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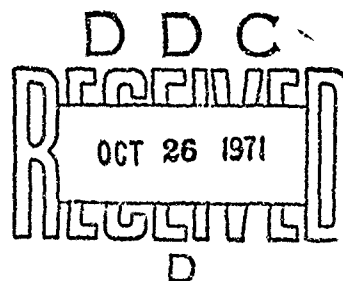
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FOIL BEARING DESIGN MANUAL

Prepared By

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L. Licht



**Ampex Corporation**  
**Research and Advanced Technology Division**

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FOIL BEARING DESIGN MANUAL

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ABSTRACT

A compendium of theoretical and experimental results, available at the present in the field of foil bearings, is presented. Design formulae and brief explanations of concepts are given for self-acting and externally pressurized foil-bearings, with reference to applications in the transport of flexible media and the support of rotating machinery.

PREFACE

Considerable effort has been expended in the past few years on the study of foil bearings. The purpose of this manual is to extract from the numerous papers and reports the information which is relevant to foil bearing design and to organize it in a form which may be easily accessed and updated.

The terminology and basic physical facts are explained in the Introduction. The user of this manual should acquaint himself with the contents of the Introduction, so that he may easily refer to the particular information required.

To make each chapter self contained, each chapter is preceded by nomenclature and followed by a list of references particular to the chapter. References are designated by numbers in brackets.

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NOMENCLATURE  
(Chapter 1)

b	Half-width of foil
e	Eccentricity of rigid-surface bearing
h	Fluid film thickness
$h_{\min}$	Minimum film thickness
$h^*$	Nominal foil-bearing film-thickness tension per unit width of foil
T	Tension per unit width of foil
W	Bearing load
$\delta$	Width of non-uniform edge zone of foil bearing
$\Theta$	Wrap angle



B. A. N.



## 1.0 INTRODUCTION

### 1.1 Definitions and Classification

Flexible webs of paper, plastic and metal under tension, transported over cylindrical rollers and guides, can be supported on thin films of air or liquid. Conversely, a high-speed rotor can be floated on a fluid film entrained between the journal and the surface of a stretched flexible foil. The term foil bearing applies to both types of support [1.1, 1.2].

A foil bearing therefore, consists of two surfaces, one sensibly rigid and the other relatively thin and flexible, separated by a fluid film. The foil bearing is capable of supporting a load by virtue of pressure induced in the lubricating film through the relative motion of surfaces, or by means of an external pressure supply. In analogy with rigid-surface, fluid-film bearings, we thus distinguish between self-acting and externally-pressurized foil bearings.

### 1.2 Applications

In the manufacture and processing of foil, plastic film and paper, the foil bearing is present, though perhaps unrecognized, at the multiplicity of rolls, drums and guides. Its function and purpose may involve minimization of friction, elimination of roll inertia and protection of surface finish and fresh coatings.

In magnetic recording, instead of maximizing the web-to-guide separation, the problem is frequently that of reducing, or of completely eliminating the air gap between the tape and the recording head.

The trend toward high-speed, high-temperature turbomachines dictates the use of unconventional lubricants and bearings, such as gas-lubricated bearings. Here, the foil bearing presents itself as one of the few, feasible alternatives, because of its attractive dynamic, thermal and wear characteristics.

### 1.3 Foil Bearings versus Rigid Surface Bearings

The most important characteristics of foil bearings and the essential differences in the behavior of foil bearings and rigid-surface bearings are a consequence of the effect of flexibility on the shape of the lubricating film. A qualitative comparison of film shapes in a rigid, partial-arc journal bearing and in a foil bearing is made in Fig. 1.2.1.

In the rigid bearing, Fig. 1.2.1a, the lubricating film consists of a converging region (1), in which the pressure is above ambient, and a diverging region (2), in the major part of which the pressure is subambient.\* In the foil bearing, the lubricating film consists of a converging entrance region (1), in which the pressure rises above ambient, an exit region (2), in which the foil diverges from the journal following a series of undulations [1.2], and a central region\*\* (3), in which both the clearance  $h^*$  and the pressure  $p$  remain virtually constant.

It will be noted that in addition to the entrance and exit regions, additional transition zones exist along the lateral edges of the foil. The latter are a consequence of the combined effect of rapid decrease of pressure

---

\*In liquid-lubricated bearings, subambient pressures cause vaporization and induce air bubbles, so that the film breaks up. In gas-lubricated bearings, subambient pressures are important in the determination of bearing load capacity.

\*\*The central region is also termed the uniformity zone.

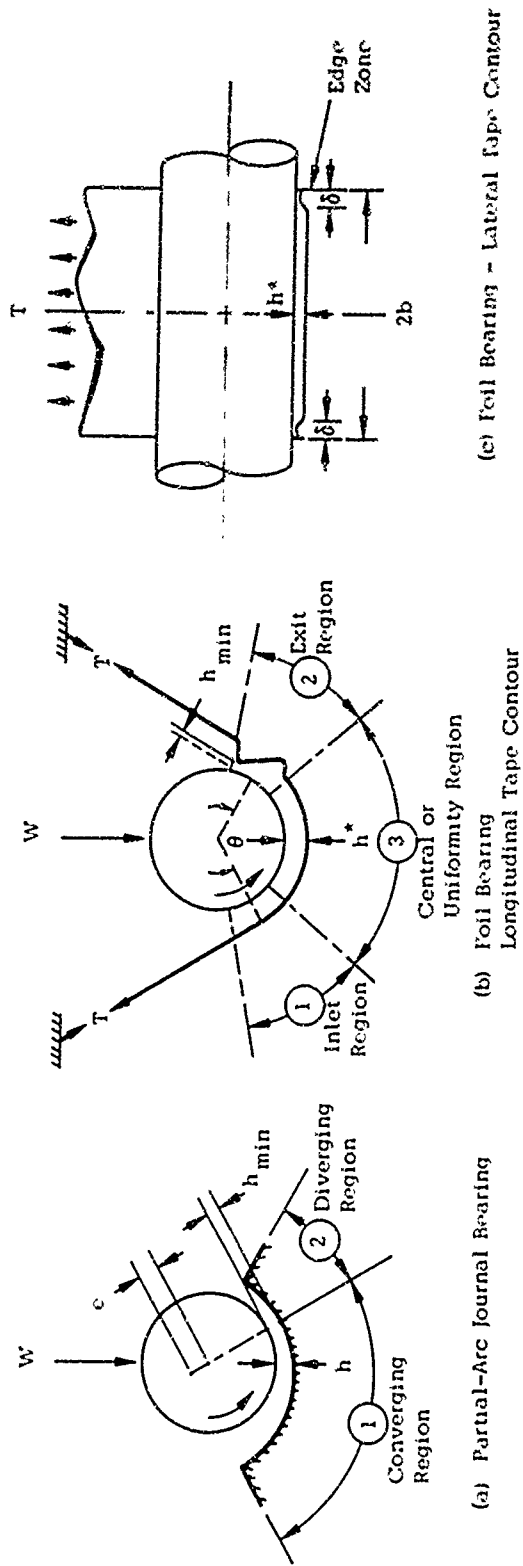


Fig. 1.1.2.1 Schematic Illustration of Clearance Distribution in a Rigid-Surface Bearing and in a Foil Bearing

along the edges and of the anticlastic edge-undulation associated with the bending of thin plates into cylindrical shapes [1.3]. The net result is the tendency of the foil to close the gap along the edges and to inhibit side leakage. Due to this phenomenon, it is possible to predict the performance of foil bearings quite accurately from formulae based on two dimensional (planar) flow.

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- 1.1 H. Blok and J.J. Van Rossum, "The Foil Bearing - A New Departure in Hydrodynamic Lubrication," Lub Eng., Vol. 9, No. 6, Dec. 1953, 316-320.
- 1.2 W. A. Gross, Gas Lubrication, John Wiley and Sons, New York, 1962, 138-141.
- 1.3 Y. C. Fung and W. H. Wittrick "A Boundary Layer Phenomenon in the Large Deflection of Thin Plates". Quart. J. Mech. and App. Math. Vol. 8, p. 191, 1955.

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NOMENCLATURE

(Chapter 2)

A ( $A, \omega$ )	Amplitude factor ( Eq. 2.3.2)
b	Foil half width
C	Compressibility parameter $p_a r_o / T$
D	Bending rigidity of foil per unit width; for foil of homogeneous cross section $D = Et^3 / [12(1-\nu^2)]$
E	Young modulus
f	Frequency
h	Local fluid film thickness
h*	Clearance in uniformity region. Film thickness at point of maximum pressure.
$h_{\mathcal{C}}$	Film thickness along centerline
$h_s$	Steady-state film thickness
H	Dimensionless clearance $H = h / (r_o \epsilon^{2/3})$
H*	Dimensionless clearance in uniformity region $H^* = h^* / (r_o \epsilon^{2/3})$
k	Bearing stiffness per unit width
$l_o$	Initial foil length (at tension $T_o$ )
I	Inertia parameter $1/2 \rho_a U^2 / (T/r_o)$
p	Local film pressure, absolute
$p_a$	Atmospheric pressure

$P_u$	Speed parameter	$P_u = \frac{U}{\sqrt{p_a/\rho_a}}$
$P_r$	Radius parameter	$P_r = \frac{r_o}{\mu/\sqrt{\rho_a p_a}}$
$P_l$	Length parameter	$P_l = \frac{r_o}{l} \frac{Et}{\sqrt{\mu^2 p_a/\rho_a}}$
$P_T$	Tension parameter	$P_T = \frac{T}{\sqrt{\mu^2 p_a/\rho_a}}$
$P_c$	Dimensionless damping per unit width	$P_c = c/\mu$
$P_h$	Dimensionless gap	$P_h = \frac{h^*}{\sqrt{\mu^2/p_a \rho_a}}$
$P_f$	Dimensionless frequency	$P_f = \mu f/p_a$
$P_k$	Dimensionless stiffness per unit width	$P_k = k/p_a$
$P_\omega$	Frequency ratio	$P_\omega = \omega r_o/U$
$S$	Stiffness Parameter, $S =$	$\frac{D}{Tr_o^2 \epsilon^{2/3}}$
$t$	Foil thickness (unless otherwise specified), also time	
$T$	Tension per unit width of foil	

$T_s$	Steady state tension per unit width
$T_o$	Preload (initial) tension per unit width
$r_l$	Radius of curvature (Fig. 2.15)
$r_o$	Radius of foil bearing journal (guide)
$U$	Algebraic sum of journal and foil surface velocities
$V_p$	Speed of propagation of disturbance
$x$	Lateral distance from edge of foil
$x_l$	Lateral distance from foil centerline
$y$	Component of displacement of rotor center
$\beta$	Edge effect number $\beta = (4D/r_o^2 \delta)^{1/4}$
$\gamma(\theta, \omega)$	Phase between tension and gap perturbation (Eq. 2.3.2)
$\delta$	Extensional rigidity of foil. For foil of homogeneous cross section $\delta = Et$
$\Delta \alpha$	Corner angle at inlet of magnetic head (Fig. 2.15)
$\Delta \theta$	Peak to trough angular distance of last exit undulation
$\Delta \xi$	Peak to trough normalized angular distance $\Delta \xi = \frac{\Delta \theta}{\epsilon^{2/3}}$
$\delta l$	Increment in foil length due to reasons other than stretching.
$\epsilon$	Foil bearing number $\epsilon = 6 \mu U/T$
$\zeta$	Dimensionless lateral distance from edge of foil $\zeta = \frac{x}{r_o \beta}$
$\Theta$	Wrap Angle
$\theta$	Angular position

- $\lambda$  Molecular mean free path
- $\Lambda$  Dimensionless molecular mean free path  $= \frac{\lambda}{r_o \epsilon^{2/3}}$
- $\mu$  Viscosity of lubricant
- $\nu$  Poisson's ratio
- $\xi$  Normalized angular position  $\xi = \frac{\theta}{\epsilon^{1/3}}$
- $\rho_a$  Density of ambient fluid
- $\tau$  Dimensionless time  $\tau = \frac{U t}{2 r_o \epsilon^{-1/3}}$  (t = time)
- $\phi_o$  Dimensionless amplitude of tension perturbation.
- $\omega$  circular frequency of excitation

## 2.0 SELF ACTING FOIL BEARINGS

### 2.1 Steady State Characteristics

#### 2.1.1 Determination of Steady State Gap in the Uniformity Region

The gap width in the region of uniformity (Fig. 1.2.1b) is given by the expression

$$h^* = H^* r_o \epsilon^{2/3} \quad (2.1.1)$$

in which the foil bearing number  $\epsilon = 6 \mu U/T$  must be small, at most the order of  $10^{-4}$ . The constant  $H^* = 0.643 \pm 3\%$ , provided the following reference conditions [2.1-2.9] are satisfied:

Stiffness	$S = \frac{D}{T r_o \epsilon^{2/3}} < 0.8$	(2.1.2)
-----------	--	---------

Compressibility	$\frac{1}{C} = \frac{T}{p_a r_o} < 0.12$	(2.1.3)
-----------------	--	---------

Inertia	$I = \frac{1/2 \rho_a U^2}{T/r_o} < 0.05$	(2.1.4)
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Wrap Angle	$\frac{\Theta}{\epsilon^{1/3}} > 6$	(2.1.5)
------------	-------------------------------------	---------

\*In this equation,  $U = |U_f + U_r|$  or  $U = |U_f - U_r|$ , according to whether the surface velocities of the foil and of the roll are in the same (sum) or in opposite (difference) directions.

Mean free path  $\frac{\lambda}{r_o \epsilon^{2/3}} < 0.005$  (2.1.6)

Foil Width  $\sqrt{tr_o} \ll 2b$  (2.1.7)  
 (where t represents foil thickness)

Continuity of slope  $\Delta \alpha = 0$  (2.1.8)

Constancy of curvature  $\frac{r_o}{r_1} = 1$  (2.1.9)

The effect of deviations from these restrictions is shown in Figs. 2.1.1 - 2.1.7. The parameters not explicitly referred to in these figures are assumed to satisfy the above reference requirements. Experimental verification, where available, is indicated on the graphs.

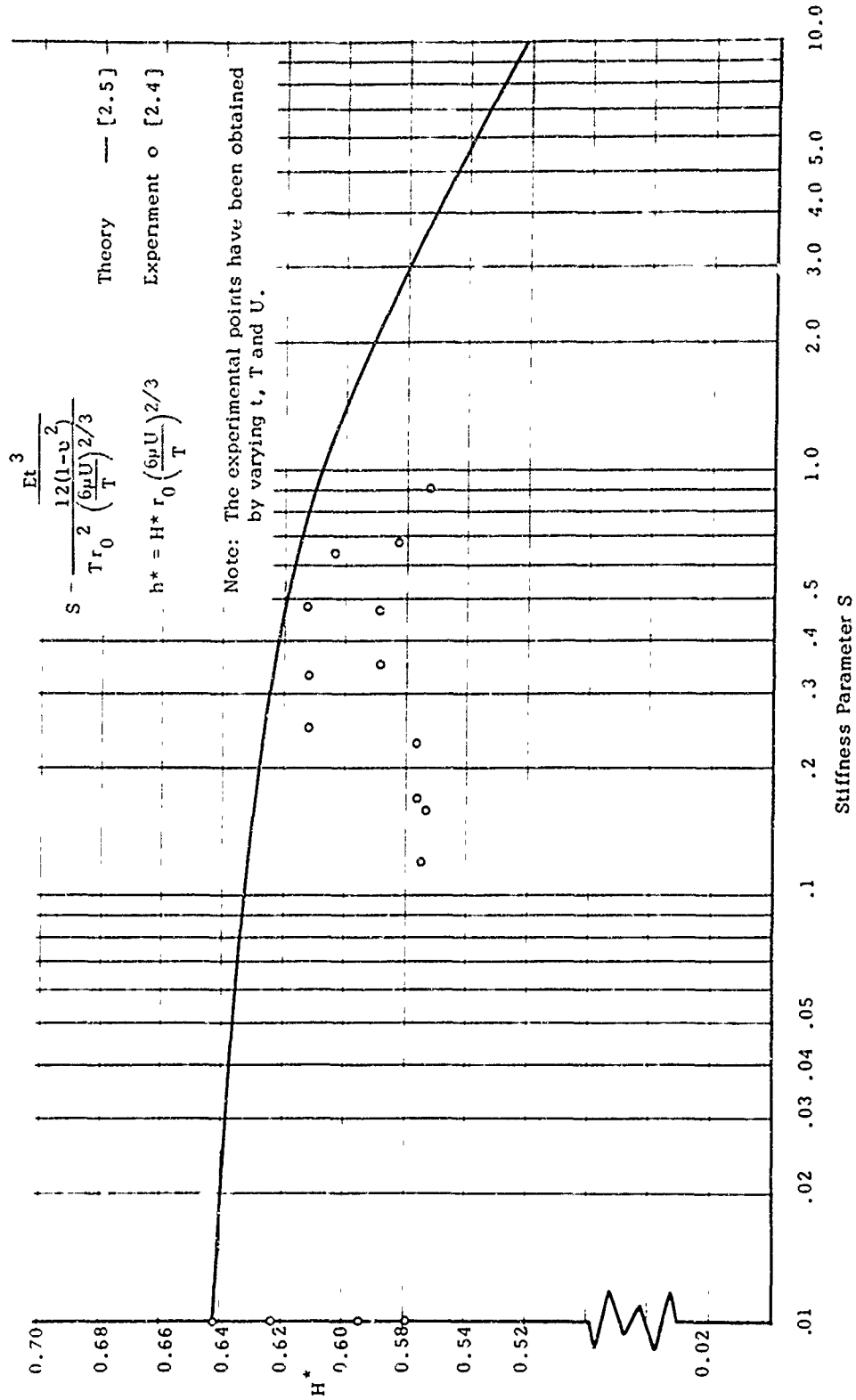


Fig. 2.1.1.1 Effect of Foil Stiffness Parameter S on the Magnitude of H\*

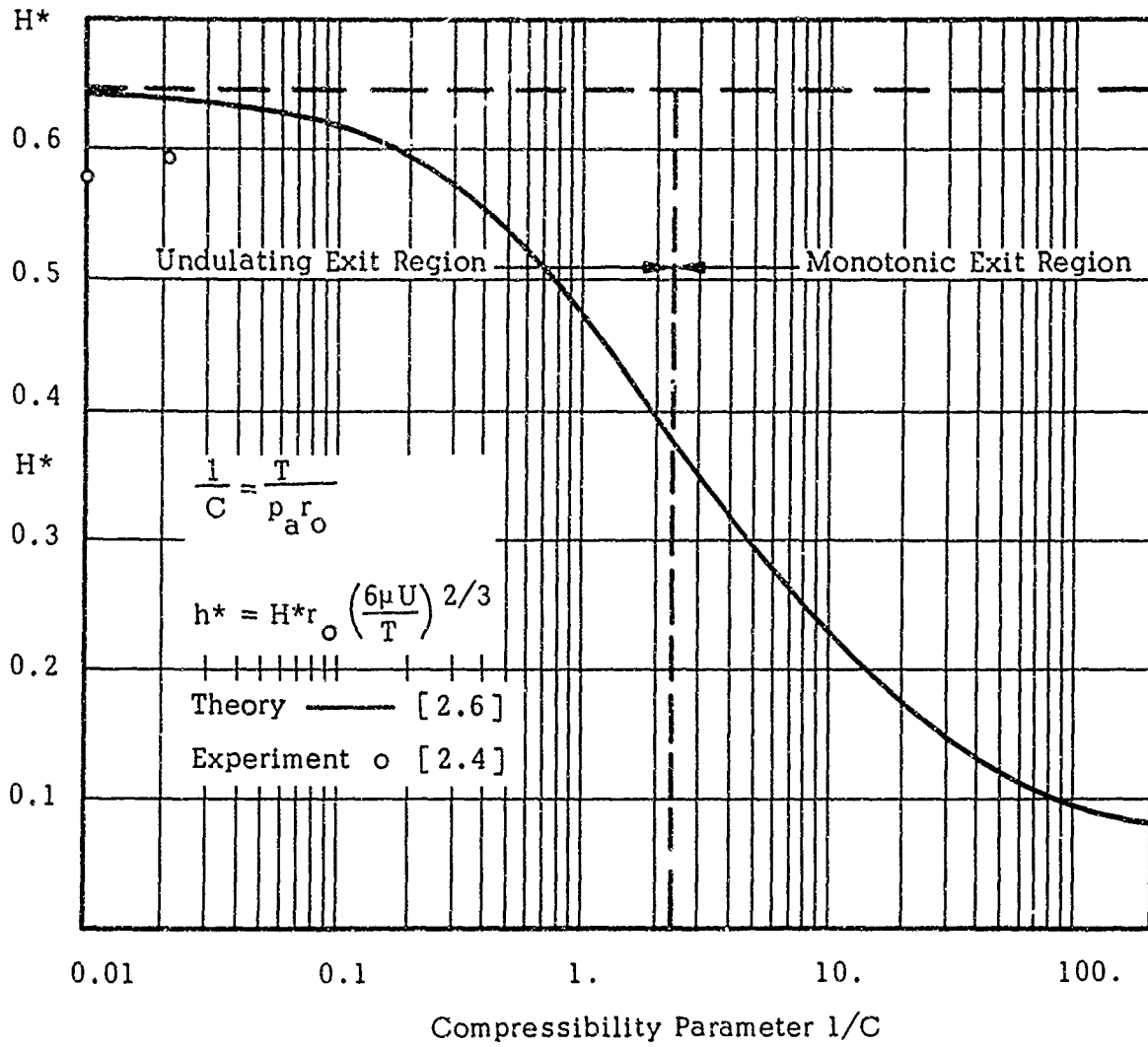


Fig. 2.1.2 Effect of the Compressibility Parameter  $1/C$  on the Magnitude of  $H^*$

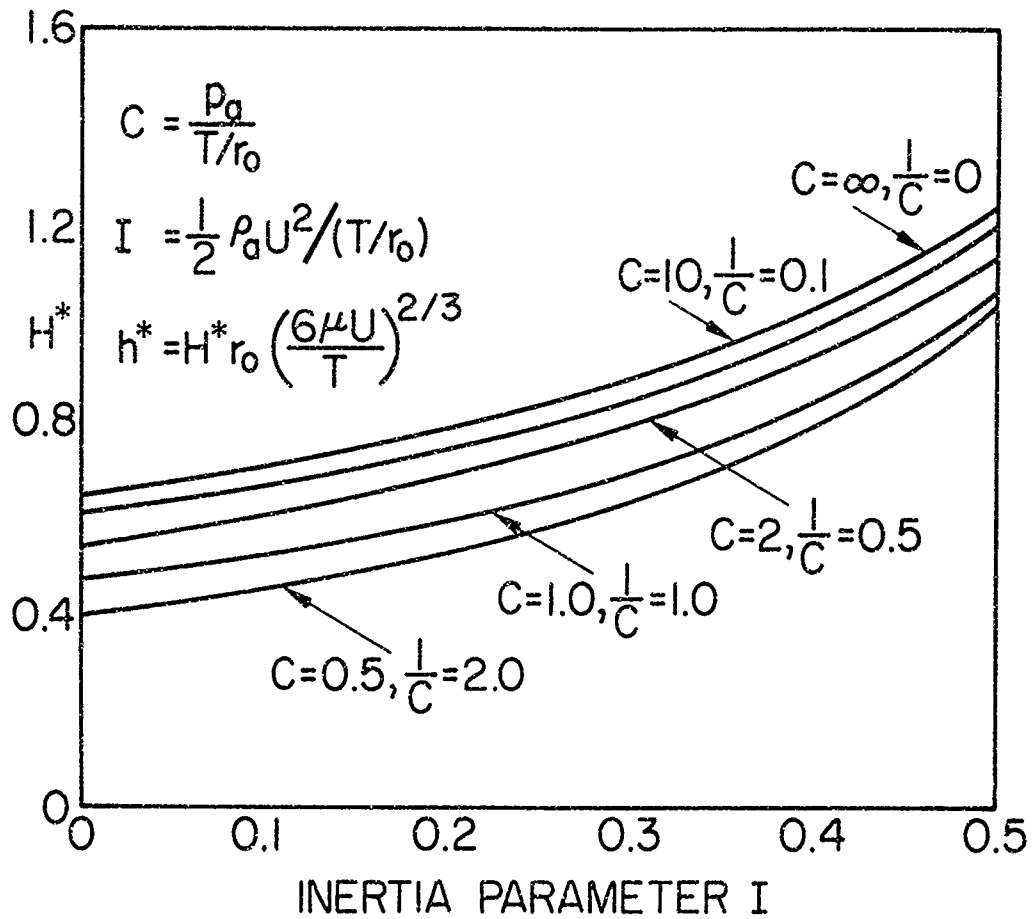


Fig. 2.1.3 Effect of the Inertia Parameter I and the Compressibility Parameter C on  $H^*$  [ 2.9 ]

