path from the source to the receiver minus the time required for the direct radiation to reach the receiver. Thus, the same set of time intervals may be used to record the time distribution at each receiver position.

The time distribution of the scattered light intensity given in Fig. 17 shows only the time distribution of the radiation received within the first 7.0 microseconds. Approximately 60% of the scattered radiation is received within 7.0 microseconds after the arrival of the direct radiation, with the remaining 40% spread over a considerably longer time period.

The comparison between the point source version of FLASH and the POLO and TPART-I results as shown in Figs. 15, 16, and 17 demonstrated an unexpected advantage of the backward Monte Carlo approach to the atmospheric scattering problem. The first attempts to calculate the scattered intensities at the receiver positions with the POLO and TPART-I programs produced results which had large statistical fluctuation and the comparison of those results with results from the point source version of FLASH was inconclusive. The statistical fluctuation of the FLASH results was found to be much smaller than the statistical fluctuations in the results obtained from the POLO and TPART-I programs. An analysis of the calculated data was made in an effort to understand why the statistical fluctuation of the POLO and TPART-I data was so much larger than that for the point source FLASH data. This analysis led to the conclusion that, if the source was located in a more dense portion of the atmosphere than was the receiver, then more accurate results would be obtained if the photon histories were started from a source located at the receiver altitude and the statistical estimates of the scattered intensities were made for the receiver located at the source altitude. The backward Monte Carlo method employed by FLASH takes advantage of this phenomenon because the photon histories were started at the receiver positions where the atmosphere is a factor of one-hundred more dense than it was at 30 km altitude where the source was positioned.

Further analysis of the problem also indicated that one could expect the same results for an isotropic source positioned at ground level and receivers located at 30 km altitude as one would obtain for the case where the source is at 30 km altitude and the receivers are at ground level. The POLO and TPART-I programs were rerun with the source positioned at ground level and the receiver positioned at 30 km altitude. The POLO and TPART-I data shown in Figs. 15, 16, and 17 are from the calculations where the source and receiver altitudes were reversed.

A series of runs were made with the point source version of FLASH to investigate the application of the program to problems involving receivers located over the horizon from the source. FLASH input data were prepared for a model atmosphere defined by the Elterman 1964 model atmosphere (Ref. 15) for wavelengths of 0.544, 0.694, and 1.065 μ . The Elterman 1964 model atmosphere is for clear atmospheric conditions having a ground level meteorological range of approximately 25 km. Water vapor and CO₂ absorption were neglected for these wavelengths. The aerosol phase matrix for each of the above mentioned wavelengths was computed with the MIE-II program for an aerosol size distribution described by Deirmendjian's haze M model (Ref. 16). An isotropic source was positioned at 4 km altitude and the receivers were placed at ground level and at ground ranges between 250 and 1750 km from the radial through the source point. All of the receiver positions were over-the-horizon from the source point.

The first FLASH runs for the above described conditions were made without the use of the option for biasing the selection of the collision positions or the directions after scattering. The results obtained for the integrated intensity at each receiver position for the 0.544 μ wavelength source when the ground albedo was 0.8 is shown in Fig. 18. No results were obtained for receivers positioned at ground ranges greater than 1500 km. The FLASH problems were rerun using the pathlength biasing option that samples distances between collisions from a linear distribution of geometric distances and using the options to sample the photon's direction after scattering along its backward path from a distribution

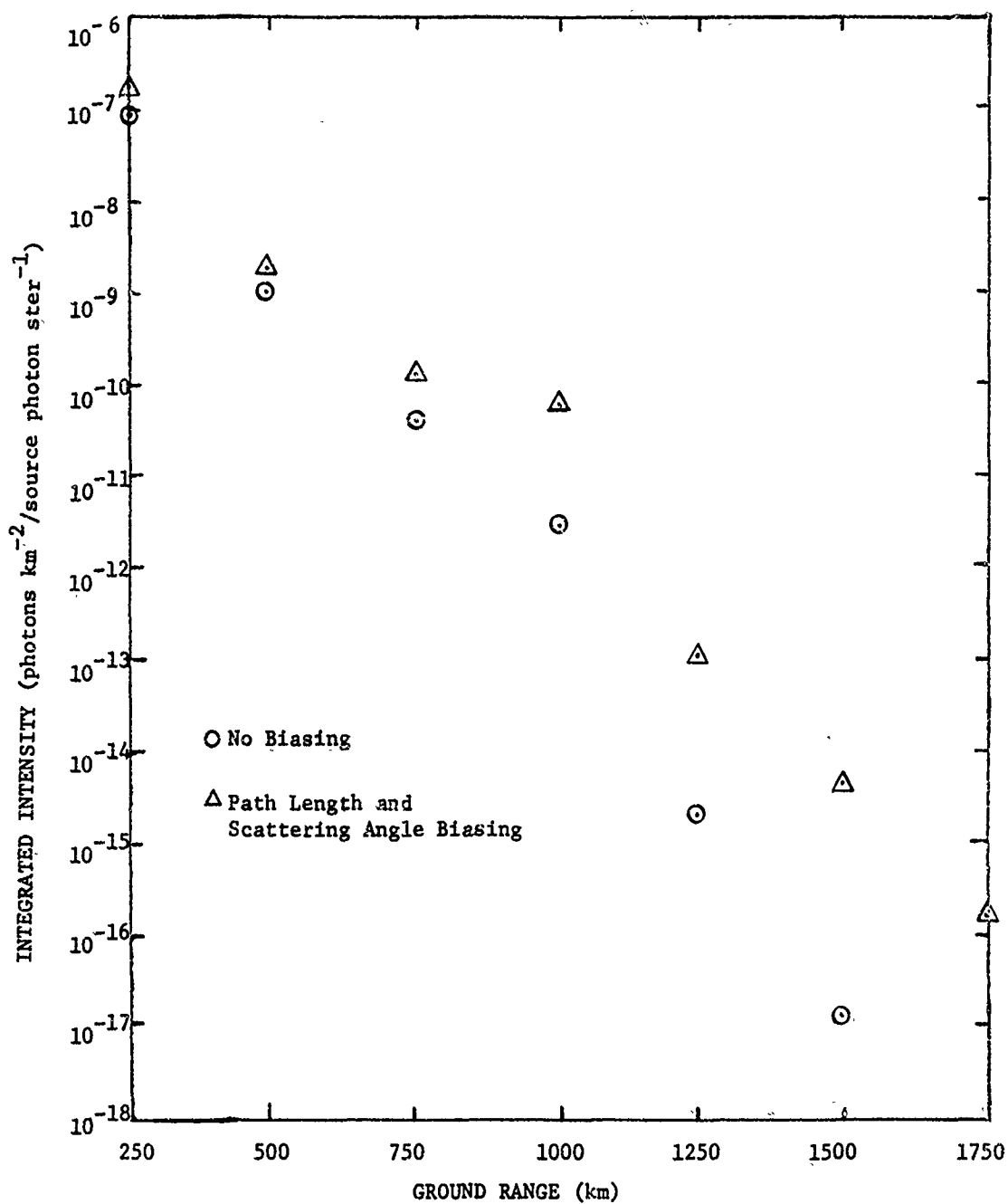


Fig. 18. Effect of Path Length and Scattering Angle Biasing on FLASH Calculations of the Integrated Scattered Intensities at Ground Range for Receivers Located Over the Horizon from a 0.544 μ Wavelength Point Isotropic Source at 4 km Altitude: Ground Albedo = 0.8

peaked in the horizontal direction and in the azimuthal direction of the source relative to the last receiver position. The biased distributions for the selection of the photons zenith and azimuthal directions after scattering are those defined in Section 3.1.8.1 using $a = 1.5$ and $Z = 0.75$. Using a value of $a = 1.5$ in the biased zenith angle distribution given by Equation (3.24) causes 50% of the directions after collision to be sampled between zenith angles of 70° and 110° while the value of 0.75 for Z in Equation (3.26) causes 50% of the azimuthal direction after collision to be sampled within $\pm 48^\circ$ from the azimuth through the source position when the collision position is given in terms of the last receiver position. The integrated scattered intensity computed using the above described biasing techniques for a wavelength of 0.544μ and a ground albedo of 0.8 are also plotted in Fig. 18. It is seen that the use of the above described biasing techniques resulted in larger integrated intensities at the receiver positions. It is felt that the results shown in Fig. 18 that were obtained without the use of biasing are underestimates of the integrated scattered intensity as a function of ground range.

The integrated scattered intensities computed for wavelengths of 0.544, 0.694, and 1.605μ by FLASH when using pathlength and scattering angle biasing for a surface albedo of 0.8 are shown in Fig. 19. The integrated scattered intensity for the 0.544μ wavelength source appears to not fall off as fast with distance as does the integrated scattered intensities for wavelengths of 0.694 and 1.605μ .

This initial application of the point source version of FLASH to over-the-horizon problems indicates that the program is applicable to problems of this nature. More studies need to be conducted to determine the extent of the biasing that should be applied to provide the optimum calculation.

