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STABILITY AND CONTROL OF LEGGED LOCOMOTION SYSTEMS

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13. ABSTRACT This report is a Ph.D. dissertation devoted to a study of stability and control in legged locomotion systems. All of the results obtained apply to a twelfth-order nonlinear differential equation model for the dynamical behavior of an animal or legged vehicle in three-dimensional space. These differential equations are linearized and necessary and sufficient conditions for stable postural control system operation are derived for bipeds and for quadrupeds. The applicability of the results obtained to the nonlinear model is verified by a vibrational mode study of the nonlinear system by means of a computer simulation. The vibrational mode analysis and synthesis techniques are then used to obtain stable feedback control laws for four different quadruped gaits and for one type of biped walk. The results presented should be useful in the design of autopilots for legged vehicles for improved off-road locomotion and in obtaining a deeper understanding of locomotion in animals and man.

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STABILITY AND CONTROL OF LEGGED LOCOMOTION SYSTEMS

DISSERTATION

Presented in Partial Fulfillment of the
Requirements for the Degree Doctor of
Philosophy in the Graduate School
of The Ohio State University

By

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* * * * *

The Ohio State University
1971

Approved by

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CHAPTER I

INTRODUCTION

1.1 General Background

Locomotion can be defined as the process of movement from one place to another. Throughout recorded history, man has been fascinated by the locomotion of various types of animals on land, under water, and in the air. For example, drawings of various animals found on the walls of cave dwellings are testimony to the interest that prehistoric civilizations had in the locomotion of animals. Locomotion studies were made around the third century B. C. by Aristotle and his associates [1]. In latter periods of history, specifically during the eighteenth and nineteenth centuries, doctors and scientists both in Europe and America, conducted various studies dealing with different aspects of legged locomotion. Some of the topics of study were: 1) Anatomical studies to determine the center of gravity of the human body [2-4], 2) Studies of the types of muscles that came into play during walking [2,5], 3) Photographic studies of human and animal gaits [6-8], 4) The kinematics and dynamics involved in human locomotion [9-12].

Recently, with the emergence of the field of Bioengineering, scientists and engineers have begun to take a fresh look at the instrumentation, the diagnostic techniques, and the types of devices used by the medical profession with a view toward their improvement. One of the areas in which such interdisplinary research is being conducted is the study of legged locomotion systems. Hopefully, the development of a sound mathematical basis for legged locomotion systems which includes system dynamics, should lead to better limb coordination control schemes. Computer Simulations of both kinematic and dynamic models of legged locomotion systems are useful in the design and development of better and more efficient prosthetic and orthotic devices, as well as in the design of automatic feedback controllers for legged vehicles.

1.2 Objectives and Limitations of the Dissertation

The objectives of this dissertation are obtain answers to some of the problems concerned with the dynamic stability and limb coordination control of legged locomotion systems. The techniques of vibrational analysis are used to obtain stable postural control of legged locomotion systems. The application of certain stability criteria on these systems is also discussed. Digital computer simulations are used to obtain dynamically stable gaits for idealized models of human and animal locomotion.

This research has the following limitations:

1) The models investigated consist of a single rigid body supported by massless legs.

2) The type of control used in the simulations presented in this dissertation produces "marching" type of limb coordination, that is, the model is assumed to move at a constant velocity on level ground, in the desired direction of motion

placing its feet regularly at specified intervals of both time and space.

Other types of control schemes and locomotion systems with leg mass are not considered in this dissertation.

1.3 Organization of the Dissertation

Chapter I provides a brief historical introduction to the general problem of legged locomotion. The objectives and limitations of the dissertation are discussed in Section 1.2. In conclusion, a summary of the contents of each chapter is given.

Chapter II contains a short survey of the extensive literature that is available on the problem of legged locomotion. Specifically, this chapter breaks down the general study of legged locomotion systems in terms of: 1) finite state models, 2) kinematic models, 3) dynamic models.

Chapter III presents the general equations of motion for an n-legged locomotion system. Also, the equations of motion of a type of inverted pendulum system that is used in this dissertation are derived.

Chapter IV provides the core of the theoretical considerations describing the method of postural control system mode analysis. After discussing the theory involved in the mode interpretation of free motion of legged locomotion systems, the small angle equations of motion are derived for a quadruped. This chapter concludes with a derivation of the linearized equations of motion of the inverted pendulum system discussed in Chapter III.

Chapter V gives a brief insight into the problem of determining the eigenvalues and eigenvectors of real, non-symmetric, square, general matrices. The chapter concludes with a discussion of the application of these methods for obtaining the eigenvalues and eigenvectors for the various modal matrices derived in Chapter IV.

Chapter VI is devoted to the problem of stability and control of the locomotion systems studied in this dissertation. The type of control used for the n-legged locomotion system, namely, model reference control is described. Then, the necessary and sufficient conditions for the small motion stability of the quadruped locomotion system are obtained using the Routh-Hurwitz criterion on the linearized equations of motion of these systems. Finally, in Section 6.7 the application of these stability criteria for obtaining stable biped and quadruped gaits is discussed.

Chapter VII describes the results of computer simulation conducted during the course of this research. In Section 7.2, the nonlinear quadruped simulation is verified by the application of vibrational analysis to the quadruped postural system. Computer output data is presented to show correspondence between the nonlinear and linearized system responses to within one part in 10^6. Details of the simulation of quadruped gaits such as the crawl, the walk, and the trot are given in Section 7.3. Section 7.4 is devoted to the results of digital computer simulation of an inverted pendulum with the mass pivoted at a distance below its center of gravity and supported by a massless leg with a "fixed" foot. Both the transient response, as well as the results of the Routh-Hurwitz analysis are presented in this section. Section 7.5 covers the application of the inverted pendulum stability criteria in the development of the simulation of a stable quadruped pace gait.

Finally, Section 7.6 discusses a type of biped gait simulated in the course of this research.

Chapter VIII summarizes the results of this research, and outlines topics for further research in the area of legged locomotion system studies.

Finally, a listing of the computer programs with explanations as well as a list of references is given in the appendices.

CHAPTER II

SURVEY OF PREVIOUS WORK

2.1 Introduction

This chapter attempts to give a brief overview of the large amount of literature that is available in the area of legged locomotion system studies. The available literature comes from the following areas among others: biomechanics, kinesiology, prosthetics, orthotics, etc. Some aspects of the general problem of legged locomotion have been mentioned in Section 1.1 which gives an idea of the historical development of this area of research.

The study of legged locomotion systems is described in terms of three types of models, 1) finite state models, 2) kinematic models, and 3) dynamic models. Section 2.2 covers the theory of finite state machines as applied to locomotion systems considering a leg as being in one of two states, either on the ground (supporting phase) or in the air (swing phase). The kinematic aspects of locomotion are discussed in Section 2.3, which presents the work being done in the area of time and motion studies of human and animal gaits. Section 2.4 surveys the work done in the modeling of legged locomotion systems including their system dynamics.

2.2 Finite State Models

The application of the theory of finite state automata to legged locomotion systems was first suggested by Tomovic and Karplus [13,14]. Based on their work, McGhee [15] developed a finite state theory for legged locomotion. He considered each leg as a sequential machine with binary output, and defined a number of basic concepts such as gait, gait matrices, duration vector, duty factor, etc. Using these basic definitions, he classified gaits into regular gaits, symmetric gaits, connected gaits, etc, and postulated theorems defining their properties. This finite state theory was used by McGhee and Frank [16] to develop criteria for selecting optimum gaits for low speed locomotion of an idealized quadruped system. They showed that the optimum gait for low speed locomotion of an idealized quadruped model corresponds to the crawl, the low speed gait preferred by most natural quadrupeds.

To test the validity of the finite state theory of legged locomotion, a small artificial quadruped with four identical legs, each with a powered hip joint, and knee joint was constructed at the University of Southern California [17]. A small special purpose digital computer was used for the coo dination of joint motion of this machine, thus proving that automatic limb coordination control of legged locomotion systems such as the quadruped could be achieved by simple finite state algorithms.

Finite state control of legged locomotion systems implies that the system is statically stable at all times. Both "synchronous" and "asynchronous" types of control were used successfully with this machine [18]. However, simple finite state control mechanisms for biped locomotion have not been emonstrated so far.

Recent work in the area of finite state aspects of locomotion has dealt with the properties of regularly realizable gait matrices. Jain [19], has applied the techniques of linear programming for determining the number of regularly realizable nonsingular gaits for bipeds and quadrupeds. He found that out of the 5040 nonsingular quadruped gaits, there were a total of 480 regularly realizable nonsingular gaits which could be reduced to 44 equivalence classes. At least 11 of these 44 equivalence classes of regularly realizable gaits were found to be the natural gait of some animal from the work of Roberts [20], and Hildebrand [21].

Legged locomotion systems possess a large number of degrees of freedom, and are by nature very nonlinear. The development of a finite state theory for legged locomotion systems was one of the first systematic approaches to the mathematical analysis of these complex systems. This study has been a natural extension of the work of earlier investigators such as, Muybridge, Roberts, Hildebrand, among others who attempted to classify human and animal locomotion by various methods.

2.3 Kinematic Models

This section reviews work done in the area of kinematic studies of legged locomotion systems. Typically, research in this area involves the study of human and animal gaits disregarding the forces associated with these motions.

Motion and time studies of human and animal locomotion have been made by man throughout recorded history. During the 17th century, Borelli [22], a professor of mathematics in Naples, Italy, determined the position of the center of gravity in man and in animals, and related it to the locomotion of various species. He drew an analogy between the bones and levers, and the muscles and the forces acting on these levers.

Marey [23] published a series of books between 1873 and 1895. He invented a pneumatic method of registering scientific experiments a short distance away from where the event takes place. The device called Marey Tambours, consisted of two lever drums connected by rubber tubing to the subject. For studying locomotion, these lever drums were used for recording the motion of various parts of the human body.

Marey also developed the chronophotographic method for recording gait [6]. In this method, successive exposures are made on the same photographic plate by means of a rotating mechanism inside the camera. The subject dressed in light clothing, walks in front of a black background. Braune and Fischer [24], used a variation of Marey's method to show only points and lines on the photograph. In their method, called geometric chronophotography, the subject was dressed entirely in black with brilliant metal buttons or shining bands attached to the clothing to mark the joints and bone segments.

Eadward Muybridge [25,26], was the first to analyze animal and human motion by photographic studies begun in 1872. His work consisted of closely spaced sequential photographs of animals and the undraped human form in motion. Muybridge's work resulted in two books, one on animal motion [7], and the other on human locomotion [8]. These books are considered as classics on the photographic analysis of the gaits of animals and human beings.

In the twentieth century, a lot of research has been conducted on various kinematic aspects of locomotion. An excellent chronological literature survey of locomotion upto 1949 has been given by Schermerhorn [27].

Kinematic studies of locomotion have involved such topics, making podograms (recording of the sequences and duration of weight-bearing on three points of the bottom of the foot) [28], investigations of the relationships between length of stride, rate, and speed, and energy consumption in level walking [29], electro-basographic method of recording gait [30], studies of normal and abnormal gait patterns [31], etc.

Very recently, engineers and scientists have begun to apply the advances in technology such as the development of modern control theory, and high speed digital computers to the analysis of the kinematics of legged locomotion systems.

Chow and Jacobsen [32], have studied human locomotion by methods of optimal programming and modern control theory. Using the techniques of optimal control theory, they analyzed biped locomotion as a multi-point boundary value problem. They were able to simulate curves of hip, knee, and ankle movements which agreed well with experimental data.

Hartrum [33], has developed a computer simulation of a kinematic model of human gait. He has used the technique of classifying gaits by their major determinants first proposed by Saunders, Inman, and Eberhart [10], and has produced a computer simulation of a stick figure of a man whose motion is determined by some 80 different kinematic parameters. Such simulations are useful in synthesizing gaits to suit experimental data. Also, they can be used as a computer aided tool to demonstrate both normal and pathological gaits to doctors and physical therapists.

Burnett and Johnson [34], have studied the development of gait in childhood. They used motion pictures of gait patterns and also studied the possibility of using electrogoniometry as a diagnostic aid in evaluating early abnormalities in children.

The research topics mentioned above should give an indication of the state-of-the-art of kinematic studies of legged locomotion systems at the present time.

2.4 Dynamic Models

Many researchers have studied the dynamics of legged locomotion systems from various angles. Some of the topics of research involved investigations such as: 1) experiments to determine the center of gravity of the human body [3,35], 2) studies of weight bearing on the foot [36], 3) force-plate method of measuring the pressure that the foot exerts against the ground [37], 4) studies of the work done by and the mechanical efficiency of human muscles during walking [38,39], etc.

Many such research efforts were qualitative in nature. Fischer [40], tried to relate data taken by chronophotography to a three axes coordinate system, and tried to determine the forces lying behind the accelerations and velocities of the pathways of gait. Using calculus, he analyzed the moving forces indirectly from pictures by establishing displacements, velocities, accelerations, and from this, the muscular efforts involved in walking.

Elftman [41-43], published many articles between 1934 and 1951 on such topics as, the measurement of the external forces in walking, the work done by the muscles in walking, experimental studies on the dynamics of human walking, etc. Using the force-plate in conjunction with motion pictures, and podograph impressions, he tried to get a total picture of what was happening to the foot during each phase of the stance. He also used graphical differentiation as a means for the determination of the external forces acting on the body from its displacements. Elftman also computed the energy transferred and the rate of work being done by the various components of the locomotion system.

In an article published in 1938, Manter [44], discussed the dynamics of quadruped walking. He recorded the external forces applied to the feet of cats as they walked over a special measuring platform, and determined the components of the acceleration of their center of gravity in a three axes coordinate system. Manter also discussed his results with those obtained by graphical differentiation, and showed that his method involved fewer approximations.

Gage [11], has discussed the accelerographic analysis of human gait. He used strain gage accelerometers mounted on subjects at the level of the second sacral vertebra to measure both linear and angular accelerations. Studies were conducted on amputees as well as non-amputees, and a comparison made between their accelerograms during slow and normal level walking, ramp descent, and stair descent. Gage used harmonic analysis to correlate gait defects with abnormalities in the individual's frequency spectrum.

From the enumeration of the various research activities in the area of the dynamic analysis of legged locomotion systems, one can see that many previous studies were qualitative in nature. Both human and animal locomotion is the result of complex processes involving many factors. However, one can get an idea of the dynamics involved in the process of legged locomotion by considering simple models for such systems. According to Bayliss [45], "A standing animal is like a box balanced on four walking-sticks, each jointed in the middle; to represent a man, the box is up-ended and balanced on two such jointed sides".

Recently, research efforts have been directed towards the development of a sound mathematical basis for the dynamics of legged locomotion systems [46]. Frank and McGhee [47], have derived the equations of motion for a four legged locomotion system consisting of a single rigid body supported by massless legs. This dynamic model was assumed to "march" with a constant velocity in the direction of motion, on level ground.

With the advent of high speed computers, and the newer techniques of modern control theory, legged locomotion systems can now be simulated so that their system dynamics can be displayed in real time. The present research uses the techniques of vibrational analysis to obtain postural control for the quadruped locomotion system of Frank and McGhee [47]. In addition, such modal analysis is used to validate the non-linear equations of motion describing the system by exposing any mistakes in the derivation.

Also, this research effort is directed toward the determination of stability criteria for quadruped locomotion systems and a type of inverted pendulum system. In addition, inverted pendulum analysis is used for the stability of both the quadruped and biped systems, and to simulate certain dynamically stable quadruped and biped gaits.

2.5 Summary

This chapter has sketched briefly the developments that have taken place in the area of analysis of legged locomotion systems. The general problem of simulation of legged locomotion systems has been outlined in terms of three specific areas, namely, finite state models, kinematic models, and dynamic models. Sections 2.2 and 2.3 have presented work done upto the present time, in the areas of finite state modeling, and kinematic system modeling of locomotion systems. Section 2.4 has surveyed the past research efforts in the analysis of the dynamic aspects of locomotion systems. In addition, this section has related the present research effort to the previous work in this area. This literature survey is by no means an exhaustive enumeration of the vast amount of literature available in the area of legged locomotion system studies.

CHAPTER III

EQUATIONS OF MOTION OF LEGGED LOCOMOTION SYSTEMS

3.1 Introduction

The objective of this chapter is to describe the dynamics associated with the motion of legged locomotion systems. In order to do this, in Section 3.2, the general equations for the translational and rotational motion of a rigid body are first derived [48]. These basic equations of motion for a rigid body are then applied to an idealized legged locomotion system consisting of a body of mass m supported by four massless legs in Section 3.3 [47]. Finally, the theory of a general inverted pendulum system is described in Section 3.4. The equations of motion of such an inverted pendulum system consisting of a mass pivoted below its center of gravity [50], will have applications to the type of biped locomotion system considered in this dissertation as well as in the stabilization of some faster quadruped gaits such as the trot and the pace.

3.2 General Equations of Motion of a Rigid Body

In general, the motion of a rigid body can be described by a translational equation and a rotational equation. Considering the general motion of a rigid body, and choosing the center of mass of the body as the reference point, if the total external force acting on the body is independent of rotational motion, and if the external moment is independent of the motion of the center of mass, then the translational and rotational equations of motion of the body can be solved separately.

Consider a rigid body with a set of body fixed coordinate axes along the principal axes of the body having its origin at the center of mass. The general rotational equations of motion of such a rigid body, also called the Euler's equations of motion, are given by [48]

$$M_x = I_{xx}\dot{\omega}_x + (I_{zz} - I_{yy})\omega_y\omega_z \tag{3-1}$$

$$M_y = I_{yy}\dot{\omega}_y + (I_{xx} - I_{zz})\omega_z\omega_x \tag{3-2}$$

$$M_z = I_{zz}\dot{\omega}_z + (I_{yy} - I_{xx})\omega_x\omega_y \tag{3-3}$$

where

$$\underline{M} = M_x\hat{i} + M_y\hat{j} + M_z\hat{k} \tag{3-4}$$

$$\underline{\omega} = \omega_x\hat{i} + \omega_y\hat{j} + \omega_z\hat{k} \tag{3-5}$$

$\underline{M}$ is the external moment acting on the rigid body, $\underline{\omega}$ is the absolute angular velocity of the rigid body, and I_{xx}, I_{yy}, and I_{zz} are the moments of inertia of the body about the x, y, and z axes respectively.

The general translational equations of motion of a rigid body of mass m are given by [48]

$$F_x = m(\dot{v}_x + v_z\omega_y - v_y\omega_z) \tag{3-6}$$

$$F_y = m(\dot{v}_y + v_x\omega_z - v_z\omega_x) \tag{3-7}$$

$$F_z = m(\dot{v}_z + v_y\omega_x - v_x\omega_y) \tag{3-8}$$

where

$$\underline{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k} \tag{3-9}$$

$$\underline{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k} \tag{3-10}$$

In the above equations, $\underline{F}$ is the total external force acting on the rigid body, $\underline{v}$ is the absolute velocity of the center of mass of the body, both being expressed interms of their instantaneous body axis components.

Equations (3-1) through (3-5), and (3-6) through (3-10) describe the rotational and translational motion of a rigid body of mass m having a system body fixed coordinate axes with its origin at the center of mass and oriented along the directions of the principal axes of the rigid body.

3.3 Equations of Motion of a Quadruped Locomotion System

3.3.1 Basic Equations

This section discusses the theory dealing with the equations of motion for a four legged locomotion system. It is to be noted that the derivations of equations (3-11) through (3-40), and (3-50) through (3-60) are from reference [47]. These results are given here for the sake of completeness because they are used extensively in the linearization techniques described in Chapter IV. Also, equation (A-29) of reference [47] is not correct, and the correct version of this result is given by equations (3-46) through (3-49). The quadruped locomotion system considered in this section is assumed to consist of a body of mass m supported by legs of negligible mass.

The following convention has been assumed for the coordinate axes. The x_E coordinate axis is directed toward the desired direction of travel, the z_E coordinate axis is in the direction of gravitational acceleration (positive downward), and the y_E axis is in the direction of the vector cross product

$$\hat{j}_E = \hat{k}_E \times \hat{i}_E \qquad (3\text{-}11)$$

The equations of motion are defined with respect to a flat, non-rotating earth, so that $\hat{i}_E$, $\hat{j}_E$, $\hat{k}_E$ are regarded as the unit vectors defining an earth fixed frame.

The total state of the locomotion system is described by the twelve element body state vector

$$\underline{x} = (x_E, y_E, z_E, u, v, w, \theta, \phi, \psi, p, q, r) \qquad (3\text{-}12)$$

where

x_E, y_E, z_E = position of the center of gravity of the system relative to an inertial frame i_E, j_E, k_E

u, v, w = components of the translational velocity of the center of gravity expressed in body coordinates

θ, ϕ, ψ = the body Euler angles

p, q, r = body rotation rates expressed in body coordinates

The body Euler angles are unambiguously defined in the following manner. A right handed body fixed coordinate system with unit vectors $\hat{i}$, $\hat{j}$, $\hat{k}$ is established with its origin fixed at the center of gravity of the model. This body fixed coordinate system is defined such that when the body angles θ, ϕ, ψ are all simultaneously reduced to zero, the $\hat{i}$, $\hat{j}$, and $\hat{k}$ axes are parallel to the $\hat{i}_E$, $\hat{j}_E$, $\hat{k}_E$ axes of the earth fixed frame respectively. The x, y, z coordinates are measured relative to the body fixed coordinate system $\hat{i}$, $\hat{j}$, $\hat{k}$ (see Figure 1).

Let the rotation from the earth fixed (x_E, y_E, z_E) system to the body fixed system (x, y, z) be accomplished by first rotating about the $\hat{k}_E$ axis (azimuth), then about the rotated $\hat{j}_E$ axis (elevation), and

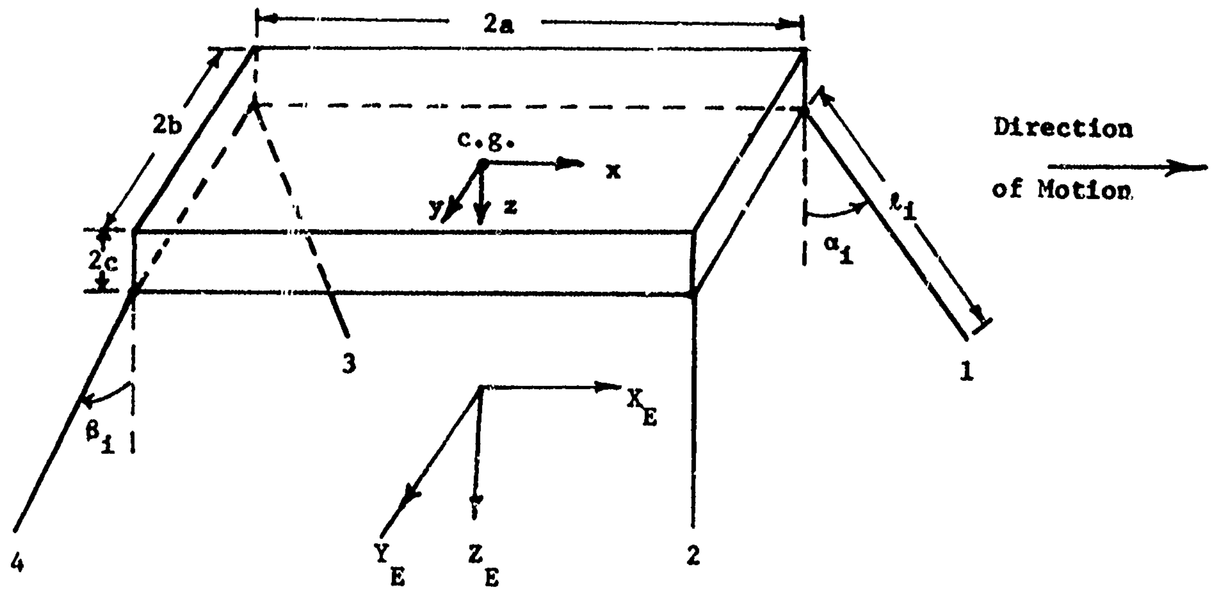


Figure 1 The Quadruped Locomotion System Consisting of a Rigid Body Supported by Four Massless Legs.

finally about the $\hat{i}$ axis (roll). Then, for any arbitrary point (x_a, y_a, z_a) in the earth fixed system, the corresponding coordinates in the body fixed coordinate system are

$$\begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix} = T_1 \begin{bmatrix} x_a - x_E \\ y_a - y_E \\ z_a - z_E \end{bmatrix} \tag{3-13}$$

where

$$T_1 = \begin{bmatrix} \cos\theta\cos\psi & \sin\psi\cos\theta & -\sin\theta \\ (\cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi) & (\cos\psi\cos\phi + \sin\psi\sin\theta\sin\phi) & \cos\theta\sin\phi \\ (\sin\psi\sin\phi + \cos\psi\sin\theta\cos\phi) & (\sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi) & \cos\phi\cos\theta \end{bmatrix} \tag{3-14}$$

For the locomotion system under consideration, if $\underline{f}$ is the vector of applied forces expressed in body coordinates

$$\underline{f} = (f_x, f_y, f_z)^T \tag{3-15}$$

and $\underline{T}$ is the vector of applied torques in the same reference frame

$$\underline{T} = (L, M, N)^T \tag{3-16}$$

and if p, q, r are the components of body rotation rate measured about the body fixed x, y, z system of axes, then, the Euler's equations of motion for this particular system become

$$I_{xx}\dot{p} = (I_{yy} - I_{zz})qr + L \tag{3-17}$$

$$I_{yy}\dot{q} = (I_{zz} - I_{xx})rp + M \tag{3-18}$$

$$I_{zz}\dot{r} = (I_{xx} - I_{yy})pq + N \tag{3-19}$$

If u, v, w are the components of body translational velocity in the x, y, z system, then, from equations (3-6) through (3-8), the three translational equations of motion for the locomotion system are given by

$$m\dot{u} = (f_x - mg\sin\theta) + mvr - mwq \qquad (3\text{-}20)$$

$$m\dot{v} = (f_y + mg\cos\theta\sin\phi) + mwp - mur \qquad (3\text{-}21)$$

$$m\dot{w} = (f_z + mg\cos\theta\cos\phi) + muq - mvp \qquad (3\text{-}22)$$

Finally, from equations (3-17) through (3-22), the following equations of motion for the locomotion system can be written

$$\dot{u} = vr - wq + f_x/m - g\sin\theta \qquad (3\text{-}23)$$

$$\dot{v} = wp - ur + f_y/m + g\cos\theta\sin\phi \qquad (3\text{-}24)$$

$$\dot{w} = uq - vp + f_z/m + g\cos\theta\cos\phi \qquad (3\text{-}25)$$

$$\dot{p} = [(I_{yy} - I_{zz})qr + L]/I_{xx} \qquad (3\text{-}26)$$

$$\dot{q} = [(I_{zz} - I_{xx})rp + M]/I_{yy} \qquad (3\text{-}27)$$

$$\dot{r} = [(I_{xx} - I_{yy})pq + N]/I_{zz} \qquad (3\text{-}28)$$

The above equations can be integrated once to get the six components of body velocity, but the determination of position requires that these velocity components be transformed to the earth fixed system.

Therefore [47]

$$\begin{bmatrix} \dot{x}_E \\ \dot{y}_E \\ \dot{z}_E \end{bmatrix} = T_1^T \begin{bmatrix} u \\ v \\ w \end{bmatrix} \qquad (3\text{-}29)$$

and

$$\begin{bmatrix} \dot{\theta} \\ \dot{\phi} \\ \dot{\psi} \end{bmatrix} = T_2 \begin{bmatrix} p \\ q \\ r \end{bmatrix} \qquad (3\text{-}30)$$

where the transformation T_2 is given by [48]

$$T_2 = \begin{bmatrix} 0 & \cos\phi & -\sin\phi \\ 1 & \tan\theta\sin\phi & \tan\theta\cos\phi \\ 0 & \sec\theta\sin\phi & \sec\theta\cos\phi \end{bmatrix} \tag{3-31}$$

Integration of equations (3-29) and (3-30) gives the desired position components of the body state vector $\underline{x}$.

3.3.2 Leg Lengths and Angles

The values of leg lengths and angles and their derivatives are necessary for the feedback control of the quadruped locomotion system. They are determined as follows.

Let the position in body coordinates of the foot of leg i, i = 1, 2, 3, 4 be given by the vector $(x_i, y_i, z_i)^T$, and let $(x_{iE}, y_{iE}, z_{iE})^T$ be the predetermined time-dependent position of foot i in the earth fixed system, then

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = T_1 \begin{bmatrix} x_{iE} & x_E \\ y_{iE} - y_E \\ z_{iE} & z_E \end{bmatrix} \tag{3-32}$$

The other end of the leg is connected to the body at the corresponding hip socket. Let the cooridnates of the hip socket for leg i in the body fixed coordinate system be given by

$$\underline{h}_i = (a_i, b_i, c_i)^T \tag{3-33}$$

The leg length, and leg hip angles are then obtained by expressing the vector

$$\underline{d}_i = \begin{bmatrix} x_i - a_i \\ y_i - b_i \\ z_i - c_i \end{bmatrix} \tag{3-34}$$

in a body fixed spherical coordinate system $(\ell_i, \alpha_i, \beta_i)$.

The length of leg i, ℓ_i, is given by

$$\ell_i = [(x_i - a_i)^2 + (y_i - b_i)^2 + (z_i - c_i)^2]^{\frac{1}{2}} \tag{3-35}$$

Angles α_i and β_i are defined by

$$\alpha_i = \tan^{-1} \frac{(x_i - a_i)}{(z_i - c_i)} \tag{3-36}$$

$$\beta_i = -\sin^{-1} \frac{(y_i - b_i)}{\ell_i} \tag{3-37}$$

where α_i is the angle which measures the forward swing of leg i about its hip axis, and β_i is the angle by which the leg moves out of the plane normal to its axis (see Figure 1).

Leg length and leg angle rates are obtained by differentiating equations (3-35) through (3-37).

$$\dot{\ell}_i = [\dot{x}_i(x_i - a_i) + \dot{y}_i(y_i - b_i) + \dot{z}_i(z_i - c_i)] \tag{3-38}$$

$$\dot{\alpha}_i = \frac{\dot{x}_i(z_i - c_i) - \dot{z}_i(x_i - a_i)}{(x_i - a_i)^2 + (z_i - c_i)^2} \tag{3-39}$$

$$\dot{\beta}_i = \frac{\dot{\ell}_i(y_i - b_i) - \dot{y}_i\ell_i}{\ell_i[(x_i - a_i)^2 + (z_i - c_i)^2]^{\frac{1}{2}}} \tag{3-40}$$

The derivatives of foot position $\dot{x}_i$, $\dot{y}_i$, and $\dot{z}_i$ appearing in the above equations are obtained by differentiating equation (3-32) as follows

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{z}_i \end{bmatrix} = \frac{dT_1}{dt} \begin{bmatrix} x_{iE} - x_E \\ y_{iE} - y_E \\ z_{iE} - z_E \end{bmatrix} + T_1 \begin{bmatrix} \dot{x}_{iE} - \dot{x}_E \\ \dot{y}_{iE} - \dot{y}_E \\ \dot{z}_{iE} - \dot{z}_E \end{bmatrix} \tag{3-41}$$

Therefore

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{z}_i \end{bmatrix} = \frac{dT_1}{dt} \begin{bmatrix} x_{iE} - x_E \\ y_{iE} - y_E \\ z_{iE} - z_E \end{bmatrix} + T_1 \begin{bmatrix} \dot{x}_{iE} \\ \dot{y}_{iE} \\ \dot{z}_{iE} \end{bmatrix} - T_1 \cdot T_1^T \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{3-42}$$

Since $T_1 \cdot T_1^T = [I]$, the identity matrix, and x_{iE}, y_{iE}, z_{iE} are constants, therefore, $\dot{x}_{iE}$, $\dot{y}_{iE}$, and $\dot{z}_{iE}$ are zero.

Hence

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{z}_i \end{bmatrix} = \frac{dT_1}{dt} \begin{bmatrix} x_{iE} - x_E \\ y_{iE} - y_E \\ z_{iE} - z_E \end{bmatrix} - \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{3-43}$$

where dT_1/dt = the derivative of matrix T_1 given in equation (3-14).

For convenience define matrix T_1 as follows

$$T_1 = \begin{bmatrix} T_{111} & T_{112} & T_{113} \\ T_{121} & T_{122} & T_{123} \\ T_{131} & T_{132} & T_{133} \end{bmatrix} \tag{3-44}$$

where T_{1ij} is defined by equation (3-14).

Then the derivative of the transformation matrix T_1 becomes

$$\frac{dT_i}{dt}=\begin{bmatrix} (-\dot\theta\sin\theta\cos\psi-\dot\psi T_{i12}) & (-\dot\theta\sin\theta\sin\psi+\dot\psi T_{i11}) & -\dot\theta\cos\theta \\ (\dot\theta\sin\phi T_{i11}+\dot\phi T_{i31}-\dot\psi T_{i22}) & (\dot\theta\sin\phi T_{i12}+\dot\phi T_{i32}+\dot\psi T_{i21} & (-\dot\theta\sin\theta\sin\phi+\dot\phi T_{i33}) \\ (\dot\theta\cos\phi T_{i11}-\dot\phi T_{i21}-\dot\psi T_{i32}) & (\dot\theta\cos\phi T_{i12}-\dot\phi T_{i22}+\dot\psi T_{i21}) & (-\dot\theta\sin\theta\cos\phi-\dot\phi T_{i23}) \end{bmatrix} \tag{3-45}$$

Finally, from equations (3-43) and (3-45), the derivatives of foot position $\dot x_i$, $\dot y_i$, $\dot z_i$ are given by

$$\begin{bmatrix} \dot x_i \\ \dot y_i \\ \dot z_i \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix} \tag{3-46}$$

where

$$A=[-(\dot\theta\sin\theta\cos\psi+\dot\psi T_{i12})(x_{iE}-x_E)-(\dot\theta\sin\theta\sin\psi-\dot\psi T_{i11})(y_{iE}-y_E)-\dot\theta\cos\theta(z_{iE}-z_E)-u] \tag{3-47}$$

$$B=[(\dot\theta\sin\phi T_{i11}+\dot\phi T_{i31}-\dot\psi T_{i22})(x_{iE}-x_E)+(\dot\theta\sin\phi T_{i12}+\dot\phi T_{i32}+\dot\psi T_{i21})(y_{iE}-y_E)- (\dot\theta\sin\theta\sin\phi-\dot\phi T_{i33})(z_{iE}-z_E)-v] \tag{3-48}$$

$$C=[\dot\theta\cos\phi T_{i11}-\dot\phi T_{i21}-\dot\psi T_{i32})(x_{iE}-x_E)+(\dot\theta\cos\phi T_{i12}-\dot\phi T_{i22}+\dot\psi T_{i31})(y_{iE}-y_E)- (\dot\theta\sin\theta\cos\phi+\dot\phi T_{i23})(z_{iE}-z_E)-w] \tag{3-49}$$

3.3.3. Control Moments and Forces

The following assumptions are made for the type of control laws considered for the locomotion system:

a) It is assumed that each foot is placed on the ground at a sequence of pre-computed positions determined prior to the start of forward motion. These foot placements are evenly spaced along the desired direction of travel and separated by one stride length for any given foot.

b) The length of each leg measured from its hip joint to its foot is allowed to vary through knee flexure.

c) The force applied by the leg to the body along a line joining the foot and the hip joint is assumed to be a linear combination of leg length and leg length rate. This means that each knee joint has a rotational spring and damper.

d) Lateral leg deflection is assumed to be controlled by a centering spring and damper.

e) The rearward rotation of each leg is controlled by applying a torque at the hip proportional to the difference between the actual angle measured from the body axis of the locomotion system to a line passing through the foot and the hip socket and the desired angle computed for an ideal constant velocity gait. Error rate damping is also included in the computation of each hip control torque.

Therefore the simulated locomotion system is controlled by applying forces along each leg and by moments applied at each hip. It is assumed that the moments are applied about the y body axis by a hip drive motor and about a lateral deflection gimbal axis by a centering spring and damper.