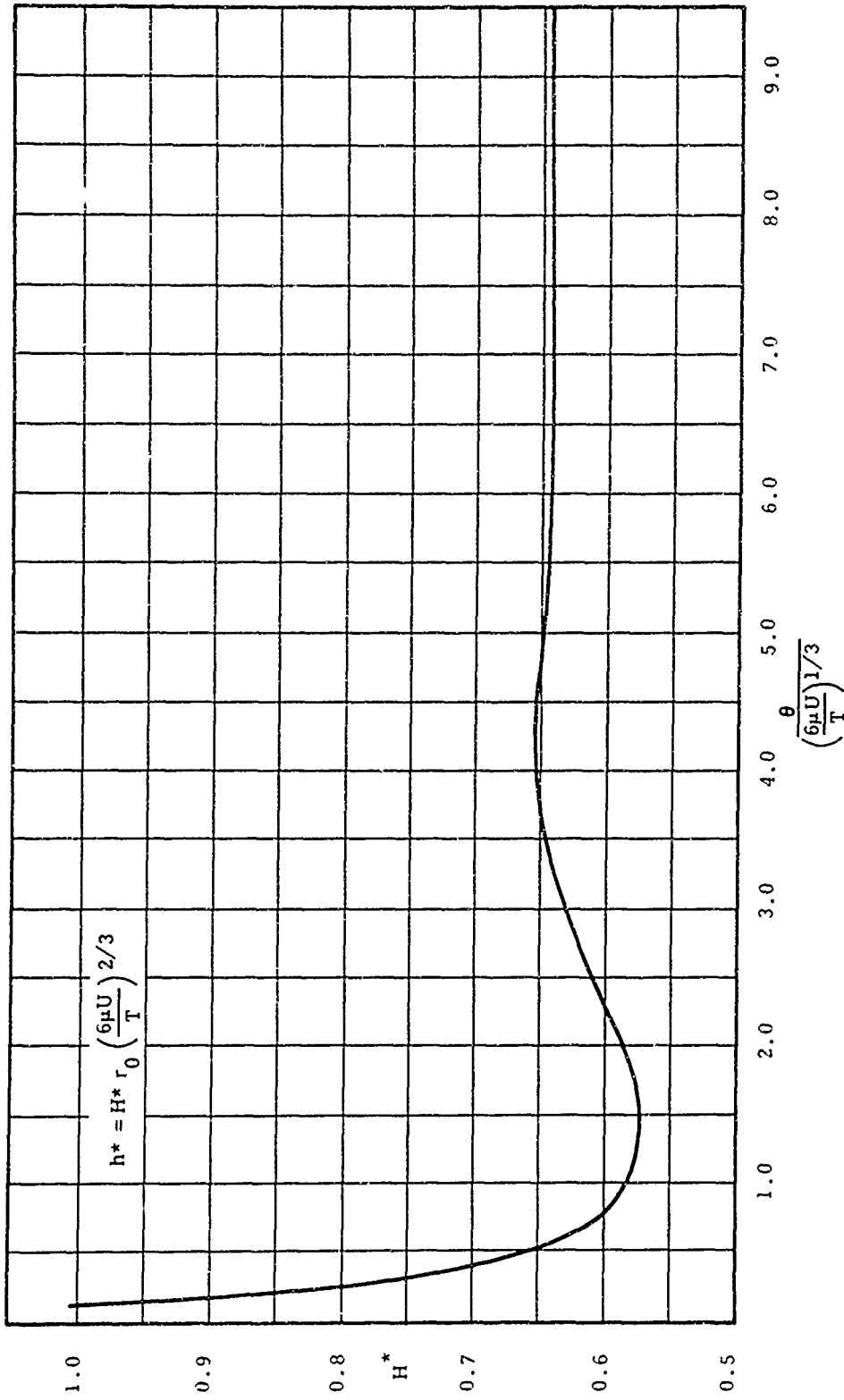


Fig. 2.1.4 Effect of Wrap Angle  $\theta$  on  $H^*$  [2.8]

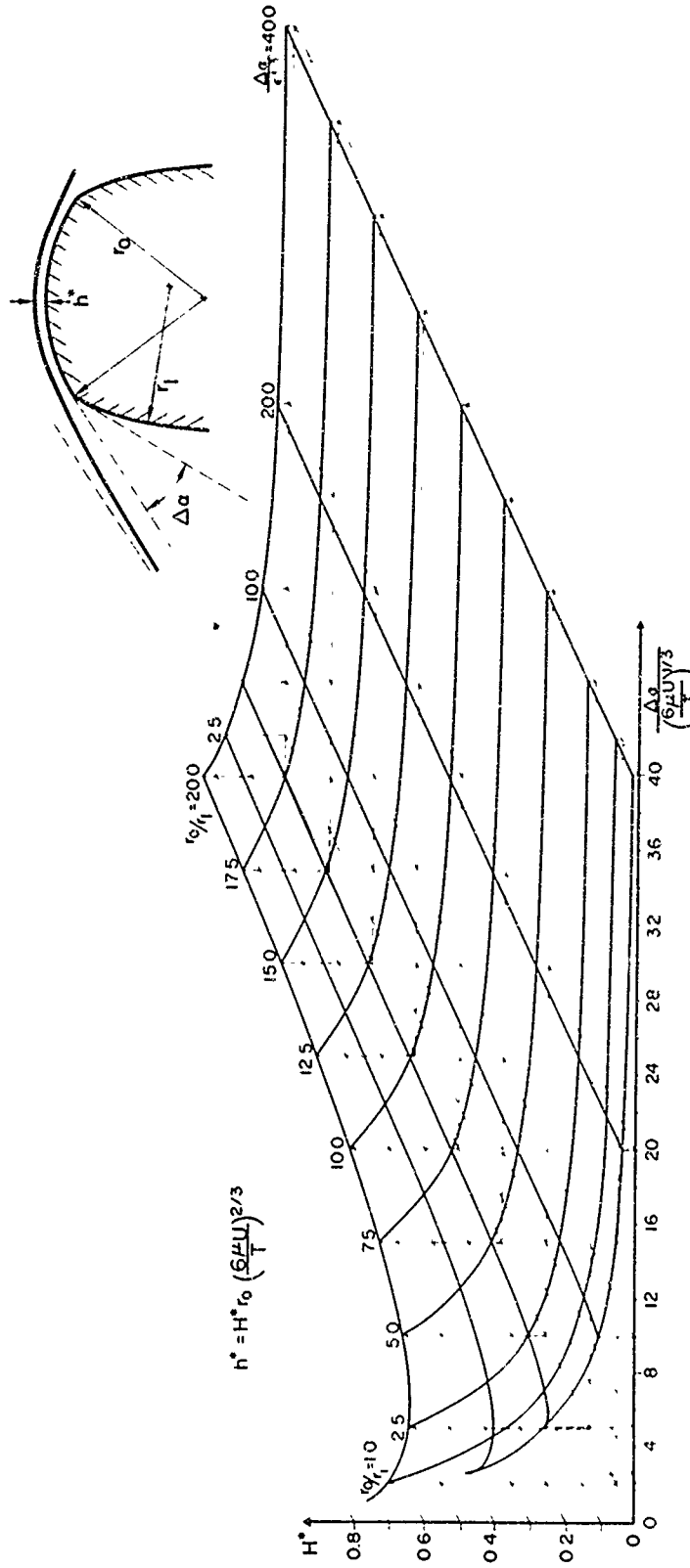


Fig. 2.1.5 Effect of Corners on  $H^*$  [2.10]

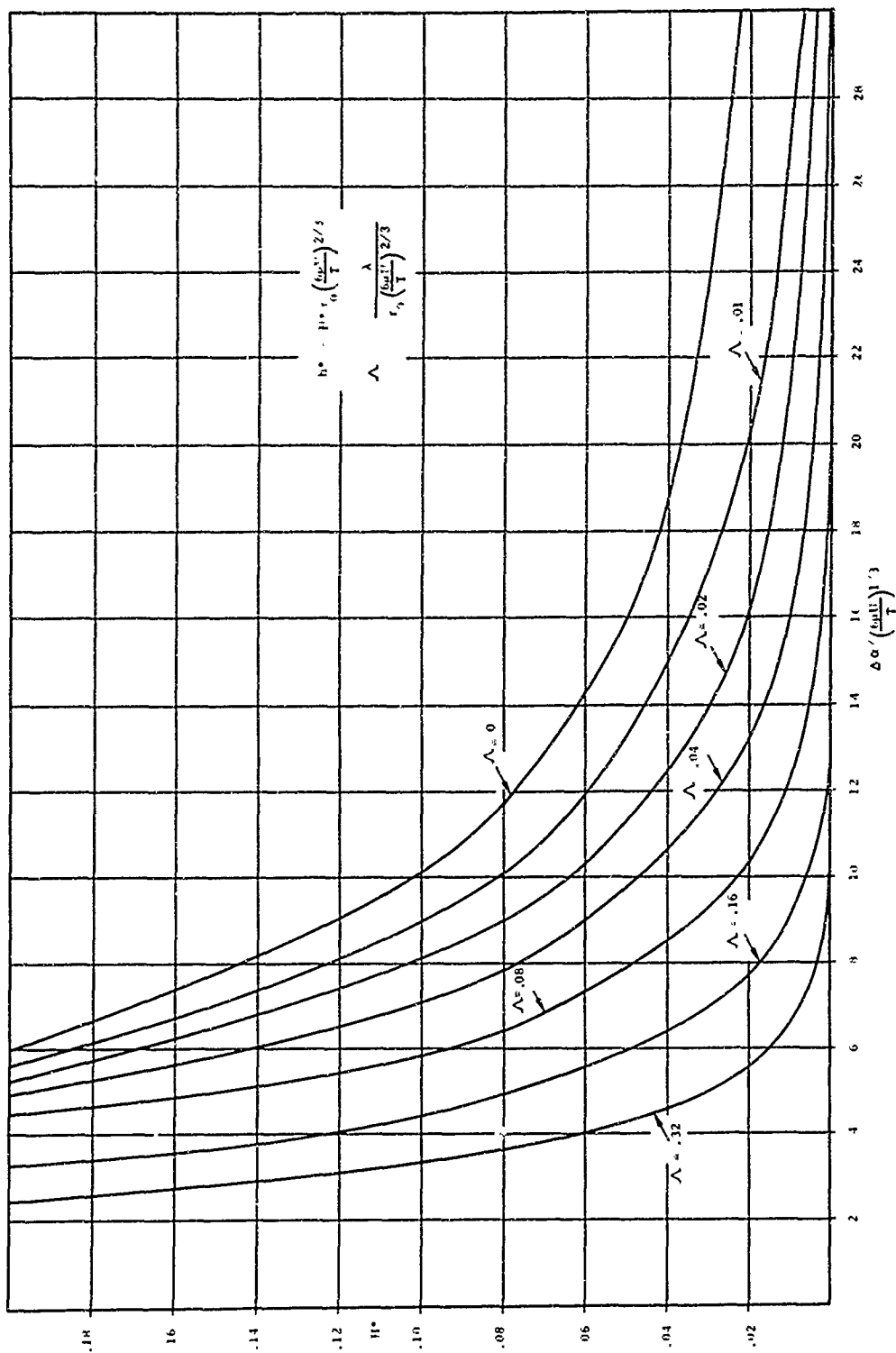


Fig. 2.1.1.6 Effect of Corners and Mean Free Path on  $H^*$  for  $r_0/r_1 = 0$

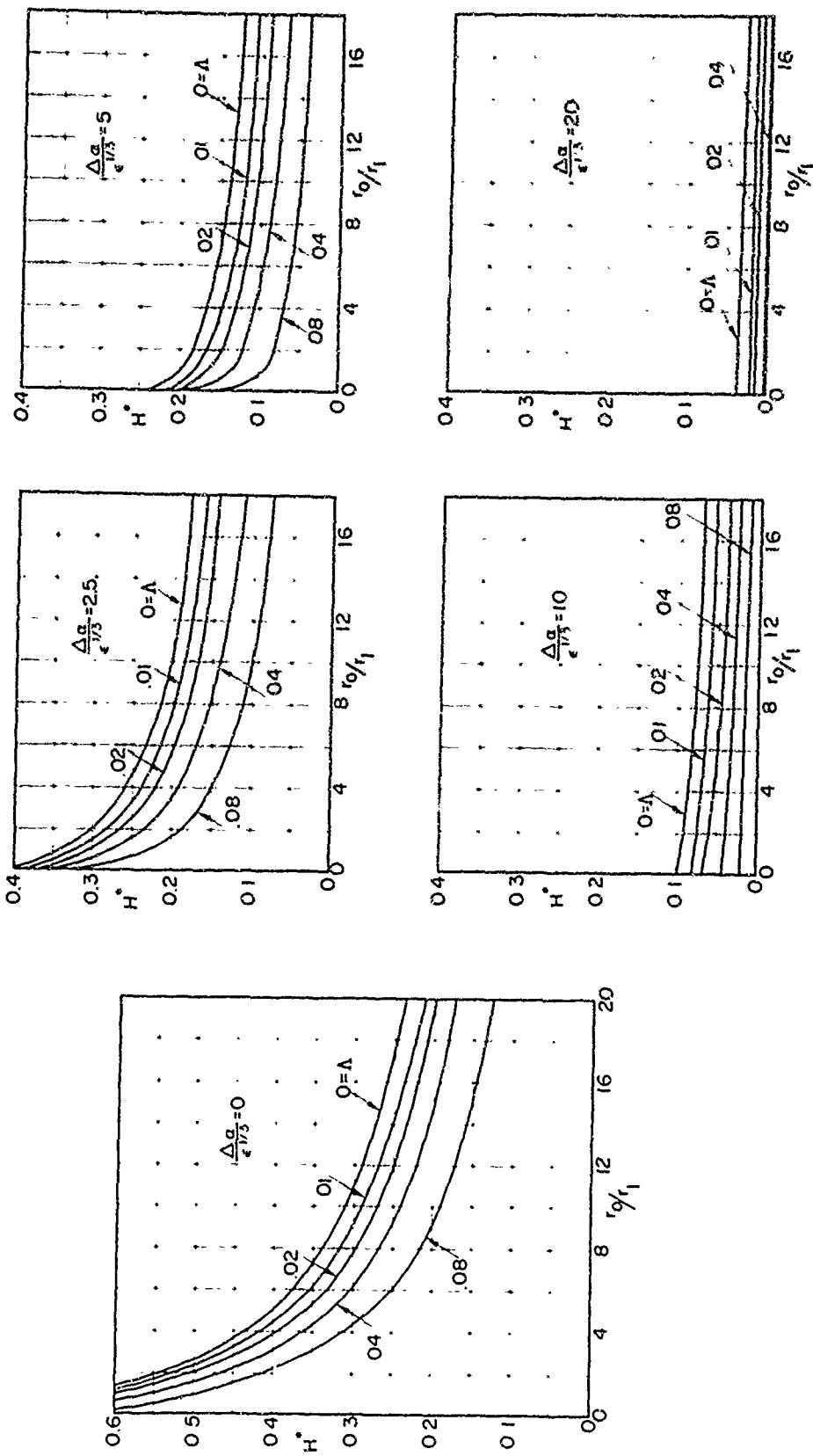


Fig. 2.1.7 Effect of Corners and of the Mean Free Path on  $H^*$  [ 2.10 ]

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Example: Find the air film in the region of uniformity of a magnetic head (Fig. 2.1.5) under the following conditions:

$$\mu = 0.265 \times 10^{-8} \text{ Lbf. sec/in}^2 \text{ -air}$$

$$\lambda = 2.5 \text{ } \mu\text{inch (air)}$$

$$U = 200 \text{ ips}$$

$$\rho_a = 1.15 \times 10^{-7} \text{ Lbf. sec}^2/\text{in}^4$$

$$p_a = 14.7 \text{ psi}$$

$$T = 0.5 \text{ Lb/in}$$

$$\Delta\alpha = 5^\circ$$

$$\Theta = 10^\circ$$

$$r_o = 0.25 \text{ in.}$$

$$r_l = \infty$$

$$b = 0.5 \text{ in.}$$

$$E = 5 \times 10^5 \text{ psi (Mylar)}$$

$$t = 1 \text{ mil (foil-thickness)}$$

$$\nu = 0.3 \text{ (Mylar)}$$

Results:

$$\epsilon = \frac{6\mu U}{T} = \frac{6 \times 0.265 \times 10^{-8} \times 200}{0.5} = 6.35 \times 10^{-6}$$

$$\frac{r_o}{r_l} = \frac{0.25}{\infty} = 0$$

$$S = \frac{Et^3}{12(1-\nu^2)T r_o^2 \epsilon^{2/3}} = \frac{5 \times 10^5 \times (10^{-3})^3}{12(1-0.3^2)0.5 \times 0.25^2 (6.35 \times 10^{-6})^{2/3}} = 4.25$$

$$\frac{\Delta \sigma}{\epsilon^{1/3}} = \frac{5 \times \pi \times 180}{(6.35 \times 10^{-6})^{1/3}} = 4.73$$

$$\frac{1}{C} = \frac{T}{\rho_a / r_o} = \frac{0.5}{14.7/0.25} = 0.0085 < 0.12$$

$$I = \frac{1/2 \rho_a U^2}{T/r_o} = \frac{1/2 \times 1.15 \times 10^{-7} \times 200^2}{0.5/0.25} = 1.15 \times 10^{-3} < 0.05$$

$$\frac{\theta}{\epsilon^{1/3}} = \frac{10 \times \pi / 180}{(6.35 \times 10^{-6})^{1/3}} = 9.45 > 6$$

$$\Lambda = \frac{\lambda}{r_o} \epsilon^{-2/3} = \frac{2.5 \times 10^{-6}}{0.25} (6.35 \times 10^{-6})^{-2/3} = 2.88 \times 10^{-2}$$

$$\sqrt{tr_o} = \sqrt{0.001 \times 0.25} = 0.0158 \ll 0.5$$

The reference conditions (2.1.2), (2.1.6), (2.1.8), (2.1.9) are violated. Assuming a perfectly flexible tape, it can be found from Fig. 2.1.7 that

$$H^* = 0.21$$

The stiffness S, has the effect of about 4% reduction in  $H^*$  (Fig. 2.1.1) which will be neglected.

The film thickness is then

$$h_o^* = r_o H^* \epsilon^{2/3} = 0.25 \times 10^6 \times 0.21 \times (6.35 \times 10^{-6})^{2/3} = 18 \mu\text{inch}$$

### 2.1.2 Determination of the Steady-State Undulations in the Exit Region

In the exit region the foil undergoes a series of undulations, which decay in the direction of the central region. When the reference conditions (Eq. 2.12-2.19) apply, the minimum gap is

$$h_{\min}/h^* = 0.716$$

and the maximum gap is

$$h_{\max}/h^* = 1.065$$

The last peak-to-trough angular distance ("half wavelength") is

$$\Delta\theta \approx 2.0 \left( \frac{6\mu U}{T} \right)^{1/3} \quad (2.1.10)$$

The effects of deviations from the reference conditions on the undulations are given in Fig's. 2.1.8 through 2.1.12. Parameters not explicitly defined in the figures, satisfy the reference conditions (Eq. 2.1.2-2.1.9).

### 2.1.3 Remarks on the Steady-State Characteristics of Finite Width Foil Bearings.

No complete, finite width theory for foil bearings is presently available. The main qualitative effects are summarized below and some sample results are presented [2.4, 2.11].

- a. Foils bent by couples into cylinders display edge-undulations (anticlastic effect), which decay rapidly toward the foil centerline, as shown in Fig. 2.1.13.