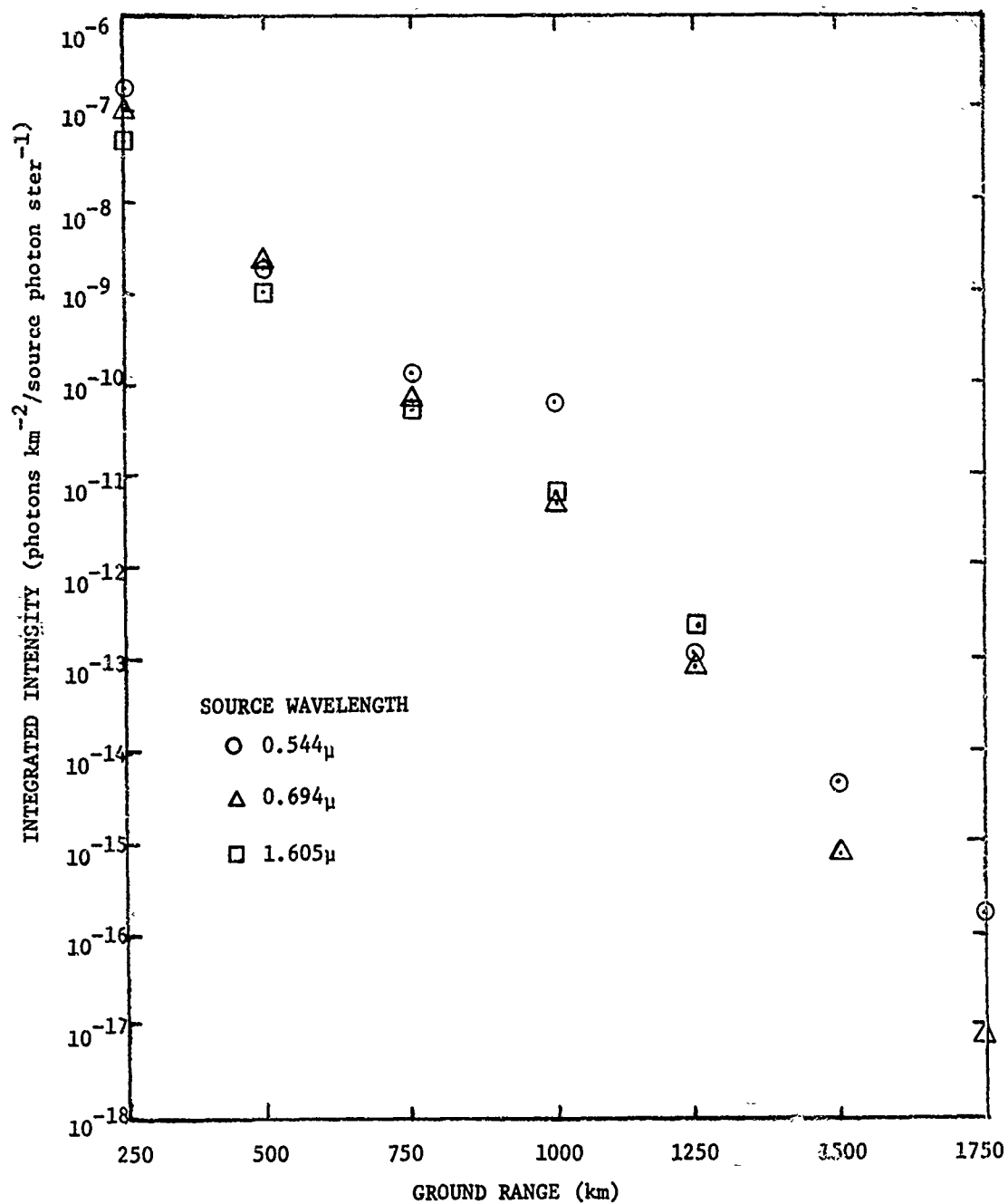


Fig. 19. Variation of the Integrated Scattered Intensity at Ground Level versus Ground Range for Receivers Located Over-the-Horizon from a Point Isotropic Source at 4 km Altitude: Ground Albedo = 0.8

5.2 Validation of the Plane Source Version of FLASH

The plane source version of FLASH has been validated through comparisons made with calculations that have been made for plane-parallel Rayleigh atmospheres (Ref. 17) and with calculations for the earth and atmospheric reflected intensities at a satellite position as given by the RRA-89 procedure (Ref. 18). Comparison of the FLASH calculations of the reflected and transmitted intensities for a spherical-shell Rayleigh atmosphere with other calculated data for a plane-parallel Rayleigh atmosphere allows one to determine the differences one would expect to find between calculations for spherical-shell and plane-parallel atmospheres.

Several FLASH calculations were run to determine the intensity and degree of polarization of the scattered light emerging from a homogeneous Rayleigh spherical-shell atmosphere for comparison with similar data reported by Coulson, et. al. (Ref. 17) for a plane-parallel homogeneous Rayleigh atmosphere. Some of the comparisons are shown in Figs. 20 through 27.

Figure 20 shows the transmitted scattered intensity in the zero azimuthal plane as a function of the receiver nadir angle for a plane parallel source incident at $\cos\theta_D = 0.92$ upon a homogeneous Rayleigh atmosphere having an optical thickness of 0.25. The triangle points shown in Fig. 20 are the data computed by FLASH for a ground albedo of 0.8 and the circled points are the data computed by FLASH for a ground albedo of 0.0. The smooth curves shown in Fig. 20 are drawn through the points computed by Coulson, et. al. (Ref. 17). The agreement with Coulson's data is excellent and indicates that, except for receiver nadir angles close to 90° , there is very little difference between the scattered intensities transmitted through the plane parallel and spherical shell atmospheres for this angle of incidence.

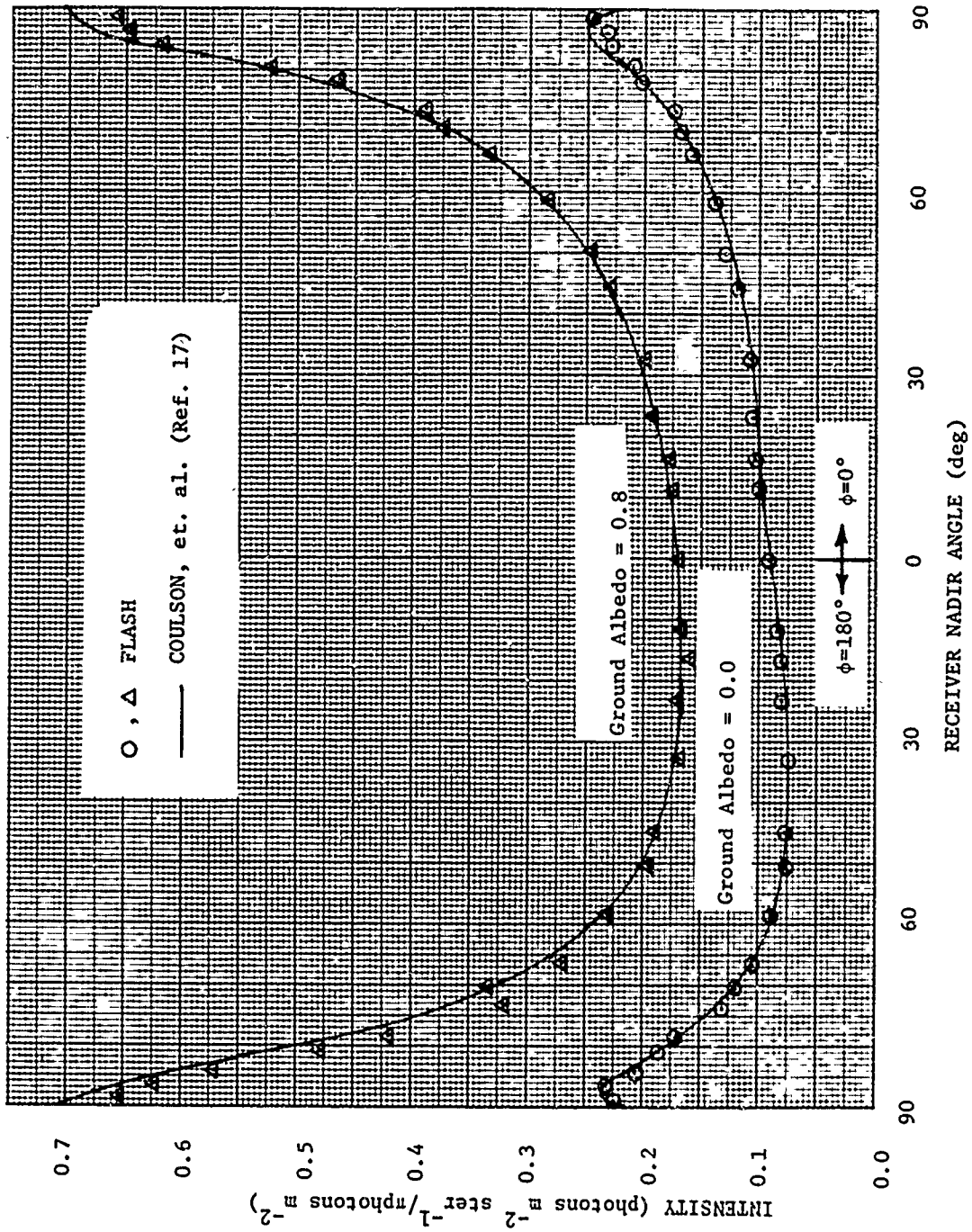


Fig. 20. Variation of the Scattered Intensity Transmitted Through a 0.25 Mean-free-path Thick Rayleigh Atmosphere in the $0^\circ, 180^\circ$ Azimuthal Plane with the Receiver Nadir Angle: Zenith Angle of Incidence of $\theta_D = \cos^{-1} 0.92$.

Figure 21 shows the degree of polarization which was computed for the transmitted radiation shown in Fig. 20. Again the agreement between the FLASH calculation and Coulson's data for the plane parallel Rayleigh atmosphere is excellent.

A difference in the results computed by FLASH and those reported by Coulson, et. al. for the scattered intensity transmitted through the 0.25 mean-free-path thick homogeneous Rayleigh atmosphere was observed for radiation incidence at $\cos\theta_D = 0.1$. Figure 22 shows a comparison of the transmitted intensities computed for several receiver nadir angles in the $30^\circ - 210^\circ$ azimuth plane. The scattered intensities transmitted through the spherical shell atmosphere as computed by FLASH are higher at all receiver nadir angles except for those receiver nadir angles larger than 79° in the 210° azimuthal direction. The scattered intensities at the large nadir angles in the 30° azimuth plane are higher than those for the plane parallel atmosphere by approximately 40%.

The degree of polarization which was computed for the radiation transmitted in the $30^\circ - 210^\circ$ azimuth plane is shown as a function of the receiver nadir angle in Fig. 23. It is difficult to determine whether the differences observed between the plane source FLASH data and Coulson's data are due to the difference in the shape of the atmosphere or just to the statistical fluctuation in the FLASH data.