Considering the body fixed coordinate system, if T_{m_i} is the torque applied to the body by the hip motor of leg i, T_{s_i} is the centering spring torque applied to the body, and $\underline{T_i}$ is the total torque applied to the body by leg i, then

$$\underline{T}_i = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} = \begin{bmatrix} T_{s_i}\cos\alpha_i \\ T_{m_i} \\ -T_{s_i}\sin\alpha_i \end{bmatrix} \tag{3-50}$$

The forces applied to the consist of the force along each leg and the reaction forces normal to each leg which are produced by the applied moments. These forces are computed in a coordinate frame attached to each leg. Let z_i' be a coordinate measured along leg i from its hip socket. If x_i' is constrained to be parallel to the x-z plane of the body fixed system, then

$$\begin{bmatrix} x_i' \\ y_i' \\ z_i' \end{bmatrix} = T_3 \begin{bmatrix} x_i - a_i \\ y_i - b_i \\ z_i - c_i \end{bmatrix} \tag{3-51}$$

where the transformation T_3 is given by [48]

$$T_3 = \begin{bmatrix} \cos\alpha_i & 0 & -\sin\alpha_i \\ \sin\beta_i\sin\alpha_i & \cos\beta_i & \sin\beta_i\cos\alpha_i \\ \cos\beta_i\sin\alpha_i & -\sin\beta_i & \cos\beta_i\cos\alpha_i \end{bmatrix} \tag{3-52}$$

If the reaction force acting on the foot of leg i is denoted by $\underline{f}_{R_i}$, then in leg coordinates

$$\underline{f}_{R_i} = (f_x', f_y', 0) \tag{3-53}$$

Since the legs are considered to be massless, the components of $\underline{f}_{R_i}$ can be obtained by noting that all forces and moments acting on any leg must sum to zero. Then, motor torque balance requires for each leg that

$$T_{m_i} = f'_{x_i} \ell_i \cos\beta_i \tag{3-54}$$

and the centering spring torque balance implies that

$$T_{s_i} = -f'_{y_i} \ell_i \tag{3-55}$$

Therefore, if $\underline{f}'_i$ is the total force applied by leg i to the body at hip socket i, expressed i leg coordinates, then

$$\underline{f}'_i = \begin{bmatrix} T_{m_i}/(\ell_i \cos\beta_i) \\ -T_{s_i}/\ell_i \\ f'_{z_i} \end{bmatrix} \tag{3-56}$$

Finally, the total leg force in body coordinates is given by

$$\underline{f}_i = T_3^T \underline{f}'_i = (f_{x_i}, f_{y_i}, f_{z_i})^T \tag{3-57}$$

3.3.4 Total Forces and Moments Applied to the Body

The total force components needed in equations (3-23) through (3-25) are obtained by summation over all leg forces. Therefore

$$\underline{f} = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{4} f_{x_i} \\ \sum_{i=1}^{4} f_{y_i} \\ \sum_{i=1}^{4} f_{z_i} \end{bmatrix} \tag{3-58}$$

The total moment vector $\underline{T}$ given by equation (3-16) is composed of the moments applied by each hip motor and spring and the moments due to forces applied at each socket h_i. Therefore

$$\underline{T} = \sum_{i=1}^{4} \underline{T}_i + \sum_{i=1}^{4} (\underline{h}_i \times \underline{f}_i) \tag{3-59}$$

where

$$\underline{M}_i = (\underline{h}_i \times \underline{f}_i) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_i & b_i & c_i \\ f_{x_i} & f_{y_i} & f_{z_i} \end{vmatrix} = \begin{bmatrix} b_i f_{z_i} - c_i f_{y_i} \\ c_i f_{x_i} - a_i f_{z_i} \\ a_i f_{y_i} - b_i f_{x_i} \end{bmatrix} \tag{3-60}$$

The equations derived above [47] are used to determine the body state vector $\underline{x}$ from its initial value and subsequently applied control torques and forces.

3.4 Equations of Motion for an Inverted Pendulum System

A simple biped locomotion system can be considered to be a mass with two massless legs attached as shown in Figure 2(a). Figure 2(b) shows the model with the torques applied about the hip and ankle joints in a longitudinal plane.

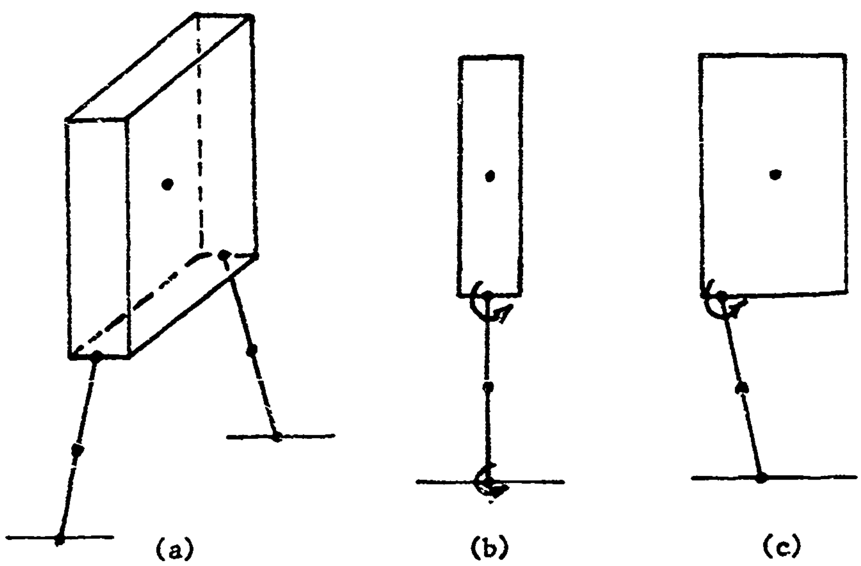


Figure 2 A Simple Biped Locomotion System.

Figure 2(c) shows a torque applied in a lateral plane to keep the body from falling when the system is being supported by one of the legs. From these figures, one can see that the stabilization of the biped system is similar in some respects to the inverted pendulum problem.

In analyzing this problem, two methods of approach are possible: 1) the application of Newton's laws of motion, and 2) the Lagrangian formulation [49]. For dynamic systems with certain constraints, the Lagrangian formulation provides an easier method for obtaining their equations of motion. The inverted pendulum system under consideration has a kinematic constraint in that its leg length ℓ is fixed. For this reason, the equations of motion of this system are derived below by the application of Lagrange's equations.

In analyzing the motion of a system by the Lagrangian approach, the first step is to choose a set of independent coordinates q_i which completely characterize the motion of the system without any redundancy. The kinetic and potential energies T and V respectively are then computed as functions of the q_i's and $\dot{q}_i$'s.

The Lagrangian function L is given by

$$L = (T - V) \tag{3-61}$$

and the equations of motion of the system are given by the Lagrange's equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \qquad i = 1,2,3,4,\ldots, n \tag{3-62}$$

where the Q_i's are nonconservative forces applied to the body. These Q_i's are calculated by producing a virtual displacement of the system and finding the virtual work done by the forces Q_i as given by

$$\delta W = \sum_{i=1}^{4} Q_i \delta q_i \tag{3-63}$$

The Lagrangian approach is now applied to the following inverted pendulum system. Figure 3 shows an inverted pendulum with a mass m supported by a massless leg of fixed length ℓ. The body has a moment of inertia I and its center of gravity is at a distance r above the hip joint. The leg is supported by a " fixed " foot, that is, the foot is attached to the ground by a frictionless hinge which permits rotation of the leg in a plane without any translation [50]. The analysis of the inverted pendulum with the mass pivoted at its center of gravity can be found in referance [51].

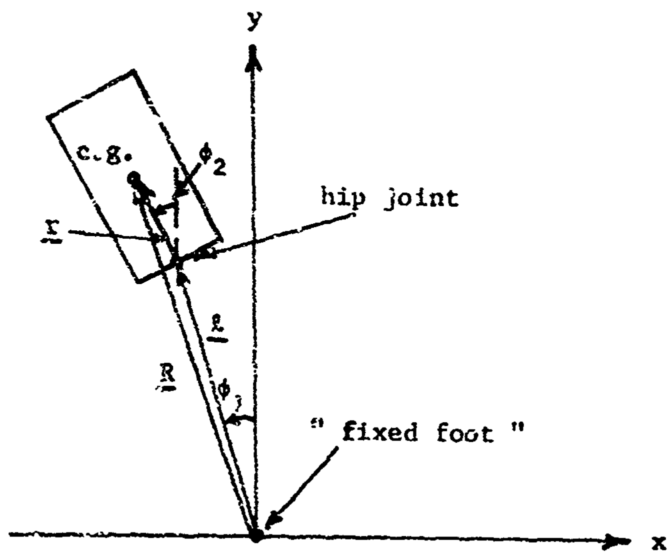


Figure 3 An Inverted Pendulum System.

Angles ϕ_1 and ϕ_2 are the two independent coordinates used in the Lagrangian formulation of the equations of motion for this system. From

Figure 3 the total kinetic energy of the system is seen to be

$$T = M|\dot{\underline{R}}|^2 + \frac{1}{2} I\dot{\phi}_2^2 \qquad (3\text{-}64)$$

and

$$\underline{R} = -(\ell\sin\phi_1 + r\sin\phi_2)\hat{i} + (\ell\cos\phi_1 + r\cos\phi_2)\hat{j} \qquad (3\text{-}65)$$

where $\hat{i}$, $\hat{j}$ are the unit vectors of the coordinate system shown in the figure above.

From equations (3-64) and (3-65), the expression for the kinetic energy becomes

$$T = \frac{1}{2} m[\ell^2\dot{\phi}_1^2 + 2\ell r\dot{\phi}_1\dot{\phi}_2\cos(\phi_1-\phi_2) + r^2\dot{\phi}_2^2] + \frac{1}{2} I\dot{\phi}_2^2 \qquad (3\text{-}66)$$

The potential energy is given by

$$V = mg(\underline{R}\cdot\hat{j}) = mg(\ell\cos\phi_1 + r\cos\phi_2) \qquad (3\text{-}67)$$

Therefore the Lagrangian function L is given by

$$L = (T-V) = \frac{1}{2} m[\ell^2\dot{\phi}_1^2+2\ell r\dot{\phi}_1\dot{\phi}_2\cos(\phi_1-\phi_2)+r^2\dot{\phi}_2^2]+ \frac{I\dot{\phi}^2}{2}2-mg\ell\cos\phi_1-mgr\cos\phi_2 \qquad (3\text{-}68)$$

The virtual work done is given by

$$\delta W = M(\delta\phi_2 - \delta\phi_1) \qquad (3\text{-}69)$$

and therefore from equations (3-63) and (3-69)

$$Q = - M \qquad (3\text{-}70)$$

$$Q = + M \qquad (3\text{-}71)$$

Solution of the Lagrange's equations (3-62) gives the following equations of motion in the independent variables ϕ_1 and ϕ_2

$$m\ell^2\ddot{\phi}_1 + m\ell r\ddot{\phi}_2\cos(\phi_1-\phi_2) + m\ell r\dot{\phi}_2^2\sin(\phi_1-\phi_2) - mg\ell\sin\phi_1 = -M \quad (3\text{-}72)$$

$$(I+mr^2)\ddot{\phi}_2 + m\ell r\ddot{\phi}_1\cos(\phi_1-\phi_2) - m\ell r\dot{\phi}_1^2\sin(\phi_1-\phi_2) - mgr\sin\phi_2 = +M \quad (3\text{-}73)$$

The above equations of motion can be rewritten in terms of state variables by using the following definitions. Let

$$x_1 = \phi_1 \quad (3\text{-}74)$$

$$x_2 = \phi_2 \quad (3\text{-}75)$$

$$x_3 = \dot{\phi}_1 = \dot{x}_1 \quad (3\text{-}76)$$

$$x_4 = \dot{\phi}_2 = \dot{x}_2 \quad (3\text{-}77)$$

Substitution of the above equations into equations (3-72) and (3-73) gives the system equations

$$\dot{x}_1 = x_3 \quad (3\text{-}78)$$

$$\dot{x}_2 = x_4 \quad (3\text{-}79)$$

$$\dot{x}_3 = \frac{[DH - FG]}{(AD - F^2)} \quad (3\text{-}80)$$

$$\dot{x}_4 = \frac{[AG - FH]}{(AD - F^2)} \quad (3\text{-}81)$$

where

$$A = m\ell^2 \quad (3\text{-}82)$$

$$B = m\ell r \quad (3\text{-}83)$$

$$C = mg\ell \quad (3\text{-}84)$$

$$D = (I + mr^2) \quad (3\text{-}85)$$

$$E = mgr \quad (3\text{-}86)$$

$$F = B\cos(x_1 - x_2) \quad (3\text{-}87)$$

$$G = [E\sin x_2 - Bx_3^2\sin(x_1 - x_2) + M] \quad (3\text{-}88)$$

$$H = [C\sin x_1 - Bx_4^2\sin(x_1 - x_2) - M] \quad (3\text{-}89)$$

Equations (3-78) through (3-81) given above provide a state variable representation of the equations of motion of the inverted pendulum system with a " fixed " foot.

3.5 Summary

In this chapter, using the Newtonian approach, the equations of motion of a general quadruped locomotion system consisting of a body of mass m supported by massless legs have been derived.

Also, the equations of motion for a type of inverted pendulum system with a " fixed " foot have been obtained. These basic equations of motion will be used in the rest of the dissertation to simulate postural control systems and to derive stable feedback control laws for various quadruped gaits as well as for a certain type of biped locomotion system.

CHAPTER IV

POSTURAL CONTROL SYSTEM MODE ANALYSIS

4.1 Introduction

From the results of the last chapter, the equations of motion of legged locomotion systems are seen to be nonlinear in nature. These equations have been programmed into a digital computer simulation which produces computer generated displays of the idealized locomotion system performing various gaits.

Now, it is well known in classical mechanics that conservative dynamic systems can be studied in terms of small vibrations about an equilibrium point [48]. The equations of motion of these systems are linearized, and any free motion of these systems can then be expressed as a superposition of " normal " modes of vibration. Each " normal " mode of vibration is sinusoidal and is characterized by its frequency. This concept of modes of free motion can be extended to any linear time-invariant differential system, even if the system is non-conservative [52].

One way to obtain a linearization of a nonlinear differential equation of the form $\underline{x} = f(\underline{x})$, is to replace $f(\underline{x})$ with a truncated Taylor series expension about an equilibrium point in which only the linear terms are retained. A theorem can be proved [53], which states that under certain fairly mild conditions on $f(\underline{x})$, such a <u>Jacobian</u>

linearization [34] yields valid information regarding the stability and damping of the small disturbance motion of the nonlinear system.

Rather than proving that the conditions of this theorem are satisfied, in this dissertation the time domain response of the linearized system equations are compared to a numerical solution of the nonlinear system differential equations for each vibrational mode. It will be seen in Chapte that the results obtained by these two methods agree to better than one part in 10^6 thereby validating both the linearization technique and both computer programs.

In line with the above objective, this chapter first deals with a description of linear system theory emphasizing the mode interpretation of the " free motion " of linear time-invariant systems.

Then, assuming that the quadruped locomotion system is in equilibrium in the postural position with its feet vertically below their respective hip socket positions, the linearized equations of motion are derived using small angle approximations, both by rigorous analysis as well as by an intuitive approach. It is shown that the rigorous approach yields a 12 x 12 system matrix which decomposes into four smaller matrices, thus giving four vibrational systems for small displacements about the equilibrium position.

Finally, in Section 4.5 the equations of motion of the inverted pendulum system derived in Chapter III are linearized and the corresponding system matrix is obtained. These results are needed to compute stabilizing control constants for the legged locomotion systems by the application of the Routh-Hurwitz stability criterion and also to obtain the eigenvalues and eigenvectors of the system matrices in order to compute their linear system response.

4.2 Mode Interpretation of the Free Motion of Linear Time-Invariant System

This section outlines briefly the theory of " free motion " of a general linear time-invariant system. The results of this section will be used to compute the linearized system response of the legged locomotion systems discussed in this dissertation. Consider a general linear time-invariant system described by the state equation

$$\dot{\underline{x}}(t) = A\underline{x}(t) + b\underline{u}(t) \tag{4-1}$$

where

A = n x n constant matrix

b = n x m constant matrix

$\underline{x}(t)$ = n rowed column vector representing the state of the system at time t

$\underline{x}(0)$ = ini . l state of the system at t = 0

$\underline{u}(t)$ = inpu. m vector

Considering " free motion " of the system, the input vector $\underline{u}(t)$ is zero for all time t, and the system state equation reduces to the form

$$\dot{\underline{x}}(t) = A\underline{x}(t) \tag{4-2}$$

Consider the general case in which λ_i, i 1, 2, 3, ... n are the n distinct eigenvalues of the matrix A. Any non-zero vector $\underline{u}_i$ such that [55]

$$A\underline{u}_i = \lambda_i\underline{u}_i \qquad \langle \underline{u}_i, \underline{u}_i \rangle = 1 \qquad i = 1, \ldots n \tag{4-3}$$

is called an eigenvector associated with the eigenvalue λ_i. Since the eigenvalues of matrix A are assumed to be distinct, their associated eigenvectors are linearly independent. Therefore, the free motion $\underline{x}(t)$ can be uniquely expressed as a linear combination of these n distinct eigenvectors of matrix A.

$$\underline{x}(t) = \text{Re} \sum_{i=1}^{n} \alpha_i(t)\underline{u}_i \qquad -\infty < t < \infty \tag{4-4}$$

The general form of $\alpha_i(t)$ is given by

$$\alpha_i(t) = C_i e^{\lambda_i t} \tag{4-5}$$

where the C_i's are constants.

Therefore

$$\underline{x}(t) = \text{Re} \sum_{i=1}^{n} C_i e^{\lambda_i t} \underline{u}_i \tag{4-6}$$

At $t = 0$, equation (4-6) yields

$$\underline{x}(0) = \text{Re} \sum_{i=1}^{n} C_i \underline{u}_i \tag{4-7}$$

The constants C_i can be found by using the reciprocal basis $\underline{r}$ defined by the scalar product

$$\langle \underline{r}_i, \underline{u}_j \rangle = \delta_{ij} \qquad (i, j = 1, \ldots n) \tag{4-8}$$

where δ_{ij} = Kronecker delta $= 1$ for $i = j$

$= 0$ for $i \neq j$

Taking the scalar product of $\underline{r}_i$ with both sides of equation (4-7) the constant C_i is found to be

$$C_i = \langle \underline{r}_i, \underline{x}(0) \rangle \tag{4-9}$$

Therefore, the " zero input state response " of a linear time-invariant system can be expressed as

$$\underline{x}(t) = \text{Re} \sum_{i=1}^{n} \langle \underline{r}_i, \underline{x}(0) \rangle e^{\lambda_i t} \underline{u}_i \tag{4-10}$$

The scalar product $\langle \underline{r}_i, \underline{x}(0) \rangle$ represents the magnitude of the i^{th} mode of the system due to the initial conditions. If the initial conditions are taken along the i^{th} eigenvector, then only the i^{th} mode is excited. The scalar products $\langle \underline{r}_j, \underline{x}(0) \rangle$ where $i \neq j$ are identically zero. Therefore for a linear, time-invariant, unforced system with distinct eigenvalues the free motion response is given by a linear weighted sum of the modes $e^{\lambda_i t}\underline{u}_i$, where λ_i is an eigenvalue of the system matrix A.

Consider the most general case in which the coefficient matrix has a complex eigenvalue λ_1, then λ_2 = complex conjugate of $\lambda_1 = \lambda_1{}^*$ is also an eigenvalue. The eigenvectors $\underline{u}_1$ and $\underline{u}_2$ corresponding to λ_1 and λ_2 are also complex conjugates such that $\underline{u}_2 = \underline{u}_1{}^*$.

Let

$$\begin{array}{lll} \lambda_1 = (\alpha_1 + j\beta_1) & \lambda_2 = (\alpha_1 - j\beta_1) & \\ \underline{u}_1 = \underline{u}_1' + j\underline{u}_1'' & \underline{u} = \underline{u}_1' - j\underline{u}_1'' & \underline{u}_1', \underline{u}_1'' \text{ real} \\ 2\underline{r}_1 = \underline{r}_1' + j\underline{r}_1' & 2\underline{r}_2 = \underline{r}_1' - j\underline{r}_1'' & \underline{r}_1', \underline{r}_1'' \text{ real} \end{array} \tag{4-11}$$

Then, from equation (4-8)

$$\langle \underline{r}_1', \underline{u}_1' \rangle = 1 \qquad \langle \underline{r}_1'', \underline{u}_1' \rangle = 0$$

$$\langle \underline{r}_1', \underline{u}_1'' \rangle = 0 \qquad \langle \underline{r}_1'', \underline{u}_1'' \rangle = 1 \tag{4-12}$$

The normalization condition for the complex eigenvectors is given by

$$\langle \underline{u}_1', \underline{u}_1' \rangle + \langle \underline{u}_1'', \underline{u}_1'' \rangle = 1 \tag{4-13}$$

Using the above results, the free motion of a system with k pairs of complex eigenvalues is given by the expression [55]

$$\underline{x}(t) = \sum_{i=1}^{k} e^{\alpha_i t}\{[\langle \underline{r}_i', \underline{x}(0)\rangle \cos\beta_i t + \underline{r}_i'', \underline{x}(0) \sin\beta_i t]\underline{u}_i' +$$

$$[\langle \underline{r}_i'', \underline{x}(0)\rangle \cos\beta_i t - \langle \underline{r}_i', \underline{x}(0)\rangle \sin\beta_i t]\underline{u}_i'' \} \tag{4-14}$$

The amplitude and phase of each oscillatory mode depends only on the initial conditions.

When the initial conditions are equal to $\underline{u}'$, that is

$$\underline{x}(0) = \underline{u}' \tag{4-15}$$

the solution of equation (4-14) gives gives mode 1 as follows

$$\underline{x}(t) = e^{\sigma_1 t} [(\cos\beta_1 t)\underline{u}_1' - (\sin\beta_1 t)\underline{u}_1''] \tag{4-16}$$

When the initial conditions are equal to $\underline{u}''$, then the solution gives mode 2 given below

$$\underline{x}(t) = e^{\alpha_1 t} [(\cos\beta_1 t)\underline{u}'' + (\sin\beta_1 t)\underline{u}'] \tag{4-17}$$

The equations derived above give the " zero input state response " of a linear time-invariant system whose coefficient matrix has some distinct complex conjugate eigenvalues and corresponding complex conjugate eigenvectors.

In general, the matrix A is real and non-symmetric for the legged locomotion systems considered in Chapter III. It has both real and complex eigenvalues which are all distinct. Therefore, the theoretical results given in this section can be used to obtain the linearized system response to small motions about an equilibrium position of these legged locomotion systems. The linearized equations of motion are derived in the next section, and these results together with the results of this section are combined to obtain the various independent vibrational modes of the quadruped locomotion system.

4.3 Small Angle Equations of Motion for the Quadruped Locomotion System

4.3.1 Basic Theory

In this section, the equations of motion derived in Chapter III for the four legged locomotion system are linearized. Using the assumptions of small motion of the system about its equilibrium position, all the equations derived in Chapter III are systematically linearized by replacing sines by the respective angles, and the cosines by unity, and ignoring terms involving products of the state variables. In addition, symmetry considerations are also used to reduce the system equations that are obtained.

It is shown that the final 12 x 12 matrix of the linearized system decomposes into four smaller matrices. These four smaller matrices describe the various independent vibrational modes of the linearized quadruped locomotion system.

The following definitions are used in the rest of this dissertation with reference to the vibrational analysis of the quadruped locomotion system:

1) Modal Matrix - The matrix that describes the various translational and rotational motions corresponding to the independent vibrational modes of the linearized locomotion system. It should be noted that this definition of a modal matrix is different from that used in linear system analysis, where a modal matrix corresponds to he matrix whose columns are the eigenvectors of the system matrix. Therefore for the linearized quadruped locomotion system in terms of the above definition there are four modal matrices.
2) X Axis Vibrational Modes - By definition the x axis vibrational modes correspond to the translational and rotational motions of the quadruped locomotion system in the y-z plane.
3) Y Axis Vibrational Modes - By definition the y axis vibrational modes correspond to the translational and rotational motions of the quadruped postural system in the x-z plane.
4) Z Axis Translational Modes - By definition the z axis translational modes correspond to the decoupled translational motion along the z axis of the quadruped postural system.
5) Z Axis Rotational Modes - By definition t z axis rotational modes correspond to the decoupled rotational motion

about the z axis of the quadruped postural system. This corresponds to a rotational motion described by the body Euler angle ψ in the x-y plane.

The linearized equations of motion are obtained assuming small motions about the equilibrium position for the various translational and rotational components of the system state vector. The approach is one of step by step linearization of all the equations of Chapter III leading finally to the linearized equations of motion for the quadruped.

Considering only small motions of the quadruped system about an equilibrium point, the following assumptions are made:

The Euler angles θ, ϕ, and ψ are small, and hence the sines of these angles can be replaced by the respective angles, and their cosines by unity.

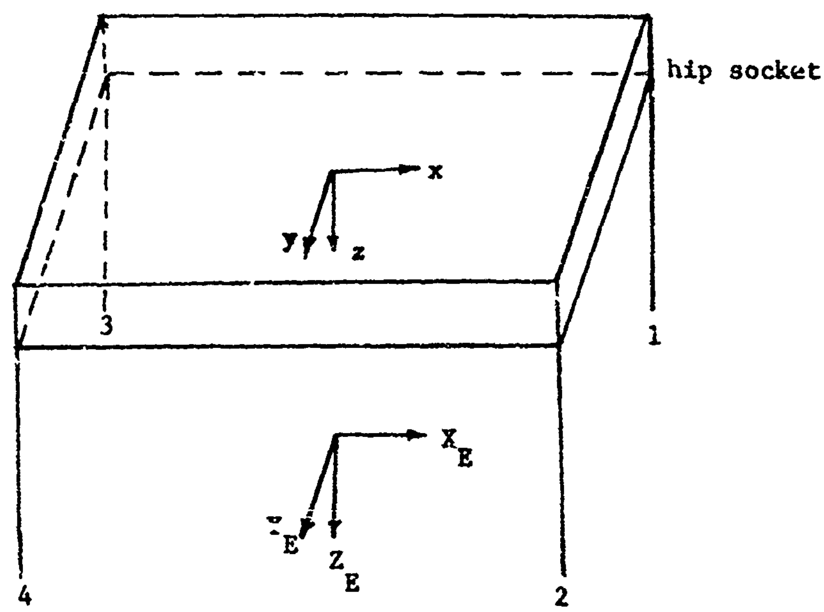
Using symmetry for the hip socket positions, the following results can be obtained (see Figure 4).

$$\begin{aligned} a_1 &= a_2 = a \\ a_3 &= a_4 = -a \\ b_1 &= b_3 = -b \\ b_2 &= b_4 = b \\ c_1 &= c_2 = c_3 = c_4 = c \end{aligned} \tag{4-18}$$

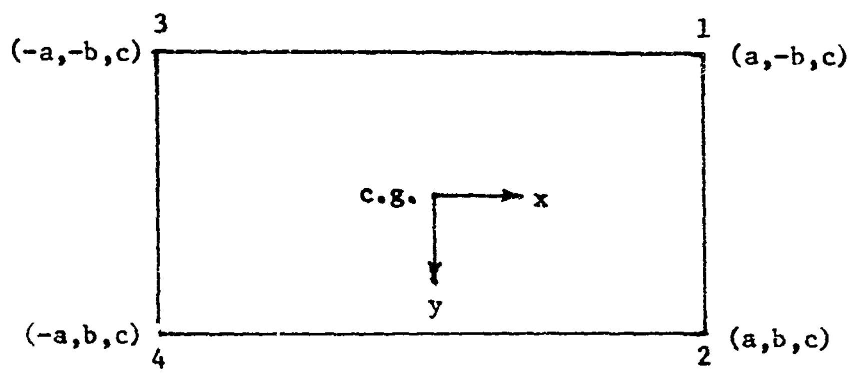
Also

$$\sum_{i=1}^{4} a_i = \sum_{i=1}^{4} b_i = \sum_{i=1}^{4} a_i b_i = 0 \tag{4-19}$$

$$\sum_{i=1}^{4} a_i^2 = 4a^2 \quad \text{and} \quad \sum_{i=1}^{4} b_i^2 = 4b^2 \tag{4-20}$$



(a) Quadruped Standing with its feet directly below the corresponding hip joints.



(b) Top View of the Postural System.

Figure 4 The Quadruped Postural System.

The foot position of leg i in the earth fixed coordinate system is given by

$$x_{iE} = a_i = \pm a$$
$$y_{iE} = b_i = \pm b$$
$$z_{iE} = 0 \qquad (4-$$

The initial position of the center of gravity of the machine in earth coordinates is given by

$$x_{E_o} = 0$$
$$y_{E_o} = 0$$
$$z_{E_o} = -(\ell_o+c_i) = -(\ell_o+c) \qquad (4-22)$$

The position of the center of gravity of the quadruped postural system in earth coordinates is $(x_E, y_E, 0)$.

From equations (3-14) and (3-31) upon replacing the sines of the Euler angles by the angles and cosines by unity, the linearized transformation matrices become

$$T_1 = \begin{bmatrix} 1 & \psi & -\theta \\ -\psi & 1 & \phi \\ \theta & -\phi & 1 \end{bmatrix} \qquad (4-23)$$

$$T_2 = \begin{bmatrix} 0 & 1 & -\phi \\ 1 & 0 & \theta \\ 0 & \phi & 1 \end{bmatrix} \qquad (4-24)$$

Assuming that Δz_E is the change in position of the z coordinate of the center of gravity of the body in the earth fixed system, equations (3-29) and (4-23) yield the components of the translational velocity of the center of gravity.

$$\begin{bmatrix} \dot{x}_E \\ \dot{y}_E \\ \Delta\dot{z}_E \end{bmatrix} = T_1^T \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{4-25}$$

From equations (3-30) and (4-24), the Euler angle rates can be expressed as

$$\begin{bmatrix} \dot{\theta} \\ \dot{\phi} \\ \dot{\psi} \end{bmatrix} = T_2 \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} q \\ p \\ r \end{bmatrix} \tag{4-26}$$

Linearizing equations (3-23) through (3-25) gives the components of the translational acceleration of the center of gravity of the body.

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} f_x/m \\ f_y/m \\ f_z/m \end{bmatrix} + \begin{bmatrix} -g & 0 & 0 \\ 0 & g & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \phi \\ \psi \end{bmatrix} \tag{4-27}$$

Finally, the body rotation rates are given by

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} L/I_{xx} \\ M/I_{yy} \\ N/I_{zz} \end{bmatrix} \tag{4-28}$$

Equations (4-25) through (4-28) can be combined to give the

12 x 12 linearized system matrix shown below.

$$
\begin{bmatrix} \dot{x}_E \\ \dot{y}_E \\ \Delta\dot{z}_E \\ \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{\theta} \\ \dot{\phi} \\ \dot{\psi} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix}
=
\begin{bmatrix}
0&0&0&1&0&0&0&0&0&0&0&0 \\
0&0&0&0&1&0&0&0&0&0&0&0 \\
0&0&0&0&0&1&0&0&0&0&0&0 \\
0&0&0&0&0&0&-g&0&0&0&0&0 \\
0&0&0&0&0&0&0&g&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&1&0 \\
0&0&0&0&0&0&0&0&0&1&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&1 \\
0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix}
\begin{bmatrix} x_E \\ y_E \\ \Delta z_E \\ u \\ v \\ w \\ \theta \\ \phi \\ \psi \\ p \\ q \\ r \end{bmatrix}
+
\begin{bmatrix}
0&0&0&0&0&0 \\
0&0&0&0&0&0 \\
0&0&0&0&0&0 \\
\frac{1}{m}&0&0&0&0&0 \\
0&\frac{1}{m}&0&0&0&0 \\
0&0&\frac{1}{m}&0&0&0 \\
0&0&0&0&0&0 \\
0&0&0&0&0&0 \\
0&0&0&0&0&0 \\
0&0&0&\frac{1}{I_{xx}}&0&0 \\
0&0&0&0&\frac{1}{I_{yy}}&0 \\
0&0&0&0&0&\frac{1}{I_{zz}}
\end{bmatrix}
\begin{bmatrix} f_x \\ f_y \\ f_z+mg \\ L \\ M \\ N \end{bmatrix}
\tag{4-29}
$$

The force components f_x, f_y, f_z and the torque components L, M, and N are obtained in terms of the 12 state variables. Then the 12 x 12 system matrix state matrix relating the first derivatives of the state variables to the state variables is computed.

From equation (3-52) using small angle approximations,

$$
T_3 \approx \begin{bmatrix} 1 & 0 & -\alpha_i \\ 0 & 1 & \beta_i \\ \alpha_i & -\beta_i & 1 \end{bmatrix} \tag{4-30}
$$

From equation (3-56) the total force applied by leg i to the body at hip socket i expressed in leg coordinates becomes

$$\underline{f}_i' = \begin{bmatrix} T_{m_i}/\ell_i \\ -T_{s_i}/\ell_i \\ f'_{z_i} \end{bmatrix} \qquad (4\text{-}31)$$

Then the total leg force is given by (see equation (3-57))

$$\underline{f}_i = \begin{bmatrix} f_{x_i} \\ f_{y_i} \\ f_{z_i} \end{bmatrix} = T_3^T \underline{f}_j' = \begin{bmatrix} (T_{m_i}/\ell_i) + \alpha_i f'_{z_i} \\ -(T_{s_i}/\ell_i) - \beta_i f'_{z_i} \\ f'_{z_i} \end{bmatrix} \qquad (4\text{-}32)$$

Let the component of the force along the z axis be of the form

$$f'_{z_i} = [\Delta f'_{z_i} - (mg/4)] \qquad (4\text{-}33)$$

Therefore from equations (4-32), (4-33) and (3-58), the total force vector components become

$$\underline{f} = \sum_{i=1}^{4} \underline{f}_i = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{4} \{(T_{m_i}/\ell_o) - (mg\alpha_i/4)\} \\ \sum_{i=1}^{4} \{(-T_{s_i}/\ell_o) + (mg\beta_i/4)\} \\ \sum_{i=1}^{4} \Delta f'_{z_i} - mg \end{bmatrix} \qquad (4\text{-}34)$$

Equation (4-34) can be written in matrix form in terms of all the components of the various forces and torques as given by equation (4-35) below.

$$\begin{bmatrix} f_x \\ f_y \\ (f_z + mg) \end{bmatrix} = \begin{bmatrix} \frac{1}{\ell_o} & \frac{1}{\ell_o} & \frac{1}{\ell_o} & \frac{1}{\ell_o} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{\ell_o} & \frac{-1}{\ell_o} & \frac{-1}{\ell_o} & \frac{-1}{\ell_o} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} T_{m_1} \\ T_{m_2} \\ T_{m_3} \\ T_{m_4} \\ T_{s_1} \\ T_{s_2} \\ T_{s_3} \\ T_{s_4} \\ \Delta f'_{z_1} \\ \Delta f'_{z_2} \\ \Delta f'_{z_3} \\ \Delta f'_{z_4} \end{bmatrix} + \begin{bmatrix} \frac{mg}{4} & \frac{mg}{4} & \frac{mg}{4} & \frac{mg}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{mg}{4} & \frac{mg}{4} & \frac{mg}{4} & \frac{mg}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix} \tag{4-35}$$

The equations giving leg lengths and angles, namely, equations (3-35) through (3-37) are now linearized using the conditions given by equations (4-18) through (4-23). Thus

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = T_1 \begin{bmatrix} x_{iE} - x_E \\ y_{iE} - y_E \\ z_{iE} - z_E \end{bmatrix} = \begin{bmatrix} 1 & \psi & -\theta \\ -\psi & 1 & \phi \\ \theta & -\phi & 1 \end{bmatrix} \begin{bmatrix} a_i - x_E \\ b_i - y_E \\ -z_E \end{bmatrix} \tag{4-36}$$

Substituting $(z_{E_o} + \Delta z_E)$ for z_E and rearranging terms in (4-36)

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = \begin{bmatrix} a_i \\ b_i \\ -z_{E_o} \end{bmatrix} - \begin{bmatrix} x_E \\ y_E \\ z_E \end{bmatrix} + \begin{bmatrix} z_{E_o} & 0 & b_i \\ 0 & -z_{E_o} & -a_i \\ a_i & -b_i & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \phi \\ \psi \end{bmatrix} \tag{4-37}$$

Assume

$$x_i = a_i + \Delta x_i$$
$$y_i = b_i + \Delta y_i \qquad (4\text{-}38)$$
$$z_i = -z_{E_o} + \Delta z_i$$

Using equations (4-37) and (4-38) and replacing z_{E_o} by $-(\ell_o+c_i) = -(\ell_o+c)$, gives the expression

$$\begin{bmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \end{bmatrix} = \begin{bmatrix} -(\ell_o+c) & 0 & b_i \\ 0 & (\ell_o+c) & -a_i \\ a_i & -b_i & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \phi \\ \psi \end{bmatrix} - \begin{bmatrix} x_E \\ y_E \\ \Delta z_E \end{bmatrix} \qquad (4\text{-}39)$$

Differentiating equation (4-39) with respect to time results in the expression

$$\begin{bmatrix} \Delta \dot{x}_i \\ \Delta \dot{y}_i \\ \Delta \dot{z}_i \end{bmatrix} = \begin{bmatrix} -(\ell_o+c) & 0 & b_i \\ 0 & (\ell_o+c) & -a_i \\ a_i & -b_i & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \phi \\ \psi \end{bmatrix} - \begin{bmatrix} \dot{x}_E \\ \dot{y}_E \\ \Delta \dot{z}_E \end{bmatrix} \qquad (4\text{-}40)$$

From equation (3-35), the leg lengths ℓ_i are approximately equal to

$$\ell_i = (z_i - c_i) \qquad (4\text{-}41)$$

The linearized expression for angle α_i is obtained from equations (3-36), (4-39) and (4-41)

$$\alpha_i = \mathrm{Tan}^{-1} \frac{(x_i-a_i)}{(z_i-c_i)} = \frac{(x_i-a_i)}{(z_i-c_i)} = \frac{\Delta x_i}{\ell_o} = \frac{1}{\ell_o}\left[-(\ell_o+c)\theta+b_i\psi-x_E\right] \qquad (4\text{-}42)$$

$$\beta_i = -\sin^{-1}\frac{(y_i-b_i)}{\ell_i} \simeq -\frac{(y_i-b_i)}{\ell_i} = -\frac{\Delta y_i}{\ell_o} = \frac{1}{\ell_o}[-(\ell_o+c)\phi + a_i\psi+y_E] \tag{4-43}$$

The time derivatives of leg lengths and angles are obtained from equations (4-41) through (4-43).

$$\dot{\ell}_i = \dot{z}_i = \Delta\dot{z}_i \tag{4-44}$$

$$\dot{\alpha}_i = \frac{\Delta\dot{x}_i}{\ell_o} = \frac{1}{\ell_o}[-(\ell_o+c)\dot{\theta}+b_i\dot{\Psi}-\dot{x}_E] \tag{4-45}$$

$$\dot{\beta}_i = \frac{\Delta\dot{y}_i}{\ell_o} = \frac{1}{\ell_o}[-(\ell_o+c)\dot{\phi} + a_i\dot{\psi} + \dot{y}_E] \tag{4-46}$$

Now the linearized expressions for the components L, M, and N of the total torque vector $\underline{T}$ are determined as follows.