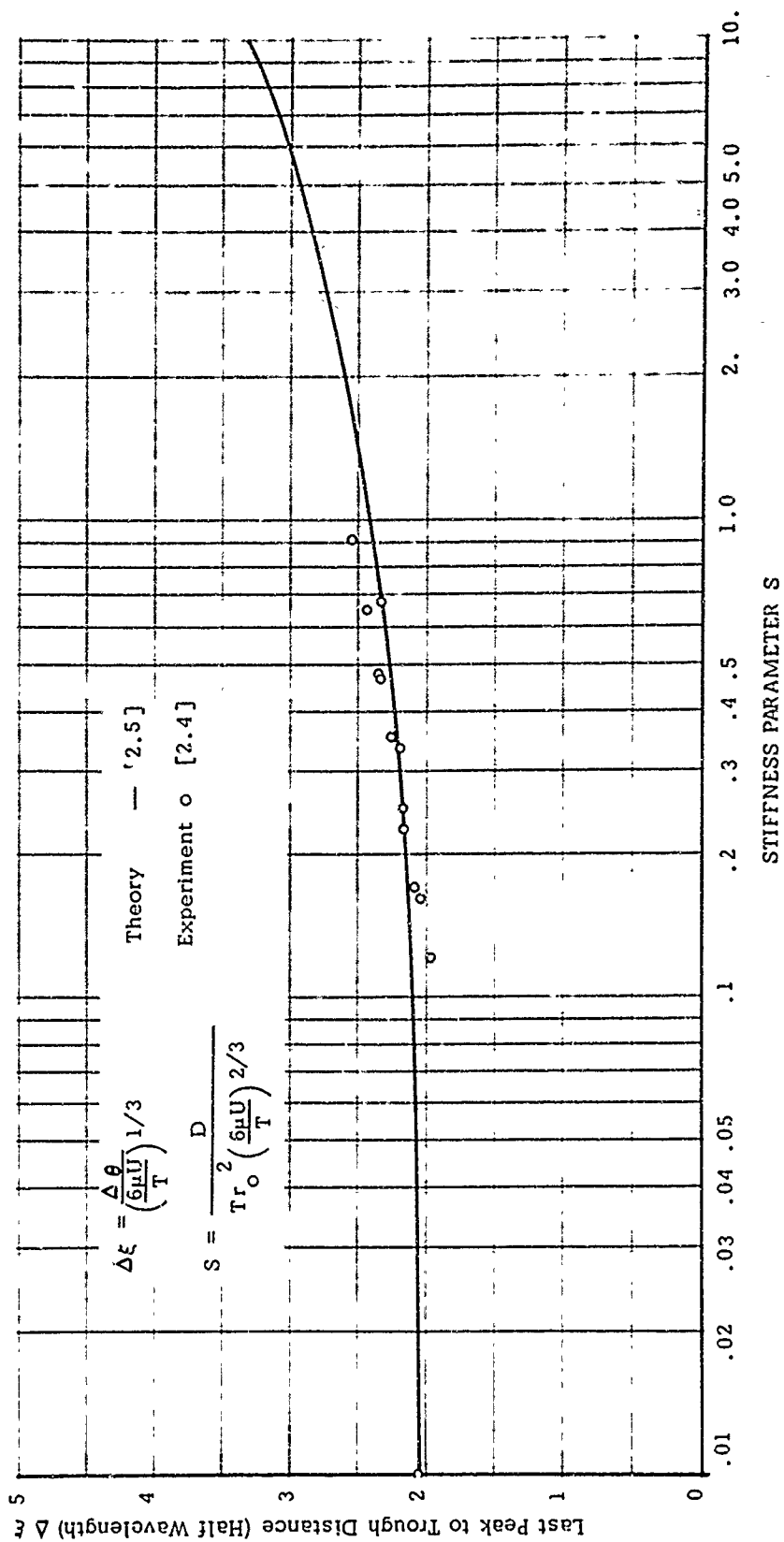


Fig. 2.1.8 Effect of the Stiffness Parameter S on the Wavelength of the Exit Undulation

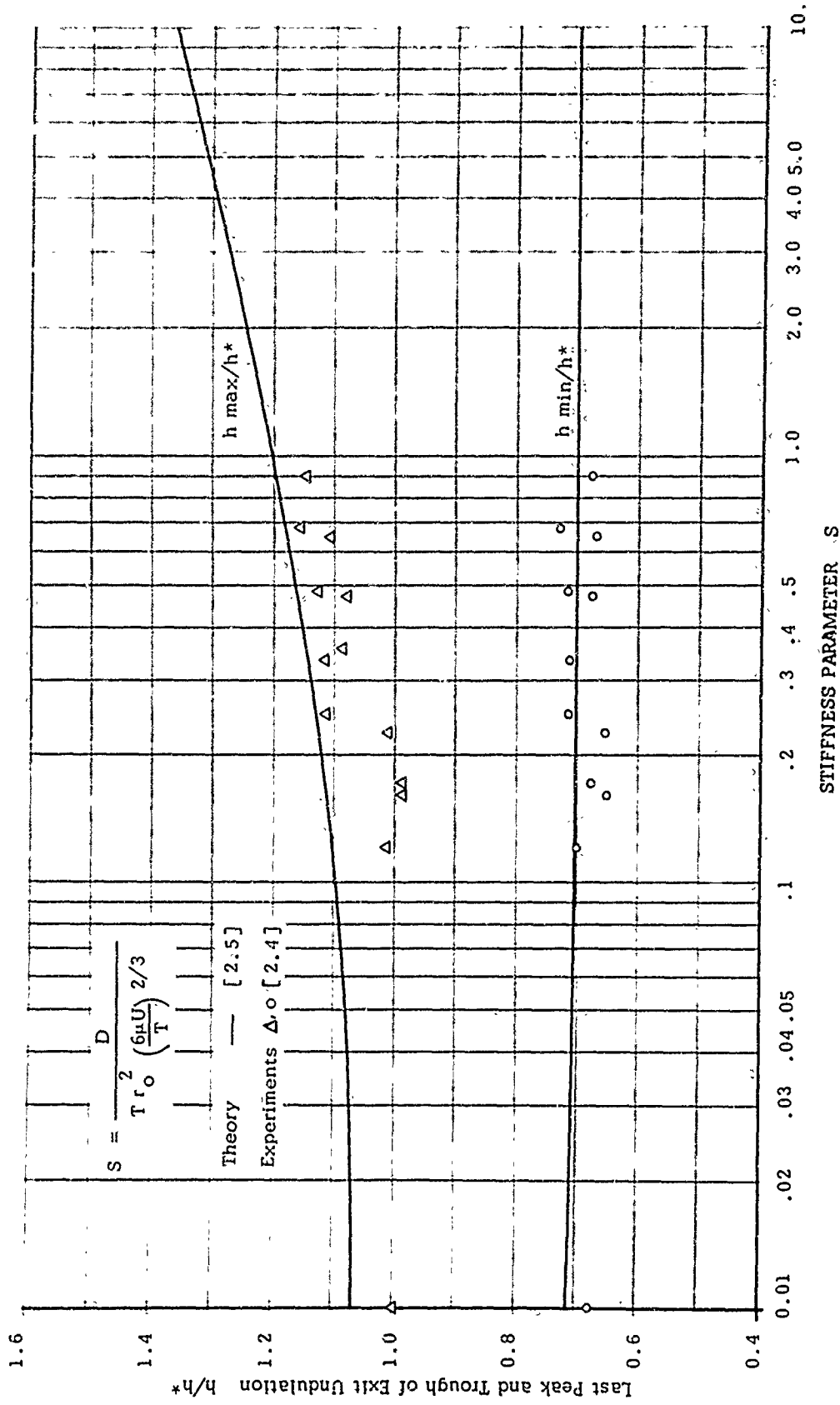


Fig. 2.1.9 Effect of the Stiffness Parameter  $S$  on the Last Peak and Trough of the Exit Undulation

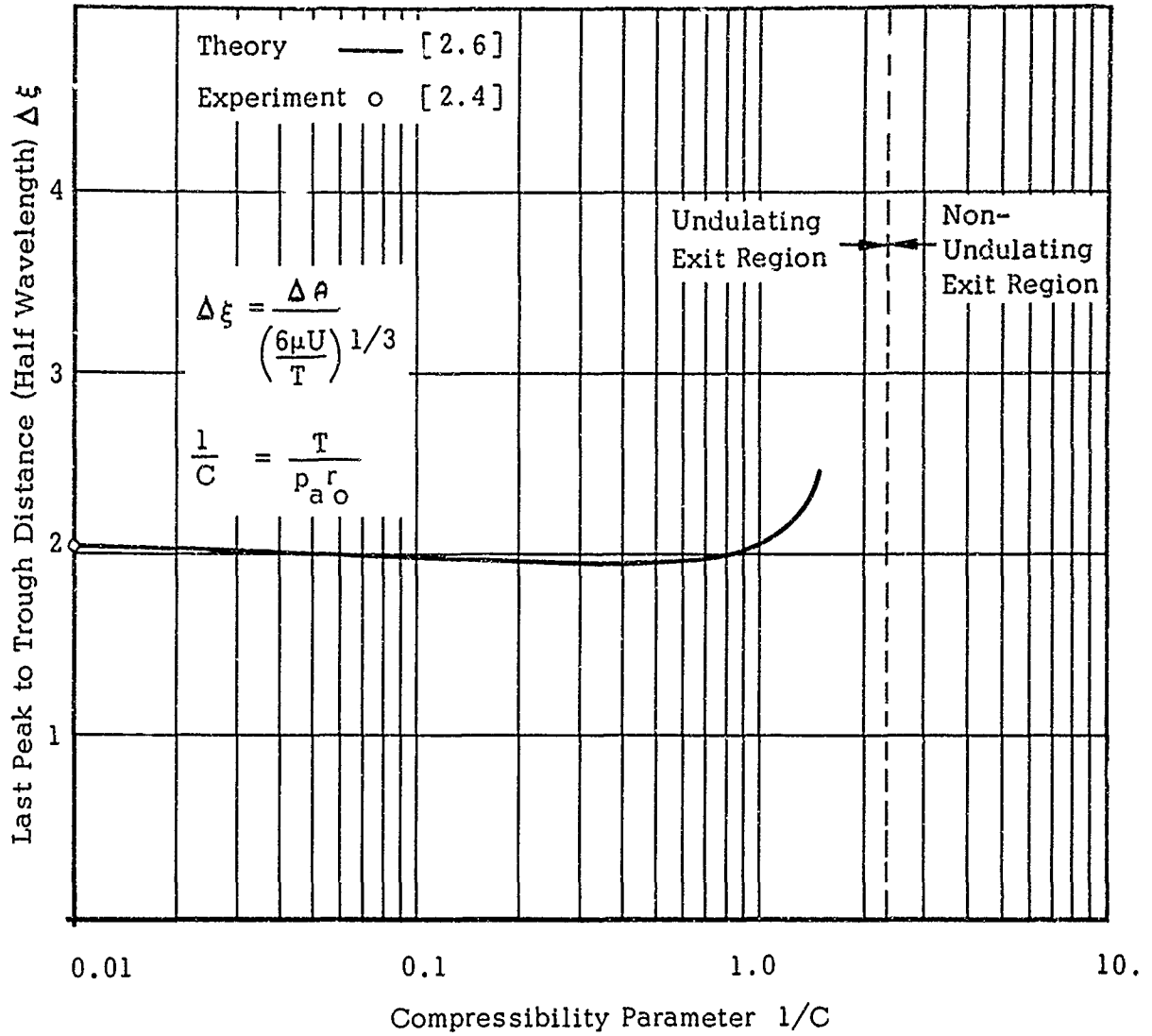


Fig. 2.1.10 Effect of the Compressibility Parameter  $1/C$  on the Wavelength of the Exit Undulation

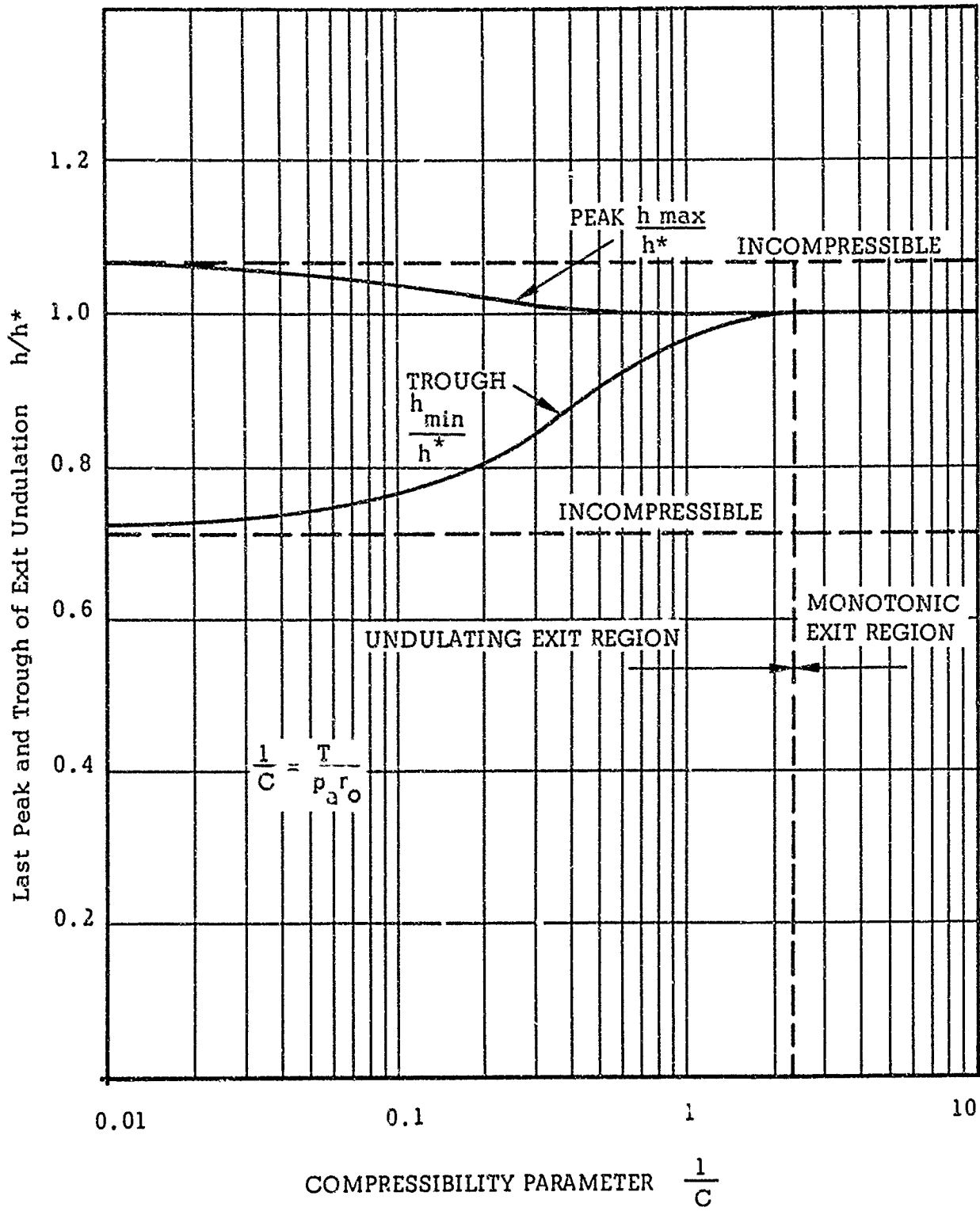


Fig. 2.1.11 Effect of Compressibility on Last Peak and Trough of Exit Undulation [ 2.6 ]

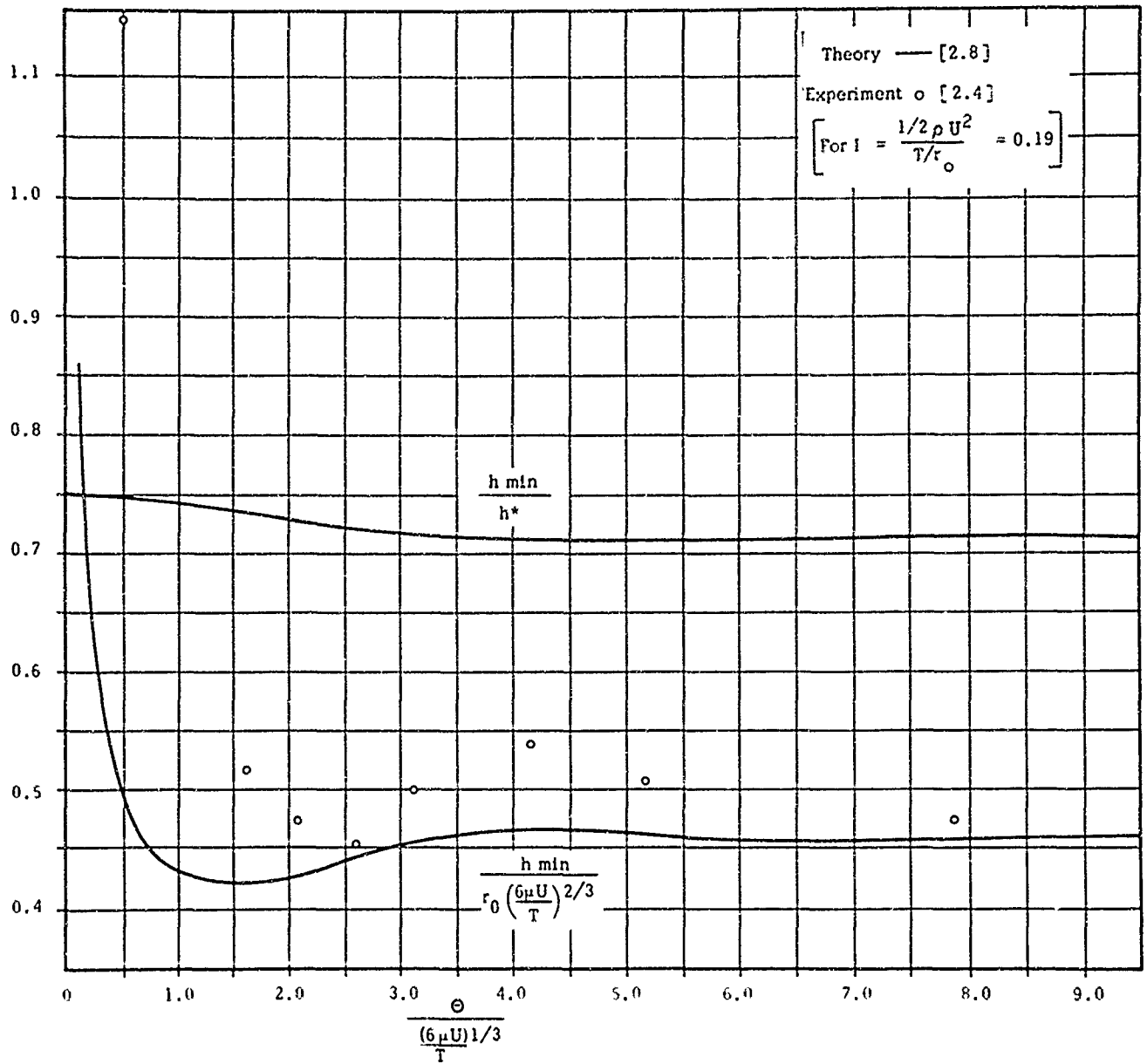


Fig. 2.1.12 Effect of Wrap Angle on the Trough of the Clearance Undulation

- b. The effect of circumferencial tension is to flatten the undulation .
- c. The effect of the rapid pressure drop to ambient in the vicinity of the foil edges causes the edge zone to sag, creating a partial or total seal.
- d. Side leakage decreases the gap width in a circumstantial direction particularly with foils for which  $b \sim \sqrt{tr_o}$   
Fig. 1.2.1c.

Fig. 2.1.14 shows typical tape contours traversed across half the width of the foil at two longitudinal stations. The upper graph shows the contour at the bisector of the 30°-wrap angle, whereas the lower graph shows the contour along the locus of minima in the exit zone. Comparisons of parts (a), (b), and (c) of the figure give an indication of the effect of foil thickness, for the same nominal clearance. The comparison of parts (c) and (d) shows the effect of increasing the nominal clearance (by increasing the speed) for the same foil thickness.

2.1.4 Film Thickness of Foil Bearings of Finite Length  
(Fixed Points of Attachment)

In applications of equation (2.1.1):

$$h^* = H^* r_o \left( \frac{6\mu U}{T_s} \right)^{2/3} \tag{2.1.11}$$

to transports of tapes and webs, the tension  $T_s$  is prescribed. In applications to rotating machinery, the tension  $T_s$  is coupled to the elastic stress-strain equation [2.12]:

$$\frac{T_s - T_o}{Et} = \frac{-\delta l - 2y \sin \frac{\Theta}{2} + \Theta h^*}{l_o} \tag{2.1.12}$$

\*  $\delta l$  is the increment in foil length due to reasons other than stretching e.g. thermal.  $y$  is the component of displacement of the rotor center, measured away from the foil, along the line which under the initial conditions ( $T_o, l_o, \Delta l = 0, h^* = 0$ ) is the foil bisector.