The scattered intensity reflected from the 0.25 mean-free-path thick Rayleigh atmosphere was also computed for several receiver zenith angles in the $30^\circ - 210^\circ$ azimuthal plane for radiation incident at $\cos\theta_D = 0.92$. A comparison of these data with those from Coulson's calculations are shown in Fig. 24. Again, the comparison between the calculated scattered intensities for the spherical-shell and plane-parallel atmospheres is good except for the very large receiver zenith angles, where the intensities reflected from the spherical shell atmosphere underpredicts that reflected from the plane-parallel atmosphere. The degree of polarization for the reflected radiation as computed with

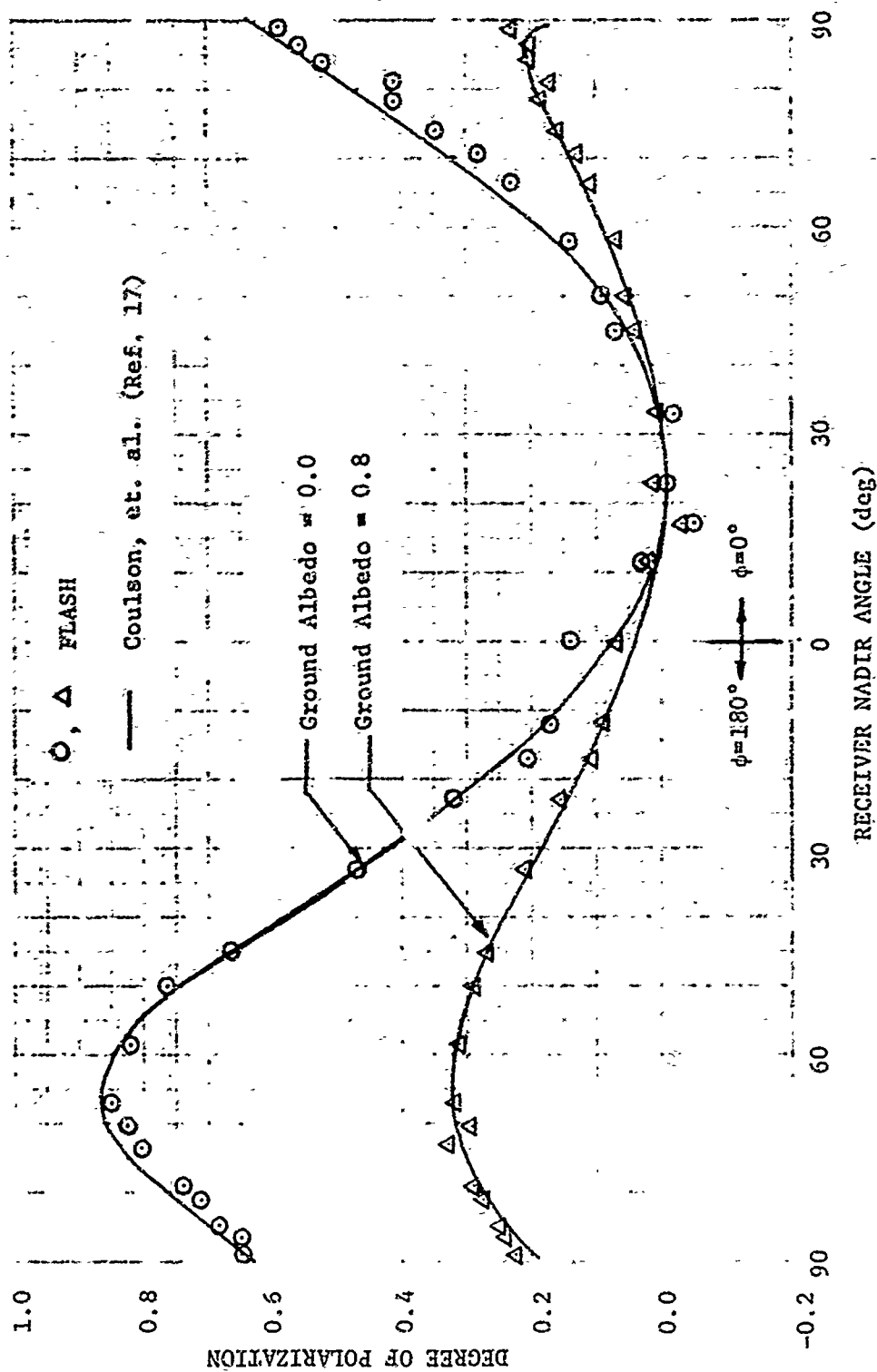


Fig. 21. Variation of the Degree of Polarization of the Scattered Intensity Transmitted through a 0.25 Mean-free-path Thick Rayleigh Atmosphere in the 0° , 180° Azimuthal Plane with the Receiver Nadir Angle: Zenith Angle of Incidence of $\theta_D = \cos^{-1} 0.92$

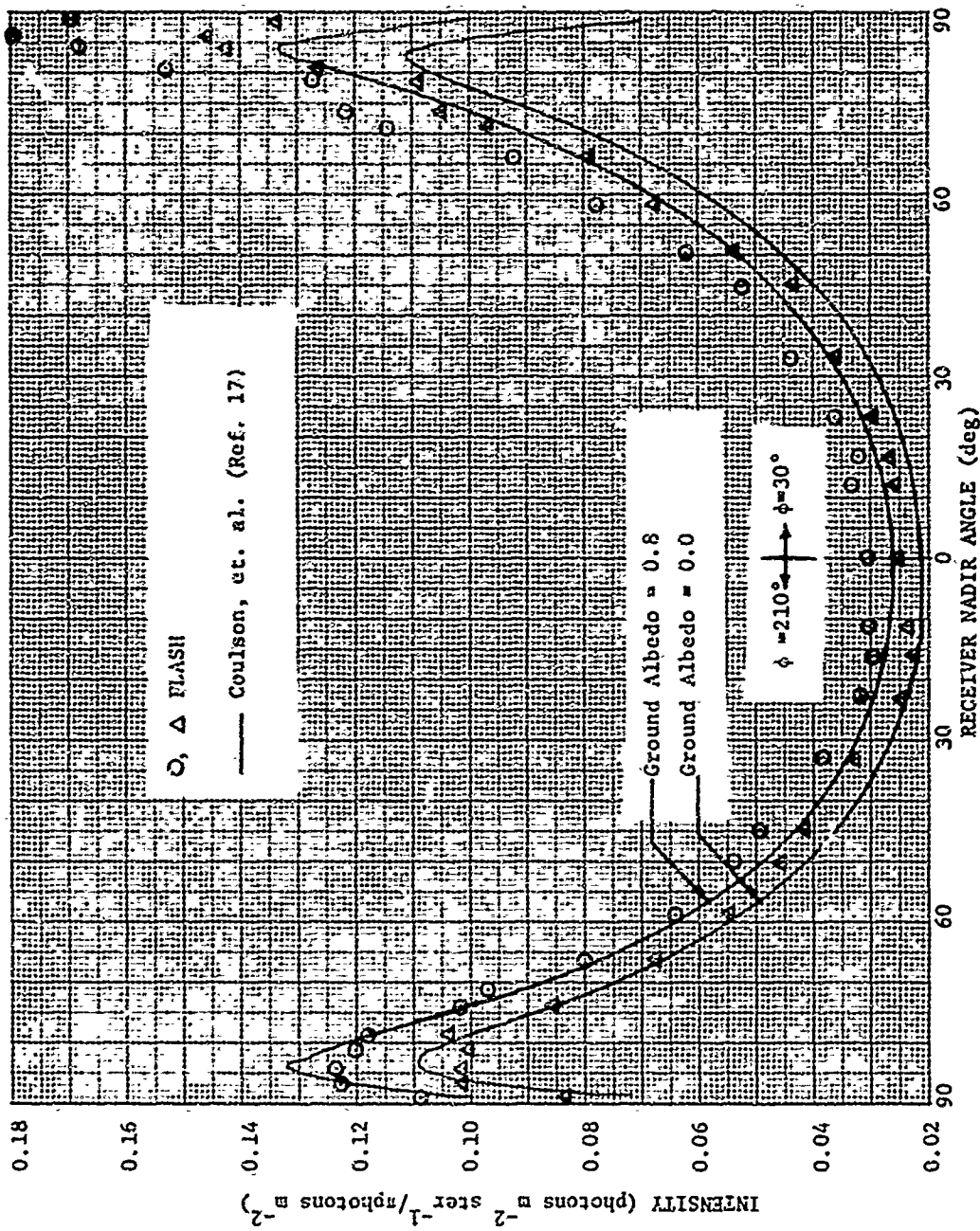


Fig. 22. Variation of the Scattered Intensity Transmitted through a 0.25 Mean-free-path Thick Rayleigh Atmosphere in the 30° - 210° Azimuthal Plane with the Receiver Nadir Angle: Zenith Angle of Incidence of $\theta_D = \cos^{-1} 0.1$

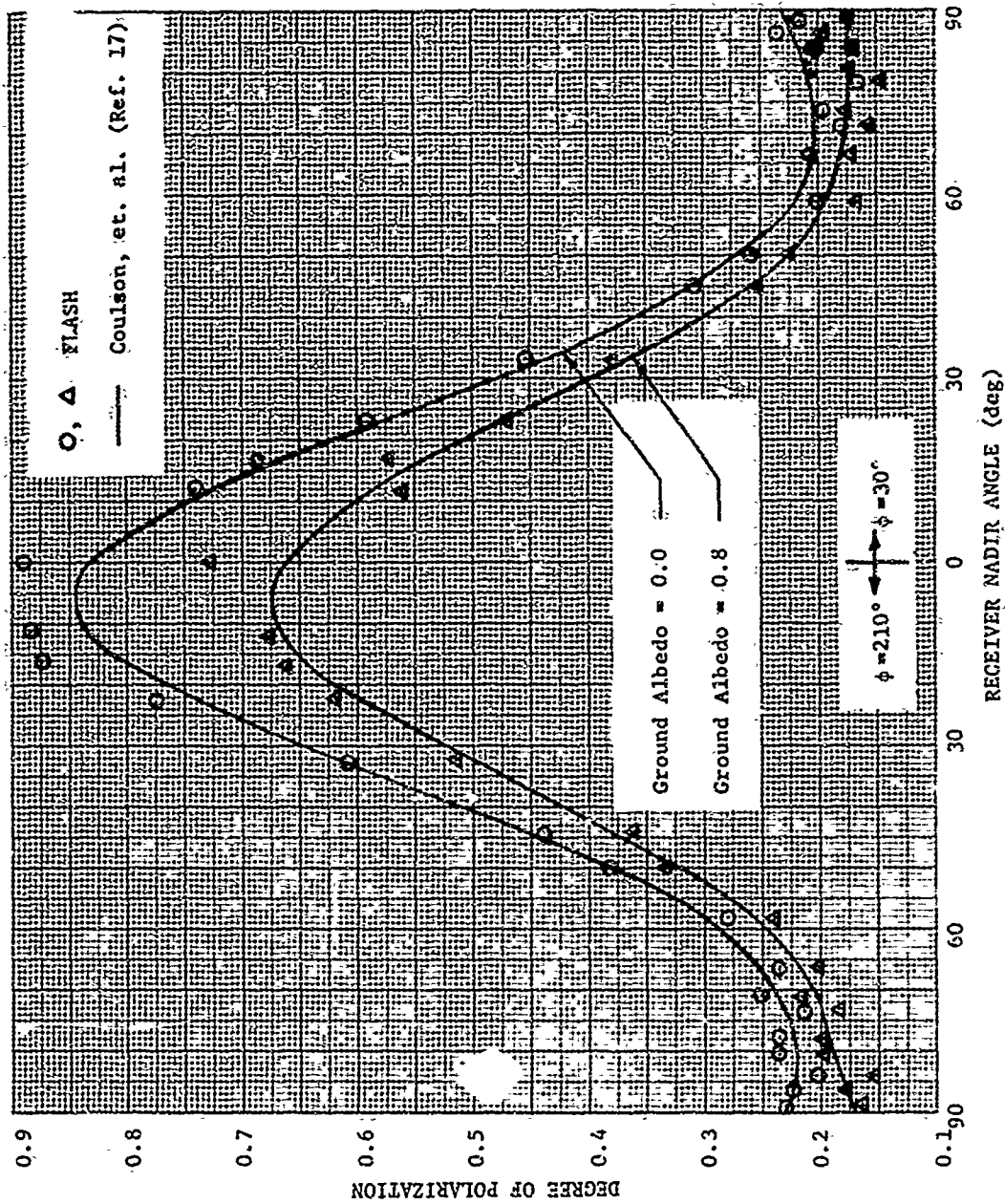


Fig. 23. Variation of the Degree of Polarization of the Scattered Intensity Transmitted Through a 0.25 Mean-free-path Thick Rayleigh Atmosphere in the $30^\circ \pm 210^\circ$ Azimuthal Plane with the Receiver Nadir Angle: Zenith Angle of Incidence of $\theta_p = \cos^{-1} 0.1$