Linearizing equation (3-50) by replacing $\sin\alpha_i$ by α_i and $\cos\alpha_i$ by unity results in the expression for $\underline{T}_i$

$$\underline{T}_i = \begin{bmatrix} T_{x_i} \\ T_{y_i} \\ T_{z_i} \end{bmatrix} = \begin{bmatrix} T_{s_i} \\ T_{m_i} \\ 0 \end{bmatrix} \tag{4-47}$$

Substituting for f_{x_i}, f_{y_i}, and f_{z_i} from equation (4-32) and for f'_{z_i} from equation (4-33) in equation (3-60) results in the linearized expression for $\underline{M}_i$ given below

$$\underline{M}_i = \begin{bmatrix} \{[\Delta f'_{z_i}-(mg/4)]b_i+c_i(T_{s_i}/\ell_o)+c_i\beta_i[\Delta f'_{z_i}-(mg/4)]\} \\ \{(c_iT_{m_i}/\ell_o)+ c_i\alpha_i\{\Delta f'_{z_i}-(mg/4)\}-a_i\{\Delta f'_{z_i}-(mg/4)\}\} \\ \\ \{-a_i(T_{s_i}/\ell_o)-a_i\beta_i[\Delta f'_{z_i}-(mg/4)]-b_i(T_{m_i}/\ell_o)-b_i\alpha_i[\Delta f'_{z_i}-\frac{mg}{4}]\} \end{bmatrix} \tag{4-48}$$

Substituting for $\underline{T}_i$ from equation (4-47) and for $\underline{M}_i$ from equation (4-48) in equation (3-60) for the total torque vector $\underline{T}$ results in the linearized expressions for the total torque vector components L, M, and N given below.

$$\underline{T} = \begin{bmatrix} L \\ M \\ N \end{bmatrix} = \sum_{i=1}^{4} \begin{bmatrix} \{[1+(c_i/\ell_o)T_{s_i} + b_i[\Delta f'_{z_i} - (mg/4)] + c_i\beta_i[\Delta f'_{z_i} - (mg/4)]\} \\ \{[1+(c_i/\ell_o)]T_{m_i} - a_i[\Delta f'_{z_i} - (mg/4)] + c_i\alpha_i[\Delta f'_{z_i} - (mg/4)]\} \\ \{(-1/\ell_o)[b_i T_{m_i} + a_i T_{s_i}] - [a_i\beta_i + b_i\alpha_i][\Delta f'_{z_i} - (mg/4)]\} \end{bmatrix} \tag{4-49}$$

4.3.2 Feedback Control Laws

The types of feedback control laws that are used in the control of the quadruped use leg angles and leg angle rates as a function of time. A simple linear postural control scheme for use with the quadruped would use the following control laws.

For vertical control, the vertical force applied by leg i to the body at hip socket i expressed in leg coordinates is given by

$$f'_{z_i} = C_\ell(\ell_i - \ell_o) + C_{\dot\ell}\dot\ell_i \tag{4-50}$$

where C_ℓ, and $C_{\dot\ell}$ are predetermined constants.

For lateral control the torque T_{s_i} produced by a centering spring and damper system is of the form

$$T_{s_i} = C_\beta(\beta_i - \beta_{c_i}) + C_{\dot\beta}\dot\beta_i + C_y y_E + C_{\dot y}\dot y_E \tag{4-51}$$

where C_β, $C_{\dot\beta}$, C_y, and $C_{\dot y}$ are predetermined constants.

Longitudinal control requires a control law giving the hip motor torque T_{m_i} required to maintain control of the machine in the longitudinal plane. The law used for longitudinal control is given below.

$$T_{m_i} = C_\alpha(\alpha_i - \alpha_{c_i}) + C_{\dot\alpha}\dot\alpha_i + C_{\dot x}\dot x_E \qquad (4\text{-}52)$$

where C_α, $C_{\dot\alpha}$, and $C_{\dot x}$ are predetermined constants and α_{c_i} is the desired hip angle for leg i, whenever leg i is in contact with ground.

4.3.3 Postural System Control Laws

For postural control the quadruped is made to stand on all its feet with the feet vertically below the respective hip joints. This condition is equivalent to the following initial values

$$\begin{aligned} \alpha_{c_i} &= 0 \\ \beta_{c_i} &= 0 \\ \ell_{c_i} &= \ell_o \end{aligned} \qquad (4\text{-}53)$$

Using the conditions of equation (4-53), and substituting for α_i , $\dot\alpha_i$ from equations (4-42) and (4-45) in equation (4-52) and using the results of equations (4-25) and (4-26), the linearized expression for the longitudinal torque simplifies to

$$T_{m_i} = \{\frac{C_\alpha}{\ell_o}[-(\ell_o+c)\theta+b_i\psi-x_E]+\frac{C_{\dot\alpha}}{\ell_o}[-(\ell_o+c)q+b_i r-u] + C_{\dot x}u\} \qquad (4\text{-}54)$$

Using the conditions of equation (4-53), and substituting for β_i, and $\dot\beta_i$ from equations (4-43) and (4-46) in equation (4-51), and simplifying the expression results in the linearized control law for the lateral torque T_{s_i} given below in equation (4-55).

$$T = \left\{\frac{C_\beta}{\ell_o}[-(\ell_o+c)\phi+a_i\psi+y_E]+\frac{C_{\dot\beta}}{\ell_o}[-(\ell_o+c)p+a_i r+v] + C_y y_E + C_{\dot y} v\right\} \tag{4-55}$$

From equation (4-50) upon simplification, the control law for the vertical force $\Delta f'_{z_i}$ reduces to the expression

$$\Delta f'_{z_i} = C_\ell[a_i\theta-b_i\phi-\Delta z_E] + C_{\dot\ell}[a_i q-b_i p-w] \tag{4-56}$$

Equations (4-54) through (4-56) describe the postural system control laws in terms of the state variables of the quadruped system.

4.3.4 Total Forces and Moments Applied to the Body

To compute the total force components f_x, f_y, f_z in terms of the state variables, substitution of the above expressions for T_{m_i}, T_{s_i}, and Δf_{z_i} into equation (4-34) and simplification by collecting terms and summing over $i = 1, \ldots 4$, using the symmetry properties of equations (4-18) through (4-20) results in

$$\underline{f} = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} \left[\frac{mg}{\ell_o}-\frac{4C_\alpha}{\ell_o^2}\right]x_E+\frac{4}{\ell_o}\left[C_{\dot x}-\frac{C_{\dot\alpha}}{\ell_o}\right]u+\left(mg-\frac{4C_\alpha}{\ell_o}\right)\frac{(\ell_o+c)}{\ell_o}\theta-4\frac{(\ell_o+c)}{\ell_o^2}C_{\dot\alpha}q \\ \left[\frac{mg}{\ell_o}-\frac{4}{\ell_o^2}(C_\beta+\ell_o C_y)\right]y_E-\frac{4}{\ell_o^2}(C_{\dot\beta}+\ell_o C_{\dot y})v-\left(mg-\frac{4C_\beta}{\ell_o}\right)\frac{(\ell_o+c)}{\ell_o}\phi+4\frac{(\ell_o+c)}{\ell_o^2}C_{\dot\beta}p \\ -4[C_\ell\Delta z_E + C_{\dot\ell}w] \end{bmatrix} \tag{4-57}$$

The components of the total torque vector $\underline{T}$, namely, L, M, N are obtained in terms of the state variables by substituting for T_{m_i}, T_{s_i}, $\Delta f'_{z_i}$, α_i, β_i, etc, in equation (4-49). After simplifying and collecting terms the following expression is obtained for the total torque vector $\underline{T}$.

$$
\underline{T} = \begin{bmatrix} L \\ \\ M \\ \\ N \end{bmatrix} = \begin{bmatrix} \begin{array}{l} \{[-\frac{mgc}{\ell_o}+4\frac{(\ell_o+c)}{\ell_o^2}(C_\beta+\ell_o C_\gamma)]y_E+[-4\frac{(\ell_o+c)^2}{\ell_o^2}C_\beta-4b^2C_\ell+\frac{mgc}{\ell_o}(\ell_o+c)]\phi \\ \qquad +[4\frac{(\ell_o+c)}{\ell_o^2}(C_{\dot\beta}+\ell_o C_{\dot\gamma})]v-[4C_{\dot\beta}\frac{(\ell_o+c)^2}{\ell_o^2}+4b^2C_{\dot\ell}]p\} \\ \{[\frac{mgc}{\ell_o}-4\frac{(\ell_o+c)}{\ell_o^2}C_\alpha]x_E-[4\frac{(\ell_o+c)}{\ell_o^2}(C_{\dot\alpha}-\ell_o C_{\dot x})]u+[mgc\frac{(\ell_o+c)}{\ell_o}-4a^2C_\ell- \\ \qquad\qquad 4\frac{(\ell_o+c)^2}{\ell_o^2}C_\alpha]\theta-4[a^2C_{\dot\ell}+\frac{(\ell_o+c)^2}{\ell_o^2}C_{\dot\alpha}]q\} \\ \{[\frac{mg}{\ell_o}(a^2+b^2)-\frac{4}{\ell_o^2}(a^2C_\beta+b^2C_\alpha)]\psi-\frac{4}{\ell_o^2}[b^2C_{\dot\alpha}+a^2C_{\dot\beta}]r\} \end{array} \end{bmatrix} \tag{4-58}
$$

4.3.5 State Variable Representation of the Small Angle Equations of Motion of the Quadruped Locomotion System

The theory of linearization of the quadruped equations of motion developed above is now used to obtain a matrix representation for this system. Upon substituting the expressions for f_x, f_y, f_z from equation (4-57), and the quantities L, M, N from equation (4-58), in the system state equations given by (4-29) and collecting terms, the system state equations are reduced to a 12 x 12 system matrix relating the components of the state vector $\underline{x}$ to their respective first derivatives with respect to time. This 12 x 12 system matrix given below in equation (4-59) decouples into four smaller matrices as indicated by the dotted lines. These four smaller matrices describe the four independent vibrational systems of the quadruped postural system.

$$\begin{bmatrix} \Delta\dot{z}_E \\ \dot{w} \\ \dot{\psi} \\ \dot{r} \\ \dot{x}_E \\ \dot{u} \\ \dot{\theta} \\ \dot{q} \\ \dot{y}_E \\ \dot{v} \\ \dot{\phi} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ A & B & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C & D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & E & F & G & H & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & J & K & L & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M & N & P & Q \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R & S & T & U \end{bmatrix} \begin{bmatrix} \Delta z_E \\ w \\ \psi \\ r \\ x_E \\ u \\ \theta \\ q \\ y_E \\ v \\ \phi \\ p \end{bmatrix} \tag{4-59}$$

where

$$A = -4\frac{C_\ell}{m} \tag{4-60}$$

$$B = -(4C_{\dot{\ell}}/m) \tag{4-61}$$

$$C = mg\frac{(a^2+b^2)}{\ell_o I_{zz}} - \frac{4(a^2C_\beta + b^2C_\alpha)}{I_{zz}\ell_o^2} \tag{4-62}$$

$$D = -(4/I_{zz}\ell_o^2)[b^2C_{\dot{\alpha}} + a^2C_{\dot{\beta}}] \tag{4-63}$$

$$E = [(g/\ell_o) - (4C_\alpha/m\ell_o^2)] \tag{4-64}$$

$$F = [-(4C_{\dot{\alpha}}/m\ell_o^2) + (4C_{\dot{x}}/m\ell_o)] \tag{4-65}$$

$$G = [(gc/\ell_o) - \{4(\ell_o+c)C_\alpha/m\ell_o^2\}] \quad (4-66)$$

$$H = [-\{4(\ell_o+c)C_{\dot{\alpha}}/m\ell_o^2\}] \quad (4-67)$$

$$I = \{-[4(\ell_o+c)C_\alpha/I_{yy}\ell_o^2] + [mgc/I_{yy}\ell_o]\} \quad (4-68)$$

$$J = \{[4(\ell_o+c)(\ell_o C_{\dot{x}} - C_{\dot{\alpha}})]/(I_{yy}\ell_o^2)\} \quad (4-69)$$

$$K = \{[mgc(\ell_o+c)/I_{yy}\ell_o] - [4a^2C_\ell/I_{yy}] - [4(\ell_o+c)^2C_\alpha/(I_{yy}\ell_o^2)]\} \quad (4-70)$$

$$L = \{-[4(\ell_o+c)^2C_{\dot{\alpha}}/(I_{yy}\ell_o^2)] - [4a^2C_{\dot{\ell}}/I_{yy}]\} \quad (4-71)$$

$$M = \{[g/\ell_o] - [4(C_\beta+\ell_o C_y)/(m\ell_o^2)]\} \quad (4-72)$$

$$N = -[4(C_{\dot{\beta}}+\ell_o C_{\dot{y}})/(m\ell_o^2)] \quad (4-73)$$

$$P = \{[4(\ell_o+c)C_\beta/(m\ell_o^2)] - (gc/\ell_o)\} \quad (4-74)$$

$$Q = [4(\ell_o+c)C_{\dot{\beta}}/(m\ell_o^2)] \quad (4-75)$$

$$R = \{[4(\ell_o+c)(C_\beta+\ell_o C_y)/(I_{xx}\ell_o^2)] - [mgc/(I_{xx}\ell_o)]\} \quad (4-76)$$

$$S = [4(\ell_o+c)(C_{\dot{\beta}}+\ell_o C_{\dot{y}})/(I_{xx}\ell_o^2)] \quad (4-77)$$

$$T = \{[mgc(\ell_o+c)/(I_{xx}\ell_o)] - [4b^2C_\ell/(I_{xx})] - [4\frac{(\ell_o+c)^2}{I_{xx}\ell_o^2}C_\beta]\} \quad (4-78)$$

$$U = -\frac{4}{I_{xx}}\{[(\ell_o+c)^2C_{\dot{\beta}}/(\ell_o^2)] + b^2C_{\dot{\ell}}\} \quad (4-79)$$

From equations (4-59) through (4-79), it is seen that the linearized equations of motion for the quadruped locomotion system decompose into basically four types of vibrational systems which are independent of each other. The upper left hand 2 x 2 matrix in equation (4-59), describes the vibrational modes of translation along the z axis. The other 2 x 2 matrix describes the vibrational system associated with rotational modes about the z axis. The next matrix in equation (4-59), which is a 4 x 4 matrix represents the vibrational system associated with the vibrational modes describing translational motion along the x axis along with rotational motion described by the Euler angle θ about the y axis. Finally, the lower right hand 4 x 4 matrix in equation (4-59) represents the vibrational system associated with modes made up of translational motion along the y axis along with rotational motion described by the Euler angle ϕ about the x axis.

4.4 Intuitive Approach to the X Axis Vibrational Modes

Assume that initially the quadruped is stand' ~ on its four legs with its feet directly below the respective hip joint. The system is then slightly disturbed from this equilibrium position such that only the vibrational modes representing translational motion along the x axis with rotational motion θ about the y axis is generated. It is intuitively assumed that such decoupling of the vibrational modes for the quadruped postural system exists. This assumption gives rise to the following conditions:

1) The Euler angles bec. ie

$$\theta = \text{small}$$

$$\phi = 0 \qquad (4\text{-}80)$$

$$\psi = 0$$

2) The position of the center of gravity of the body relative the earth fixed frame has coordinates

$$x_E = \text{small}$$
$$y_E = 0 \qquad (4\text{-}81)$$
$$z_E = -z_{E_o}$$

where z_{E_o} = intial value of the z coordinate of the center of gravity of the body in the earth fixed frame.

3) The initial position of the center of gravity of the body in earth fixed coordinates is

$$x_{E_o} = 0$$
$$y_{E_o} = 0 \qquad (4\text{-}82)$$
$$z_{E_o} = -(\ell_o+c)$$

where ℓ_o = initial length of all four legs of the quadruped, and c = z coordinate of all four hip sockets of the body.

4) The foot position of leg i, i=1,...4, in earth fixed coordinates is

$$x_{iE} = a_i$$
$$y_{iE} = b_i \qquad (4\text{-}83)$$
$$z_{iE} = 0$$

5) The leg angles have values

$$\alpha_i = \text{small}$$
$$\beta_i = 0 \qquad (4\text{-}84)$$

6) Since the quadruped is assumed to be excited in the particular vibrational modes under consideration here, the total moment vector $\underline{T}$ defined by equation (3-16) becomes

$$\underline{T} = [0,\ M,\ 0]^T \tag{4-85}$$

7) The total force vector $\underline{f}$ (see equation (3-58)), reduces to

$$\underline{f} = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{4} f_{x_i} \\ 0 \\ -mg \end{bmatrix} \tag{4-86}$$

since the total z axis component of the total force equals the negative of the weight of the body.

Assuming that $\sin\theta$ can be replaced by θ, and $\cos\theta$ equals unity, and substituting equations (4-85) and (4-86) into the equations of motion derived in Chapter III, namely, equations (3-23) through (3-28), the body translational velocity components and the components of body rotational rate measured about the x, y, z axes become

$$\dot{u} = (f_x/m) - g\theta \tag{4-87}$$

$$\dot{v} = 0 \tag{4-88}$$

$$\dot{w} = 0 \tag{4-89}$$

$$\dot{p} = 0 \tag{4-90}$$

$$\dot{q} = (M/I_{yy}) \tag{4-91}$$

$$\dot{r} = 0 \tag{4-92}$$

Using the assumptions outlined in equations (4-80) through (4-86) the equations derived in Chapter III are systematically linearized to get expressions for f_x and M, and thus for $\dot{u}$ and $\dot{q}$ in terms of the state variables.

Simplification of the transformation matrices T_1, T_2 and T_3 yields [see equations (3-14), (3-31), and (3-52)]

$$T_1 = \begin{bmatrix} 1 & 0 & -\theta \\ 0 & 1 & 0 \\ \theta & 0 & 1 \end{bmatrix} \tag{4-93}$$

$$T_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & \theta \\ 0 & 0 & 1 \end{bmatrix} \tag{4-94}$$

$$T_3 = \begin{bmatrix} 1 & 0 & -\alpha_i \\ 0 & 1 & 0 \\ \alpha_i & 0 & 1 \end{bmatrix} \tag{4-95}$$

The position of the foot of leg i in body coordinates becomes

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = T_1 \begin{bmatrix} (x_{iE}-x_E) \\ (y_{iE}-y_E) \\ (z_{iE}-z_E) \end{bmatrix} = \begin{bmatrix} a_i-(\ell_o+c)\theta-x_E \\ b_i \\ a_i\theta+(\ell_o+c) \end{bmatrix} \tag{4-96}$$

From equation (4-96)

$$\begin{bmatrix} (x_i-a_i) \\ (y_i-b_i) \\ (z_i-c_i) \end{bmatrix} \approx \begin{bmatrix} -[x_E + (\ell_o+c)\theta] \\ 0 \\ [a_i\theta + \ell_o] \end{bmatrix} \tag{4-97}$$

From equations (4-97) and (3-35), neglecting the term $[-\theta(\ell_o+c)+x_E]^2$ as compared to $[\ell_o+a_i\theta]^2$, the leg lengths and their time derivatives are given by

$$\ell_i = [\ell_o + a_i\theta] \tag{4-98}$$

$$\dot{\ell}_i = a_i \dot{\theta} \tag{4-99}$$

From equations (3-36), (4-98), and (4-99) linearization results in the following expression for leg angle α_i

$$\alpha_i = -\{[x_E+(\ell_o+c)\theta]/[\ell_o+a_i\theta]\} \tag{4-100}$$

Using a Taylor's series expression of the form

$$\alpha_i = \left(\frac{\partial \alpha_i}{\partial x_E}\right)_{\substack{x_E=0\\ \theta=0}} x_E + \left(\frac{\partial \alpha_i}{\partial \theta}\right)_{\substack{x_E=0\\ \theta=0}} \theta \tag{4-101}$$

on equation (4-100), the leg angle α_i and its time derivative are given by the expressions

$$\alpha_i = -\frac{1}{\ell_o}[x_E+(\ell_o+c)\theta] \tag{4-102}$$

$$\dot{\alpha}_i = -\frac{1}{\ell_o}[\dot{x}_E+(\ell_o+c)\dot{\theta}] \tag{4-103}$$

Substitution of the expressions for α_i, $\dot{\alpha}_i$, x_i from the above equations (4-102), (4-103) and (4-96), and $\dot{x}_i = -[\dot{x}_E+(\ell_o+c)\dot{\theta}]$ in equation (4-52) for the lateral control torque T_{m_i} results in the linearized expression

$$T_{m_i} = -\frac{C_\alpha}{\ell_o}[x_E+(\ell_o+c)\theta] - \frac{C_{\dot{\alpha}}}{\ell_o}[\dot{x}_E+(\ell_o+c)\dot{\theta}] + C_{\dot{x}}\dot{x}_E \tag{4-104}$$

From equation (3-56), the total force applied by leg i to the body at hip socket i is given by

$$\underline{f}_i' = [\, T_{m_i}/\ell_i \;\; 0 \;\; f_{z_i}' \,]^T \tag{4-105}$$

Also

$$\frac{T_{m_i}}{\ell_i} = \frac{T_{m_i}}{\ell_o}\,\frac{\ell_o}{\ell_i} = \frac{T_{m_i}}{\ell_o}\,\frac{\ell_o}{(\ell_o + a_i\theta)} \approx \frac{T_{m_i}}{\ell_o} \tag{4-106}$$

The z component of the total force applied by leg i to the body is assumed to be given by the expression

$$f'_{z_i} = [C_\ell(\ell_i - \ell_o) + C_{\dot{\ell}}\dot{\ell}_i - (mg/4)] \tag{4-107}$$

The total leg force vector components in body coordinates then become (see equations(3-57), (3-58))

$$\underline{f} = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{4}[(T_{m_i}/\ell_o) + \alpha_i f'_{z_i}] \\ 0 \\ \sum_{i=1}^{4} f'_{z_i} \end{bmatrix} \tag{4-108}$$

Simplification of equation (4-108) using equations (4-102), (4-104), (4-107), and the symmetry conditions of equations (4-18) through (4-20) results in the linearized equation for $\underline{f}$

$$\underline{f} = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} -\frac{4C_\alpha}{\ell_o^2}[(\ell_o+c)\theta + x_E] - \frac{4C_{\dot{\alpha}}}{\ell_o^2}[(\ell_o+c)\dot{\theta} + \dot{x}_E] + \frac{4C_{\dot{x}}\dot{x}_E}{\ell_o} + \frac{mg}{\ell_o}[x_E + (\ell_o+c)\theta] \\ 0 \\ -mg \end{bmatrix} \tag{4-109}$$

From equations (3-50), (3-59), and (3-60) the component M of the total torque vector $\underline{T}$ finally reduces to the expression

$$M = \left\{\frac{[x_E + (\ell_o+c)\theta]}{\ell_o^2}[mgc\ell_o - 4C_\alpha(\ell_o+c)] - 4C_{\dot{\alpha}}\frac{(\ell_o+c)}{\ell_o^2}[\dot{x}_E + (\ell_o+c)\dot{\theta}] + 4C_{\dot{x}}\frac{(\ell_o+c)}{\ell_o}\dot{x}_E - 4a^2(C_\ell\theta + C_{\dot{\ell}}\dot{\theta})\right\} \tag{4-110}$$

From equations (3-29), (3-30), (4-93), and (4-94)

$$\dot{x}_E = u \tag{4-111}$$

$$\dot{\theta} = q \tag{4-112}$$

Application of the expressions for f_x and M from equations (4-109) and (4-110) in the equations of motion (4-87) and (4-91) results in the equations expressing $\dot{u}$ and $\dot{q}$ in terms of the state variables.

The translational acceleration along the x axis is given by the relationship

$$\dot{u} = \{[(g/\ell_o) - (4C_\alpha/m\ell_o^2)]x_E + [(4/m\ell_o^2)(C_{\dot{x}}\ell_o - C_{\dot{\alpha}})]u + [(gc/\ell_o) - \{4C_\alpha(\ell_o+c)/m\ell_o^2\}]\theta - [4C_{\dot{\alpha}}(\ell_o+c)/(m\ell_o^2)]q\} \tag{4-113}$$

The angular acceleration of the body about the y axis becomes equal to

$$\dot{q} = \{(1/I_{yy}\ell_o^2)[mgc\ell_o - 4(\ell_o+c)C_\alpha]x_E + [4(\ell_o+c)/(I_{yy}\ell_o^2)][\ell_o C_{\dot{x}} - C_{\dot{\alpha}}]u + (1/I_{yy}\ell_o^2)[mgc\ell_o(\ell_o+c) - 4a^2\ell_o^2 C_\ell - 4C_\alpha(\ell_o+c)^2]\theta - (4/I_{yy}\ell_o^2)[a^2\ell^2 C_{\dot{\ell}} + (\ell_o+c)^2 C_{\dot{\alpha}}]q\} \tag{4-114}$$

From equations (4-111) through (4-114), the linearized system matrix that describes the x axis vibrational modes for small disturbances of the system from its equilibrium position can be written down. This matrix is given below.

$$\begin{bmatrix} x_E \\ u \\ \theta \\ q \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ E & F & G & H \\ 0 & 0 & 0 & 1 \\ I & J & K & L \end{bmatrix} \begin{bmatrix} x_E \\ u \\ \theta \\ q \end{bmatrix} \tag{4-115}$$

In equation (4-115), the quantities E, F, G, ... L, are given by expressions from equations (4-64) through (4-71) respectively.

Equation (4-115) describes the linearized system for small motions associated with the independent vibrational modes of translation along the x axis together with rotation θ about the y axis, that is the modes associated with motions in the y-z plane. Comparing this modal matrix derived intuitively considering that the x axis modes are decoupled from the other vibrational systems, with the results obtained by rigorous analysis (see equation (4-60)), it is seen that the two results are identical. Using a similar intuitive approach, it is possible to derive the modal matrices associated with the translational and rotational motions with respect to the y axis as well as the z axis. For dynamical systems whose vibrational modes do not decouple, the linearized system matrix should be obtained by rigorous analysis.

4.5 Small Angle Equations of Motion for the Inverted Pendulum System

The equations of motion for the inverted pendulum system derived in Section 3.4 are now linearized and a state variable representation for the small angle equations of motion for this system is obtained.

Consider the following system of state variables

$$x_1 = \phi_1 \tag{4-116}$$

$$x_2 = \phi_2 \qquad (4\text{-}117)$$

$$x_3 = \dot{\phi}_1 = \dot{x}_1 \qquad (4\text{-}118)$$

$$x_4 = \dot{\phi}_2 = \dot{x}_2 \qquad (4\text{-}119)$$

Substituting these state variables in the equations of motion of the inverted pendulum system given by equations (3-72) and (3-73) results in the following system of equations

$$m\ell^2\dot{x}_3 + m\ell r\dot{x}_4\cos(x_1-x_2) + m\ell r x_4^2\sin(x_1-x_2) - mg\ell\sin x_1 = -M \qquad (4\text{-}120)$$

$$(I+mr^2)\dot{x}_4 + m\ell r\dot{x}_3\cos(x_1-x_2) - m\ell r x_3^2\sin(x_1-x_2) - mgr\sin x_2 = M \qquad (4\text{-}121)$$

Replacing the cosines by unity, and the sines by their angles, and ignoring terms containing products of the state variables, the linearized equations for the inverted pendulum system are obtained.

$$m\ell^2\dot{x}_3 + m\ell r\dot{x}_4 - mg\ell x_1 = -M \qquad (4\text{-}122)$$

$$(I+mr^2)\dot{x}_4 + m\ell r\dot{x}_3 - mgrx_2 = M \qquad (4\text{-}123)$$

In equations (4-122), and (4-123), m = mass of the body, r = distance of the pivot below the center of gravity of the body, ℓ = constant length of the supporting leg, g = acceleration due to gravity, and M = control torque applied to stabilize the system.

Assume a control torque of the form

$$M = K_1x_2 + K_2x_4 + K_3x_1 + K_4x_3 \qquad (4\text{-}124)$$

Substituting equation (4-124) in equations (4-122) and (4-123), and simplifying the resulting expressions, the equations of the linearized system are obtained in terms of the state variables and the control constants K_1 through K_4..

$$m\ell^2\dot{x}_3 + m\ell r\dot{x}_4 + (K_3 - mg\ell)x_1 + K_1x_2 + K_4x_3 + K_2x_4 = 0 \qquad (4\text{-}125)$$

$$m\ell r\dot{x}_3 + (I + mr^2)\dot{x}_4 - K_3x_1 - (K_1 + mgr)x_2 - K_4x_3 - K_2x_4 = 0 \qquad (4\text{-}126)$$

The above equations can be further simplified by using the following substitutions.

Let

$$A_1 = m\ell^2 \qquad (4\text{-}127)$$

$$A_2 = (I + mr^2) \qquad (4\text{-}128)$$

$$A_3 = m\ell r \qquad (4\text{-}129)$$

$$A_4 = (K_3 - mg\ell) \qquad (4\text{-}130)$$

$$A_5 = (K_1 + mgr) \qquad (4\text{-}131)$$

Then, equations (4-125) and (4-126) can be expressed in the form

$$A_1\dot{x}_3 + A_3\dot{x}_4 + A_4x_1 + K_1x_2 + K_4x_3 + K_2x_4 = 0 \qquad (4\text{-}132)$$

$$A_3\dot{x}_3 + A_2\dot{x}_4 - K_3x_1 - A_5x_2 - K_4x_3 - K_2x_4 = 0 \qquad (4\text{-}133)$$

Solving these equations simultaneously for $\dot{x}_3$ and $\dot{x}_4$, the linearized system state equations become

$$\dot{x}_1 = x_3 \qquad (4\text{-}134)$$

$$\dot{x}_2 = x_4 \qquad (4\text{-}135)$$

$$\dot{x}_3 = -\frac{[(A_2A_4 + A_3K_3)x_1 + (A_2K_1 + A_3A_5)x_2 + (A_2 + A_3)K_4x_3 + (A_2 + A_3)K_2x_4]}{[A_1A_2 - A_3^2]} \qquad (4\text{-}136)$$

$$\dot{x}_4 = \frac{[(A_1K_3+A_3A_4)x_1+(A_1A_5+A_3K_1)x_2+(A_1+A_3)K_4x_3+(A_1+A_3)K_2x_4]}{[A_1A_2 - A_3^2]} \tag{4-137}$$

Equations (4-134) through (4-137) are the linearized system state equations for the inverted pendulum system with a " fixed " foot, and a massless leg with the mass pivoted below its center of gravity.

Equations (4-134) through (4-137) can be put in the following matrix form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ A & B & C & D \\ E & F & G & H \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \tag{4-138}$$

where

$$A = -[(A_2A_4+A_3K_3)/(A_1A_2-A_3^2)] \tag{4-139}$$

$$B = -[(A_3A_5+A_2K_1)/(A_1A_2-A_3^2)] \tag{4-140}$$

$$C = -[(A_2+A_3)K_4/(A_1A_2-A_3^2)] \tag{4-141}$$

$$D = -[(A_2+A_3)K_2/(A_1A_2-A_3^2)] \tag{4-142}$$

$$E = [(A_3A_4+A_1K_3)/(A_1A_2-A_3^2)] \tag{4-143}$$

$$F = [(A_1A_5+A_3K_1)/(A_1A_2-A_3^2)] \tag{4-144}$$

$$G = [(A_1+A_3)K_4/(A_1A_2-A_3^2)] \tag{4-145}$$

$$H = [(A_1+A_3)K_2/(A_1A_2-A_3^2)] \tag{4-146}$$

The state variable representation of the linearized equations of motion given above will be used to obtain the linear system response of the inverted pendulum system for small motions about its equilibrium position. This matrix representation is also used for the application of the Routh-Hurwitz test to determine stabilizing control constants for this system.

4.6 Summary

In this chapter, the application of vibrational analysis to legged locomotion systems has been described, and the basic equations for the linearized systems of these dynamical systems have been derived.

It is shown that the system equations of the quadruped after linearization yield a 12 x 12 system matrix which decomposes into four smaller matrices. These smaller matrices describe the various independent vibrational systems of the quadruped postural system corresponding to the translational and rotational motions associated with the x, y, and z axis respectively.

Next, using intuitive assumptions of a decoupled system, the linearized equations for the vibrational modes associated with motions in the y-z plane (x axis modes) are derived separately. A comparison between the approaches, namely, the rigorous analysis, and the intuitive approach, shows that their results are identical.

The chapter concludes with the derivation of the linearized equations of motion for the inverted pendulum system discussed in Section 3.4. These linearized equations will be used in later chapters for computing the eigenvalues and eigenvectors of the linear system and hence obtaining the linear system response for small perturbations. Then, the nonlinear system is excited along the same independent vibrational modes, and its response compared with the linearized system response. It is shown in Chapter VII that these two responses agree for small motions to the desired degree of accuracy, thereby verifying the nonlinear quadruped simulation, as well as the linearization techniques.

CHAPTER V

EIGENVALUES AND EIGENVECTORS

5.1 Introduction

This chapter discusses the problem of finding the eigenvalues and eigenvectors of the real, non-symmetric modal matrices obtained for the linearized locomotion systems in Chapter IV. Eigenvalues and their corresponding eigenvectors are needed to compute the linear system response.

At the present time, practical computer programs for determining the eigenvalues and eigenvectors of general, non-symmetric, real, square matrices are not many in number, and numerical analysis techniques in this area are still being improved. The purpose of this chapter is mainly to outline the applicaiton of some recently published " state-of-the-art " computational subroutines in this area on the linearized locomotion system matrices. Accordingly, Section 5.2 discusses briefly some of the methods for solving the complete eigenvalue problem for non-symmetric, real matrices. Section 5.3 outlines the use of subroutines NSEVB and EIGENP for finding the eigenvalues and eigenvectors of the linearized system matrices of both the quadruped as well as the inverted pendulum systems.

5.2 The Complete Eigenproblem for Non-Symmetric, Real Matrices

There are different methods available in the literature for finding the eigenvalues and eigenvectors of non-symmetric, real matrices [56-59]. Some of these methods are discussed briefly in this section. In these methods, a series of simularity transformations are performed in order to reduce the non-symmetric matrix A to either a tridiagonal matrix, or a Hessenberg matrix with eigenvalues which are the same as those of A but more easily computable.

5.2.1 Method of Lanczos [60]

In this method, a similarity transformation is used to reduce the non-symmetric, real matrix into a tridiagonal matrix. The eigenvalues and eigenvectors of this transformed matrix are then computed.

5.2.2 The LR Transformation [61]

This method can be applied to an arbitrary matrix, however it is particularly useful for matrices in the tridiagonal or the Hessenberg form. This method is based on the successive decomposition of a sequence of matrices $\{A_k\}$ all of which have the same form as the original matrix. The process of deriving the sequence $\{A_k\}$ from A by successive triangular decompositions is called the Left-Right (LR) transformation. This method converges for a large class of matrices including all symmetric, positive definite matrices, many matrices with distinct, real eigenvalues, and many matrices with real eigenvalues which satisfy neither of these two conditions. This method may not converge for some matrices with complex eigenvalues. Also, triangular decomposition may not always be numerically stable even for matrices having only real eigenvalues.