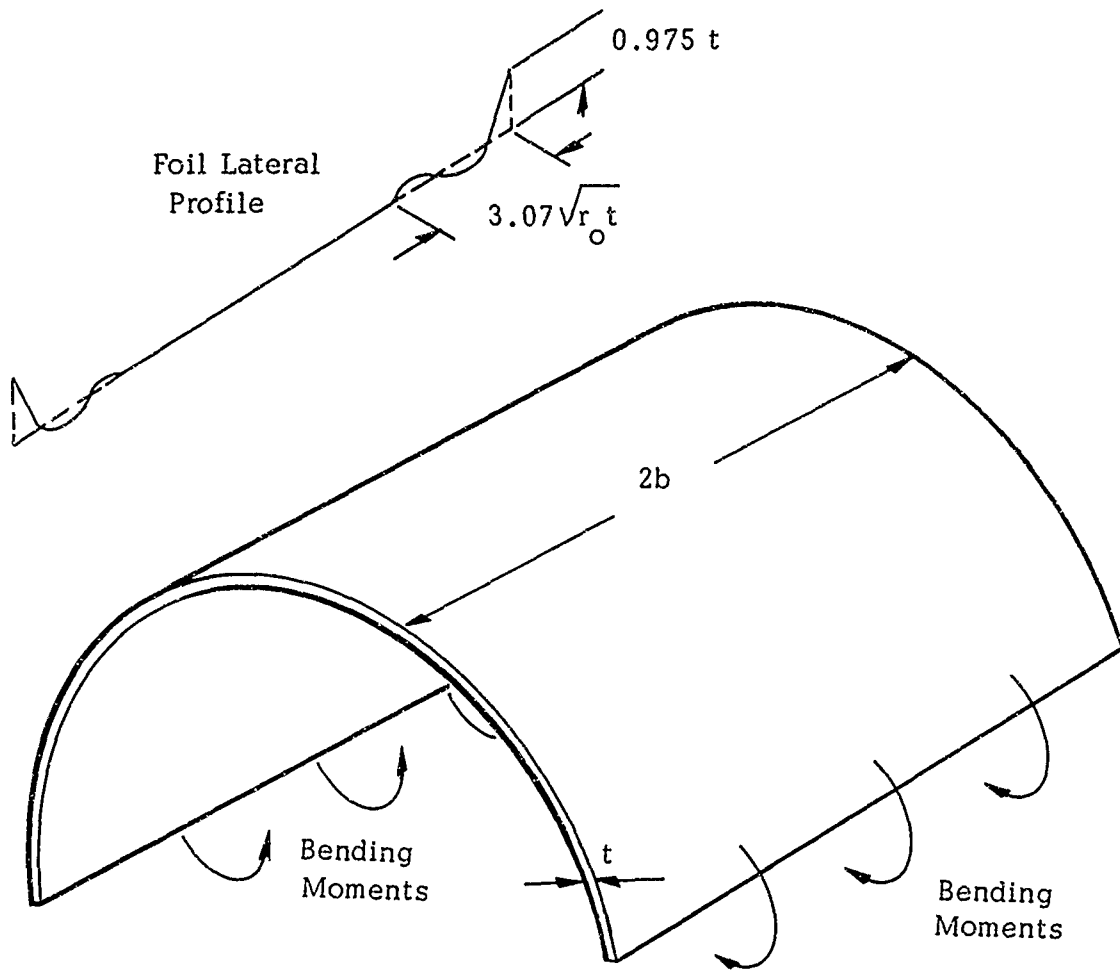


Fig. 2.1.13 Lateral Foil Profile Due to the Effect of Pure Bending

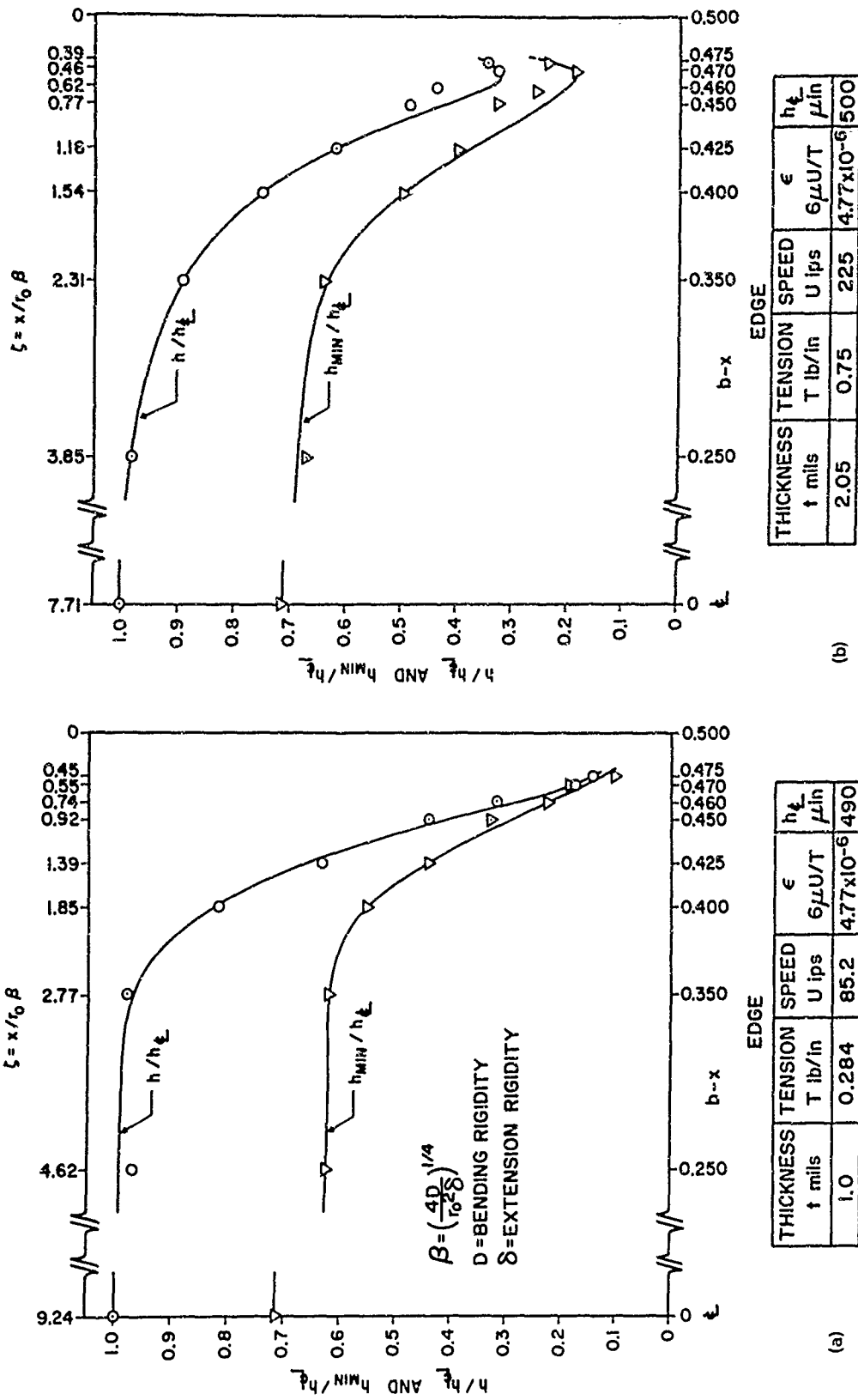
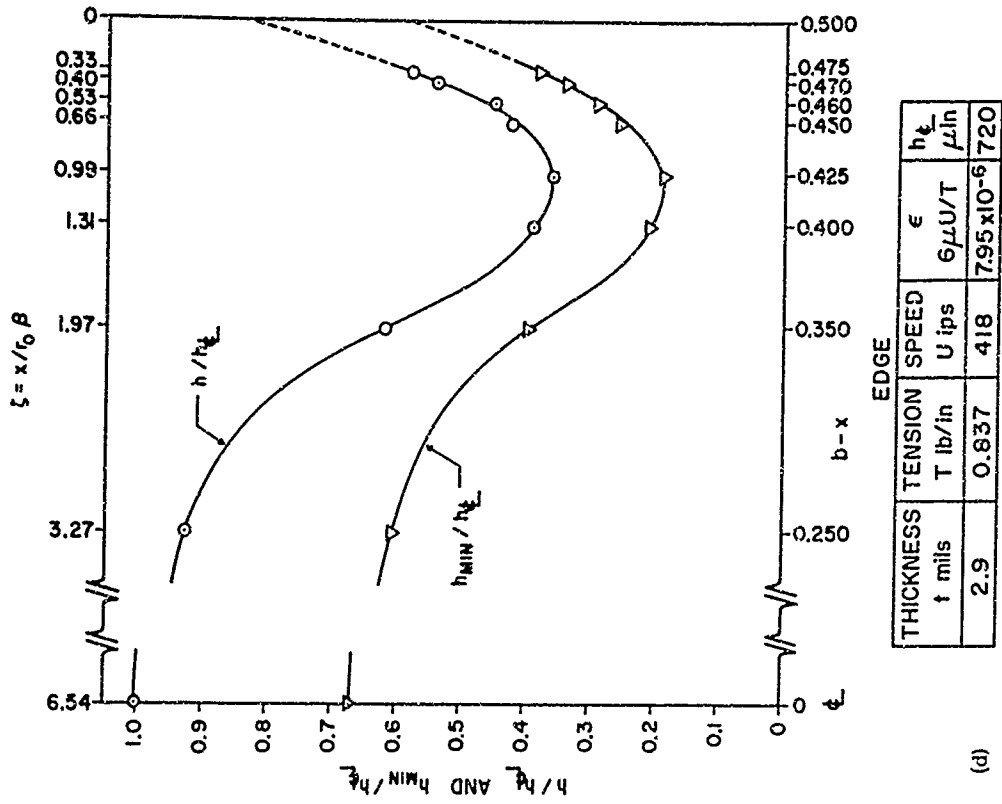
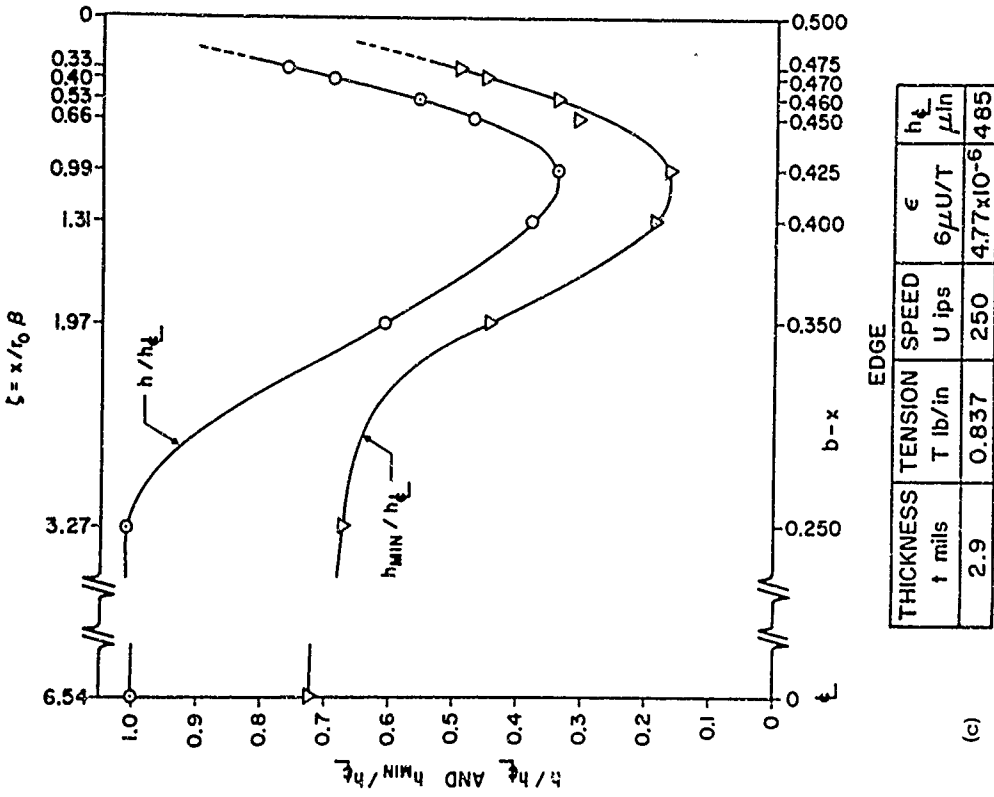


Fig. 2.1.14 Lateral Profiles of Elastic Foils: Edge Characteristics [2.4].



(d)



(c)

Fig. 2.1.14 Lateral Profiles of Elastic Foils: Edge Characteristics [ 2.4 ] (cont)

The above is a consequence of the fact that foil lengths in continuous transport are very large, whereas the elongation of finite, fixed-end foil segments, used in the support of rotors, has an appreciable effect on the tension.

The design curves in Figs. 2.1.15 and 2.1.16 give the clearance  $h^*$  and the tension  $T_s$  for the case when  $y = 0$  (concentric rotor position) and  $\delta l = 0$  (elongation due to stretching only).

These curves are presented here only as a guide. The user is cautioned that the following effects are not included in the curves:

1. Foil Slack
2. Fluid inertia
3. Thermal elongation
4. Slippage of the foil at points of attachment.

Comparisons with film thicknesses measured in the particular design of Licht<sup>[2.13]</sup> show that Fig. 2.1.15 underestimates  $h^*$  by a factor of 2. When corrections for the above factors are included the predicted values are in reasonable agreement with the experimental results.

## 2.2 Response to Sudden Disturbances

Disturbances in the contour of the foil, as well as the response of the foil contour to tension step-changes, propagate downstream, from entrance to exit, at approximately <sup>[2.14 - 2.16]</sup>

$$V_p = \frac{U}{2} \tag{2.2.1}$$

This speed of propagation is not appreciably affected by stiffness and compressibility.

Computer simulation of the response is shown for two typical cases in Figs. 2.2.1, 2.2.2. The film thickness distributions at successive time intervals are shown by displacing upward by one division the datum for

consecutive curves. (The scale is indicated for the initial film thickness distribution only.)

The speed of propagation of a disturbance produced by an impact of the foil was measured by Licht [ 2.15 ] who verified the theory [ Fig. 2.2.3 ].

### 2.3 Response to Periodic Excitation

If a sinusoidal variation of tension,

$$T(t) = T_s (1 + \varphi_o \cos \omega t) \quad (2.3.1)$$

in which  $\varphi_o \ll 1$ , is imposed on the foil, the steady-state response of the film thickness can be shown to be of the form:

$$h(\Theta, t) = h_s(\Theta) \left\{ 1 + \frac{\varphi_o A(\Theta, \omega)}{H^*} \cos [\omega t + \gamma(\Theta, \omega)] \right\} \quad (2.3.2)$$

in which  $A(\theta, \omega)$ , and  $\gamma(\theta, \omega)$  are the space and frequency-dependent amplitude factor and phase [ 2.17 ].

Typical spatial distribution of the film thickness perturbation is shown in Fig. 2.3.1. One should note the decay in amplitude and the travelling-wave character of the phenomenon. The spatial and frequency dependence of  $A$  and  $\gamma$  is illustrated in Fig. 2.3.2.

The physical interpretation of variation of  $A$  and  $\gamma$  with increasing frequency of tension pulsation is as follows. The tension variation in the foil can be assumed to be spatially uniform, with the gap responding first in the entrance zone and the front of the change washed downstream at nearly  $U/2$ . At the same time, for a compressible fluid, local instantaneous compression is synchronous with changes of tension.

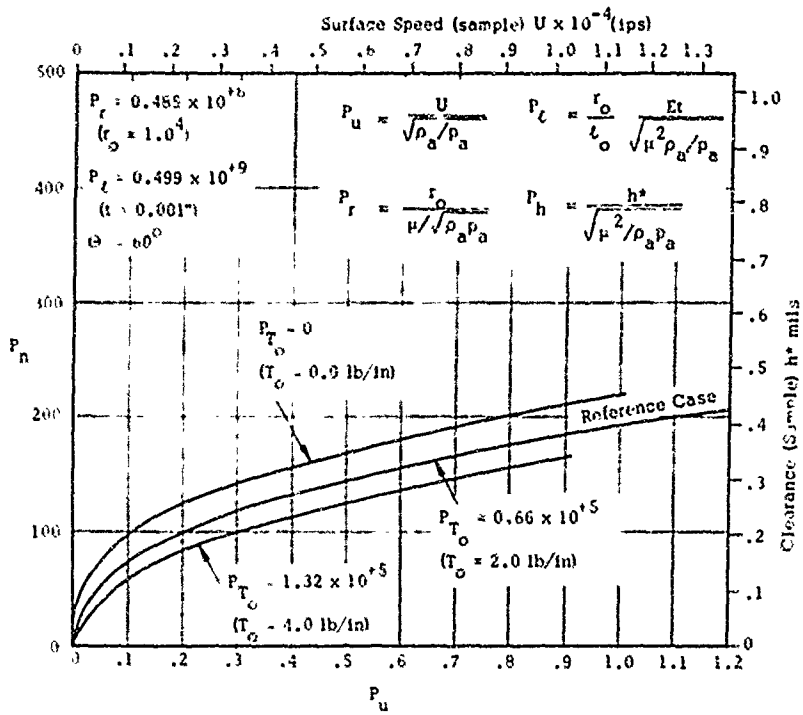


Fig. 2.1.15a Effect of Initial Tension of Steady State Clearance [2.12]

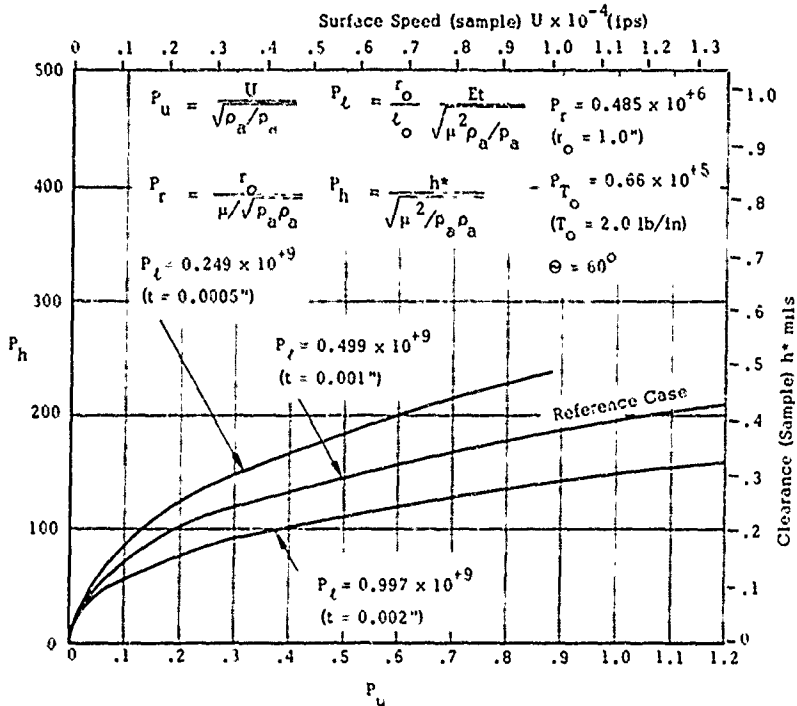


Fig. 2.1.15b Effect of Foil Thickness on Steady State Clearance [2.12]

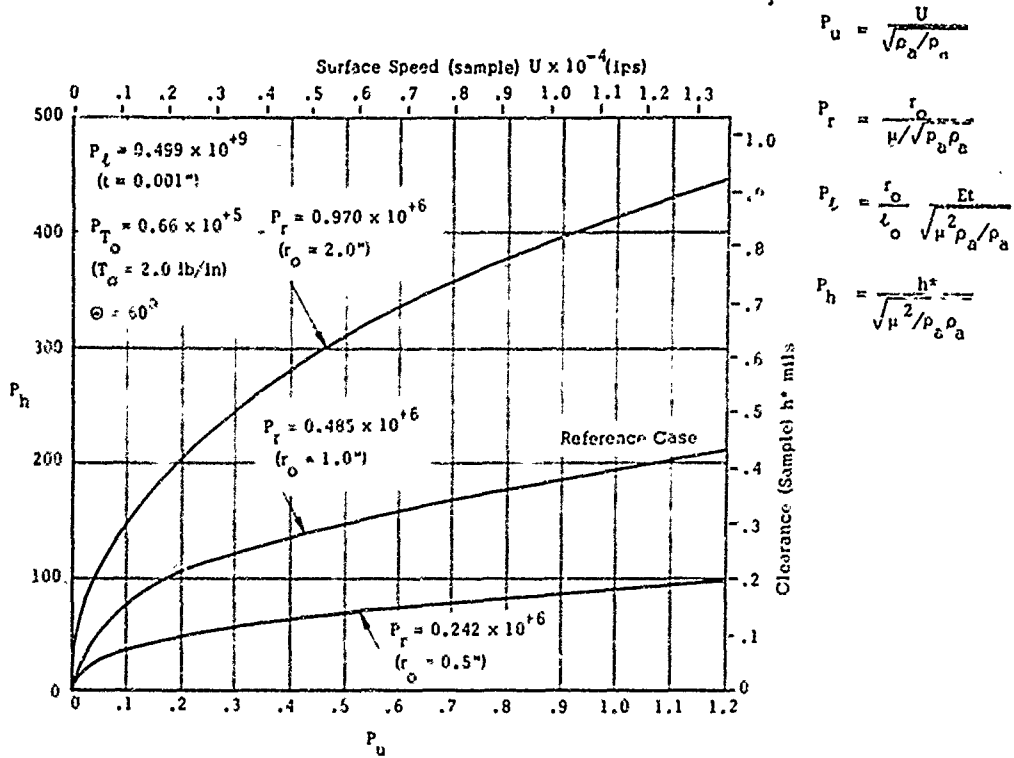


Fig. 2.1.15c Effect of Journal Radius on Steady State Clearance [ 2.12 ]

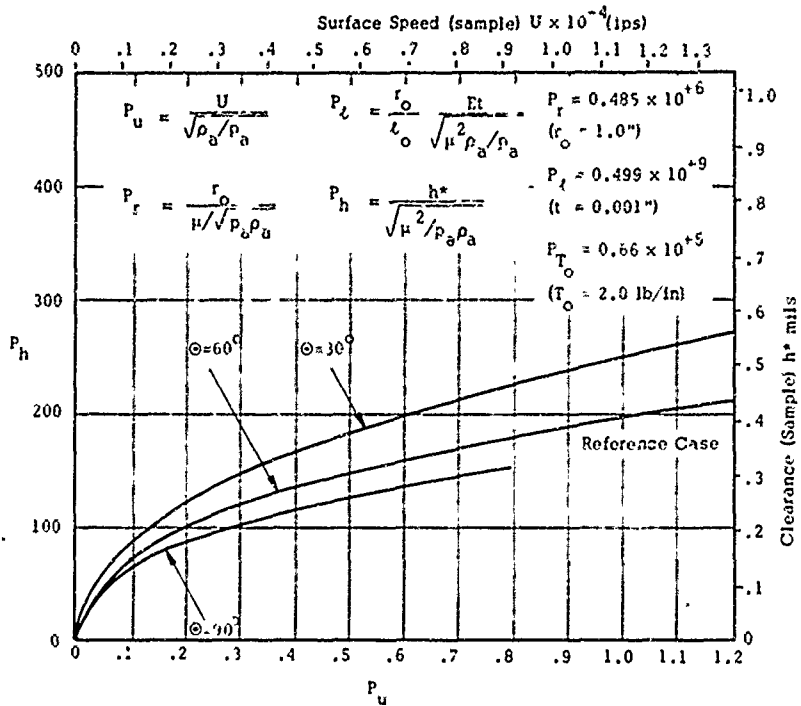


Fig. 2.1.15d Effect of Wrap Angle on Steady State Clearance [ 2.12 ]

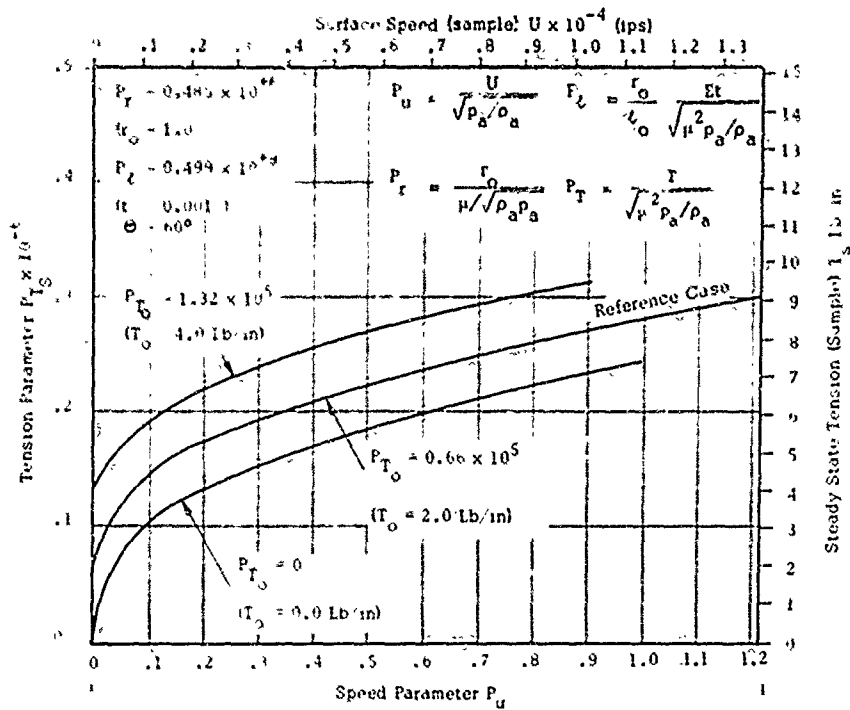


Fig. 2.1.16a Effect of Initial Tension on Steady State Tension [ 2.12 ]

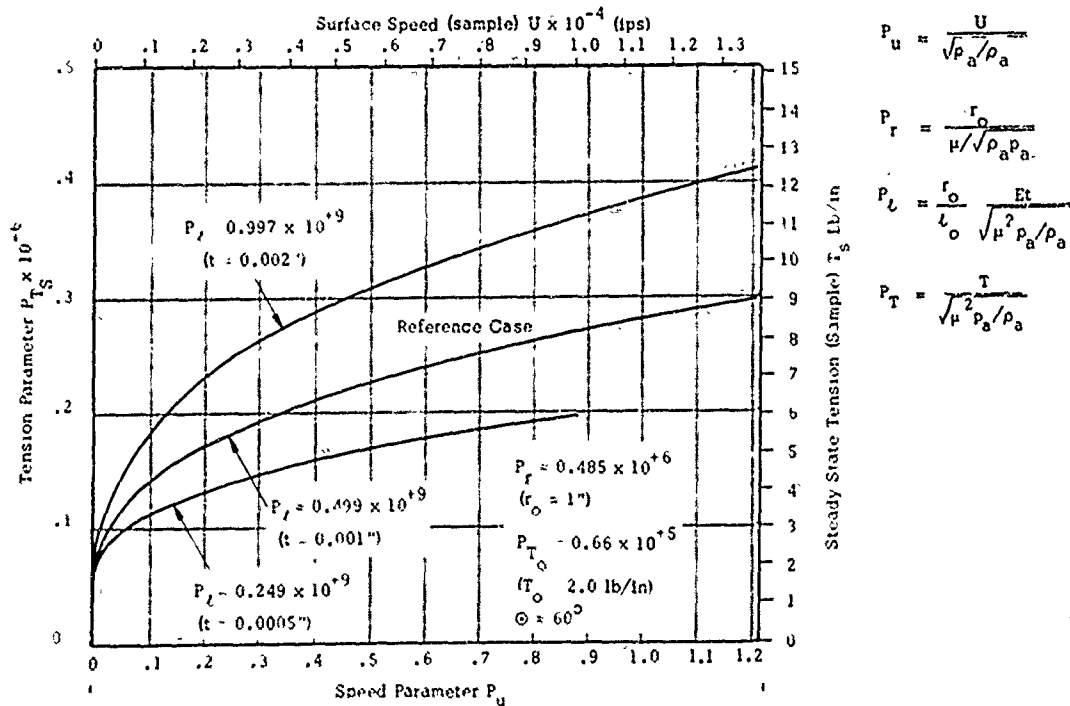


Fig. 2.16b Effect of Foil Thickness on Steady State Tension [ 2.12 ]