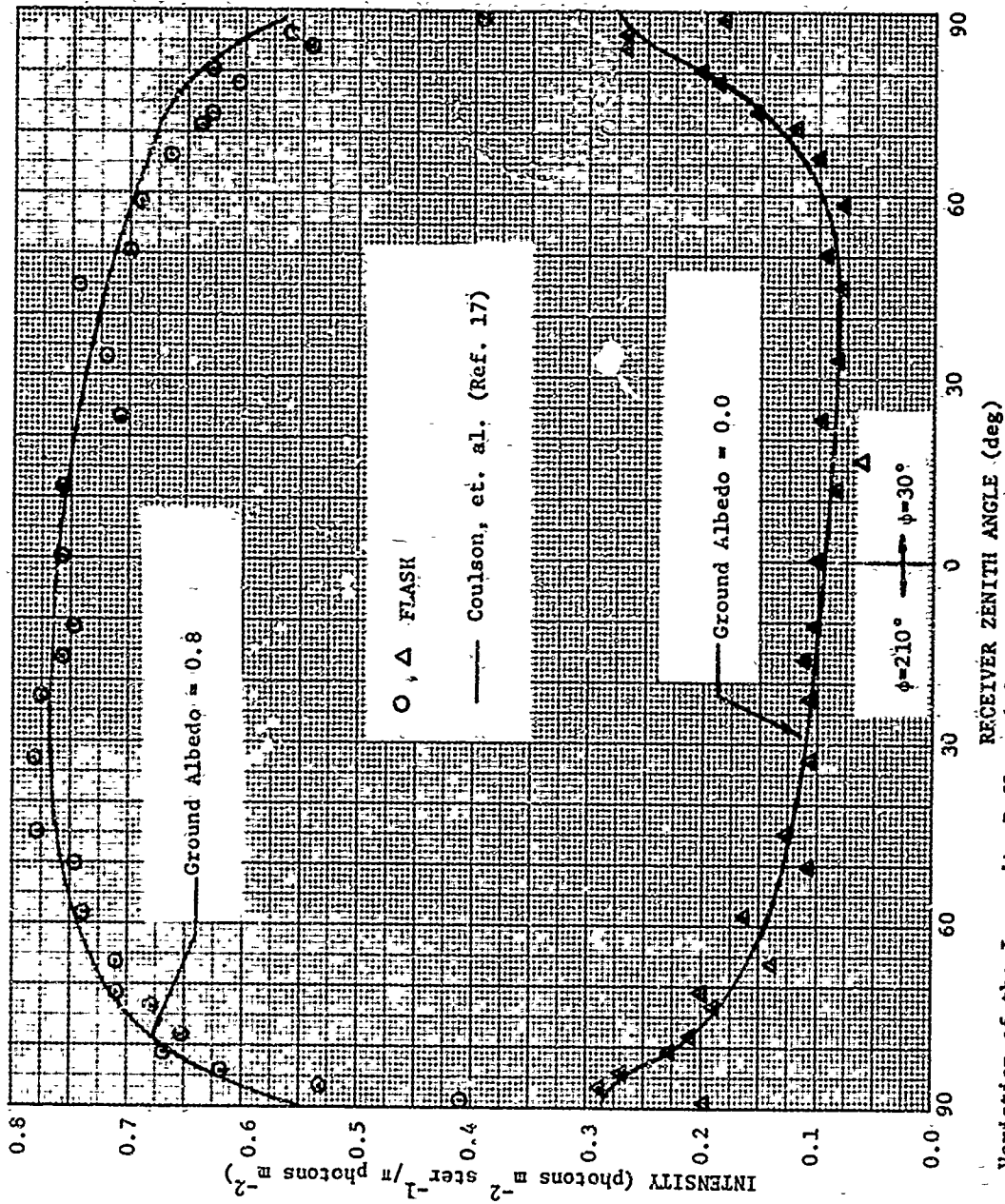


Fig. 24. Variation of the Intensity Reflected from a 0.25 Mean-free-path Thick Rayleigh Atmosphere in the 30°, 210° Azimuthal plane with the Receiver Zenith Angle: Zenith Angle of Incidence of $\theta_D = \cos^{-1} 0.92$

FLASH is compared with Coulson's calculation of the degree of polarization in Fig. 25. It is seen in Fig. 25 that there are no apparent differences in the degree of polarization computed for the two atmospheres for the radiation incident at $\cos\theta_D = 0.92$.

The calculated values of the intensity reflected from the plane-parallel and spherical-shell atmospheres for radiation incident at $\cos\theta_D = 0.1$ are compared in Fig. 26. The reflected intensity in the 0° - 180° azimuth plane is shown for several receiver zenith angles. The reflected intensity for the radiation incident upon the spherical-shell atmosphere at an angle of $\theta_D = \cos^{-1}0.1$ tends to slightly over-predict that reflected from the plane-parallel atmosphere at all but the extremely large receiver zenith angles. The degree of polarization of the reflected radiation for light incident at $\cos\theta_D = 0.1$ is shown in Fig. 27. The degree of polarization computed for the radiation reflected from the spherical-shell atmosphere in the zenith direction appears to be slightly higher than that computed for the plane-parallel atmosphere.

The comparisons of the FLASH calculations of the angular distribution of the scattered intensity and degree of polarization of the radiation transmitted and reflected from a spherical-shell homogeneous Rayleigh atmosphere with similar data from Coulson's calculations for a plane-parallel atmosphere indicate the validity of the plane source version of FLASH. The comparisons also indicate that one should expect slightly different results for radiation emitted from the spherical-shell atmosphere for zenith angles of incidence greater than $\theta_D = \cos^{-1}0.1$ than the results one would get for a plane-parallel atmosphere.

As a further checkout of the calculational methods used in the plane source version of the FLASH program, calculations were run to determine the scattered intensities reflected from an inhomogeneous atmosphere containing aerosols to a satellite positioned 300 nautical miles above the earth's surface. These FLASH calculations were then compared with similar results computed with the RRA-89 procedure (Ref. 18). The atmospheric model was taken to be that for a clear maritime atmosphere having a ground level meteorological range of 25 km. The

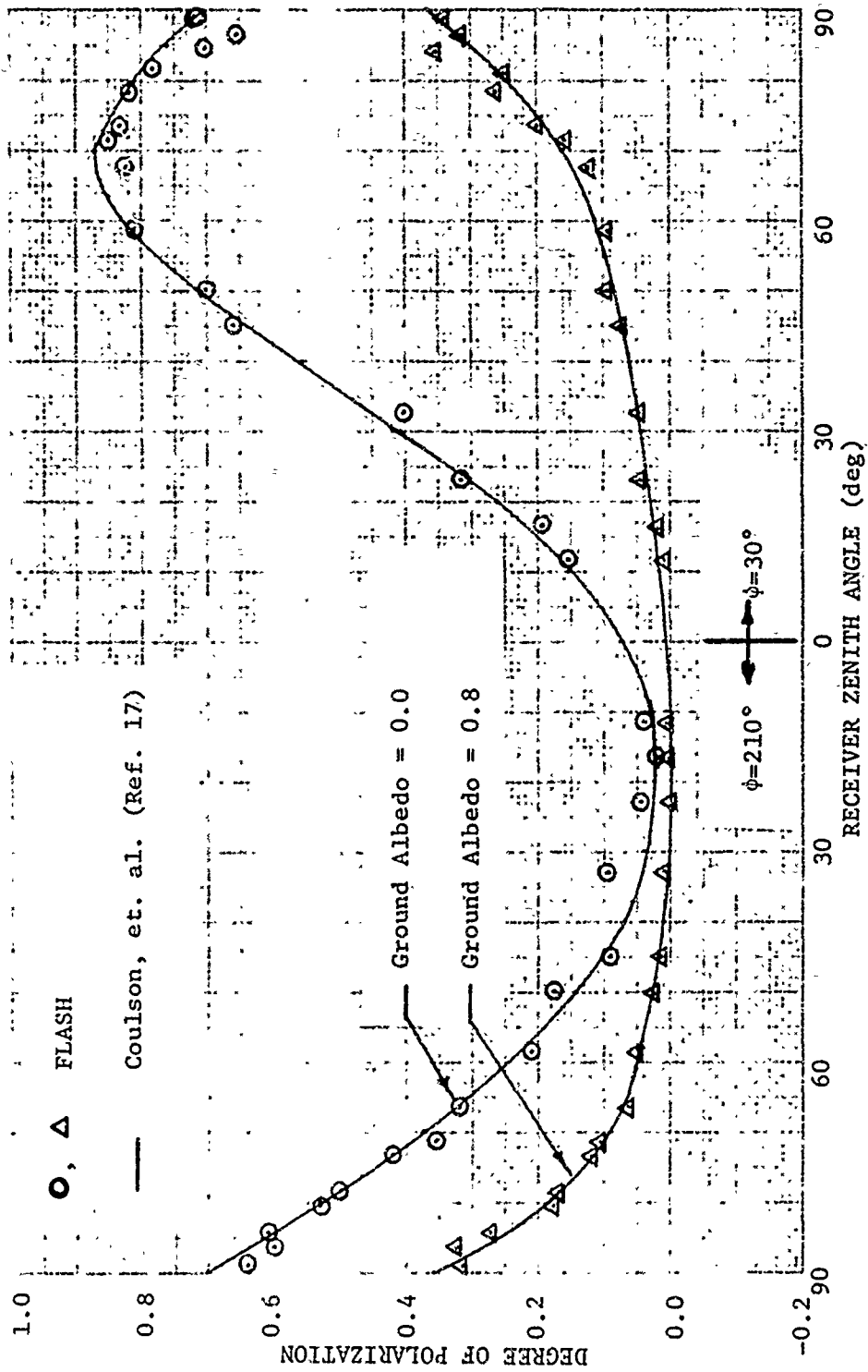


Fig. 25. Variation of the Degree of Polarization of the Intensity Reflected from a 0.25 Mean-free-path Thick Rayleigh Atmosphere in the 30° , 210° Azimuthal Plane with the Receiver Zenith Angle: Zenith Angle of Incidence of $\theta_D = \cos^{-1} 0.92$