5.2.3 The QR Transformation [62]

This method is analogous to the LR transformation but is more numerically stable since it makes use of orthogonal transformations rather than triangular decompsition. This method becomes too laborious for arbitrary matrices, and is used mostly on special matrices such as the Hessenberg matrix or symmetric-band matrices.

A good general purpose scheme for solving the complete eigenvalue problem for non-symmetric, real matrices is to first reduce the matrix to the Hessenberg form using the Gaussian elemination technique, and then to use the QR transformation on the reduced matrix to calculate the eigenvalues. The eigenvectors can then be computed by methods such as the inverse-iteration procedure.

5.3 Determination of the Eigenvalues and Eigenvectors of the Linearized Locomotion System Matrices

The linearized locomotion system matrices for the quadruped vibrational modes and the inverted pendulum system were derived in Chapter IV. From the results of Chapter IV, the types of matrices under consideration are of the following forms:

$$\begin{bmatrix} 0 & 1 \\ A & B \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 0 & 0 \\ A & B & C & D \\ 0 & 0 & 0 & 1 \\ E & F & G & H \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ A & B & C & D \\ E & F & G & H \end{bmatrix}$$

where A, B,..., H are all scalars. Therefore, these non-symmetric, real matrices may have real as well as complex conjugate eigenvalues and correspondingly, real and complex conjugate eigenvectors.

After a survey of practical methods for determining the eigenvalues and eigenvectors for non-symmetric, real matrices, two " state-of-the-art " methods that are particularly applicable to the problem at hand were found [63,65]. At the present time, the problem of finding the eigenvalues and eigenvectors of general non-symmetric, real matrices is receiving a lot of attention, and in the near future more practical computer algorithms should become available.

In the method of Grad and Brebner [63], namely subroutine EIGENP, the following steps are carried out. The matrix is scaled by a sequence of similarity transformations so that the absolute sums of the corresonding rows and columns are roughly equal. Then, the scaled matrix is normalized so that the square of the Euclidean norm is equal to unity. This matrix is then reduced to an upper-Hessenberg form by means of similarity transformations (Householder's method). Then the eigenvalues are computed by the QR double-step method and the corresponding eigenvectors by inverse iteration.

Subroutine NSEVB [64,65], is another program that can be used for the numerical solution of the non-symmetric eigenproblem. In this subroutine, the real general matrix is first reduced to an almost upper-triangular form by stabilized, elementary, similarity transformations. Then, the QR double-step algorithm is applied to the reduced matrix. The eigenvectors are then computed by Wielandt's inverse iteration method. For the case of well separated eigenvalues, rigorous machine bounds are given for the computed eigensystem using Gerschgorin's theorem.

The Fortran subroutine EIGENP and a Fortran version of the original Algol subroutine NSEVB were used on an IBM 360/75 computer to determine the eigenvalues and eigenvectors of the non-symmetric, real matrices of the linearized locomotion systems of the type discussed above. These matrices yielded both real and complex conjugate eigenvalues and eigenvectors which were then used to compute the linear system response as discussed in Section 4.2.

5.4 Summary

This chapter has briefly outlined the methods available for solving the eigenproblem for general, non-symmetric, real matrices. The computational algorithms used in this research for the determination of the eigenvalues and eigenvectors of the linearized locomotion system matrices, namely subroutines EIGENP and NSEVB have also been described.

In this chapter, no attempt has been made to present the computational details of any of these methods. The numerical results obtained on an IBM 360/75 computer are given in Chapter VII.

In conclusion, this chapter represents a brief introduction to the complete eigenvalue problem for real, non-symmetric matrices encountered in the course of this research.

CHAPTER VI

STABILITY AND CONTROL OF LEGGED LOCOMOTION SYSTEMS

6.1 Introduction

The problem of stability and control of legged locomotion systems has been studied from several viewpoints in the past few years [16, 47, 51]. There are two aspects of the stability problem for legged locmotion systems, namely, static stability and dynamic stability [66]. For some of the slower gaits such as the quadruped crawl during which the machine is statically stable at all times, a simple type of finite state control has been successfully used to produce stable locomotion [15]. For higher speed gaits such as the quadruped trot and the pace which are statically unstable in all their phases, other means of control need to be used to produce dynamic stability.

This chapter discusses one such type of control, namely, model reference control. Then the necessary and sufficient conditions for the small motion stability of the quadruped and inverted pendulum systems are derived. Finally, the application of the problem of stability of the inverted pendulum system to produce stable gaits such as the quadruped pace and a form of the biped walk is discussed. It is suggested that stability for the biped is obtained by two types of mechanisms, the body torquing mechanism for controlling attitude and lateral position, and the ability to choose the position in which the feet are placed to maintain stability in the direction of motion [66]. The last section of this chapter discusses the body torquing method of control for producing a type of stable biped walk with the biped " marching " at a constant velocity in the direction of motion with a predetermined fixed stride length. Stability of the biped by moving the position of the feet to produce alternating fall and recovery phases is not covered in this dissertation.

6.2 Model Reference Control

As explained in the introduction to this chapter, the dynamic stability problem for legged locomotion systems becomes difficult for the faster gaits such as the quadruped walk, trot, and the pace in comparison with the finite state algorithmic type of control which can be used for the quadruped crawl. The faster quadruped walk contains some statically unstable phases, while the trot and pace are statically unstable at all times. Therefore, for stabilization of the more complicated quadruped gaits, another type of control called " model reference control " is used for the locomotion systems discussed in this dissertation.

Figure 5 shows the block diagram of a model reference adaptive control system [67]. In this " closed loop " control scheme, the adaptive controller is adjusted to minimize the performance criterion. The output of the plant $c(t)$ is compared with the desired output $c_d(t)$ of the reference model and the error $e(t) = c(t) - c_d(t)$ is minimized by the adaptive mechanism so that $c(t)$ approximates $c_d(t)$ despite time variations in the plant parameters. The reference model is assumed to be

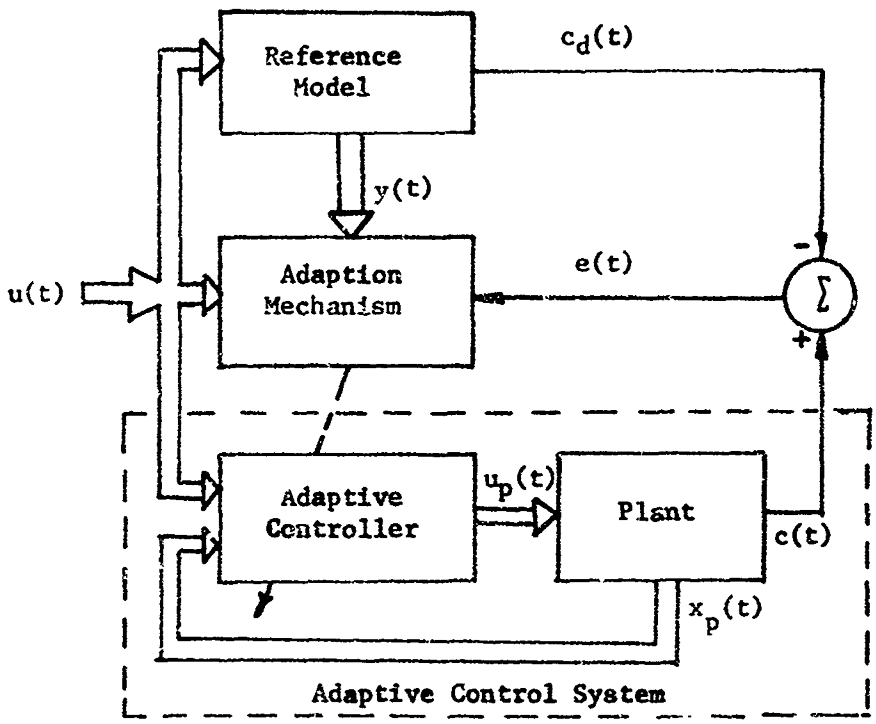


Figure 5 Model Referenced Adaptive Control System.

free from any disturbing influences. The type of model reference control scheme used in the simulation for obtaining both postural control and stable locomotion gaits for the quadruped system uses a simplified version of the adaptive control scheme shown in Figure 5. The controller used in the simulation computes the control inputs u(t) as linear functions of the error signal. While more complicated nonlinear control schemes ought to yield systems with better performance they are not considered in this dissertation.

For each type of gait an ideal kinematic model is assumed. For example, for postural control, the reference model of the quadruped is assumed to be standing on all its four legs with the feet vertically below the respective hip sockets. The leg lengths of the model are all equal to ℓ_o, the initial length and the angles α_{c_i} and β_i are all equal to zero. The output of this ideal kinematic model for the standing quadruped is then compared at each instant of time with the actual plant and the controller outputs are regulated to minimize the error between the plant and the ideal model of the system. Similarly, for the case when the quadruped is moving using a particular gait, the ideal kinematic model for that gait is used as a reference model.

In this dissertation the only type of control that has been considered for producing stable locomotion can be described as " marching " type of control. That is, the ideal kinematic model moves with constant velocity in the direction of motion, placing its feet in the desired sequence along predetermined points on level ground [47]. The actual parameters of the ideal kinematic models describing the different gaits and the control constants used for each type of gait are given in Chapter VII.

6.3 Necessary and Sufficient Conditions for the Small Motion Stability of the Quadruped Locomotion System

one of the methods that can be used to determine the range of control constants to assure small motion stability of the linearized legged locomotion system is the Routh-Hurwitz criterion [68]. This test shows the number of right-half plane zeros of the characteristic equation of a linear system. If the characteristic equation of a linear system contains control constants whose values can be changed, the Routh-Hurwitz test gives a set of inequality constraints on these control constants which must be satisfied to obtain a stable system. The detailed derivation of the Routh-Hurwitz stability criterion and its applications can be found in the literature [68,69], and will not be given here. The method will be used below to compute the control constants which produce small motion stability for the quadruped system as well as for the inverted pendulum system.

The characteristic equation for the linearized quadruped system can be obtained from the 12 x 12 quadruped system matrix given by equation (4-59). But from the results of equation (4-59), the 12 x 12 system matrix is seen to be decomposed into four smaller independent modal matrices. Therefore the Routh test can be applied to each of these modal matrices separately, and should yield constraints on the control constants for stability of the system when the system is excited to produce these vibrational modes. Of the four modal matrices, two are of dimension 2 x 2, and the other two are 4 x 4 matrices.

Consider the modal matrix for translation along the z axis (see equation (4-59)).

$$[M_1] = \begin{bmatrix} 0 & 1 \\ A & B \end{bmatrix} \tag{6-1}$$

where

$$A = \frac{-4C_\ell}{m} \tag{6-2}$$

$$B = \frac{-4C_{\dot{\ell}}}{m} \tag{6-3}$$

The characteristic equation for this mode becomes

$$\det [M - \lambda I] = \lambda^2 - B\lambda - A = 0 \tag{6-4}$$

or

$$\lambda^2 + \frac{4C_{\dot{\ell}}\lambda}{m} + \frac{4C_\ell}{m} = 0 \tag{6-5}$$

The Routh array can be written as

$$\begin{array}{|ll} 1 & -A \\ -B & \\ -A & \end{array}$$

From the above, the Routh-Hurwitz criterion gives the following inequality constraints on the control constants for stability,

$$-A > 0 \quad \text{or} \quad \frac{4C_\ell}{m} > 0 \rightarrow C_\ell > 0 \tag{6-6}$$

$$-B > 0 \quad \text{or} \quad \frac{4C_{\dot{\ell}}}{m} > 0 \rightarrow C_{\dot{\ell}} > 0 \tag{6-7}$$

The modal matrix for rotation about the z axis is given by

$$[M\ ] = \begin{bmatrix} 0 & 1 \\ C & D \end{bmatrix} \tag{6-8}$$

where

$$C = \frac{mg(a^2 + b^2)}{I_{zz}\ell_o} - \frac{4(b^2 C_\alpha + a^2 C_\beta)}{I_{zz}\ell_o^2} \tag{6-9}$$

$$D = \frac{-4(b^2 C_{\dot{\alpha}} + a^2 C_{\dot{\beta}})}{I_{zz}\ell_o^2} \tag{6-10}$$

The characteristic equation is given by

$$\lambda^2 - D\lambda - C = 0 \tag{6-11}$$

The Routh test gives the inequality conditions

$$C < 0 \tag{6-12}$$

$$D < 0 \tag{6-13}$$

Since a, b, ℓ_o, I_{zz} are all positive for the quadruped system equations (6-9) and (6-12) yield the inequality

$$[b^2 C_\alpha + a^2 C_\beta] > \frac{mg(a^2 + b^2)}{4} \tag{6-14}$$

which the control constants C_α and C_β must satisfy.

The inequality of equation (6-13) along with equation (6-10) yield the condition

$$\frac{C_{\dot\alpha}}{C_{\dot\beta}} > -\frac{a^2}{b^2} \tag{6-15}$$

which control constants $C_{\dot\alpha}$ and $C_{\dot\beta}$ must satisfy.

The 4 x 4 modal matrices describing the translational and rotational modes associated with the x and y axes respectively, contain elements consisting of rather long expressions as can be seen from equations (4-59), and (4-64) through (4-79). Therefore the Routh conditions for these matrices will be derived in terms of the symbolic representations of these elements.

The modal matrix for the x axis modes is

$$[M_3] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ E & F & G & H \\ 0 & 0 & 0 & 1 \\ I & J & K & L \end{bmatrix} \tag{6-16}$$

where E, F, G, H, I, J, K, L are given equations (4-64) through (4-71) respectively. The characteristic equation for this matrix M_3 becomes

$$\lambda^4 - (F+L)\lambda^3 + (\text{[illegible]}-HJ-E-K)\lambda^2 + (FK-GJ+LE-HI)\lambda + (KE-GI) = 0 \tag{6-17}$$

This characteristic equation can be written as

$$R_0\lambda^4 + R_1\lambda^3 + R_2\lambda^2 + R_3\lambda + R_4 = 0 \qquad (6\text{-}18)$$

where the coefficients R_0 through R_4 are given by

$$R_0 = 1 \qquad (6\text{-}19)$$

$$R_1 = -(F+L) \qquad (6\text{-}20)$$

$$R_2 = (LF-HJ-E-K) \qquad (6\text{-}21)$$

$$R_3 = (KF+LE-GJ-HI) \qquad (6\text{-}22)$$

$$R_4 = (KE-GI) \qquad (6\text{-}23)$$

The Routh array for this equation becomes

$$\begin{array}{|lll}
R_0 & R_2 & R_4 \\
R_1 & R_3 & \\
R_5 = \dfrac{R_1R_2-R_0R_3}{R_1} & R_4 & \\
R_6 = \dfrac{R_5R_3-R_1R_4}{R_5} & & \\
R_4 & &
\end{array}$$

Since $R_0 = 1 > 0$, for stability

$$R_i > 0, \quad i = 1,2,3,4 \qquad (6\text{-}24)$$

Also

$$R_5 > 0 \qquad (6\text{-}25)$$

$$R_6 > 0 \qquad (6\text{-}26)$$

The conditions $R_5 > 0$ and $R_1 > 0$ yield the inequality

$$(LF-HJ) > \frac{(GJ+HI+EF+KL)}{(L+F)} \qquad (6\text{-}27)$$

From equations (6-26) and (6-27) the following condition is obtained

$$(FK+LE-GJ-HI)[(LF-HJ)(L+F)-(GJ+HI+EF+KL)]-(F+L)^2(KE-GI) < 0 \qquad (6\text{-}28)$$

Substitution of equations (4-65) and (4-71) for the quantities F and L into the condition $R_1 > 0$ gives after simplification the expression

$$[C_{\dot{x}}I_{yy} - ma^2\ell_o^2C_{\dot{\ell}}] < [I_{yy}+m(\ell_o+c)^2]\frac{C_{\dot{\alpha}}}{\ell_o} \qquad (6\text{-}29)$$

which $C_{\dot{x}}$, $C_{\dot{\alpha}}$, and $C_{\dot{\ell}}$ should satisfy.

Simplification of the condition $R_2 > 0$ by substituting expressions for E, F, H, J and K yields the inequality

$$4a^2C_{\dot{\ell}}(C_{\dot{\alpha}}-\ell_oC_{\dot{x}})+C_\alpha[I_{yy}+m(\ell_o+c)^2] + m\ell_o^2a^2C_\ell > \frac{[I_{yy}+mc(\ell_o+c)]mg\ell_o}{4}$$

(6-30)

The condition $R_3 > 0$, reduces to the inequality

$$a^2[C_\alpha C_{\dot{\ell}}+C_{\dot{\alpha}}C_\ell]-a^2\ell_oC_\ell C_{\dot{x}}-mg[a^2\ell_oC+(\ell_o+c)C_{\dot{\alpha}}] > 0 \qquad (6\text{-}31)$$

Finally from equation (6-24) the inequality condition $R_4 > 0$ yields

$$mga^2\ell_oC_\ell + C_\alpha[mg(\ell_o+c)-4a^2C_\ell] < \frac{m^2g^2\ell_oc}{4} \qquad (6\text{-}32)$$

In the above results, if some of the control constants are either zero or proportional to each other for particular gaits, these equations can be simplified further and should give a more meaningful set of inequality constraints on the control constants from which the allowable range for these constants can be determined.

The matrix for the y axis modes is given by

$$[M_4] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ M & N & P & Q \\ 0 & 0 & 0 & 1 \\ R & S & T & U \end{bmatrix} \tag{6-33}$$

where M, N, P, Q, R, S, T, and U are defined by equations (4-72) through (4-79).

The characteristic equation for this matrix M_4 is given by

$$S_0\lambda^4 + S_1\lambda^3 + S_2\lambda^2 + S_3\lambda + S_4 = 0 \tag{6-34}$$

where

$$S_0 = 1 \tag{6-35}$$

$$S_1 = -(N+U) \tag{6-36}$$

$$S_2 = (NU-QS-T-M) \tag{6-37}$$

$$S_3 = (NT+MU-PS-QR) \tag{6-38}$$

$$S_4 = (MT-PR) \tag{6-39}$$

Since $S_0 = 1 > 0$, for stability

$$S_i > 0; \quad i = 1,2,3,4 \tag{6-40}$$

Also

$$S_5 > 0 \tag{6-41}$$

$$S_6 > 0 \tag{6-42}$$

An expression similar to equation (6-27) is obtained from the Routh array using equation (6-41) and the condition $S_1 > 0$.

Thus

$$(NU-QS) > \frac{(PS+QR+MN+TU)}{(N+U)} \tag{6-43}$$

The condition $S_6 > 0$ and equation (6-43) yield an expression similar to equation (6-28) for this particular vibrational system represented by matrix $\mathbf{M}_4$.

$$\{(NT+MU-PS-QR)[(NU-QS)(N+U)-(PS+QR+MN+TU)] - (N+U)^2(MT-PR)\} < 0 \tag{6-44}$$

A set of results similar to equations (6-29) through (6-32) can be derived for the modal matrix M_4 describing the y axis vibrational modes. From the condition $S_1 > 0$ one gets upon simplification the inequality

$$[C_{\dot{y}} I_{xx} + mb^2 \ell_o C_{\dot{\ell}}] < [I_{xx} + m(\ell_o + c)^2] \frac{C_{\dot{\beta}}}{\ell_o} \tag{6-45}$$

The condition $S_2 > 0$ yields the inequality

$$4b^2 C_{\dot{\ell}}(C_{\dot{\beta}} + \ell_o C_{\dot{y}}) + C_\beta[m(\ell_o+c)^2 + I_{xx}] + mb^2 \ell_o^2 C_\ell + I_{xx} \ell_o C_y > [I_{xx} + mc(\ell_o+c)] \frac{mg\ell_o}{4} \tag{6-46}$$

Similarly the condition $S_3 > 0$ reduces after simplification to the condition

$$b^2[C_\ell C_{\dot{\beta}} + C_{\dot{\ell}} C_\beta] + b^2 \ell_o [C_\ell C_{\dot{y}} + C_{\dot{\ell}} C_y] - [(\ell_o+c) C_{\dot{\beta}} + b^2 \ell_o C_{\dot{\ell}}] \frac{mg}{4} > 0 \tag{6-47}$$

Finally, $S_4 > 0$ results in the condition

$$mgb^2 \ell_o C_\ell + C_\beta[mg(\ell_o+c) - 4b^2 \ell_o C_\ell] - 4b^2 \ell_o C_\ell C_y < \frac{m^2 g^2 \ell_o c}{4} \tag{6-48}$$

The above inequalities are the necessary and sufficient conditions for the small motion stability of the quadruped locomotion system.

In the next section, algorithms are derived to choose a set of control constants which produce a stable system and the controllability of the system is thus proved.

6.4 Controllability of the Quadruped Locomotion System

In this section algorithms are developed for obtaining at least one solution to the Routh-Hurwitz inequalities derived in Section 6.3. The existence of at least one set of stabilizing constants proves the controllability of the linearized quadruped system.

To simplify the analysis, it is assumed in the following that all the control constants are equal to C. This gives the condition

$$C_\alpha = C_\beta = C_{\dot\alpha} = C_{\dot\beta} = C_\ell = C_{\dot\ell} = C_{\dot x} = C_y = C_{\dot y} = C \qquad (6\text{-}49)$$

From this assumption, the Routh-Hurwitz inequalities for the z axis translational mode (see equations (6-6) and (6-7)) are satisfied if $C > 0$.

Applying equation (6-49) to equations (6-14) and (6-15), the inequalities associated with the z axis rotational modes, results in the conditions

$$C > \frac{mg}{4} \qquad (6\text{-}50)$$

$$b^2 > -a^2 \qquad (6\text{-}51)$$

The inequalities of equations (6-29) through (6-32) yield upon substitution of equation (6-49) the expressions

$$I_{yy}(\ell_o - 1) < [m(\ell_o + c)^2 + ma^2\ell_o^2\ ] \qquad (6\text{-}52)$$

$$4a^2(1-\ell_o)C^2 + [I_{yy} + m(\ell_o+c)^2 + ma^2\ell_o^2]C > [I_{yy} + mc(\ell_o+c)]\ \frac{mg\ell_o}{4} \qquad (6\text{-}53)$$

$$C > \frac{mg[\ell_o(a^2-1)-c]}{4a^2(2-\ell_o)} \qquad (6\text{-}54)$$

$$C^2 - \frac{mg(a^2\ell_o+\ell_o+c)}{4a^2} C > -\frac{m^2g^2\ell_o c}{16a^2} \qquad (6\text{-}55)$$

An algorigthm can be developed by choosing C to satisfy equation (6-50) as well as conditions given by equations (6-52) through (6-55) for particular values of a, b, c, m, and ℓ_o.

For the case when C = 0, a similar set of inequality conditions can be derived for the modal matrix associated with translationa and rotational motion about the y axis. From the above analysis a particular set of control constants can be found such that all the inequality conditions of the Routh-Hurwitz test are satisfied. This proves the controllability of the quadruped locomotion system for small motions about its equilibrium position.

A computer program was written to obtain sets of control constants C_α, $C_{\dot\alpha}$, C_β, $C_{\dot\beta}$, C_ℓ, $C_{\dot\ell}$, $C_{\dot x}$, C_y, and $C_{\dot y}$ that satisfy the Routh-Hurwitz test for all the modes of vibration of the Quadruped system.

6.5 Stability Criteria for the Inverted Pendulum System

The Routh-Hurwitz stability test is now applied to the linearized equations of motion of the inverted pendulum system derived in Chapter IV. From equation (4-138), the characteristic equation of this system is given by

$$\lambda^4-(C+H)\lambda^3+(CH-DG-F-A)\lambda^2+(CF-BG+AH-DE)\lambda+(AF-BE) = 0 \qquad (6\text{-}56)$$

where the quantities A, B, C, D, E, F, G and H are defined by equations

(4-139) through (4-146) respectively. Substituting for the quantities A through H the expressions given in equations (4-139) through (4-146) and simplifying the resulting expression yields the characteristic equation for the inverted pendulum system in terms of the control constants and the parameters m, g, ℓ, r, and I as follows

$$R_0\lambda^4 + R_1\lambda^3 + R_2\lambda^2 + R_3\lambda + R_4 = 0 \tag{6-57}$$

where

$$R_0 = I\ell^2 \tag{6-58}$$

$$R_1 = [(I/m)+r^2+\ell r]K_4 - K_2\ell(\ell+r) \tag{6-59}$$

$$R_2 = [(I/m)+r^2+\ell r]K_3 - K_1\ell(\ell+r) - (g\ell I+mg\ell r^2+mg\ell^2 r) \tag{6-60}$$

$$R_3 = g(K_2\ell - K_4 r) \tag{6-61}$$

$$R_4 = g(mg\ell r + K_1\ell - K_3 r) \tag{6-62}$$

Since $R_0 = I\ell^2$ is positive, for the stability of the system

$$R_i > 0, \qquad i = 1,\ldots 4 \tag{6-63}$$

Also from the first column of the Routh array

$$R_5 = \frac{(R_1R_2 - R_0R_3)}{R_1} > 0 \tag{6-64}$$

$$R_6 = \frac{(R_5R_3 - R_1R_4)}{R_5} > 0 \tag{6-65}$$

From equation (6-63) observing that R should be greater than zero, equation (6-64) reduces to the condition

$$R_1R_2 - R_0R_3 > 0 \quad \text{or} \quad R_2 > \frac{R_0R_3}{R_1} \tag{6-66}$$

Similarly from equations (6-63) and (6-64) and the condition $R_6 > 0$ the following inequality is obtained.

$$(R_1R_2 - R_0R_3)R_3 - R_1R_4 > 0 \tag{6-67}$$

If the inequality constraints given by equations (6-63), (6-66) and (6-67) can be satisfied by a set of control constants, then such a set of constants will produce a stable system. In the next section an algorithm is given for obtaining these stabilizing control constants for the inverted pendulum system.

6.6 Algorithm for Obtaining Stabilizing Control Constants for the Inverted Pendulum System

The Routh-Hurwitz criterion for the inverted pendulum system considered in this dissertation can be written down in the form of the inequality conditions of equations (6-63) through (6-65) given in Section 6.5.

An algorithm is now developed for obtaining at least one set of control constants that satisfy all the above conditions, thereby proving the controllability of the inverted pendulum system.

From equation (6-63) using the conditions $R_1 > 0$ and $R_3 > 0$ results in the following

$$R_1 = [(I/m)+r^2+\ell r]K_4 - (\ell^2+\ell r)K_2 > 0 \rightarrow K_4 > \frac{\ell(\ell+r)}{[(I/m)+r^2+\ell r]} K_2 \tag{6-68}$$

$$R_3 = g(K_2\ell - K_4 r) > 0 \rightarrow K_4 < \frac{\ell}{r} K_2 \tag{6-69}$$

Therefore the constraints on K_4 and K_2 are

$$\frac{a}{b} K_2 < K_4 < \frac{\ell}{r} K_2 \qquad \text{or} \qquad \frac{a}{b} < \frac{K_4}{K_2} < \frac{\ell}{r} \tag{6-70}$$

where

$$a = \ell(\ell+r) \tag{6-71}$$

$$b = [(I/m)+r^2+\ell r] \tag{6-72}$$

6.6.1 A Particular Solution

To derive a particular set of control constants that will produce a stable system, choose

$$(1)\ K_4 = \mu K_2 \tag{6-73}$$

where

$$\mu = \frac{1}{2}[(a/b) + (\ell/r)] \tag{6-74}$$

$$(2)\ K_1 = \frac{r}{\ell} K_3 \tag{6-75}$$

The constraint $R_5 > 0$ yields

$$R_5 = R_2 - \frac{R_0 R_3}{R_1} = (R_2 - \eta) > 0 \tag{6-76}$$

where

$$\eta = \frac{R_0 R_3}{R_1} \tag{6-77}$$

The constraint $R_6 > 0$ can be written as

$$R_3 > \frac{R_1 R_4}{[R_2 - (R_0 R_3/R_1)]} = \frac{R_1 R_4}{(R_2 - \eta)} \tag{6-78}$$

A particular value of η can be obtained by using equation (6-73) in the equation defining η, namely equation (6-77). This particular value of η, namely η_p is then given by the expression

$$\eta_p = \frac{I\ell^2 g(\ell - r\mu)}{(a\mu - b)} \tag{6-79}$$

Therefore from equation (6-76)

$$R_2 > \eta \tag{6-80}$$

From equation (6-78)

$$R_3 > \frac{R_1 R_4}{(R_2 - \eta_p)} \tag{6-81}$$

Substituting the condition $K_1 = \frac{r}{\ell} K_3$ in the inequality condition $R_4 > 0$ results in the condition

$$R_4 = mg^2 r\ell + g\ell K_1 - grK_3 = mg^2 r\ell > 0 \tag{6-82}$$

for any value of K_3. Since R_0, R_1, R_3, and R_4 are all greater than zero for the particular values of the control constants chosen above, from equation (6-81) the only other condition that needs to be satisfied is

$$R_2 > \eta_p \tag{6-83}$$

Upon substitution for R_2 and η_p from equations (6-60) and (6-79) into equation (6-83) and simplifying results in the condition

$$K_3 > [(m^2 g\ell b/I) + (mg\ell^2 r/b)] \tag{6-84}$$

where a and b are defined by equations (6-71) and (6-72) respectively.

Therefore a particular solution that yields a stable inverted pendulum system can be chosen by an algorithm as follows:

1) Choose $K_4 = \mu K_2$ where $\mu = \dfrac{[(a/b) + (\ell/r)]}{2}$.

2) Choose K_3 such that it satisfies the inequality of equation (6-84).

3) Choose $K_1 = \frac{r}{\ell} K_3$

These steps should yield a particular set of control constants K_1, K_2, K_3, and K_4 which produce a stable system, thus proving the controllability of the inverted pendulum system.

6.6.2 General Algorithm for Computing Stabilizing Control Constants

A more general algorithm which describes the procedure for obtaining stabilizing control constants can be obtained as an extension of the theory developed so far.

Choose $K_4 = \mu K_2$ where μ can now have any value within the limits set by the constraints of equation (6-70). Then the conditions $R_0 > 0$, $R_1 > 0$, and $R_3 > 0$ are satisfied.

The constraint $R_5 > 0$ yields after simplification the condition

$$K_3 > \{(b/a)K_1 - (mg\ell b/a) + \frac{I\ell^2 g(\ell - r\mu)}{a(a\mu - b)}\} \tag{6-85}$$

The condition $R_4 > 0$ yields in this general case the inequality

$$K_3 < [K_1(\ell/r) + mg\ell] \tag{6-86}$$

Therefore a general algorithm for finding the control constants that satisfy the Routh-Hurwitz test for the inverted pendulum system consists of the following steps:

1) Choose any value of K_2.

2) Choose μ such that $\frac{a}{b} < \mu < \frac{\ell}{r}$.

3) Choose $K_4 = \mu K_2$.

4) Substitute this value of μ into the inequality (6-85).

5) Choose any values of K_1 and K_3 which satisfy the inequality constraints (6-85) and (6-86).

This algorithm has been programmed on a digital computer and the designer can get a set of control constants that satisfy all the conditions of the Routh-Hurwitz test. A listing of this program is given in the appendix.

6.7 Stabilizing Control Mechanisms

The general form of the control schemes used in the quadruped simulation has been discussed in Chapter IV. In this section the details of these control laws for the different gaits are discussed.

As pointed out in Section 6.2, the model reference control system is used for the control of the quadruped s lation. The model used for the various gaits is assumed to be moving with a constant velocity in the desired direction of travel placing its feet at predetermined points along the way. This type of an idealized walking system corresponds to a " marching " type of locomotion system. The model reference control system compares the various translational and rotational components of the actual system with the idealized model, and produces correction torques and forces.

The type of control laws used for the quadruped simulation have been stated in Chapter IV and are repeated here.

The torque applied to the body by the hip motor of leg i is given by the relationship

$$T_{m_i} = C_\alpha(\alpha_i - \alpha_{c_i}) + C_{\dot\alpha}\dot\alpha_i + C_{\dot x}u \qquad (6\text{-}87)$$

The lateral control torque is derived from a centering spring and and damper system and has the form

$$T_{s_i} = C_\beta(\beta_i - \beta_{c_i}) + C_{\dot\beta}\dot\beta_i + C_y y_E + C_{\dot y}v \qquad (6\text{-}88)$$

Vertical control is obtained by using the control law

$$f'_z = C_\ell(\ell_i - \ell_o) + C_{\dot\ell}\dot\ell_i - C_g \qquad (6\text{-}89)$$

in which the constant C_g describes the effect due to gravity.

Equations (6-87) through (6-89) describe the general type of control laws used in the quadruped simulation.

For the following cases, namely, the quadruped postural system, the quadruped crawl (which is a slow speed gait with no statically unstable phases), and the quadruped walk (which has some statically unstable phases), the control laws given above were used with the constants $C_{\dot x}$, C_y, and $C_{\dot y}$ all equal to zero.

For the quadruped trot, which is a faster gait than either the crawl or the walk, and which has no statically stable phases, it was necessary to use the general equations with the lateral torque T_{s_i} dependent on both y_E and $\dot{y}_E$ in addition to the usual β and $\dot{\beta}$ terms.

For the quadruped pace, which is a faster gait than the trot, and in which the quadruped uses its legs on the same side alternately, it was necessary to incorporate a different type of control law for lateral control.

From equation (4-124), the control law used for the inverted pendulum system is of the form

$$T = K_1\phi_2 + K_2\dot{\phi}_2 + K_3\phi_1 + K_4\dot{\phi}_1 \tag{6-90}$$

As viewed from the y-z plane, the quadruped looks like an inverted pendulum. Therefore a correspondence can be established between the inverted pendulum system and the lateral control mechanism of the quadruped. The control law that is used for the lateral control of the pacing quadruped is of the form

$$T_{s_i} = \frac{1}{2}\,[K_1\phi + K_2p + K_3(\phi+\beta) + K_4(p+\dot{\beta})] \tag{6-91}$$

where K_1, K_2, K_3, and K_4 are the control constants computed for the inverted pendulum system and ϕ, p, β, $\dot{\beta}$ are the parameters of the quadruped locomotion system.

6.8 Summary

This chapter has discussed the type of feedback control systems as well as the stability criteria used to produce digital computer simulations of idealized stable quadruped and biped gaits. For the quadruped system, the modal matrices derived in Chapter IV were used to obtain the characteristic equation for each mode after which the Routh-Hurwitz criterion was applied to get sets of inequality constraints amon the control constants. Then the above procedure was repeated for the type of inverted pendulum system considered in this research.

Algorithms were developed for both systems to compute sets of control constants which satisfied all the constraints of the Routh-Hurwitz test. Finally, the control mechanisms used for the various quadruped gaits and the correspondence between the inverted pendulum system control law and that of the quadruped pace was described.

Chapter VII gives detailed descriptions of the various quadruped and biped gaits, the different vibrational modes of the quadruped postural system, and the inverted pendulum system.

CHAPTER VII

COMPUTER SIMULATION

7.1 Introduction

The objective of this chapter is to present details of the digital computer simulations of the different quadruped and biped gaits as well as the results of the application of vibrational analysis on the quadruped locomotion system. Accordingly, Section 7.2 discusses the results of the vibrational analysis techniques used in verifying the nonlinear quadruped locomotion system. Section 7.3 outlines the details of the kinematic and dynamic parameters used to obtain stable quadruped gaits such as the crawl, the walk, and the trot. The next section describes the results of computer simulation of the inverted pendulum system along with the application of the Routh-Hurwitz analysis for obtaining stabilizing control constants. Section 7.5 details the application of inverted pendulum analysis for simulating a stable quadruped pace. Finally, Section 7.6 covers the simulation of a certain type of biped walk. This chapter contains only the results of the various simulations. The actual computer programs used are listed in the appendix.

7.2 Vibrational Analysis

This section discusses the results of the verification of the nonlinear quadruped locomotion system simulation by vibrational analysis. The results of Section 3.3 were used in programming a nonlinear digital computer simulation of the quadruped locomotion system consisting of a mass supported by four massless legs. By using different types of reference models for the ideal quadruped locomotion system in this simulation, both postural control as well as stable quadruped gaits were simulated.

In vibrational analysis, the quadruped was assumed to stand on all its four legs, with the feet vertically below their respective hip socket joints. Figure 6 is a photograph of the computer generated display of the quadruped postural system. Figures 7 through 9 show the digital computer displays of the vibrational modes excited in the nonlinear simulation of the quadruped locomotion system. These figures give a qualitative idea of the translational and rotational motions produced about the x, y, and z axes for the quadruped when postural system is excited to produce the various independent vibrational modes. However, a quantitative idea about the response of the quadruped simulation to these vibrational modes is needed.

From the results of Section 4.3 (equations (4-59) through (4-79)), the linearized system matrix is seen to decompose into four smaller matrices. These matrices describe the independent vibrational modes of the standing quadruped locomotion system. The following procedure is adopted for the vibrational analysis of this system:

1) Subroutine NSEVB is used to determine the eigenvalues and corresponding eigenvectors (real or complex) of the real, nonsymmetric modal matrices of the linearized quadruped locomotion system.