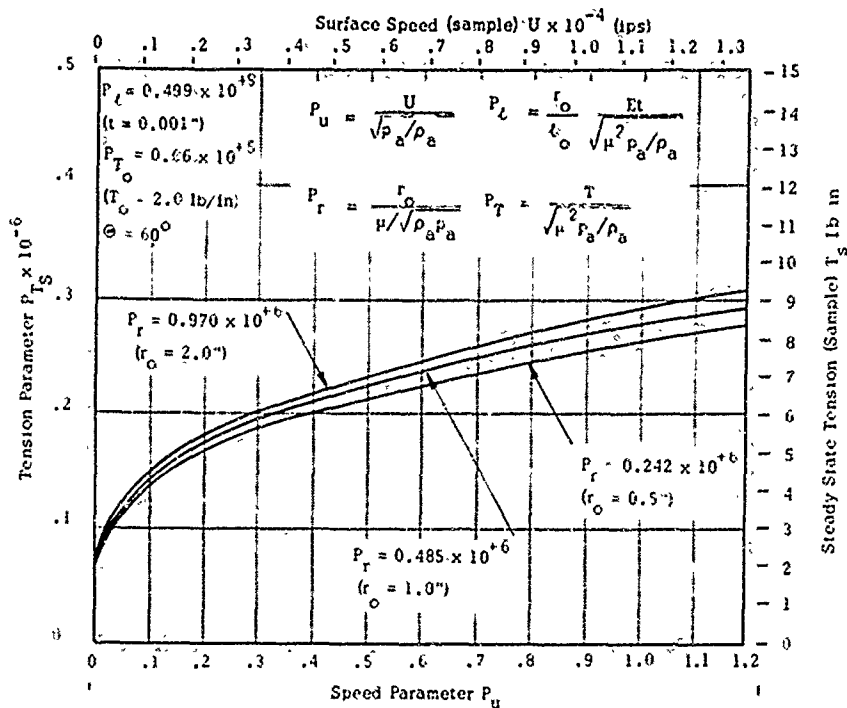


Fig. 2.1.16c Effect of Journal Radius on Steady State Tension [ 2.12 ]

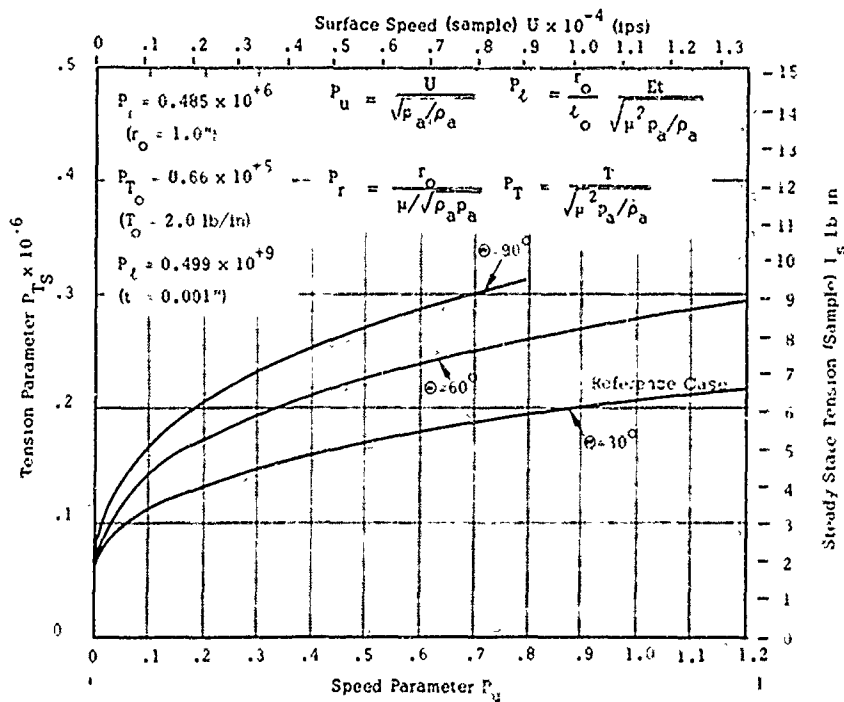


Fig. 2.1.16d Effect of Wrap-Angle on Steady State Tension [ 2.12 ]

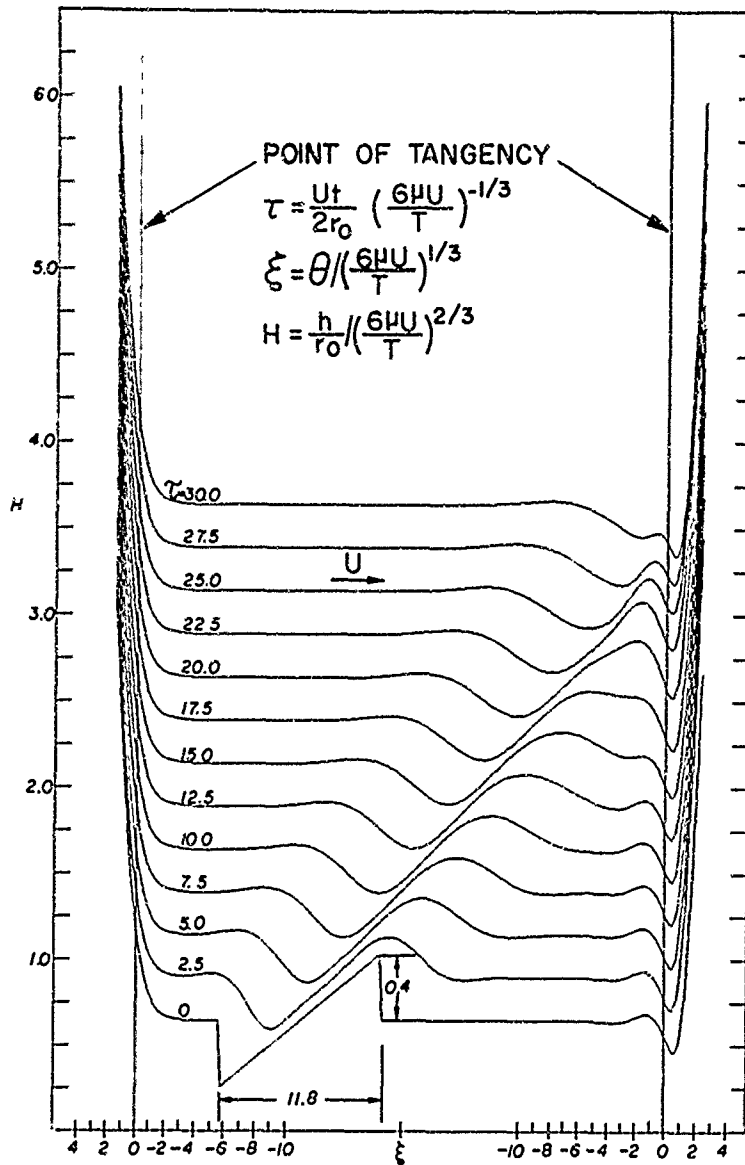


Fig. 2.2.1 Computer Simulation of the dynamic response of a foil bearing to a saw tooth wavelet of width  $2a = 11.8$  and height  $A = 0.4$  (Rapid initial smoothing of the disturbance occurs initially and is followed by a downstream movement of the disturbance at  $U/2$ ) [2.14]. Note: In this figure  $t$  represents time.

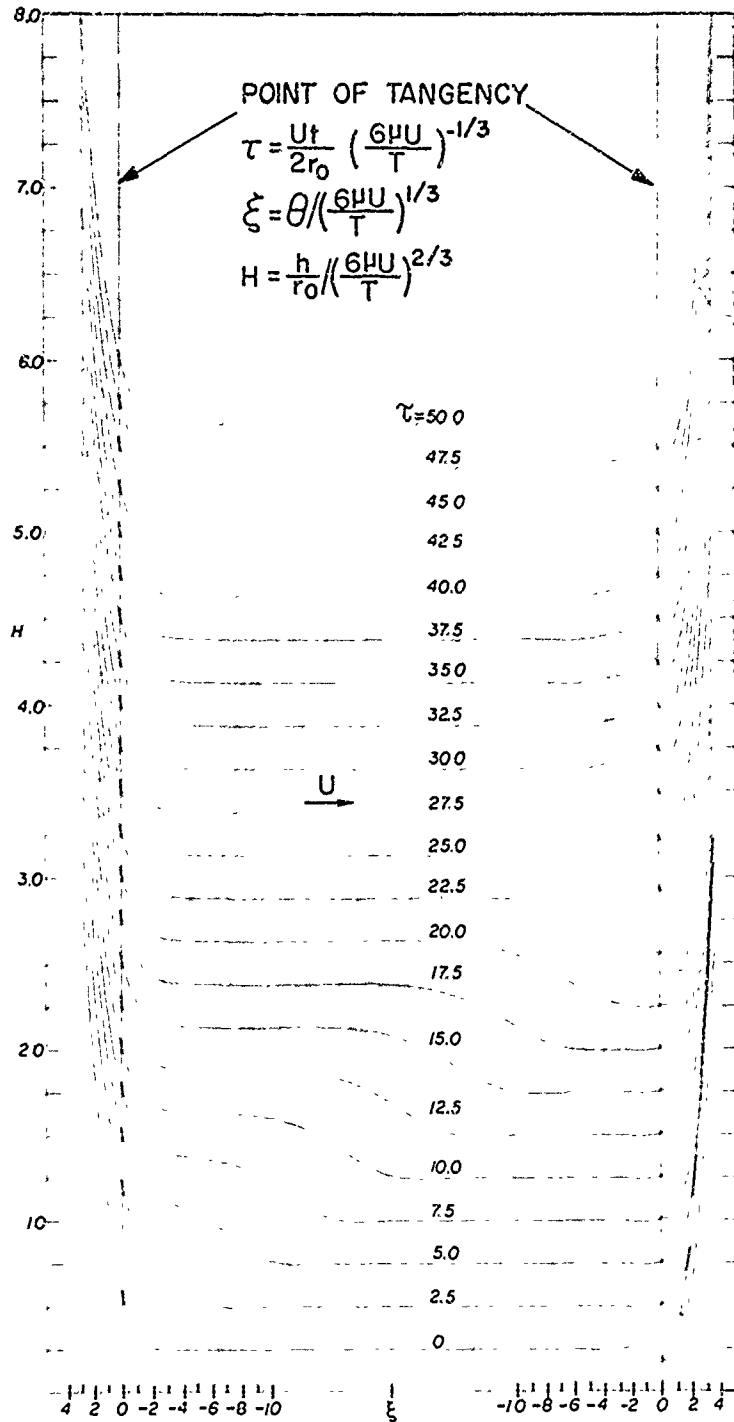


Fig. 2.2.2 Computer simulation of the dynamic response of a foil bearing to a step reduction in tension. (Greater film thickness is established by a front propagating from the inlet towards the exit at  $U/2$ ). [2.14]. Note: In this figure  $t$  represents time.

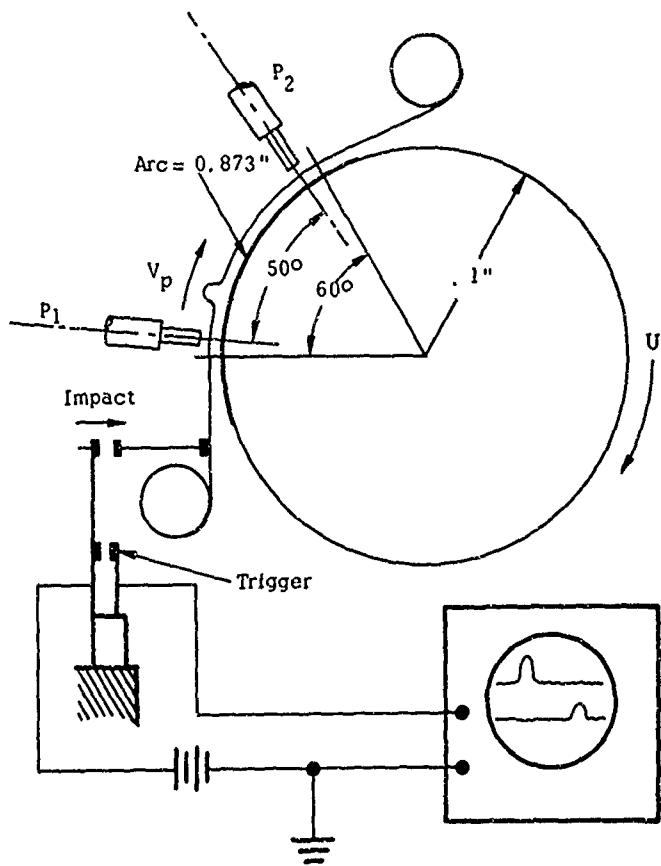
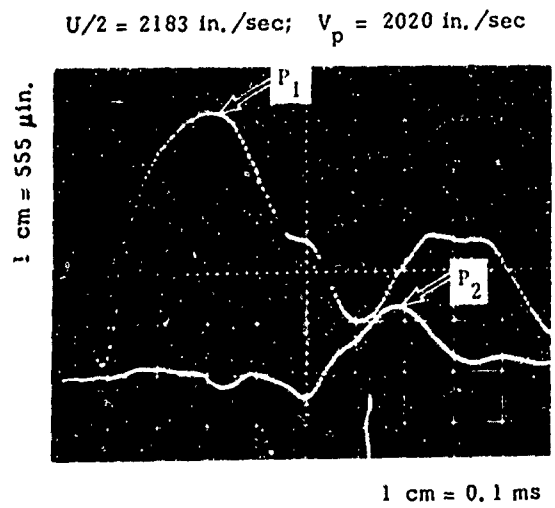
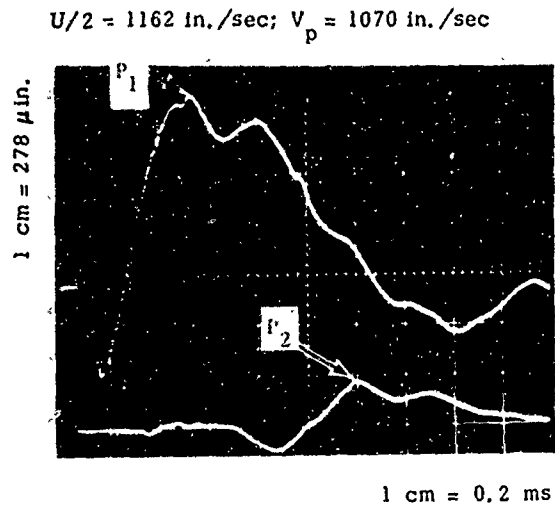
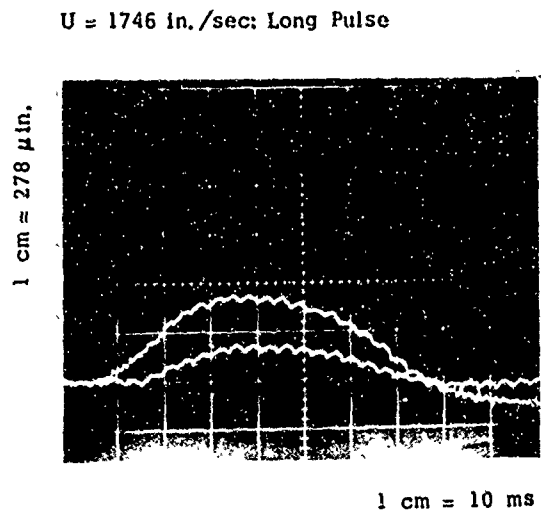
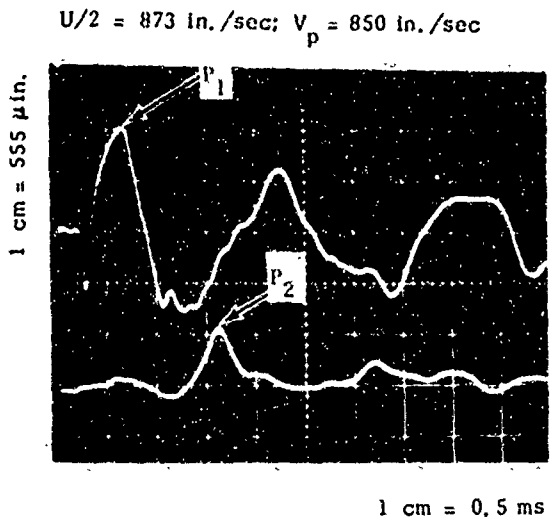


Fig. 2.2.3 Comparison of Measured Velocity of Propagation of a Disturbance Along a Foil  $V_p$  with the Theoretically Predicted Value of  $U/2$  [2.16]

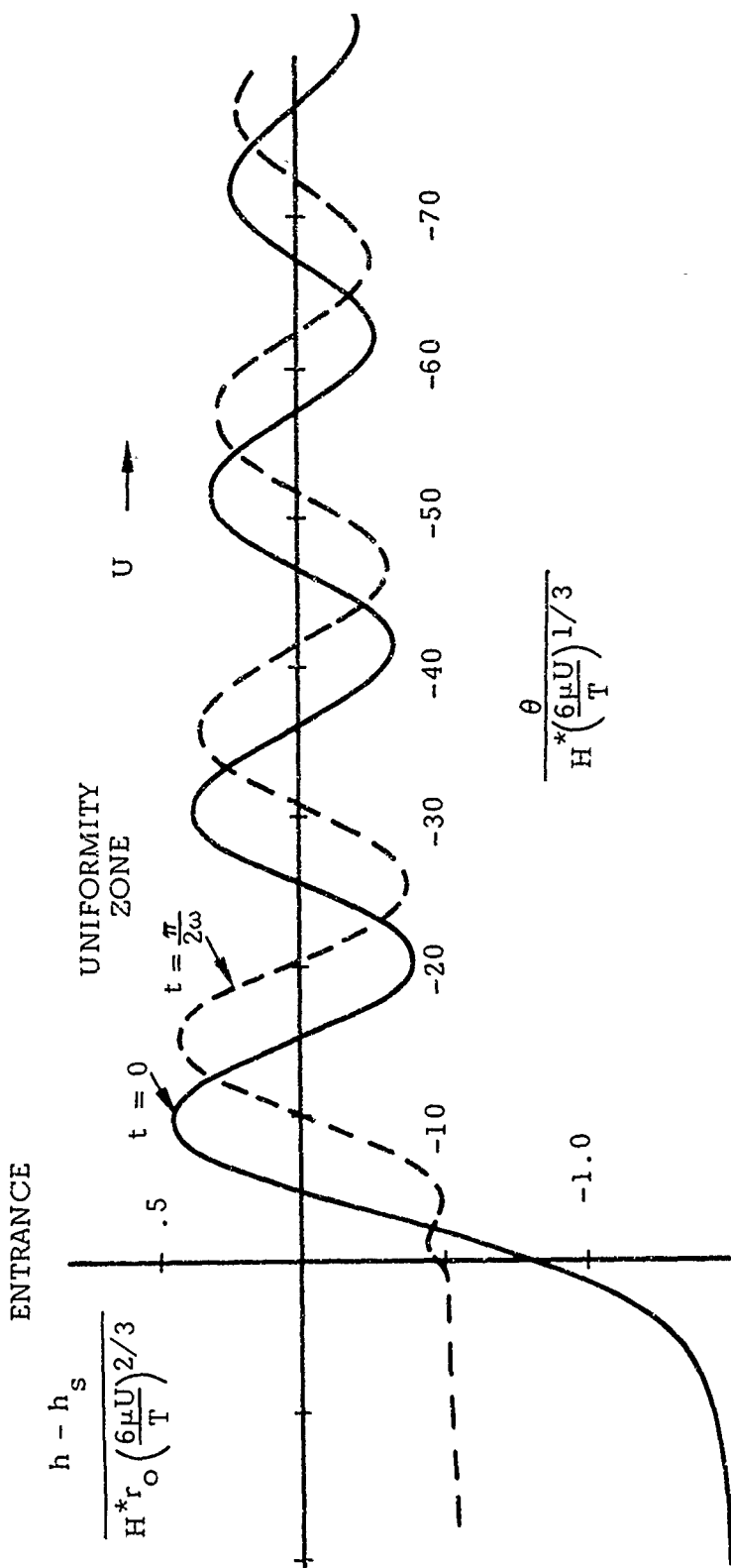


Fig. 2.3.1 Two typical spatial distributions of the film thickness perturbation for frequency ratio  $P_\omega = \frac{0.3}{2H^* \left(\frac{6\mu U}{T}\right)^{1/3}}$ . (A quarter of a cycle apart timewise [ 2.17 ] ).

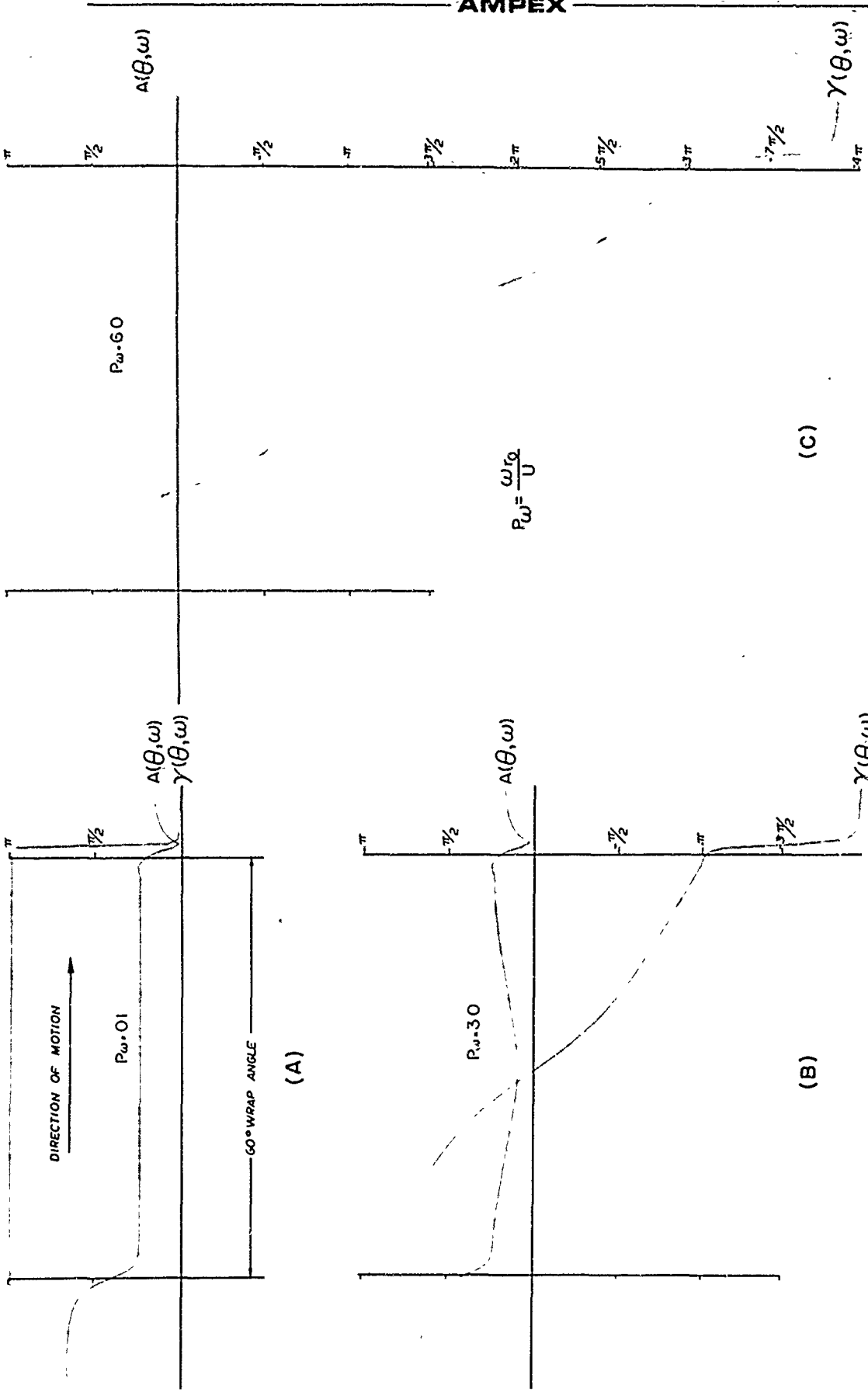


Fig. 2.3.2 Amplitude factor and phase distributions of response to periodic tension perturbations [2.2]

Thus, when the frequency is low and the wavelength large, the displacement of the foil in the region of wrap is nearly uniform ( $P_{\omega} = 0.1$ ). With increasing frequency, the number of travelling undulations in the region of wrap increases. With regard to the phase  $\gamma$ , low frequencies correspond to quasi-static changes of gap, associated with negligible lag. As the frequency increases, the delay for the effect of the tension-pulse to reach a point downstream of the entrance increases. For an incompressible fluid, the increase is linear, but compressibility introduces a periodic component in the  $\gamma$  curve.