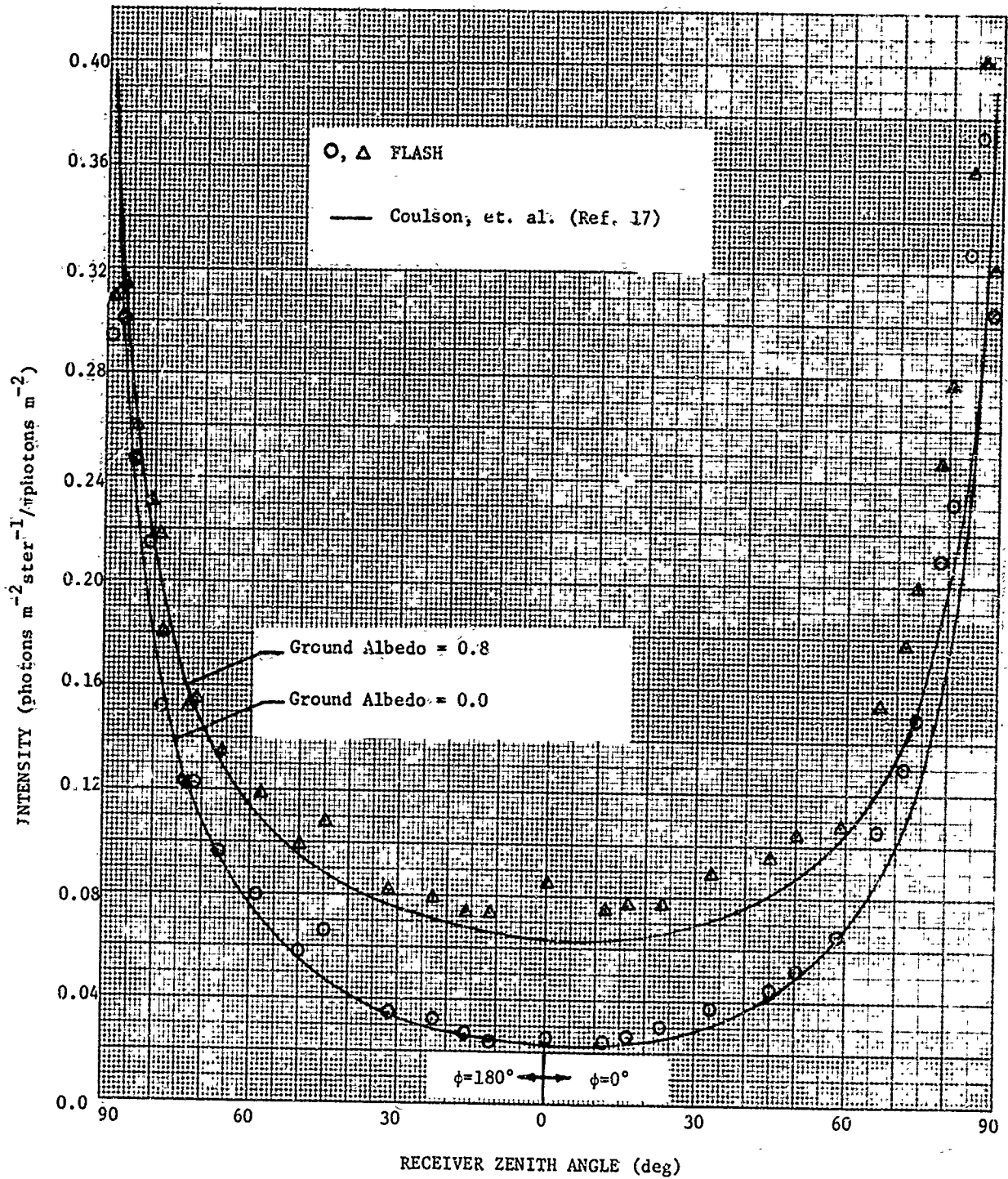


Fig. 26. Variation of the Intensity Reflected from a 0.25 Mean-free-path Thick Rayleigh Atmosphere in the 0° , 180° Azimuthal Plane with the Receiver Zenith Angle: Zenith Angle of Incidence of $\theta_D = \cos^{-1} 0.1$

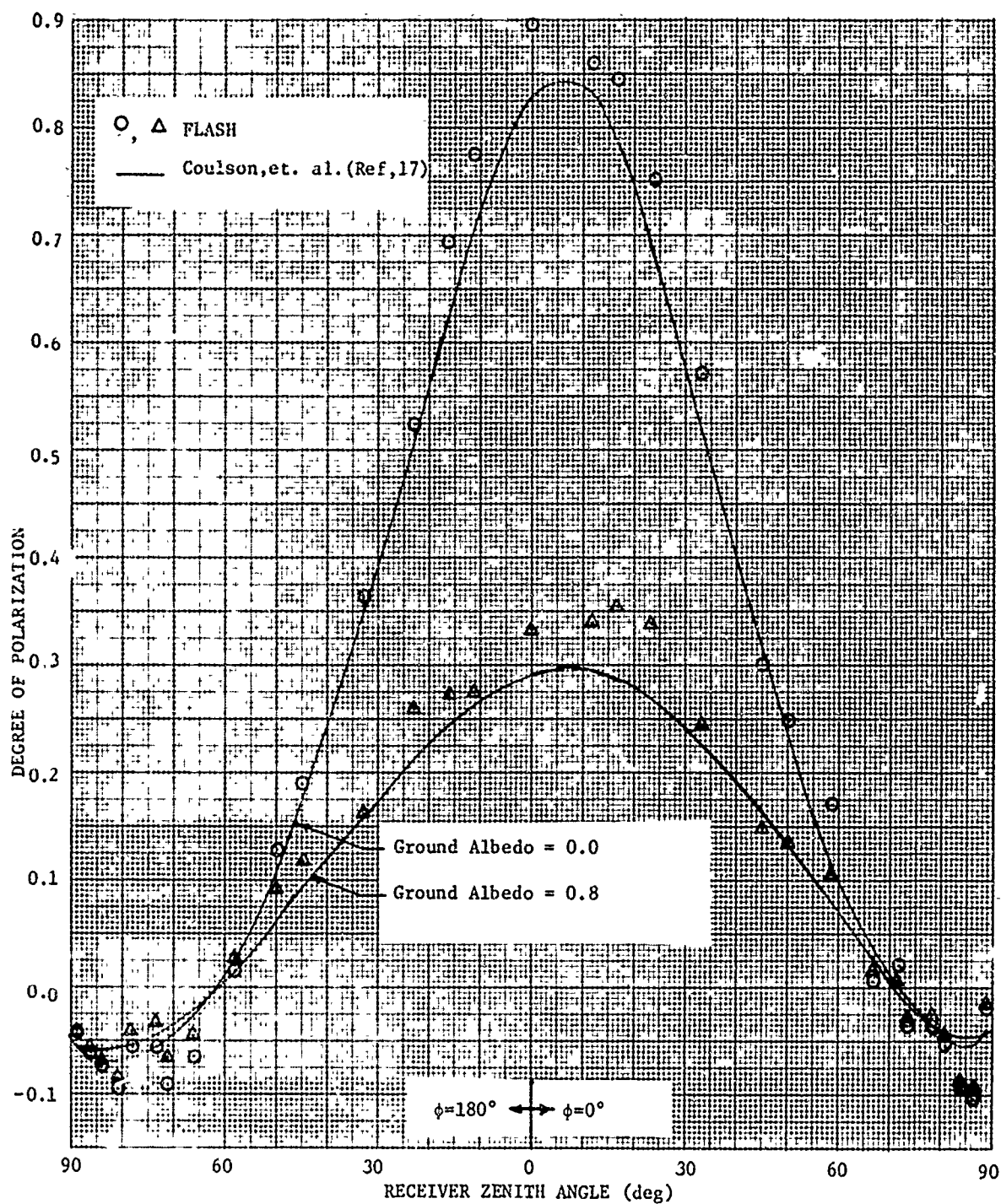


Fig. 27. Variation of the Degree of Polarization of the Intensity Reflected from a 0.25 Mean-free-path Thick Rayleigh Atmosphere in the 0°, 180° Azimuthal Plane with the Receiver Zenith Angle: Zenith Angle of Incidence of $\theta_D = \cos^{-1} 0.1$

variation of the aerosol number density with altitude was defined by the Elterman 1968 model atmosphere (Ref. 14). The aerosol size distribution was assumed to be Deirmendjian's Haze M model (Ref. 16). FLASH calculations were made for several satellite positions, where the satellite position is defined in terms of the earth angle which is the angle θ_D between the earth radials through the satellite position and the sun's position. The reflected intensities for 0.37μ and 0.78μ wavelength light were computed for the above described model atmosphere. The scattering calculations considered the ground surface to be a Lambert reflection surface having an albedo of 0.0 and 0.8. The results of the FLASH calculations giving the integrated intensity at the receiver as a function of the earth angle are compared in Figs. 28 and 29 with similar results from the RRA-89 program for wavelengths of 0.37 and 0.78μ , respectively.