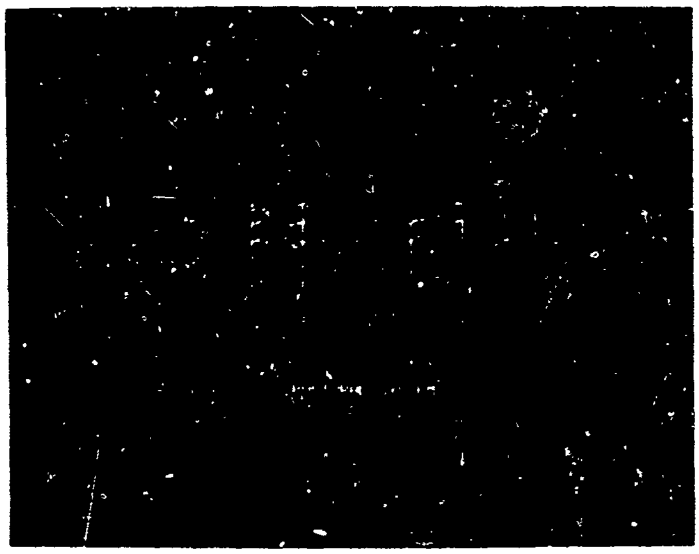


Figure 6. Computer Generated Display of the Quadruped Postural System.

Figure 7. X Axis Vibrational Mode of the Quadruped Postural System.

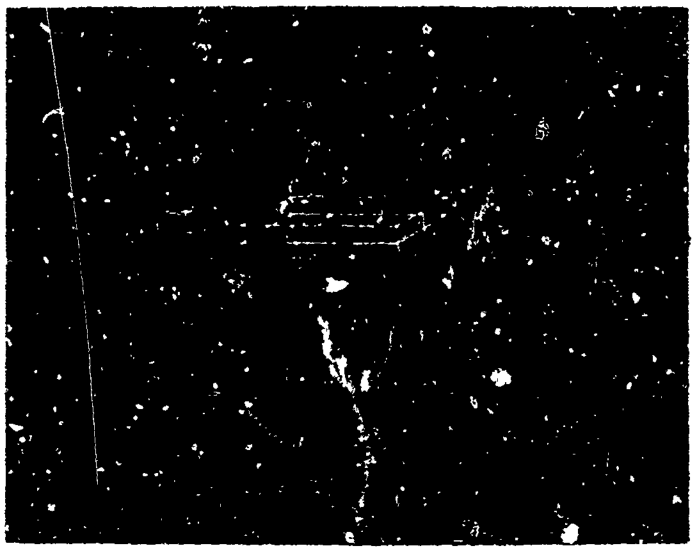


Figure 8. Y Axis Vibrational Mode of the Quadruped Postural System.

Figure 9. Z Axis Vibrational Modes of the Quadruped Postural System.

2) The small motion response of the linearized system for the various modes is then computed using subroutine LINEAR. Subroutine LINEAR computes the response of the linearized system for each mode by perturbing the system using one-tenth of the actual values of the components of each of the twelve eigenvectors (real or complex) of the linearized quadruped system computed in step 1 above.

3) The nonlinear simulation is then excited along each of the same twelve eigenvectors using the same values as in step 2 above, and its small motion response recorded.

Tables 1 through 4 list the four modal matrices, their computed eigenvalues with error bounds, and corresponding eigenvectors for one particular set of control constants, namely, $C_\alpha = 9000.0$, $C_{\dot\alpha} = 200.0$, $C_\beta = 9000.0$, $C_{\dot\beta} = 200.0$, $C_\ell = 1500.0$, $C_{\dot\ell} = 100.0$, $C_{\dot x} = C_y = C_{\dot y} = 0.0$.

Figures 10 and 11 show the transient response of the quadruped postural system for certain x and y axis modes. Figures 12, and 13 describe the response of system to the translational and rotational motions of the z axis modes respectively.

Table 5 is a sample of the linear system response when the system is excited along the first eigenvector of the x axis modal matrix given in Table 1. Table 6 gives the response of the nonlinear quadruped system for the same excitation.

From the transient response of the quadruped postural system to the different vibrational modes (Figures 10 through 13), as well as from the sample sets of data given (Tables 5 and 6), it is seen that the nonlinear and linearized quadruped system responses agree to within

TABLE 1

Modal Matrix, Eigenvalues, and Eigenvectors for the X Axis Vibrational Modes

DIMENSIONS OF BODY

A= 2.50 B= 1.00 C= 0.25 LENGTH= 3.00 MASS=20.00

MOMENTS OF INERTIA

IXX= 7.0833 IYY=42.0833 IZZ=48.3333

CONTROL CONSTANTS

CA=	9000.0000	DCA=	200.0000	CB=	9000.0000	DCB=	200.0000
CL=	1500.0000	DCL=	100.0000	DCX=	0.0	CY=	0.0
		DCY=	0.0				

X-AXIS TRANSLATIONAL & ROTATIONAL MODAL MATRIX

0.0	1.000000	0.0	0.0
-189.266663	-4.444444	-647.316650	-14.444444
0.0	0.0	0.0	1.000000
-307.635498	-6.864691	-1890.905273	-81.716187

COMPUTED EIGENVALUES

NO.	REAL PART	IMAG. PART	ERROR BOUND
1	-2.140625954	8.860431671	0.000393874
2	-2.140625954	-8.860431671	0.000393874
3	-40.939682007	15.313523293	0.002375914
4	-40.939682007	-15.313523293	0.002375914

COMPUTED EIGENVECTOR NO.= 1

REAL PART	IMAG. PART	ERROR BOUND
-0.025762886	-0.106637120	0.000002869
1.000000000	0.0	0.000020361
0.007656872	0.016778924	0.000004073
-0.165058970	0.031925749	0.000126384

TABLE 1 Continued

COMPUTED EIGENVECTOR NO.= 2

REAL PART	IMAG. PART	ERROR BOUND
-0.025762886	0.106637120	0.000002869
1.000000000	0.0	0.000020361
0.007656872	-0.016778924	0.000004073
-0.165058970	-0.031925749	0.000126384

COMPUTED EIGENVECTOR NO.= 3

REAL PART	IMAG. PART	ERROR BOUND
-0.001992267	0.001950445	0.000002338
0.051694594	-0.110359192	0.0000250[illegible]
-0.021478082	-0.008015189	0.000001025
1.000000000	0.0	0.000003253

COMPUTED EIGENVECTOR NO.= 4

REAL PART	IMAG. PART	ERROR BOUND
-0.001992267	-0.001950445	0.000002338
0.051694594	0.110359192	0.000025075
-0.021478082	0.008015189	0.000001025
1.000000000	0.0	0.000003253

LARGEST RESIDUAL=0.4580798180-01

TABLE 2

Modal Matrix, Eigenvalues, and Eigenvectors for the Y Axis Vibrational Modes

DIMENSIONS OF BODY

A= 2.50 B= 1.00 C= 0.25 LENGTH= 3.00 W [illegible]=20.00

MOMENTS OF INERTIA

IXX= 7.0833 IYY=42.0833 IZZ=48.3333

CONTROL CONSTANTS

CA= 9000.0000 DCA= 200.0000 CB= 9000.0000 DCB= 200.0000
CL= 1500.0000 DCL= 100.0000 DCX= 0.0 CY= 0.0
DCY= 0.0

Y-AXIS TRANSLATIONAL & ROTATIONAL MODAL MATRIX

0.0	1.000000	0.0	0.0
-189.266663	-4.444444	647.316650	14.444444
0.0	0.0	0.0	1.000000
1827.718750	40.784332	-6787.140625	-139.019684

COMPUTED EIGENVALUES

NO.	REAL PART	IMAG. PART	ERROR BOUND
1	-0.745484173	3.754101753	0.000611827
2	-0.745484173	-3.754101753	0.000611827
3	-48.164916992	0.0	0.009812821
4	-143.808242798	0.0	0.010169104

COMPUTED EIGENVECTOR NO.= 1

REAL PART	IMAG. PART	ERROR BOUND
-0.050889663	-0.256269693	0.000021063
1.000000000	0.0	0.000012168
-0.015198555	-0.069005013	0.000011925
0.270382166	-0.005614769	0.000364130

TABLE 2 Continued

COMPUTED EIGENVECTOR NO.= 2

REAL PART	IMAG. PART	ERROR BOUND
-0.[illegible]663	0.256269693	0.000021063
1.000000000	0.0	0.000012168
-0.015198555	0.069005013	0.000011929
0.270382166	0.005614769	0.000364130

COMPUTED EIGENVECTOR NO.= 3

REAL PART	IMAG. PART	ERROR BOUND
0.000437835	0.0	0.000005041
-0.021088269	0.0	0.000023514
-0.020761997	0.0	0.000002083
1.000000000	0.0	0.000107283

COMPUTED EIGENVECTOR NO.= 4

REAL PART	IMAG. PART	ERROR BOUND
0.000491485	0.0	0.000001752
-0.070679545	0.0	0.000007792
-0.006953701	0.0	0.000002673
1.000000000	0.0	0.000107757

LARGEST RESIDUAL=0.6944913120-01

TABLE 3

Modal Matrix, Eigenvalues, and Eigenvectors
for the Z Axis Translational
Modes of Vibration

DIMENSIONS OF BODY

A= 2.50 B= 1.00 C= 0.25 LENGTH= 3.00 MASS=20.00

MOMENTS OF INERTIA

IXX= 7.0833 IYY=42.0833 IZZ=48.3333

CONTROL CONSTANTS

CA= 9000.0000 DCA= 200.0000 CB= 9000.0000 DCB= 200.0000
CL= 1500.0000 DCL= 100.0000 DCX= 0.0 CY= 0.0
DCY= 0.0

Z-AXIS TRANSLATIONAL MODAL MATRIX

0.0	1.000000
-300.000000	-20.000000

COMPUTED EIGENVALUES

NO.	REAL PART	IMAG. PART	ERROR BOUND
1	-10.000000000	14.142135620	0.000051282
2	-10.000000000	-14.142135620	0.000051282

COMPUTED EIGENVECTOR NO.= 1

REAL PART	IMAG. PART	ERROR BOUND
-0.033333331	-0.047140453	0.000000085
1.000000000	0.0	0.000000000

COMPUTED EIGENVECTOR NO.= 2

REAL PART	IMAG. PART	ERROR BOUND
-0.033333331	0.047140453	0.000000085
1.000000000	0.0	0.000000000

LARGEST RESIDUAL=0.3178914390-08

TABLE 4

Modal Matrix, Eigenvalues, and Eigenvectors
for the Z Axis Rotational
Modes of Vibration

DIMENSIONS OF BODY

A= 2.50 B= 1.00 C= 0.25 LENGTH= 3.00 MASS=20.00

MOMENTS OF INERTIA

IXX= 7.0833 IYY=42.0833 IZZ=48.3333

CONTROL CONSTANTS

CA= 9000.0000 DCA= 200.0000 CB= 9000.0000 DCB= 200.0000
CL= 1500.0000 DCL= 100.0000 DCX= 0.0 CY= 0.0
DCY= 0.0

Z-AXIS ROTATIONAL MODAL MATRIX

0.0	1.000000
-567.800049	-13.333341

COMPUTED EIGENVALUES

NO.	REAL PART	IMAG. PART	ERROR BOUND
1	-6.666671753	22.876968384	0.000108706
2	-6.666671753	-22.876968384	0.000108706

COMPUTED EIGENVECTOR NO.= 1

REAL PART	IMAG. PART	ERROR BOUND
-0.011741225	-0.040290516	0.000000096
1.000000000	0.0	0.000000000

COMPUTED EIGENVECTOR NO.= 2

REAL PART	IMAG. PART	ERROR BOUND
-0.011741225	0.040290516	0.000000096
1.000000000	0.0	0.000000000

LARGEST RESIDUAL=0.2692596160-07

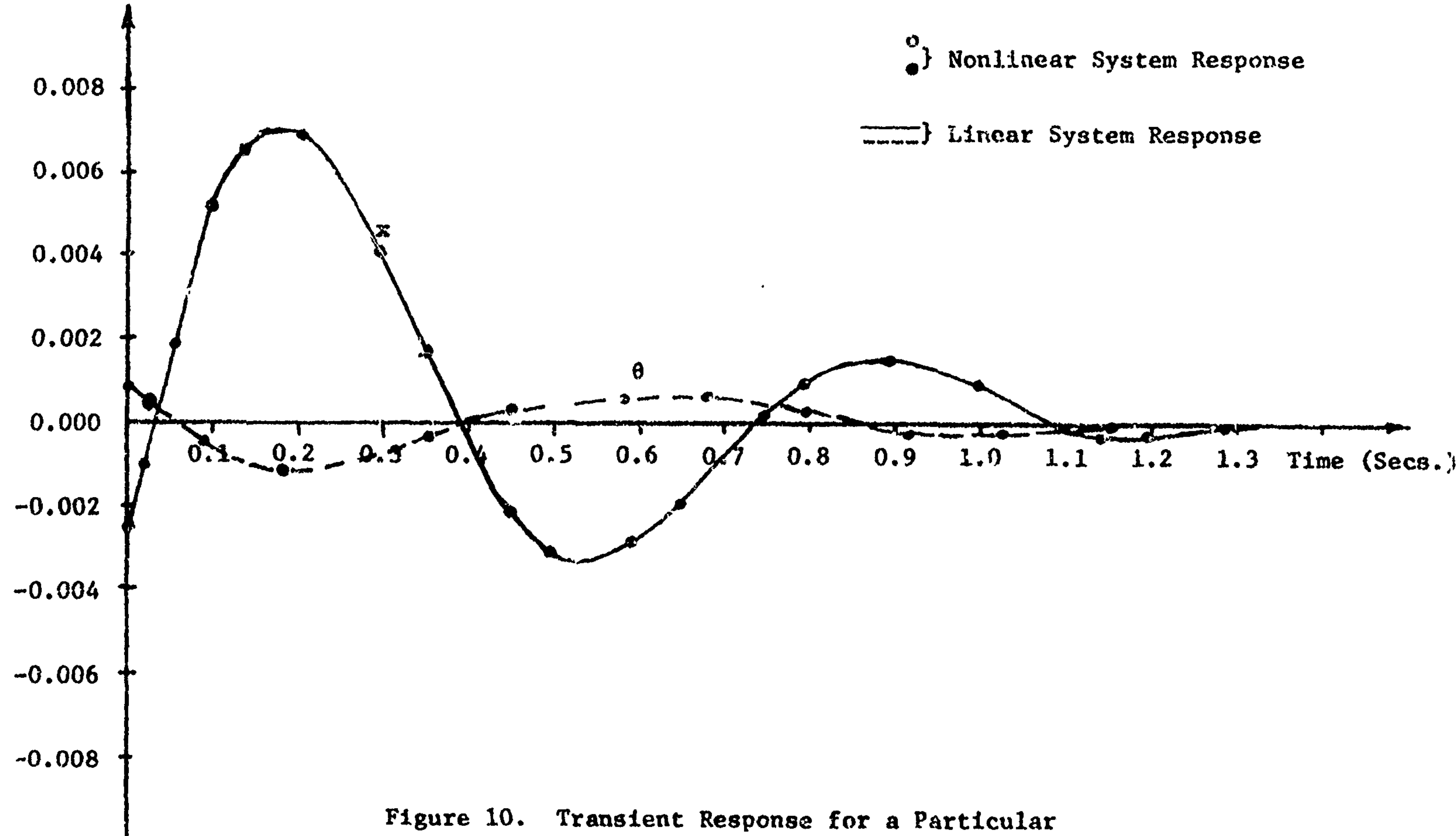


Figure 10. Transient Response for a Particular X Axis Vibrational Mode.

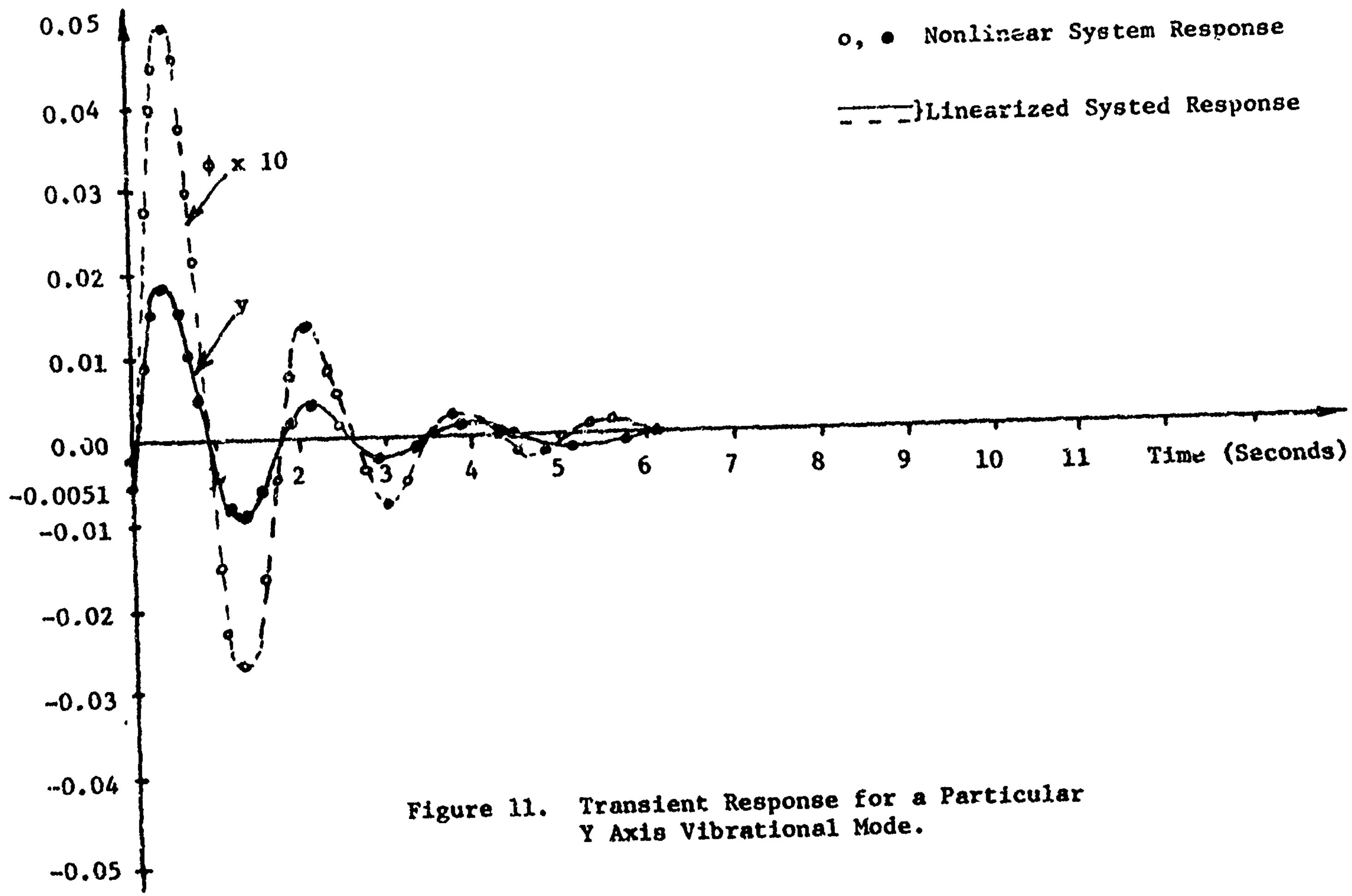


Figure 11. Transient Response for a Particular Y Axis Vibrational Mode.

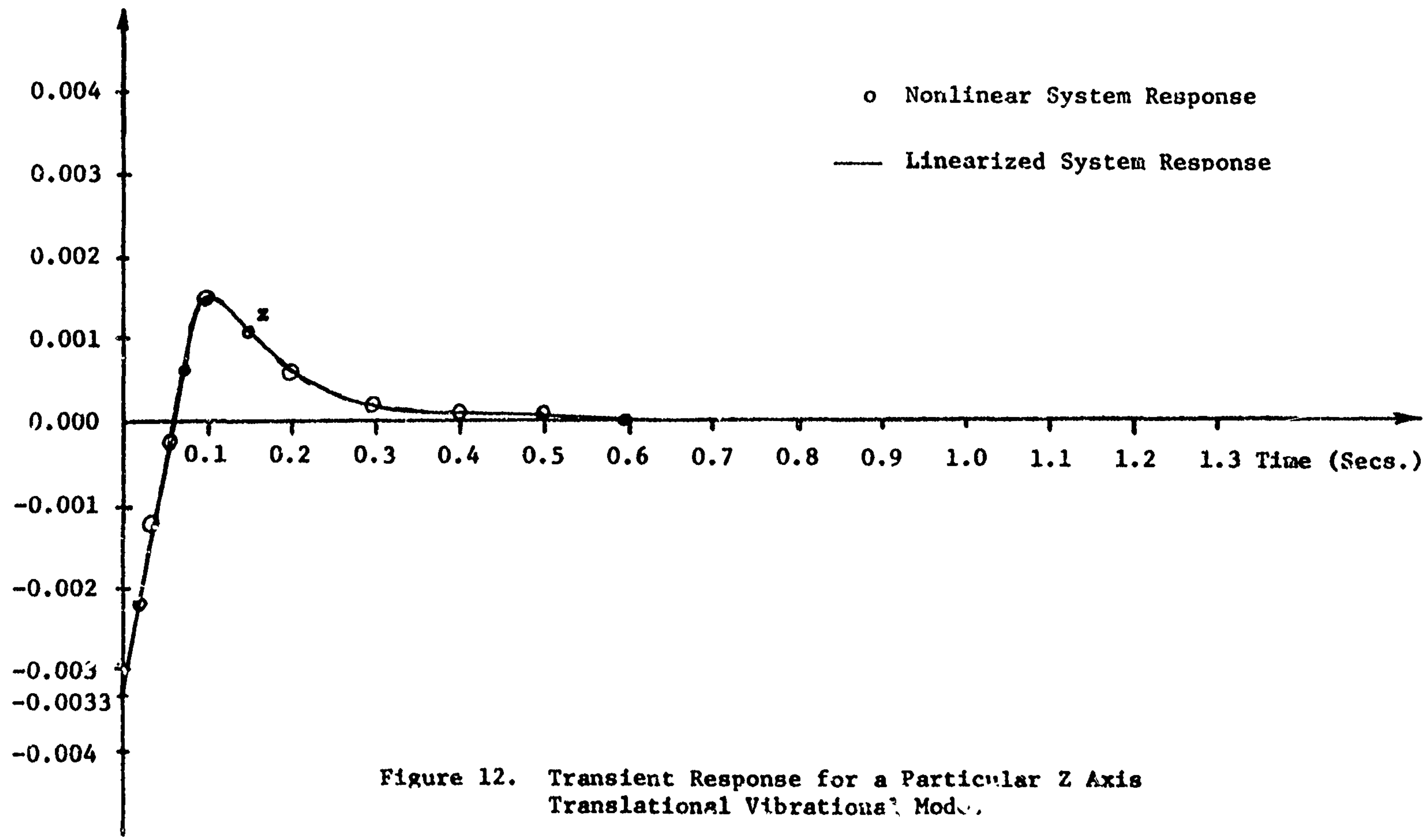


Figure 12. Transient Response for a Particular Z Axis Translational Vibrational Mode.

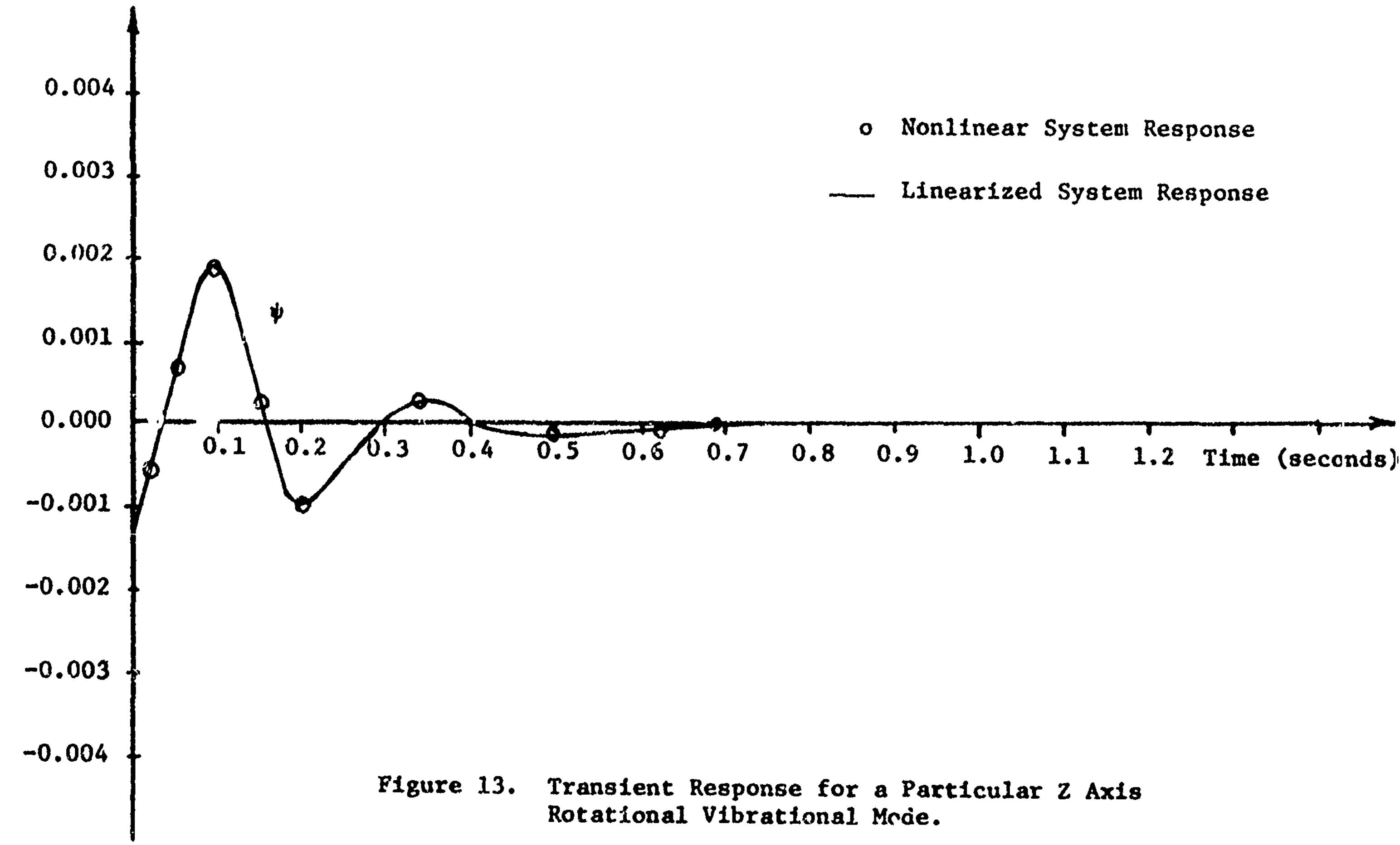


Figure 13. Transient Response for a Particular Z Axis Rotational Vibrational Mode.

TABLE 5

Linear System Response of the Quadruped Locomotion System for a Particular X Axis Vibrational Mode

DIMENSIONS OF BODY

A= 2.50 B= 1.00 C= 0.25 LENGTH= 3.00 MASS=20.00

MOMENTS OF INERTIA

IXX= 7.0833 IYY=42.0833 IZZ=48.3333

CONTROL CONSTANTS

CA= 9000.0000 DCA= 200.0000 CB= 9000.0000 DCB= 200.0000
CL= 1500.0000 DCL= 100.0000 DCX= 0.0 CY= 0.0
DCY= 0.0

X-AXIS TRANSLATIONAL & ROTATIONAL MODAL MATRIX

0.0	1.000000	0.0	0.0
-189.266663	-4.444444	-647.316650	-14.444444
0.0	0.0	0.0	1.000000
-307.635498	-6.864691	-1890.905273	-81.716187

COMPUTED EIGENVALUES

NO.	REAL PART	IMAG. PART	ERROR BOUND
1	-2.140625954	8.860431671	-1.000000000
2	-2.140625954	-8.860431671	-1.000000000
3	-40.939682007	15.313523293	-1.000000000
4	-40.939682007	-15.313523293	-1.000000000

COMPUTED EIGENVECTOR NO.= 1

REAL PART	IMAG. PART	ERROR BOUND
-0.0[illegible]764219	-0.106632531	-1.000000000
0.999999940	-0.000000000	-1.000000000
0.007656947	0.016776852	-1.000000000
-0.165051877	0.031954452	-1.000000000

TABLE 5 Continued

COMPUTED EIGENVECTOR NO.= 2

REAL PART	IMAG. PART	ERROR BOUND
-0.025764219	0.106632531	-1.000000000
0.999999940	0.000000000	-1.000000000
0.007656947	-0.016776452	-1.000000000
-0.165051877	-0.031954452	-1.000000000

COMPUTED EIGENVECTOR NO.= 3

REAL PART	IMAG. PART	ERROR BOUND
-0.001989994	0.001950331	-1.000000000
0.051681701	-0.110332429	-1.000000000
-0.021427996	-0.008013181	-1.000000000
0.999999881	0.000000000	-1.000000000

COMPUTED EIGENVECTOR NO.= 4

REAL PART	IMAG. PART	ERROR BOUND
-0.001989994	-0.001950331	-1.000000000
0.051681701	0.110332429	-1.000000000
-0.021427996	0.008013181	-1.000000000
0.999999881	-0.000000000	-1.000000000

LARGEST RESIDUAL=0.658079719D-01

EIGENVALUE

ALPHA= -2.1406260 OMEGA= 8.8604317

0.1*COMPUTED EIGENVECTOR NO.=1

REAL PART	IMAG. PART
-.257642101E-02	-.106632486E-01
0.999999066E-01	-.355271241E-15
0.765694305E-03	0.167768449E-02

TABLE 5 Continued

LINEAR SYSTEM RESPONSE

TIME	X	$\dot{X}$	θ	$\dot{\theta}$
0.0	-0.002576421	0.059999905	0.000765694	-0.016505182
0.1000	0.005352337	0.051060088	-0.000658113	-0.010425717
0.2000	0.007144954	-0.013030227	-0.001171093	0.000110147
0.3000	0.003808168	-0.046583876	-0.000767011	0.006907232
0.4000	-0.000767671	-0.039079394	-0.000020037	0.006981898
0.5000	-0.003265847	-0.009547848	0.000479422	0.002628280
0.6000	-0.002834774	0.015718896	0.000502646	-0.001866303
0.7000	-0.000766429	0.022274811	0.000200849	-0.003618805
0.8000	0.001064826	0.012502614	-0.000122481	-0.002479206
0.9000	0.001586908	-0.001749458	-0.000255978	-0.000173288
1.0000	0.000926575	-0.009934880	-0.000181580	0.001438810
1.1000	-0.000088009	-0.009005375	-0.000018603	0.001582257
1.2000	-0.000693757	-0.002721440	0.000099345	0.000678088
1.3000	-0.000651108	0.003089895	0.000113575	-0.000338732
1.4000	-0.000212770	0.004929077	0.000051237	-0.000787850
1.5000	0.000207066	0.003019796	-0.000021697	-0.000583787
1.6000	0.000350123	-0.000128596	-0.000055549	-0.000082703
1.7000	0.000222596	-0.002099423	-0.000042587	0.000296018
1.8000	-0.000000869	-0.002060124	-0.000007287	0.000356193
1.9000	-0.000145960	-0.000735557	0.000020314	0.000170824
2.0000	-0.000148489	0.000591496	0.000025494	-0.000057697
2.1000	-0.000056512	0.001083416	0.000012795	-0.000170250
2.2000	0.000039065	0.000720897	-0.000003549	-0.000136258
2.3000	0.000076724	0.000030084	-0.000011963	-0.000028189
2.4000	0.000052890	-0.000439108	-0.000009904	0.000060016
2.5000	0.000004008	-0.000468023	-0.000002317	0.000079660
2.6000	-0.000030377	-0.000191769	0.000004088	0.000042235
2.7000	-0.000033633	0.000109192	0.000005685	-0.000008786
2.8000	-0.000014549	0.000236487	0.000003141	-0.000036498
2.9000	0.000007062	0.000170339	-0.000000497	-0.000031546
3.0000	0.000016694	0.000019824	-0.000002555	-0.000008428
3.1000	0.000012445	-0.000090770	-0.000002285	0.000011953
3.2000	0.000001829	-0.000105614	-0.000000668	0.000017699
3.3000	-0.000006243	-0.000048697	0.000000807	0.000010284
3.4000	-0.000007568	0.000019103	0.000001259	-0.000001033
3.5000	-0.000003659	0.000051245	0.000000760	-0.000007757
3.6000	0.000001195	0.000039882	-0.000000044	-0.000007249
3.7000	0.000003605	0.000007329	-0.000000541	-0.000002347
3.8000	0.000002903	-0.000018508	-0.000000523	0.000002328
3.9000	0.000000615	-0.000023676	-0.000000182	0.000003906
4.0000	-0.000001264	-0.000012116	0.000000155	0.000002472
4.1000	-0.000001692	0.000003057	0.000000277	-0.000000021
4.2000	-0.000000904	0.000011019	0.000000182	-0.000001633
4.3000	0.000000180	0.000009260	0.000000005	-0.000001654
4.4000	0.000000772	0.000002275	-0.000000113	-0.000000625
4.5000	0.000000672	-0.000003712	-0.000000119	0.000000440
4.6000	0.000000182	-0.000005273	-0.000000048	0.000000856
4.7000	-0.000000251	-0.000002966	0.000000029	0.000000588
4.8000	-0.000000376	0.000000408	0.000000061	0.000000042
4.9000	-0.000000220	0.000002349	0.000000043	-0.000000340
5.0000	0.000000020	0.000002133	0.000000004	-0.000000375

TABLE 5 Continued

5.1000	0.000000164	0.000000647	-0.000000023	-0.000000161
5.2000	0.000000154	-0.000000729	-0.000000027	0.000000080
5.3000	0.000000051	-0.000001167	-0.000000012	0.000000186
5.4000	-0.000000049	-0.000000716	0.000000005	0.000000138
5.5000	-0.000000083	0.000000029	0.000000013	0.000000020
5.6000	-0.000000053	0.000000496	0.000000010	-0.000000070
5.7000	0.000000000	0.000000488	0.000000002	-0.000000084
5.8000	0.000000035	0.000000175	-0.000000005	-0.000000041
5.9000	0.000000035	-0.000000140	-0.000000006	0.000000014
6.0000	0.000000013	-0.000000256	-0.000000003	0.000000040
6.1000	-0.000000009	-0.000000171	0.000000001	0.000000032
6.2000	-0.000000018	-0.000000007	0.000000003	0.000000007
6.3000	-0.000000013	0.000000104	0.000000002	-0.000000014
6.4000	-0.000000001	0.000000111	0.000000001	-0.000000019
6.5000	0.000000007	0.000000046	-0.000000001	-0.000000010
6.6000	0.000000008	-0.000000026	-0.000000001	0.000000002
6.7000	0.000000003	-0.000000056	-0.000000001	0.000000009
6.8000	-0.000000002	-0.000000040	0.000000000	0.000000007
6.9000	-0.000000004	-0.000000005	0.000000001	0.000000002
7.0000	-0.000000003	0.000000021	0.000000001	-0.000000003
7.1000	-0.000000000	0.000000025	0.000000000	-0.000000004
7.2000	0.000000001	0.000000012	-0.000000000	-0.000000002
7.3000	0.000000002	-0.000000004	-0.000000000	0.000000000
7.4000	0.000000001	-0.000000012	-0.000000000	0.000000002
7.5000	-0.000000000	-0.000000009	0.000000000	0.000000002
7.6000	-0.000000001	-0.000000002	0.000000000	0.000000001
7.7000	-0.000000001	0.000000004	0.000000000	-0.000000001
7.8000	-0.000000000	0.000000006	0.000000000	-0.000000001
7.9000	0.000000000	0.000000003	-0.000000000	-0.000000001
8.0000	0.000000000	-0.000000001	-0.000000000	0.000000000
8.1000	0.000000000	-0.000000001	-0.000000000	0.000000000
8.2000	-0.000000000	-0.000000002	-0.000000000	0.000000000
8.3000	-0.000000000	-0.000000001	0.000000000	0.000000000
8.4000	-0.000000000	0.000000001	0.000000000	-0.000000000
8.5000	-0.000000000	0.000000001	0.000000000	-0.000000000
8.6000	0.000000000	0.000000001	-0.000000000	-0.000000000
8.7000	0.000000000	-0.000000000	-0.000000000	-0.000000000
8.8000	0.000000000	-0.000000001	-0.000000000	0.000000000
8.9000	-0.000000000	-0.000000001	-0.000000000	0.000000000
9.0000	-0.000000000	-0.000000000	0.000000000	0.000000000
9.1000	-0.000000000	0.000000000	0.000000000	-0.000000000
9.2000	-0.000000000	0.000000000	0.000000000	-0.000000000
9.3000	0.000000000	0.000000000	-0.000000000	-0.000000000
9.4000	0.000000000	-0.000000000	-0.000000000	-0.000000000
9.5000	0.000000000	-0.000000000	-0.000000000	0.000000000
9.6000	-0.000000000	-0.000000000	-0.000000000	0.000000000
9.7000	-0.000000000	-0.000000000	0.000000000	0.000000000
9.8000	-0.000000000	0.000000000	0.000000000	-0.000000000
9.9000	-0.000000000	0.000000000	0.000000000	-0.000000000

TABLE 6

Nonlinear System Response of the Quadruped Locomotion System for a Particular X Axis Vibrational Mode

X-AXIS MODE NO.1

INPUT PARAMETERS

VALUE FOR FOOT NUMBER- 1	2	3	4	NAME	DESCRIPTION
0.00000	0.00000	0.00000	0.00000	BETA	FOOT DUTY CYCLE
0.00000	0.00000	0.00000	0.00000	GAMMA	X-COORD. OF FOOT AT TOUCH-DOWN
0.00000	0.00000	0.00000	0.00000	PHI	PHASE OF FOOT ACTION - WHEN FOOT IS SET DOWN
2.50000	2.50000	-2.50000	-2.50000	H(1,-)	X-COORD. OF HIP)
-1.00000	1.00000	-1.00000	1.00000	H(2,-)	Y-COORD. OF HIP) IN BODY COORD.
0.25000	0.25000	0.25000	0.25000	H(3,-)	Z-COORD. OF HIP)
-1.00000	1.00000	-1.00000	1.00000	YF	Y-COORD. OF FOOT IN BODY COORD.