The foil approaches its straight-line asymptotes from above, away from the entrance zone, and from below, away from the exit zone. An increase in tension causes the foil to move toward the asymptotes, essentially parallel to itself. Thus, the foil excursion leads the tension by 180° in the approach to the entrance, while the foil perturbation downstream of the exit lags by 180° the motion at the end of the angle of wrap.

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AMPEX

*B. ad.*



NOMENCLATURE  
(Chapter 3)

$a_o$	Orifice cross-sectional area
$a'_o$	Effective orifice cross-sectional area ( $a'_o = c_d a_o$ )
$a_c$	Cross sectional area of capillary
$b$	Foil half width
$c_d$	Discharge coefficient
$H_1^*$	Film thickness upstream of pressurization groove
$H_2^*$	Film thickness downstream of pressurization groove
$l_c$	Length of capillary
$l_g$	Circumferential length of pressurization groove
$L_g$	Dimensionless groove length = $\frac{l_g \epsilon^{1/3}}{h_1^*}$
$p_s$	Supply pressure
$p_g$	Groove pressure
$r_o$	Radius of foil bearing journal (guide)
$Q_1$	Inflow through entrance zone
$Q_2$	Outflow from bearing
$Q_g$	Inflow through groove restrictor
$T$	Foil tension per unit width
$U$	Algebraic sum of journal and foil surface velocities
$V_p$	Speed of propagation of disturbance

- $\Delta h^*$  Change in nominal film-thickness due to pressurization
- $\Delta P$  Pressure parameter,  $\Delta P = h_1^* \left[ r_o (p_g - p_a) - 1 \right] / (r_o \epsilon^{2/3})$
- $\epsilon$  Foil bearing number  $\epsilon = \frac{6\mu U}{T}$

### 3.0 EXTERNALLY PRESSURIZED FOIL BEARINGS

#### 3.1 Steady State Characteristics

##### 3.1.1 Interaction of Self-Acting and Pressurization Effects

In a variety of applications, it may be desirable to supplement the film thickness generated by self acting effects, by means of external pressurization. Conversely, it may be necessary to withdraw fluid from the lubricating film.

A representative configuration is illustrated in Fig. 3.1.1 showing a lateral groove in the region of wrap, extending over the major portion of foil width. This type of foil bearing is characterized by two regions of uniformity. Here, in addition to the entrance and exit region at the extremities of the angle of wrap, secondary exit and entrance regions form in the vicinity of the groove. The flow through the region  $h^*_1$  is augmented or diminished by the flow through the groove restrictor, so that the clearance  $h^*_2$  in the downstream region of uniformity can be greater, or less than the clearance  $h^*_1$ .

The design of this bearing can be accomplished with the aid of Fig. 3.1.2. The gap width in the upstream region of uniformity can be evaluated from Eq. 2.1.1. Assume that it is required to increase the gap width in the downstream region to  $h^*_2 = h^*_1 + \Delta h^*$ . We enter the design chart in Fig. 3.1.2 along the ordinate of the required value of  $\Delta h^*/h^*_1$  and locate the intersection with a  $(L_g = \epsilon_g^{1/3}/h^*_1)$ -curve, corresponding to a selected groove width  $\epsilon_g$ . Proceeding now horizontally, we determine the groove pressure  $p_g$  from the value of the pressure parameter

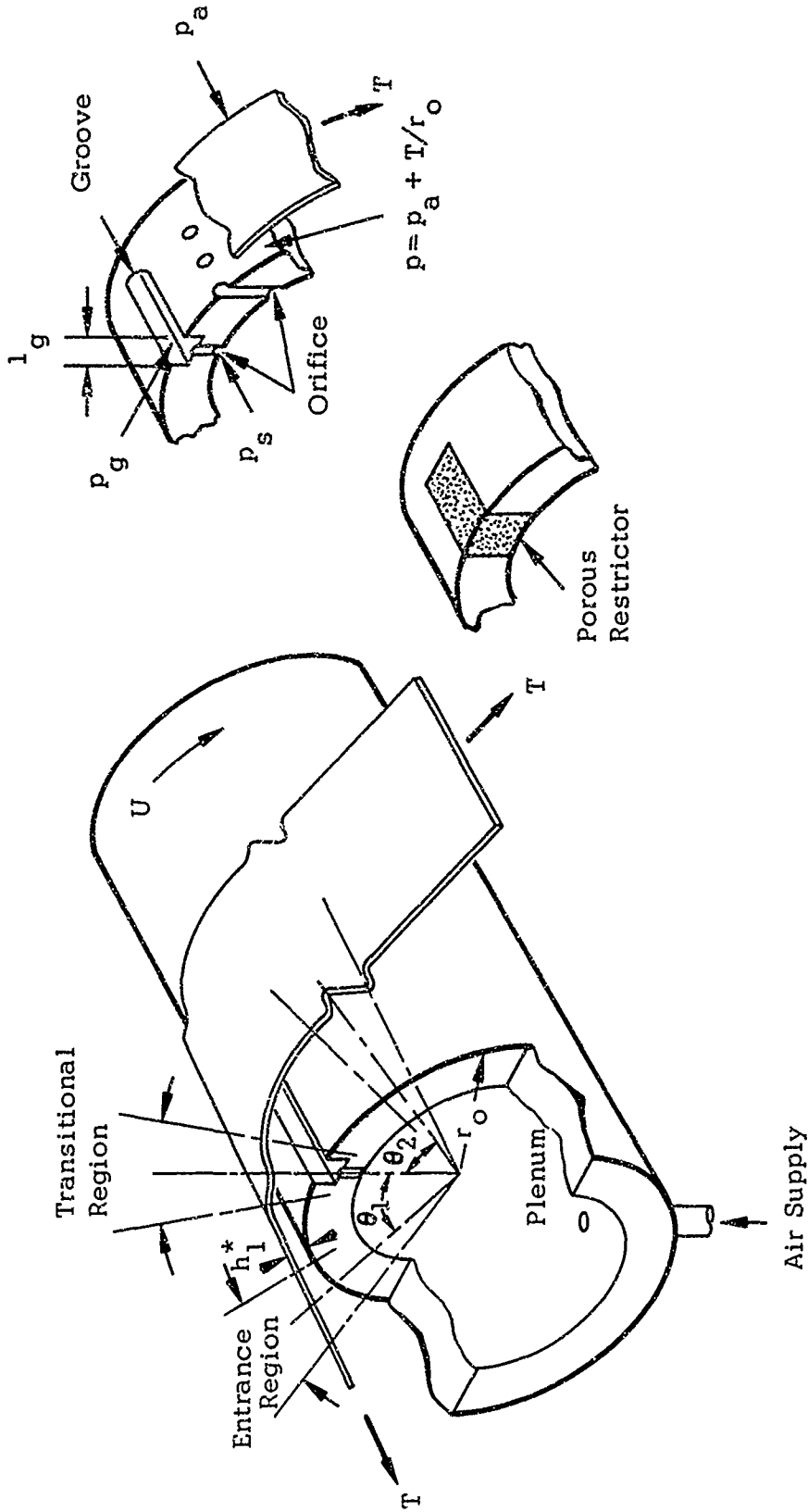


Fig. 3.1.1.1 Externally pressurized foil bearing, with foil moving over stationary guide.

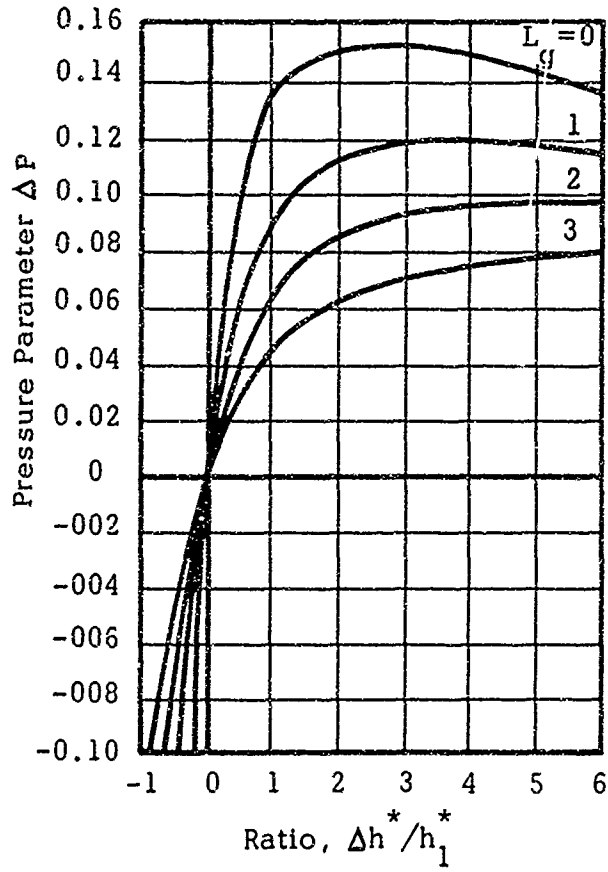


Fig. 3.1.2 Computer Simulation of the Clearance Response of a Foil Bearing to a Moving Supply Source

(3.1.1)

The second part of the design procedure involves the determination of the supply pressure and type of restrictor compatible with the bearing flow rate. The outflow from the bearing  $Q_2$  is the sum  $Q_1 + Q_g$  of inflows through the entrance zone and through the groove restrictor (assuming side leakage to be negligible).

Symbolically:

$$Q_2 = Q_1 + Q_g \quad (3.1.2)$$

or

$$2b(Uh_2^*/2) = 2b(Uh_1^*/2) + Q_g \quad (3.1.3)$$

so that the flow entering from the groove is:

$$Q_g = b U \Delta h^* \quad (3.1.4)$$

in which  $U$  and  $\Delta$  and  $\Delta h^* = h_2^* - h_1^*$  are known and  $b$  is the foil half-width.

If the groove restrictor is an orifice:

$$Q_g = c_d a_o \sqrt{2(p_s - p_g)/\rho} \quad (3.1.5)$$

With  $Q_g$  known and  $p_g$  previously determined, the supply pressure  $p_s$  can be selected for an effective orifice flow area

$$a_o' = c_d a_o \quad (3.1.6)$$

If the groove restrictor is a capillary,

$$Q_g = a_c d_c^2 (p_s - p_g) / 32 \mu l_c \quad (3.1.7)$$

Again, with  $Q_g$  and  $p_g$  known, the supply pressure  $p_s$  and the dimensions of the capillary,  $a_c$  and  $t_c$ , can be selected.

### 3.2 Response to Step Disturbances

When a sudden tension increase (or reduction) is imposed on a self acting foil bearing, a corresponding reduction (or increase) of film thickness occurs immediately in the inlet region. This change, then propagates downstream approximately at  $V_p = U/2$ . If a pressurization groove, separating two uniformity zones is present, wave-fronts, travelling downstream at nearly  $U/2$ , will be initiated in the entrance zone and at the trailing edge of the groove.

When the inlet disturbance reaches the groove it becomes a "new" disturbance for the inlet of the second zone of uniformity and is washed downstream again at a speed  $V_p \approx U/2$  Fig. 3.2.1 [3.2].

### 3.3 Response to Moving Sources

In the application of foil bearings to the support of high speed rotors there arises sometime the need for external pressurization. If pressurization is through the interior of a rotor, the system corresponds to a series of moving sources. The response of the clearance to a rotating pressurization groove is shown qualitatively in Fig. 3.2.2 [3.2]. To begin with, the clearance profile is that of the self acting bearing. As the recess approaches the inlet zone, the foil dips since the recess pressure, initially atmospheric, must rise to  $p_a + T/r_o$  in the region of wrap. When the recess is filled with gas to the bearing pressure-level, then, as it proceeds downstream, the fluid supplied causes the foil to bulge. The front of the bulge travels downstream at the speed of the source  $U$ , whereas the rear of the bulge travels downstream at approximately  $U/2$ . Thus, the length of the bulge increases at a rate nearly equal to  $U/2$  also, until the source eventually leaves the zone of wrap.

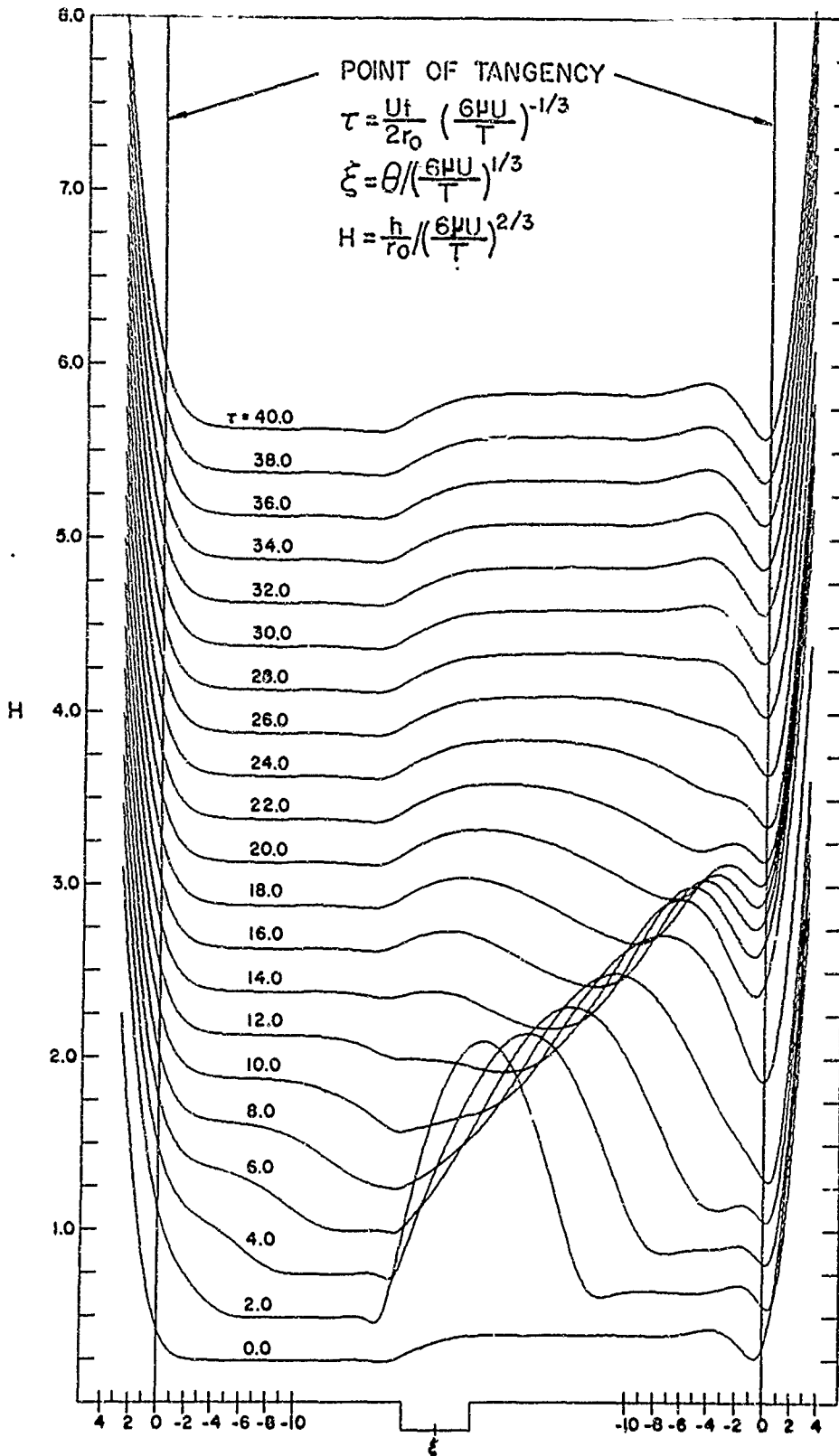


Fig. 3.2.1 Computer Simulation of Dynamic Response of an Externally Pressurized Foil Bearing to a Step Reduction in Tension

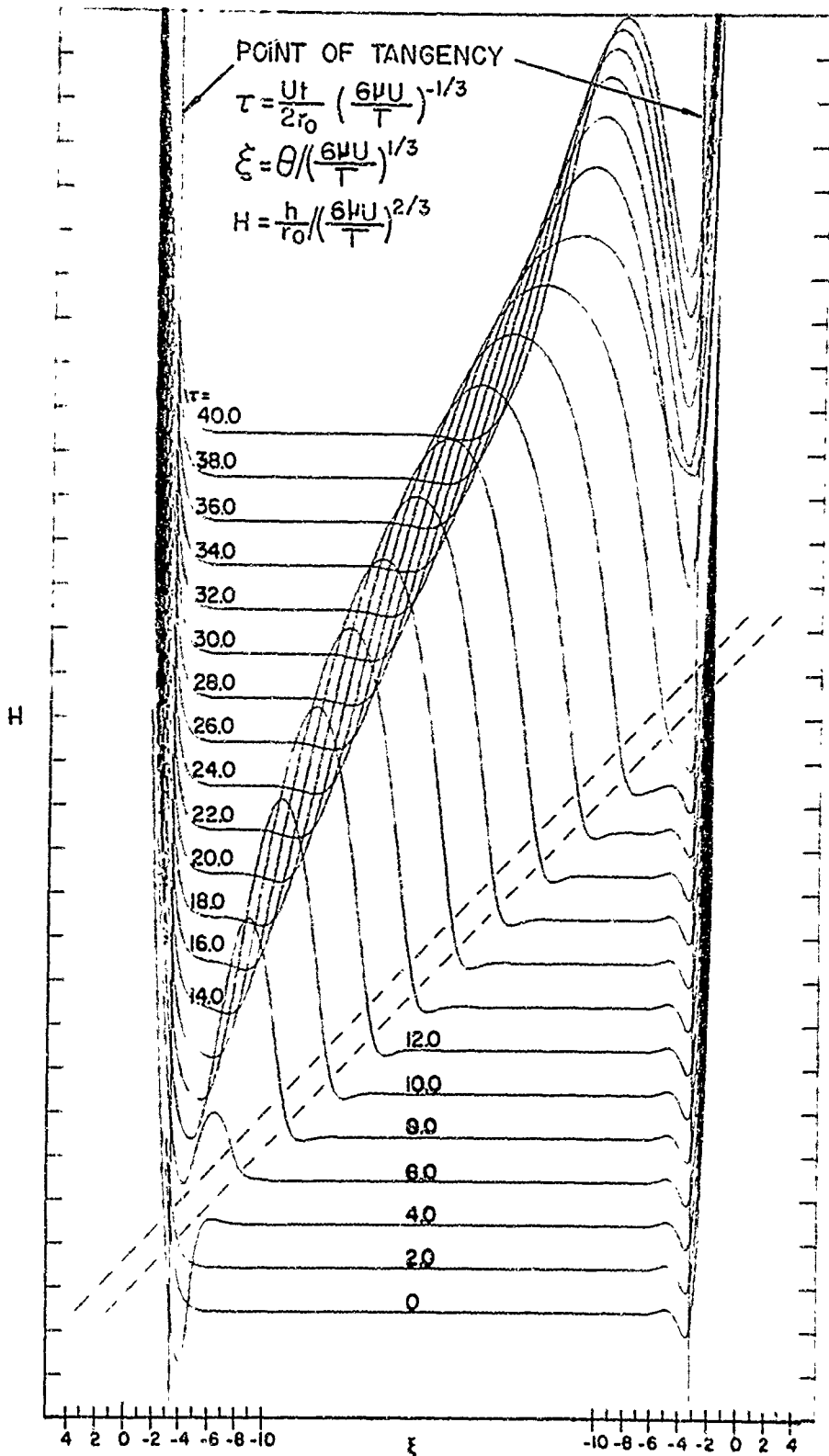


Fig. 3.2.2 Computer Simulation of Dynamic Response of a Foil Bearing to a Moving External Pressurization Source (marked with broken lines)

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In preparation 1971.

NOMENCLATURE

(Section 4)

E	Young's Modulus	
$f_e$	Frequency of excitation	
$f_n$	Undamped natural frequency	$f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$
$F_x, F_y$	Resultant force components on rotor in x, y directions respectively	
$G_{x,y}$	Amplitudes of transverse and in-line excitation (in g's)	
$h^*$	Film thickness of foil bearing in uniformity zone	
$H^*$	Dimensionless film thickness in uniformity zone	$H^* = h^*/(r_o \epsilon^{2/3})$ (See Chapter 2)
$I_p$	Polar moment of inertia of rotor	
$I_t$	Transverse moment of inertia of motor	
k	Subscript denoting kth foil sector. Bearing stiffness per unit width.	
$l_o$	Initial foil length (at $T_o$ )	
m	Rotor mass per unit width of foil	
N	Rotational frequency	
$P_u$	Speed parameter	$P_u = \frac{U}{\sqrt{p_a / \rho_a}}$
$P_r$	Radius parameter	$P_r = \frac{r_o}{\mu / \sqrt{p_a \rho_a}}$
$P_l$	Length parameter	$P_l = \frac{r_o}{l_o} \frac{Et}{\sqrt{\mu^2 p_a / \rho_a}}$

$P_T$	Tension parameter	$P_T = \frac{T}{\sqrt{\mu^2 p_a / \rho_a}}$
$P_C$	Dimensionless damping per unit width	$P_C = c/\mu$
$P_h$	Dimensionless gap	$P_h = \frac{h^*}{\sqrt{\mu^2 / \rho_a p_a}}$
$P_f$	Dimensionless frequency	$P_f = \mu f_e / p_a$
$P_k$	Dimensionless stiffness per unit width	$P_k = k/p_a$
$P_\omega$	Frequency ratio	$P_\omega = \omega r_0 / U$
$p_a$	Ambient pressure, absolute	
$r_0$	Journal radius	
$T_k$	Tension per unit width in k th foil sector	
$T_0$	Preload tension per unit width	
$t$	Foil thickness	
$U$	Journal surface speed	
$W$	Rotor Weight	
$x, y$	Components of displacement of rotor (Fig. 4.2.1)	
$y_k$	Component of displacement of rotor away from and along the bisector of the k th foil sector. (Fig. 4.2.1)	
$\gamma_k$	Angular position of k th foil sector relative to the first foil sector (Fig. 4.2.1).	
$\delta l_k$	Elongation of k th foil due to reasons other than stretching (e.g. thermal).	