The RRA-89 procedure was originally developed to provide a method of computing the monochromatic light intensity above model atmospheres that results from atmospheric scattering and diffuse reflection by the ground surface. The RRA-89 program utilizes as a basic set of data the angular distributions of the light intensity reflected from a plane-parallel atmosphere for each of a set of incident directions such as Coulson et. al's. data for a Rayleigh atmosphere (Ref. 17) or the data from the LITE-II program (Ref. 19) for aerosol atmospheres. The intensity at a satellite position is computed by RRA-89 by dividing the surface of the upper atmosphere below the satellite position into a number of area increments. For each area increment the angle of incidence of the sunlight and the reflection direction toward the satellite are computed. It is assumed that the intensity reflected from the area increment is equivalent to the intensity reflected per unit area from a plane-parallel atmosphere for the corresponding incident and reflection directions times the size of the area increment. By summing the intensity reflected from all areas covering the surface of the upper atmosphere seen by the satellite and irradiated by the sun, the total intensity reflected from the atmosphere is obtained.

The comparisons seen in Figs. 28 and 29 between the RRA-89 data and that from the plane source FLASH program show good agreement in most

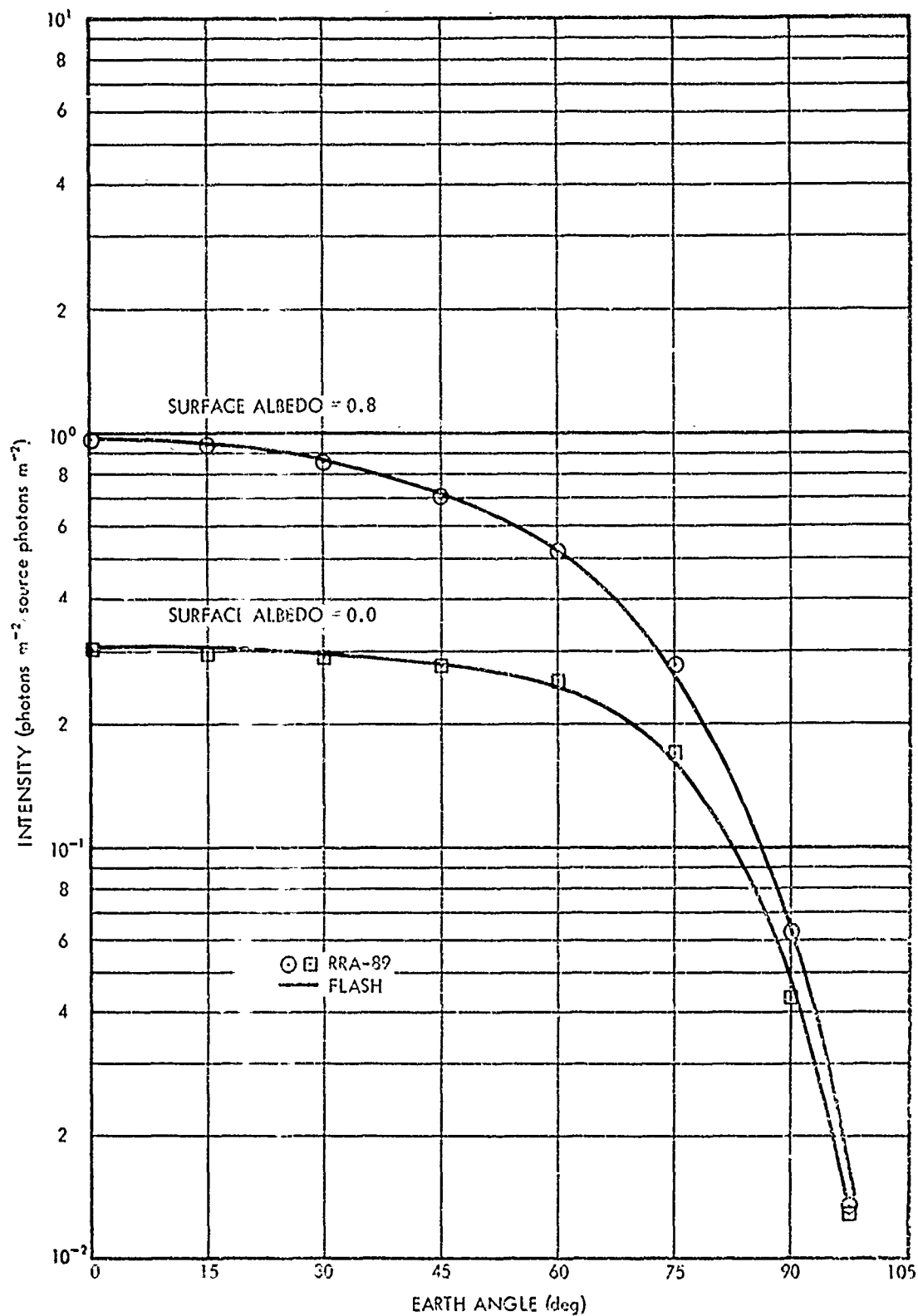


Fig. 28. Reflected Intensity at 300 Nautical Miles Altitude as a Function of the Earth Angle: $\lambda = 0.37\mu$

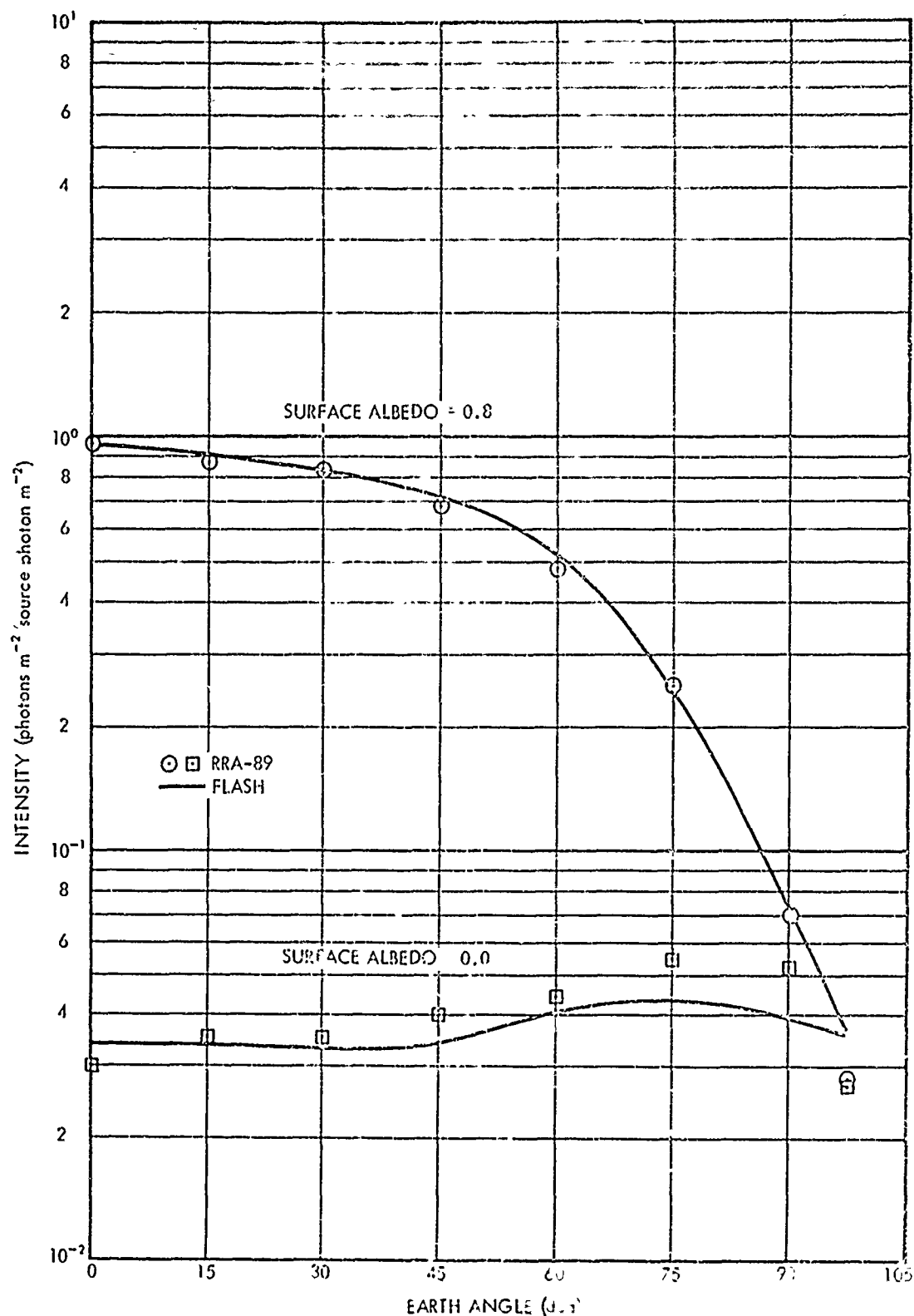


Fig. 29. Reflected Intensity at 300 Nautical Miles Altitude as a Function of the Earth Angle: $\lambda = 0.78\mu$

cases for earth angles less than 90 degrees. It appears that for the satellite at 97.5° the plane source FLASH program gives slightly higher results than the RRA-89 procedure. This is not unexpected because the plane-parallel atmospheric data used in the RRA-89 program were limited to angles of incidence of less than 90° while the plane source FLASH program treats reflection from areas on the upper atmospheric surface that are not irradiated by the sun.

5.3 Validation of the Volume Source Version of FLASH

The volume source version of the FLASH program is designed to compute both the direct and scattered light intensity at each of a set of view angles defined about the receiver positions. The direct intensity for each receiver solid angle of view that looks at the source volume is computed for the direction passing through the center of the receiver solid angle of view. The accuracy of the direct intensity computed in this manner depends upon the variation of the direct intensity within the receiver solid angle of view. If the emission of the volume source does not vary radically with spatial position and the receiver solid angle of view is relatively small, then the direct intensity as computed with the volume source FLASH program should be accurate.