STRIDE = 0.00 FT.   PERIOD = 0.00 SEC.   DESIRED Z-COORD. OF BODY = 3.00 FT.

INTEGRATION TIME INTERVAL = 0.0001 SEC.   PERIOD OF STUDY = 20.00 SEC.

INITIAL CONDITIONS

X= -0.022576 FT Y= 0.000000 FT Z= -3.250000 FT Ẋ= 0.136000F/S Ẏ= 0.000000F/S Ż= 0.000000F/S

θ= 0.002766 RAD φ= 0.000000 RAD ψ= 0.000000 RAD θ̇= 0.000000R/S φ̇= -0.016505R/S ψ̇= 0.000020R/S

BODY INERTIAL PROPERTIES

MASS = 20.00   MOMENTS OF INERTIA -- X-AXIS = 7.00300   Y-AXIS = 42.00300   Z-AXIS = 48.33000

CONTROL CONSTANTS

K1	K2	K3	K4	K5	K6	K7	K8	K9
9000.0000	200.0000	9000.0000	200.0000	1500.0000	100.0000	0.0000	0.0000	0.0000

OUTPUT CONSTANTS

PRINT INTERVAL = 0.100 SEC.   DISPLAY INTERVAL = -0.070 SEC. (IF .LT. 0.0, W/ AUTO. PHOTO.)

TABLE 6 Continued

X-AXIS MODE NO.1

PRINTED OUTPUT FORMAT

TIME	x	$\dot{x}$	θ	$\dot{\theta}$
0.0010	-0.002476395	0.099782199	0.000749184	-0.016498180
0.1000	0.005352720	0.051059124	-0.000658253	-0.010424134
0.2000	0.007145131	-0.013031146	-0.001171211	0.000112243
0.3000	0.003876015	-0.046583465	-0.000767025	0.006908264
0.4000	-0.002767939	-0.039077571	-0.000019971	0.006901428
0.5000	-0.003265963	-0.009545999	0.000479490	0.002627061
0.6000	-0.002834663	0.015719448	0.000502663	-0.001867219
0.7000	-0.000766215	0.022273673	0.000200817	-0.003618858
0.8000	0.001264960	0.012500891	-0.000122523	-0.002478636
0.9000	0.001566879	-0.001750546	-0.000255996	-0.000172668
1.0000	0.000397436	-0.029934722	-0.000181560	0.001439086
1.1000	-0.000[illegible]138	-0.009024254	-0.000018575	0.001582059
1.2000	-0.000693781	-0.002720371	0.000099362	0.000677710
1.3000	-0.000651036	0.003090249	0.000113573	-0.000339002
1.4000	-0.000217677	0.004928619	0.000051223	-0.000777859
1.5000	0.0002[illegible]7106	0.003019044	-0.000021709	-0.000583583
1.6000	0.000357399	-0.000129075	-0.000055352	-0.000082574
1.7000	0.000272539	-0.002399347	-0.000342580	0.000296082
1.8000	-0.000[illegible]912	-0.002659712	-0.000557278	0.000356113
1.9010	-0.000146695	-0.000719915	0.000020487	0.000168285
2.0010	-0.000147867	0.000601381	0.000025434	-0.000059628
2.1010	-0.000055400	0.001883270	0.000012619	-0.000178562
2.2010	0.000039795	0.000714327	-0.000003657	-0.000135313
2.3010	0.000276741	0.000023474	-0.000011990	-0.000027013
2.4010	0.000057430	-0.000441564	-0.000009841	0.000060611
2.5010	0.0000[illegible]3530	-0.000466209	-0.000002235	0.000079456
2.6010	-0.000034569	-0.000180358	0.000004130	0.000041681
2.7010	-0.000033515	0.000111571	0.000005675	-0.000029236
2.8010	-0.000014304	0.000236637	0.000003103	-0.000036576
2.9010	0.000017236	0.000168967	-0.000009539	-0.000031361
3.0010	0.000016711	0.000018250	-0.000002563	-0.000008205
3.1010	0.000012345	-0.000091436	-0.000002272	0.000012105
3.2010	0.000001713	-0.000105323	-0.000000647	0.000017646
3.3010	-0.000006300	-0.000047934	0.000000819	0.000016169
3.4010	-0.000007551	0.000019741	0.000001257	-0.000001168
3.5010	-0.000003595	0.000051454	0.000000751	-0.000000775
3.6010	0.000001252	0.000039559	-0.000000054	-0.000007226
3.7010	0.000003619	0.000006859	-0.000000545	-0.000002282
3.8010	0.000002874	-0.000018806	-0.000000528	0.000002378
3.9010	0.000000574	-0.000022615	-0.000000175	0.000003912
4.0010	-0.000001286	-0.000011050	0.000000168	0.000002427
4.1010	-0.000001687	0.000005321	0.000000278	-0.000000109
4.2010	-0.000000802	0.000012006	0.000000179	-0.000001605
4.3010	0.000000199	0.000009193	0.000000303	-0.000001400
4.4010	0.000000774	0.000002115	-0.000000115	-0.000000995
4.5010	0.000000668	-0.000003646	-0.000000122	0.000000479
4.6000	0.000000192	-0.000003[illegible]	-0.000000049	0.000000845
4.7000	-0.000000240	-0.000002632	0.000000027	0.000000862
4.8000	-0.000000371	0.000000[illegible]	0.000000061	-0.000000022

TABLE 6 Continued

4.9000	-0.000002223	0.000002163	0.000000047	-0.000000429
5.0000	0.000002093	0.000002016	0.000000013	-0.000000192
5.1000	0.000000143	0.000000568	-0.000000022	-0.000000402
5.2000	0.000000146	-0.000000425	-0.000000027	0.000000135
5.3000	0.000000066	-0.000000907	-0.000000022	0.000000069
5.4000	0.000000003	-0.000000496	-0.000000012	0.000000042
5.5000	-0.000000038	-0.000000334	-0.000000009	0.000000029
5.6000	-0.000000062	-0.000000003	-0.000000003	0.000000204
5.7000	-0.000000045	0.000000124	0.000000004	-0.000000015
5.8000	-0.000000038	0.000000088	0.000000003	-0.000000015
5.9001	-0.000000030	0.000000063	0.000000001	-0.000000215
6.0001	-0.000000024	0.000000060	-0.000000000	-0.000000015
6.1001	-0.000000018	0.000000058	-0.000000002	-0.000000015
6.2001	-0.000000012	0.000000061	-0.000000003	-0.000000015
6.3001	-0.000000035	0.000000067	-0.000000005	-0.000000015
6.4001	0.000000021	0.000000374	-0.000000006	-0.000000015
6.5001	0.000000009	0.000000053	-0.000000008	-0.000000015
6.6001	0.000000018	0.000000093	-0.000000009	-0.000000015
6.7001	0.000000027	0.000000103	-0.000000011	-0.000000215
6.8001	0.000000038	0.000000113	-0.000000012	-0.000000015
6.9001	0.000000250	0.000000124	-0.000000014	-0.000000015
7.0001	0.000000360	-0.000000093	-0.000000018	-0.000000194
7.1001	0.000000063	0.000000058	-0.000000020	-0.000000015
7.2001	0.000000071	0.000000108	-0.000000021	-0.000000015
7.3001	0.000000084	0.000000144	-0.000000023	-0.000000015
7.4001	0.000000085	-0.000000150	-0.000000022	0.000000035
7.5001	0.000000074	-0.000000060	-0.000000021	0.000000015
7.6001	0.000000071	-0.000000005	-0.000000019	0.000000015
7.7001	0.000000073	0.000000034	-0.000000019	-0.000000011
7.8001	0.000000079	0.000000085	-0.000000020	-0.000000013
7.9001	0.000000089	0.000000126	-0.000000021	-0.000000015
8.0001	0.000000082	-0.000000084	-0.000000021	0.000000015
8.1001	0.000000077	-0.000000018	-0.000000020	0.000000015
8.2001	0.000000078	0.000000026	-0.000000019	0.000000005
8.3001	0.000000082	0.000000067	-0.000000019	-0.000000007
8.4001	0.000000090	-0.000000046	-0.000000022	-0.000000165
8.5001	0.000000056	-0.000000010	-0.000000022	0.000000015
8.6001	0.000000587	0.000000041	-0.000000021	0.000000001
8.7001	0.000000088	-0.000000019	-0.000000023	0.000000015

one part in 10^6 for small motions about the equilibrium point. This proves the validity of the nonlinear simulation of the quadruped locomotion system as well as that of the linearization scheme.

7.3 Simulation of Quadruped Gaits

This section describes the simulations of quadruped gaits such as the crawl, the walk, and the trot. As mentioned in Section 7.2, the nonlinear equations of motion of the quadruped have been programmed into a digital computer simulation which displays the various gaits on a cathode ray tube display system connected to the computer. For producing stable gaits, the model reference type of approach discussed in Chapter VI is used. The ideal kinematic reference model is assumed to walk in a straight line with constant velocity, placing its feet periodically at precomputed points a stride length apart along the direction of motion, namely the positive x axis.

The kinematic model for each gait is described by the following parameters:

1) The duty factor - the relative amount of time spent on the ground by each leg during one locomotion cycle.
2) The relative phase - the amount by which the motion of leg i, i = 2, 3, 4, lags behind that of leg 1 expressed as a fraction of the time required to complete one locomotion cycle.
3) The stride length - the constant distance by which the body is translated in one complete locomotive cycle of the gait.
4) The period - time required for one complete locomotive cycle of the gait.

5) The initial foot position for leg i - the coordinates (x_i, y_i, z_i) of the position of the foot of leg i at the reference leg, namely leg 1 first touches the supporting surface in any locomotion cycle. These coordinates are measured in the body fixed coordinate system x,y,z with its origin at center of gravity of the locomotion system.

6) The desired z coordinate of the body - the constant height of the center of gravity of the ideal kinematic model of the locomotion system above a horizontal plane supporting surface on which the reference model walks in a straight line in the direction of motion with its legs cycling periodically in both space and time.

The ideal kinematic model for each quadruped gait is specified by the above parameters.

In the computer simulation, simple linear feedback control laws based on the difference between the actual and desired values of leg lengths, angles and their time derivatives, are used to obtain stable gaits. The simulation of the various quadruped gaits is considered in greater detail below. For each case, a general description of the gait, a list of the kinematic and dynamic parameters used, as well as a photograph of the computer display is given. During any phase of the locomotion cycle, legs which are off the supporting surface are not displayed.

7.3.1 The Quadruped Crawl

This is a slow speed gait during which the quadruped has alternately either three or four legs on the ground at all times during a locomotion cycle. All the phases of this gait are statically stable because the center of gravity of the body is within the " support pattern " [16] at all times. Therefore this gait is easily stabilized. The crawl gait is thus well suited for low speed locomotion. [illegible] preferred by natural quadrupeds as well as animals with more than four legs for low speed terrestrial locomotion.

Figure 14 is a photograph of the computer simulation display output of the quadruped with all its four legs on the ground at the beginning of its crawl gait cycle. Table 7 lists parameters for the crawl.

7.3.2 The Quadruped Walk

This is a faster gait during which the quadruped employs alternately three and two legs for its support. During the fraction of the locomotion cycle when there are only two legs on the supporting surface, the quadruped is statically unstable. The parameters used for the simulation of this gait are listed in Table 8, and a photograph of the quadruped at the beginning of its locomotion cycle while performing the walk is shown in Figure 15.

7.3.3 The Quadruped Trot

This is a higher speed gait that quadrupeds employ. While trotting, the quadruped uses alternately diagonally opposite pairs of legs to support itself. Therefore, this gait is characterized by the fact that all its phases are statically unstable. Dynamic stability of the quadruped during a trot gait is more difficult because it is necessary to incorporate feedback terms proportional to the body translational velocity normal to the nominal direction of motion. Thus terms proportional to y_E and y_E were needed to stabilize this gait.

Table 9 lists the parameters used for the simulation of this

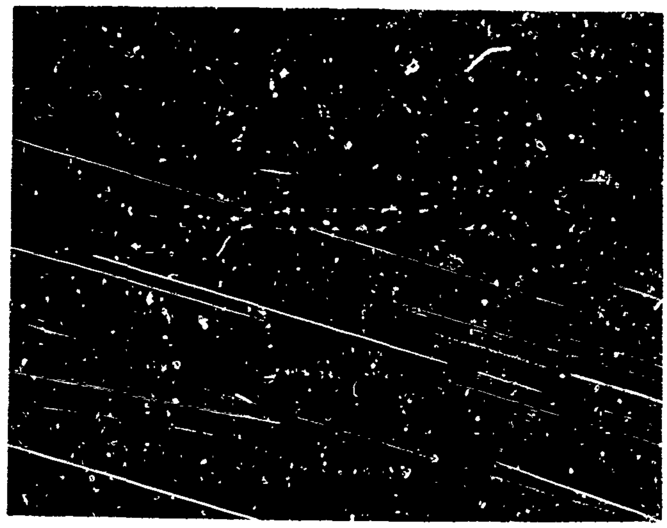


Figure 14. The Quadruped Crawl.

Figure 15. The Quadruped Walk.

TABLE 7

Parameters Used for the Quadruped Crawl Gait Simulation

QUADRUPED CRAWL

VALUE FOR FOOT NUMBER-				INPUT PARAMETERS	
1	2	3	4	NAME	DESCRI.TION
0.91670	0.91670	0.91670	0.91670	BETA	FOOT DUTY CYCLE
0.91670	0.91670	0.00000	0.00000	GAMMA	X-COORD. OF FOOT AT TOUCH-DOWN
0.00000	0.50000	0.75000	0.25000	PHI	PHASE OF FOOT ACTION - WHEN FOOT IS SET DOWN
2.50000	2.50000	-2.50000	-2.50000	H(1,-)	X-COORD. OF HIP) IN BODY COORD.
-1.00000	1.00000	-1.00000	1.00000	H(2,-)	Y-COORD. OF HIP) IN BODY COORD.
0.25000	0.25000	0.25000	0.25000	H(3,-)	Z-COORD. OF HIP) IN BODY COORD.
-1.00000	1.00000	-1.50000	1.00000	YF	Y-COORD. OF FOOT IN BODY COORD.

STRIDE = 3.45 FT. PERIOD = 2.00 SEC. DESIRED Z-COORD. OF BODY = 3.00 FT.

INTEGRATION TIME INTERVAL = 0.0200 SEC. PERIOD OF S.UDY = 12.00 SEC.

INITIAL CONDITIONS

X= 0.000000 FT Y= 0.000000 FT Z= -3.250000 FT Ẋ= 2.725000F/S Ẏ= 0.000000F/S Ż= 0.000000F/S

Ø= 0.000000 RAD Ø= 0.000000 RAD Ø= 0.000000 RAD Ø̇= 0.000000R/S Ø̇= 0.000000R/S Ø̇= 0.000000R/S

BODY INERTIAL PROPERTIES

MASS = 20.30 MOMENTS OF INERTIA -- X-AXIS = 7.08300 Y-AXIS = 42.00000 Z-AXIS = 48.33000

CONTROL CONSTANTS

K1	K2	K3	K4	K3	K6	K7	K8	K9
10000.0000	200.000	10000.0000	200.0000	3000.0000	100.0000	0.0000	0.0000	2.0200

OUTPUT CONSTANTS

PRINT INTERVAL = 0.500 SEC. DISPLAY INTERVAL = -0.020 SEC. (IF .LT. 0.0. W/ AUTO. PHOTO.)

TABLE 8

Parameters Used for the Quadruped Walk Gait Simulation

QUADRUPED WALK

VALUE FOR FOOT NUMBER- 1	2	3	4	NAME	DESCRIPTION (INPUT PARAMETERS)
0.70000	0.70000	0.70000	0.70000	BETA	FOOT DUTY CYCLE
0.70000	0.70000	0.00000	0.00000	GAMMA	X-COORD. OF FOOT AT TOUCH-DOWN
0.00000	0.50000	0.75000	0.25000	PHI	PHASE OF FOOT ACTION - WHEN FOOT IS SET DOWN
2.50000	2.50000	-2.50000	-2.50000	H(1,-)	X-COORD. OF HIP (IN BODY COORD.)
-1.00000	1.00000	-1.00000	1.00000	H(2,-)	Y-COORD. OF HIP (IN BODY COORD.)
0.25000	0.25000	0.25000	0.25000	H(3,-)	Z-COORD. OF HIP (IN BODY COORD.)
-1.00000	1.00000	-1.00000	1.00000	YF	Y-COORD. OF FOOT IN BODY COORD.

STRIDE = 7.14 FT. PERIOD = 2.00 SEC. DESIRED Z-COORD. OF BODY = 3.00 FT.

INTEGRATION TIME INTERVAL = 0.0100 SEC. PERIOD OF STUDY = 12.00 SEC.

INITIAL CONDITIONS

X= 0.000000 FT Y= 0.000000 FT Z= -3.250000 FT $\dot{X}$= 3.571000F/S $\dot{Y}$= 0.000000F/S $\dot{Z}$= 0.000000F/S

ϕ= 0.000000 RAD θ= 0.000000 RAD ψ= 0.000000 RAD $\dot{\phi}$= 0.000000R/S $\dot{\theta}$= 0.000000R/S $\dot{\psi}$= 0.000000R/S

BODY INERTIAL PROPERTIES

MASS = 29.00 MOMENTS OF INERTIA -- X-AXIS = 7.00000 Y-AXIS = 42.10000 Z-AXIS = 48.30000

CONTROL CONSTANTS

K1	K2	K3	K4	K5	K6	K7	K8	K9
40000.0000	250.0000	30000.0000	250.0000	4000.0000	200.0000	0.0000	0.0000	2.0000

OUTPUT CONSTANTS

PRINT INTERVAL = 0.100 SEC. DISPLAY INTERVAL = -0.030 SEC. (IF .LT. 0.0, W/ AUTO. PHOTO.)

gait, while Figure 16 shows the computer generated display of the quadruped simulation performing the trot. From Figure 16, one can see that the quadruped has legs 1 and 4 on the ground, while legs 2 and 3 which are lefted off the supporting surface are not displayed. Also it is apparent that the trotting quadruped resembles an inverted pendulum system as far as its diagonally opposite support pattern is concerned.

In the trot gaits described by Muybridge [7], there are phases during which the animal lifts all its four feet off the ground in between the times when it has diagonally opposite legs on the ground. But in the computer simulation, the quadruped is assumed to shift its support from one pair of diagonally opposite legs to another instantaneously. This is indicated by the foot duty cycle which is seen to be 0.5 for all the legs (see Table 7).

7.4 The Inverted Pendulum System Simulation

The inverted pendulum system described earlier was programmed into a digital computer simulation. A photograph of the computer display of this system is shown in Figure 17. Table 10 gives the values of the parameters used for the inverted pendulum including a set of stabilizing control constants which were obtained using the Routh-Hurwitz algorithms derived for this system in Chapter VI. The inverted pendulum system with the pivot at the center of gravity of the body, that is, with $r = 0$ is easily stabilized. However, the case with $r \neq 0$ is harder to stabilize. Figure 18 shows the transient response of the inverted pendulum system consisting of a mass pivoted at a distance r below its center of gravity, and supported on a massless leg of constant length ℓ. A simple linear

TABLE 9

Parameters Used for the Quadruped Trot Gait Simulation

QUADRUPED TROT

VALUE FOR FOOT NUMBER-				INPUT PARAMETERS	
1	2	3	4	NAME	DESCRIPTION
0.50000	0.50000	0.50000	0.50000	BETA	FOOT DUTY CYCLE
1.50000	1.50000	-1.00000	-1.00000	GAMMA	X-COORD. OF FOOT AT TOUCH-DOWN
0.00000	0.50000	0.50000	0.00000	PHI	PHASE OF FOOT ACTION - WHEN FOOT IS SET DOWN
2.50000	2.50000	-2.50000	-2.50000	H(1,-)	X-COORD. OF HIP) IN BODY COORD.
-1.00000	1.00000	-1.00000	1.00000	H(2,-)	Y-COORD. OF HIP) IN BODY COORD.
0.25000	0.25000	0.25000	0.25000	H(3,-)	Z-COORD. OF HIP) IN BODY COORD.
-1.00000	1.00000	-1.00000	1.00000	YF	Y-COORD. OF FOOT IN BODY COORD.

STRIDE = 2.00 FT. PERIOD = 1.00 SEC. DESIRED Z-COORD. OF BODY = 3.00 FT.

INTEGRATION TIME INTERVAL = 0.0050 SEC. PERIOD OF STUDY = 12.00 SEC.

INITIAL CONDITIONS

X= 0.000000 FT Y= 0.000000 FT Z= -3.250000 FT $\dot{X}$= 2.000000F/S $\dot{Y}$= 0.000000F/S $\dot{Z}$= 0.000000F/S

θ= 0.000000 RAD ϕ= 0.000000 RAD ψ= 0.000000 RAD $\dot{\theta}$= 0.000000R/S $\dot{\phi}$= 0.000000R/S $\dot{\psi}$= 0.000000R/S

BODY INERTIAL PROPERTIES

MASS = 22.00 MOMENTS OF INERTIA -- X-AXIS = 7.00000 Y-AXIS = 42.10000 Z-AXIS = 46.32000

CONTROL CONSTANTS

K1	K2	K3	K4	K5	K6	K7	K8	K9
10000.0000	300.0000	10000.0000	400.0000	2000.0000	430.0306	0.0000	3000.0000	2200.0000

OUTPUT CONSTANTS

PRINT INTERVAL = 0.100 SEC. DISPLAY INTERVAL = -0.030 SEC. (IF .LT. 0.0, W/ AUTO. PHOTO.)

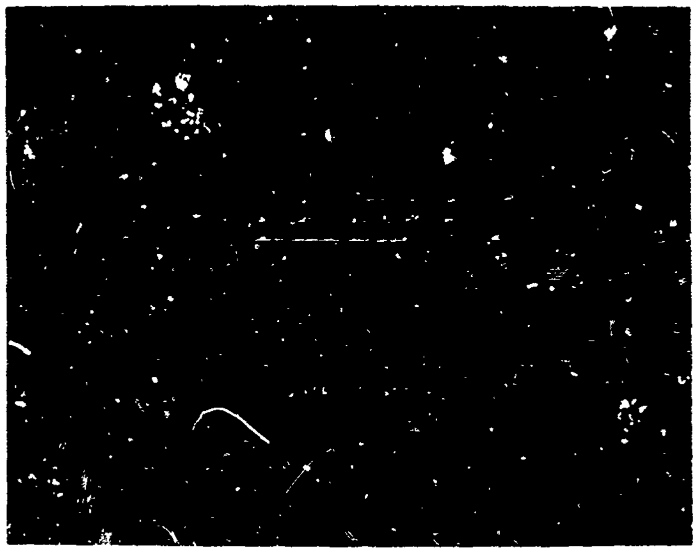


Figure 16. The Quadruped Trot.

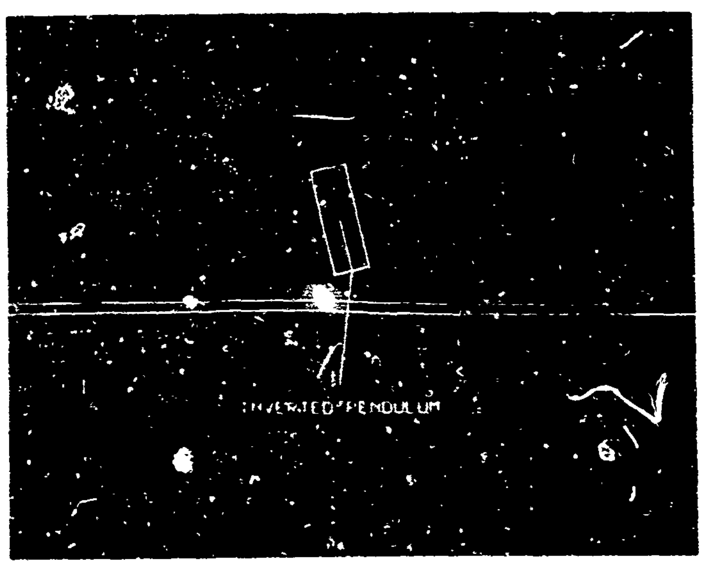


Figure 17. An Inverted Pendulum System.

TABLE 10

Parameters Used for the Simulation of the Inverted Pendulum System

INVERTED PENDULUM SYSTEM

MASS, INERTIA, LEG LENGTH, LENGTH XR, CALCINC, PRINTINC, ENDTIME, CRTINC

5.0 15.0 3.0 1.500 0.010 0.1000 20.0 -0.030

CONTROL CONSTANTS

K1	K2	K3	K4
5200.000	950.000	10000.000	1500.000

INITIAL STATE VECTOR

-0.0500 0.0100 0.0000 0.0000

TIME	STATE VECTOR X1	X2	X3	X4	TORQUE
0.01	-0.029131	0.008784	0.163142	-0.228387	220.283184
0.10	0.022671	-0.064018	0.735538	-1.037751	25.182770
0.20	0.079141	-0.144367	0.325552	-0.471190	110.946320
0.30	0.089697	-0.160477	-0.063435	0.078607	74.102831
0.40	0.077499	-0.144066	-0.133285	0.103397	28.754204
0.50	0.068134	-0.131013	-0.047922	0.068738	19.586184
0.60	0.066659	-0.128619	0.006164	-0.023876	29.077815
0.70	0.067262	-0.128853	-0.001182	0.009048	35.199021
0.80	0.065985	-0.126001	-0.025189	0.045388	33.995728
0.90	0.062653	-0.120302	-0.037299	0.064966	30.381621
1.00	0.058013	-0.113614	-0.038494	0.068916	27.783522
1.10	0.055021	-0.106678	-0.037997	0.070101	26.223510
1.20	0.051142	-0.099502	-0.039548	0.073810	24.689467
1.30	0.047053	-0.091005	-0.041980	0.078458	22.767114
1.40	0.042772	-0.083854	-0.043696	0.081826	20.600061
1.50	0.038350	-0.075577	-0.044399	0.083503	18.422010
1.60	0.033910	-0.067192	-0.044468	0.084033	16.272600
1.70	0.029475	-0.058794	-0.044103	0.083825	14.143890
1.80	0.025094	-0.050449	-0.043581	0.082950	12.028236
1.90	0.020771	-0.042227	-0.042628	0.081301	9.943206
2.00	0.016572	-0.034195	-0.041332	0.079144	7.912930
2.10	0.012514	-0.026410	-0.039736	0.076319	5.955150
2.20	0.008631	-0.018948	-0.037807	0.072990	4.082020
2.30	0.004944	-0.011834	-0.035822	0.069226	2.303623
2.40	0.001473	-0.005115	-0.033572	0.065080	0.629412
2.50	-0.001765	0.001174	-0.031169	0.060639	-0.932206
2.60	-0.004757	0.007004	-0.028645	0.055937	-2.374421
2.70	-0.007491	0.012355	-0.026030	0.051045	-3.692132
2.80	-0.009961	0.017209	-0.023356	0.046020	-4.881752
2.90	-0.012161	0.021556	-0.020651	0.040917	-5.941076
3.00	-0.014091	0.025331	-0.017942	0.035700	-6.869254
3.10	-0.015751	0.028714	-0.015255	0.030604	-7.660703
3.20	-0.017143	0.031530	-0.012613	0.025650	-8.335057
3.30	-0.018275	0.033848	-0.010038	0.020738	-8.877084
3.40	-0.019154	0.035601	-0.007551	0.015963	-9.296593
3.50	-0.019789	0.037047	-0.005169	0.011302	-9.598268
3.60	-0.020192	0.037965	-0.002900	0.007019	-9.787617
3.70	-0.020375	0.038459	-0.000700	0.002981	-9.870880
3.80	-0.020353	0.038554	0.001203	-0.000951	-9.854863
3.90	-0.020143	0.038278	0.003033	-0.004518	-9.746792
4.00	-0.019751	0.037660	0.004703	-0.007786	-9.554355
4.10	-0.019234	0.036731	0.006208	-0.010747	-9.285412
4.20	-0.018515	0.035521	0.007547	-0.013393	-8.948825

TABLE 10 Continued

TIME	X1	X2	X3	X4	TORQUE
4.30	-0.017721	0.034063	0.008718	-0.015722	-8.558346
4.40	-0.016777	0.032387	0.009722	-0.017736	-8.100475
4.50	-0.015762	0.030526	0.010562	-0.019437	-7.626378
4.60	-0.014670	0.028510	0.011243	-0.020832	-7.075922
4.70	-0.013510	0.026369	0.011730	-0.021921	-6.516747
4.80	-0.012321	0.024133	0.012105	-0.022744	-5.936159
4.90	-0.011094	0.021830	0.012300	-0.023265	-5.341153
5.00	-0.009857	0.019485	0.012402	-0.023568	-4.730370
5.10	-0.008602	0.017124	0.012400	-0.023609	-4.134024
5.20	-0.007361	0.014770	0.012322	-0.023425	-3.533900
5.30	-0.006141	0.012446	0.012078	-0.023034	-2.943342
5.40	-0.004949	0.010170	0.011738	-0.022455	-2.367190
5.50	-0.003795	0.007960	0.011312	-0.021706	-1.809842
5.60	-0.002689	0.005834	0.010819	-0.020808	-1.275177
5.70	-0.001636	0.003803	0.010242	-0.019776	-0.766603
5.80	-0.000643	0.001882	0.009619	-0.018633	-0.287035
5.90	0.000255	0.000080	0.008950	-0.017396	0.161080
6.00	0.001146	-0.001595	0.008244	-0.016084	0.575795
6.10	0.001934	-0.003135	0.007511	-0.014714	0.955564
6.20	0.002648	-0.004536	0.006760	-0.013304	1.290313
6.30	0.003256	-0.005795	0.005998	-0.011860	1.606371
6.40	0.003847	-0.006909	0.005235	-0.010424	1.876463
6.50	0.004333	-0.007880	0.004476	-0.008984	2.109681
6.60	0.004743	-0.008707	0.003729	-0.007562	2.306447
6.70	0.005079	-0.009393	0.003001	-0.006170	2.467512
6.80	0.005344	-0.009942	0.002296	-0.004620	2.593980
6.90	0.005539	-0.010359	0.001620	-0.003520	2.686886
7.00	0.005659	-0.010648	0.000976	-0.002279	2.747978
7.10	0.005736	-0.010817	0.000369	-0.001106	2.778879
7.20	0.005744	-0.010872	-0.000198	-0.000005	2.781439
7.30	0.005694	-0.010820	-0.000722	0.001017	2.757654
7.40	0.005621	-0.010671	-0.001203	0.001956	2.709632
7.50	0.005459	-0.010432	-0.001638	0.002910	2.639547
7.60	0.005275	-0.010112	-0.002027	0.003577	2.549633
7.70	0.005055	-0.009719	-0.002368	0.004256	2.442153
7.80	0.004803	-0.009264	-0.002664	0.004846	2.319362
7.90	0.004524	-0.008753	-0.002913	0.005349	2.183513
8.00	0.004272	-0.008197	-0.003117	0.005765	2.036814
8.10	0.003902	-0.007603	-0.003277	0.006098	1.881416
8.20	0.003569	-0.006980	-0.003395	0.006349	1.719397
8.30	0.003224	-0.006336	-0.003472	0.006523	1.552737
8.40	0.002875	-0.005678	-0.003512	0.006623	1.383387
8.50	0.002523	-0.005013	-0.003515	0.006654	1.213000
8.60	0.002173	-0.004349	-0.003486	0.006622	1.043499
8.70	0.001827	-0.003691	-0.003425	0.006526	0.876108
8.80	0.001489	-0.003045	-0.003337	0.006378	0.712557
8.90	0.001158	-0.002417	-0.003224	0.006180	0.553382
9.00	0.000845	-0.001811	-0.003088	0.005938	0.401302
9.10	0.000543	-0.001231	-0.002933	0.005658	0.255817
9.20	0.000259	-0.000680	-0.002762	0.005344	0.118290
9.30	-0.000008	-0.000153	-0.002576	0.005003	-0.010555
9.40	-0.000256	0.000319	-0.002380	0.004638	-0.130125
9.50	-0.000484	0.000764	-0.002175	0.004257	-0.239755
9.60	-0.000691	0.001170	-0.001965	0.003862	-0.339706
9.70	-0.000877	0.001536	-0.001751	0.003459	-0.429160
9.80	-0.001041	0.001862	-0.001536	0.003052	-0.508209
9.90	-0.001184	0.002147	-0.001321	0.002646	-0.576050
10.00	-0.001305	0.002391	-0.001110	0.002243	-0.635179

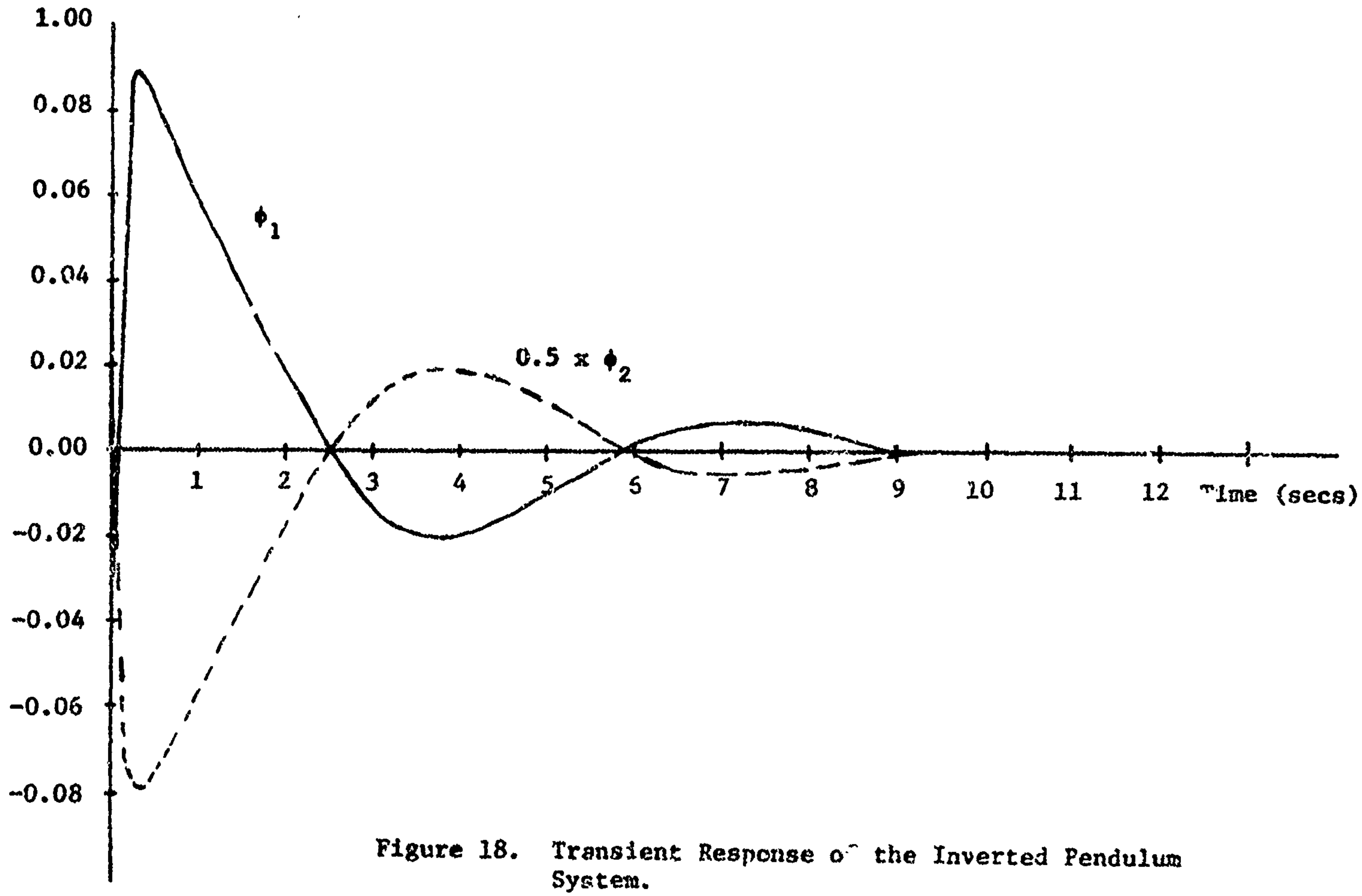


Figure 18. Transient Response of the Inverted Pendulum System.

feedback control law is used for the stabilization of this system.