0

- $\mu$       Viscosity of lubricant
- $\rho_a$      Density of lubricant at ambient pressure
- $\Theta$       Wrap angle

Blank

## 4.0 FOIL ROTOR SUPPORT

### 4.1 Advantages of Foil Bearings

The application of foil bearing technology to the support of small, high-speed rotors has been shown to be practical and to offer certain advantages over other types of gas bearings. [4.1-4.3]. These advantages are:

1. No whirl instability.
2. No sensitivity to fractional frequency excitation.
3. Self alignment.
4. Simplicity of construction and low requirements of manufacturing accuracy.
5. Accommodation to thermal gradients.
6. Excellent wipe-wear characteristics and tolerance of foreign particles.

### 4.2 Steady State Characteristics

The procedure of determining the load capacity, the tensions and the gaps for a given displacement  $(x,y)$  of the rotor is as follows.

1. Find  $y_k$  ( $k = 1, 2, 3$ ; Fig. 4.2.1)

$$y_k = x \sin \gamma_k + y \cos \gamma_k \quad (4.2.1)$$

2. Solve for  $T_k$  and  $h_k^*$  simultaneously by iteration of the following equations.

$$\frac{T_k - T_o}{Et} \cong \frac{-\delta l_k - 2y_k \sin \frac{\Theta}{2} + \Theta h_k^*}{l_o} \quad (4.2.2)$$

$$h_k^* = H_k^* r_o \left( \frac{6\mu U}{T_k} \right)^{2/3} \quad (\text{see Sec. 2.1.1}) \quad (4.2.3)$$

$\delta l_k$  = elongation of the k th foil due to reasons other than stretching e.g. thermal.

3. The forces on the rotor are

$$F_x = \sqrt{3} \sin \frac{\Theta}{2} (T_3 - T_2) \quad (4.2.2)$$

$$F_y = 2 \sin \frac{\Theta}{2} \left[ T_1 - \frac{T_2}{2} - \frac{T_3}{2} \right] \quad (4.2.3)$$

### 4.3 Stiffness and Damping Coefficients

Theoretical curves of the stiffness and damping coefficients for a self acting foil rotor support of simplified geometry, shown in Fig. 4.2.1 are given in this section. These curves apply to the case when no radial load is applied to the journal. Though they do not constitute a complete map, they include a range of parameters which is of practical interest.

The following simplifications pertain, to curves presented here-with:

- a) The clamping method of foil ends is idealized, as shown in Fig. (4.2.1).

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\*The length of the foil is approximated in this formula as composed of two straight sections and a circular arc of radius  $r_o + h^*$ .

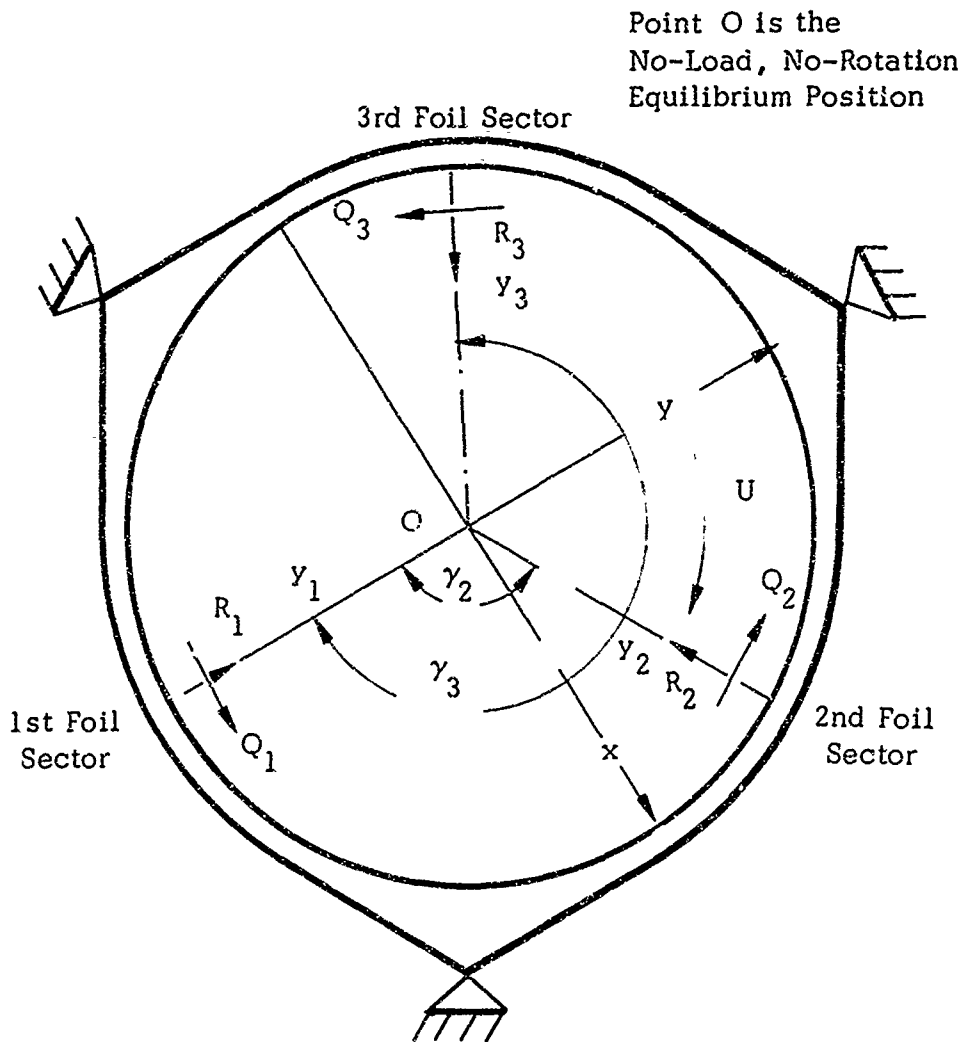


Fig. 4.2.1 Schematic Diagram of Foil Bearing Configuration

- b) Fluid inertia effects are neglected.
- c) The relaxation of foil tension due to temperature increase with speed is not taken into account.
- d) The slack remaining in the foil due to its finite stiffness, is neglected.

The effects of foil thickness  $t$ , journal radius  $r_o$ , wrap angle  $\Theta$  preload tension  $T_o$ , rotational speed  $N$ , and excitation frequency  $f$  upon the stiffness coefficient  $k$  and the damping coefficient  $c$  are shown in Figs. (4.3.1) - (4.3.4).

The stiffness coefficient may be estimated on a quasi static basis by means of the formula:

$$k = \frac{Et}{r_o} \frac{6 \sin^2 \frac{\Theta}{2}}{\frac{l_o}{r_o} + \frac{Et}{r_o} \frac{\partial \delta l}{\partial T} + \frac{\Theta}{T/Et} \left( \frac{6\mu U}{T} \right)^{2/3} \left( \frac{2}{3} H^* - \frac{\partial H^*}{\partial T} T \right)} \quad (4.3.1)$$

Results based on this formula are indicated along the  $P_k$ -axis ( $P_f = 0$ ) of Figs. 4.3.1 and 4.3.2. The damping coefficient cannot be estimated on a quasi static basis.

A comparison of theoretical and experimental results is shown in Fig. 4.3.5. The curve marked theoretical I is based on stiffness calculated from Eq. 4.3.1 with  $\frac{\partial \delta l}{\partial T}$  and  $\frac{\partial H^*}{\partial T}$  taken as zero. The next three curves are obtained by sequentially removing the idealization (a), (b), and (c) above.

#### 4.4 Design Considerations Related to the Bearing Frequency-Response

It is noteworthy that for a foil bearing the damping coefficient vanishes whenever the ratio of excitation frequency  $f_e$  and rotational frequency  $N$  is an integral multiple of  $\pi/\Theta$ , where  $\Theta$  is the wrap angle (Fig. 4.3.3, 4.3.4). The bearing stiffness, however, is not sensitive to either the excitation frequency, or to the speed. The following points out some design implications due to the existence of these "critical" frequencies, at which the damping vanishes.

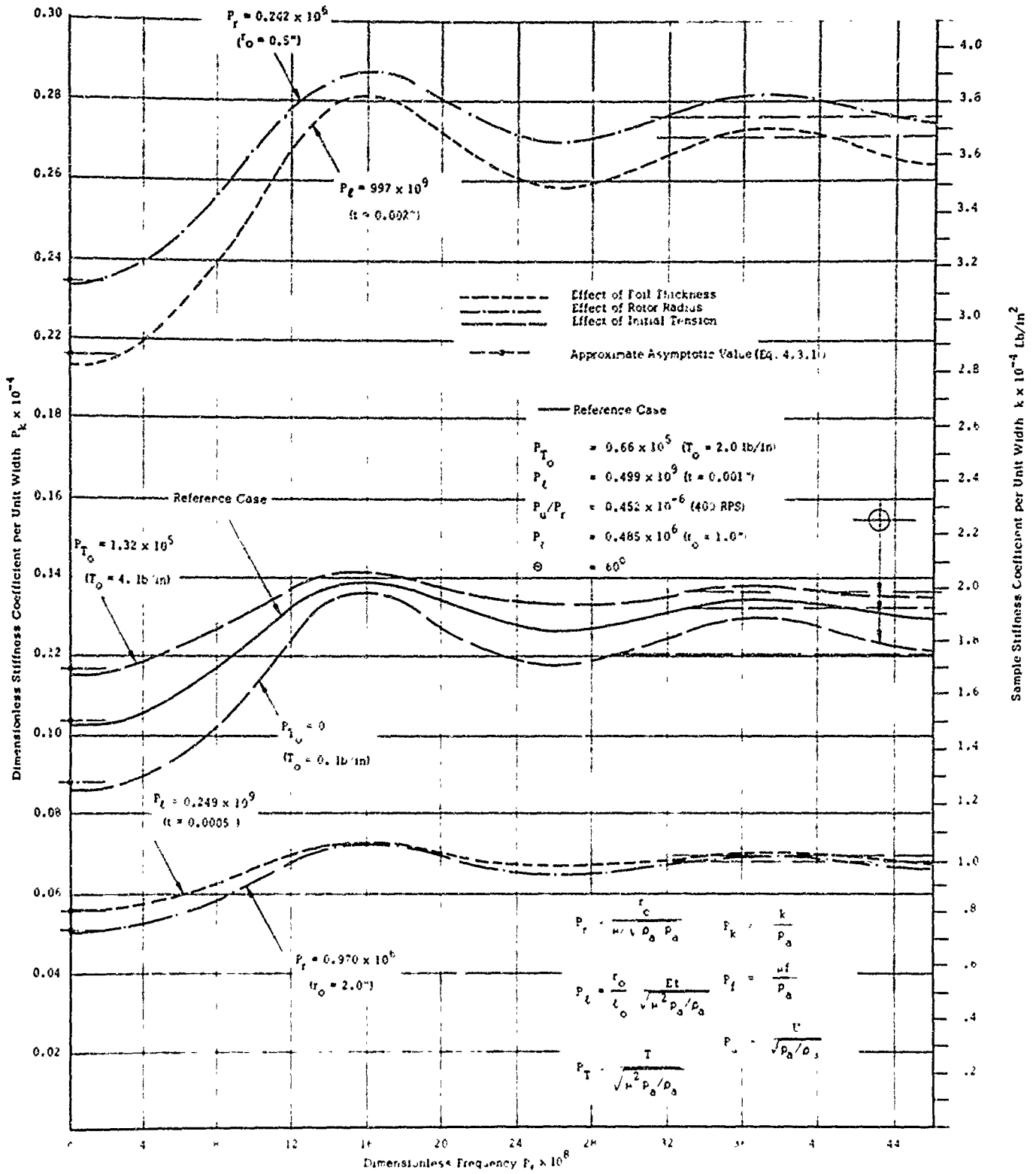


Fig. 4.3.1 Stiffness Coefficient of a Foil Rotor Support with Zero Radial Load as a Function of Excitation Frequency, Foil Thickness, Rotor Radius and Initial Tension

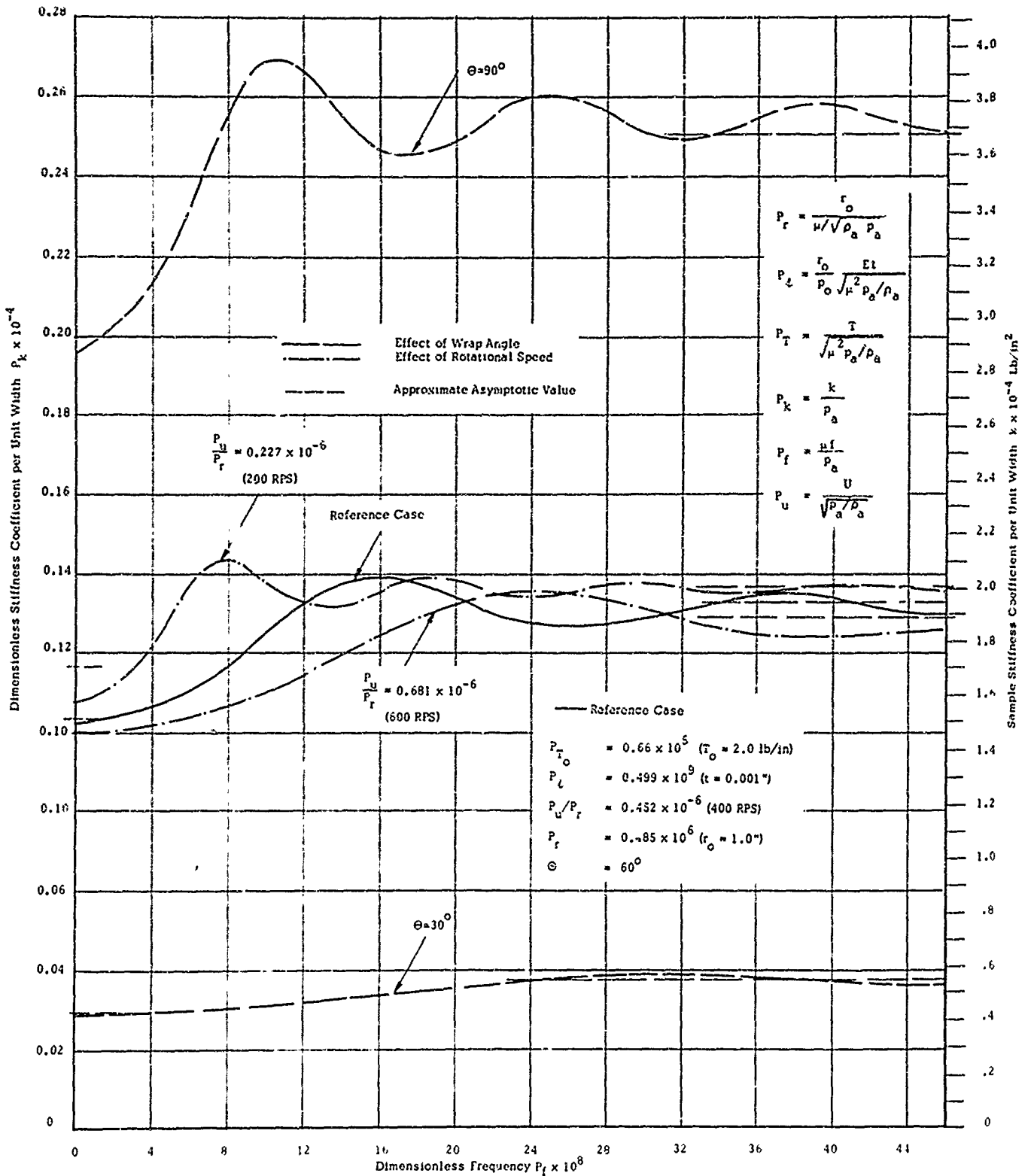


Fig. 4.3.2 Stiffness Coefficient of a Foil Rotor Support with Zero Radial Load as a Function of Excitation Frequency, Wrap Angle and Rotational Frequency

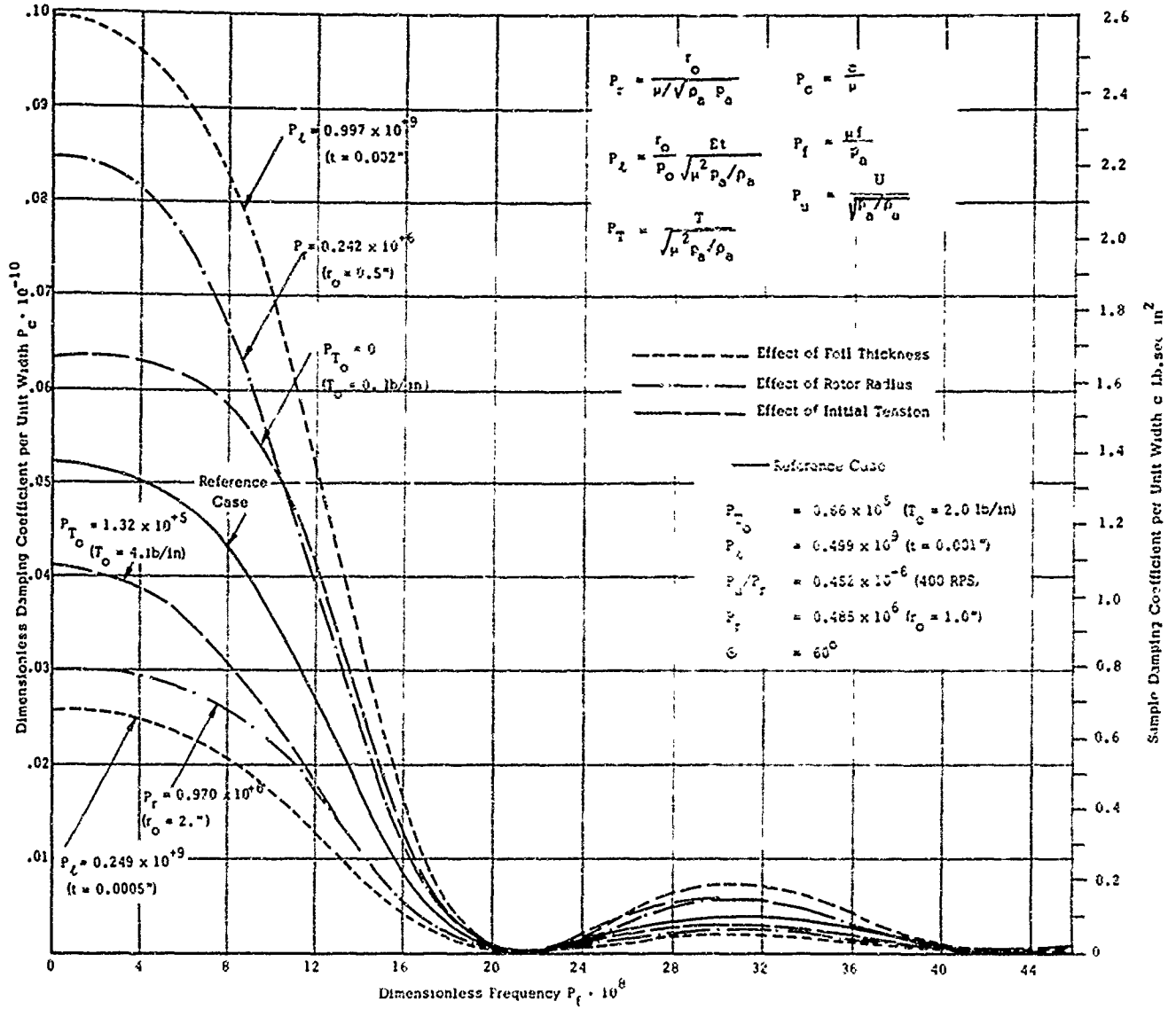


Fig. 4.3.3 Damping Coefficient of a Foil Rotor Support with Zero Radial Load as a Function of Excitation Frequency, Foil Thickness, Rotor Radius and Initial Tension [4.3]

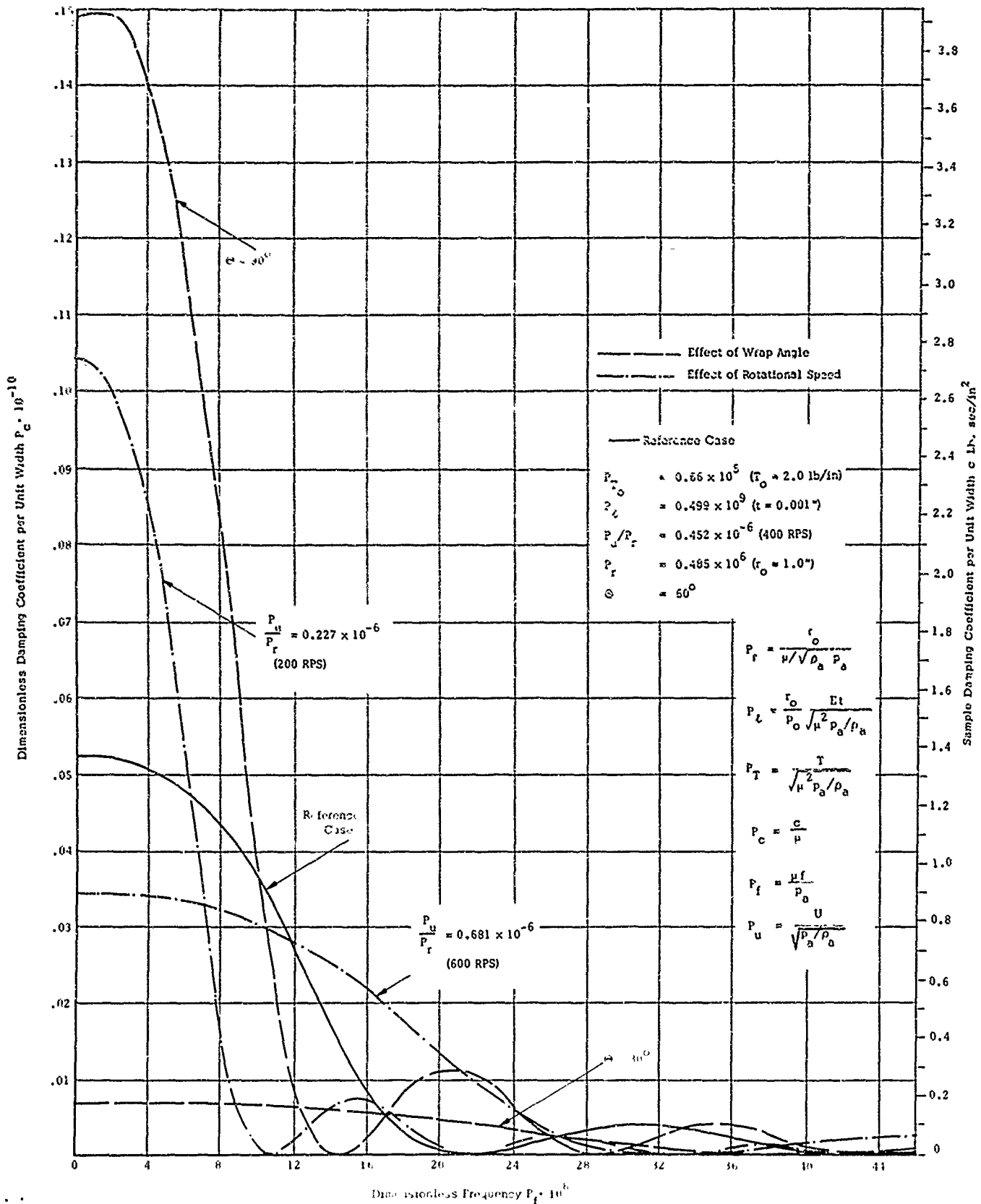


Fig. 4.3.4 Damping Coefficient of a Foil Rotor Support with Zero Radial Load as a Function of Excitation Frequency, Wrap Angle and Rotational Frequency [ 4.3 ]