A test problem was run with FLASH to compute the angular distribution of the direct intensity escaping from the bottom of a homogeneous atmosphere which emits radiation isotropically and uniformly throughout the atmosphere. A homogeneous spherical-shell atmosphere with an optical thickness of 0.8 was defined to encompass the earth. In FLASH the atmosphere was taken to be 50 km in thickness. An extremely large spherical volume source was defined with its center at the mid-altitude of the atmosphere, Fig. 30. The spherical volume source was assumed to emit $0.916 \text{ photons ster}^{-1} \text{ per km of altitude}$. The source radiation was emitted isotropically and was distributed uniformly within the source volume illustrated in Fig. 30 which lies within the spherical-shell atmosphere. A receiver was positioned on a radial through the center of the source volume at the lower surface of the atmosphere and the direct intensity was computed for several view angles at the receiver position. The direct intensity as a function of the angle between the emerging direct radiation and a radial through the receiver position is compared in

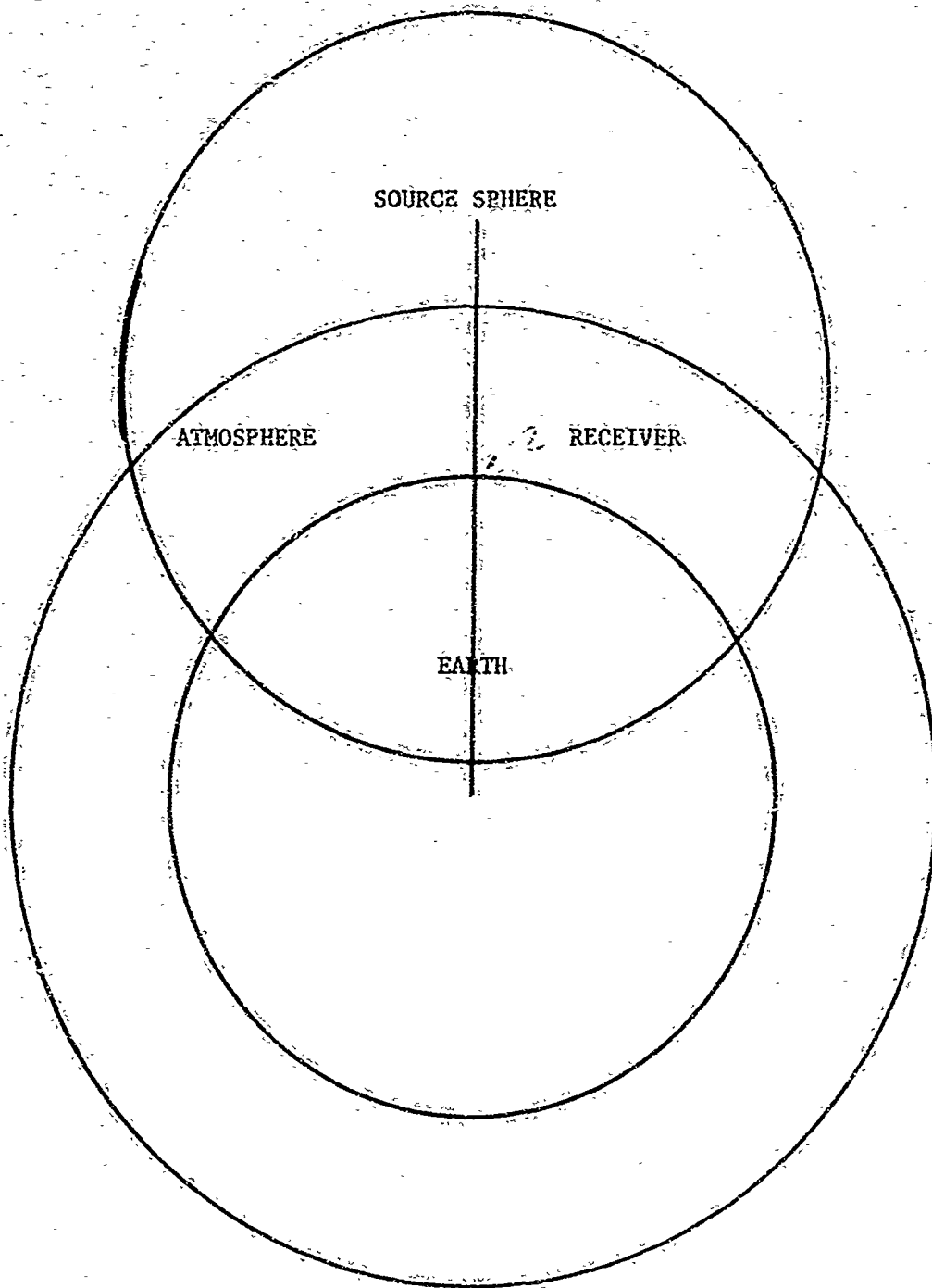


Fig. 30. Geometry for Spherical Volume Source

Table III with Horak's calculation (Ref. 20) of the direct intensity emerging from a uniform emitting plane parallel atmosphere. The FLASH results agree to the second significant digit with Horak's data indicating the direct intensity routine in the FLASH program is computing the emerging direct intensity correctly.

FLASH calculations were made to determine the polar angle distribution of the scattered radiation emerging from the above described uniform emitting atmosphere. For the scattering calculation the atmosphere was assumed to have a single scattering albedo of 0.5 and to scatter radiation isotropically. The scattered intensity emerging from the bottom of the atmosphere as given by FLASH calculations are compared in Table IV with scattering calculations reported by Horak (Ref. 20) for a plane-parallel atmosphere. The FLASH data given in Table IV were computed with two separate program runs. The first run used the spherical volume source routines and the second run used the lense type volume source routines to describe the emitting source. In the calculations using the lense type source, the source volume was contained between the two spherical surfaces bounding the upper and lower surfaces of the atmosphere and within a 30° half angle conic section with a vertex at the earth's center.

The agreement between the two different FLASH calculations of the emerging scattered intensities and Horak's calculations, as seen in Table IV, is not as good as the agreement noted previously for the emerging direct intensities because of the statistical fluctuations in the FLASH data. Even so, the agreement is good enough to indicate that the volume source versions of FLASH is computing the scattered intensities correctly.

One of the objectives in developing the volume source FLASH program, and the main reason for including the capability for treating spherical volume and spherical surface sources was to apply the program to the analysis of photographic data. Test problems were run to test the application of the FLASH program to problems of this nature. For the first test problem a 10 km radius spherical volume source emitting radiation at a wavelength of 0.43μ isotropically and uniformly with volume was located

TABLE III

Direct Intensity Emerging From a Uniform
Emitting Atmosphere as a Function of Angle

Emerging Angle	FLASH Spherical Volume	Horak's Exact Calculation
5° 54'	0.55361	0.552582
13° 31'	0.56225	0.560801
21° 07'	0.57704	0.575829
28° 38'	0.59897	0.598067
36° 00'	0.62842	0.628033
43° 12'	0.66601	0.666256
50° 09'	0.71167	0.713047
56° 48'	0.76504	0.768035
63° 06'	0.82478	0.829323
68° 57'	0.88522	0.892168
74° 17'	0.93934	0.947771
78° 59'	0.97744	0.984810
82° 59'	0.99505	0.998558
86° 09'	0.99944	0.999993
88° 25'	0.99995	1.000000
89° 41'	0.99999	1.000000

TABLE IV
 Scattered Intensity Emerging From a Uniform
 Emitting Atmosphere as a Function of Angle

Emerging Angle	FLASH Spherical Volume	FLASH Lense Volume	Horak's Exact Calculation
5°54'	.211	.212	.213
13°31'	.219	.205	.216
21°07'	.221	.231	.222
28°38'	.245	.233	.230
36°00'	.237	.250	.242
43°12'	.246	.257	.256
50°09'	.272	.238	.274
56°48'	.280	.328	.294
63°06'	.304	.294	.316
68°57'	.316	.356	.338
74°17'	.343	.347	.355
78°59'	.381	.324	.361
82°59'	.337	.317	.352
86°09'	.321	.292	.330
88°25'	.302	.306	.305
89°41'	.322	.316	.283

with its center at 21.3 km altitude. The source was embedded within the Elterman 1964 model clear atmosphere (Ref. 15) having a ground meteorological range of 25 km. The aerosol scattering distribution was defined as that for the Diermendjian Haze C model (Ref. 16). A ground level receiver was located at 53.2 km horizontal range or 57.3 km slant range from the center of the spherical source volume. A set of discrete directions for the radiation incident at the receiver was defined such that the directions correspond to 15 points distributed over the visible disk of the source as viewed from the receiver. Figure 31 shows the direct (upper number) and the scattered intensities (lower number) computed for each of the 15 points on the source disk. The units on the ordinate and abscissa in Fig. 31 are dependent upon the size of the photographic plate and are related to the view angles used in the FLASH program through the focal length of the receiver instrument. Since the 10 km radius source subtends a solid angle with an approximately 10° half angle at the receiver position, one unit of X or Y in Fig. 31 would correspond to a receiver polar angle of approximately 10° with respect to the line through the center of the source volume and the receiver position.

It is interesting to note the relationship between the scattered and direct radiation as a function of position on the source disk. The direct radiation has its maximum intensity at the center of the source disk but does not decrease for points in the upper part of the disk as rapidly as in the lower part. This is understandable because the direct radiation at the top of the disk has penetrated a thinner atmosphere than that at the bottom of the disk. The higher value at the center of the disk is due to the larger thickness of the source region along a diameter. This phenomenon is better observed by looking at a constant elevation across the source disk where the atmospheric attenuation is constant. The direct intensity decreases as the point nears the edge of the source disk. The scattered intensity along the vertical at $X = 0$ is seen to vary less with position than does the direct intensity. It is thought that the scattered intensities at the point $X = \pm 0.5$ and $Y = 0.0$ are too high due to statistical fluctuation. There is no physical