7.5 The Quadruped Pace

This is a high speed gait that some animals employ. It is faster than the trot. During the pace, the animal uses two legs on the same side to support itself. This gait has no statically stable phases in its locomotion cycle. Dynamic stability is achieved by alternate fall and recovery using the two legs on the same side. Muybridge [7] has shown that the pacing horse has all its feet off the ground for a small fraction of the locomotion cycle in between the times the horse switches its supporting legs.

The pace is more difficult to stabilize than the trot. Since the system looks like an inverted pendulum in a direction perpendicular to the direction of motion, the following correspondence was established with the inverted pendulum system Routh-Hurwitz analysis.

Stabilizing control constants were computed using the dynamic parameters of the quadruped in the Routh-Hurwitz algorithms for the inverted pendulum system. Then, one half of the values of the control constants K_1 through K_4 computed for the inverted pendulum system were used in the feedback control law for the lateral centering torque T_{s_i}, given by equation (6-114). Table 11 shows the parameters used for the simulation of the quadruped pace. For this particular gait, the dimensions of the body were 5' x 2' x 2'.

Figure 19 shows a photograph of the quadruped pace that was displayed by the computer simulation. For display purposes the two legs that are on the ground have been shown at the center of the center of the body. Use of the inverted pendulum system Routh-Hurwitz analysis

TABLE 11

Parameters Used for the Simulation of the Quadruped Pace Gait

QUADRUPED PACE

VALUE FOR FOOT NUMBER-				INPUT PARAMETERS	
1	2	3	4	NAME	DESCRIPTION
0.50000	0.50000	0.50000	0.50000	BETA	FOOT DUTY CYCLE
1.50000	1.50000	-1.00000	-1.00000	GAMMA	X-COORD. OF FOOT AT TOUCH-DOWN
0.00000	0.50000	0.00000	0.50000	PHI	PHASE OF FOOT ACTION - WHEN FOOT IS SET DOWN
2.50000	2.50000	-2.50000	-2.50000	H(1,-)	X-COORD. OF HIP)
0.00000	0.00000	0.00000	0.00000	H(2,-)	Y-COORD. OF HIP) IN BODY COORD.
1.00000	1.00000	1.00000	1.00000	H(3,-)	Z-COORD. OF HIP)
0.30000	0.00000	0.00000	0.00000	YF	Y-COORD. OF FOOT IN BODY COORD.

STRIDE = 2.00 FT.   PERIOD = 1.00 SEC.   DESIRED Z-COORD. OF BODY = 3.00 FT.

INTEGRATION TIME INTERVAL = 0.0040 SEC.   PERIOD OF STUDY = 20.00 SEC.

INITIAL CONDITIONS

X= 0.000000 FT Y= 0.000000 FT Z= -4.000000 FT $\dot{X}$= 2.000000F/S $\dot{Y}$= 0.000000F/S $\dot{Z}$= 0.000000F/S

θ= 0.000000 RAD θ= 0.000000 RAD θ= 0.000000 RAD $\dot{\theta}$= 0.000000R/S $\dot{\theta}$= 0.000000R/S $\dot{\theta}$= 0.000000R/S

BODY INERTIAL PROPERTIES

MASS = 20.00   MOMENTS OF INERTIA -- X-AXIS = 13.33000   Y-AXIS = 48.33000   Z-AXIS = 48.33000

CONTROL CONSTANTS

K1	K2	K3	K4	K5	K6	K7	K8	K9
10000.0000	400.0000	5000.0000	675.0000	2000.0000	430.0000	0.0000	2500.0000	250.0000

OUTPUT CONSTANTS

PRINT INTERVAL = 0.150 SEC.   DISPLAY INTERVAL = -0.030 SEC. (IF .LT. 0.0, W/ AUTO. PHOTO.)

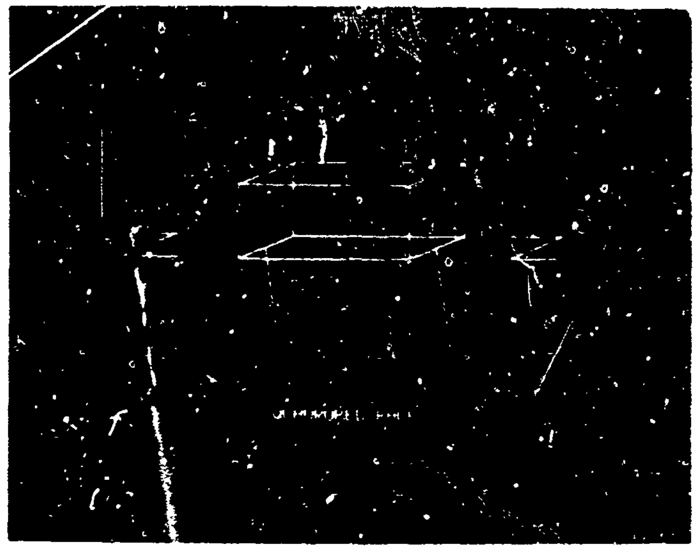


Figure 19. The Quadruped Pace.

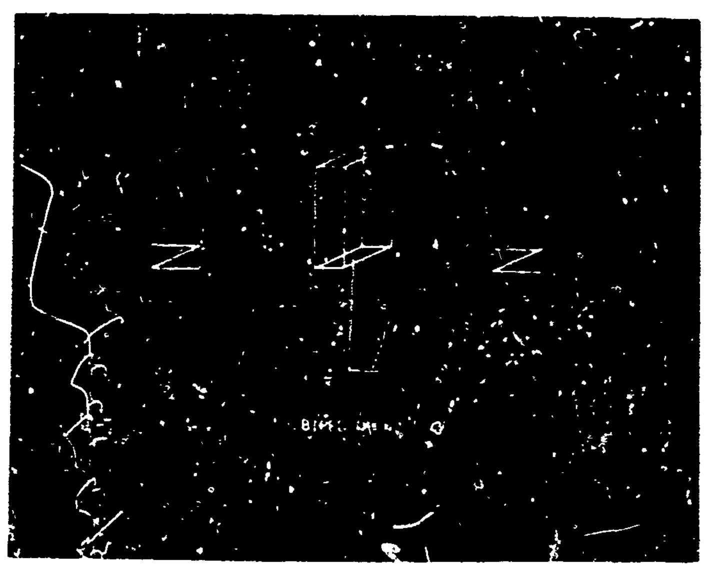


Figure 20. A Type of Biped Walk.

for the lateral control of the quadruped locomotion system produced a stable pace.

7.6 A Type of Biped Walk

There are basically two types of mechanisms that bipeds employ for stabilization during walking. One of them is body torquing for lateral control, and the other is foot placement for longitudinal control in the direction of motion. As an example, body torquing is by tight rope walkers as a stabilization mechanism. They use long poles to effectively increase their moment of inertia during their walk on the tight rope. Foot placement is important for example, for a stilt walker who maintains stability by alternate fall and recovery by placing the stilts in the right position.

In this research only the body torquing mechanism has been investigated in the simulation of a type of biped walk. The idealized biped moves with a constant velocity in the direction of motion over a horizontal supporting surface.

Table 12 shows a complete set of input data used for the biped walk. Figure 20 is a photograph of the cathode ray tube display of the biped gait simulated during the course of this research. For this simulation the following assumption was made. Two parallel massless legs supporting the body with the distance of the body width separating them as in the quadruped pace display (see Figure 19), was equivalent to support by one leg with a foot whose length was equal to the width of the body. The result is the display of Figure 20 in which the supporting leg is shown at the center of the body for convenience and the leg that is in the air is not shown. Stabilizing control constants

TABLE 12

Parameters Used for the Simulation of the Biped Walk

BIPED WALK

VALUE FOR FOOT NUMBER- 1	2	3	4	NAME	INPUT PARAMETERS DESCRIPTION
0.50000	0.50000	0.50000	0.50000	BETA	FOOT DUTY CYCLE
0.50000	0.50000	0.00000	0.00000	GAMMA	X-COORD. OF FOOT AT TOUCH-DOWN
0.00000	0.50000	0.00000	0.50000	PHI	PHASE OF FOOT ACTION - WHEN FOOT IS SET DOWN
0.50000	0.50000	-0.50000	-0.50000	H(1,-)	X-COORD. OF HIP } IN BODY COORD.
0.00000	0.00000	0.00000	0.00000	H(2,-)	Y-COORD. OF HIP } IN BODY COORD.
1.50000	1.50000	1.50000	1.50000	H(3,-)	Z-COORD. OF HIP } IN BODY COORD.
0.00000	0.00000	0.00000	0.00000	YF	Y-COORD. OF FOOT IN BODY COORD.

STRIDE = 2.00 FT. PERIOD = 0.50 SEC. DESIRED Z-COORD. OF BODY = 3.00 FT.

INTEGRATION TIME INTERVAL = 0.0040 SEC. PERIOD OF STUDY = 20.00 SEC.

INITIAL CONDITIONS

X= 0.000000 FT Y= 0.000000 FT Z= -4.500000 FT $\dot{X}$= 4.000000F/S $\dot{Y}$= 0.000000F/S $\dot{Z}$= 0.000000F/S

θ= 0.000000 RAD θ= 0.000000 RAD θ= 0.000000 RAD $\dot{\theta}$= 3.600000R/S $\dot{\theta}$= 0.000000R/S $\dot{\theta}$= 0.000000R/S

BODY INERTIAL PROPERTIES

MASS = 5.00 MOMENTS OF INERTIA -- X-AXIS = 5.47000 Y-AXIS = 4.17000 Z-AXIS = 2.06000

CONTROL CONSTANTS

K1	K2	K3	K4	K5	K6	K7	K8	K9
10230.0002	400.0000	5000.0000	475.0000	4000.0000	400.0000	0.0050	2500.0000	250.0000

OUTPUT CONSTANTS

PRINT INTERVAL = 0.100 SEC. DISPLAY INTERVAL = -0.030 SEC. (IF .LT. 0.0. W/ AUTO. PHOTO.)

for the biped walk were once again obtained by the application of the Routh-Hurwitz algorithm for the inverted pendulum system for the lateral control of this locomotion system using a feedback control law similar to that described in the previous section for the quadruped pace.

7.7 Summary

This chapter has described the experimental results obtained from digital computer simulations of the idealized locomotion systems considered in this dissertation.

In Section 7.2 the results of vibrational analysis on the quadruped locomotion system were given and the validity of the non-linear simulation as well as that of the linearization techniques used shown by the correspondence between their outputs to one part in 10^6. Then the actual parameters used, and the computer generated displays of the various quadruped gaits such as the crawl, the walk and the trot were described. Section 7.4 outlines the parameters used to obtain a stable inverted pendulum system. The application of inverted pendulum system stability criteria for the simulation of a stable quadruped pace gait was covered in Section 7.5. Finally, the simulation of a type of biped walk using the body torquing mechanism for stability was discussed in Section 7.6.

CHAPTER VIII

CONCLUSIONS AND FURTHER TOPICS FOR RESEARCH

8.1 Results and Contributions of this Dissertation

The following results and contributions have been made to the study of legged locomotion systems:

1) The techniques of linearization and modal analysis have been applied for the first time to the nonlinear equations of motion of idealized dynamic models of legged locomotion systems. From this, a design tool has been obtained for the determination of stabilizing control constants for the postural control of these systems. In addition, these techniques have provided a method for determining the validity of the nonlinear equations of motion describing the dynamics of legged locomotion systems.

2) The use of the above techniques on the nonlinear equations of motion of the legged locomotion system developed by Frank and McGhee [47] has shown that some of their assumptions and certain equations were incorrect. The appropriate modifications were made, and for the first time a completely authenticated dynamic simulation of a massless leg quadruped locomotion system has been obtained.

3) A nonlinear simulation of an inverted pendulum system with the mass pivoted below its center of gravity on a massless leg of constant length, supported by a " fixed " foot has been developed. This simulation has been used to show that such a system can be stabilized for small motions about its equilibrium position by torquing the mass using a simple linear feedback control law.

4) Stability criteria for the controllability of both the quadruped locomotion system, and the inverted pendulum system have been established.

5) Four dynamically stable gaits, namely, the crawl, the walk, the trot, and the pace have been simulated for the quadruped locomotion system.

6) A type of biped walk using the body torquing mechanism for stability has been simulated. This proves that the techniques of inverted pendulum stabilization can be applied for the lateral stability of a simple biped model consisting of a mass supported on two massless legs.

8.2 Topics for Further Research

Some of the areas where more work is needed include:

1) Development of the equations of motion of legged locomotion systems taking into account leg mass.

2) Stability and control of the simulation of a biped model consisting of a body supported by massless legs and using both the body torquing as well as the foot placement mechanisms.

3) The system dynamics incorporated in the simulations obtained during the course of this research have assumed that the models had fixed stride lengths and gait periods, and were travelling on level ground. This work needs to be extended to simulations of legged locomotion systems producing stable locomotion over uneven terrain using variable stride lengths and gait periods.

4) The application of pole assignment techniques [70] to the linearized equations of motion of legged locomotion systems for obtaining desired transient response.

5) Digital computer simulations of kinematic models of human gait [33] with applications to the study of pathological gaits [34]. Such simulations are also useful as a computer aided instructional tool for demonstrating normal and pathological gaits to medical personnel.

8.3 Conclusions

In a real life situation, human and animal locomotion is a complex process dependent upon many factors such as: 1) visual inputs, 2) proprioreceptive sensing of muscle dynamics, 3) angular acceleration feedback from the vestibular system, etc. Simulations which include all these different aspects of system dynamics are difficult, if not impossible to achieve. However, this research has produced idealized dynamic models of legged locomotion systems. These models have simulated " marching " type of quadruped and biped gaits using a " model reference " type of control system. The dynamic theory of legged locomotion systems needs extensive further development.

The next step is to extend the work to the simulation of locomotion systems which take into such factors as leg mass, the nature of the terrain, variable foot placement, etc. Such systems would probably need adaptive controllers as well as inertial guidance systems for their stability because the locomotion system would have to change its gait automatically to take care of variations in the terrain.

If leg mass is included, the equations of motion of a legged locomotion system become quite complicated. For example, if a biped is modeled as a rigid body with two arms, and two legs, with each limb having a two degree of freedom hip (shoulder) joint, and a single degree of freedom knee (elbow) joint, then this model has eighteen degrees of freedom, and is therefore described by a 36th order differential equation.

The type of biped gait simulated in this research has used only the body torquing mechanism for its stability. A more comprehensive treatment of biped dynamics should include both the body torquing and the foot placement (base motion) mechanisms for stability in the lateral as well as the longitudinal directions respectively.

Even though legged locomotion systems are inherently very complex, this research has shown that it is possible to construct a mathematical basis for their system dynamics by the application of the laws of mechanics, modern control theory, and computer simulation.

APPENDIX

COMPUTER PROGRAMS WITH EXPLANATION

A.1 Introduction

This appendix lists the computer programs used in the course of this research along with an explanation covering both the various symbols used as well as a description of the program. Section A.2 discusses the main program for the quadruped gaits. This program is used to simulate the quadruped crawl, the walk, and the trot gaits. Section A.3 discusses the modification of the main quadruped program for use in the vibrational analysis of the quadruped locomotion system.

Section A.4 covers the computer programs used for obtaining the linear system response of the quadruped locomotion system. This section lists the main program for obtaining the eigenvalues and eigenvectors of the real non-symmetric modal matrices of the linearized quadruped locomotion system, as well as the subroutine LINEAR used to compute its linear system response.

Section A.5 describes the program used for the simulation of the inverted pendulum system considered in the dissertation. It also includes a listing of the algorithm used for computing the stabilizing control constants using the Routh-Hurwitz analysis.

Finally, Section A.6 describes the modifications made to the main walking program of Seciton A.2 to simulate the quadruped pace as well as a type of biped walk.

A.2 Computer Program of the Quadruped Locomotion System

This program is used to simulate the following quadruped gaits: 1) the crawl, 2) the walk, and 3) the trot. The main program listed in Figure 21 should be used with the Macro subroutine PIC given by Figure 22 for use on the PDP-9 computer of the Digital Equipment Corporation.

For use with any other digital computer, the main program should be used with the proper display subroutine after making the appropriate changes in subroutine PICTUR to modify it to suit the particular display package used. This program uses the Euler Predictor-Corrector method of integration.

The following is a partial listing of the symbols used in this program:

A(4) = α the actual forward angle of the leg at the hip

AC(4) = α_c the desired forward angle of the leg at the hip

B(4) = β the actual lateral angle subtended at the hip by the leg

BETA(4) = the foot duty cycle, percentage of the cycle the foot is on the ground

CPH = cosine ϕ

CT = fraction of the present period which has elapsed

CTH = cosine θ

DA(4) = derivative of angle α

DAC(4) = time derivative of the desired angle α_c

DB(4) = derivative of angle β

DL(4) = time derivative of the leg length ℓ

DLC(4) = time derivative of the desired leg length ℓ_o

DOTX = stride/period = desired x-directed velocity

DT = differential time increment for integration

DTP = time between printouts

DX(12) = time derivative of the state vector $\underline{x}$

F(3) = force vector $\underline{f}$ applied to the body

FZP(4) = force in the z direction due to each leg

G(4) = PHI(4) + BETA(4) - 1.0 = instant when foot is lifted in a cycle. If G is greater than zero, foot is lifted in next cycle. If G is less than zero, then the foot is lifted in this cycle.

GAMMA(4) = position of the foot when it first touches the ground relative to the body in stride normalized coordinates

H(3,4) = matrix of hip positions

I,J = general indexing variables

M(4) = indicator of the foot position: M = 1 (on ground), M = 0 (off ground)

PER = period of the walking cycle

PHI(4) = ϕ_i, phase of the foot action relative to the first foot action

STH = sin θ

STD = length of the stride

T(3) = torque vector $\underline{T}$ applied to the body

TIME = total elapsed time of the gait

TM(4) = motor torque applied at the hip (T_{m_i})

TMAX = maximum time of the gait

TP = next scheduled printing time

TS(4) = lateral torque T_{s_i} at the hip

T1(3,3) = the matrix of transformation T_1

T2(3,3) = the matrix of transformation T_2

T3T(3,3) = the transposed matrix of transformation T_3^T

X(12) = the body state vector $\underline{x}$

XF(4) = the next position of the foot in ground coordinates

XI(3) = moment of inertia vector = $[I_{xx}, I_{yy}, I_{zz}]^T$

XK(10) = control constants K_1 through K_{10}

XL(4) = leg lengths ℓ_i

XLC(4) = magnitude of the leg length projected on the x-z plane

XMASS = mass of the body

XS(12) = previous value of the state vector $\underline{x}$ saved for the computation of $\dot{\underline{x}}$ using the Euler Predictor-Corrector method of integration

YF(4) = y coordinate of the foot

ZR = z coordinate of the body

The following symbols are used in subroutine FANDM:

CA = cosine α

CB = cosine β

FPX,FPY,FPZ = forces applied to the body by leg i at the hip

FX,FY,FZ = total leg force in body coordinates

SA = sine α

SB = sine β

TX,TY,TZ = torque applied to the body by the legs

XM,YM,ZM = moments due to forces applied at the hip sockets

The following symbols are used with subroutine MODEL:

CG = fraction of the present stride which has been executed

CT = fraction of present cycle which has elapsed

NT = number of elapsed periods of gait cycle

XDNF(4) = next position of the feet in body coordinate system

XNF(4) = next position of the feet in normalized body coordinates

The following symbols are used with subroutine LEG:

CPS = cosine ψ

```
C     FORTRAN.
C     THIS PROGRAM IS FOR THE SIMULATION OF THE QUADRUPED LOCOMOTION
C     SYSTEM GAITS.  THIS PROGRAM SHOULD BE USED WITH THE MACRO SUB-
C     ROUTINE PIC FOR DISPLAY PURPOSES.  THE QUADRUPED GAITS SIMULATED
C     BY THIS PROGRAM ARE: THE CRAWL, THE WALK AND THE TROT.
C     THIS PROGRAM CAN BE USED WITH THE PROPER DISPLAY SUBROUTINES
C     ON ANY DIGITAL COMPUTER.
C
C     PSEUDO-DIMENSIONED VARIABLES (E.G.: X(3) = X1, X2, + X3)
C    1BETA(4), D(3), DXL(3), F(3), T(3), T1(3,3), T2(3,3)
C    2T3T(3,3), XB(3), XI(3), XT(3)
      LOGICAL LOOP, M(4), HEADNG, WOF
      DIMENSION A(4), AC(4), B(4), DA(4), DAC(4), DB(4), DL(4),
     1DLC(4), DX(12), FX(4), FY(4), FZ(4), FZP(4), G(4), GAMMA(4),
     2H1(4), H2(4), H3(4), PHI(4), TH(4), TS(4), TX(4),
     3TY(4), TZ(4), X(12), XF(4), XK(10), XL(4), XLC(4),
     4XM(4), XS(12), YF(4), YM(4), ZM(4), EENPUT(60), EXECUT(60),
     5TITLE(4), IHX(4), IHY(8), IFX(4), IFY(4)
      COMMON DX1, DX2, DX3, DX4, DX5, DX6, DX7, DX8, DX9, DX10,
     1DX11, DX12, FX1, FX2, FX3, FX4, FY1, FY2, FY3, FY4, FZ1,
     2FZ2, FZ3, FZ4, G1, G2, G3, G4, TX1, TX2, TX3, TX4, TY1,
     3TY2, TY3, TY4, TZ1, TZ2, TZ3, TZ4, XM1, XM2,
     4XM3, XM4, YM1, YM2, YM3, YM4, ZM1, ZM2, ZM3, ZM4
      COMMON /INPUT/ BETA1, BETA2, BETA3, BETA4, GAMMA,
     1PHI1, PHI2, PHI3, PHI4, H1, H2, H3, YF, STD, PER, ZR, DT,
     2YMAX, X1, X2, X3, X4, X5, X6, X7, X8, X9, X10,
     3X11, X12, XMAS, XI1, XI2, XI3, XK1, XK2, XK3, XK4, XK5,
     4XK6, XK7, XK8, XK9, DTP, DTD, T111, T112, T113, T121, T122,
     5T123, T131, T132, T133, CG, XF, M, IADR, WOF,
     6TITLE1, TITLE2, TITLE3,TITLE4,XK10
      COMMON /DISP/ IHX1, IHX2, IHX3, IHX4, IHY1, IHY2, IHY3,
     1IHY4,IHY5,IHY6,IHY7,IHY8
      EQUIVALENCE (IHX(1),IHX1), (IHY(1),IHY1)
      EQUIVALENCE (DX(1),DX1), (FX(1),FX1),
     1(FY(1),FY1), (FZ(1),FZ1), (G(1),G1),
     2(PHI(1),PHI1), (TX(1),TX1), (TY(1),TY1),
     3(TZ(1),TZ1), (X(1),X1), (XK(1),XK1), (XM(1),XM1), (YM(1),YM1),
     4(ZM(1),ZM1), (EXECUT(1),BETA1), (TITLE(1),TITLE1)
      DATA HEADNG/.FALSE./,BLANKS/5H     /,XONE/1.0/
C
C     *** FIRST HALF OF MAIN PROGRAM ***
C
      IADR = IADDR(TITLE(1))
100   IF (HEADNG) GO TO 110
      HEADNG = .TRUE.
110   WRITE(4,600)
      CALL ACCEPT (4, K)
      IF (K) 170,240,120
120   DO 150 L = 1, K
      CALL ACCEPT (4, I, DEN)
      IF (I) 150, 150, 130
130   IF (I - 61) 140, 160, 150
140   EENPUT (I) = DEN
150   CONTINUE
      GO TO 110
160   READ (4,580) TITLE
      GO TO 110
```

Figure 21. The Main Quadruped Nonlinear Gait Program

```
170   IF (K + 2) 230, 200, 180
180   DO 190 I = 1, 57, 4
      CALL ACCEPT (5, BETA1, BETA2, BETA3, BETA4)
      EENPUT(I)   = BETA1
      EENPUT(I+1) = BETA2
      EENPUT(I+2) = BETA3
190   EENPUT(I+3) = BETA4
      TITLE1 = BLANKS
      TITLE2 = BLANKS
      TITLE3 = BLANKS
      TITLE4 = BLANKS
      GO TO 110
200   DO 210 I = 1, 60
210   EXECUT(I) = EENPUT(I)
      DO 220 I = 1, 3
      DO 220 L = 1, 4
      J = 4 * I + L + 8
      K = 3 * L + I + 9
220   EXECUT(J) = EENPUT(K)
      WRITE (6,570) TITLE, EXECUT
      GO TO 110
230   WRITE (7,550) (EENPUT(I), I = 1, 60)
      GO TO 110
240   WRITE (4, 610)
      CALL ACCEPT(4,XK10)
      DO 250 I = 1, 60
250   EXECUT(I) = EENPUT(I)
      DO 260 I = 1, 3
      DO 260 L = 1, 4
      J = 4 * I + L + 8
      K = 3 * L + I + 9
260   EXECUT(J) = EENPUT(K)
      WRITE(6,570) TITLE, EXECUT, BLANKS, TITLE
      IF (DTD) 270,290,280
270   WOF = .FALSE.
      DTD = -DTD
      GO TO 290
280   WOF = .TRUE.
290   CONTINUE
      G1 = PHI1 + BETA1 - 1.
      G2 = PHI2 + BETA2 - 1.
      G3 = PHI3 + BETA3 - 1.
      G4 = PHI4 + BETA4 - 1.
      TP = -0.00001
      TD = -0.00001
      TIME = 0.
      DOTX = STD / PER
      LOOP = .TRUE.
C
C     *** SUBROUTINE 'MODEL' ***
C
300   NT = INT(TIME / PER)
      FNT = FLOAT (NT)
      CT = (TIME - FNT * PER) / PER
      CC = (FNT + CT) * STD
      DO 390 I = 1,4
      IF (G(I)) 310, 310,320
```

Figure 21. Continued

```
310   IF (CT - PHI(I)) 330, 340, 340
320   IF (CT - PHI(I)) 360, 350, 350
330   M(I) = .FALSE.
      XNF = GAMMA(I) * STD
      GO TO 380
340   IF (CT - G(I) - 1.0) 350, 350, 330
350   M(I) = .TRUE.
      XNF = (GAMMA(I) + PHI(I) - CT) * STD
      GO TO 380
360   IF (CT - G(I)) 370, 370, 330
370   M(I) = .TRUE.
      XNF = (GAMMA(I) + PHI(I) - CT - 1.0) * STD
380   XF(I) = XNF + CG
      DEN = XNF - H1(I)
      AC(I) = ATAN(DEN/ZR)
      XLC(I) = SQRT(DEN*DEN + ZR*ZR)
      DAC(I) = -DOTX * ZR / (XLC(I)*XLC(I))
390   DLC(I) = -DOTX * DEN / XLC(I)
C
C     *** SUBROUTINE 'LEG' ***
C
      CTH = COS(X7)
      STH = SIN(X7)
      CPH = COS(X8)
      SPH = SIN(X8)
      CPS = COS(X9)
      SPS = SIN(X9)
      T111 = CTH * CPS
      T112 = CTH * SPS
      T113 = -STH
      T121 = CPS * STH * SPH - SPS * CPH
      T122 = CPS * CPH + SPS * SPH * STH
      T123 = CTH * SPH
      T131 = SPS * SPH + STH * CPS * CPH
      T132 = SPS * STH * CPH - CPS * SPH
      T133 = CPH * CTH
      T211 = 0.0
      T212 = CPH
      T213 = -SPH
      T221 = 1.0
      T222 = STH * SPH / CTH
      T223 = STH * CPH / CTH
      T231 = 0.0
      T232 = SPH / CTH
      T233 = CPH / CTH
      DX7  = T211 * X10 + T212 * X11 + T213 * X12
      DX8  = T221 * X10 + T222 * X11 + T223 * X12
      DX9  = T231 * X10 + T232 * X11 + T233 * X12
      DO 400 I = 1, 4
      XT1 = XF(I) - X1
      XT2 = YF(I) - X2
      XT3 = -X3
      XB1 = XT1 * T111 + XT2 * T112 + XT3 * T113
      XB2 = XT1 * T121 + XT2 * T122 + XT3 * T123
      XB3 = XT1 * T131 + XT2 * T132 + XT3 * T133
      D1 = XB1 - H1(I)
      D2 = XB2 - H2(I)
```

Figure 21. Continued

```
      D3 = XB3 - H3(I)
      XL(I) = SQRT(D1*D1 +D2*D2 + D3*D3)
      A(I) = ATAN(D1/D3)
      DEN = D2/XL(I)
      DEN = DEN/SQRT(1. - DEN*DEN)
      B(I) = -ATAN(DEN)
      DXL1 = (-DX7*STH*CPS-DX9*T112)*XT1 + (DX9*T111-DX7*STH*SPS)
     1*XT2 - DX7*CTH*XT3 - X4
      DXL2 = (DX7*T111*SPH+DX8*T131-DX9*T122)*XT1 + (DX7*
     1T112*SPH+DX8*T132+DX9*T121)*XT2 - (DX7*STH*SPH-DX8*T133)*
     2XT3 - X5
      DXL3 = (DX7*T111*CPH-DX8*T121-DX9*T132)*XT1 + (DX7*T112*CPH
      X6*T122+DX9*T131)*XT2 - (DX7*STH*CPH+DX8*T123)*XT3 - X6
      DL(I) = (DXL1 * D1 + DXL2 * D2 + DXL3 * D3)/ XL(I)
      DEN = D1*D1 + D3*D3
      DA(I) = (DXL1 * D3 - DXL3 * D1) / DEN
400   DB(I) = (DL(I) * D2 - XL(I) * DXL2) / (XL(I) * SQRT(DEN))
C
C     *** SUBROUTINE 'CONTRL' ***
C
      DO 420 I = 1, 4
      IF (M(I)) GO TO 410
      TM(I) = 0.
      TS(I) = 0.
      FZP(I) = 0.
      GO TO 420
410   TM(I) = XK1*(A(I)-AC(I)) + XK2*(DA(I)-DAC(I)) + XK7*X4
      TS(I) = XK3*B(I) + XK4*DB(I) + XK8*X2 + XK9*X5
      FZP(I) = XK5*(XL(I)-XLC(I)) + XK6*(DL(I)-DLC(I)) - XK10
420   CONTINUE
C
C     *** SUBROUTINE 'FANDM' ***
C
      DO 430 I = 1, 4
      CA = COS(A(I))
      SA = SIN(A(I))
      CB = COS(B(I))
      SB = SIN(B(I))
      TX(I) = TS(I) * CA
      TY(I) = TM(I)
      TZ(I) = -TS(I) * SA
      T3T11 = CA
      T3T12 = SA * SB
      T3T13 = CB * SA
      T3T21 = 0.
      T3T22 = CB
      T3T23 = -SB
      T3T31 = -SA
      T3T32 = SB * CA
      T3T33 = CB * CA
      FPX = TM(I) / (XL(I) * CB)
      FPY = -TS(I) / XL(I)
      FPZ = FZP(I)
      FX(I) = T3T11 * FPX + T3T12 * FPY + T3T13 * FPZ
      FY(I) = T3T21 * FPX + T3T22 * FPY + T3T23 * FPZ
      FZ(I) = T3T31 * FPX + T3T32 * FPY + T3T33 * FPZ
      XM(I) = H2(I) * FZ(I) - H3(I) * FY(I)
```

Figure 21. Continued

```
      YM(I) = H3(I) * FX(I) - H1(I) * FZ(I)
430   ZM(I) = H1(I) * FY(I) - H2(I) * FX(I)
      F1 = FX1 + FX2 + FX3 + FX4
      F2 = FY1 + FY2 + FY3 + FY4
      F3 = FZ1 + FZ2 + FZ3 + FZ4
      T1 = TX1 + TX2 + TX3 + TX4 + XM1 + XM2 + XM3 + XM4
      T2 = TY1 + TY2 + TY3 + TY4 + YM1 + YM2 + YM3 + YM4
      T3 = TZ1 + TZ2 + TZ3 + TZ4 + ZM1 + ZM2 + ZM3 + ZM4
C
C     *** SUBROUTINE 'XDOT' ***
C
      DX4 = X5 * X12 - X6 * X11 + F1 / XMAS - 32.2 * STH
      DX5 = X6 * X10 - X4 * X12 + F2 / XMAS + 32.2 * CTH * SPH
      DX6 = X4 * X11 - X5 * X10 + F3 / XMAS + 32.2 * CTH * CPH
      DX10 = ((XI2 - XI3) * X11 * X12 + T1) / XI1
      DX11 = ((XI3 - XI1) * X10 * X12 + T2) / XI2
      DX12 = ((XI1 - XI2) * X10 * X11 + T3) / XI3
      DX1 = T111 * X4 + T121 * X5 + T131 * X6
      DX2 = T112 * X4 + T122 * X5 + T132 * X6
      DX3 = T113 * X4 + T123 * X5 + T133 * X6
C
C     *** SECOND HALF OF MAIN PROGRAM ***
C
      IF (LOOP) GO TO 520
      LOOP = .TRUE.
      DO 440 I = 1, 12
440   X(I) = (XS(I) + DX(I) * DT + X(I)) / 2.
      IF (TIME - TD) 490, 450, 450
450   TD = TD + DTD
C
C     *** SUBROUTINE 'PICTUR' ***
C
      XMCG = X1 - CG
      DO 460 I = 1, 4
      XX = T111 * H1(I) + T121 * H2(I) + T131 * H3(I) + XMCG
      YY = T112 * H1(I) + T122 * H2(I) + T132 * H3(I) + X2
      ZZ = T113 * H1(I) + T123 * H2(I) + T133 * H3(I) + X3
      IHX(I) = INT(40.0 * XX - 32.0 * YY) + 265
      IHY(I) = INT(-12.0 * YY - 40.0 * ZZ) + 90
      IHY(I+4) = IHY(I) + INT(80.0 * H3(I))
      IF (.NOT. M(I)) GO TO 460
      YEF = X2 - YF(I)
      IFX(I) = INT((XF(I) - X1) * 40.0 + YEF * 32.0) + 265
      IFY(I) = INT(YEF * 12.0) + 90
460   CONTINUE
      CALL ERASE
      CALL POINT (0, 400, 0)
      CALL LINE (500, 400)
      CALL POINT (0, 222, 0)
      CALL LINE (64, 222)
      CALL LINE (0, 198)
      CALL LINE (64, 198)
      CALL POINT (447, 222, 0)
      CALL LINE (511, 222)
      CALL LINE (447, 198)
      CALL LINE (511, 198)
      CALL POINT (265, 200, 0)
```

Figure 21. Continued

```
      ICGX = INT(XMCG * 40.0 - X2 * 32.0) + 265
      ICGY = INT(-X2 * 12.0 - X3 * 40.0) + 90
      CALL POINT(ICGX,ICGY,1)
      CALL POINT(IHX1,IHY1,0)
      CALL LINE (IHX2, IHY2)
      CALL LINE (IHX4, IHY4)
      CALL LINE (IHX3, IHY3)
      CALL LINE (IHX1, IHY1)
      CALL LINE (IHX1,IHY5)
      CALL LINE (IHX2,IHY6)
      CALL LINE (IHX2,IHY2)
      CALL POINT(IHX2,IHY6,0)
      CALL LINE (IHX4,IHY8)
      CALL LINE (IHX4,IHY4)
      CALL POINT(IHX4,IHY8,0)
      CALL LINE (IHX3,IHY7)
      CALL LINE (IHX3,IHY3)
      CALL POINT(IHX3,IHY7,0)
      CALL LINE (IHX1,IHY5)
      DO 470 I = 1, 4
      IF (.NOT. M(I)) GO TO 470
      CALL POINT (IHX(I), IHY(I), 0)
      CALL LINE (IFX(I), IFY(I))
470   CONTINUE
      CALL POINT (155, 25, 0)
      CALL SYMBOL (IADR, 4)
      CALL POINT (0, 400, 0)
      IF (NOF) GO TO 490
C
C     *** TRIGGER ANIMATION CAMERA ***
C
      CALL ON
      DO 480 I = 1, 50
480   XX = XONE ** 2
      CALL OFF
490   CONTINUE
C
C     *** REMAINING PORTION OF SECOND HALF OF MAIN PROGRAM ***
C
      IF (TIME - TP) 510, 500, 500
500   TP = TP + DTP
      WRITE (6,560) TIME, F1, F2, F3, T1, T2, T3
      WRITE (6,590) X, TM, TS, F2P, A, B, XL
510   IF (TIME - TMAX) 300, 300, 100
520   LOOP = .FALSE.
      DO 530 I = 1, 12
      XS(I) = X(I)
530   X(I) = XS(I) + DX(I) * DT
      TIME = TIME + DT
      GO TO 300
540   READ (5,580)
      STOP
C
C     *** FORMATS ***
C
550   FORMAT (4(1X, G10.4))
560   FORMAT (/12(F9.3, 1X))
```