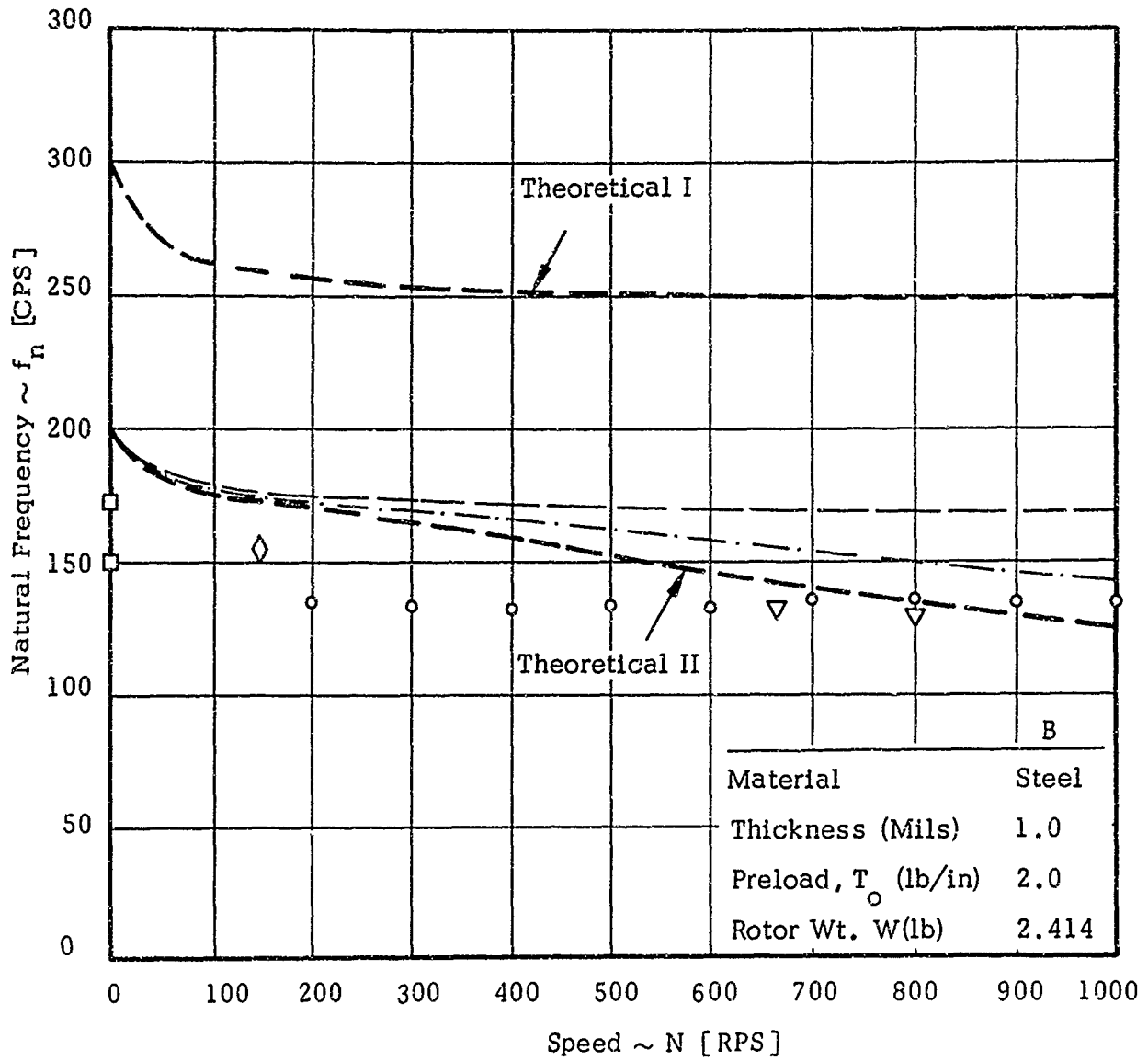


Fig. 4.3.5 "Natural Frequencies" of Foil-Bearing Supported Rotor; - Comparison of Theory and Experiment [4.4]

If both the resonant frequency  $f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ , and the excitation frequency  $f_e$  occur at a rotational frequency  $N$  such that

$$f_n = f_e = \frac{\pi}{\Theta} N i \quad i = 1, 2 \dots \quad (4.4.1)$$

there will be no damping to limit the resonant amplitude of the system.

When the excitation is synchronous, the foregoing condition arises only when  $\Theta = \pi$ , which is a wrap angle impractical for design purposes.

For any given operating speed  $N$ , this coincidence is avoided if the rotor mass is such that  $f_n$  is considerably less than  $\frac{\pi N}{\Theta}$ .

During acceleration and deceleration, however, points at which

$$f_n = \left( \frac{\pi}{\Theta} N \right) : \quad i = 1, 2 \dots$$

are always encountered. If the bearing is excited at  $f_e = f_n$  at these speeds a dangerous situation may occur.

Example

Assume  $\Theta = 60^\circ = \pi/3$  rad.  
 $N = 400$  rps  
 $k = 10,000$  lb/in/in  
 $m = 3$  lb/in

The resonant frequency in the planar mode is

$$f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{1000}{3/386}} \cong 180 \text{ cps}$$

The excitation frequencies with zero damping are  $f_e = 1200$  cps, 2400 cps etc.

At the operating speed, therefore, no danger exists. On the other hand during acceleration or deceleration, an external excitation at  $f_e = f_n = 180$  cps will be dangerous, whenever the frequency of rotation is

$$N = \frac{f_n}{\frac{\pi}{\Theta} i} = \frac{180}{\pi i} = 60,30 \dots \text{ rps}$$

where  $i = 1, 2 \dots$

#### 4.5 Example of a Turbo-Alternator Bearing Design

Fig. 4.5.1 shows the essential components of a 21-lb simulator supported by foil bearings [4.2]. Starting and stopping is aided by external pressurization, through the interior of the rotor and rows of orifices. This particular rotor was operated stably at speeds in excess of 50,000 rpm in both the vertical and horizontal attitudes.

Fig. 4.5.2 shows a scan of response of the rotor to rotating imbalance. A scan of reponse to unidirectional excitation is shown in Fig. 4.5.3. The motion of the rotor at various levels of excitation is illustrated in Fig. 4.5.4.

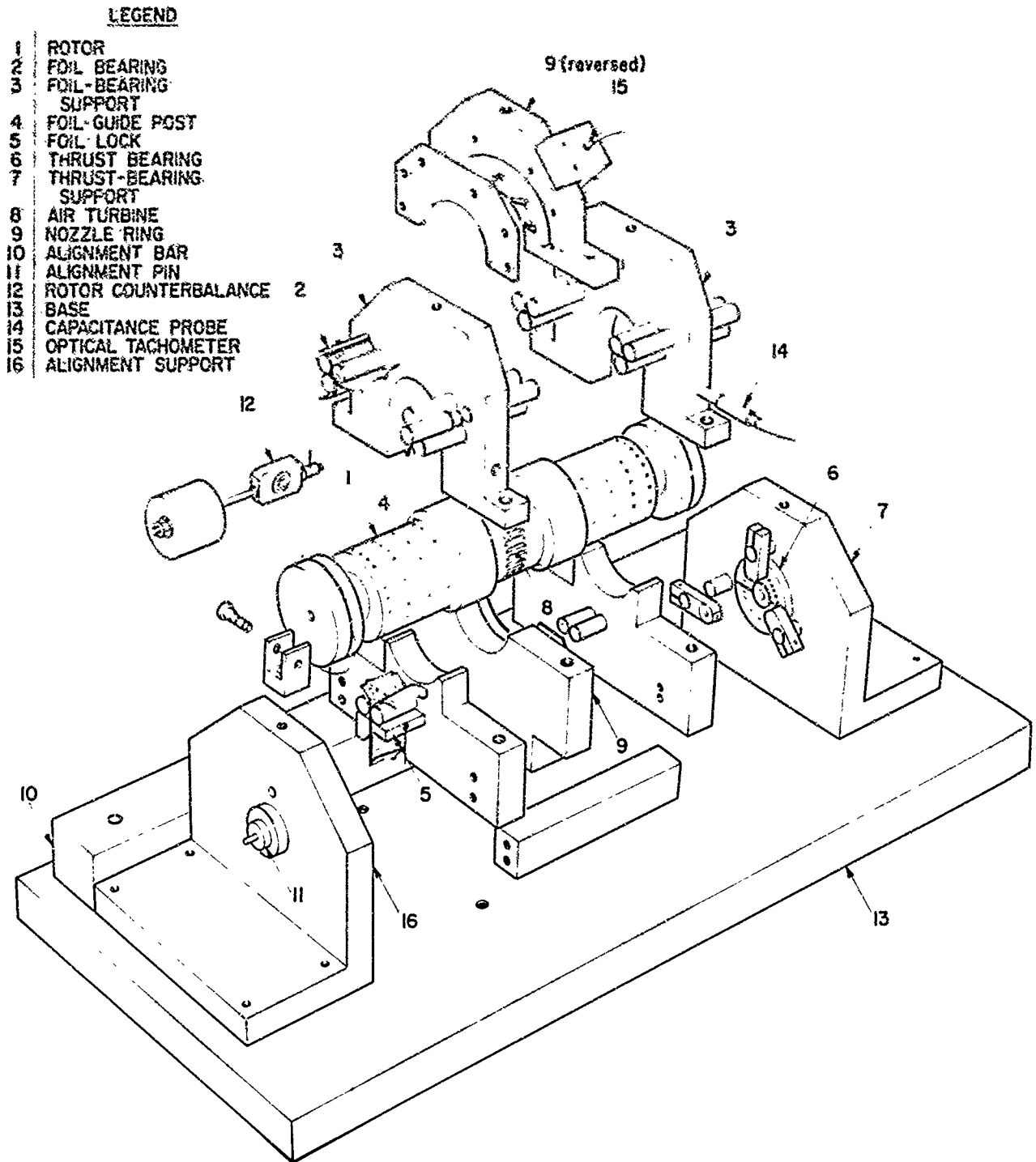


Fig. 4.5.1 Schematic Diagram of Turbo-Alternator Simulator [ 4.2 ]

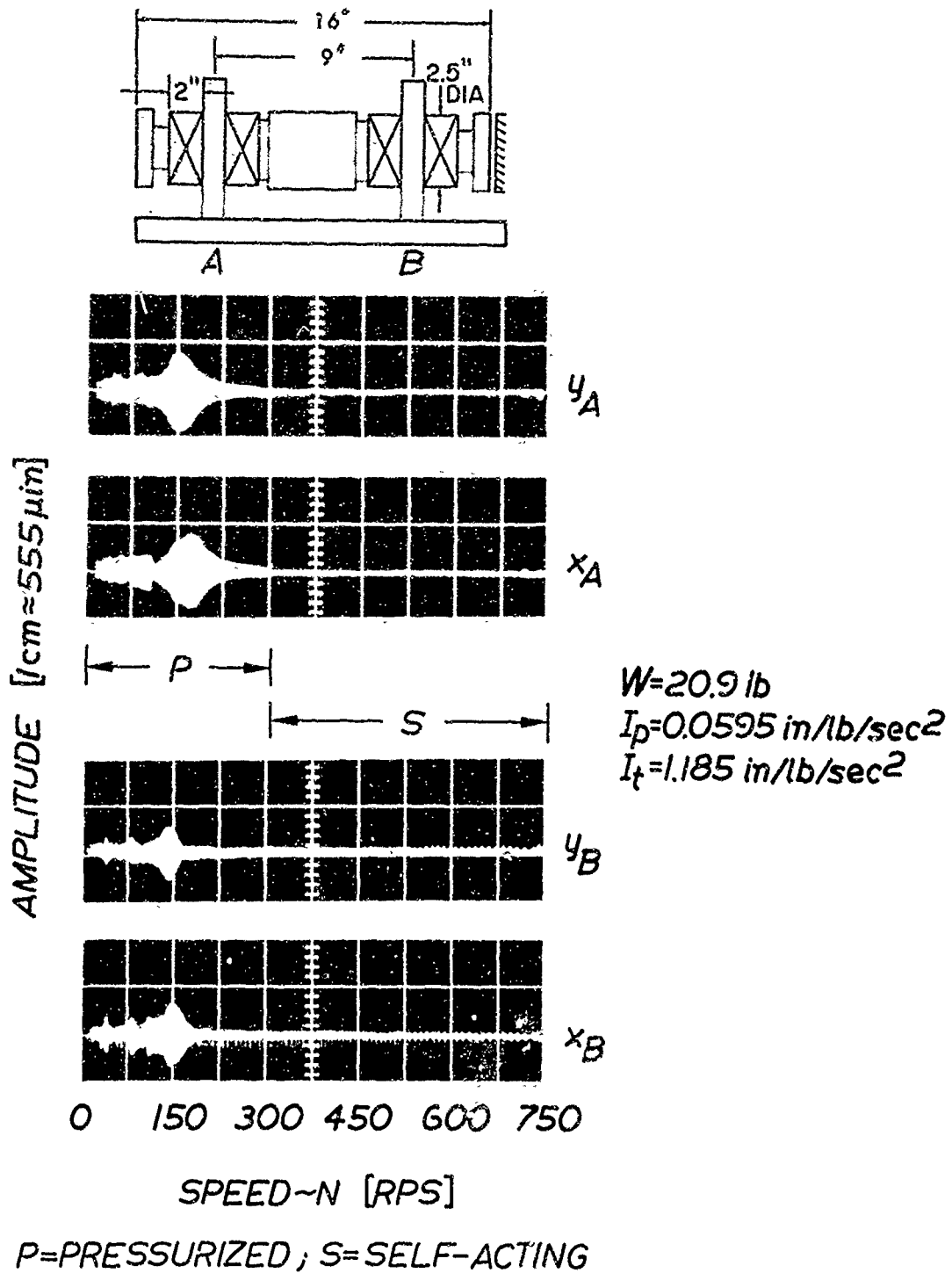


Fig. 4.5.2 Scan of Response to Remanent Imbalance in Vertical Attitude (Decreasing Speed) [4.2]

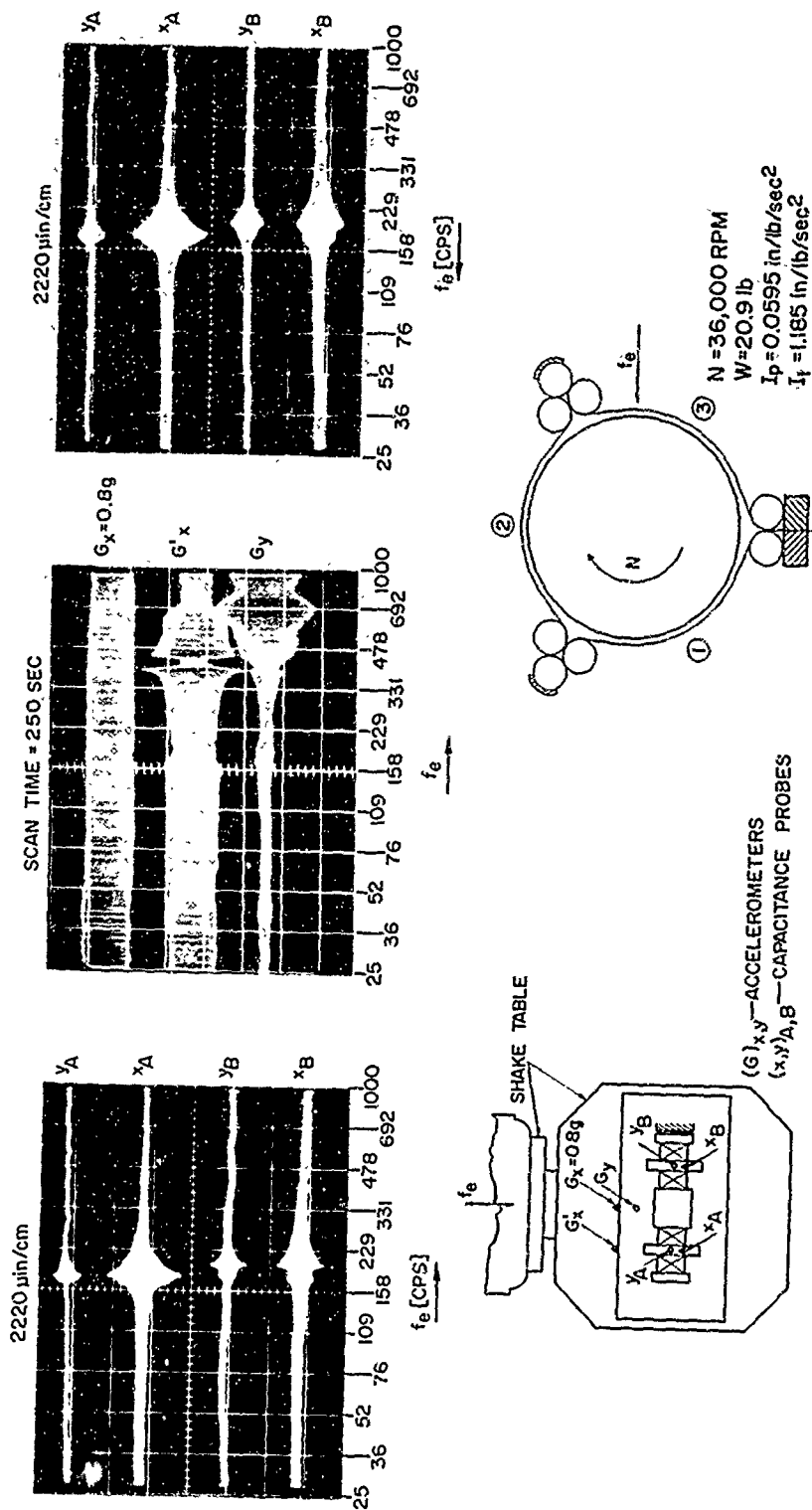


Fig. 4.5.3 Scan of Response to Unidirectional Excitation  
 (Horizontal Attitude;  $N = 36,000 \text{ RPM}$ ;  
 $G_x = 0.8 \text{ g}$ ) [ 4.2 ]

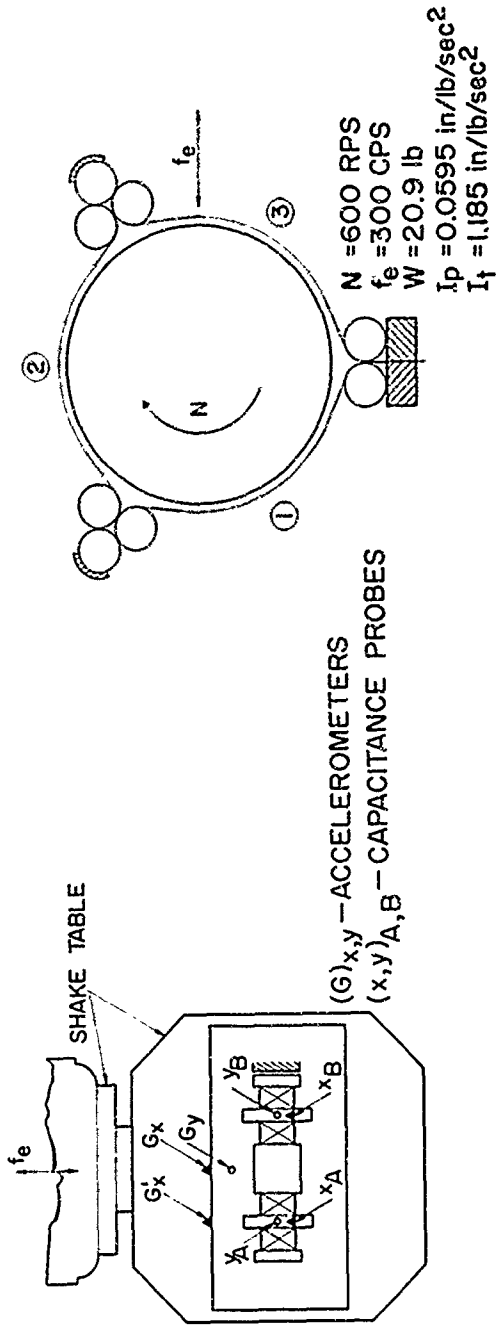
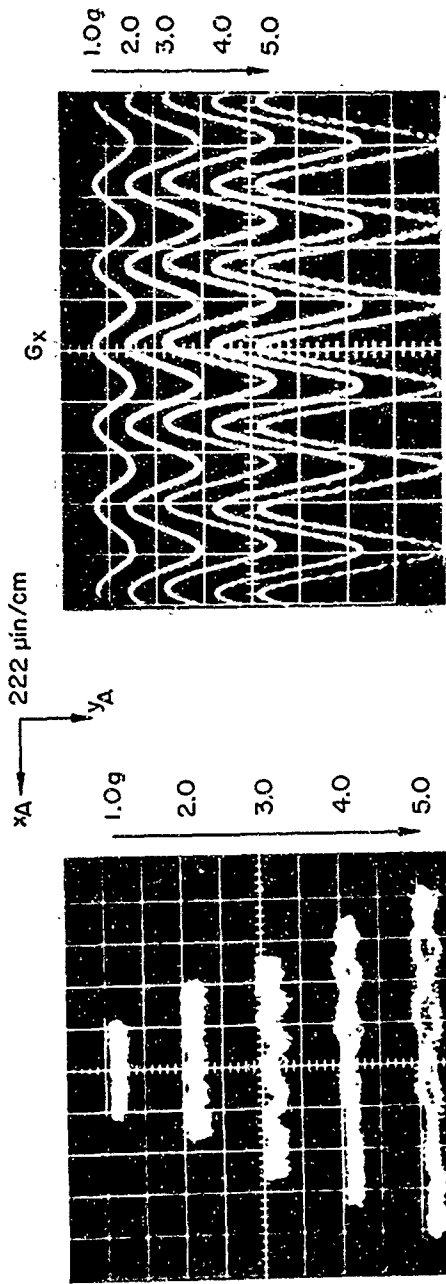


Fig. 4.5.4 Response of Rotor at Variable Level of Excitation (Horizontal Attitude;  $N = 600 \text{ RPS}$ ;  $f_e = 300 \text{ CPS}$ ;  $1 \leq G_x \leq 5 \text{ g}$ ) [4.2]

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