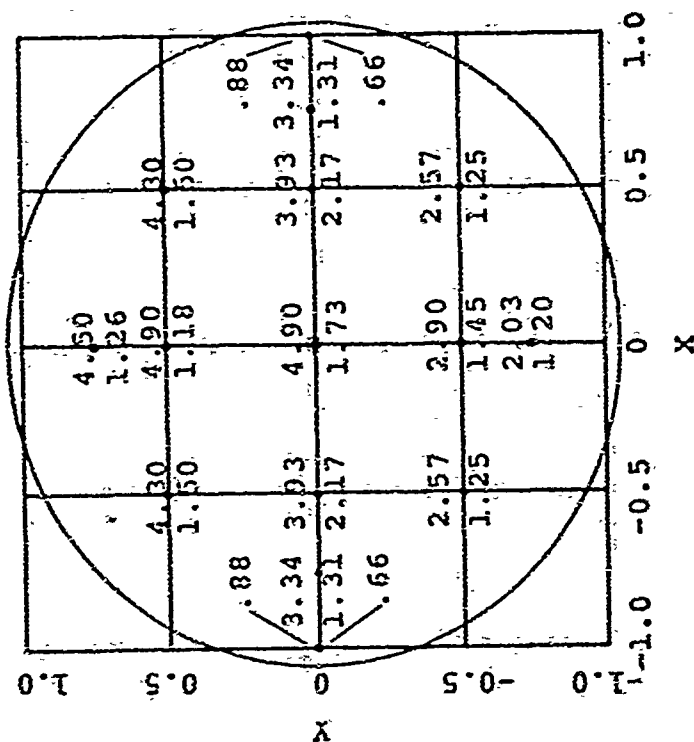
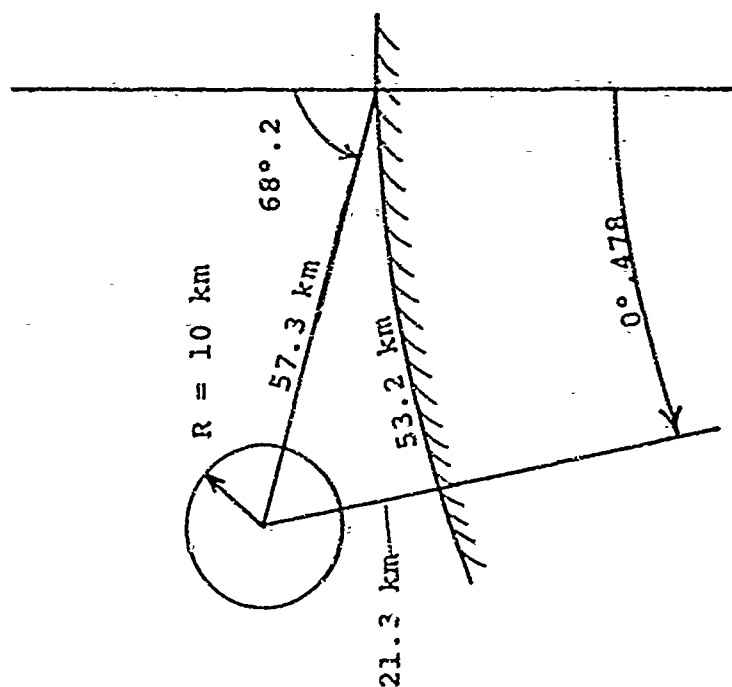


Fig. 31. Variation of the Direct and Air Scattered Intensities with Position on a Photographic Plate at Ground Level Viewing a 9.43μ Wavelength Spherical Volume Source Located at 53.2 km Ground Range and at 21.3 km Altitude -3 (Units of Intensities are photons $\text{km}^{-2} \text{ster}^{-1} \text{source photon ster}^{-1} \text{km}^{-2}$)

reason for the scattered intensity being higher at these points than at the center of the source disk.

The diffuse light at $4300\text{\AA}$ (which is near the wavelength of maximum response for an ordinary photographic emulsion) accounts for some 20-40% of the total intensity and therefore is a significant portion of the total intensity.

A second test problem was run for a 10 km radius spherical surface source centered at an altitude of 21.3 km above the ground surface. The source was assumed to be located within a homogeneous atmosphere with a mean-free-path thickness of 0.8 and a scattering-to-extinction coefficient ratio of 0.5. Scattering was assumed to be isotropic. The atmosphere was assumed to be 50 km in thickness. A receiver was positioned at ground level and at a ground range of 53.2 km from the radial through the center of the source. Figure 32 shows the direct and scattered intensities that were computed for receiver view angles corresponding to 15 points distributed over the visible disks of the source. There is a slight variation in the direct intensities from points at equal distances from the center of the source disk. It is felt that this slight variation probably resulted from round-off errors in defining the mean-free-path distance as a function of altitude and the receiver zenith and azimuthal angles for each of the 15 directions in the problem input data. The scattered intensities are less than 5% of the direct intensities. The low values of the scattered intensities are probably due to the high absorption coefficient which was 50% of the extinction coefficient.

The results of the test problems described above show that the volume source version of FLASH can be used to analyze photographic data. The use of the backward Monte Carlo method in FLASH allows one to confine the scattering calculations to only those receiver view angles of interest.

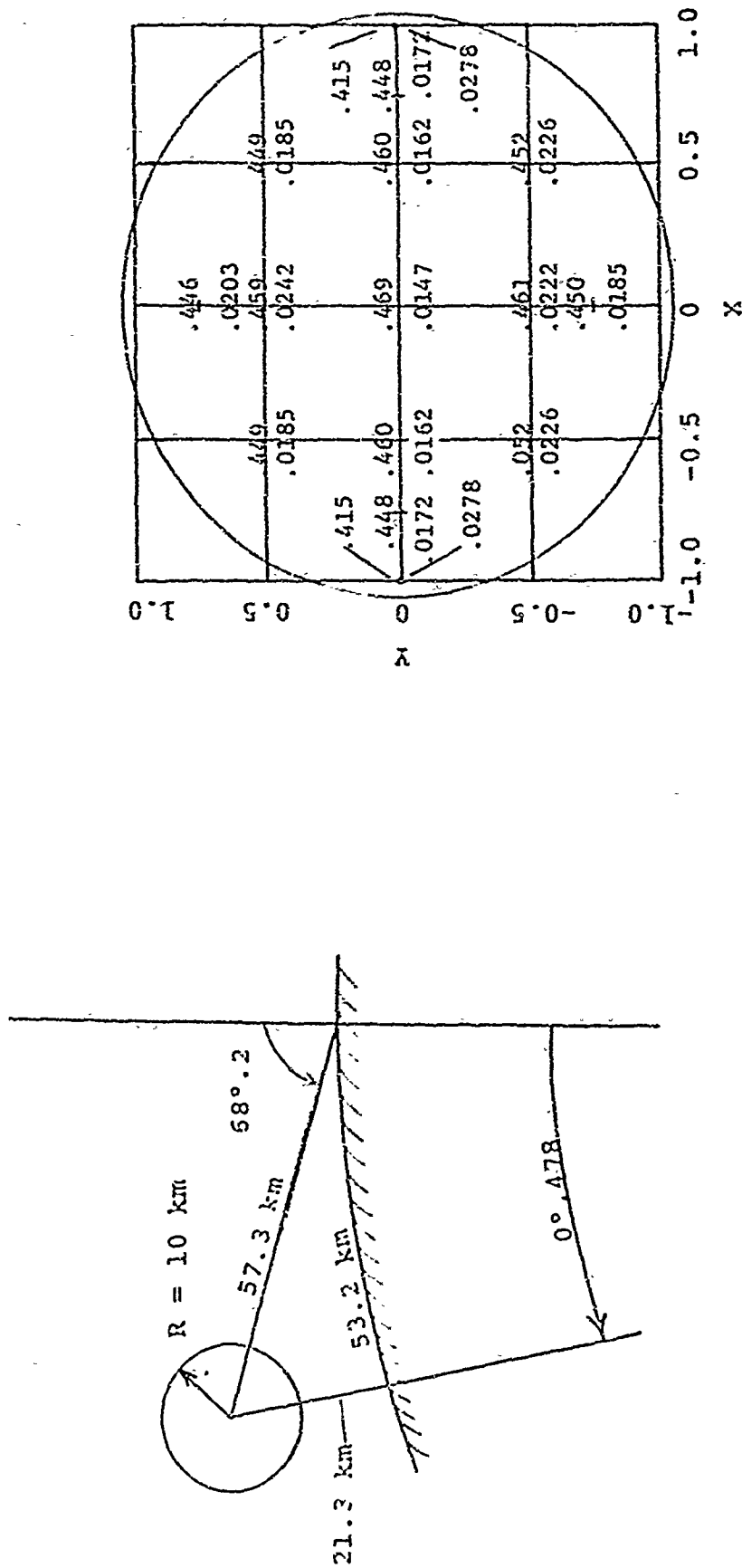


Fig. 32. Variation of the Direct and Air Scattered Intensities with Position on a Photographic Plate at Ground Level Viewing a Spherical Surface Source Located at 53.2 km Ground Range and at 21.3 km Altitude
(Units of Intensities are photons $\text{km}^{-2} \text{ster}^{-1}$ / source photon $\text{km}^{-2} \text{ster}^{-1}$)

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Unclassified

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing or abstract - the entire document is classified)

1. ORIGINATING ACTIVITY (Corporate author)		2. SECURITY CLASSIFICATION	
RADIATION RESEARCH ASSOCIATES, INC. 3550 Hulen Street Fort Worth, Texas 76107		UNCLASSIFIED	
3. REPORT TITLE			
Computer Procedure for Calculating Time Dependent Light Scattering in Spherical-Shell Atmospheres			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates)			
Final Report (1 July 1968-31 January 1971)			
5. AUTHOR(S) (First name, middle initial, last name)			
Dave G. Collins and Michael B. Wells			
6. REPORT DATE		7a. TOTAL NO OF PAGES	7b. NO OF REFS
April 1971		135	20
8a. CONTRACT OR GRANT NO.		9a. ORIGINATOR'S REPORT NUMBER(S)	
DASA01-68-C-0169		RRA-T7017	
b. PROJECT NO.		9b. OTHER REPORT NUMBER(S) (Any other numbers that may be assigned this report)	
NWER XAXH		DASA 2640	
c. TASK AND SUBTASK C062			
d. WORK UNIT 06			
10. DISTRIBUTION STATEMENT			
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11. SUPPLEMENTARY NOTES		12. SPONSORING MILITARY ACTIVITY	
		DIRECTOR Defense Atomic Support Agency Washington, D.C. 20305	
13. ABSTRACT			
This report describes three Monte Carlo procedures that were developed for the purpose of providing calculational tools that could be used to study the transport of visible and infrared light in spherical-shell atmospheres. These procedures, designated as FLASH, treat atmospheric scattering problems for plane-parallel, point, volume and spherical surface sources. The FLASH procedures consider the effect of Rayleigh and aerosol scattering and ozone, water vapor, CO₂, and aerosol particle absorption on the transport of light. The procedures also consider the changes in polarization that occurs at each scattering event. The input formats of the codes allows one to describe the scattering and absorption properties of the atmosphere as a function of altitude. Provisions are incorporated for treating the reflection of light from ground and cloud surfaces with an albedo method. The output of the FLASH procedures gives the scattered and direct light intensities at each receiver position as a function of a polar and azimuthal angle and time. The FLASH procedures were written in FORTRAN-IV language for both the CDC-6600 and IBM Direct Couple System. The codes were verified through comparison with the results of other calculations of light transport in the atmosphere. Also described are the MIE-II and SHINE procedures. The MIE-II procedure uses Mie theory to generate aerosol particle scattering and absorption coefficients and the four elements of the aerosol phase matrix for use as input data to the FLASH procedures. The SHINE procedure was developed to integrate the results of FLASH calculations for point monochromatic sources over an arbitrary wavelength and time dependent source spectrum.			

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14 - F - W - R - S	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Atmospheric Transmission						
Atmospheric Scattering						
Monte Carlo						
Point Sources						
Plane Parallel Sources						
Volume Sources						
Spherical Surface Sources						
Spherical Shell Atmospheres						

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