Figure 21. Continued

```
570    FORMAT(1H1,48X,4A5//
      118X,22HVALUE FOR FOOT NUMBER-.42X,16HINPUT PARAMETERS
      1//6X,1H1,14X,1H2,14X,1H3,14X,1H4,13X,4HNAME,4X,11HDESCRIPTION//
      21X,4(F12.5,3X),3X,4HBETA,5X,15HFOOT DUTY CYCLE//1X,4(F12.5,3X),3X,
      35HGAMMA,4X,30HX-COORD. OF FOOT AT TOUCH-DOWN//1X,4(F12.5,3X),3X,
      43HPHI,6X,44HPHASE OF FOOT ACTION - WHEN FOOT IS SET DOWN//
      51X,4(F12.5,3X),3X,6HH(1,-),3X,17HX-COORD. OF HIP )/89X,1H)/
      61X,4(F12.5,3X),3X,6HH(2,-),3X,32HY-COORD. OF HIP ) IN BODY COORD.
      7/89X,1H)/1X,4(F12.5,3X),3X,6HH(3,-),3X,17HZ-COORD. OF HIP )//
      81X,4(F12.5,3X),3X,2HYF,7X,31HY-COORD. OF FOOT IN BODY COORD.//
      91X,9HSTRIDE = ,F5.2,4H FT.,5X,9HPERIOD = ,F5.2,5H SEC.,5X,
      127HDESIRED Z-COORD. OF BODY = ,F5.2,4H FT.//1X,
      126HINTEGRATION TIME INTERVAL = ,F5.4,5H SEC.,5X,
      218HPERIOD OF STUDY = ,F5.2,5H SEC.///58X,18HINITIAL CONDITIONS/
      959X,1H.,2(17X,1H.)/
      31X,2HX=,F12.6,3H FT,1X,2HY=,F12.6,3H FT,1X,2HZ=,
      4F12.6,3H FT,1X,2HX=,F12.6,3HF/S,1X,2HY=,F12.6,3HF/S,
      51X,2HZ=,F12.6,3HF/S/58X,1H.,2(17X,1H.)/
      61X,2HO=,F12.6,4H RAD,1X,2HO=,F12.6,4H RAD,1X,2HU=,
      7F12.6,4H RAD,1X,2HO=,F12.6,3HR/S,1X,2HO=,F12.6,3HR/S,
      81X,2HU=,F12.6,3HR/S/1H+,1H-,18X,1HI,18X,1HI,18X,1HI,
      917X,1H-,17X,1HI
      9///47X,24HBODY INERTIAL PROPERTIES//
      21X,7HMASS = ,F7.2,10X,31HMOMENTS OF INERTIA -- X-AXIS = ,
      1F7.2,10X,9HY-AXIS = ,F7.2,10X,9HZ-AXIS = ,F7.2///
      250X,17HCONTROL CONSTANTS//6X,1H1,12X,1H2,12X,1H3,12X,1H4,
      312X,1H5,12X,1H6,12X,1H7,12X,1H8,12X,1H9/9(1X,F12.4)///
      450X,16HOUTPUT CONSTANTS//1X,17HPRINT INTERVAL = ,F7.3,
      55H SEC.,10X,19HDISPLAY INTERVAL = ,F7.3,36H SEC. (
      61F .LT. 0.0, W/ AUTO. PHOTO.)/1H1,A5,43X,4A5//40X,21HPRINTED
      7 OUTPUT FORMAT//2X,6H(TIME),6X,22H(FORCES ON BODY X-Y-Z),3X,
      823H(TORQUES ON BODY X-Y-Z)/26X,3(9X,1H.),30X,3(9X,1H.)/
      92(5X,1HX,9X,1HY,9X,1HZ,4X),2(5X,1HO,9X,1HO,9X,1HU,4X)/
      31H+,64X,1H-,3(9X,1HI),9X,1H-,9X,1HI//7X,26H(HIP MOTOR
      1 TORQUE 1-2-3-4),14X,27H(HIP SPRING TORQUE 1-2-3-4),12X,
      228H(VERTICAL LEG FORCE 1-2-3-4)//7X,27H(FORWARD HIP ANGLE 1-2-3-4)
      3,13X,27H(LATERAL HIP ANGLE 1-2-3-4),14X,20H(LEG LENGTH 1-2-3-4)//)
580    FORMAT (4A5)
590    FORMAT(/12(F8.4,1X))
600    FORMAT(19H READY FOR INPUT N=)
610    FORMAT(16H TTY INPUT XK10=)
       END
```

Figure 21. Continued

```
/       MACRO.
/       SUBROUTINE PIC - ROUTINE TO ESTABLISH THE PROPER LEVEL OF
/       INDIRECT ADDRESSING FOR CALLING THE SUBROUTINE SYMBOL AND
/       FOR TRIGGEREING THE ANIMATION CAMERA VIA THE A/D CLOCK F/F.
/
        .GLOBL  IADDR
        .GLOBL    ON,OFF
ADCLON=701624
ADCLOF=701644
IADDR   0
        LAC     IADDR
        TAD     (1
        DAC     X
        LAC*    X
        JMP*    IADDR
X       0
ON      0
        ADCLON
        JMP*  ON
OFF     0
        ADCLOF
        JMP*    OFF
        .END
```

Figure 22. Subroutine PIC - Macro Language Subroutine for Interphasing the CRT Display System to the PDP-9 Computer.

D(3) = position of a leg in body coordinates

DEN = intermediate computational variable

DXL(3) = derivative of foot position in body coordinates

SPS = sine ψ

XB(3) = position of a leg in body coordinates

XT(3) = position of a foot in ground coordinates

The input to the quadruped gait program is by means of paper tape and consists of the values of 70 parameters which specify the body dimensions, control constants, kinematic parameters of the particular gait, etc. Each gait has a separate paper tape, and in addition any parameter value can be changed if desired, and a new paper tape punched out.

Values of the various quantities can be printed on a line printer if desired, and a visual display of the locomotion system is produced on a cathode ray tube display system attached to the computer.

This quadruped gait program was also put on the faster PDP-10 computer using a different display package (subroutine DISPL). The faster computational time of this computer and its better display system produced displays of the quadruped gaits that approached simulation in real time.

A.3 Vibrational Analysis of the Quadruped Locomotion System

The computer program described in Section A.2 is the main nonlinear quadruped simulation for producing quadruped gaits. This program needs to be modified to produce a postural quadruped locomotion system. For vibrational analysis, the quadruped is assumed to stand on all its four feet with its hips vertically above each leg. Therefore, the only changes that are needed, are in the subroutine MODEL. That is, the reference model needs to be changed from one describing " marching " type of motion to a " postural " type.

Accordingly, subroutine MODEL in the program listing of the main quadruped gait program (see Section A,2) should be replaced by subroutine POSTUR given below:

```
C
C      *** SUBROUTINE POSTUR ***
C
       DO 300 I = 1, 4
       AC(I) = 0.0
       DAC(I) = 0.0
       XLC(I) = ZR
       DLC(I) = 0.0
       M(I) = .TRUE.
300    XF(I) = H1(I)
```

This converts the program to a quadruped postural system simulation. No other changes are necessary for obtaining the nonlinear system response to the different vibrational modes.

This quadruped postural system simulation was used to compute the nonlinear system response by exciting it with each of the twelve eigenvectors computed from the linearized quadruped locomotion system modal matrices using subroutine NSEVB. The nonlinear system response to an eigenvector describing one of the x axis vibrational modes is given in Table 6.

A.4 Linearized System Response of the Quadruped Locomotion System

Figure 23 is the main computer program used to determine the linearized system for the quadruped locomotion system. The expressions derived in Chapter IV for the linearized quadruped system (see equations (4-59) through (4-79)), are used to compute the eigenvalues and eigenvectors of each modal matrix. After this, the linear system response is computed from subroutine LINEAR. Figure 24 is a listing of subroutine LINEAR. Table 5 gives the linear system response for one of the x axis vibrational modes.

A.5 Inverted Pendulum System Computer Programs

This section describes the programs used for the simulation of the inverted pendulum system. Figure 25 is a listing of the program used to compute stabilizing control constants for the small motion stability of the inverted pendulum system with the mass pivoted a distance $r \neq 0$ below its center of gravity. This program uses the general algorithm derived in Section 6.6.2. The program allows the designer to select the required control constants to satisfy the Routh-Hurwitz criterion.

Figure 26 shows the listing used for the simulation of the inverted pendulum system. This program uses the nonlinear system equations described in Section 3.4.

For the PDP-9 computer, this program should be used with the Macro subroutine PIC for CRT display purposes. This program can be used with any digital computer if the necessary modifications are made to take into account the particular display package used. This inverted pendulum simulation was also programmed on the faster PDP-10 computer using the display package DISPL.

A.6 Quadruped Pace and Biped Walk Programs

For the simulation of the quadruped pace and also the type of biped walk considered in this research, the main quadruped gait program of Section A.2 was modified as follows.

```
C      FORTRAN PROGRAM.
C      PROGRAM TO FIND THE EIGENVALUES AND EIGENVECTORS OF THE REAL NON-
C      SYMMETRIC MATRICES DESCRIBING THE VARIOUS VIBRATIONAL MODES OF THE
C      QUADRUPED POSTURAL SYSTEM.  THIS PROGRAM ALSO COMPUTES THE "FREE
C      MOTION" OF THE LINEARIZED SYSTEM.
C      THE MAIN PROGRAM USES BODY DIMENSIONS AND CONTROL CONSTANTS AS
C      INPUTS AND COMPUTES THE MOMENTS OF INERTIA OF THE BODY ALONG THE
C      X, Y, Z AXES.  THE MODAL MATRICES ARE THEN COMPUTED.  MAIN THEN
C      CALLS SUBROUTINE NSEVB WHICH COMPUTES THE EIGENVALUES AND THE
C      CORRESPONDING EIGENVECTORS FOR EACH MODAL MATRIX.  FINALLY, THE
C      LINEAR SYSTEM RESPONSE IS COMPUTED FOR EACH EIGENVECTOR BY
C      SUBROUTINE LINEAR.
       DIMENSION A(20,20), ER(20), EI(20), VR(20,20), VI(20,20),
      1EBND(20),VBND(20,20),B(20,20)
       DOUBLE PRECISION RES, SR, SI
       REAL IXX,IYY,IZZ,MA,LO
       NMAX=20
       READ(5,10) MM
 10    FORMAT(I3)
       LCOUNT=0
 15    IF(LCOUNT.EQ.MM) GO TO 370
       READ(5,20) AA,BB,C,LO,MA
 20    FORMAT(8F10.4)
C      COMPUTE THE MOMENTS OF INERTIA FOR A RECTANGULAR PRISM WITH SIDES
C      2A, 2B, 2C.
       IXX=((MA*(BB*BB+C*C))/3.0)
       IYY=((MA*(AA*AA+C*C))/3.0)
       IZZ=((MA*(AA*AA+BB*BB))/3.0)
       WRITE(6,30) AA, BB, C, LO, MA, IXX, IYY, IZZ
 30    FORMAT (/17X,'DIMENSIONS OF BODY',//,' A=',F5.2,2X,'B=',F5.2,2X,'C
      1=',F5.2,2X,'LENGTH=',F5.2,2X,'MASS=',F5.2, //17X,'MOMENTS OF INERT
      2IA',//5X,'IXX=',F7.4,2X,'IYY=',F7.4,2X,'IZZ=',F7.4,/)
       READ(5,40) M
 40    FORMAT(I5)
       KOUNT=0
 45    IF(KOUNT.EQ.M) GO TO 360
       READ(5,50) CA,DCA,CB,DCB,CL,DCL,DCX,CY,DCY
 50    FORMAT(8F10.4)
       WRITE(6,60) CA,DCA,CB,DCB,CL,DCL,DCX,CY,DCY
 60    FORMAT(/17X,'CONTROL CONSTANTS',//,' CA=',F10.4,2X,'DCA=',F10.4,2X
      1,'CB=',F10.4,2X,'DCB=',F10.4,/,' CL=',F10.4,2X,'DCL=',F10.4,1X,'DC
      2X=',F10.4,3X,'CY=',F10.4,/,16X,'DCY=',F10.4,//)
       NCOUNT=1
       N=4
C      COMPUTE THE SYSTEM MATRIX FOR THE X - MODE.
       B(1,1)=0.0
       B(1,2)=1.0
       B(1,3)=0.0
```

Figure 23. Main Program for Computing the Modal Matrices, and the Linear System Response of the Quadruped Locomotion System.

```
      B(1,4)=0.0
      B(2,1) = (32.2/LO) - ((4.0*CA)/(MA*LO*LO))
      B(2,2) = -((4.0*DCA)/(MA*LO*LO)) + ((4.0*DCX)/(MA*LO))
      B(2,3) = ((32.2*C)/LO) - ((4.0*(LO+C)*CA)/(MA*LO*LO))
      B(2,4) = -((4.0*(LO+C)*DCA)/(MA*LO*LO))
      B(3,1)=0.0
      B(3,2)=0.0
      B(3,3)=0.0
      B(3,4)=1.0
      B(4,1) =((32.2*MA*C)/(IYY*LO)) - ((4.0*CA*(LO+C))/(IYY*LO*LO))
      B(4,2) = ((4.0*(LO+C)*DCX)/(IYY*LO)) - ((4.0*(LO+C)*DCA)/(IYY*LO*
     1LO))
      B(4,3) = -((4.0*(LO+C)*(LO+C)*CA)/(IYY*LO*LO)) + ((32.2*MA*C*(LO+C
     1))/(IYY*LO)) - ((4.0*AA*AA*CL)/IYY)
      B(4,4) = -((4.0*(LO+C)*(LO+C)*DCA)/(IYY*LO*LO)) - ((4.0*AA*AA*DCL)
     1/(IYY))
      WRITE(6,70)
 70   FORMAT(3X,'**X-AXIS TRANSLATIONAL & ROTATIONAL MODAL MATRIX**',//)
      GO TO 1000
900   NCOUNT=2
      N=4
C     COMPUTE THE SYSTEM MATRIX FOR THE Y - MODE.
      B(1,1)=0.0
      B(1,2)=1.0
      B(1,3)=0.0
      B(1,4)=0.0
      B(2,1) = (32.2/LO) - ((4.0*CB)/(MA*LO*LO)) - ((4.0*CY)/(MA*LO))
      B(2,2) = -((4.0*DCB)/(MA*LO*LO)) - ((4.0*DCY)/(MA*LO))
      B(2,3) =-((32.2*C)/LO) + ((4.0*(LO+C)*CB)/(MA*LO*LO))
      B(2,4) =  ((4.0*(LO+C)*DCB)/(MA*LO*LO))
      B(3,1)=0.0
      B(3,2)=0.0
      B(3,3)=0.0
      B(3,4)=1.0
      B(4,1)=-((32.2*MA*C)/(IXX*LO)) + ((4.0*CB*(LO+C))/(IXX*LO*LO)) + (
     1(4.0*(LO+C)*CY)/(IXX*LO))
      B(4,2) = ((4.0*(LO+C)*DCY)/(IXX*LO)) + ((4.0*(LO+C)*DCB)/(IXX*LO*
     1LO))
      B(4,3) = -((4.0*(LO+C)*(LO+C)*CB)/(IXX*LO*LO)) + ((32.2*MA*C*(LO+C
     1))/(IXX*LO)) - ((4.0*BB*BB*CL)/IXX)
      B(4,4) = -((4.0*(LO+C)*(LO+C)*DCB)/(IXX*LO*LO)) - ((4.0*BB*BB*DCL)
     1/(IXX))
      WRITE(6,30) AA, BB, C, LO, MA, IXX, IYY, IZZ
      WRITE(6,40) CA,DCA,CB,DCB,CL,DCL,DCX,CY,DCY
      WRITE(6,100)
100   FORMAT(3X,'**Y-AXIS TRANSLATIONAL & ROTATIONAL MODAL MATRIX**',//)
      GO TO 1000
800   NCOUNT=3
```

Figure 23. Continued

```
      N=2
C     COMPUTE THE SYSTEM MATRIX FOR THE TRANSLATIONAL Z - AXIS MODE.
      B(1,1)=0.0
      B(1,2)=1.0
      B(2,1)=-(4.0*CL)/MA
      B(2,2)=-(4.0*DCL)/MA
      WRITE(6,30) AA, BB, C, LO, MA, IXX, IYY, IZZ
      WRITE(6,60) CA,DCA,CB,DCB,CL,DCL,DCX,CY,DCY
      WRITE(6,130)
130   FORMAT(8X,'**Z-AXIS TRANSLATIONAL MODAL MATRIX**',//)
      DO 135 I = 1,N
135   WRITE(6,210) (B(I,J),J=1,N)
      GO TO 220
700   NCOUNT=4
      N=2
C     COMPUTE THE ROTATIONAL Z - AXIS MODE.
      B(1,1)=0.0
      B(1,2)=1.0
      B(2,1) = ((32.2*MA*(AA*AA+BB*BB))/(IZZ*LO)) - ((6.0*(AA*AA*CB+BB*
     1BB*CA))/(IZZ*LO*LO))
      B(2,2)=-4.0*(BB*BB*DCA+AA*AA*DCB)/(IZZ*LO*LO)
      WRITE(6,30) AA, BB, C, LO, MA, IXX, IYY, IZZ
      WRITE(6,60) CA,DCA,CB,DCB,CL,DCL,DCX,CY,DCY
      WRITE(6,170)
170   FORMAT(8X,'**Z-AXIS ROTATIONAL MODAL MATRIX**',//)
      DO 175 I = 1,N
175   WRITE(6,210) (B(I,J),J=1,N)
      GO TO 220
1000  CONTINUE
      DO 190 I = 1, N
190   WRITE(6,200) (B(I,J),J=1,N)
200   FORMAT('0',4F13.6)
210   FORMAT('0',13X,2F13.6)
220   DO 230 I =1, N
      DO 230 J =1, N
230   A(I,J) = B(I,J)
      BIG = 0.0
      DO 240 I =1, N
      DO 240 J =1, N
240   BIG = AMAX1(BIG,ABS(A(I,J)))
      CALL NSEVB(A,NMAX,N,ER,EI,VR,VI,EBND,VBND)
      WRITE(6,250)
250   FORMAT(1H0,16X,'COMPUTED EIGENVALUES',//)
      WRITE(6,260)
260   FORMAT(1X,'NO.',3X,'REAL PART',7X,'IMAG. PART',6X,'ERROR BOUND',/)
      DO 270 I =1, N
270   WRITE(6,280) I, ER(I), EI(I), EBND(I)
280   FORMAT(1X,I2,3(1X,F15.9),/)
```

Figure 23. Continued

```
      DO 310 J =1, N
      WRITE(6,290) J
290   FORMAT(/,13X,'COMPUTED EIGENVECTOR NO.=',I1,/)
      WRITE(6,300)
300   FORMAT(13X,'REAL PART',7X,'IMAG. PART',6X,'ERROR BOUND',/)
      DO 310 I =1, N
310   WRITE(6,320) VR(I,J),VI(I,J),VBND(I,J)
320   FORMAT(1X,3(1X,F15.9),/)
      RES=0.0D0
      DO 340 I= 1, N
      DO 340 J= 1, N
      SR=-(VR(I,J)*ER(J)-VI(I,J)*EI(J))
      SI=-(VR(I,J)*EI(J)+ER(J)*VI(I,J))
      DO 330 K =1, N
      SR=SR+A(I,K)*VR(K,J)
330   SI = SI + A(I,K) * VI(K,J)
340   RES = DMAX1(RES,DSQRT(SR**2 + SI**2))
      RES=RES/BIG
      WRITE(6,350) RES
350   FORMAT(//,1X,'LARGEST RESIDUAL=',E15.9,//)
      CALL LINEAR(N,ER,EI,VR,VI)
      GO TO (900,800,700), NCOUNT
      KOUNT=KOUNT+1
      GO TO 45
360   CONTINUE
      LCOUNT=LCOUNT+1
      GO TO 15
370   STOP
      END
```

Figure 23. Continued

```
      SUBROUTINE LINEAR(N,ER,EI,VR,VI)
      DIMENSION ER(20),EI(20),VR(20,20),VI(20,20),X(200,4),Y(200,4),
     1T(110),D(20,20),E(20,20)
      DO 30 J=1,N
      DO 30 K=1,N
      D(J,K)=0.1*VR(J,K)
30    E(J,K)=0.1*VI(J,K)
      DO 101 K=1,N
      WRITE(6,49)
      FORMAT(1X,'EIGENVALUE')
      WRITE(6,50) (ER(K),EI(K))
50    FORMAT(/,1X,'ALPHA=',F12.7,3X,'OMEGA=',F12.7,/)
      WRITE(6,52) K
52    FORMAT(/,16X,'0.1*COMPUTED EIGENVECTOR NO.=',I1,//,18X,'REAL PART'
     1,7X,'IMAG. PART',/)
      WRITE(6,51) (D(L,K),E(L,K),L=1,N)
51    FORMAT(15X,E15.9,2X,E15.9,/)
      WRITE(6,31)
31    FORMAT(/,22X,'LINEAR SYSTEM RESPONSE',/)
      WRITE(6,35)
35    FORMAT(31X,'.',/,1X,'TIME',11X,'X',14X,'X',14X,'0',14X,'0',
     1/1H+,45X,'-',14X,'-',//)
      DO 100 I=1,101
      DO 100 J=1,N
      T(I) = (I-1)/10.0
      DEN=ER(K)*T(I)
      IF(DEN.GT.100.0) GO TO 101
      Q1=EXP(DEN)
      Q2=COS(EI(K)*T(I))
      Q3=SIN(EI(K)*T(I))
      A=Q1*Q2
      B=Q1*Q3
      X(I,J)=A*D(J,K)-B*E(J,K)
100   Y(I,J)=B*D(J,K)+A*E(J,K)
      WRITE(6,104) (T(M),X(M,1),X(M,2),X(M,3),X(M,4),M=1,101)
104   FORMAT(F7.4,4F15.9)
101   CONTINUE
      RETURN
      END
```

Figure 24. Subroutine LINEAR - Fortran Subroutine for Computing the Linear System Response for each Eigenvector of a Modal Matrix of the Quadruped Postural System.

```
C       FORTRAN
C       PROGRAM TO FIND STABILIZING CONTROL CONSTANTS FOR AN
C       INVERTED PENDULUM SYSTEM WITH THE MASS PIVOTED BELOW THE
C       THE BODY, AND HAVING ONE MASSLESS LEG OF CONSTANT LENGTH,
C       SUPPORTED BY A " FIXED " FOOT.
C       XM = MASS OF THE BODY
C       XL = LEG LENGTH
C       XR = DISTANCE OF THE BODY CENTER OF GRAVITY ABOVE THE PIVOT
C       XI = MOMENT OF INERTIA OF THE BODY
C       XK1 THROUGH XK4 = CONTROL CONSTANTS
C       R0 THROUGH R6 = ROUTH COEFFICIENTS
C       X1 = ANGLE OF LEG TO THE VERTICAL
C       X2 = ANGLE OF BODY (LONG SIDE) TO THE HORIZONTAL
C       GAMMA = K4 / K2
10      WRITE(5, 20)
20      FORMAT(1X,21H TTY INPUT M, L, R, I )
        CALL ACCEPT(4, XM, XL, XR, XI)
        WRITE(4,30) XM, XL, XR, XI
30      FORMAT(6X,1HM,7X,1HL,7X,1HR,7X,1HI/4X,4F7.3/)
        XA = XL * (XL + XR)
        XB = (XI/XM) + XR * XR + XL * XR
        XC = XA/XB
        IF (XR.EQ.0.0) GO TO 50
        XD = XL/XR
        WRITE(4,40) XD
40      FORMAT(1X,4HL/R=/)
50      CONTINUE
        WRITE(4,60)
60      FORMAT(1X,5HA/B =,F7.3/)
        WRITE(4,70)
70      FORMAT(62H CHOOSE GAMMA SO THAT IT IS GREATER THAN A/B
       1BUT LESS THAN L/R/)
        WRITE(4,80)
80      FORMAT(22H CHOOSE GAMMA ON TTY )
        CALL ACCEPT(4, GAMMA)
        WRITE(4,90)
90      FORMAT(34HTORGUE=XK1*X2+XK2*X4+XK3*X1+XK4*X3 )
100     WRITE(4,110)
110     FORMAT(21H CHOOSE K2, K3 ON TTY/)
        CALL ACCEPT(4, XK2, XK3)
        XK4 = GAMMA * XK2
        XE = (XK3/XC) - ((32.2*XM*XL*XL)/XC) - ((32.2*XI*XL*XL*
       1(XL-XR*GAMMA))/(XB*(XA*GAMMA-XB)))
        XF = (XR*XK4/XL) - 32.2 * XM * XR
120     WRITE(4,130)
130     FORMAT(1X,3HK2=,F10.4,4H K3=,F10.4,4H K4=,F10.4,
       14H XE=,F10.4,4H XF=,F10.4/)
        WRITE(4,140)
140     FORMAT(44H CHOOSE K1 LESS THAN XE, BUT GREATER THAN XF/)
        CALL ACCEPT(4, XK1)
        R0 = XI * XL * XL
        R1 = XA * XK4 - XB * XK2
        R2 = XA * (XK3 - XM * XL *32.2) - XB * XK1
        R3 = 32.2 * (XK2 * XL - XK4 * XR)
        R4 = 32.2 * (32.2 * XM * XR * XL + XK1 * XL - XK3 * XR)
        R5 = ((R1 * R2 - R0 * R3)/R1)
```

Figure 25. Program for Computing Stabilizing Control Constants for the Small Motion Stability of the Inverted Pendulum System.

```
      R6 = ((R3 * R5 - R1 * R4)/R5)
      WRITE(5,150) R0, R1, R2, R3, R4
150   FORMAT(1X,3HR0=,F10.4,4X,4HR1 =,F10.4,4X,4HR2 =,F10.4/6X,
     14HR3 =,F10.4,5X,4HR4 =,F10.4/)
      WRITE(4,160) R5, R6
160   FORMAT(1X,4HR5 =,F10.3,5X,4HR6 =,F10.3/)
      WRITE(4,170) XK1, XK2, XK3, XK4
170   FORMAT(1X,4H K1=,F10.4,5H K2 =,F10.4,5H K3 =,F10.4,5H K4 =,F10.4/)
      WRITE(4,180)
180   FORMAT(41H TYPE -1,0,1 TO CHANGE K1,(K2,K3),RESTART/)
      IF (K) 140,110,10
      STOP
      END
```

Figure 25. Continued

```
C      FORTRAN
C      ROUTINE TO STABILIZE A SELF-TORQUING INVERTRD PENDULUM
C      WITH THE MASS PIVOTED BELOW THE BODY, AND HAVING ONE MASSLESS
C      LEG OF CONSTANT LENGTH, WITH A " FIXED " FOOT.
C      THIS PROGRAM USES THE EULER-PREDICTOR CORRECTOR METHOD OF
C      INTEGRATION.
C      THIS PROGRAM CAN BE USED WITH ANY DIGITAL COMPUTER PROVIDED
C      THE PROPER MODIFICATIONS ARE MADE TO SUBROUTINE PICTUR CORRES-
C      PONDING TO THE PARTICULAR DIPLAY PACKAGE USED.
C      XM = MASS OF THE BODY
C      XL = LENGTH OF THE LEG
C      XR = DISTANCE OF THE CENTER OF GRAVITY ABOVE THE PIVOT
C      XI = MOMENT OF INERTIA OF THE BODY
C      XK1, XK2, XK3, XK4 = CONTROL CONSTANTS
C      X1 = ANGLE OF LEG TO THE VERTICAL
C      X2 = ANGLE OF THE BODY(LONG SIDE) TO THE HORIZONTAL
C      IX1,IY1 = FIXED DISPLAY COORDINATES OF THE FOOT
C      DIAG = 1/2 OF BODY DIAGONAL (FT) * SCALE FACTOR =1/2*FT*40.0
C      THETA = ANGLE OF BODY DIAGONAL TO BODY LONG SIDE
       LOGICAL NOCRT, LOOP, HEADNG
       DIMENSION XS(4), X(4), DX(4), IX(8), IY(8), TITLE(4), A(6),
      1 C(3), XK(4)
       COMMON TORK,A1,A2,A3,A4,A5,A6,C1,C2,C3,DX1,DX2,DX3,DX4
       COMMON/INPUT/DIAG,THETA,XM,XI,XL,XR,CT,PT,TEND,CRTINC,
      1XK1,XK2,XK3,XK4,X1,X2,X3,X4,TITLE1,TITLE2,TITLE3,TITLE4
       EQUIVALENCE (X(1),X1),(DX(1),DX1),(TITLE(1),TITLE1)
       EQUIVALENCE (IX(1),IX1),(IY(1),IY1),(XK(1),XK1),(A(1),A1)
      1,(C(1),C1)
       DATA HEADNG/.FALSE./,IX1,IY1/255,50/,XONE/1.0/
       IADR = IADR(TITLE(1))
100    IF(HEADNG) GO TO 110
       HEADNG = .TRUE.
110    WRITE(4,120)
120    FORMAT(1X,24H TTY INPUT DIAG., THETA=)
       CALL ACCEPT (4,DIAG,THETA)
       WRITE(4,130)
130    FORMAT(28H TTY INPUT K=1 FOR NEW TITLE )
       CALL ACCEPT(4,K)
       IF(K.EQ.0) GO TO 150
       READ(4,140) TITLE
140    FORMAT(4A5)
150    CONTINUE
       WRITE(4,160)
160    FORMAT(1X,77HMASS, INERTIA,LEG LENGTH,LENGTH XR, CALCINC,
      1 PRINTINC, ENDTIME, CRTINC/)
       CALL ACCEPT(4, XM, XI, XL, XR, CT, PT, TEND, CRTINC)
       WRITE(6,170) XM, XI, XL, XR, CT, PT, TEND, CRTINC
170    FORMAT(1X,F4.1,2X,F5.1,4X,F5.1,6X,F5.1,5X,F5.1,4X,F7.4,3X,F5.1,
      1 4X,F6.3/)
       IF (CRTINC) 180, 190, 190
180    CRTINC = - CRTINC
       NOCRT = .FALSE.
       GO TO 200
190    NOCRT = .TRUE.
200    WRITE(4,210)
210    FORMAT(1X,17HCONTROL CONSTANTS/)
```

Figure 26. Main Program for the Nonlinear Inverted Pendulum System Simulation.

```
      CALL ACCEPT(4,XK1,XK2,XK3,XK4)
      WRITE(6,220) XK1, XK2, XK3, XK4
220   FORMAT(1X,4(1X,F10.3)/)
230   WRITE(4,240)
240   FORMAT(1X,20HINITIAL STATE VECTOR/)
      CALL ACCEPT (4, X1, X2, X3, X4)
      WRITE(6,250) X1, X2, X3, X4
250   FORMAT(1X,4F10.4/)
      WRITE(6,260)
260   FORMAT(2X,4HTIME,13X,12HSTATE VECTOR,18X,6HTORQUE/15X,
     12HX1,7X,2HX2,8X,2HX3,8X,2HX4/)
C
C  ***FIRST HALF OF THE MAIN PROGRAM***
C
      TIME = 0.0
      PTIME = -0.00001
      TD = -0.20001
      LOOP = .TRUE.
270   TORK = XK1*X2 + XK2*X4 + XK3*X1 + XK4*X3
      A1 = XM * XL * XL
      A2 = XM * XL * XR * COS(X1 -X2)
      A3 = XM * XL * XR * SIN(X1 -X2)
      A4 = 32.2 * XM * XL * SIN(X1)
      A5 = 32.2 * XM * XR * SIN(X2)
      A6 = (XI + XM * XR * XR)
      C1 = (A4 - A3 * X4 * X4 - TORK)
      C2 = (A5 + A3 * X3 * X3 + TORK)
      C3 = (A1 * A6 - A2 * A2)
      DX1 = X3
      DX2 = X4
      DX3 = ((A6 * C1 - A2 * C2)/C3)
      DX4 = ((A1 * C2 - A2 * C1)/C3)
      IF (LOOP) GO TO 370
      LOOP = .TRUE.
      DO 280 I = 1, 4
289   X(I) = (X(I) + DX(I) * CT + XS(I))/2.0
      IF(TIME - TD) 320, 290, 290
290   TD = TD + CRTINC
C
C  ***SUBROUTINE PICTURE***
C
      XX = X2 + THETA
       Y = X2 - THETA
      IAL = DIAG * COS(XX)
      IBL = DIAG * COS(Y)
      ICL = DIAG * SIN(XX)
      IDL = DIAG * SIN(Y)
      IX2 = INT(-XL * SIN(X1) * 40.0) + IX1
      IY2 = INT( XL * COS(X1) * 40.0) + IY1
      IX3 = INT(-XR * SIN(X2) * 40.0) + IX2
      IY3 = INT( XR * COS(X2) * 40.0) + IY2
      IX4 = IAL + IX3
      IY4 = ICL + IY3
      IX5 = IBL + IX3
      IY5 = IDL + IY3
      IX6 = -IAL + IX3
      IY6 = -ICL + IY3
```

Figure 26. Continued

```
      IX7 = -IBL + IX3
      IY7 = -IDL + IY6
      CALL ERASE
      CALL POINT(0, 400, 0)
      CALL LINE(200, 400)
      CALL POINT(IX1, IY1, 0)
      CALL LINE(IX2, IY2)
      CALL LINE(IX3, IY3)
      CALL POINT(IX4, IY4, 0)
      CALL LINE(IX5, IY5)
      CALL LINE(IX6, IY6)
      CALL LINE(IX7, IY7)
      CALL LINE(IX4, IY4)
      CALL POINT(155, 25, 0)
      CALL SYMBOL(IADR, 4)
      CALL POINT(0, 400, 0)
      IF (NOCMT) GO TO 320
C
C   ***TRIGGER ANIMATION CAMERA***
C
      DO 295 I = 1, 50
295   XQ = XONE ** 2
      CALL ON
      DO 300 I = 1, 70
300   XQ = XONE ** 2
      CALL OFF
      DO 310 I = 1, 100
310   XQ = XONE ** 2
320   CONTINUE
C
C   *** SECOND HALF OF THE MAIN PROGRAM***
C
330   IF (TIME - PTIME) 360, 340, 340
340   PTIME = PTIME + FT
      WRITE(6,350) TIME, X1, X2, X3, X4, TORK
350   FORMAT(1X,F5.2,3X,4F10.6,3X,F10.3)
360   IF (TIME - TEND) 270, 270, 390
370   LOOP = .FALSE.
      DO 380 I = 1, 4
      XS(I) = X(I)
380   X(I) = XS(I) + DX(I) * CT
      TIME = TIME + CT
      GO TO 270
390   WRITE(4,400)
400   FORMAT(1X,45HDO YOU WISH TO CHANGE ANY PHYSICAL CONSTANTS? )
      CALL ACCEPT(4, XA)
      IF (XA .EQ. 1.0) GO TO 100
      WRITE(4,410)
410   FORMAT(1X,44HDO YOU WISH TO CHANGE ANY CONTROL CONSTANTS? )
      CALL ACCEPT(4, XB)
      IF (XB .EQ. 1.0) GO TO 200
      GO TO 230
      STOP
      END
```

Figure 26. Continued

The expression for the lateral control torque TS(I) in subroutine CONTRL corresponding to equation (6-111) was replaced by the expression

TS(I) = XK3 * X8 + XK4 * B(I) + XK8 * (X8+B(I)) + XK9 * (X10+DB(I))

corresponding to equation (6-114).

In addition, the subroutine PICTUR was appropriately modified to display the quadruped pace and the biped walk as shown in Figures 19 and 20 respectively. The values of the control constants K_3, K_4, K_8, and K_9 were obtained by taking one half of the values of the stabilizing control constants K_1 through K_4 obtained by applying the inverted pendulum Routh-Hurwitz analysis algorithm to the body in its lateral (y-z) plane.

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